Title: PSI 17/18 - Quantum Field Theory II - Lecture 15

Date: Nov 24, 2017 09:00 AM

URL: http://pirsa.org/17110025

Abstract:

Non-abelian Gauge Theories : continued

Yang-Mills Theory $G = SU(2)$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$

$S_{YM} + S_{\substack{\text{GAUGE} \\ \text{FIXING TERM}}}$ $\quad A = (A_\mu^a)$ $\quad a = 1, 3$ group indices

$\longrightarrow$ Adj Repr. of G

$$S_{YM} = -\frac{1}{2g^2}\int d^4x \, \mathrm{Tr}\left[F_{\mu\nu} F^{\mu\nu}\right]$$

$$S_{GF} =$$

Non-abelian Gauge Theories : continued

Yang-Mills Theory $G = SU(2)$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$

$S[A, c, \bar{c}, B] = S_{YM} + S_{\text{GAUGE FIXING TERM}}$

$A = (A_\mu^a)$ $a = 1, 3$ group indices Spin 1

$\searrow$ Adj Repr. of G

$S_{YM} = -\frac{1}{2g^2} \int d^4x \, \mathrm{Tr}[F_{\mu\nu} F^{\mu\nu}]$

$C = (C^a)$ ghost field

$\bar{C} = (\bar{C}_a)$ anti-ghost "

Spin 0

$S_{GF} = \int d^4x \, \bar{C} \, \partial^\mu D_\mu C + \int d^4x \, B \cdot \partial^\mu A_\mu$, B Lagrange multiplier

Gauge Theories : continued

$G = SU(2)$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$

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$\rightarrow$ Adj Repr. of G

GAUGE FIXING TERM

$\int \mathrm{Tr}[F_{\mu\nu} F^{\mu\nu}]$

$C = (C^a)$ ghost field

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Spin 0 fermions

$\partial^\mu D_\mu C + \int d^4x \, B \cdot \partial^\mu A_\mu$, B Lagrange multiplier $\partial^\mu A_\mu = 0$ Landau Gauge

$\int D[A] D[$

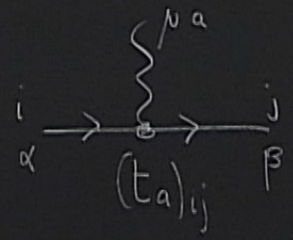
+ Add m

Action for the D

$$\int \mathcal{D}[A]\,\mathcal{D}[\bar{c},c]\,\mathcal{D}[B]\,\exp\left(i\,S[A,c,\bar{c},B]\right) \longrightarrow \text{perturbation theory}$$

+ Add matter: Dirac Fields $\psi = (\psi_i^\alpha)$, $\bar{\psi} = (\bar{\psi}_i^\alpha)$ $\quad i = 1,2 \in$ Fund Rep; $\alpha = 1 \ldots 4$ Dirac indices

Action for the Dirac Field $S_{Dirac} = \int d^4x\ \bar{\psi}(i\not{D} - m)\psi$ $\quad \not{D} = \gamma^\mu D_\mu = \gamma^\mu(\partial_\mu + i A_\mu)$



t_a generators of the group

Landau Gauge

70's Gauge theories are renormalizable

counterterms to g charge (+ Field renormalizations)

- dimensional regularization $d = 4 \rightarrow d = 4 - \epsilon$ UV $\rightarrow$ poles $\frac{1}{\epsilon}$

  do not spoil gauge invariance

- Effective action (1 loop) $\Gamma\left[A_\mu^{\text{background}}\right] = S_{YM}\left[A_\mu^{\text{bkgr}}\right] + \frac{1}{2} \operatorname{Tr} \log\left[S''_{YM}\right]$

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gauge fix $A_\mu = A_\mu^{\text{bkgr}} + \tilde{A}_\mu$ ↳ quantum fluctuation

gauge fixing: background Landau gauge $D_\mu^{\text{bkgr}} \tilde{A}_\mu = 0$ $D_\mu^{\text{bkgr}} \circ = \partial_\mu \circ - i\left[A_\mu^{\text{bkgr}}, \circ\right]$

theories are renormalizable

no calculation here

to g charge (+ Field renormalization)

regularization $d = 4 \rightarrow d = 4 - \epsilon$ UV → poles $\frac{1}{\epsilon}$

gauge invariance

(1 loop) $\Gamma\left[A_\mu^{\text{background}}\right] = S_{YM}\left[A_\mu^{\text{bkgr}}\right] + \frac{1}{2} Tr \log\left[S''_{YM}\right]$

↳ 1 loop

$A_\mu = A_\mu^{\text{bkgr}} + \hat{A}_\mu$

↳ quantum fluctuation

background Landau gauge $D_\mu^{\text{bkgr}} \hat{A}_\mu = 0$ $D_\mu^{\text{bkgr}} \cdot = \partial_\mu - i\left[A_\mu^{\text{bkgr}}, \cdot\right]$

$g_R = g + g^2 \cdot \frac{1}{\epsilon} + \ldots \Rightarrow$ R.G Beta function $\alpha = g^2$

$$\beta_g(g) = g^3 \frac{1}{(4\pi)^2}\left(-\frac{11}{3} C_2 + \frac{4}{3} N_F C_F\right)$$

minus sign

$$\beta_\alpha(\alpha) = \alpha^2 \frac{2}{(4\pi)^2}\left(\text{same coeff.}\right)$$

$C_2(G) = 2$ $SU(2)$

$= N$ $SU(N)$

$=$ 2nd Casimir of G

$N_F =$ # families of Fermions

$C_F(G) = \frac{1}{2}$ fundamental repr

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't Hooft, Gross + Wilczek, Politzer for SU(3)

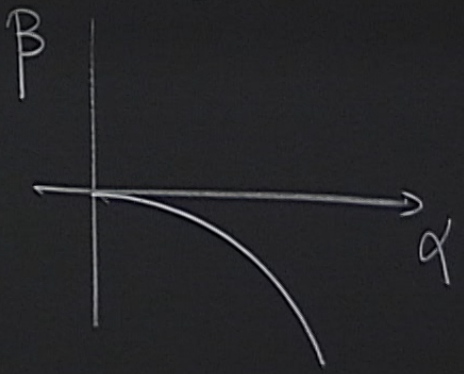
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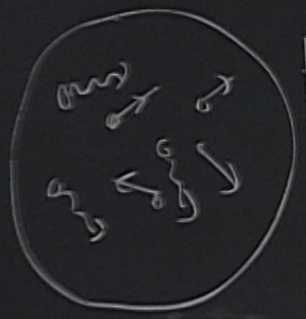
Asymptotic Freedom (at High Energies)

$$E\frac{d}{dE}\alpha_{eff}(E) = \beta(\alpha_{eff}(E))$$

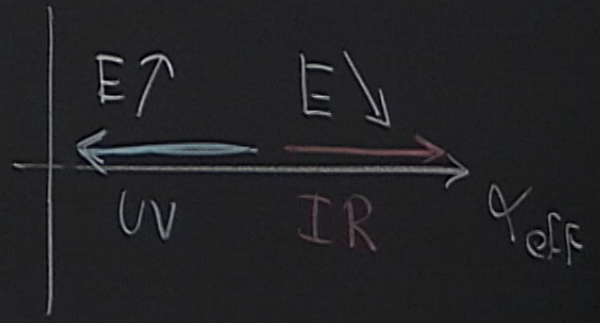


High Energy interaction becomes negligible

"Free gas" of gauge bosons "gluons"
fermions "quarks"



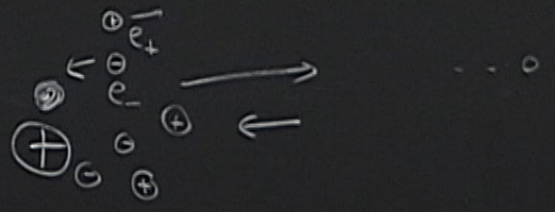
Hadron ⟸ parton model



Strong coupling at low energies

QED $\beta(\alpha) = +\alpha^2$ $\alpha(E) \searrow$ $E \searrow 0$

screening effect



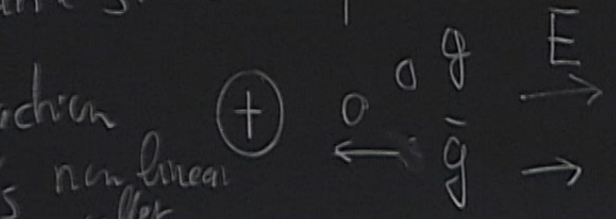
QCD SU(3)

anti screening effect

Peskin S. semi-quantitative description

backreaction

$E \times B$'s non linear effect



energy scale E_{IR} such that $\alpha_{eff}(E_{IR}) \nearrow \infty$

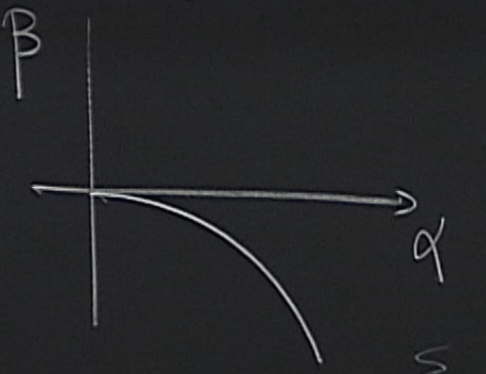
is it a problem

renormalization energy $\mu_0 \longrightarrow$ coupling α_0

$$E_{IR} = \mu_0 \exp\left(-\frac{c}{\alpha}\right)$$

dom (at High Energies)

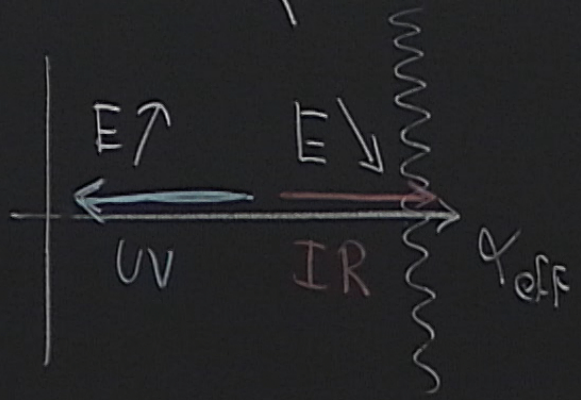
$= \beta(\alpha_{eff}(E))$



ca becomes negligible

"gluons"

"quarks"



Parton model

Strong coupling at low

QED $\beta(\alpha) = +\alpha^2$

screening effect



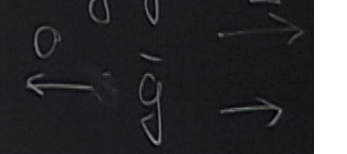
QCD SU(3)

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backreaction

E × B's non-linear effect



③

energy scale E_{IR} such that $\alpha_{eff}(E_{IR}) \nearrow \infty$

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renormalization energy $\mu_0 \longrightarrow$ coupling α_0

$E_{IR} = \mu_0 \exp\left(-\frac{c}{\alpha_0}\right)$ invisible in perturbation theory

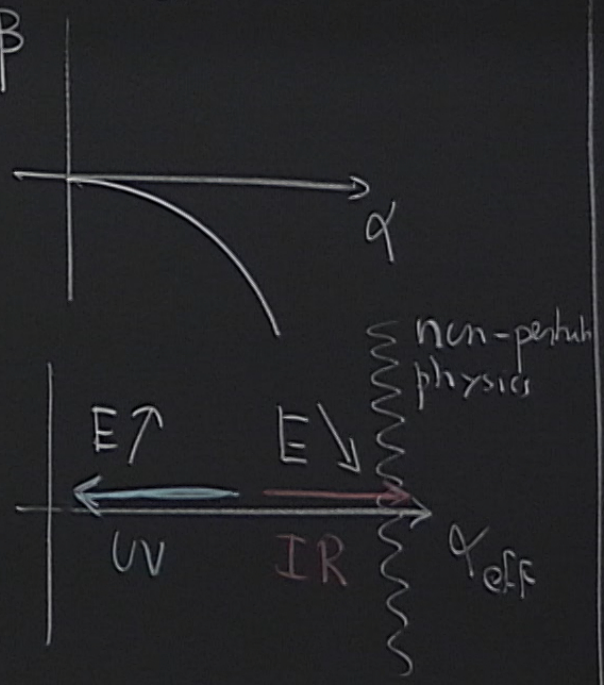
$$E\frac{d}{dE}\alpha = -c\alpha^2$$

$$\frac{d}{d}\left(\frac{1}{\alpha}\right) = -c\frac{dE}{E} \quad -\frac{1}{\alpha} + \frac{1}{\alpha_0} = -c\log\left(\frac{E}{\mu_0}\right)$$

$$\boxed{E = \mu_0 \exp\left(-\frac{1}{c\alpha_0}\right)}$$

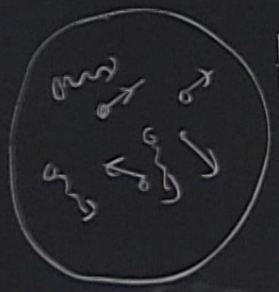
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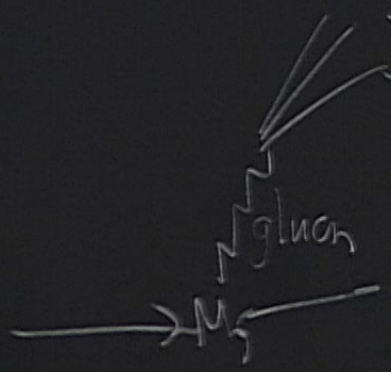
Hadron ⟸ parton model

expect that $\text{mass}_{\text{bound states}} \propto E_{IR}$

Spectrum of Y.M. theories ⟵ non-perturbative methods

Confinement — no non-color singlets (neutral bound-states) can be observed

can be conducted as asymptotic states



QCD Sum Rules (SVZ)

Idea: Add to perturbation theory terms proportional to non-perturbative v.e.v of operators

$\langle 0 | T_r(F_{\mu\nu} F^{\mu\nu})(x) | 0 \rangle$ $\quad dim = E^4$

$\langle 0 | \phi | 0 \rangle$ in ϕ^4 theory $\neq 0$ Spont sym breaking

QCD Sum Rules (SVZ) : physical quantities = perturb

Idea: Add to perturbation theory terms proportional to non-perturbative v.e.v of operators

signal of non-perturbative physics

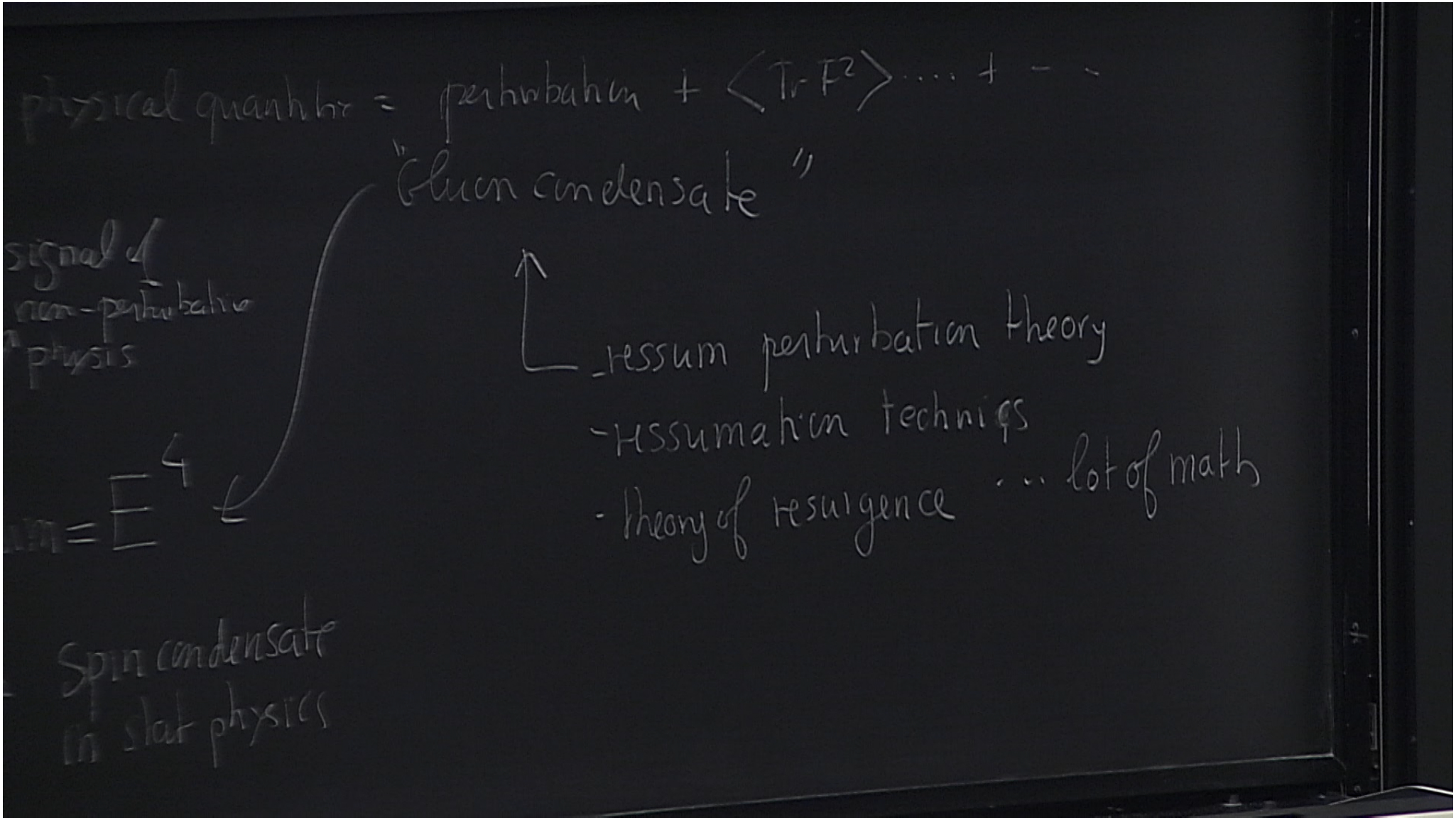
"Gluon c

$\langle 0 | \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu})(x) | 0 \rangle = E^4_{IR} \neq 0$ dim $= E^4$

maybe sense of this

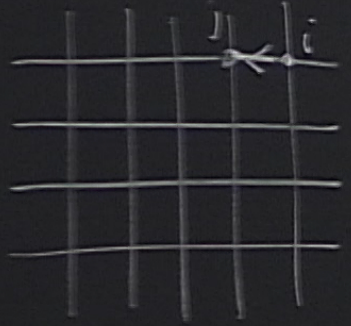
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Spin condensate in stat physics


physical quantity = perturbation + $\langle Tr\, F^2 \rangle$ ... + - -
"Gluon condensate"
signal of
non-perturbative
physics
$= E^4$
Spin condensate
in stat physics
resum perturbation theory
- resummation techniqs
- theory of resurgence ... lot of math

Lattice Gauge Theories ← K. Wilson 74 (+ Wegner)

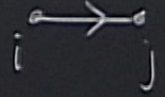
Euclidean space time → Lattice (square lattice) → Functional integral and can we compute.



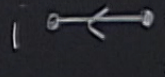
- scalar field $\phi(x) \longrightarrow$ vertices ϕ_i
- gauge field $A_\mu(x)\,dx^\mu \longrightarrow$ edges e of the lattice
  is a connection → connect two points
  how to move on the lattice
- gauge transformations local → vertices

oriented edge $e = (i, j) \longrightarrow g(e) \in G$ element of the group.

gauge field



$e^{-1} = (j, i) \longrightarrow g(e^{-1}) = g(e)^{-1}$



G-connection on the lattice

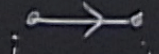
$g \in SU(2)$

Covariant derivative : scalar field ϕ vertex $i \longrightarrow \phi(i) \in$ Fund Representation of G

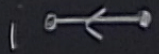
$\phi \in \mathbb{R}^2$

oriented edge $e = (i,j) \longrightarrow g(e) \in G$ element of the group

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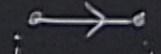
Covariant derivative : scalar field ϕ vertex $i \longrightarrow \phi(i) \in$ Fund Representation of G

$\phi \in \mathbb{C}^2$

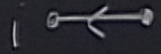
$e = (i,j)$ $D_e \phi$

derivative of ϕ along edge e

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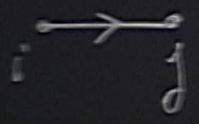
$e = (i,j) \qquad D_e \phi = \phi(j) - \phi(i)$

derivative of ϕ along edge e

- Gauge Transformations h = collection of
  at each vertex i ← independent transformation $h(i) \in G$

$\phi \to \phi_h$ $\quad \phi_h(i) = h(i) \cdot \phi(i)$ $\quad$ Like in the continuum theory

- covariant derivative



$\phi_h(i) = h(i)\,\phi(i)$

$\phi_h(j) = h(j)\,\phi(j)$

$D_e \phi_h = \phi_h(j) - g(e)\,\phi_h(i) = h(j) \cdot \phi(j) - g(e)\, h(i$

$= h(j)\,(D_e \phi_h)\, h^{-1}(i)$

= collection of

dependent $h(i) \in G$

nsformation

$g(e) \to g_h(e)$

$$\boxed{g_h(e) = h(j)\, g(e)\, h^{-1}(i)}$$

(i) Like in the continuum theory

$$D_e \phi_h = \phi_h(j) - g(e)\,\phi_h(i) = h(j)\cdot\phi(j) - g_h(e)\, h(i)\,\phi(i)$$

$$= h(i)\,(D_e \phi_h)\, h^{-1}(i)$$

- Gauge Transformations h = collection of at each vertex i ← independent transformation $h(i) \in G$

$g(e) \to g_h(e)$

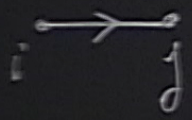
$$g_h(e) = h(j)\, g(e)$$

$\phi \to \phi_h$

$$\phi_h(i) = h(i) \cdot \phi(i)$$

Like in the continuum theory

- covariant derivative



$$\phi_h(i) = h(i)\,\phi(i)$$
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$$D_e \phi_h = \phi_h(j) - g(e)\,\phi_h(i) = h(j)\cdot\phi(j) - g_h(e)$$
$$= h(j)\,(D_e \phi)\,h^{-1}(i)$$

h = collection of

- independent $h(i) \in G$ transformation

$g(e) \to g_h(e)$

$$g_h(e) = h(j)\, g(e)\, h^{-1}(i)$$

$\phi(i) - \phi(i)$

Like in the continuum theory

$\phi(i)$

$\phi(j)$

$$D_e \phi_h^{(j)} = \phi_h(j) - g(e)\phi_h(i) = h(j)\cdot\phi(j) - g_h(e)\, h(i)\, \phi(i)$$

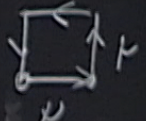
$$= h(j)\,(D_e \phi)(j)$$

Action for the connection

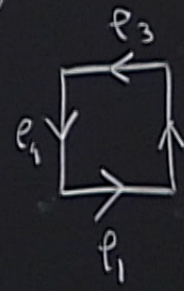
$$\exp\left(-\frac{1}{g^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right]\right)$$

$$F_{\mu\nu} = i\left[D_\mu, D_\nu\right]$$

curvature tensor



plaquette p

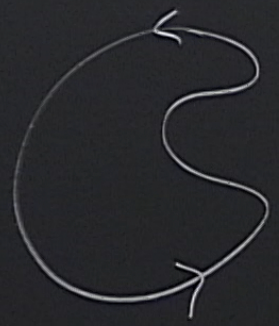


$g(p) = g(e_4)g(e_3)g(e_2)g(e_1) = \prod_{e\in p} g(e)$ element of the group

gauge invariant

$$\prod_{\text{plaquette}} \mathrm{Re}\left[\mathrm{Tr}\left[g(p)\right]\right] \leftarrow \exp\left(-S_{YM}[A]\right)$$

$A_\nu dx^\nu$)



U(1) theory

Mass = Flux

Phase of Aharonov-Bohm Effect

at distance L $\sim \frac{1}{L}$ at small distances Coulomb

- Gauge

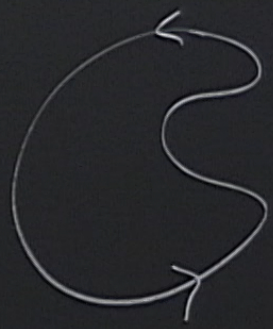
at ea

$\phi \rightarrow$

- covarian

$i \rightarrow$

Wilson Loop $\longleftarrow$ $\exp\left(i \oint A_\mu dx^\mu\right)$



U(1)

Masr

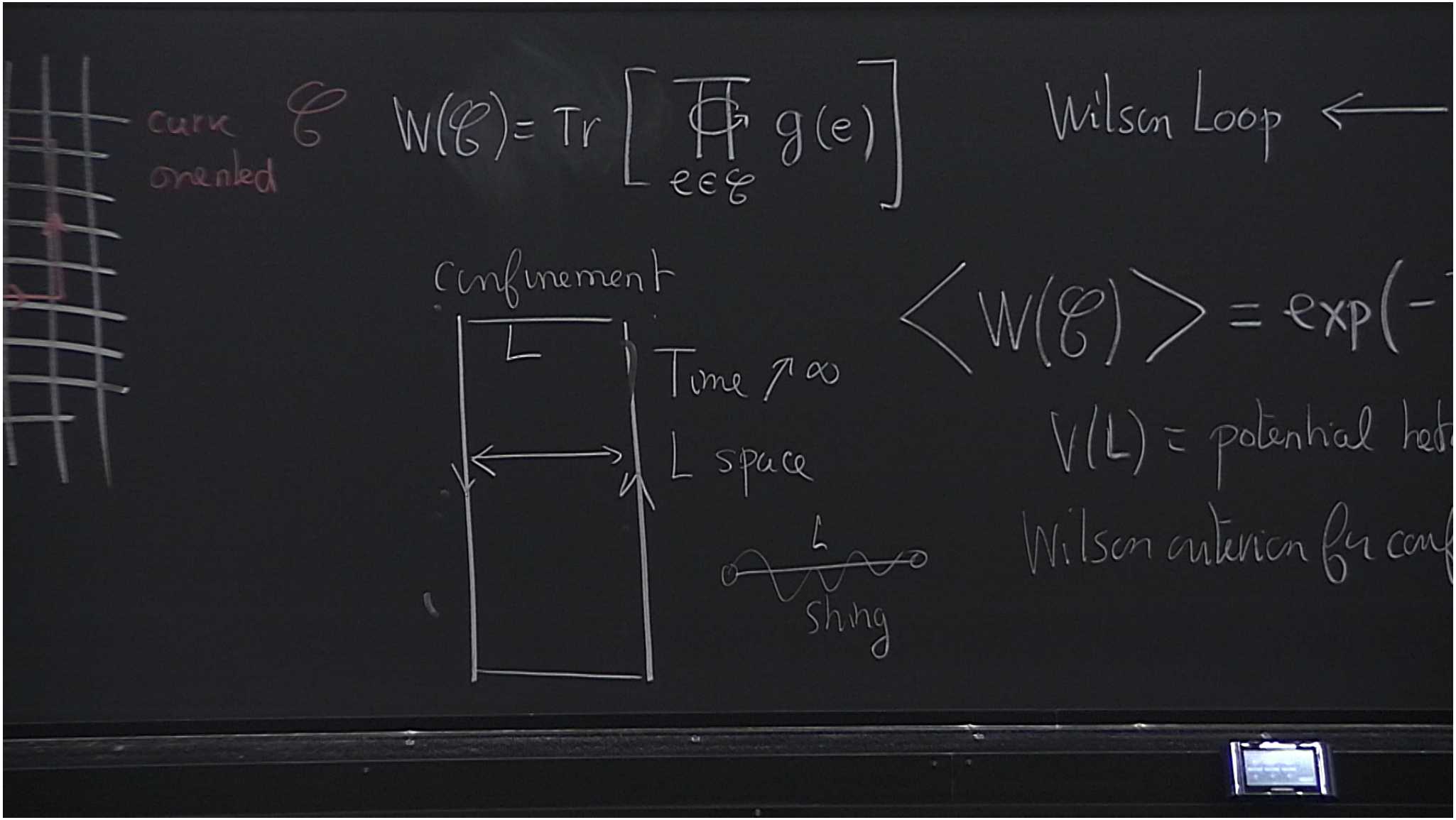
Phase of

$\langle W(\mathcal{C}) \rangle = \exp(-T \times V(L))$

$V(L)$ = potential between two charges at distance L $\sim \frac{1}{L}$

Wilson criterion for confinement $W(L) \sim \alpha_{\text{String}} L$ at large distance

string tension


curve
oriented
$\mathcal{C}$
$W(\mathcal{C}) = \mathrm{Tr}\left[\prod_{e \in \mathcal{C}} g(e)\right]$
Wilson Loop
confinement
L
Time $\nearrow \infty$
L space
L
String
$\langle W(\mathcal{C}) \rangle = \exp(-$
$V(L)$ = potential
Wilson criterion for

$\prod_{eee} g(e)$

Wilson Loop $\longleftarrow$ $\exp\left(i \oint A_\mu dx^\mu\right)$

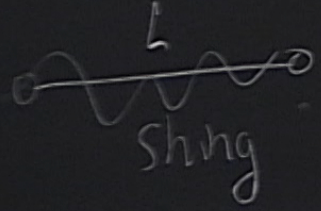
$\langle W(\mathcal{C}) \rangle = \exp(-T \times V(L))$

Time $\nearrow \infty$

L space

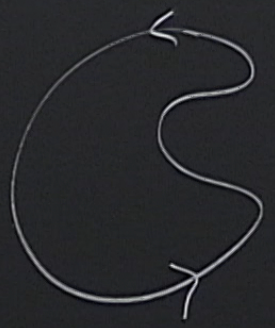
$V(L)$ = potential between two charges at distance L

Wilson criterion for confinement $W(L) \sim \alpha_{String} L$ at



String

$\exp\left(i \oint A_\mu dx^\mu\right)$



U(1) theory

Magn. = Flux

Phase of Aharonov-Bohm Effect

$\Gamma \times V(L))$

...een two charges at distance L

$\sim \sqrt{\frac{(\text{charge})^2}{L}}$ at short distances
Coulomb

...rement $V(L) \sim \alpha_{\text{String}} L$ at large distance
string tension