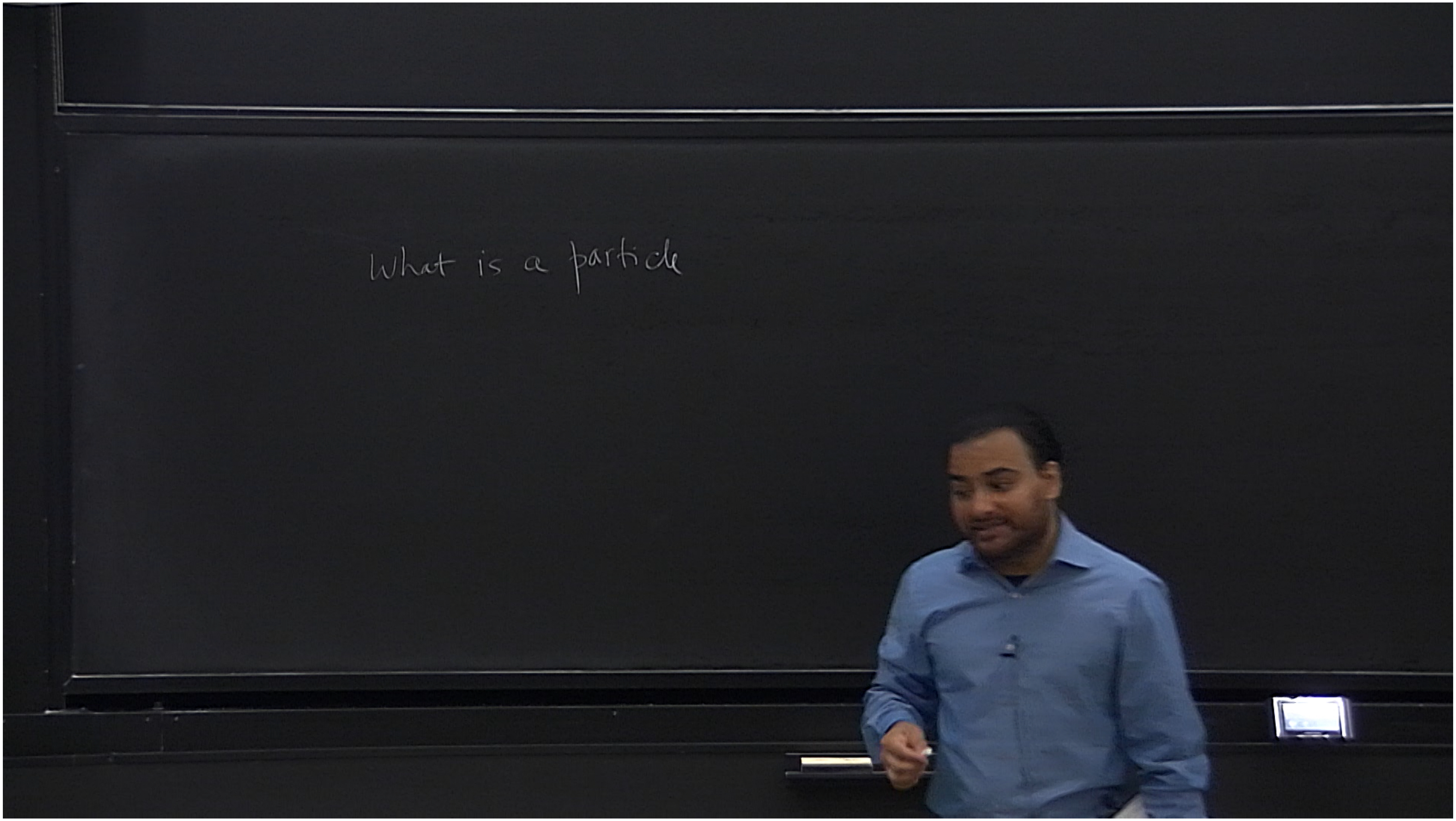
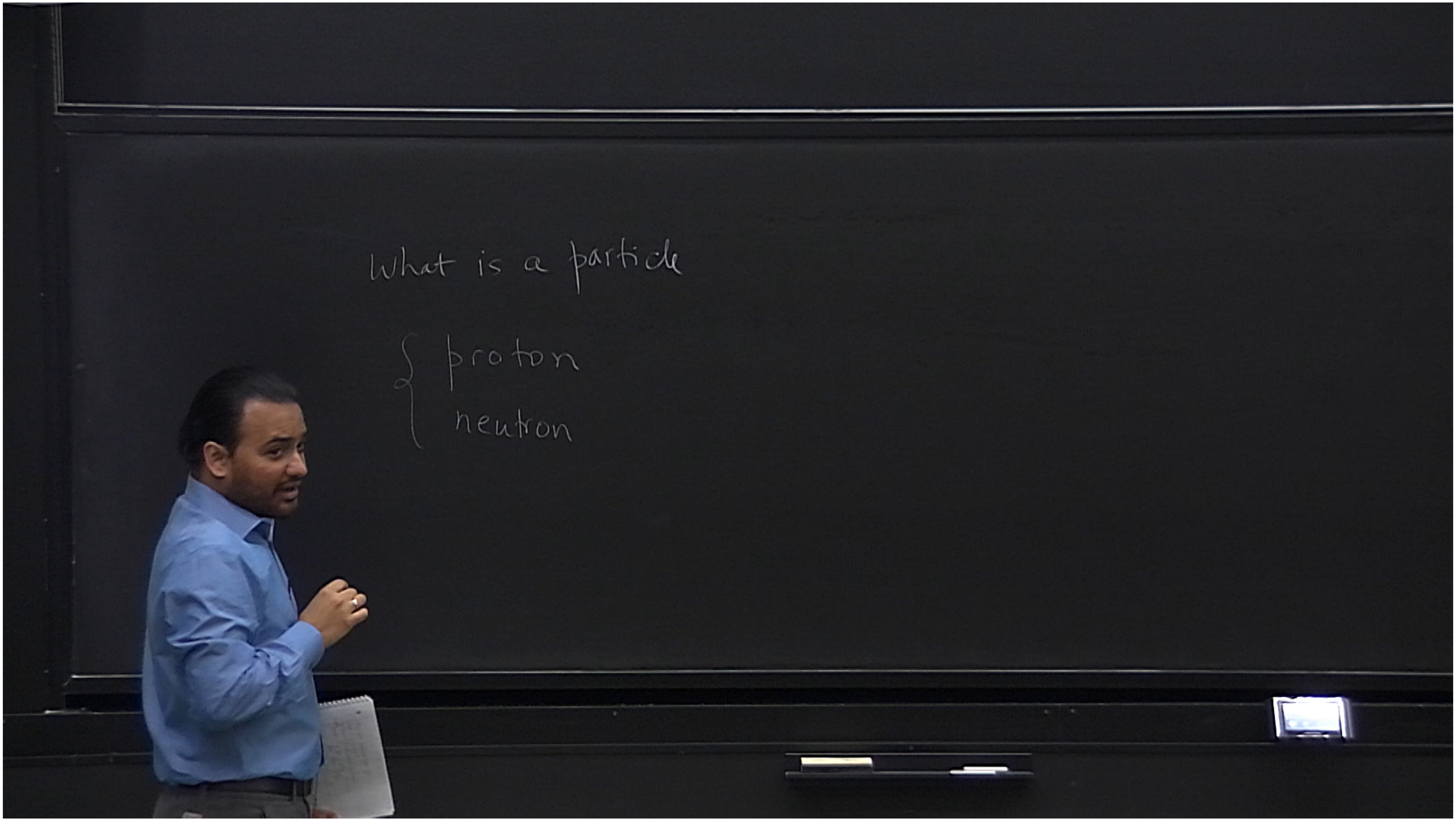
Title: PSI 17/18 - Condensed Matter - Lecture 1

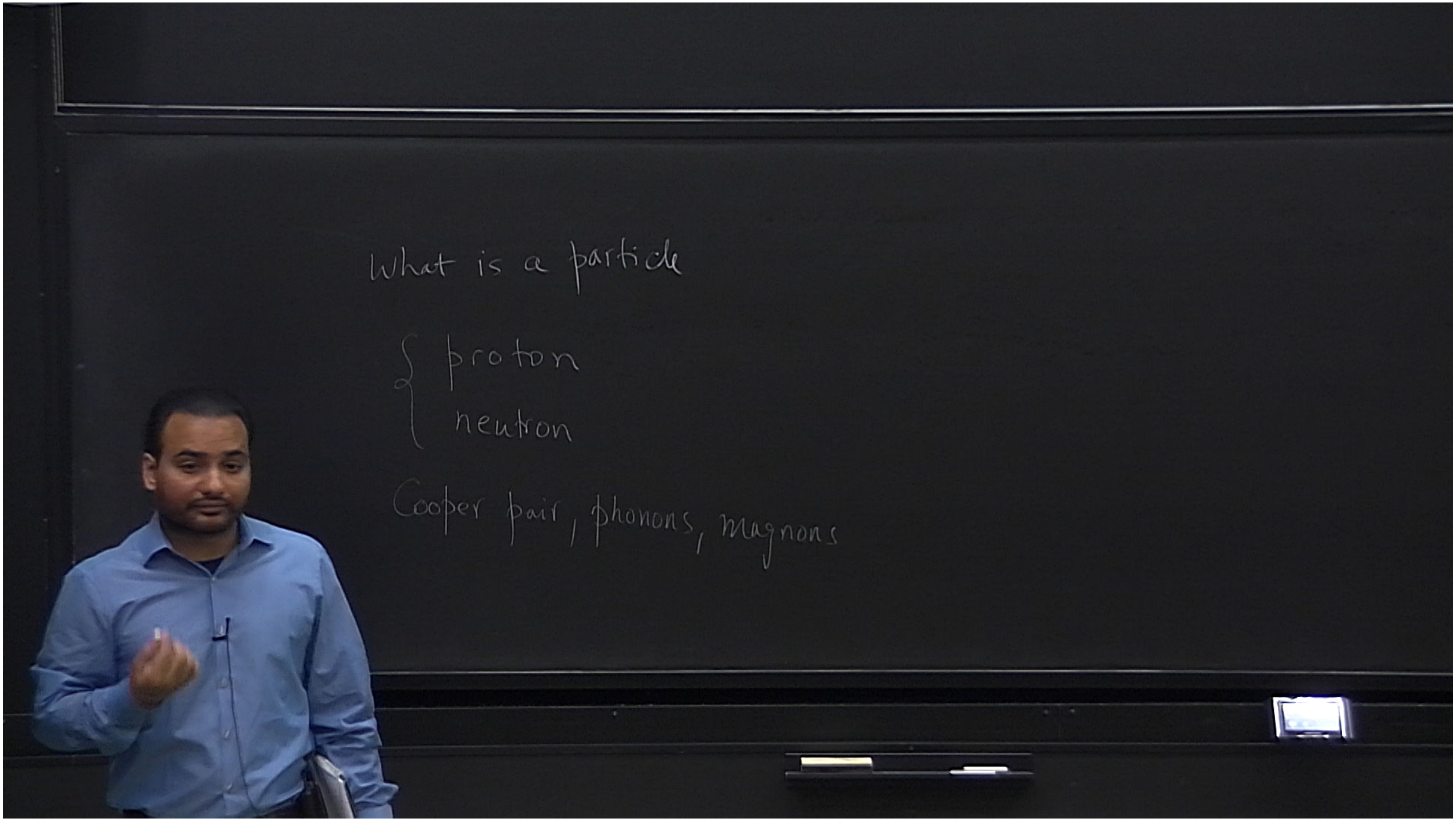
Date: Nov 06, 2017 10:45 AM

URL: http://pirsa.org/17110026

Abstract:

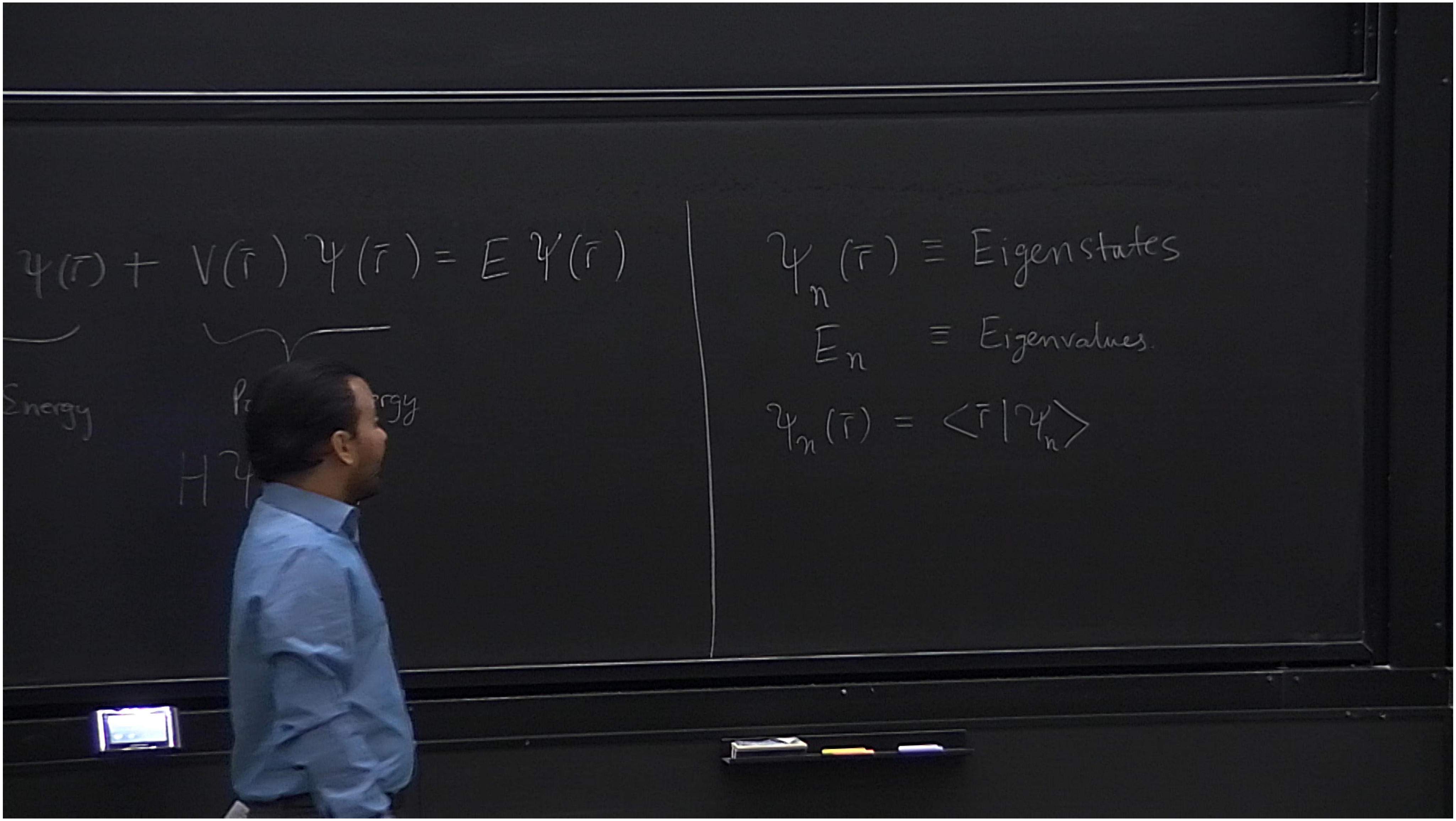

What is a particle


What is a particle
proton
neutron


What is a particle
proton
neutron
Cooper pair, phonons, magnons

$$\underbrace{-\frac{\hbar^2}{2m}\nabla^2\psi(\bar{r})}_{\text{Kinetic Energy}} + \underbrace{V(\bar{r})\,\psi(\bar{r})}_{\text{Potential Energy}} = E\,\psi(\bar{r})$$

$$H\psi = E\psi$$


$\psi(\bar r) + V(\bar r)\,\psi(\bar r) = E\,\psi(\bar r)$
Energy
$\psi_n(\bar r) \equiv$ Eigenstates
$E_n \equiv$ Eigenvalues
$\psi_n(\bar r) = \langle \bar r | \psi_n \rangle$

$$\underbrace{\psi(\bar{r})}_{\text{Energy}} + \underbrace{V(\bar{r})\,\psi(\bar{r})}_{\text{Potential Energy}} = E\,\psi(\bar{r})$$

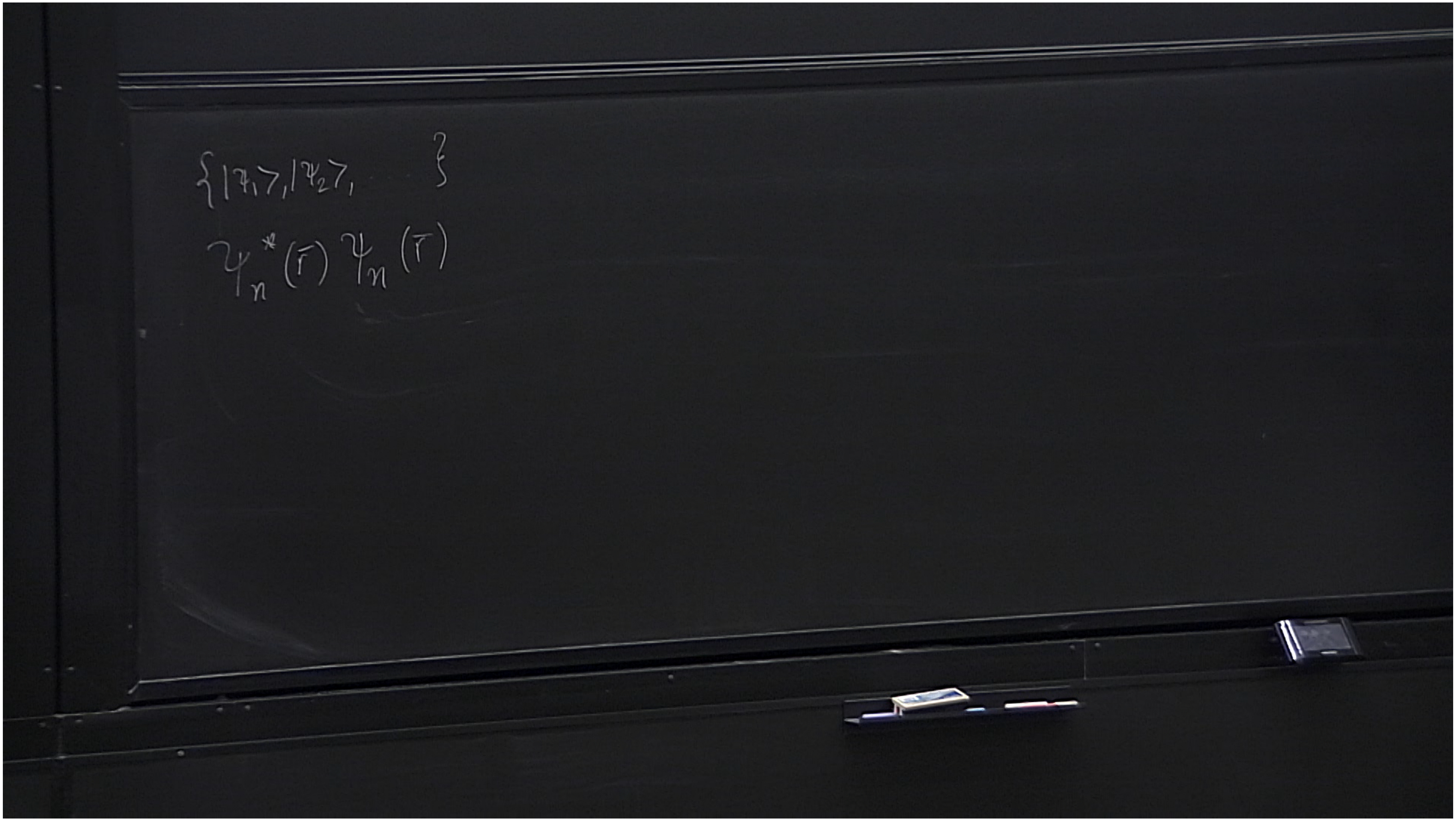
$$H\psi = E\psi$$

$\psi_n(\bar{r}) \equiv$ Eigenstates

$E_n \equiv$ Eigenvalues.

$$\psi_n(\bar{r}) = \langle \bar{r} | \psi_n \rangle$$

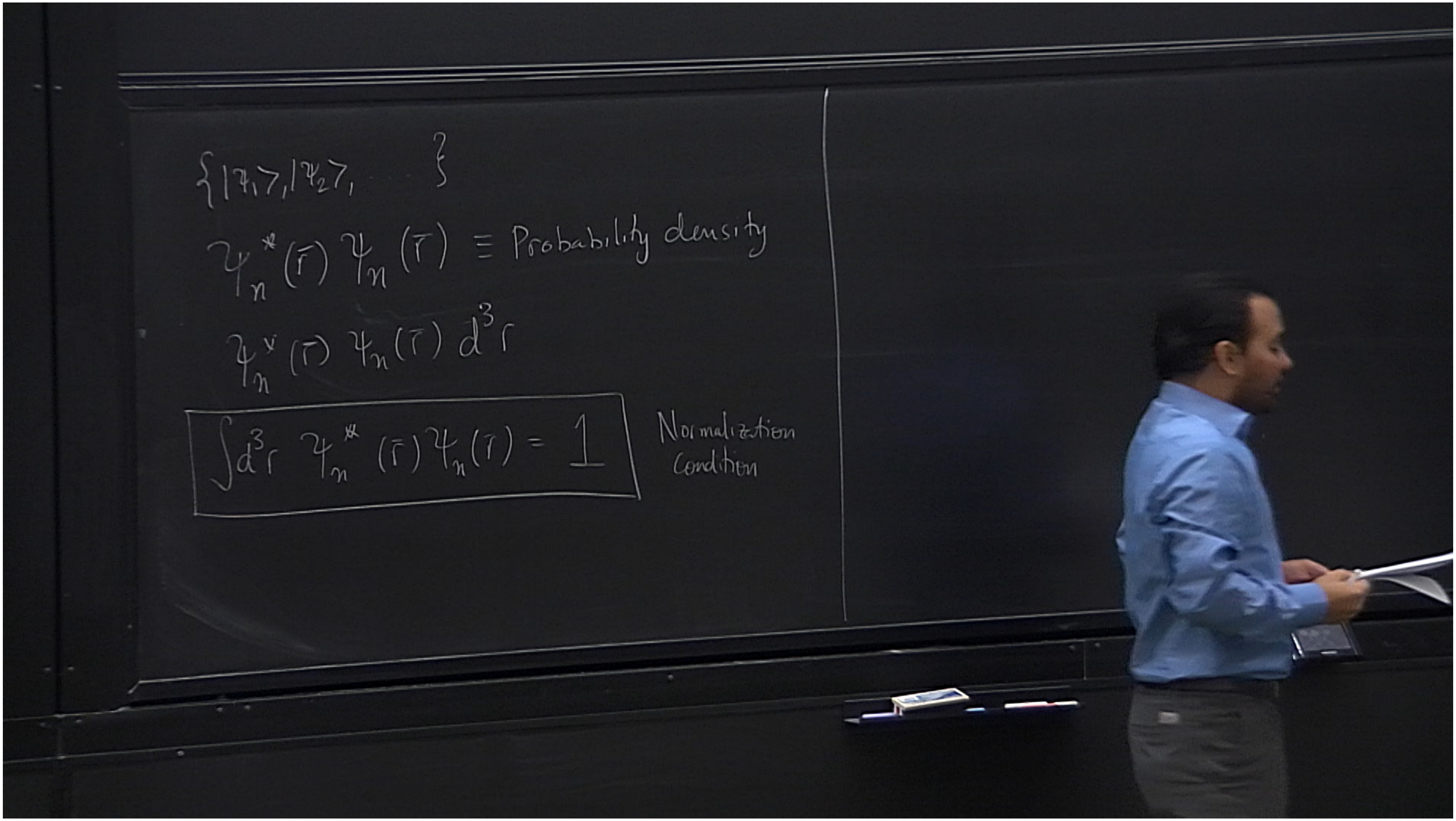
$$\int d^3r\, \psi_m^*(\bar{r})\, \psi_n(\bar{r}) = \langle \psi_m | \psi_n \rangle$$

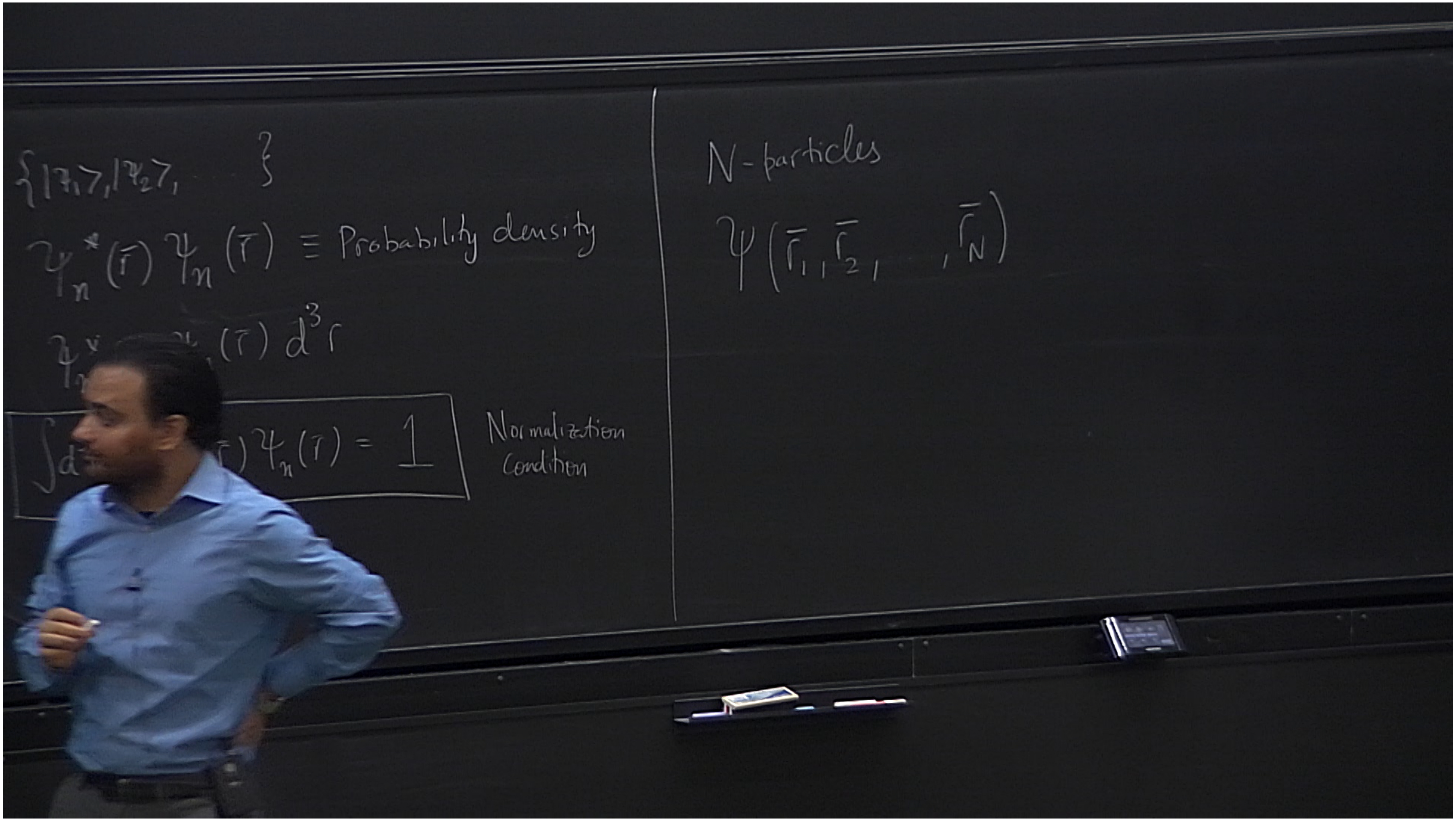

$\{|\psi_1\rangle, |\psi_2\rangle, \quad \}$
$\psi_n^*(\bar{r})\, \psi_n(\bar{r})$

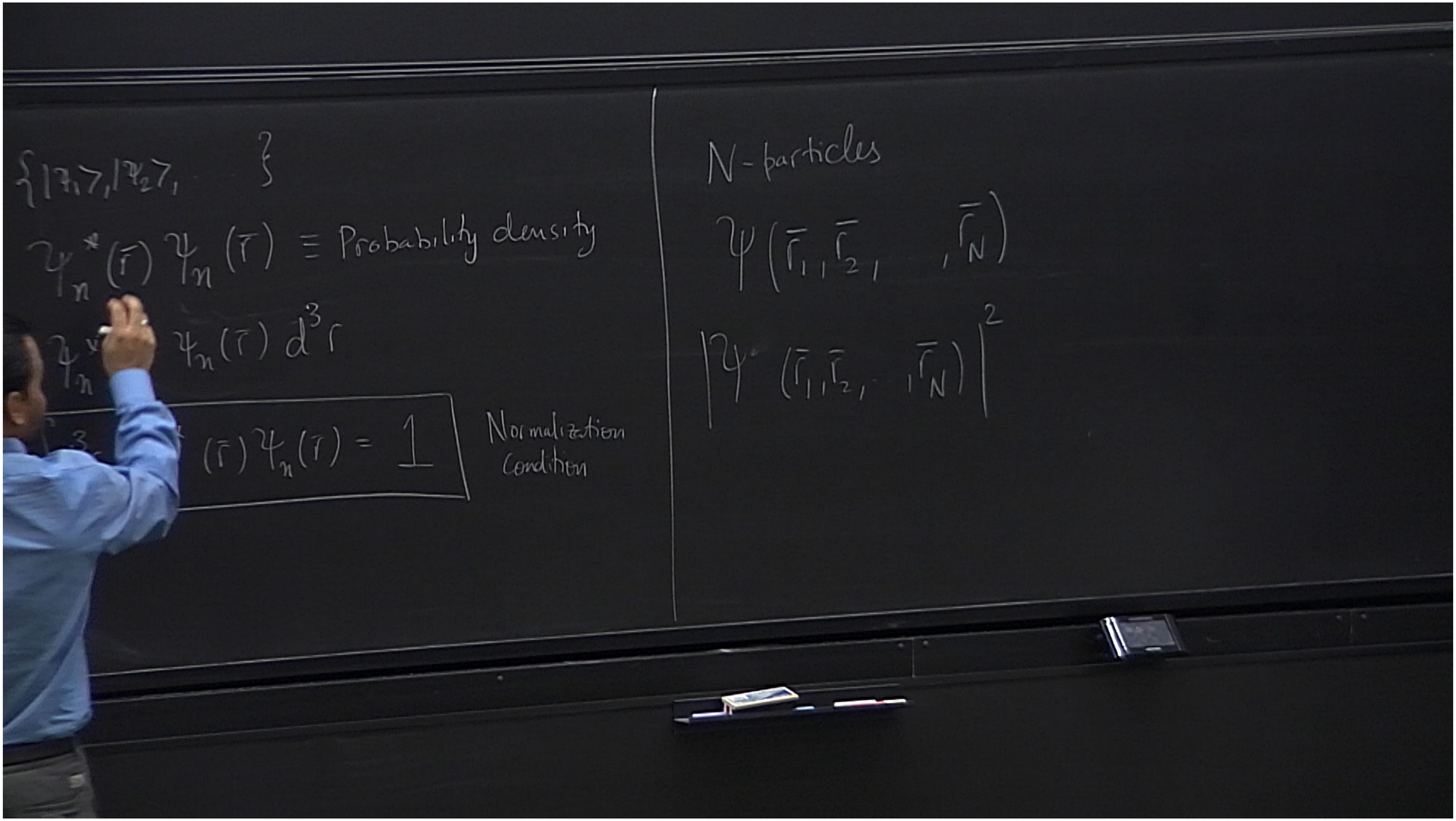
$$\{ |\psi_1\rangle, |\psi_2\rangle, \quad \}$$

$$\psi_n^*(\bar{r})\, \psi_n(\bar{r}) \equiv \text{Probability density}$$

$$\psi_n^*(\bar{r})\, \psi_n(\bar{r})\, d^3 r$$


$\{|\psi_1\rangle, |\psi_2\rangle, \ldots\}$
$\psi_n^*(\bar{r})\,\psi_n(\bar{r}) \equiv$ Probability density
$\psi_n^*(\bar{r})\,\psi_n(\bar{r})\,d^3r$
$\boxed{\int d^3r\, \psi_n^*(\bar{r})\,\psi_n(\bar{r}) = 1}$
Normalization condition


≡ Probability density
Normalization
Condition
N-particles


$\{|\psi_1\rangle, |\psi_2\rangle, \ldots\}$
$\psi_n^*(\bar{r})\,\psi_n(\bar{r}) \equiv$ Probability density
$\psi_n(\bar{r})\, d^3r$
$(\bar{r})\,\psi_n(\bar{r}) = 1$
Normalization Condition
N-particles
$\psi(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_N)$
$|\psi(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_N)|^2$

$\{|\psi_1\rangle, |\psi_2\rangle, \ \ \}$

$\psi_n^*(\bar{r})\,\psi_n(\bar{r}) \equiv$ Probability density

$\psi_n^*(\bar{r})\,\psi_n(\bar{r})\, d^3r$

$$\boxed{\int d^3r\ \psi_n^*(\bar{r})\,\psi_n(\bar{r}) = 1}$$

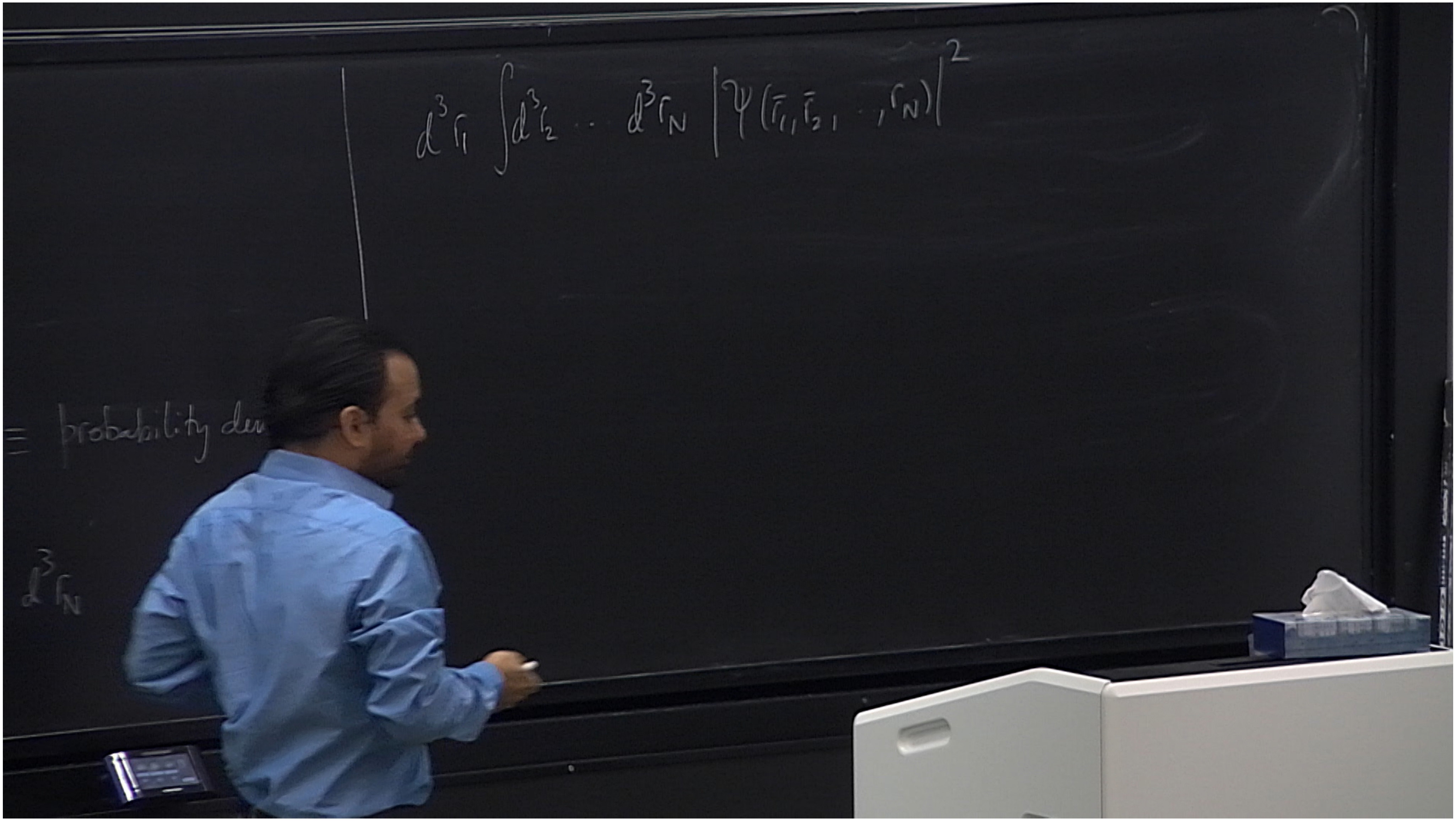
Normalization condition

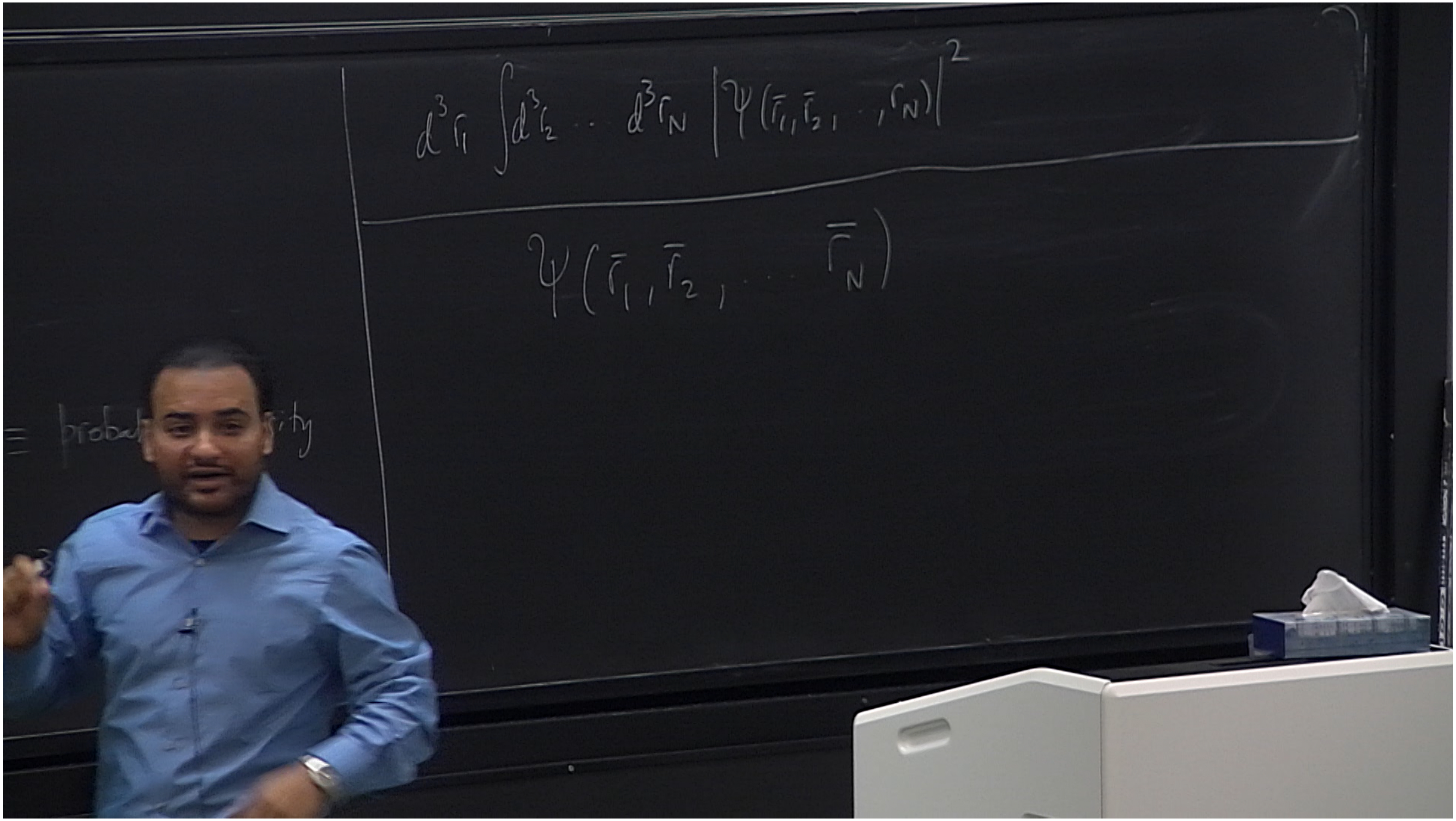
N-particles

$\psi(\bar{r}_1, \bar{r}_2, \ \ , \bar{r}_N)$

$|\psi(\bar{r}_1, \bar{r}_2, \ \ , \bar{r}_N)|^2 \equiv$ probability density

$|\psi(\bar{r}_1, \bar{r}_2, \ \ , \bar{r}_N)|^2\, d^3r_1\, d^3r_2 \ \ d^3r_N$


d³r₁ ∫d³r₂ … d³r_N |ψ(r̄₁, r̄₂, …, r_N)|²
= probability den
d³r_N


$d^3 r_1 \int d^3 r_2 \cdots d^3 r_N \left|\psi(\bar{r}_1, \bar{r}_2, \ldots, r_N)\right|^2$
$\psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N)$

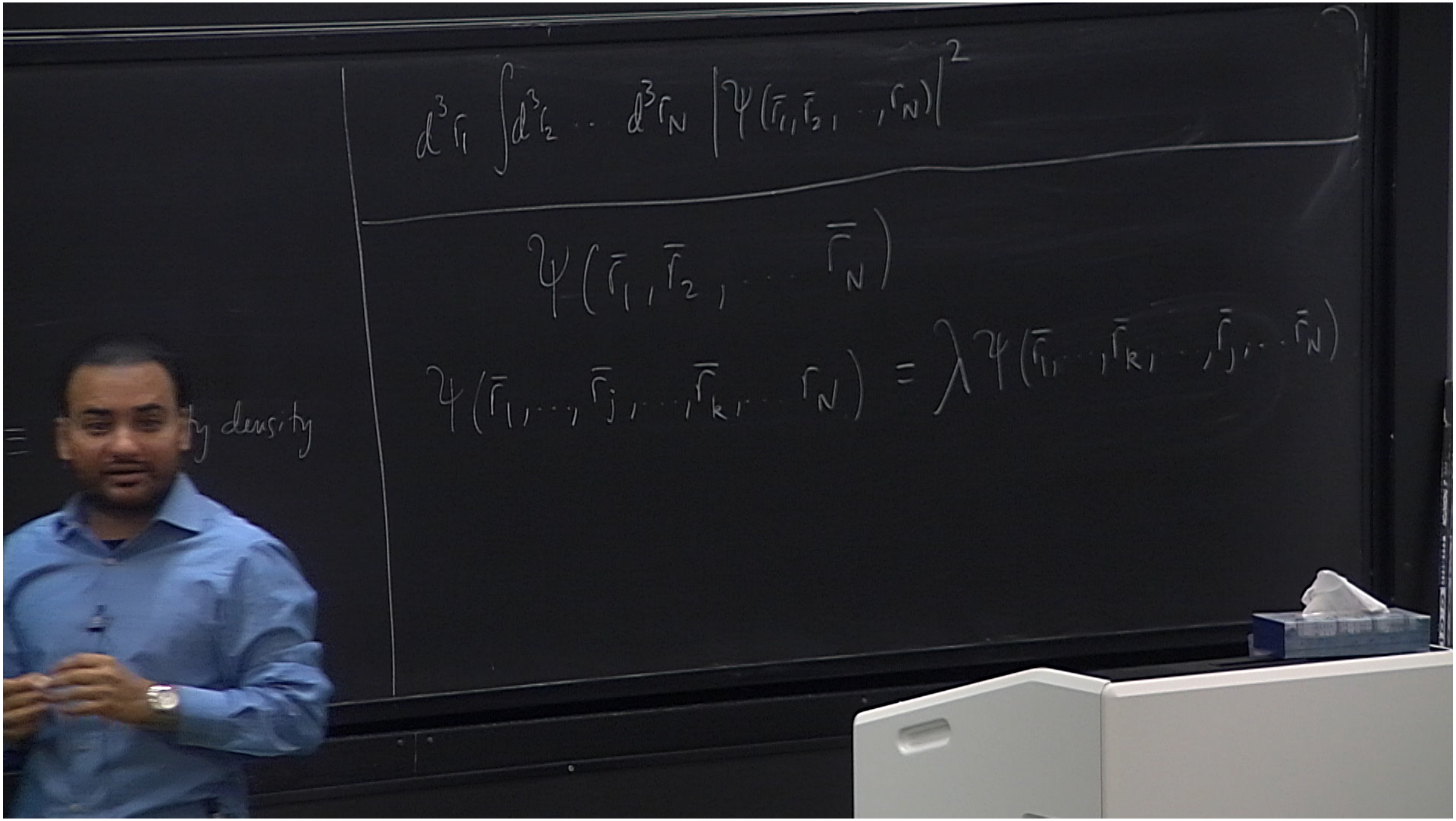
$$d^3 r_1 \int d^3 r_2 \cdots d^3 r_N \left|\psi(\bar{r}_1, \bar{r}_2, \ldots, r_N)\right|^2$$

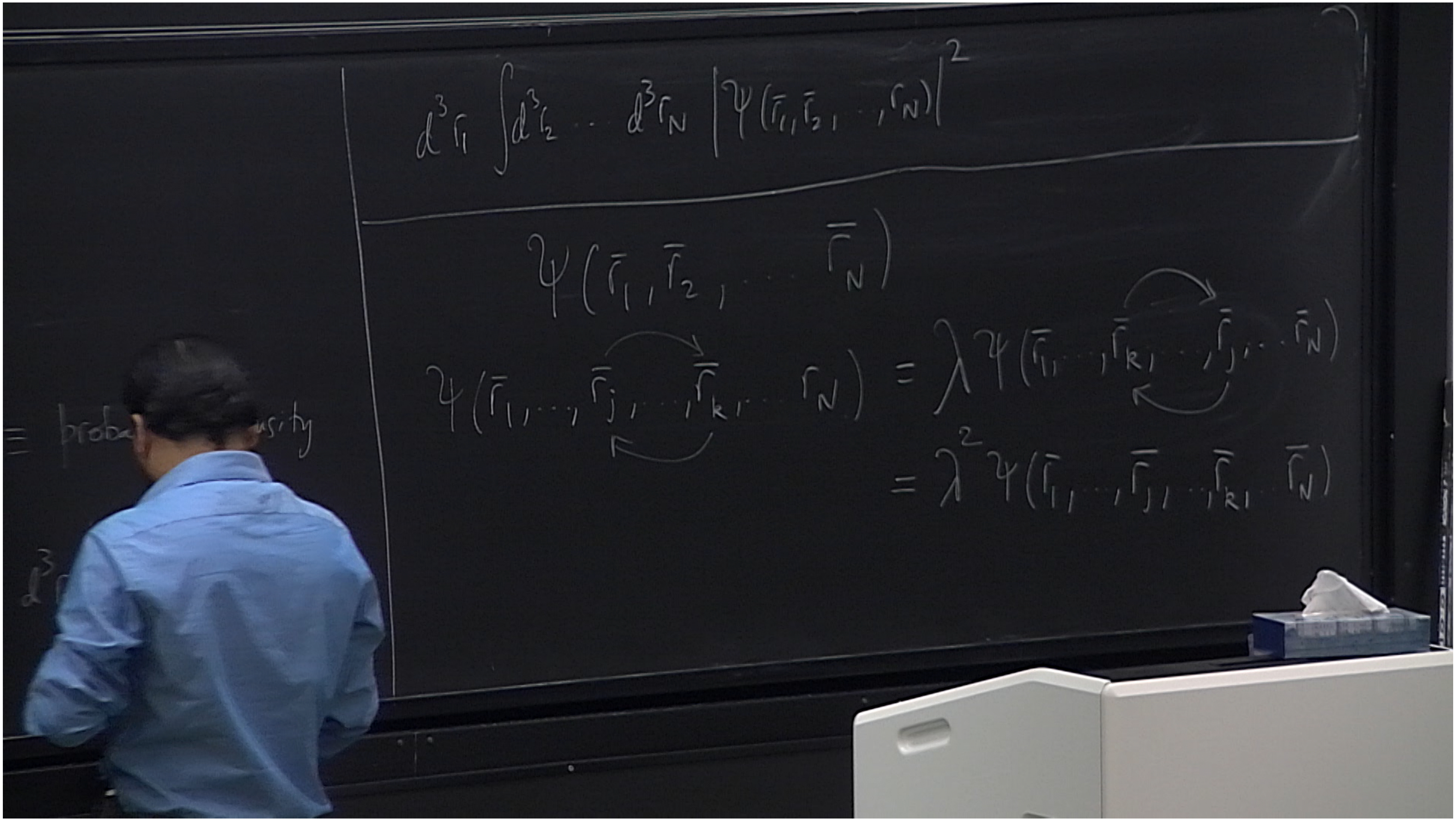
$$\psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N)$$

$$\psi(\bar{r}_1, \ldots, \bar{r}_j, \ldots, \bar{r}_k, \ldots r_N) \qquad \psi(\bar{r}_1 \ldots, \bar{r}_k, \ldots, \bar{r}_j, \ldots \bar{r}_N)$$

= probability density

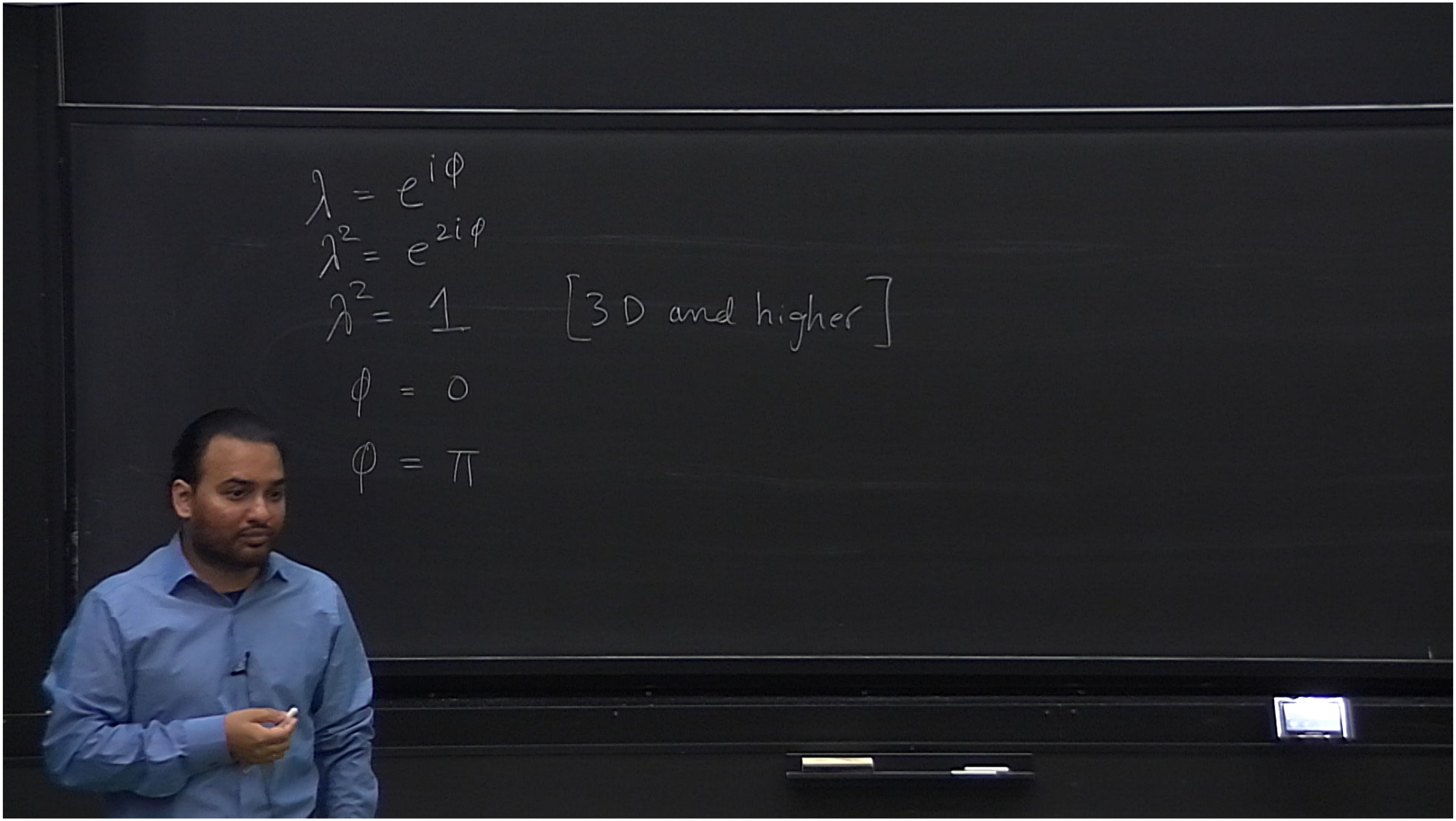
$d^3 r_N$

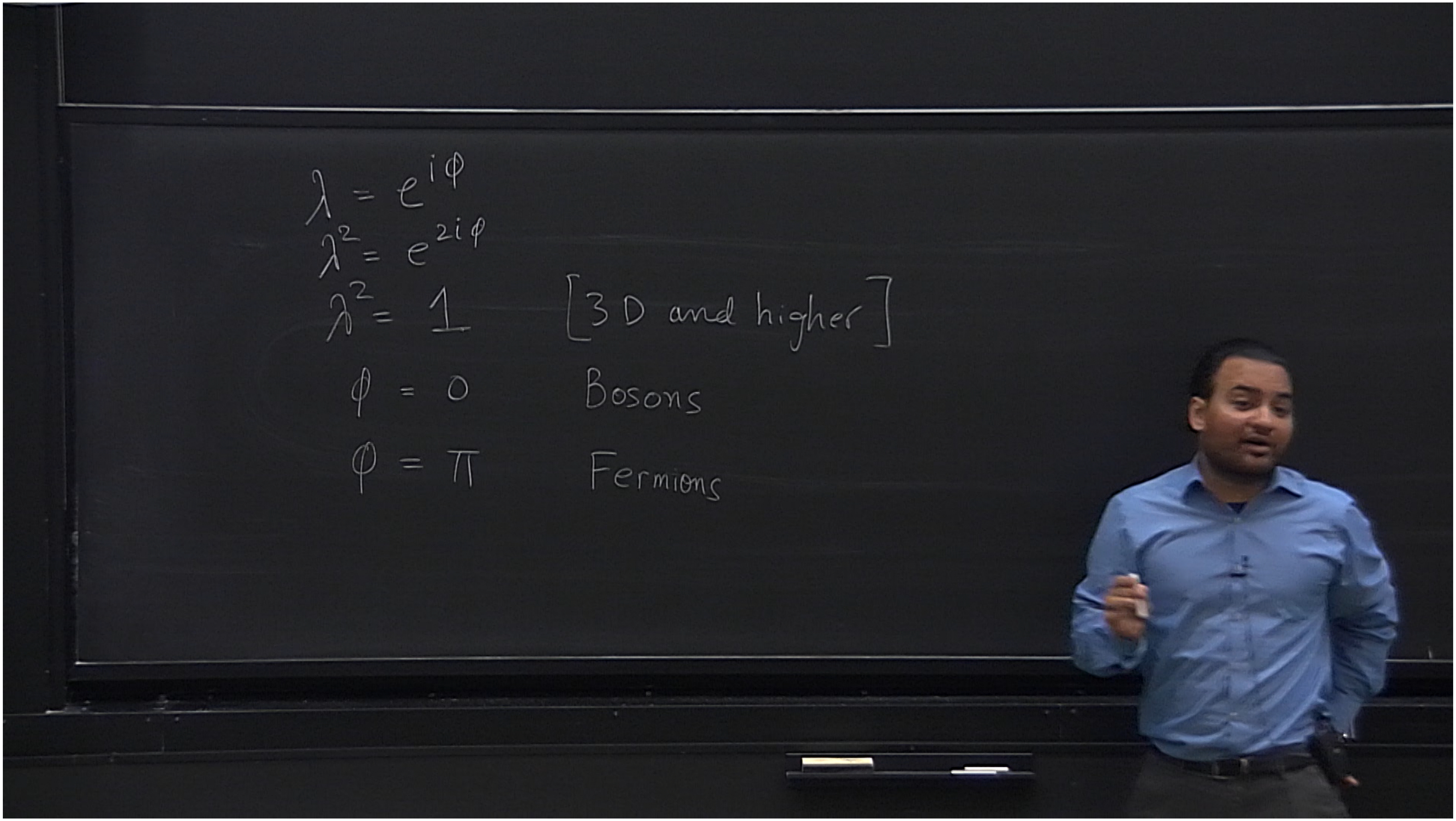

$d^3 r_1 \int d^3 r_2 \cdots d^3 r_N \left| \psi(\bar{r}_1, \bar{r}_2, \ldots, r_N) \right|^2$
$\psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N)$
$\psi(\bar{r}_1, \ldots, \bar{r}_j, \ldots, \bar{r}_k, \ldots r_N) = \lambda \psi(\bar{r}_1, \ldots, \bar{r}_k, \ldots, \bar{r}_j, \ldots \bar{r}_N)$
density

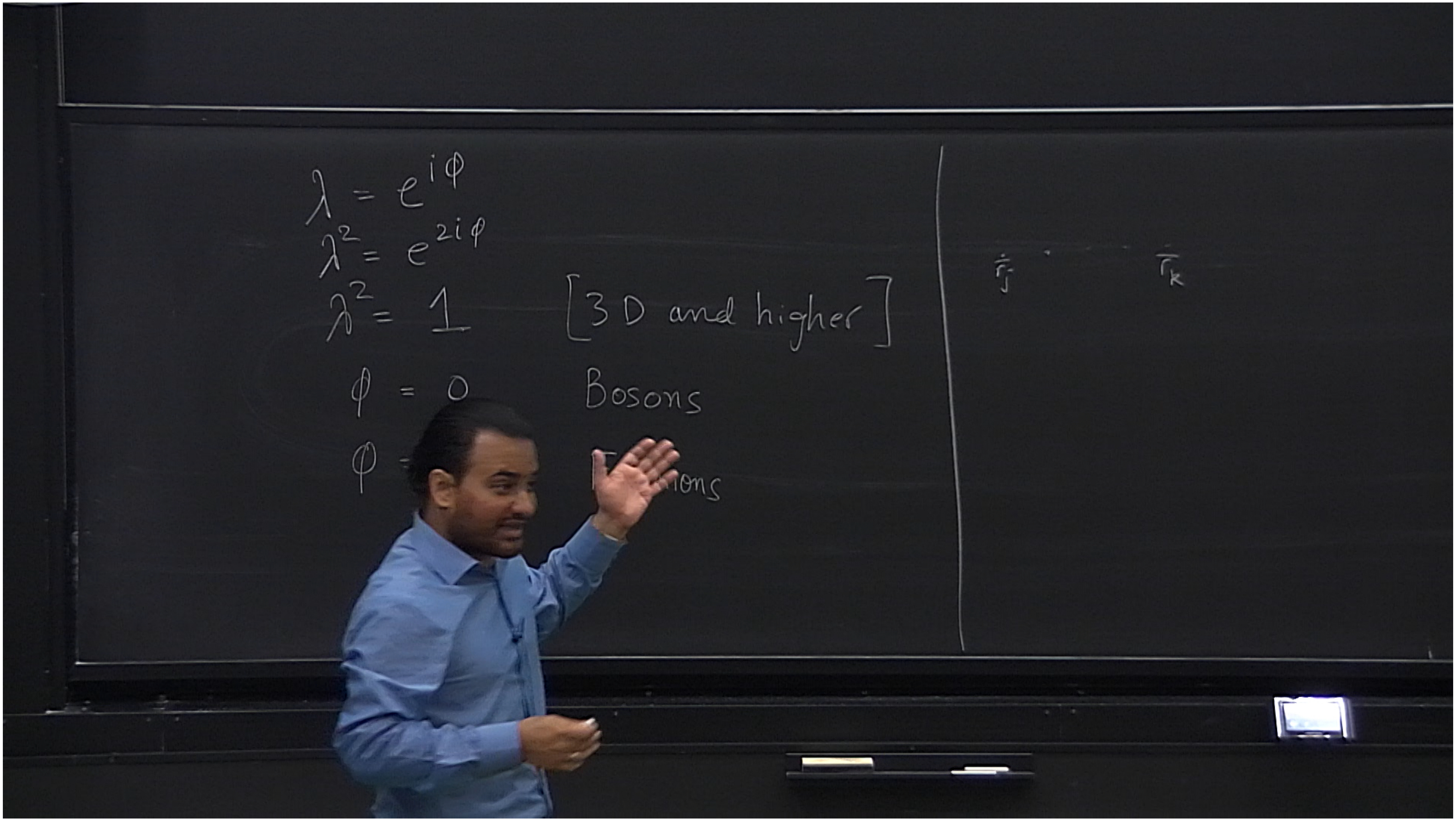

$d^3 r_1 \int d^3 r_2 \cdots d^3 r_N \left|\psi(\bar{r}_1, \bar{r}_2, \ldots, r_N)\right|^2$
$\psi(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N)$
$\psi(\bar{r}_1, \ldots, \bar{r}_j, \ldots, \bar{r}_k, \ldots r_N) = \lambda \psi(\bar{r}_1 \ldots, \bar{r}_k, \ldots, \bar{r}_j, \ldots \bar{r}_N)$
$= \lambda^2 \psi(\bar{r}_1, \ldots, \bar{r}_j, \ldots, \bar{r}_k \ldots \bar{r}_N)$

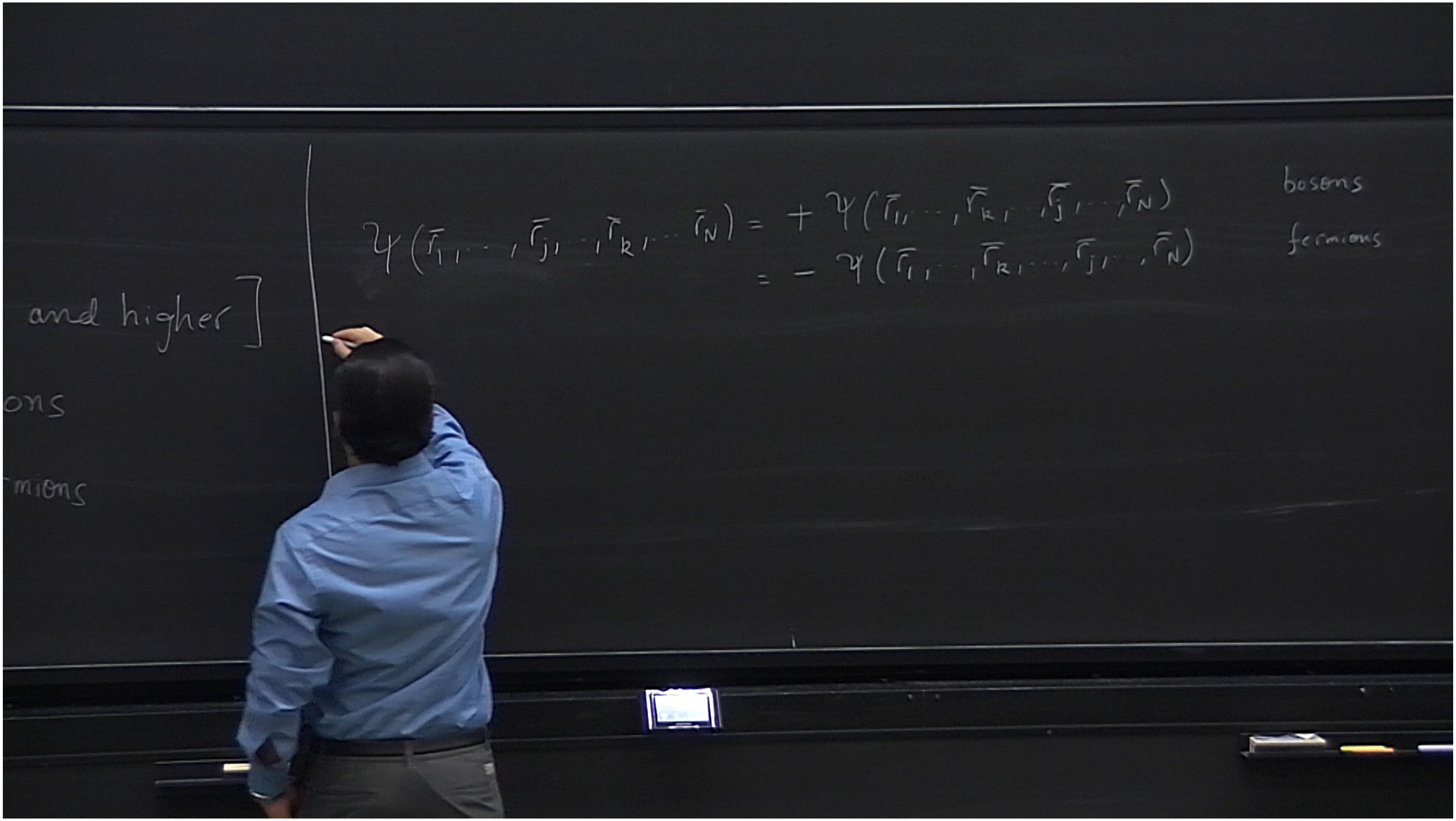
$$\lambda = e^{i\phi}$$

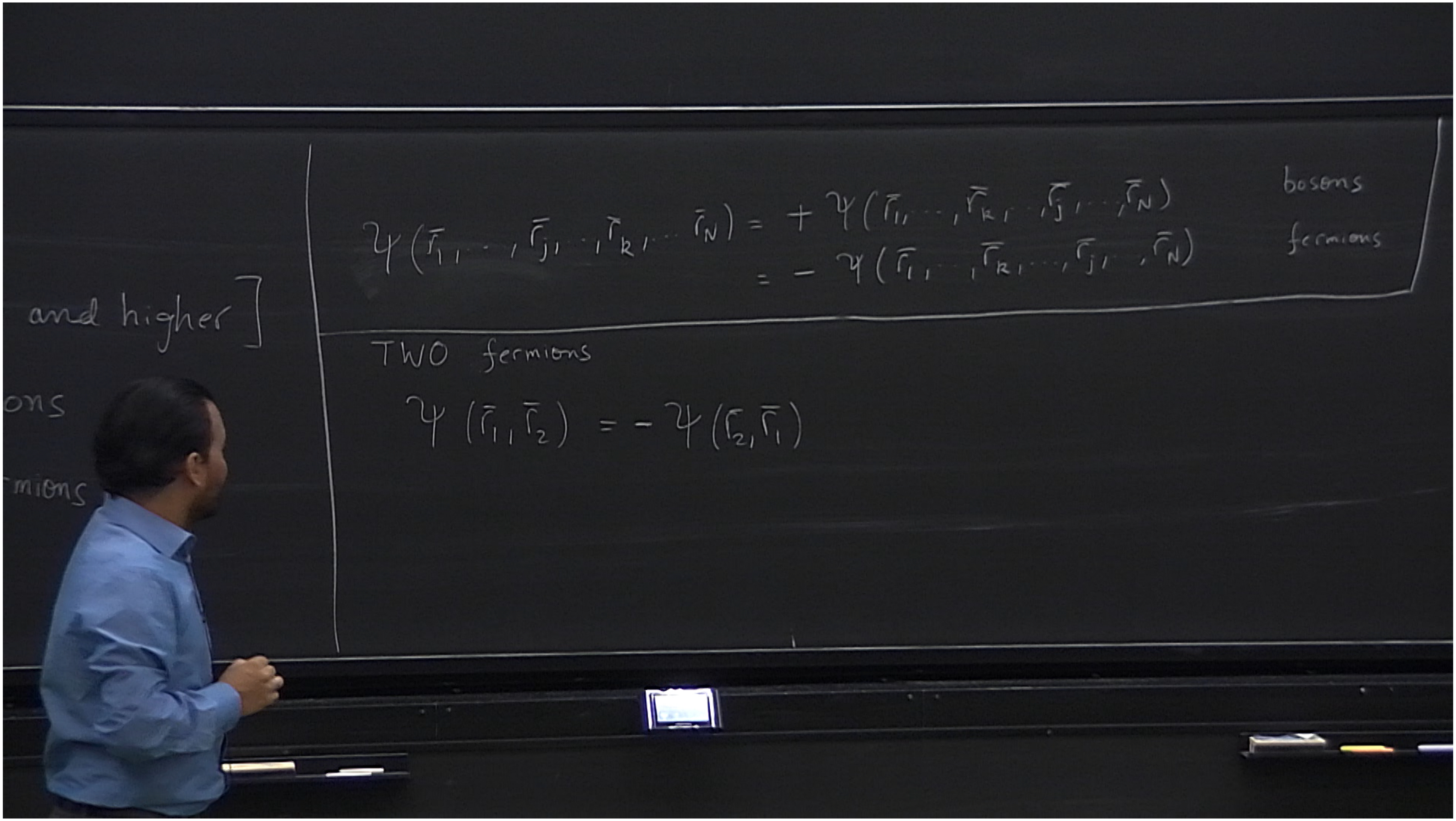
$$\lambda^2 = e^{2i\phi}$$


$\lambda = e^{i\phi}$
$\lambda^2 = e^{2i\phi}$
$\lambda^2 = 1$
[3D and higher]
$\phi = 0$
$\phi = \pi$


$\lambda = e^{i\phi}$
$\lambda^2 = e^{2i\phi}$
$\lambda^2 = 1$
[3D and higher]
$\phi = 0$
Bosons
$\phi = \pi$
Fermions


$\lambda = e^{i\phi}$
$\lambda^2 = e^{2i\phi}$
$\lambda^2 = 1$
[3D and higher]
$\phi = 0$
Bosons
ϕ
$\bar{r}_j$
$\bar{r}_k$


$\psi(\bar{r}_1, \ldots, \bar{r}_j, \ldots, \bar{r}_k, \ldots \bar{r}_N) = +\psi(\bar{r}_1, \ldots, \bar{r}_k, \ldots \bar{r}_j, \ldots, \bar{r}_N)$ bosons
$= -\psi(\bar{r}_1, \ldots, \bar{r}_k, \ldots, \bar{r}_j, \ldots, \bar{r}_N)$ fermions
and higher]
ons
rmions


and higher]
ons
rmions
$\psi(\bar{r}_1, \ldots, \bar{r}_j, \ldots, \bar{r}_k, \ldots \bar{r}_N) = + \psi(\bar{r}_1, \ldots, \bar{r}_k, \ldots, \bar{r}_j, \ldots, \bar{r}_N)$ bosons
$= - \psi(\bar{r}_1, \ldots, \bar{r}_k, \ldots, \bar{r}_j, \ldots, \bar{r}_N)$ fermions
TWO fermions
$\psi(\bar{r}_1, \bar{r}_2) = - \psi(\bar{r}_2, \bar{r}_1)$

and higher]

ons

rmions

$$\psi(\bar r_1,\dots,\bar r_j,\dots,\bar r_k,\dots,\bar r_N) = +\psi(\bar r_1,\dots,\bar r_k,\dots,\bar r_j,\dots,\bar r_N) \quad \text{bosons}$$

$$= -\psi(\bar r_1,\dots,\bar r_k,\dots,\bar r_j,\dots,\bar r_N) \quad \text{fermions}$$

TWO fermions

$$\psi(\bar r_1,\bar r_2) = -\psi(\bar r_2,\bar r_1)$$

$$\psi(\bar r,\bar r) = -\psi(\bar r,\bar r) \Rightarrow \psi(\bar r,\bar r)=0$$

$$\ldots, \bar{r}_k, \ldots \bar{r}_N) = +\Psi(\bar{r}_1, \ldots, \bar{r}_k, \ldots, \bar{r}_j, \ldots, \bar{r}_N) \quad \text{bosons}$$

$$= -\Psi(\bar{r}_1, \ldots, \bar{r}_k, \ldots, \bar{r}_j, \ldots, \bar{r}_N) \quad \text{fermions}$$

ns

$$) = -\Psi(\bar{r}_2, \bar{r}_1)$$

$$= -\Psi(\bar{r}, \bar{r}) \Rightarrow \Psi(\bar{r}, \bar{r}) = 0$$

$$\Psi(\bar{r}_1, \bar{r}_2) = -\Psi(\bar{r}_2, \bar{r}_1)$$

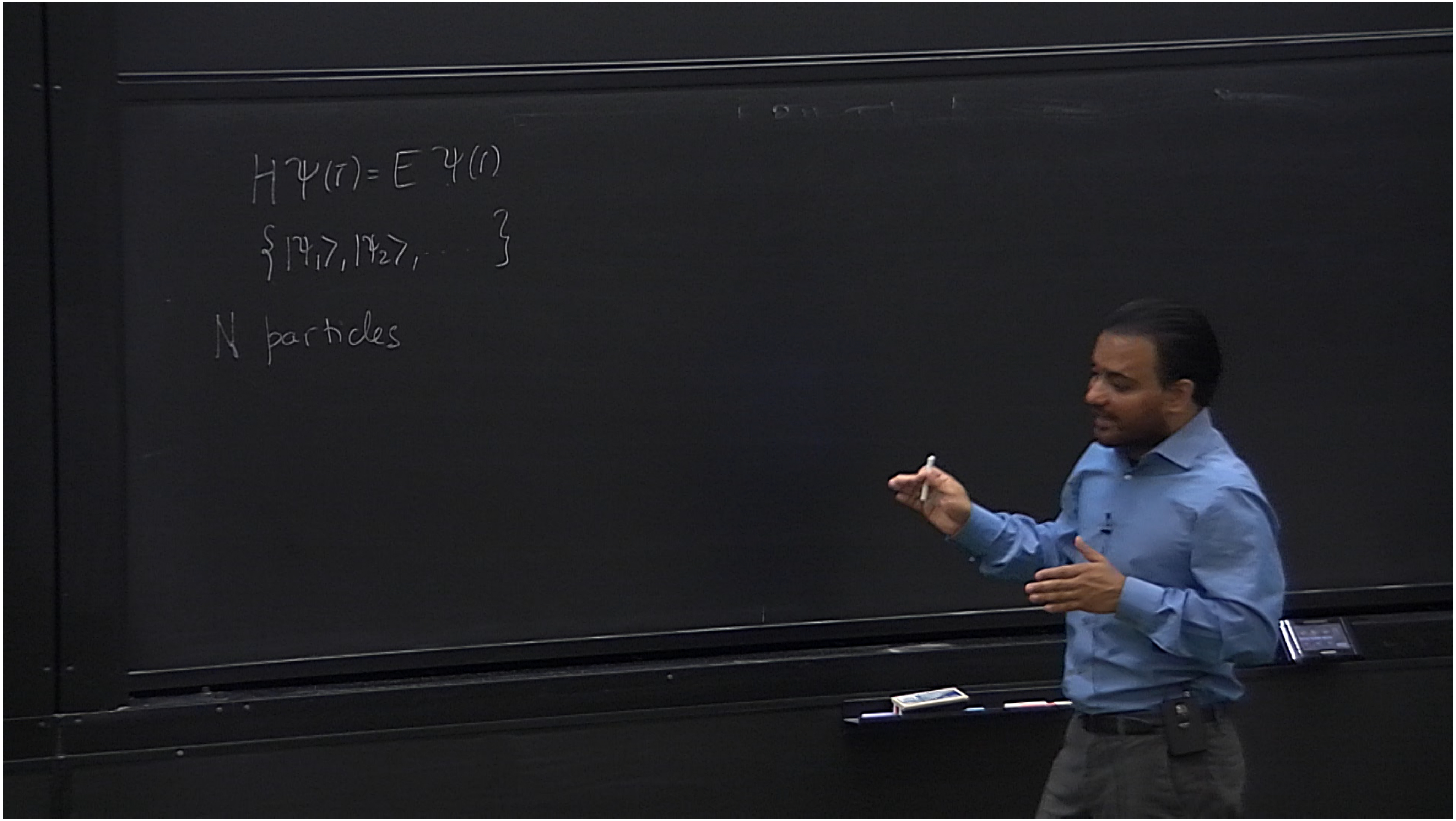
$\Uparrow$

Fermions are Antisymmetric under exchange

Bosons " Symmetric " "

$\Downarrow$

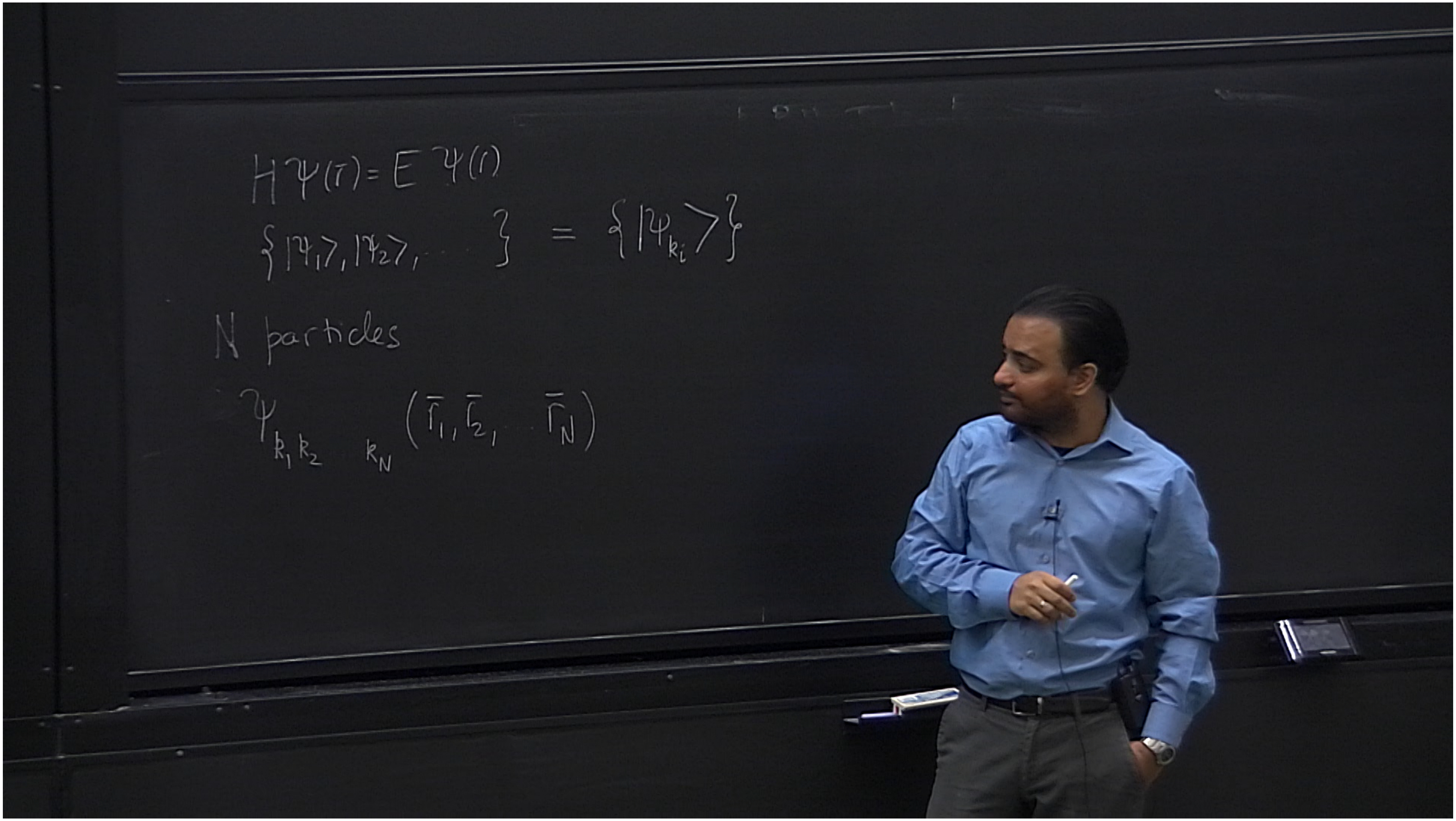
$$\Psi(\bar{r}_1, \bar{r}_2) = \Psi(\bar{r}_2, \bar{r}_1)$$


$H\Psi(r) = E\Psi(r)$
$\{|\Psi_1\rangle, |\Psi_2\rangle, \ldots\}$
N particles

$$H\Psi(\vec{r}) = E\Psi(\vec{r})$$

$$\{|\psi_1\rangle, |\psi_2\rangle, \ldots\} = \{|\psi_{k_i}\rangle\}$$

N particles


H ψ(r) = E ψ(r)
{|ψ1>, |ψ2>, ... } = {|ψki>}
N particles
ψk1 k2 ... kN (r1, r2, ... rN)

$$H\psi(\bar{r}) = E\,\psi(r)$$

$$\{|\psi_1\rangle, |\psi_2\rangle, \dots\} = \{|\psi_{k_i}\rangle\}$$

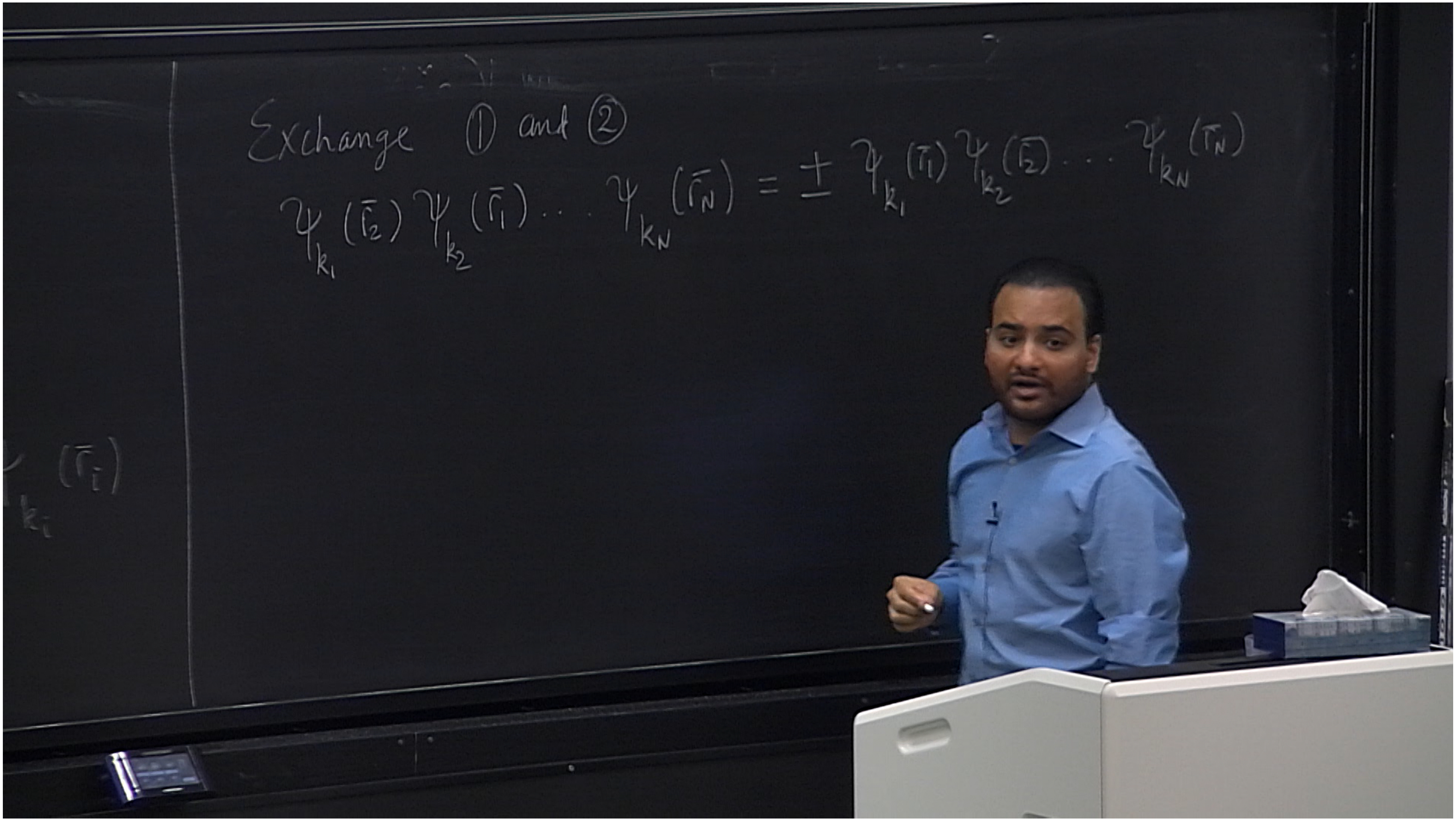
N particles

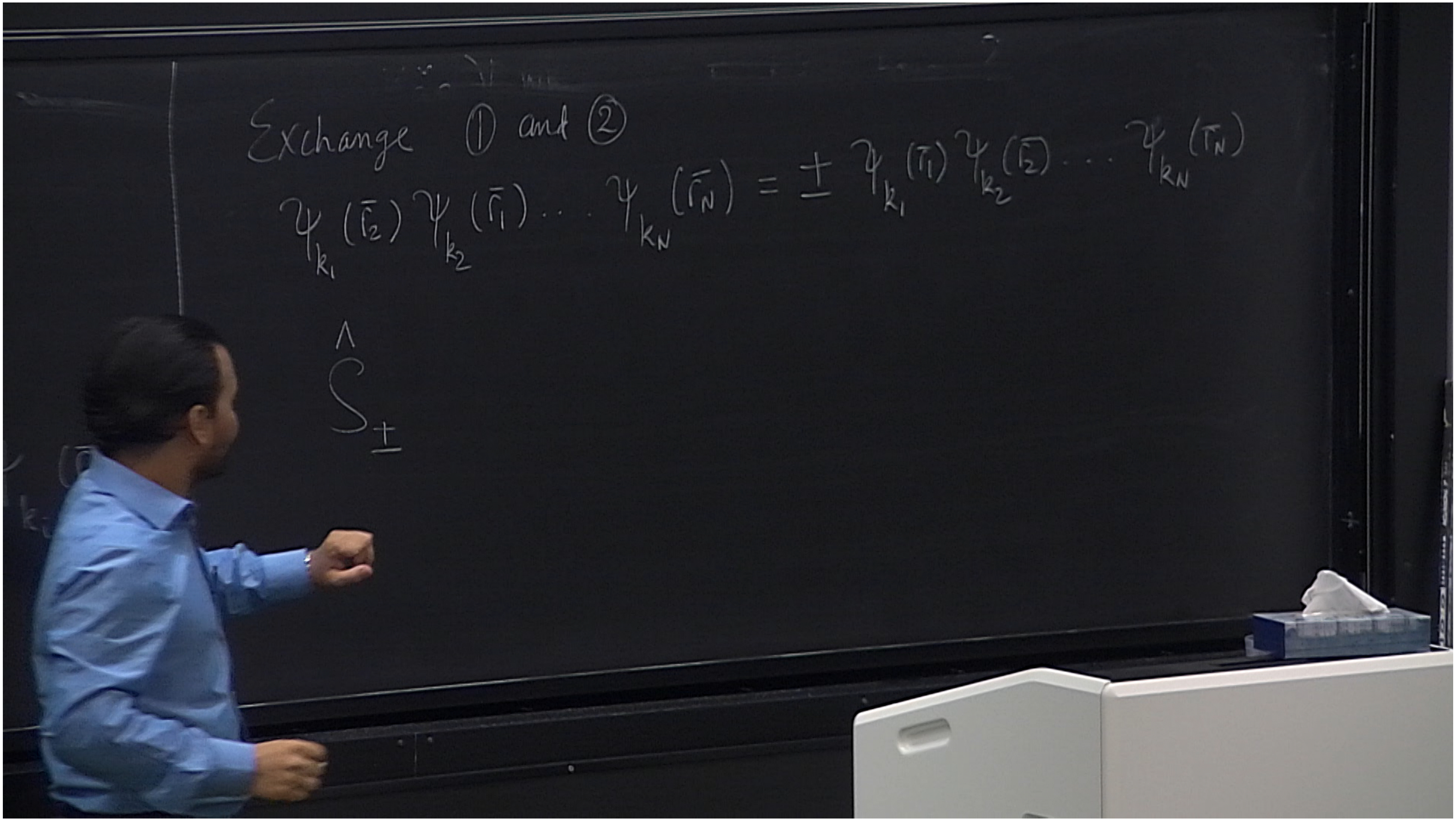
$$\psi_{k_1 k_2 \dots k_N}(\bar{r}_1, \bar{r}_2, \dots \bar{r}_N) = \psi_{k_1}(\bar{r}_1)\,\psi_{k_2}(\bar{r}_2)\dots\psi_{k_N}(\bar{r}_N) = \prod_{k_i} \psi_{k_i}(\bar{r}_i)$$

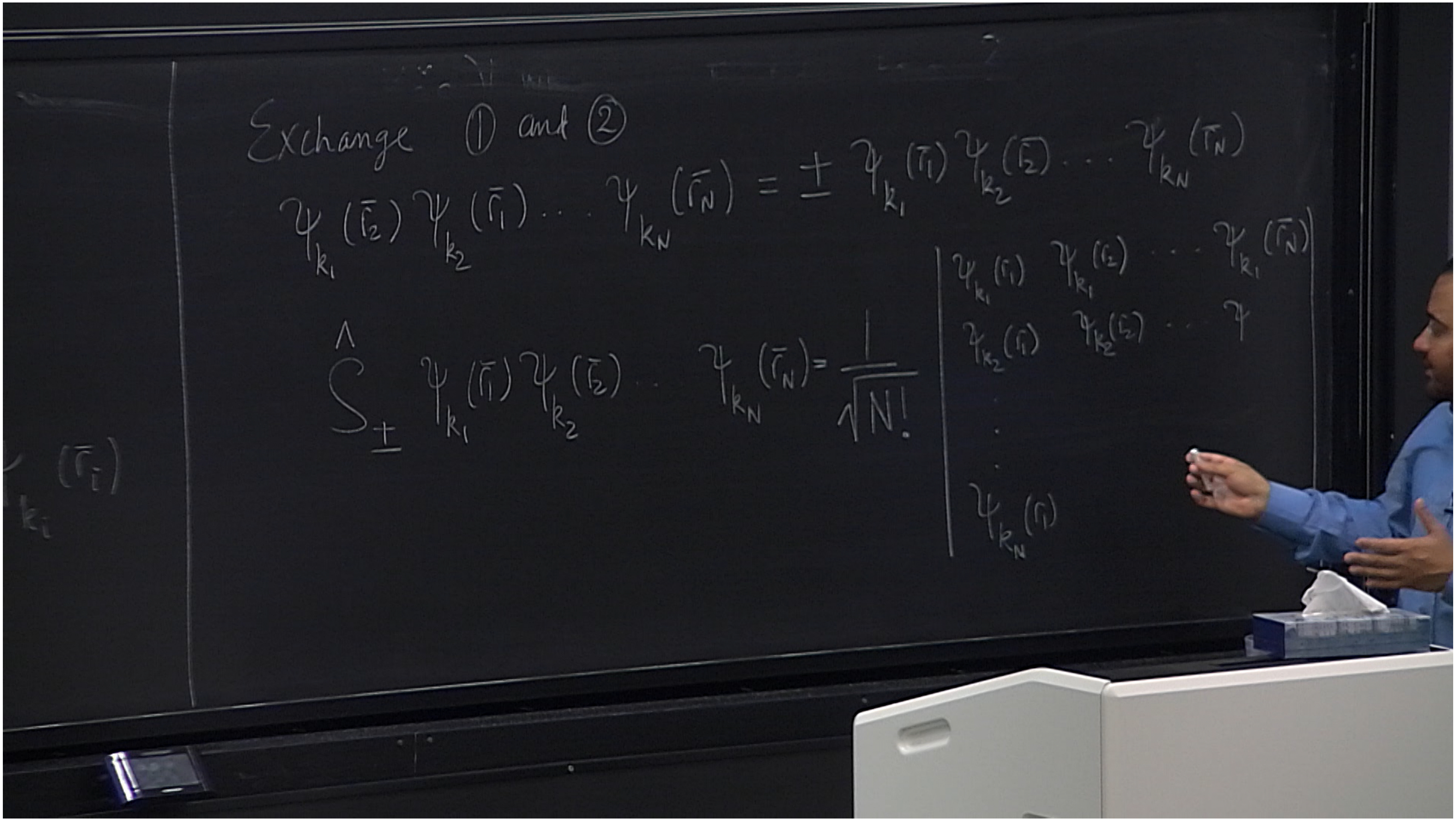
Exchange 1 and 2

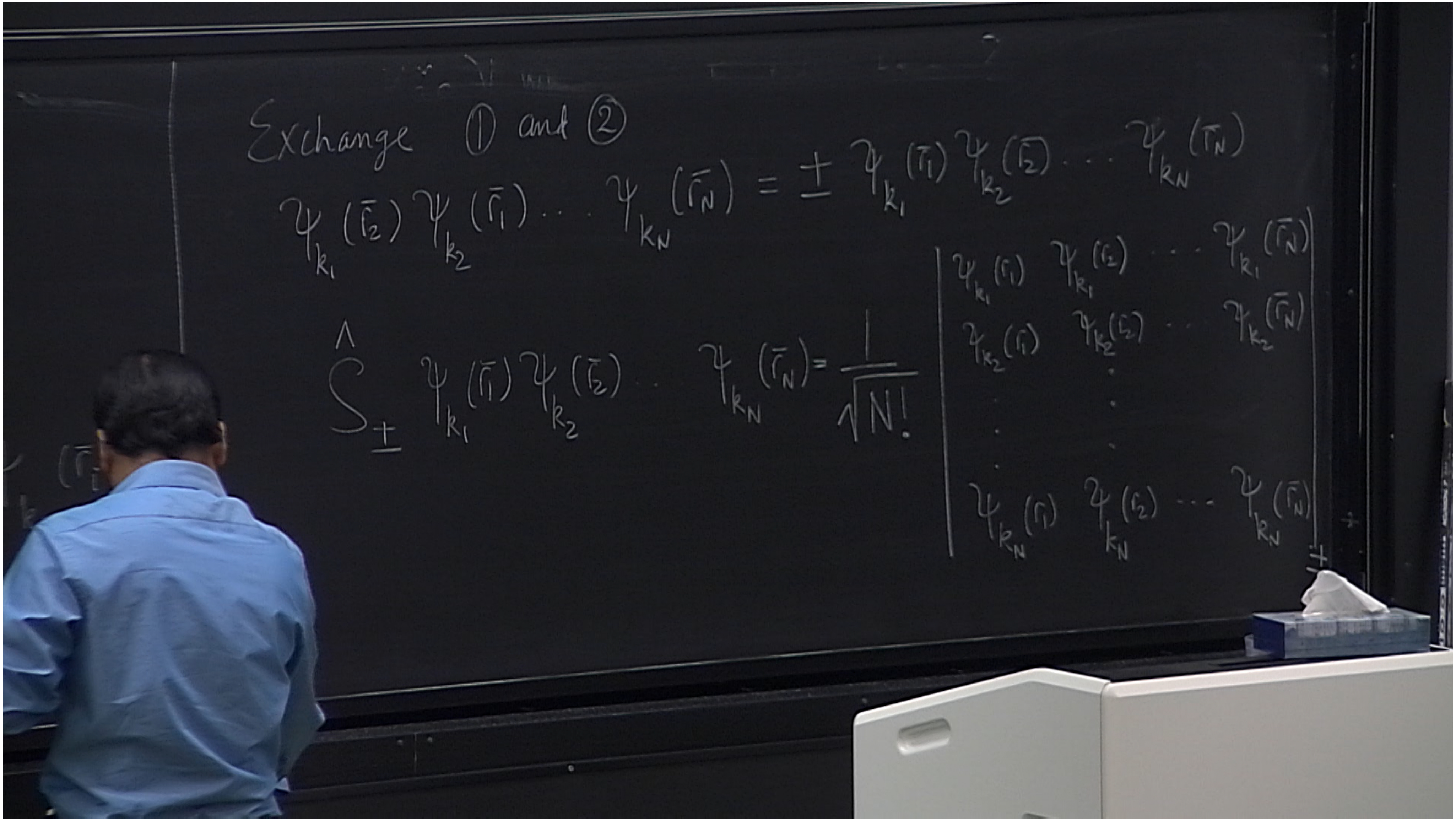
$$\psi_{k_1}(\bar{r}_2)\,\psi_{k_2}(\bar{r}_1)\ldots\psi_{k_N}(\bar{r}_N)$$

$\psi_{k_i}(\bar{r}_i)$


Exchange 1 and 2
$\psi_{k_1}(\bar{r}_2)\,\psi_{k_2}(\bar{r}_1)\ldots\psi_{k_N}(\bar{r}_N) = \pm\,\psi_{k_1}(\bar{r}_1)\,\psi_{k_2}(\bar{r}_2)\ldots\psi_{k_N}(\bar{r}_N)$
$\psi_{k_i}(\bar{r}_i)$


Exchange 1 and 2
$\psi_{k_1}(\bar{r}_2)\psi_{k_2}(\bar{r}_1)\ldots\psi_{k_N}(\bar{r}_N) = \pm\psi_{k_1}(\bar{r}_1)\psi_{k_2}(\bar{r}_2)\ldots\psi_{k_N}(\bar{r}_N)$
$\hat{S}_{\pm}$


Exchange 1 and 2
$\psi_{k_1}(\bar{r}_2)\psi_{k_2}(\bar{r}_1)\ldots\psi_{k_N}(\bar{r}_N) = \pm\,\psi_{k_1}(\bar{r}_1)\psi_{k_2}(\bar{r}_2)\ldots\psi_{k_N}(\bar{r}_N)$
$\hat{S}_{\pm}\,\psi_{k_1}(\bar{r}_1)\psi_{k_2}(\bar{r}_2)\ldots\psi_{k_N}(\bar{r}_N) = \frac{1}{\sqrt{N!}}\begin{vmatrix}\psi_{k_1}(\bar{r}_1) & \psi_{k_1}(\bar{r}_2) & \ldots & \psi_{k_1}(\bar{r}_N)\\ \psi_{k_2}(\bar{r}_1) & \psi_{k_2}(\bar{r}_2) & \ldots & \psi\\ \vdots & & & \\ \psi_{k_N}(\bar{r}_1) & & & \end{vmatrix}$



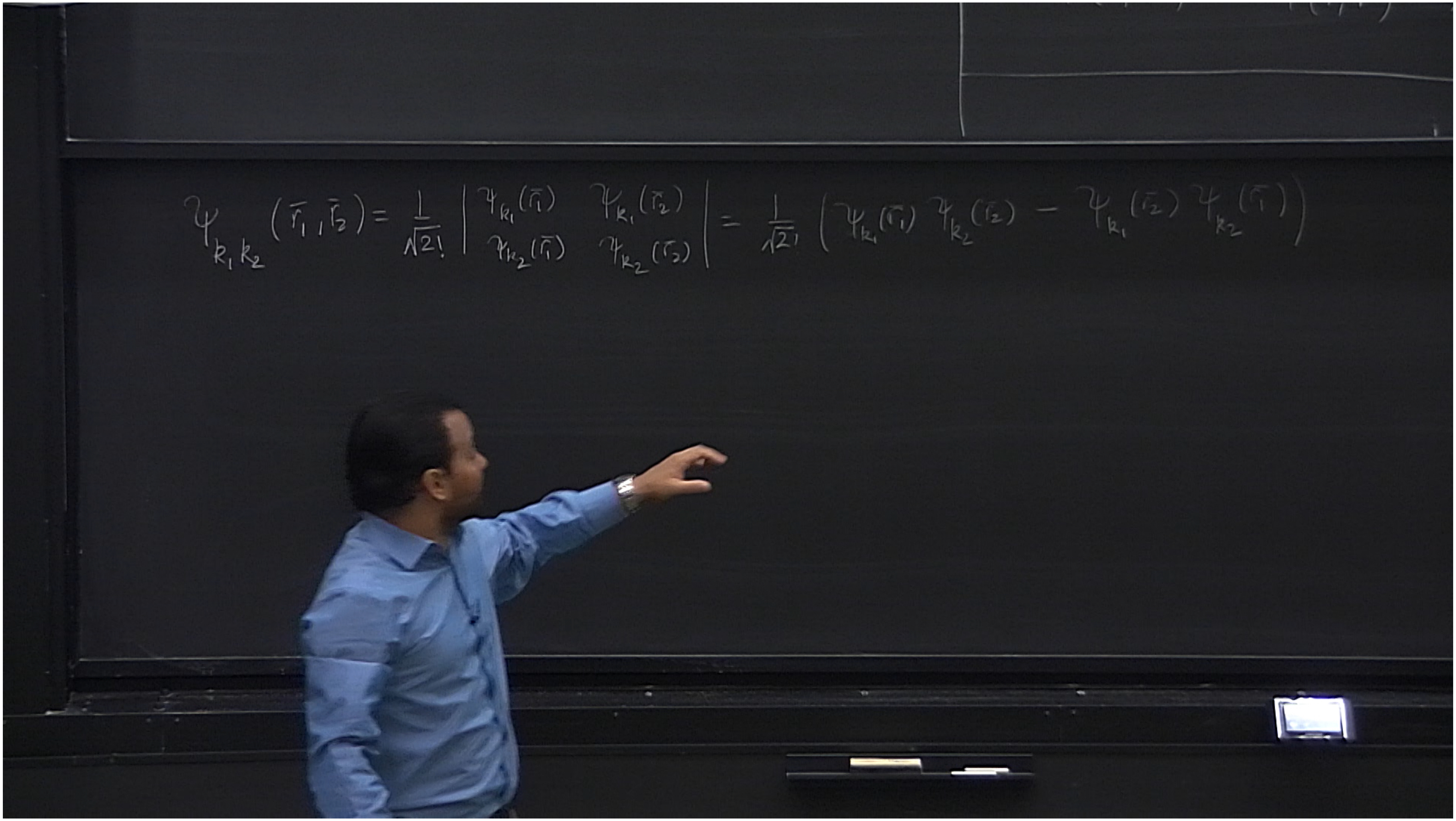
Exchange ① and ②
$\psi_{k_1}(\bar{r}_2)\psi_{k_2}(\bar{r}_1)\ldots\psi_{k_N}(\bar{r}_N) = \pm\,\psi_{k_1}(\bar{r}_1)\psi_{k_2}(\bar{r}_2)\ldots\psi_{k_N}(\bar{r}_N)$
$\hat{S}_\pm\,\psi_{k_1}(\bar{r}_1)\psi_{k_2}(\bar{r}_2)\ldots\psi_{k_N}(\bar{r}_N) = \frac{1}{\sqrt{N!}}\begin{vmatrix} \psi_{k_1}(\bar{r}_1) & \psi_{k_1}(\bar{r}_2) & \cdots & \psi_{k_1}(\bar{r}_N) \\ \psi_{k_2}(\bar{r}_1) & \psi_{k_2}(\bar{r}_2) & \cdots & \psi_{k_2}(\bar{r}_N) \\ \vdots & \vdots & & \\ \psi_{k_N}(\bar{r}_1) & \psi_{k_N}(\bar{r}_2) & \cdots & \psi_{k_N}(\bar{r}_N) \end{vmatrix}_\pm$

Exchange (1) and (2)

$$\psi_{k_1}(\bar{r}_2)\,\psi_{k_2}(\bar{r}_1)\ldots\psi_{k_N}(\bar{r}_N) = \pm\,\psi_{k_1}(\bar{r}_1)\,\psi_{k_2}(\bar{r}_2)\ldots\psi_{k_N}(\bar{r}_N)$$

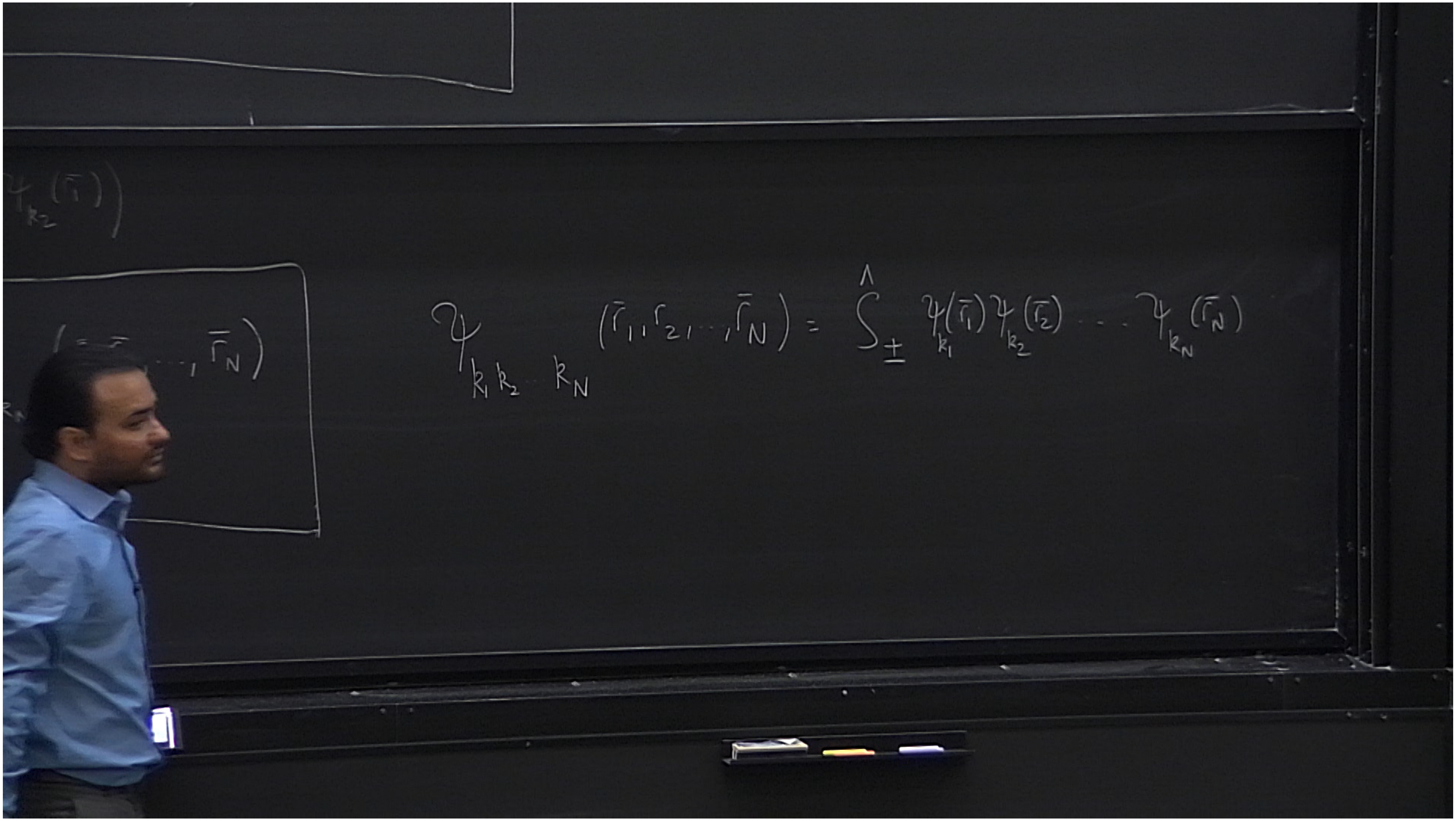
$$\hat{S}_{\pm}\,\psi_{k_1}(\bar{r}_1)\,\psi_{k_2}(\bar{r}_2)\ldots\psi_{k_N}(\bar{r}_N) = \frac{1}{\sqrt{N!}}\begin{vmatrix} \psi_{k_1}(\bar{r}_1) & \psi_{k_1}(\bar{r}_2) & \ldots & \psi_{k_1}(\bar{r}_N) \\ \psi_{k_2}(\bar{r}_1) & \psi_{k_2}(\bar{r}_2) & \ldots & \psi_{k_2}(\bar{r}_N) \\ \vdots & \vdots & & \\ \psi_{k_N}(\bar{r}_1) & \psi_{k_N}(\bar{r}_2) & \ldots & \psi_{k_N}(\bar{r}_N) \end{vmatrix}_{\pm}$$

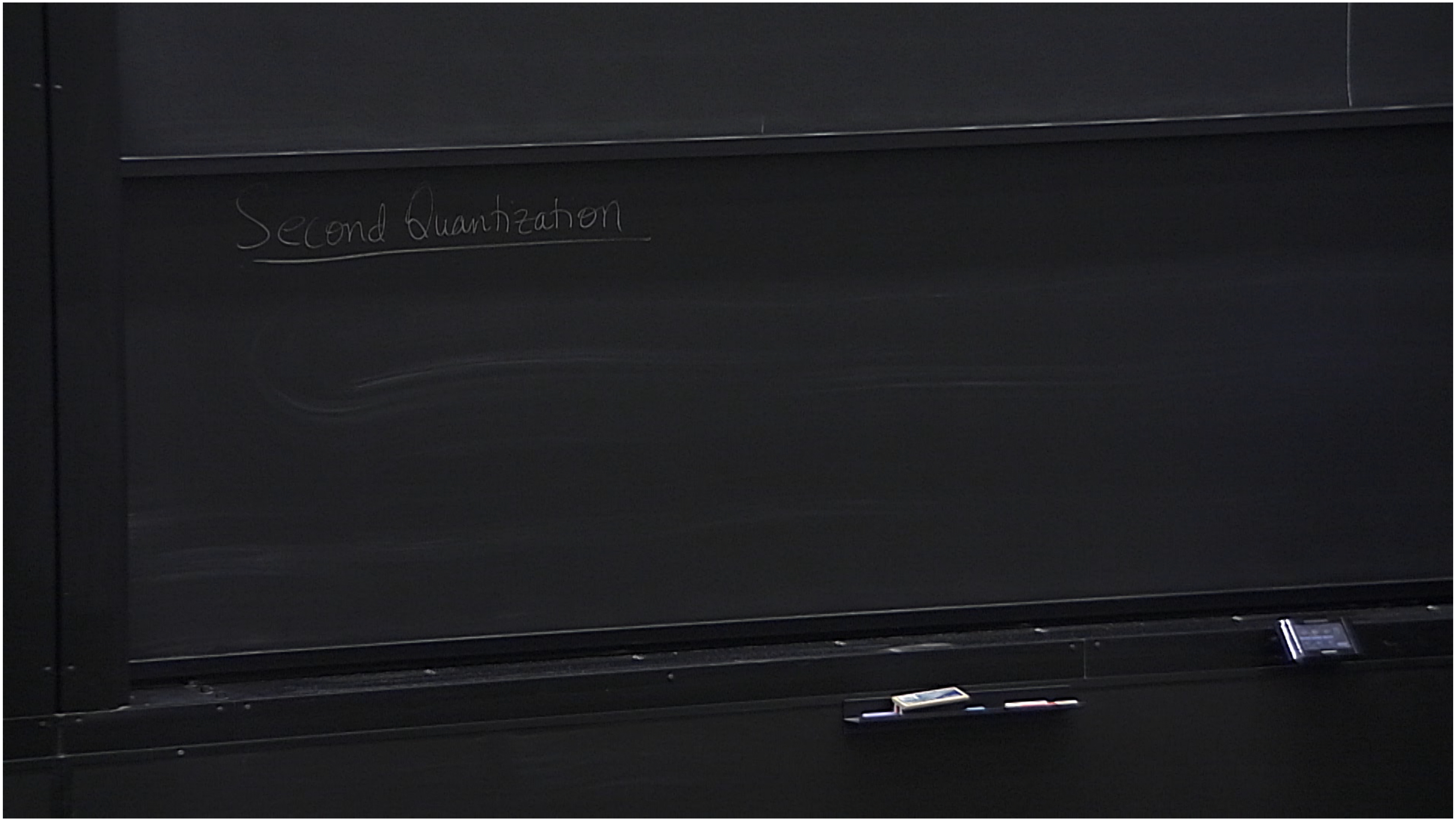
$\psi_{k_i}(\bar{r}_i)$

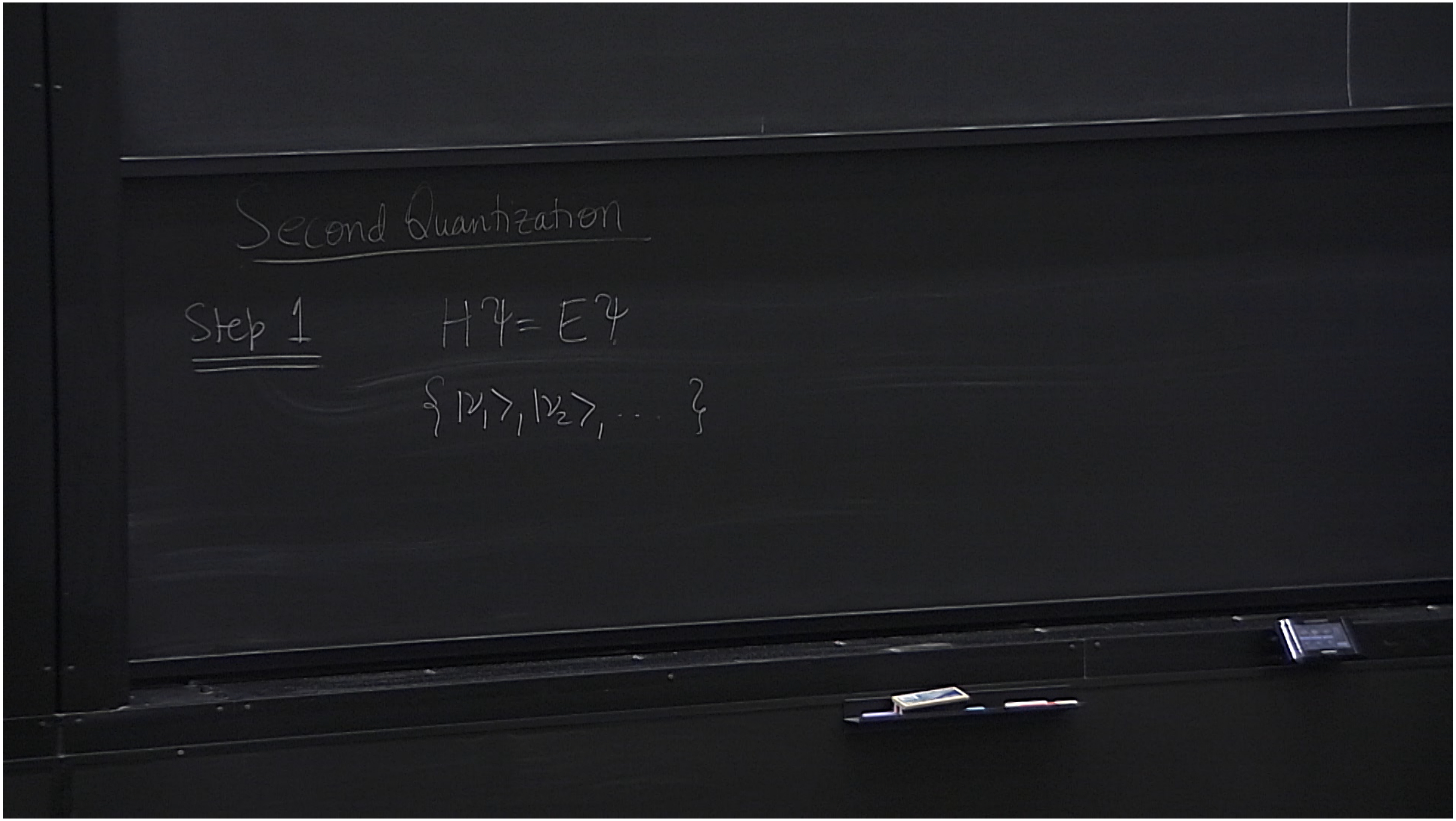

$\psi_{k_1 k_2}(\bar{r}_1, \bar{r}_2) = \frac{1}{\sqrt{2!}} \begin{vmatrix} \psi_{k_1}(\bar{r}_1) & \psi_{k_1}(\bar{r}_2) \\ \psi_{k_2}(\bar{r}_1) & \psi_{k_2}(\bar{r}_2) \end{vmatrix} = \frac{1}{\sqrt{2!}} \left(\psi_{k_1}(\bar{r}_1)\psi_{k_2}(\bar{r}_2) - \psi_{k_1}(\bar{r}_2)\psi_{k_2}(\bar{r}_1) \right)$

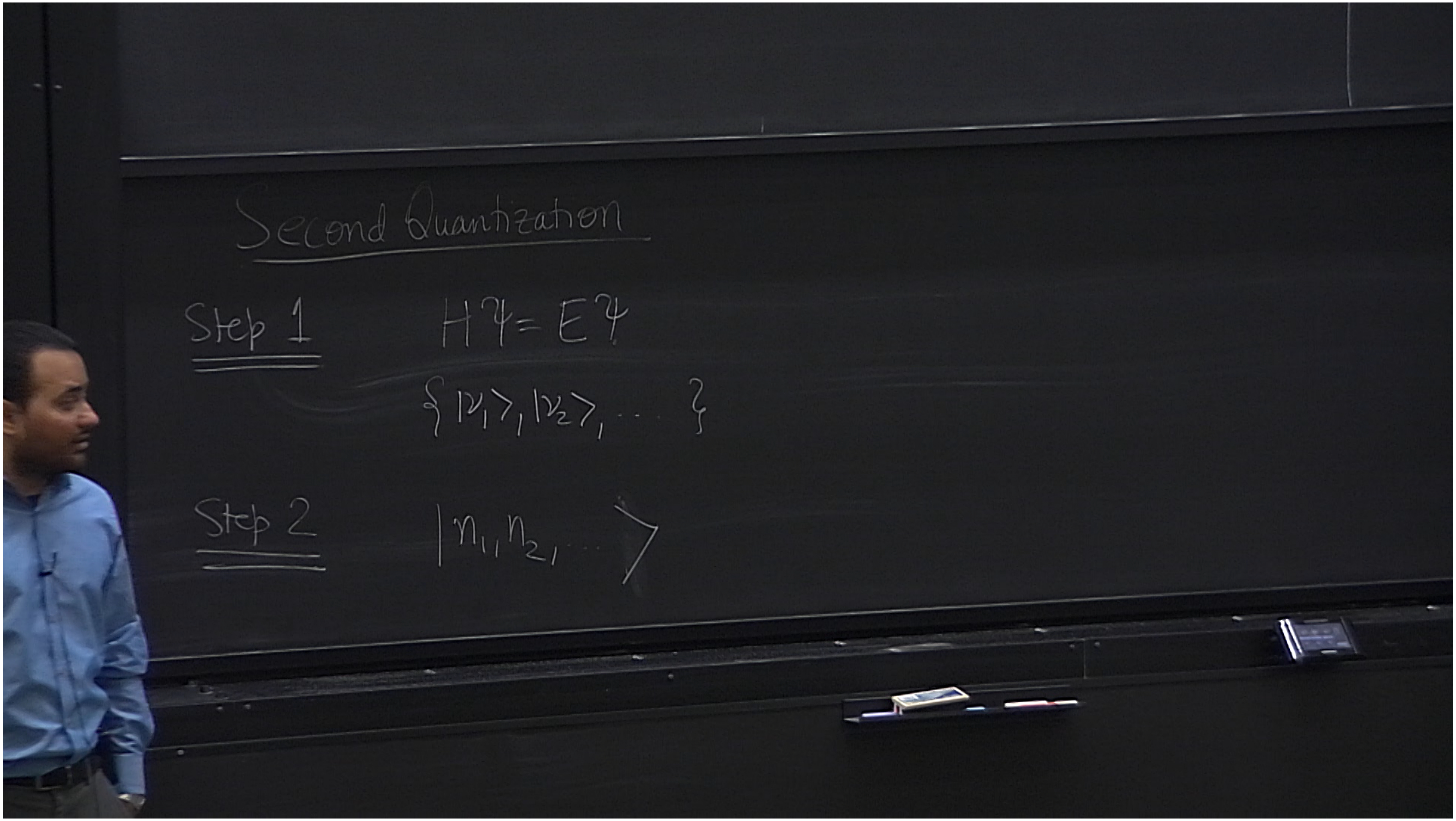
$$\psi_{k_1 k_2}(\bar r_1, \bar r_2) = \frac{1}{\sqrt{2!}} \begin{vmatrix} \psi_{k_1}(\bar r_1) & \psi_{k_1}(\bar r_2) \\ \psi_{k_2}(\bar r_1) & \psi_{k_2}(\bar r_2) \end{vmatrix} = \frac{1}{\sqrt{2!}} \left(\psi_{k_1}(\bar r_1)\, \psi_{k_2}(\bar r_2) - \psi_{k_1}(\bar r_2)\, \psi_{k_2}(\bar r_1) \right)$$

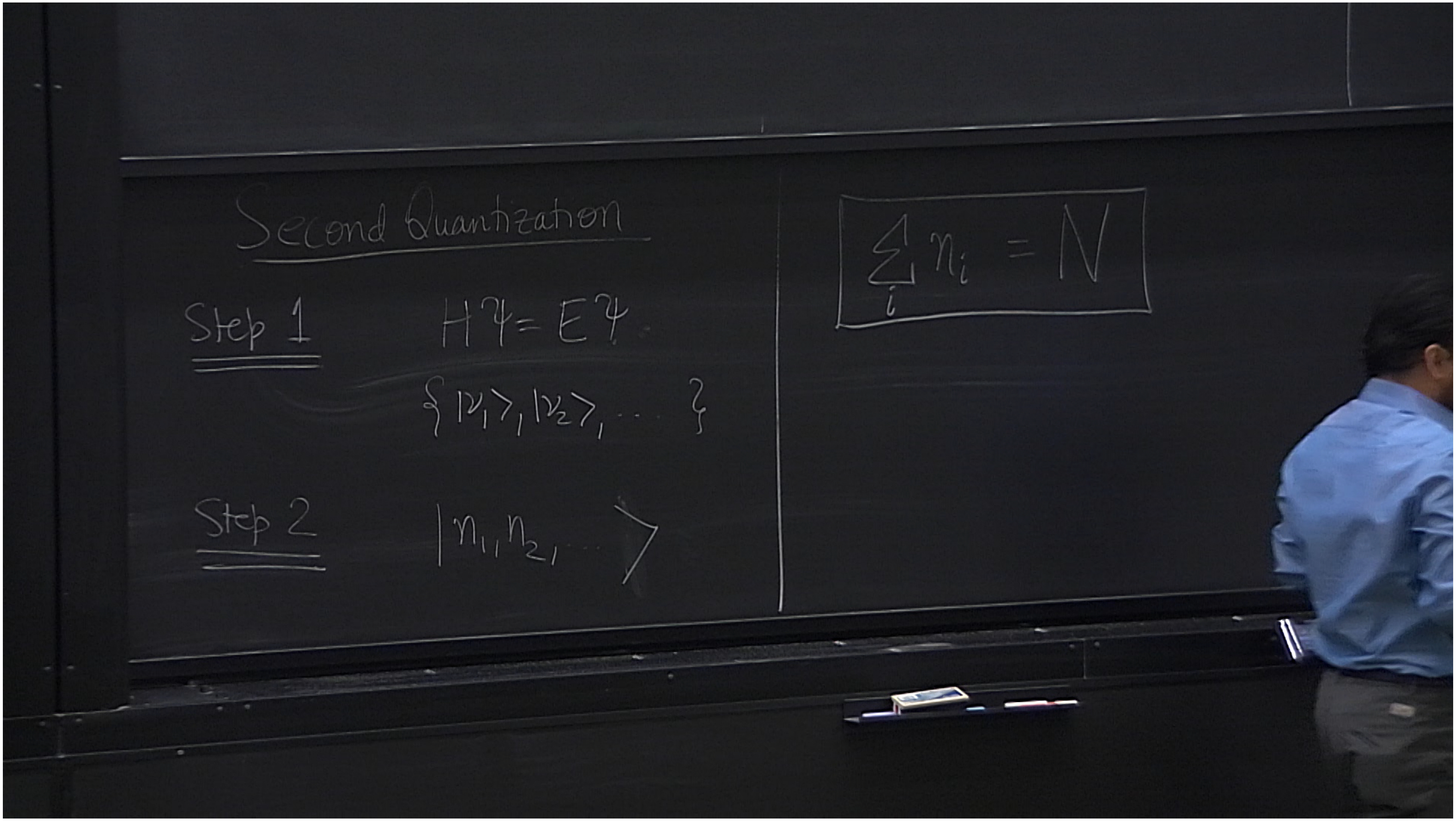
$$\boxed{\Psi(\bar r_1, \bar r_2, \dots, \bar r_N) = \sum_{k_1, k_2, \dots, k_N} \alpha_{k_1 k_2 \dots k_N} \, \Psi_{k_1 k_2 \dots k_N}(\bar r_1, \bar r_2, \dots, \bar r_N)}$$

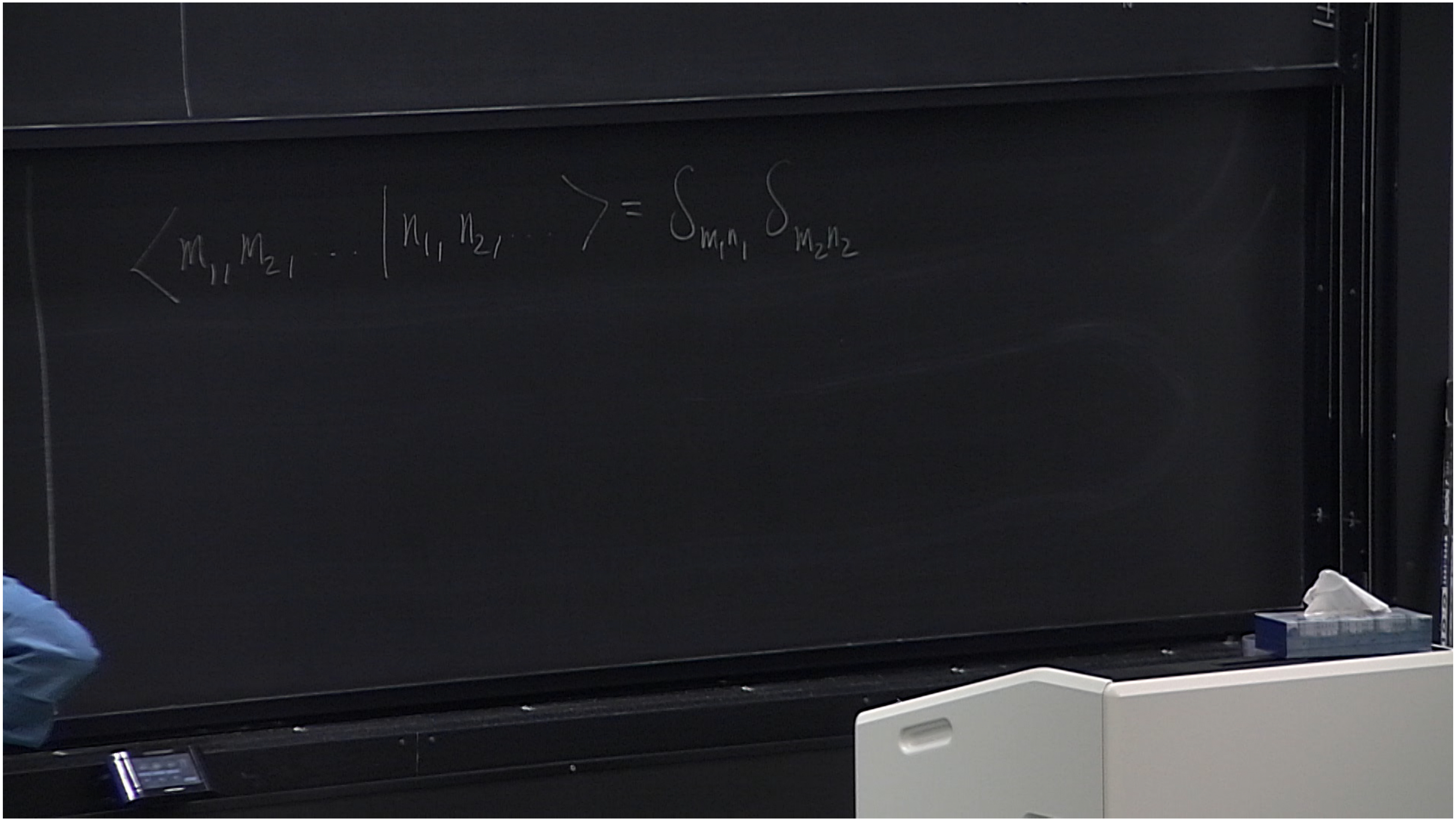

$\psi_{k_1 k_2 \cdots k_N}(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_N) = \hat{S}_{\pm}\, \psi_{k_1}(\bar{r}_1)\, \psi_{k_2}(\bar{r}_2) \cdots \psi_{k_N}(\bar{r}_N)$


Second Quantization


Second Quantization
Step 1
$H\Psi = E\Psi$
$\{|\nu_1\rangle, |\nu_2\rangle, \dots\}$


Second Quantization
Step 1
$H\Psi = E\Psi$
$\{|\nu_1\rangle, |\nu_2\rangle, \dots\}$
Step 2
$|n_1, n_2, \dots\rangle$


Second Quantization
Step 1
$H\Psi = E\Psi$
$\{|\nu_1\rangle, |\nu_2\rangle, \ldots\}$
Step 2
$|n_1, n_2, \ldots\rangle$
$\sum_i n_i = N$


$\langle m_1, m_2, \ldots | n_1, n_2, \ldots \rangle = \delta_{m_1 n_1} \delta_{m_2 n_2}$