

Title: PSI 17/18 - Condensed Matter - Lecture 11

Date: Nov 20, 2017 10:45 AM

URL: <http://pirsa.org/17110036>

Abstract:



$$\psi(\vec{r}_1, \vec{r}_2) \otimes \left( \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \right)$$

$$-\frac{\hbar^2}{m} \nabla_r^2 \psi(\vec{r}_1, \vec{r}_2) + V(\vec{r}_1, \vec{r}_2) \psi(\vec{r}_1, \vec{r}_2) = (2\epsilon_F + E) \psi(\vec{r}_1, \vec{r}_2)$$



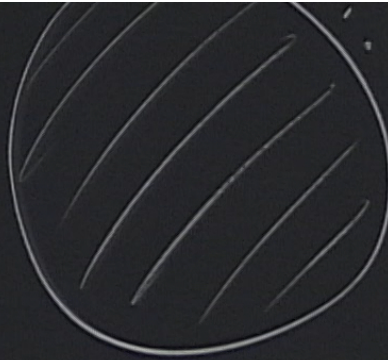


$$\psi(\vec{r}_1 - \vec{r}_2) \otimes \left( \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \right)$$

$$-\frac{\hbar^2}{m} \nabla_r^2 \psi(\vec{r}_1 - \vec{r}_2) + V(\vec{r}_1 - \vec{r}_2) \psi(\vec{r}_1 - \vec{r}_2) = (2\epsilon_F + E) \psi(\vec{r}_1 - \vec{r}_2)$$

↓ Fourier transform

$$\frac{\hbar^2 k^2}{m} \psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2\epsilon_F + E) \psi_k$$



$$V_{kk'} = \frac{1}{\Omega} \int d^3r V(\vec{r}) e^{i(\vec{k}' - \vec{k}) \cdot \vec{r}}$$

$$\psi(\vec{r}_1 - \vec{r}_2) \otimes \left( \frac{|\uparrow\downarrow\rangle}{\sqrt{2}} \right)$$

$$-\frac{\hbar^2}{m} \nabla_r^2 \psi(\vec{r}_1 - \vec{r}_2)$$

$$\frac{\hbar^2 k^2}{m} \psi_k +$$



$$\psi(\vec{r}_1, \vec{r}_2) \otimes \left( \frac{| \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle }{\sqrt{2}} \right)$$

$$-\frac{\hbar^2}{m} \nabla_r^2 \psi(\vec{r}_1, \vec{r}_2) + V(\vec{r}_1, \vec{r}_2) \psi(\vec{r}_1, \vec{r}_2) = (2\epsilon_F + E) \psi(\vec{r}_1, \vec{r}_2)$$

↓ Fourier transform

$$\frac{\hbar^2 k^2}{m} \psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2\epsilon_F + E) \psi_k \Rightarrow$$

$$(E - 2\epsilon_{k'}) \psi_k = \sum_{k'} V_{kk'} \psi_{k'}$$

$$\epsilon_{k'} = \frac{\hbar^2 k'^2}{2m} - \epsilon_F$$



$$\psi(\vec{r}_1 - \vec{r}_2) \otimes \left( \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \right)$$

$$-\frac{\hbar^2}{m} \nabla_r^2 \psi(\vec{r}_1 - \vec{r}_2) + V(\vec{r}_1 - \vec{r}_2) \psi(\vec{r}_1 - \vec{r}_2) = (2\epsilon_F + E) \psi(\vec{r}_1 - \vec{r}_2)$$

↓ Fourier transform

$$\frac{\hbar^2 k^2}{m} \psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2\epsilon_F + E) \psi_k \Rightarrow$$

$$(E - 2\epsilon_F)$$

$$\psi_k = \sum_{k'} V_{k'}$$

$$\psi_k = \frac{\sum_{k'} V_{kk'} \psi_{k'}}{E - 2\varepsilon_{sk}}$$

$$(-2\varepsilon_{sk}) \psi_k = \sum_{k'} V_{kk'} \psi_{k'}$$

$$\varepsilon_{sk} = \frac{\hbar^2 k^2}{2m} - \varepsilon_F$$



$$\psi_k = \frac{\sum_{k'} V_{kk'} \psi_{k'}}{E - \epsilon_k}$$

Onshell Approximation

$$V_{kk'} = \begin{cases} -V & \text{If } |\epsilon_k - \epsilon_F| \leq \hbar\omega_c \\ & \text{and } |\epsilon_{k'} - \epsilon_F| \leq \hbar\omega_c \\ 0 & \text{otherwise} \end{cases}$$

$$\psi_k = \sum_{k'} V_{kk'} \psi_{k'}$$

$$\begin{aligned} \epsilon_k &= \frac{\hbar^2 k^2}{2m} - \epsilon_F \\ &= \epsilon_k - \epsilon_F \end{aligned}$$



$$\psi_k = \frac{\sum_{k'} V_{kk'} \psi_{k'}}{E - 2\epsilon_k}$$

$$\psi_k = \frac{V}{2\epsilon_k - E} \sum_{k'}^* \psi_{k'}$$

Onshell Approximation

$$V_{kk'} = \begin{cases} -V \\ 0 \end{cases}$$

If  $|\epsilon_k - \epsilon_F| \leq \hbar\omega_c$   
and  $|\epsilon_{k'} - \epsilon_F| \leq \hbar\omega_c$   
otherwise

$$\psi_k = \sum_{k'} V_{kk'} \psi_{k'}$$

$$\begin{aligned} \epsilon_k &= \frac{\hbar^2 k^2}{2m} - \epsilon_F \\ &= \epsilon_k - \epsilon_F \end{aligned}$$



$$\psi_k = \frac{\sum_{k'} V_{kk'} \psi_{k'}}{E - 2\epsilon_{jk}}$$

$$\psi_k = \frac{V}{2\epsilon_{jk} - E} \sum_{k'}^* \psi_{k'}$$

$$\sum_k^* \psi_k = \sum_k^* \frac{V}{2\epsilon_{jk} - E} \sum_{k'}^* \psi_{k'}$$

$$\psi_k = \sum_{k'} V_{kk'} \psi_{k'}$$

$$\begin{aligned} \epsilon_{jk} &= \frac{\hbar^2 k^2}{2m} - \epsilon_F \\ &= \epsilon_k - \epsilon_F \end{aligned}$$

Onshell Approximation

$$V_{kk'} = \begin{cases} -V \\ 0 \end{cases}$$

If  $|\epsilon_k - \epsilon_F| \leq \hbar\omega_c$   
and  $|\epsilon_{k'} - \epsilon_F| \leq \hbar\omega_c$   
otherwise



$$\psi_k = \frac{\sum_{k'} V_{kk'} \psi_{k'}}{E - 2\varepsilon_{jk}}$$

$$\psi_k = \frac{V}{2\varepsilon_{jk} - E} \sum_{k'}^* \psi_{k'}$$

$$\sum_k^* \psi_k = \sum_k^* \frac{V}{2\varepsilon_{jk} - E} \sum_{k'}^* \psi_{k'}$$

$$\psi_k = \sum_{k'} V_{kk'} \psi_{k'}$$

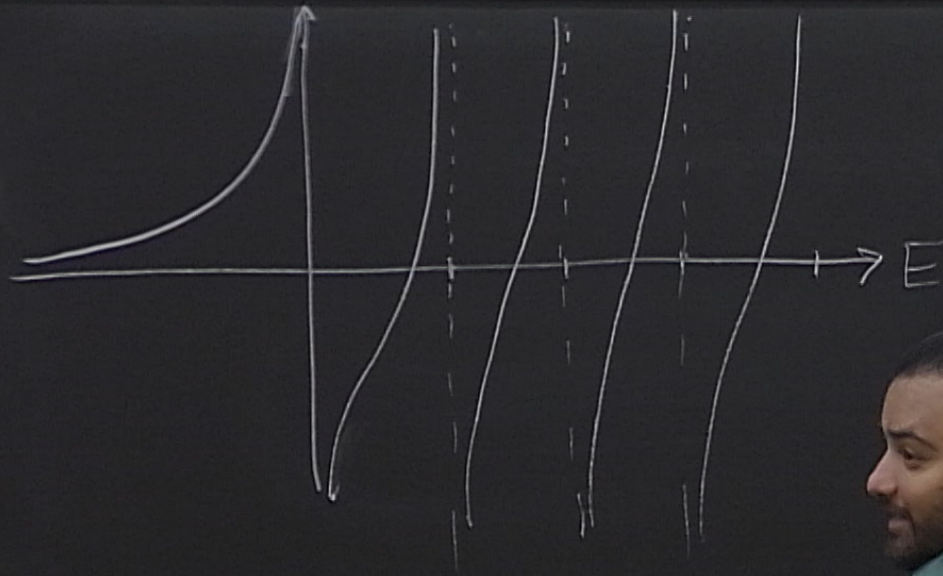
$$\begin{aligned} \varepsilon_{jk} &= \frac{\hbar^2 k^2}{2m} - \epsilon_F \\ &= \epsilon_k - \epsilon_F \end{aligned}$$

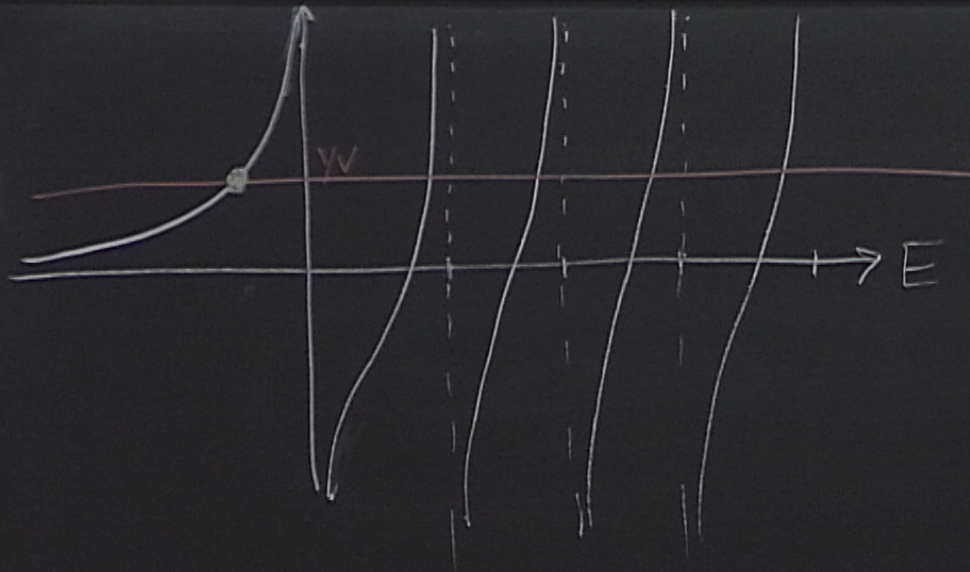
Onshell Approximation

$$V_{kk'} = \begin{cases} -V & \text{If } |\epsilon_k - \epsilon_F| \leq \hbar\omega_c \\ & \text{and } |\epsilon_{k'} - \epsilon_F| \leq \hbar\omega_c \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{1}{V} = \sum_k^* \frac{1}{2\varepsilon_{jk} - E}$$

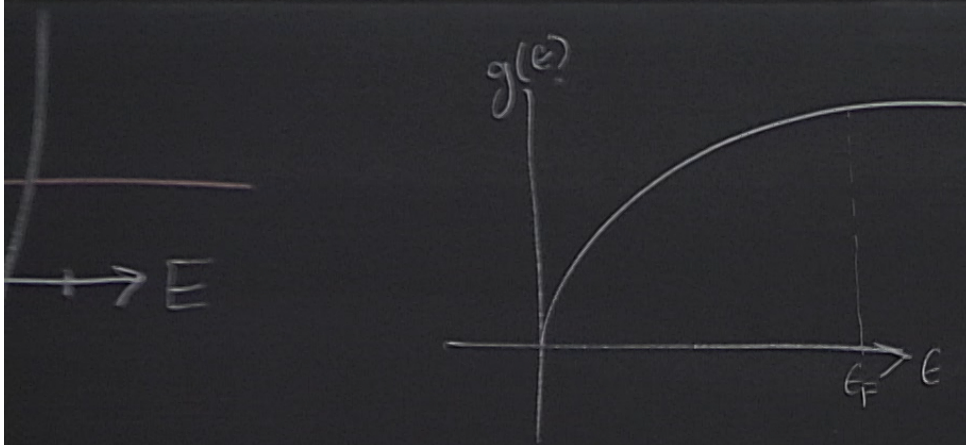




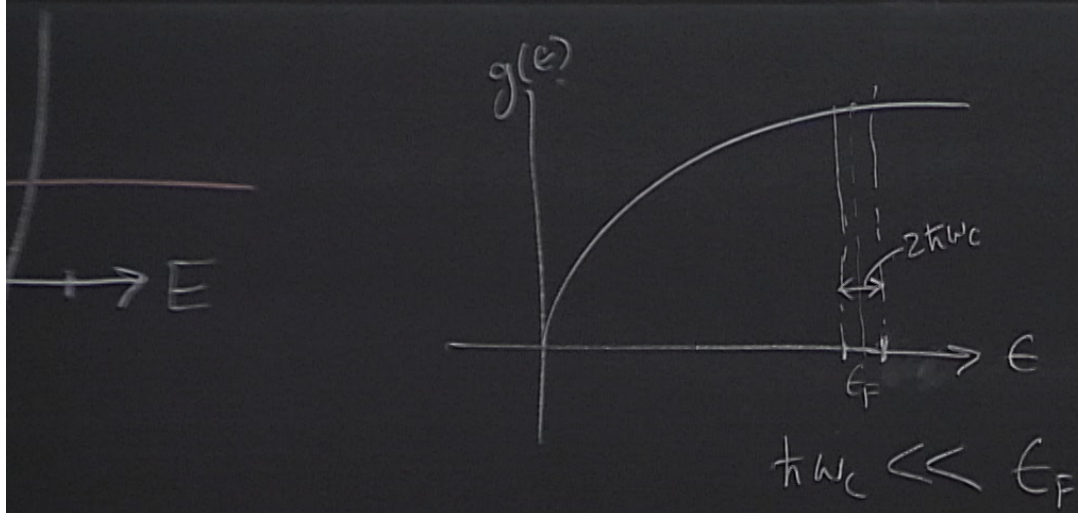


$$\frac{1}{V} = \int d\xi \, g(\xi) \frac{1}{2\xi - E}$$





$$\frac{1}{2g - E}$$



$$\frac{1}{2\xi_F - E} \approx g(E_F) \int d\xi_g \frac{1}{2\xi_g - E}$$



$$\begin{array}{c} \text{---} \text{---} \text{---} \rightarrow E \\ E_F \end{array}$$

$$\hbar \omega_c \ll E_F$$

$$\frac{1}{2E_F - E} = \frac{g(E_F)}{2} \ln \left( \frac{E - 2\hbar\omega_c}{E} \right)$$

$$E = 2\hbar\omega_c \left(1 - e^{\frac{2}{g(E_F)V}}\right)^{-1} = 2\hbar\omega_c / \left(1 - e^{\frac{2}{g(E_F)V}}\right)$$

$$\approx -2\hbar\omega_c \exp\left(-\frac{2}{g(E_F)V}\right)$$

$$g(E_F)V \ll 1$$

Weak Coupling  
limit



$$= 2\hbar\omega_c (1 - e^{\frac{2}{g(E_F)V}})^{-1} = 2\hbar\omega_c / (1 - e^{\frac{2}{g(E_F)V}}) \quad g(E_F)V \ll 1 \quad \text{Weak Coupling limit}$$

$$\approx -2\hbar\omega_c \exp\left(-\frac{2}{g(E_F)V}\right)$$

$$|\psi_0\rangle = \sum_{k > k_F} \psi_k c_{k\uparrow}^+ c_{-k\downarrow}^+ |F\rangle$$

$$|\psi_0\rangle = \sum_{k > k_F} \psi_k c_{k\uparrow}^+ c_{-k\downarrow}^+ |F\rangle$$

$$|\psi^{(N)}\rangle = \left( \sum_k \psi_k c_{k\uparrow}^+ c_{-k\downarrow}^+ \right)^{N/2} |\text{vacuum}\rangle$$

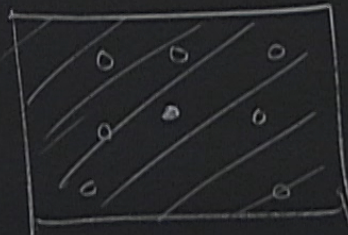


$k$  can take  $M$  values

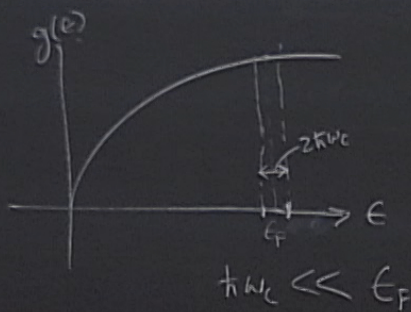
$$\frac{M!}{\frac{N}{2}! (M - \frac{N}{2})!} \sim 10^{10^{20}}$$



$\equiv$



$\overline{N}$



$$\Rightarrow E = 2\hbar\omega_c (1 - e^{-\frac{2}{g(E_F)V}})^{-1} = 2\hbar\omega_c / (1 - e^{-\frac{2}{g(E_F)V}}) \quad g(E_F)V \ll 1 \quad \text{Weak Coupling limit}$$

$$\approx 2\hbar\omega_c \exp\left(-\frac{2}{g(E_F)V}\right)$$

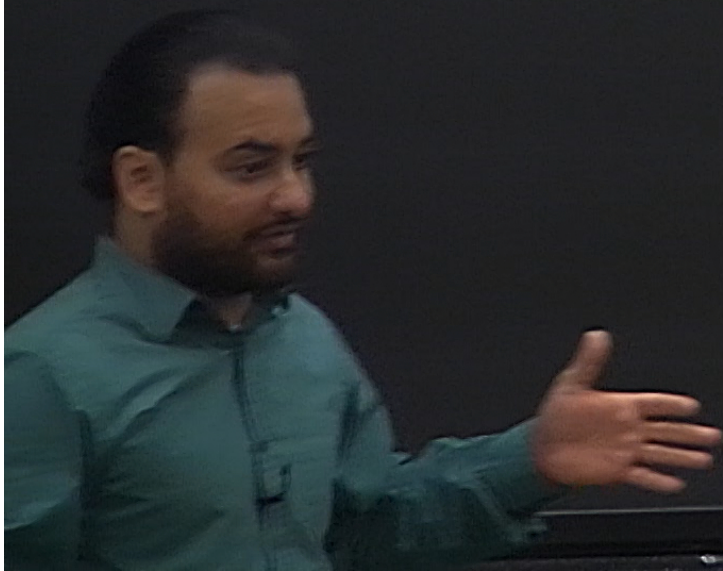
$$\approx g(E_F) \int_0^{\hbar\omega_c} d\xi \frac{1}{2\xi - E} = \frac{g(E_F)}{2} \ln\left(\frac{E - 2\hbar\omega_c}{E}\right)$$

$$|\psi_0\rangle = \sum_{k > k_F} \psi_k c_{k\uparrow}^+ c_{-k\downarrow}^+ |F\rangle$$

$$|\psi^{(N)}\rangle = \left( \sum_k g_k c_{k\uparrow}^+ c_{-k\downarrow}^+ \right)^{N/2} |\text{vacuum}\rangle$$



$$|\psi_G\rangle = \exp\left(\sum_{\mathbf{k}} g_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}\right) |\text{vacuum}\rangle$$



$$|\psi_G\rangle = \exp\left(\sum_{\mathbf{k}} g_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}\right) |\text{vacuum}\rangle$$

$$= \prod_{\mathbf{k}} \exp(g_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}) |\text{vacuum}\rangle$$

$$= \prod_{\mathbf{k}} \left(1 + g_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}\right) |\text{vacuum}\rangle$$

$$c_{\mathbf{k}\sigma}^{\dagger 2} = 0$$

$$= \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}\right) |\text{vacuum}\rangle$$

$$|u_{\mathbf{k}}|^2 \quad (\mathbf{k}\uparrow, -\mathbf{k}\downarrow) \text{ empty}$$

$$|v_{\mathbf{k}}|^2 \quad (\mathbf{k}\uparrow, -\mathbf{k}\downarrow) \text{ occupied}$$



$$|\psi_G\rangle = \exp\left(\sum_{\mathbf{k}} g_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}\right) |\text{vacuum}\rangle$$

$$= \prod_{\mathbf{k}} \exp(g_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger}) |\text{vacuum}\rangle$$

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$$|u_{\mathbf{k}}|^2 \quad (\mathbf{k}\uparrow, -\mathbf{k}\downarrow) \text{ empty}$$

$$|v_{\mathbf{k}}|^2 \quad (\mathbf{k}\uparrow, -\mathbf{k}\downarrow) \text{ occupied}$$

$$\boxed{|u_{\mathbf{k}}|^2 + |v_{\mathbf{k}}|^2 = 1}$$

$$-C_k - C_F$$

$$|00\rangle_k = (k\uparrow, -k\downarrow) \quad \text{Empty.}$$

$$|11\rangle_k = (k\uparrow, -k\downarrow) \quad \text{Occupied.}$$

$|m\rangle$

$|C_{\text{even}}\rangle$

$$|u\rangle^2 + |u_R\rangle^2 = 1$$



$$|00\rangle_k = (k\uparrow, -k\downarrow) \quad \text{empty}$$

$$|11\rangle_k = (k\uparrow, -k\downarrow) \quad \text{Occupied}$$

$$|\psi_g\rangle = \prod_k (u_k |00\rangle_k + v_k |11\rangle_k)$$



$$|00\rangle_k = (k\uparrow, -k\downarrow) \text{ Empty}$$

$$|11\rangle_k = (k\uparrow, -k\downarrow) \text{ Occupied}$$

$$|\psi_g\rangle = \prod_k (u_k |00\rangle_k + v_k |11\rangle_k)$$

$$|\text{vacuum}\rangle = \prod_k |00\rangle_k$$

$$\langle \psi_g | \psi_g \rangle = \prod_k (|u_k|^2 + |v_k|^2) = 1$$

$$|u_k|^2 + |v_k|^2 = 1$$



$$\bar{N} = \langle N \rangle = \langle \psi_G | \sum_{\mathbf{k}} (c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\uparrow} + c_{-\mathbf{k}\downarrow}^\dagger c_{-\mathbf{k}\downarrow}) | \psi_G \rangle$$

$$= 2 \sum_{\mathbf{k}} \langle \psi_G | c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\uparrow} | \psi_G \rangle = 2 \sum_{\mathbf{k}} |v_{\mathbf{k}}|^2$$

$$\bar{N} = \langle N \rangle = \langle \psi_G | \sum_k (c_{k\uparrow}^\dagger c_{k\uparrow} + c_{-k\downarrow}^\dagger c_{-k\downarrow}) | \psi_G \rangle$$

$$= 2 \sum_k \langle \psi_G | c_{k\uparrow}^\dagger c_{k\uparrow} | \psi_G \rangle = 2 \sum_k |v_k|^2$$

$$\langle N^2 \rangle = 4 \sum_k |v_k|^2 + 4 \sum_{k, k' \neq k} |v_k|^2 |v_{k'}|^2$$



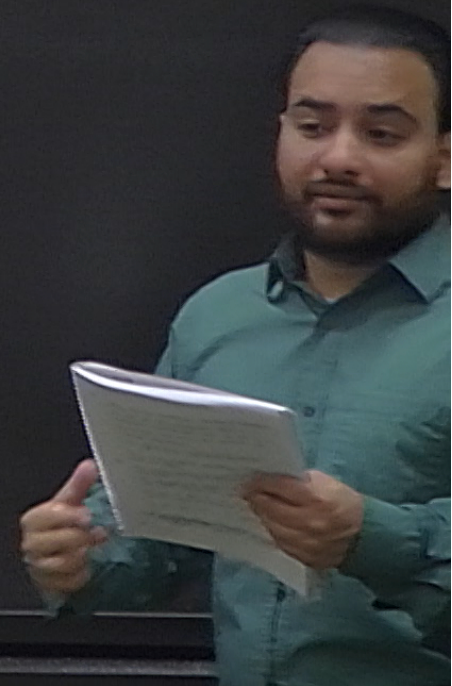
$$\bar{N} = \langle N \rangle = \langle \psi_G | \sum_{\mathbf{k}} (c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\uparrow} + c_{-\mathbf{k}\downarrow}^\dagger c_{-\mathbf{k}\downarrow}) | \psi_G \rangle$$

$$= 2 \sum_{\mathbf{k}} \langle \psi_G | c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\uparrow} | \psi_G \rangle = 2 \sum_{\mathbf{k}} |v_{\mathbf{k}}|^2$$

$$\langle N^2 \rangle = 4 \sum_{\mathbf{k}} |v_{\mathbf{k}}|^2 + 4 \sum_{\mathbf{k}, \mathbf{k}' \neq \mathbf{k}} |v_{\mathbf{k}}|^2 |v_{\mathbf{k}'}|^2$$

$$\langle (\delta N)^2 \rangle = \langle (N - \bar{N})^2 \rangle = \langle N^2 \rangle - \langle \bar{N} \rangle^2$$

$$= 4 \sum_{\mathbf{k}} |v_{\mathbf{k}}|^2 - |v_{\mathbf{k}}|^4 = 4 \sum_{\mathbf{k}} |v_{\mathbf{k}}|^2 |u_{\mathbf{k}}|^2$$



$\langle G \rangle$   
 $|V_k|^2$

$\sqrt{2}$   
 $\sqrt{2}$

$$\mathcal{H} = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k'} V_{kk'} c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger c_{-k'\downarrow} c_{k'\uparrow}$$

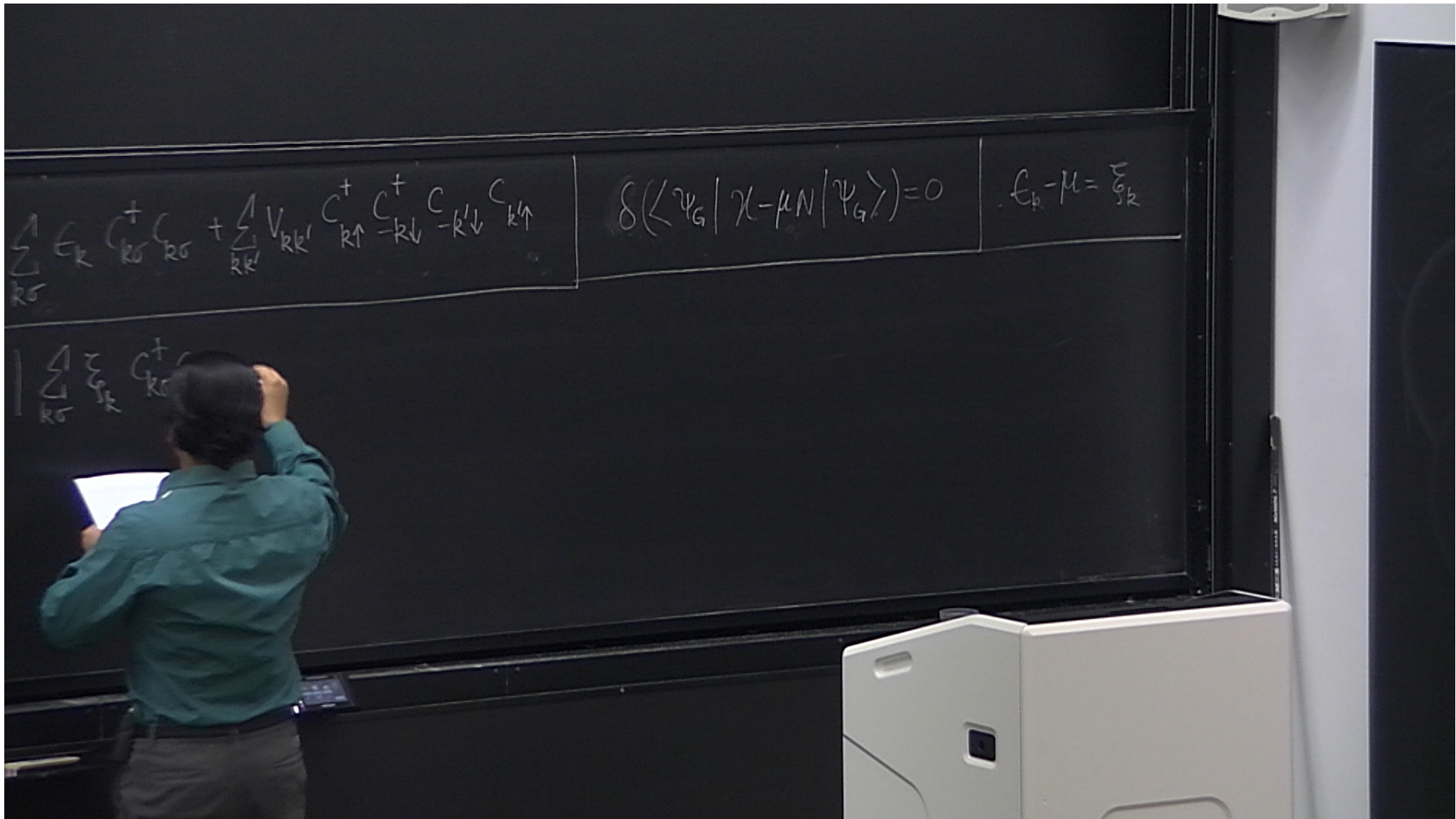


$$\mathcal{H} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

$$\delta(\langle \Psi_G | \mathcal{H} - \mu N | \Psi_G \rangle) = 0$$

$$\langle \Psi_G | \sum_{\mathbf{k}\sigma}$$







$$\delta(\langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle) = 0$$

$$E_k - \mu = \bar{\epsilon}_{Gk}$$

$$\mathcal{H} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

$$\delta \langle \psi_G | \mathcal{H} -$$

$$\langle \psi_G | \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} | \psi_G \rangle$$

$$|\psi\rangle$$

$$|\psi\rangle$$



$$c_{-k'\downarrow} c_{k'\uparrow}$$

$$\delta \langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle = 0$$

$$E_k - \mu = \epsilon_{Gk}$$

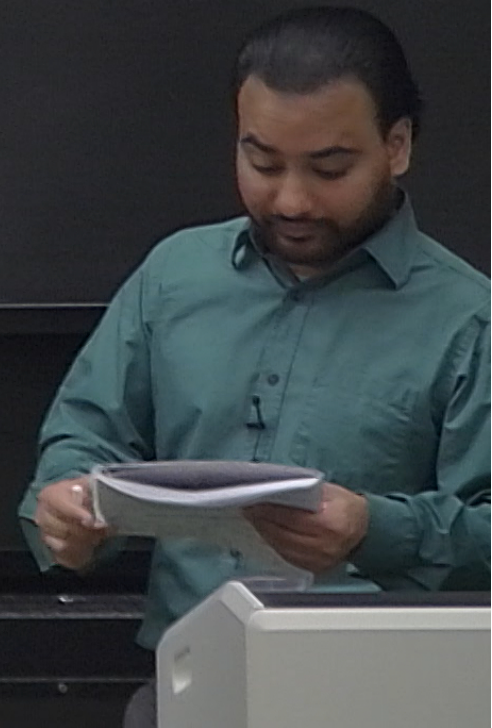
$$|\psi_G\rangle = (u_{k_1} |00\rangle_{k_1} + v_{k_1} |11\rangle_{k_1}) \dots (u_k |00\rangle_k + v_k |11\rangle_k)$$



$$\langle \psi_G | \sum_{k\sigma} \epsilon_{jk} c_{k\sigma}^\dagger c_{k\sigma} | \psi_G \rangle$$

$$= 2 \sum_k \epsilon_{jk} |v_k|^2$$

$$|\psi_G\rangle = (u_{k_1} |00\rangle_{k_1} + v_{k_1} |11\rangle_{k_1})$$





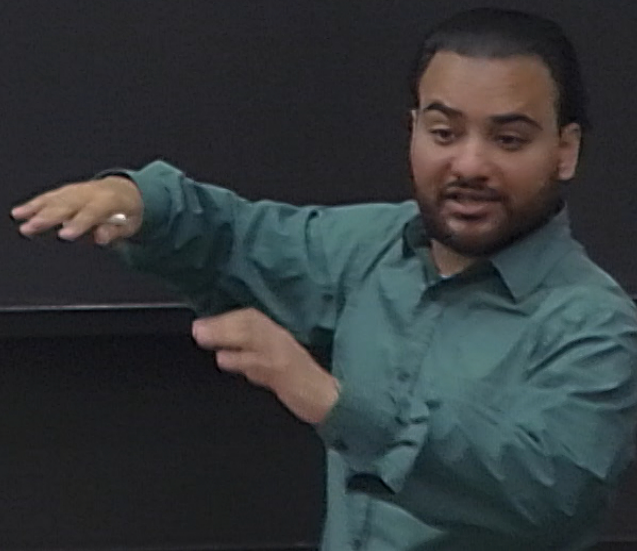
$$c_{-k\downarrow}^+ c_{-k'\downarrow} c_{k'\uparrow}$$

$$\delta \langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle = 0$$

$$\epsilon_k - \mu = \epsilon_{Gk}$$

$$|\psi_G\rangle = (u_{k_1} |00\rangle_{k_1} + v_{k_1} |11\rangle_{k_1}) \dots (u_k |00\rangle_k + v_k |11\rangle_k)$$

$$c_{-k'\downarrow} c_{k'\uparrow} |\psi_G\rangle = (u_{k_1} |00\rangle_{k_1} + v_{k_1} |11\rangle_{k_1}) \dots (v_{k'} |00\rangle_{k'}) \dots$$





$$c_{k\uparrow}^+ c_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

$$\delta \langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle = 0$$

$$\epsilon_k - \mu = \epsilon_k$$

$$|\psi_G\rangle = (u_{k_1} |00\rangle_{k_1} + v_{k_1} |11\rangle_{k_1}) \dots (u_k |00\rangle_k + v_k |11\rangle_k)$$

$$c_{-k'\downarrow} c_{k'\uparrow} |\psi_G\rangle = (u_{k_1} |00\rangle_{k_1} + v_{k_1} |11\rangle_{k_1}) \dots (v_{k'} |00\rangle_{k'}) \dots$$

$$c_{k\uparrow}^+ c_{-k\downarrow}^+ c_{-k'\downarrow} c_{k'\uparrow} |\psi_G\rangle = (u_{k_1} |00\rangle_{k_1} + v_{k_1} |11\rangle_{k_1}) \dots (u_k |11\rangle_k) \dots (v_{k'} |00\rangle_{k'})$$



$$\mathcal{H} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

$$\delta \langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle$$

$$\begin{aligned} \langle \psi_G | \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} | \psi_G \rangle \\ = 2 \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} |V_{\mathbf{k}}|^2 \end{aligned}$$

$$|\psi_G\rangle = (u_{\mathbf{k}_1} |00\rangle_{\mathbf{k}_1} + v_{\mathbf{k}_1} |11\rangle_{\mathbf{k}_1}) \dots (u_{\mathbf{k}} |$$

$$\begin{aligned} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} |\psi_G\rangle &= (u_{\mathbf{k}_1} |00\rangle_{\mathbf{k}_1} + v_{\mathbf{k}_1} |11\rangle_{\mathbf{k}_1}) \dots \\ c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} |\psi_G\rangle &= (u_{\mathbf{k}_1} |00\rangle_{\mathbf{k}_1} + v_{\mathbf{k}_1} |11\rangle_{\mathbf{k}_1}) \dots \end{aligned}$$

$$\delta \langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle = \delta \left( 2 \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} |V_{\mathbf{k}}|^2 + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} u_{\mathbf{k}} v_{\mathbf{k}}^* u_{\mathbf{k}'}^* v_{\mathbf{k}'} \right) = 0$$

$$\sum_k |u_k|^2 = \sum_k |v_k|^2$$

$$\delta \langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle = 0$$

$$u_k = \sin \theta_k \quad v_k = \cos \theta_k$$

$$\langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle = \sum_k \epsilon_k (1 + \cos 2\theta_k) + \frac{1}{4} \sum_{kk'} V_{kk'} \sin 2\theta_k \sin 2\theta_{k'}$$

$$\frac{\partial}{\partial \theta_{k''}} \langle \psi_G | \mathcal{H} - \mu N | \psi_G \rangle = 0 \Rightarrow -2 \epsilon_{k''} \sin 2\theta_{k''} + \frac{1}{4} \sum_{k'} V_{k''k'} (2 \cos 2\theta_{k''}) \sin 2\theta_{k'}$$

$$\tan 2\theta_k = \frac{\sum_{k'} V_{kk'} \sin 2\theta_{k'}}{2 \epsilon_k}$$



$$\delta \langle \Psi_G | X - \mu N | \Psi_G \rangle = \delta \left( 2 \sum_k \epsilon_k |V_k|^2 + \sum_{k,k'} V_{kk'} U_k V_k^* U_{k'}^* V_{k'} \right) = 0$$

$$V_{kk'} \sin 2\theta_k \sin 2\theta_{k'}$$

$$+ \frac{1}{4} \sum_{k'} V_{kk'} (2 \cos 2\theta_{k''}) \sin 2\theta_{k'} + \frac{1}{4} \sum_k V_{kk'} (\sin 2\theta_k) (2 \cos 2\theta_{k''}) = 0$$