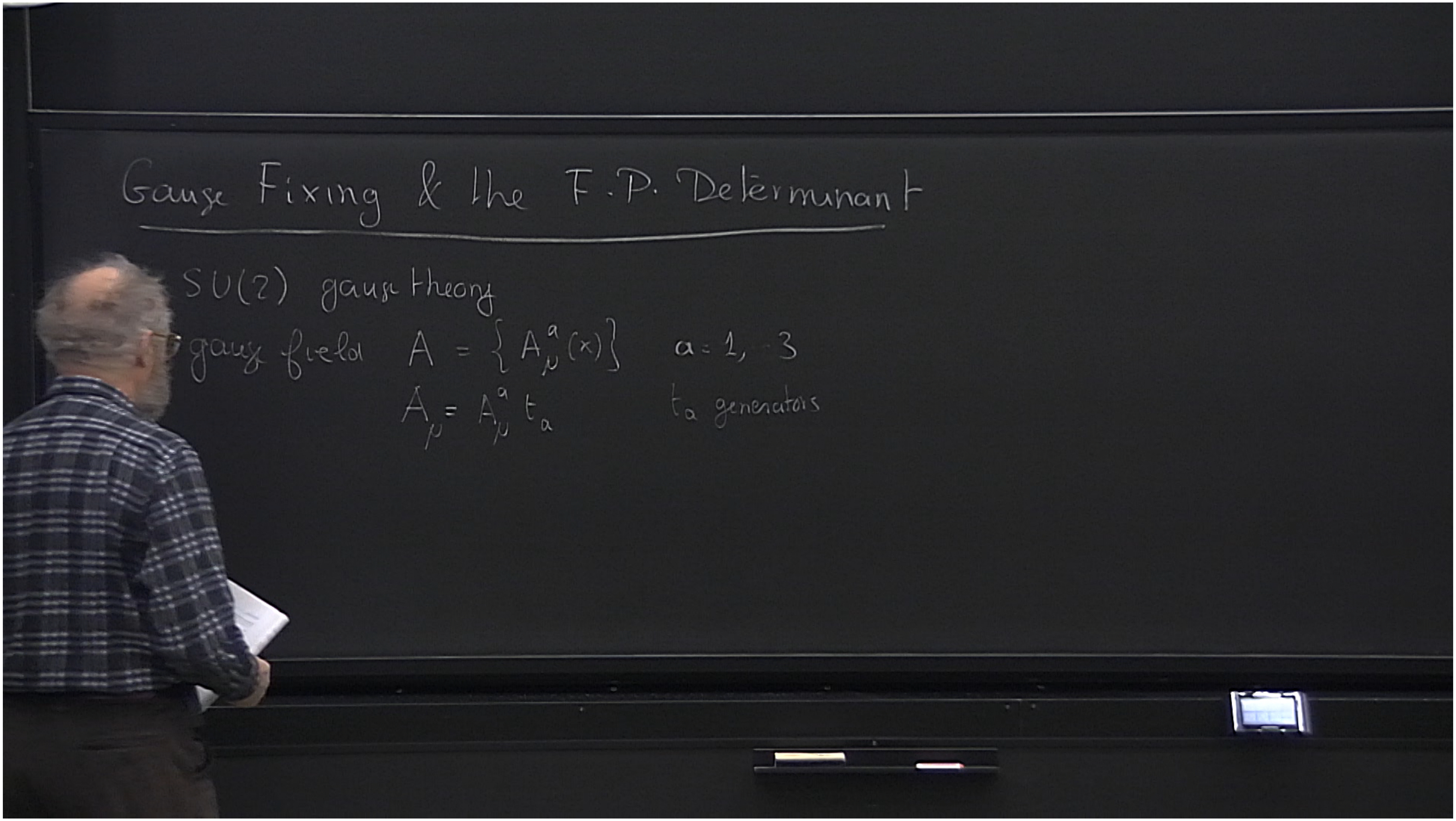
Title: PSI 17/18 - Quantum Field Theory II - Lecture 14

Date: Nov 23, 2017 09:00 AM

URL: http://pirsa.org/17110024

Abstract:

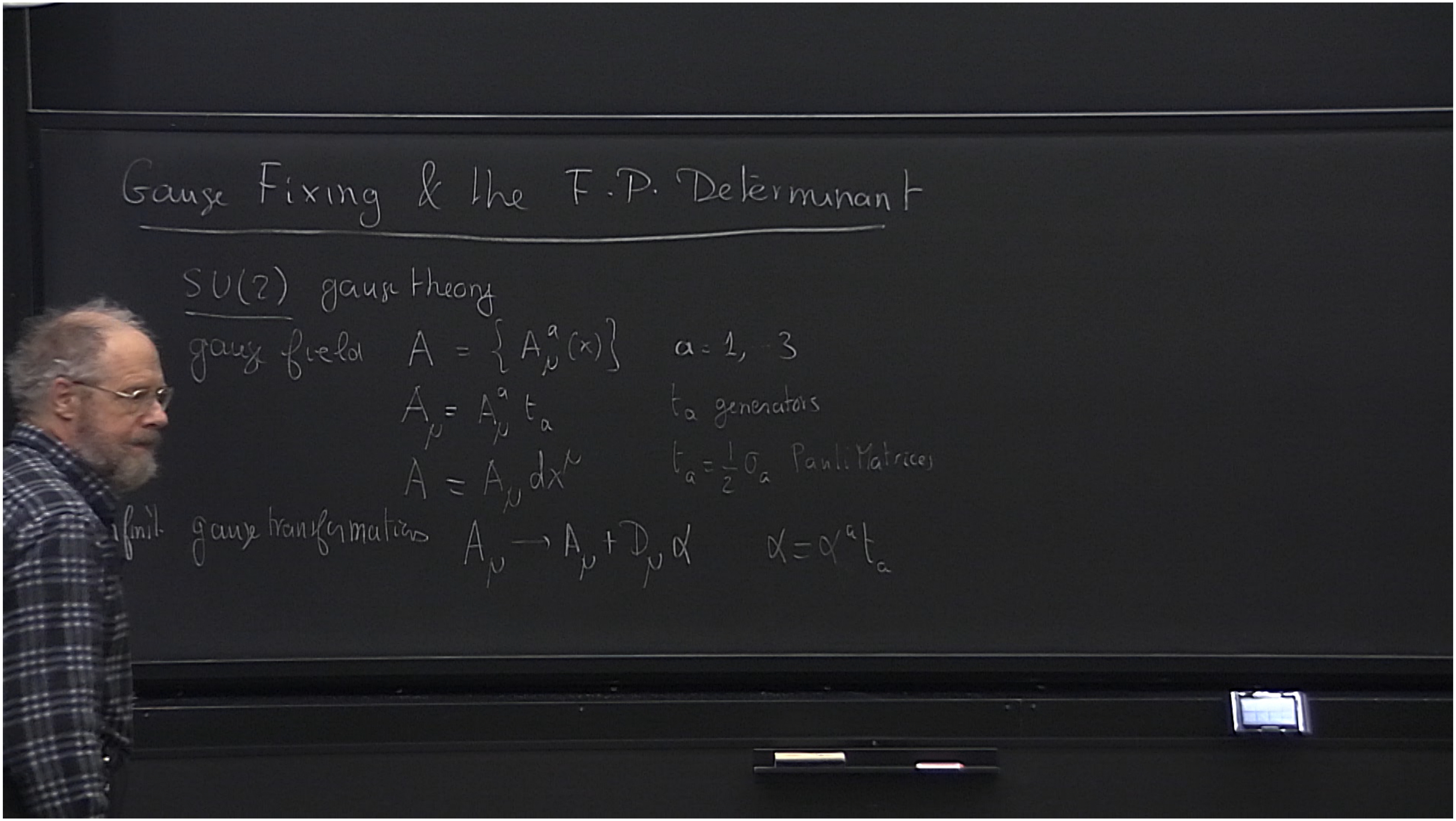


Gauge Fixing & the F.P. Determinant

SU(2) gauge theory

gauge field $A = \{ A^a_\mu(x) \}$ $a = 1, \ldots 3$

$A_\mu = A^a_\mu t_a$ t_a generators



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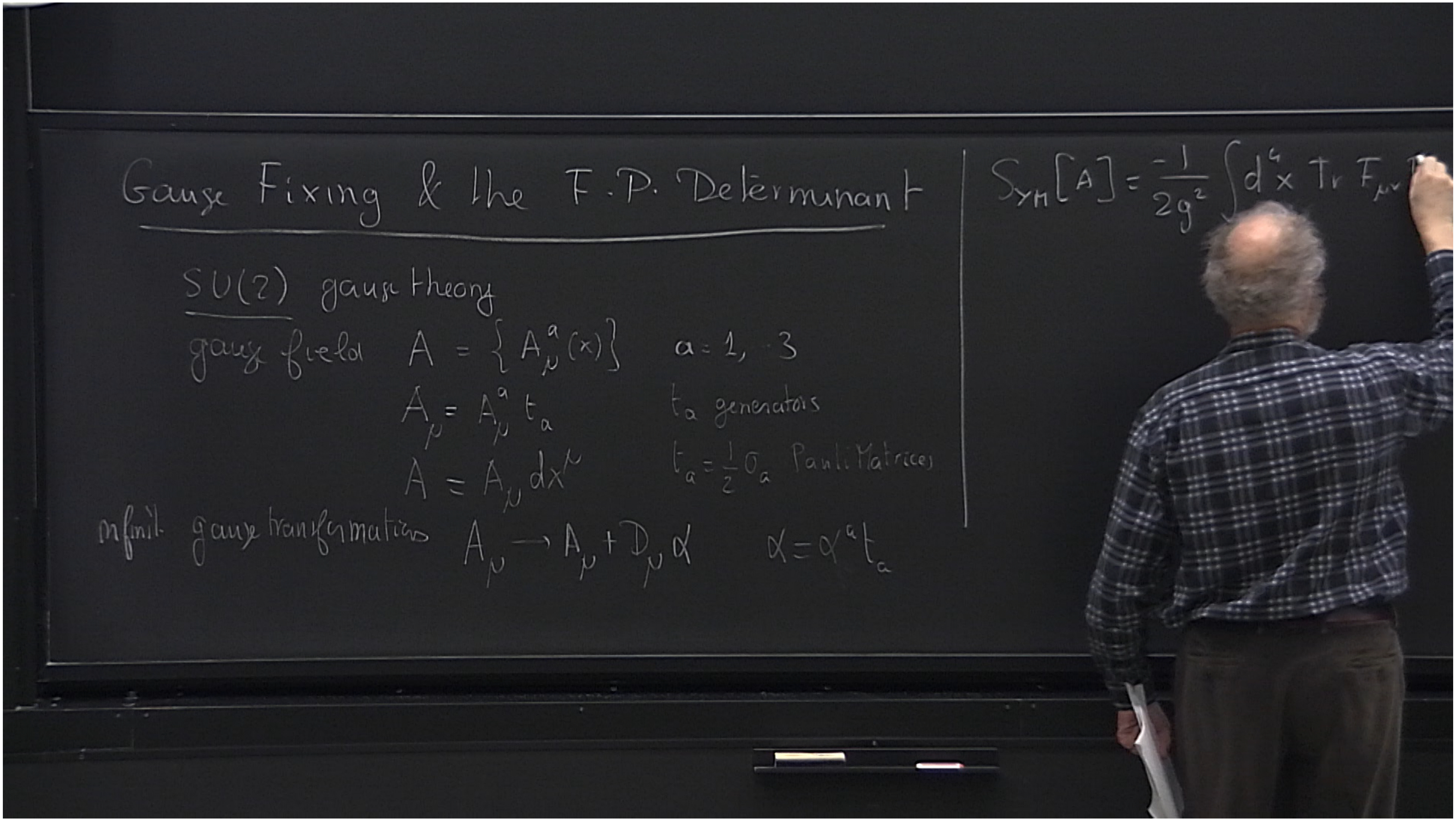
$A = A_\mu dx^\mu$ $t_a = \frac{1}{2}\sigma_a$ Pauli Matrices

finit. gauge transformations $A_\mu \longrightarrow A_\mu + D_\mu \alpha$ $\alpha = \alpha^a t_a$



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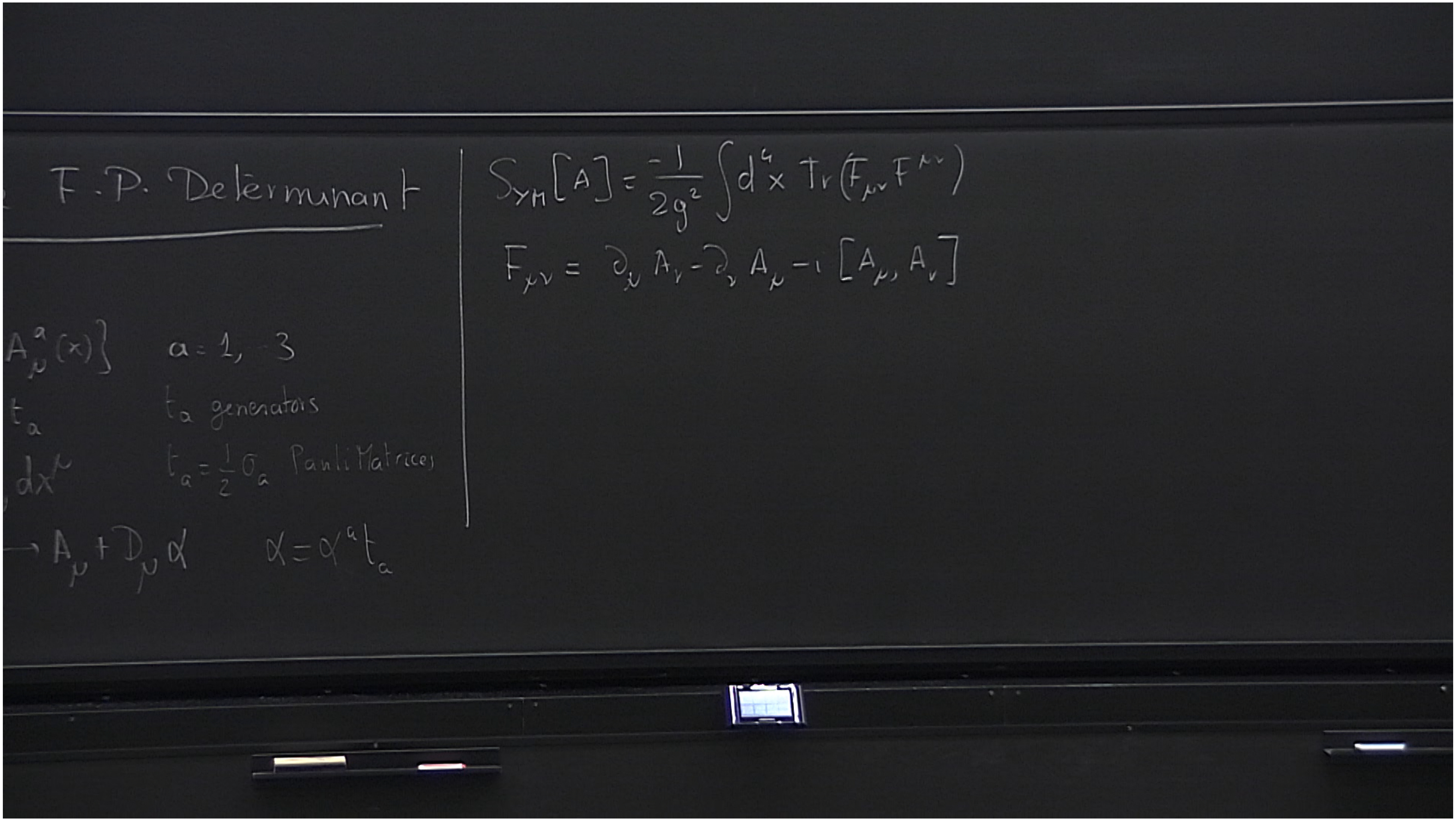
$S_{YM}[A] = \frac{-1}{2g^2}\int d^4x \, \mathrm{Tr}\, F_{\mu\nu}$



F.P. Determinant

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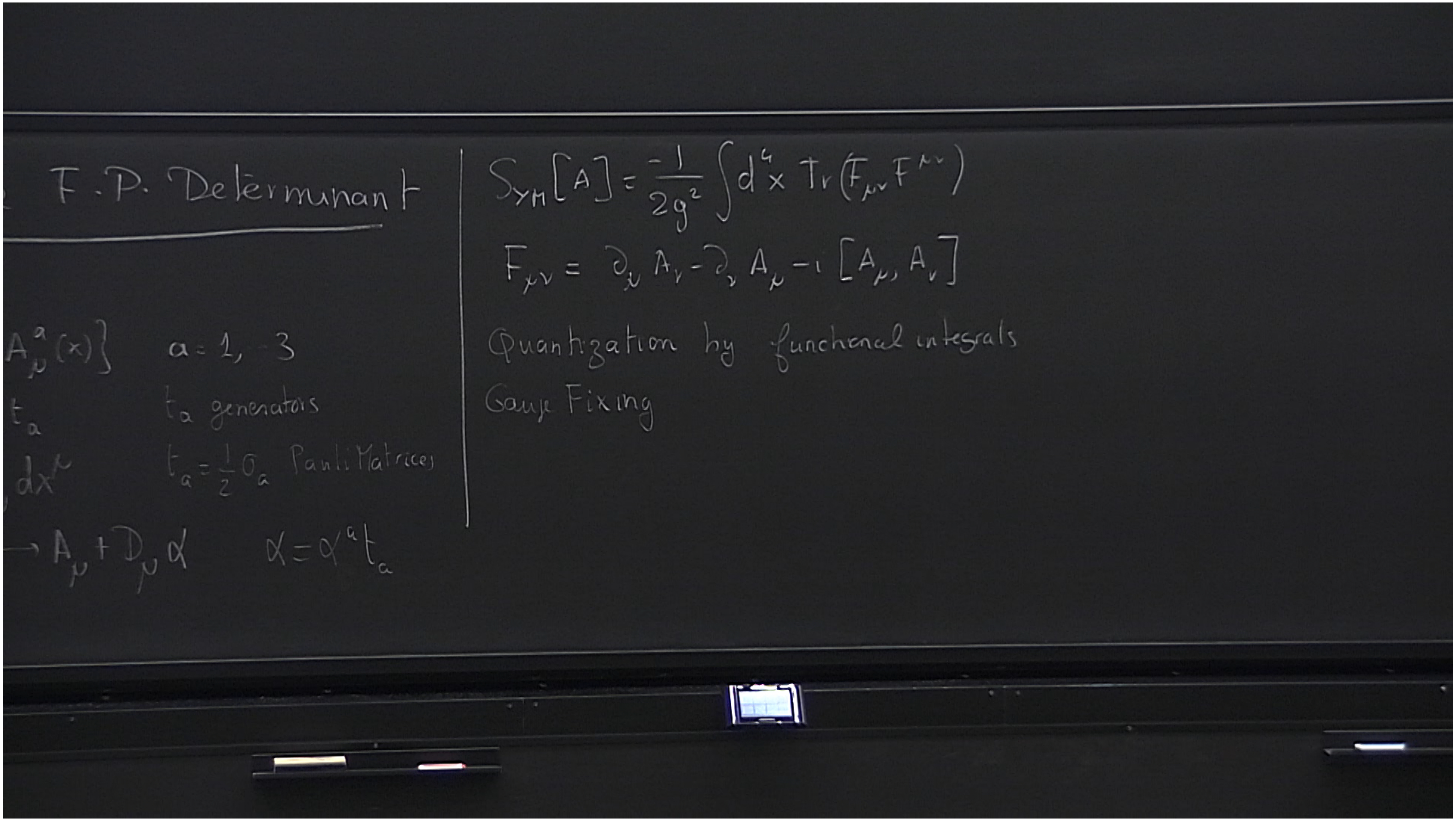
$\longrightarrow A_\mu + D_\mu \alpha$ $\alpha = \alpha^a t_a$

$$S_{YM}[A] = \frac{-1}{2g^2}\int d^4x \, \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$



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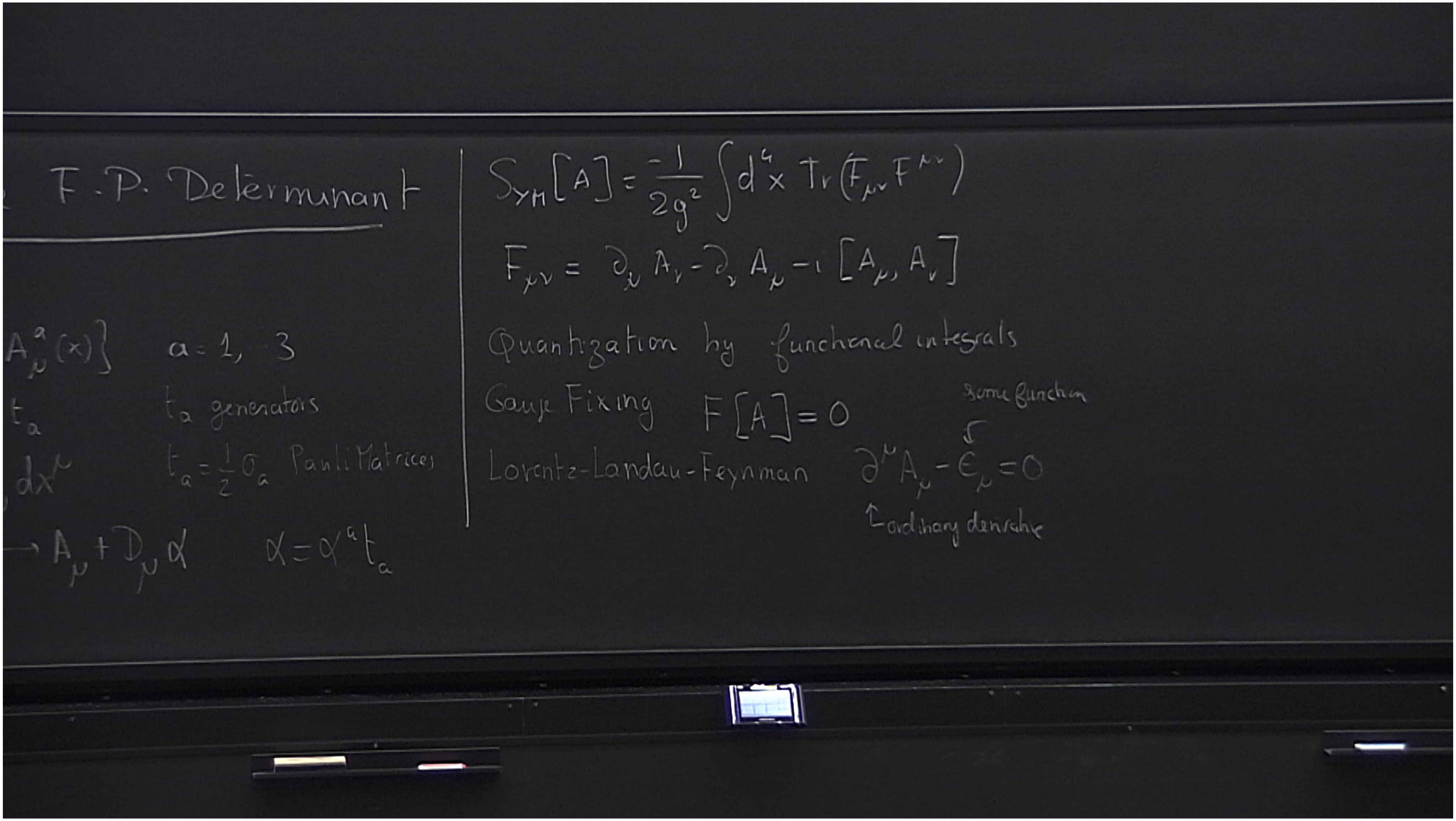
Quantization by functional integrals

Gauge Fixing



F.P. Determinant

$A^a_\mu(x)\}$ $\quad a = 1, \cdot\cdot 3$

t_a $\quad t_a$ generators

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$\to A_\mu + D_\mu \alpha \qquad \alpha = \alpha^a t_a$

$$S_{YM}[A] = \frac{-1}{2g^2}\int d^4x \, Tr\left(F_{\mu\nu}F^{\mu\nu}\right)$$

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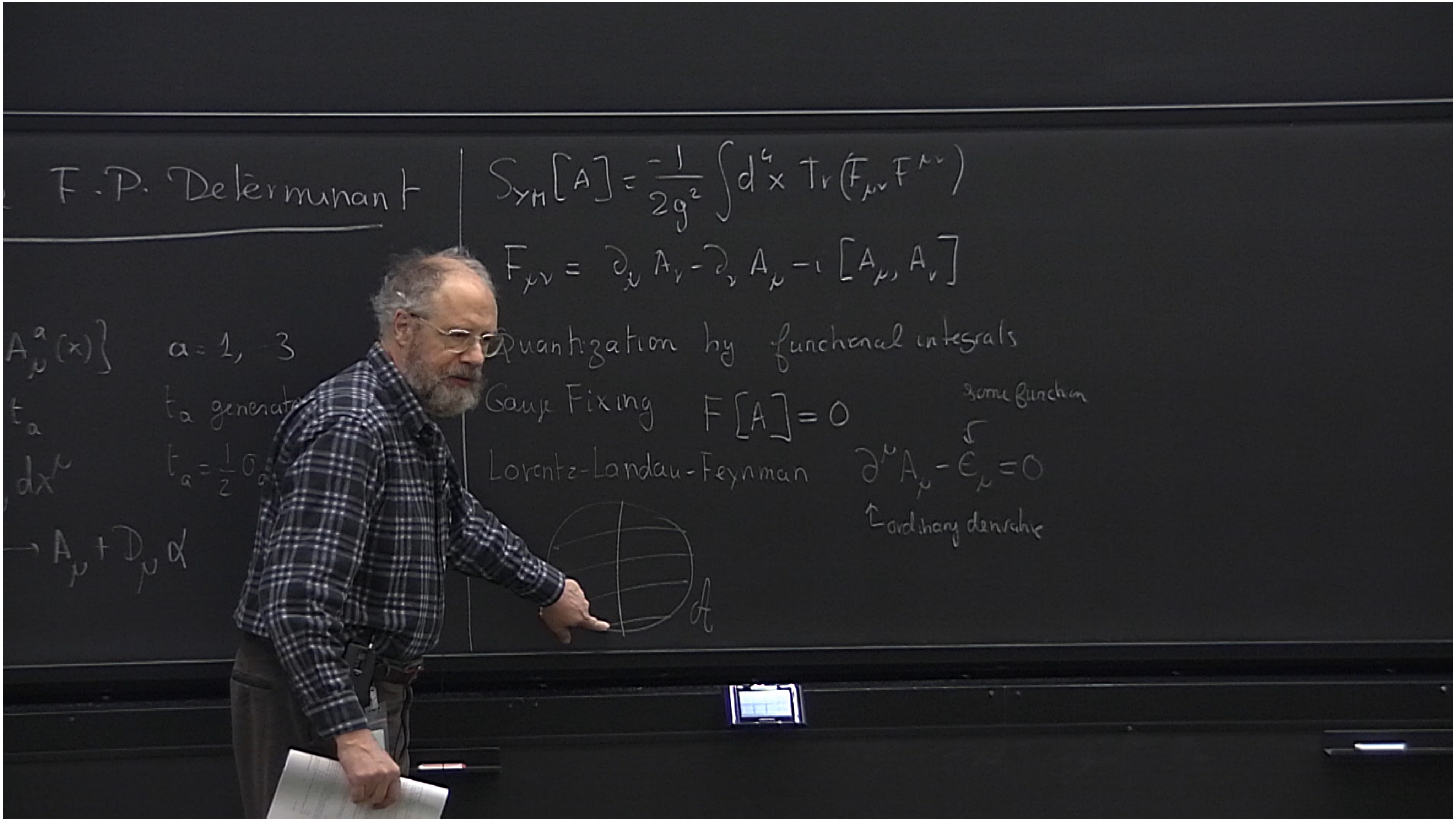
Quantization by functional integrals

Gauge Fixing $\quad F[A] = 0$

some function

Lorentz-Landau-Feynman $\quad \partial^\mu A_\mu - \epsilon_\mu = 0$

ordinary derivative



F.P. Determinant

$A^a_\mu(x)\}$ $a = 1, \cdot\, 3$

t_a t_a generat

dx^μ $t_a = \frac{1}{2}\sigma_a$

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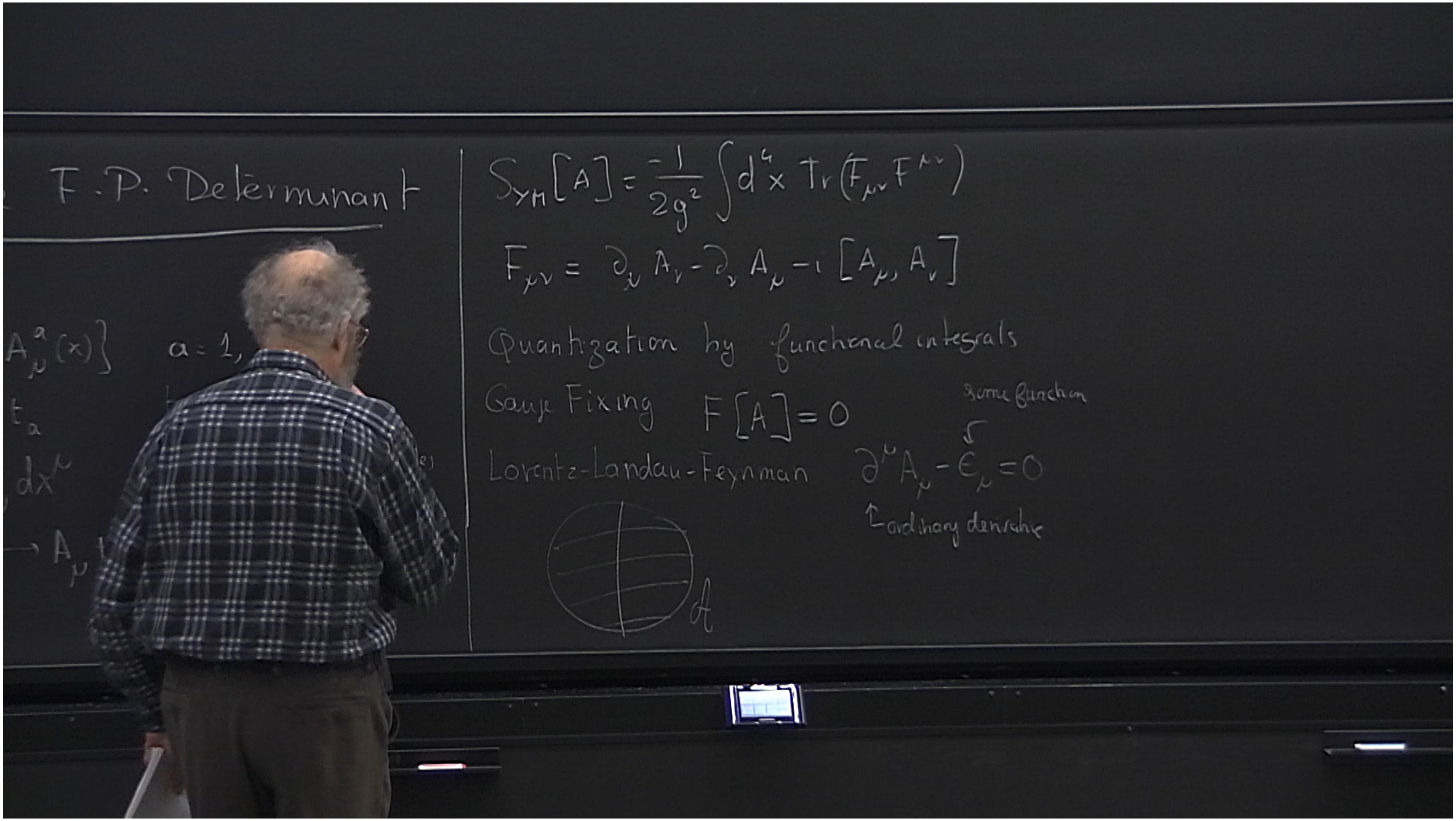

F.P. Determinant

$\{A_\mu^a(x)\}$ $a = 1,$

t_a

dx^μ

$\to A_\mu$

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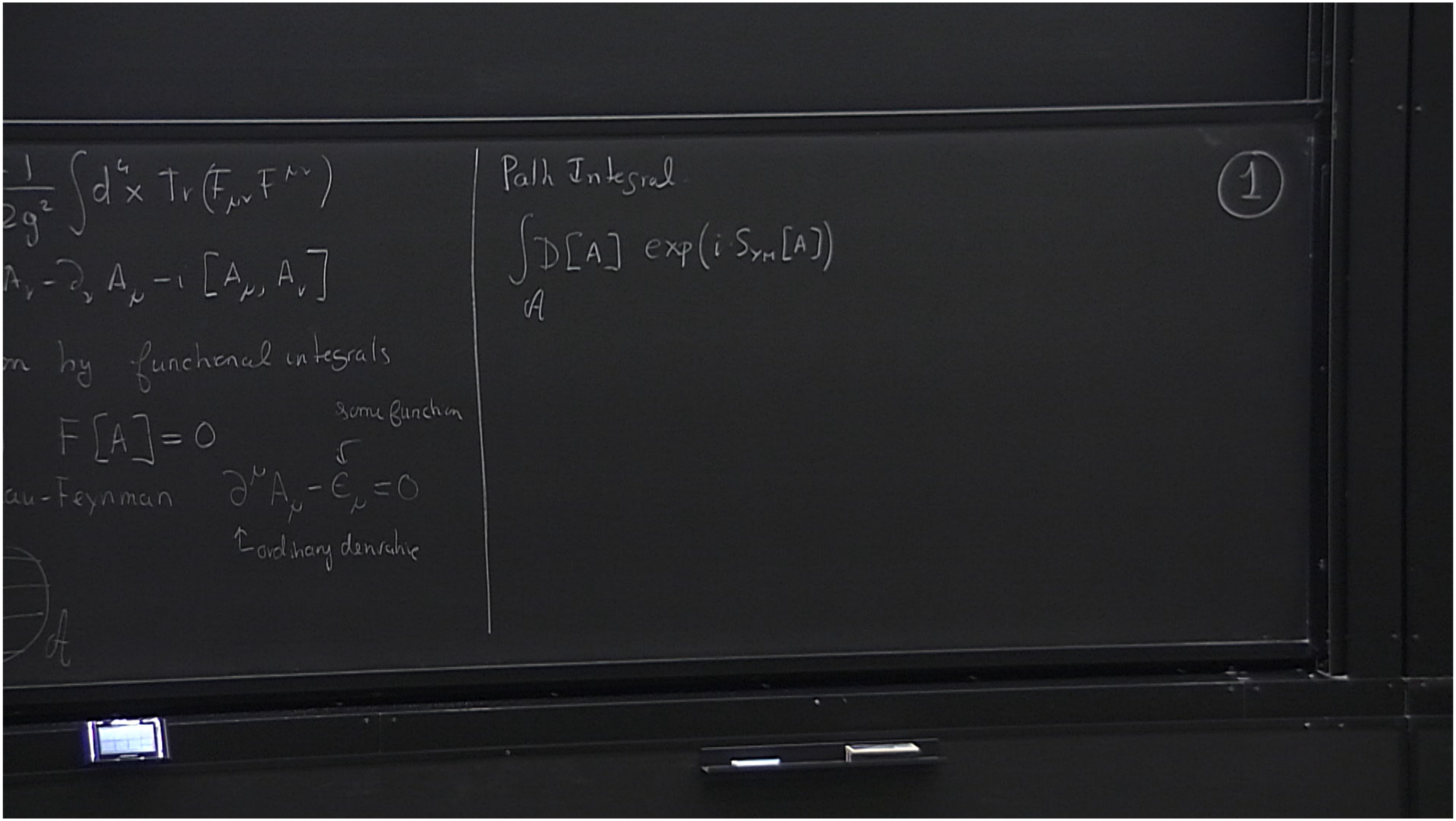


$$\frac{1}{2g^2}\int d^4x \, \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

$$A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]$$

on by functional integrals

$$F[A] = 0$$

some function

au-Feynman $\partial^\mu A_\mu - \epsilon_\mu = 0$

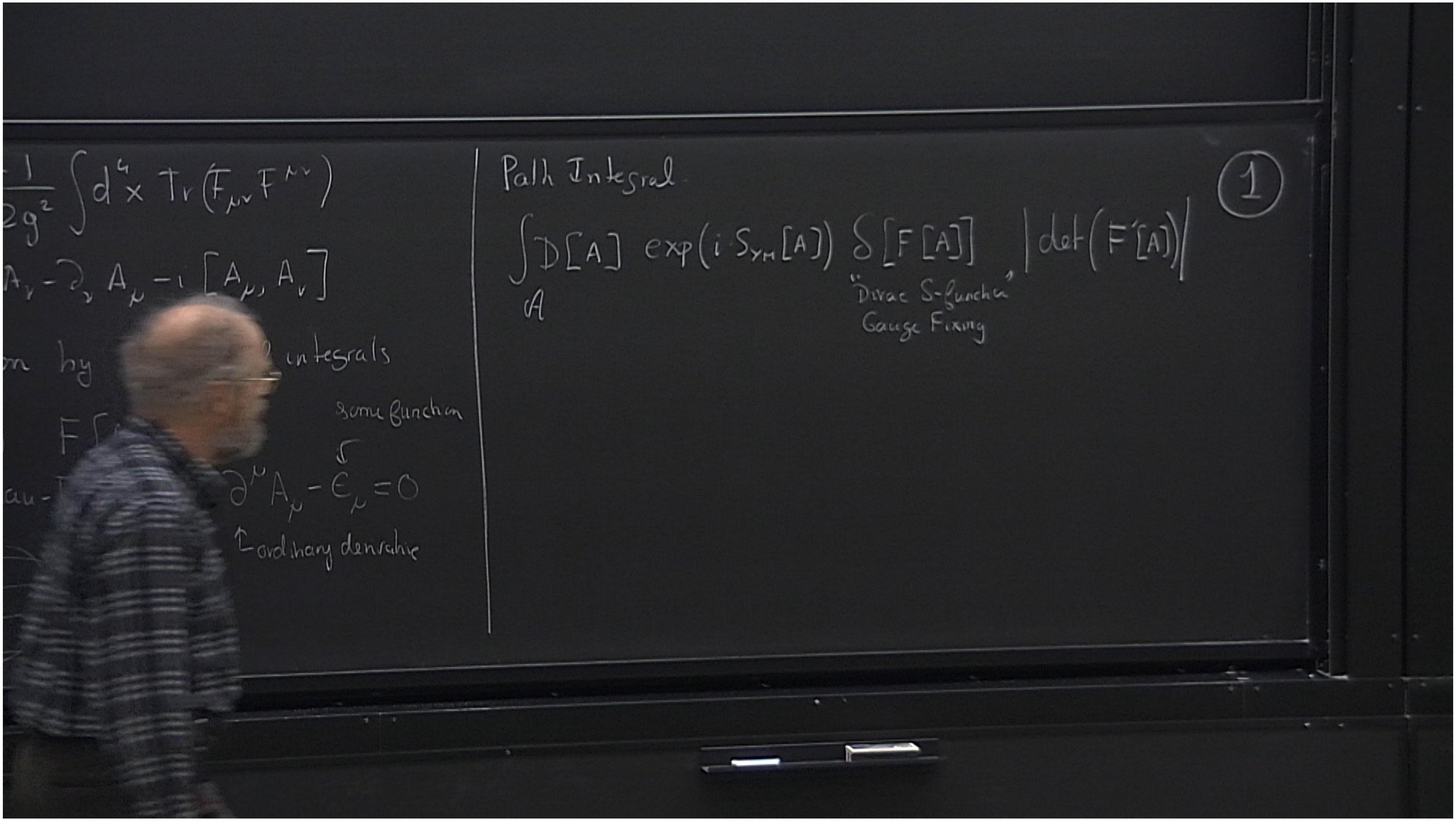
↑ ordinary derivative

$\mathcal{A}$

①

Path Integral

$$\int_{\mathcal{A}} D[A] \, \exp(i \cdot S_{YM}[A])$$



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$$A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]$$

on by ... integrals

some function

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$$\partial^\mu A_\mu - \epsilon_\mu = 0$$

↑ ordinary derivative

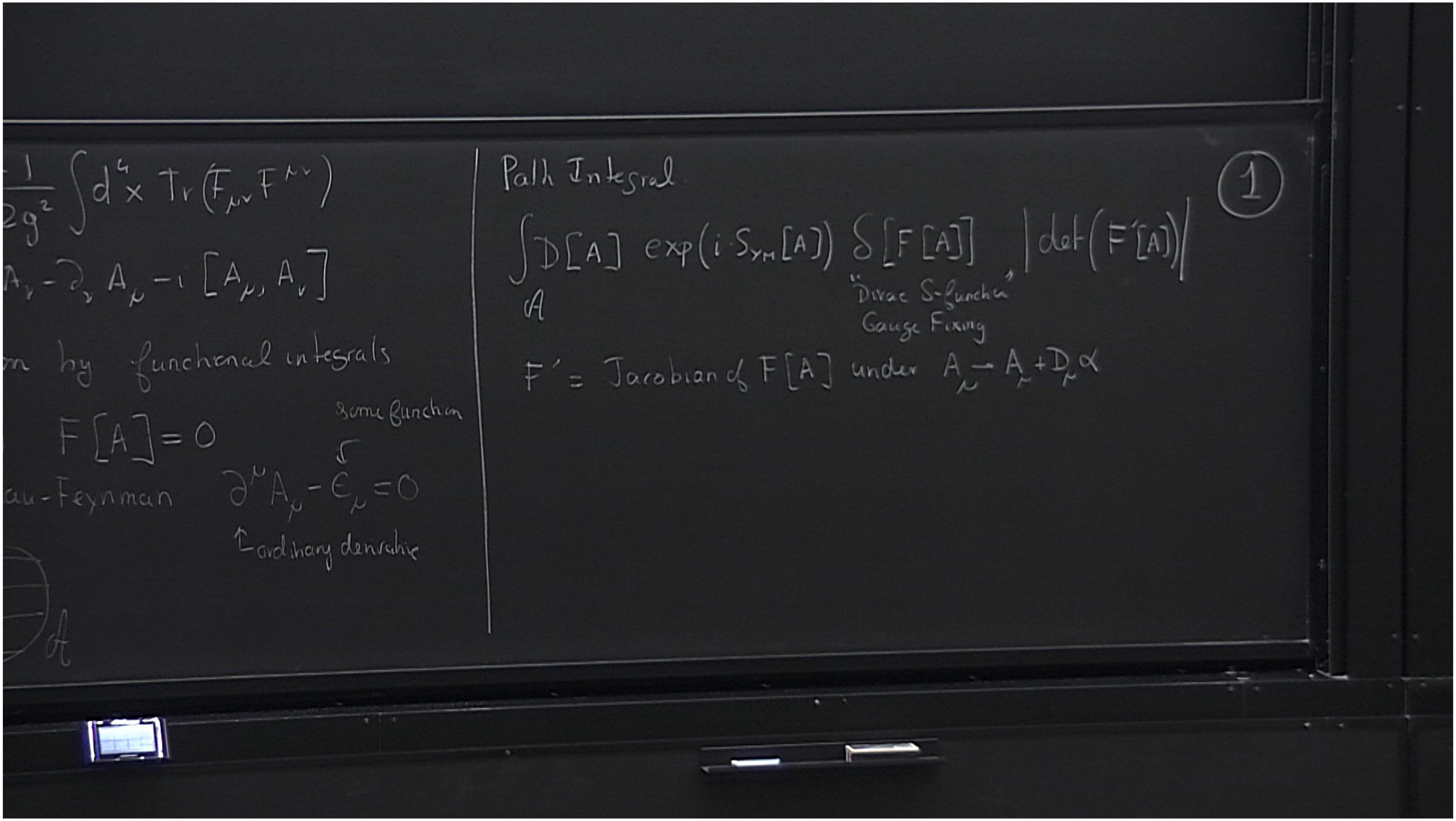
Path Integral

①

$$\int_{A} D[A] \, \exp(i\, S_{YM}[A]) \, \delta[F[A]] \, \left|\det(F'[A])\right|$$

"Dirac δ-function"

Gauge Fixing



$\frac{1}{2g^2}\int d^4x \, \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$

$A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]$

on by functional integrals

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Path Integral ①

$$\int_{\mathcal{A}} D[A] \; \exp(i\, S_{YM}[A]) \; \delta[F[A]] \, , \; \left|\det(F'[A])\right|$$

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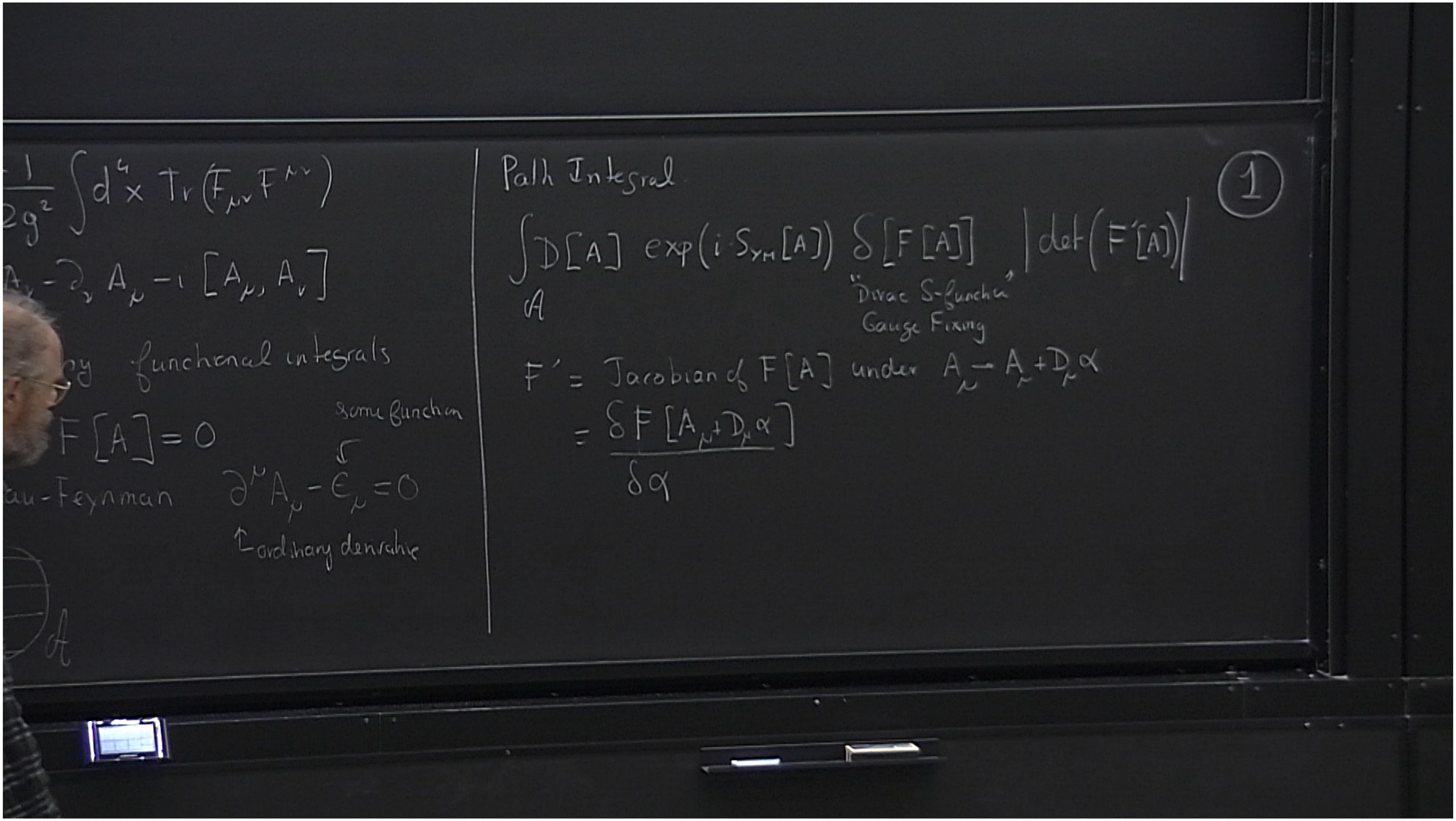
Gauge Fixing

$F' =$ Jacobian of $F[A]$ under $A_\mu \to A_\mu + D_\mu \alpha$



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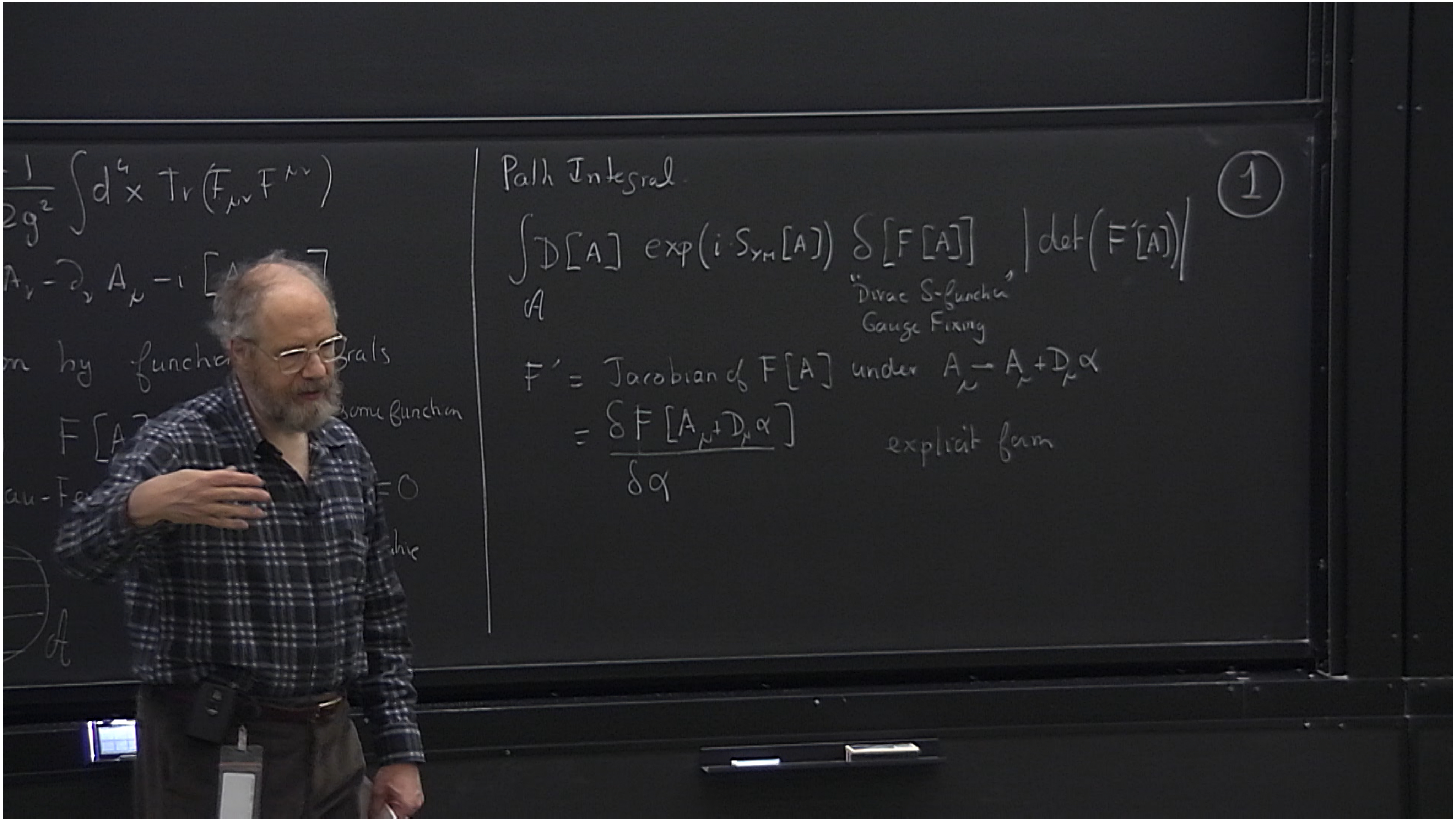
Gauge Fixing

$F' =$ Jacobian of $F[A]$ under $A_\mu \to A_\mu + D_\mu \alpha$

$$= \frac{\delta F[A_\mu + D_\mu \alpha]}{\delta \alpha}$$



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$A_\nu - \partial_\nu A_\mu - i\,[A$

on by functi grals

some function

$F[A$

au-Fe $= 0$

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Path Integral ①

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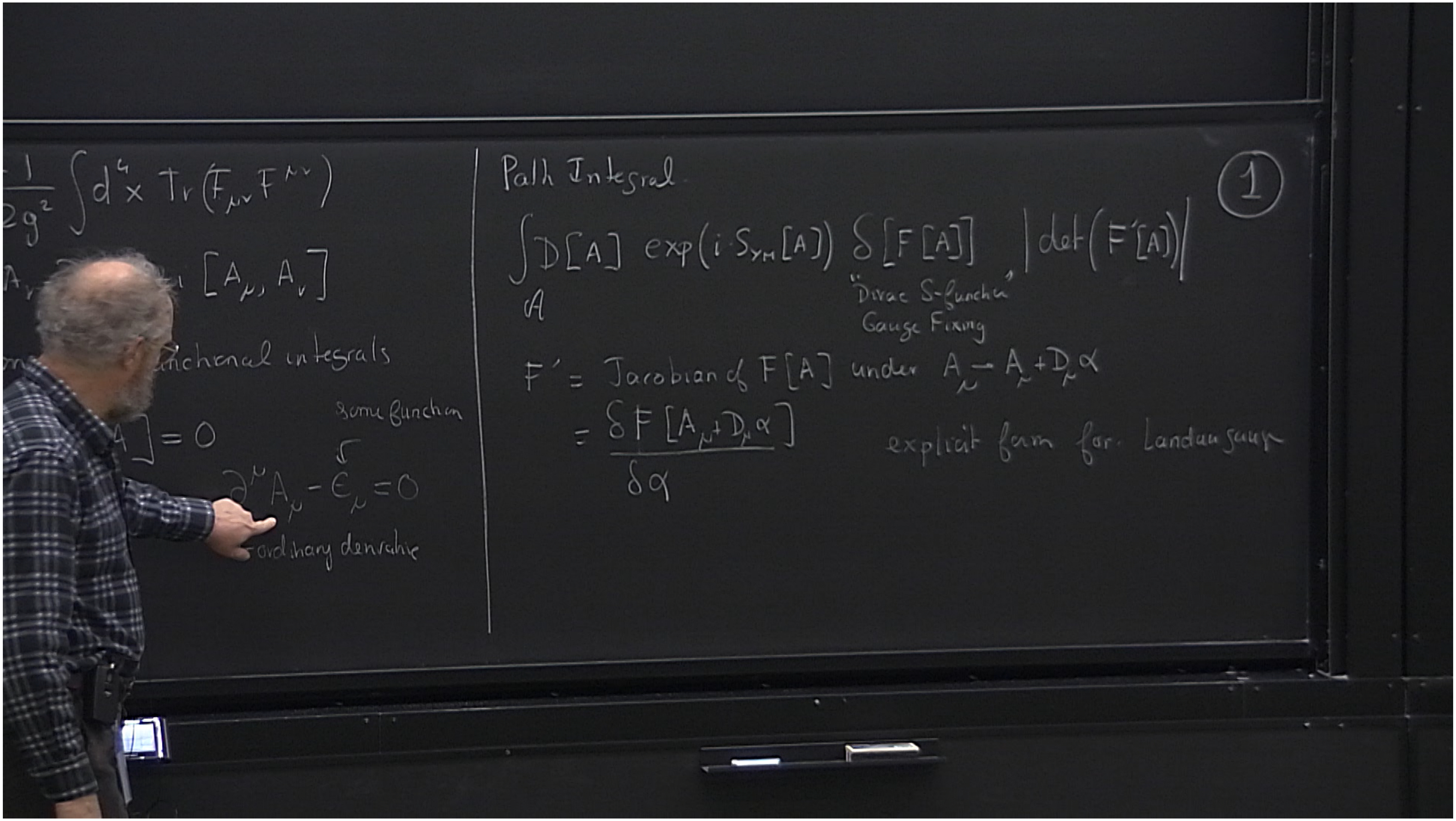
explicit form



$$\frac{1}{2g^2}\int d^4x \, \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

$$A_\nu \quad [A_\mu, A_\nu]$$

nchional integrals

$$A] = 0$$

some function

$$\partial^\mu A_\mu - \overset{\uparrow}{\epsilon_\mu} = 0$$

ordinary derivative

Path Integral

①

$$\int_A D[A] \exp(i S_{YM}[A]) \, \delta[F[A]] \, \left|\det(F'[A])\right|$$

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Gauge Fixing

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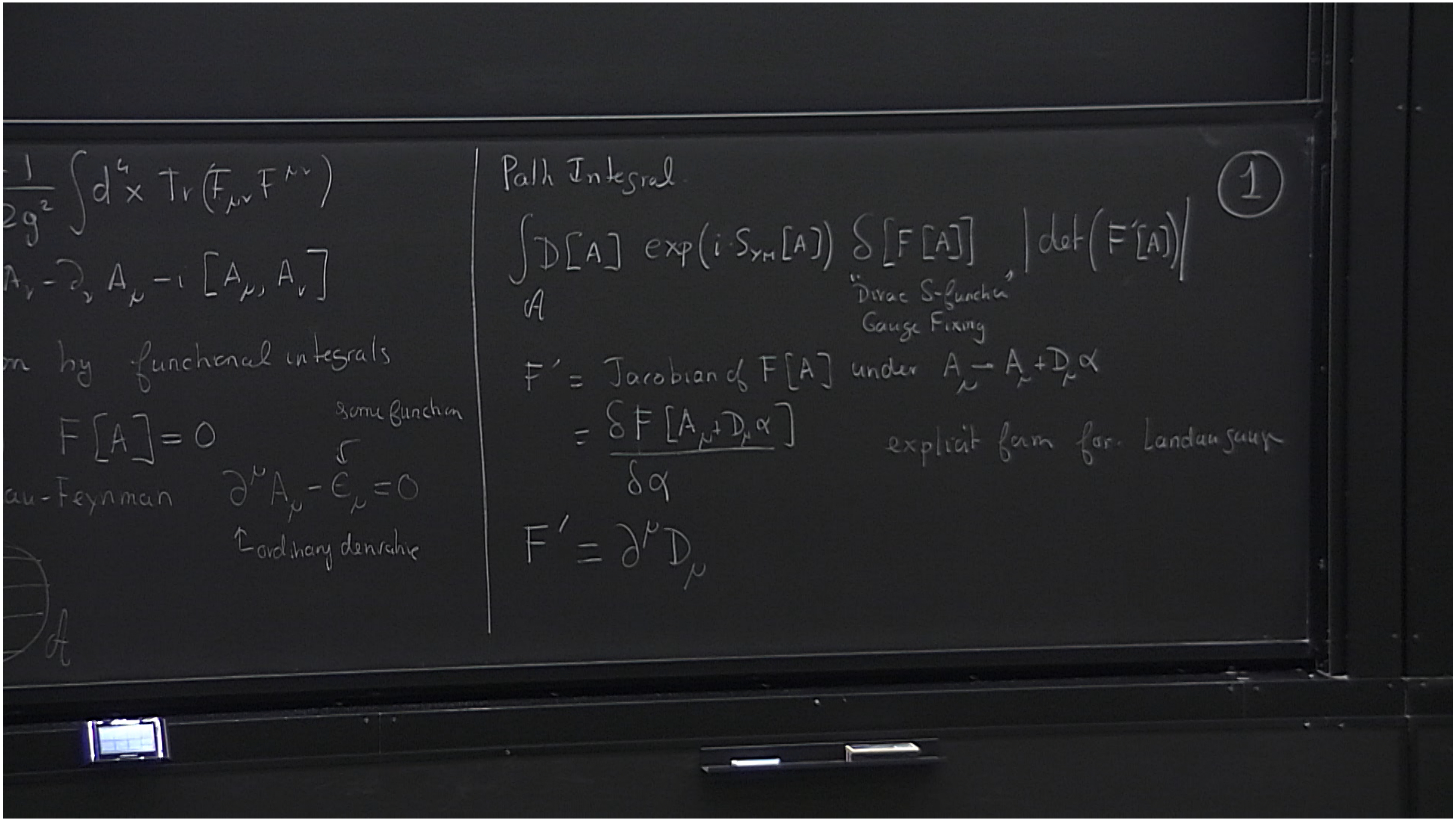
explicit form for Landau gauge



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$A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]$

n by functional integrals

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Path Integral ①

$$\int_{\mathcal{A}} D[A]\, \exp(i\, S_{YM}[A])\, \delta[F[A]]\, |\det(F'[A])|$$

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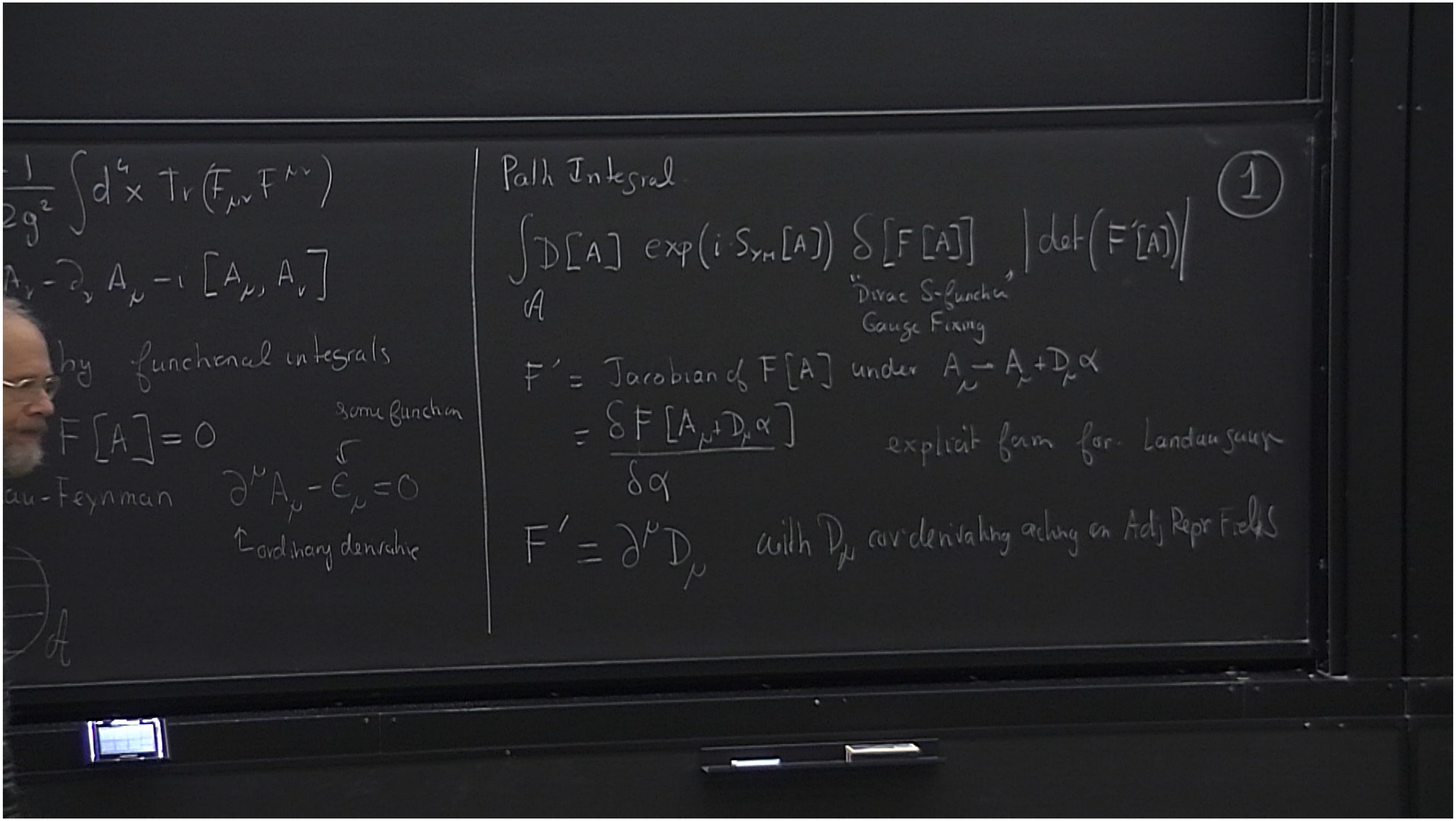

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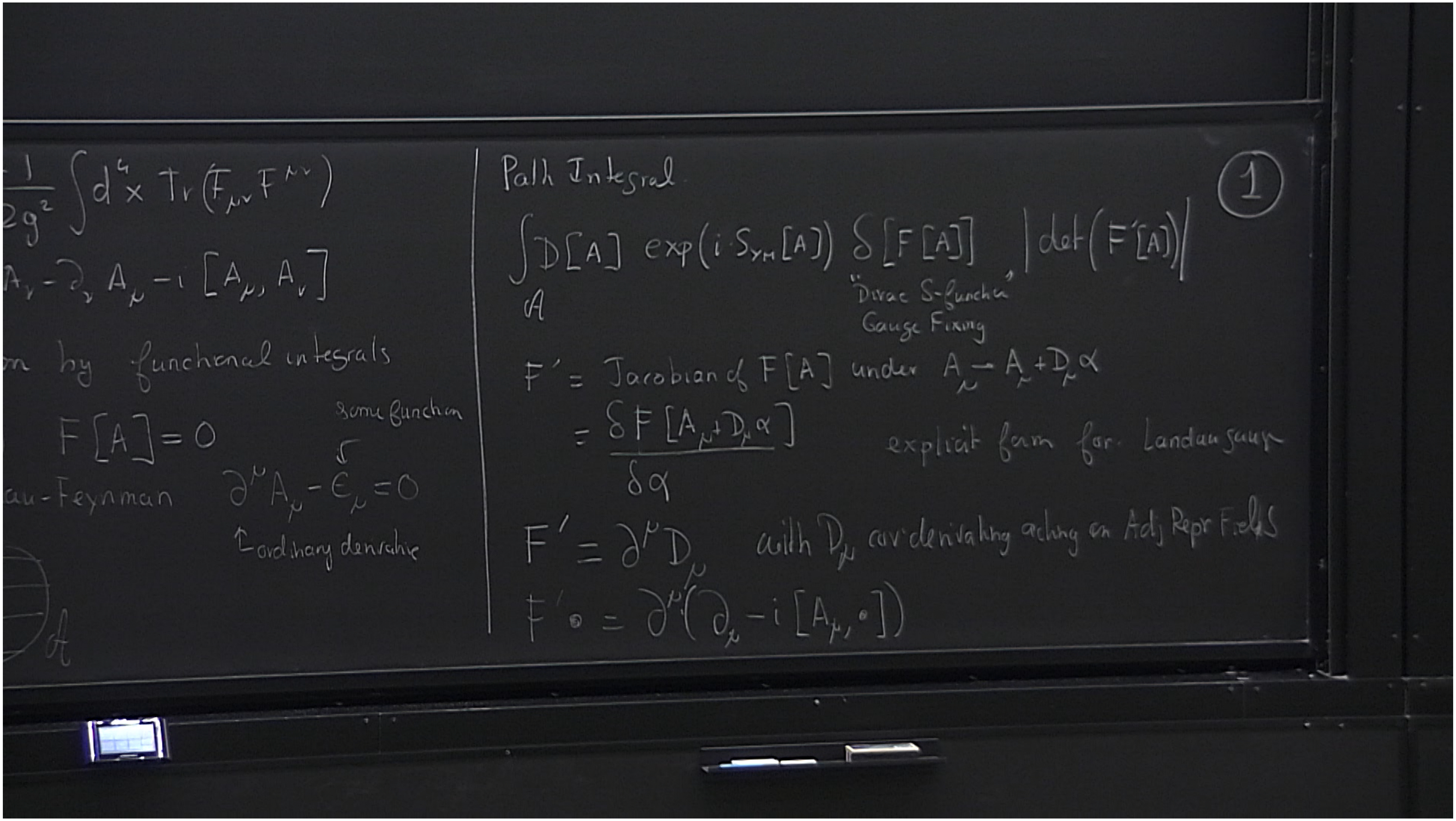
Gauge Fixing

$F' =$ Jacobian of $F[A]$ under $A_\mu \to A_\mu + D_\mu \alpha$

$= \frac{\delta F[A_\mu + D_\mu \alpha]}{\delta \alpha}$ explicit form for Landau gauge

$F' = \partial^\mu D_\mu$ with D_μ cov. derivative acting on Adj Repr Fields



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Path Integral

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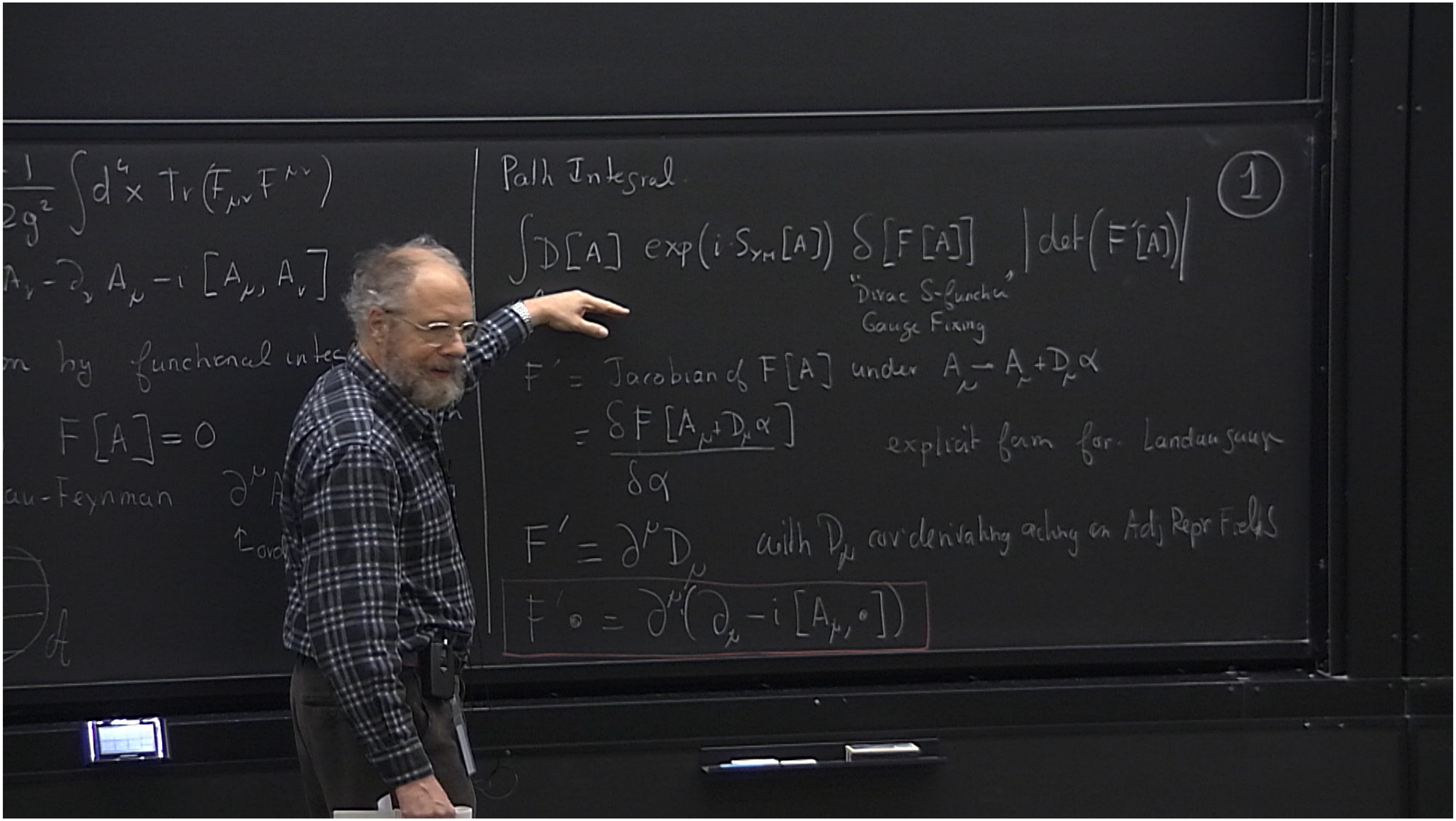

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au-Feynman $\partial^\mu A$

↑ ord

Path Integral ①

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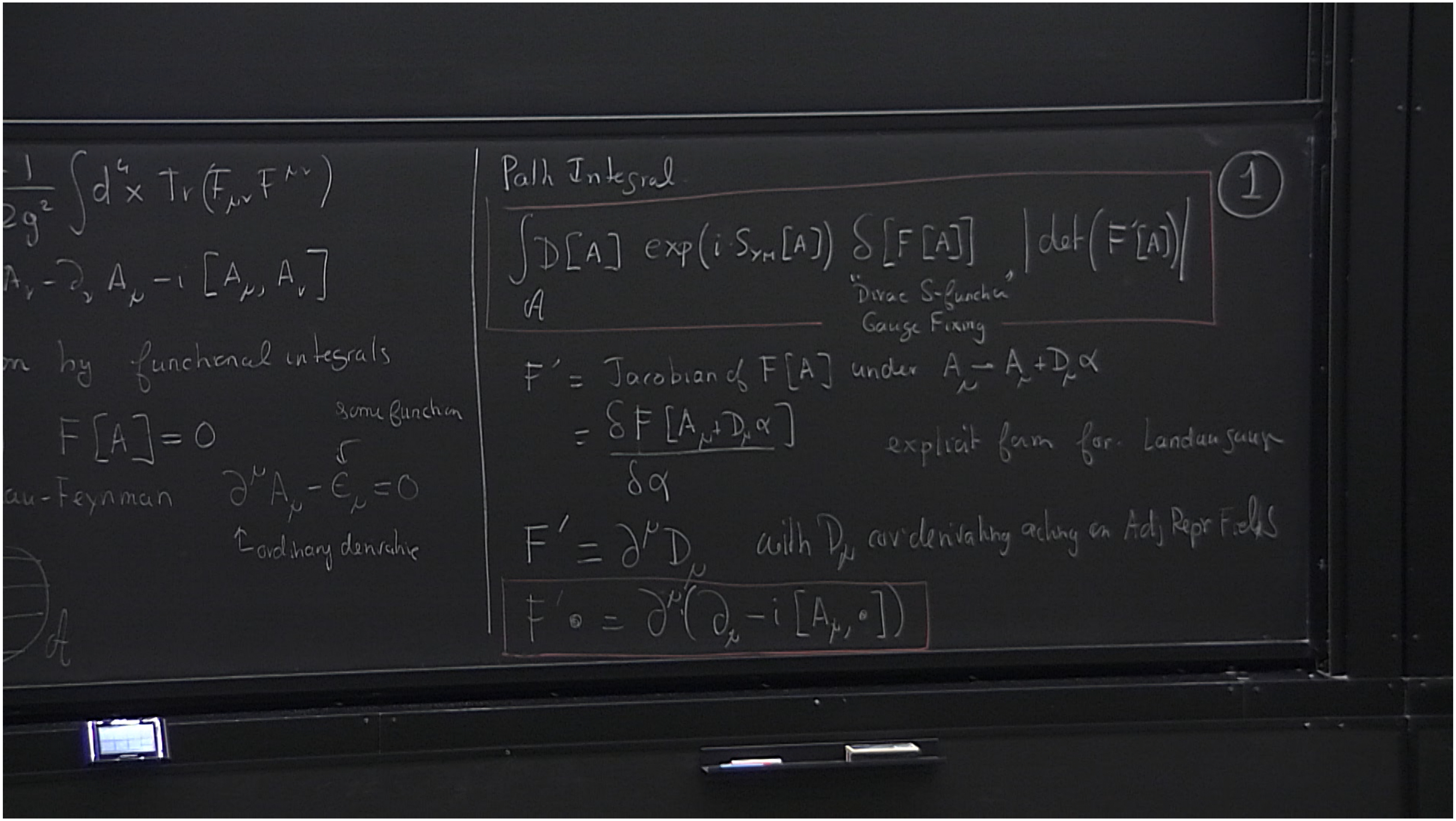

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↑ ordinary derivative

$\mathcal{A}$

Path Integral ①

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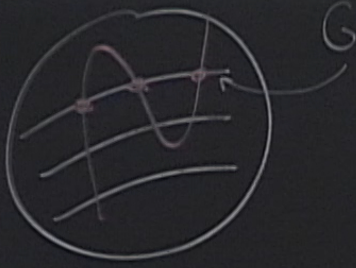

Gribov copies

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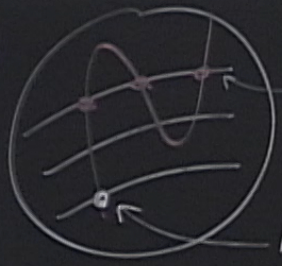
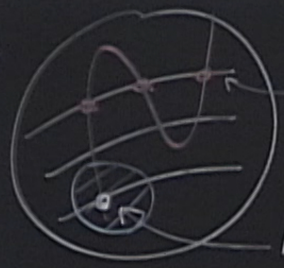


Gribov copies

one has to be more carefull

$A_\mu=0$ "classical vacuum"

* close our eyes in perturbation theory

* 

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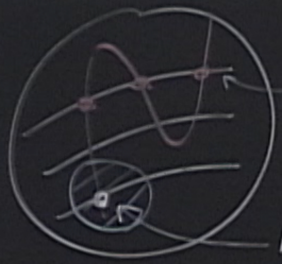

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∘ Fadeev + Popov

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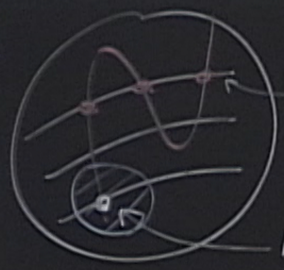

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- Fadeev + Popov Ghost Fields

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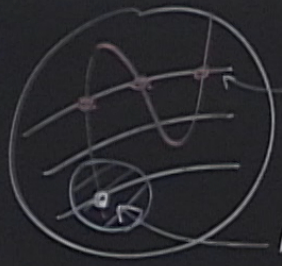

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- Fadeev + Popov Ghost Fields

Berezin calculus Gaussian Integral → Det.

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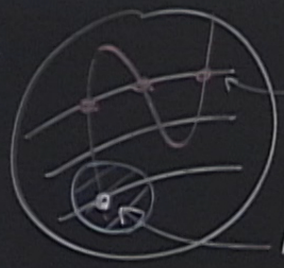

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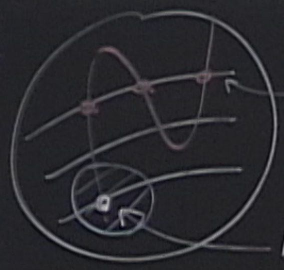
Berezin calculus   Gaussian Integral → Det.

det F'

*



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- Fadeev + Popov Ghost Fields

Berezin calculus Gaussian Integral → Det.

$$\det(F') = \int D[\bar{c}, c] \exp(i \bar{c} \cdot F' \cdot c)$$

↳ Grassman a.c. fields

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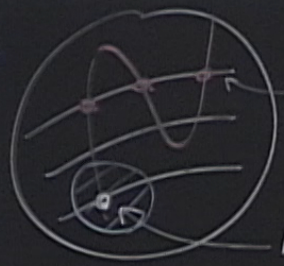

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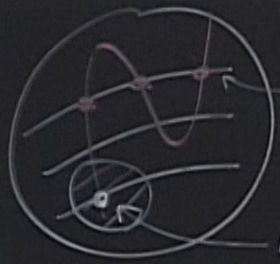

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$A_\mu = 0$ "classical vacuum"

$$C(x) = C^a(x)\, t_a$$

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Berezin calculus   Gaussian Integral → Det.

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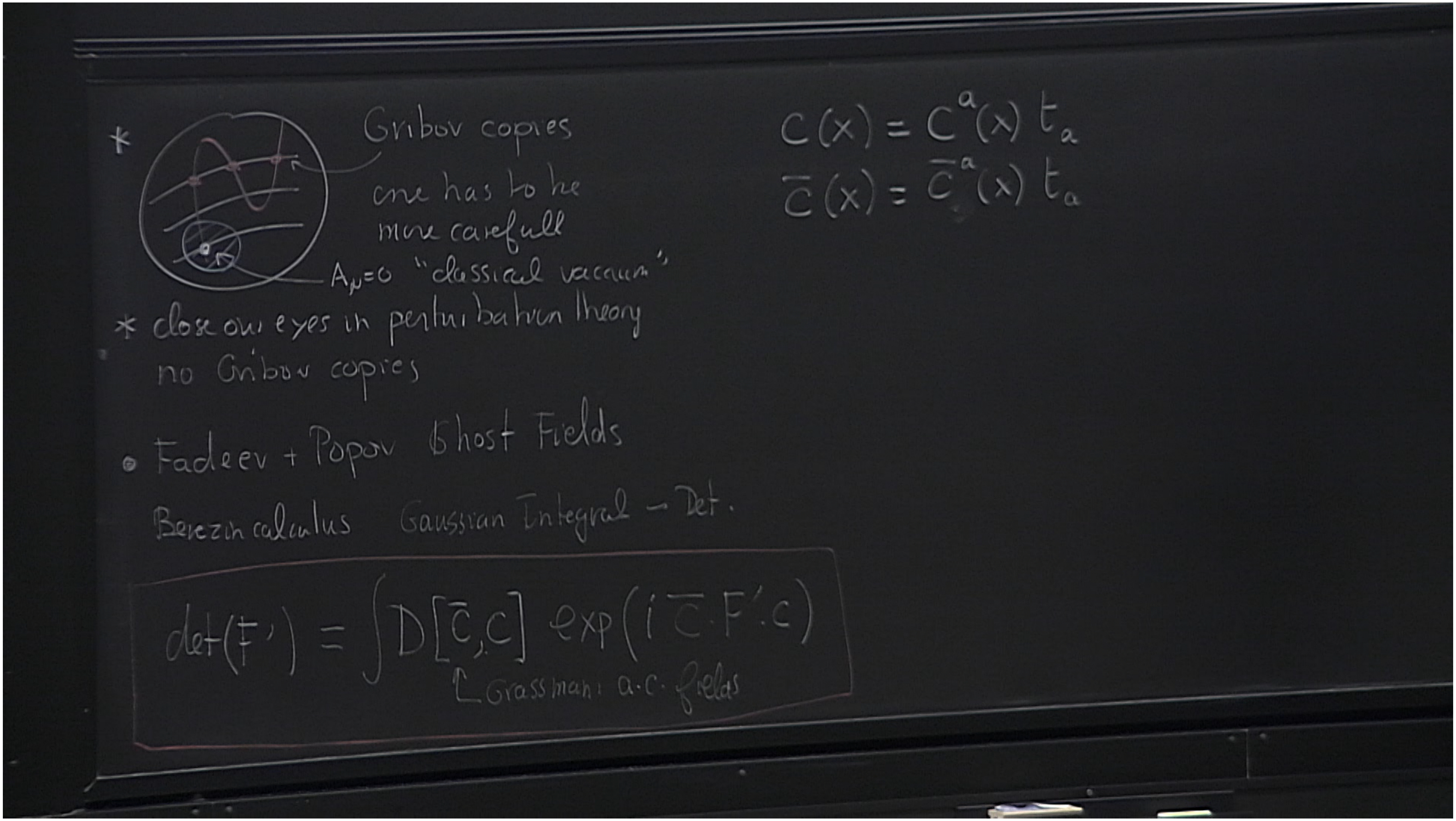

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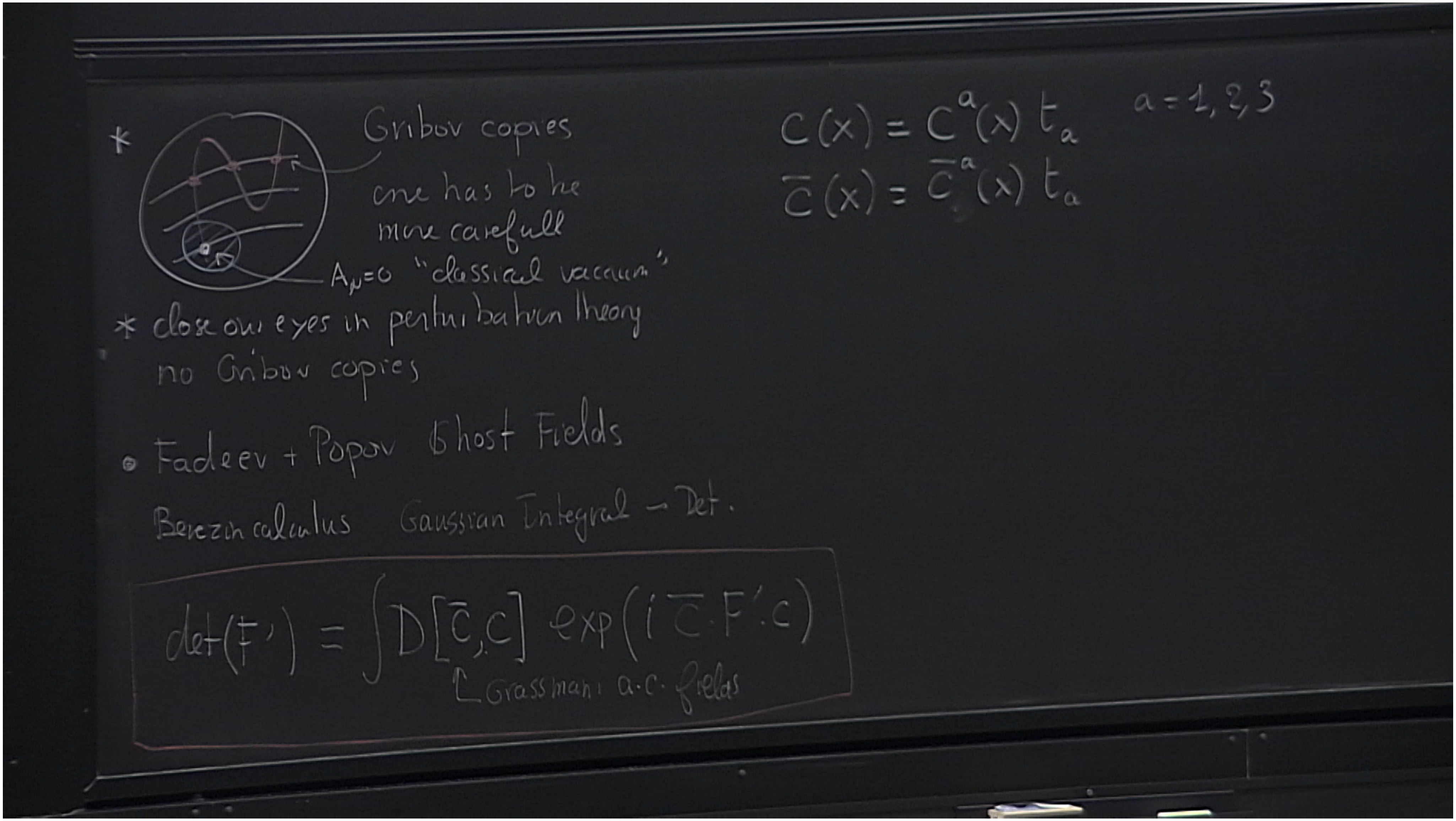
⌞ Grassman: a.c. fields

$$c(x) = c^a(x)\, t_a$$
$$\bar{c}(x) = \bar{c}^a(x)\, t_a$$



* Gribov copies

one has to be more carefull

$A_\mu = 0$ "classical vacuum"

* close our eyes in perturbation theory

no Gribov copies

- Fadeev + Popov Ghost Fields

Berezin calculus   Gaussian Integral → Det.

$$\boxed{\det(F') = \int D[\bar{c}, c] \exp(i \bar{c} \cdot F' \cdot c)}$$

↳ Grassmann a.c. fields

$$C(x) = C^a(x)\, t_a \qquad a = 1, 2, 3$$

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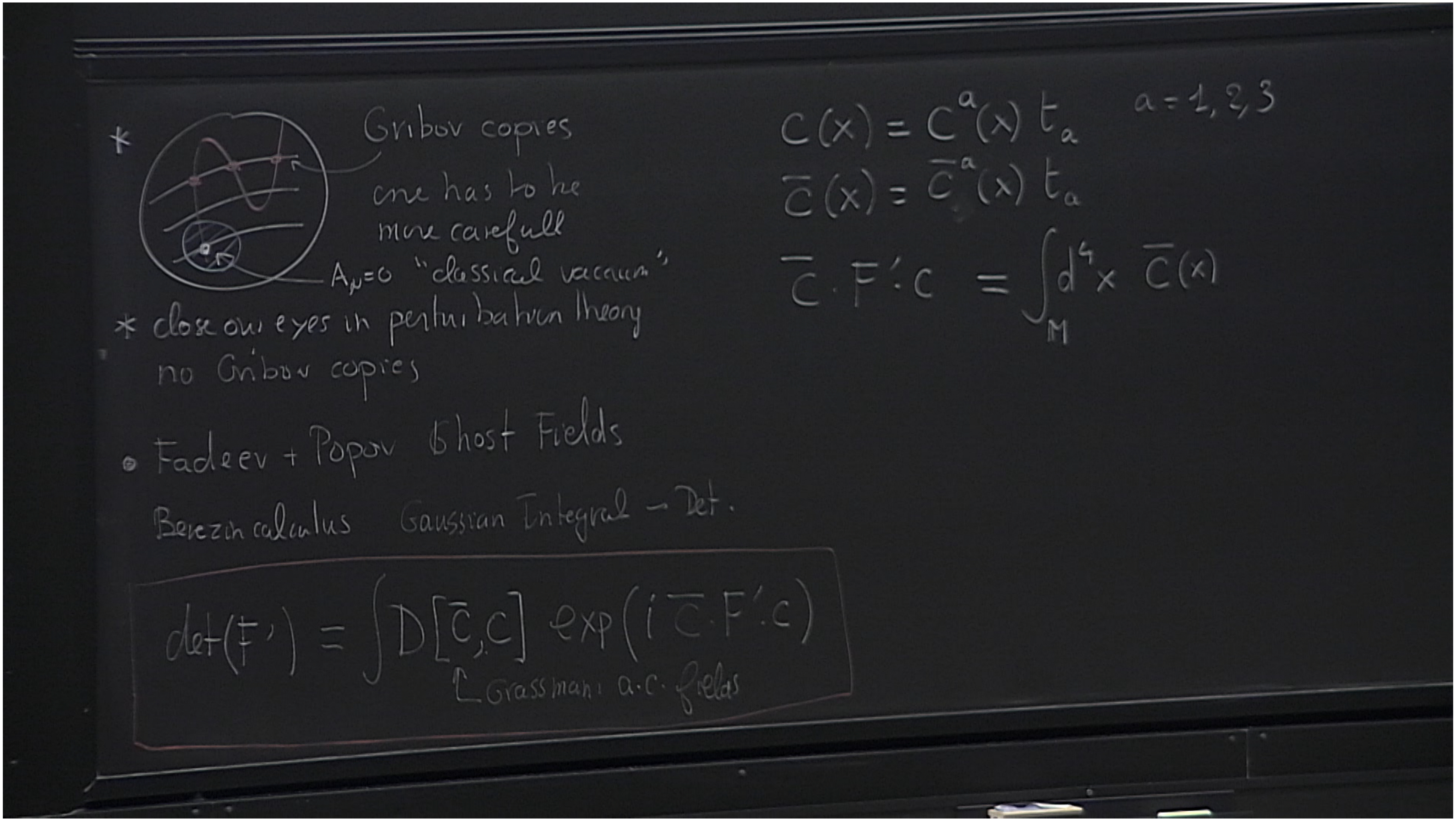

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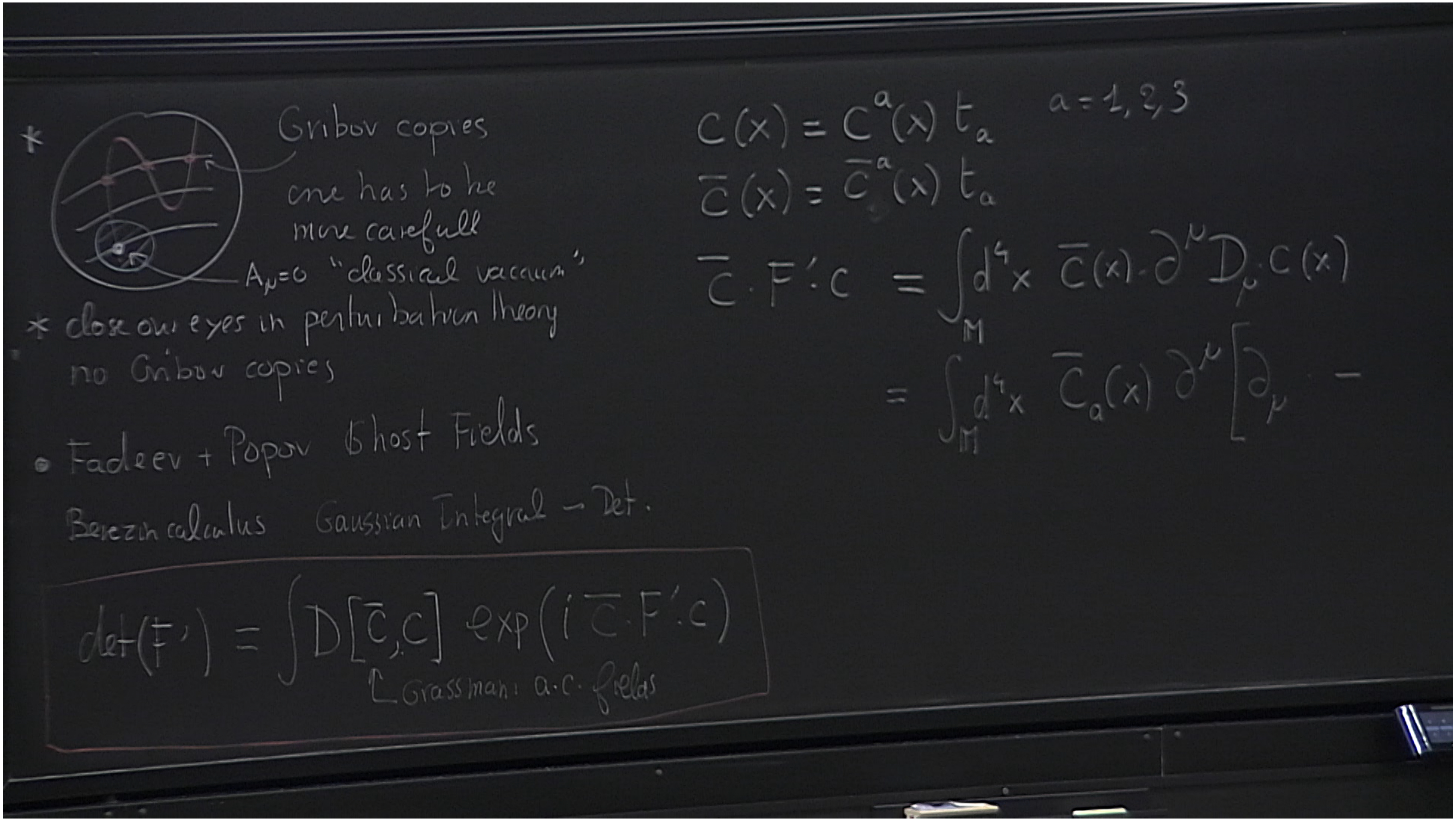

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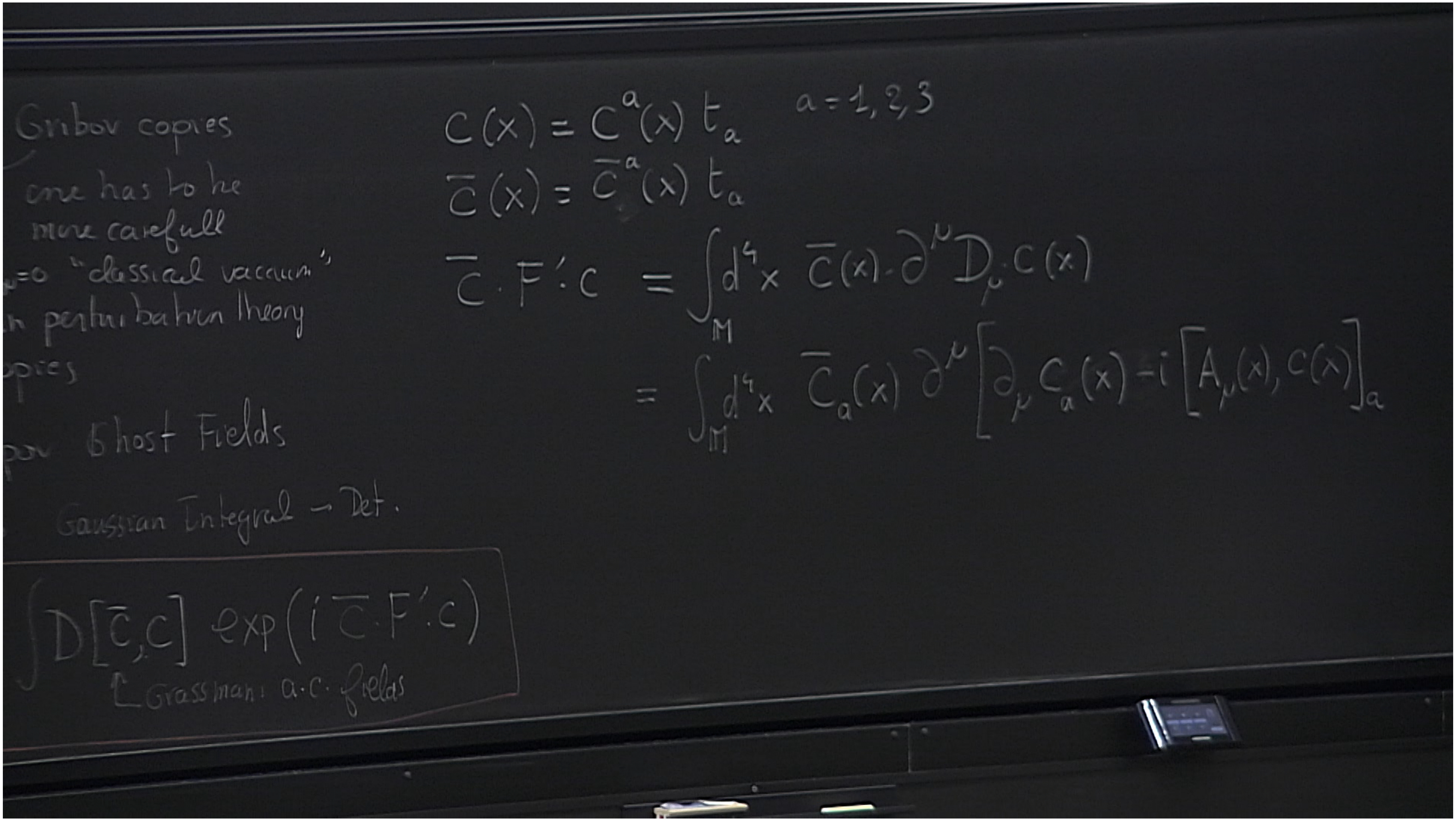
$$= \int_M d^4x\, \bar{C}_a(x)\, \partial^\mu \left[\partial_\mu \cdot - \right.$$



Gribov copies

one has to be more carefull

$_\mu=0$ "classical vacuum"

n perturbation theory

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pov Ghost Fields

Gaussian Integral → Det.

$$\boxed{\int D[\bar{C},C]\, \exp(i\,\bar{C}\cdot F'\cdot c)}$$

↑ Grassman: a.c. fields

$$C(x) = C^a(x)\, t_a \qquad a = 1,2,3$$

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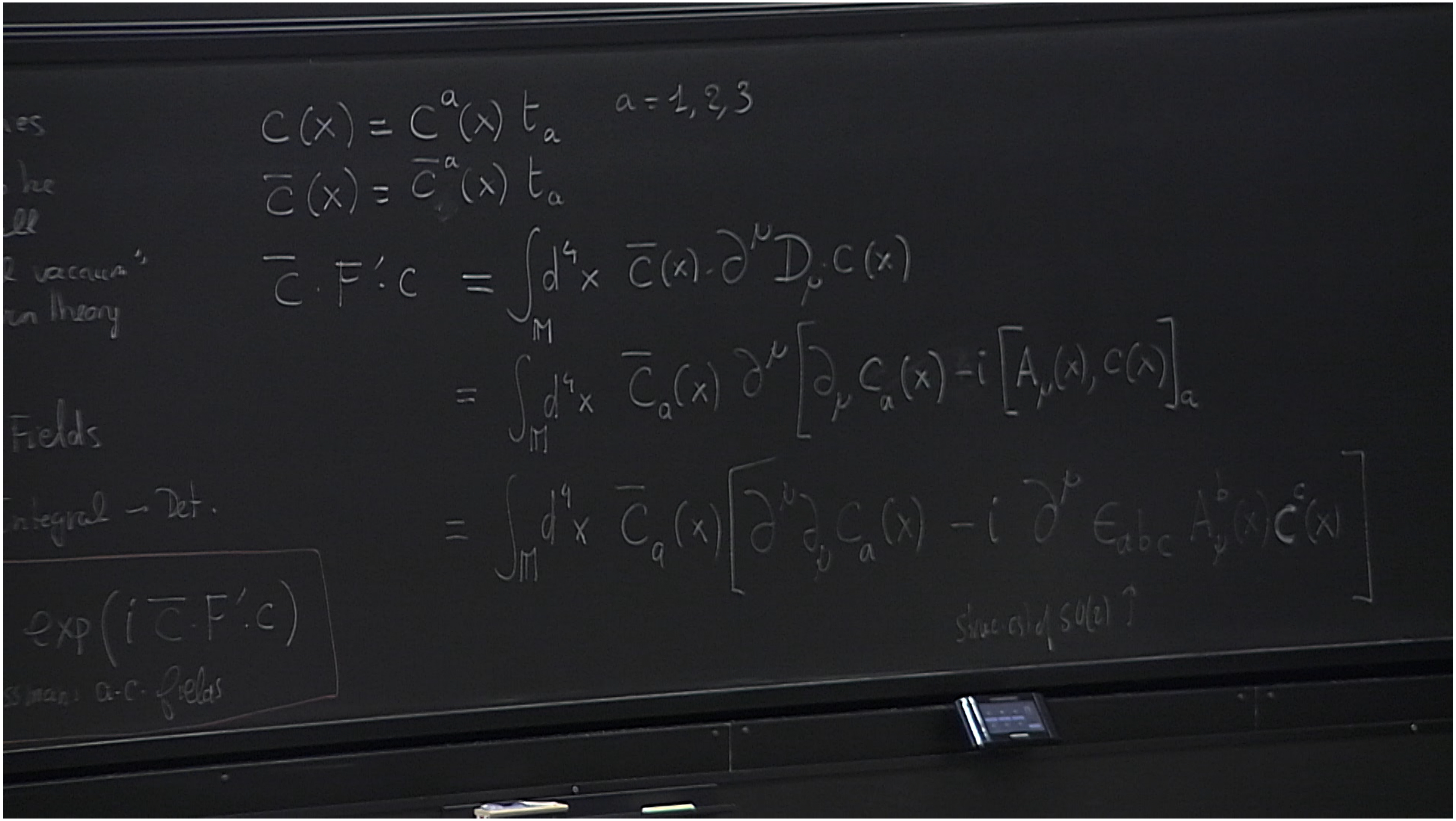
$$\bar{C}\cdot F'\cdot C = \int_M d^4x\; \bar{C}(x)\cdot \partial^\mu D_\mu C(x)$$

$$= \int_M d^4x\; \bar{C}_a(x)\, \partial^\mu \left[\partial_\mu C_a(x) - i\left[A_\mu(x), C(x)\right]_a\right]$$



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vacuum"

n theory

Fields

ntegral → Det.

$$\boxed{\exp\left(i\,\bar{c}\cdot F'\cdot c\right)}$$

ssman a-c- fields

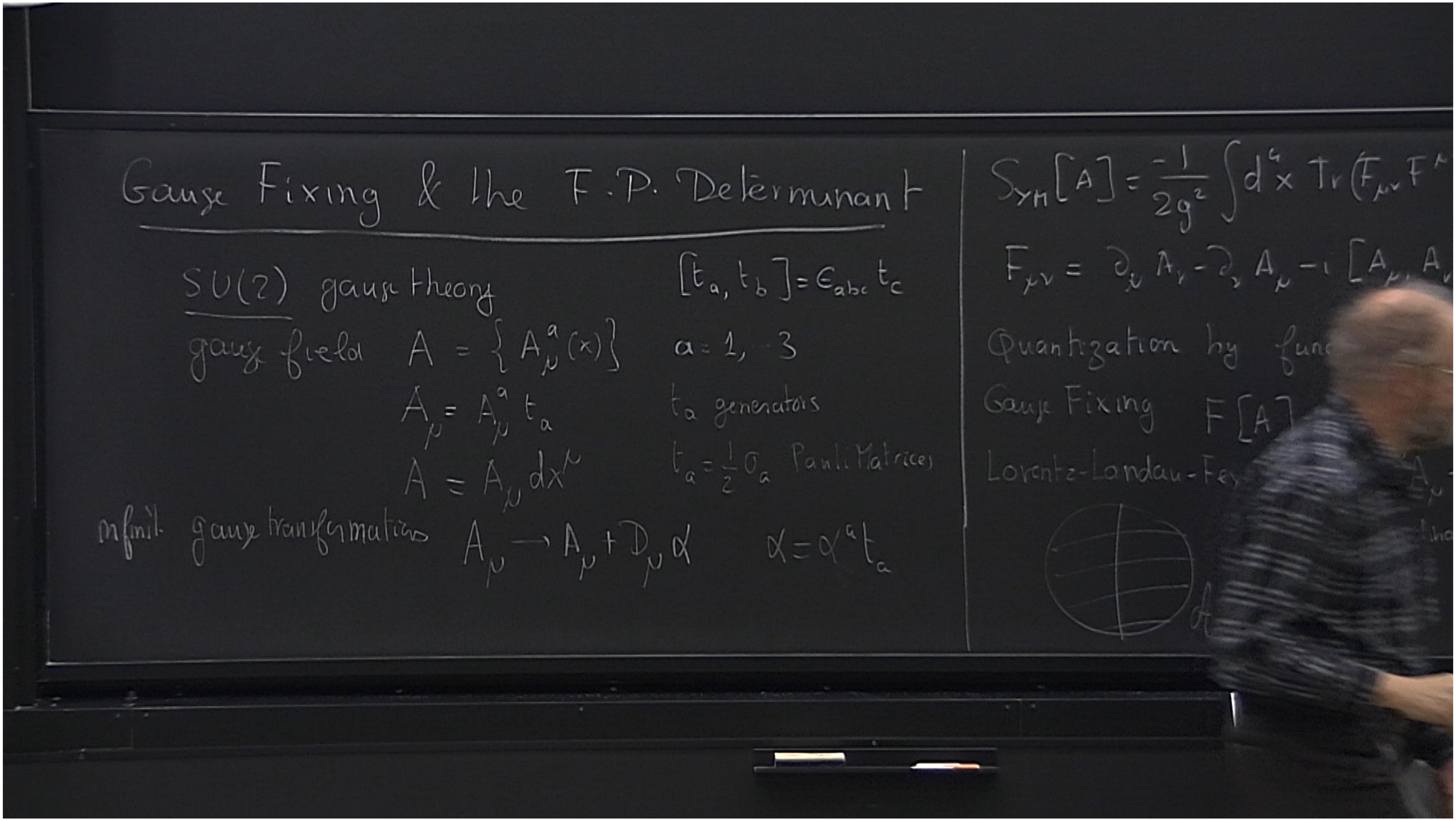
$$c(x) = c^a(x)\,t_a \qquad a = 1, 2, 3$$

$$\bar{c}(x) = \bar{c}^a(x)\,t_a$$

$$\bar{c}\cdot F'\cdot c = \int_M d^4x\ \bar{c}(x)\cdot\partial^\mu D_\mu\cdot c(x)$$

$$= \int_M d^4x\ \bar{c}_a(x)\,\partial^\mu\Big[\partial_\mu c_a(x) - i\,\big[A_\mu(x), c(x)\big]_a$$

$$= \int_M d^4x\ \bar{c}_a(x)\Big[\partial^\nu\partial_\nu c_a(x) - i\,\partial^\mu\,\underset{\text{struc. cst. of } SU(2)\ \uparrow}{\epsilon_{abc}}\,A^b_\mu(x)\,c^c(x)\Big]$$



Gauge Fixing & the F.P. Determinant

SU(2) gauge theory $[t_a, t_b] = \epsilon_{abc} t_c$

gauge field $A = \{A^a_\mu(x)\}$ $a = 1, \cdot 3$

$A_\mu = A^a_\mu t_a$ t_a generators

$A = A_\mu dx^\mu$ $t_a = \frac{1}{2}\sigma_a$ Pauli Matrices

infinit. gauge transformations $A_\mu \longrightarrow A_\mu + D_\mu \alpha$ $\alpha = \alpha^a t_a$

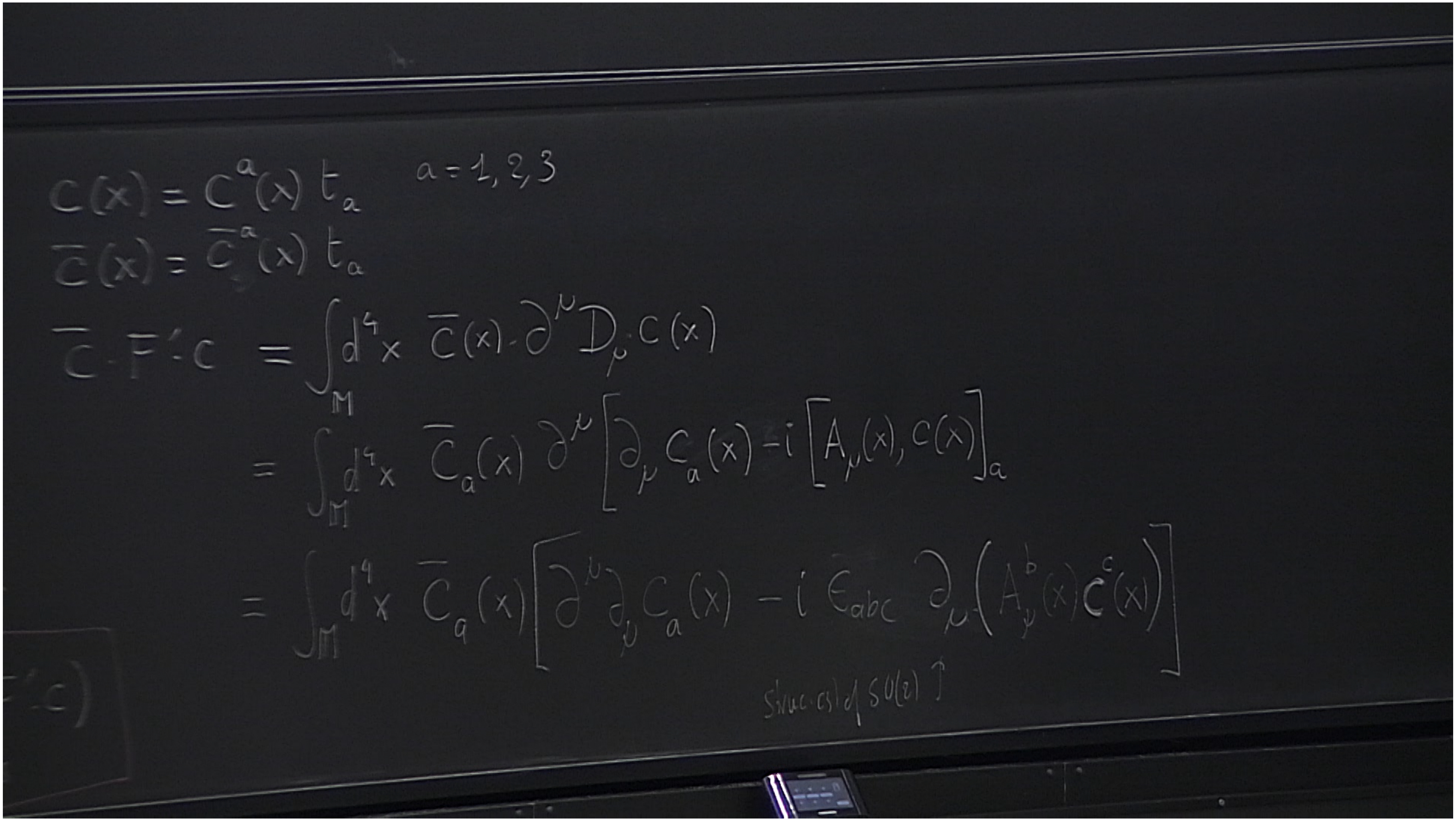
$S_{YM}[A] = \frac{-1}{2g^2}\int d^4x \, \mathrm{Tr}(F_{\mu\nu} F^{\mu}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu$

Quantization by fun

Gauge Fixing $F[A]$

Lorentz-Landau-Fey A_μ



$$C(x) = C^a(x)\, t_a \qquad a = 1, 2, 3$$

$$\bar{C}(x) = \bar{C}^a(x)\, t_a$$

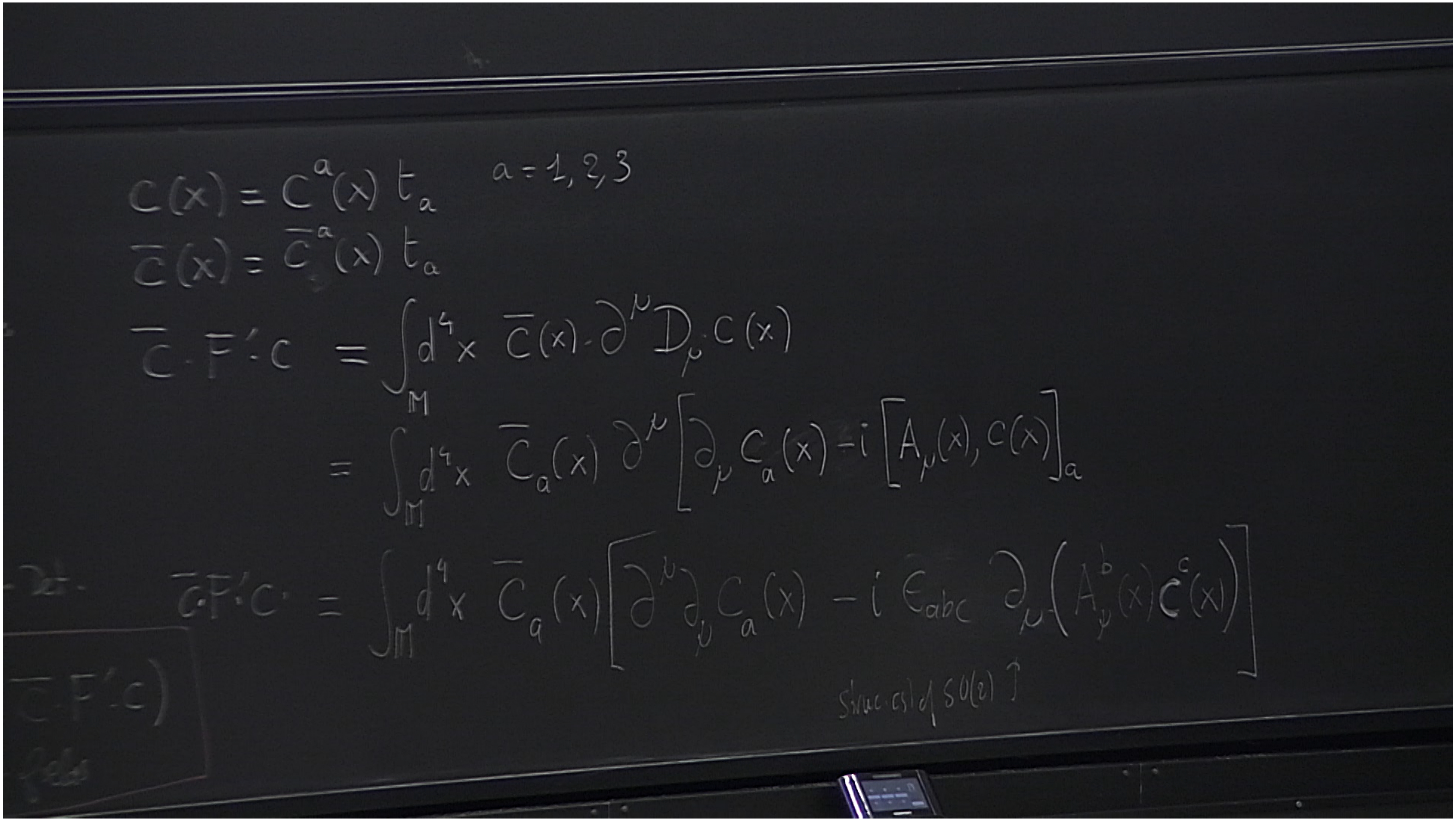
$$\bar{C}\cdot F'\cdot C = \int_M d^4x\ \bar{C}(x)\cdot \partial^\mu D_\mu \cdot C(x)$$

$$= \int_M d^4x\ \bar{C}_a(x)\, \partial^\mu \left[\partial_\mu C_a(x) - i \left[A_\mu(x), C(x) \right]_a \right]$$

$$= \int_M d^4x\ \bar{C}_a(x) \left[\partial^\mu \partial_\mu C_a(x) - i\, \epsilon_{abc}\, \partial_\mu \left(A^b_\mu(x)\, C^c(x) \right) \right]$$

struc. cst of SU(2) ↑

$F'\cdot c)$



$$C(x) = C^a(x)\, t_a \qquad a = 1, 2, 3$$

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$$\bar{C} \cdot F' \cdot C = \int_M d^4x\ \bar{C}(x) \cdot \partial^\mu D_\mu \cdot C(x)$$

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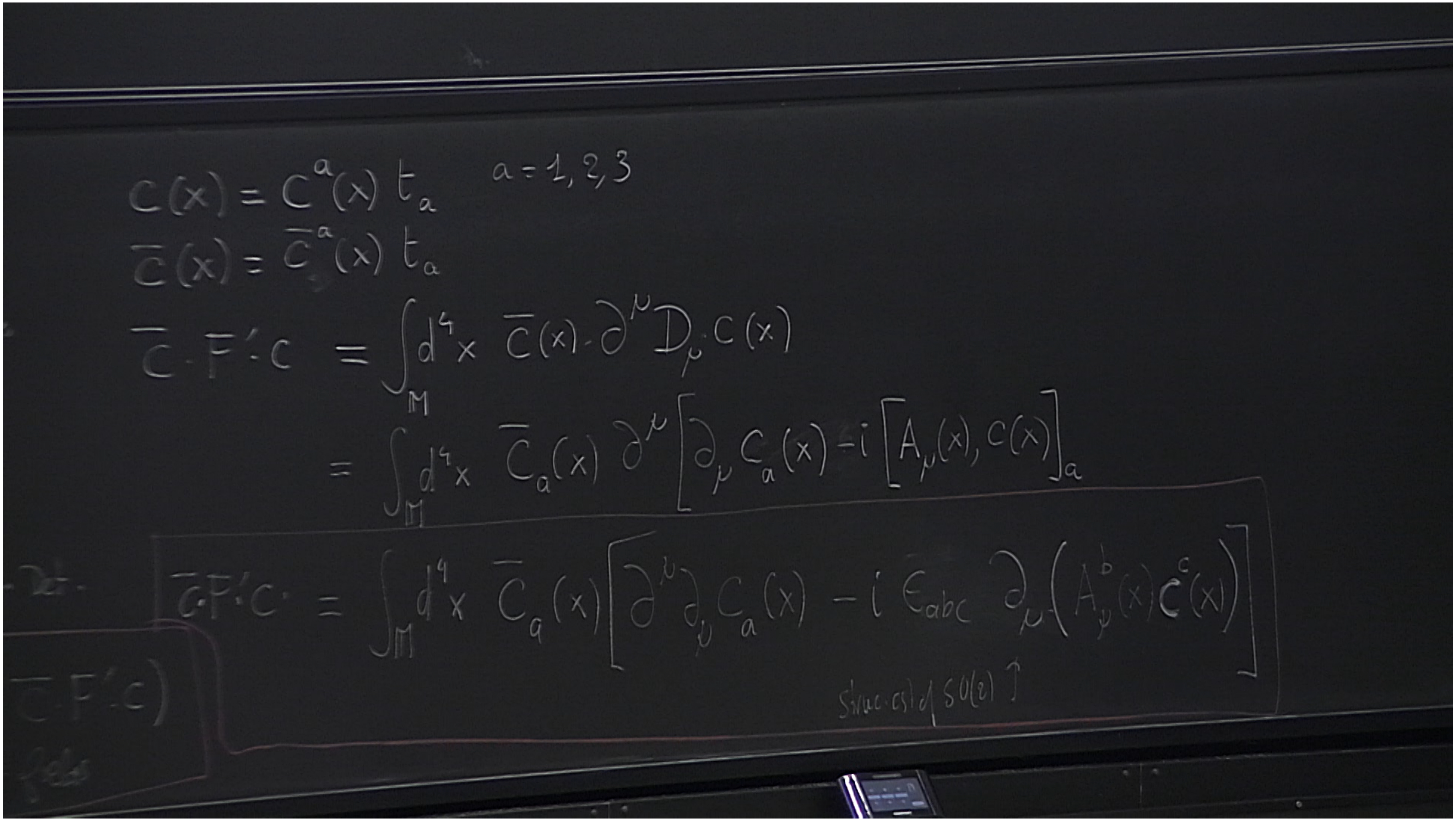
-Det

$$\bar{C} \cdot F' \cdot C = \int_M d^4x\ \bar{C}_a(x) \left[\partial^\mu \partial_\mu C_a(x) - i\, \epsilon_{abc}\, \partial_\mu \left(A^b_\mu(x)\, C^c(x) \right) \right]$$

struc. cst. of SO(3) ↑

$\bar{C} \cdot F' \cdot C)$

fields



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$$= \int_M d^4x\; \bar{C}_a(x)\, \partial^\mu \Big[\partial_\mu C_a(x) - i\, [A_\mu(x), C(x)]_a$$

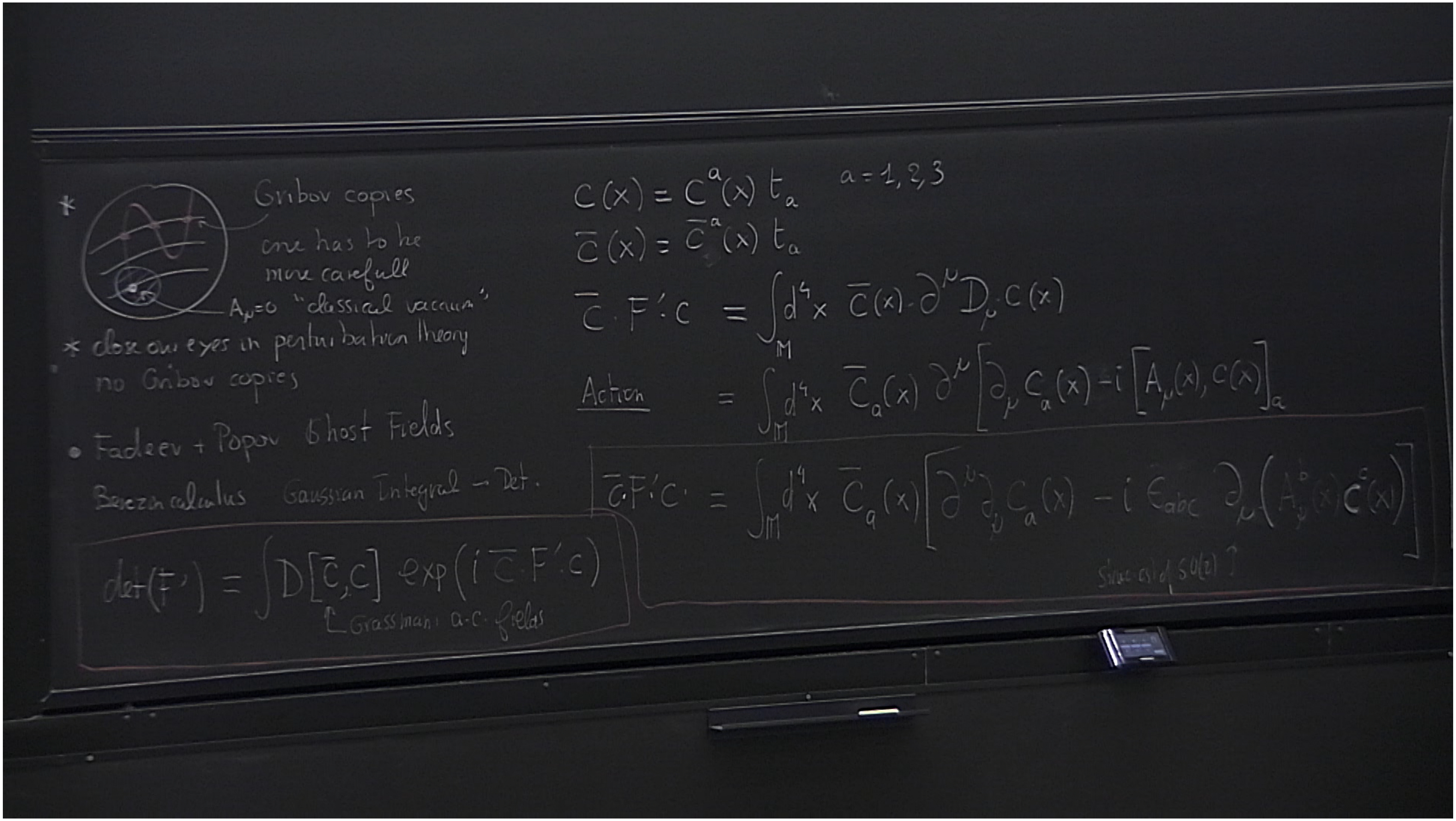
Det.

$$\boxed{\bar{C}\cdot F'\cdot C = \int_M d^4x\; \bar{C}_a(x) \Big[\partial^\mu \partial_\mu C_a(x) - i\, \epsilon_{abc}\, \partial_\mu \big(A^b_\mu(x) C^c(x) \big) \Big]}$$

struc. cst. of SU(2) ↑

$(\bar{C}\cdot F'\cdot C)$

fels



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- Fadeev + Popov Ghost Fields

Berezin calculus   Gaussian Integral → Det.

$$\boxed{\det(F') = \int D[\bar{c},c]\, \exp(i\, \bar{c}\cdot F'\cdot c)}$$

Grassman a.c. fields

$$c(x) = c^a(x)\, t_a \qquad a = 1,2,3$$

$$\bar{c}(x) = \bar{c}^a(x)\, t_a$$

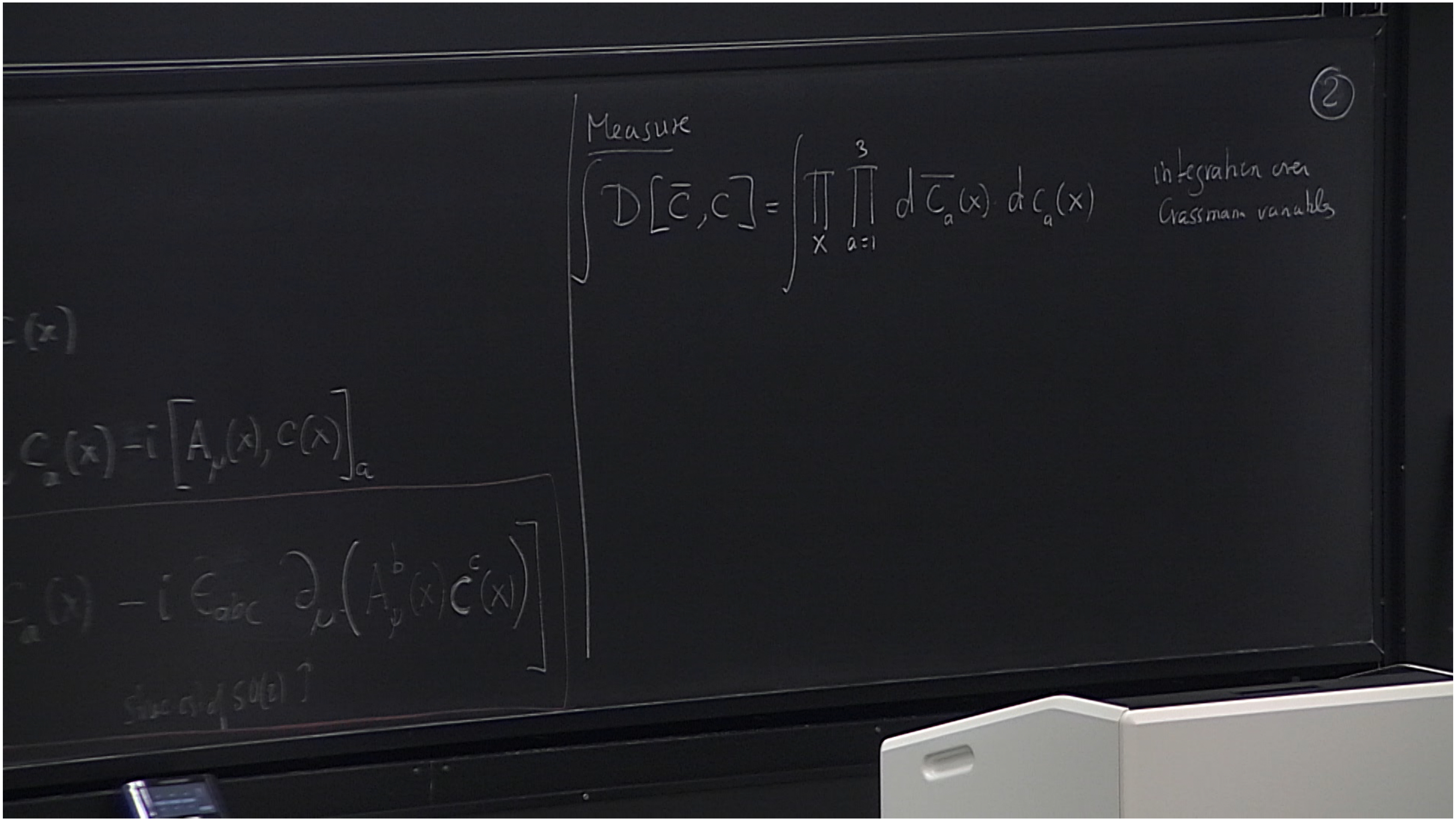
$$\bar{c}\cdot F'\cdot c \equiv \int_M d^4x\; \bar{c}(x)\cdot \partial^\mu D_\mu \cdot c(x)$$

Action

$$= \int_M d^4x\; \bar{c}_a(x)\, \partial^\mu \left[\partial_\mu c_a(x) - i\, [A_\mu(x), c(x)]_a \right]$$

$$\boxed{\bar{c}\cdot F'\cdot c = \int_M d^4x\; \bar{c}_a(x) \left[\partial^\mu \partial_\mu c_a(x) - i\, \epsilon_{abc}\, \partial_\mu \left(A^b_\mu(x)\, c^c(x) \right) \right]}$$

Struc. cst. of SU(2) ?



$c(x)$

$C_a(x) - i\left[A_\mu(x), c(x)\right]_a$

$C_a(x) - i\,\epsilon_{abc}\,\partial_\mu\left(A^b_\nu(x)\,c^c(x)\right)\Big]$

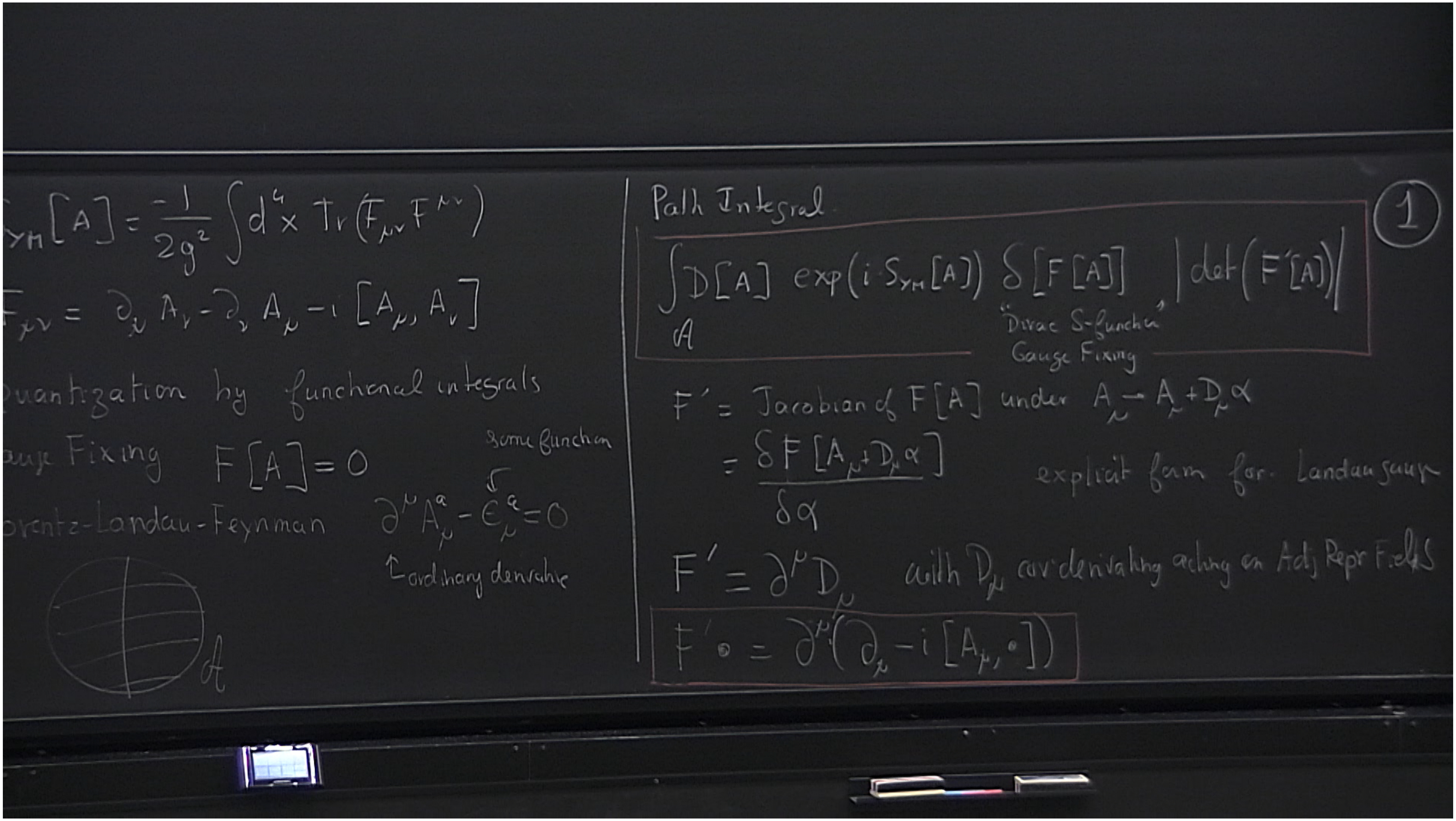
[illegible] of SU(2)]

②

Measure

$$\int D[\bar{c}, c] = \int \prod_x \prod_{a=1}^{3} d\,\bar{c}_a(x)\, d\,c_a(x)$$

integration over Grassmann variables



$$_{YM}[A] = \frac{-1}{2g^2}\int d^4x\, \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

$$_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]$$

uantization by functional integrals

auge Fixing $F[A] = 0$

some function

orentz-Landau-Feynman $\partial^\mu A^a_\mu - \overset{\uparrow}{\epsilon}{}^a_\mu = 0$

$\uparrow$ ordinary derivative

$\mathcal{A}$

Path Integral

①

$$\boxed{\int_{\mathcal{A}} D[A]\, \exp(i\, S_{YM}[A])\, \delta[F[A]]\, \left|\det(F'[A])\right|}$$

"Dirac δ-function"

Gauge Fixing

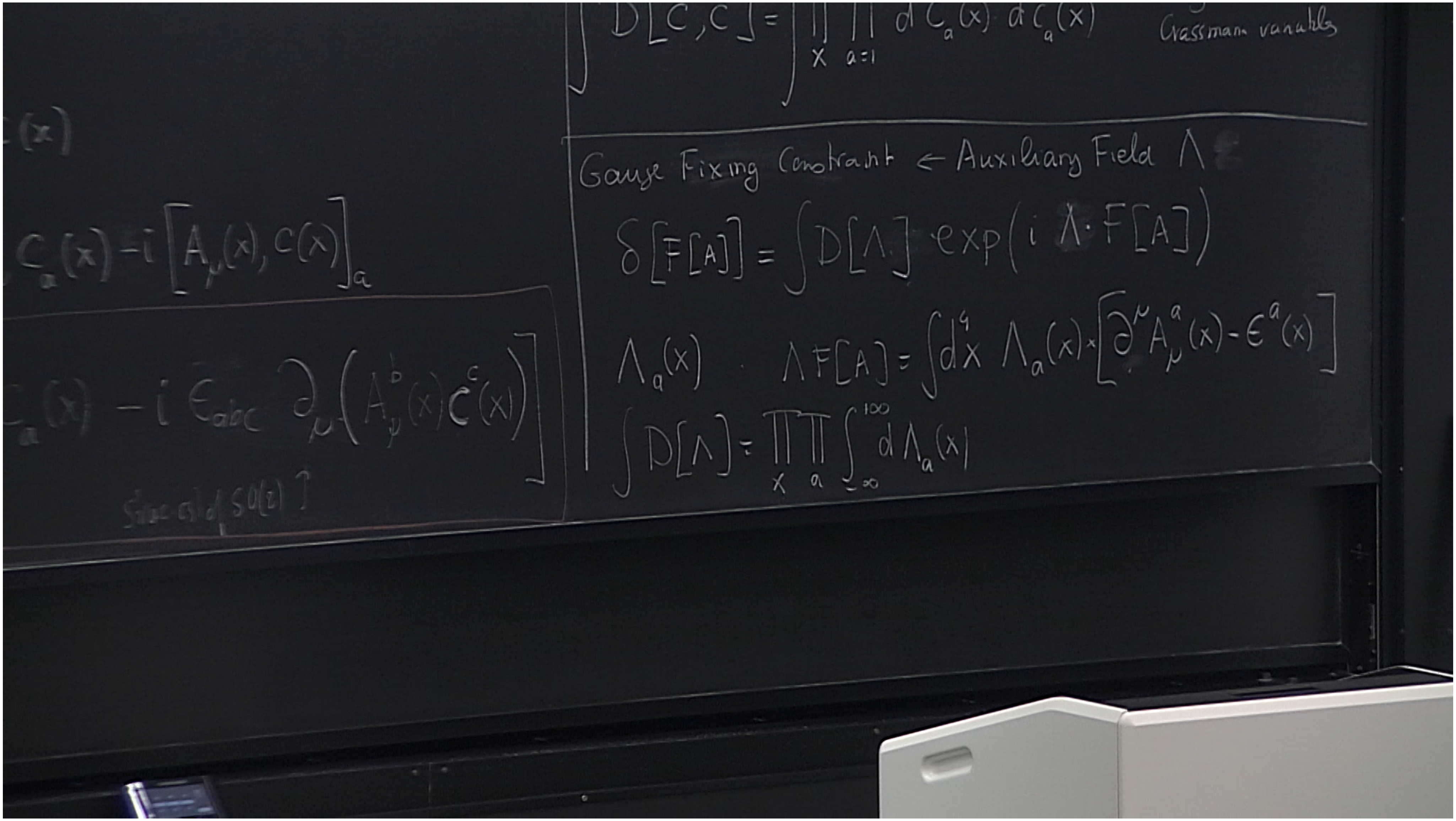
$F' =$ Jacobian of $F[A]$ under $A_\mu \to A_\mu + D_\mu\alpha$

$$= \frac{\delta F[A_\mu + D_\mu\alpha]}{\delta\alpha}$$

explicit form for Landau gauge

$F' = \partial^\mu D_\mu$ with D_μ cov. derivative acting on Adj Repr Fields

$$\boxed{F' \cdot = \partial^\mu(\partial_\mu - i\,[A_\mu, \cdot\,])}$$



$$\int D[\bar{c},c] = \int \prod_{x} \prod_{a=1} d\bar{c}_a(x)\, dc_a(x)$$

Grassmann variables

Gauss Fixing Constraint ← Auxiliary Field Λ

$$\delta\left[F[A]\right] = \int D[\Lambda]\, \exp\left(i\, \Lambda \cdot F[A]\right)$$

$$\Lambda_a(x) \qquad \Lambda F[A] = \int d^4x\, \Lambda_a(x) \times \left[\partial^\mu A^a_\mu(x) - \epsilon^a(x)\right]$$

$$\int D[\Lambda] = \prod_{x} \prod_{a} \int_{-\infty}^{+\infty} d\Lambda_a(x)$$

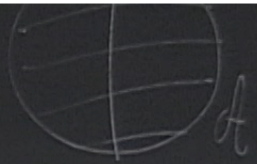
$$c(x)$$

$$c_a(x) - i\left[A_\mu(x), c(x)\right]_a$$

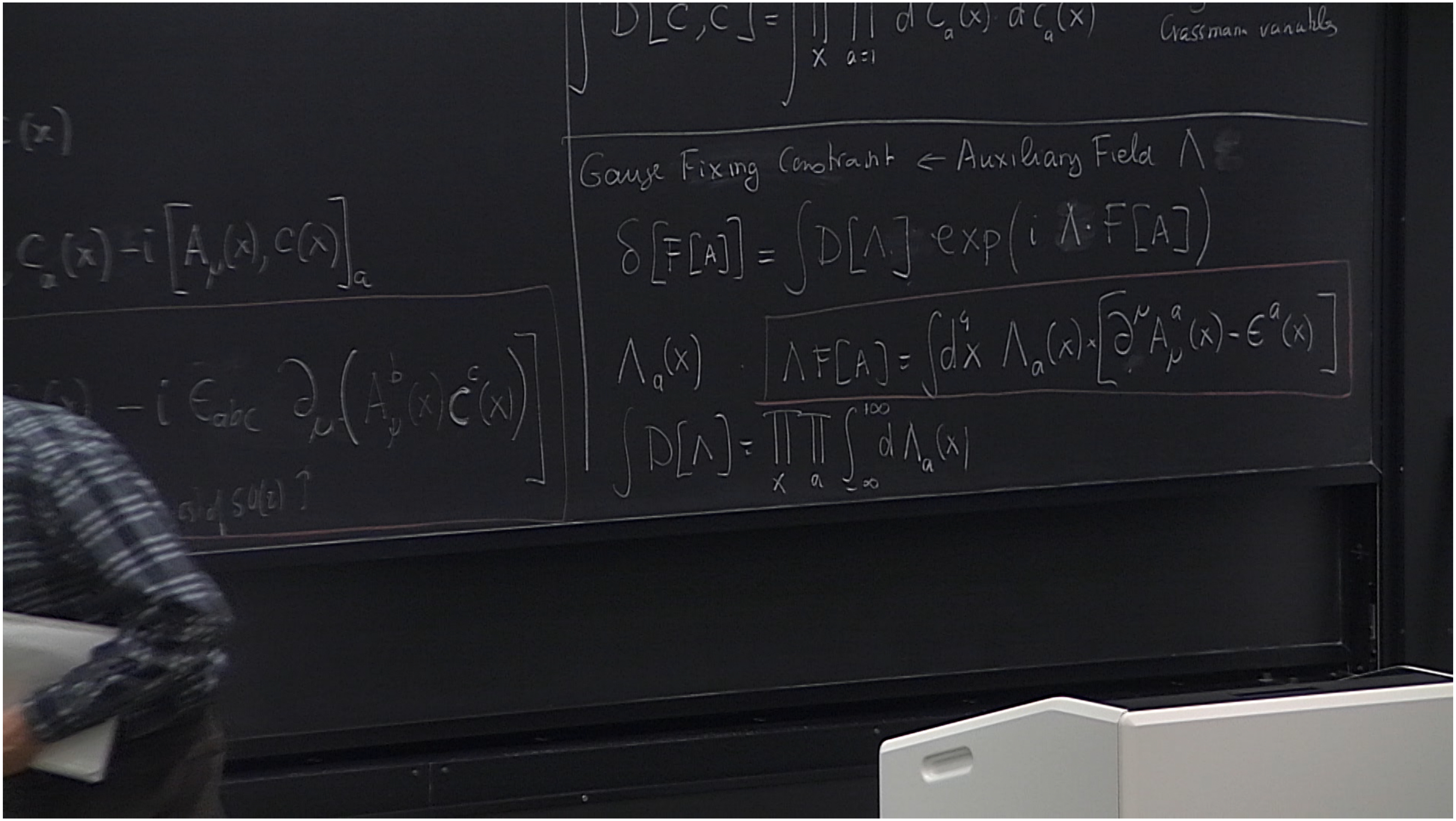
$$c_a(x) - i\, \epsilon_{abc}\, \partial_\mu\left(A^b_\mu(x)\, c^c(x)\right)\Big]$$

Structure const of SO(3) ↑

$A_\mu \longrightarrow A_\mu + D_\mu \alpha \qquad \alpha = \alpha^a t_a$



$$\int D[A]\, D[\bar{c}, c]\, D[\Lambda]\, \exp\left(i\left(S_{YM}[A] + \bar{c}\cdot F'[A]c + \Lambda\cdot F[A]\right)\right)$$



$c(x)$

$C_a(x) - i\left[A_\mu(x), c(x)\right]_a$

$-i\,\epsilon_{abc}\,\partial_\mu\left(A^b_\mu(x)\,c^c(x)\right)\Big]$

sid SO(2) ↑

$$\int D[\bar{c}, c] = \int \prod_x \prod_{a=1} d\bar{c}_a(x)\cdot dc_a(x)$$

Grassmann variables

Gauge Fixing Constraint ← Auxiliary Field Λ

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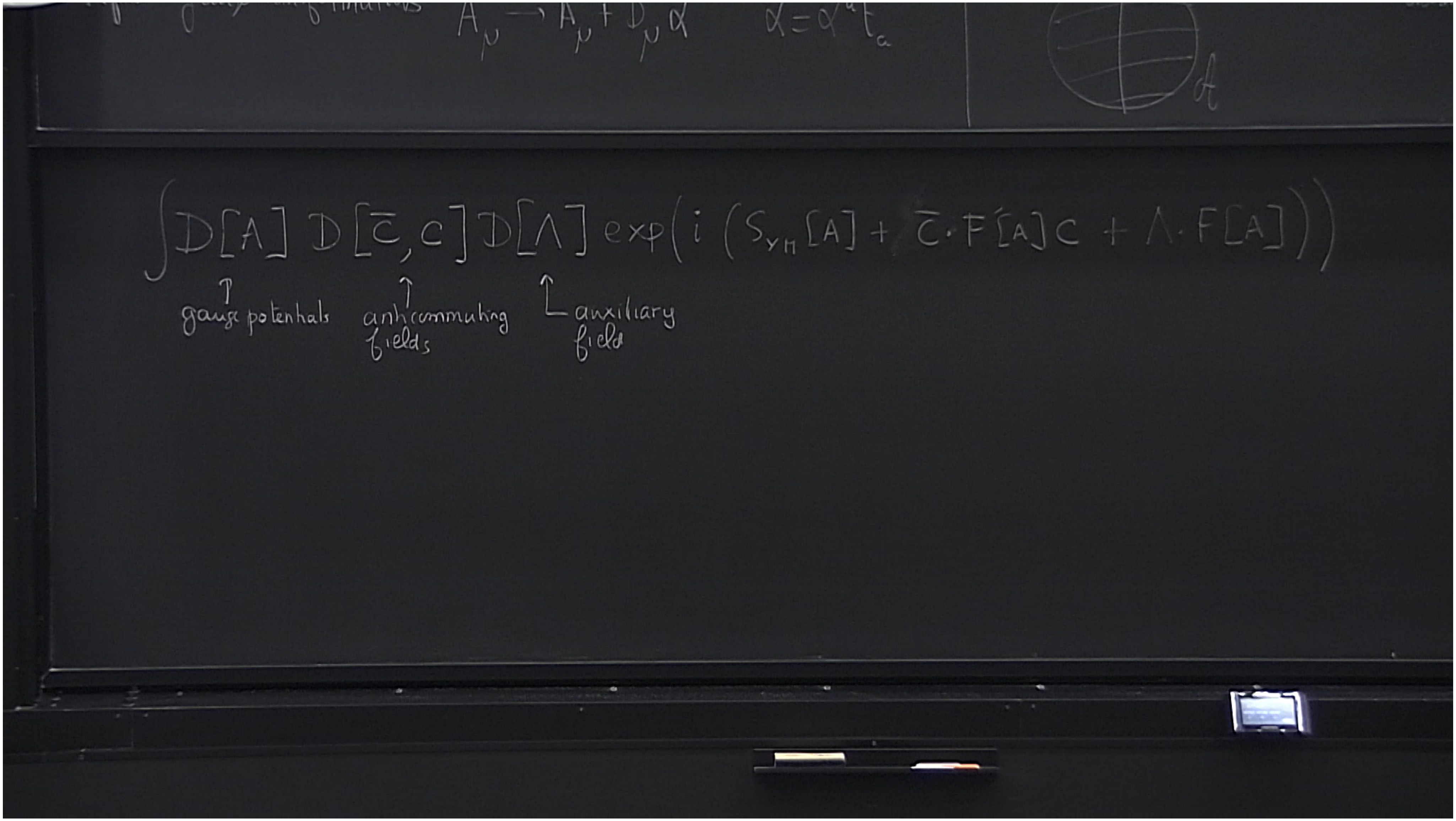
$$\Lambda_a(x) \qquad \boxed{\Lambda F[A] = \int d^4x\, \Lambda_a(x) \times \left[\partial^\mu A^a_\mu(x) - \epsilon^a(x)\right]}$$

$$\int D[\Lambda] = \prod_x \prod_a \int_{-\infty}^{+\infty} d\Lambda_a(x)$$



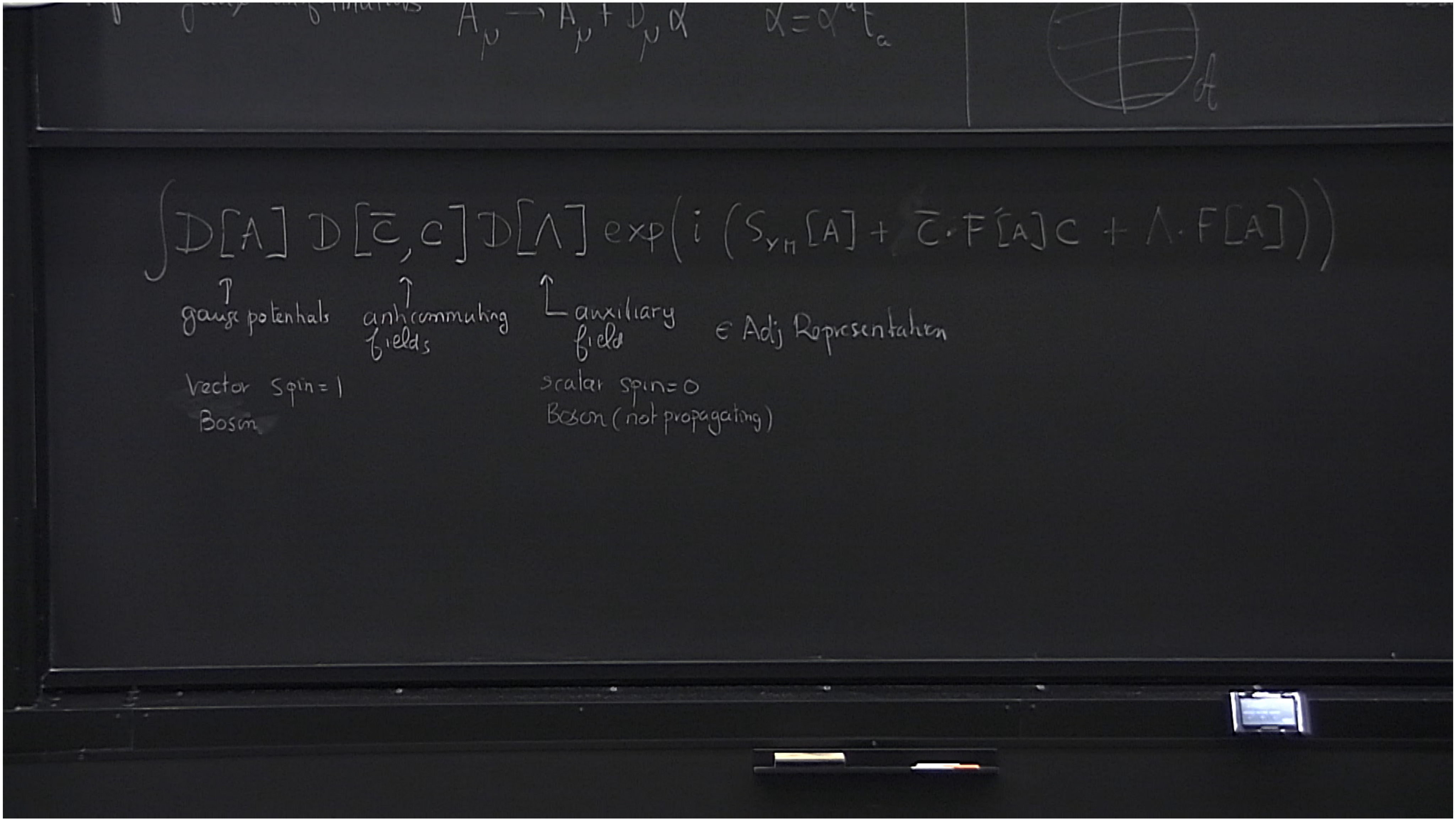
gauge transformations $A_\mu \longrightarrow A_\mu + D_\mu \alpha$ $\quad \alpha = \alpha^a t_a$

$$\int D[A]\, D[\bar{c}, c]\, D[\Lambda]\, \exp\Big(i\,\big(S_{YM}[A] + \bar{c}\cdot F'[A]\,c + \Lambda\cdot F[A]\big)\Big)$$

gauge potentials

anticommuting fields

auxiliary field



$$A_\mu \longrightarrow A_\mu + D_\mu \alpha \qquad \alpha = \alpha^a t_a$$

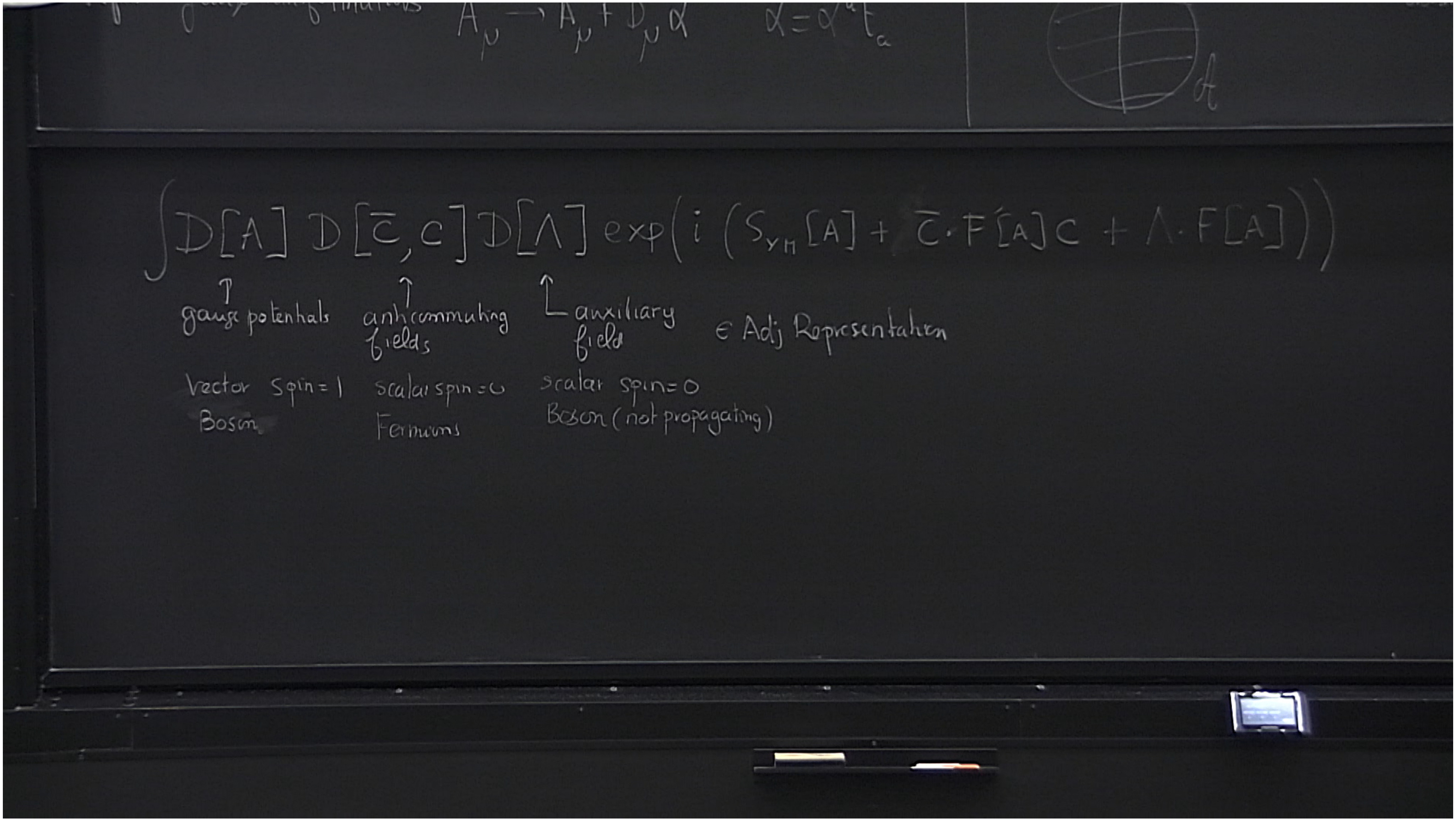
$$\int D[A]\, D[\bar{c}, c]\, D[\Lambda]\, \exp\left(i\left(S_{YM}[A] + \bar{c}\cdot F'[A]\, c + \Lambda \cdot F[A]\right)\right)$$

gauge potentials — Vector spin = 1 Boson

anticommuting fields

auxiliary field — scalar spin = 0 Boson (not propagating)

∈ Adj Representation



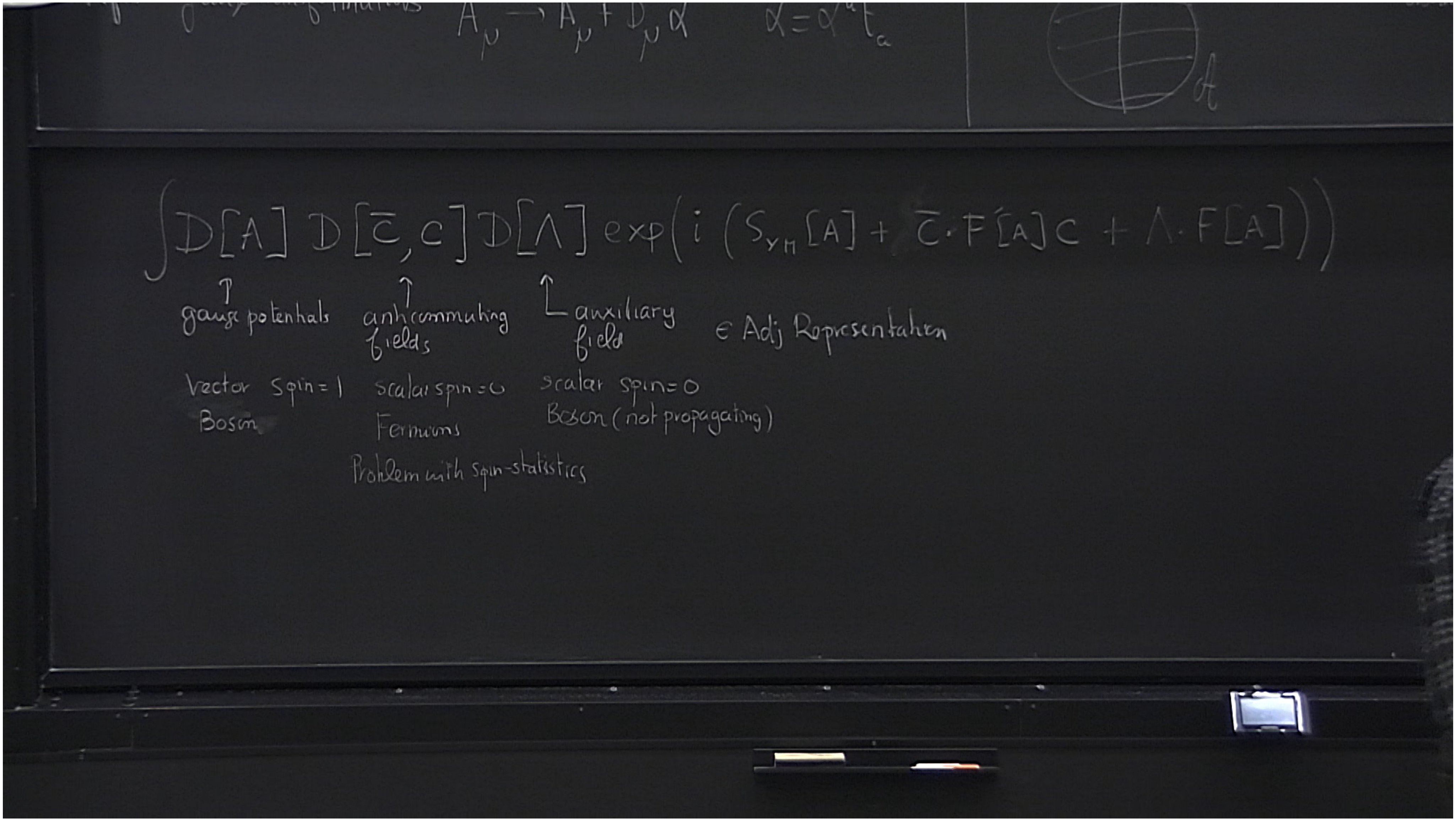
gauge transformations $A_\mu \to A_\mu + D_\mu \alpha$ $\alpha = \alpha^a t_a$

$\mathcal{G}$

$$\int D[A]\, D[\bar{c}, c]\, D[\Lambda]\, \exp\left(i\left(S_{YM}[A] + \bar{c} \cdot F'[A]\, c + \Lambda \cdot F[A]\right)\right)$$

gauge potentials — anticommuting fields — auxiliary field — $\in$ Adj Representation

Vector spin = 1 Boson — scalar spin = 0 Fermions — scalar spin = 0 Boson (not propagating)



gauge transformations $A_\mu \to A_\mu + D_\mu \alpha \qquad \alpha = \alpha^a t_a$

$$\int D[A]\, D[\bar{c}, c]\, D[\Lambda]\, \exp\Big(i\,\big(S_{YM}[A] + \bar{c}\cdot F'[A]\,c + \Lambda\cdot F[A]\big)\Big)$$

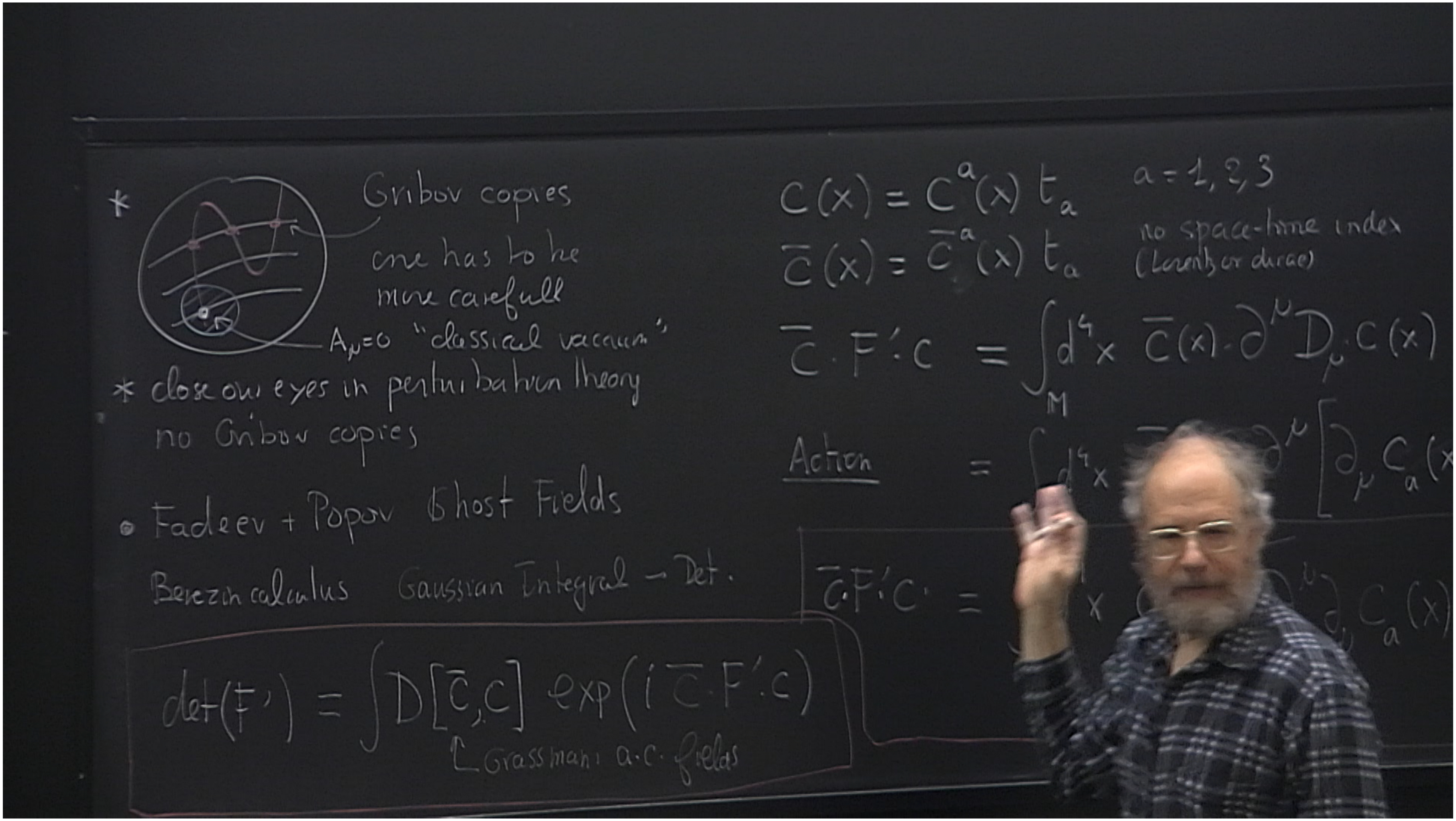
gauge potentials — anticommuting fields — auxiliary field — ∈ Adj Representation

vector spin = 1 Boson

scalar spin = 0 Fermions

scalar spin = 0 Boson (not propagating)

Problem with spin-statistics



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one has to be more carefull

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Berezin calculus   Gaussian Integral → Det.

$$\boxed{\det(F') = \int D[\bar{c}, c] \, \exp\left(i \, \bar{c} \cdot F' \cdot c\right)}$$

Grassman a.c. fields

$$c(x) = c^a(x) \, t_a \qquad a = 1, 2, 3$$

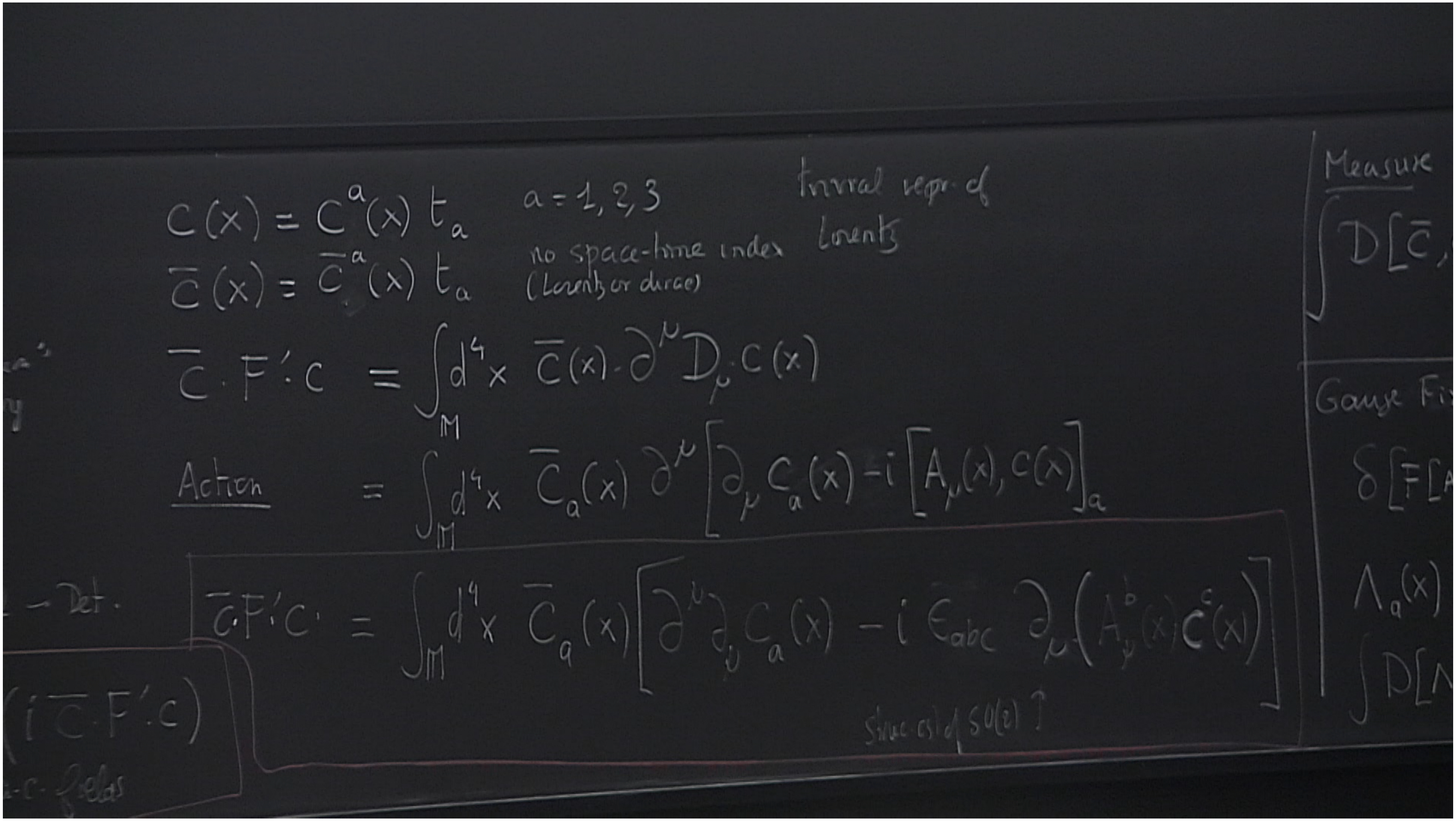
$$\bar{c}(x) = \bar{c}^a(x) \, t_a$$

no space-time index
(Lorentz or dirac)

$$\bar{c} \cdot F' \cdot c = \int_M d^4x \; \bar{c}(x) \cdot \partial^\mu D_\mu \cdot c(x)$$

Action $= \int d^4x \quad \partial^\mu \left[\partial_\mu c_a(x \right.$

$$\bar{c} \cdot F' \cdot c = \quad x \quad \partial^\mu \partial_\mu c_a(x)$$



$$C(x) = C^a(x)\, t_a \qquad a = 1, 2, 3$$

$$\bar{C}(x) = \bar{C}^a(x)\, t_a$$

no space-time index (Lorentz or dirac)

trivial repr. of Lorentz

$$\bar{C} \cdot F' \cdot C = \int_M d^4x\ \bar{C}(x) \cdot \partial^\mu D_\mu \cdot C(x)$$

Action

$$= \int_M d^4x\ \bar{C}_a(x)\, \partial^\mu \left[\partial_\mu C_a(x) - i \left[A_\mu(x), C(x) \right]_a \right.$$

$$\boxed{\bar{C} \cdot F' \cdot C = \int_M d^4x\ \bar{C}_a(x) \left[\partial^\mu \partial_\mu C_a(x) - i\, \epsilon_{abc}\, \partial_\mu \left(A^b_\mu(x)\, C^c(x) \right) \right]}$$

struc. cst. of SO(3) ↑

→ Det.

$$\left(i\, \bar{C} \cdot F' \cdot C \right)$$

..-c- fields

Measure

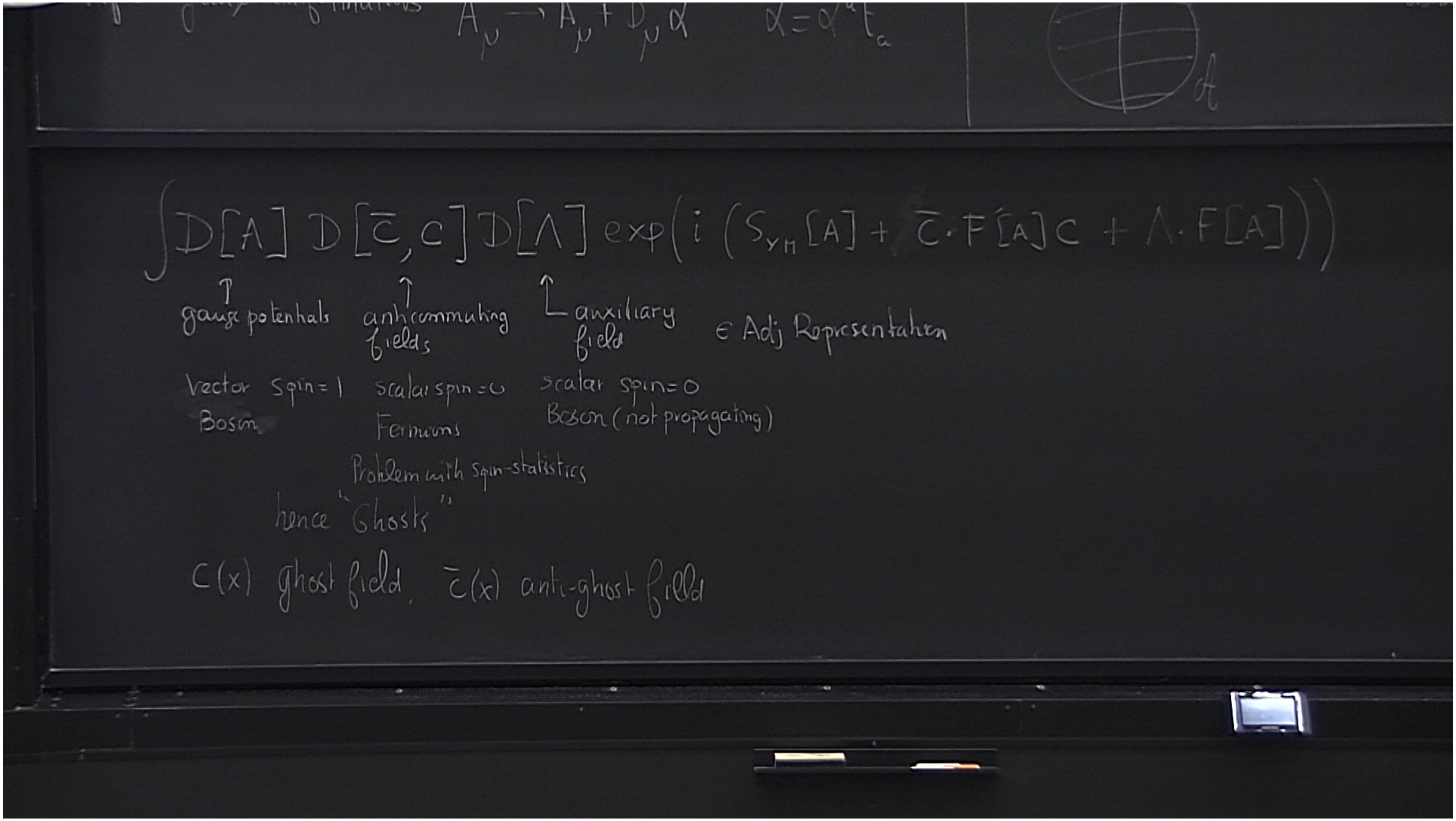
$$\int D[\bar{C},$$

Gauge Fi

$$\delta[F[A$$

$$\Lambda_a(x)$$

$$\int D[\Lambda$$



$A_\mu \to A_\mu + D_\mu \alpha \qquad \alpha = \alpha^a t_a$

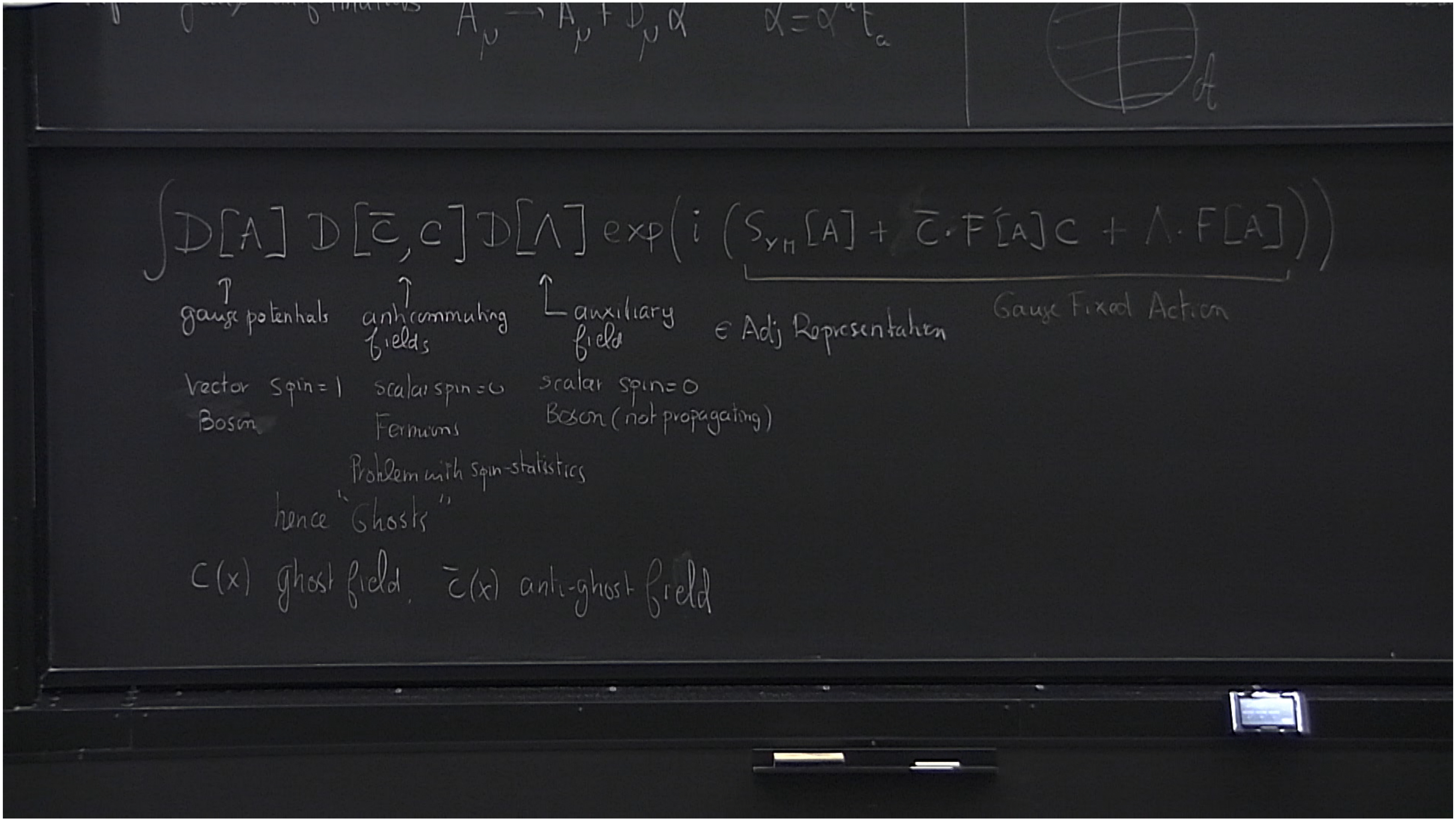
$$\int D[A]\, D[\bar{c}, c]\, D[\Lambda]\, \exp\left(i \left(S_{YM}[A] + \bar{c} \cdot F'[A] c + \Lambda \cdot F[A] \right) \right)$$

gauge potentials	anticommuting fields	auxiliary field	∈ Adj Representation
vector spin = 1	scalar spin = 0	scalar spin = 0	
Boson	Fermions	Boson (not propagating)	

Problem with spin-statistics

hence "Ghosts"

$c(x)$ ghost field, $\bar{c}(x)$ anti-ghost field



$A_\mu \to A_\mu + D_\mu \alpha \qquad \alpha = \alpha^a t_a$

$$\int \mathcal{D}[A]\, \mathcal{D}[\bar{c}, c]\, \mathcal{D}[\Lambda]\, \exp\Big(i\, \big(\underbrace{S_{YM}[A] + \bar{c}\cdot F'[A]\, c + \Lambda \cdot F[A]}_{\text{Gauge Fixed Action}}\big)\Big)$$

gauge potentials	anticommuting fields	auxiliary field	∈ Adj Representation
Vector spin = 1	scalar spin = 0	scalar spin = 0	
Boson	Fermions	Boson (not propagating)	
	Problem with spin-statistics		

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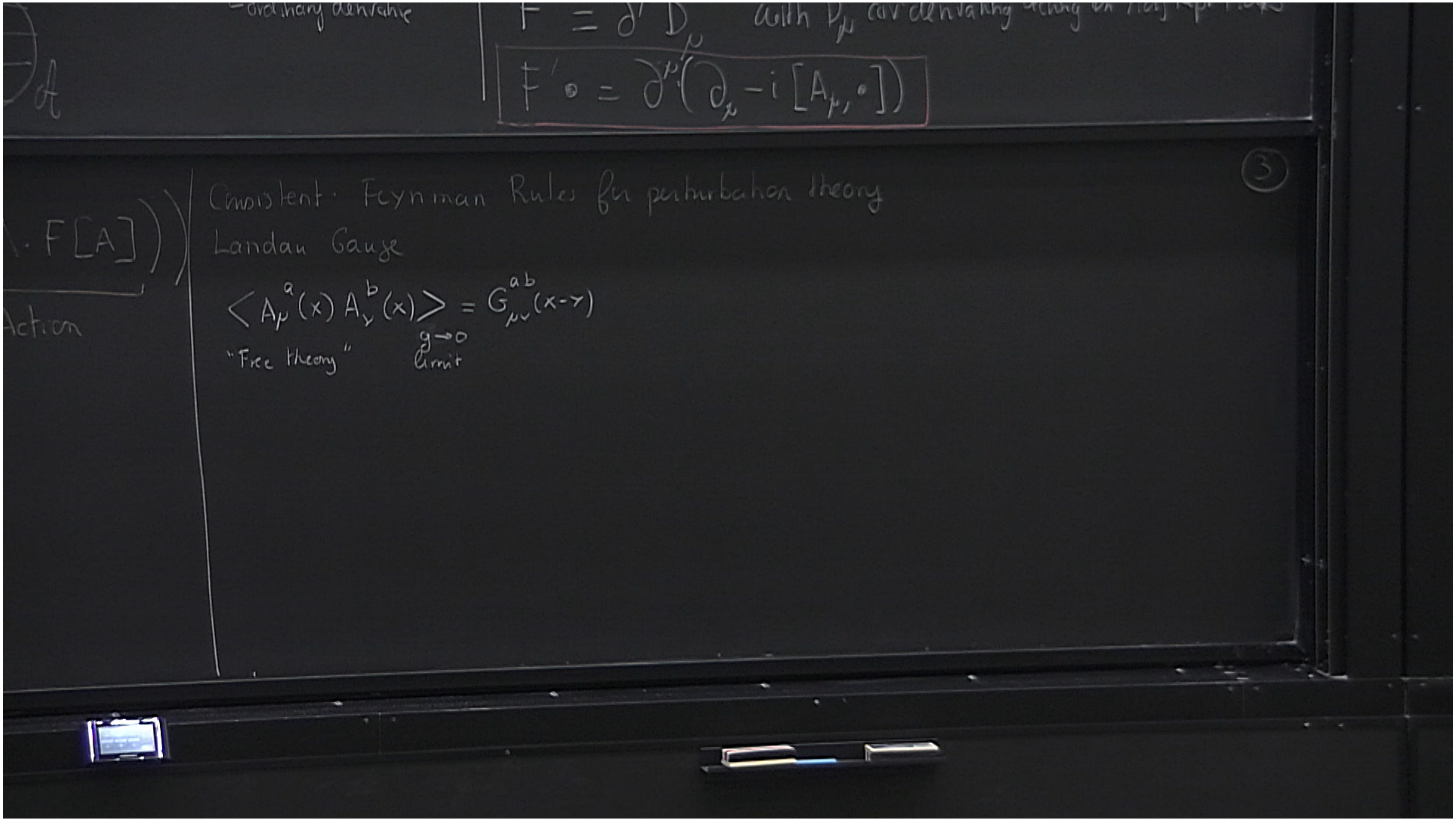
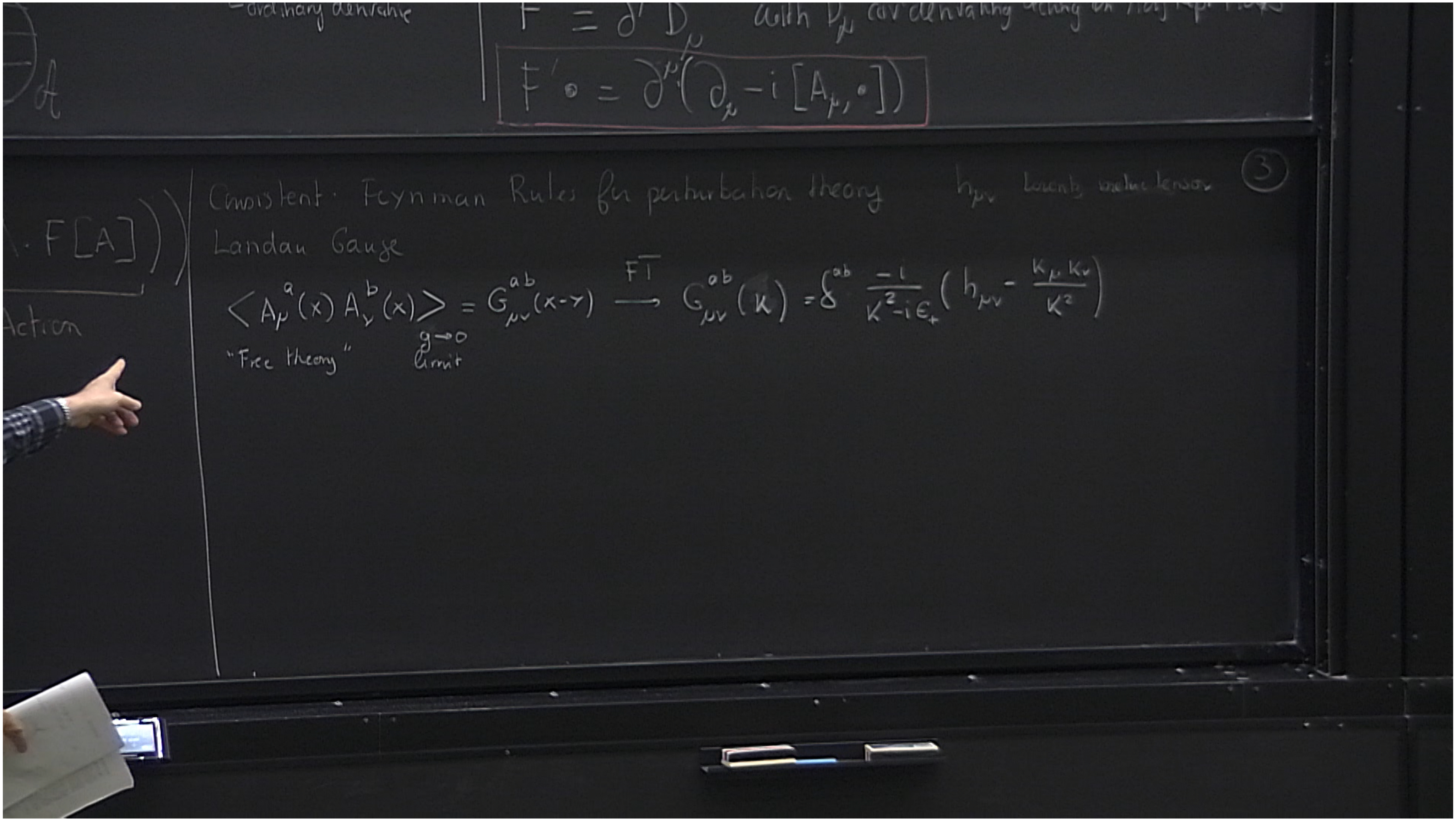




– ordinary derivative

$F = \partial^{\mu} D_{\mu}$ with D_{μ} covariant derivative acting on adj rep fields

$$\boxed{F' \cdot = \partial^{\mu}(\partial_{\mu} - i[A_{\mu}, \cdot])}$$

$\cdot F[A]))$

Action

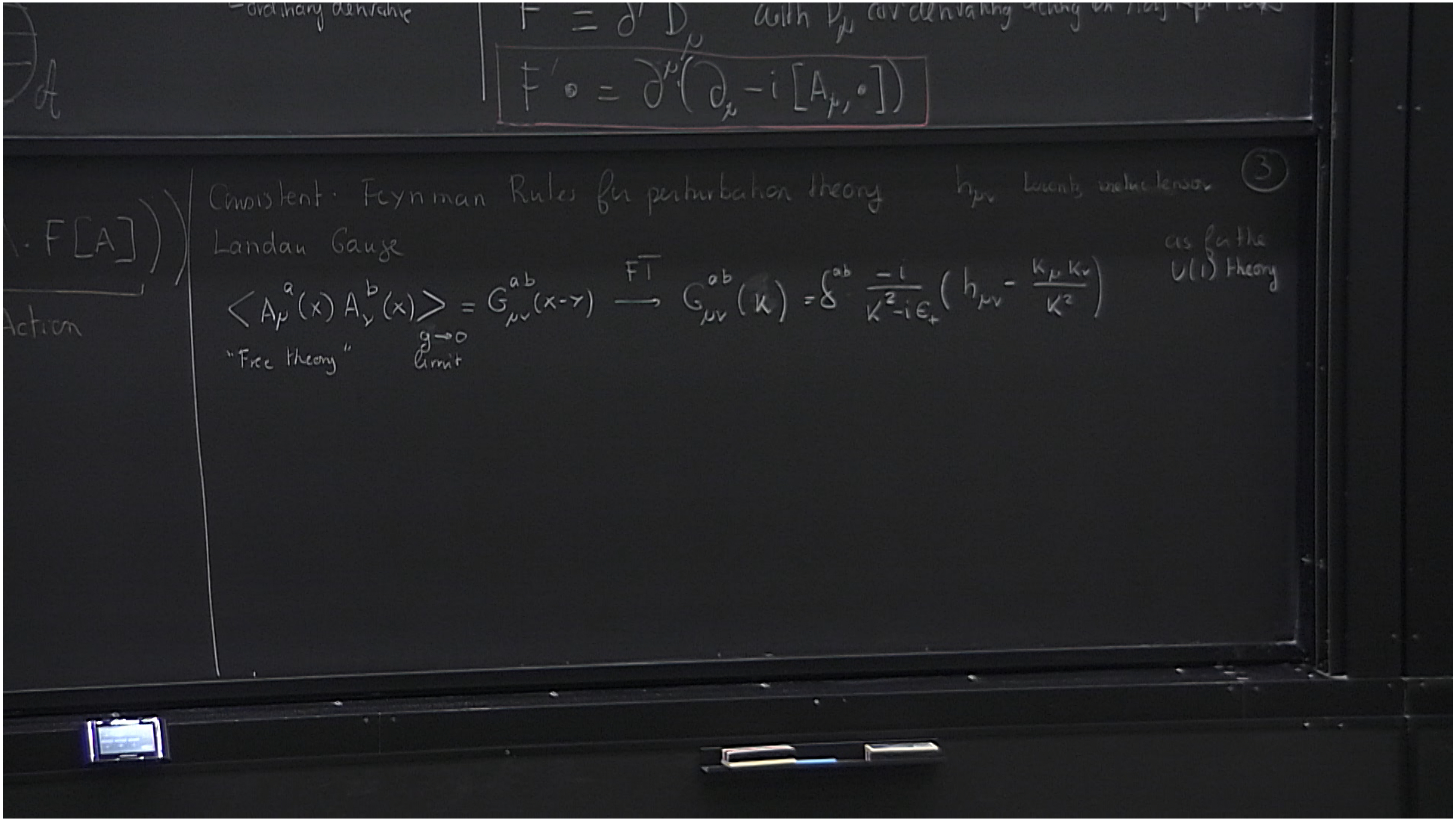
③

Consistent · Feynman Rules for perturbation theory $h_{\mu\nu}$ Lorentz metric tensor

Landau Gauge

$$\langle A_{\mu}^{a}(x) A_{\nu}^{b}(x) \rangle \underset{\substack{g \to 0 \\ \text{limit}}}{=} G_{\mu\nu}^{ab}(x-y) \xrightarrow{FT} G_{\mu\nu}^{ab}(k) = \delta^{ab} \frac{-i}{k^2 - i\epsilon_+}\left(h_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}\right)$$

"Free theory"



ordinary derivative

$F = \partial^\mu D_\mu$ with D_μ cov derivative acting on adj rep

$$\boxed{F' \bullet = \partial^\mu \left(\partial_\mu - i [A_\mu, \bullet] \right)}$$

$\cdot F[A]))$

Action

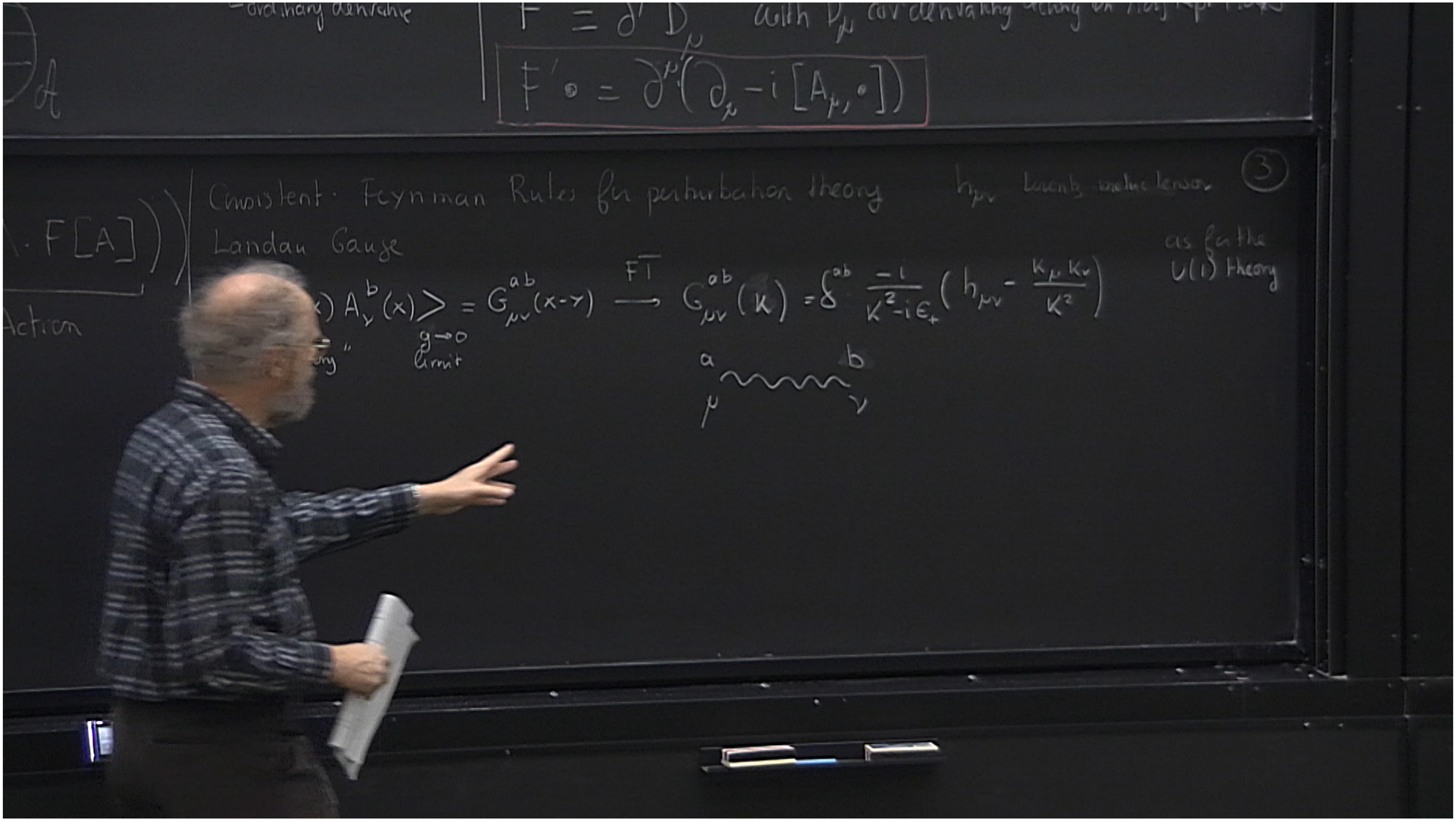
Consistent Feynman Rules for perturbation theory $\quad h_{\mu\nu}$ Lorentz metric tensor ③

Landau Gauge

$$\langle A^a_\mu(x) A^b_\nu(x) \rangle = G^{ab}_{\mu\nu}(x-y) \xrightarrow{FT} G^{ab}_{\mu\nu}(k) = \delta^{ab} \frac{-i}{k^2 - i\epsilon_+} \left(h_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$$

as for the U(1) theory

"Free theory" $\quad g \to 0$ limit



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$$\boxed{F' \cdot = \partial^\mu (\partial_\mu - i [A_\mu, \cdot])}$$

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Action

Consistent Feynman Rules for perturbation theory $h_{\mu\nu}$ Lorentz metric tensor ③

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$$x) A^b_\nu(x) \underset{\substack{g\to 0\\ \text{limit}}}{>} = G^{ab}_{\mu\nu}(x-y) \xrightarrow{FT} G^{ab}_{\mu\nu}(k) = \delta^{ab} \frac{-i}{k^2 - i\epsilon_+}\left(h_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$$

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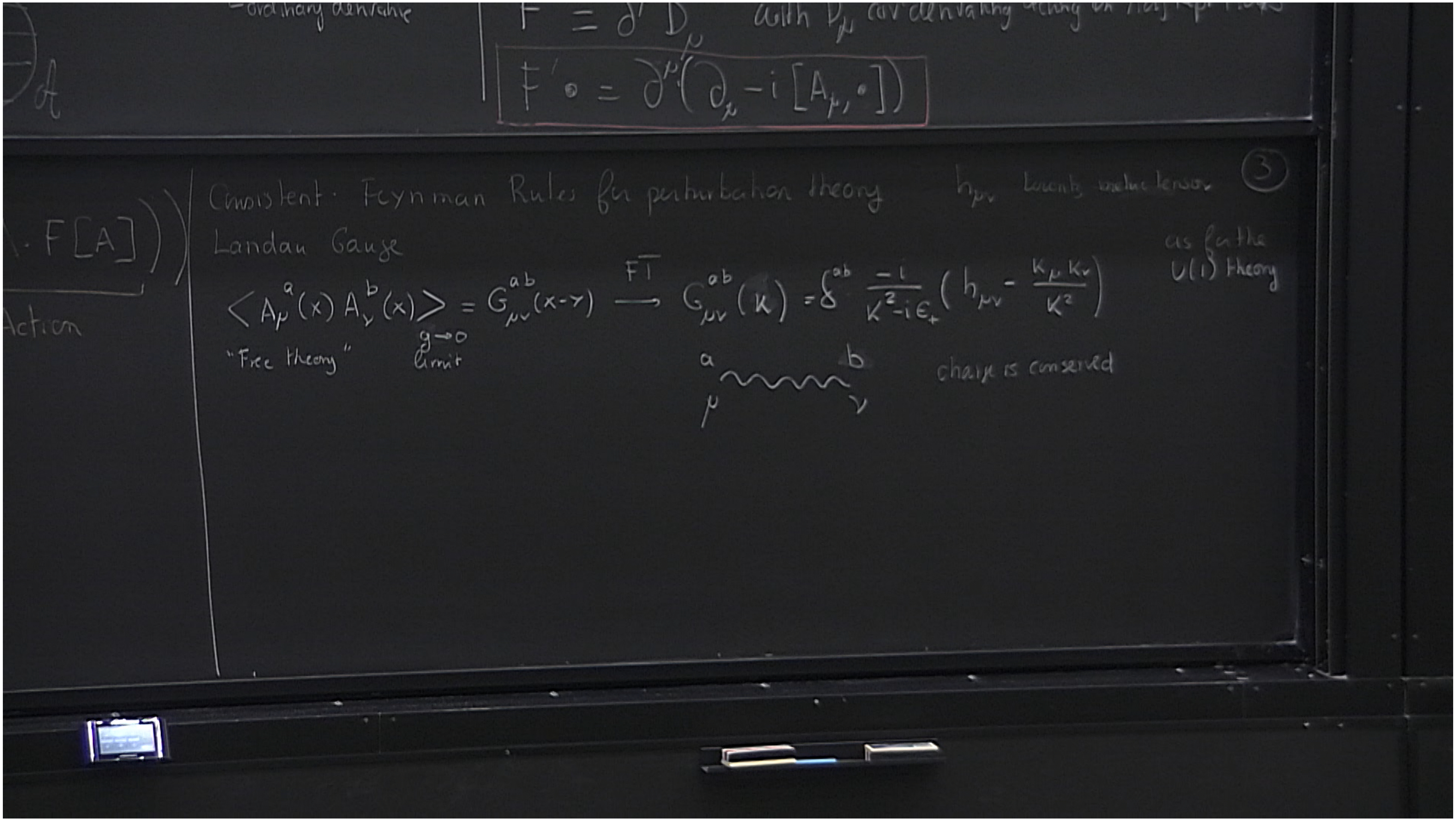

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$\cdot F[A]))$

Action

③

Consistent · Feynman Rules for perturbation theory $h_{\mu\nu}$ Lorentz invariant tensor

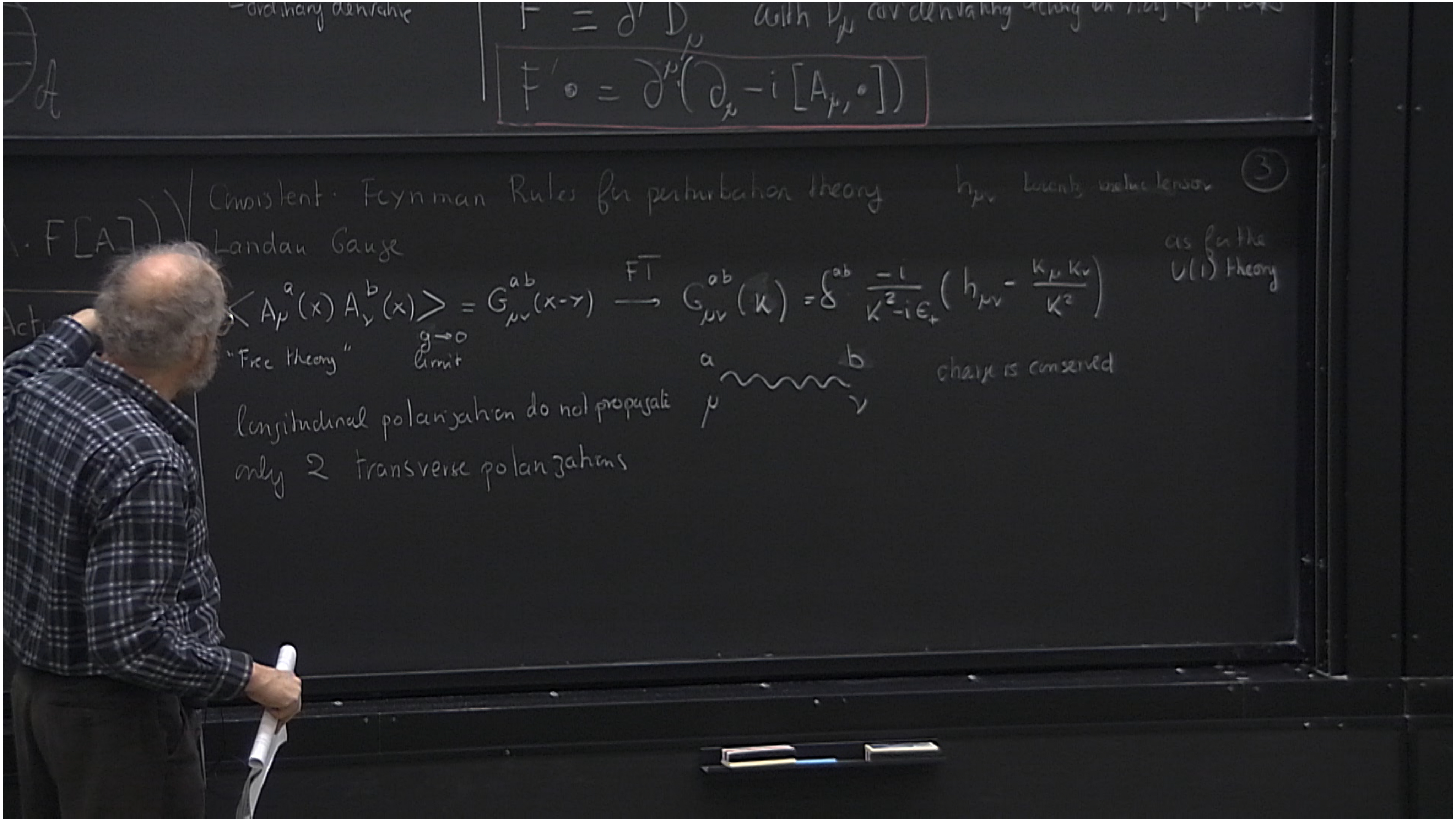
Landau Gauge

$$\langle A^a_\mu(x) A^b_\nu(x) \rangle \underset{g \to 0 \text{ limit}}{=} G^{ab}_{\mu\nu}(x-y) \xrightarrow{FT} G^{ab}_{\mu\nu}(k) = \delta^{ab} \frac{-i}{k^2 - i\epsilon_+} \left(h_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$$

"Free theory"

as for the U(1) theory

charge is conserved



ordinary derivative

$F = \partial^\mu D_\mu$ with D_μ cov derivative acting on

$$\boxed{F'\cdot = \partial^\mu(\partial_\mu - i\,[A_\mu, \cdot])}$$

$\cdot F[A]$

Act

③

Consistent · Feynman Rules for perturbation theory

$h_{\mu\nu}$ Lorentz invariant tensor

Landau Gauge

$$\langle A^a_\mu(x)\, A^b_\nu(x)\rangle \underset{g\to 0}{=} G^{ab}_{\mu\nu}(x-y) \xrightarrow{FT} G^{ab}_{\mu\nu}(k) = \delta^{ab}\,\frac{-i}{k^2 - i\epsilon_+}\left(h_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

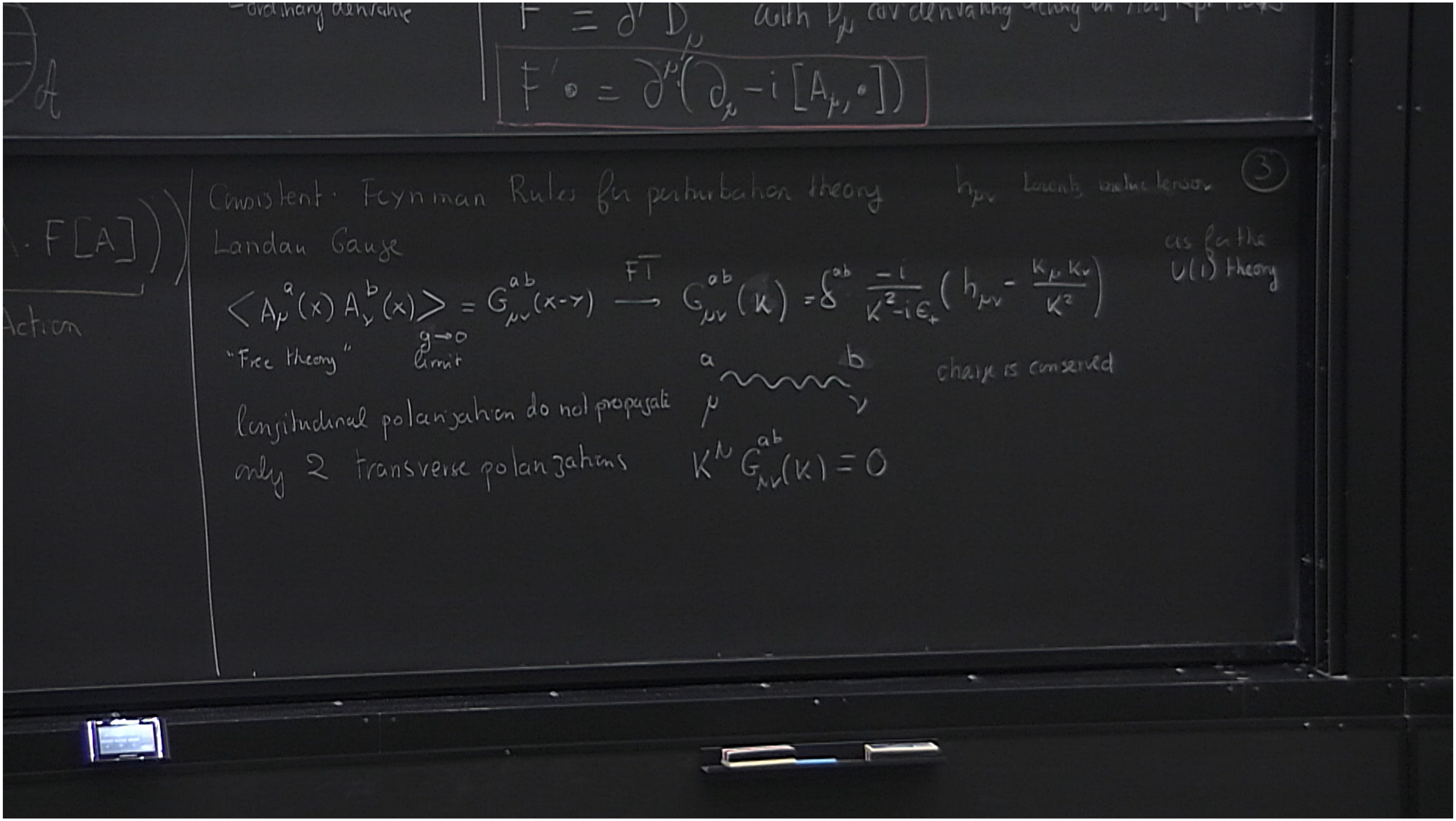
as for the U(1) theory

"Free theory" limit

charge is conserved

longitudinal polarization do not propagate

only 2 transverse polarizations



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Action

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Consistent · Feynman Rules for perturbation theory $h_{\mu\nu}$ Lorentz metric tensor

Landau Gauge

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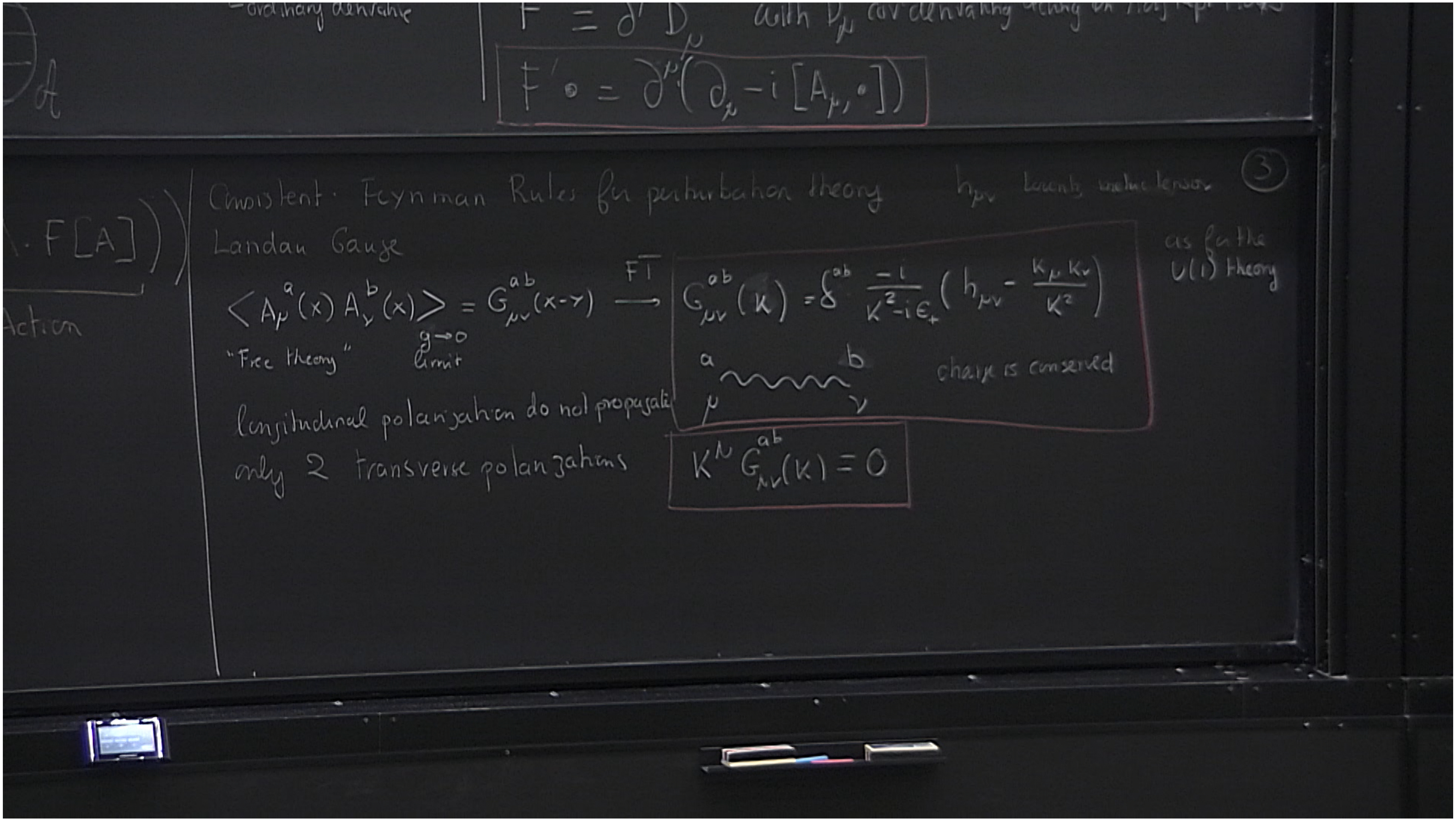

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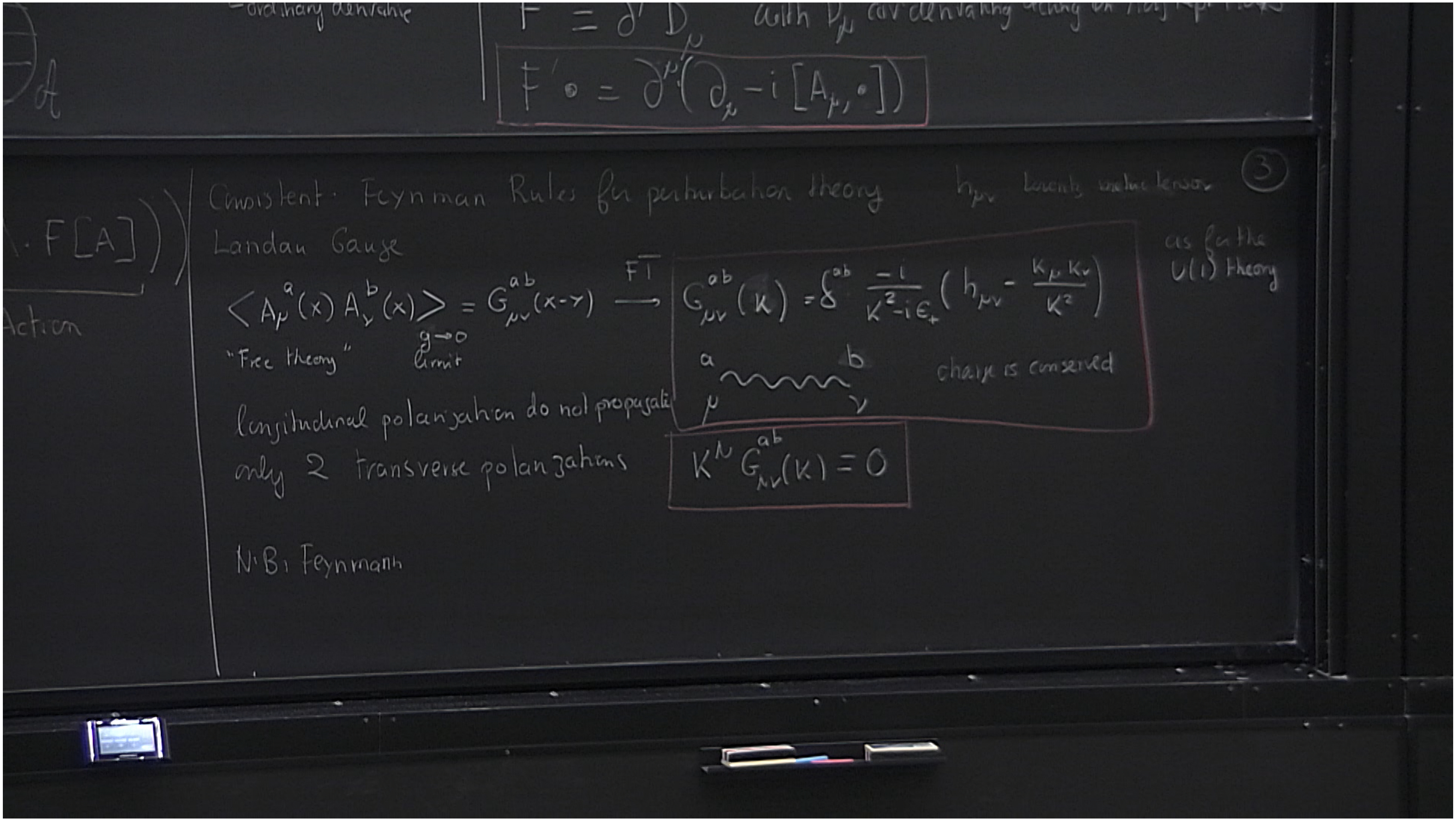

– ordinary derivative

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A

$\cdot F[A]))$

Action

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"Free theory"   limit

as for the U(1) theory

a b

μ ν

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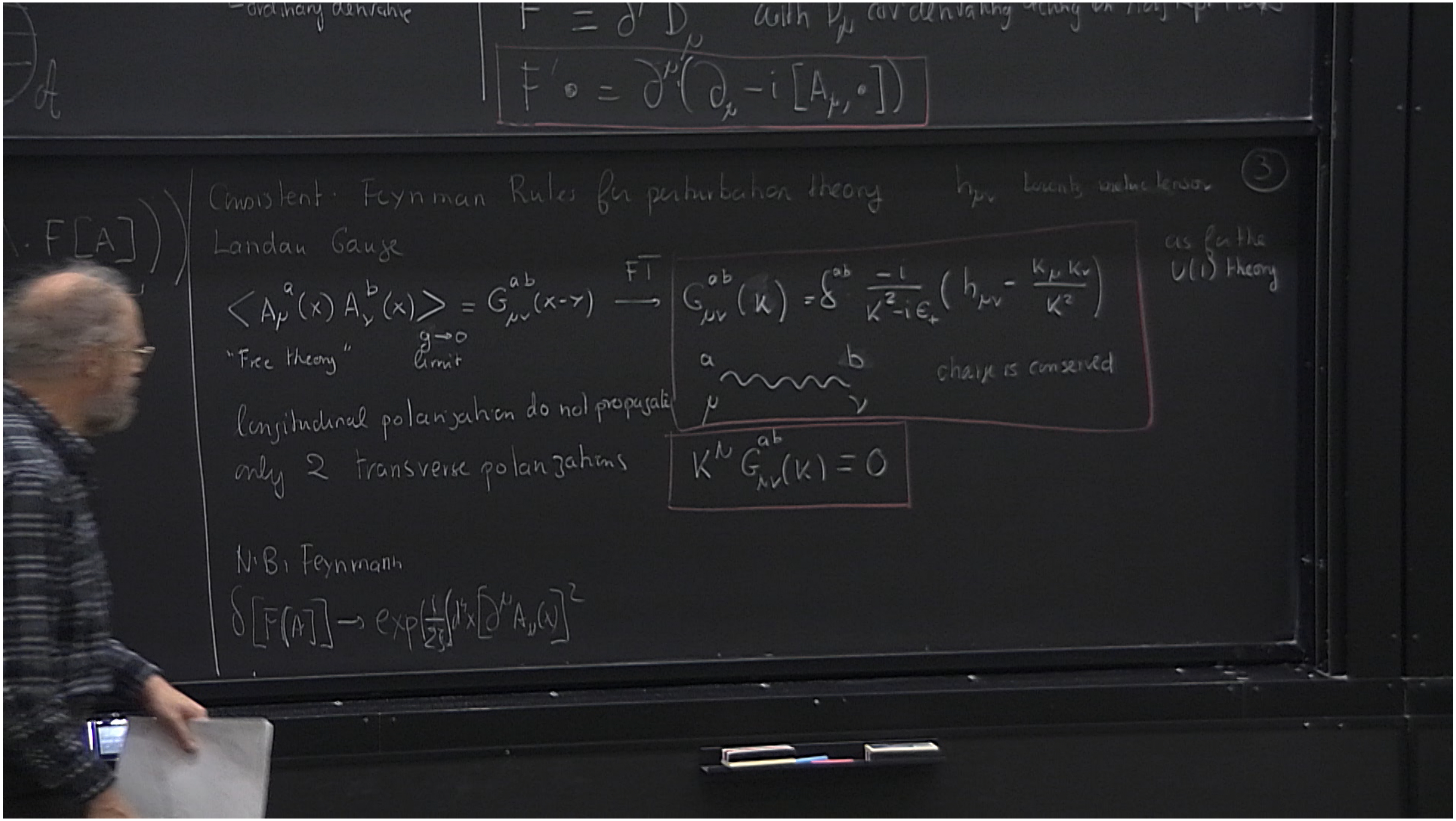

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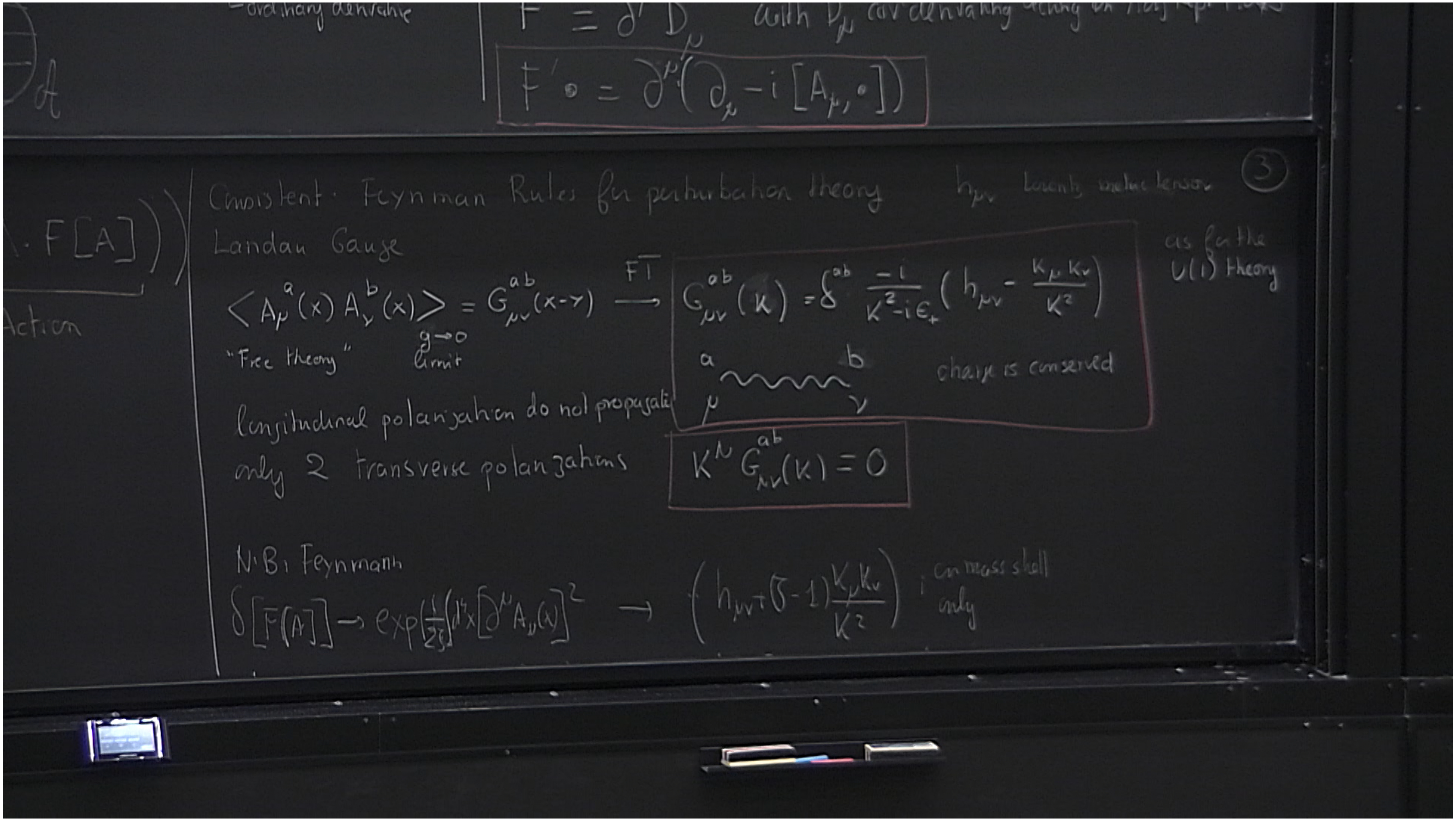
$$\delta[F(A)] \to \exp\left(\frac{i}{2\xi}\int d^4x \left[\partial^\mu A_\mu(x)\right]^2\right.$$



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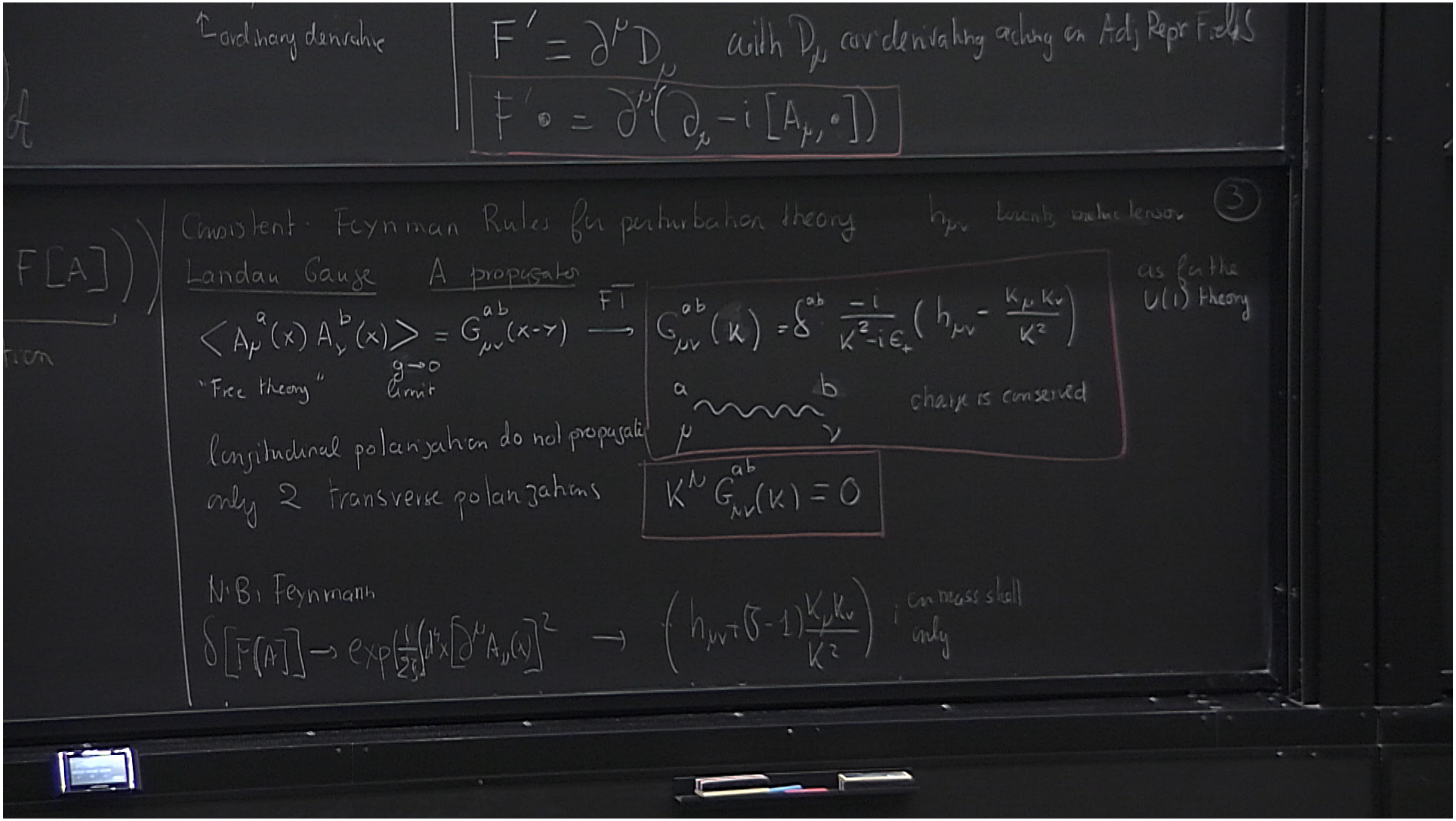
; on mass shell only



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$$\boxed{F' \bullet = \partial^\mu (\partial_\mu - i [A_\mu, \bullet])}$$

F[A]))

tion

③

Consistent Feynman Rules for perturbation theory $h_{\mu\nu}$ Lorentz metric tensor

Landau Gauge A propagator

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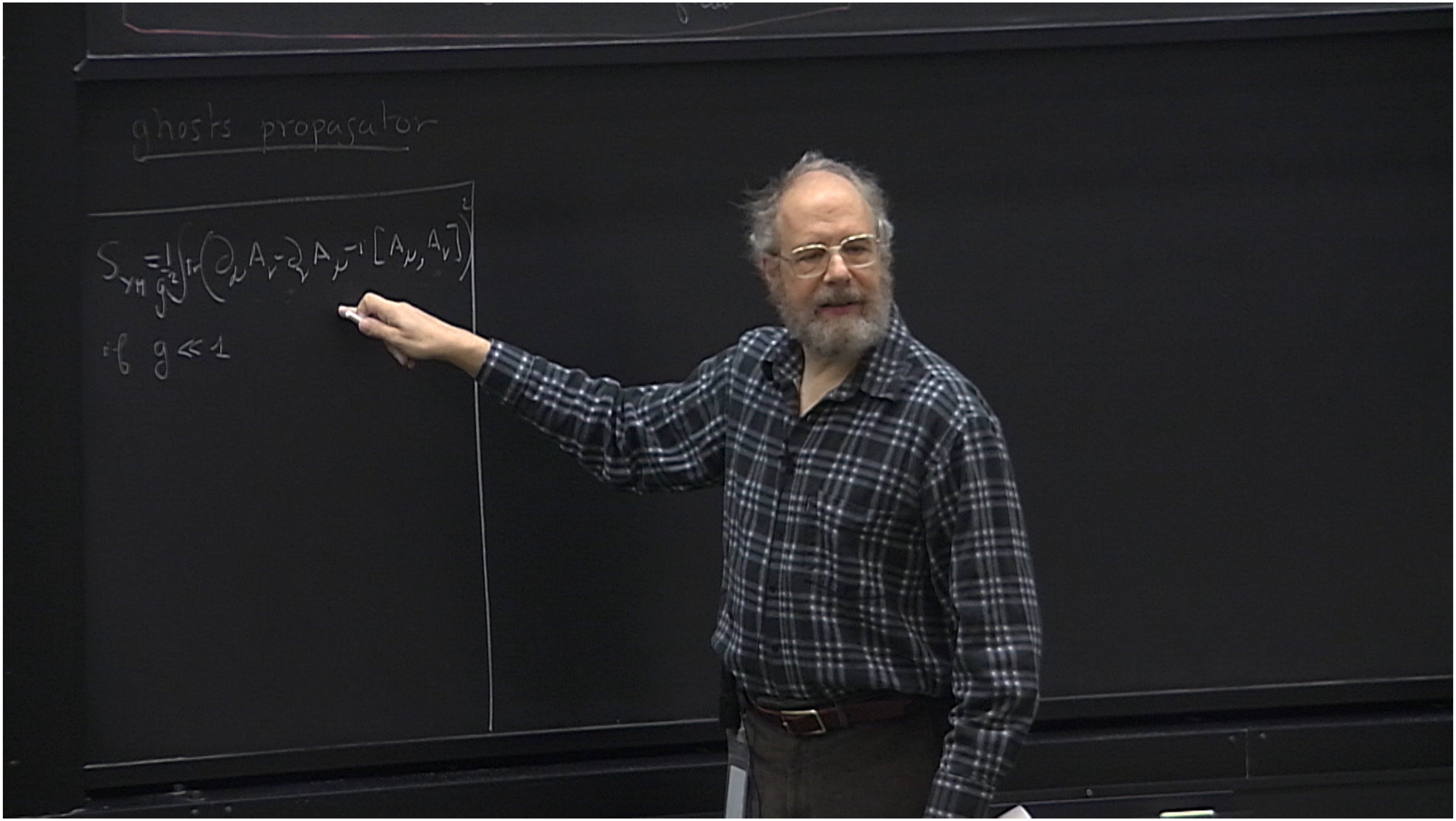
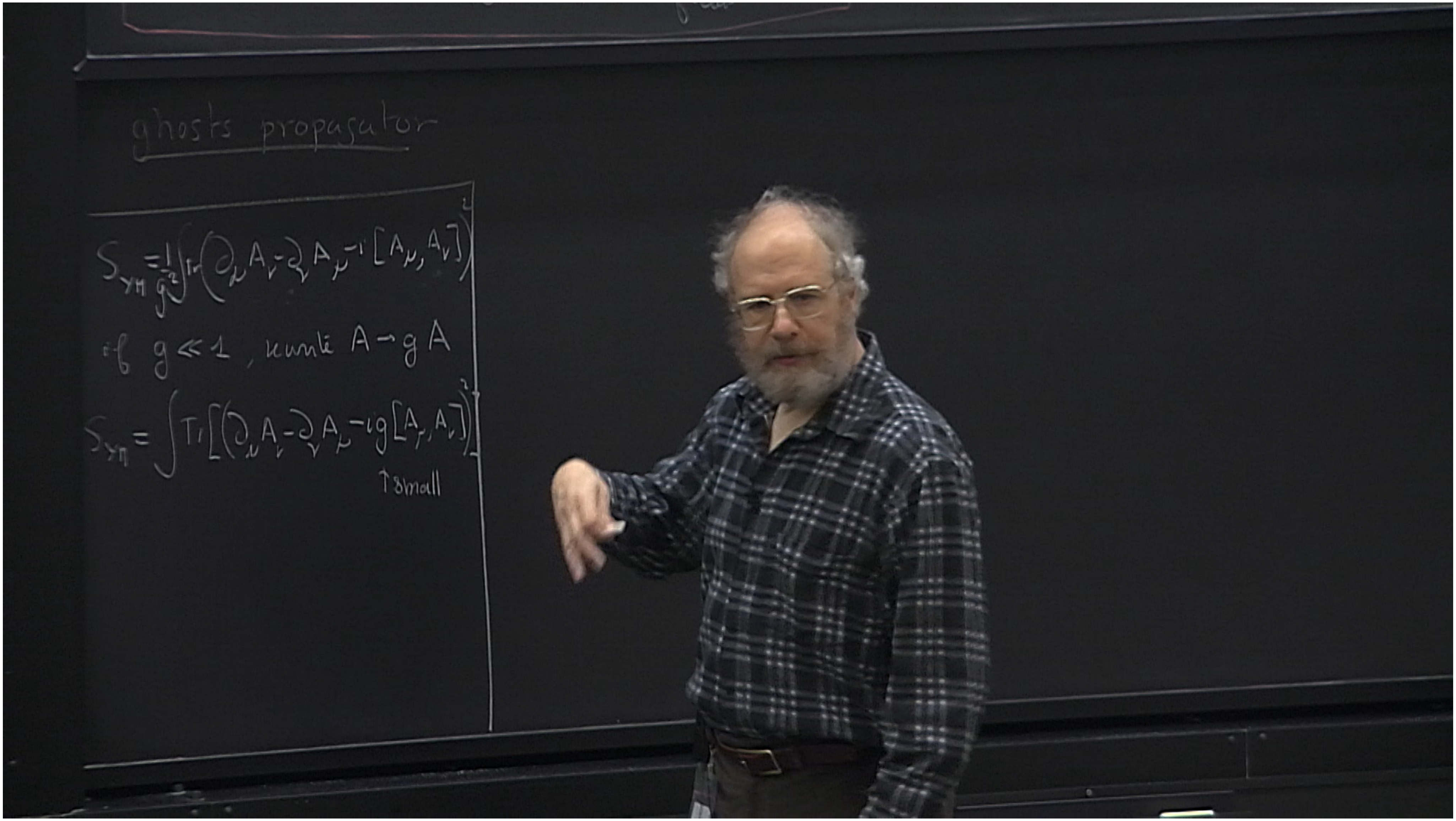

ghosts propagator
$S_{YM} = \frac{1}{g^2}\int \mathrm{tr}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]\right)^2$
if $g \ll 1$



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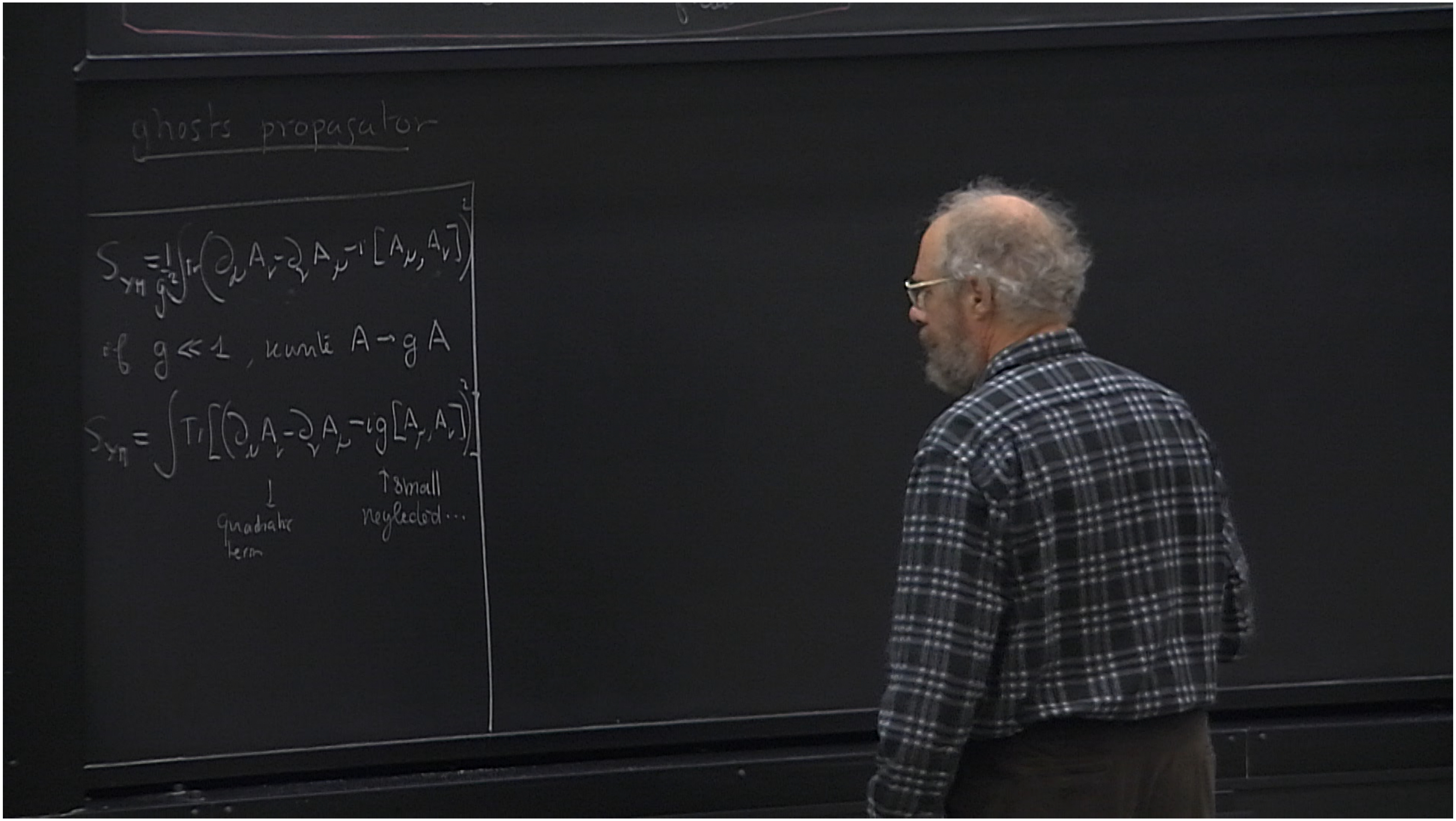
if $g \ll 1$, rescale $A \to g A$

$$S_{YM} = \int \mathrm{Tr}\left[\left(\partial_\mu A_\nu - \partial_\nu A_\mu - \underset{\uparrow \text{small}}{ig}[A_\mu, A_\nu]\right)^2\right]$$



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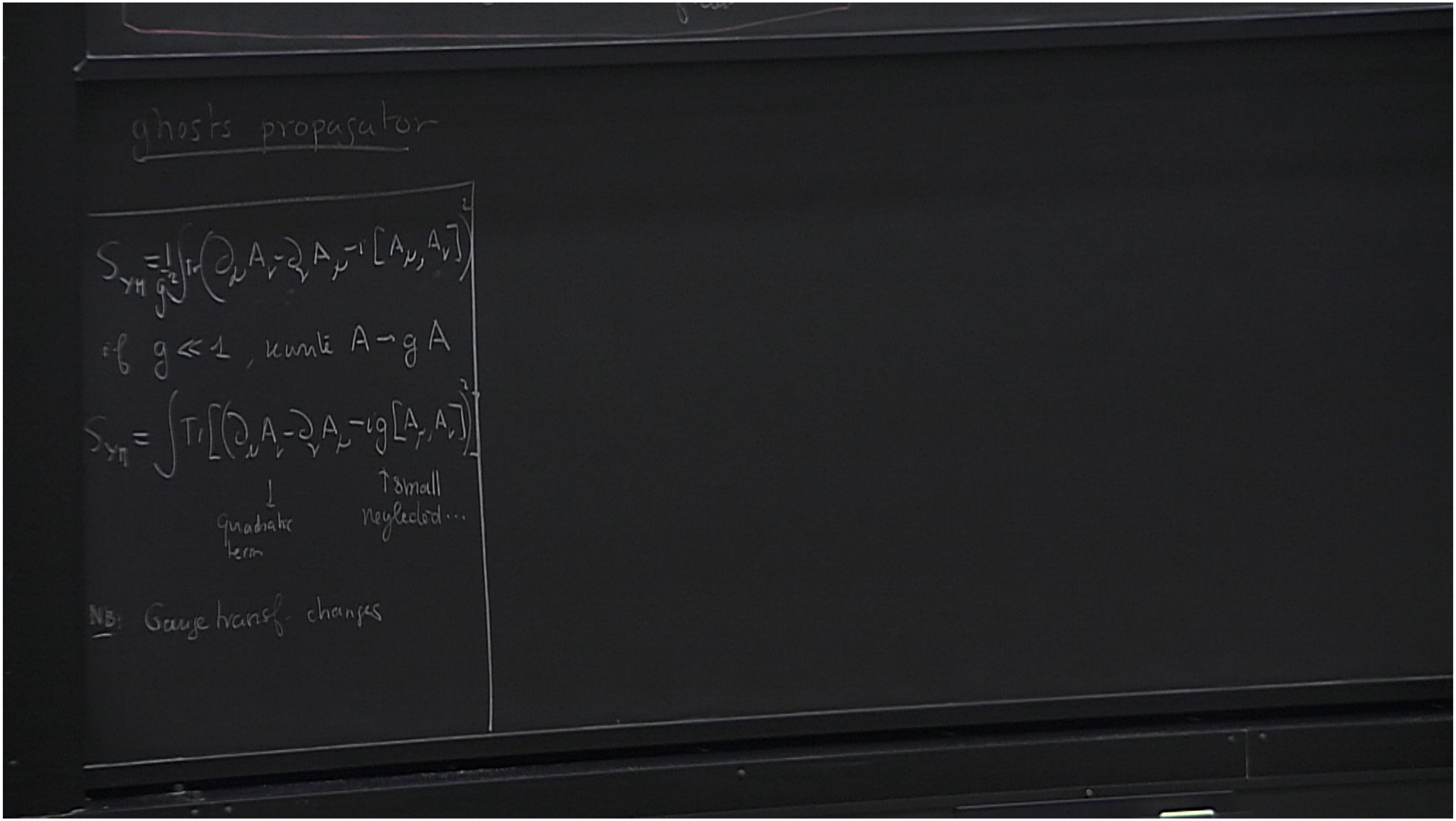

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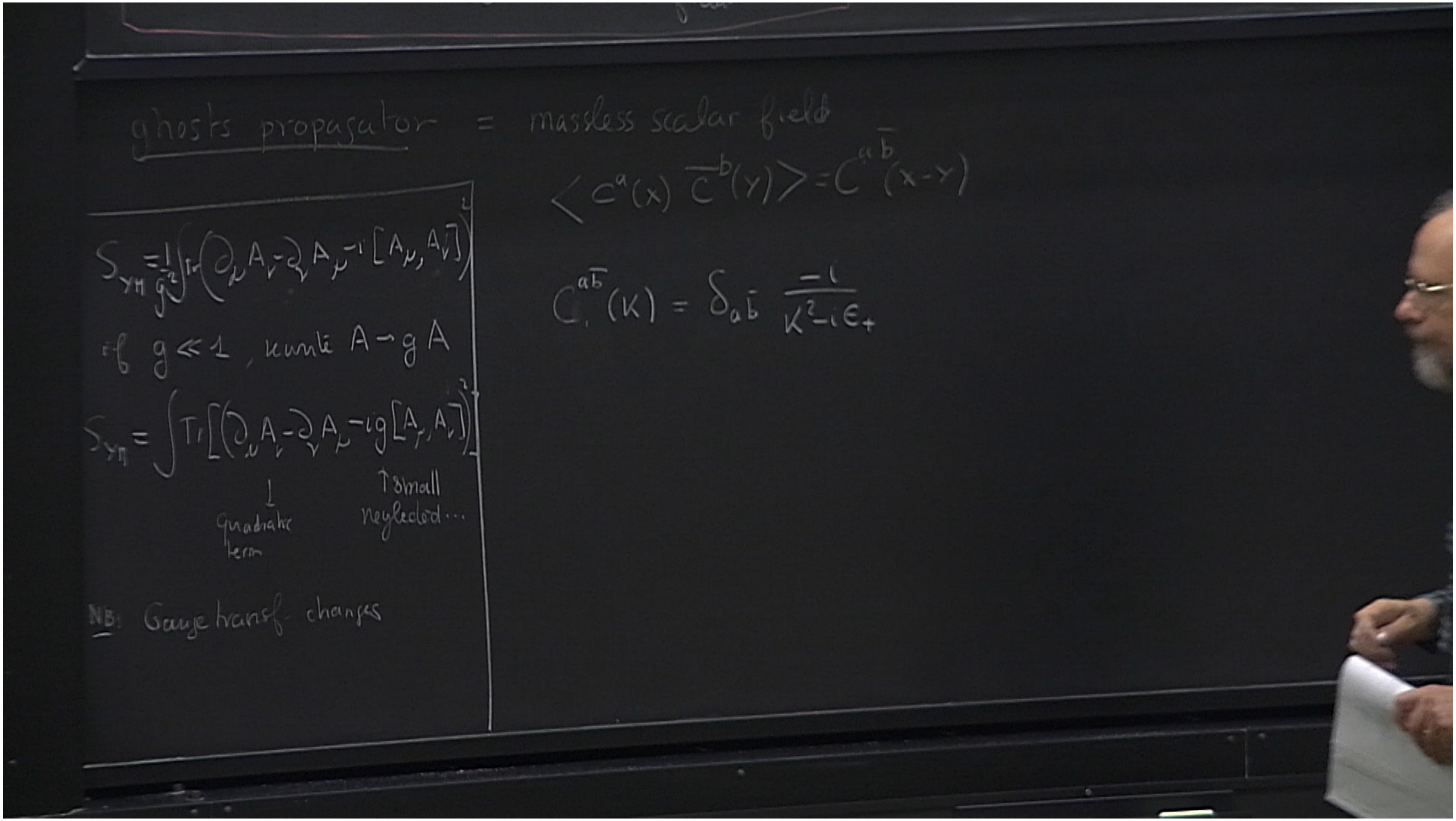
NB: Gauge transf. changes



ghosts propagator = massless scalar field

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$$S_{YM} = \int \mathrm{Tr} \Big[\big(\underset{\substack{\uparrow \\ \text{quadratic} \\ \text{term}}}{\partial_\nu A_\nu - \partial_\nu A_\mu} - \underset{\substack{\uparrow \text{small} \\ \text{neglected} \dots}}{ig [A_\mu, A_\nu]} \big)^2 \Big]$$

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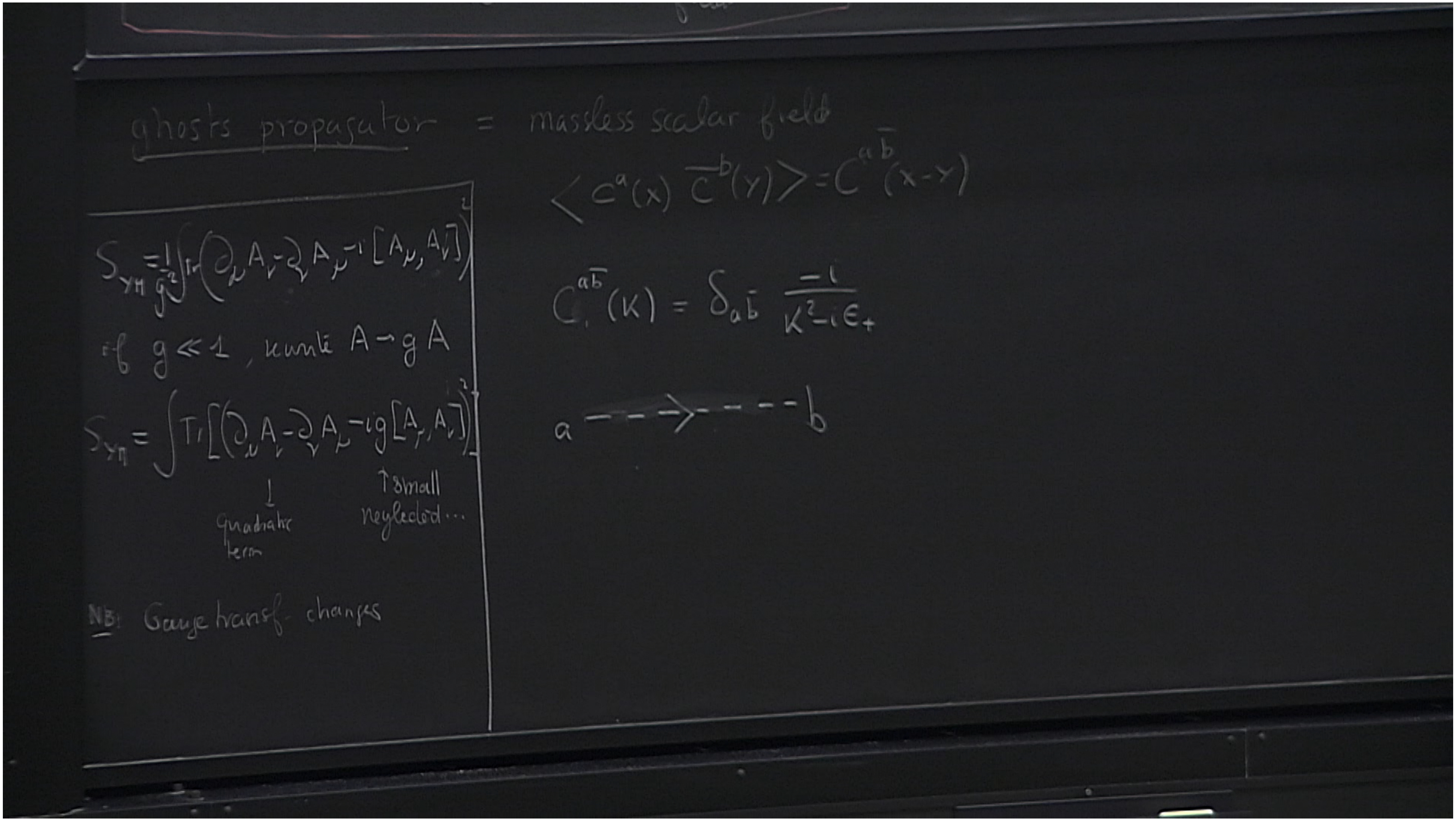
$$\langle c^a(x)\, \bar{c}^b(y) \rangle = C^{a\bar{b}}(x-y)$$

$$C^{a\bar{b}}(K) = \delta_{a\bar{b}} \frac{-i}{K^2 - i\epsilon_+}$$



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$$a \;- - \rightarrow - - - \; b$$

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if $g \ll 1$, rescale $A \to g A$

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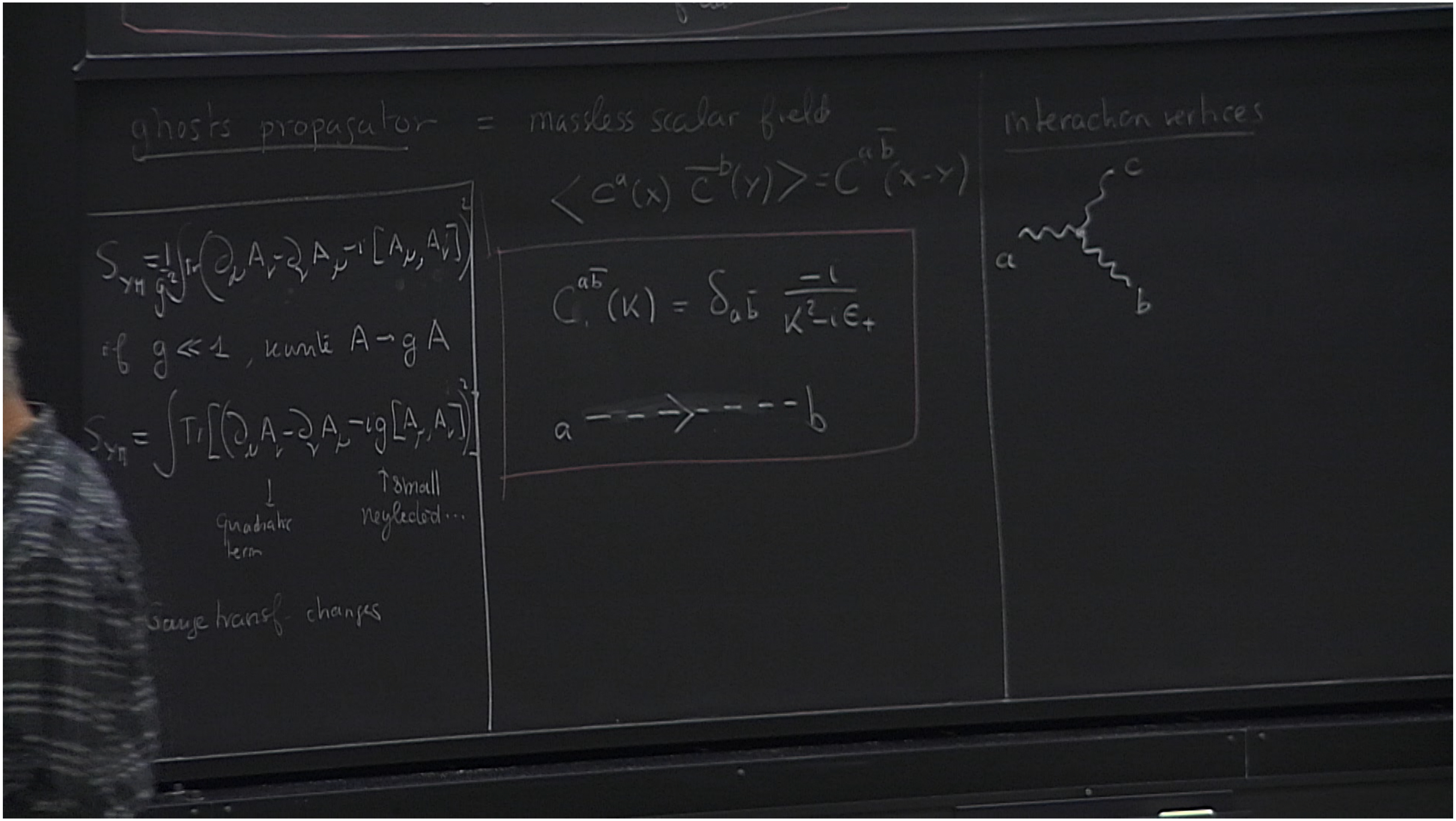

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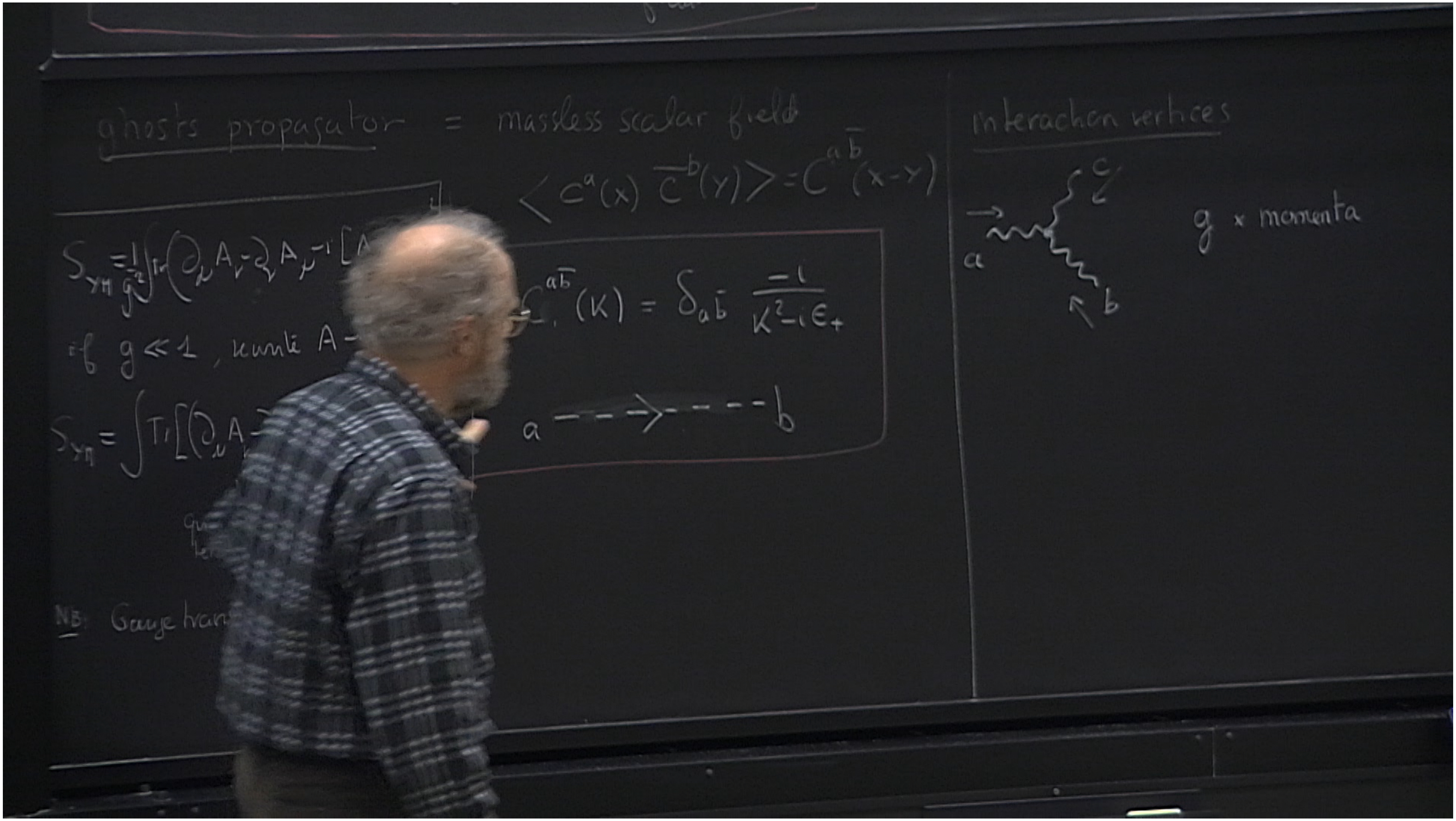
interaction vertices



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if $g \ll 1$, rescale $A \to$

$S_{YM} = \int \mathrm{Tr}[(\partial_\mu A_$

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ke

NB: Gauge tran

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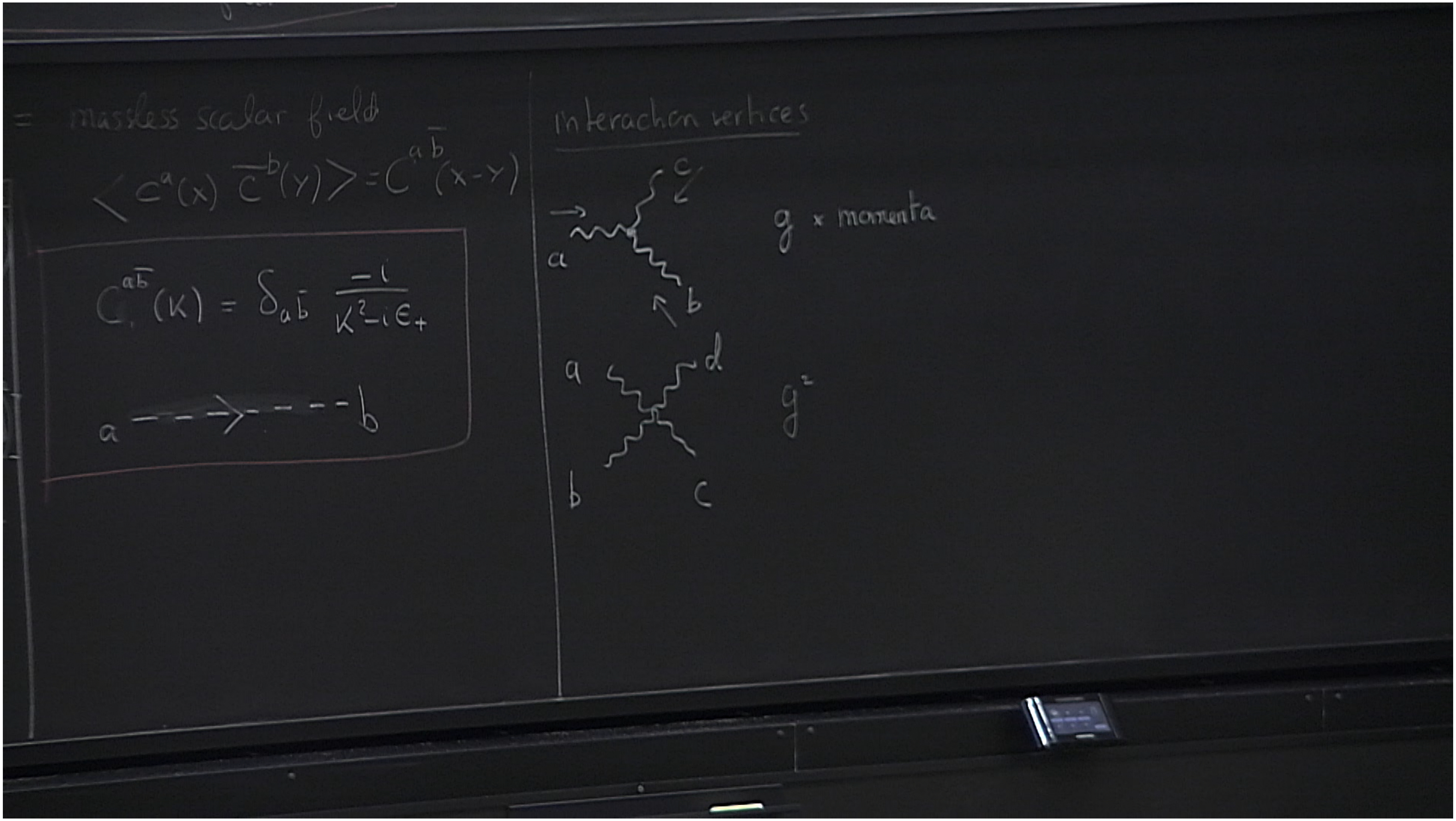
a - - - -> - - - - b

interaction vertices

g × momenta



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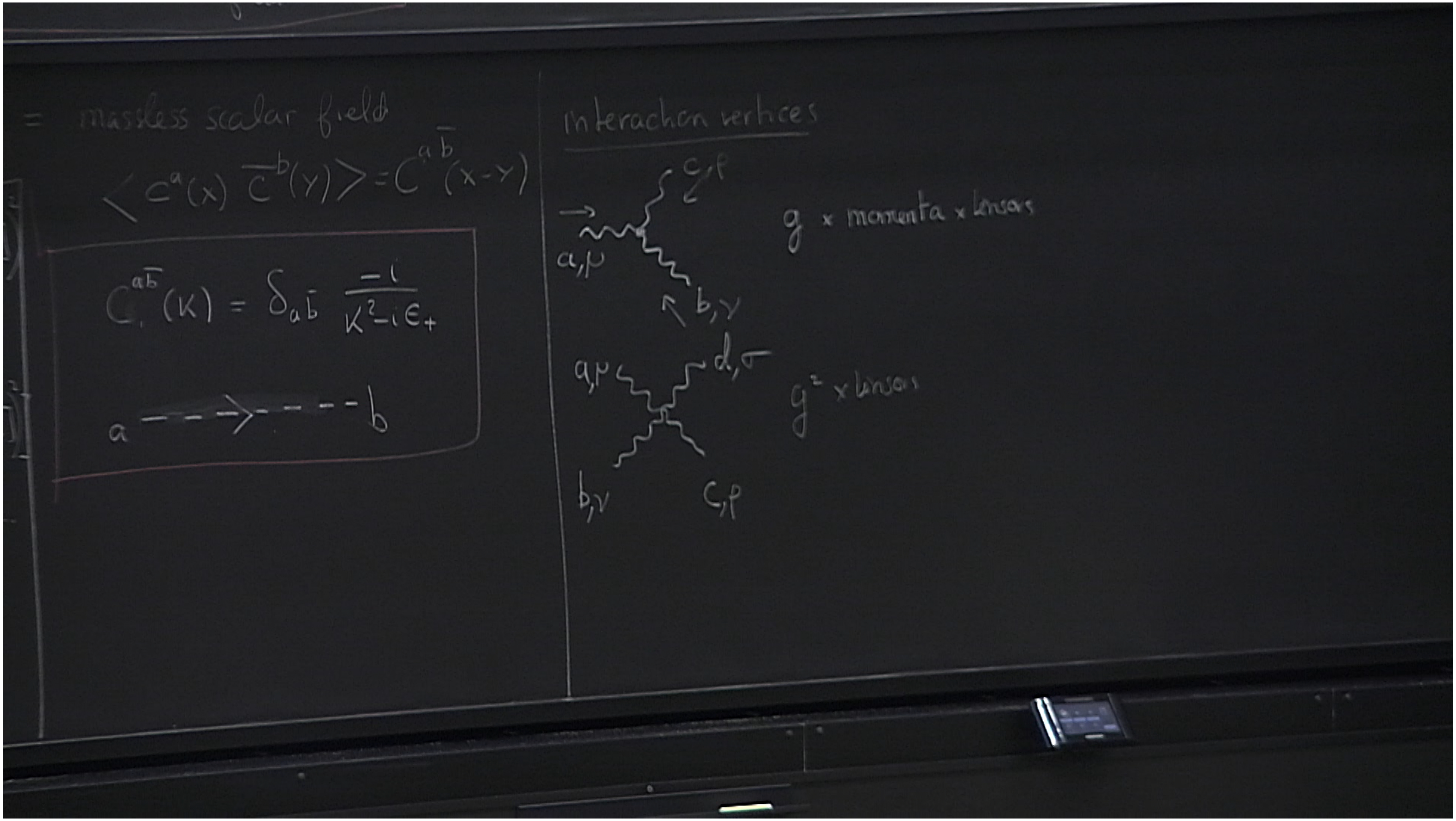
interaction vertices

g × momenta

g^2



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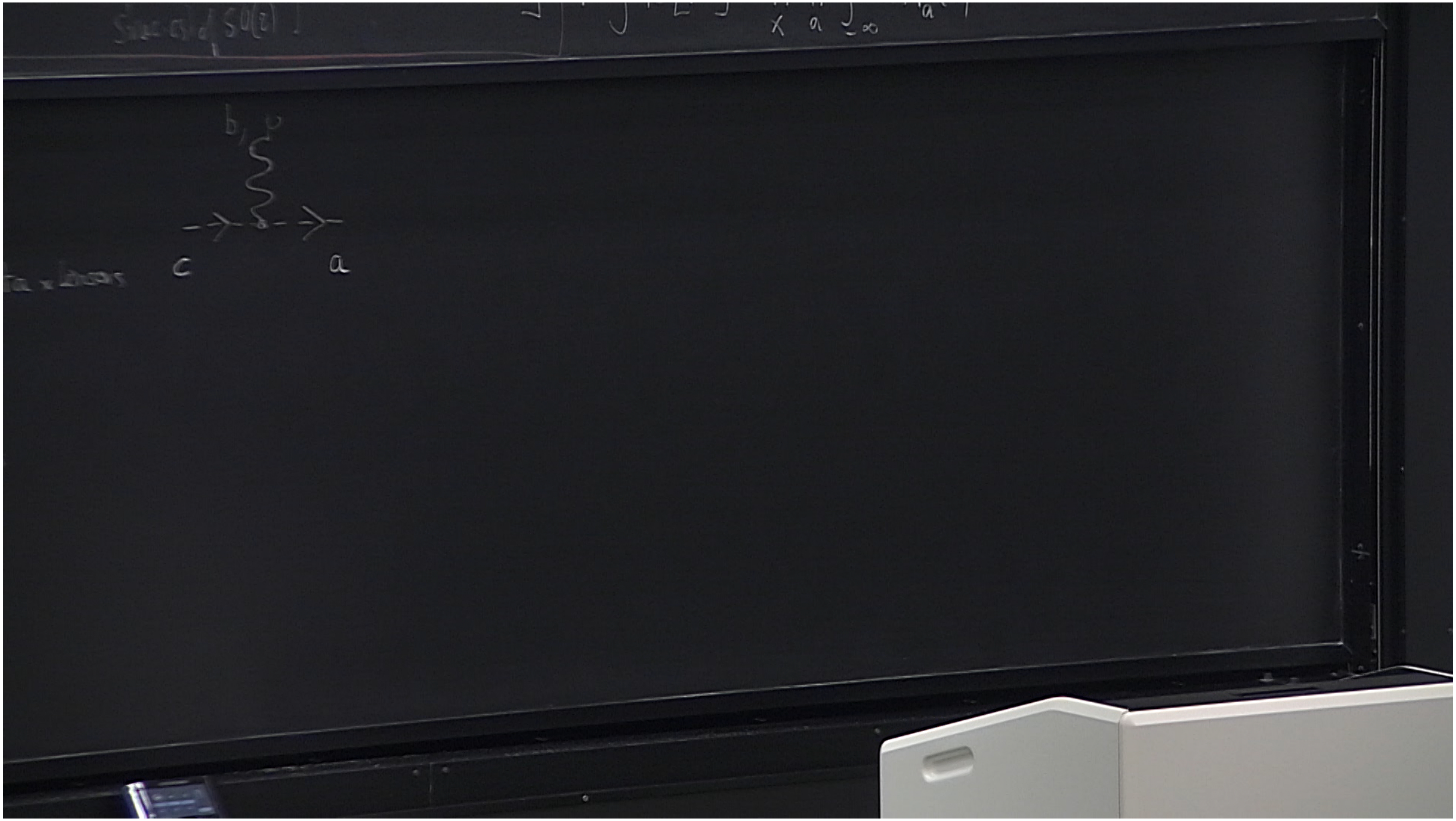
Interaction vertices

g × momenta × tensors

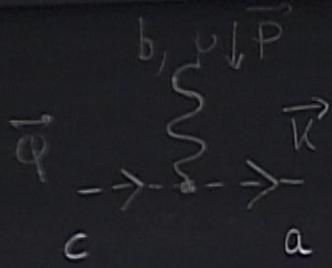
g^2 × tensors


b, ν
c
a

Structure of SO(2) ?

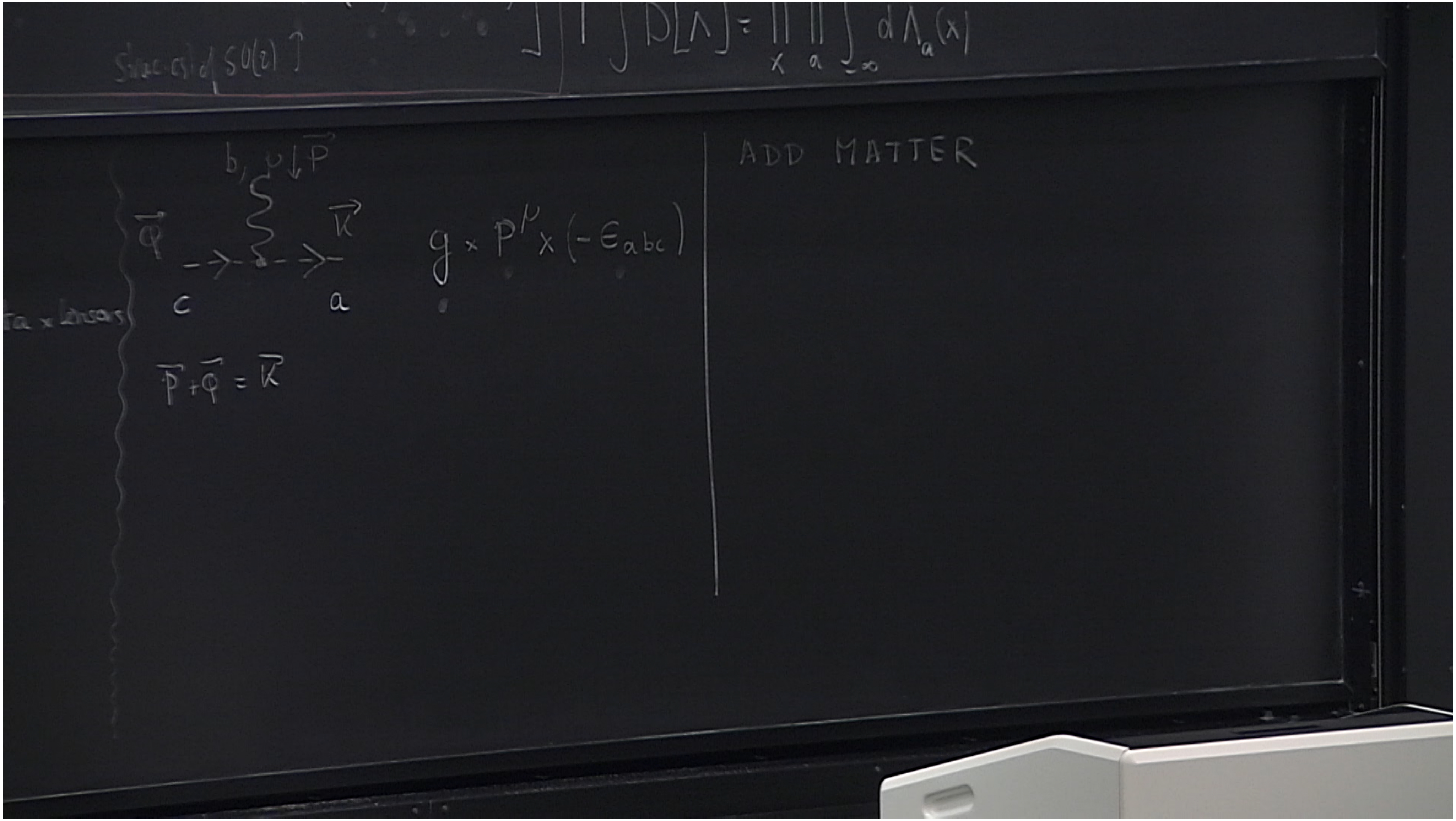
$$\int D[\Lambda] = \prod_x \prod_a \int_{-\infty} d\Lambda_a(x)$$

ta × tensors



$$g \times p^{\nu} \times (-\epsilon_{abc})$$

$$\vec{p} + \vec{q} = \vec{k}$$



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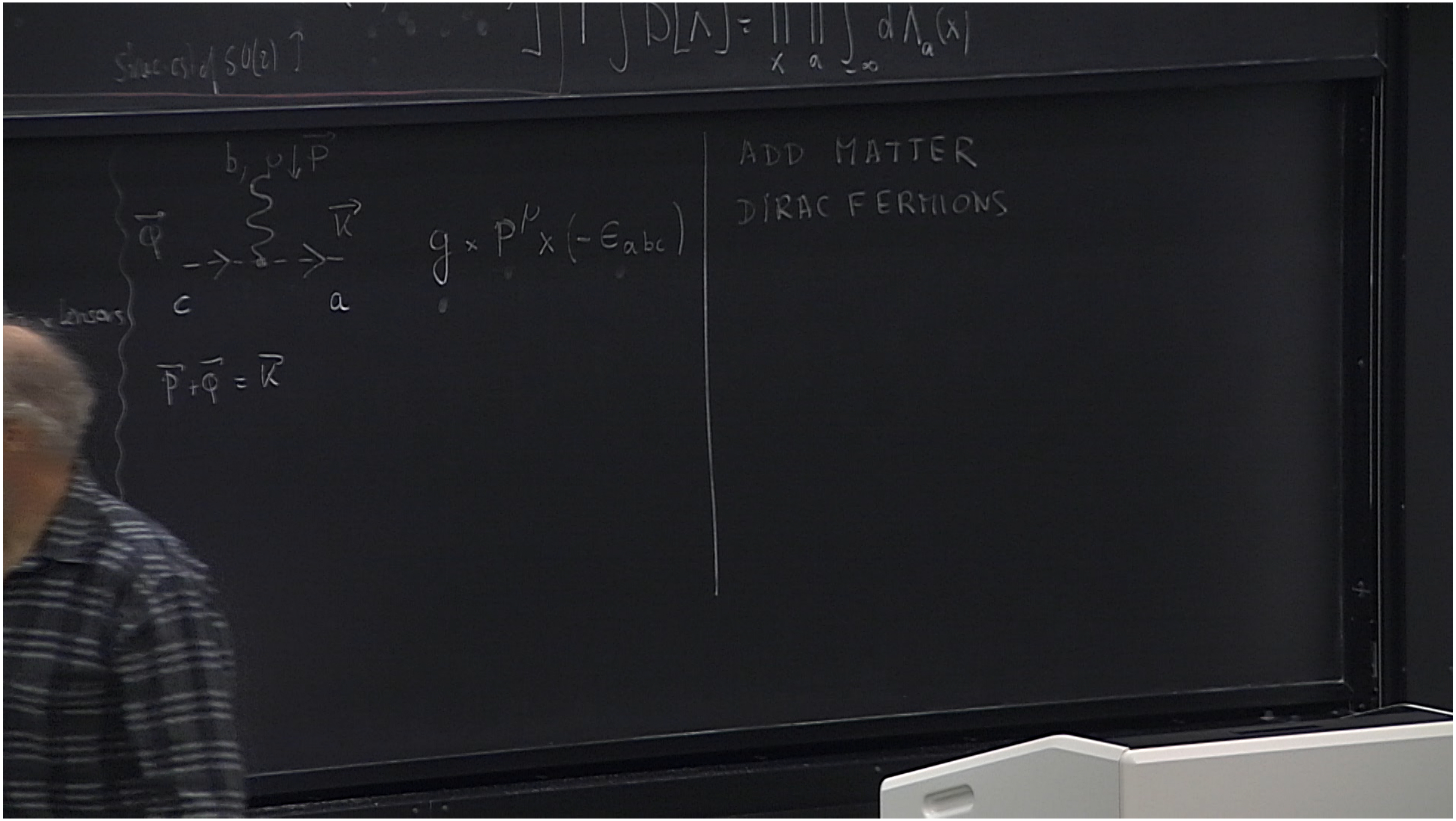

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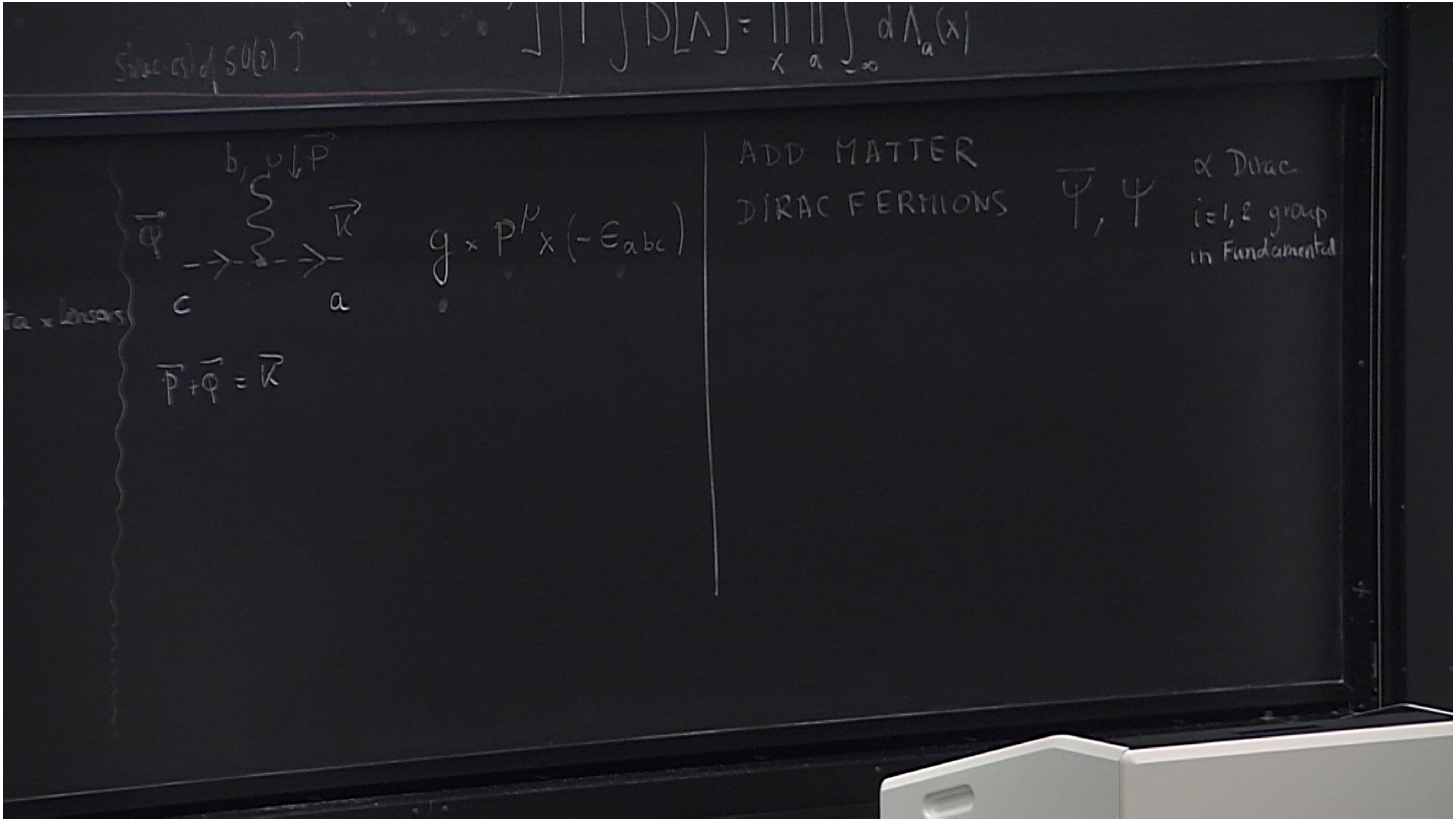
ADD MATTER

DIRAC FERMIONS



Since cst'd SO(2) ?

$\int D[\Lambda] = \prod_x \prod_a \int_{-\infty} d\Lambda_a(x)$

ta × tensors

$b, \nu \downarrow \vec{p}$

$\vec{q}$ $\vec{k}$

c a

$g \times p^{\nu} \times (-\epsilon_{abc})$

$\vec{p} + \vec{q} = \vec{k}$

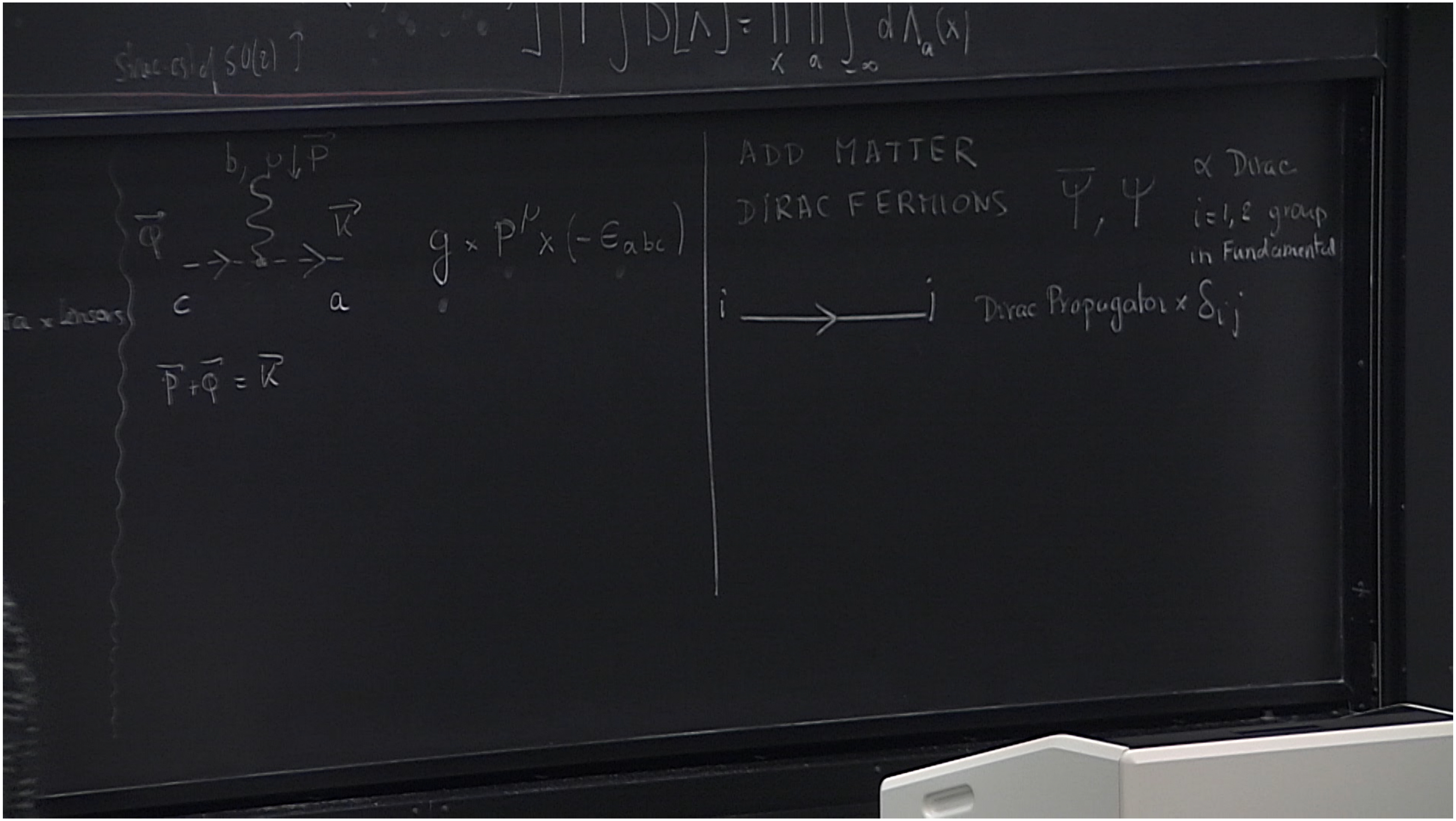
ADD MATTER

DIRAC FERMIONS $\overline{\Psi}, \Psi$

α Dirac

$i = 1, 2$ group

in Fundamental



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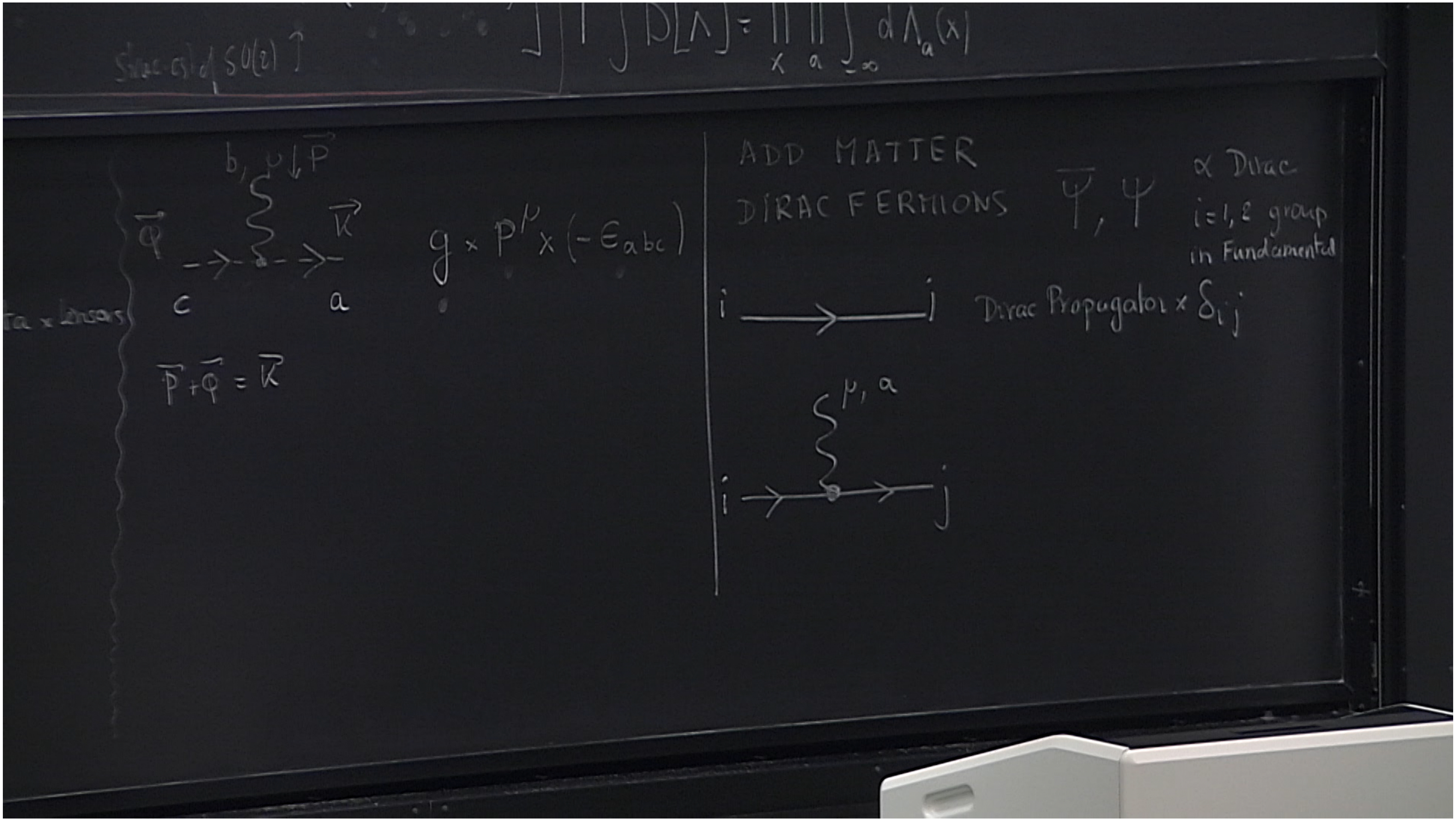
$\vec{p}+\vec{q}=\vec{k}$

ADD MATTER

DIRAC FERMIONS $\bar{\Psi}, \Psi$

α Dirac
i=1,2 group
in Fundamental

Dirac Propagator × δ_{ij}



Spac-csld SO(2)]

$$\Big] \; \Big| \; \int D[\Lambda] = \prod_x \prod_a \int_{-\infty} d\Lambda_a(x)$$

ta × tensors

$b, \mu \downarrow \vec{P}$

$\vec{q}$ $\quad$ $\vec{k}$

c $\quad$ a

$$g \times P^{\mu} \times (-\epsilon_{abc})$$

$$\vec{P} + \vec{q} = \vec{k}$$

ADD MATTER

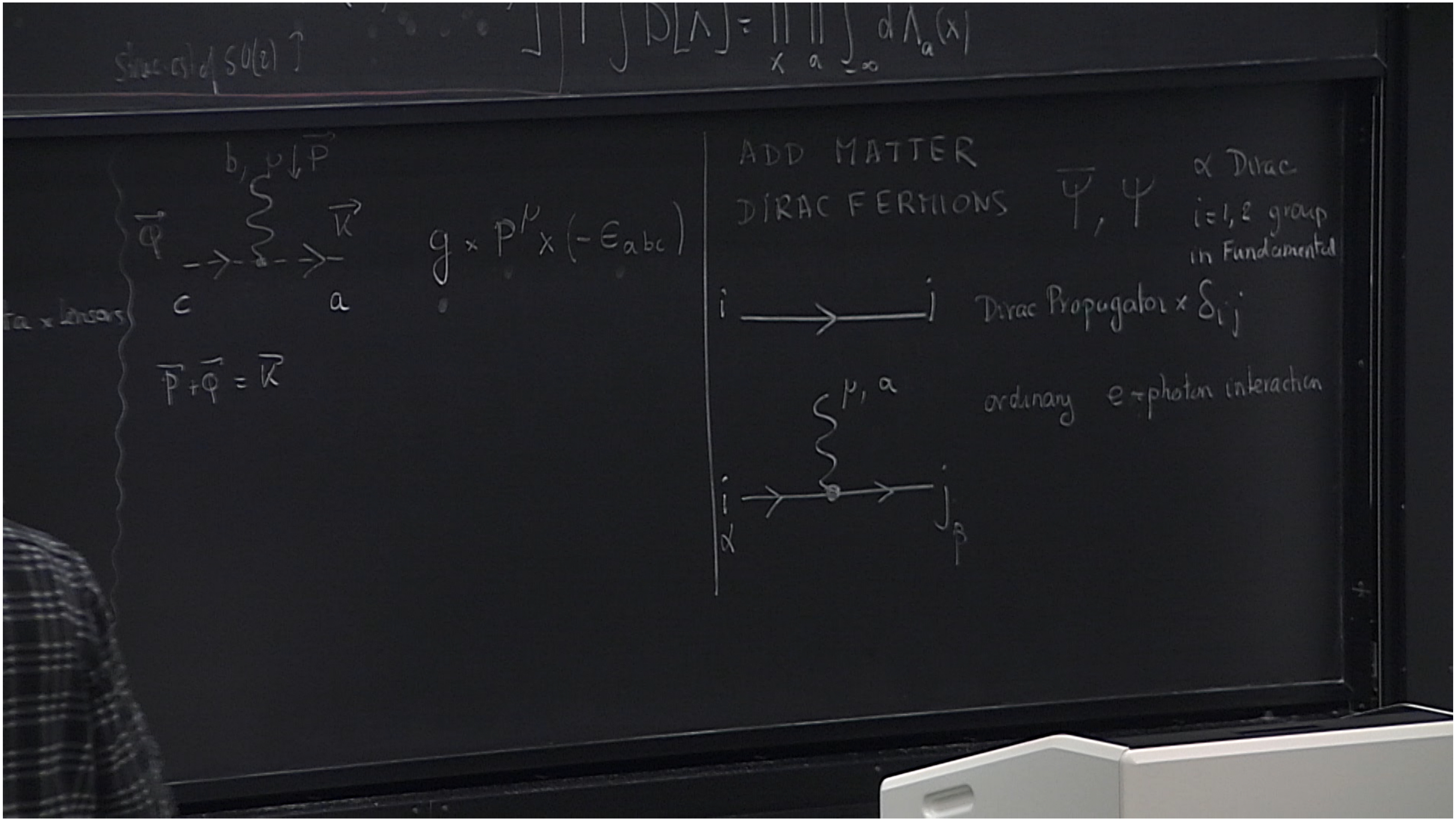
DIRAC FERMIONS $\bar{\Psi}, \Psi$

α Dirac
$i = 1, 2$ group
in Fundamental

$i \longrightarrow j$ Dirac Propagator × δ_{ij}

μ, a

$i \rightarrow \rightarrow j$



$\int D[\Lambda] = \prod_x \prod_a \int_{-\infty} d\Lambda_a(x)$

$g \times P^\mu \times (-\epsilon_{abc})$

$\vec{P} + \vec{q} = \vec{k}$

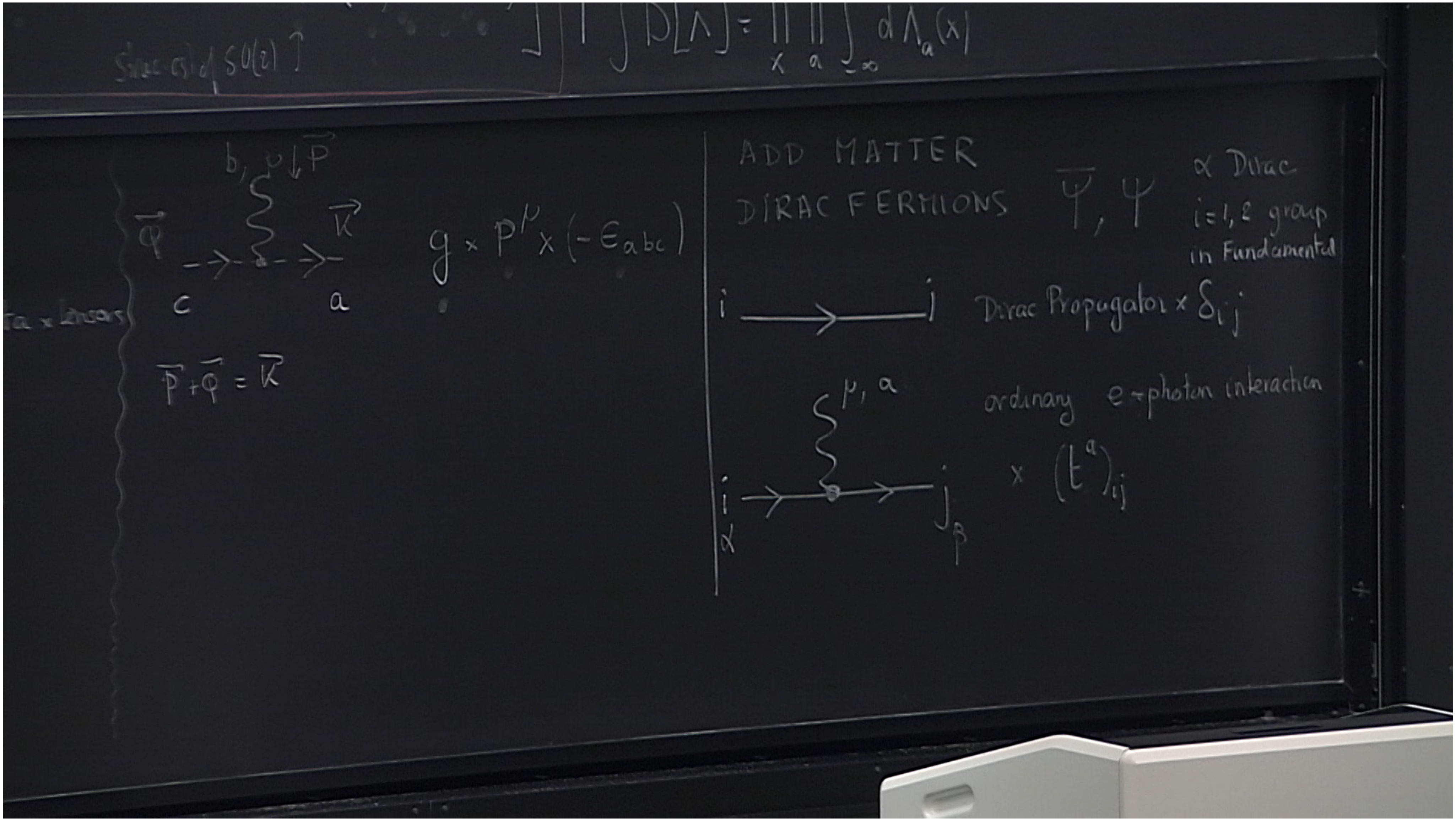
ADD MATTER

DIRAC FERMIONS $\bar{\Psi}, \Psi$

α Dirac
i=1,2 group
in Fundamental

Dirac Propagator × δ_{ij}

ordinary e-photon interaction



$\int D[\Lambda] = \prod_x \prod_a \int_{-\infty} d\Lambda_a(x)$

Since es... SO(2) ?

$b, \nu, \vec{p}$

$\vec{q}$ $\vec{k}$

c a

$g \times p^\nu \times (-\epsilon_{abc})$

$\vec{p} + \vec{q} = \vec{k}$

ta × tensors

ADD MATTER

DIRAC FERMIONS $\bar{\Psi}, \Psi$

α Dirac

$i = 1, 2$ group in Fundamental

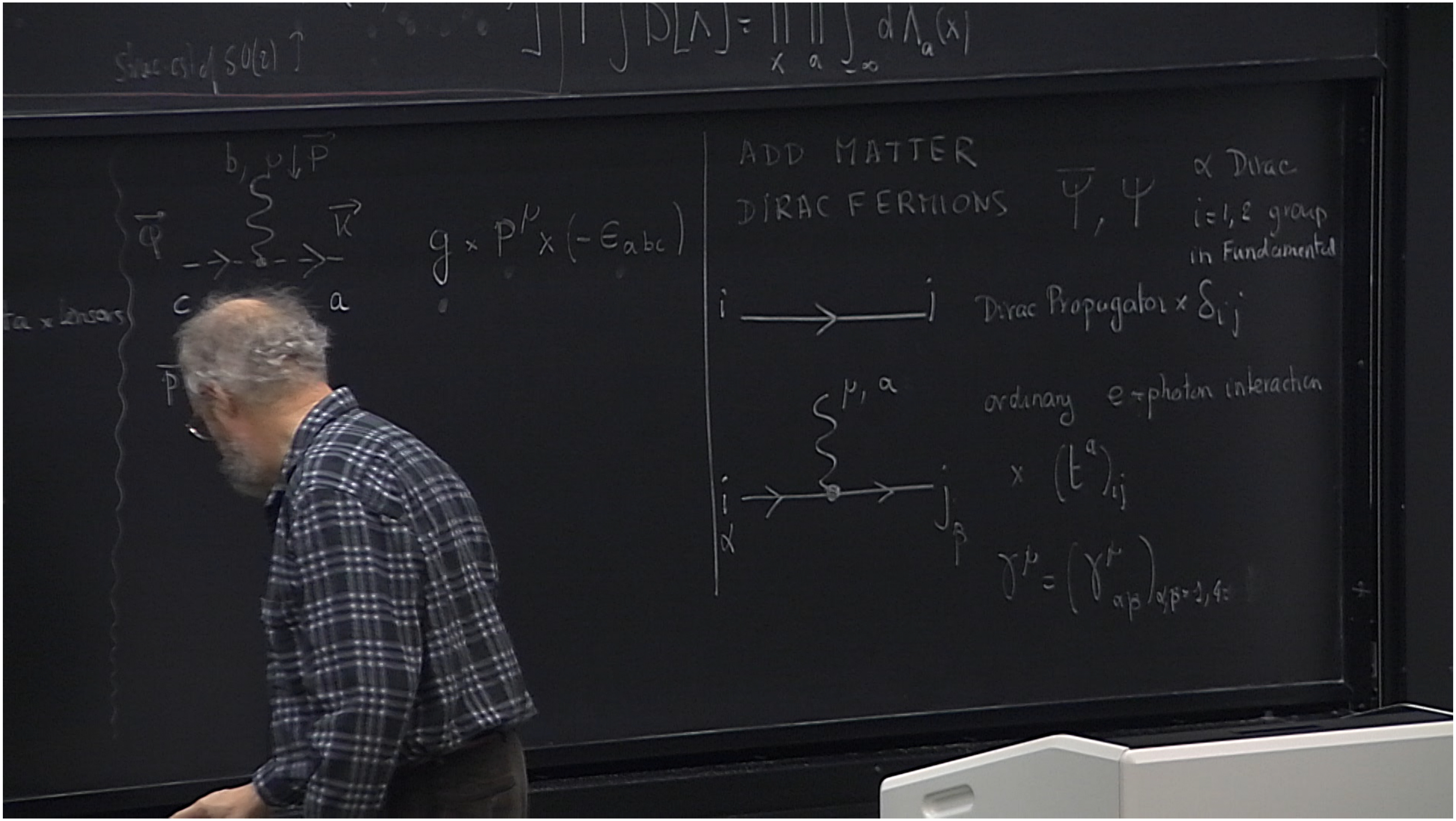
$i \longrightarrow j$ Dirac Propagator $\times \delta_{ij}$

μ, a

$i_\alpha \quad j_\beta$

ordinary e−photon interaction

$\times (t^a)_{ij}$



Sinc csid SO(2) ?

$\int D[\Lambda] = \prod_x \prod_a \int_{-\infty} d\Lambda_a(x)$

ta × tensors

$g \times P^\mu \times (-\epsilon_{abc})$

ADD MATTER

DIRAC FERMIONS $\bar{\Psi}, \Psi$

α Dirac

i=1,2 group

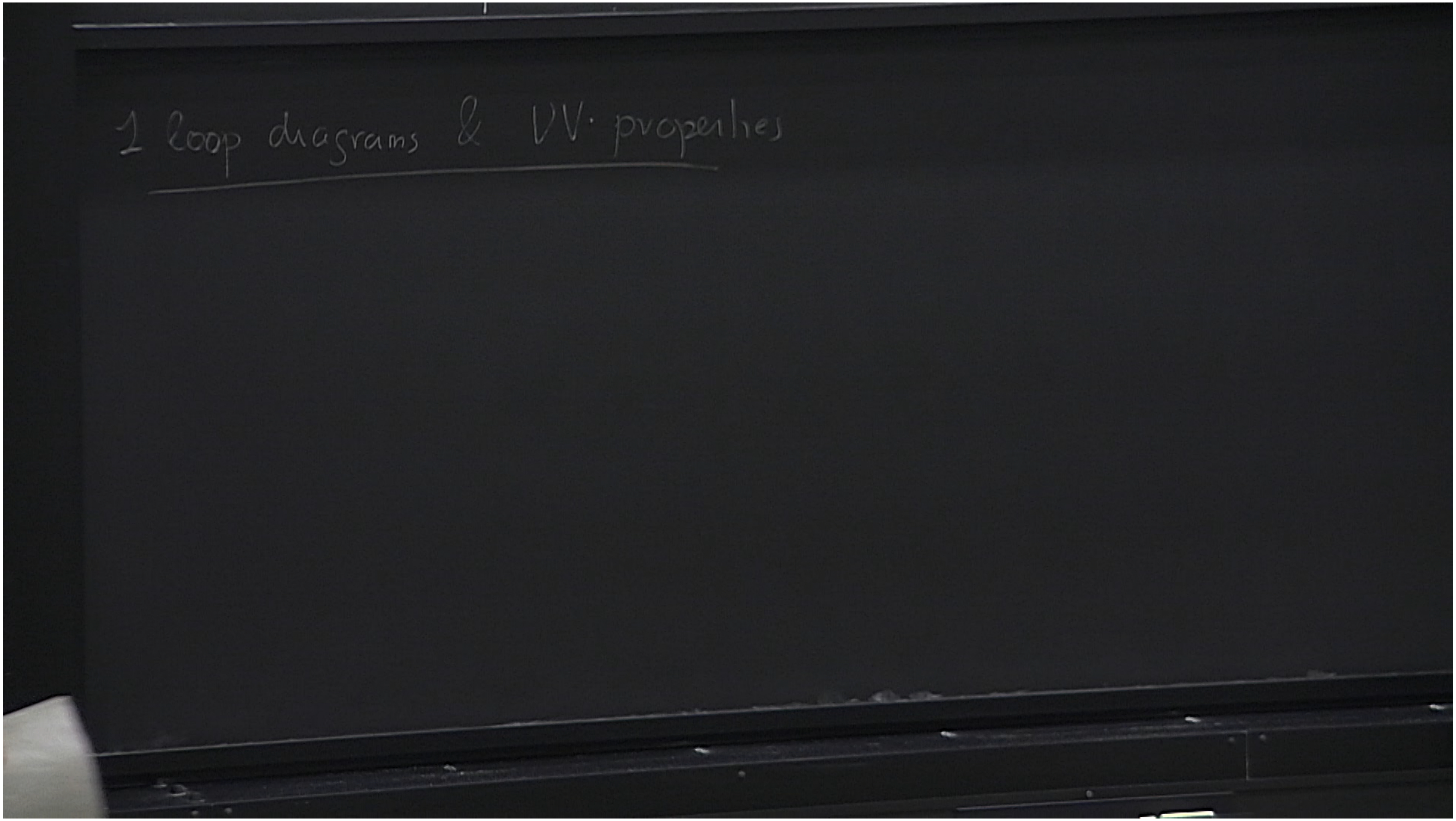
in Fundamental

i → j Dirac Propagator × δ_{ij}

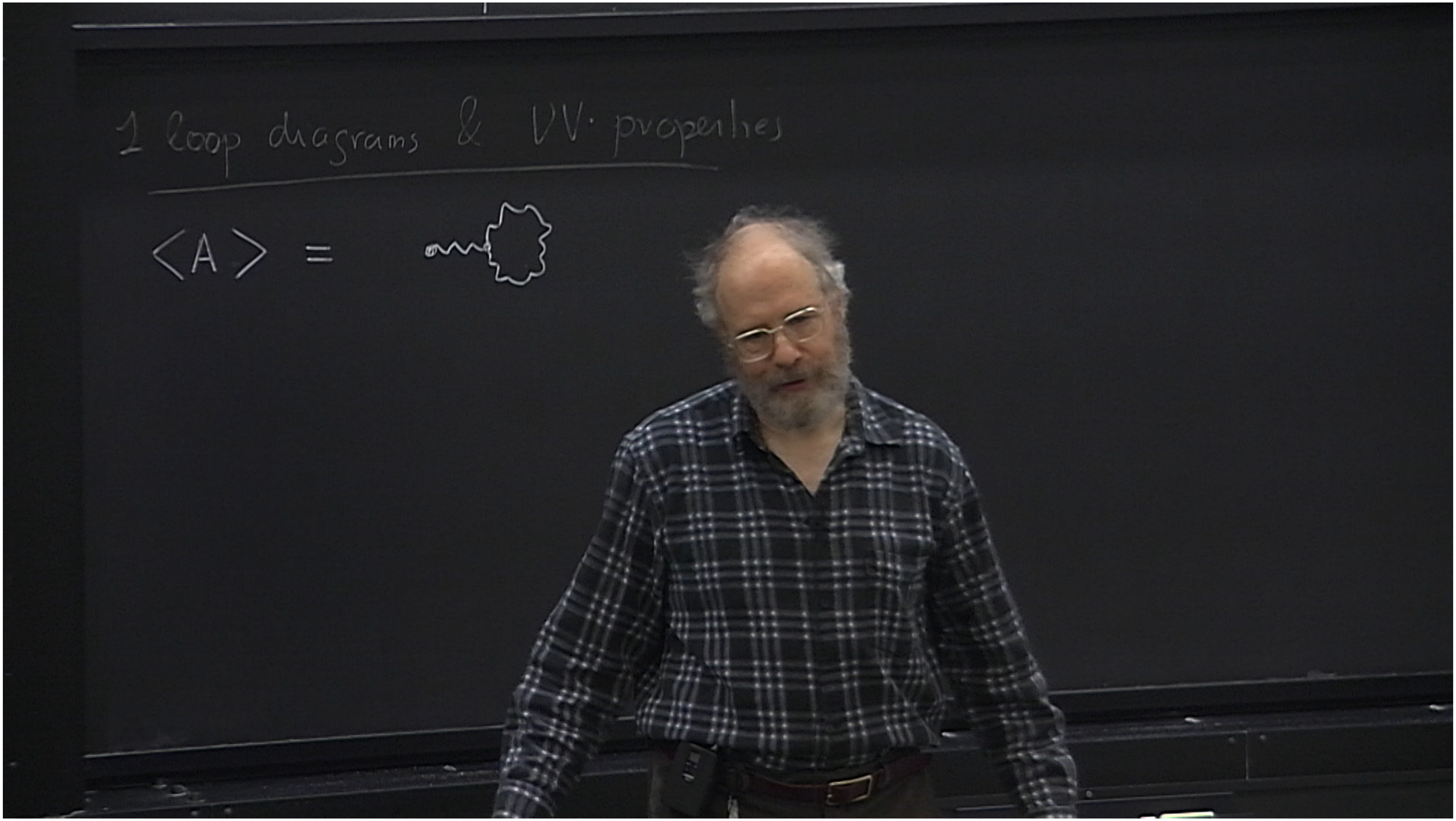
ordinary e-photon interaction

× $(t^a)_{ij}$

$\gamma^\mu = (\gamma^\mu_{\alpha\beta})_{\alpha,\beta=1,4}$



1 loop diagrams & UV· properties

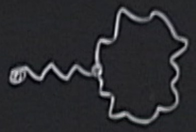


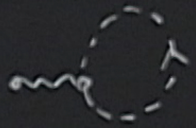
1 loop diagrams & UV properties

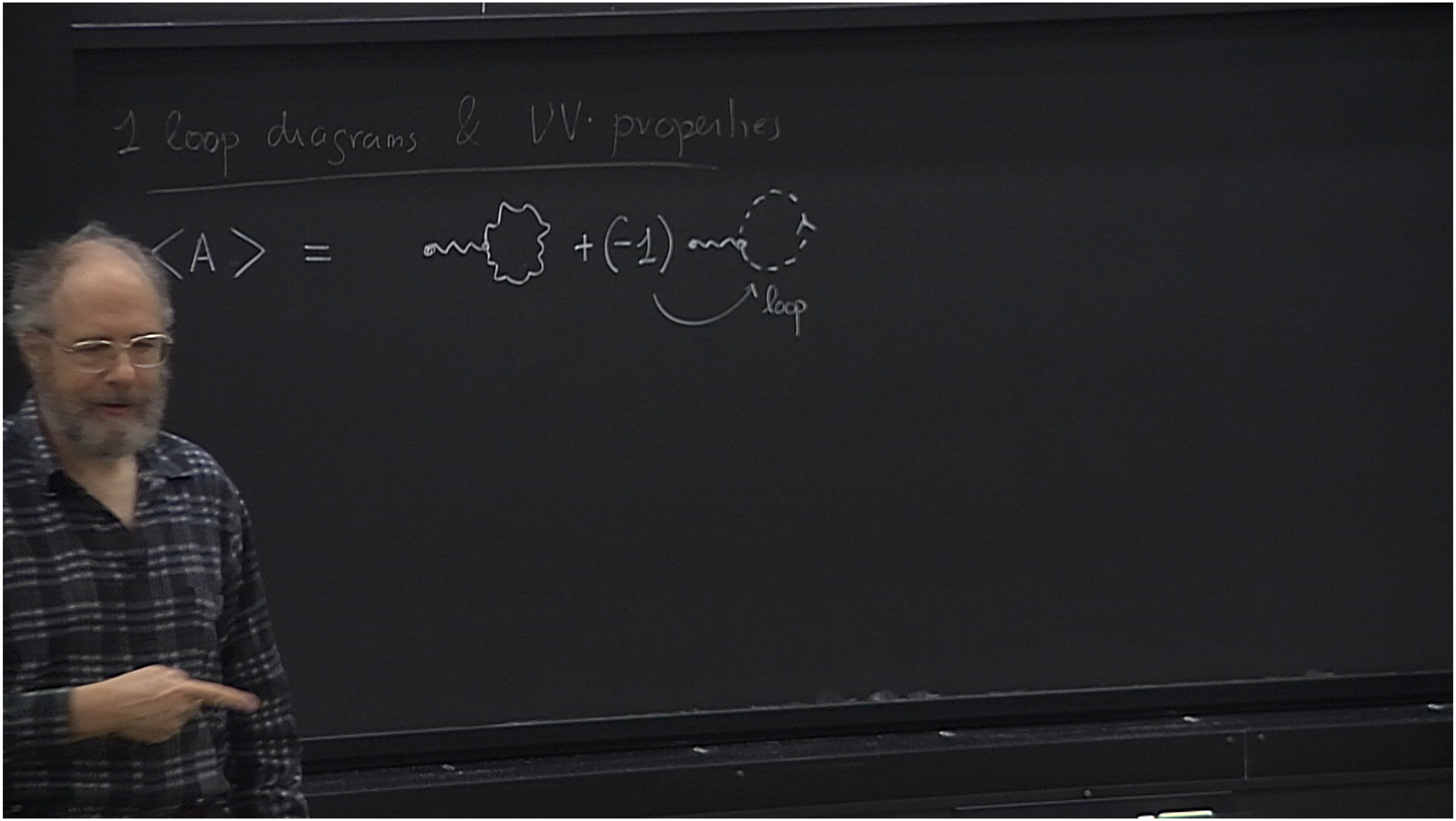
$\langle A \rangle =$

1 loop diagrams & UV properties

$\langle A \rangle =$



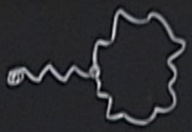
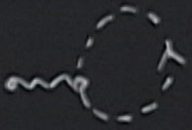




1 loop diagrams & UV properties

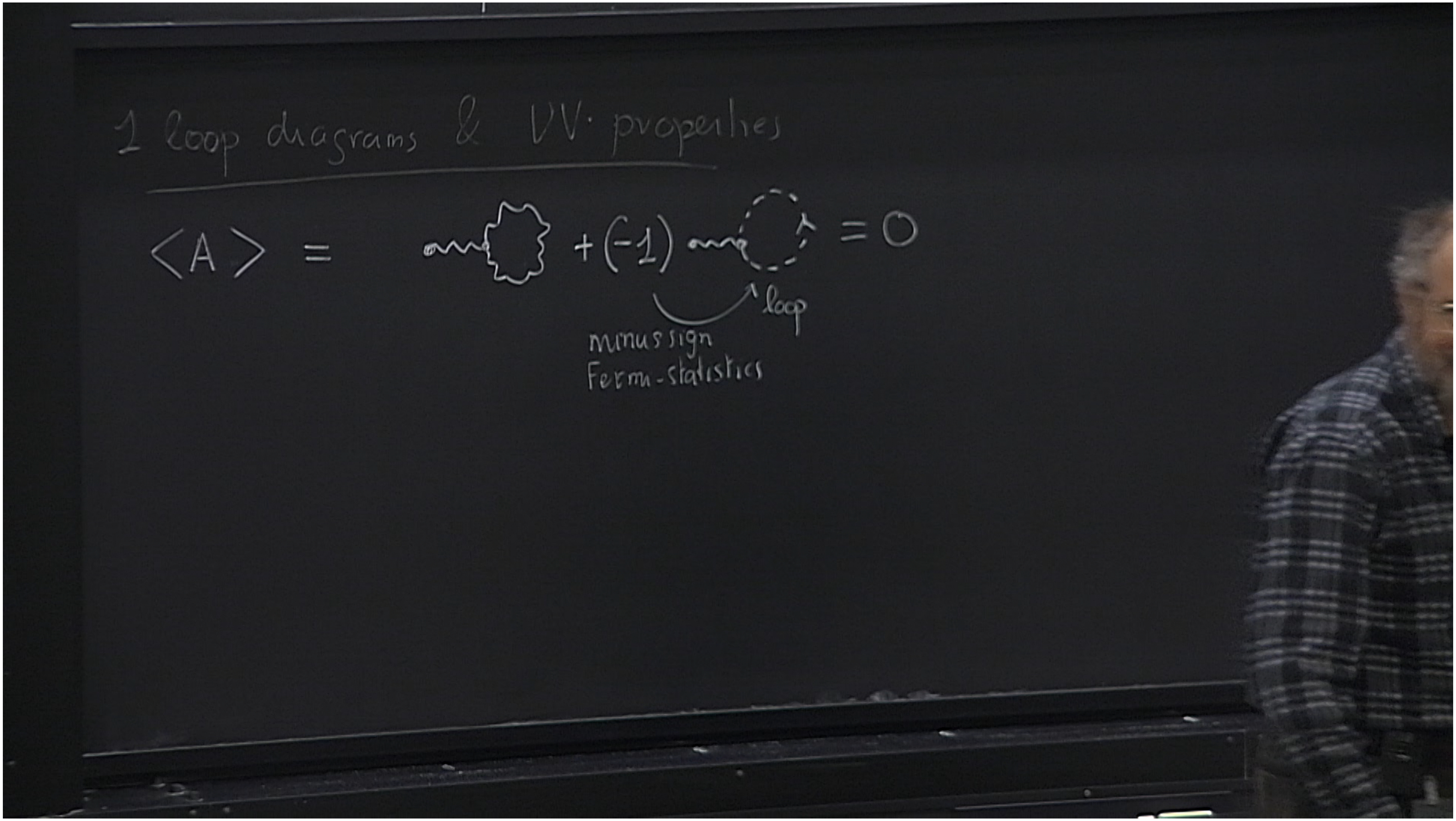
$\langle A \rangle = $

1 loop diagrams & UV· properties

$\langle A \rangle =$  $+ (-1)$ 

loop

minus sign
Ferm-statistics

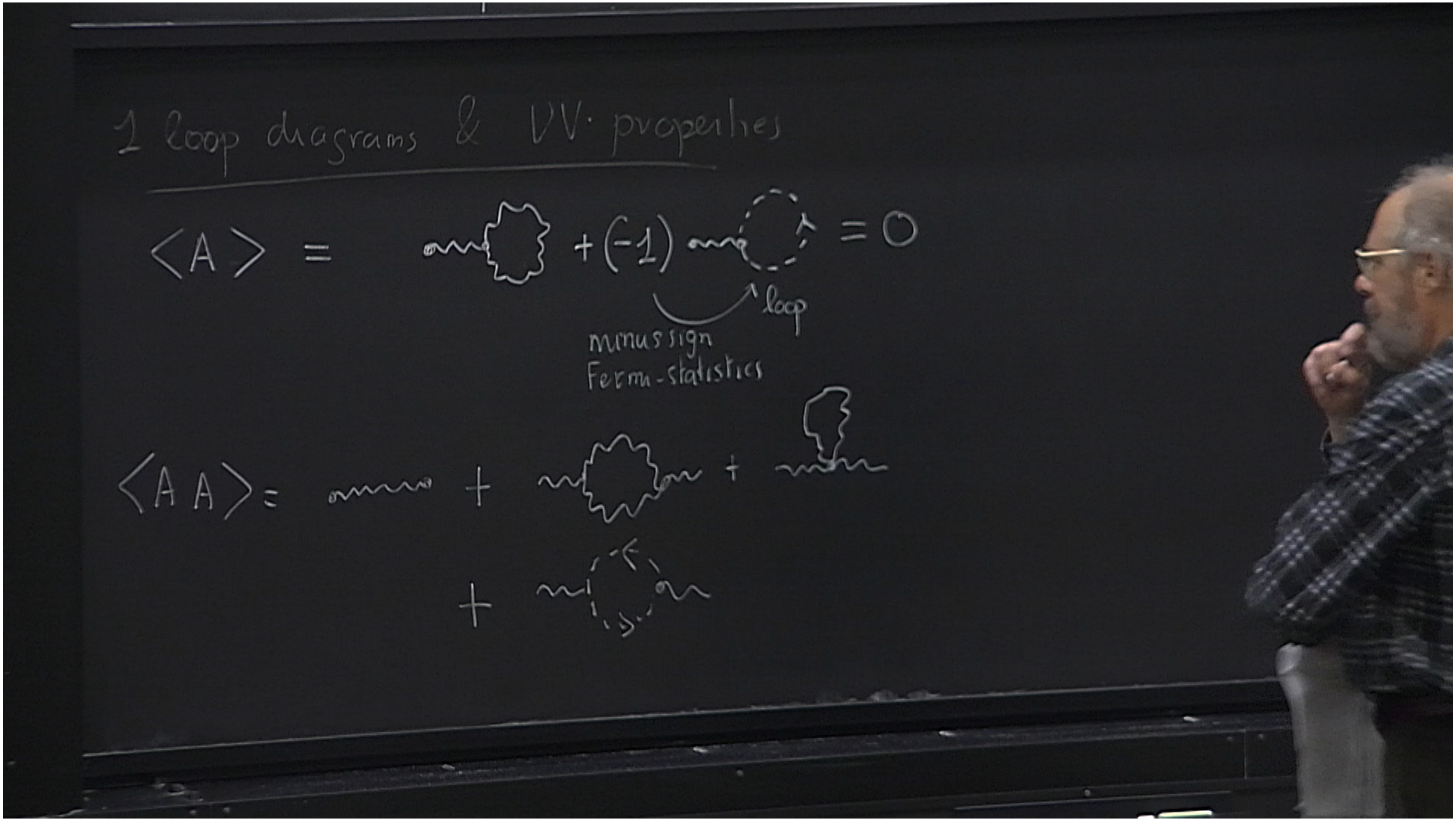


1 loop diagrams & UV· properties

$$\langle A \rangle = \quad + (-1) \quad = 0$$

loop

minus sign
Ferm-statistics



1 loop diagrams & UV· properties

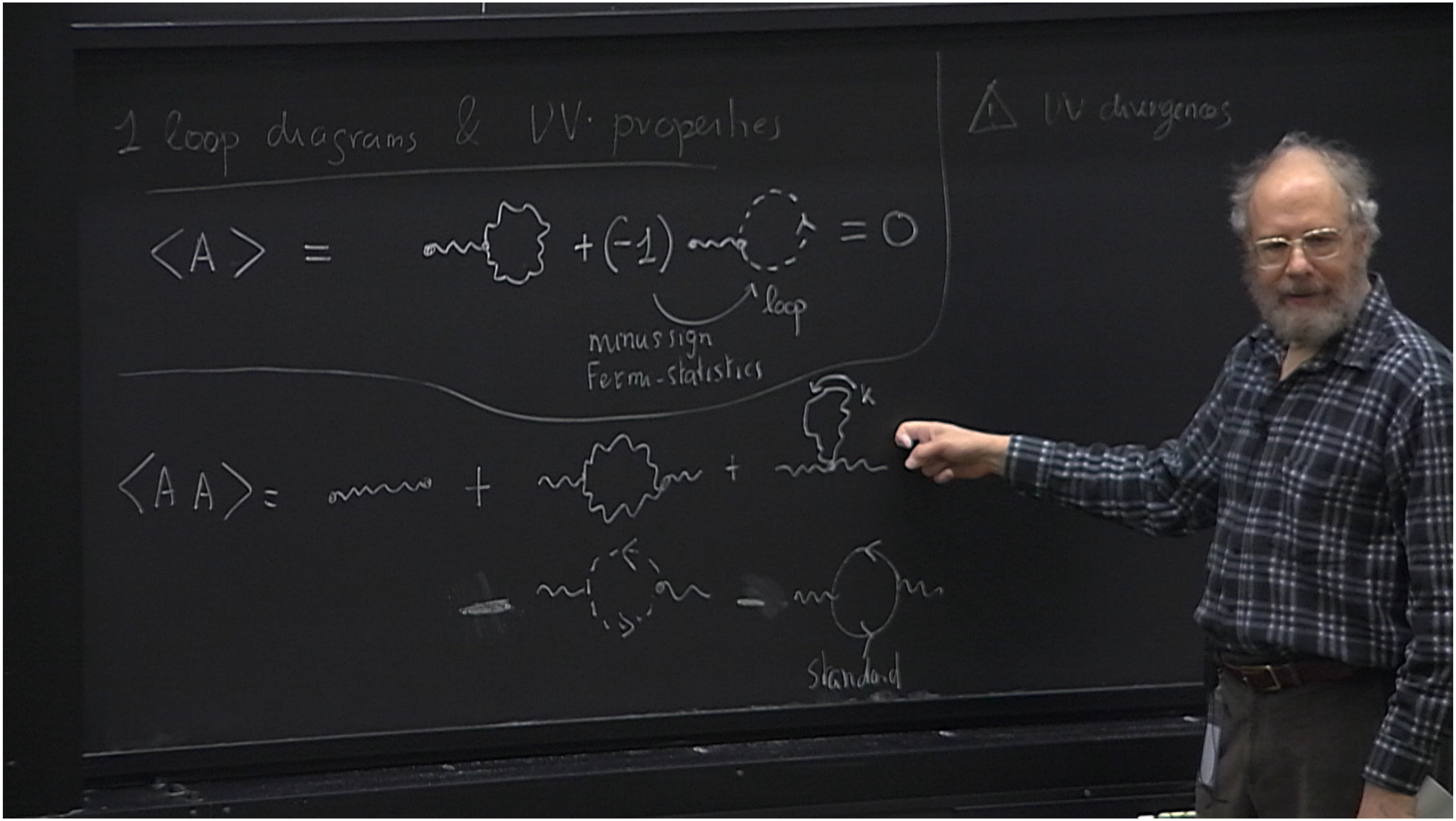
$\langle A \rangle =$ + (-1) $= 0$

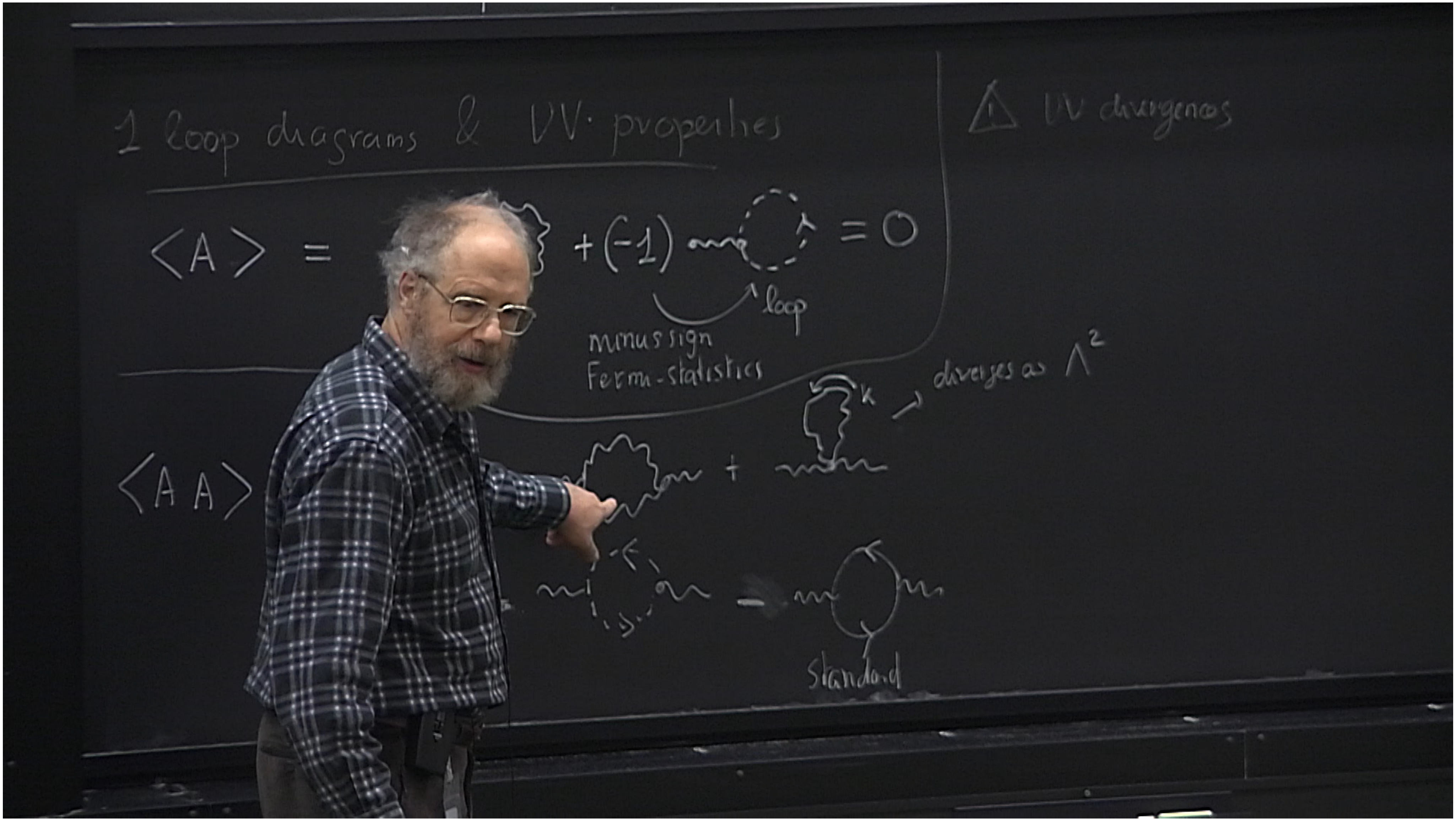
loop

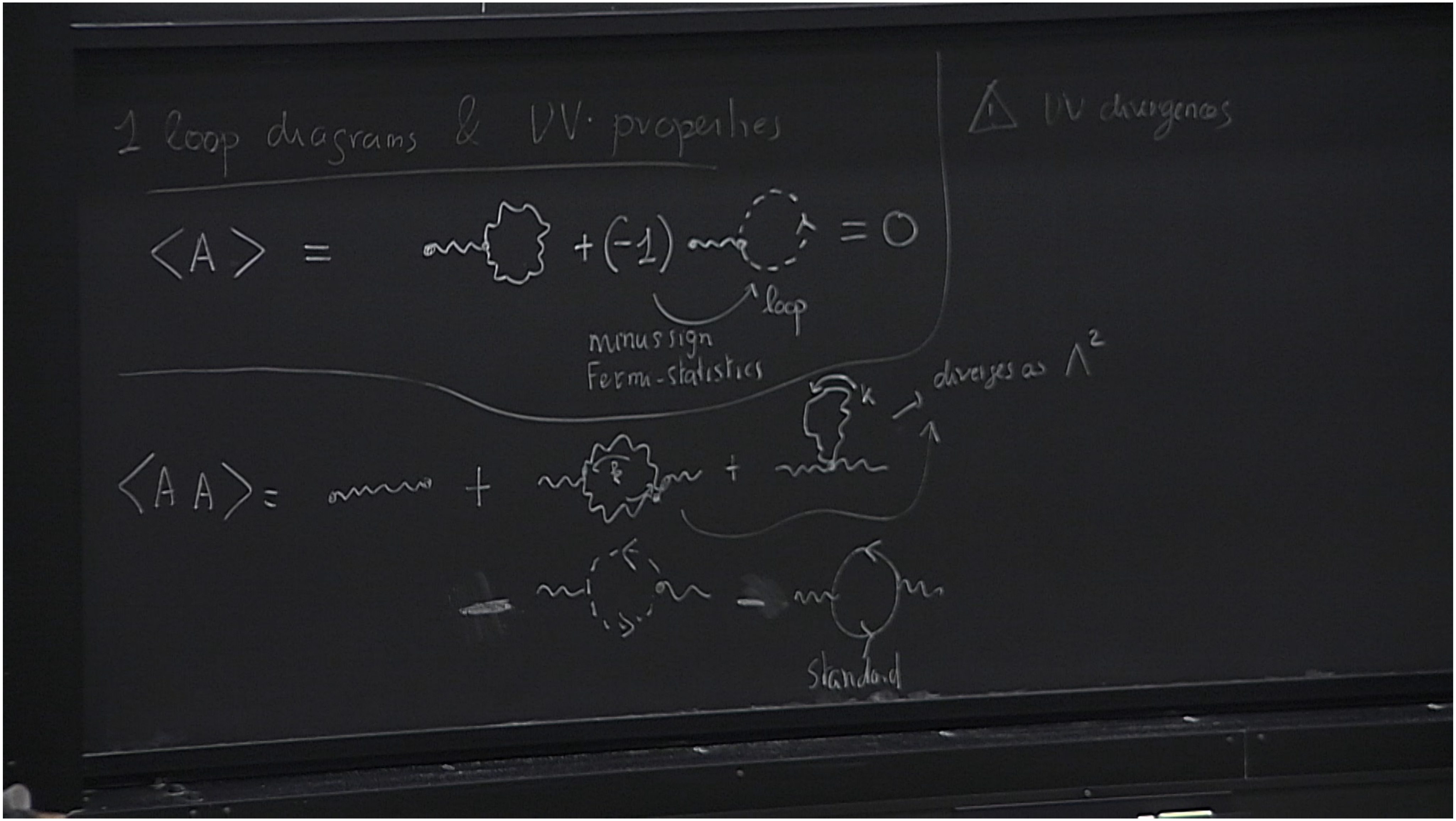
minus sign
Fermi-statistics

$\langle A\, A \rangle =$ + +

+


1 loop diagrams & UV properties
⚠ UV divergences
⟨A⟩ =
+ (−1)
= 0
loop
minus sign
Fermi-statistics
k
⟨A A⟩ =
+
+
−
−
standard


1 loop diagrams & UV· properties
UV divergences
⟨A⟩ =
+ (−1)
= 0
loop
minus sign
Fermi-statistics
k
diverges as Λ²
⟨A A⟩
+
−
standard



1 loop diagrams & UV properties

$\langle A \rangle =$ + (−1) = 0

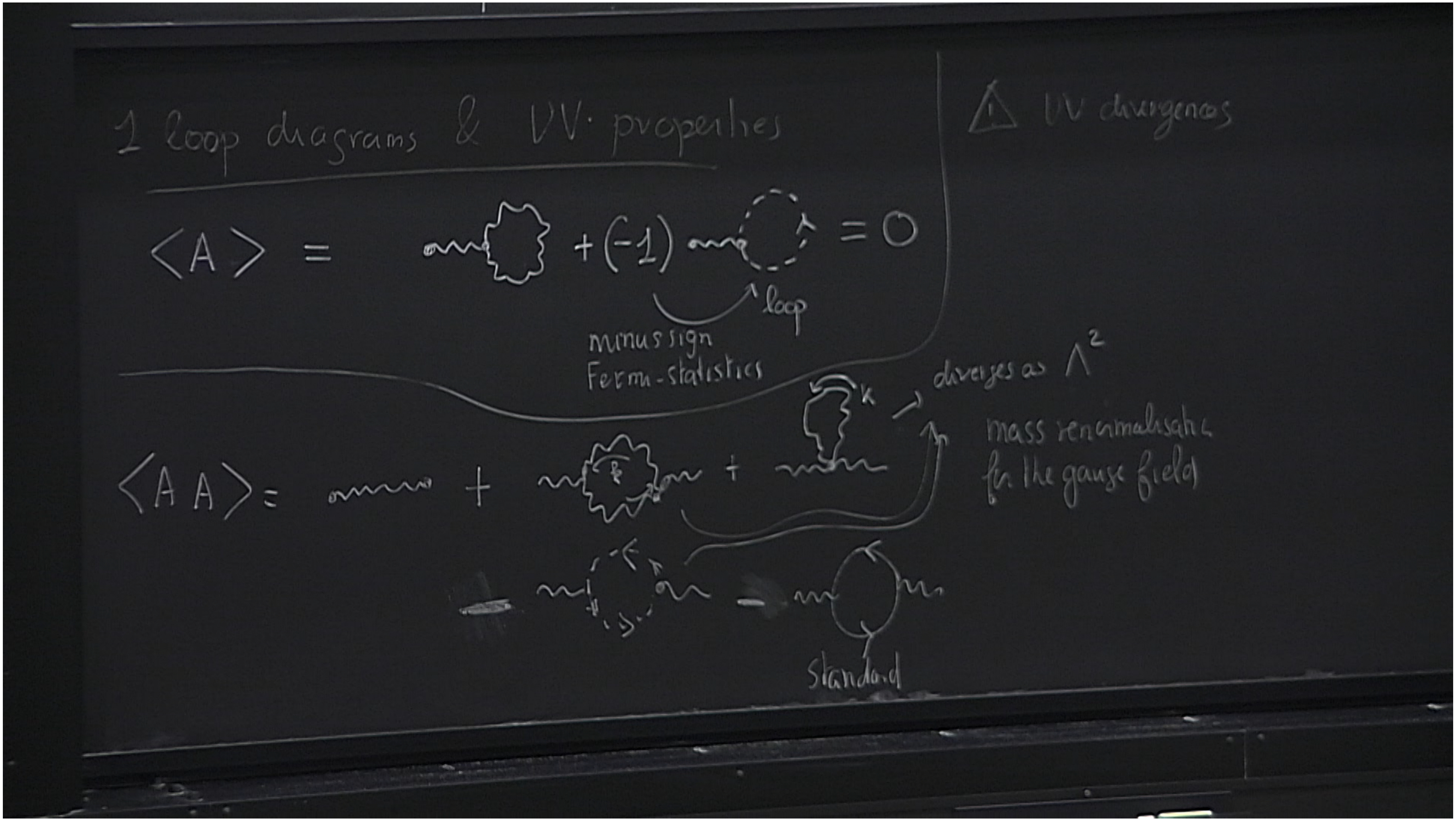
loop

minus sign
Fermi-statistics

⚠ UV divergences

$\langle A A \rangle =$ + + → diverges as Λ^2

− −

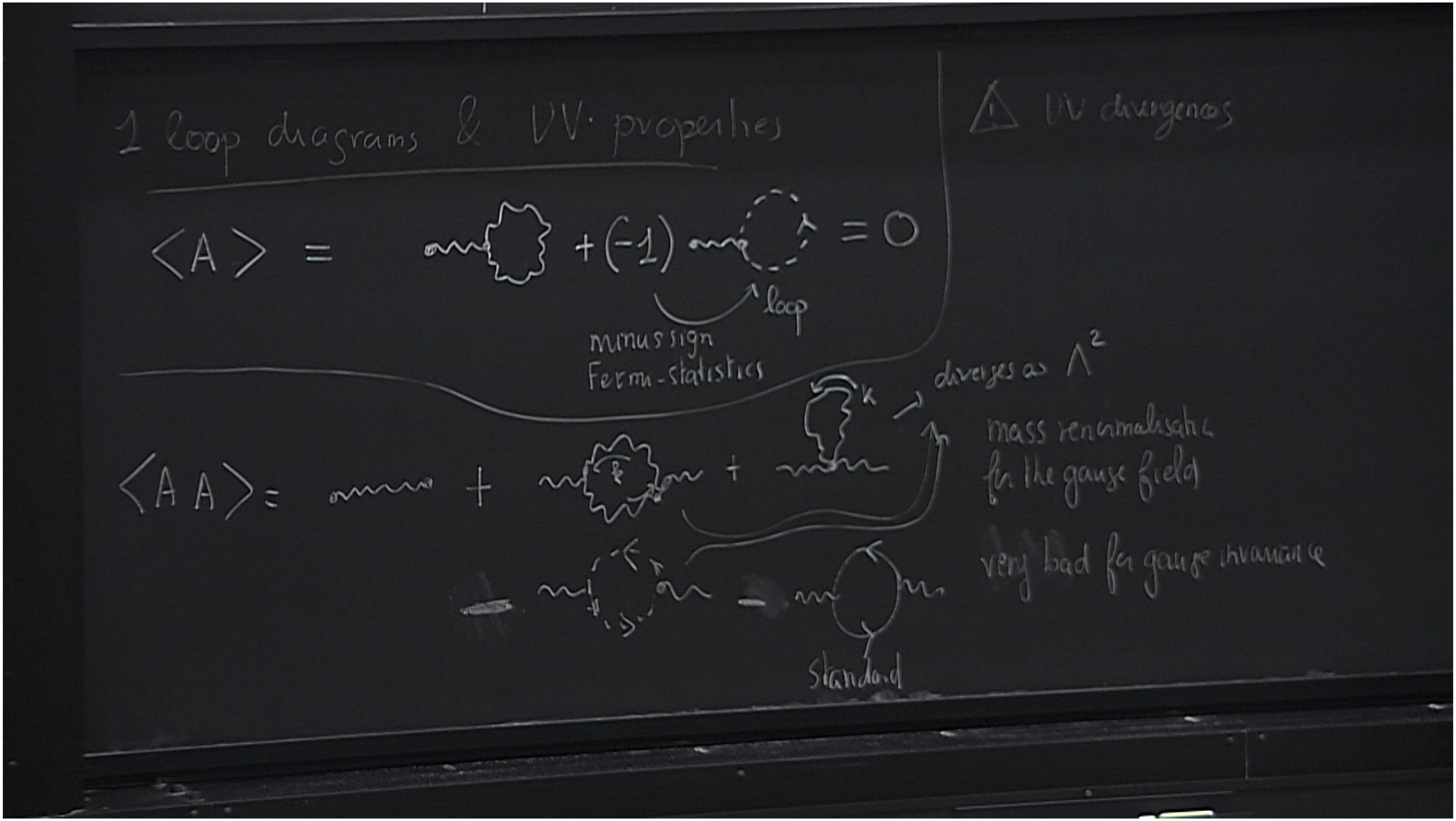


1 loop diagrams & UV properties

⚠ UV divergences

$\langle A \rangle = \; + (-1) \; = 0$

$\langle A A \rangle = \; + \; + \; - \; -$



1 loop diagrams & UV· properties

$\langle A \rangle =$ + (−1) $= 0$

loop

minus sign
Fermi-statistics

⚠ UV divergences

$\langle A A \rangle =$ + +

k

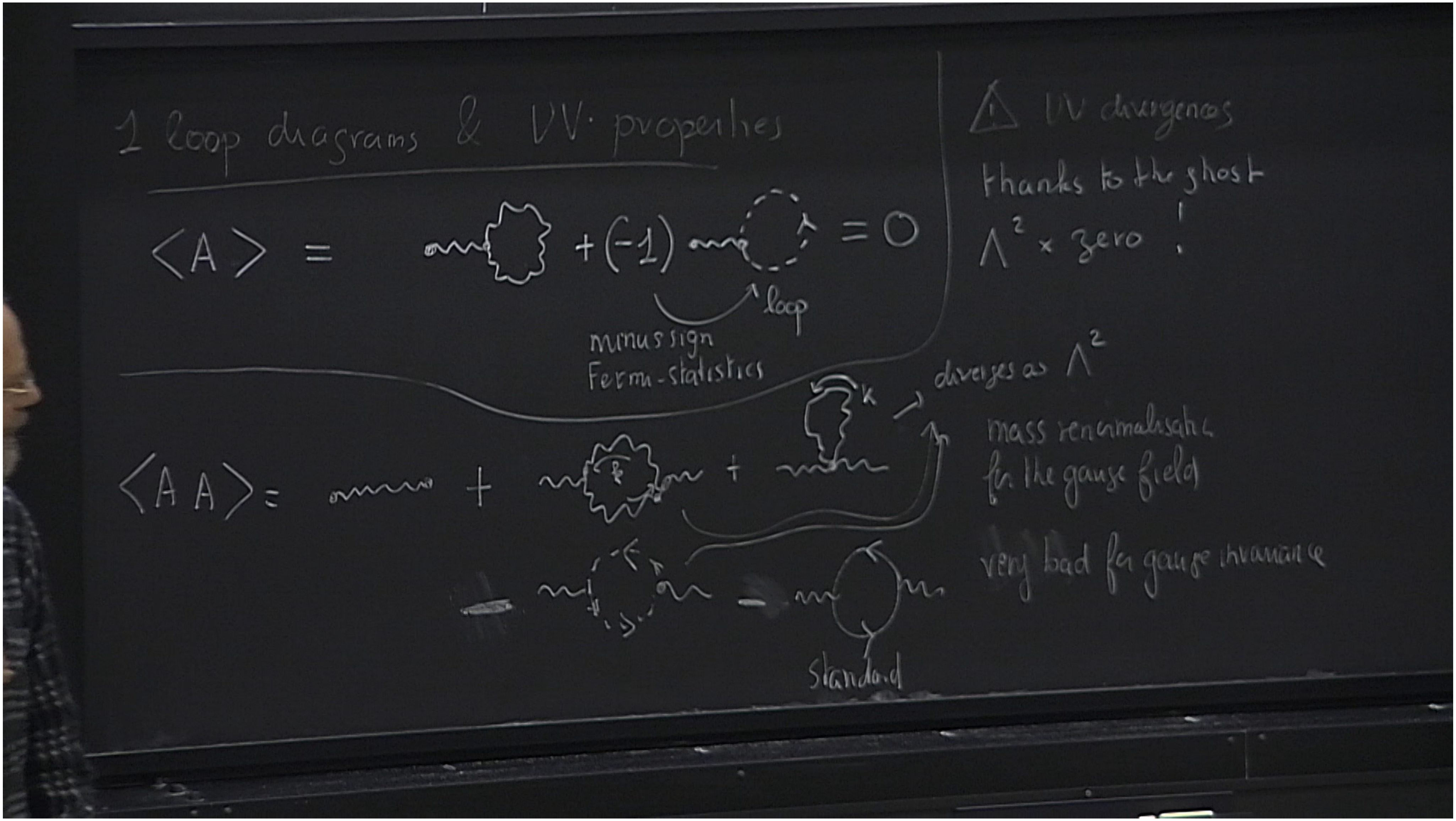
diverges as Λ^2

mass renormalisation
for the gauge field

very bad for gauge invariance

− −

standard



1 loop diagrams & UV· properties

$\langle A \rangle =$ + (−1) $= 0$

loop

minus sign
Fermi-statistics

⚠ UV divergences

thanks to the ghost

$\Lambda^2 \times$ zero !

$\langle A\,A \rangle =$ + + − −

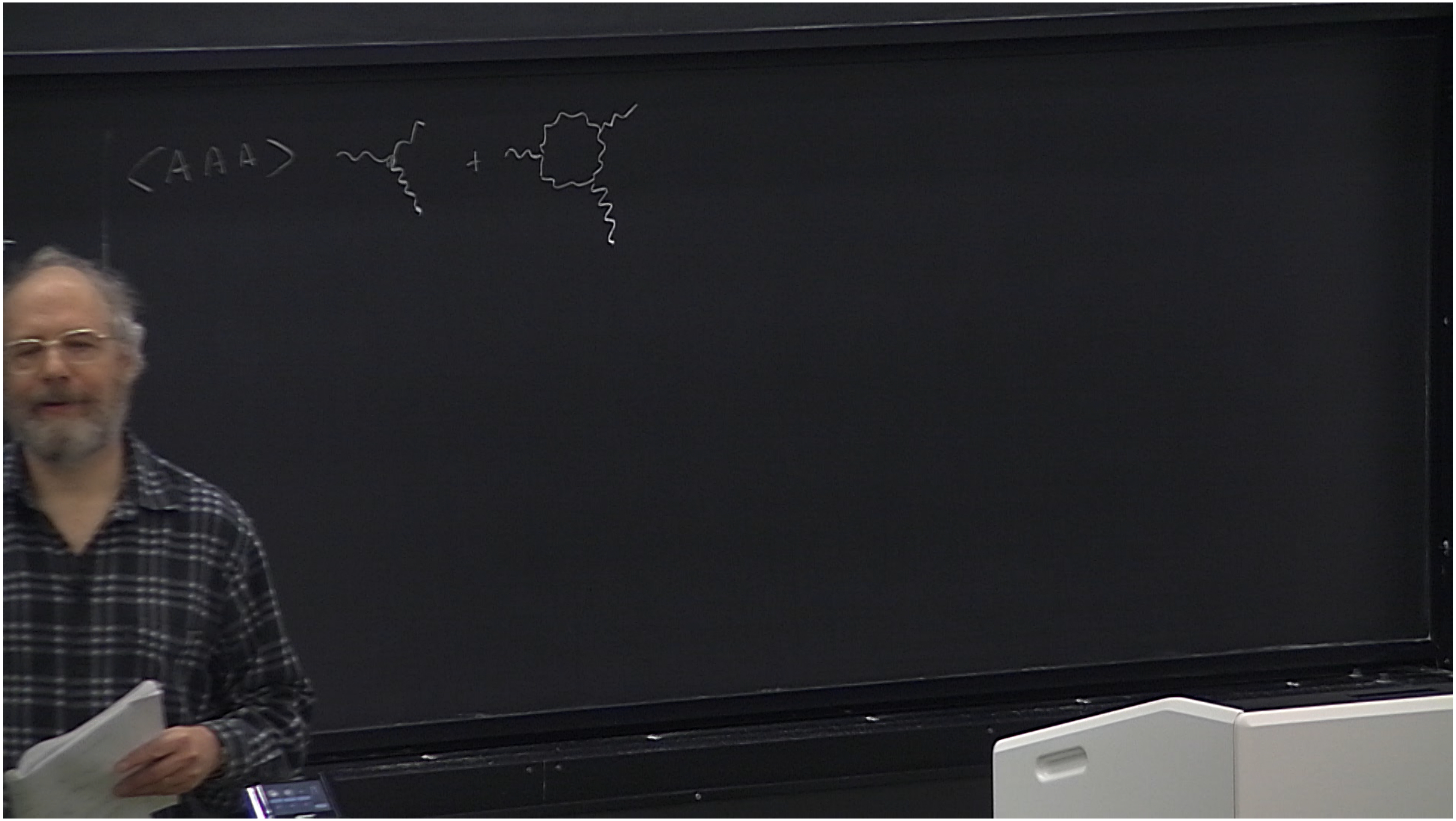
k

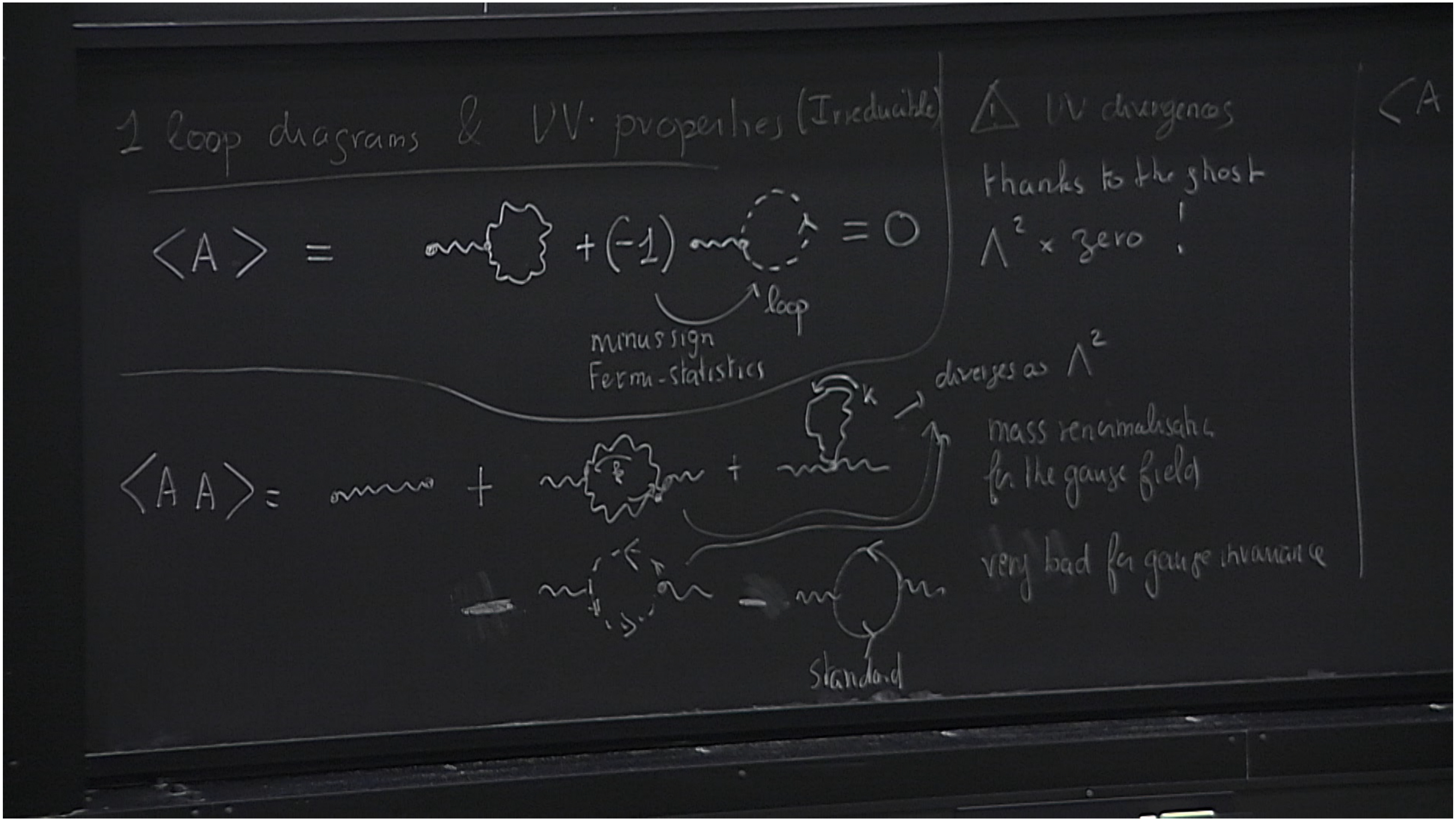
standard

diverges as Λ^2

mass renormalisation
for the gauge field

very bad for gauge invariance


⟨AAA⟩
+



1 loop diagrams & UV· properties (Irreducible)

$\langle A \rangle =$ $+ (-1)$ $= 0$

loop

minus sign
Fermi-statistics

$\langle A A \rangle =$ $+$ $+$ $-$ $-$

k

standard

⚠ UV divergences

thanks to the ghost

$\Lambda^2 \times$ zero !

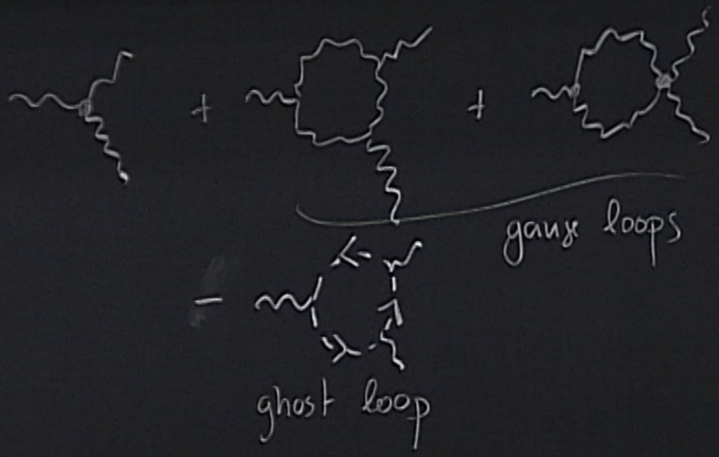
diverges as Λ^2

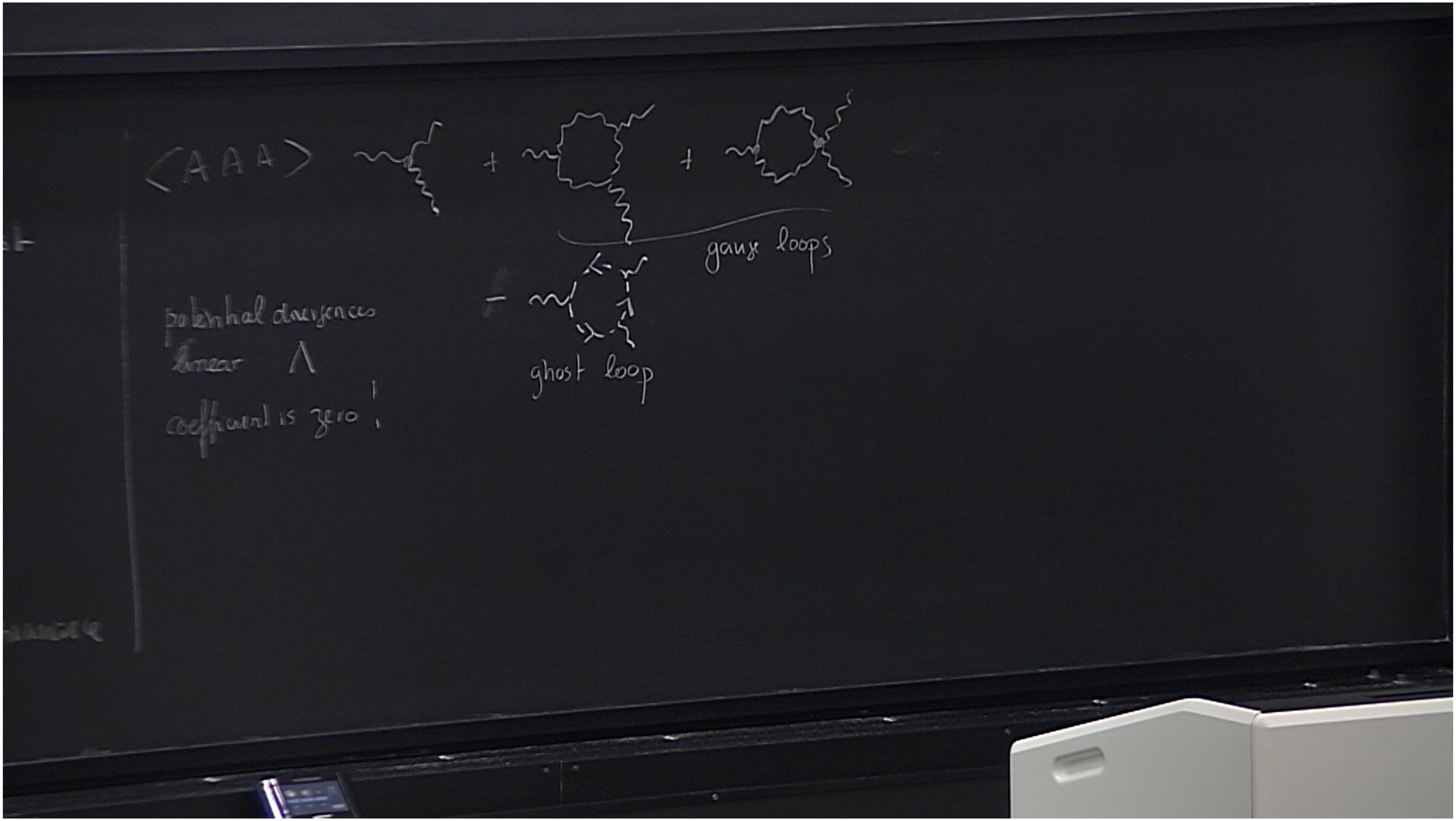
mass renormalisation for the gauge field

very bad for gauge invariance

$\langle A$

$\langle AAA \rangle$



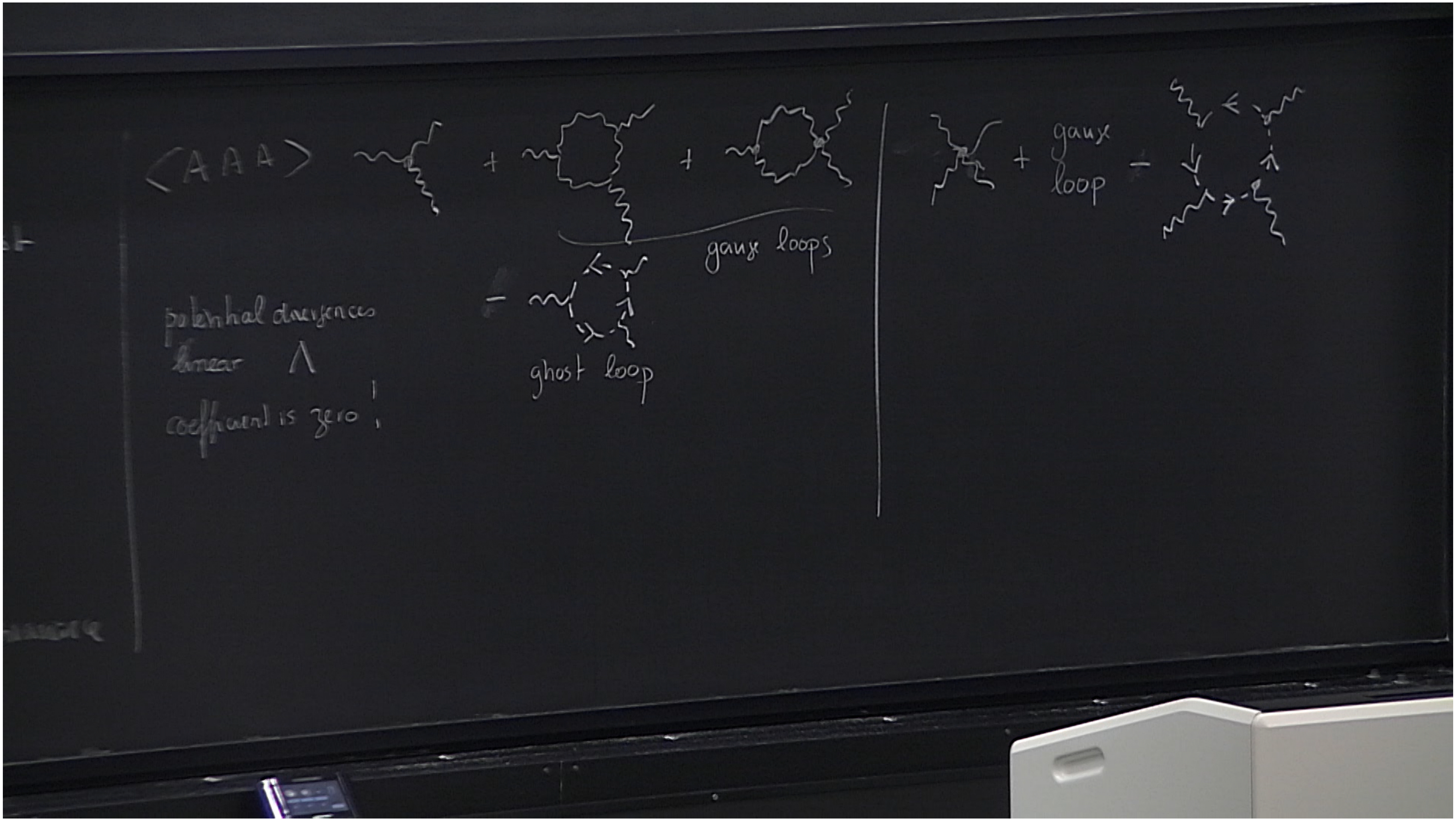


$\langle AAA \rangle$

potential divergences

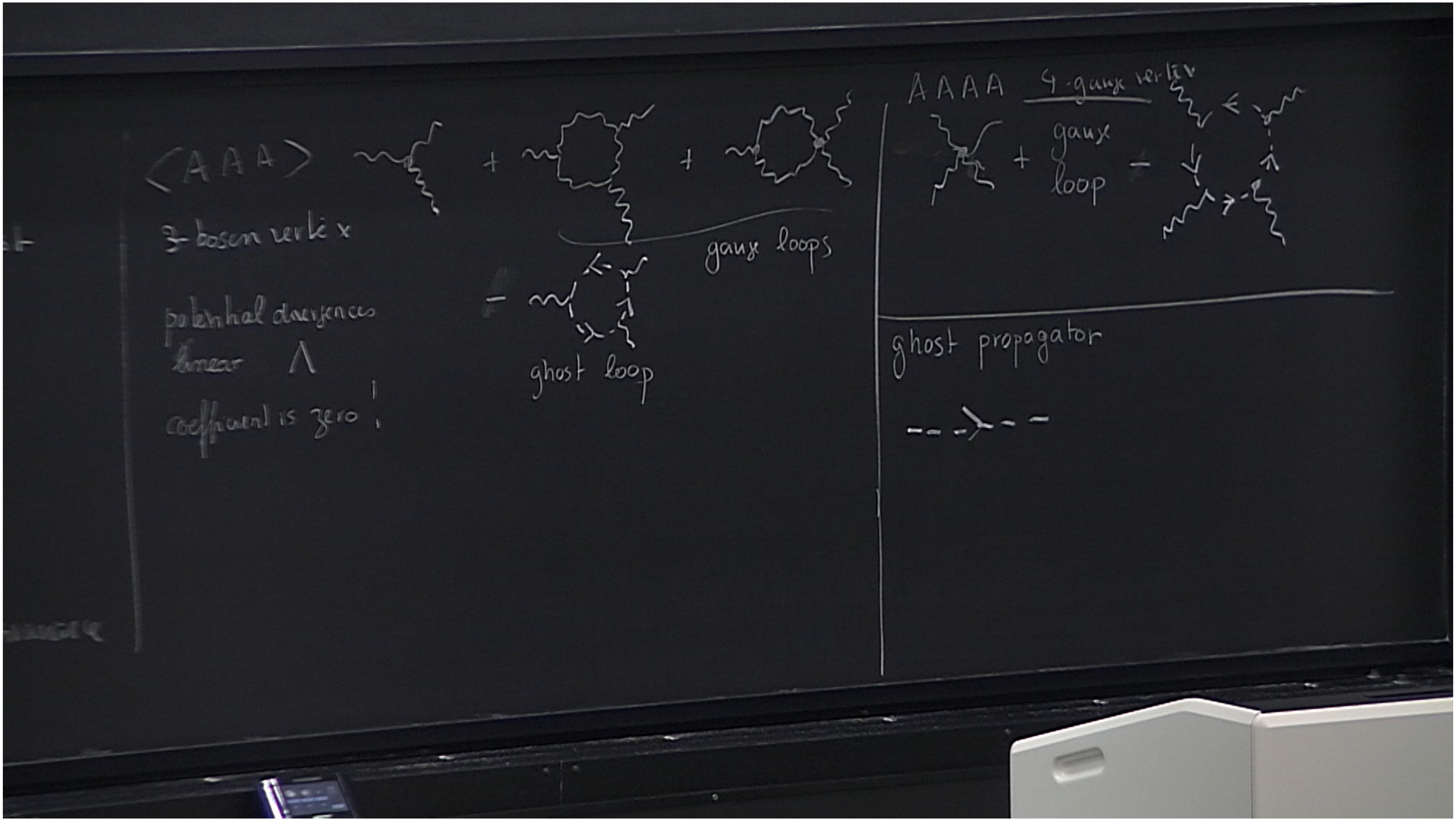
linear Λ

coefficient is zero!



$\langle AAA \rangle$

potential divergences
linear Λ
coefficient is zero!



$\langle AAA \rangle$

3-boson vertex

potential divergences

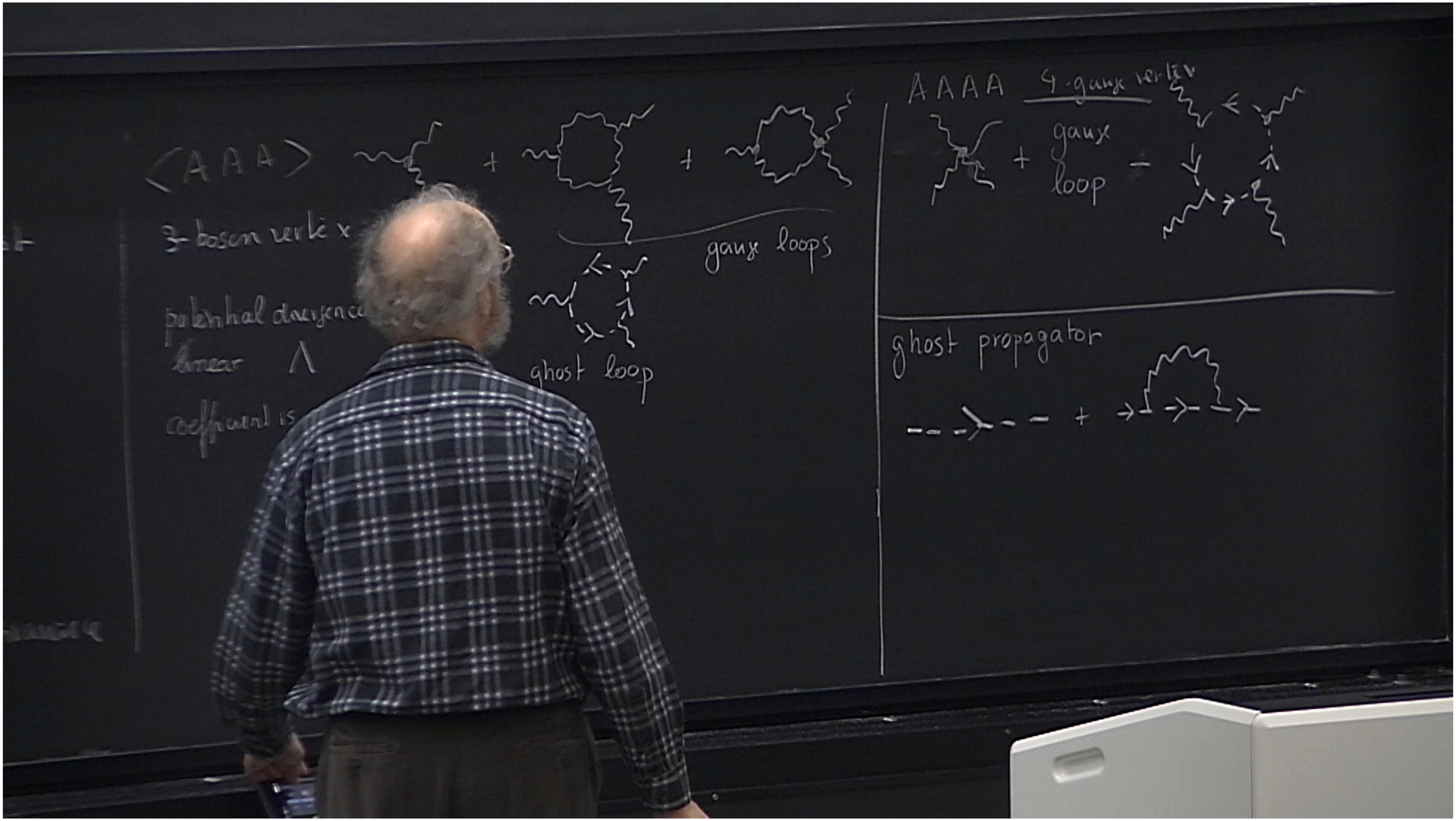
linear Λ

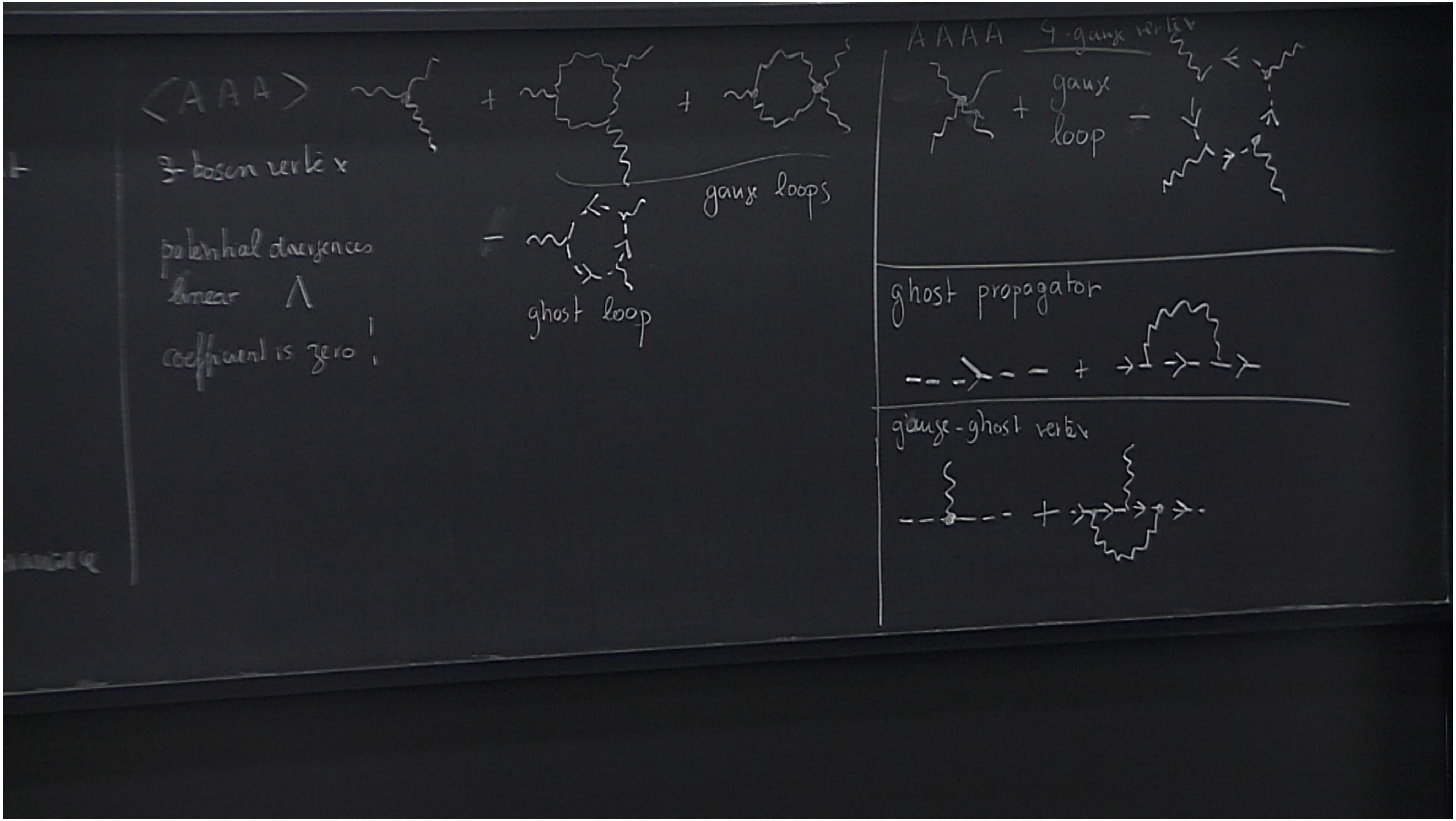
coefficient is zero!

AAAA

4-gauge vertex

ghost propagator


⟨AAA⟩
+
+
3-boson vertex
gauge loops
potential divergence
linear Λ
coefficient is
ghost loop
AAAA
4-gauge vertex
+
gauge
loop
ghost propagator
+



$\langle AAA \rangle$

3-boson vertex

potential divergences

linear Λ

coefficient is zero!

gauge loops

ghost loop

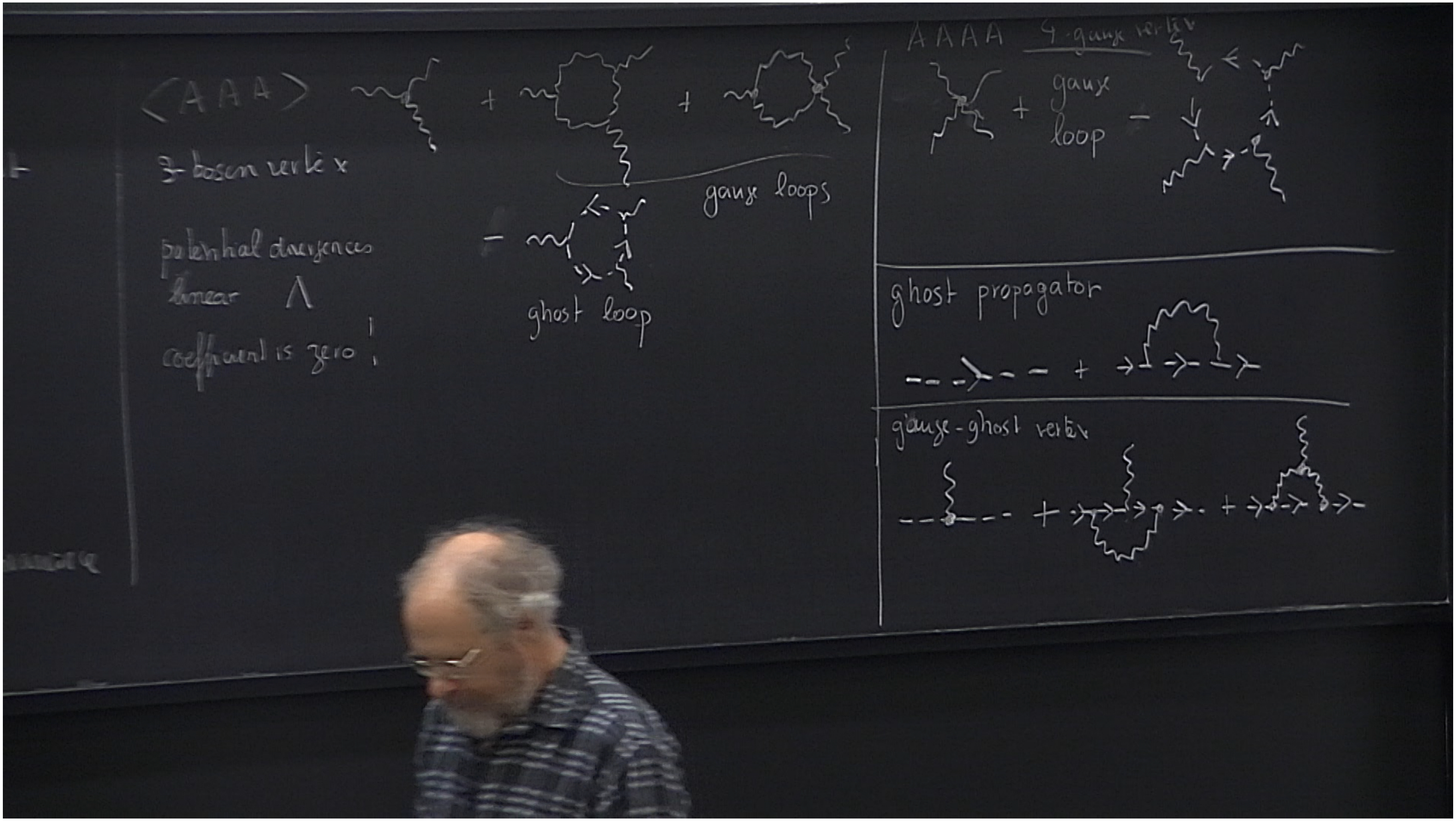
AAAA

4-gauge vertex

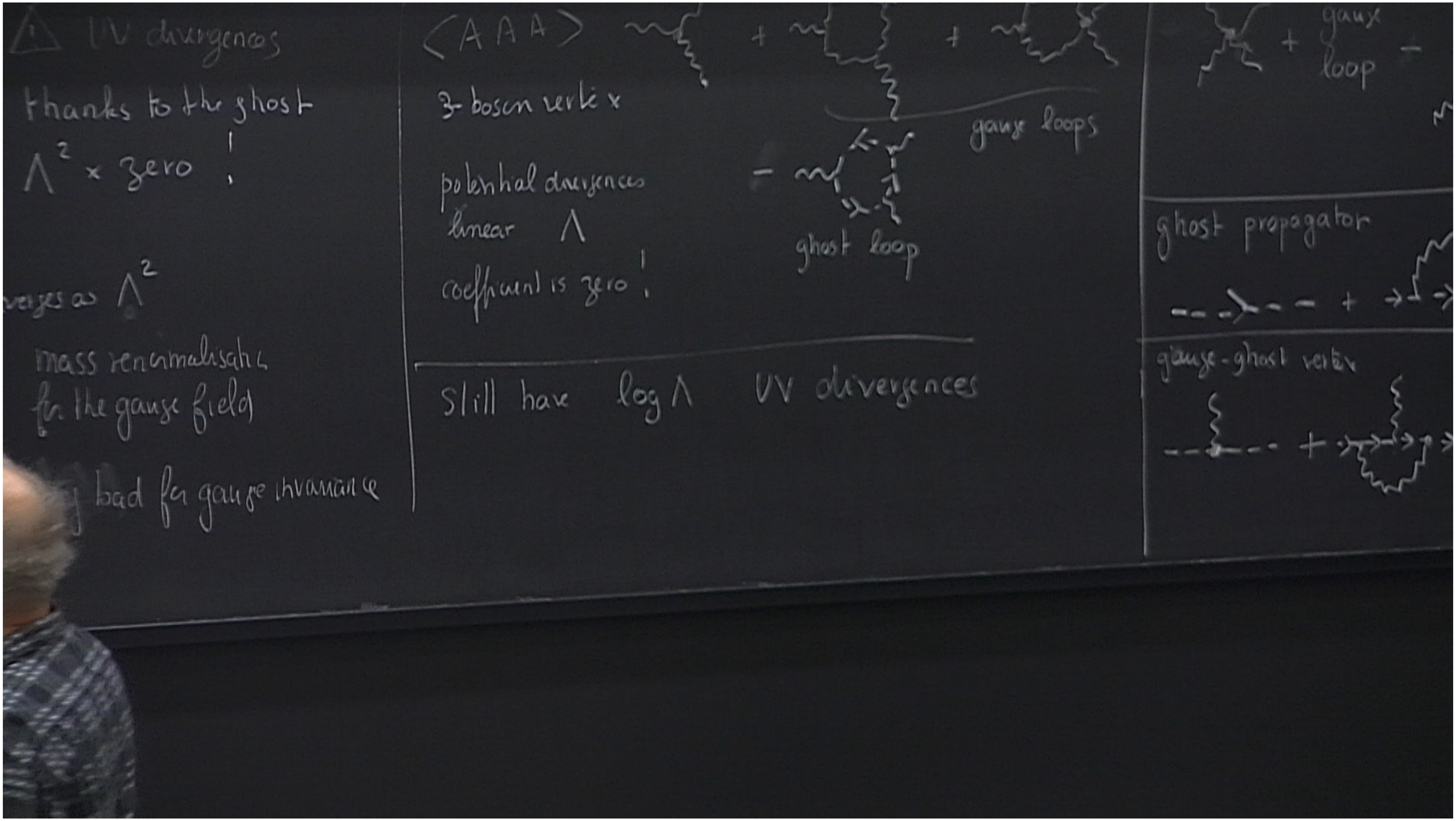
gauge loop

ghost propagator

gauge-ghost vertex



⟨AAA⟩
+
+
3-boson vertex
gauge loops
potential divergences
linear Λ
coefficient is zero!
−
ghost loop
AAAA
4-gauge vertex
+
gauge loop
−
ghost propagator
+
gauge-ghost vertex
+
+



⚠ UV divergences

thanks to the ghost

$\Lambda^2 \times$ zero !

verges as Λ^2

mass renormalisation
for the gauge field

bad for gauge invariance

$\langle AAA \rangle$

3-boson vertex

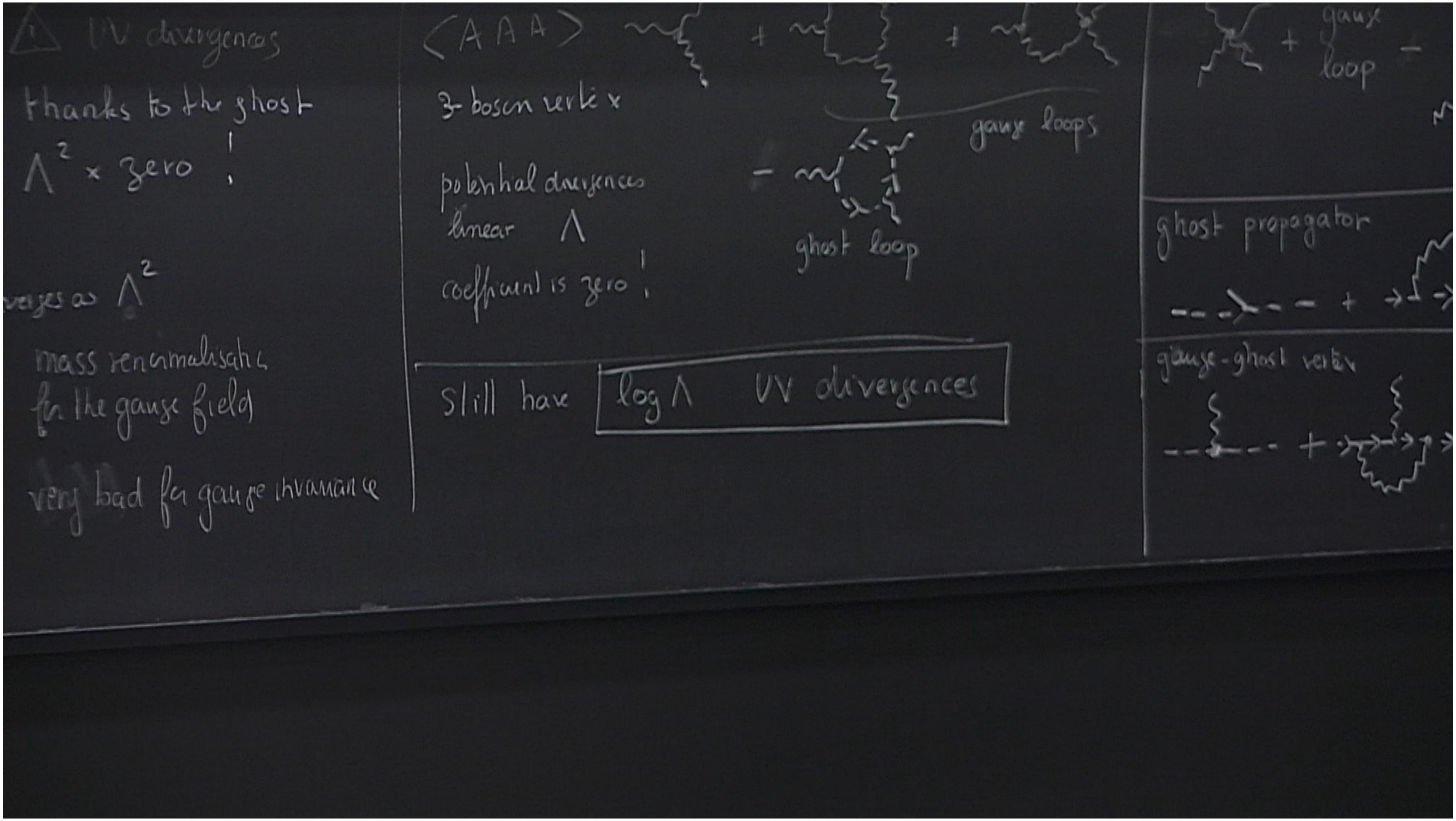
potential divergences
linear Λ

coefficient is zero !

Still have $\log \Lambda$ UV divergences

ghost propagator

gauge-ghost vertex



⚠ UV divergences

Thanks to the ghost

$\Lambda^2 \times$ zero !

verges as Λ^2

mass renormalisation for the gauge field

very bad for gauge invariance

$\langle AAA \rangle$ + +

3-boson vertex

gauge loops

potential divergences linear Λ

coefficient is zero !

−

ghost loop

Still have $\boxed{\log\Lambda \quad \text{UV divergences}}$

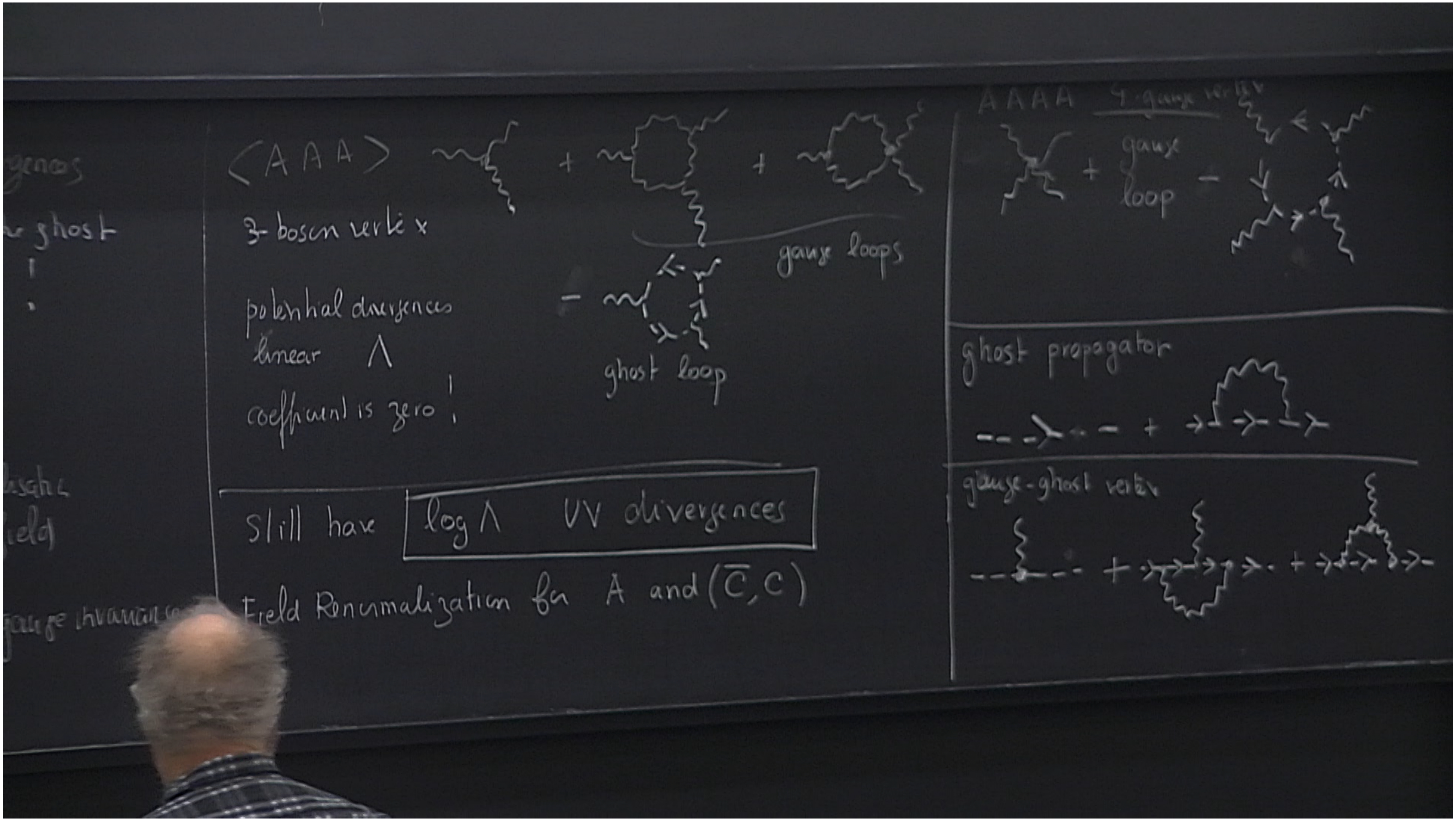
+ gauge loop −

ghost propagator

+

gauge-ghost vertex

+



rgences

ghost

istic

ield

gauge invarian

$\langle A A A \rangle$ + +

3-boson vertex

gauge loops

potential divergences

linear Λ

$-$

ghost loop

coefficient is zero !

Still have $\boxed{\log \Lambda \quad \text{UV divergences}}$

Field Renormalization for A and $(\bar{C}, C)$

AAAA 4-gauge vertex

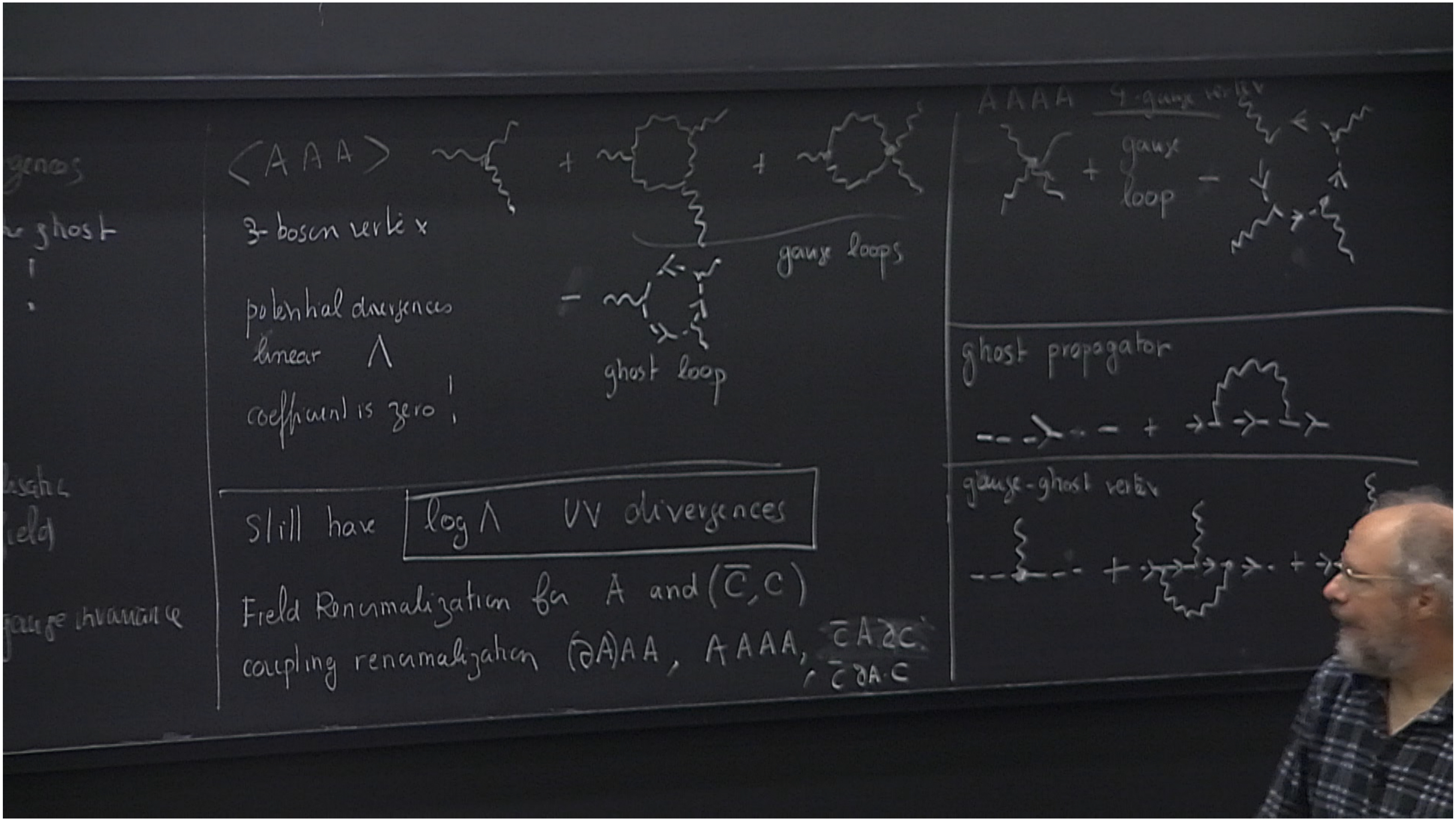
+ gauge loop $-$

ghost propagator

+

gauge-ghost vertex

+ +



...gences

...r ghost

...astic

...ield

gauge invariance

$\langle A\,A\,A \rangle$ + + −

3-boson vertex

gauge loops

potential divergences

linear Λ

ghost loop

coefficient is zero!

Still have $\boxed{\log\Lambda \quad \text{UV divergences}}$

Field Renormalization for A and $(\bar{C}, C)$

coupling renormalization $(\partial A)AA$, $AAAA$, $\bar{C}A\partial C$, $\bar{C}\,\partial A\cdot C$

AAAA 4-gauge vertex

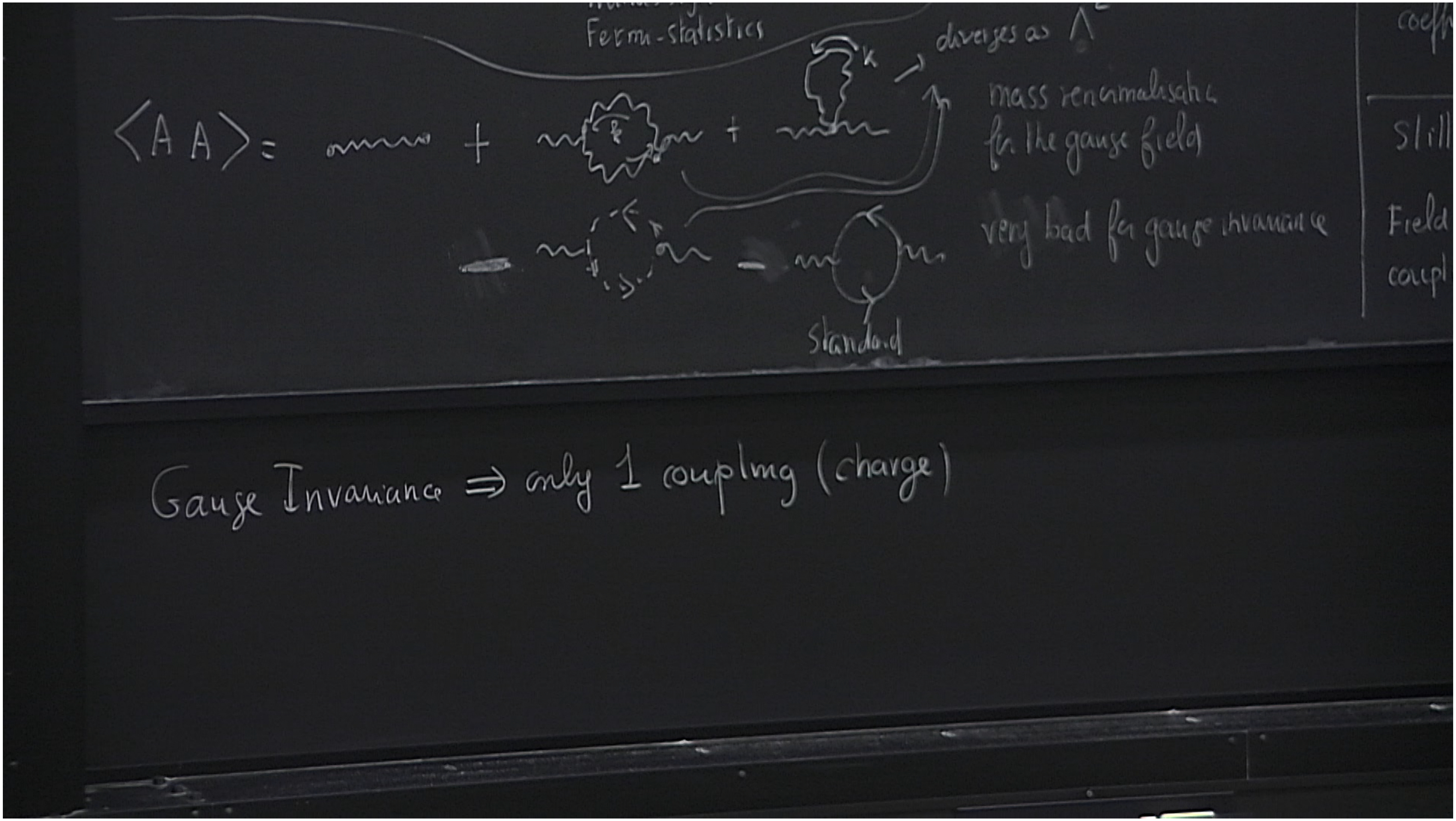
\+ gauge loop −

ghost propagator

\+

gauge-ghost vertex

\+ +



Fermi-statistics

$\langle A A \rangle =$ + + diverges as Λ^2

mass renormalisation for the gauge field

− − very bad for gauge invariance

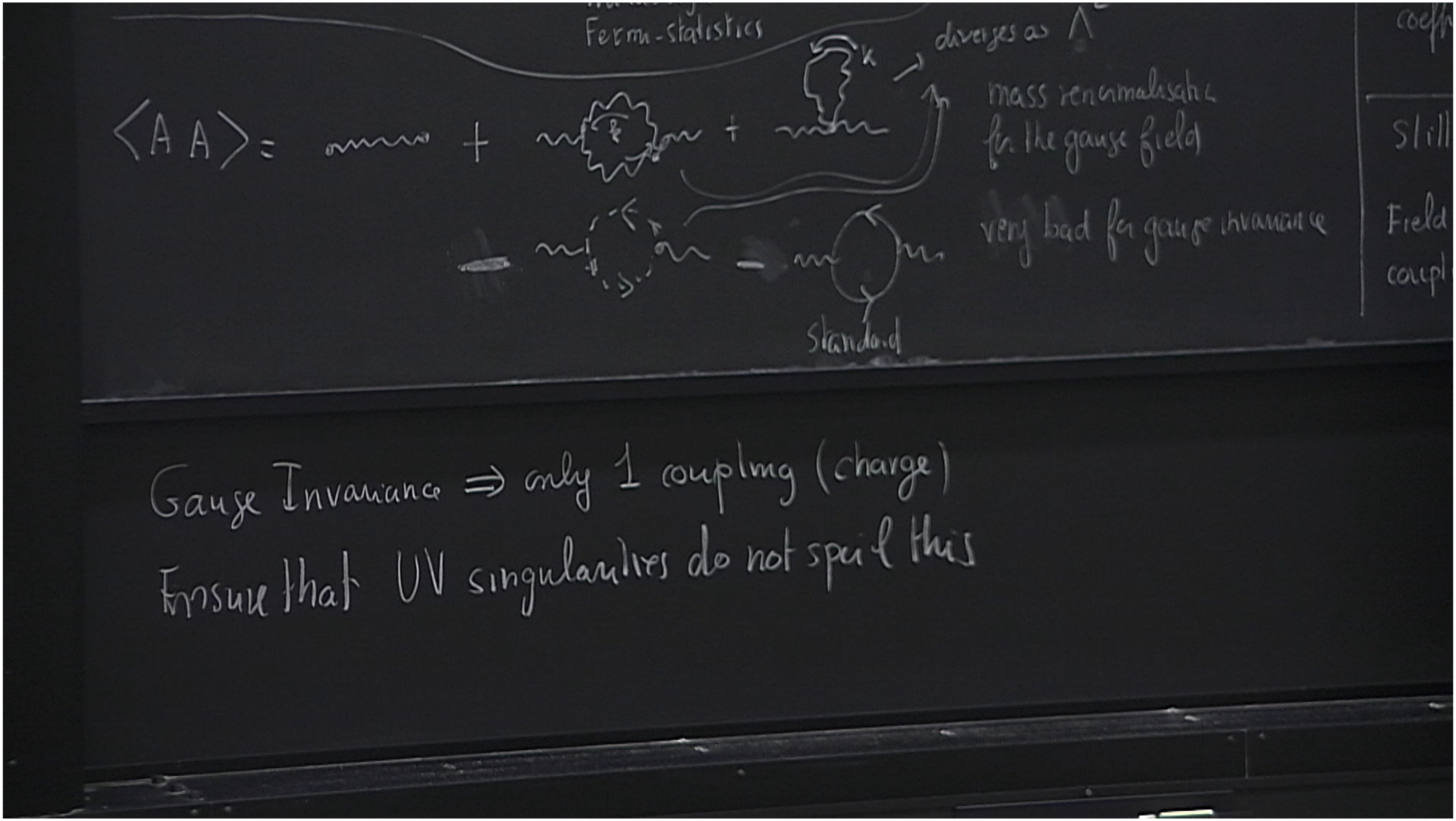
standard

coeff

Still

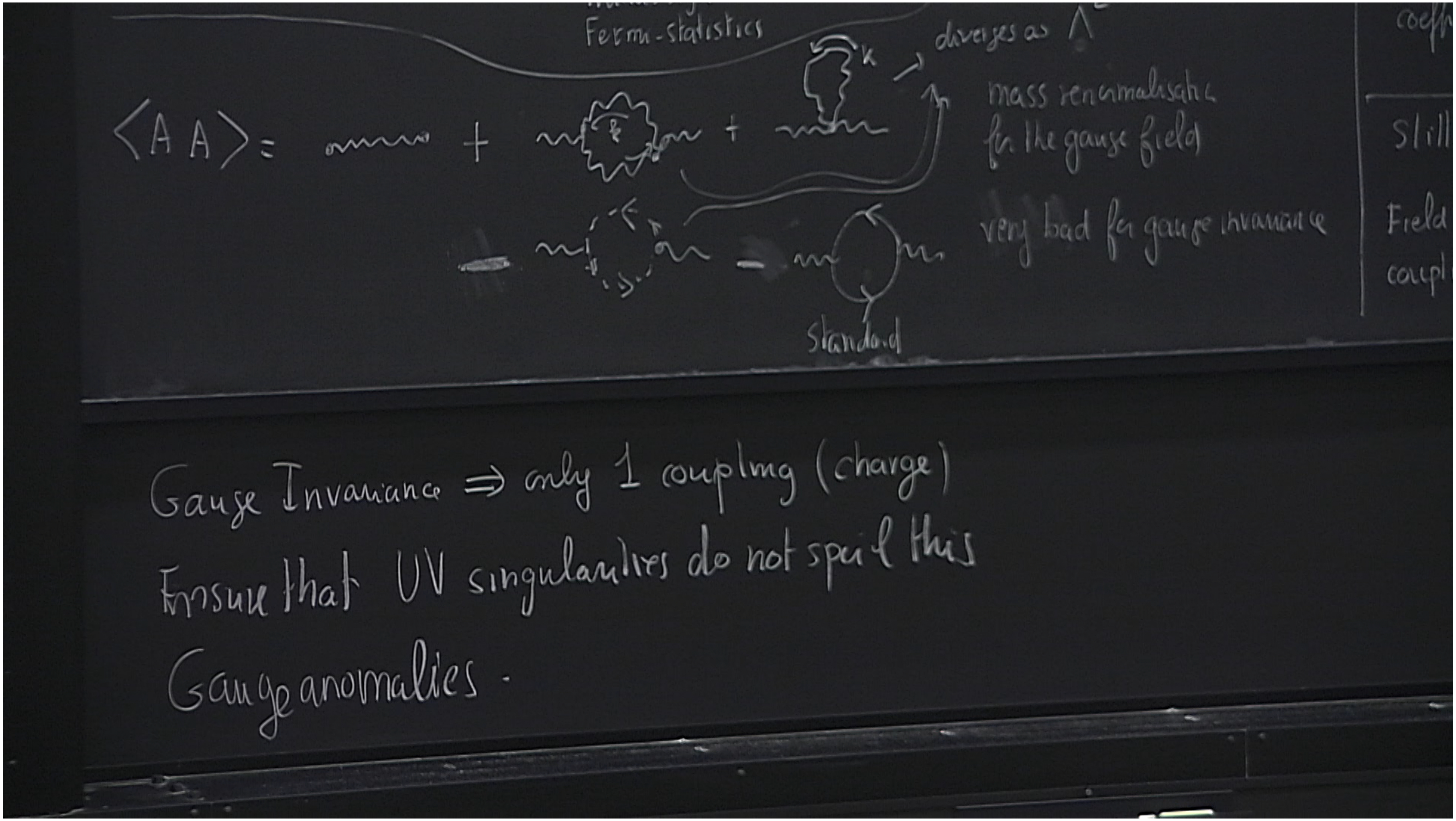
Field coupl

Gauge Invariance ⇒ only 1 coupling (charge)



Gauge Invariance ⇒ only 1 coupling (charge)

Ensure that UV singularities do not spoil this



$\langle A\ A \rangle =$ + +

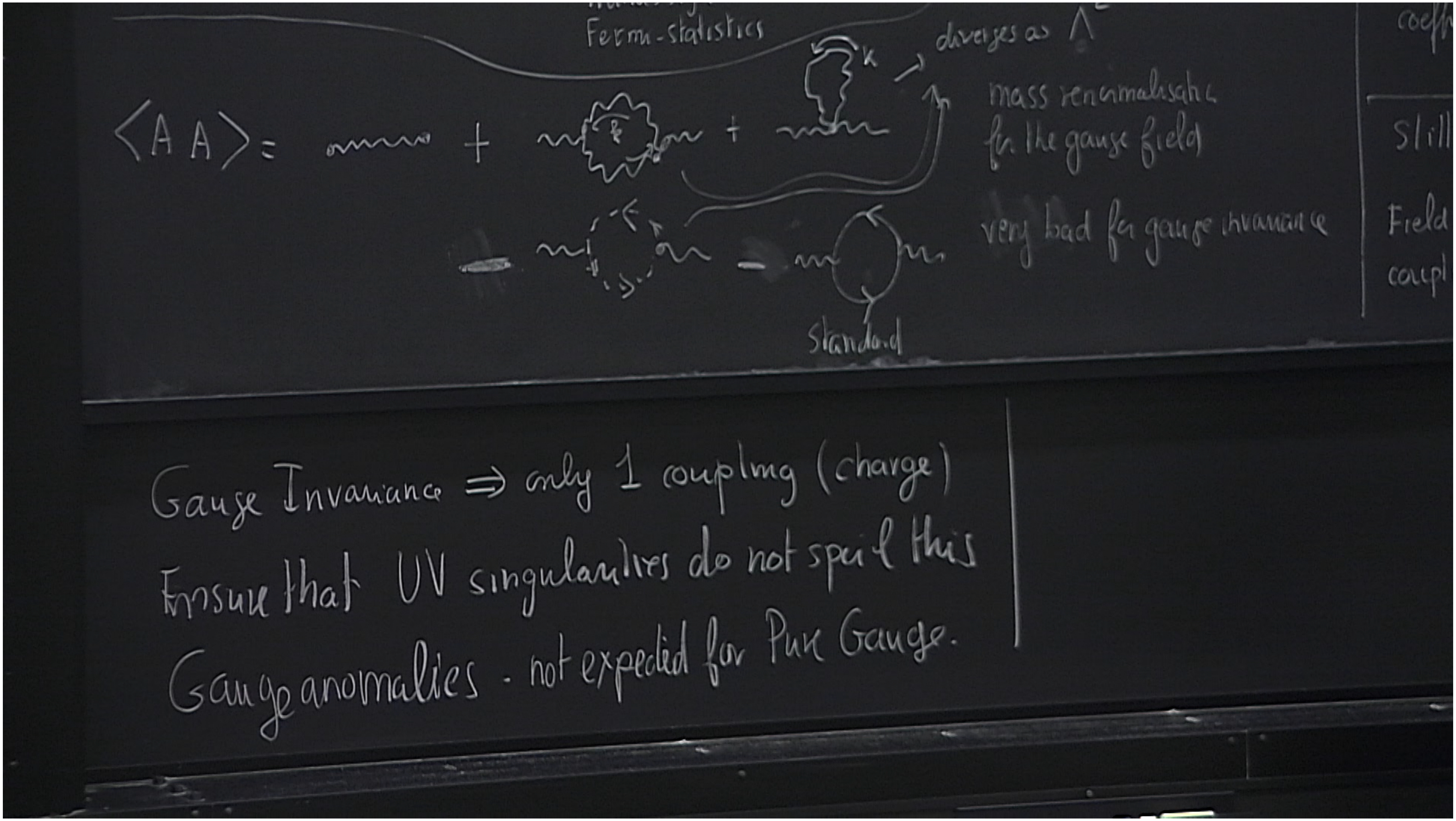
mass renormalisation for the gauge field

very bad for gauge invariance

Gauge Invariance ⇒ only 1 coupling (charge)

Ensure that UV singularities do not spoil this

Gauge anomalies.



Gauge Invariance ⇒ only 1 coupling (charge)

Ensure that UV singularities do not spoil this

Gauge anomalies - not expected for Pure Gauge.