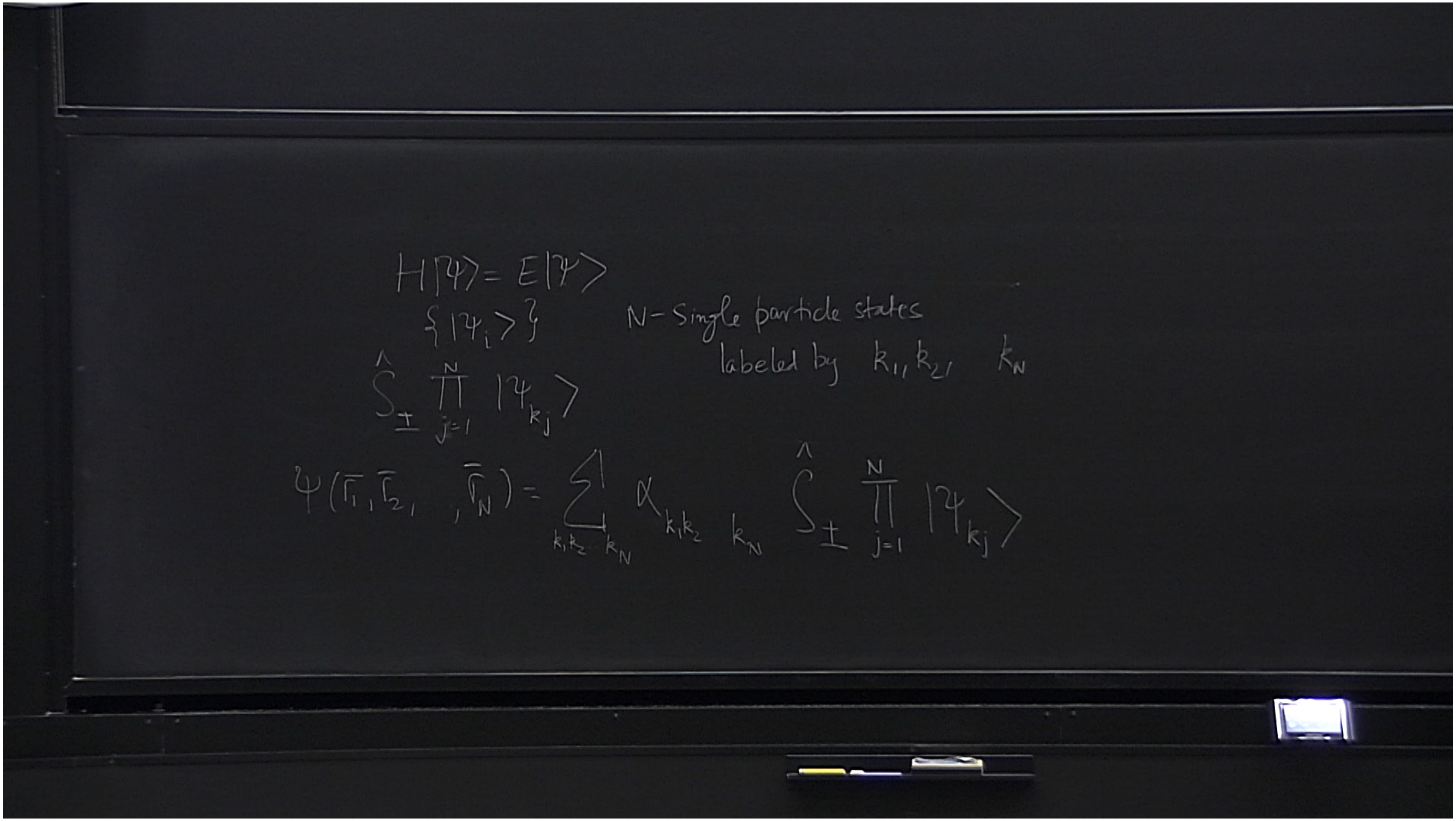
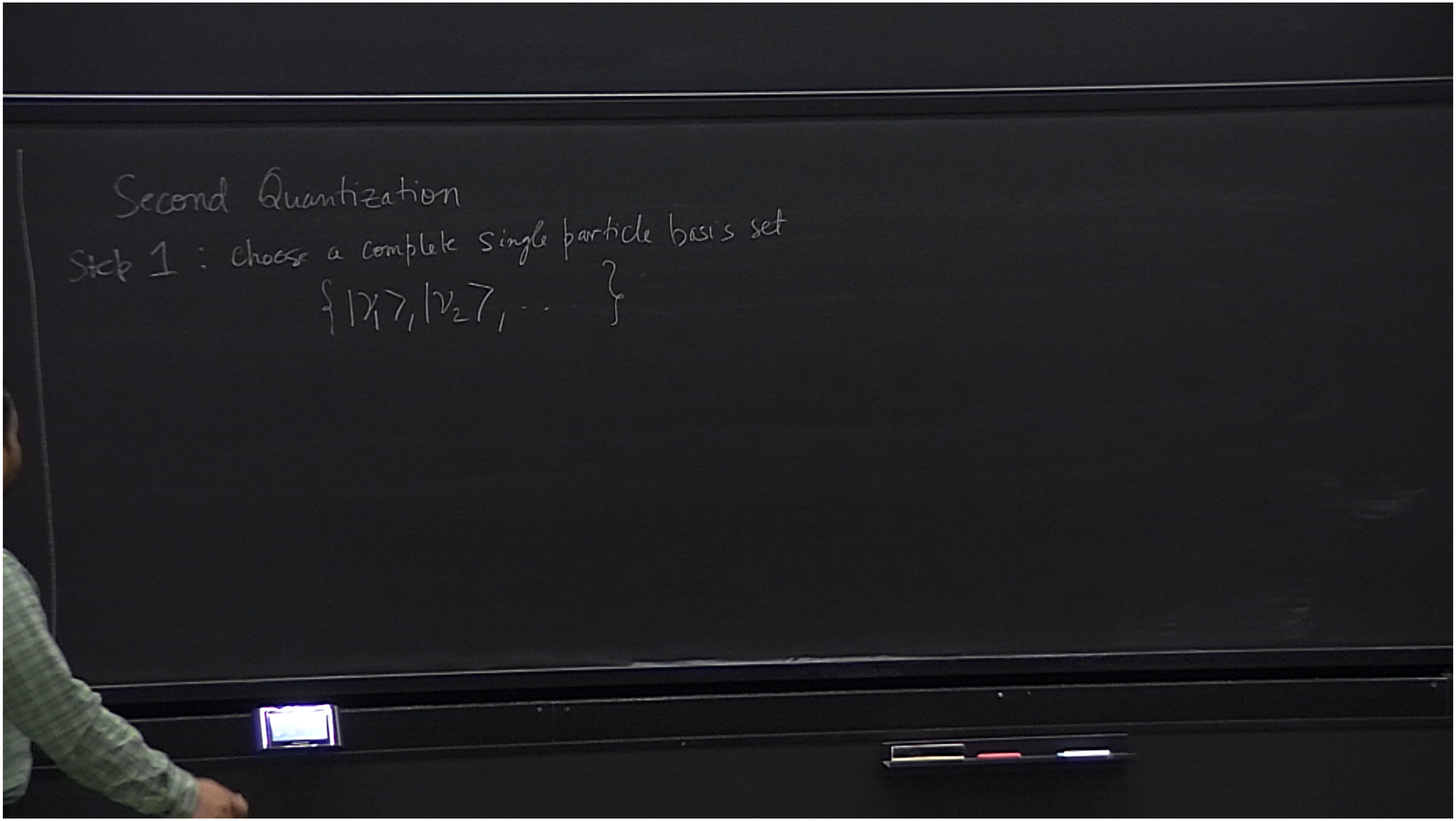
Title: PSI 17/18 - Condensed Matter - Lecture 2

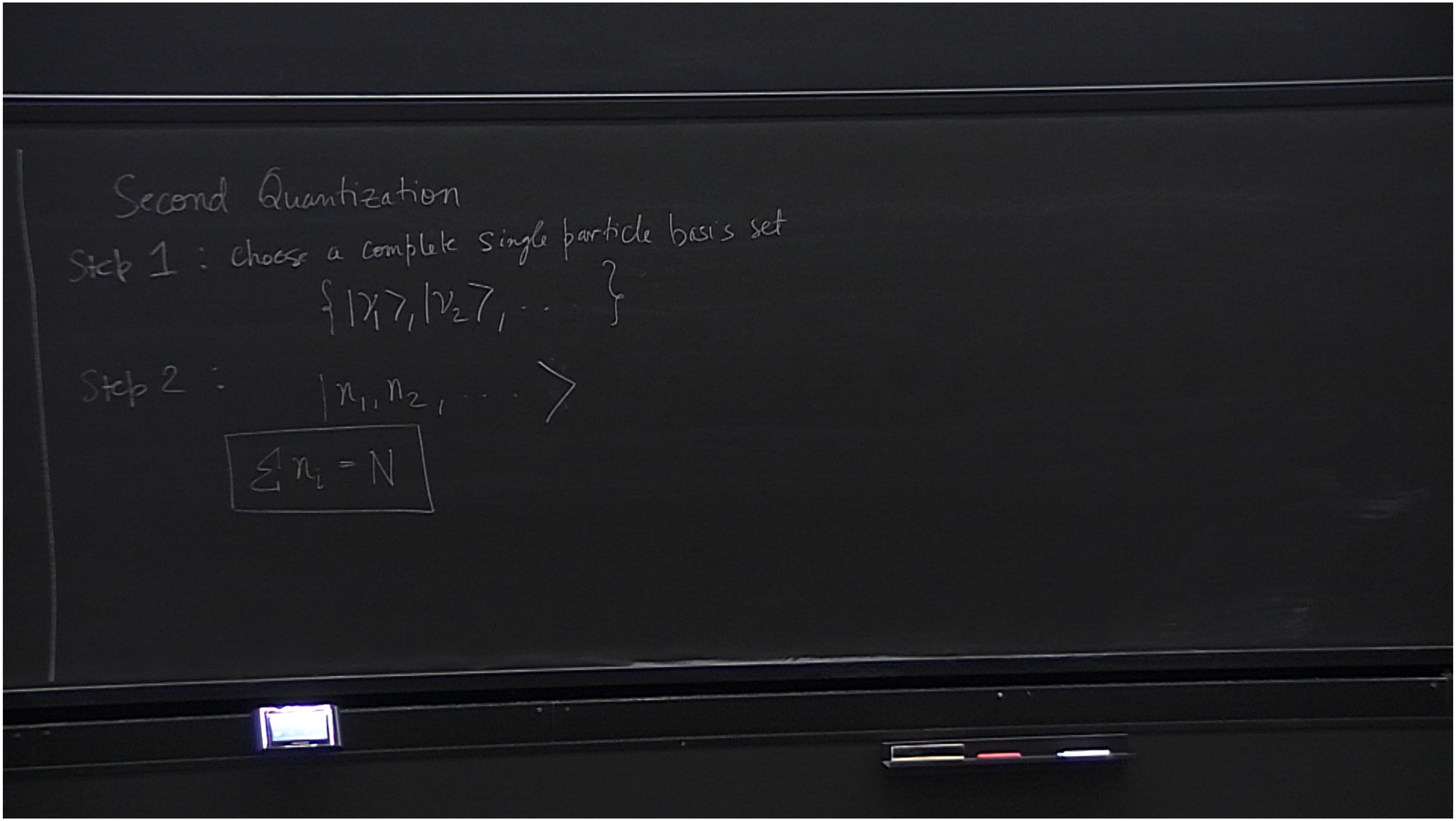
Date: Nov 07, 2017 10:45 AM

URL: http://pirsa.org/17110027

Abstract:


$H|\Psi\rangle = E|\Psi\rangle$
$\{|\Psi_i\rangle\}$
N-Single particle states
labeled by $k_1, k_2, \; k_N$
$\hat{S}_{\pm} \prod_{j=1}^{N} |\Psi_{k_j}\rangle$
$\Psi(\bar{r}_1, \bar{r}_2, \; , \bar{r}_N) = \sum_{k_1 k_2 \; k_N} \alpha_{k_1 k_2 \; k_N} \hat{S}_{\pm} \prod_{j=1}^{N} |\Psi_{k_j}\rangle$


Second Quantization
Step 1 : choose a complete single particle basis set
$\{|\nu_1\rangle, |\nu_2\rangle, \ldots\}$


Second Quantization
Step 1 : choose a complete single particle basis set
$\{|\nu_1\rangle, |\nu_2\rangle, \ldots\}$
Step 2 :
$|n_1, n_2, \ldots\rangle$
$\sum n_i = N$

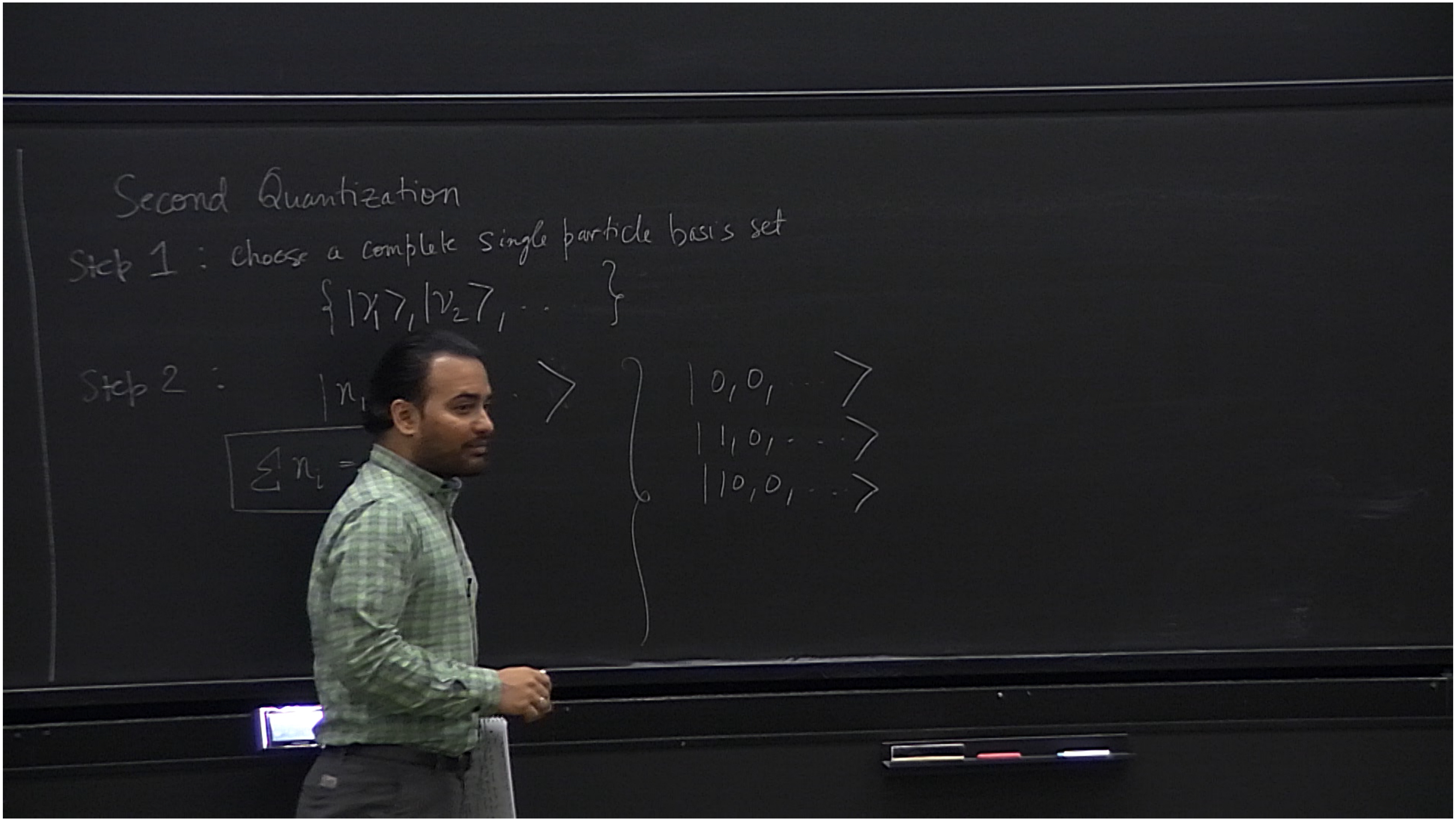
Second Quantization

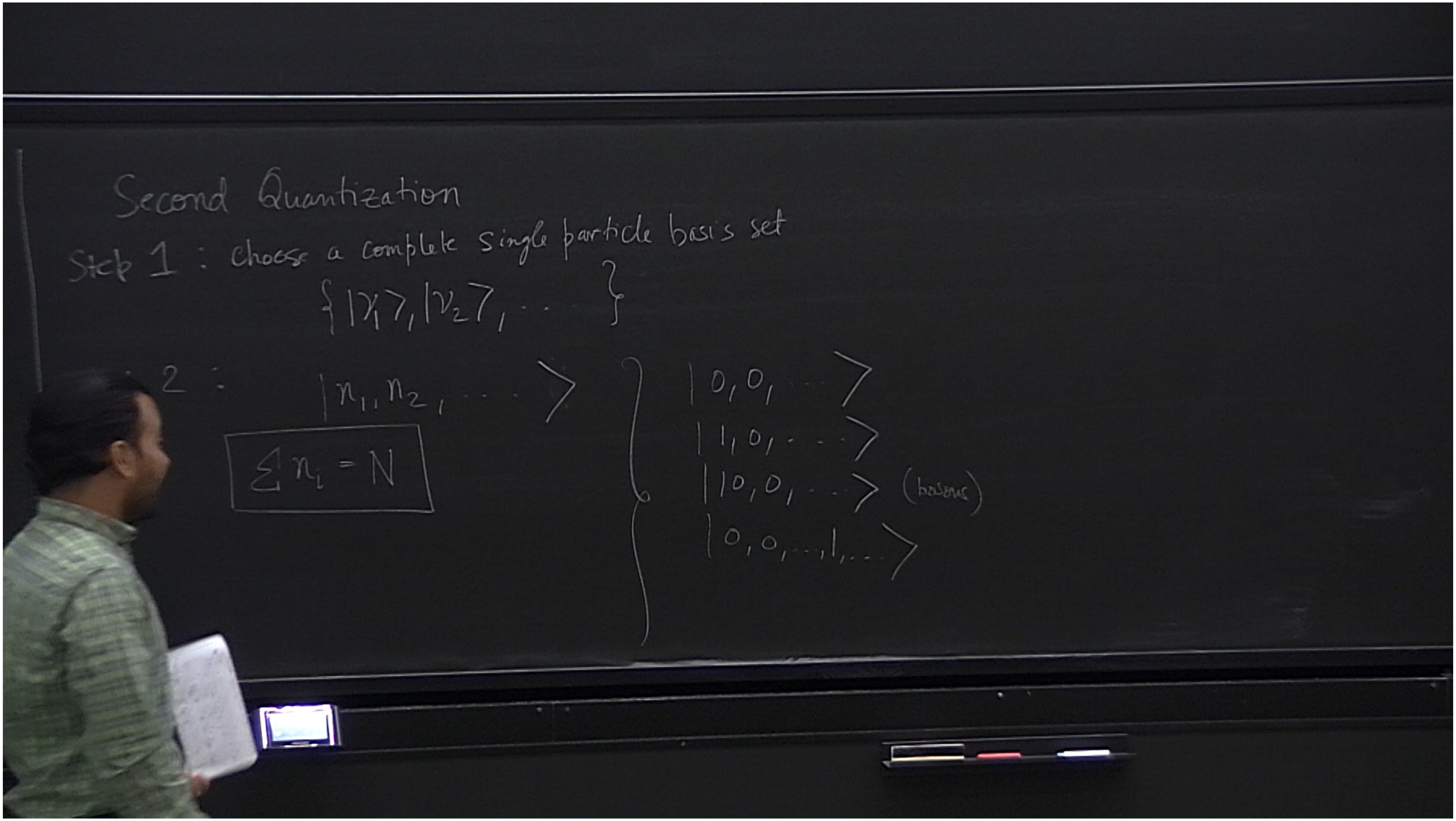
Step 1 : choose a complete single particle basis set

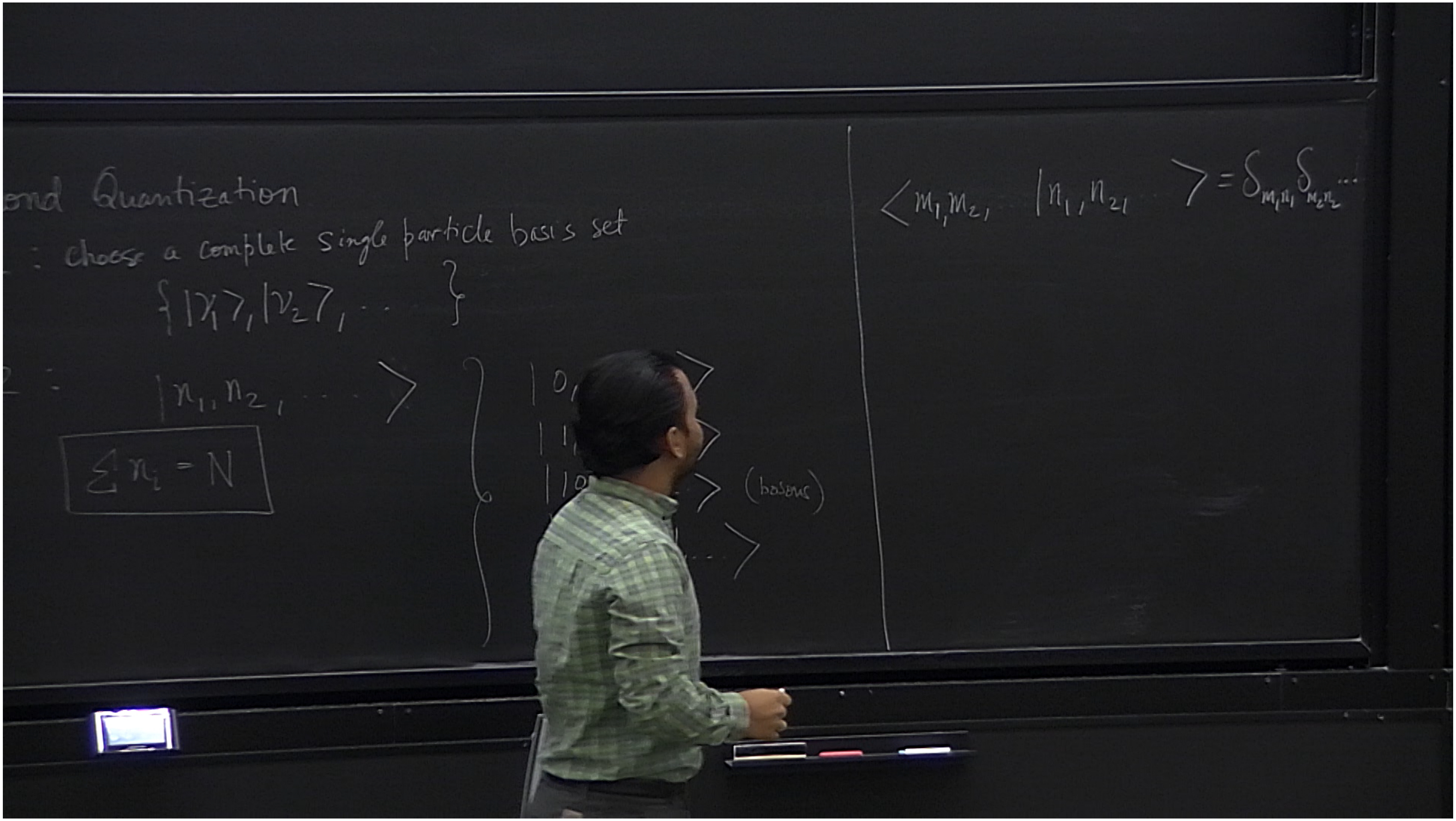
$$\{ |\nu_1\rangle, |\nu_2\rangle, \ldots \}$$

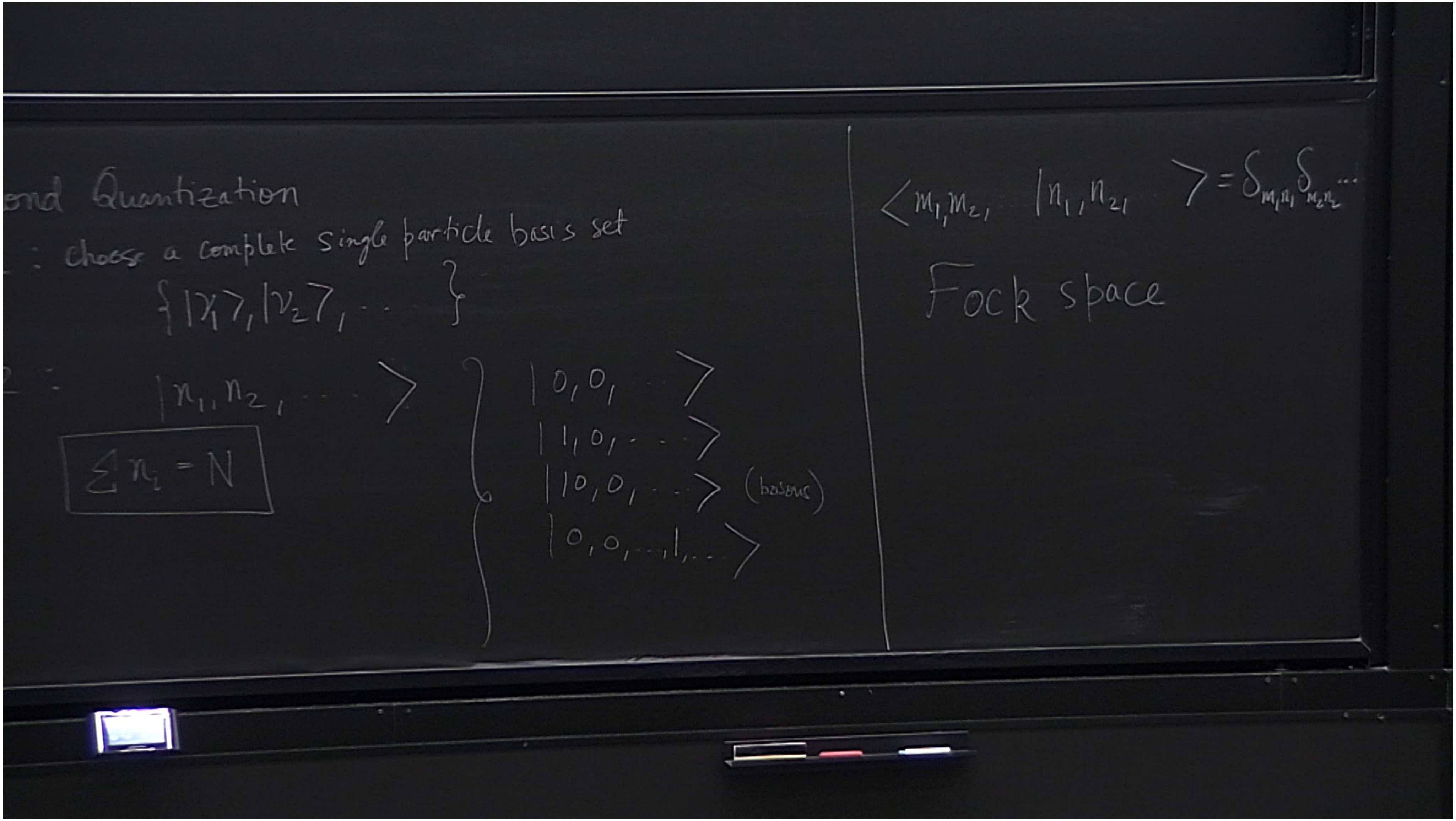
Step 2 : $|n_1, n_2, \ldots \rangle$ $\{$ $|0, 0, \ldots \rangle$

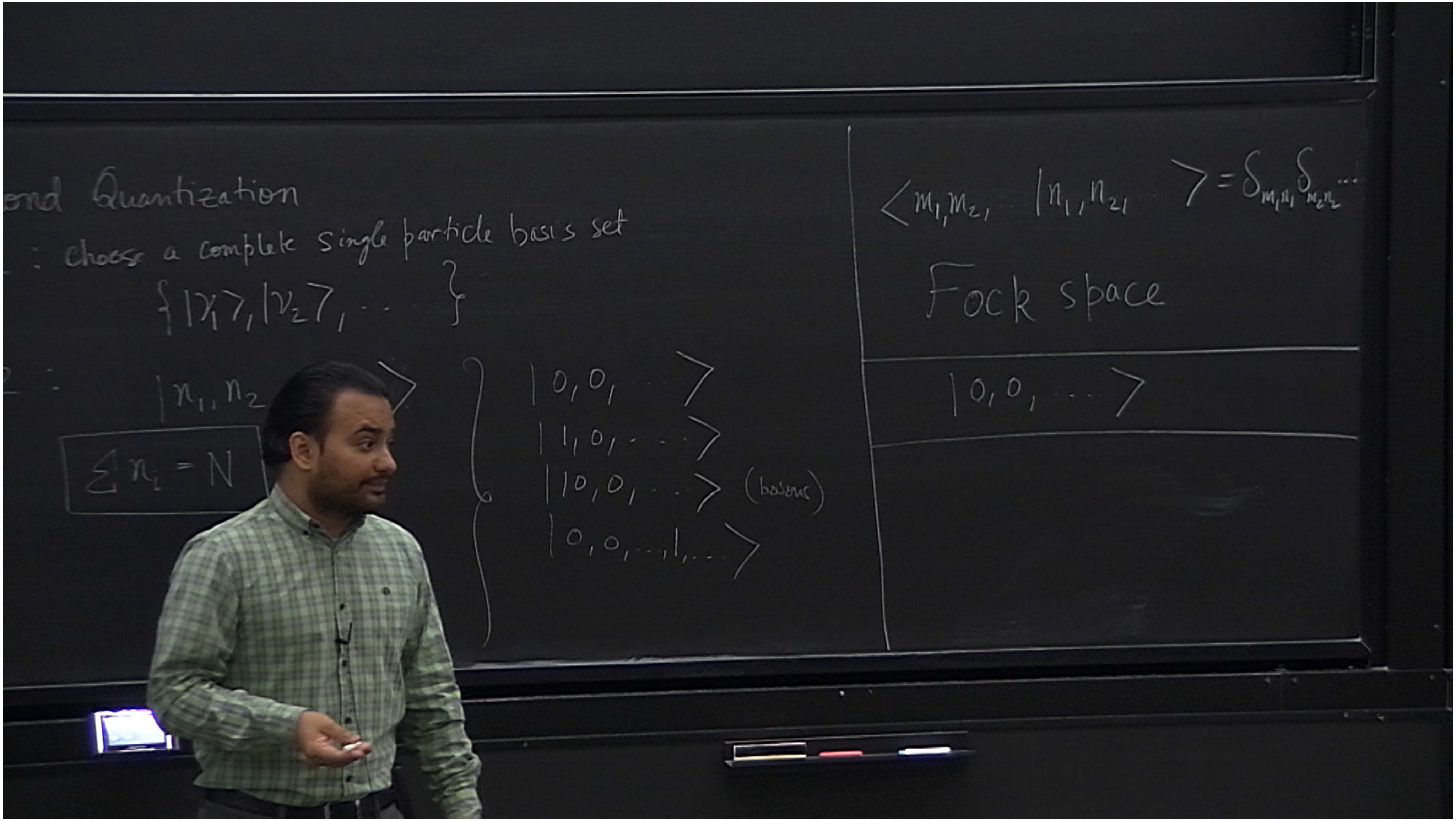
$$\boxed{\sum n_i = N}$$

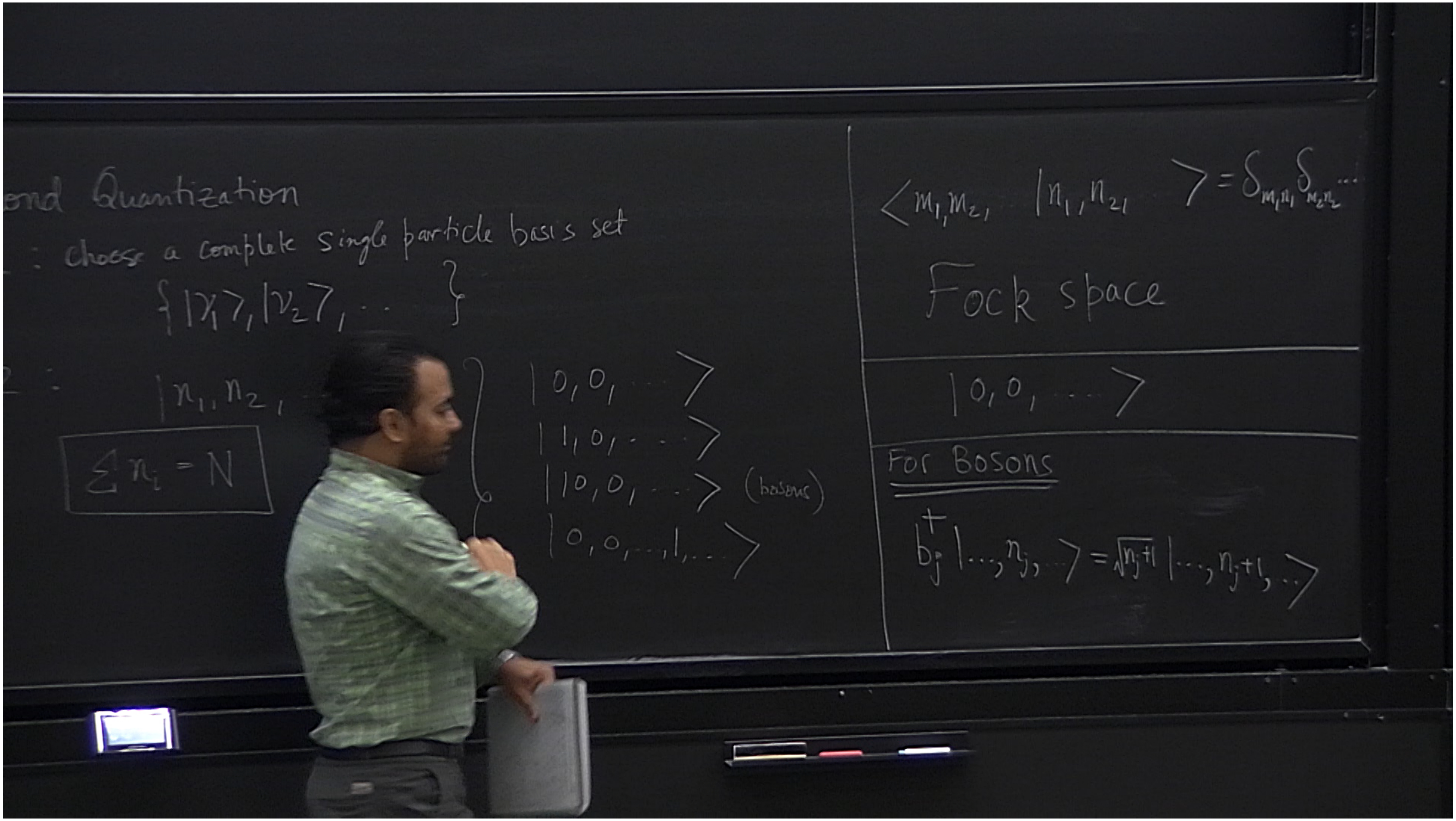

Second Quantization
Step 1 : choose a complete single particle basis set
Step 2 :


Second Quantization
Step 1 : choose a complete single particle basis set
{|ν1>, |ν2>, . . . }
2 : |n1, n2, . . . >
Σ ni = N
|0, 0, . . . >
|1, 0, . . . >
|10, 0, . . . > (bosons)
|0, 0, . . . , 1, . . . >


ond Quantization
: choose a complete single particle basis set
{|ν1>, |ν2>, . . . }
|n1, n2, ... >
Σ ni = N
(bosons)
< m1, m2, | n1, n2, > = δm1n1 δm2n2 . . .


ond Quantization
: choose a complete single particle basis set
{|ν1>, |ν2>, ... }
|n1, n2, ... >
Σ ni = N
|0, 0, ... >
|1, 0, ... >
|10, 0, ... >
(bosons)
|0, 0, ..., 1, ... >
<m1, m2, ... | n1, n2, ... > = δm1n1 δm2n2 ...
Fock space


ond Quantization
choose a complete single particle basis set
Fock space
(bosons)


ond Quantization
: choose a complete single particle basis set
{|ν₁>, |ν₂>, ... }
|n₁, n₂, ...>
Σ nᵢ = N
|0, 0, ...>
|1, 0, ...>
|10, 0, ...>
(bosons)
|0, 0, ..., 1, ...>
<m₁, m₂, ... | n₁, n₂, ...> = δ_{m₁n₁} δ_{m₂n₂} ...
Fock space
|0, 0, ...>
For Bosons
b_j† |..., n_j, ...> = √(n_j+1) |..., n_j+1, ...>

$$b_j \,|\ldots, n_j, \ldots\rangle = \sqrt{n_j}\,|\ldots, n_j - 1, \ldots\rangle$$

$$b_j \,|\ldots, n_j, \ldots\rangle = \sqrt{n_j}\,|\ldots, n_j - 1, \ldots\rangle$$

$$b_j \,|\ldots, 0, \ldots\rangle = 0$$

$$b_j |\ldots, n_j, \ldots\rangle = \sqrt{n_j}\, |\ldots, n_j - 1, \ldots\rangle$$

$$b_j |\ldots, 0, \ldots\rangle = 0$$

$$[b_j, b_j^\dagger] |\ldots, n_j, \ldots\rangle = (b_j b_j^\dagger - b_j^\dagger b_j) |\ldots, n_j, \ldots\rangle$$

$$= (n_j + 1)$$

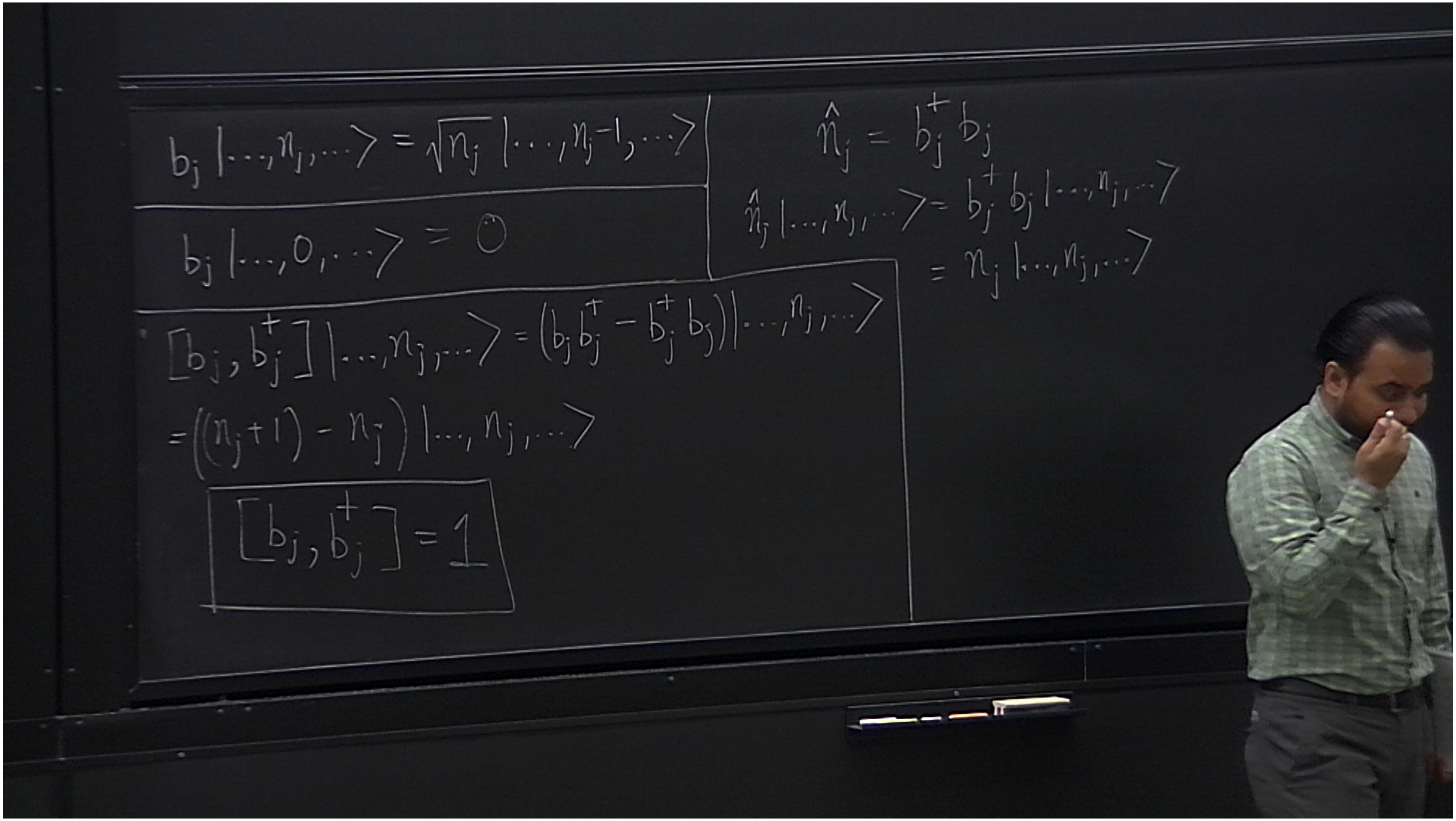
$$b_j |\ldots, n_j, \ldots\rangle = \sqrt{n_j}\, |\ldots, n_j - 1, \ldots\rangle$$

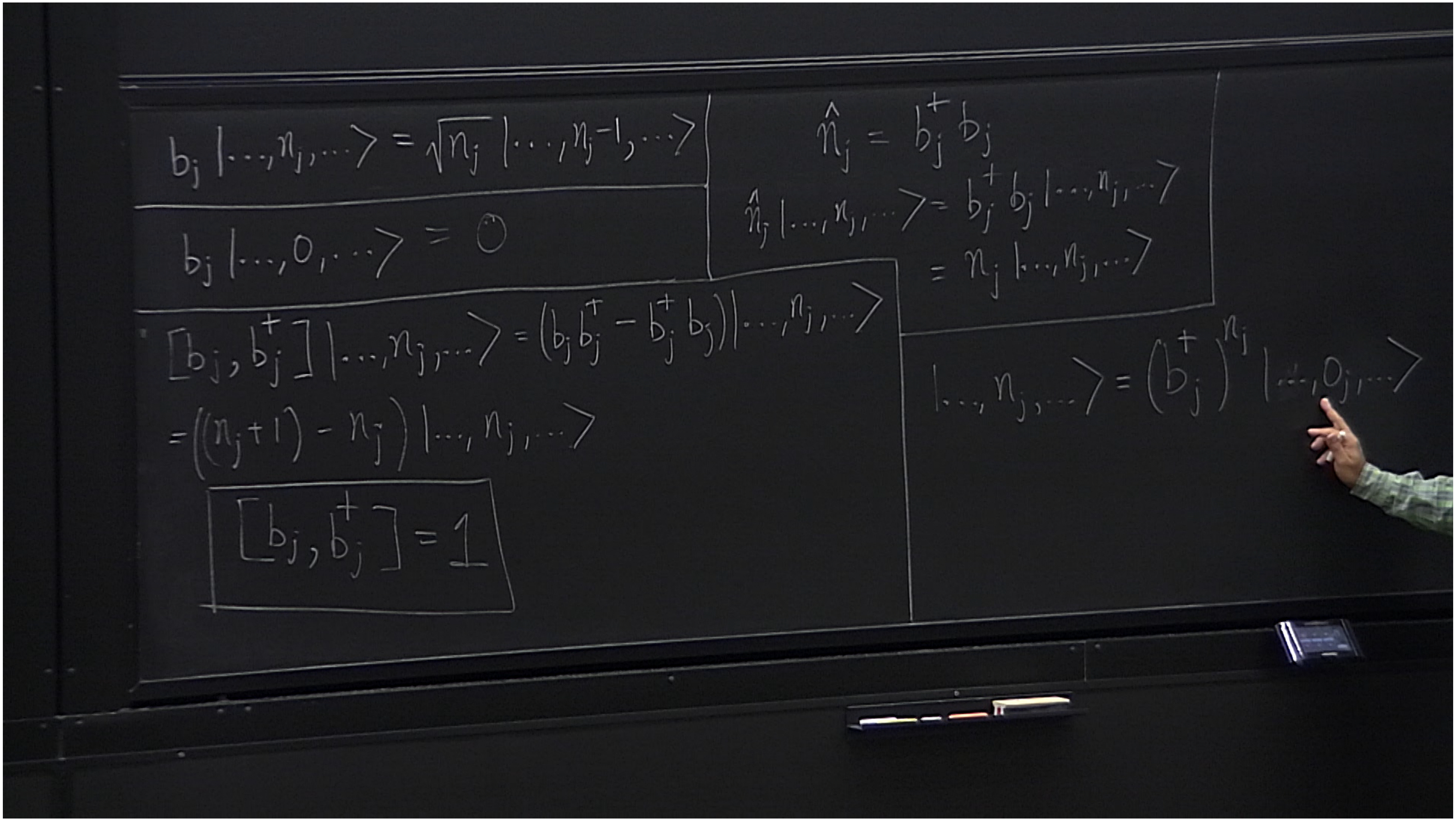
$$b_j |\ldots, 0, \ldots\rangle = 0$$

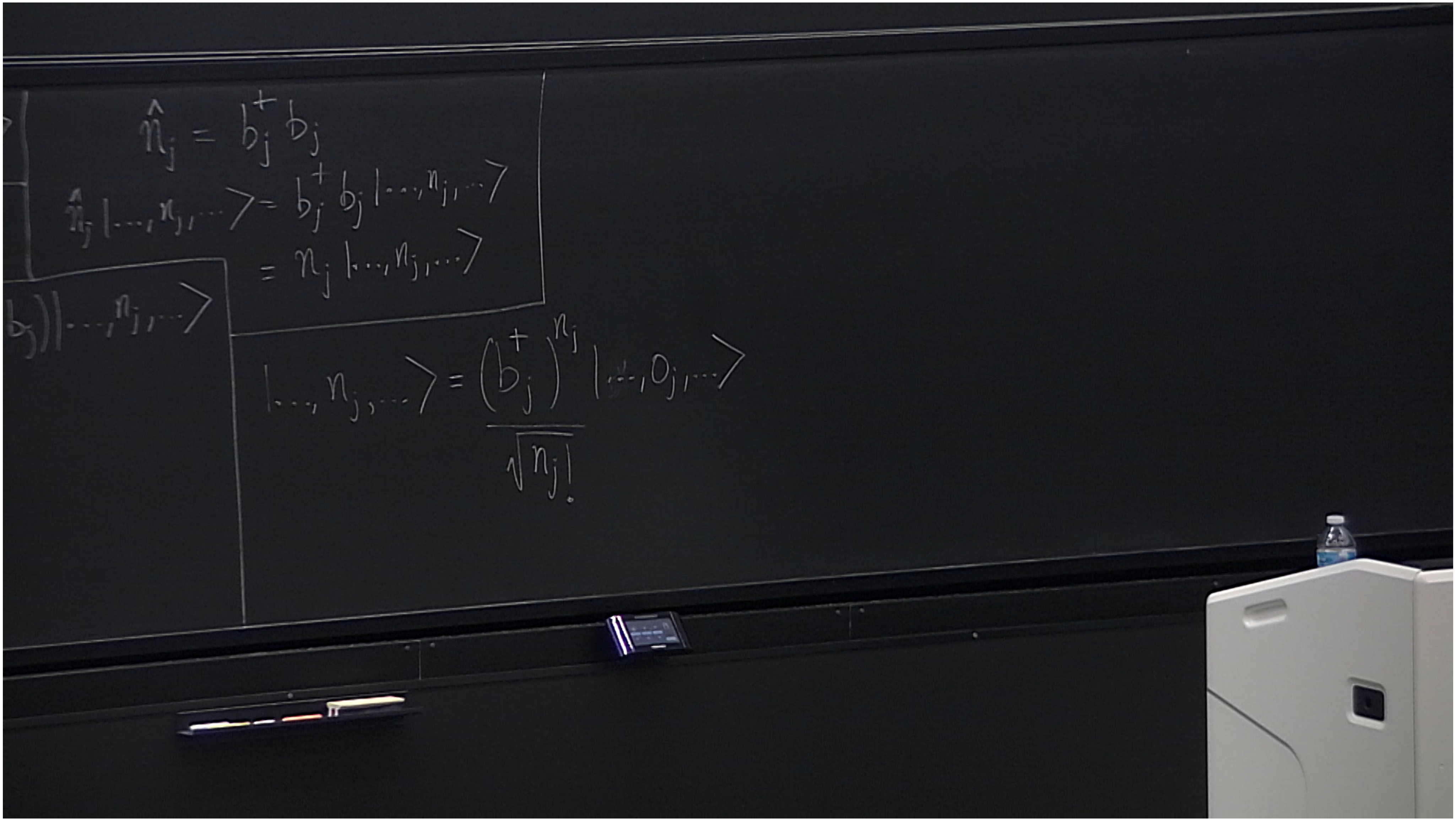
$$[b_j, b_j^\dagger]\, |\ldots, n_j, \ldots\rangle = (b_j b_j^\dagger - b_j^\dagger b_j)\, |\ldots, n_j, \ldots\rangle$$

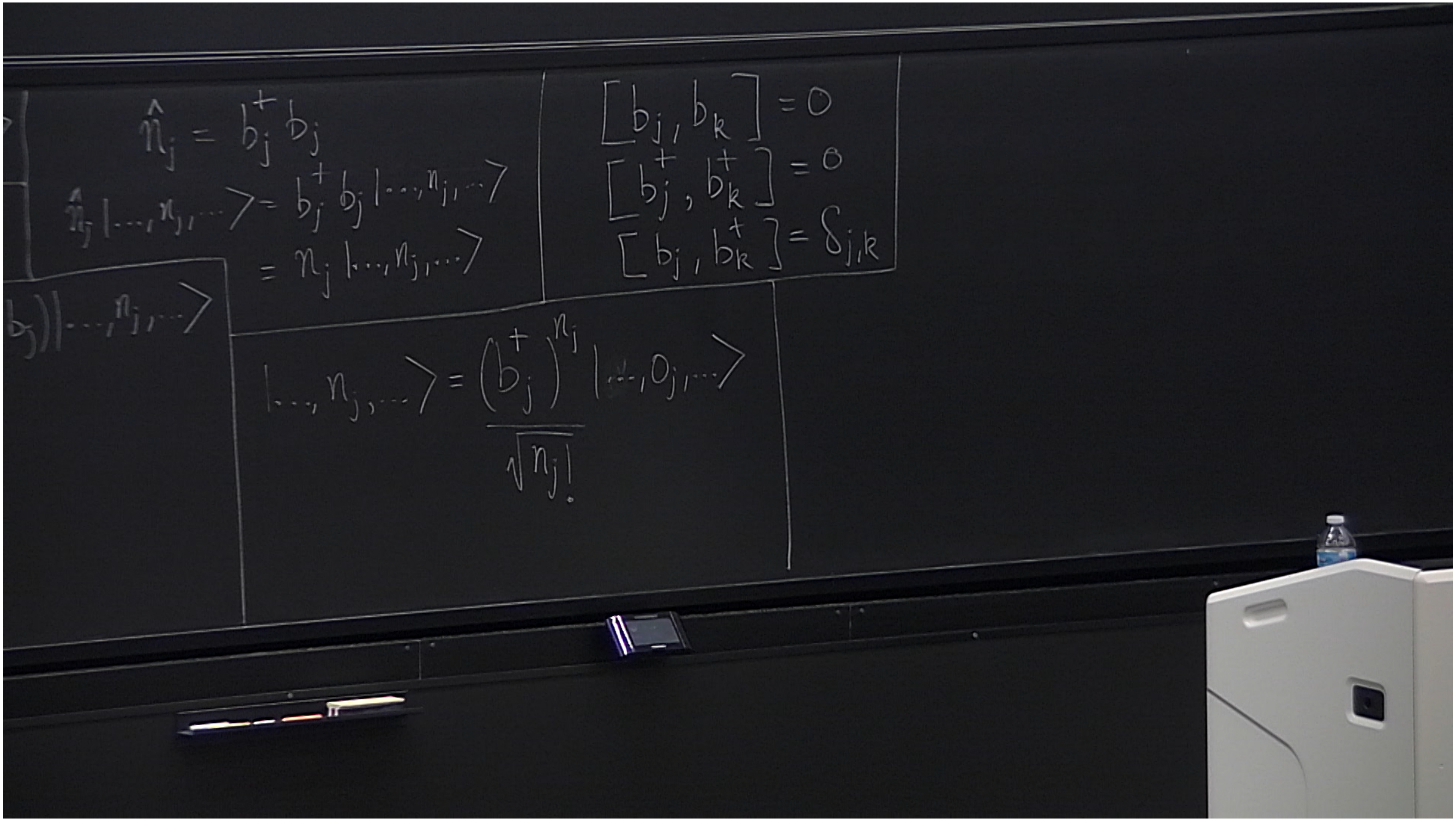
$$= \left((n_j + 1) - n_j\right) |\ldots, n_j, \ldots\rangle$$

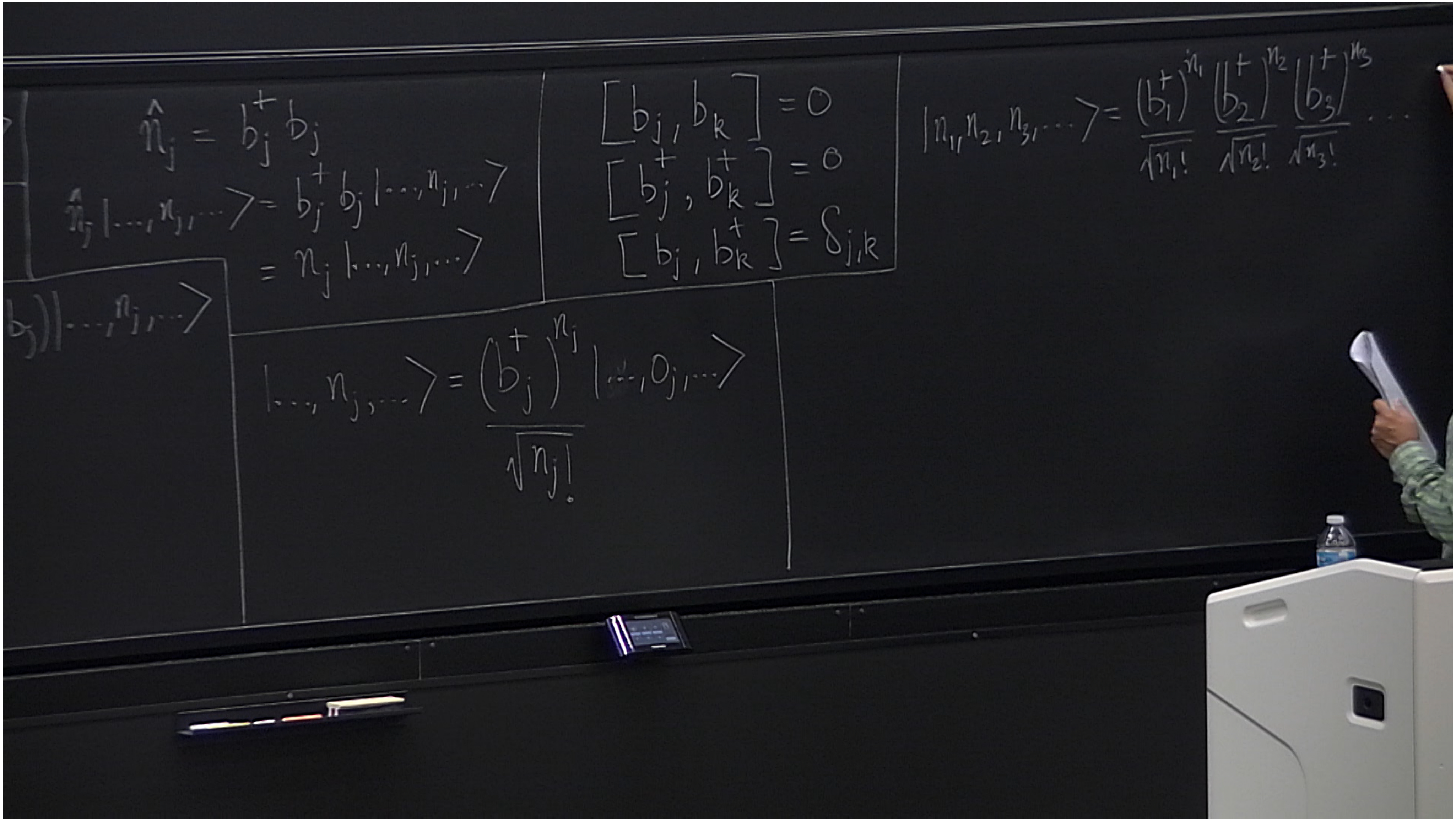
$$\boxed{[b_j, b_j^\dagger] = 1}$$

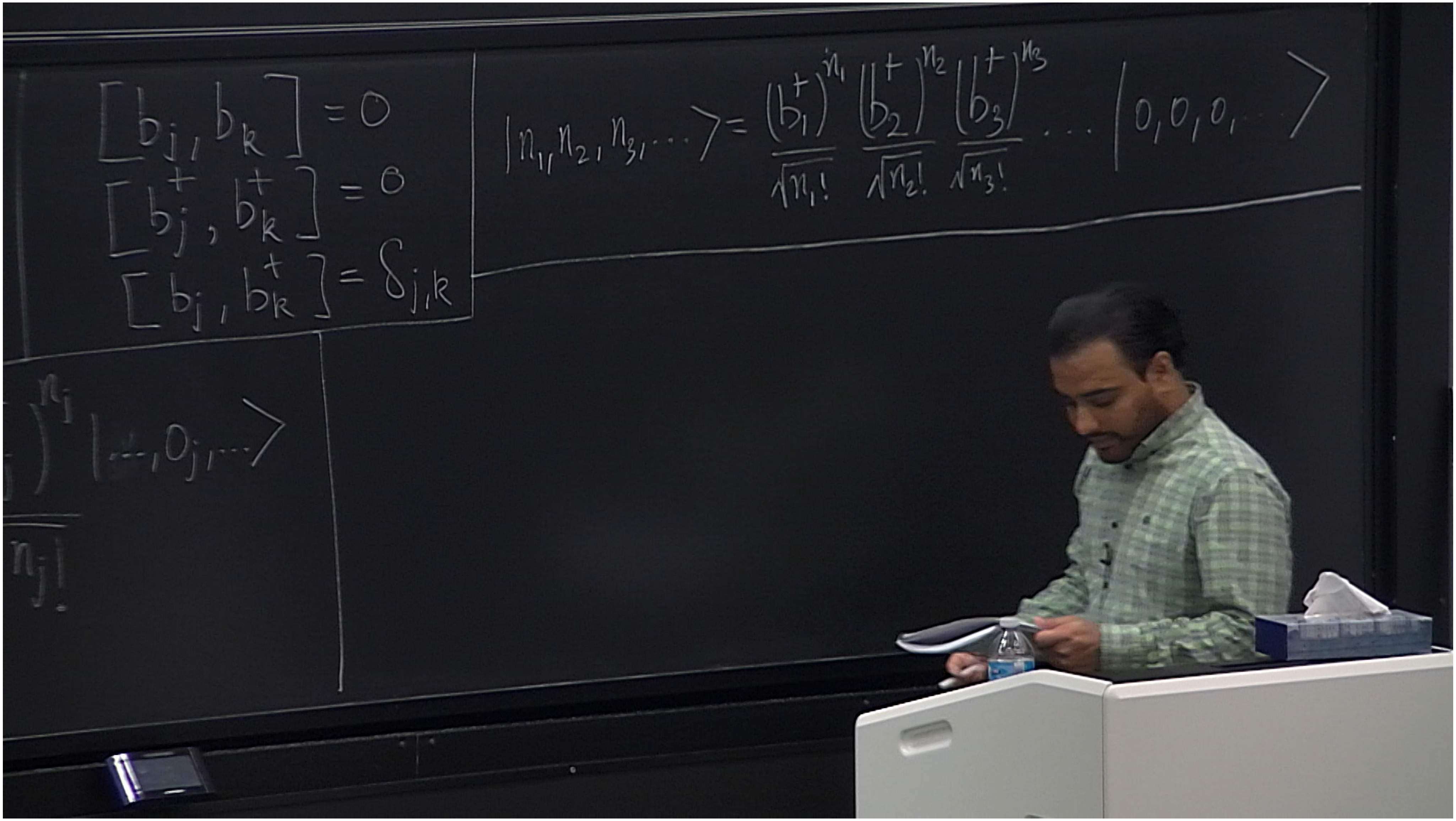

$b_j |\ldots, n_j, \ldots\rangle = \sqrt{n_j} |\ldots, n_j - 1, \ldots\rangle$
$b_j |\ldots, 0, \ldots\rangle = 0$
$[b_j, b_j^\dagger] |\ldots, n_j, \ldots\rangle = (b_j b_j^\dagger - b_j^\dagger b_j) |\ldots, n_j, \ldots\rangle$
$= ((n_j + 1) - n_j) |\ldots, n_j, \ldots\rangle$
$[b_j, b_j^\dagger] = 1$
$\hat{n}_j = b_j^\dagger b_j$
$\hat{n}_j |\ldots, n_j, \ldots\rangle = b_j^\dagger b_j |\ldots, n_j, \ldots\rangle$
$= n_j |\ldots, n_j, \ldots\rangle$


$b_j |\ldots, n_j, \ldots\rangle = \sqrt{n_j}\, |\ldots, n_j - 1, \ldots\rangle$
$b_j |\ldots, 0, \ldots\rangle = 0$
$\hat{n}_j = b_j^\dagger b_j$
$\hat{n}_j |\ldots, n_j, \ldots\rangle = b_j^\dagger b_j |\ldots, n_j, \ldots\rangle$
$= n_j |\ldots, n_j, \ldots\rangle$
$[b_j, b_j^\dagger] |\ldots, n_j, \ldots\rangle = (b_j b_j^\dagger - b_j^\dagger b_j) |\ldots, n_j, \ldots\rangle$
$= ((n_j + 1) - n_j) |\ldots, n_j, \ldots\rangle$
$[b_j, b_j^\dagger] = 1$
$|\ldots, n_j, \ldots\rangle = (b_j^\dagger)^{n_j} |\ldots, 0_j, \ldots\rangle$

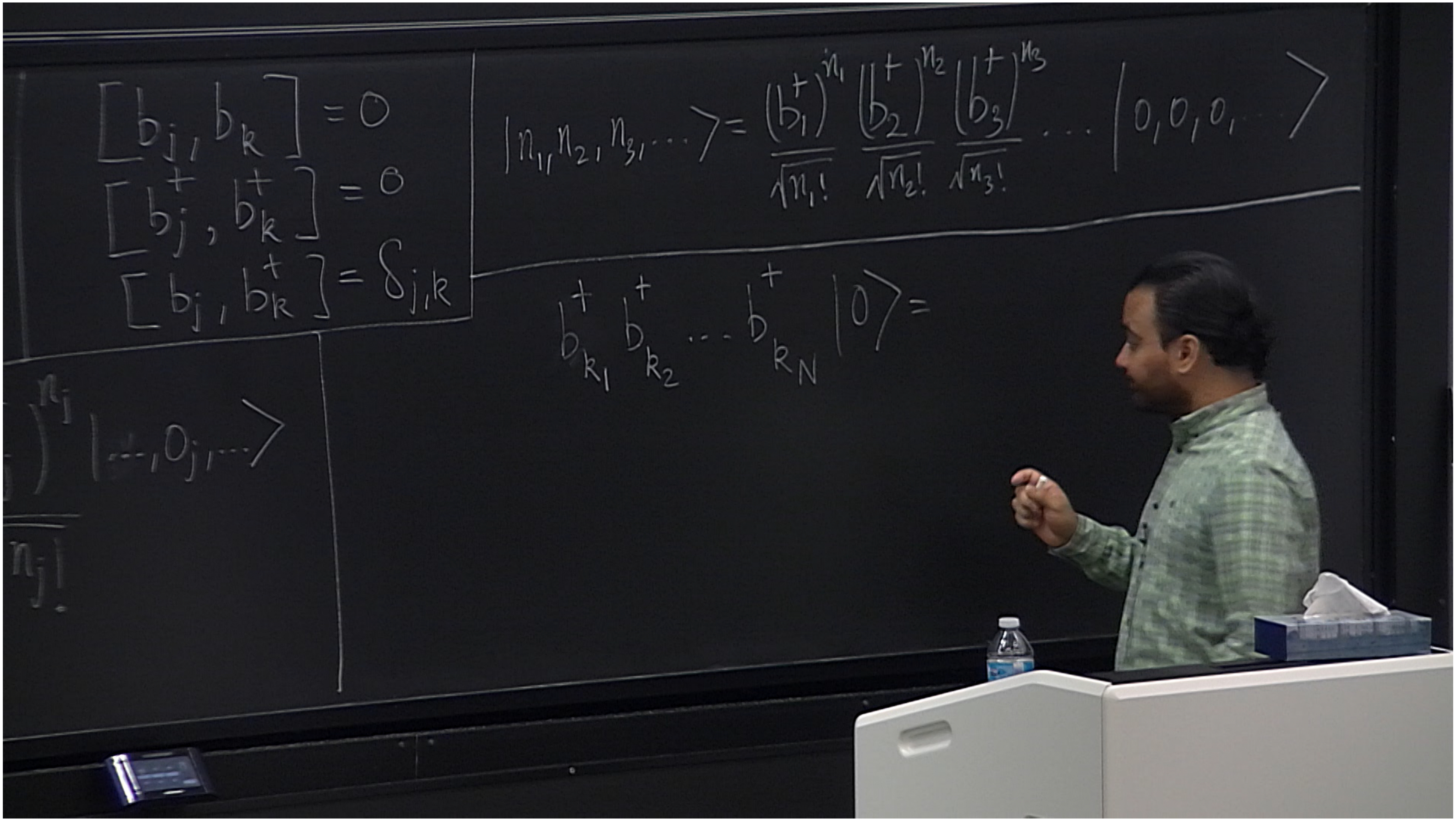

$\hat{n}_j = b_j^{+} b_j$
$\hat{n}_j |\ldots, n_j, \ldots\rangle = b_j^{+} b_j |\ldots, n_j, \ldots\rangle$
$= n_j |\ldots, n_j, \ldots\rangle$
$b_j))|\ldots, n_j, \ldots\rangle$
$|\ldots, n_j, \ldots\rangle = \frac{(b_j^{+})^{n_j}}{\sqrt{n_j!}} |\ldots, 0_j, \ldots\rangle$

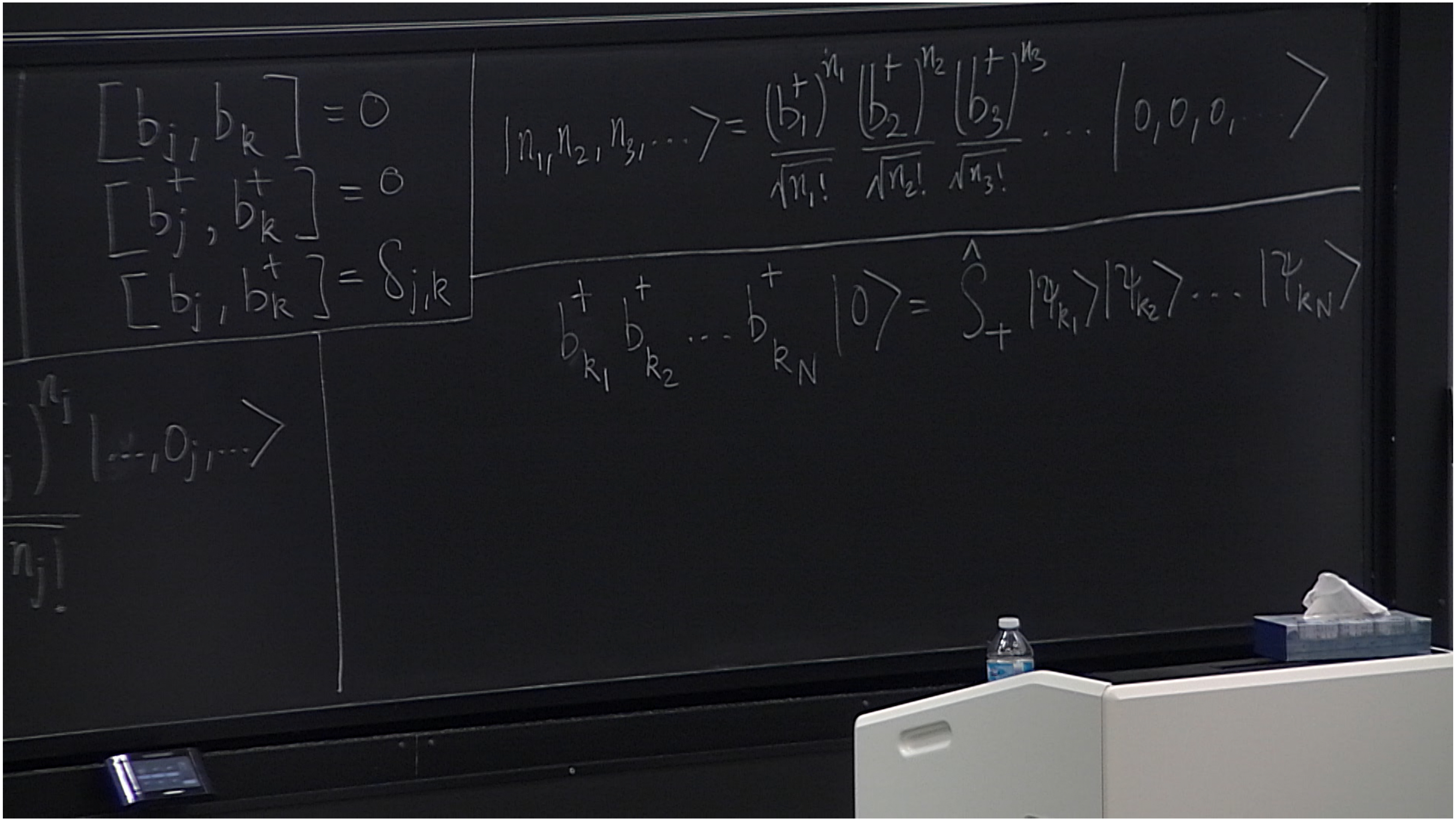

$\hat{n}_j = b_j^\dagger b_j$
$\hat{n}_j |\ldots, n_j, \ldots\rangle = b_j^\dagger b_j |\ldots, n_j, \ldots\rangle$
$= n_j |\ldots, n_j, \ldots\rangle$
$b_j))|\ldots, n_j, \ldots\rangle$
$[b_j, b_k] = 0$
$[b_j^\dagger, b_k^\dagger] = 0$
$[b_j, b_k^\dagger] = \delta_{j,k}$
$|\ldots, n_j, \ldots\rangle = \frac{(b_j^\dagger)^{n_j}}{\sqrt{n_j!}} |\ldots, 0_j, \ldots\rangle$

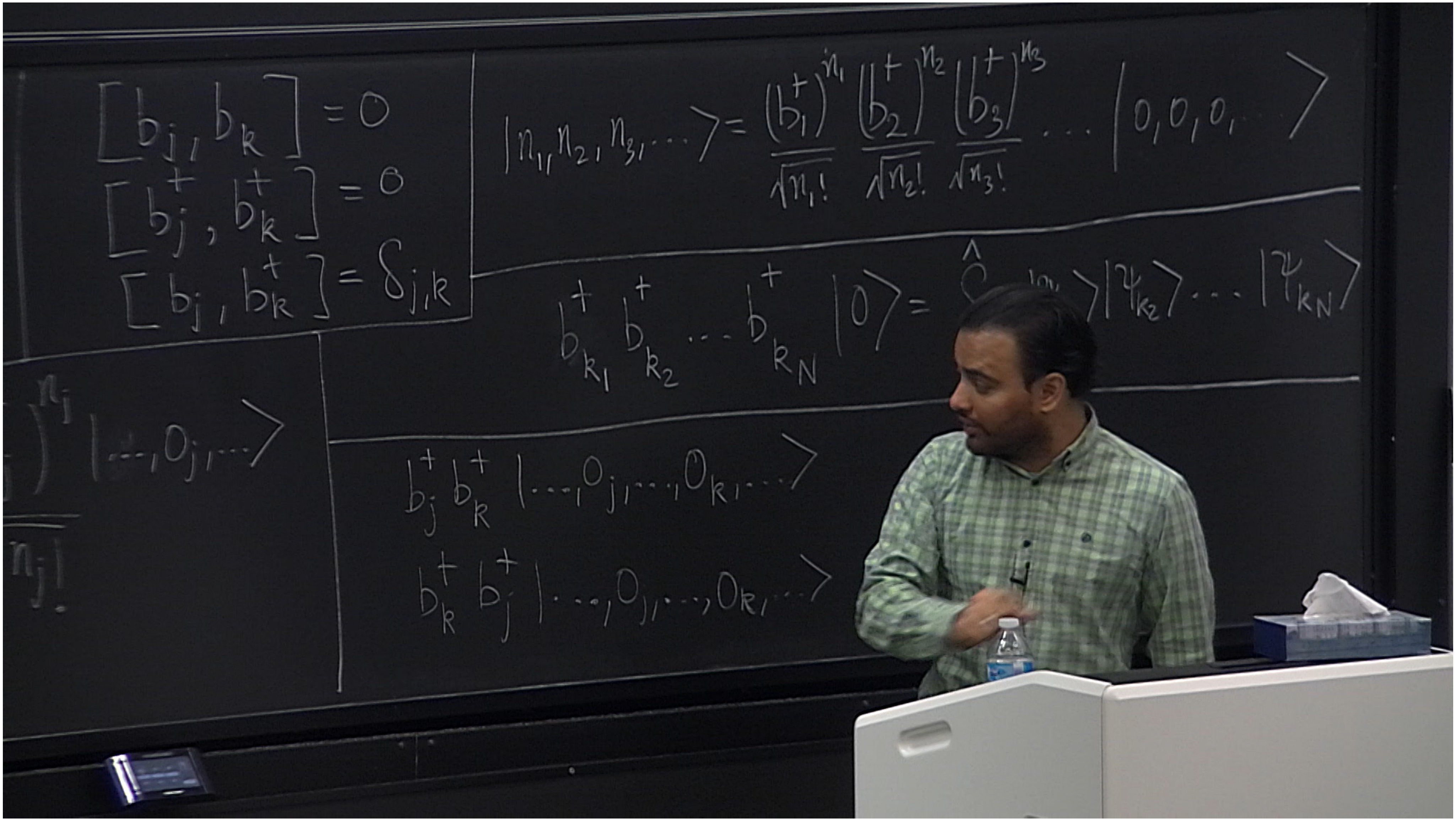

$\hat{n}_j = b_j^\dagger b_j$
$\hat{n}_j |\ldots, n_j, \ldots\rangle = b_j^\dagger b_j |\ldots, n_j, \ldots\rangle$
$= n_j |\ldots, n_j, \ldots\rangle$
$b_j)) |\ldots, n_j, \ldots\rangle$
$[b_j, b_k] = 0$
$[b_j^\dagger, b_k^\dagger] = 0$
$[b_j, b_k^\dagger] = \delta_{j,k}$
$|\ldots, n_j, \ldots\rangle = \frac{(b_j^\dagger)^{n_j}}{\sqrt{n_j!}} |\ldots, 0_j, \ldots\rangle$
$|n_1, n_2, n_3, \ldots\rangle = \frac{(b_1^\dagger)^{n_1}}{\sqrt{n_1!}} \frac{(b_2^\dagger)^{n_2}}{\sqrt{n_2!}} \frac{(b_3^\dagger)^{n_3}}{\sqrt{n_3!}} \ldots$

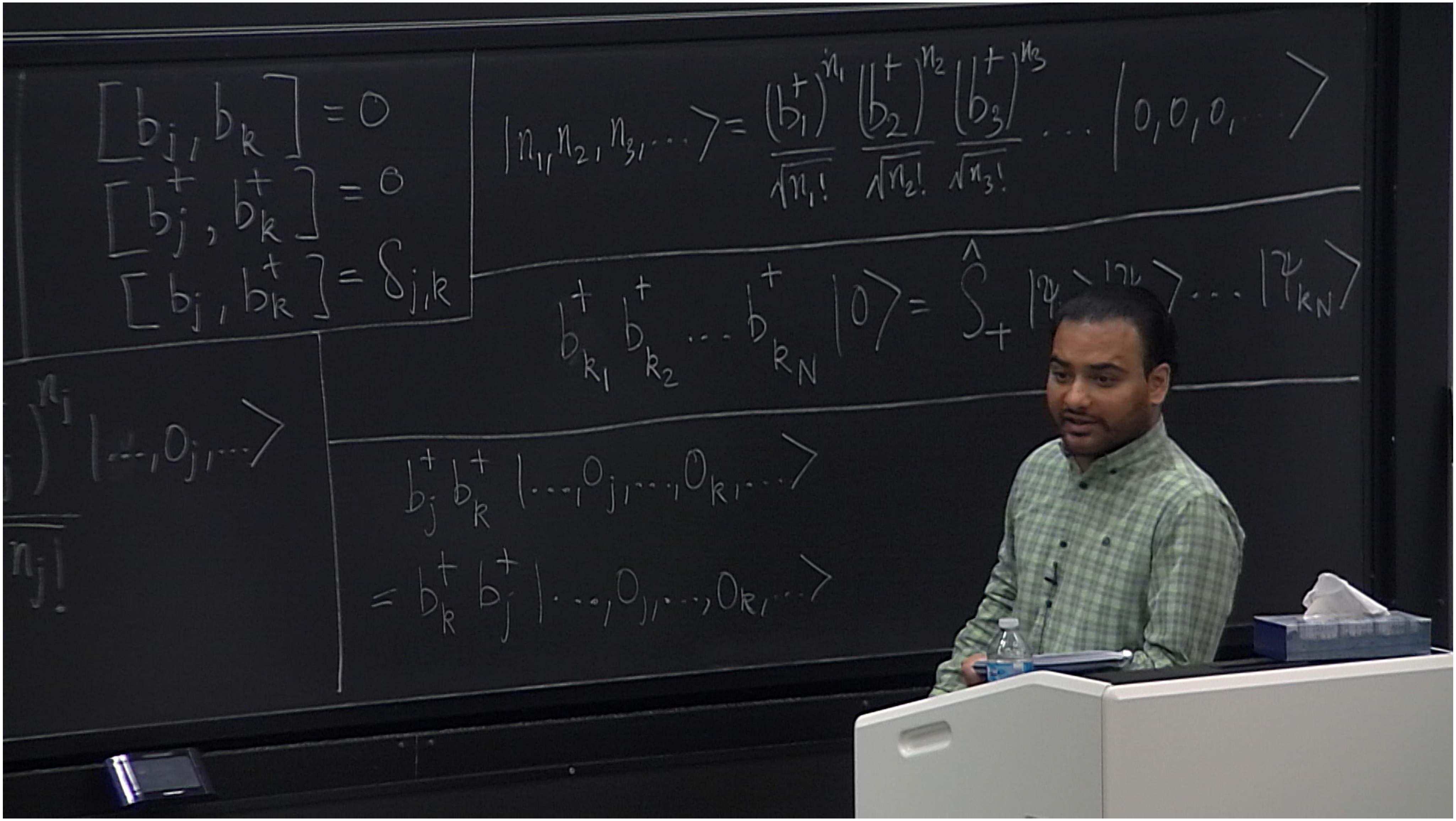


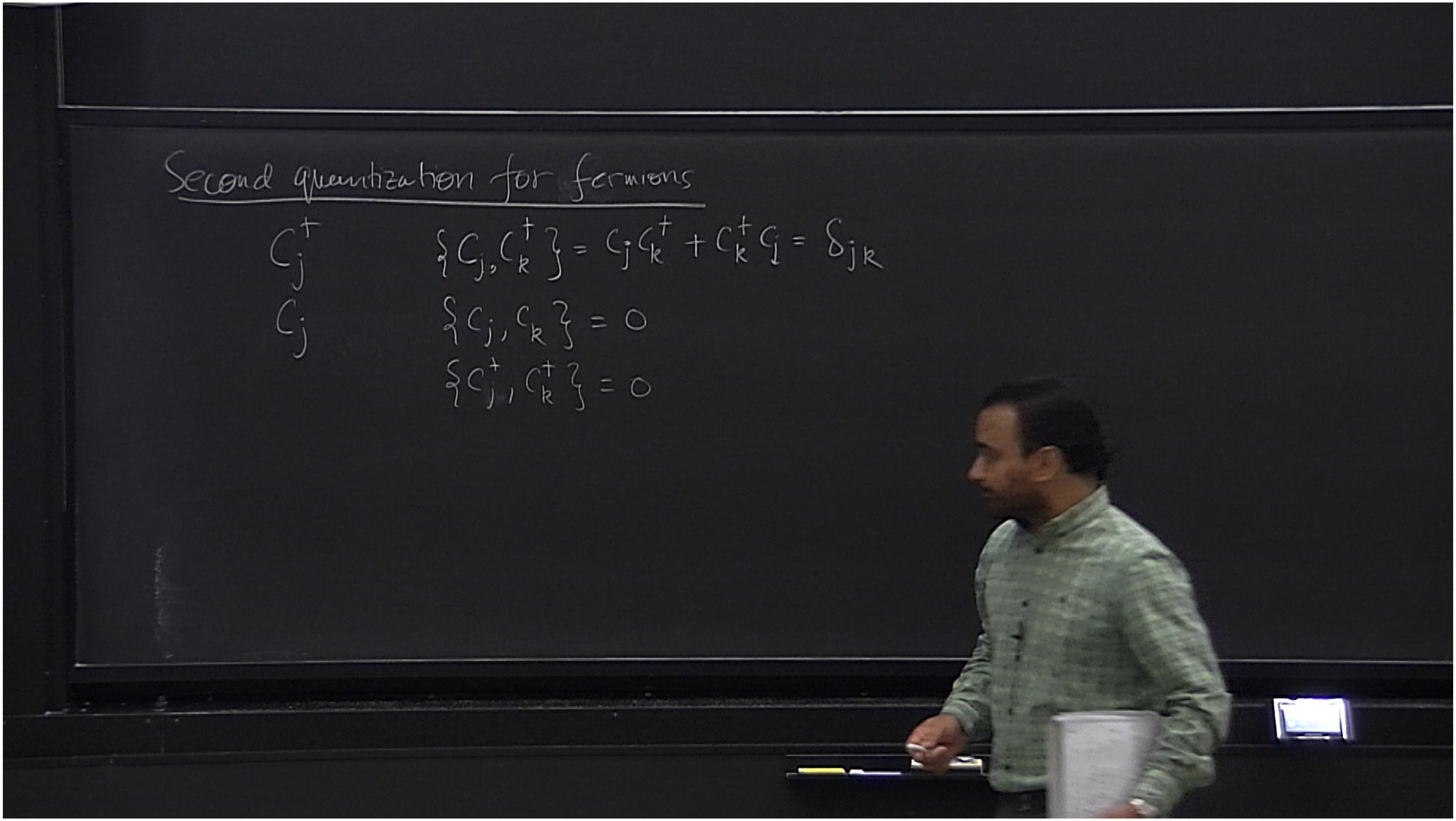
$[b_j, b_k] = 0$
$[b_j^+, b_k^+] = 0$
$[b_j, b_k^+] = \delta_{j,k}$
$|n_1, n_2, n_3, \ldots\rangle = \frac{(b_1^+)^{n_1}}{\sqrt{n_1!}} \frac{(b_2^+)^{n_2}}{\sqrt{n_2!}} \frac{(b_3^+)^{n_3}}{\sqrt{n_3!}} \cdots |0, 0, 0, \ldots\rangle$

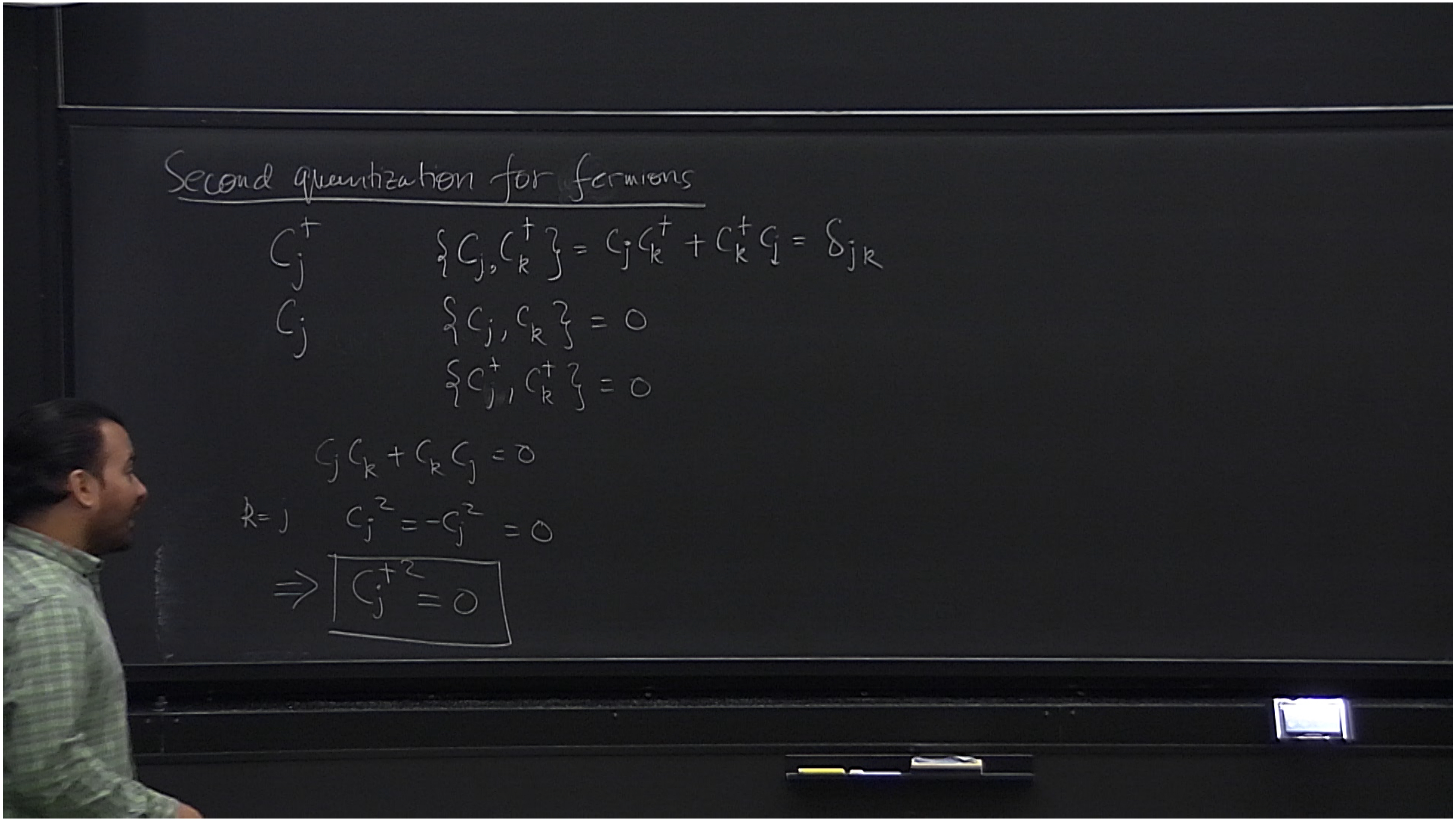

[b_j, b_k] = 0
[b_j^†, b_k^†] = 0
[b_j, b_k^†] = δ_{j,k}
|n_1, n_2, n_3, ...> = (b_1^†)^{n_1}/√(n_1!) (b_2^†)^{n_2}/√(n_2!) (b_3^†)^{n_3}/√(n_3!) ... |0, 0, 0, ...>
b_{k_1}^† b_{k_2}^† ... b_{k_N}^† |0> =
|..., 0_j, ...>

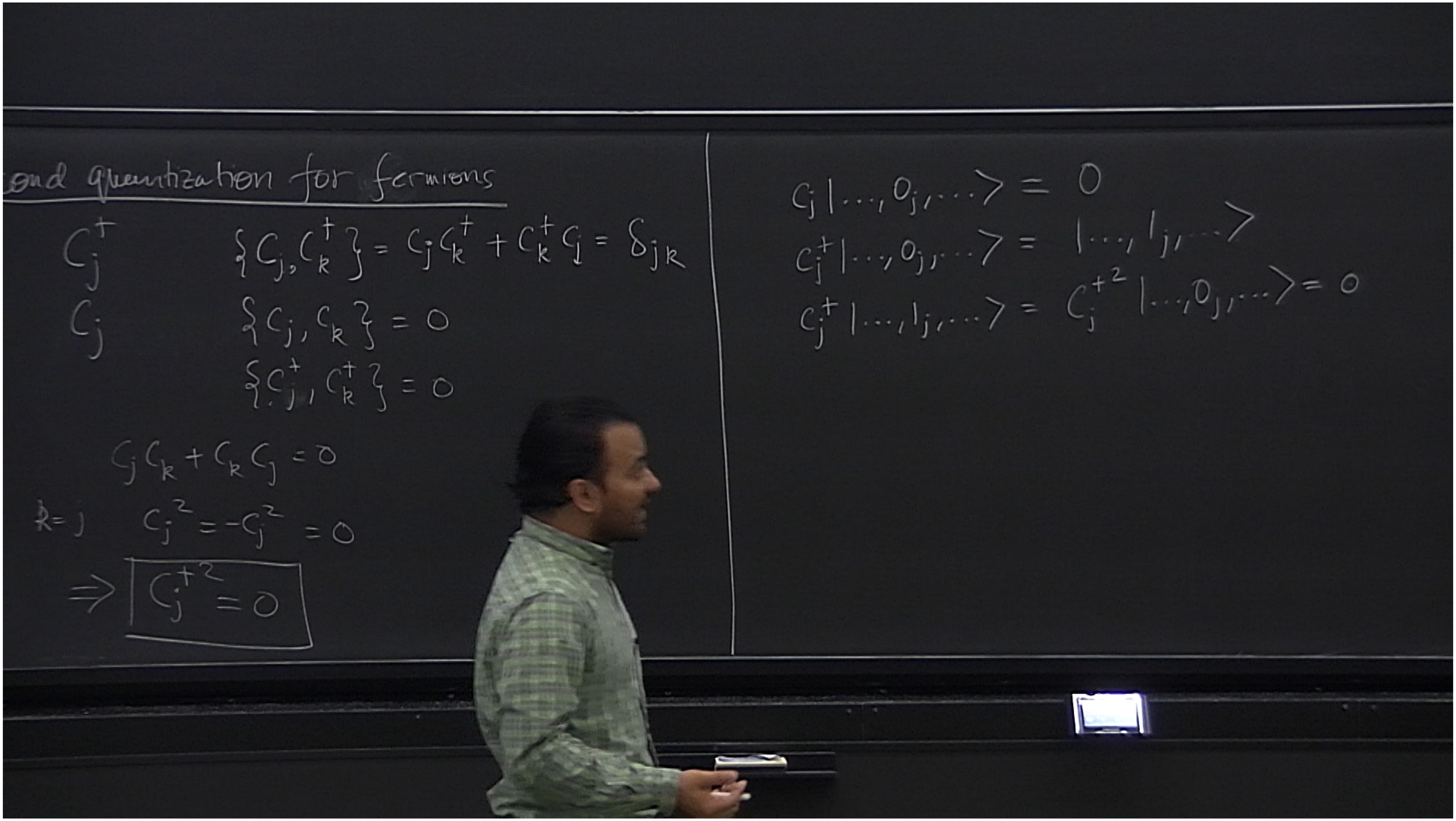

[b_j, b_k] = 0
[b_j^+, b_k^+] = 0
[b_j, b_k^+] = δ_{j,k}
|n_1, n_2, n_3, ...> = (b_1^+)^{n_1}/√(n_1!) (b_2^+)^{n_2}/√(n_2!) (b_3^+)^{n_3}/√(n_3!) ... |0, 0, 0, ...>
b_{k_1}^+ b_{k_2}^+ ... b_{k_N}^+ |0> = Ŝ_+ |ψ_{k_1}> |ψ_{k_2}> ... |ψ_{k_N}>
)^{n_j} |..., 0_j, ...>
n_j!



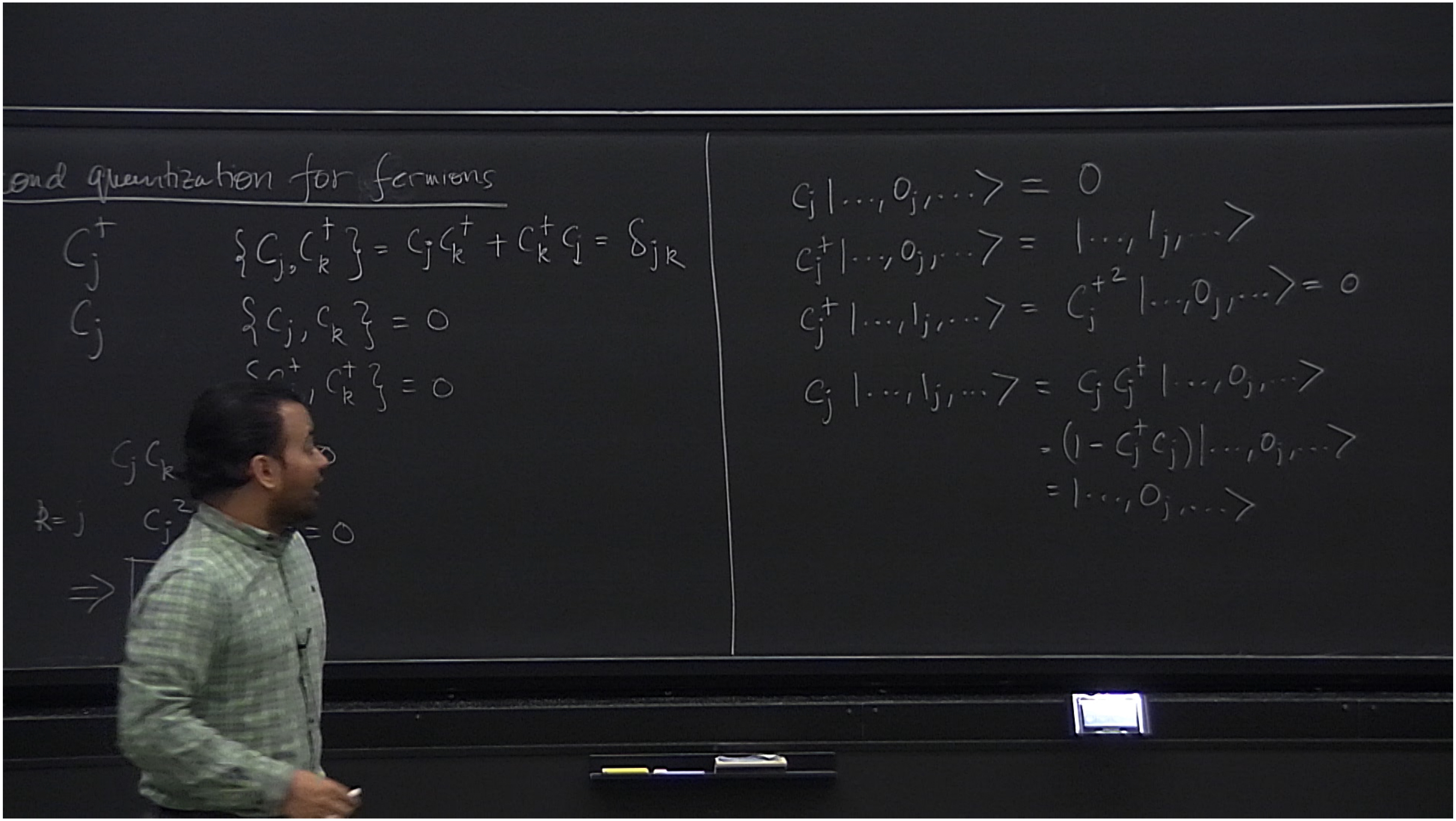

[b_j, b_k] = 0
[b_j^+, b_k^+] = 0
[b_j, b_k^+] = δ_{j,k}
|n_1, n_2, n_3, ...> = (b_1^+)^{n_1}/√(n_1!) (b_2^+)^{n_2}/√(n_2!) (b_3^+)^{n_3}/√(n_3!) ... |0, 0, 0, ...>
b_{k_1}^+ b_{k_2}^+ ... b_{k_N}^+ |0> = Ŝ_+ |ψ_{k_1}> |ψ_{k_2}> ... |ψ_{k_N}>
b_j^+ b_k^+ |..., 0_j, ..., 0_k, ...>
= b_k^+ b_j^+ |..., 0_j, ..., 0_k, ...>

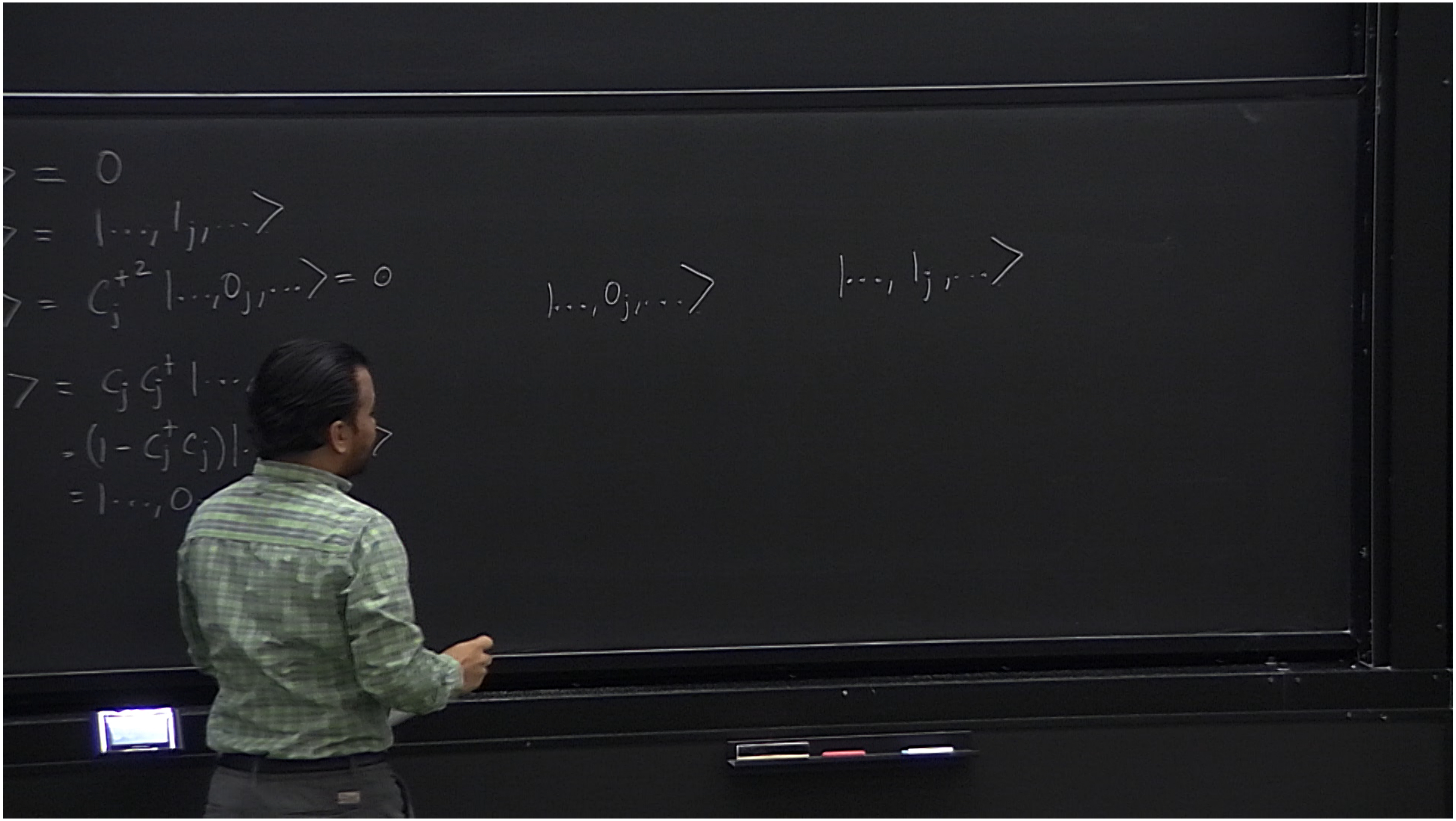

Second quantization for fermions
$c_j^\dagger$
c_j
$\{c_j, c_k^\dagger\} = c_j c_k^\dagger + c_k^\dagger c_j = \delta_{jk}$
$\{c_j, c_k\} = 0$
$\{c_j^\dagger, c_k^\dagger\} = 0$

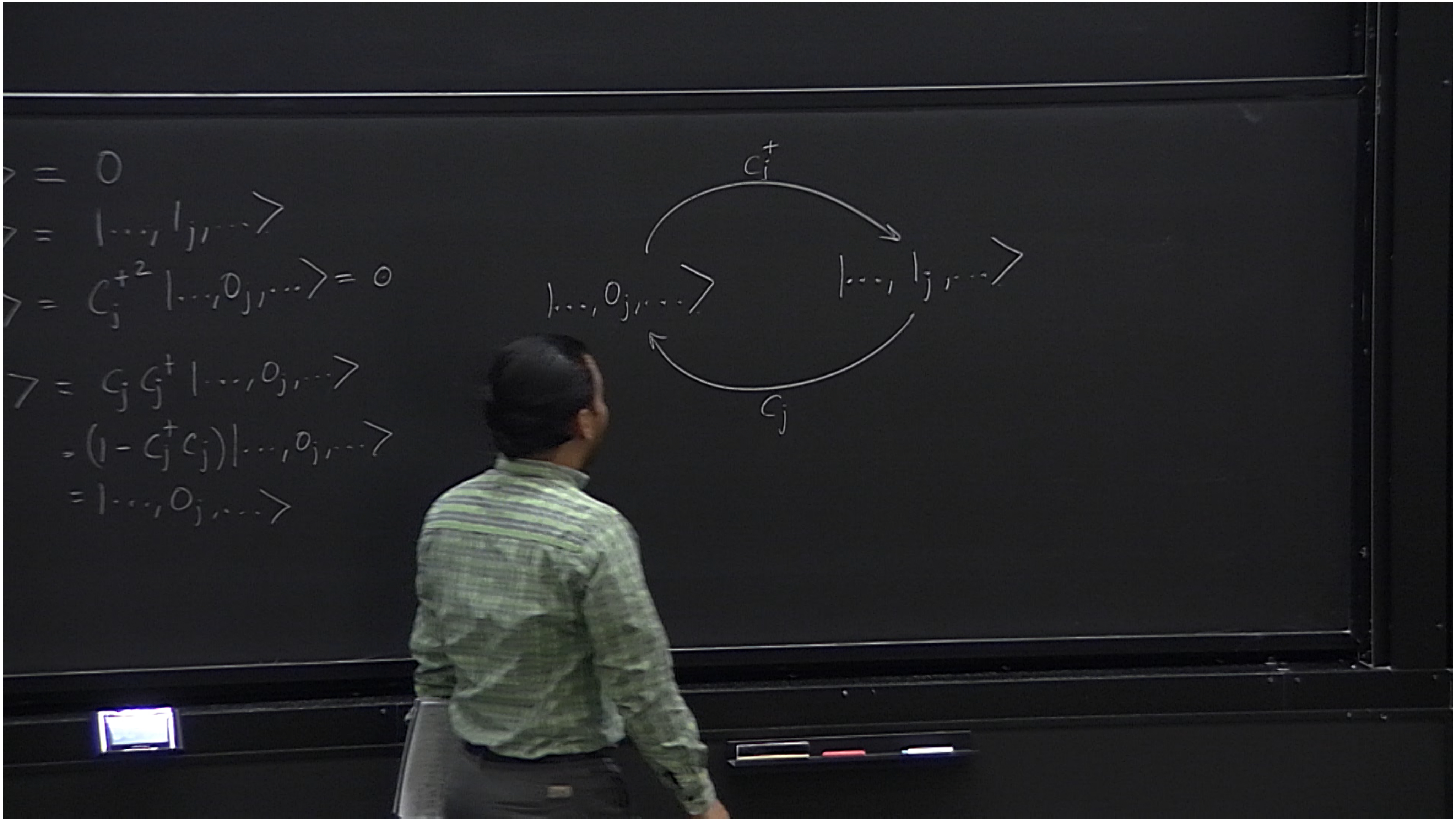

Second quantization for fermions
$c_j^\dagger$
$\{c_j, c_k^\dagger\} = c_j c_k^\dagger + c_k^\dagger c_j = \delta_{jk}$
c_j
$\{c_j, c_k\} = 0$
$\{c_j^\dagger, c_k^\dagger\} = 0$
$c_j c_k + c_k c_j = 0$
$k = j$
$c_j^2 = -c_j^2 = 0$
$\Rightarrow c_j^{\dagger 2} = 0$

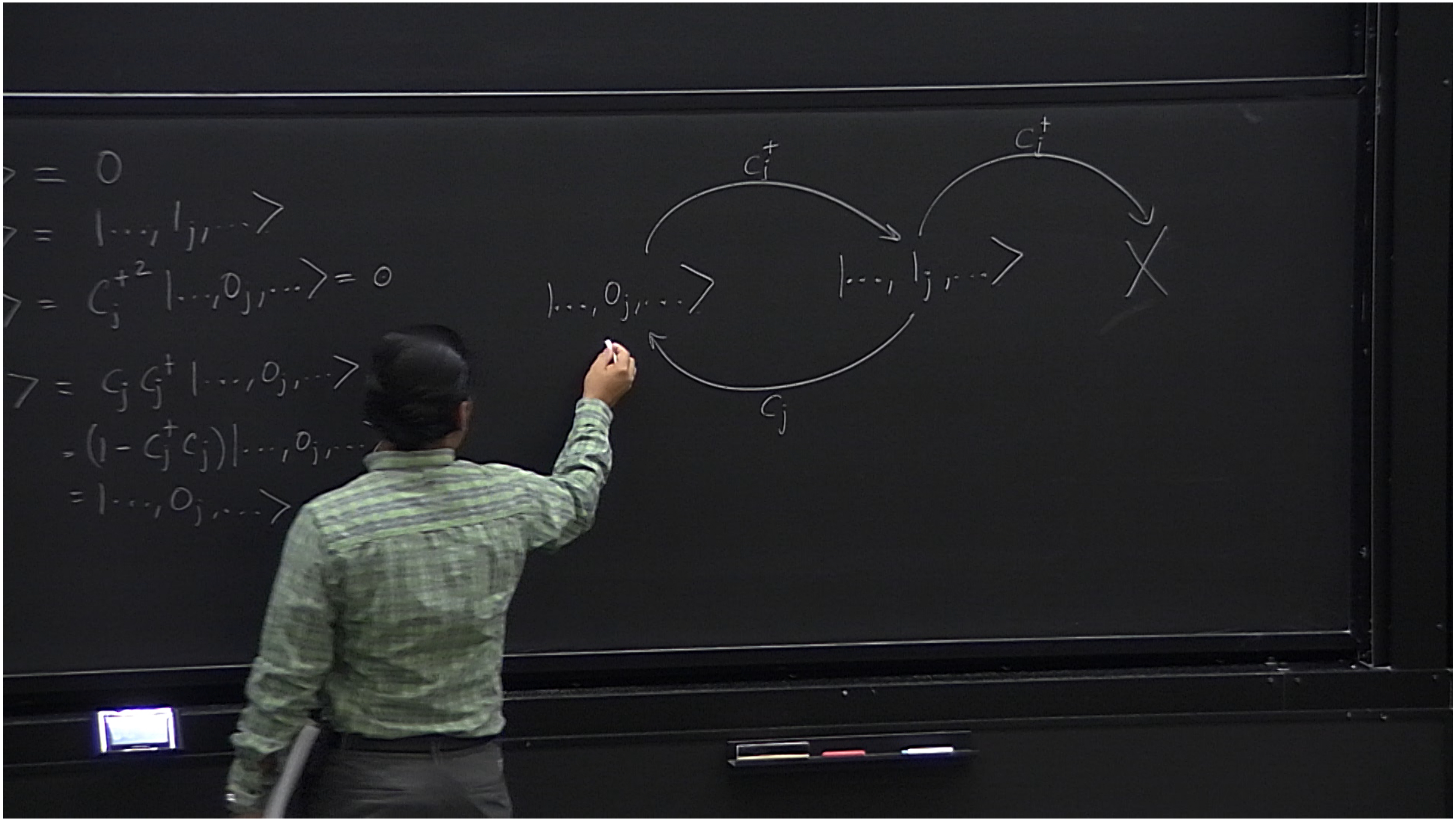


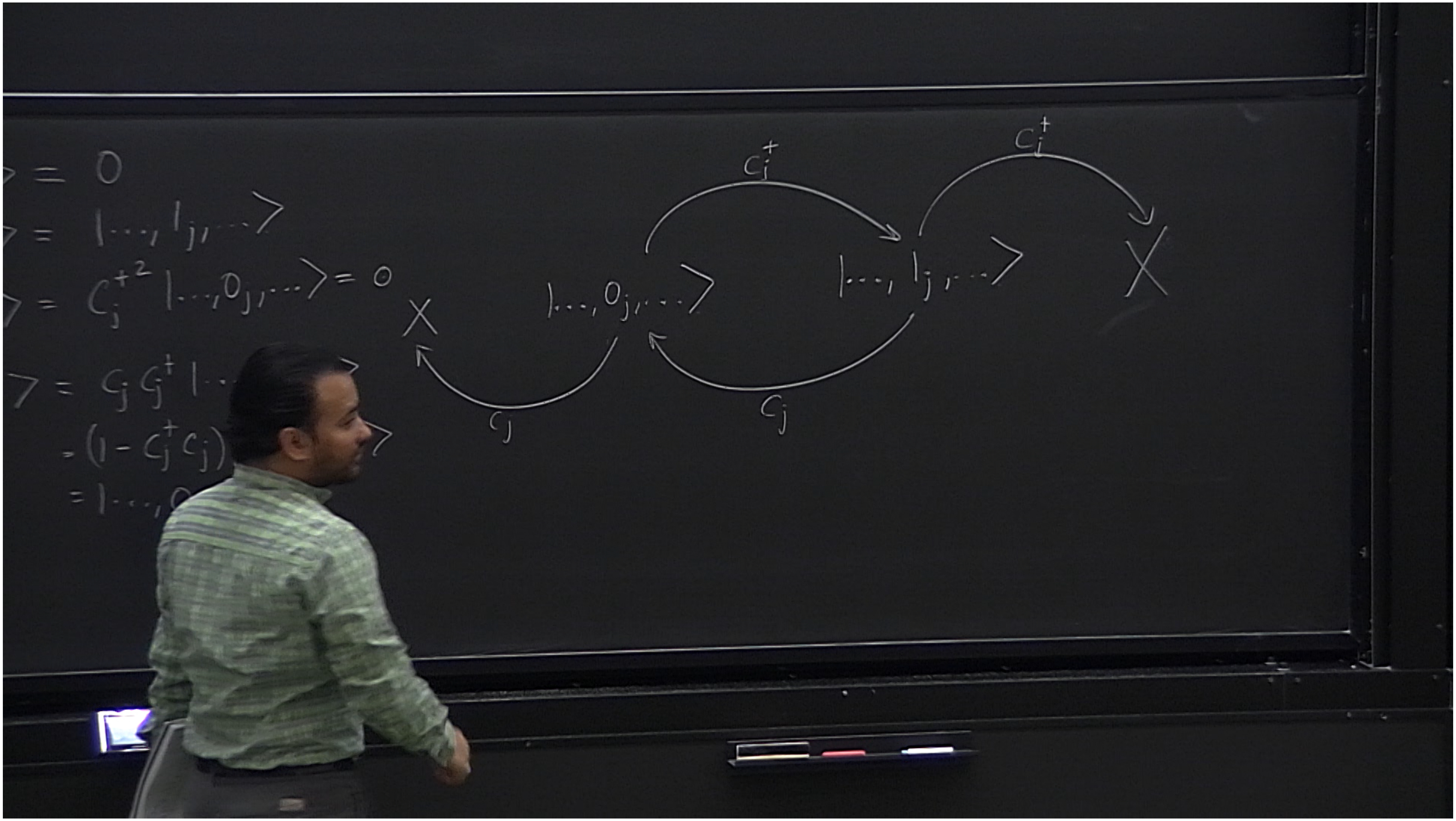
cond quantization for fermions
$c_j^\dagger$
c_j
$\{c_j, c_k^\dagger\} = c_j c_k^\dagger + c_k^\dagger c_j = \delta_{jk}$
$\{c_j, c_k\} = 0$
$\{c_j^\dagger, c_k^\dagger\} = 0$
$c_j c_k + c_k c_j = 0$
$k=j \quad c_j^2 = -c_j^2 = 0$
$\Rightarrow c_j^{\dagger 2} = 0$
$c_j |\ldots, 0_j, \ldots\rangle = 0$
$c_j^\dagger |\ldots, 0_j, \ldots\rangle = |\ldots, 1_j, \ldots\rangle$
$c_j^\dagger |\ldots, 1_j, \ldots\rangle = c_j^{\dagger 2} |\ldots, 0_j, \ldots\rangle = 0$


ond quantization for fermions
$c_j^\dagger$
c_j
$\{c_j, c_k^\dagger\} = c_j c_k^\dagger + c_k^\dagger c_j = \delta_{jk}$
$\{c_j, c_k\} = 0$
$c_k^\dagger\} = 0$
$k = j$
$= 0$
$c_j|\ldots, 0_j, \ldots\rangle = 0$
$c_j^\dagger|\ldots, 0_j, \ldots\rangle = |\ldots, 1_j, \ldots\rangle$
$c_j^\dagger|\ldots, 1_j, \ldots\rangle = c_j^{\dagger 2}|\ldots, 0_j, \ldots\rangle = 0$
$c_j|\ldots, 1_j, \ldots\rangle = c_j c_j^\dagger|\ldots, 0_j, \ldots\rangle$
$= (1 - c_j^\dagger c_j)|\ldots, 0_j, \ldots\rangle$
$= |\ldots, 0_j, \ldots\rangle$

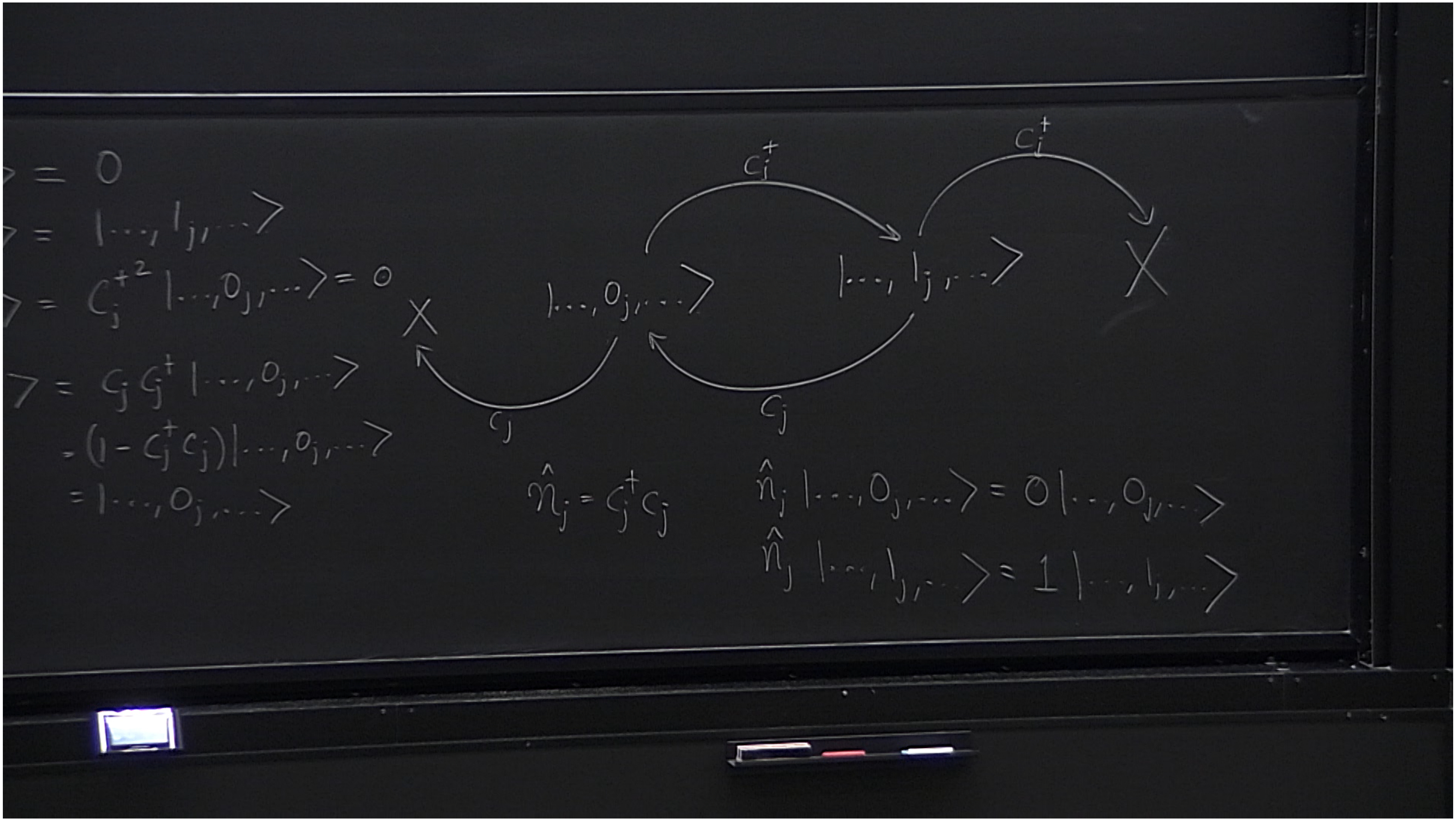

= 0
= |..., 1j, ...>
= cj+2 |..., 0j, ...> = 0
|..., 0j, ...>
|..., 1j, ...>


$c_j^\dagger$
$|\ldots, 0_j, \ldots\rangle$
$|\ldots, 1_j, \ldots\rangle$
c_j
$= 0$
$= |\ldots, 1_j, \ldots\rangle$
$= c_j^{\dagger 2} |\ldots, 0_j, \ldots\rangle = 0$
$= c_j c_j^\dagger |\ldots, 0_j, \ldots\rangle$
$= (1 - c_j^\dagger c_j) |\ldots, 0_j, \ldots\rangle$
$= |\ldots, 0_j, \ldots\rangle$


= 0
= |..., 1j, ...>
= cj†² |..., 0j, ...> = 0
= cj cj† |..., 0j, ...>
= (1 - cj† cj) |..., 0j, ...
= |..., 0j, ...>
cj†
|..., 0j, ...>
|..., 1j, ...>
cj
cj†
X



|..., 0_j, ...>
|..., 1_j, ...>
c_j^†
c_j^†
c_j
c_j

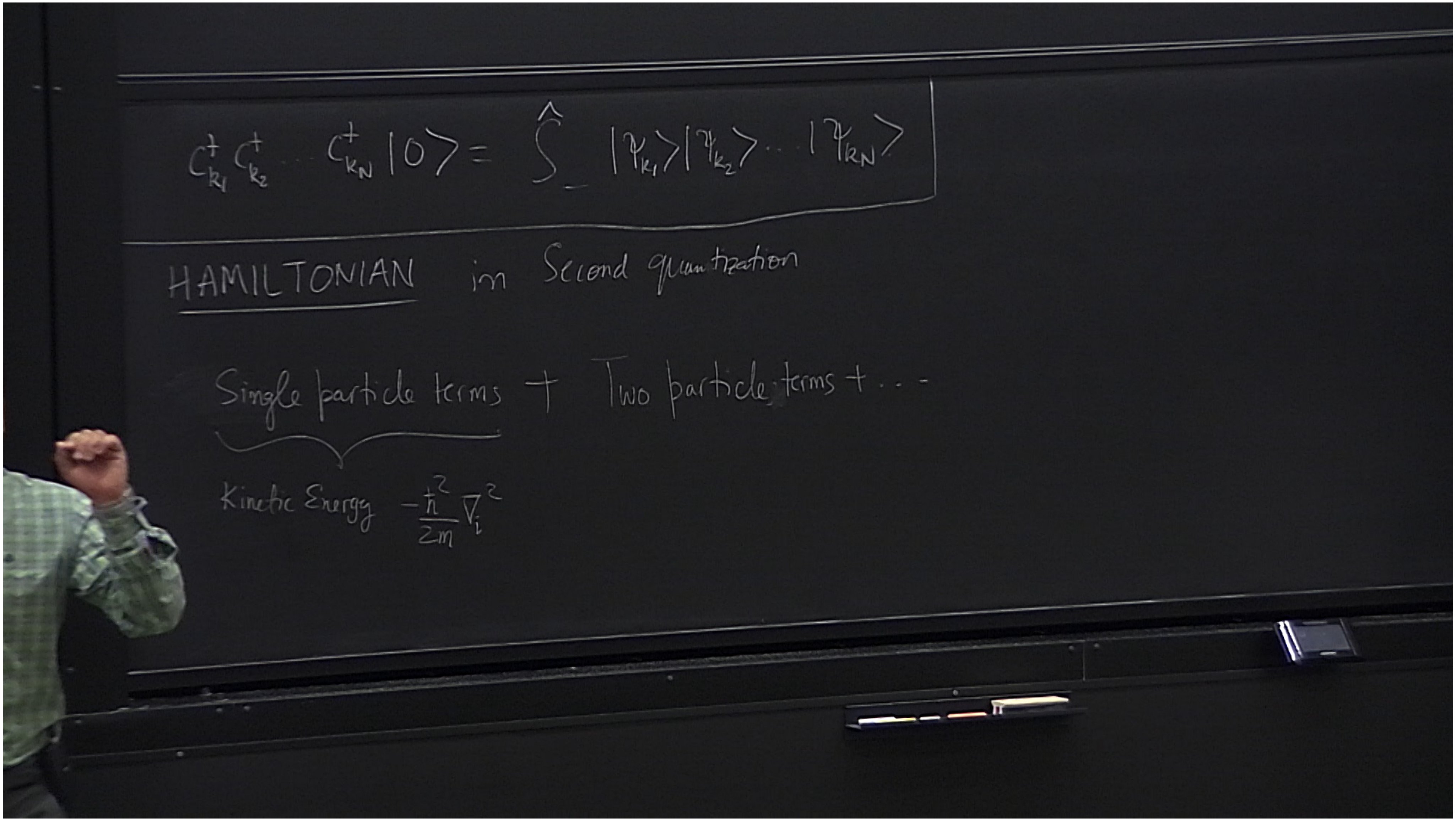

$= 0$
$= |\ldots, 1_j, \ldots\rangle$
$= c_j^{\dagger 2} |\ldots, 0_j, \ldots\rangle = 0$
$= c_j c_j^\dagger |\ldots, 0_j, \ldots\rangle$
$= (1 - c_j^\dagger c_j) |\ldots, 0_j, \ldots\rangle$
$= |\ldots, 0_j, \ldots\rangle$
$c_j^\dagger$
$c_j^\dagger$
X
$|\ldots, 0_j, \ldots\rangle$
$|\ldots, 1_j, \ldots\rangle$
X
c_j
c_j
$\hat{n}_j = c_j^\dagger c_j$
$\hat{n}_j |\ldots, 0_j, \ldots\rangle = 0 |\ldots, 0_j, \ldots\rangle$
$\hat{n}_j |\ldots, 1_j, \ldots\rangle = 1 |\ldots, 1_j, \ldots\rangle$

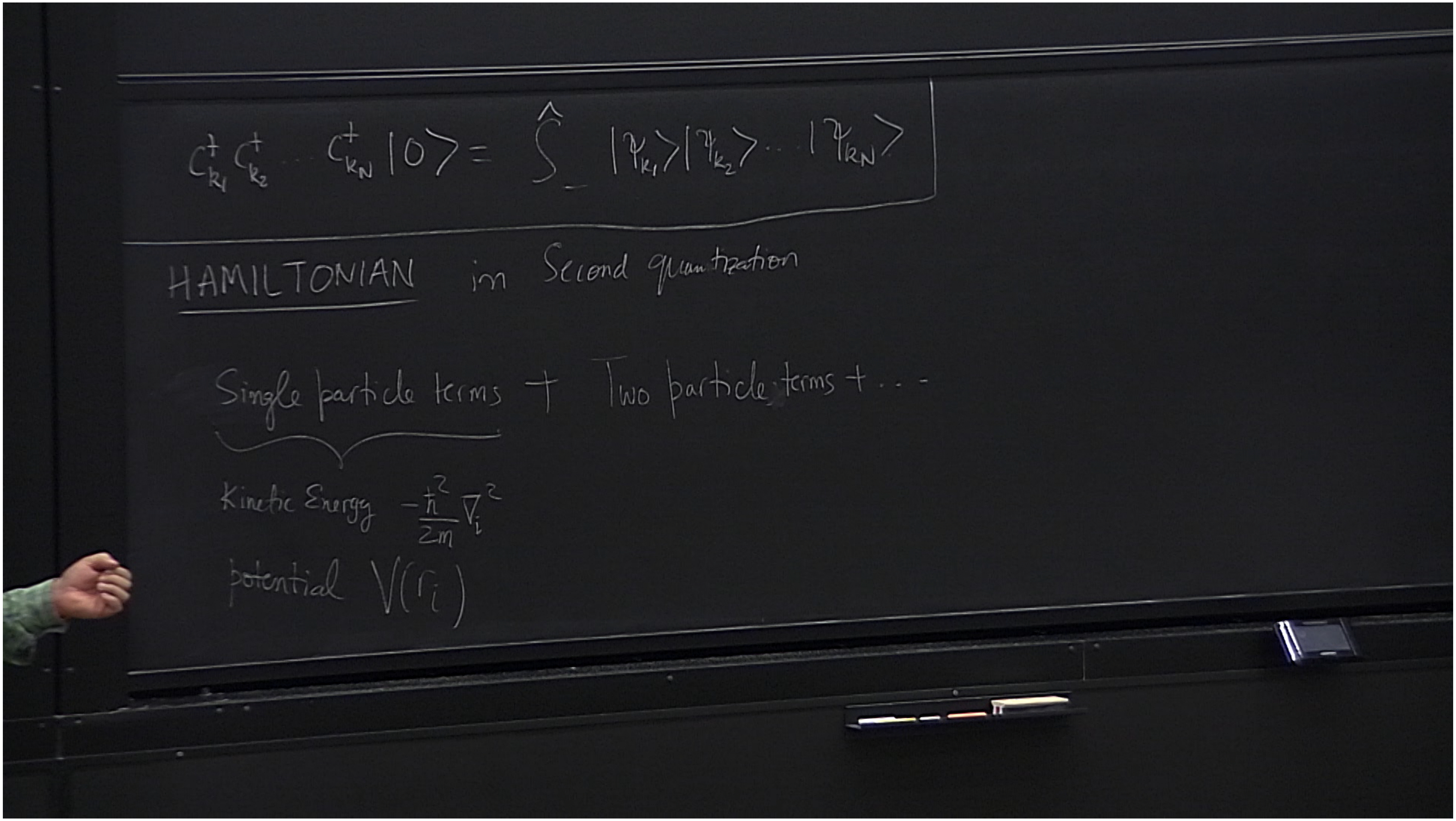
$$c^{\dagger}_{k_1} c^{\dagger}_{k_2} \cdots c^{\dagger}_{k_N} |0\rangle = \hat{S}_{-} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \cdots |\psi_{k_N}\rangle$$

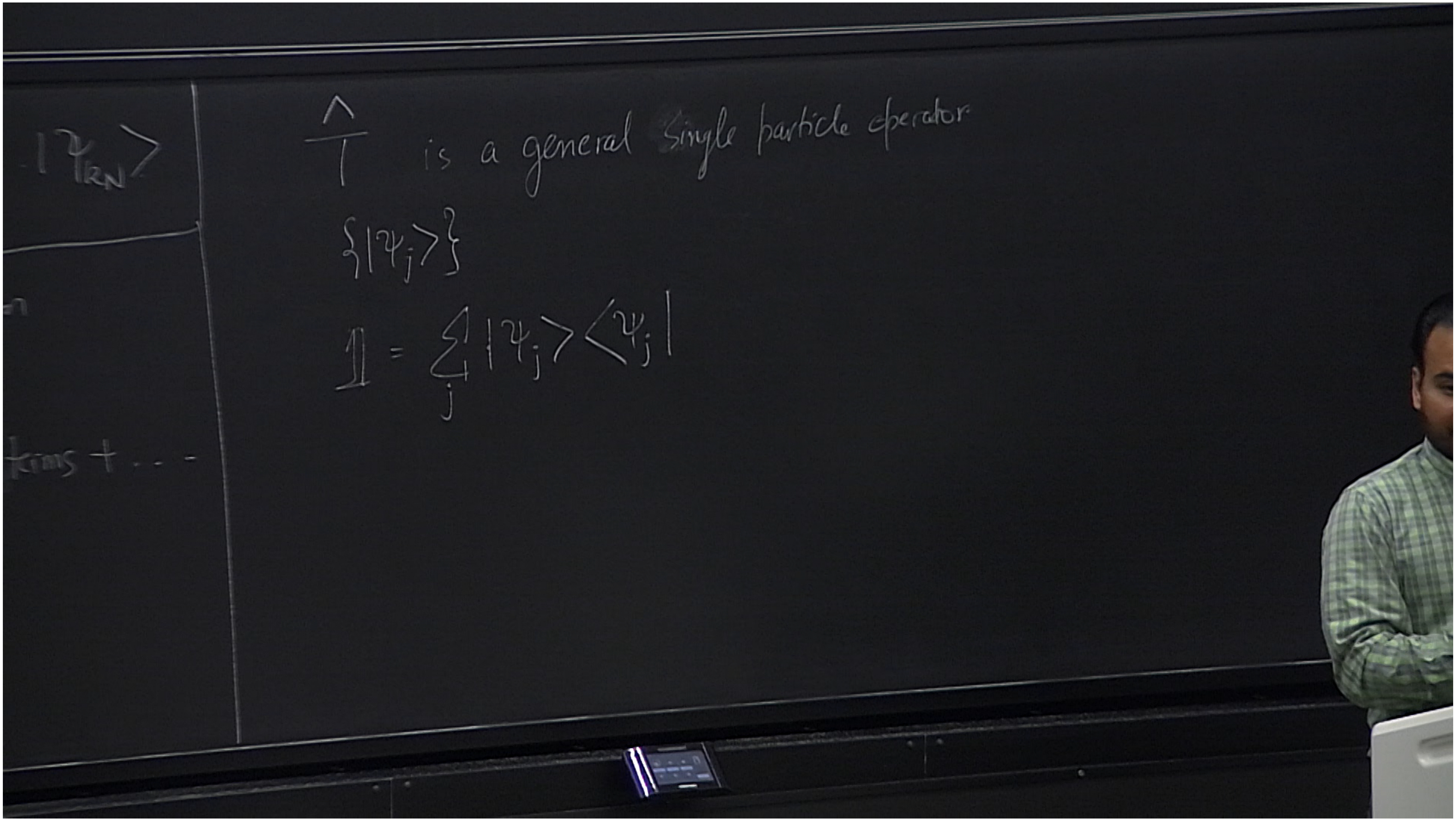
$$c^\dagger_{k_1} c^\dagger_{k_2} \cdots c^\dagger_{k_N}|0\rangle = \hat{S}_- |\psi_{k_1}\rangle|\psi_{k_2}\rangle\cdots|\psi_{k_N}\rangle$$

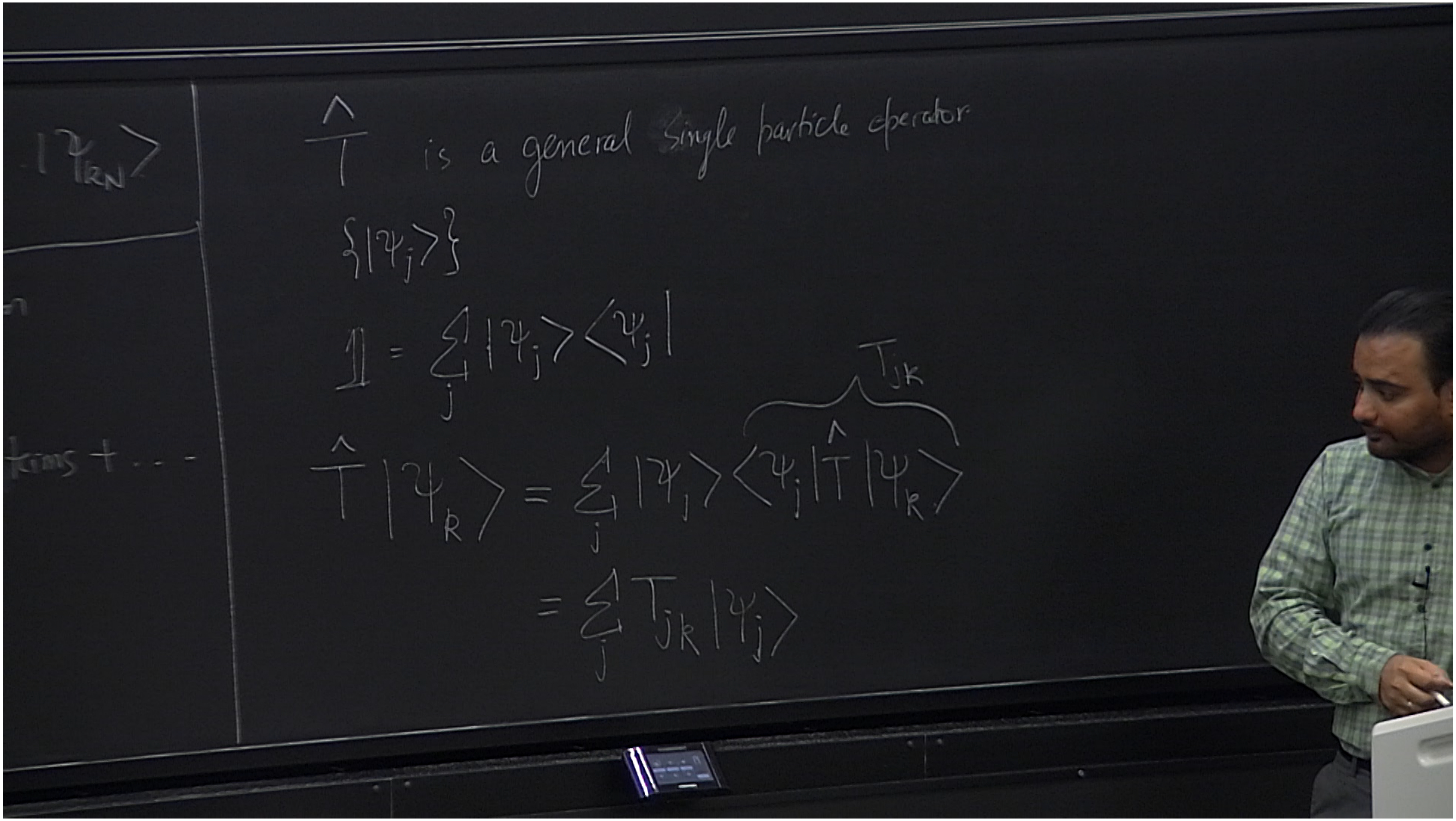
HAMILTONIAN in Second quantization

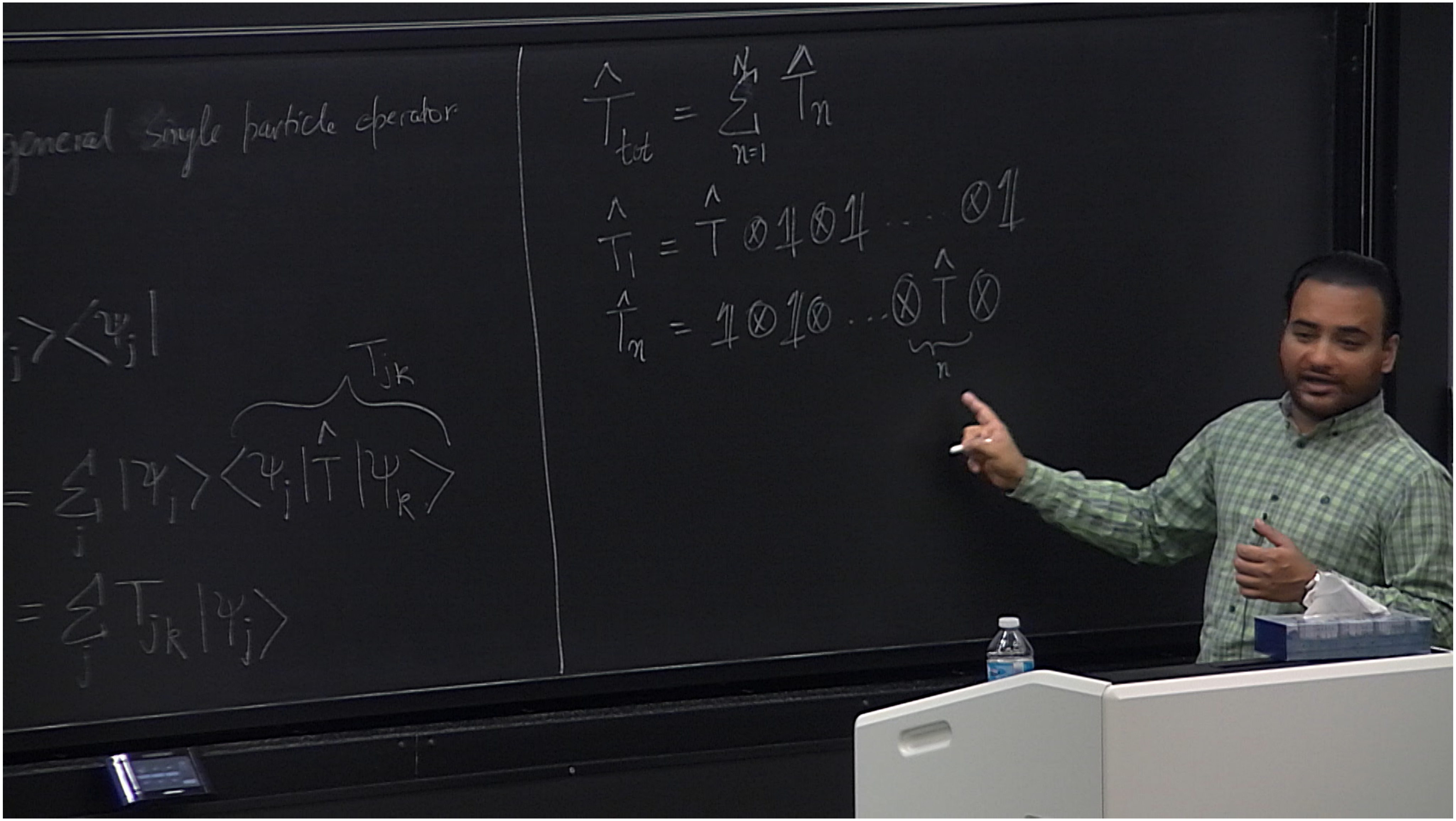
Single particle terms + Two particle terms + . . .

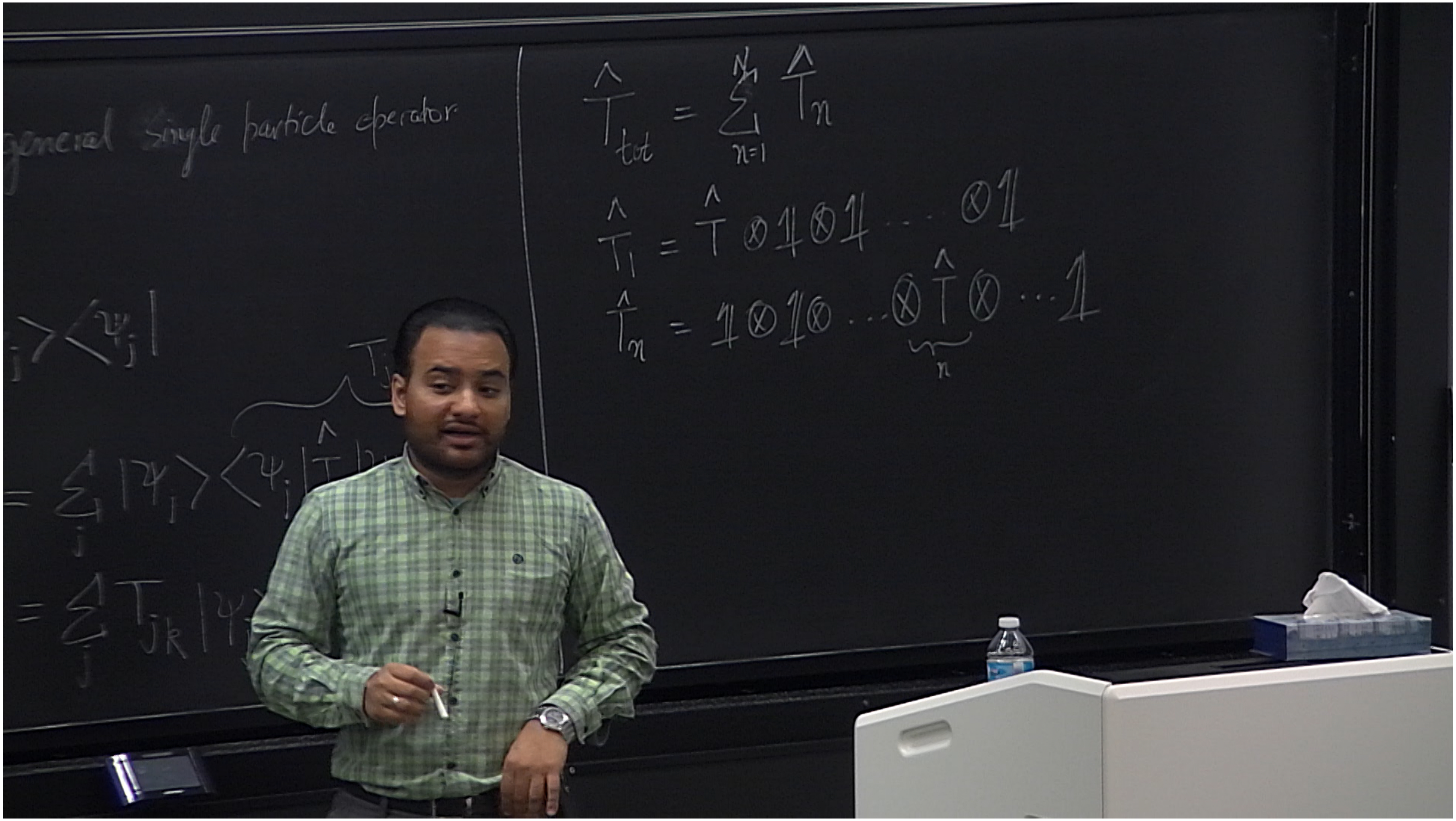

$c^\dagger_{k_1} c^\dagger_{k_2} \cdots c^\dagger_{k_N}|0\rangle = \hat{S}_- |\psi_{k_1}\rangle|\psi_{k_2}\rangle \cdots |\psi_{k_N}\rangle$
HAMILTONIAN in Second quantization
Single particle terms + Two particle terms + . . .
Kinetic Energy $-\frac{\hbar^2}{2m}\nabla_i^2$

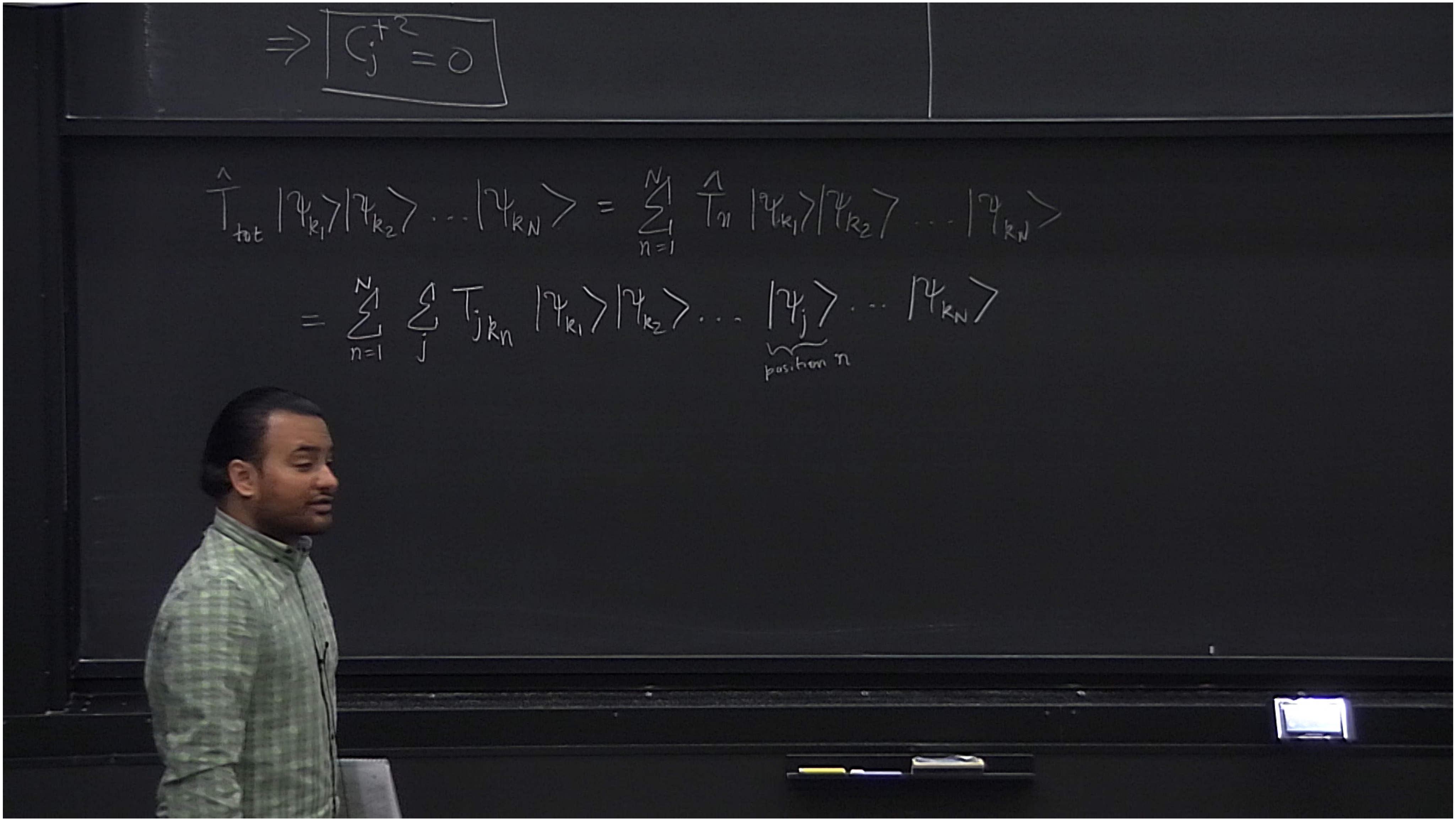

$c^\dagger_{k_1} c^\dagger_{k_2} \cdots c^\dagger_{k_N} |0\rangle = \hat{S}_- |\psi_{k_1}\rangle |\psi_{k_2}\rangle \cdots |\psi_{k_N}\rangle$
HAMILTONIAN in Second quantization
Single particle terms + Two particle terms + . . .
Kinetic Energy $-\frac{\hbar^2}{2m}\nabla_i^2$
potential $V(r_i)$

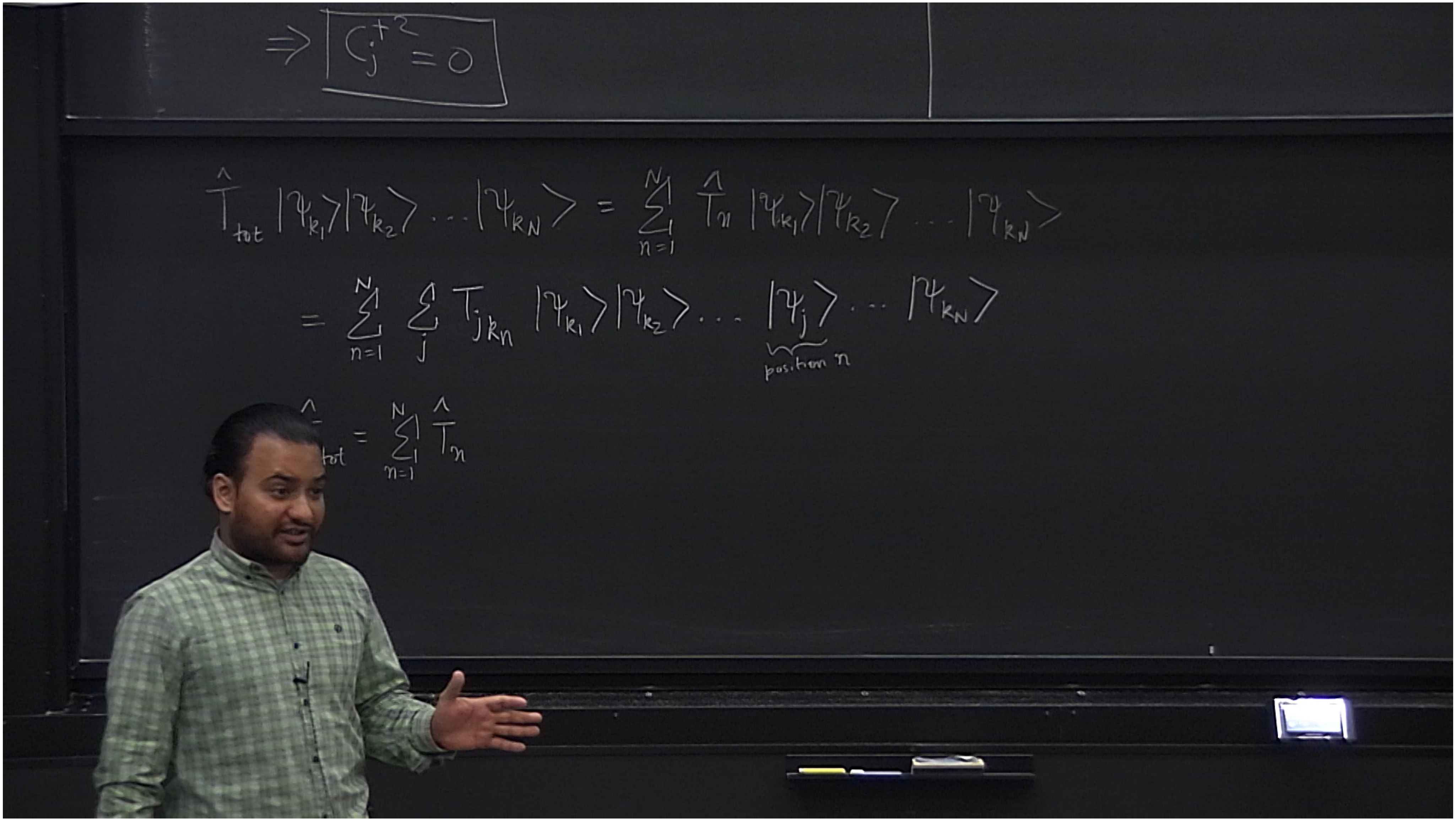

$\hat{T}$ is a general single particle operator
$\{|\psi_j\rangle\}$
$\mathbb{1} = \sum_j |\psi_j\rangle\langle\psi_j|$

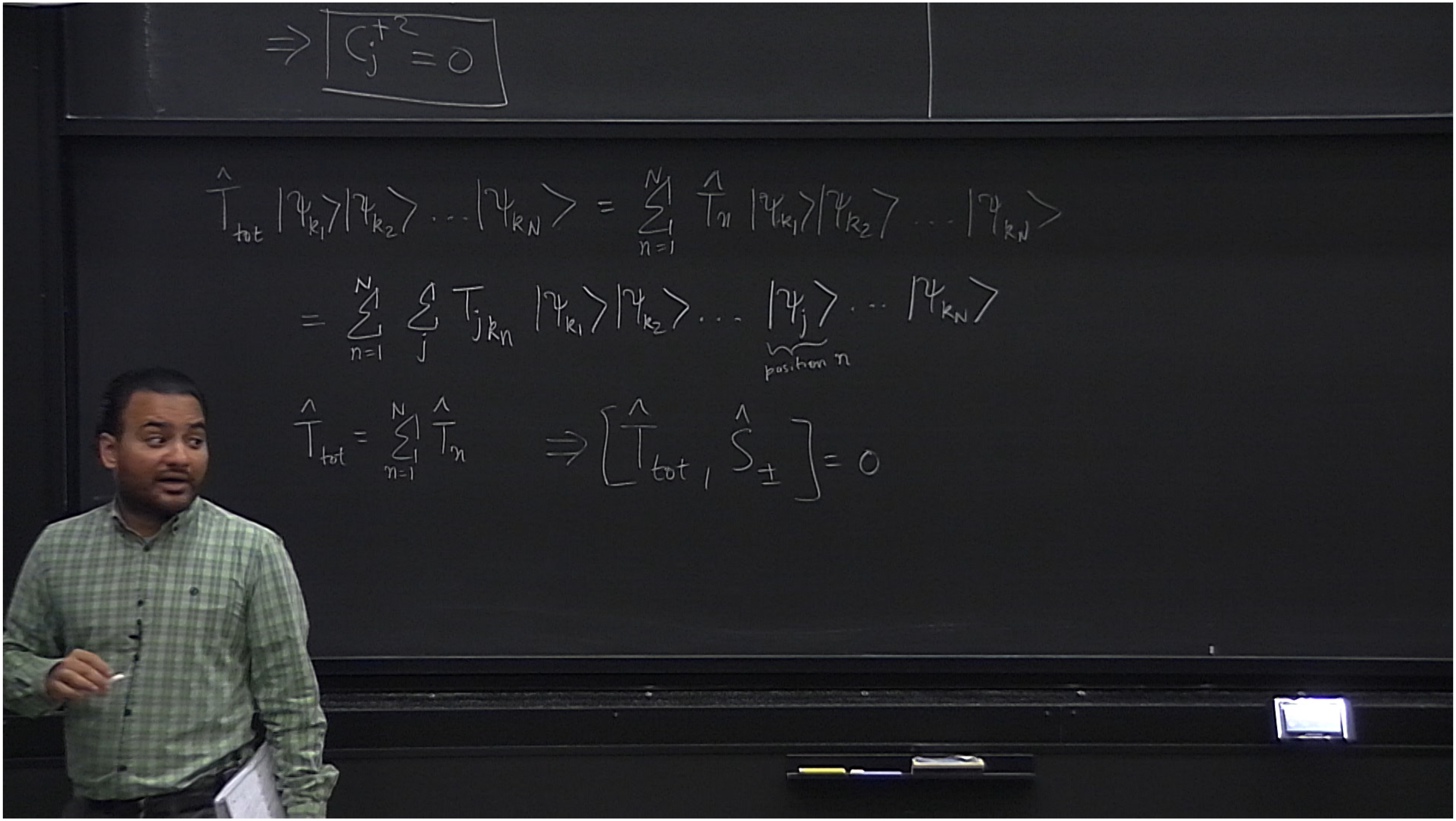

T̂ is a general single particle operator
{|ψ_j⟩}
𝟙 = Σ_j |ψ_j⟩⟨ψ_j|
T_jk
T̂|ψ_k⟩ = Σ_j |ψ_j⟩⟨ψ_j|T̂|ψ_k⟩
= Σ_j T_jk |ψ_j⟩
terms + . . .

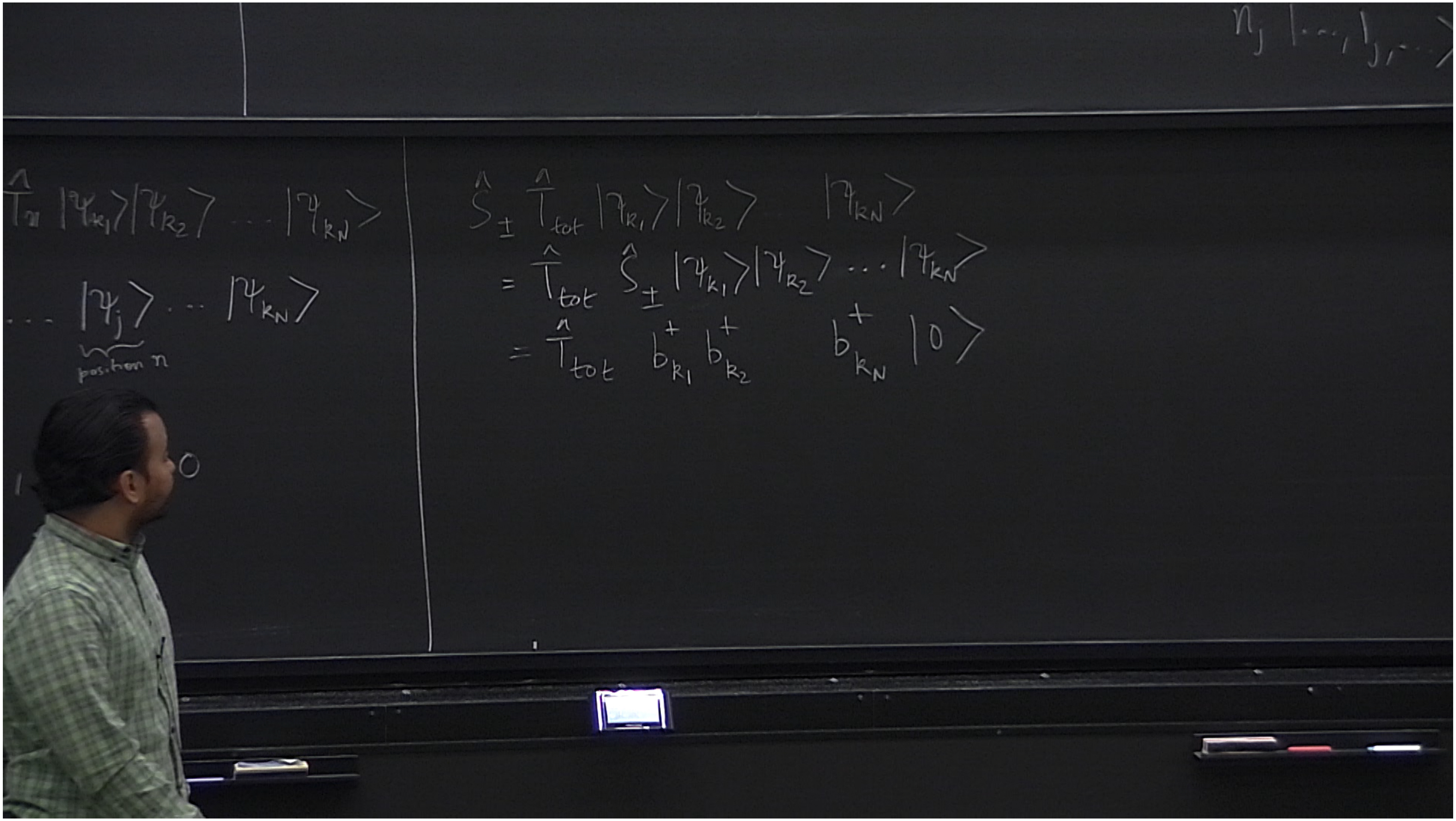

general single particle operator
$\hat{T}_{tot} = \sum_{n=1}^{N} \hat{T}_n$
$\hat{T}_1 = \hat{T} \otimes 1 \otimes 1 \ldots \otimes 1$
$\hat{T}_n = 1 \otimes 1 \otimes \ldots \otimes \hat{T} \otimes$
$= \sum_j |\psi_j\rangle \langle \psi_j | \hat{T} | \psi_k \rangle$
$= \sum_j T_{jk} |\psi_j\rangle$

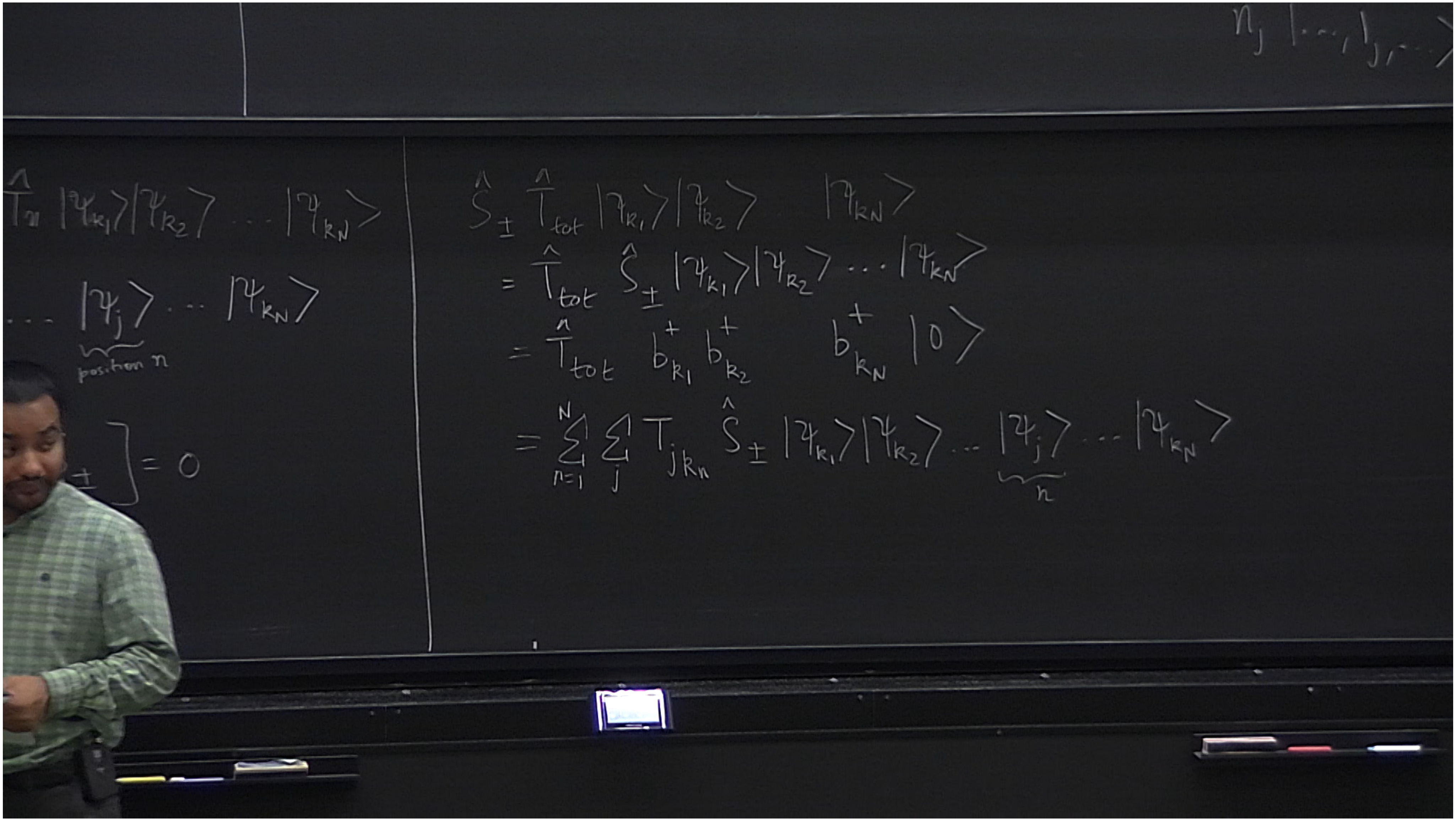

general single particle operator
$\hat{T}_{tot} = \sum_{n=1}^{N} \hat{T}_n$
$\hat{T}_1 = \hat{T} \otimes 1 \otimes 1 \ldots \otimes 1$
$\hat{T}_n = 1 \otimes 1 \otimes \ldots \otimes \hat{T} \otimes \ldots 1$

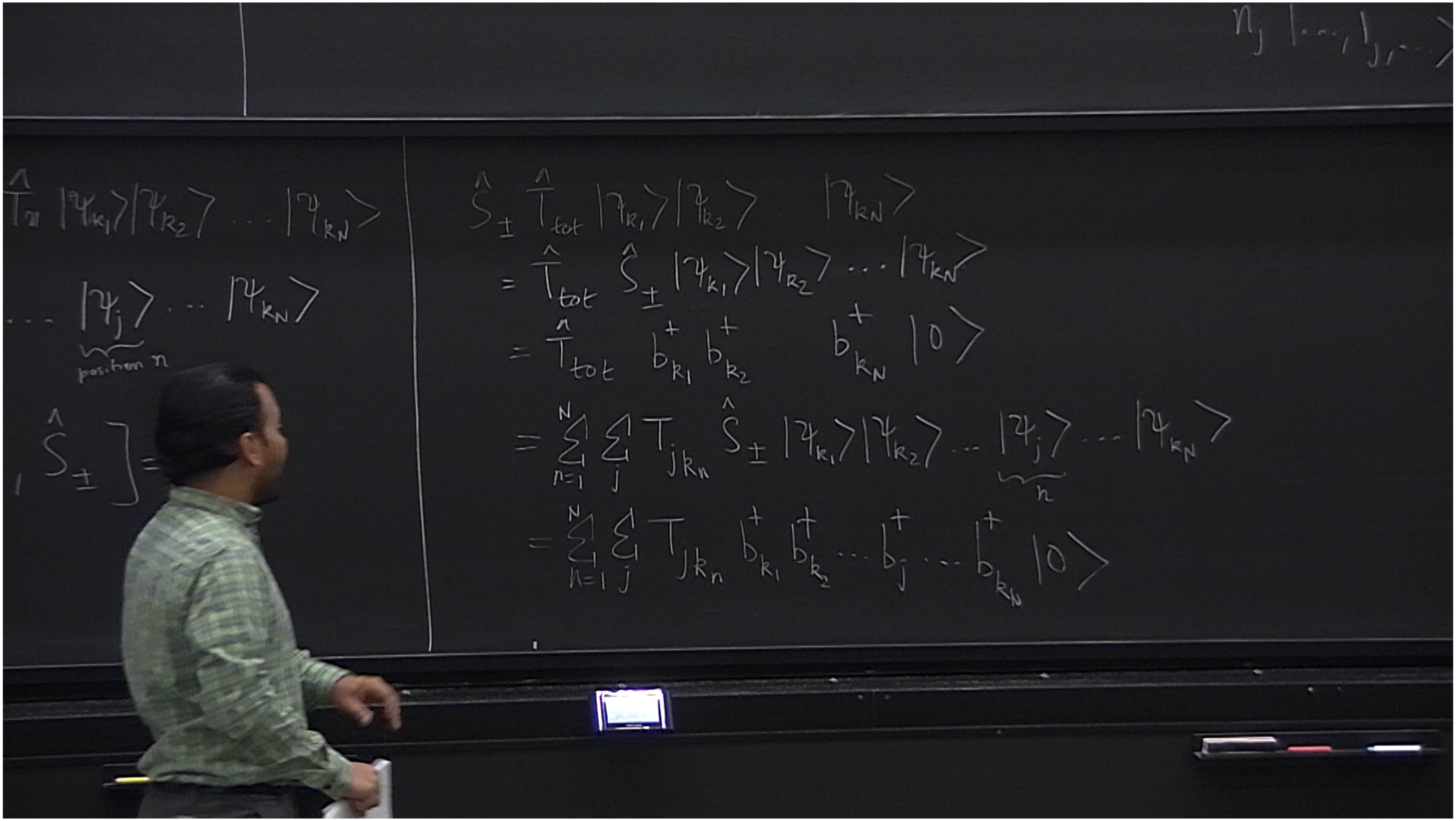

$\Rightarrow \boxed{c_j^{\dagger 2} = 0}$
$\hat{T}_{tot} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle = \sum_{n=1}^{N} \hat{T}_n |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \sum_{n=1}^{N} \sum_j T_{j k_n} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_j\rangle \ldots |\psi_{k_N}\rangle$
position n

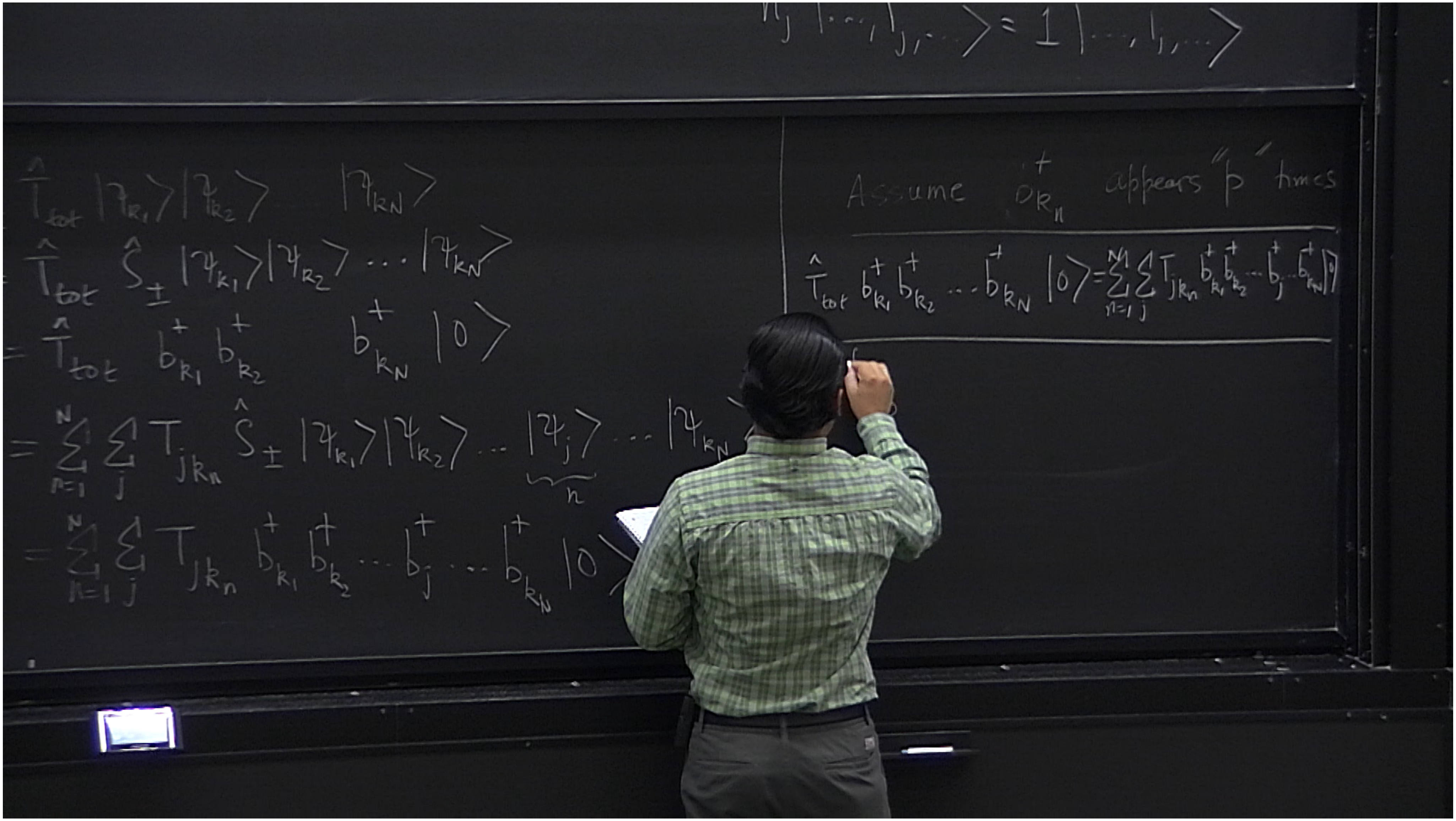

$\Rightarrow \; c_j^{\dagger 2} = 0$
$\hat{T}_{tot} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle = \sum_{n=1}^{N} \hat{T}_n |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \sum_{n=1}^{N} \sum_j T_{j k_n} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_j\rangle \ldots |\psi_{k_N}\rangle$
position n
$\hat{T}_{tot} = \sum_{n=1}^{N} \hat{T}_n$

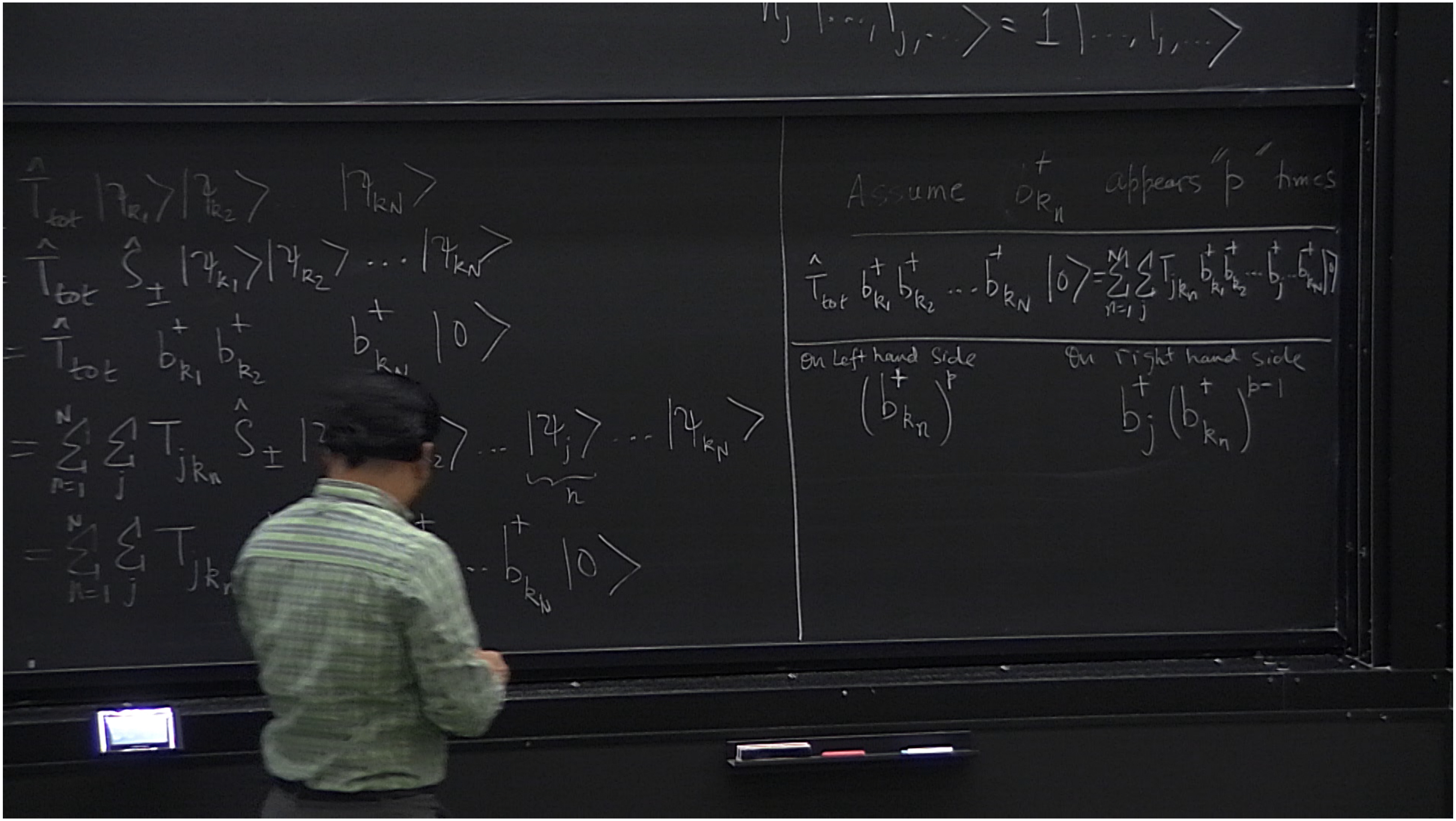

$\Rightarrow C_j^{\dagger 2} = 0$
$\hat{T}_{tot} |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle = \sum_{n=1}^{N} \hat{T}_n |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \sum_{n=1}^{N} \sum_j T_{j k_n} |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_j\rangle \ldots |\psi_{k_N}\rangle$
position n
$\hat{T}_{tot} = \sum_{n=1}^{N} \hat{T}_n$
$\Rightarrow [\hat{T}_{tot}, \hat{S}_\pm] = 0$

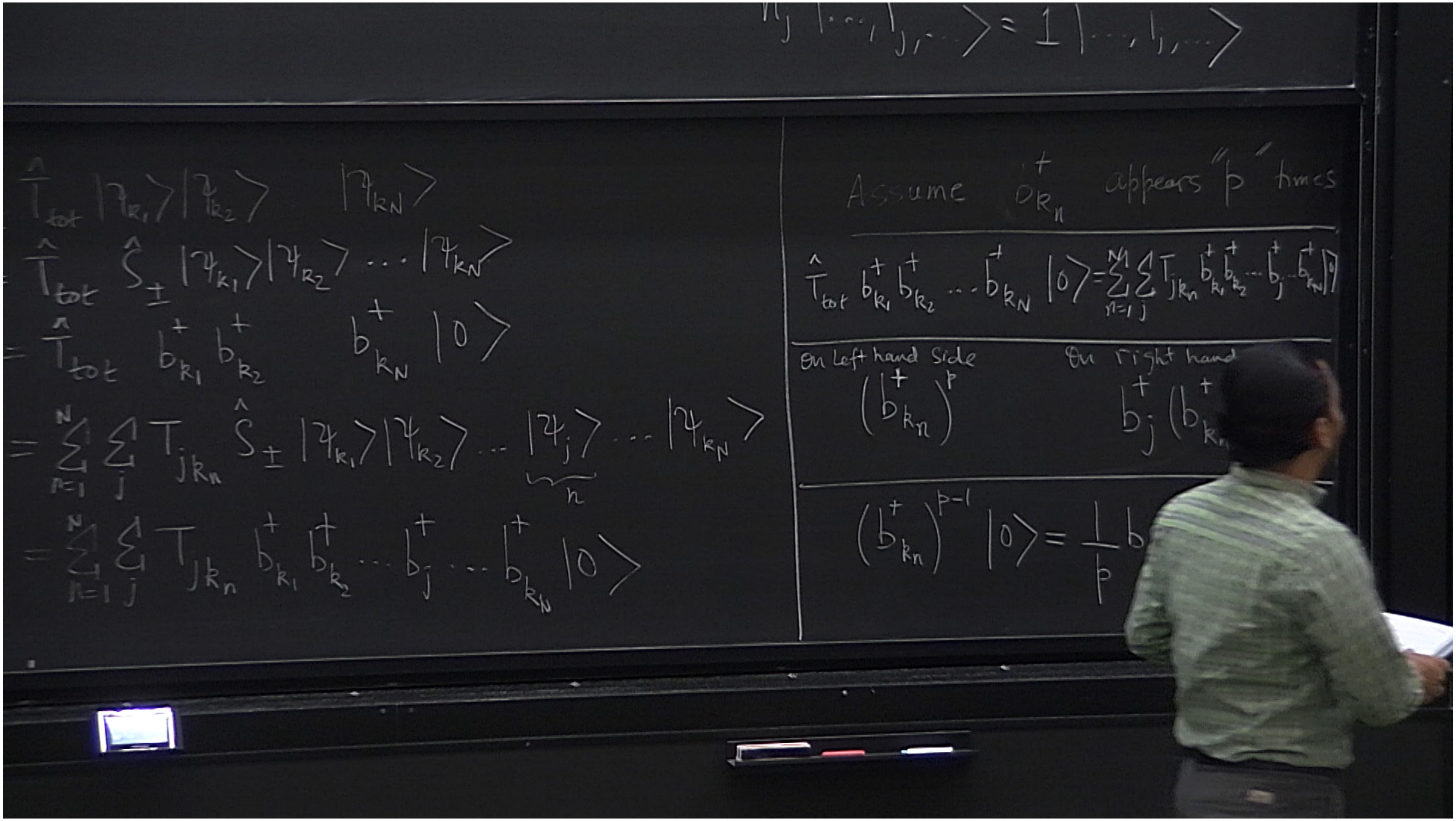

n_j |..., 1_j, ...>
T̂_n |ψ_k1>|ψ_k2> ... |ψ_kN>
... |ψ_j> ... |ψ_kN>
position n
Ŝ_± T̂_tot |ψ_k1>|ψ_k2> |ψ_kN>
= T̂_tot Ŝ_± |ψ_k1>|ψ_k2> ... |ψ_kN>
= T̂_tot b+_k1 b+_k2 b+_kN |0>


$n_j |\ldots, 1_j, \ldots\rangle$
$\hat{T}_{tot} |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$\ldots |\psi_j\rangle \ldots |\psi_{k_N}\rangle$
position n
$] = 0$
$\hat{S}_\pm \hat{T}_{tot} |\psi_{k_1}\rangle|\psi_{k_2}\rangle \quad |\psi_{k_N}\rangle$
$= \hat{T}_{tot} \hat{S}_\pm |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \hat{T}_{tot} b^+_{k_1} b^+_{k_2} \quad b^+_{k_N} |0\rangle$
$= \sum_{n=1}^{N} \sum_j T_{jk_n} \hat{S}_\pm |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_j\rangle \ldots |\psi_{k_N}\rangle$
n

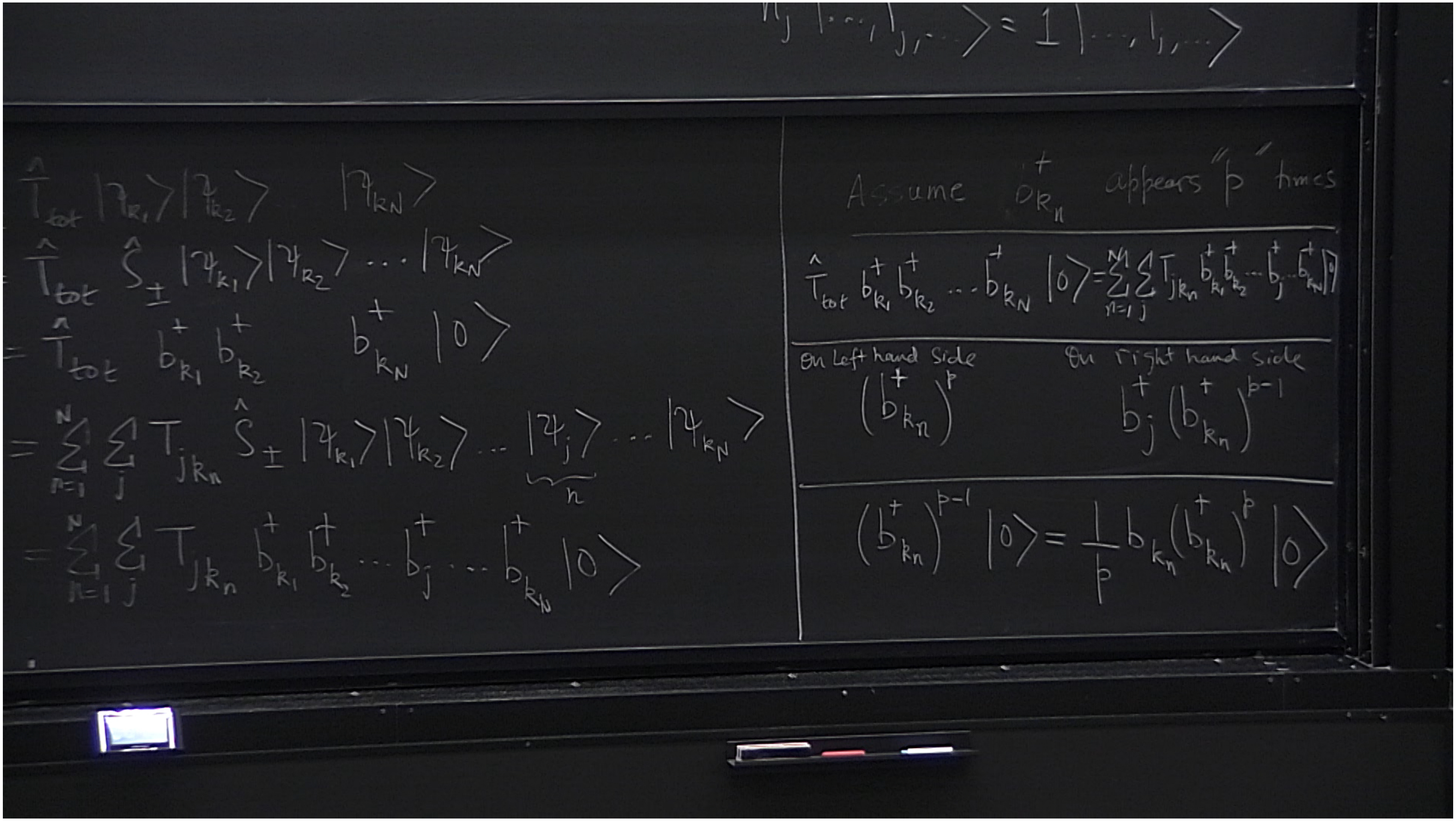

$n_j |\ldots, 1_j, \ldots\rangle$
$\hat{T}_n |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$\ldots |\psi_j\rangle \ldots |\psi_{k_N}\rangle$
position n
$\hat{S}_\pm \hat{T}_{tot} |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \hat{T}_{tot} \hat{S}_\pm |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \hat{T}_{tot} b^\dagger_{k_1} b^\dagger_{k_2} \ldots b^\dagger_{k_N} |0\rangle$
$= \sum_{n=1}^{N} \sum_j T_{jk_n} \hat{S}_\pm |\psi_{k_1}\rangle|\psi_{k_2}\rangle \ldots |\psi_j\rangle \ldots |\psi_{k_N}\rangle$
$= \sum_{n=1}^{N} \sum_j T_{jk_n} b^\dagger_{k_1} b^\dagger_{k_2} \ldots b^\dagger_j \ldots b^\dagger_{k_N} |0\rangle$


Assume b†_{k_n} appears "p" times

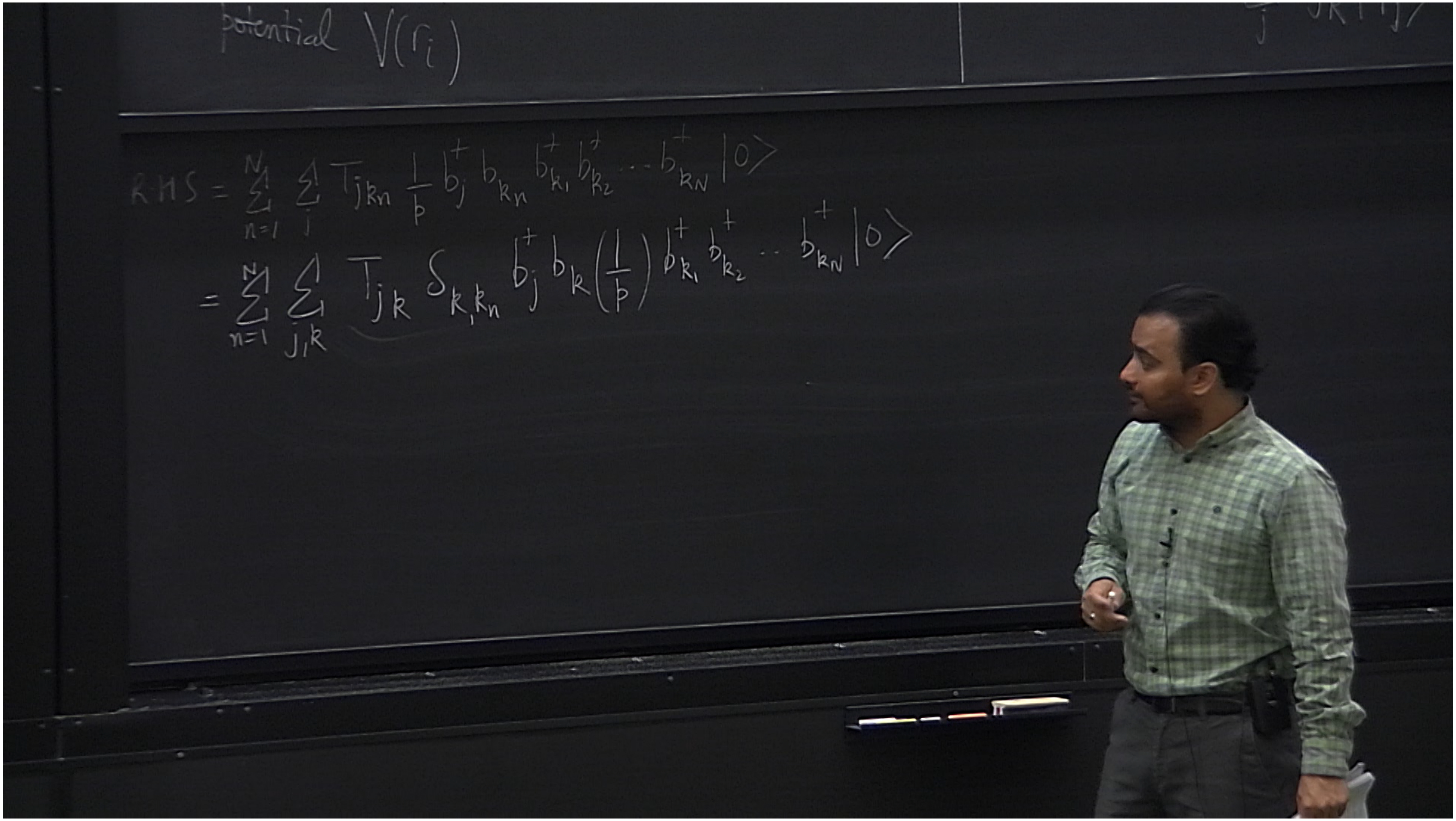

Assume
appears "p" times
On Left hand Side
On right hand side

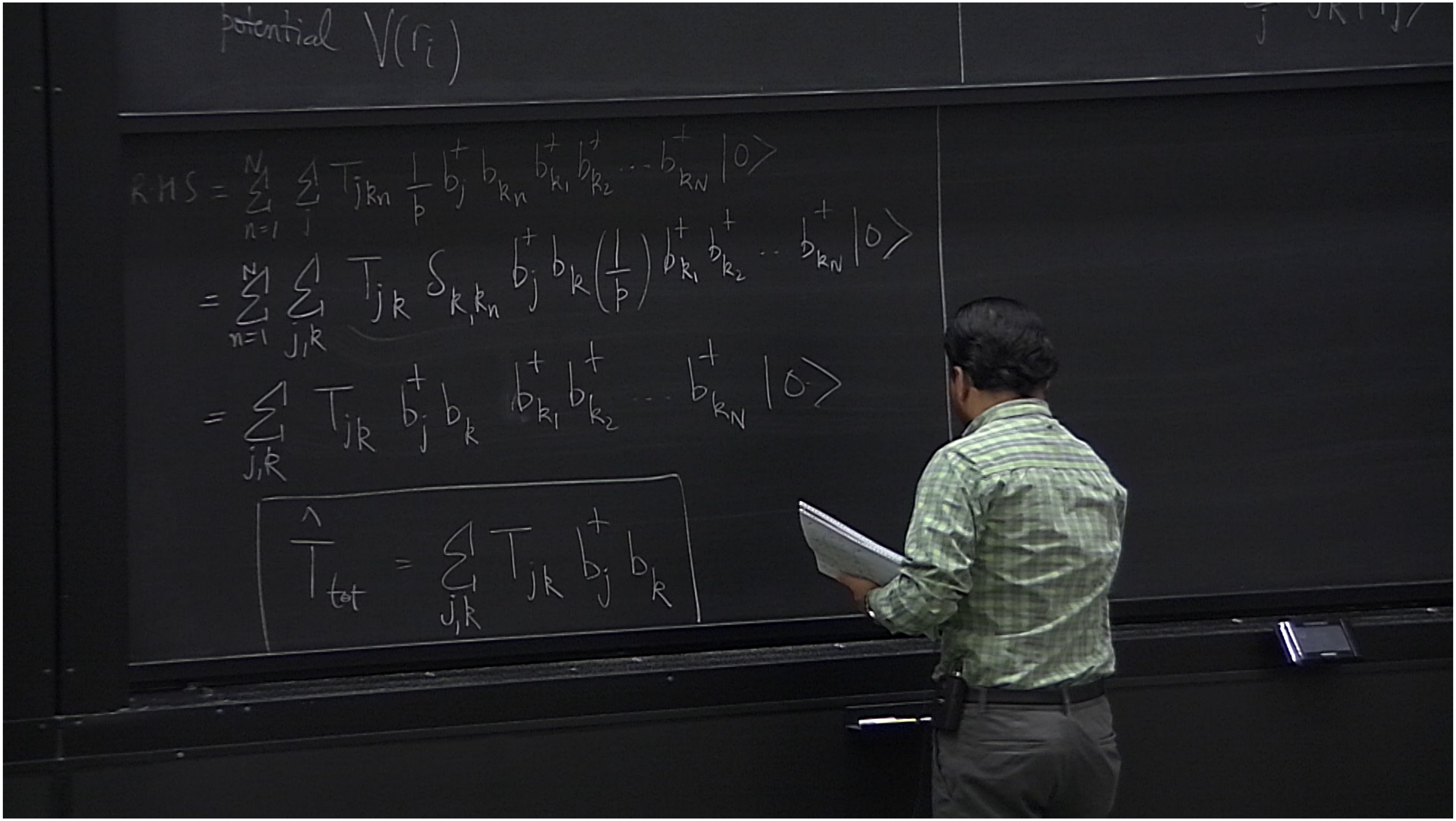


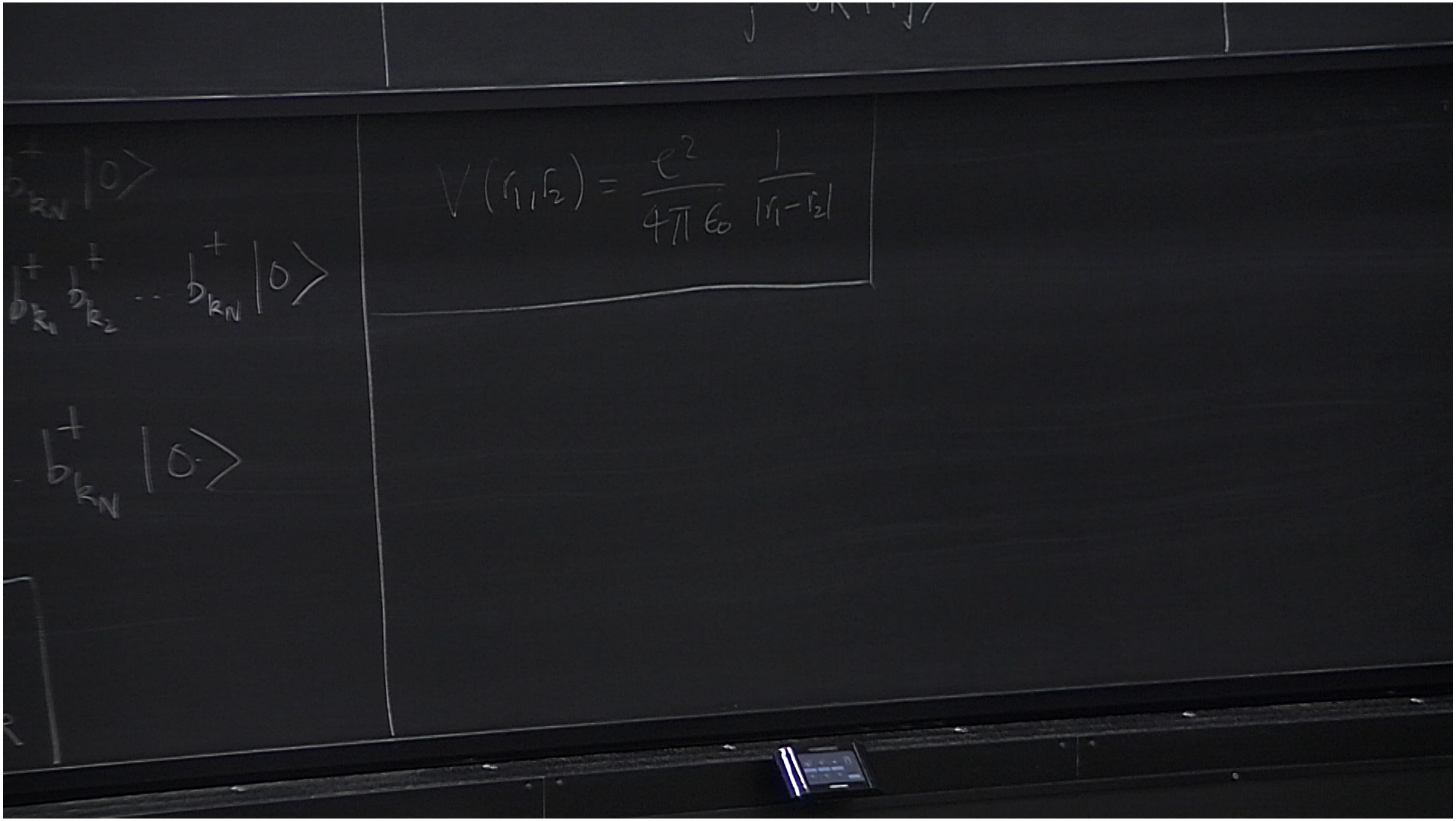
$n_j |\ldots, 1_j, \ldots\rangle = 1 |\ldots, 1_j, \ldots\rangle$
$\hat{T}_{tot} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \quad |\psi_{k_N}\rangle$
$\hat{T}_{tot} \hat{S}_\pm |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \hat{T}_{tot} \, b^\dagger_{k_1} b^\dagger_{k_2} \quad b^\dagger_{k_N} |0\rangle$
$= \sum_{n=1}^{N} \sum_j T_{jk_n} \hat{S}_\pm |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots \underbrace{|\psi_j\rangle}_{n} \ldots |\psi_{k_N}\rangle$
$= \sum_{n=1}^{N} \sum_j T_{jk_n} b^\dagger_{k_1} b^\dagger_{k_2} \ldots b^\dagger_j \ldots b^\dagger_{k_N} |0\rangle$
Assume $b^\dagger_{k_n}$ appears "p" times
$\hat{T}_{tot} b^\dagger_{k_1} b^\dagger_{k_2} \ldots b^\dagger_{k_N} |0\rangle = \sum_{n=1}^{N} \sum_j T_{jk_n} b^\dagger_{k_1} b^\dagger_{k_2} \ldots b^\dagger_j \ldots b^\dagger_{k_N} |0\rangle$
On Left hand Side
$(b^\dagger_{k_n})^p$
On right hand
$(b^\dagger_{k_n})^{p-1} |0\rangle = \frac{1}{p}$

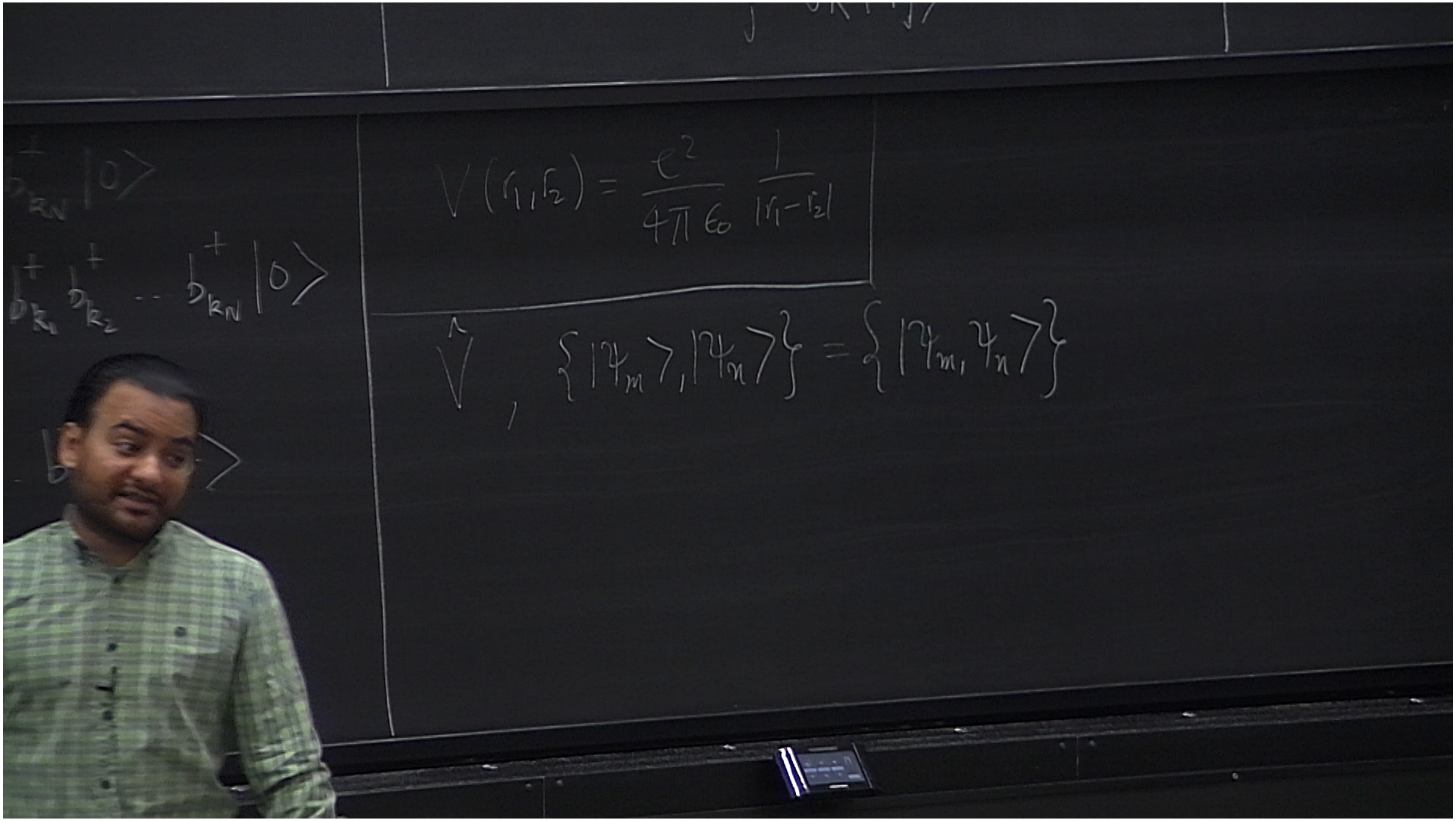


$n_j |\ldots, 1_j, \ldots\rangle = 1 |\ldots, 1_j, \ldots\rangle$
$\hat{T}_{tot} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \hat{T}_{tot} \hat{S}_{\pm} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_{k_N}\rangle$
$= \hat{T}_{tot} b^{+}_{k_1} b^{+}_{k_2} \ldots b^{+}_{k_N} |0\rangle$
$= \sum_{n=1}^{N} \sum_{j} T_{jk_n} \hat{S}_{\pm} |\psi_{k_1}\rangle |\psi_{k_2}\rangle \ldots |\psi_j\rangle \ldots |\psi_{k_N}\rangle$
n
$= \sum_{n=1}^{N} \sum_{j} T_{jk_n} b^{+}_{k_1} b^{+}_{k_2} \ldots b^{+}_{j} \ldots b^{+}_{k_N} |0\rangle$
Assume $b^{+}_{k_n}$ appears "p" times
$\hat{T}_{tot} b^{+}_{k_1} b^{+}_{k_2} \ldots b^{+}_{k_N} |0\rangle = \sum_{n=1}^{N} \sum_{j} T_{jk_n} b^{+}_{k_1} b^{+}_{k_2} \ldots b^{+}_{j} \ldots b^{+}_{k_N} |0\rangle$
On Left hand Side
$(b^{+}_{k_n})^{p}$
On right hand side
$b^{+}_{j} (b^{+}_{k_n})^{p-1}$
$(b^{+}_{k_n})^{p-1} |0\rangle = \frac{1}{p} b_{k_n} (b^{+}_{k_n})^{p} |0\rangle$

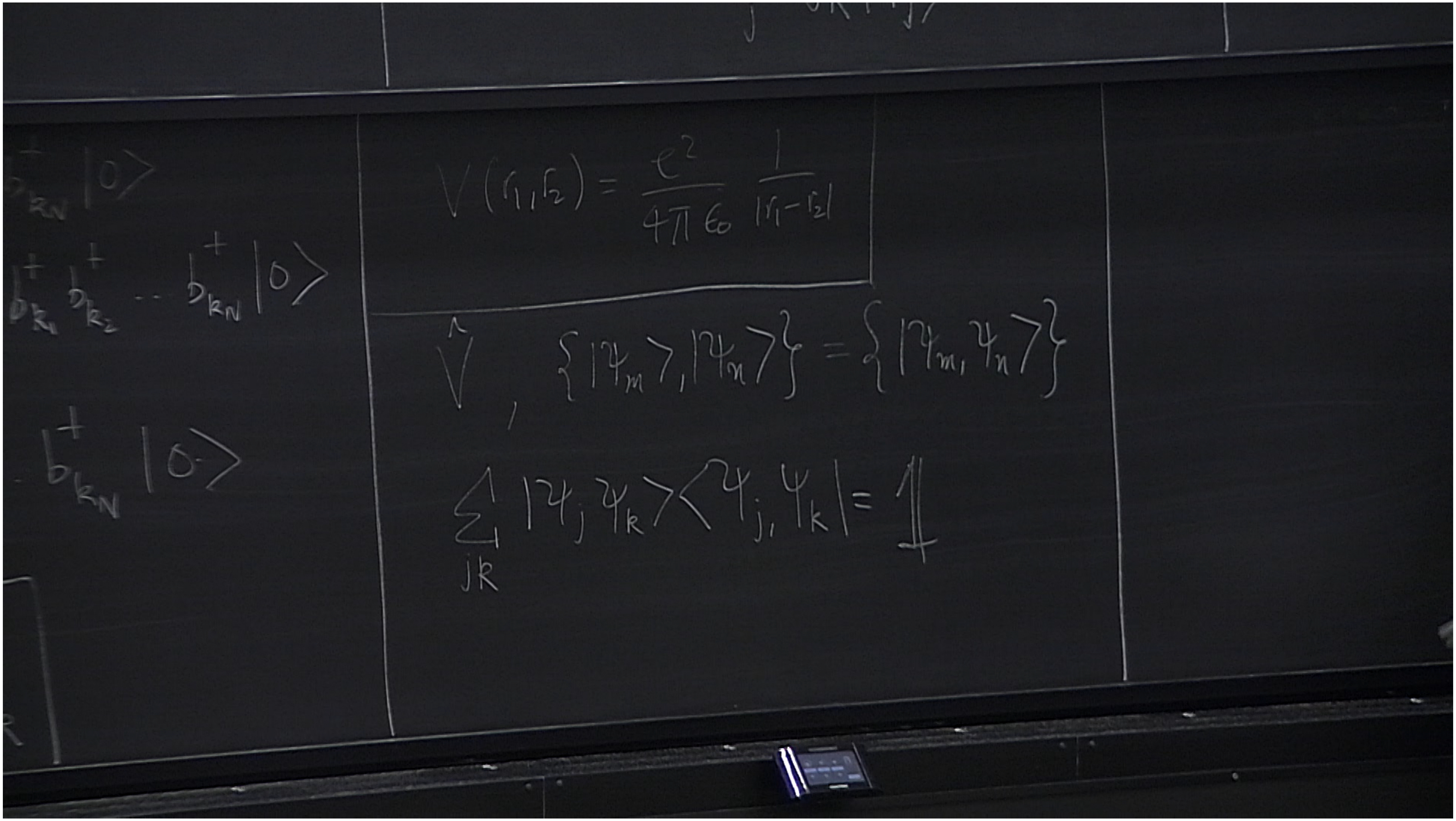

potential $V(r_i)$
$R.H.S = \sum_{n=1}^{N} \sum_{j} T_{jk_n} \frac{1}{p} b_j^{+} b_{k_n} b_{k_1}^{+} b_{k_2}^{+} \dots b_{k_N}^{+} |0\rangle$
$= \sum_{n=1}^{N} \sum_{j,k} T_{jk} \delta_{k,k_n} b_j^{+} b_k \left(\frac{1}{p}\right) b_{k_1}^{+} b_{k_2}^{+} \dots b_{k_N}^{+} |0\rangle$

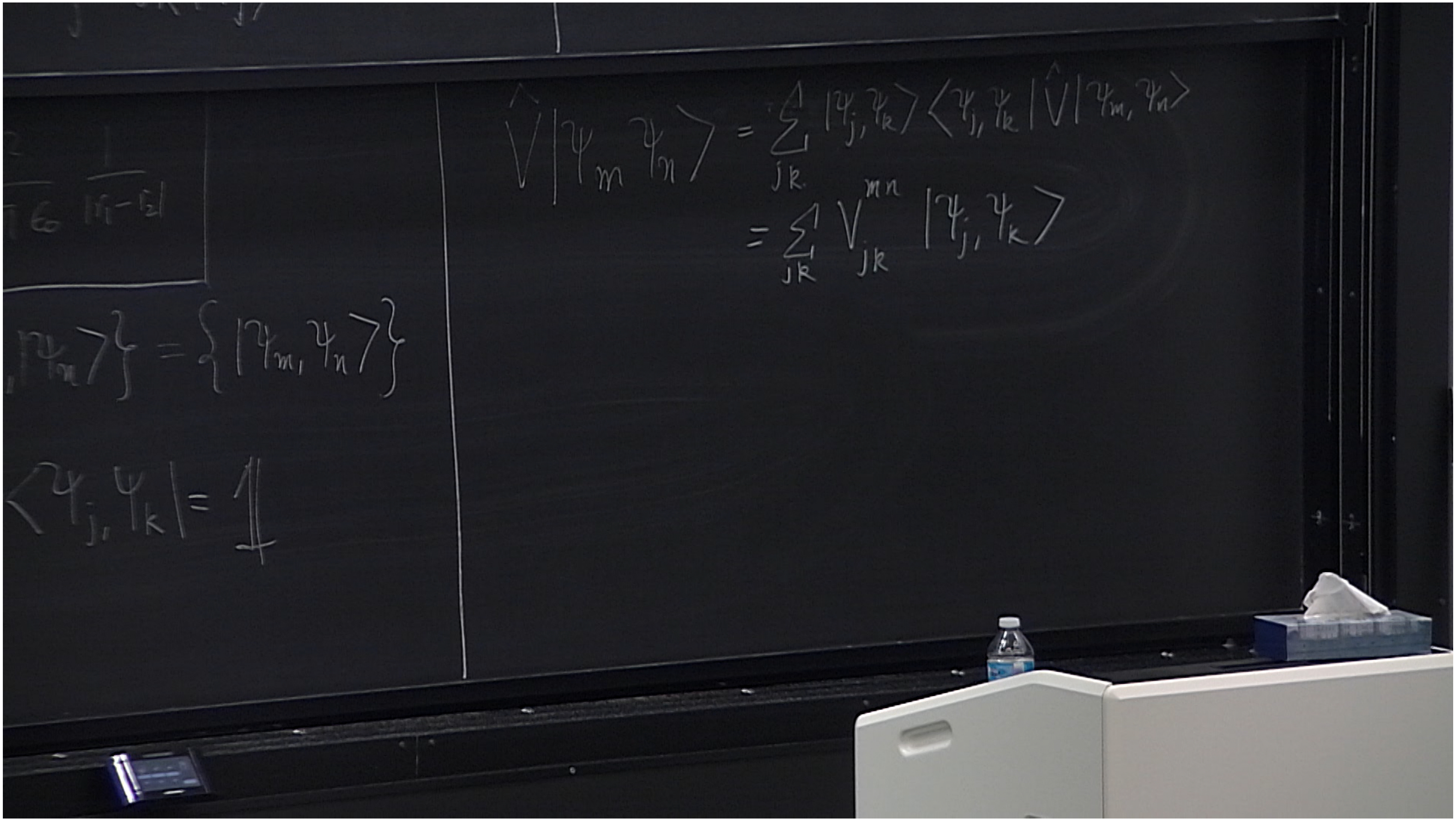

potential V(r_i)
R.H.S = Σ_{n=1}^{N} Σ_j T_{jk_n} (1/p) b_j^+ b_{k_n} b_{k_1}^+ b_{k_2}^+ ... b_{k_N}^+ |0>
= Σ_{n=1}^{N} Σ_{j,k} T_{jk} δ_{k,k_n} b_j^+ b_k (1/p) b_{k_1}^+ b_{k_2}^+ ... b_{k_N}^+ |0>
= Σ_{j,k} T_{jk} b_j^+ b_k b_{k_1}^+ b_{k_2}^+ ... b_{k_N}^+ |0>
T̂_tot = Σ_{j,k} T_{jk} b_j^+ b_k

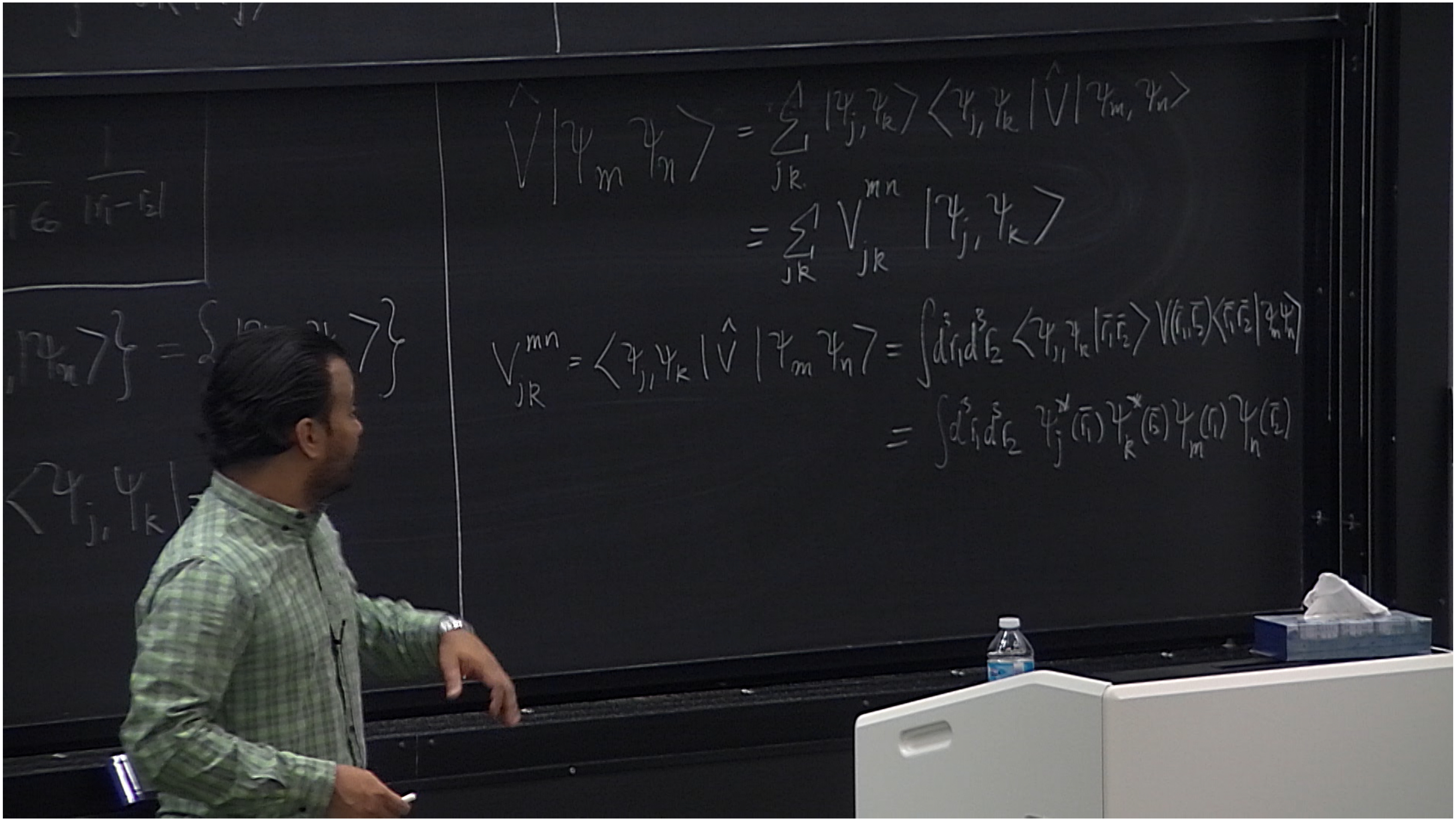

$V(r_1, r_2) = \frac{e^2}{4\pi\epsilon_0} \frac{1}{|r_1 - r_2|}$
$b^\dagger_{k_N} |0\rangle$
$b^\dagger_{k_1} b^\dagger_{k_2} \dots b^\dagger_{k_N} |0\rangle$
$b^\dagger_{k_N} |0\rangle$



$b^\dagger_{k_N}|0\rangle$
$b^\dagger_{k_1} b^\dagger_{k_2} \dots b^\dagger_{k_N}|0\rangle$
$V(r_1, r_2) = \frac{e^2}{4\pi\epsilon_0} \frac{1}{|r_1 - r_2|}$
$\hat{V}, \{|\psi_m\rangle, |\psi_n\rangle\} = \{|\psi_m, \psi_n\rangle\}$


$b^{\dagger}_{k_N} |0\rangle$
$b^{\dagger}_{k_1} b^{\dagger}_{k_2} \dots b^{\dagger}_{k_N} |0\rangle$
$\dots b^{\dagger}_{k_N} |0\rangle$
$V(r_1, r_2) = \frac{e^2}{4\pi \epsilon_0} \frac{1}{|r_1 - r_2|}$
$\hat{V}, \{|\psi_m\rangle, |\psi_n\rangle\} = \{|\psi_m, \psi_n\rangle\}$
$\sum_{jk} |\psi_j \psi_k\rangle\langle\psi_j, \psi_k| = \mathbb{1}$


$\hat{V}|\psi_m \psi_n\rangle = \sum_{jk} |\psi_j, \psi_k\rangle\langle\psi_j, \psi_k|\hat{V}|\psi_m, \psi_n\rangle$
$= \sum_{jk} V_{jk}^{mn} |\psi_j, \psi_k\rangle$
$= \{|\psi_m, \psi_n\rangle\}$
$\langle\psi_j, \psi_k| = \mathbb{1}$


$\hat{V}|\psi_m \psi_n\rangle = \sum_{jk} |\psi_j, \psi_k\rangle \langle \psi_j, \psi_k | \hat{V} | \psi_m, \psi_n \rangle$
$= \sum_{jk} V_{jk}^{mn} |\psi_j, \psi_k\rangle$
$V_{jk}^{mn} = \langle \psi_j, \psi_k | \hat{V} | \psi_m \psi_n \rangle = \int d^3r_1 d^3r_2 \langle \psi_j, \psi_k | \bar{r}_1 \bar{r}_2 \rangle V(\bar{r}_1, \bar{r}_2) \langle \bar{r}_1 \bar{r}_2 | \psi_m \psi_n \rangle$
$= \int d^3r_1 d^3r_2 \, \psi_j^*(\bar{r}_1) \psi_k^*(\bar{r}_2) \psi_m(\bar{r}_1) \psi_n(\bar{r}_2)$