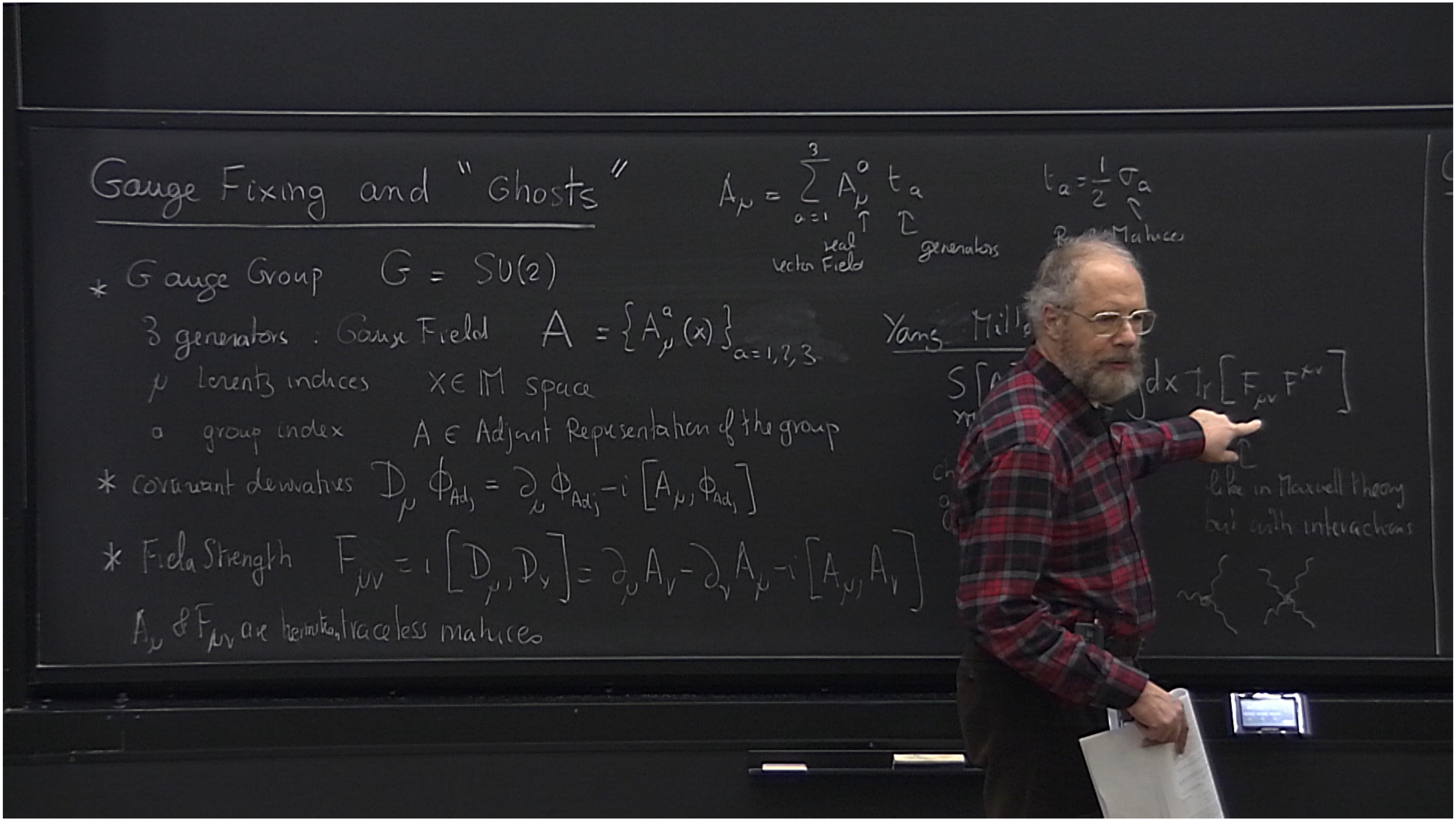
Title: PSI 17/18 - Quantum Field Theory II - Lecture 13

Date: Nov 22, 2017 09:00 AM

URL: http://pirsa.org/17110023

Abstract:


Gauge Fixing and "Ghosts"
$A_\mu = \sum_{a=1}^{3} A_\mu^a \, t_a$
real vector Field
generators
$t_a = \frac{1}{2}\sigma_a$
* Gauge Group $G = SU(2)$
3 generators : Gauge Field $A = \{A_\mu^a(x)\}_{a=1,2,3}$
μ Lorentz indices $x \in M$ space
a group index $A \in$ Adjoint Representation of the group
* covariant derivatives $D_\mu \phi_{Adj} = \partial_\mu \phi_{Adj} - i[A_\mu, \phi_{Adj}]$
* Field Strength $F_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
A_μ & $F_{\mu\nu}$ are hermitian, traceless matrices
Yang Mills
like in Maxwell theory
but with interactions

Gauge Fixing and "Ghosts"

$A_\mu = \sum_{a=1}^{3} A_\mu^a \, t_a$ real vector field, generators

$t_a = \frac{1}{2}\sigma_a$ Pauli Matrices

* Gauge Group $G = SU(2)$

3 generators : Gauge Field $A = \{A_\mu^a(x)\}_{a=1,2,3}$

μ Lorentz indices $x \in M$ space

a group index $A \in$ Adjoint Representation of the group

* covariant derivatives $D_\mu \phi_{Adj} = \partial_\mu \phi_{Adj} - i\left[A_\mu, \phi_{Adj}\right]$

* Field Strength $F_{\mu\nu} = i\left[D_\mu, D_\nu\right] = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]$

A_μ & $F_{\mu\nu}$ are hermitian traceless matrices

Yang Mills Action

$$S_{YM}[A] = -\frac{1}{4g^2}\int_M dx \, Tr\left[F_{\mu\nu}F^{\mu\nu}\right]$$

charge of the gauge fields

like in Maxwell theory but with interactions

Quantize ? Functional Integral

$$\int_{\mathcal{A}} \mathcal{D}[A] \exp\left(-S_{YM}[A]\right)$$

does this makes sense ?

can we extract perturbation theory ?

A is a gauge potential configuration

$\mathcal{A}$ = "space of all gauge configurations"

measure

$$\mathcal{D}[A] = \prod_{x\in M} \prod_{\mu} \prod_{a} dA^a_\mu(x)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \iff \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial B}{\partial t} = 0 \iff \vec{E}$$

comes from A

non linear too

set $\hbar = 1$

loop expansion is also a g^2 expansion

Gauge theory on a lattice

Wilson 1974

makes sense of this

theory

...ctions

Gauge transformation (local)

$$x \in M \longrightarrow g(x) \in G = SU(2)$$

$$\mathcal{G} = G^{\otimes M} = \{g(x)\}_{x \in M} =$$ set of all local gauge transformation

↑ Big Group $g_1, g_2 \in \mathcal{G}$ $g = g_1 \cdot g_2$ such that $g(x) = g_1(x)\, g_2(x)$ for any $x \in M$

$g \in \mathcal{G}$: $g = \{g(x)\}_{x \in M}$ → gauge transformation

$\mathrm{Lie}(\mathcal{G}) \ni \alpha$ $\alpha = \{\alpha(x)\}_{x \in M}$ $g(x) = \exp(i\alpha(x))$ all $x \in M$

infinitesimal gauge transf.

...mation

...thod $g(x) = g_1(x)\, g_2(x)$

...ry $x \in M$

$\exp(i\alpha(x))$

$\in M$

... gauge transf.

- Measure on $\mathcal{G}$ (gauge transformations)

Haar measure on $G = SU(2)$, $d_{HAAR}(g)$

$$d_{HAAR}(g) = \prod_x d_{HAAR}(g(x))$$

- Action of $\mathcal{G}$ on $\mathcal{A}$

"smooth"

$$A_\mu(x) \longrightarrow g(x) A_\mu(x) g^{-1}(x) + i\, g(x) \partial_\mu g^{-1}(x)$$

$$= A_{g,\mu}(x)$$

compact way

$$\boxed{A \in \mathcal{A},\ g \in \mathcal{G} \quad A \to A_g}$$

Gauge invariance

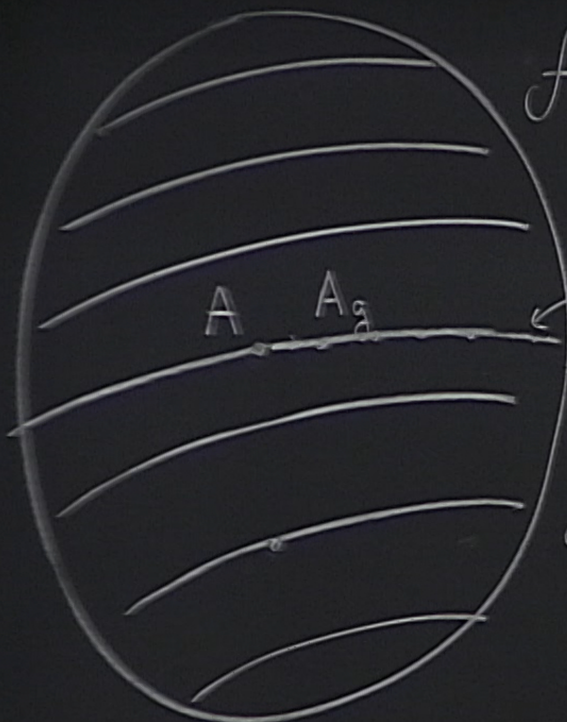
$$S[A_g] = S[A] \quad , \quad \mathcal{D}[A_g] = \mathcal{D}[A]$$

$\uparrow$ Flat directions

A , A_g they are physically equivalents (as in $U(1)$ Maxwell)

Gauge symmetry is not a "physical symmetry" but a redundancy in the description of the system by gauge potentials

2



$\mathcal{A}$

$A \in \mathcal{A}$

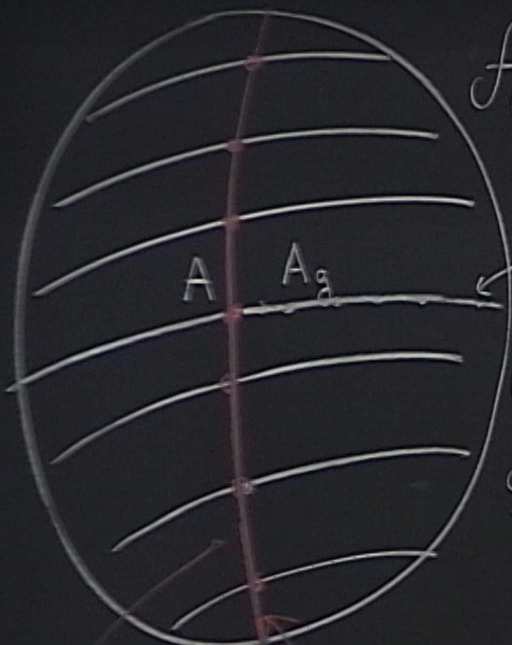
$g \in \mathcal{G}$

subspace : orbit of A by $\mathcal{G}$

$$\mathcal{G}(A) = \{ A' = A_g \text{ for some } g \in \mathcal{G} \}$$

Space of orbits = space of physical configurations = $\mathcal{C} = \mathcal{A}/\mathcal{G}$

quotient of $\mathcal{A}$ by the action of $\mathcal{G}$



$\mathcal{A}$

$A \in \mathcal{A}$

$g \in \mathcal{G}$

subspace : orbit of A by $\mathcal{G}$

$\mathcal{G}(A) = \{ A' = A_g \text{ for some } g \in \mathcal{G} \}$

Space of orbits = space of physical configurations = $\mathcal{C} = \mathcal{A}/\mathcal{G}$

quotient of $\mathcal{A}$ by the action of $\mathcal{G}$

Gauge fixing Function of A = 0

$F(A) = 0$

How $\int_{\mathcal{G}} D[A]$ —

pick an el

$F[A]$ ga

$$D[A] = \prod_{x \in M} \prod_{\mu} \prod_{a} dA^a_\mu(x)$$

$\int_{\mathcal{G}} D[A] \longrightarrow$ integral over $\mathcal{C} = \mathcal{A}/\mathcal{G}$?

pick an element A in each orbit $\Leftrightarrow$ fix a gauge by a gauge condition

$F[A] = \{ F^a(A_\mu(x), \partial_\nu A_\mu$

Gauge Fixing condition

$F[A]$ gauge fixing function $\quad F[A] = 0 \Longrightarrow$ gauge slice

<u>Maxwell theory</u>

$\vec{\nabla} \cdot \vec{A}(x) = 0$ Coulomb $\qquad \partial^\mu \cdot A_\mu(x) = 0$ Lorentz gauge

$A_0(x) = 0$ Axial gauge

SU(2) gauge theory

$\partial^\mu A^a_\mu(x) = \epsilon^a(x)$ Lorentz-Landau-Feynman

$a = 1, 3$ $\qquad$ arbitrary fixed function

$$D[A] = \prod_{x \in M} \prod_{\mu} \prod_{a} dA^a_\mu(x)$$

① ③

...n tegral over $\mathcal{C} = \mathcal{A}/\mathcal{G}$?

$$F[A] = \{ F^a(A_\mu(x), \partial_\nu A_\mu(x), \dots, x) \}_{\substack{x \in M \\ a=1,3}}$$

Gauge Fixing condition

... A in each orbit $\Longleftrightarrow$ fix a gauge by a gauge condition

...ixing function $F[A]=0 \Longrightarrow$ gauge slice

SU(2) gauge theory

$$\partial^\mu A^a_\mu(x) = \epsilon^a(x)$$

$a=1,3$

arbitrary fixed function

Lorentz-Landau-Feynman

$$F_\epsilon[A] = \{ \partial^\mu A^a_\mu - \epsilon^a(x) \}$$

Coulomb $\quad \partial^\mu \cdot A_\mu(x) = 0$ Lorentz gauge

Axial gauge

The Measure problem: we must find a measure independant on the choice of gauge slice

non-abelian symmetry ← gauge fixing independent results.
makes things more complicated

<u>The trick</u>: Fadeev-Popov & Feynman, Dewitt

choose some $F[A]=0$ gauge fixing

$$Z = \int_{\mathcal{A}} D[A] \exp(-S[A])$$

For each A, there is a $g_F \in \mathcal{G}$ such that A_g satisfy the gauge fixing condition $F[A]=0$

Let us assume there is only one

unique $g = g(A)$ s.that $F[A_{g_F(A)}]=0$

$$Z = \int_{\mathcal{A}} D[A] \exp\left(-S_{YM}[A]\right) \int_{g \in \mathcal{G}} D[g]\, \delta\left[g - g_F[A]\right]$$

↑ Dirac delta function on the group $\mathcal{G}$

$g_F[A]$ such that $F\left[A_{g_F[A]}\right] = 0$

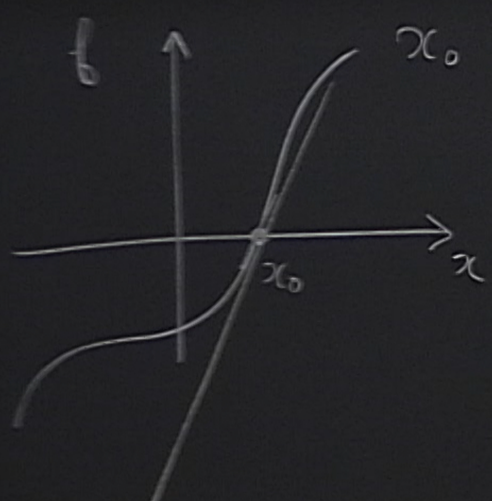
$$\boxed{\int_{\mathcal{G}} D[g]\, \delta\left[F[A_g]\right] \times \left|\det\left[F'[A_g]\right]\right|}$$

↑ Dirac delta function on $\mathrm{Lie}(\mathcal{G})$

Jacobian

unique $g = g(A)$ s.that $F[A_{g_F(A)}] = 0$

Jacobian. System with a constraint $\quad 1 = \int_{-\infty}^{+\infty} dx\, \delta(x - x_0) = \int_{-\infty}^{+\infty} dx\, \delta(f(x))\, f'(x_0)$



x_0 is determined by $f(x_0) = 0$

N-dim. $\quad \int_{\mathbb{R}^N} d^N X\, \delta(X - X_0)$ $\qquad X = (x_1, \dots x_N)$

X_0 such that $(f_1(X), \dots f_N(X)) = (0 \dots 0)$

N-conditions

$$\| $$

$$\int d^N X \prod_{a=1}^{N} \delta(f_a(X)) \left| \det \frac{\partial f_a}{\partial x_b}(X) \right|$$

Jacobian

$F[A]$ is a trace class operator in X $\quad F'[A](X,Y)$ $\quad 3\times 3$ matrices

Before: use <u>gauge invariance</u>

$$Z = \int_{g} D[g] \int_{A} D[A]\, \delta\big[F[A_g]\big] \times \Big|\det\big[F'[A_g]\big]\Big| \exp\big(-S_{YM}[A]\big)$$

↑ change of variable $A \to A_g$

$$Z = \frac{\int_{g} [D[g]]}{\mathrm{Vol}(\mathcal{G})} \int_{A} D[A]\, \delta\big[F[A]\big] \times \Big|\det\big[F'[A]\big]\Big| \exp\big(-S_{YM}[A]\big)$$

↑ enforce the gauge fixing condition

$F(A)=0$

Let us look at what is thus $F'[A]$ For Landau-Feynman Gauge

$F[A]$ $\qquad F[A]^a(x) = \partial^\mu A^a_\mu(x) - \epsilon^a(x)$ $\qquad \delta F[A]^a(x) = ($

$F'[A]$ derivatives under infinitesimal gauge transformations $\delta\alpha$

$g(x) = 1 + i\,\delta\alpha^a(x)\, t_a$

for SU(2)

$$A^a_\mu(x) \to A^a_\mu(x) + D_\mu \delta\alpha^a(x) = A^a_\mu(x) + \underbrace{\partial_\mu \delta\alpha^a(x) + \epsilon^a{}_{bc} A^b_\mu(x)\,\delta\alpha^c(x)}_{\delta A^a_\mu(x)}$$

$$A^a_\mu(x) + \delta A^a_\mu(x)$$

=0 Axial gauge

a=1,3 arbitrary fixed function

Gauge

Maxwell

New, depends on A_μ

$$\delta F[A]^a(x) = (\partial_\mu \partial^\mu)\delta\alpha^a(x) + \epsilon^a{}_{bc}\,\partial^\mu\left(A^b_\mu(x)\,\delta\alpha^c(x)\right)$$

Laplacian

$$F'[A] = \frac{\delta F[A]^a(x)}{\delta\alpha^b(y)} = \text{Kernel of a differential operator} \left[\Delta_x \delta^{ab} - \epsilon^a{}_{cb}\,\overleftarrow{\partial^\mu} A^c_\mu\right](x,y)$$

Functional derivative

$$\boxed{\left(F'[A]\,\psi\right)^a(x) = \Delta_x \psi^a(x) + \epsilon^{abc}\frac{\partial}{\partial x_\mu} A^b_\mu(x)\,\psi^c(x)}$$