Title: PSI 17/18 - Condensed Matter - Lecture 4

Date: Nov 09, 2017 10:45 AM

URL: http://pirsa.org/17110029

Abstract:

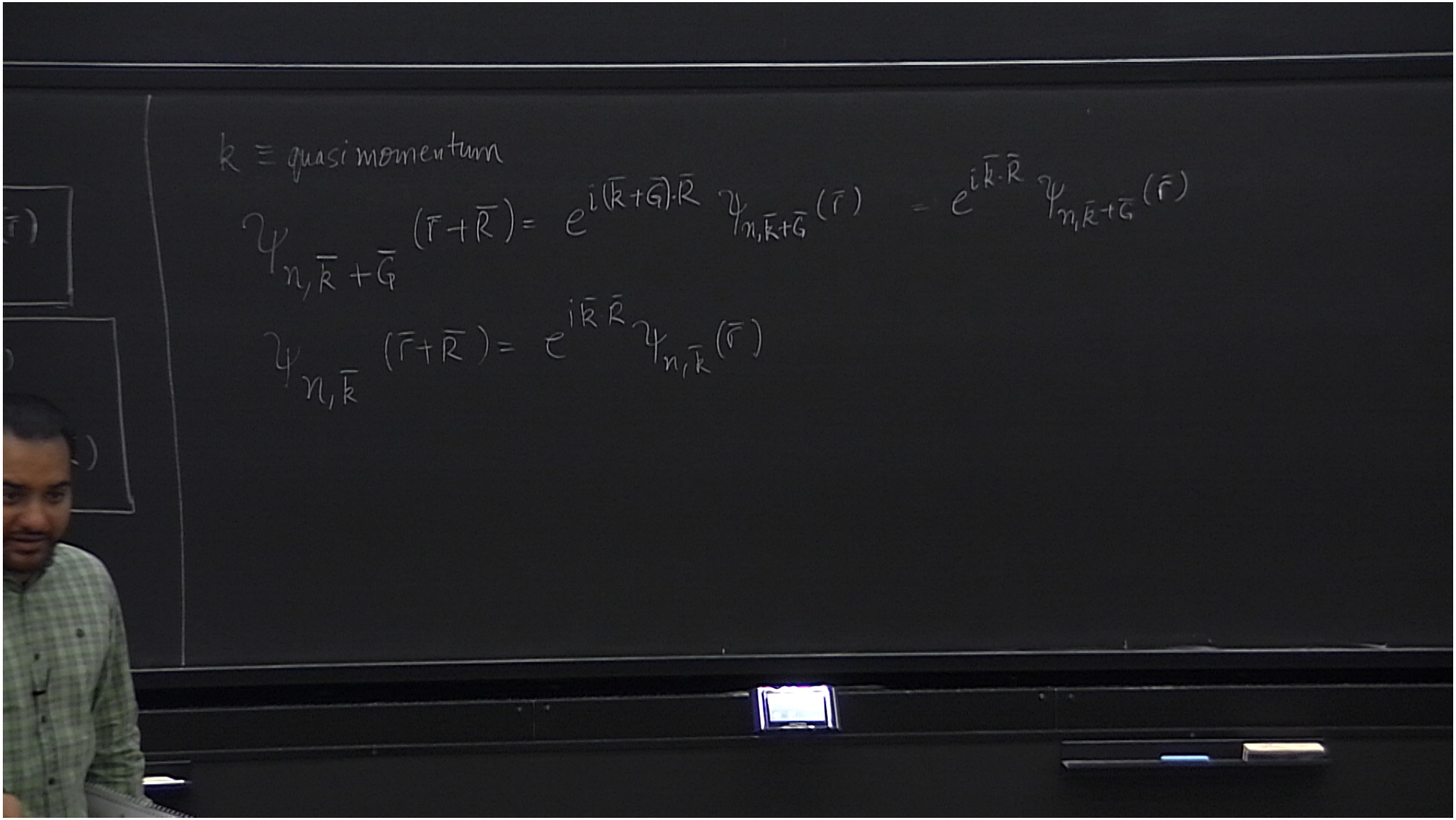
Bloch's theorem

$$\psi_{n,\bar{k}}(\bar{r}+\bar{R}) = e^{i\bar{k}\cdot\bar{R}}\,\psi_{n,\bar{k}}(\bar{r})$$

and

$$\psi_{n,\bar{k}}(\bar{r}) = e^{i\bar{k}\cdot\bar{r}}\,u_{n,\bar{k}}(\bar{r})$$

where $u_{n,\bar{k}}(\bar{r}) = u_{n,\bar{k}}(\bar{r}+\bar{R})$

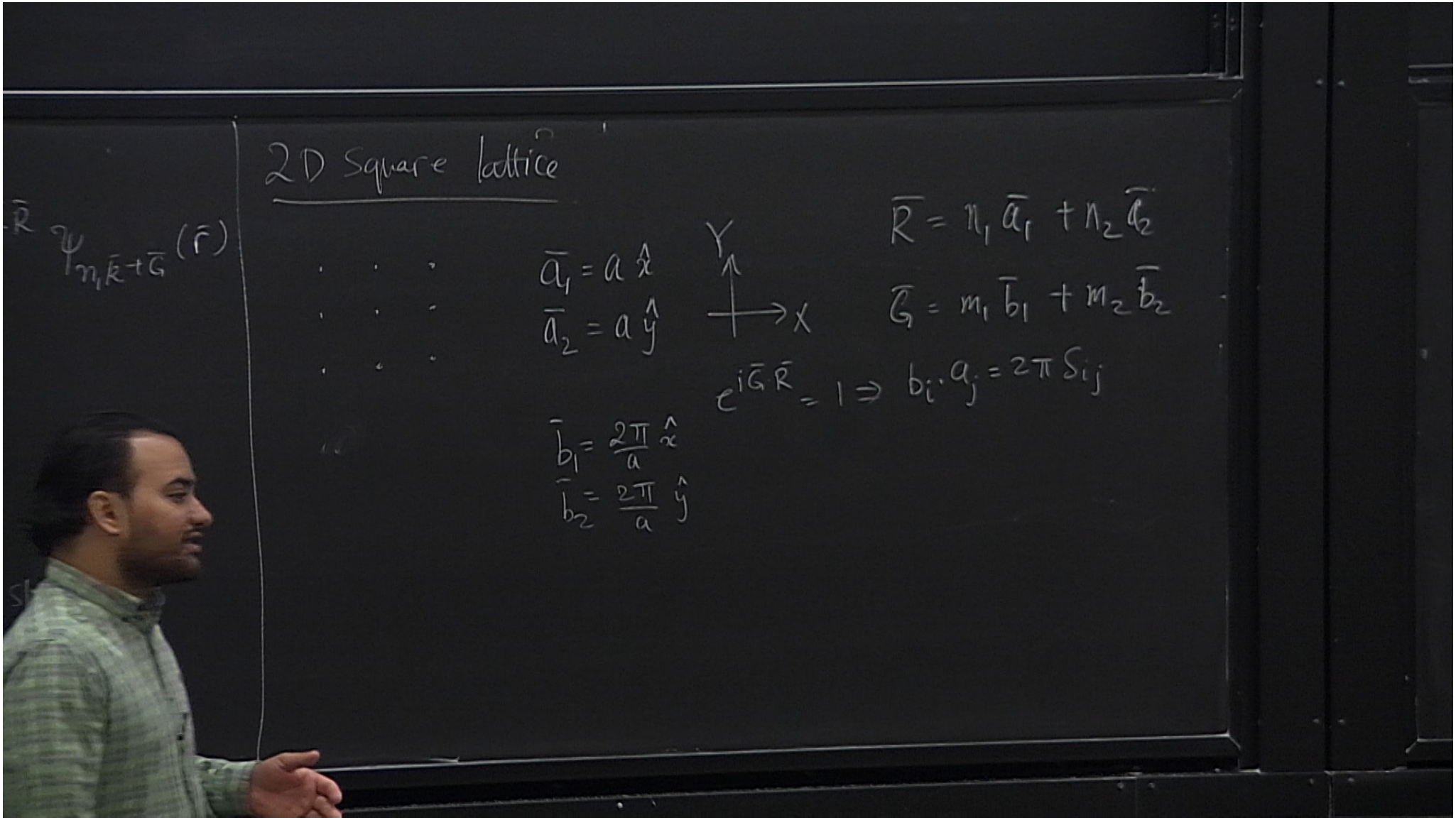

k ≡ quasi momentum
$\psi_{n,\bar{k}+\bar{G}}(\bar{r}+\bar{R}) = e^{i(\bar{k}+\bar{G})\cdot\bar{R}}\,\psi_{n,\bar{k}+\bar{G}}(\bar{r}) = e^{i\bar{k}\cdot\bar{R}}\,\psi_{n,\bar{k}+\bar{G}}(\bar{r})$
$\psi_{n,\bar{k}}(\bar{r}+\bar{R}) = e^{i\bar{k}\bar{R}}\,\psi_{n,\bar{k}}(\bar{r})$

$k \equiv$ quasi momentum

$$\psi_{n,\bar{k}+\bar{G}}(\bar{r}+\bar{R}) = e^{i(\bar{k}+\bar{G})\cdot\bar{R}}\,\psi_{n,\bar{k}+\bar{G}}(\bar{r}) = e^{i\bar{k}\cdot\bar{R}}\,\psi_{n,\bar{k}+\bar{G}}(\bar{r})$$

$$\psi_{n,\bar{k}}(\bar{r}+\bar{R}) = e^{i\bar{k}\bar{R}}\,\psi_{n,\bar{k}}(\bar{r})$$

First Brillouin Znone (FBZ) : Region in Reciprocal space which is closer to $\bar{G}=0$ than any other $\bar{G}$


2D Square lattice
$\bar{a}_1 = a\hat{x}$
$\bar{a}_2 = a\hat{y}$
Y
X
$\bar{R} = n_1\bar{a}_1 + n_2\bar{a}_2$
$\bar{G} = m_1\bar{b}_1 + m_2\bar{b}_2$
$e^{i\bar{G}\bar{R}} = 1 \Rightarrow b_i \cdot a_j = 2\pi\delta_{ij}$
$\bar{b}_1 = \frac{2\pi}{a}\hat{x}$
$\bar{b}_2 = \frac{2\pi}{a}\hat{y}$
$\psi_{n_1\bar{k}+\bar{G}}(\bar{r})$

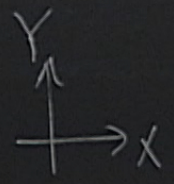
$\bar{R}$ $\psi_{n_1\bar{k}+\bar{G}}(\bar{r})$

Space

2D Square lattice

$\bar{a}_1 = a\hat{x}$

$\bar{a}_2 = a\hat{y}$



$\bar{R} = n_1\bar{a}_1 + n_2\bar{a}_2$

$\bar{G} = m_1\bar{b}_1 + m_2\bar{b}_2$

$e^{i\bar{G}\bar{R}} = 1 \Rightarrow b_i \cdot a_j = 2\pi\delta_{ij}$

$\bar{b}_1 = \frac{2\pi}{a}\hat{x}$

$\bar{b}_2 = \frac{2\pi}{a}\hat{y}$

(0,1) (0,0) (1,0)

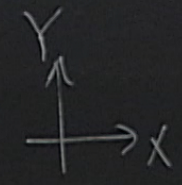
$\bar{R}$ $\psi_{n,\bar{k}+\bar{G}}(\bar{r})$

Space

2D Square lattice

$\bar{a}_1 = a\hat{x}$

$\bar{a}_2 = a\hat{y}$



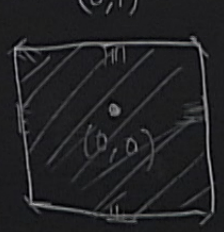
$\bar{R} = n_1\bar{a}_1 + n_2\bar{a}_2$

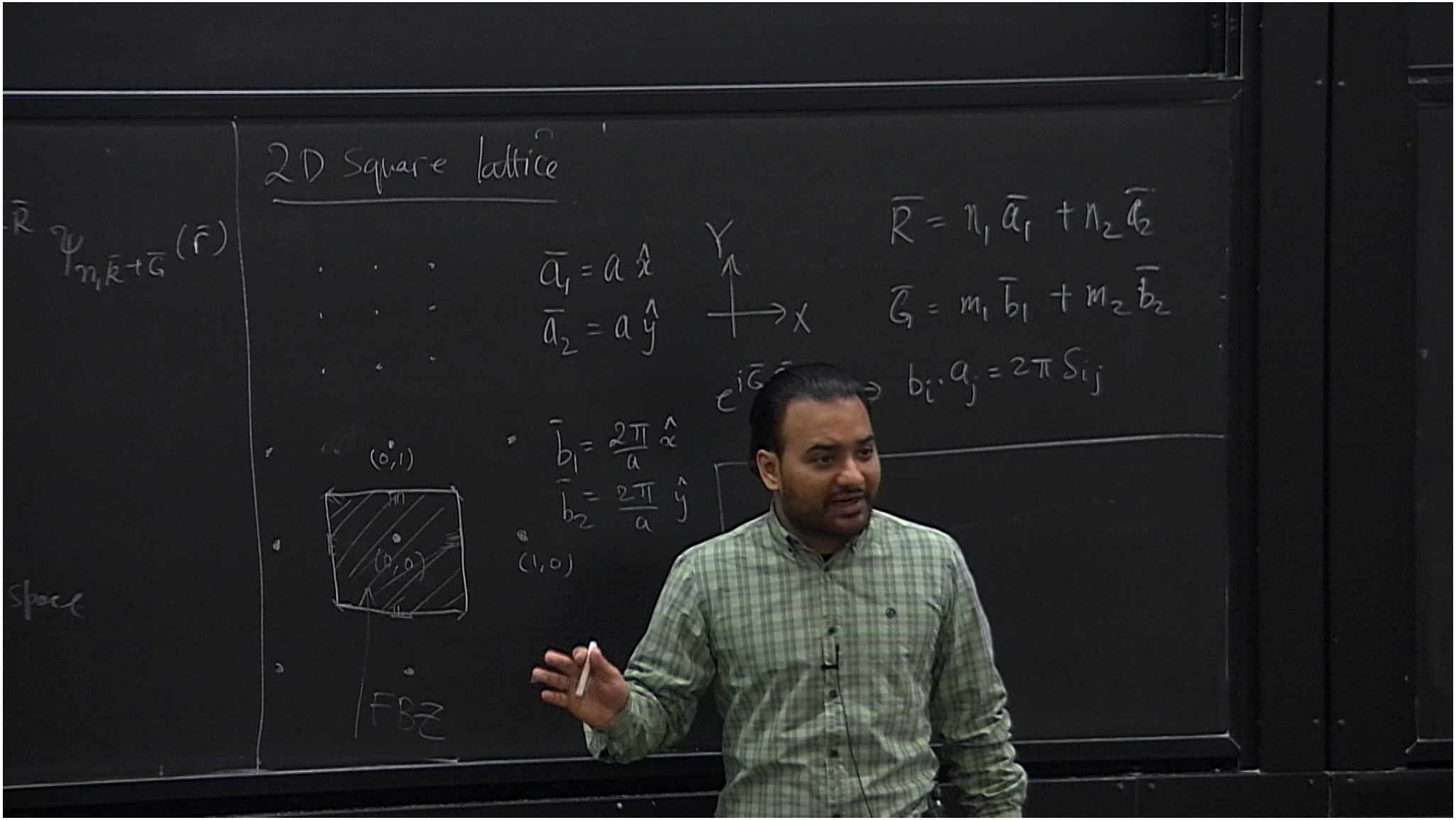
$\bar{G} = m_1\bar{b}_1 + m_2\bar{b}_2$

$e^{i\bar{G}\bar{R}} = 1 \Rightarrow \bar{b}_i \cdot \bar{a}_j = 2\pi\delta_{ij}$

$\bar{b}_1 = \frac{2\pi}{a}\hat{x}$

$\bar{b}_2 = \frac{2\pi}{a}\hat{y}$




2D Square lattice
$\bar{a}_1 = a\hat{x}$
$\bar{a}_2 = a\hat{y}$
$\bar{R} = n_1\bar{a}_1 + n_2\bar{a}_2$
$\bar{G} = m_1\bar{b}_1 + m_2\bar{b}_2$
$b_i \cdot a_j = 2\pi\delta_{ij}$
$\bar{b}_1 = \frac{2\pi}{a}\hat{x}$
$\bar{b}_2 = \frac{2\pi}{a}\hat{y}$
(0,1)
(0,0)
(1,0)
FBZ
Y
X
Space

$\bar{R}\ \psi_{n,\bar{k}+\bar{G}}(\bar{r}) = e^{i\bar{k}\cdot\bar{R}}\,\psi_{n,\bar{k}+\bar{G}}(\bar{r})$

$\psi_{n,\bar{k}}(\bar{r})$

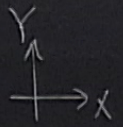
: Region in Reciprocal Space

any other $\bar{G}$

2D Square lattice

$\bar{a}_1 = a\,\hat{x}$

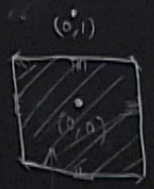
$\bar{a}_2 = a\,\hat{y}$



$\bar{R} = n_1\bar{a}_1 + n_2\bar{a}_2$

$\bar{G} = m_1\bar{b}_1 + m_2\bar{b}_2$

$e^{i\bar{G}\bar{R}} = 1 \Rightarrow b_i \cdot a_j = 2\pi\,\delta_{ij}$



$\bar{b}_1 = \frac{2\pi}{a}\hat{x}$

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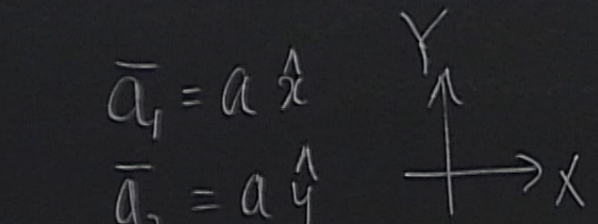
$\forall\, q$ in Reciprocal Space

$\exists\, k$ in FBZ and a $\bar{G}$

such that $\bar{q} = \bar{k} + \bar{G}$

$\bar{a}_1 = a\hat{x}$

$\bar{a}_2 = a\hat{y}$



$\bar{R} = n_1\bar{a}_1 + n_2\bar{a}_2$

$\bar{G} = m_1\bar{b}_1 + m_2\bar{b}_2$

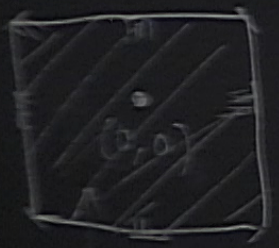
$e^{i\bar{G}\cdot\bar{R}} = 1 \Rightarrow b_i \cdot a_j = 2\pi\delta_{ij}$

$\bar{b}_1 = \frac{2\pi}{a}\hat{x}$

$\bar{b}_2 = \frac{2\pi}{a}\hat{y}$

(0,1)

(1,0)



FBZ

$\forall\, q$ in Reciprocal space

$\exists\, k$ in FBZ and a G

Such that $\bar{q} = \bar{k} + \bar{G}$

$$\psi_{n,\bar{q}}(\bar{r}) = \psi_{n,\bar{k}+\bar{G}}(\bar{r})$$

Tight. binding Model

1 D Lattice

. . . .

$$H = -\sum_{ij}\left(t_{ij}\, c_i^{+} c_j + t_{ji}\, c_j^{+} c_i\right) + \sum_i V_i\, c_i^{+} c_i$$

⋮

$$\psi_{n,\bar{q}}(\bar{r}) = \psi_{n,\bar{k}+\bar{G}}(\bar{r})$$

Tight binding Model

1 D Lattice

$$H = -\sum_{ij}\left(t_{ij}\, c_i^{+} c_j + t_{ji}\, c_j^{+} c_i\right) + \sum_i V_i\, c_i^{+} c_i$$

V_i = Constant = V

$$V \sum_i c_i^{+} c_i = V N_e$$



$$\psi_{n,\bar{q}}(\bar{r}) = \psi_{n,\bar{k}+\bar{G}}(\bar{r})$$

Tight. binding Model

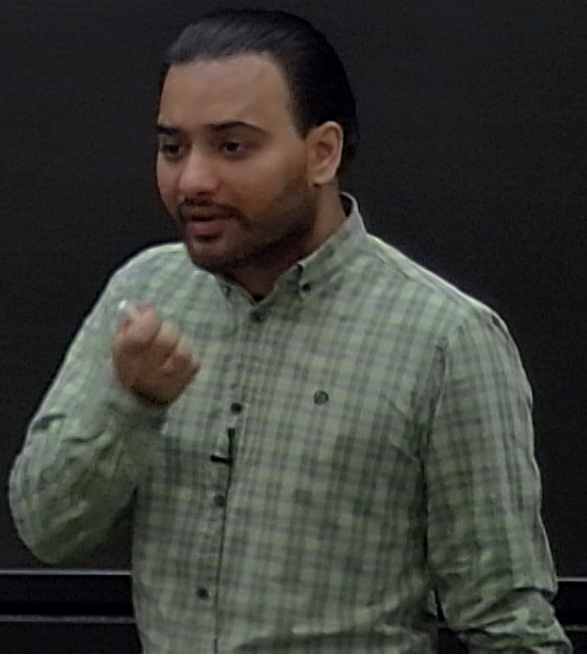
1 D Lattice

. . . . .

$$H = -\sum_{ij}\left(t_{ij}\, c_i^{+} c_j + t_{ji}\, c_j^{+} c_i\right) + \sum_i V_i\, c_i^{+} c_i$$

V_i = Constant = V

$$V \sum_i c_i^{+} c_i = V N_e$$



$$\psi_{n,\bar{q}}(\bar{r}) = \psi_{n,\bar{k}+\bar{G}}(\bar{r})$$

Tight. binding Model

1 D Lattice

. . . . .

$$H = -\sum_{ij}\left(t_{ij}\, c_i^{\dagger} c_j + t_{ji}\, c_j^{\dagger} c_i\right) + \sum_i V_i\, c_i^{\dagger} c_i$$

V_i = Constant = V

$V \sum_i c_i^{\dagger} c_i = V N_e$

$$\approx -\sum_{\langle ij \rangle} t_{ij}\, c_i^{\dagger} c_j + t_{ji}\, c_j^{\dagger} c_i$$

$$\psi_{n,\bar{q}}(\bar{r}) = \psi_{n,\bar{k}+\bar{G}}(\bar{r})$$

$H = H^{\dagger} \Rightarrow$

Tight. binding Model

1 D Lattice

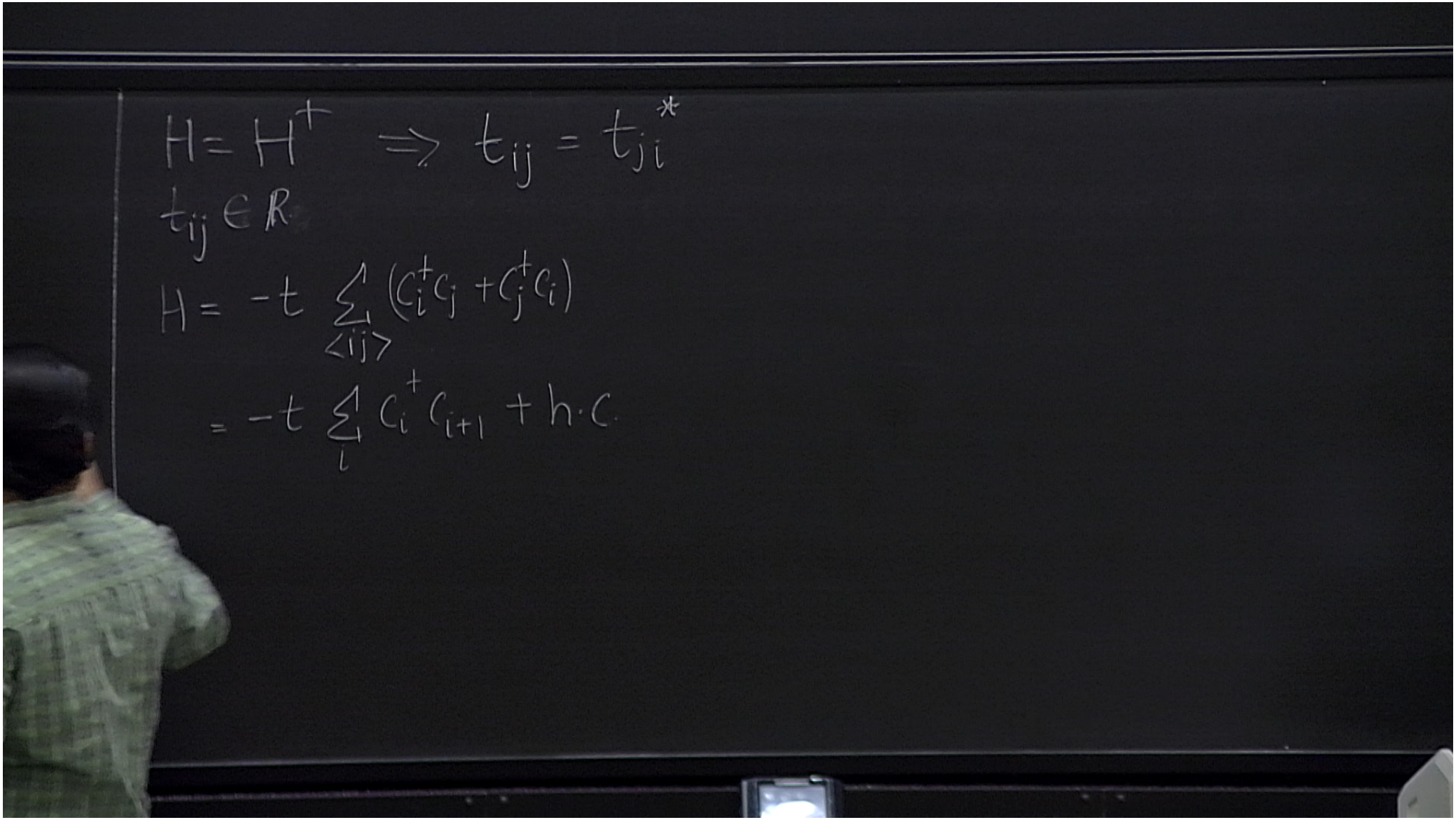
. . . . .

$$H = -\sum_{ij}\left(t_{ij}\, c_i^{\dagger} c_j + t_{ji}\, c_j^{\dagger} c_i\right) + \sum_i V_i\, c_i^{\dagger} c_i$$

V_i = Constant = V

$$V \sum_i c_i^{\dagger} c_i = V N_e$$

$$\rightsquigarrow -\sum_{\langle ij \rangle}\left(t_{ij}\, c_i^{\dagger} c_j + t_{ji}\, c_j^{\dagger} c_i\right)$$


$H = H^\dagger \Rightarrow t_{ij} = t_{ji}^*$
$t_{ij} \in \mathbb{R}$
$H = -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + c_j^\dagger c_i)$
$= -t \sum_i c_i^\dagger c_{i+1} + h.c.$

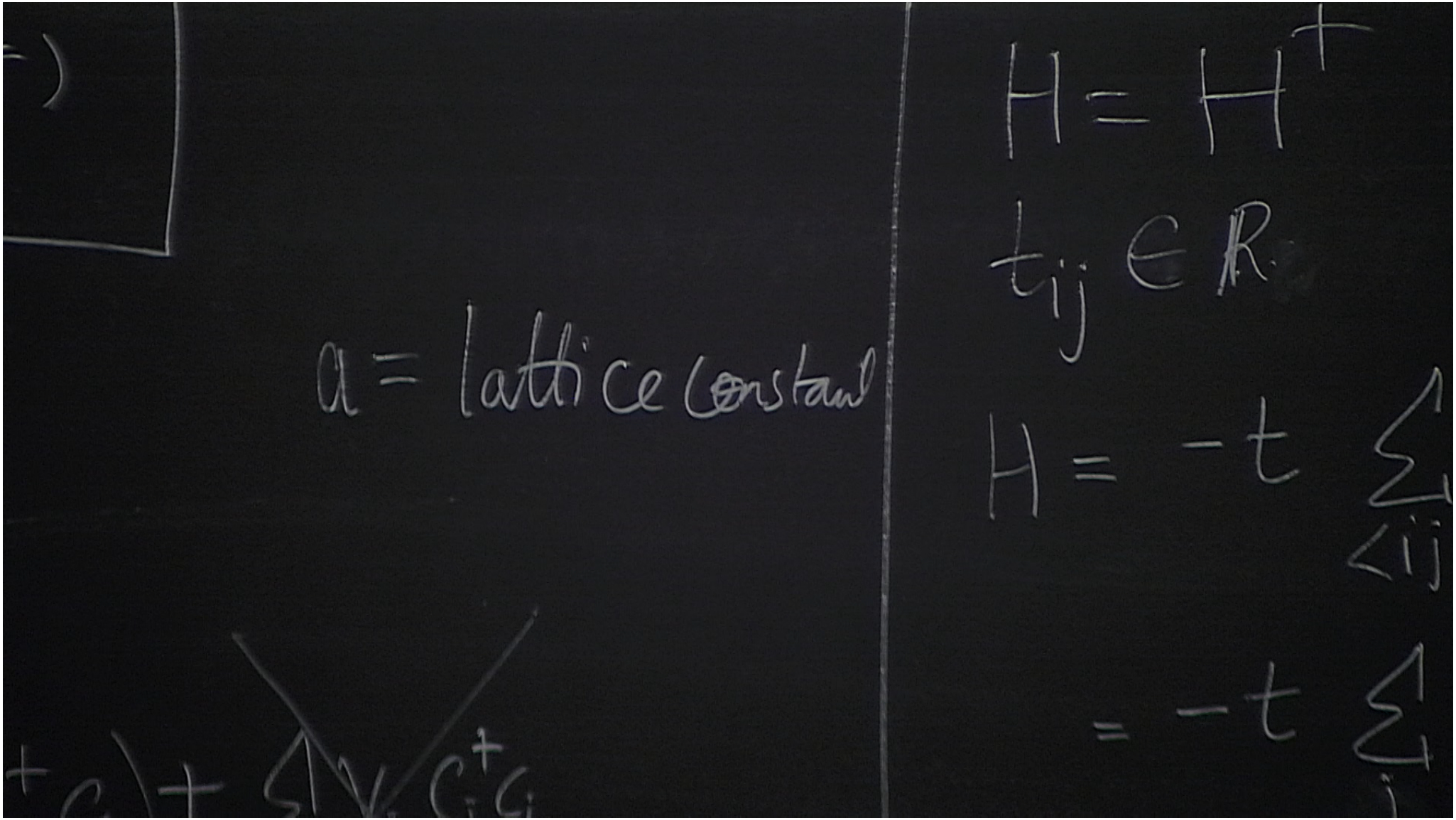
$$H = H^{\dagger} \Rightarrow t_{ij} = t_{ji}^{*}$$

$t_{ij} \in \mathbb{R}$

$$H = -t \sum_{\langle ij \rangle} (c_i^{\dagger} c_j + c_j^{\dagger} c_i)$$

$$= -t \sum_j c_j^{\dagger} c_{j+1} + h.c.$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k e^{ikja} c_k$$


a = lattice constant
$H = H^\dagger$
$t_{ij} \in \mathbb{R}$
$H = -t \sum_{\langle ij \rangle}$
$= -t \sum_{i}$

$$H = -\frac{t}{N} \sum_{j} \sum_{kk'} e^{-ikja} e^{ik'(j+1)a} c_k^\dagger c_{k'} + h.c.$$

$$\frac{1}{N} \sum_{j} e^{-i(k-k')ja} = \delta_{kk'}$$

a = lattice constant

$+ \sum_i V_i c_i^\dagger c_i$

V_i = Constant = V

$V \sum_i c_i^\dagger c_i = VNe$

$c_j^\dagger c_i$)

$H = H^\dagger \Rightarrow t_{ij} = t_{ji}^*$

$t_{ij} \in \mathbb{R}$

$$H = -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + c_j^\dagger c_i)$$

$$= -t \sum_j c_j^\dagger c_{j+1} + h.c.$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k e^{ikja} c_k$$

N = total # of sites

$H = \frac{-t}{N} \sum_j \sum_{kk}$

$\frac{1}{N} \sum_j e^{-i(k}$

$$H = -\frac{t}{N} \sum_{j} \sum_{kk'} e^{-ikja} e^{ik'(j+1)a} c_k^\dagger c_{k'} + h.c.$$

$$\frac{1}{N} \sum_{j} e^{-i(k-k')ja} = \delta_{kk'}$$

$$H = -t \sum_{k} \left(e^{ika} + e^{-ika}\right) c_k^\dagger c_k$$

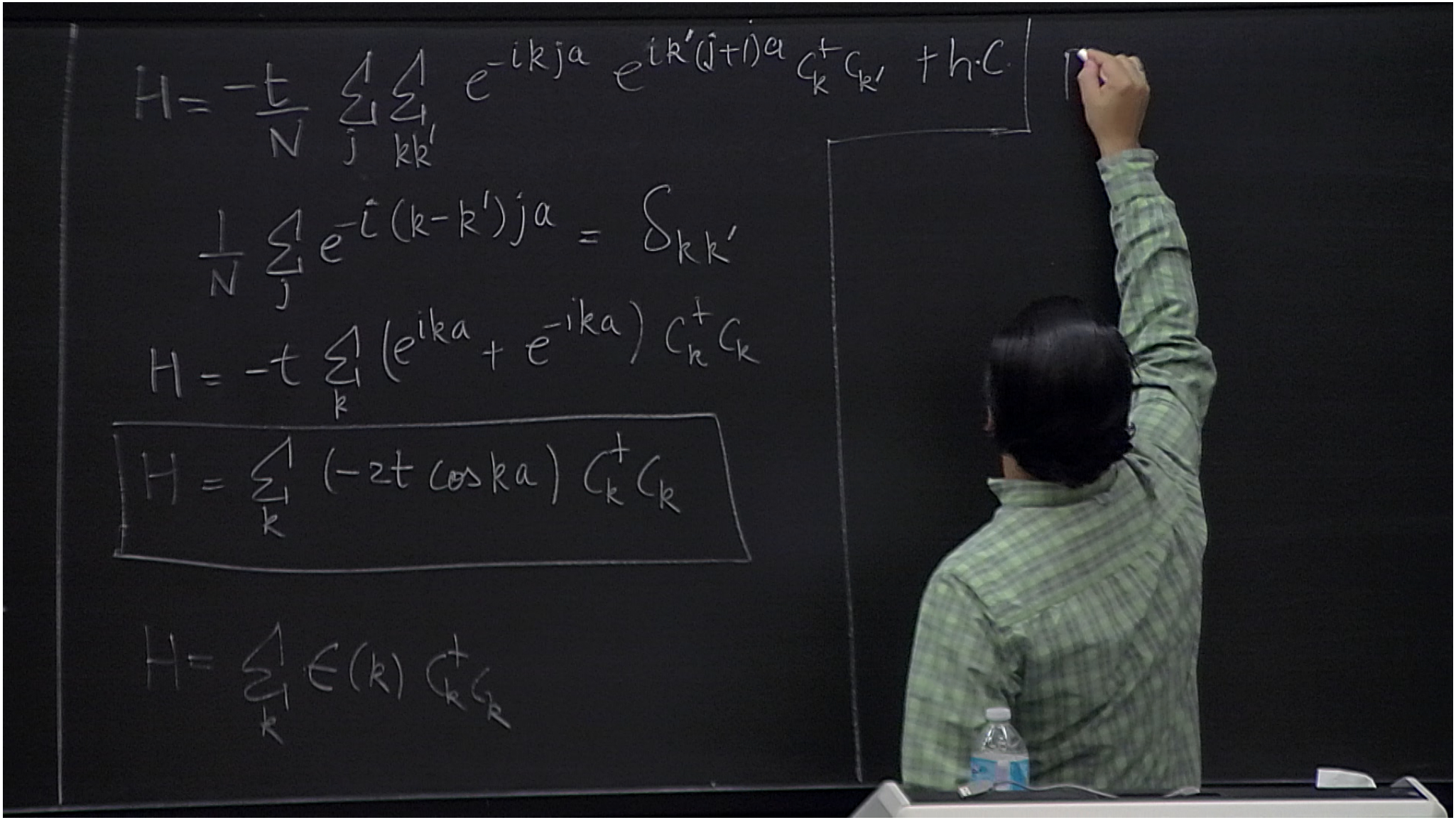
$$= \sum_{k} \left(-2t \cos ka\right) c_k^\dagger c_k$$

$$H = -\frac{t}{N} \sum_{j} \sum_{kk'}' e^{-ikja} e^{ik'(j+1)a} c_k^\dagger c_{k'} + h.c.$$

$$\frac{1}{N} \sum_{j} e^{-i(k-k')ja} = \delta_{kk'}$$

$$H = -t \sum_{k}' \left(e^{ika} + e^{-ika}\right) c_k^\dagger c_k$$

$$\boxed{H = \sum_{k}' \left(-2t \cos ka\right) c_k^\dagger c_k}$$


$H = -\frac{t}{N} \sum_j \sum_{kk'} e^{-ikja} e^{ik'(j+1)a} c_k^\dagger c_{k'} + h.c.$
$\frac{1}{N} \sum_j e^{-i(k-k')ja} = \delta_{kk'}$
$H = -t \sum_k \left(e^{ika} + e^{-ika}\right) c_k^\dagger c_k$
$H = \sum_k \left(-2t \cos ka\right) c_k^\dagger c_k$
$H = \sum_k \epsilon(k)\, c_k^\dagger c_k$


$e^{ik'(j+l)a} c_k^\dagger c_{k'}$ + h.c.
Free particles
$\epsilon(k) = \frac{\hbar^2 k^2}{2m}$
$= \delta_{kk'}$
$ka) c_k^\dagger c_k$
$c_k^\dagger c_k$

$e^{ik'(j+1)a} c_k^\dagger c_{k'}$ + h.c.

$= \delta_{kk'}$

$ka)\, c_k^\dagger c_k$

$c_k^\dagger c_k$

Free particles $\epsilon(k) = \frac{\hbar^2 k^2}{2m}$

$\vec{R} = n_1 \vec{a}_1$ $\quad \vec{a}_1 = a\hat{x}$

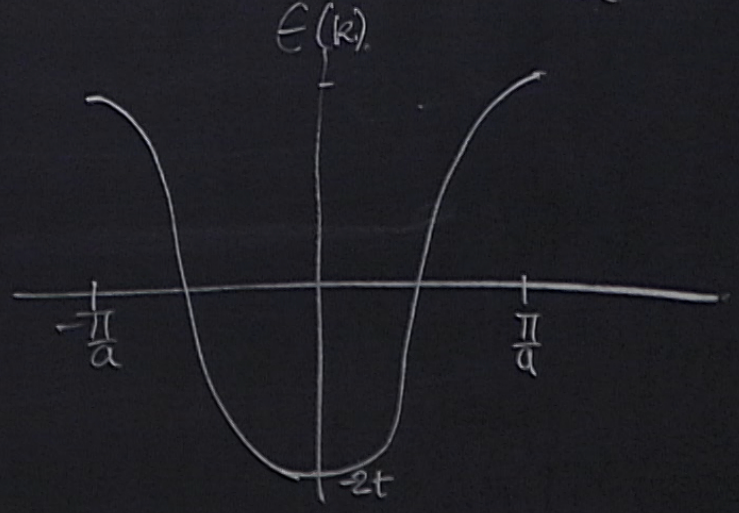
$e^{i\vec{G}\cdot\vec{R}} = 1 \Rightarrow \vec{G} = m_1 \vec{b}_1$

$b_1 = \frac{2\pi}{a}\hat{x}$

$-\frac{2\pi}{a}$ $-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$ $\frac{2\pi}{a}$

$k \in FBZ$

$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$



$e^{ik(j+1)a} c_k^\dagger c_{k'}$ + h.c.

Free particles $\epsilon(k) = \frac{\hbar^2 k^2}{2m}$

$= \delta_{kk'}$

$-ika) c_k^\dagger c_k$

$c_k^\dagger c_k$

$\vec{R} = n_1 \vec{a}_1$ $\quad \vec{a}_1 = a\hat{x}$

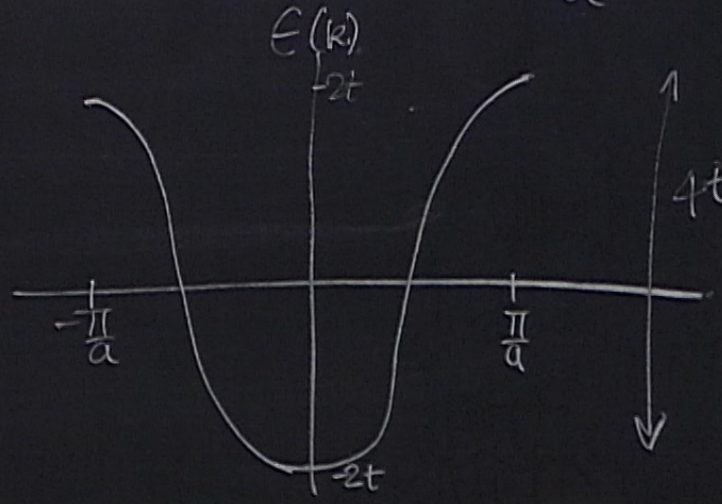
$e^{i\vec{G}\cdot\vec{R}} = 1 \Rightarrow \vec{G} = m_1 \vec{b}_1$

$\vec{b}_1 = \frac{2\pi}{a}\hat{x}$

$-\frac{2\pi}{a}$ $-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$ $\frac{2\pi}{a}$ $\longrightarrow x$

$k \in FBZ$

$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$



Nearly free electrons

electrons in a weak periodic potential

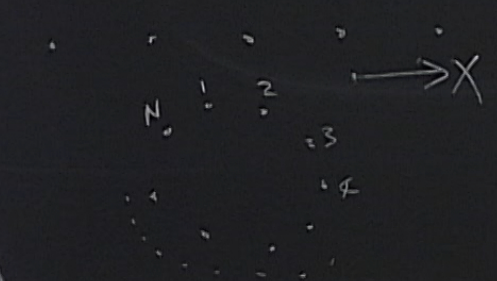
→ Perturbation theory

Nearly free electrons

electrons in a weak periodic potential

→ Perturbation theory

Periodic Boundary Conditions



$$\psi_{\bar{k}}(\bar{r} + N\bar{a}) = \psi_{\bar{k}}(\bar{r}) = e^{i\bar{k}\cdot N\bar{a}}\,\psi_{\bar{k}}(\bar{r})$$

$$e^{i\bar{k}\cdot N\bar{a}} = 1$$

$$\Rightarrow \bar{k}\cdot\bar{a} = \frac{2\pi}{N}n \quad , n \in \mathbb{Z}$$

$$\Rightarrow \bar{k} = k\hat{x} \Rightarrow k = \frac{2\pi}{Na} = \frac{2\pi}{L}$$

ons

periodic potential

ary Conditions

$$\psi_{\bar{k}}(\bar{r}+N\bar{a}) = \psi_{\bar{k}}(\bar{r}) = e^{i\bar{k}\cdot N\bar{a}}\,\psi_{\bar{k}}(\bar{r})$$

$$e^{i\bar{k}\cdot N\bar{a}} = 1$$

$$\Rightarrow \bar{k}\cdot\bar{a} = \frac{2\pi}{N}n \quad , n\in\mathbb{Z}$$

$$\Rightarrow \bar{k} = k\hat{x} \Rightarrow k = \frac{2\pi}{Na}n = \frac{2\pi}{L}n$$

FBZ

$$k\in\left(-\frac{\pi}{a},\frac{\pi}{a}\right)$$

$$n\in\left(-\frac{N}{2},\frac{N}{2}\right)$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_{p,k}(x) + U(x)\,\psi_{p,k}(x) = E_p(k)\,\psi_{p,k}(x)$$

$$U(x) = U(x+a)$$

$$\psi_{p,k}(x) = e^{ikx}\,u_{p,k}(x)$$

where $u_{p,k}(x) = u_{p,k}(x+a)$

$$-\frac{\hbar^2}{2m}\left(-k^2 u_{p,k}(x) + 2ik\,u'_{p,k}(x) + u''_{p,k}(x)\right) + U(x)\,u_{p,k}(x) = E_p(x)\,u_{p,k}(x)$$

$$U(x) = \frac{1}{L}\sum_q e^{iqx}\,\tilde{U}_q \quad ; \quad u_{p,k}(x) = \frac{1}{\sqrt{L}}\sum_q e^{iqx}\,\tilde{u}_{p,k}(q)$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_{p,k}(x) + U(x)\,\psi_{p,k}(x) = E_p(k)\,\psi_{p,k}(x)$$

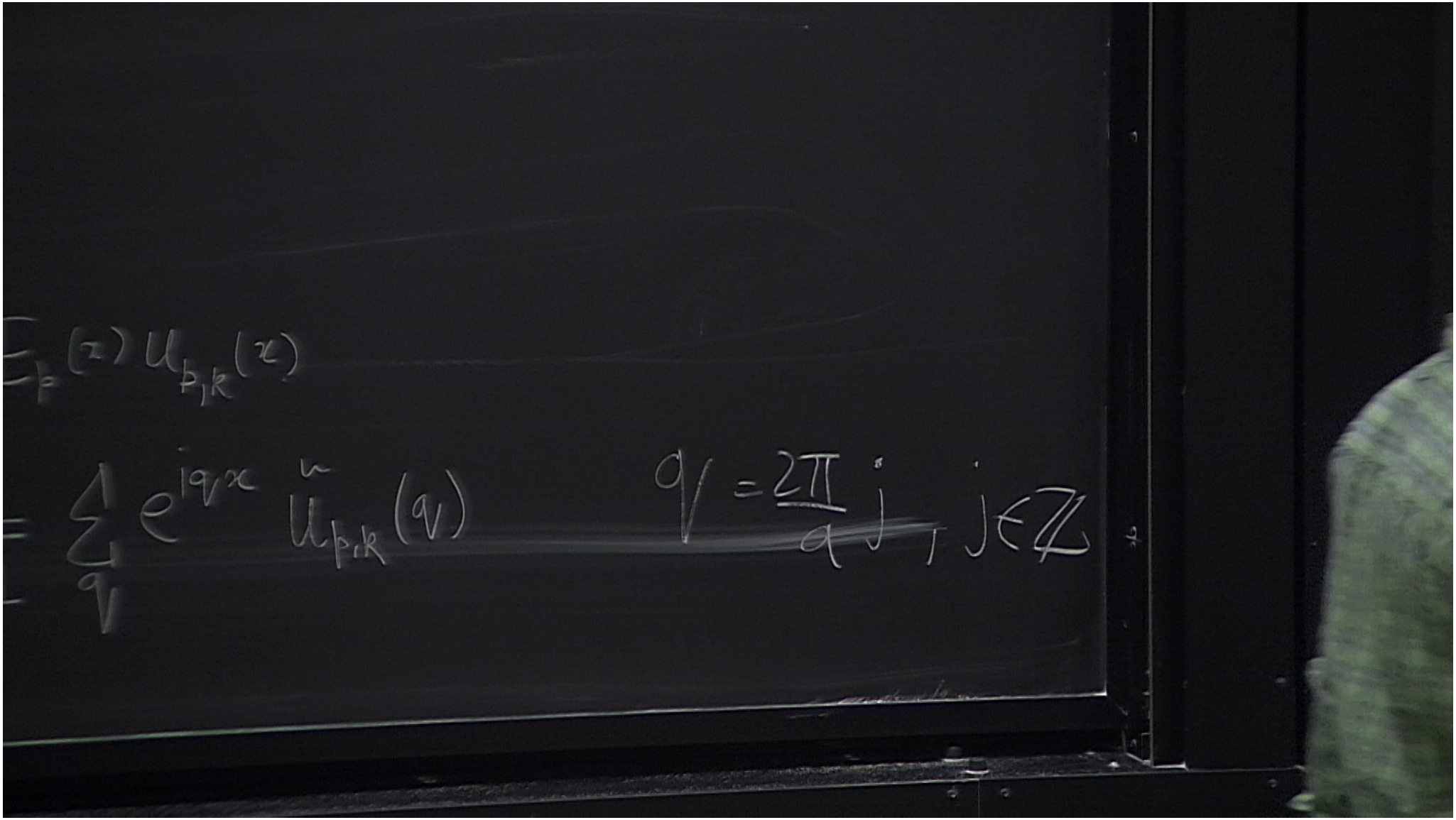
$$U(x) = U(x+a)$$

$$\psi_{p,k}(x) = e^{ikx}\,u_{p,k}(x)$$

where $u_{p,k}(x) = u_{p,k}(x+a)$

$$\Rightarrow -\frac{\hbar^2}{2m}\left(-k^2 u_{p,k}(x) + 2ik\,u'_{p,k}(x) + u''_{p,k}(x)\right) + U(x)\,u_{p,k}(x) = E_p(x)\,u_{p,k}(x)$$

$$U(x) = \frac{1}{L}\sum_q e^{iqx}\,\tilde{U}_q \quad ; \quad u_{p,k}(x) = \frac{1}{\sqrt{L}}\sum_q e^{iqx}\,\tilde{u}_{p,k}(q)$$


e^{iqx}
$q = \frac{2\pi}{a} j$, $j \in \mathbb{Z}$

$$-\frac{\hbar^2}{2mNL}\sum_{q}^{1}\left[-k^2 e^{iqx}\tilde{u}_{p,k}(q)+2ik(iq)\,e^{iqx}\tilde{u}_{p,k}(q)-q^2\tilde{u}_{p,k}(q)\right]+\frac{1}{L^{3/2}}\sum_{qq'}^{1}e^{iq'x}e^{iqx}\tilde{U}(q')\,\tilde{u}_{p,k}(q)$$



$$\sum_{qq'} e^{iq'x} e^{iqx} \tilde{U}(q') \tilde{u}_{pk}(q) = E_p(k) \frac{1}{\sqrt{L}} \sum_q e^{iqx} \tilde{u}_{pk}(q)$$

$$\sum_{q q'} e^{iq'x} e^{ivx} \tilde{U}(q') \tilde{u}_{p,k}(q) = E_p(k) \frac{1}{\sqrt{L}} \sum_{q} e^{ivx} \tilde{u}_{p,k}(q)$$

$$q'' = q' + q$$

$$-\frac{\hbar^2}{2m\sqrt{L}} \sum_q' \left[-k^2 e^{iqx} \tilde{u}_{b,k}(q) + 2ik(iq)\, e^{iqx} \tilde{u}_{b,k}(q) - q^2 \tilde{u}_{b,k}(q) \right] + \frac{1}{L^{3/2}} \sum_{qq'}' e^{iq'x} e^{iqx} \tilde{U}(q') \tilde{u}_{b,k}(q)$$

$$q'' = q' + q$$

$$\sum_q' \left(\frac{\hbar^2}{2m} (k+q)^2 \tilde{u}_{b,k}(q) + \frac{1}{L} \sum_{q'}' \tilde{U}(q')\, u_{b,k}(q-q') - E_b(k)\, \tilde{u}_{b,k}(q) \right) = 0$$

$$-\frac{\hbar^2}{2mNL}\sum_{q}\left[-k^2 e^{iqx}\tilde{u}_{b,k}(q)+2ik(iq)\,e^{iqx}\tilde{u}_{b,k}(q)-q^2\tilde{u}_{b,k}(q)\right]+\frac{1}{L^{3/2}}\sum_{q'q''} e^{iq'x}e^{iqx}\tilde{U}(q'')$$

$$q''=q'+q$$

$$\sum_{q}\left(\frac{\hbar^2}{2m}(k+q)^2\tilde{u}_{b,k}(q)+\frac{1}{L}\sum_{q'}\tilde{U}(q')\,u_{b,k}(q-q')-E_b(k)\,\tilde{u}_{b,k}(q)\right)=0$$

$$\Downarrow \quad \forall\, q$$

$$-\frac{\hbar^2}{2m\mathcal{V}} \sum_{q}' \left[-k^2 e^{iqx}\, \tilde{u}_{p,k}(q) + 2ik(iq)\, e^{ivx}\, \tilde{u}_{p,k}(q) - q^2 \tilde{u}_{p,k}(q) \right] + \frac{1}{L^{3/2}} \sum_{qq'}' e^{iq'x}\, e^{ivx}\, \tilde{U}(\quad$$

$$q'' = q' + q$$

$$\sum_{v}' \left(\frac{\hbar^2}{2m} (k+q)^2\, \tilde{u}_{p,k}(q) + \frac{1}{L} \sum_{q'}' \tilde{U}(q')\, u_{p,k}(q-q') - E_p(k)\, \tilde{u}_{p,k}(q) \right) = 0$$

$$\Big\| \quad \forall\, q$$

Unperturbed Solutions $U(x) = 0$

$$\Rightarrow \left[\frac{\hbar^2}{2m} (k+q)^2 - E_p^{(0)}(k) \right] \tilde{u}^{(0)}_{p,k}(q) = 0$$

$$-\frac{\hbar^2}{2m\sqrt{L}} \sum_q' \left[-k^2 e^{iqx} \tilde{u}_{p,k}(q) + 2ik(iq)\, e^{iqx} \tilde{u}_{p,k}(q) - q^2 \tilde{u}_{p,k}(q) \right] + \frac{1}{L^{3/2}} \sum_{q q'} e^{iq'x} e^{iqx} \tilde{U}(\quad$$

$$q'' = q' + q$$

$$\sum_q' \left(\frac{\hbar^2}{2m} (k+q)^2 \tilde{u}_{p,k}(q) + \frac{1}{L} \sum_{q'}' \tilde{U}(q')\, u_{p,k}(q-q') - E_p(k)\, \tilde{u}_{p,k}(q) \right) = 0$$

$$\| \quad \forall\, q$$

$$0$$

Perturbed Solutions $U(x) = 0$

$$\left[(k+q)^2 - E_p^{(0)}(k) \right] \tilde{u}^{(0)}_{p,k}(q) = 0 \quad \forall\, q$$

$$\ddot{u}_{b,k}(q) - q^2 \ddot{u}_{b,k}(q)\Big] + \frac{1}{L^{3/2}} \sum_{q q'} e^{iq'x} e^{iqx} \ddot{U}(q') \ddot{u}_{b,k}(q) = E_b(k) \frac{1}{\sqrt{L}} \sum_{q} e^{iqx} \ddot{u}_{b,k}(q)$$

$q'' = q' + q$

$$\ddot{u}_{b,k}(q-q') - E_b(k)\, \ddot{u}_{b,k}(q)\Big) = 0$$

$$E_p^{(0)}(k) = \frac{\hbar^2}{2m}(k+q)^2$$

$+ q$

$U(x) = 0$

$k(q) =$

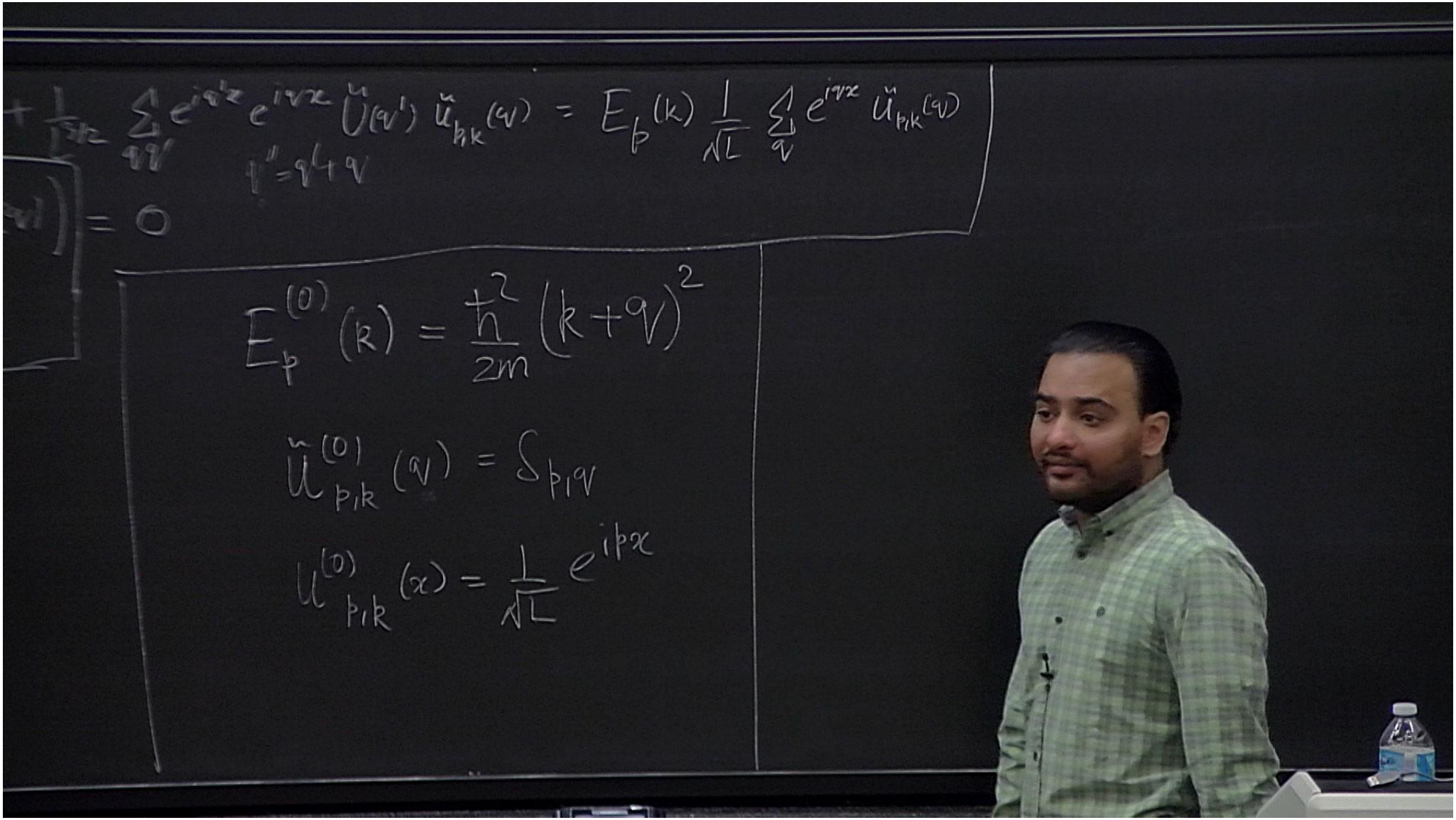
$$+\frac{1}{L^{3/2}}\sum_{q q'} e^{iq'z} e^{iqz}\, \tilde{U}(q')\, \tilde{u}_{p,k}(q) = E_p(k)\,\frac{1}{\sqrt{L}}\sum_{q} e^{iqz}\, \tilde{u}_{p,k}(q)$$

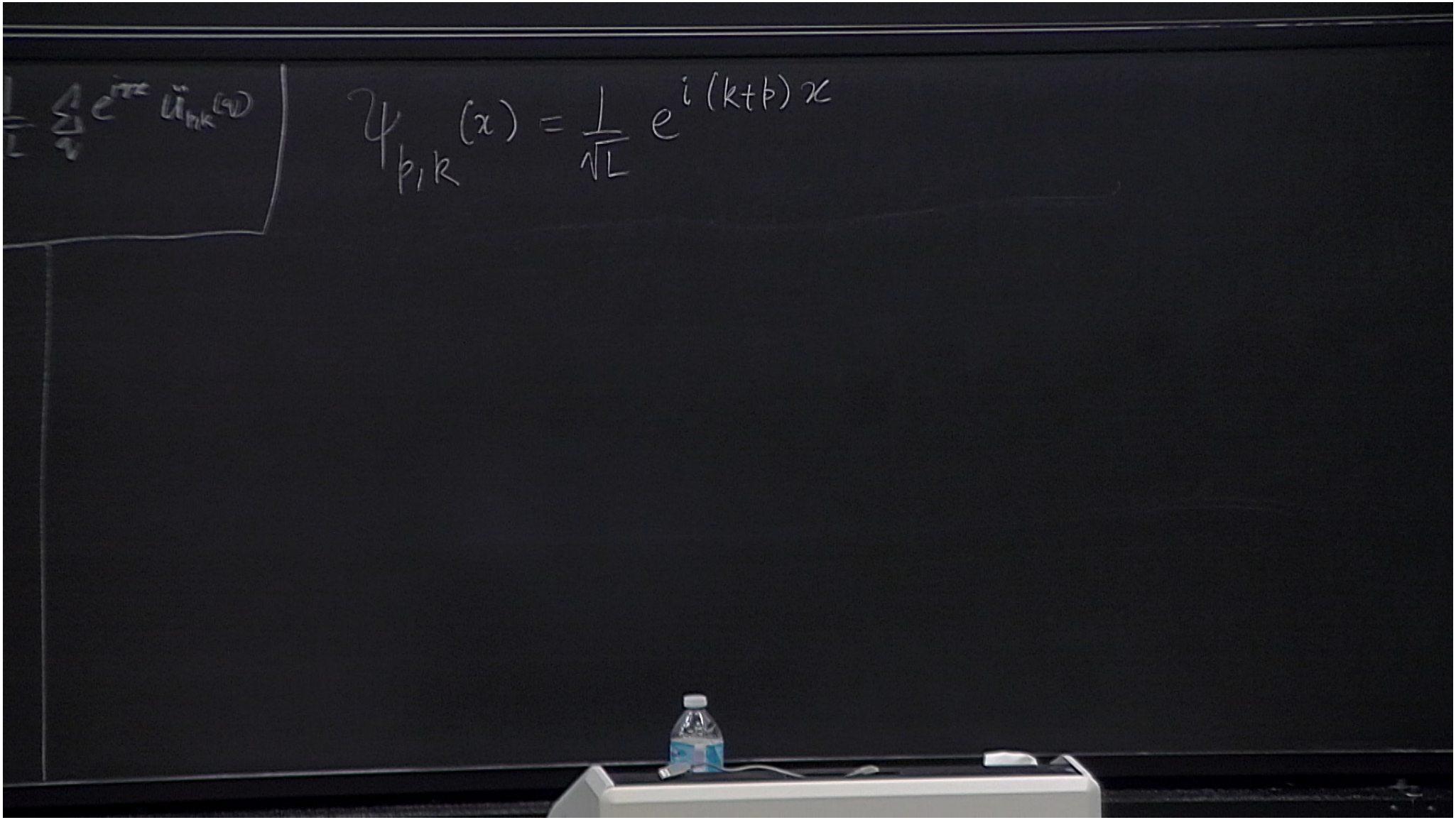
$$q'' = q' + q$$

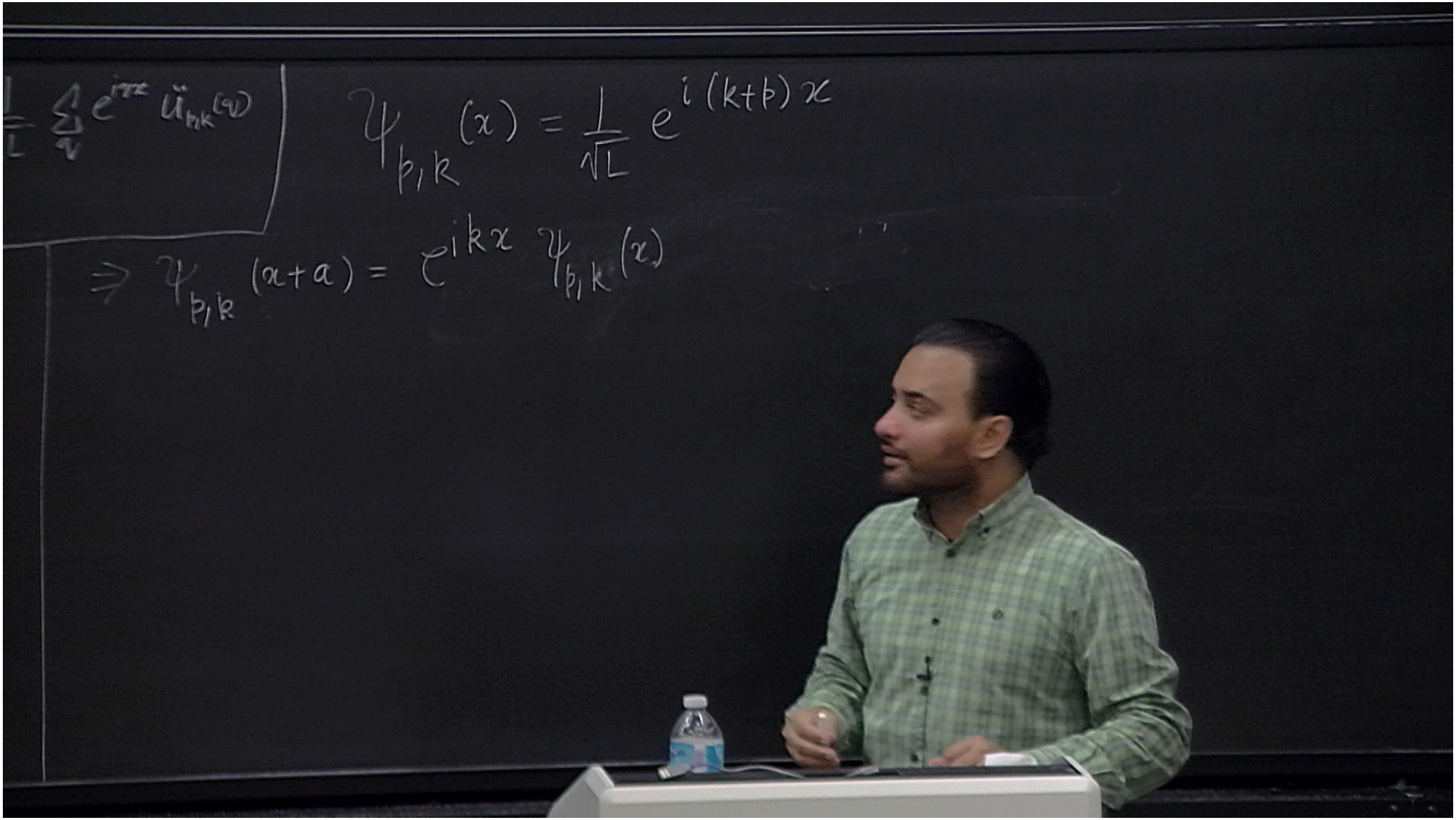
$$= 0$$

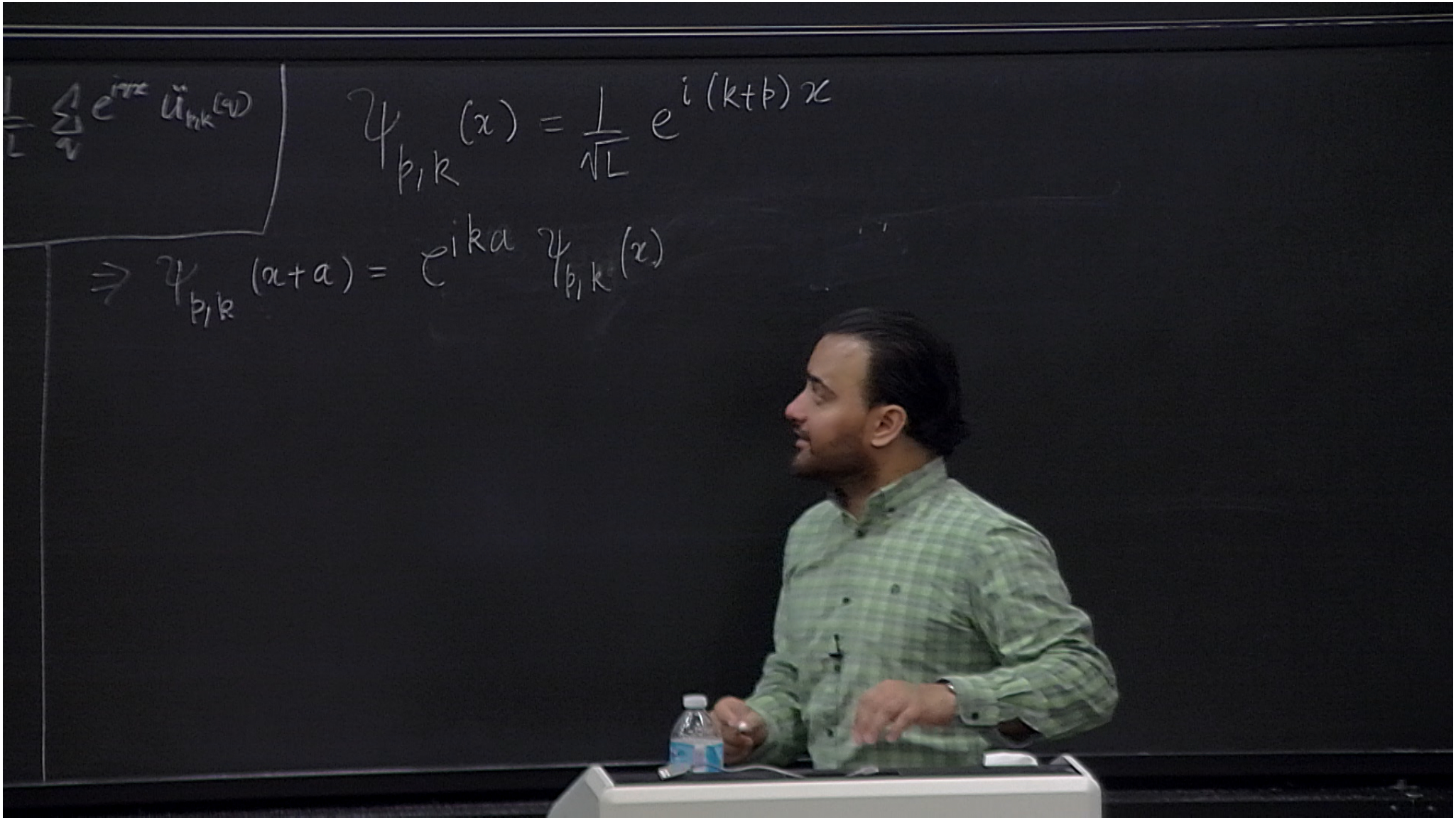
$$E_p^{(0)}(k) = \frac{\hbar^2}{2m}(k+q)^2$$

$$\tilde{u}^{(0)}_{p,k}(q) = \delta_{p,q}$$


$\frac{1}{\sqrt{L}} \sum_{q q'} e^{iq'x} e^{iqx} \tilde{U}(q') \tilde{u}_{p,k}(q) = E_p(k) \frac{1}{\sqrt{L}} \sum_{q} e^{iqx} \tilde{u}_{p,k}(q)$
$q'' = q' + q$
$= 0$
$E_p^{(0)}(k) = \frac{\hbar^2}{2m}(k+q)^2$
$\tilde{u}^{(0)}_{p,k}(q) = \delta_{p,q}$
$u^{(0)}_{p,k}(x) = \frac{1}{\sqrt{L}} e^{ipx}$


$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x}$


$\frac{1}{L}\sum_{q} e^{iqx}\,\ddot{u}_{pk}(q)$
$\psi_{p,k}(x) = \frac{1}{\sqrt{L}}\, e^{i(k+p)x}$
$\Rightarrow \psi_{p,k}(x+a) = e^{ikx}\,\psi_{p,k}(x)$


$\frac{1}{L}\sum_{\nu} e^{i\nu x}\,\ddot{u}_{pk}(q)$
$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x}$
$\Rightarrow \psi_{p,k}(x+a) = e^{ika}\,\psi_{p,k}(x)$

$\sum_q e^{iqx} \tilde{u}_{nk}(q)$

$$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x} = \frac{1}{\sqrt{L}} e^{ikx} e^{ipx}$$

$\therefore \Rightarrow \frac{e^{ipx}}{\sqrt{L}}$ Should have same periodicity as that of the lattice

$$\Rightarrow \psi_{p,k}(x+a) = e^{ika} \psi_{p,k}(x)$$

$$\Rightarrow e^{ip(x+a)} = e^{ip(x)} \Rightarrow e^{ipa} = 1 \Rightarrow p = \frac{2\pi}{a} n, \; n \in \mathbb{Z}$$

$\sum_q e^{iqx} \tilde{u}_{pk}(q)$

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$$\int dx \, \psi^{(0)*}_{p,k}(x) \, \psi^{(0)}_{p',k'}(x)$$

$$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x} = \frac{1}{\sqrt{L}} e^{ikx} e^{ipx}$$

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$$\int dx \, \psi^{(0)*}_{p,k}(x) \, \psi^{(0)}_{p',k'}(x) = \delta_{k+p,\, k'+p'}$$

$$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x} = \frac{1}{\sqrt{L}} e^{ikx} e^{ipx}$$

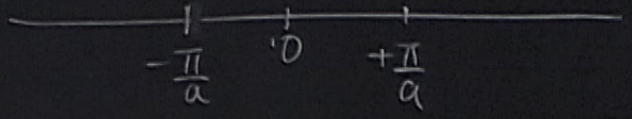
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$k \in$ FBZ

$$\int dx \, \psi^{(0)*}_{p,k}(x) \, \psi^{(0)}_{p',k'}(x) = \delta_{k+p,\, k'+p'}$$



$$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x} = \frac{1}{\sqrt{L}} e^{ikx} e^{ipx}$$

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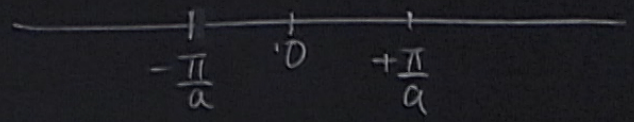
$$\Rightarrow \psi_{p,k}(x+a) = e^{ika} \psi_{p,k}(x)$$

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$k \in$ FBZ

$$\int dx \, \psi^{(0)*}_{p,k}(x) \, \psi^{(0)}_{p',k'}(x) = \delta_{k+p,\, k'+p'}$$

$p = \frac{2\pi}{a} n$



$\sum_q e^{iqx} \tilde{u}_{pk}(q)$

$$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x} = \frac{1}{\sqrt{L}} e^{ikx} e^{ipx}$$

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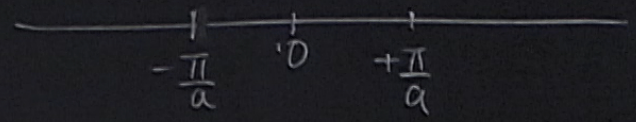
$$\Rightarrow \psi_{p,k}(x+a) = e^{ika} \psi_{p,k}(x)$$

$$\Rightarrow e^{ip(x+a)} = e^{ipx} \Rightarrow e^{ipa} = 1 \Rightarrow p = \frac{2\pi}{a} n, \; n \in \mathbb{Z}$$

$$\int dx \, \psi^{(0)*}_{p,k}(x) \, \psi^{(0)}_{p',k'}(x) = \delta_{k+p,\, k'+p'}$$

$k \in FBZ$

$p = \frac{2\pi}{a} n$, if $p = 0$, $k' = k - p'$



$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x} = \frac{1}{\sqrt{L}} e^{ikx} e^{ipx}$

$\Rightarrow \frac{e^{ipx}}{\sqrt{L}}$ Should have same periodicity as that of the lattice

$\Rightarrow \psi_{p,k}(x+a) = e^{ika} \psi_{p,k}(x)$

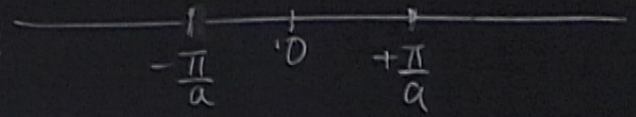
$\Rightarrow e^{ip(x+a)} = e^{ip(x)} \Rightarrow e^{ipa} = 1 \Rightarrow p = \frac{2\pi}{a} n, \; n \in \mathbb{Z}$

$k \in FBZ$

$\int dx \, \psi^{(0)*}_{p,k}(x) \, \psi^{(0)}_{p',k'}(x) = \delta_{k+p,\, k'+p'}$

$p = \frac{2\pi}{a} n$, if $p=0$, $k' = k - p'$, $k' \in FBZ$

if $p' \neq 0$ $k' \notin FBZ$



$\sum_q e^{iqx} \tilde{u}_{nk}(q)$

$$\psi_{p,k}(x) = \frac{1}{\sqrt{L}} e^{i(k+p)x} = \frac{1}{\sqrt{L}} e^{ikx} e^{ipx}$$

$$\Rightarrow \frac{e^{ipx}}{\sqrt{L}}$$ Should have same periodicity as that of the lattice

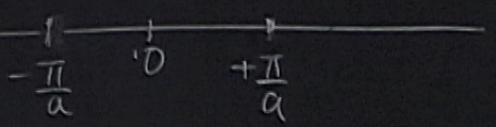
$$\Rightarrow \psi_{p,k}(x+a) = e^{ika} \psi_{p,k}(x)$$

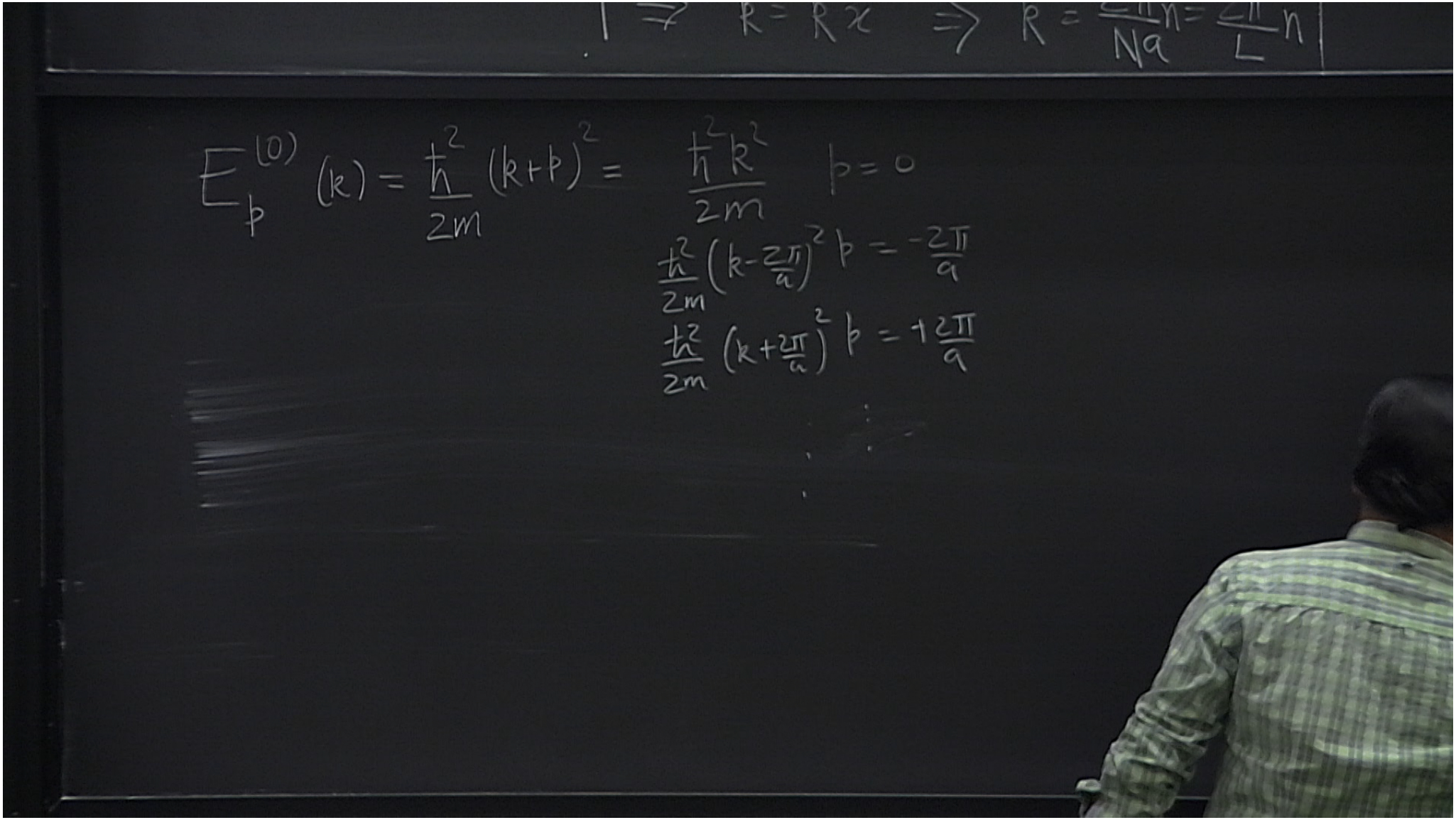
$$\Rightarrow e^{ip(x+a)} = e^{ip(x)} \Rightarrow e^{ipa} = 1 \Rightarrow p = \frac{2\pi}{a} n, \; n \in \mathbb{Z}$$

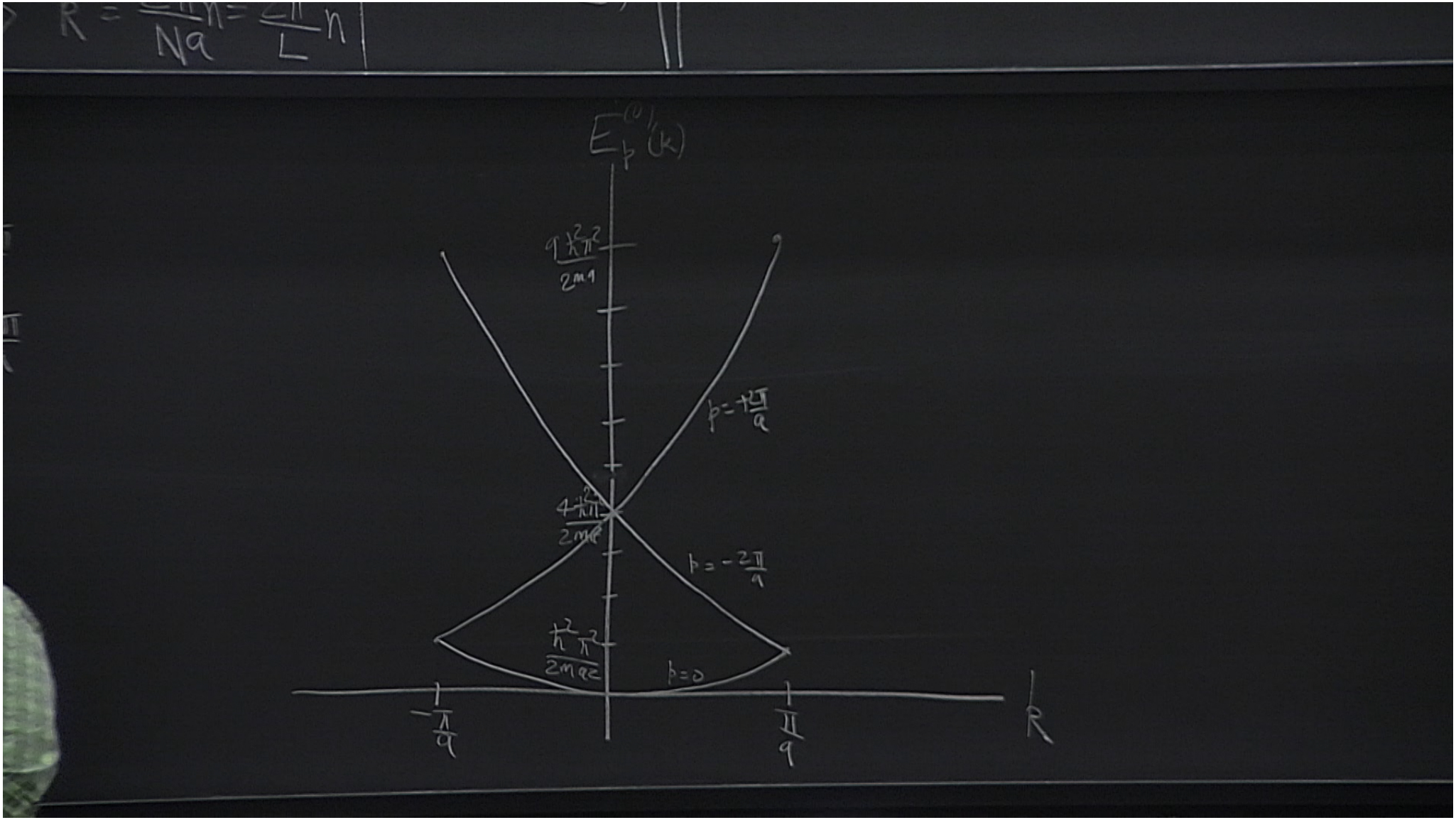
$$\int dx \, \psi^{(0)*}_{p,k}(x) \, \psi^{(0)}_{p',k'}(x) = \delta_{k+p,\,k'+p'} = \delta_{kk'} \delta_{pp'} \quad k \in FBZ$$

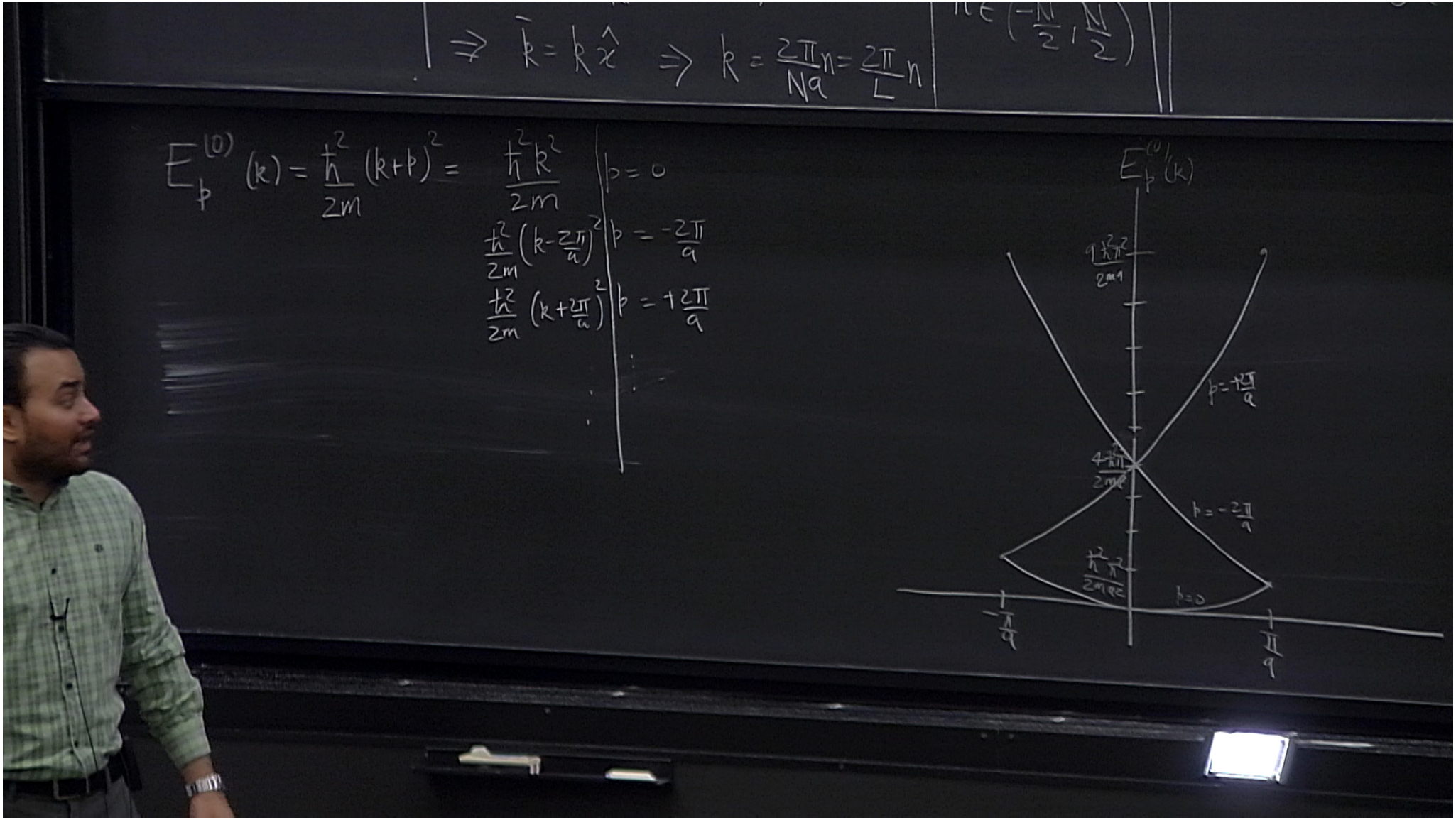
$p = \frac{2\pi}{a} n$, if $p = 0$, $k' = k - p'$, $k' \in FBZ$

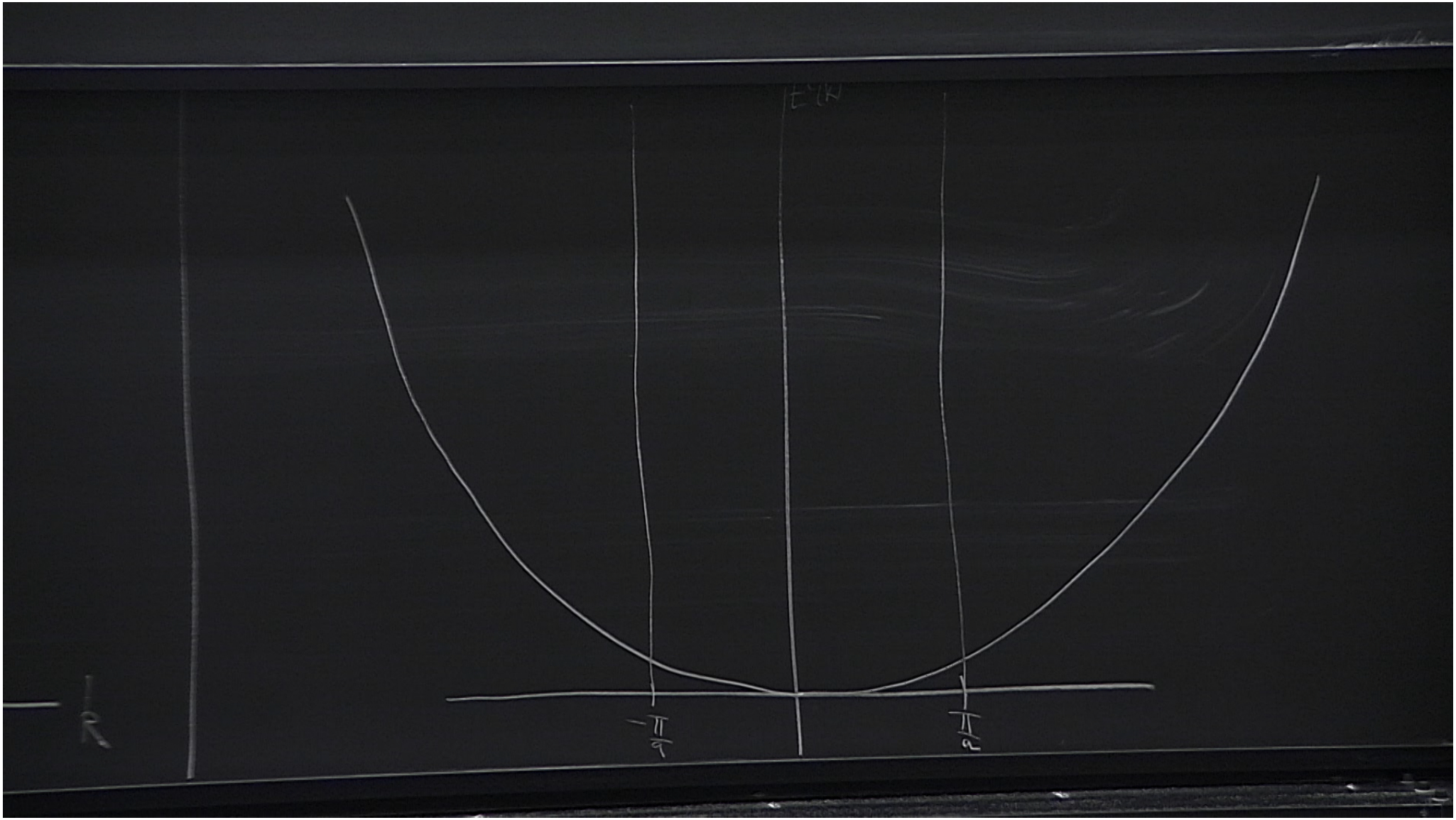
if $p' \neq 0$ $k' \notin FBZ$

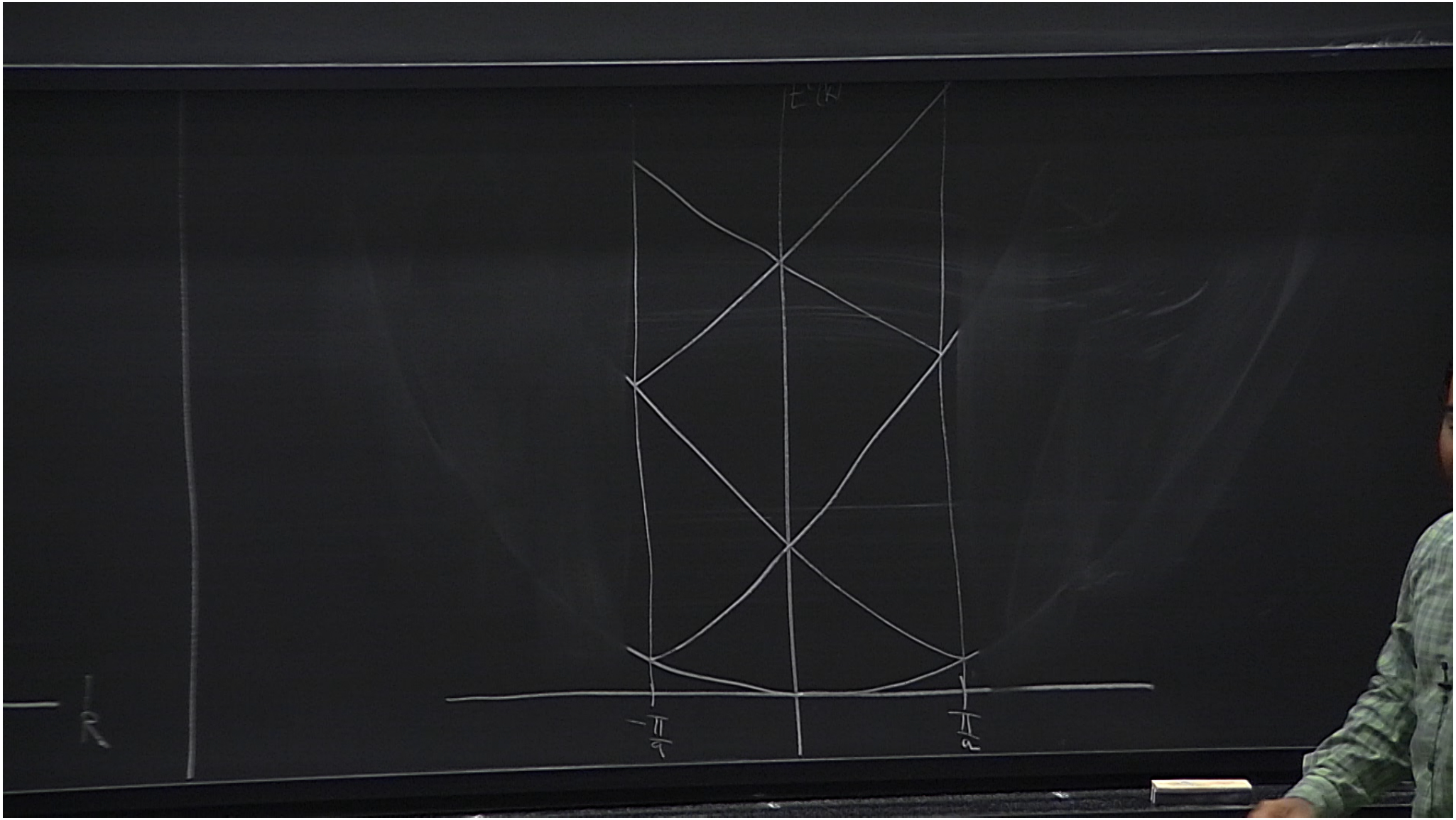


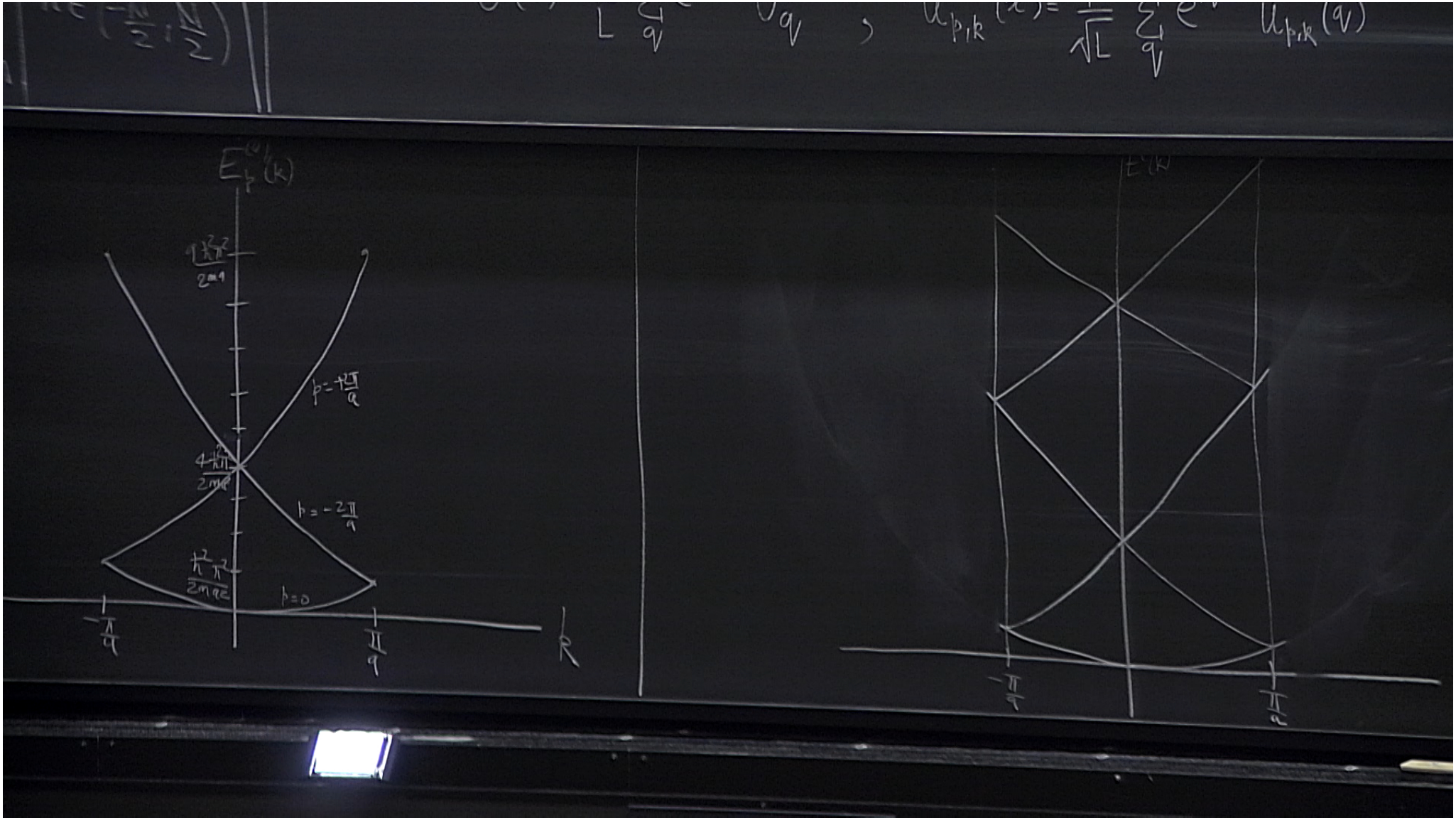

$E_p^{(0)}(k) = \frac{\hbar^2}{2m}(k+p)^2 = \frac{\hbar^2 k^2}{2m}$ $p=0$
$\frac{\hbar^2}{2m}\left(k-\frac{2\pi}{a}\right)^2 p = -\frac{2\pi}{a}$
$\frac{\hbar^2}{2m}\left(k+\frac{2\pi}{a}\right)^2 p = +\frac{2\pi}{a}$


$E_p^{(0)}(k)$
$\frac{\hbar^2\pi^2}{2ma}$
$\frac{4\hbar^2\pi^2}{2ma^2}$
$\frac{\hbar^2\pi^2}{2ma^2}$
$p=\pm\frac{2\pi}{a}$
$p=-\frac{2\pi}{a}$
$p=0$
$-\frac{\pi}{a}$
$\frac{\pi}{a}$
k


$\Rightarrow \vec{k} = k\hat{x} \Rightarrow k = \frac{2\pi}{Na} n = \frac{2\pi}{L} n$
$n \in \left(-\frac{N}{2}, \frac{N}{2}\right)$
$E_p^{(0)}(k) = \frac{\hbar^2}{2m}(k+p)^2 = \frac{\hbar^2 k^2}{2m}$ $p = 0$
$\frac{\hbar^2}{2m}\left(k - \frac{2\pi}{a}\right)^2$ $p = -\frac{2\pi}{a}$
$\frac{\hbar^2}{2m}\left(k + \frac{2\pi}{a}\right)^2$ $p = +\frac{2\pi}{a}$
$E_p^{(0)}(k)$
$\frac{\hbar^2\pi^2}{2ma^2}$
$p = +\frac{2\pi}{a}$
$\frac{4\hbar^2\pi^2}{2ma^2}$
$p = -\frac{2\pi}{a}$
$\frac{\hbar^2\pi^2}{2ma^2}$
$p = 0$
$-\frac{\pi}{a}$
$\frac{\pi}{a}$


$\frac{1}{R}$
$-\frac{\pi}{q}$
$\frac{\pi}{q}$


$\frac{\pi}{q}$
$\frac{\pi}{q}$


$E_p^{(1)}(k)$
$p=\pm\frac{2\pi}{a}$
$p=-\frac{2\pi}{a}$
$p=0$
$-\frac{\pi}{a}$
$\frac{\pi}{a}$
k
$E(k)$
$-\frac{\pi}{a}$
$\frac{\pi}{a}$