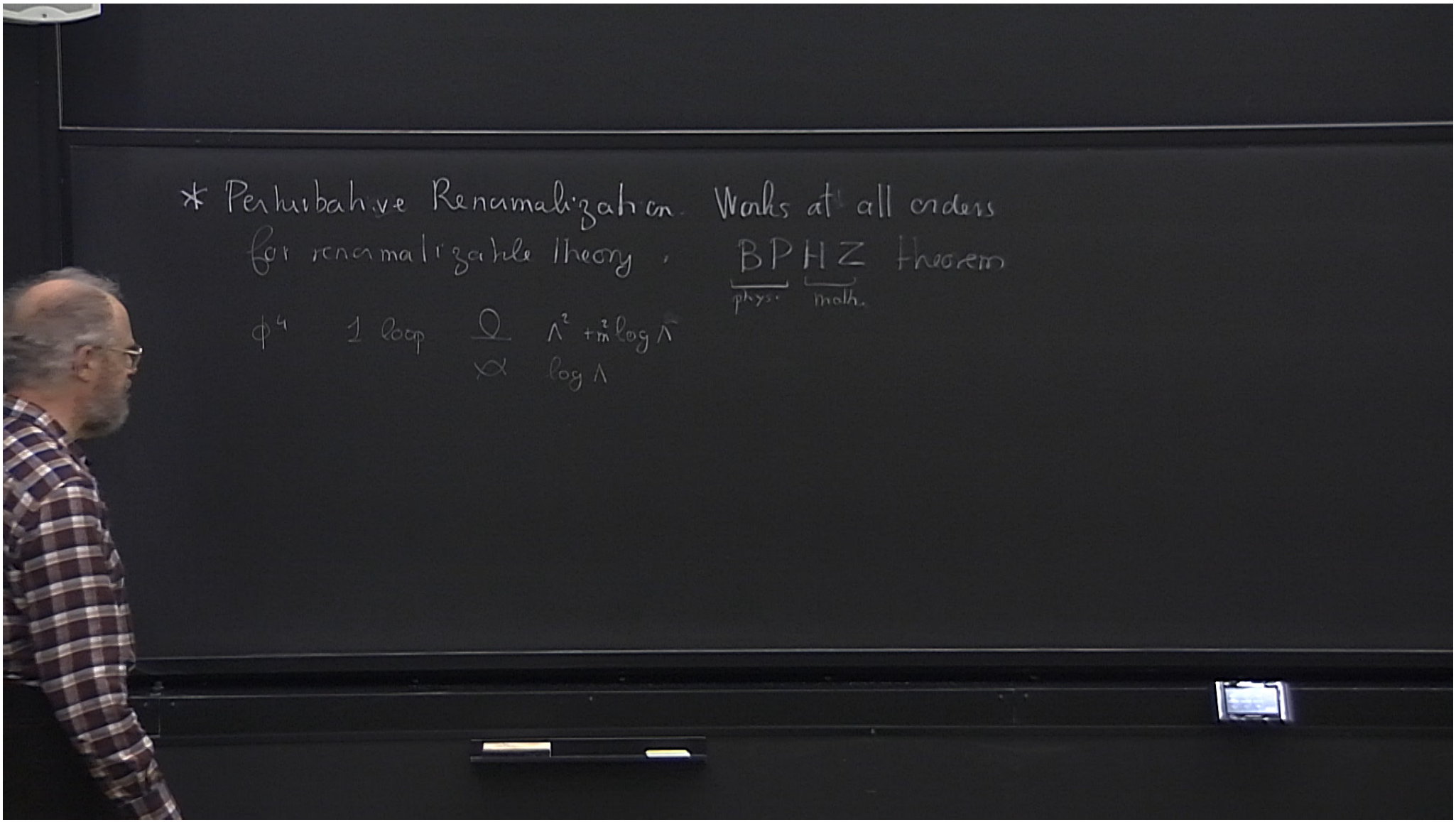
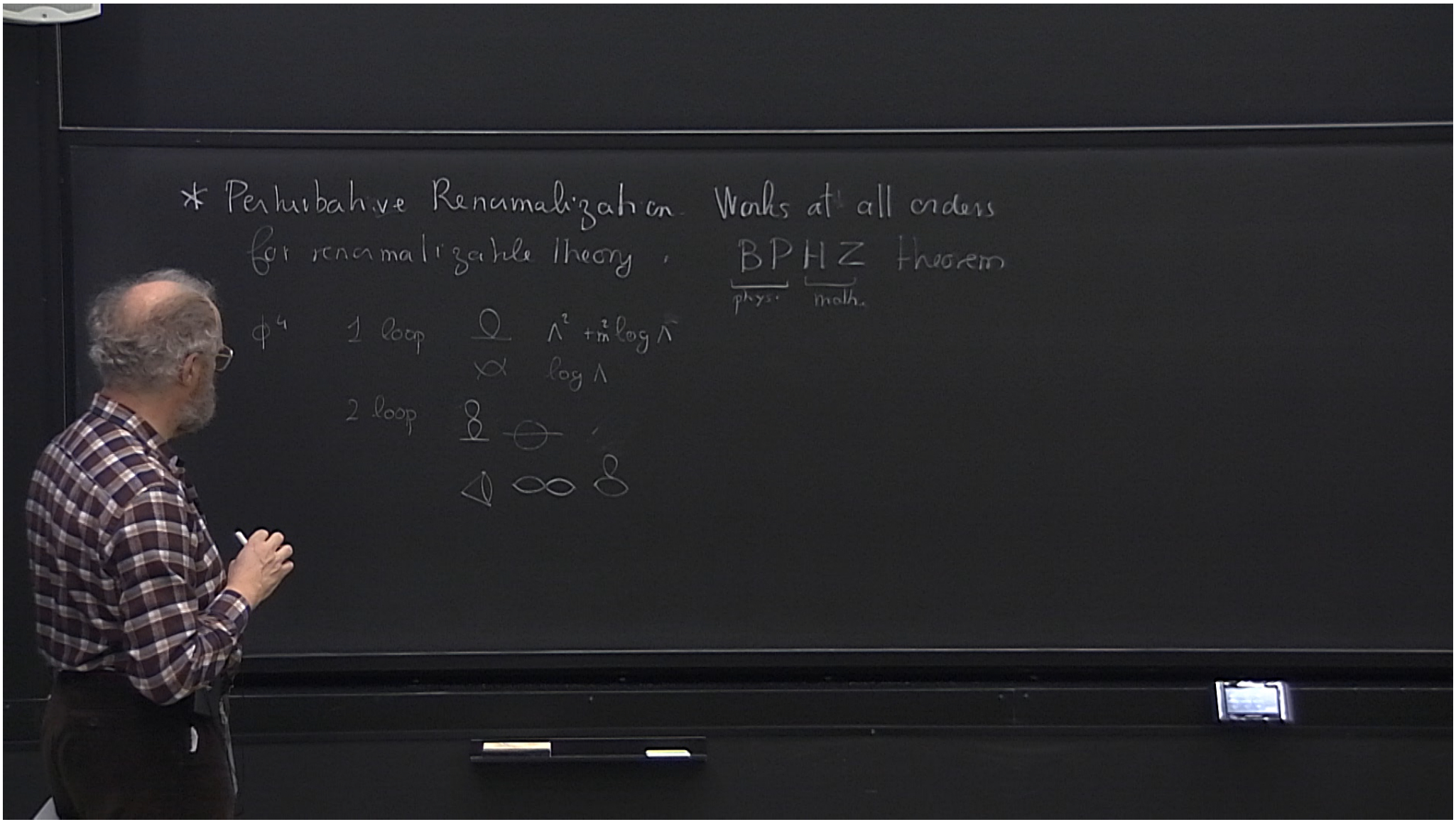
Title: PSI 17/18 - Quantum Field Theory II - Lecture 10

Date: Nov 17, 2017 09:00 AM

URL: http://pirsa.org/17110020

Abstract:


* Perturbative Renormalization. Works at all orders
for renormalizable theory. BPHZ theorem
phys. math.
ϕ^4 1 loop Ω $\Lambda^2 + m^2 \log \Lambda$
$\propto$ $\log \Lambda$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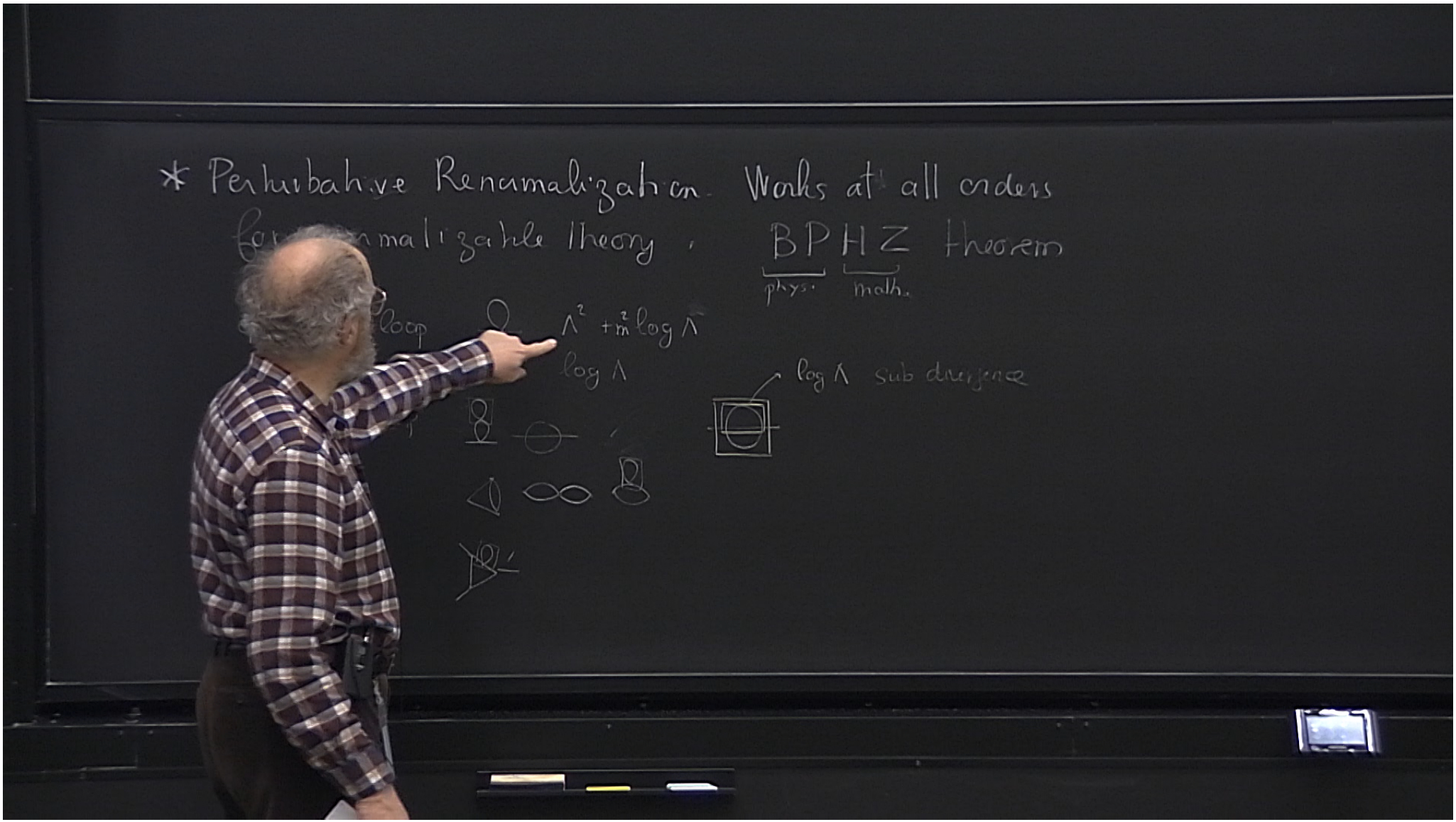
$\log \Lambda$
2 loop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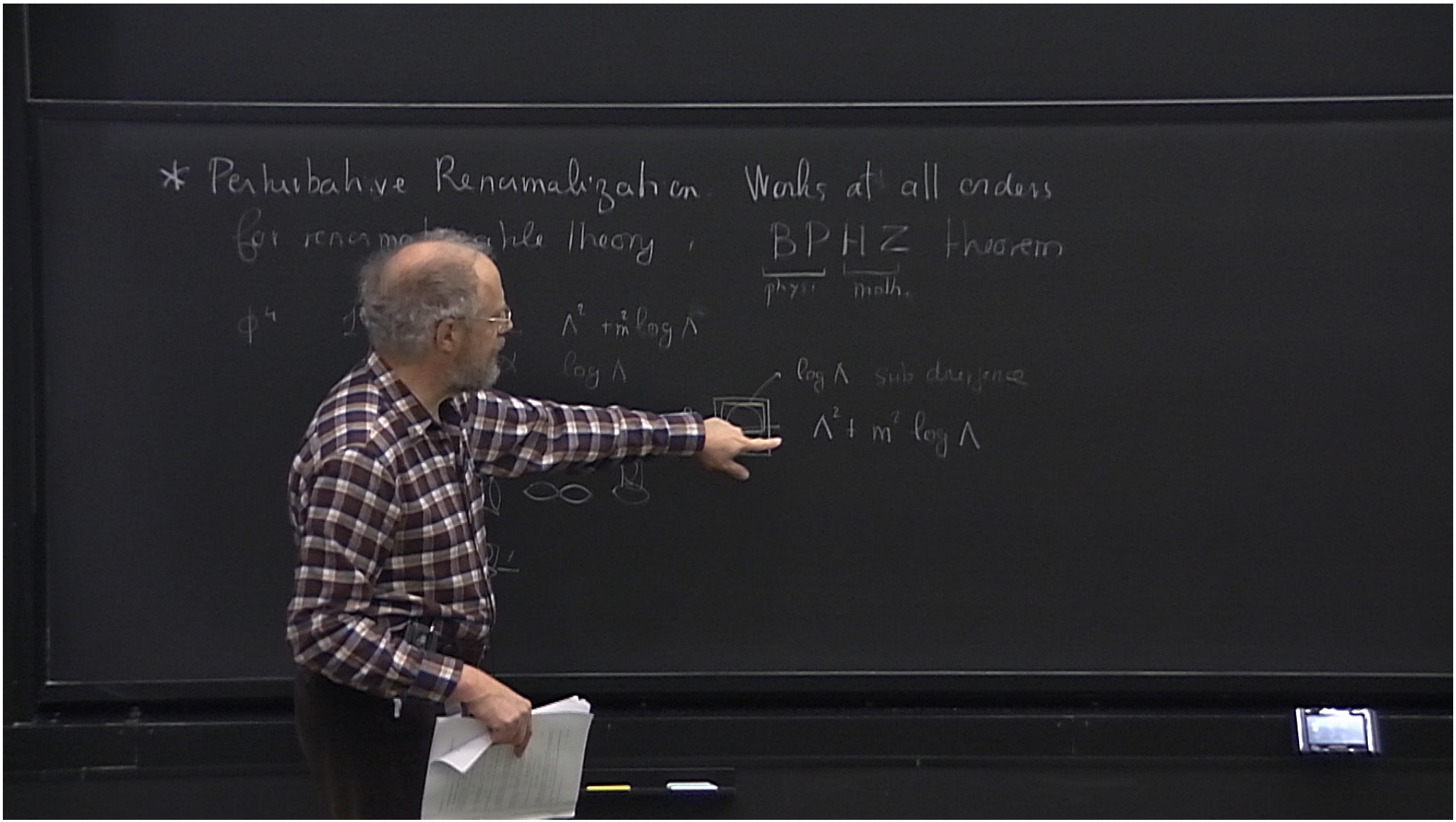
loop
$\Lambda^2 + m^2 \log \Lambda$
$\log \Lambda$
$\log \Lambda$ sub divergence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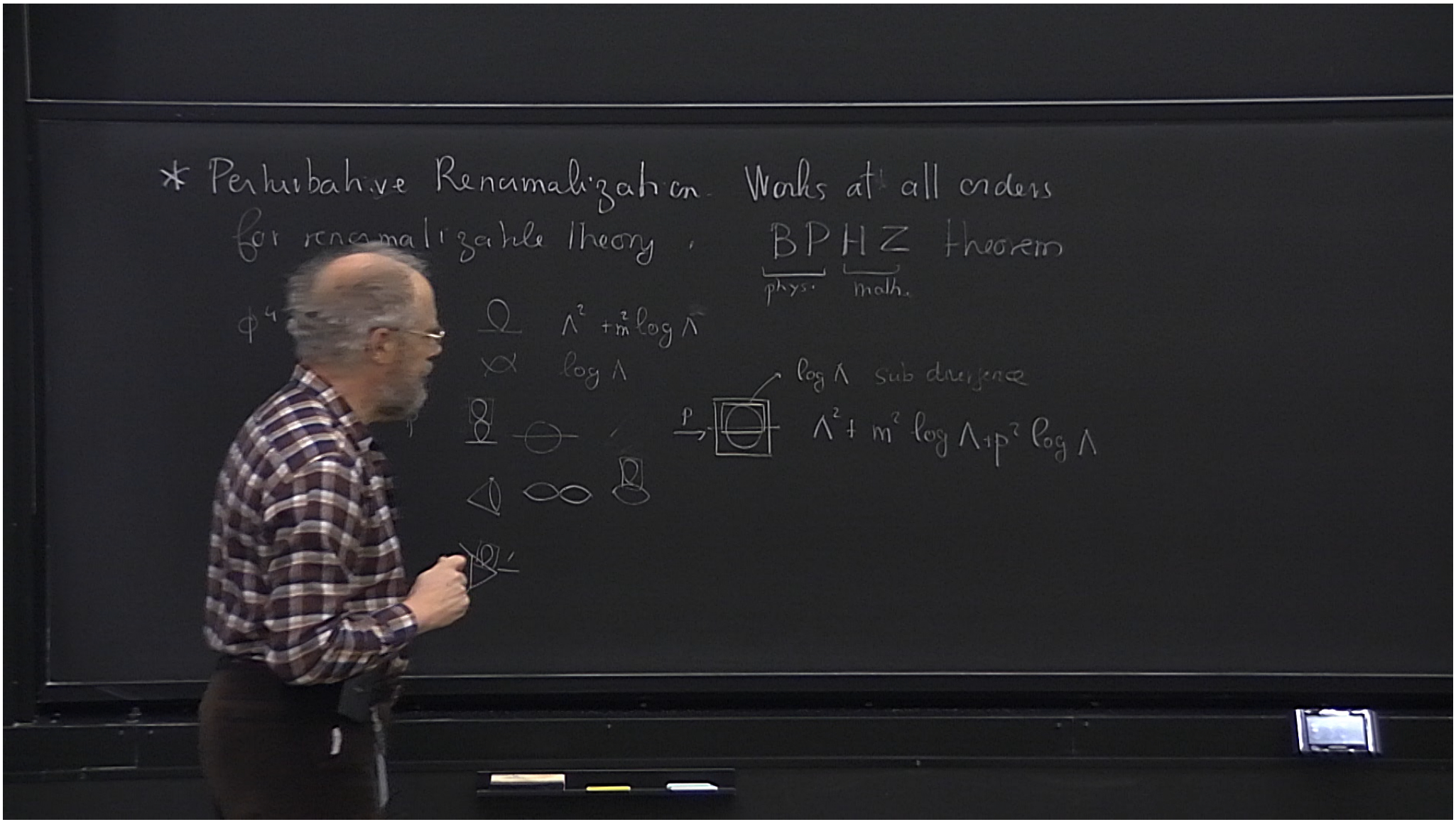
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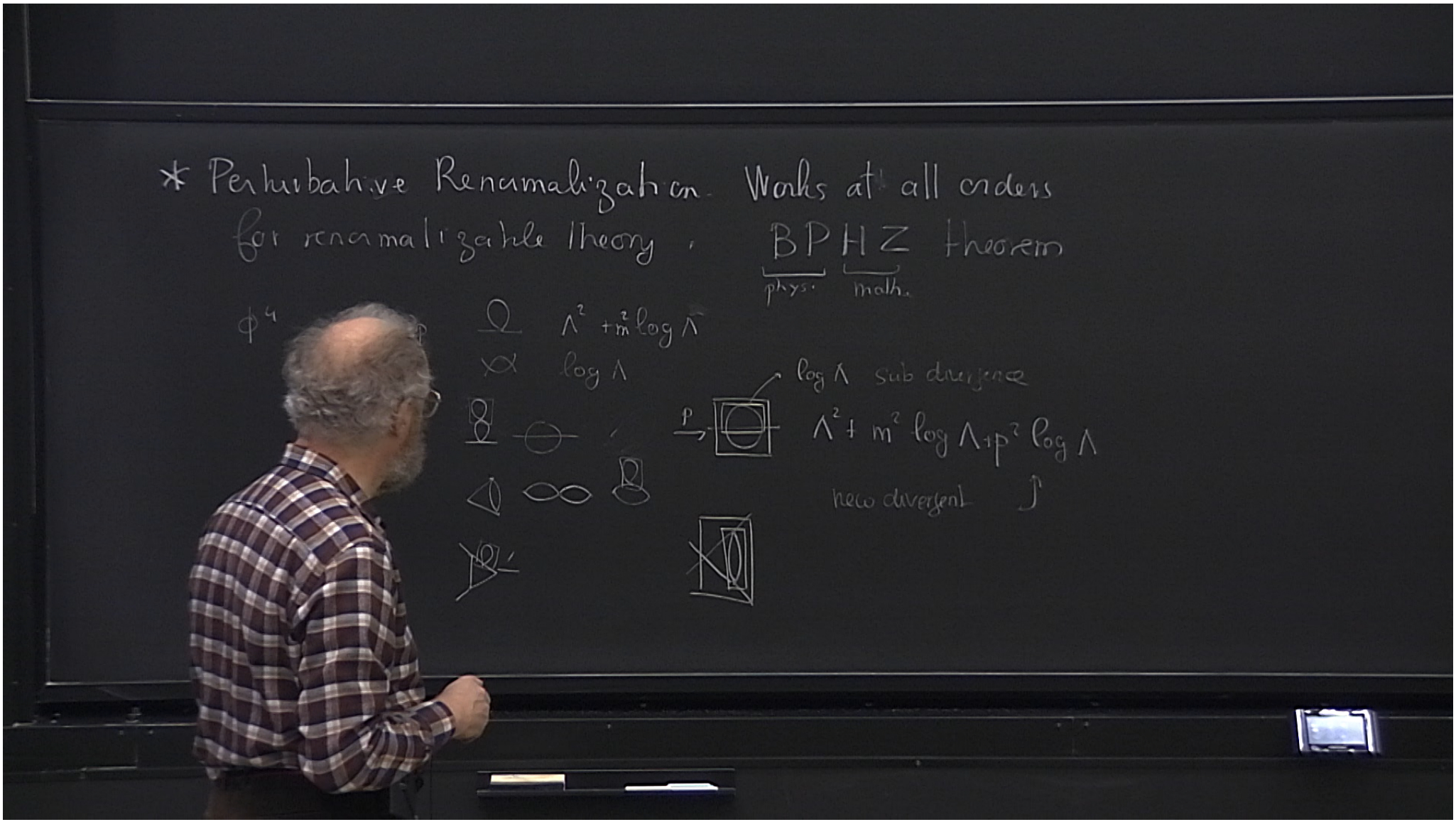
$\log \Lambda$ sub divergence
p
$\Lambda^2 + m^2 \log \Lambda + p^2 \log \Lambda$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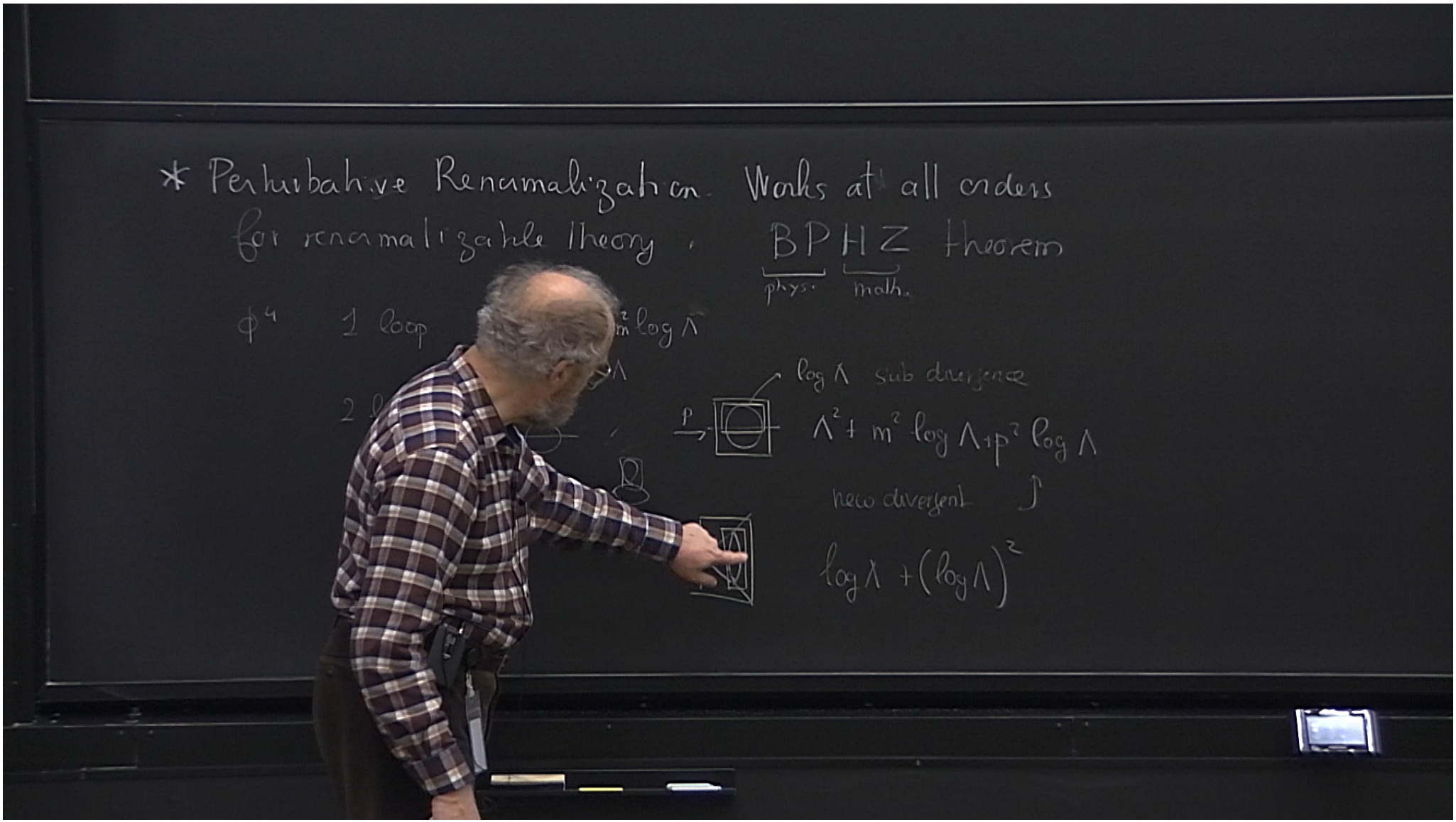
$\Lambda^2 + m^2 \log \Lambda + p^2 \log \Lambda$
new divergent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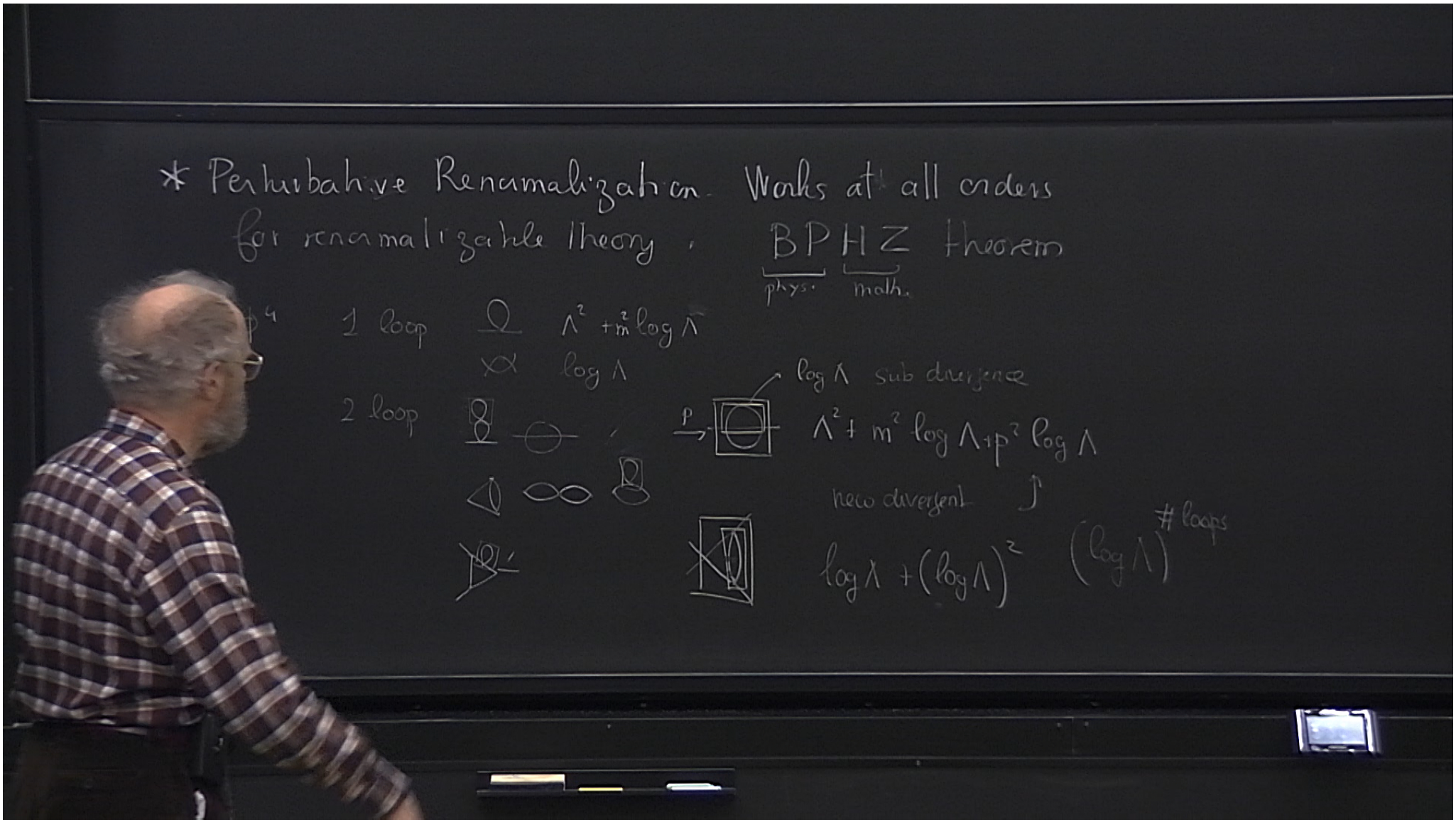
ϕ^4 1 loop
$m^2 \log \Lambda$
2 l
$\log \Lambda$ sub divergence
p
$\Lambda^2 + m^2 \log \Lambda + p^2 \log \Lambda$
new divergent
$\log \Lambda + (\log \Lambda)^2$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$\Lambda^2 + m^2 \log \Lambda + p^2 \log \Lambda$
new divergent
$\log \Lambda + (\log \Lambda)^2$
$(\log \Lambda)^{\#\text{loops}}$

nalization. Works at all orders

e theory, $\underbrace{\text{BP}}_{\text{phys.}}\underbrace{\text{HZ}}_{\text{math.}}$ theorem

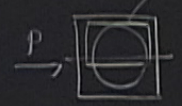
2 loop. new terms in $\mathcal{A}$, $\mathcal{B}$ & $\mathcal{C}$

$$\mathcal{A} = 1 + g_R^2 \cdot \log(\Lambda/\mu)$$

$\Lambda^2 + m^2 \log \Lambda$

$\log \Lambda$

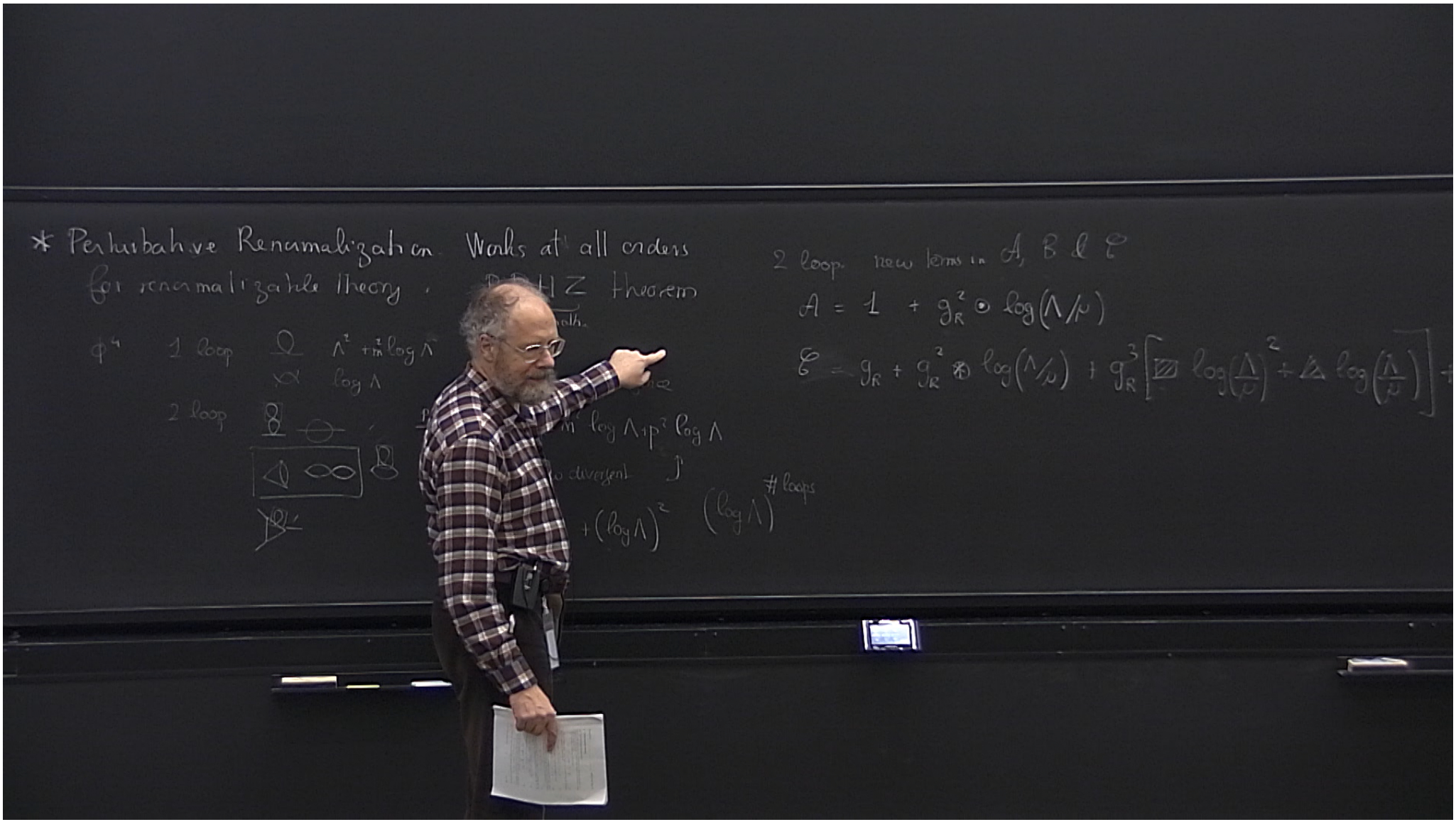
$\log \Lambda$ sub divergence



$$\Lambda^2 + m^2 \log \Lambda + p^2 \log \Lambda$$

new divergent

$$\log \Lambda + (\log \Lambda)^2 \quad (\log \Lambda)^{\#\text{ loops}}$$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for renormalizable theory
2 loop new terms in A, B & C
A = 1 + g_R^2 ⊙ log(Λ/μ)
φ^4 1 loop Λ^2 + m^2 log Λ
log Λ
2 loop
log Λ + p^2 log Λ
+ (log Λ)^2
(log Λ)^# loops

ders

orem

nce

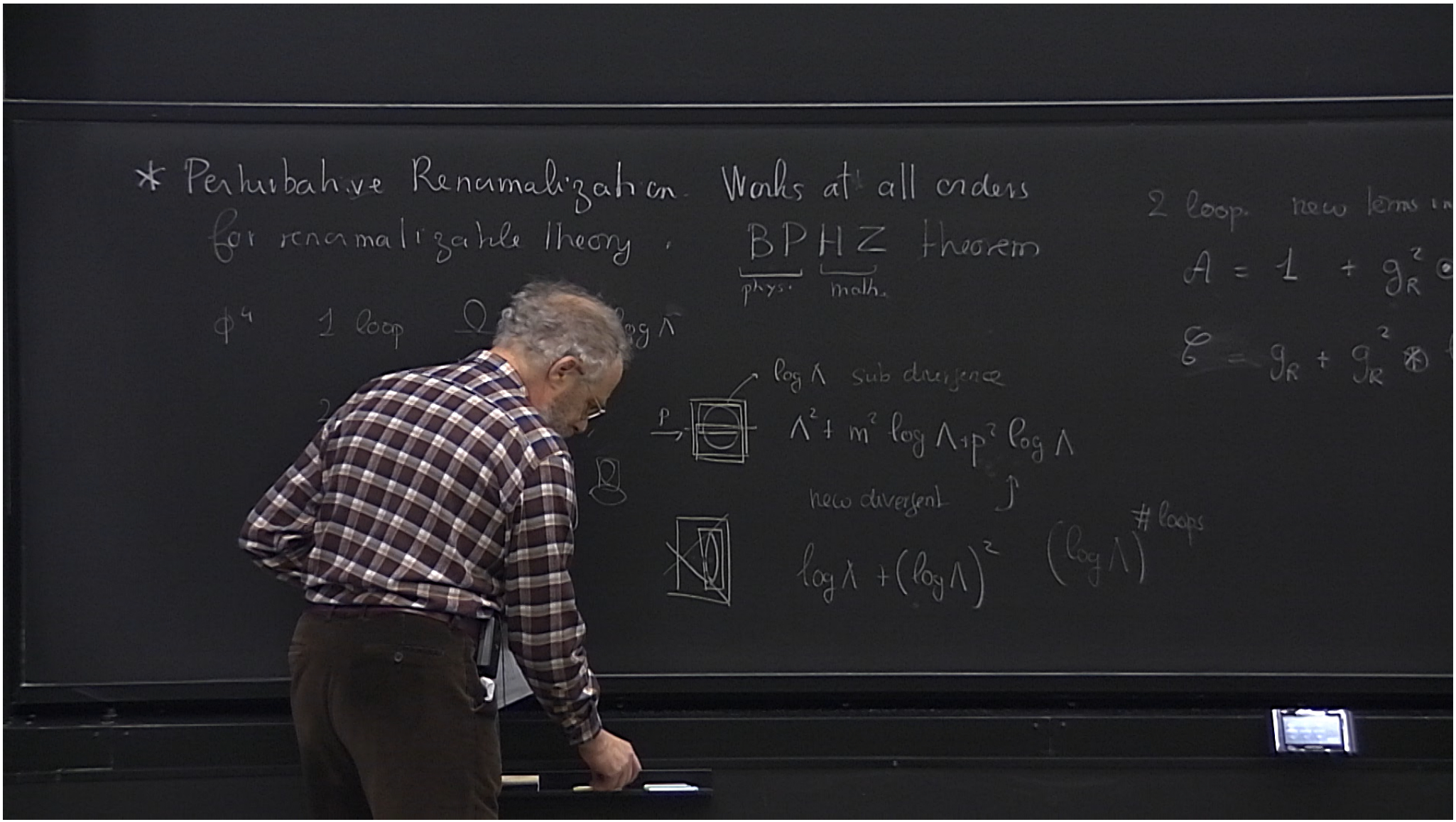
$+p^2 \log \Lambda$

$\Lambda)^2 \quad (\log \Lambda)^{\#\,\text{loops}}$

2 loop. new terms in $\mathcal{A}$, $\mathcal{B}$ & $\mathcal{C}$

$$\mathcal{A} = 1 + g_R^2 \odot \log(\Lambda/\mu)$$

$$\mathcal{C} = g_R + g_R^2 \circledast \log(\Lambda/\mu) + g_R^3 \left[\log\left(\frac{\Lambda}{\mu}\right)^2 + \log\left(\frac{\Lambda}{\mu}\right) \right] + \text{etc} \ldots$$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$\Lambda^2 + m^2 \log\Lambda + p^2 \log\Lambda$
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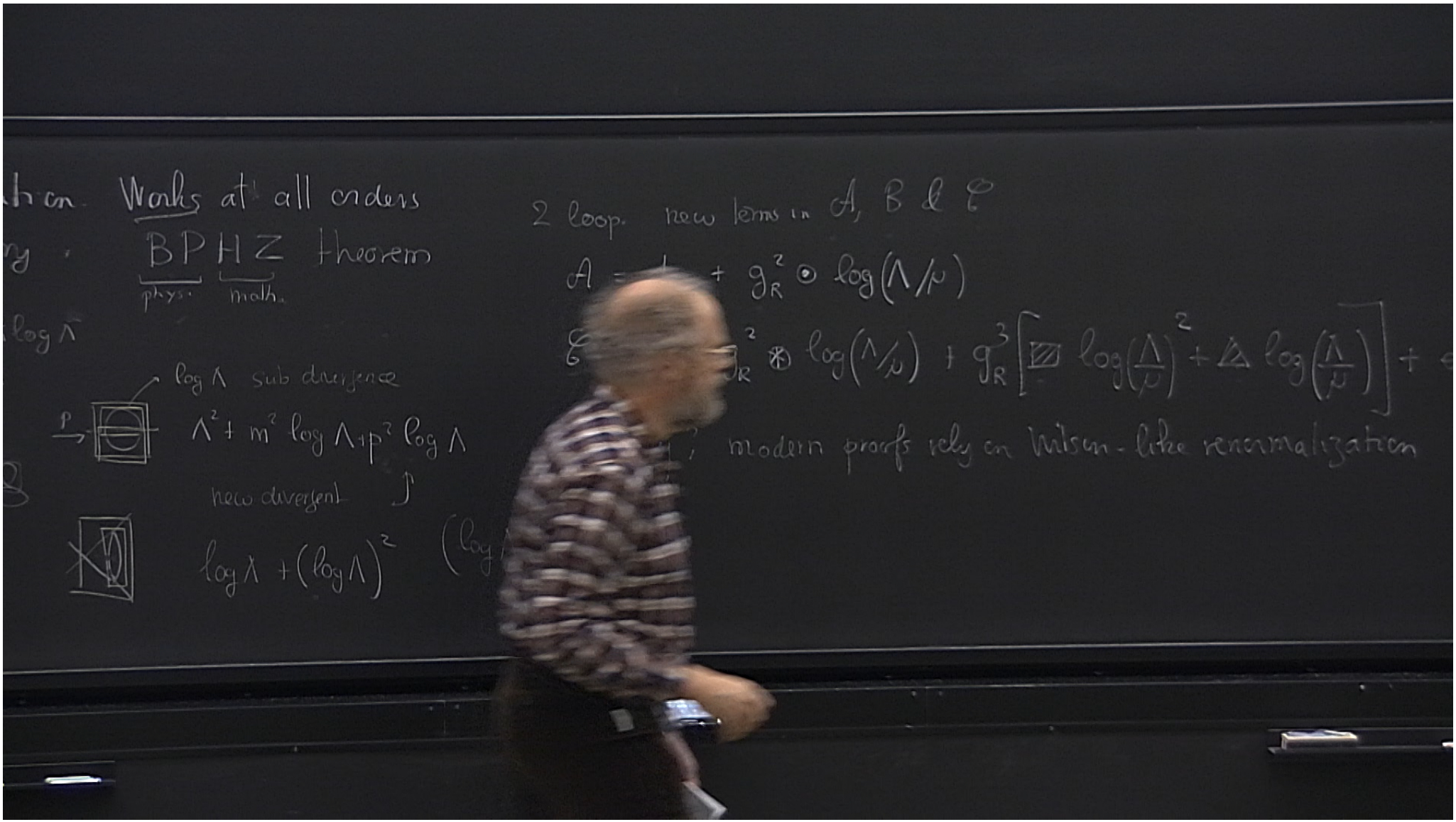
$\log\Lambda + (\log\Lambda)^2$ $(\log\Lambda)^{\#\text{loops}}$
2 loop. new terms
$A = 1 + g_R^2$
$\mathcal{E} = g_R + g_R^2$


Works at all orders
BPHZ theorem
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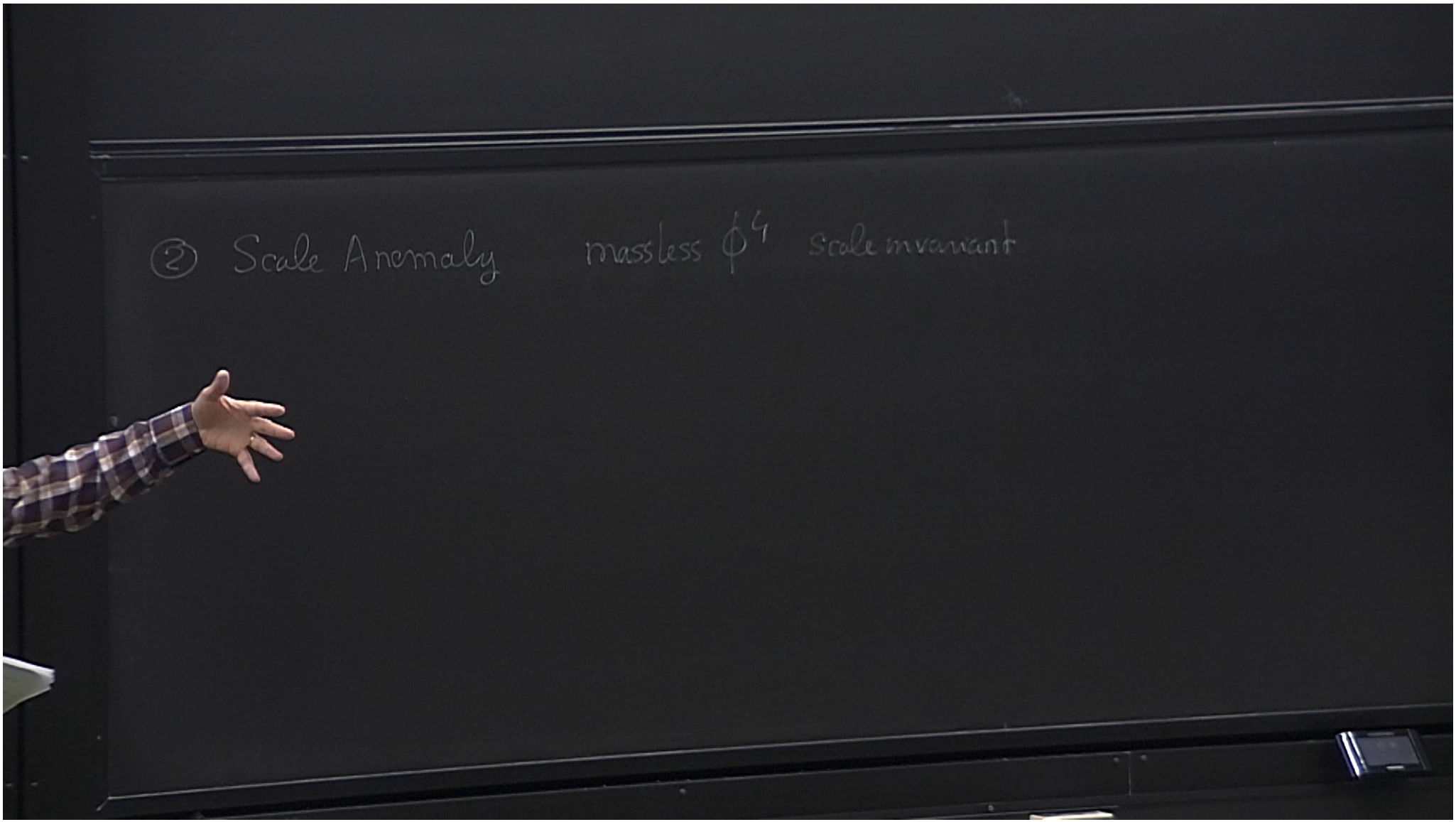
log Λ
log Λ sub divergence
Λ² + m² log Λ + p² log Λ
new divergent
log Λ + (log Λ)²
2 loop. new terms in A, B & C
A = + g_R² ⊙ log(Λ/μ)
modern proofs rely on Wilson-like renormalization


② Scale Anomaly
massless ϕ^4 scale invariant

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massless ϕ^4 scale invariant symmetry in $d = 4$

$$\phi(x) \to \phi_\lambda(x) = \lambda \phi(\lambda x)$$

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current conserved

$$J^\mu_{scale} =$$

② Scale Anomaly massless ϕ^4 scale invariant symmetry in $d=4$

$$\phi(x) \to \phi_\lambda(x) = \lambda\phi(\lambda x)$$

current conserved

$$J^\mu_{scale} = T^\mu{}_\nu x^\nu + \phi\partial^\mu\phi$$

↑ energy momentum

$$T^{\mu\nu} = \partial^\mu\phi\,\partial^\nu\phi - \delta^{\mu\nu}\mathcal{L}$$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$$

② Scale Anomaly massless ϕ^4 scale invariant symmetry in $d=4$

$$\phi(x) \to \phi_\lambda(x) = \lambda\phi(\lambda x)$$

current conserved

$$J^\mu_{scale} = T^\mu{}_\nu x^\nu + \phi\partial^\mu\phi \qquad \partial_\mu J^\mu_{scale} = 0$$

e.o.m of ϕ^4

$$+ \partial_\mu T^{\mu\nu} = 0$$

↑ energy momentum

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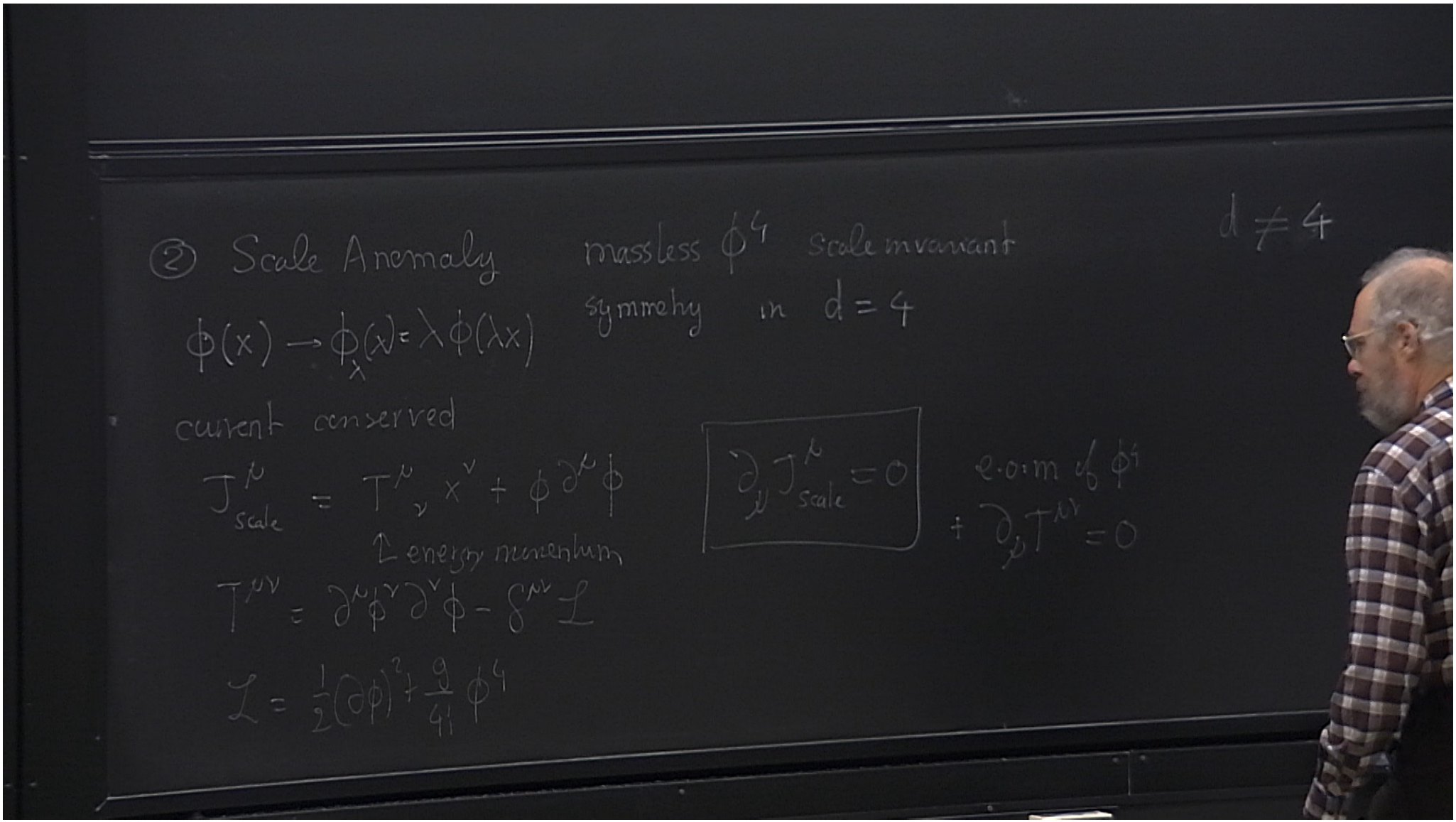
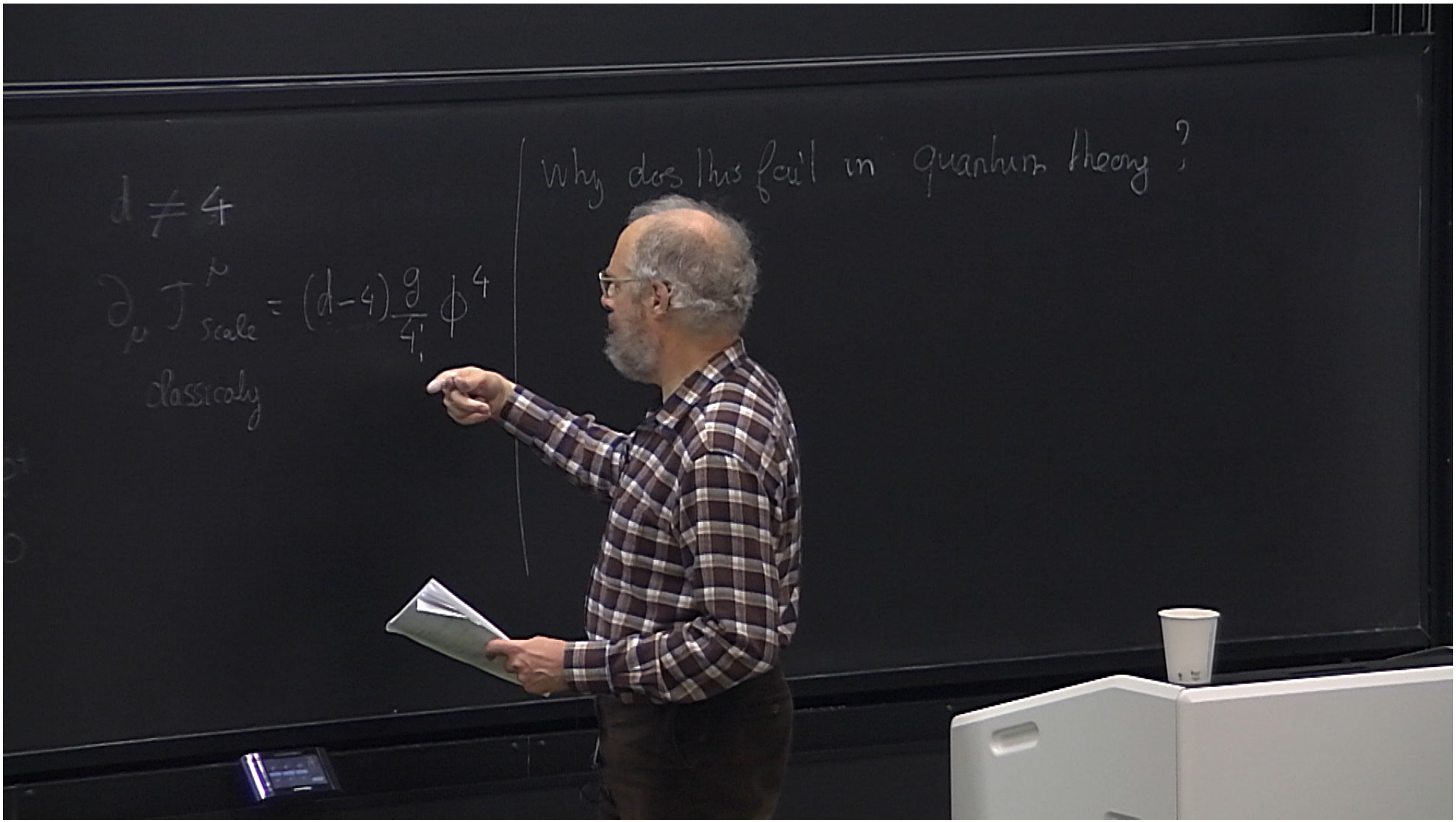

② Scale Anomaly
massless ϕ^4 scale invariant symmetry in $d=4$
$d \neq 4$
$\phi(x) \to \phi_\lambda(x) = \lambda\phi(\lambda x)$
current conserved
$J^\mu_{scale} = T^\mu{}_\nu x^\nu + \phi\partial^\mu\phi$
energy momentum
$T^{\mu\nu} = \partial^\mu\phi\,\partial^\nu\phi - g^{\mu\nu}\mathcal{L}$
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$
$\partial_\mu J^\mu_{scale} = 0$
e.o.m of ϕ^4
$+ \partial_\mu T^{\mu\nu} = 0$

$d \neq 4$

$$\partial_\mu J^\mu_{\text{scale}} = (d-4)\frac{g}{4!}\phi^4$$

classically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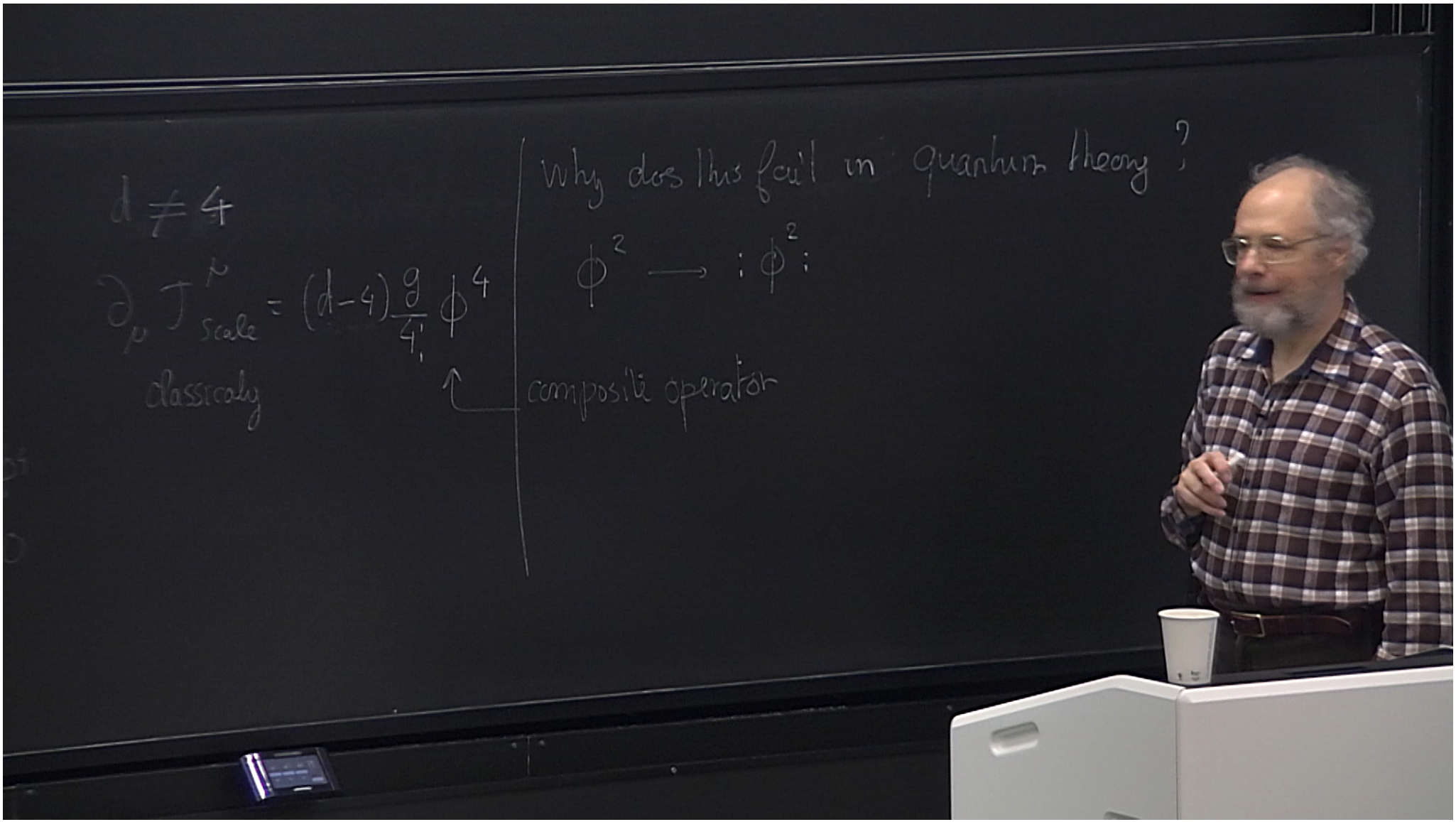
Why does this fail in quantum theory?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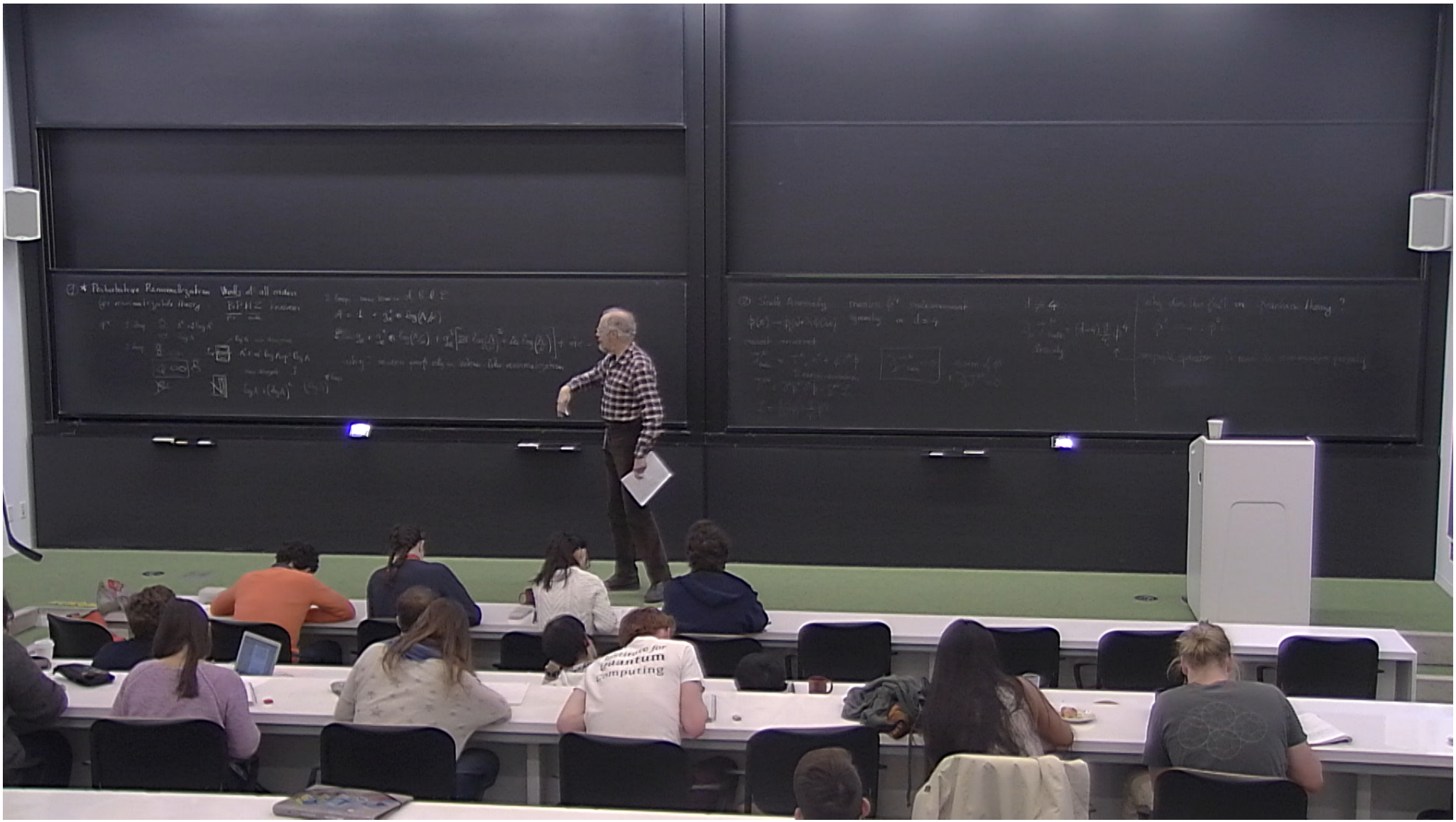
$\phi^2 \longrightarrow :\phi^2:$
composite operator


Institute for
Quantum
Computing

$d \neq 4$

$$\partial_\mu J^\mu_{\text{scale}} = (d-4)\frac{g}{4!}\phi^4$$

classically

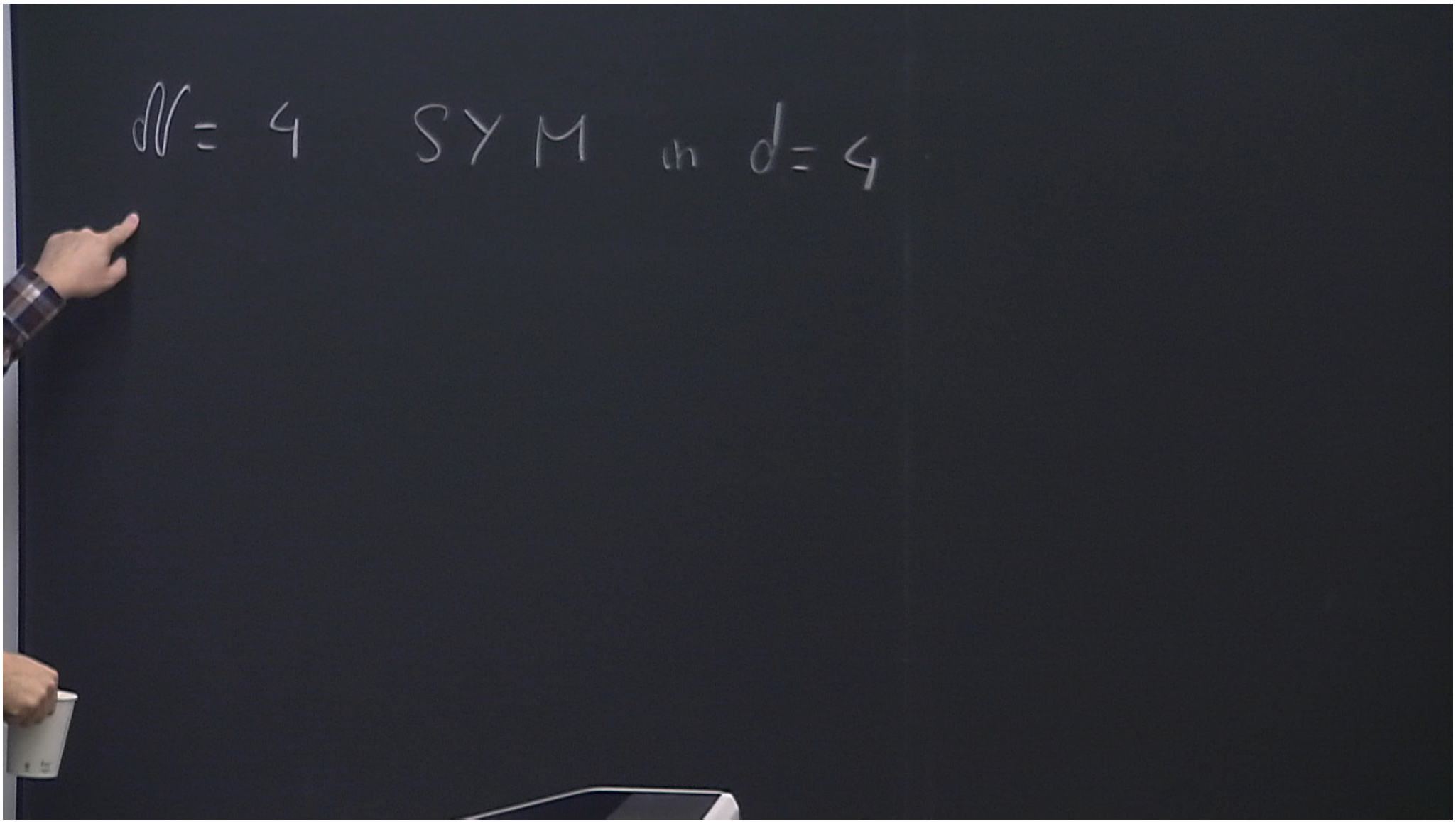
Why does this fail in quantum theory?

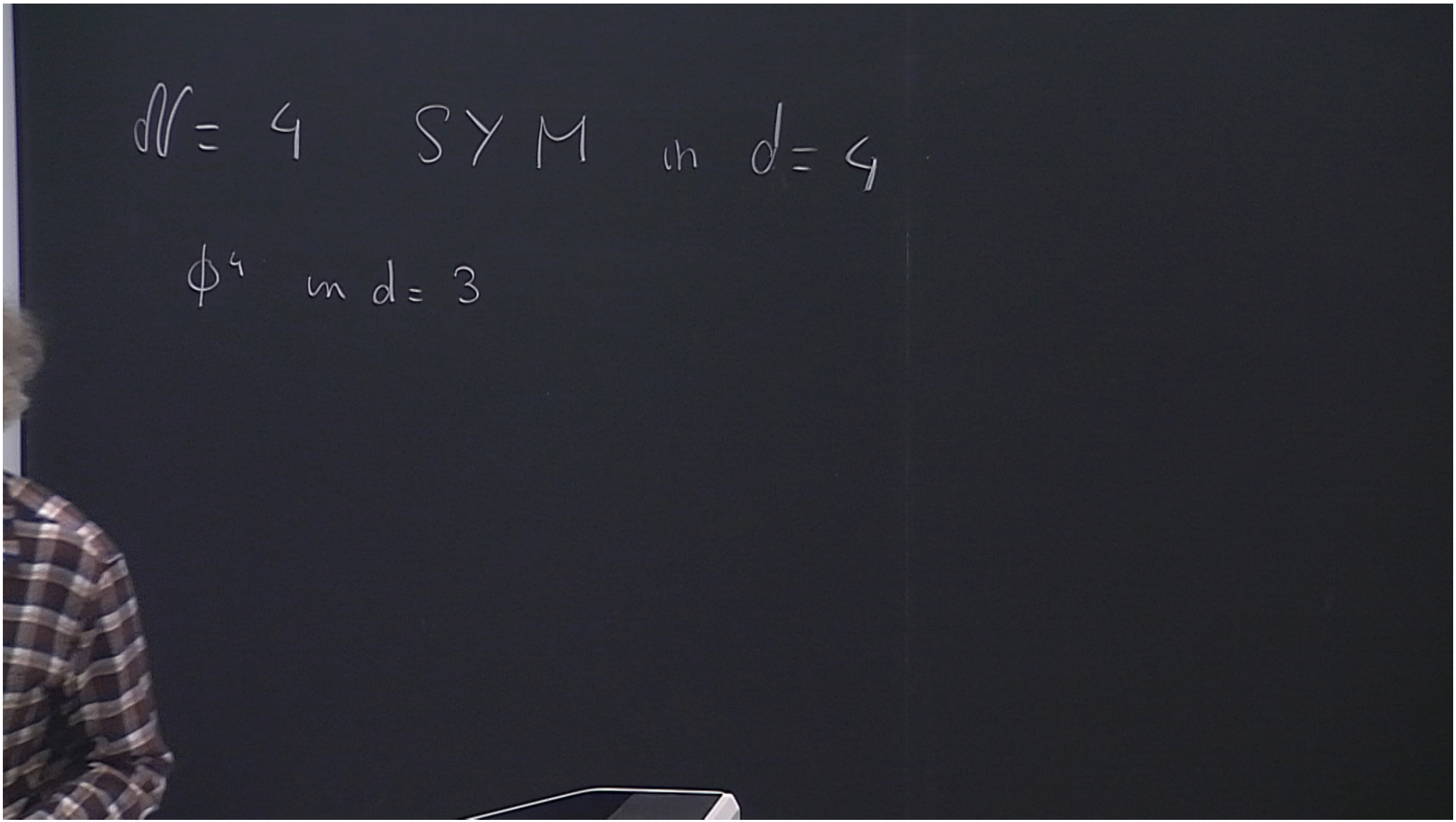
$$\phi^2 \longrightarrow :\phi^2:$$

composite operator it must be renormalized properly

$$\boxed{\partial_\mu J^\mu_{\substack{\text{scale}\\ \text{quantum}}} = \beta(g)\frac{1}{4!}\phi^4_{\text{quantum}}}$$

anomaly


$\mathcal{N} = 4$ SYM in $d = 4$


$\mathcal{N} = 4$ SYM in $d = 4$
ϕ^4 in $d = 3$

$d \neq 4$

$$\partial_\mu J^\mu_{\text{scale}} = (d-4)\frac{g}{4!}\phi^4$$

classically

$$\beta(g_R) = (d-4)\, g_R + O(g_R^2)$$

classical

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anomaly

$\mathcal{N} = 4$ SYM in $d = 4$

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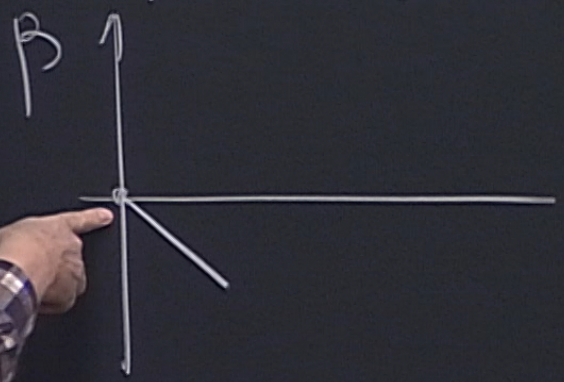
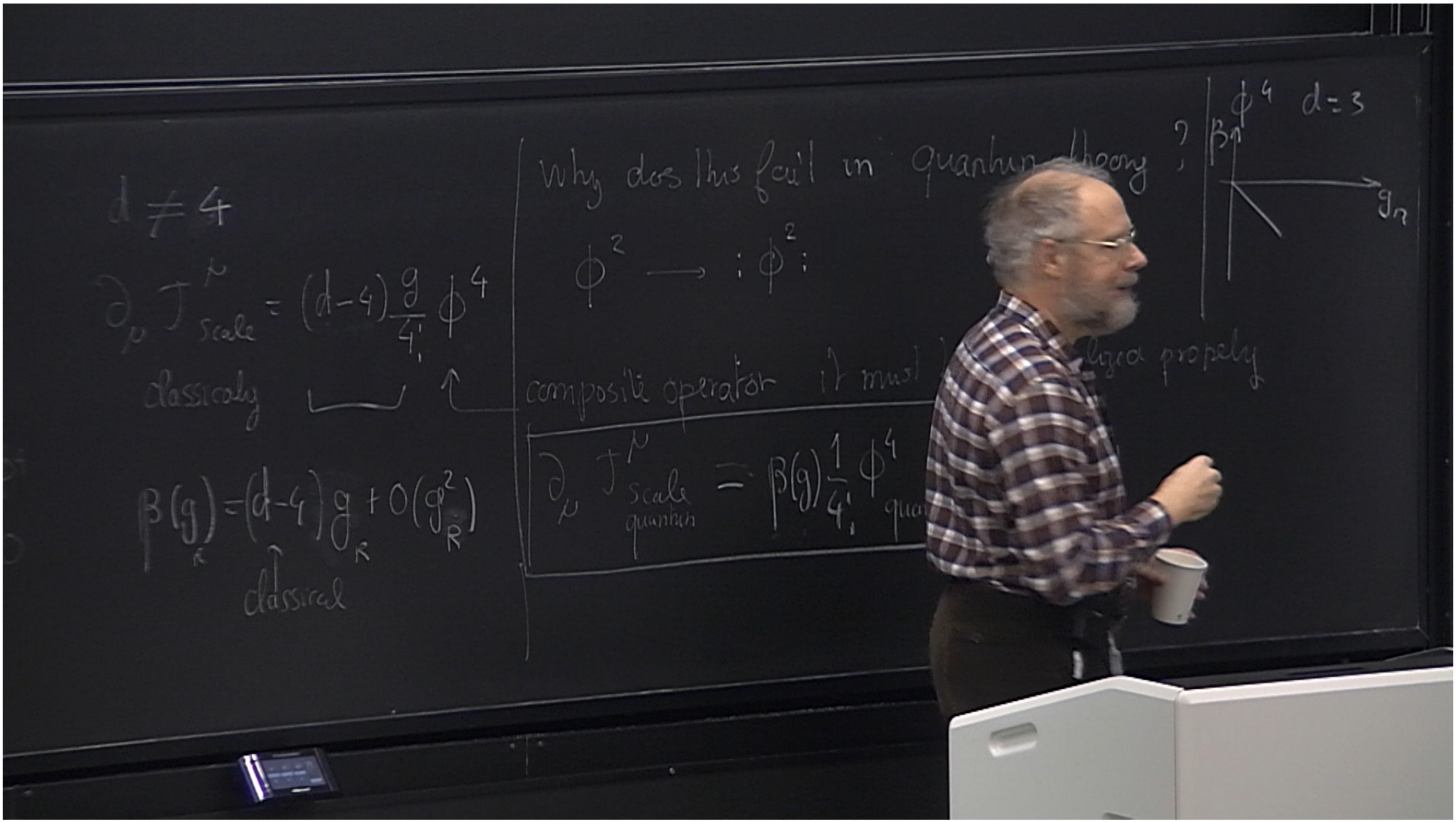
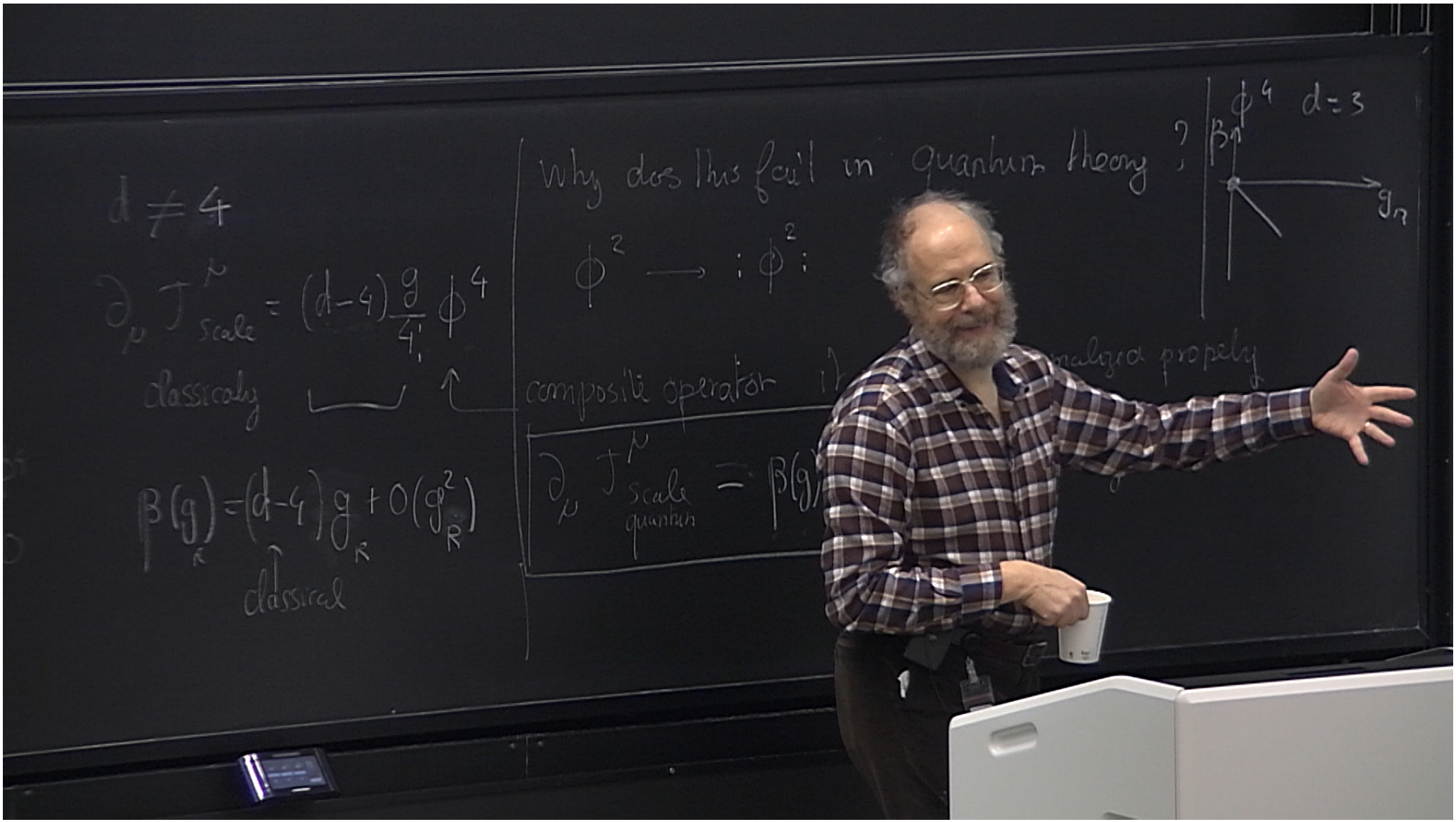



d ≠ 4
∂_μ J^μ_scale = (d−4) g/4! φ^4
classicaly
β(g_R) = (d−4) g_R + O(g_R^2)
classical
Why does this fail in quantum theory ?
φ^2 → :φ^2:
composite operator it must
lyza properly
∂_μ J^μ_scale quantum = β(g) 1/4! φ^4
φ^4 d=3
β
g_R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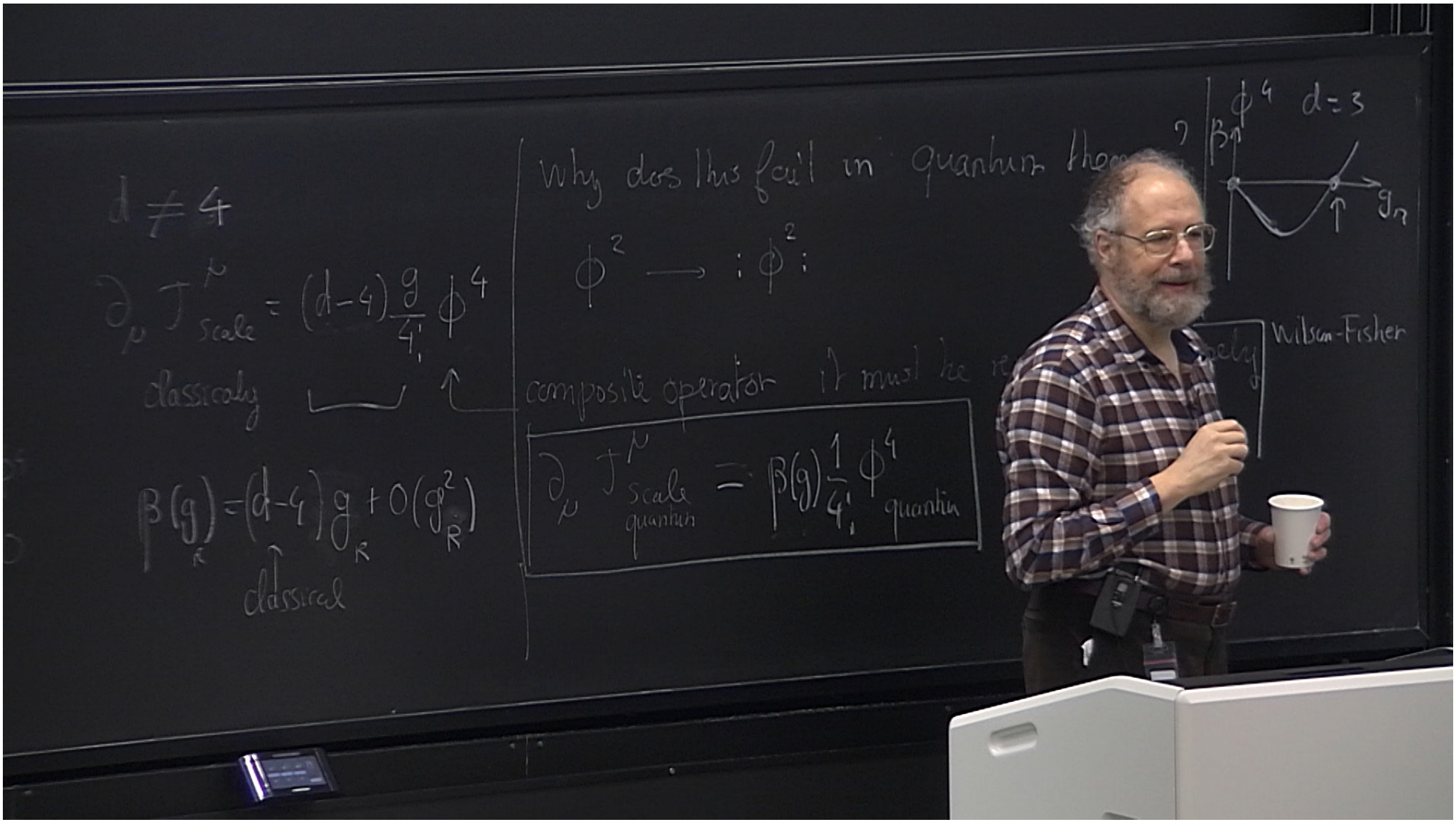
$d \neq 4$
$\partial_\mu J^\mu_{scale} = (d-4)\frac{g}{4!}\phi^4$
classically
$\beta(g_R) = (d-4) g_R + O(g_R^2)$
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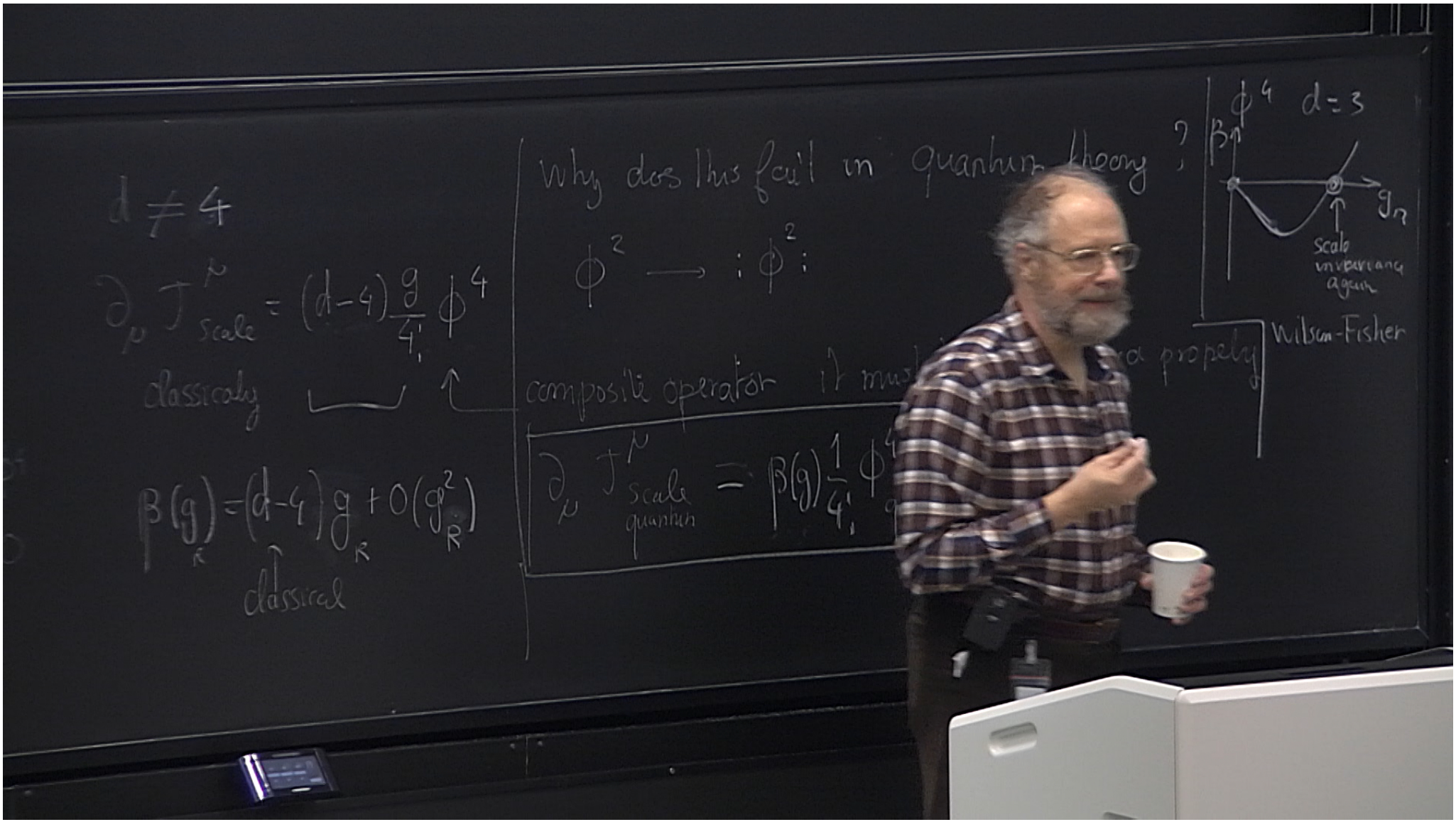
Why does this fail in quantum theory?
$\phi^2 \longrightarrow :\phi^2:$
composite operator it must be re
$\partial_\mu J^\mu_{scale\ quantum} = \beta(g)\frac{1}{4!}\phi^4_{quantum}$
ϕ^4 $d=3$
β
g_R
Wilson-Fisher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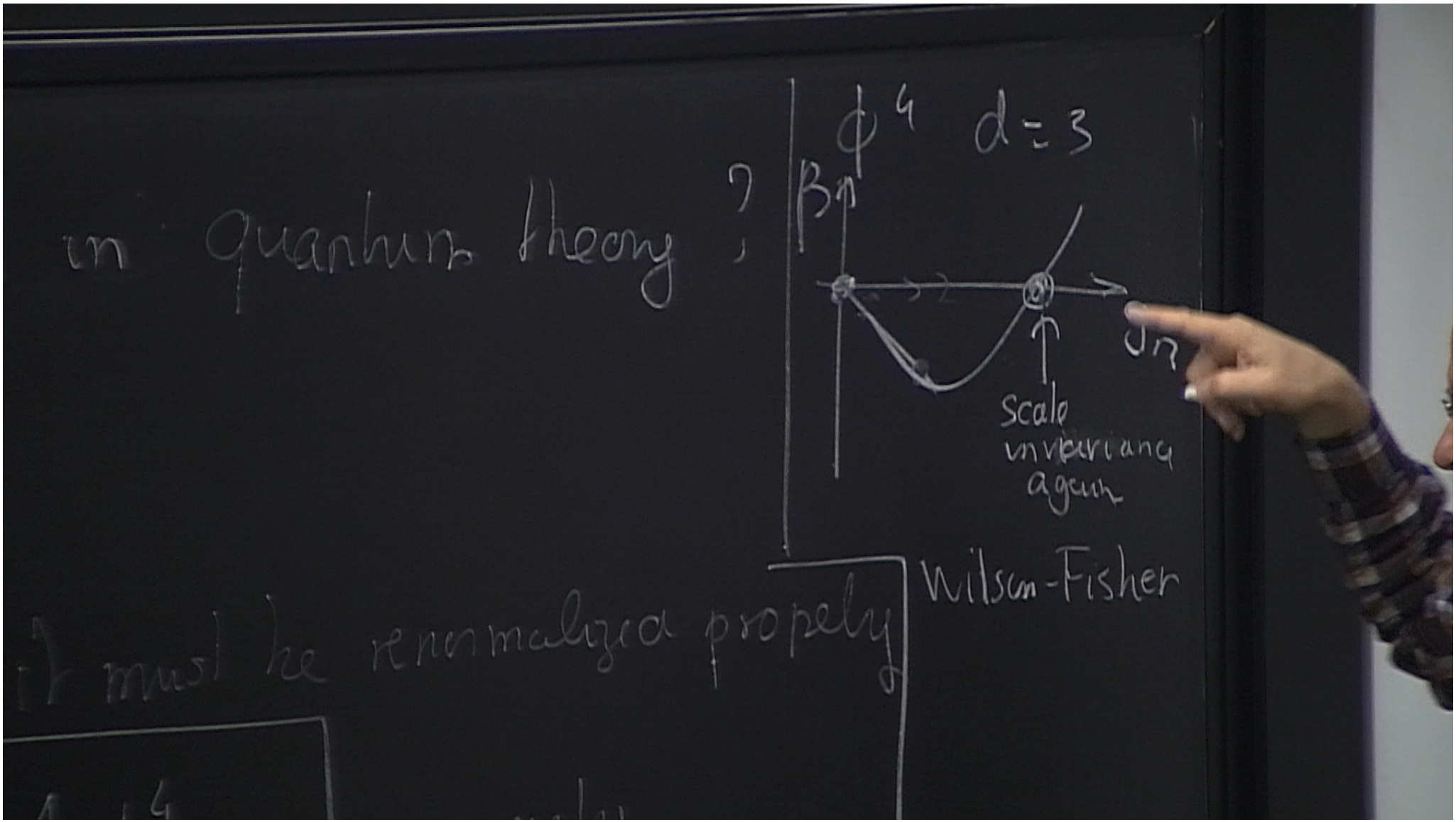
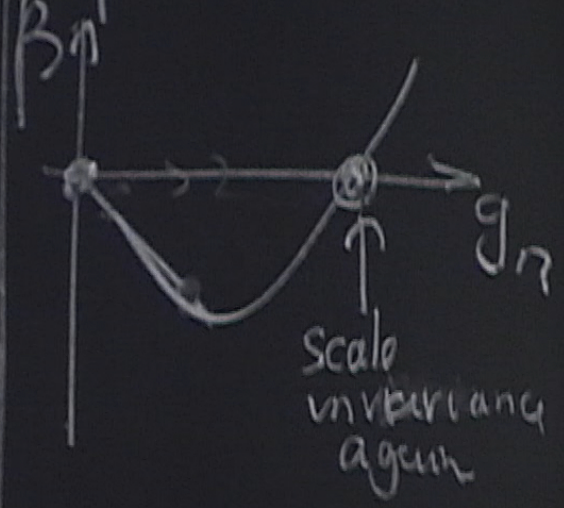
∂_μ J^μ_scale quantum = β(g) 1/4! φ^4
φ^4 d=3
β
g_R
Scale invariance again
Wilson-Fisher


in quantum theory ?
ϕ^4 $d=3$
β
u
Scale invariance again
Wilson-Fisher
it must be renormalized properly

in quantum theory ?

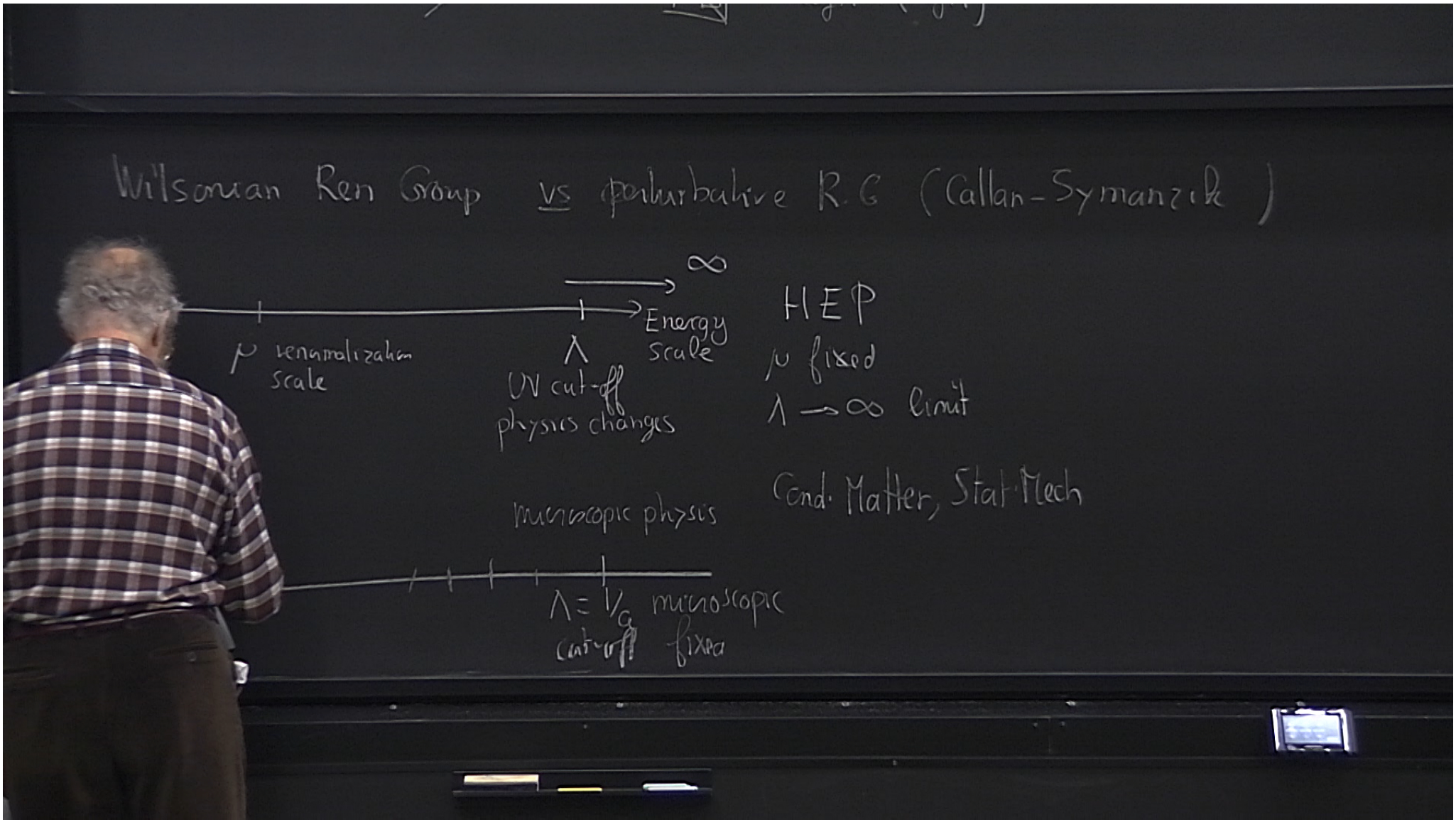
ϕ^4 $d=3$

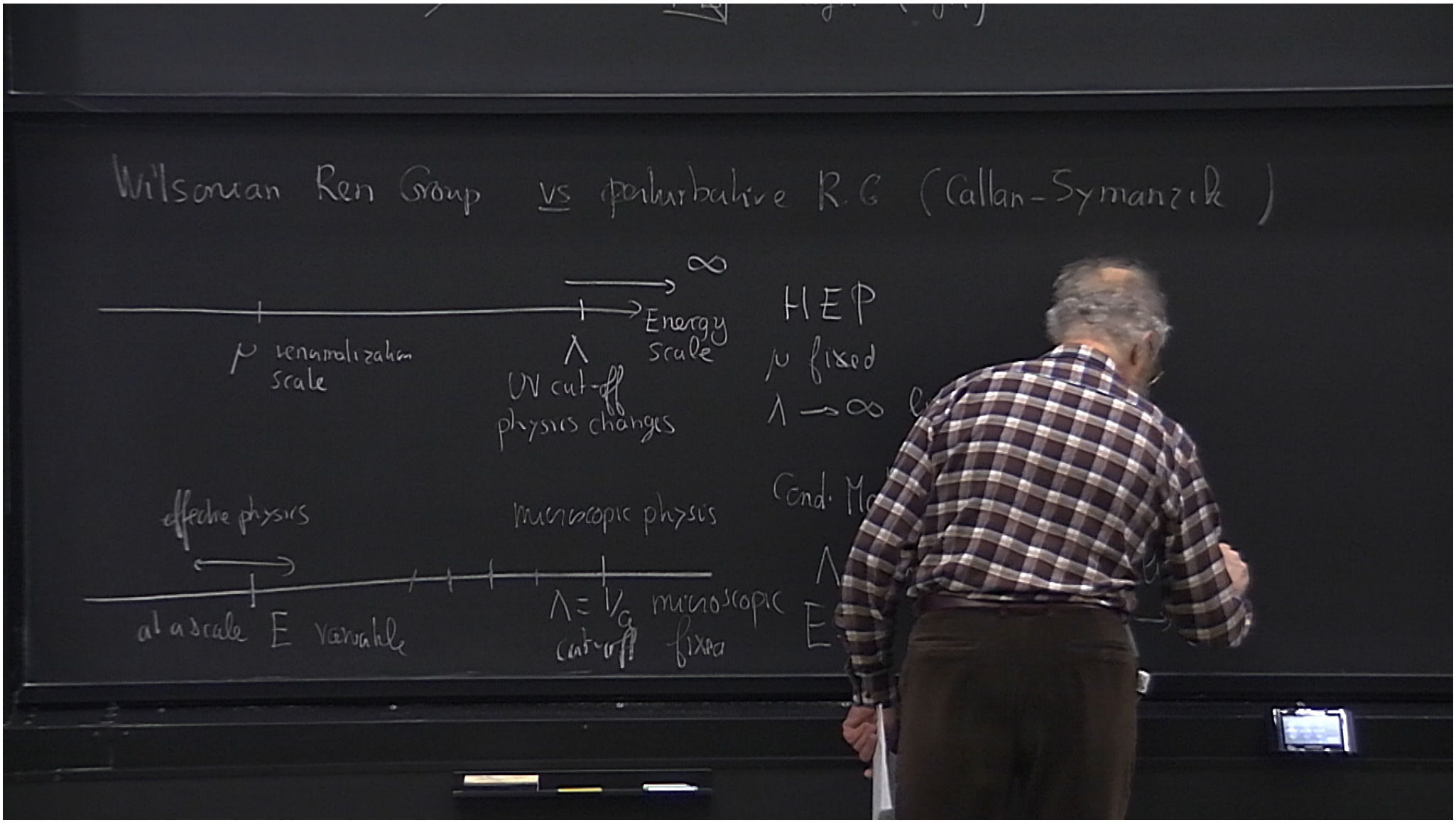


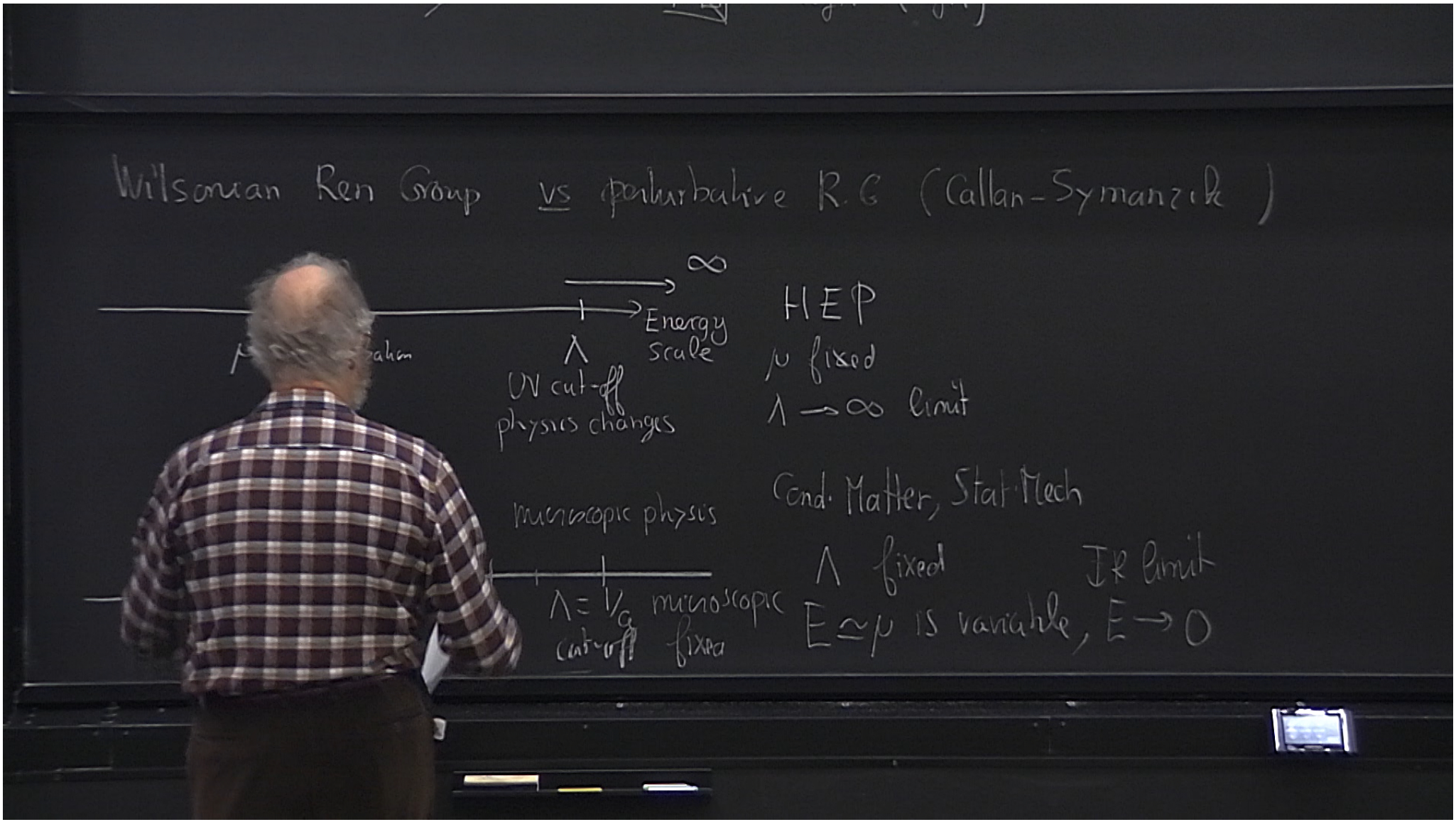
Wilson-Fisher

it must be renormalized properly

Wilsonian Ren Group vs perturbative R.G (Callan-Symanzik)


Wilsonian Ren Group vs perturbative R.G (Callan-Symanzik)
∞
Energy scale
μ renormalization scale
Λ
UV cut-off
physics changes
HEP
μ fixed
Λ → ∞ limit
Cond. Matter, Stat. Mech
microscopic physics
Λ = 1/a
cutoff
microscopic fixed


Wilsonian Ren Group vs perturbative R.G (Callan-Symanzik)
∞
Energy scale
μ renormalization scale
Λ
UV cut-off
physics changes
HEP
μ fixed
Λ → ∞
effective physics
microscopic physics
at a scale E variable
Λ = 1/a
cutoff
microscopic fixed


Wilsonian Ren Group vs perturbative R.G (Callan-Symanzik)
∞
Energy scale
Λ
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physics changes
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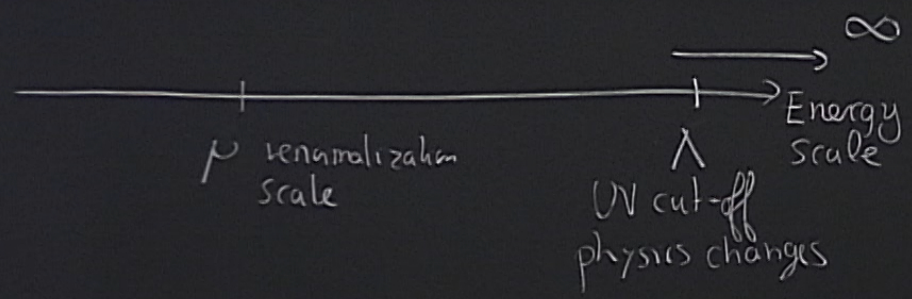
Λ → ∞ limit
microscopic physics
Cond. Matter, Stat. Mech
Λ fixed
IR limit
Λ = 1/a
cut-off
microscopic fixed
E ≃ μ is variable, E → 0

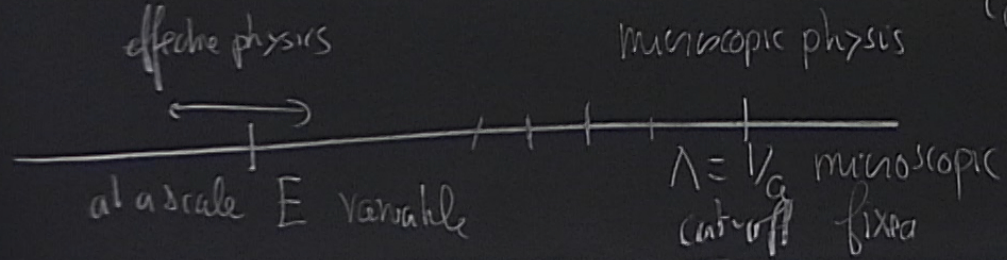
Wilsonian Ren Group vs perturbative R.G (Callan-Symanzik)



HEP

μ fixed

$\Lambda \to \infty$ limit

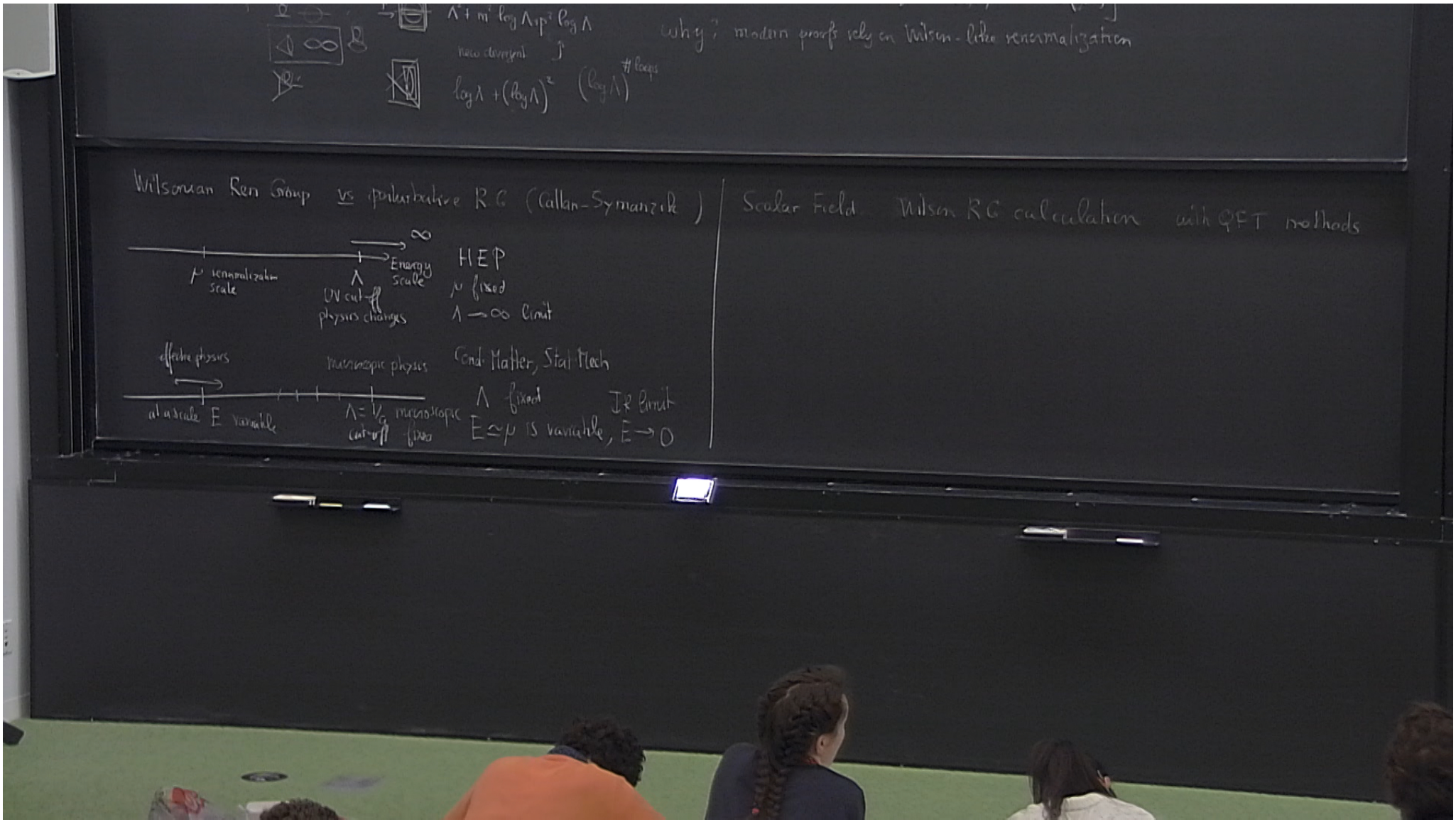


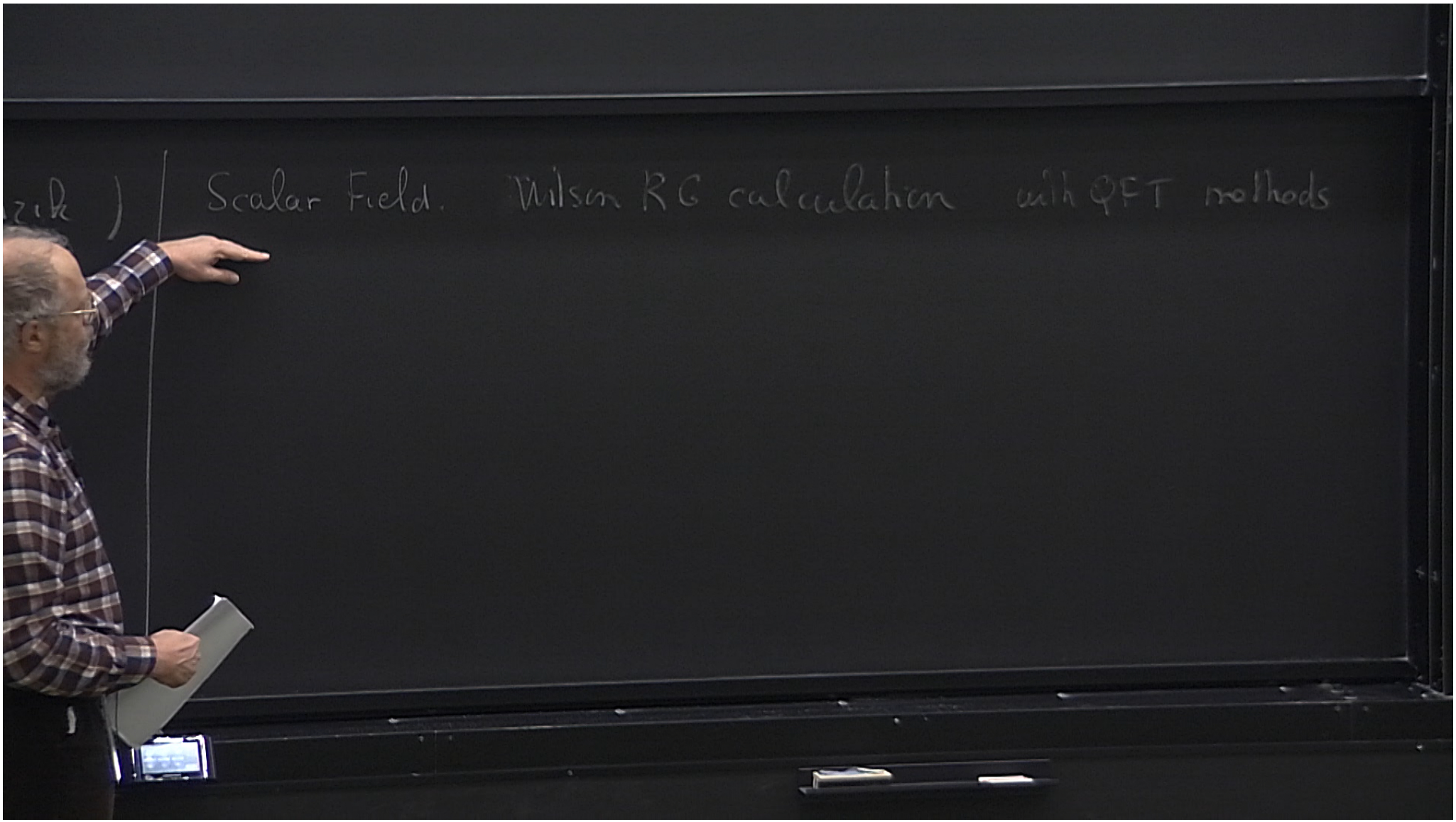
Cond. Matter, Stat. Mech

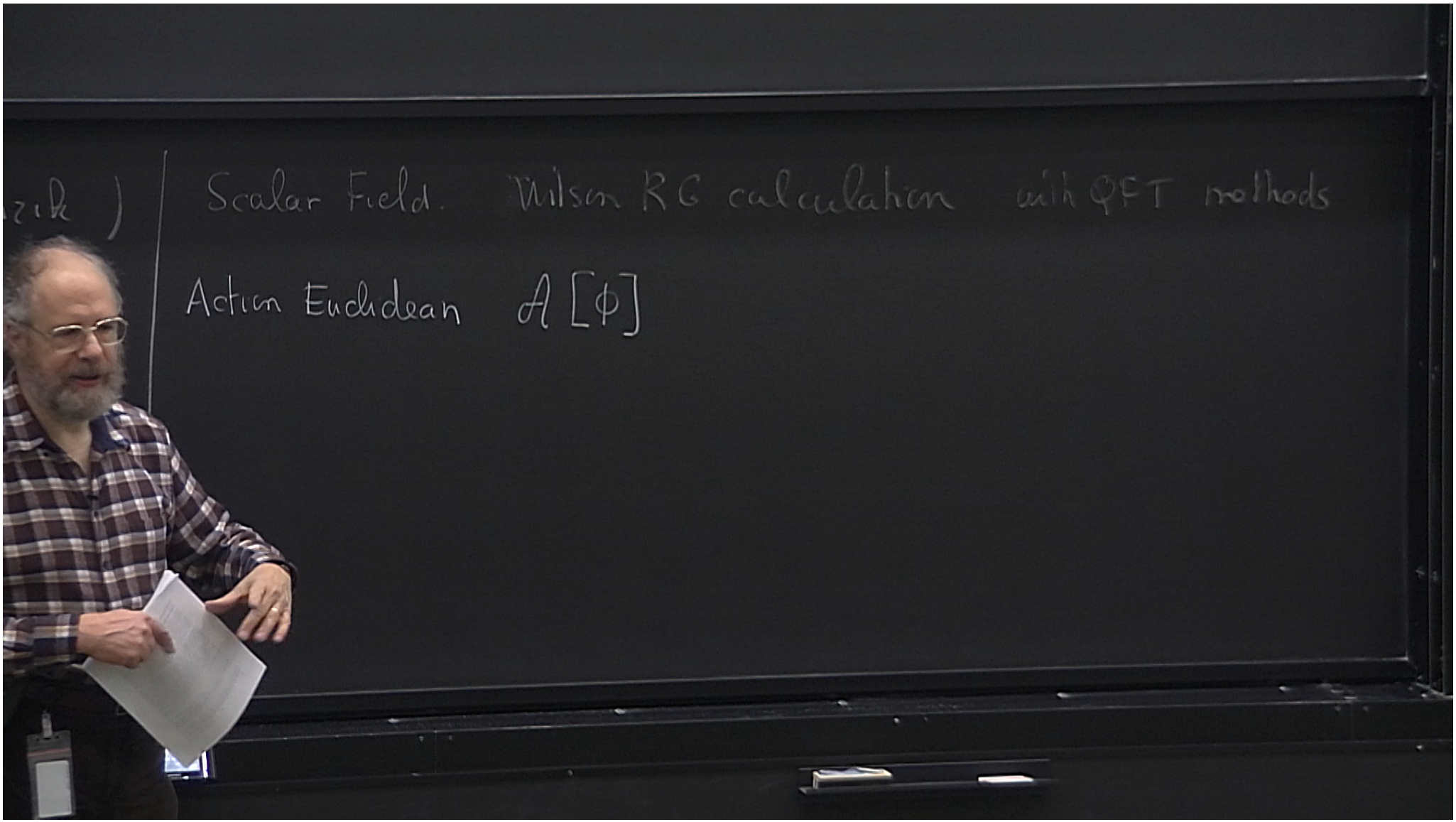
Λ fixed

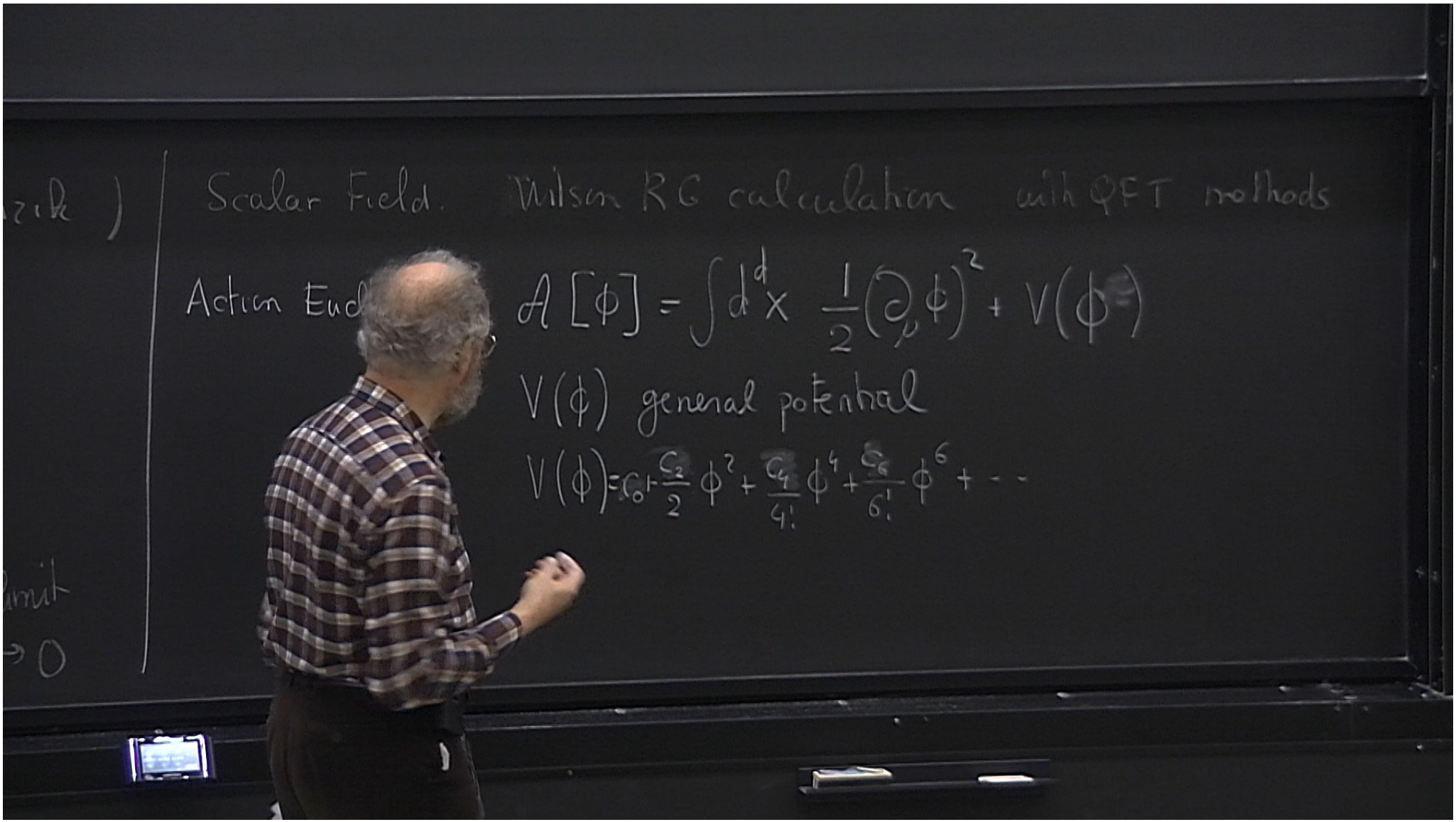
IR limit

$E \simeq \mu$ is variable, $E \to 0$


Λ + m² log Λ + p² log Λ
new divergent
log Λ + (log Λ)² (log Λ)^# loops
why? modern proofs rely on Wilson-like renormalization
Wilsonian Ren Group vs perturbative R.G. (Callan-Symanzik)
∞
μ renormalization scale
Λ
UV cut-off
physics changes
Energy scale
HEP
μ fixed
Λ → ∞ limit
effective physics
macroscopic physics
Cond. Matter, Stat. Mech
at a scale E variable
Λ = 1/a macroscopic cut-off fixed
Λ fixed
IR limit
E ≃ μ is variable, E → 0
Scalar Field Wilson RG calculation with QFT methods


Scalar Field. Wilson RG calculation with QFT methods


Scalar Field. Wilson RG calculation with QFT methods
Action Euclidean $\mathcal{A}[\phi]$


Scalar Field. Wilson RG calculation with QFT methods
Action Euc
$\mathcal{A}[\phi] = \int d^d x \, \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$
$V(\phi)$ general potential
$V(\phi) = c_0 + \frac{c_2}{2}\phi^2 + \frac{c_4}{4!}\phi^4 + \frac{c_6}{6!}\phi^6 + \cdots$

Scalar Field. Wilson RG calculation with QFT methods

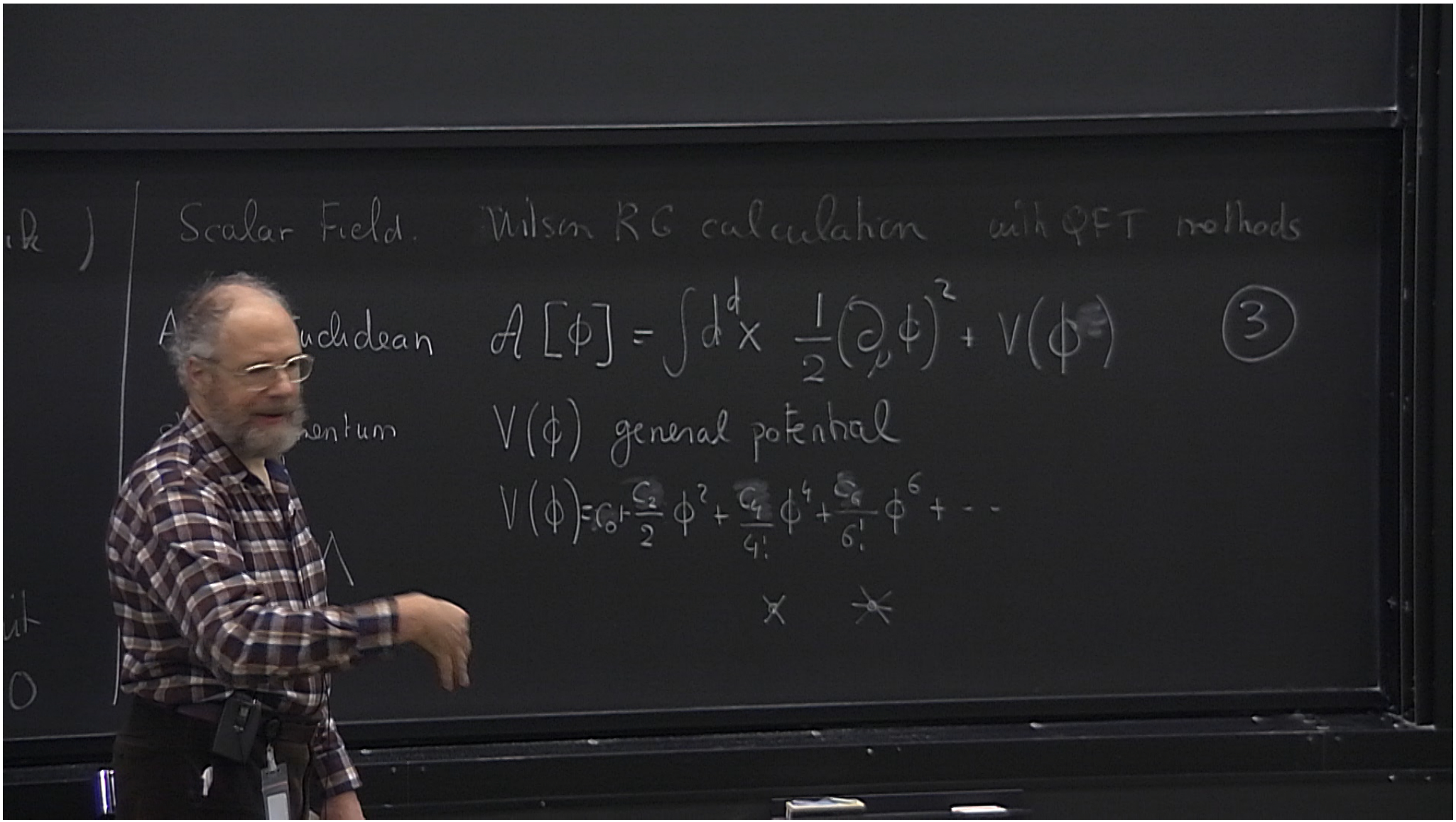
Action Euclidean $\mathcal{A}[\phi] = \int d^d x \, \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$

sharp momentum cut-off

$|k| < \Lambda$

$V(\phi)$ general potential

$V(\phi) = c_0 + \frac{c_2}{2}\phi^2 + \frac{c_4}{4!}\phi^4 + \frac{c_6}{6!}\phi^6 + \cdots$


Scalar Field. Wilson RG calculation with QFT methods
A[φ] = ∫ d^d x (1/2)(∂_μ φ)^2 + V(φ)
3
V(φ) general potential
V(φ) = c_0 + (c_2/2) φ^2 + (c_4/4!) φ^4 + (c_6/6!) φ^6 + ...

Scalar Field. Wilson RG calculation with QFT methods

Action Euclidean $\mathcal{A}[\phi] = \int d^d x \, \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$ (3)

sharp momentum cut-off

$|k| < \Lambda$

$V(\phi)$ general potential

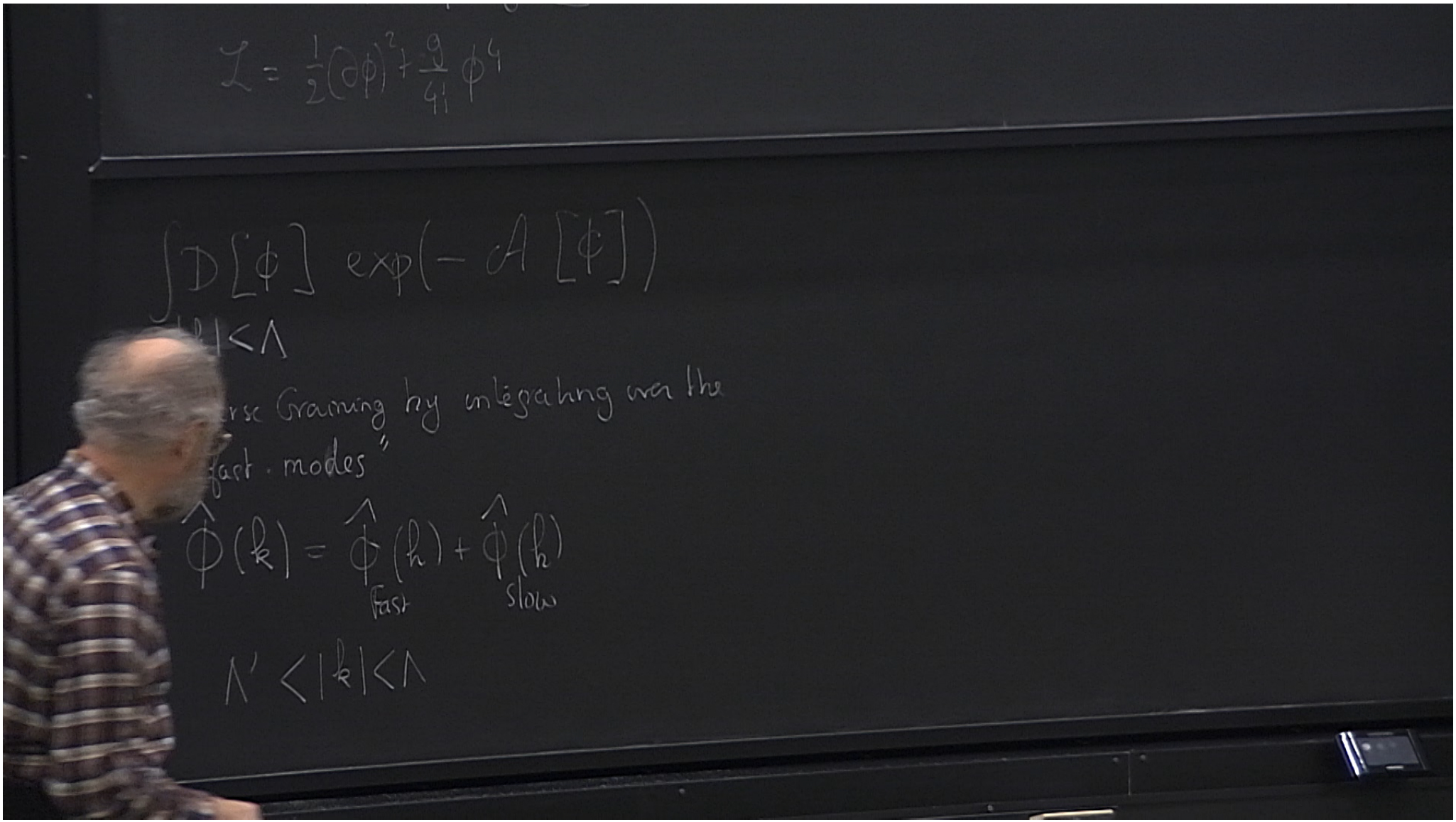
$V(\phi) = c_0 + \frac{c_2}{2}\phi^2 + \frac{c_4}{4!}\phi^4 + \frac{c_6}{6!}\phi^6 + \cdots$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$$

$$\int_{|k|<\Lambda} \mathcal{D}[\phi] \exp(-\mathcal{A}[\phi])$$

- Coarse Graining by integrating over the "fast modes"

$$\hat{\phi}(k)$$


$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$
$\int D[\phi] \exp(-\mathcal{A}[\phi])$
$|k| < \Lambda$
rse Graining by integrating over the
fast modes"
$\hat{\phi}(k) = \hat{\phi}(k) + \hat{\phi}(k)$
Fast
Slow
$\Lambda' < |k| < \Lambda$

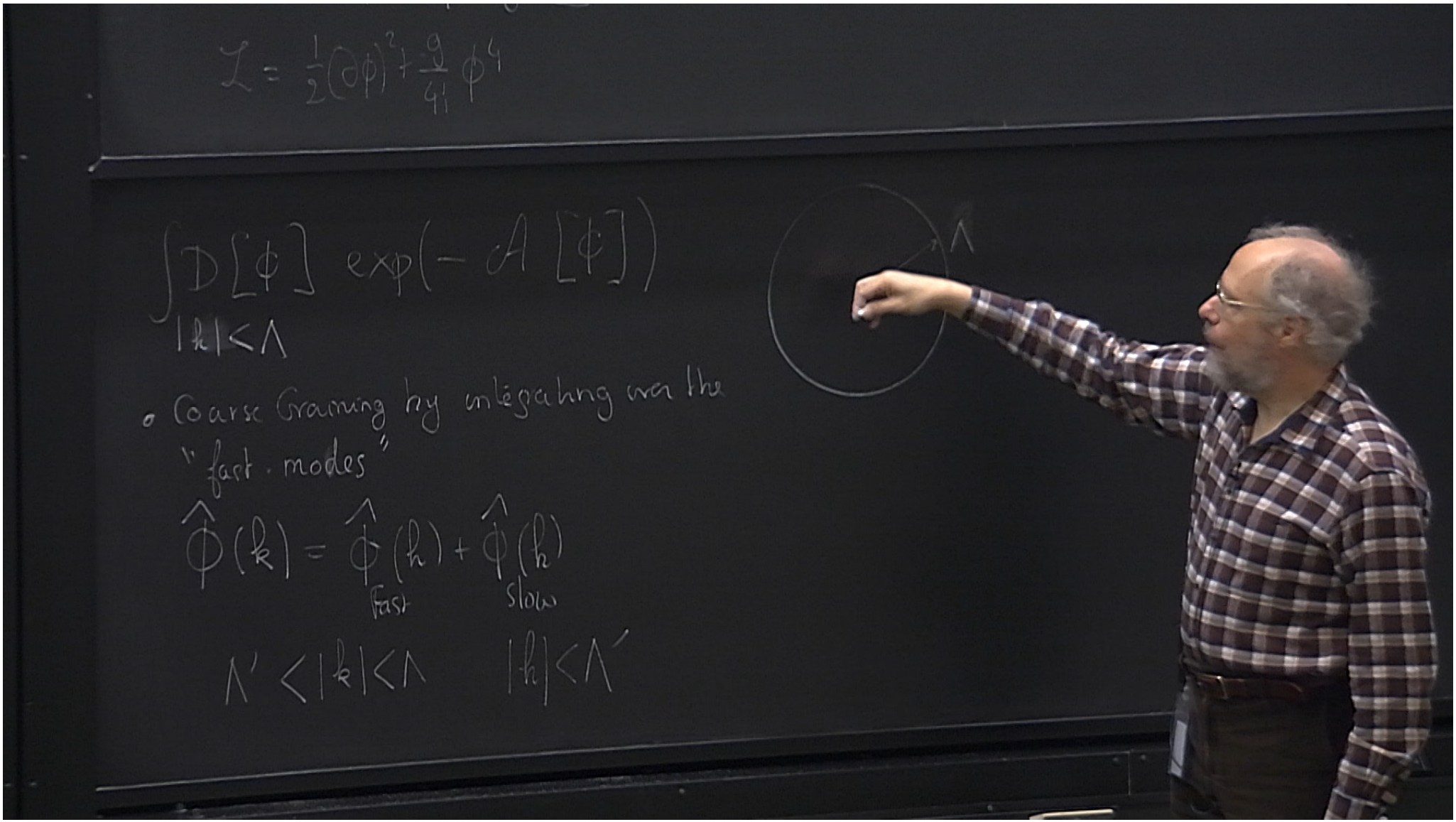
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$$

$$\int_{|k|<\Lambda} D[\phi] \exp(-\mathcal{A}[\phi])$$

- Coarse Graining by integrating over the "fast modes"

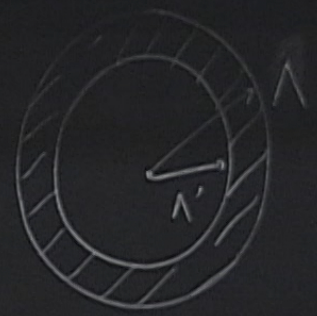
$$\hat{\phi}(k) = \underset{\text{Fast}}{\hat{\phi}(k)} + \underset{\text{Slow}}{\hat{\phi}(k)}$$

$$\Lambda' < |k| < \Lambda \qquad |k| < \Lambda$$


$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$
$\int_{|k|<\Lambda} D[\phi] \exp(-\mathcal{A}[\phi])$
Λ
Coarse Graining by integrating out the "fast modes"
$\hat{\phi}(k) = \hat{\phi}(k)_{\text{Fast}} + \hat{\phi}(k)_{\text{Slow}}$
$\Lambda' < |k| < \Lambda$
$|k| < \Lambda'$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$$

$$\int_{|k|<\Lambda} \mathcal{D}[\phi] \exp(-\mathcal{A}[\phi])$$



- Coarse Graining by integrating out the "fast modes"

$$\hat{\phi}(k) = \underbrace{\hat{\phi}(k)}_{\text{Fast}} + \underbrace{\hat{\phi}(k)}_{\text{Slow}}$$

$$\Lambda' < |k| < \Lambda \qquad |k| < \Lambda'$$

$$\int_{\Lambda' < |k| < \Lambda} D[\phi_{fast}] \exp(-\mathcal{A}[\phi]) = \exp(-\mathcal{A}'[\phi_{slow}])$$

$\mathcal{A}'[\phi_{slow}]$: effective action for the slow modes

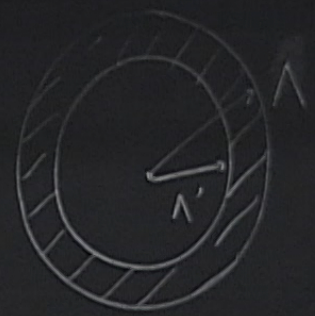
$$\int_{\Lambda' < |k| < \Lambda} D[\phi_{fast}] \exp(-\mathcal{A}[\phi]) = \exp(-\mathcal{A}'[\phi_{slow}])$$

$\mathcal{A}'[\phi_{slow}]$: effective action for the slow modes

$\phi_{slow} \quad 0 < |k| < \Lambda'$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$$

$$\int_{|k|<\Lambda} D[\phi] \exp(-\mathcal{A}[\phi])$$



$$\int_{\Lambda'<|k|<\Lambda} D[\phi_{fast}] \exp(-\mathcal{A}[\phi]$$

1. Coarse Graining by integrating over the "fast modes"

$$\hat{\phi}(k) = \hat{\phi}_{Fast}(k) + \hat{\phi}_{slow}(k)$$

$$\Lambda' < |k| < \Lambda \qquad |k| < \Lambda'$$

step 2: rescale momenta and field

$$k = k'\frac{\Lambda'}{\Lambda} \qquad \phi_{slow} = \left(\frac{\Lambda'}{\Lambda}\right.$$

$$\int_{\Lambda'<|k|<\Lambda} D[\phi_{fast}] \exp(-\mathcal{A}[\phi]) = \exp(-\mathcal{A}_{eff}[\phi_{slow}])$$

$\mathcal{A}_{eff}[\phi_{slow}]$: effective action for the slow modes

ϕ_{slow} $0<|k|<\Lambda'$

step ②: rescale momenta and field

$$k = \frac{k'\Lambda'}{\Lambda} \qquad \phi_{slow} \overset{?}{=} \left(\frac{\Lambda'}{\Lambda}\right)\phi'$$

$$= \exp(-\mathcal{A}$$

$$\int_{\Lambda' < |k| < \Lambda} D[\phi_{fast}] \exp(-\mathcal{A}[\phi]) = \exp(-\mathcal{A}_{eff}[\phi_{slow}])$$

$\mathcal{A}_{eff}[\phi_{slow}]$: effective action for the slow modes

ϕ_{slow} $0 < |k| < \Lambda'$

step ②: rescale momenta and field

$$k = k' \frac{\Lambda'}{\Lambda} \qquad \phi_{slow} \overset{?}{=} \left(\frac{\Lambda'}{\Lambda}\right) \phi'$$

renormalized fields

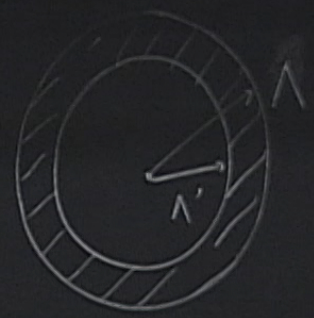
$$= \exp(-\mathcal{A}'[\phi'])$$

$\mathcal{A}'[\phi']$ renormalized action after a coarse graining step

ϕ' $0 < |k'| < \Lambda$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4$$

$$\int_{|k|<\Lambda} D[\phi] \exp(-\mathcal{A}[\phi])$$



$$\int_{\Lambda'<|k|<\Lambda} D[\phi_{fast}] \exp(-\mathcal{A}[\phi]) =$$

Coarse Graining by integrating over the "fast modes"

$$\hat{\phi}(k) = \hat{\phi}_{fast}(k) + \hat{\phi}_{slow}(k)$$

$\Lambda' < |k| < \Lambda$ $\quad$ $|k| < \Lambda'$

step ②: rescale momenta and field

$$k = k'\frac{\Lambda'}{\Lambda} \qquad \phi_{slow} = \left(\frac{\Lambda'}{\Lambda}\right)^{?}\phi'$$

renormalized field

step ③ Iterate

$\Lambda' < |k| < \Lambda \qquad |h| < \Lambda'$

step ③ Iterate

renormalized

Local Potential Approximation $\leftarrow$ Renormalized effective potential method (

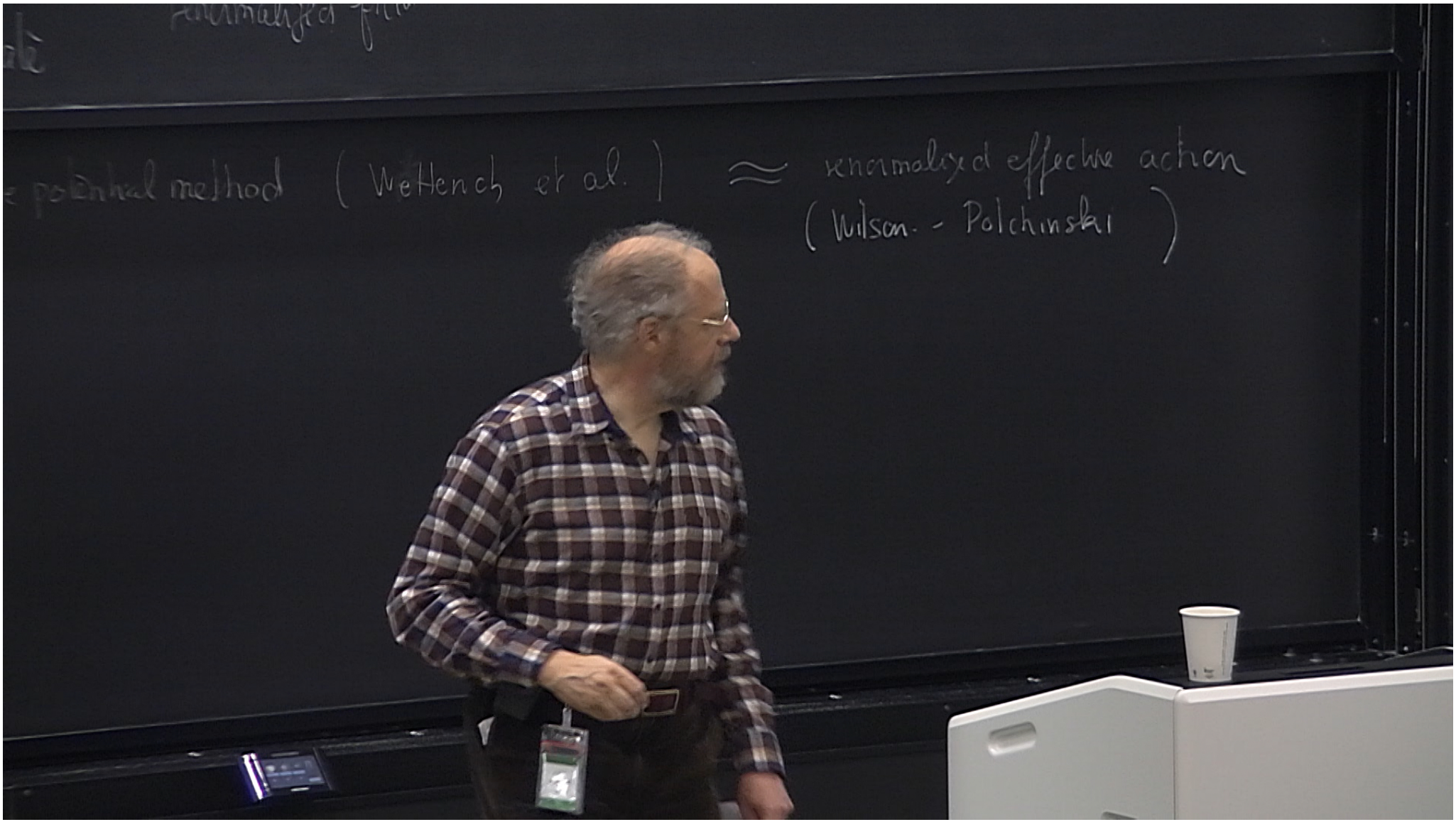
$\varphi(k)$ Fast + $\varphi(k')$ Slow

$\Lambda' < |k| < \Lambda$ $\quad |k'| < \Lambda'$

step ③ Iterate

Λ $\quad \varphi(\Lambda)$ renormalized fields

Local Potential Approximation $\leftarrow$ Renormalized effective potential method (Wetterich et al.)


potential method (Wetterich et al.) ≈ renormalized effective action
(Wilson - Polchinski)

$\Lambda' < |k| < \Lambda \qquad |k| < \Lambda'$

step ③ Iterate

Renormalized f

Local Potential Approximation $\leftarrow$ Renormalized effective potential method (V

$\mathcal{A}[\phi] \longrightarrow$ 1 loop effective action $\Gamma[\varphi]$

$\Lambda' < |k| < \Lambda \qquad |h| < \Lambda'$

step ③ Iterate

renormalized fields

Local Potential Approximation $\leftarrow$ Renormalized effective potential method (Wetter

$\mathcal{A}[\phi] \longrightarrow$ 1 loop effective action $\Gamma[\varphi] = \mathcal{A}[\varphi] + \frac{1}{2} \mathrm{Tr}\left(\mathrm{Log}\left[\mathcal{A}''[\varphi]\right]\right)$

$\Lambda' < |k| < \Lambda \qquad |k| < \Lambda'$

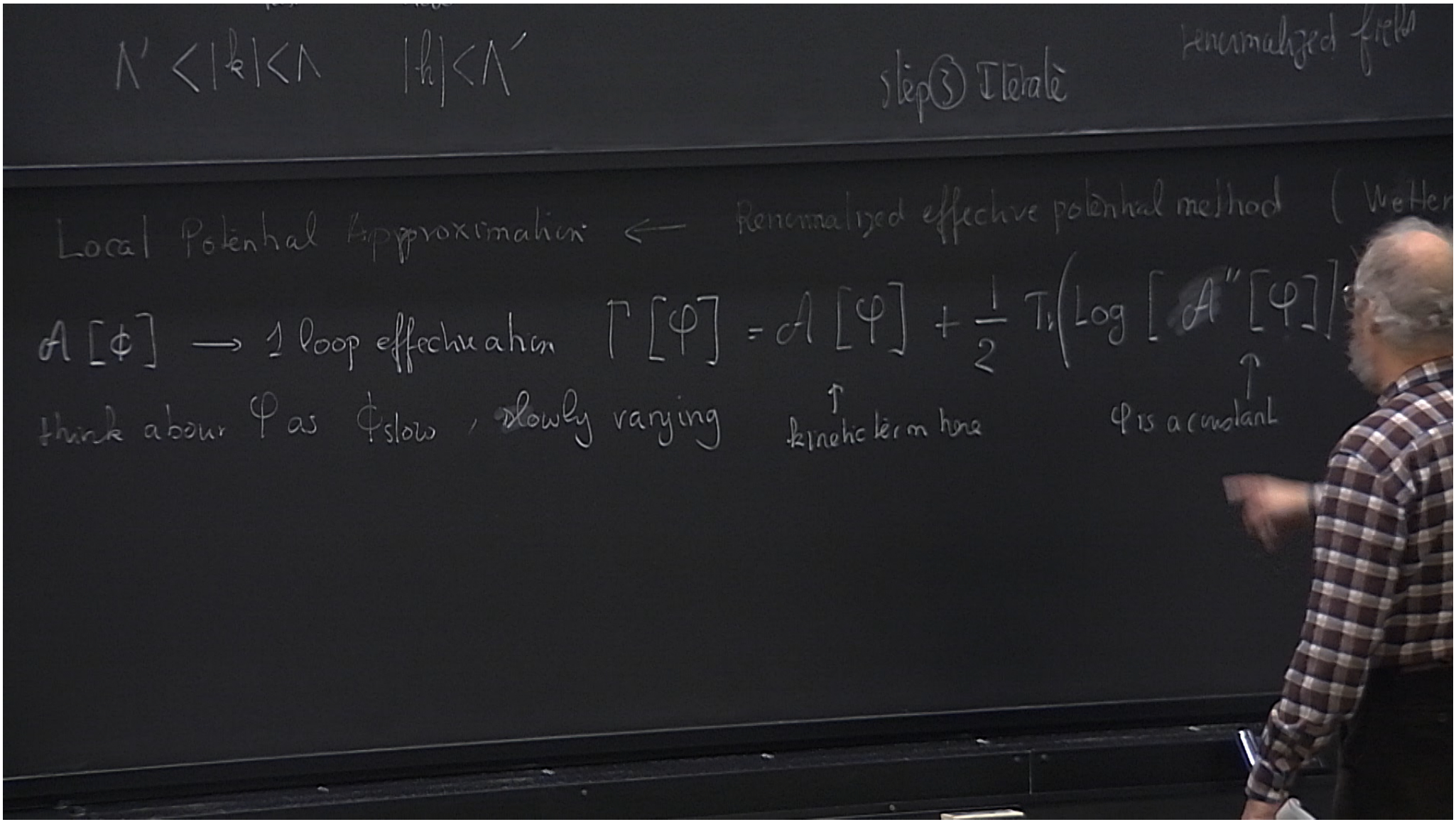
step ③ Iterate

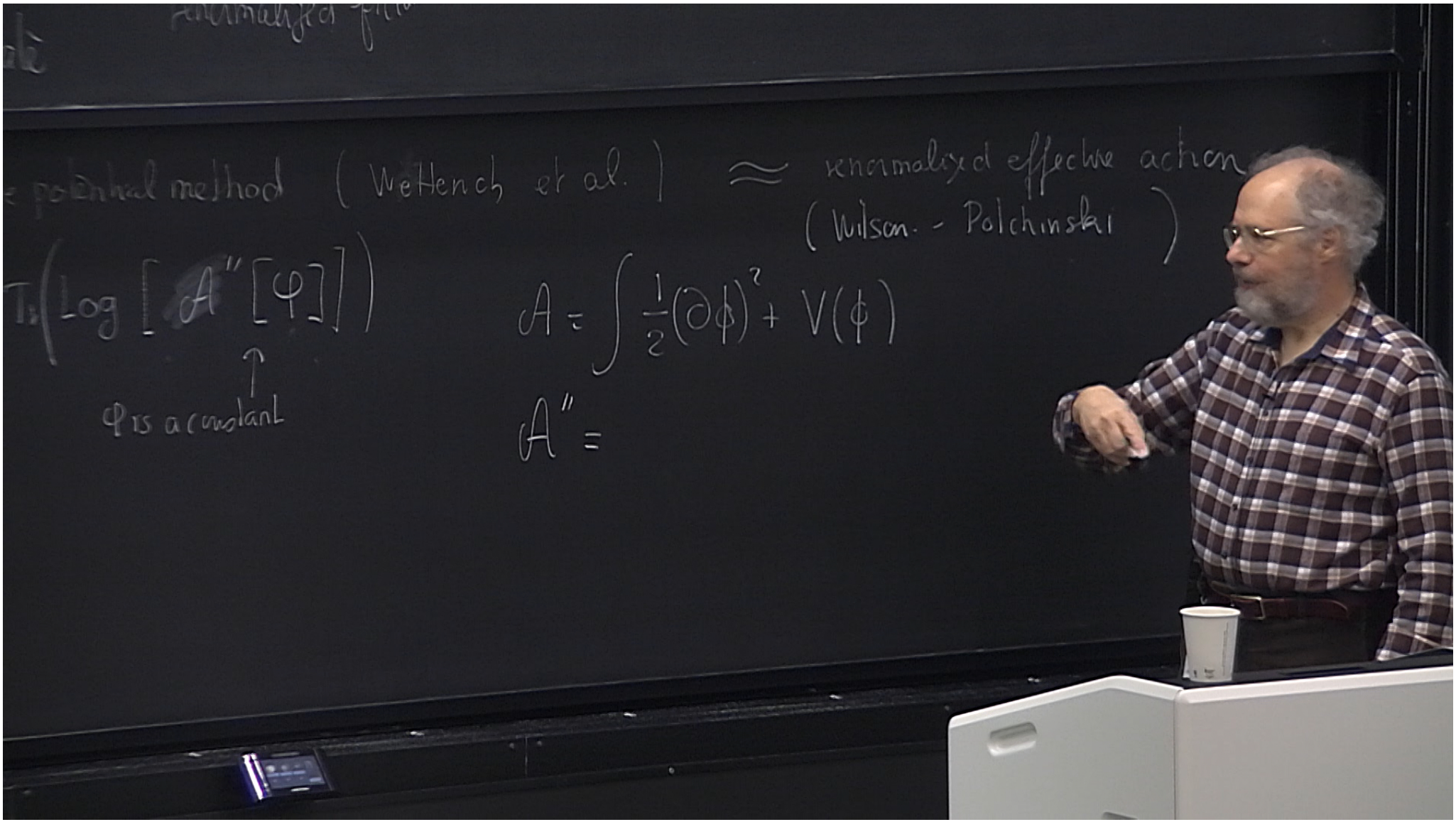
renormalized fields

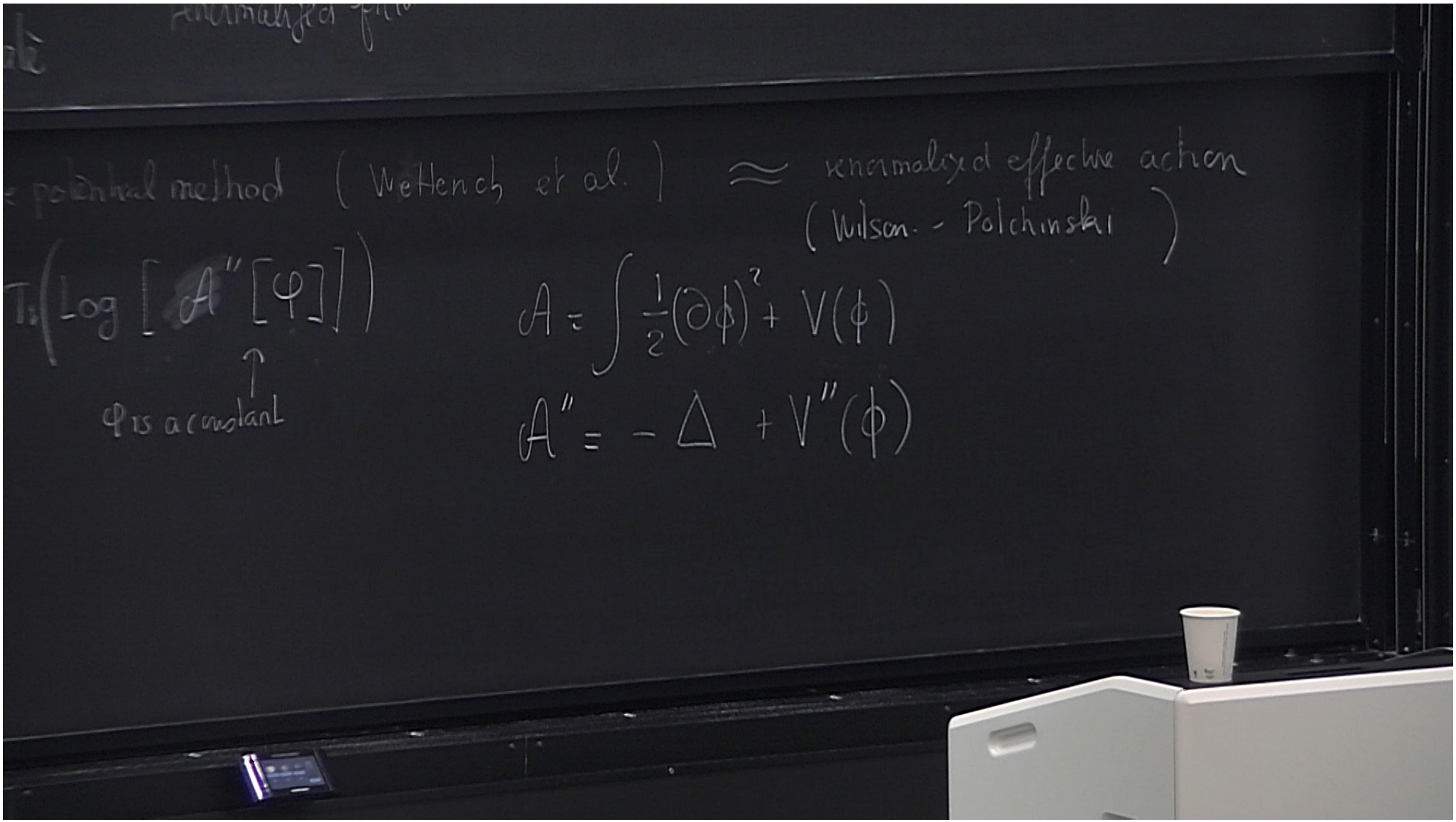
Local Potential Approximation $\leftarrow$ Renormalized effective potential method (Wetter

$\mathcal{A}[\phi] \longrightarrow$ 1 loop effective action $\Gamma[\varphi] = \mathcal{A}[\varphi] + \frac{1}{2} \mathrm{Tr}\left(\mathrm{Log}\left[\mathcal{A}''[\varphi]\right]\right)$

think about φ as ϕ_{slow}, slowly varying


$\Lambda' < |k| < \Lambda$
$|k| < \Lambda'$
step ③ Iterate
renormalized fields
Local Potential Approximation ← Renormalized effective potential method (Wetter
$\mathcal{A}[\phi] \longrightarrow$ 1 loop effective action
$\Gamma[\varphi] = \mathcal{A}[\varphi] + \frac{1}{2} \mathrm{Tr}\left(\mathrm{Log}\left[\mathcal{A}''[\varphi]\right]\right)$
think about φ as ϕ_{slow}, slowly varying
kinetic term here
φ is a constant


potential method (Wetterich et al.) ≈ renormalized effective action
(Wilson - Polchinski)
Tr(Log[A''[φ]])
φ is a constant
A = ∫ ½(∂φ)² + V(φ)
A'' =


potential method (Wetterich et al.) ≈ renormalized effective action
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Tr(Log[A''[φ]])
φ is a constant
A = ∫ ½(∂φ)² + V(φ)
A'' = − Δ + V''(φ)

Scalar Field. Wilson RG calculation with QFT methods

Action Euclidean $\mathcal{A}[\phi] = \int d^d x \, \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$ (3)

sharp momentum cut-off

$|k| < \Lambda$

$V(\phi)$ general potential

$$V(\phi) = c_0 + \frac{c_2}{2}\phi^2 + \frac{c_4}{4!}\phi^4 + \frac{c_6}{6!}\phi^6 + \cdots$$

$$V''(\phi) = c_2 + \frac{c_4}{2}\phi^2 + \frac{c_6}{4!}\phi^4 + \cdots$$

potential method (Wetterich et al.) $\approx$ renormalized effective action (Wilson - Polchinski)

$$\mathrm{Tr}\left(\mathrm{Log}\left[\mathcal{A}''[\varphi]\right]\right)$$

φ is a constant

$$\mathcal{A} = \int \frac{1}{2}(\partial\phi)^2 + V(\phi)$$

multiplicative by a function

$$\mathcal{A}'' = -\Delta + V''(\phi)$$

Laplace diff. operator

$\Lambda' < |k| < \Lambda \qquad |k| < \Lambda'$

step ③ Iterate

renormalized f

Local Potential Approximation $\leftarrow$ Renormalized effective potential method (W

$\mathcal{A}[\phi] \longrightarrow$ 1 loop effective action $\Gamma[\varphi] = \mathcal{A}[\varphi] + \frac{1}{2} \mathrm{Tr}\left(\mathrm{Log}\left[\mathcal{A}''[\varphi]\right]\right)$

think about φ as ϕ_{slow}, slowly varying

kinetic term here

φ is a constant

in this approximation

$\mathrm{Tr}\,\mathrm{Log}\left(-\Delta + V''(\phi)\right)$

$\Lambda' < |k| < \Lambda \qquad |k| < \Lambda'$

step ③ Iterate

renormalized f

Local Potential Approximation $\leftarrow$ Renormalized effective potential method (V

$\mathcal{A}[\phi] \longrightarrow$ 1 loop effective action $\Gamma[\varphi] = \mathcal{A}[\varphi] + \frac{1}{2} \mathrm{Tr}\left(\mathrm{Log}\left[\mathcal{A}''[\varphi]\right]\right)$

think about φ as ϕ_{slow}, slowly varying

kinetic term here

φ is a constant

in this approximation

$$\mathrm{Tr}\,\mathrm{Log}\left(-\Delta + V''(\phi)\right) = \int d^d x \langle x | \log(-\Delta + V''(\phi)) | x \rangle = \int d^d x \int_{|k|<\Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi)\right)$$

$\Lambda' < |k| < \Lambda \qquad |k| < \Lambda'$

step ③ Iterate

renormalized f

Local Potential Approximation $\leftarrow$ Renormalized effective potential method (V

$\mathcal{A}[\phi] \longrightarrow$ 1 loop effective action $\Gamma[\varphi] = \mathcal{A}[\varphi] + \frac{1}{2} \mathrm{Tr}\left(\mathrm{Log}\left[\mathcal{A}''[\varphi]\right]\right)$

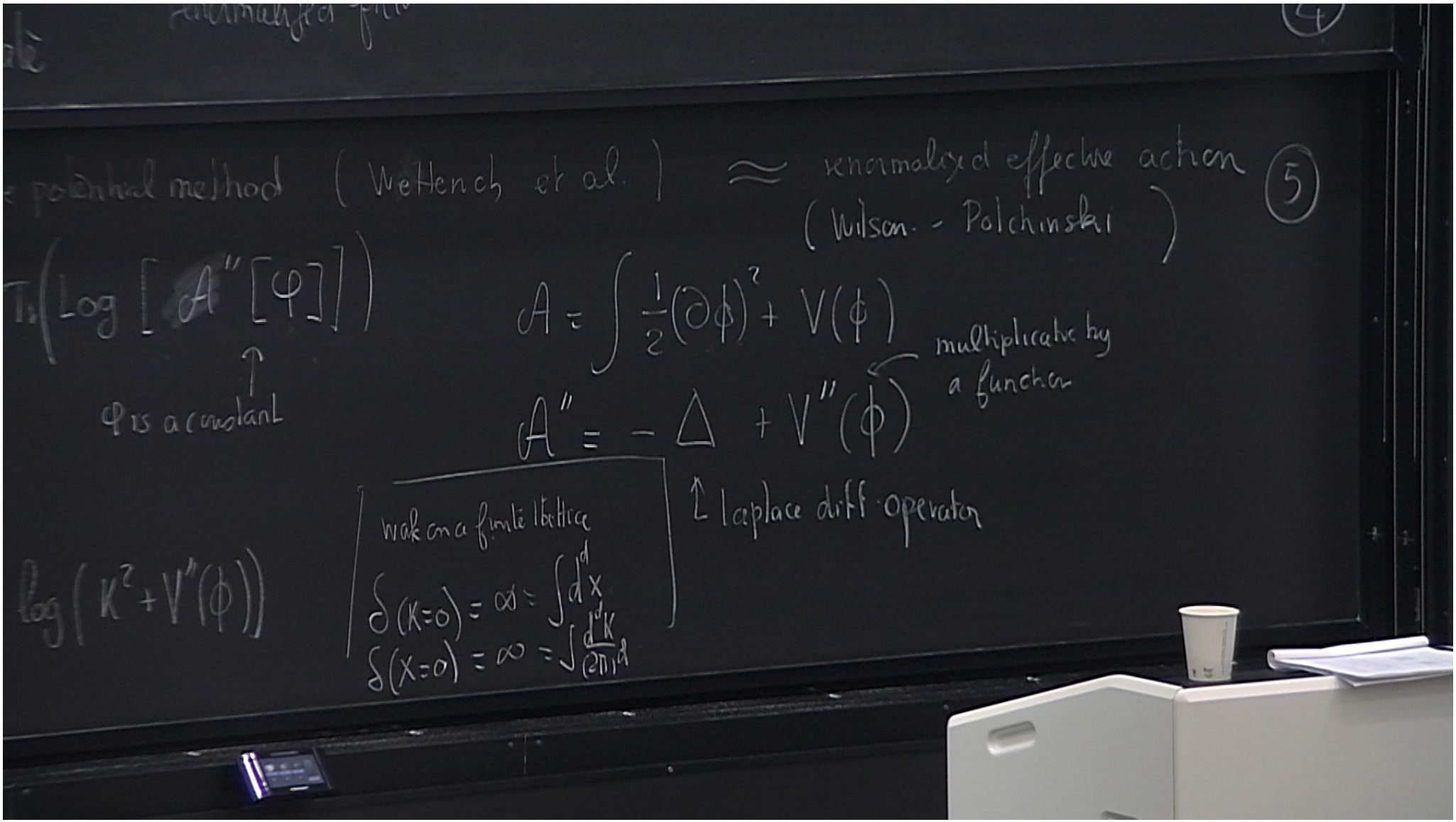
think about φ as ϕ_{slow}, slowly varying

kinetic term here

φ is a constant

in this approximation

$$\mathrm{Tr}\,\mathrm{Log}\left(-\Delta + V''(\phi)\right) = \int d^d x \langle x | \mathrm{Log}\left(-\Delta + V''(\phi)\right) | x \rangle = \int d^d x \int_{|k|<\Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi)\right)$$


potential method (Wetterich et al.) ≈ renormalized effective action
(Wilson - Polchinski)
5
Tr(Log [A''[φ]])
φ is a constant
A = ∫ ½(∂φ)² + V(φ)
multiplicative by a function
A'' = − Δ + V''(φ)
Laplace diff. operator
log(K² + V''(φ))
work on a finite lattice
δ(K=0) = ∞ = ∫ d^d x
δ(X=0) = ∞ = ∫ d^d K / (2π)^d

al a scale E variable

$\Lambda = 1/a$ microscopic cutoff fixed

$E \simeq \mu$ is variable, $E \to 0$

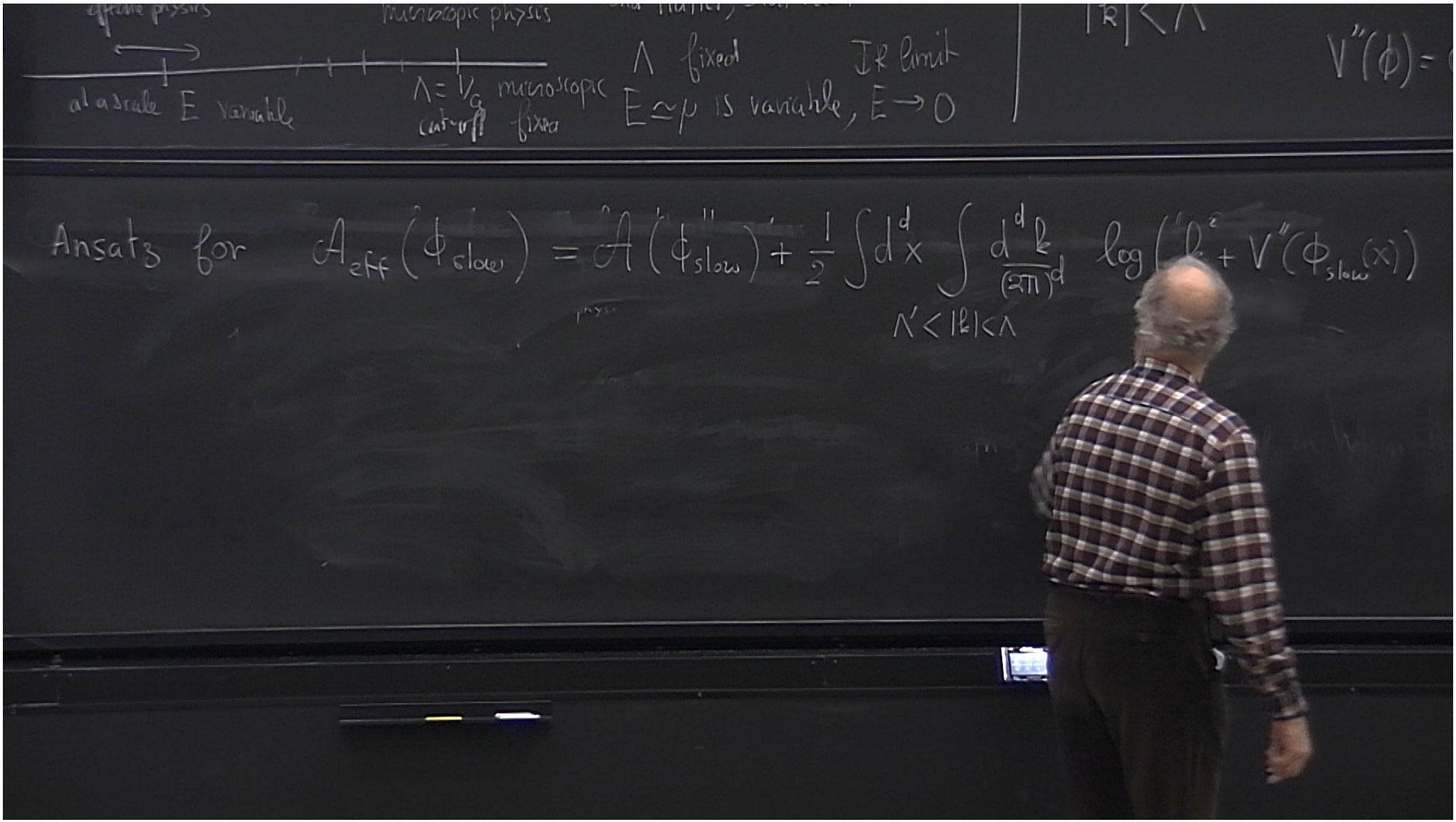
Ansatz for $\mathcal{A}_{eff}(\phi_{slow})$

at a scale E variable

$\Lambda = 1/a$ microscopic cutoff fixed

$E \simeq \mu$ is variable, $E \to 0$

Ansatz for $\mathcal{A}_{eff}(\phi_{slow}) = \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^d x \int_{\Lambda' < |k| < \Lambda} \frac{d^d k}{(2\pi)^d}$


effective physics
microscopic physics
at a scale E variable
Λ = 1/a microscopic cutoff fixed
Λ fixed
IR limit
E ≃ μ is variable, E → 0
|k| < Λ
V''(φ) =
Ansatz for $\mathcal{A}_{eff}(\phi_{slow}) = \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^dx \int_{\Lambda' < |k| < \Lambda} \frac{d^dk}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$

effective physics — microscopic physics

at a scale E variable — $\Lambda = 1/a$ microscopic cutoff fixed

Λ fixed — IR limit

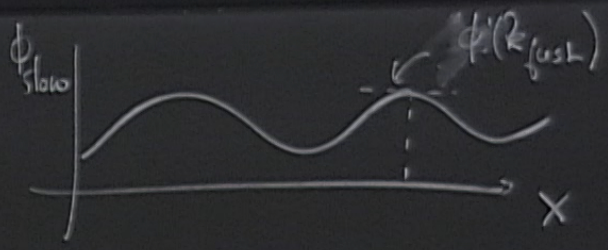
$E \simeq \mu$ is variable, $E \to 0$

$|k| < \Lambda$

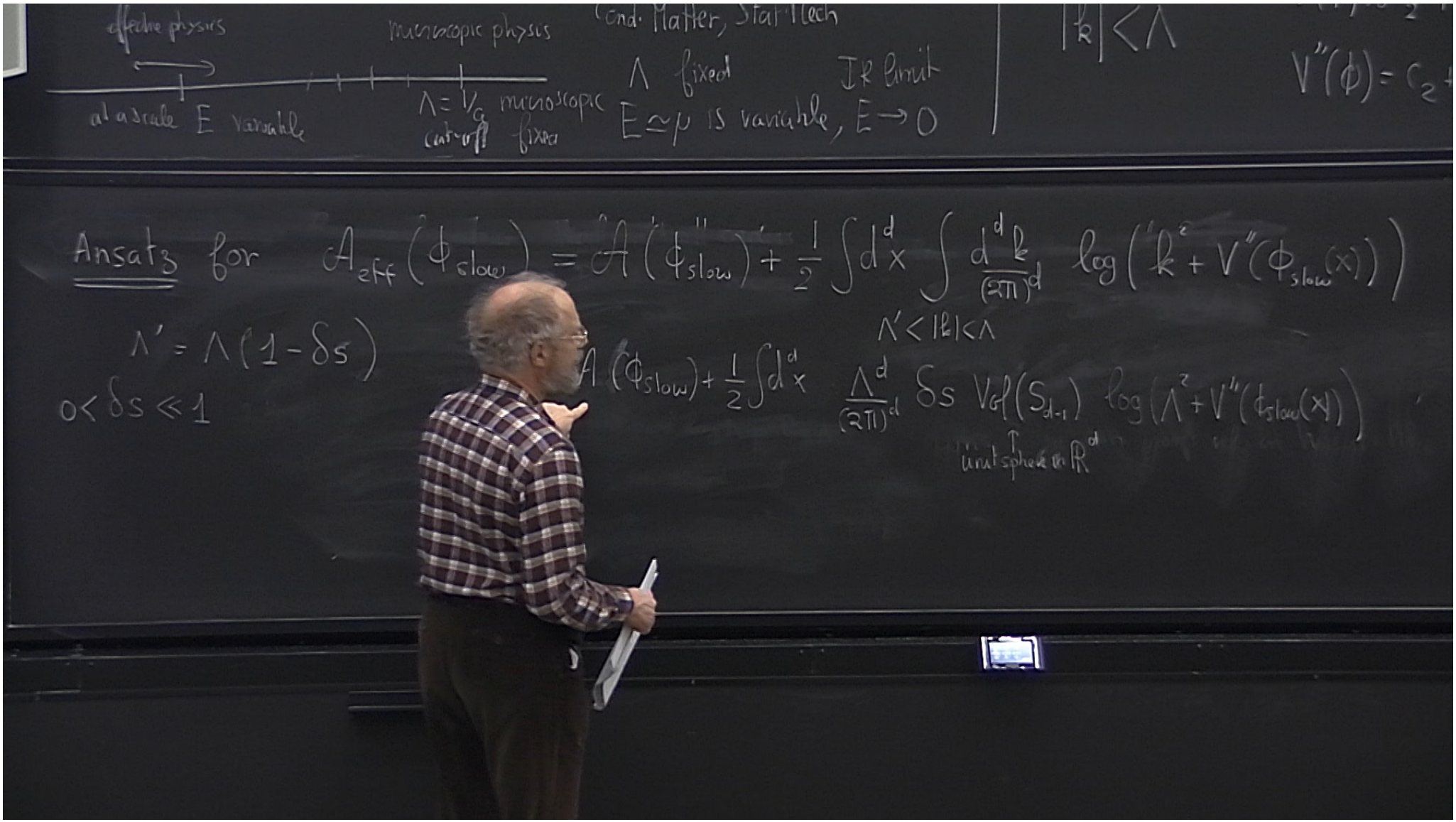
$V''(\phi) =$

Ansatz for $\mathcal{A}_{eff}(\phi_{slow}) = \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^d x \int_{\Lambda' < |k| < \Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$

$$\int_{\Lambda' < |k| < \Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x)) \right)$$



6


effective physics
microscopic physics
Cond. Matter, Stat Mech
at a scale E variable
Λ = 1/a microscopic cutoff fixed
Λ fixed
IR limit
E ≃ μ is variable, E → 0
|k| < Λ
V''(φ) = c2
Ansatz for A_eff(φ_slow) = A(φ_slow) + 1/2 ∫ d^d x ∫ d^d k/(2π)^d log(k^2 + V''(φ_slow(x)))
Λ' < |k| < Λ
Λ' = Λ(1 − δs)
0 < δs ≪ 1
A(φ_slow) + 1/2 ∫ d^d x Λ^d/(2π)^d δs Vol(S_{d-1}) log(Λ^2 + V''(φ_slow(x)))
unit sphere in R^d

effective physics — microscopic physics

at a scale E variable — $\Lambda = 1/a$ microscopic cutoff fixed

Cond. Matter, Stat Mech

Λ fixed — IR limit

$E \simeq \mu$ is variable, $E \to 0$

$|k| < \Lambda$

$V''(\phi) = c_2 +$

<u>Ansatz</u> for $\mathcal{A}_{eff}(\phi_{slow}) = \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^d x \int_{\Lambda' < |k| < \Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$

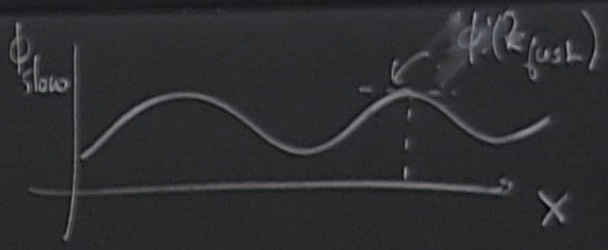
$\Lambda' = \Lambda(1 - \delta s)$

$0 < \delta s \ll 1$

$$\doteq \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^d x \, \frac{\Lambda^d}{(2\pi)^d} \, \delta s \, \mathrm{Vol}(S_{d-1}) \log\left(\Lambda^2 + V''(\phi_{slow}(x))\right)$$

unit sphere in $\mathbb{R}^d$

$$\int_{\Lambda' < |k| < \Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$$



6

$$\delta S \; \mathrm{Vol}(S_{d-1}) \log\left(\Lambda^2 + V''(\phi_{slow}(x))\right) = \int d^d x \; \frac{1}{2}(\partial \phi_{slow})^2 + V_{eff}(\phi_{slow})$$

unit sphere in $\mathbb{R}^d$

at a scale E variable $\Lambda = 1/a$ microscopic cutoff fixed $E \simeq \mu$ is variable, $E \to 0$

<u>Ansatz</u> for $\mathcal{A}_{eff}(\phi_{slow}) = \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^d x \int_{\Lambda' < |k| < \Lambda} \frac{d^d k}{(2\pi)^d} \log$

$\Lambda' = \Lambda(1 - \delta s)$ $\quad = \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^d x \, \frac{\Lambda^d}{(2\pi)^d} \delta s \, \mathrm{Vol}(S_{d-1})$

$0 < \delta s \ll 1$

- renormalization of the local potential only

unit sphere in $\mathbb{R}^d$

al a scale E variable $\Lambda = 1/a$ microscopic cutoff fixed $E \simeq \mu$ is variable, $E \to 0$

<u>Ansatz</u> for $\mathcal{A}_{eff}(\phi_{slow}) = \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^dx \int_{\Lambda' < |k| < \Lambda} \frac{d^dk}{(2\pi)^d} \log$

$\Lambda' = \Lambda(1-\delta s)$ $\quad = \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^dx \, \frac{\Lambda^d}{(2\pi)^d}\, \delta s \, \mathrm{Vol}(S_{d-1})$

$0 < \delta s \ll 1$

unit sphere in $\mathbb{R}^d$

renormalization of the local potential only

$$V_{eff} = V + \delta s \, \frac{1}{2} \frac{\Lambda^d}{(2\pi)^d} \mathrm{Vol}(S_{d-1}) \log(\Lambda^2 + V'')$$

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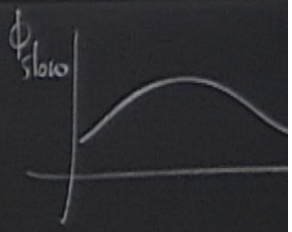
Step ② rescale ϕ_{slow} the k and the x

$E \simeq \mu$ is variable, $E \to 0$

$$= \mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^d x \int_{\Lambda' < |k| < \Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$$



$$\mathcal{A}(\phi_{slow}) + \frac{1}{2}\int d^d x \; \frac{\Lambda^d}{(2\pi)^d} \delta s \, \mathrm{Vol}(S_{d-1}) \log\left(\Lambda^2 + V''(\phi_{slow}(x))\right) = \int d^d x \, \frac{1}{2}(\partial \phi_{slow})^2 +$$

unit sphere in $\mathbb{R}^d$

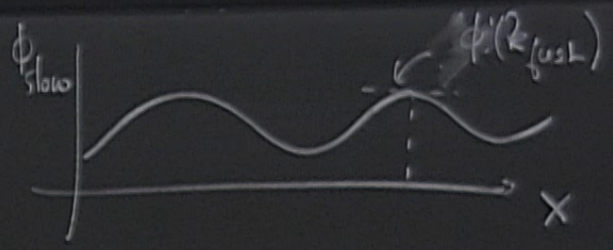
$$\boxed{(\Lambda^2 + V'')}$$

step ② rescale ϕ_{slow}, the k and the x

new field ϕ and a new renormalized potential $V_{REN}(\phi) =$

$$\int \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$$

$< |k| < \Lambda$



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$$\delta s \, \mathrm{Vol}(S_{d-1}) \log\left(\Lambda^2 + V''(\phi_{slow}(x))\right) = \int d^d x \, \frac{1}{2}(\partial \phi_{slow})^2 + V_{eff}(\phi_{slow})$$

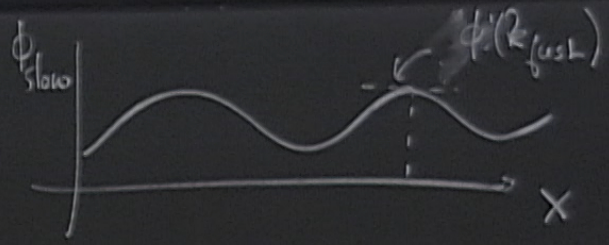
unit sphere in $\mathbb{R}^d$

ϕ_{slow} the k and the x

and a new renormalized potential

$$V_{REN}(\phi) = V(\phi) + ds\left[dV(\phi) - \frac{d-2}{2}\phi V'(\phi)\right]$$

$$\int_{<|k|<\Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$$



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unit sphere in $\mathbb{R}^d$

ϕ_{slow} the k and the x

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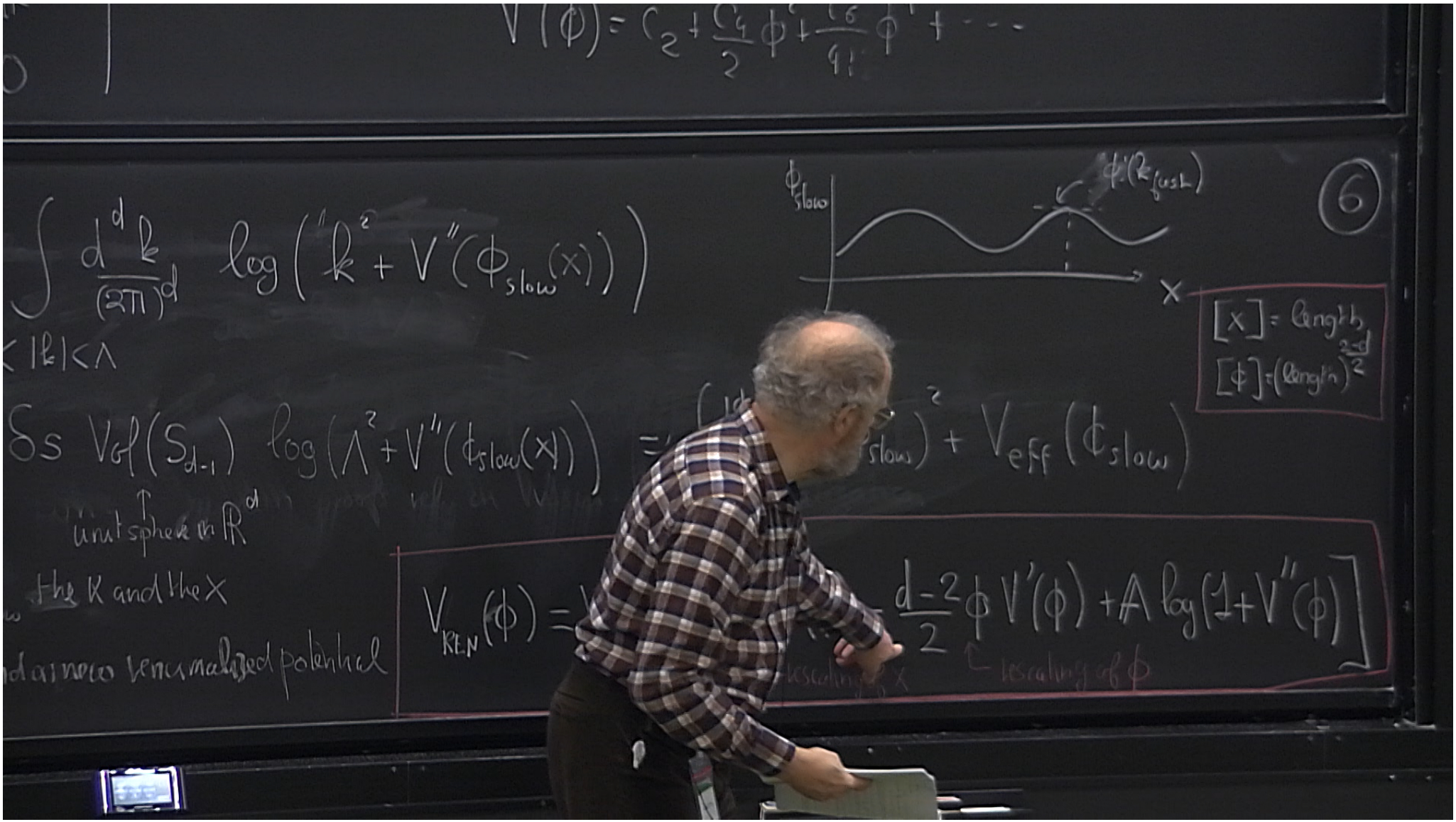
$$\boxed{V_{REN}(\phi) = V(\phi) + ds\left[d\,V(\phi) - \frac{d-2}{2}\phi V'(\phi) + A \log(1 + V''(\phi))\right]}$$

rescaling of x — rescaling of ϕ


$V(\phi) = c_2 + \frac{c_4}{2}\phi^2 + \frac{c_6}{4!}\phi^4 + \cdots$
$\int \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$
ϕ_{slow}
$\phi'(k_{fast})$
x
6
$[x] = \text{length}$
$[\phi] = (\text{length})^{\frac{2-d}{2}}$
$\delta s \, \mathrm{Vol}(S_{d-1}) \log\left(\Lambda^2 + V''(\phi_{slow}(x))\right)$
unit sphere in $\mathbb{R}^d$
the k and the x
$V_{REN}(\phi) = \frac{d-2}{2}\phi V'(\phi) + A \log(1 + V''(\phi))$
rescaling of x
rescaling of ϕ

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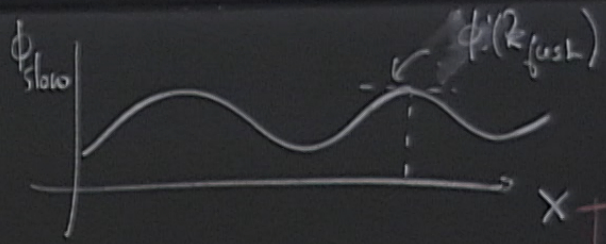
$$\int_{<|k|<\Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$$



$[x] = \text{length}$

$[\phi] = (\text{length})^{\frac{2-d}{2}}$

$$\delta s \, \mathrm{Vol}(S_{d-1}) \log\left(\Lambda^2 + V''(\phi_{slow}(x))\right) = \int d^d x \, \frac{1}{2}(\partial\phi_{slow})^2 + V_{eff}(\phi_{slow})$$

unit sphere in $\mathbb{R}^d$

the k and the x

and a new renormalized potential

$$\boxed{V_{REN}(\phi) = V(\phi) + \delta s\left[d\,V(\phi) - \frac{d-2}{2}\phi V'(\phi) + A\log(1 + V''(\phi))\right]}$$

rescaling of x — rescaling of ϕ — 1 loop

$$\mathrm{Tr}\,\mathrm{Log}\left(-\Delta+V''(\phi)\right) = \int d^d x \,\langle x|\mathrm{Log}\left(-\Delta+V''(\phi)\right)|x\rangle = \int d^d x \int_{|k|<\Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2+V''(\phi)\right)$$

$$\delta(k=0) = \infty = \int d^d x$$

$$\delta(x=0) = \infty = \int \frac{d^d k}{(2\pi)^d}$$

Step ③ iterate $\Lambda \longrightarrow \Lambda/S$ $\quad S$ scale factor $\quad S = (1+\delta s)^N \gg 1$

$$V(\phi) = c_2 + \frac{c_4}{2}\phi^2 + \frac{c_6}{4!}\phi^4 + \cdots$$

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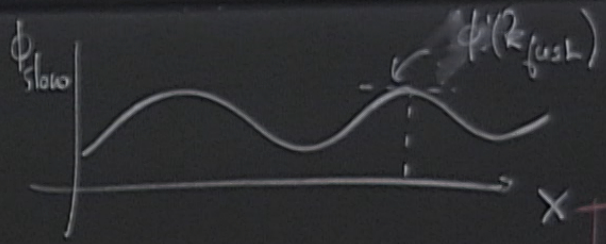

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unit sphere in $\mathbb{R}^d$

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rescaling of x — rescaling of ϕ — 1 loop

Step ③ iterate $\Lambda \longrightarrow \Lambda/S$ S scale factor $S = (1+\delta s)$

$V_{REN}(\phi, s)$

Step ③ iterate $\Lambda \longrightarrow \Lambda/S$ S scale factor $S = (1+\delta s)$

$$V_{REN}(\phi, s) = V_S(\phi)$$

$$S\frac{\partial}{\partial S} V_S(\phi) = d V_S(\phi) + \frac{2-d}{2}\phi \cdot \frac{\partial}{\partial \phi} V_S(\phi)$$

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rescale V so that $\Lambda = 1$

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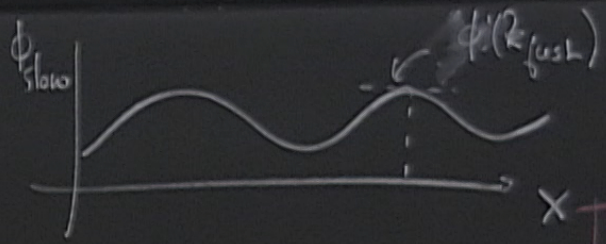
rescale V so that $\boxed{\Lambda = 1}$ and k

R.G. Flow equation in the space of potentials $V(\phi)$

$$V(\phi) = c_2 + \frac{c_4}{2}\phi^2 + \frac{c_6}{4!}\phi^4 + \cdots$$

$$\int_{<|k|<\Lambda} \frac{d^d k}{(2\pi)^d} \log\left(k^2 + V''(\phi_{slow}(x))\right)$$



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$[x] = \text{length}$, and $[\phi] = (\text{length})^2$

$$\delta s \, \mathrm{Vol}(S_{d-1}) \log\left(\Lambda^2 + V''(\phi_{slow}(x))\right) = \int d^d x \, \frac{1}{2}(\partial\phi_{slow})^2 + V_{eff}(\phi_{slow})$$

unit sphere in $\mathbb{R}^d$

the k and the x

a new renormalized potential

$$\boxed{V_{REN}(\phi) = V(\phi) + \delta s\left[d\,V(\phi) - \frac{d-2}{2}\phi V'(\phi) + A\log\left(\Lambda^2 + V''(\phi)\right)\right]}$$

rescaling of x — rescaling of ϕ — 1 loop

RG in the local potential approximation

Start from bare action for theory with cut-off $|k| < \Lambda$

$$\mathcal{A}_B(\phi_B) = \int d^4x_B \; \frac{1}{2}(\partial\phi_B)^2 + V_B(\phi_B)$$

Reexpress in dimensionless variables

$$\phi_B = \Lambda\phi \;, \quad x_B = \Lambda^{-1}x \;, \quad V_B = \Lambda^4 V$$

Perform renormalisation group transformation up to physical scale μ

$$S\frac{\partial}{\partial S}V_S = 4V_S - \phi\frac{\partial}{\partial\phi}V_S + \mathbb{A}\log\left(1 + \frac{\partial^2}{\partial\phi^2}V_S\right)$$

Reexpress in physical units to obtain the renormalized parameters

$$\mu = \Lambda/S \;, \quad \phi = \mu^{-1}\phi_R \;, \quad x = \mu x_R \;, \quad V_S = \mu^{-4}V_R$$

Local potential approximation

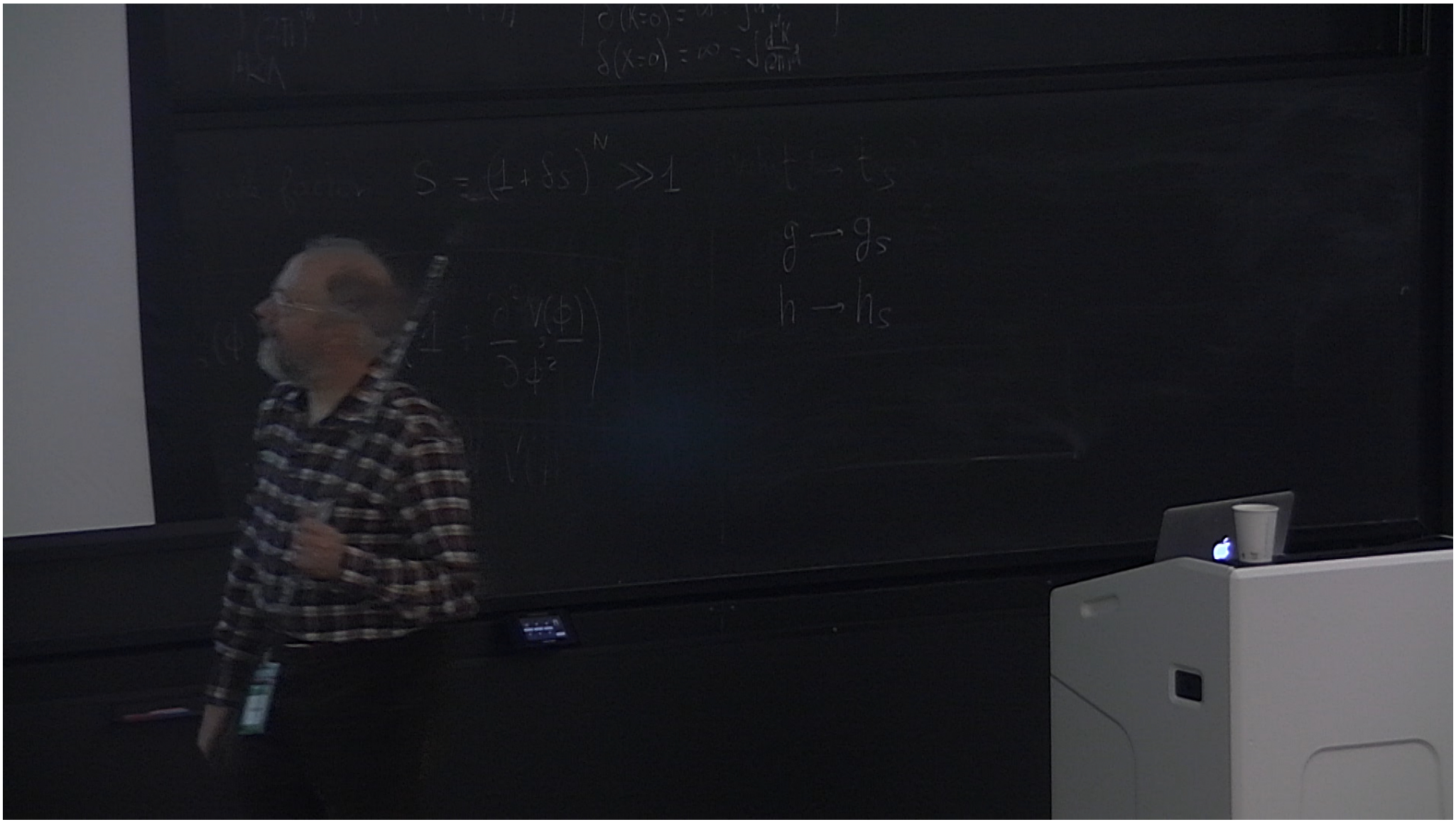
RG flow equations at 1 loop and order ϕ^6

$$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$$

$$S\frac{\partial}{\partial S}t = -\beta_t = 2t + \frac{g}{16\pi^2(t+1)}$$

$$S\frac{\partial}{\partial S}g = -\beta_g = -\frac{3g^2}{16\pi^2(t+1)^2} + \frac{h}{16\pi^2(t+1)^2} + \frac{ht}{16\pi^2(t+1)^2}$$

$$S\frac{\partial}{\partial S}h = -\beta_h = -2h + \frac{15g^3}{8\pi^2(t+1)^3} - \frac{15ght}{16\pi^2(t+1)^3} - \frac{15gh}{16\pi^2(t+1)^3}$$


$\delta(k=0) = \infty = \int \frac{d^4k}{(2\pi)^4}$
$S = (1+\delta_s)^N \gg 1$
$t \to t_s$
$g \to g_s$
$h \to h_s$

Local potential approximation

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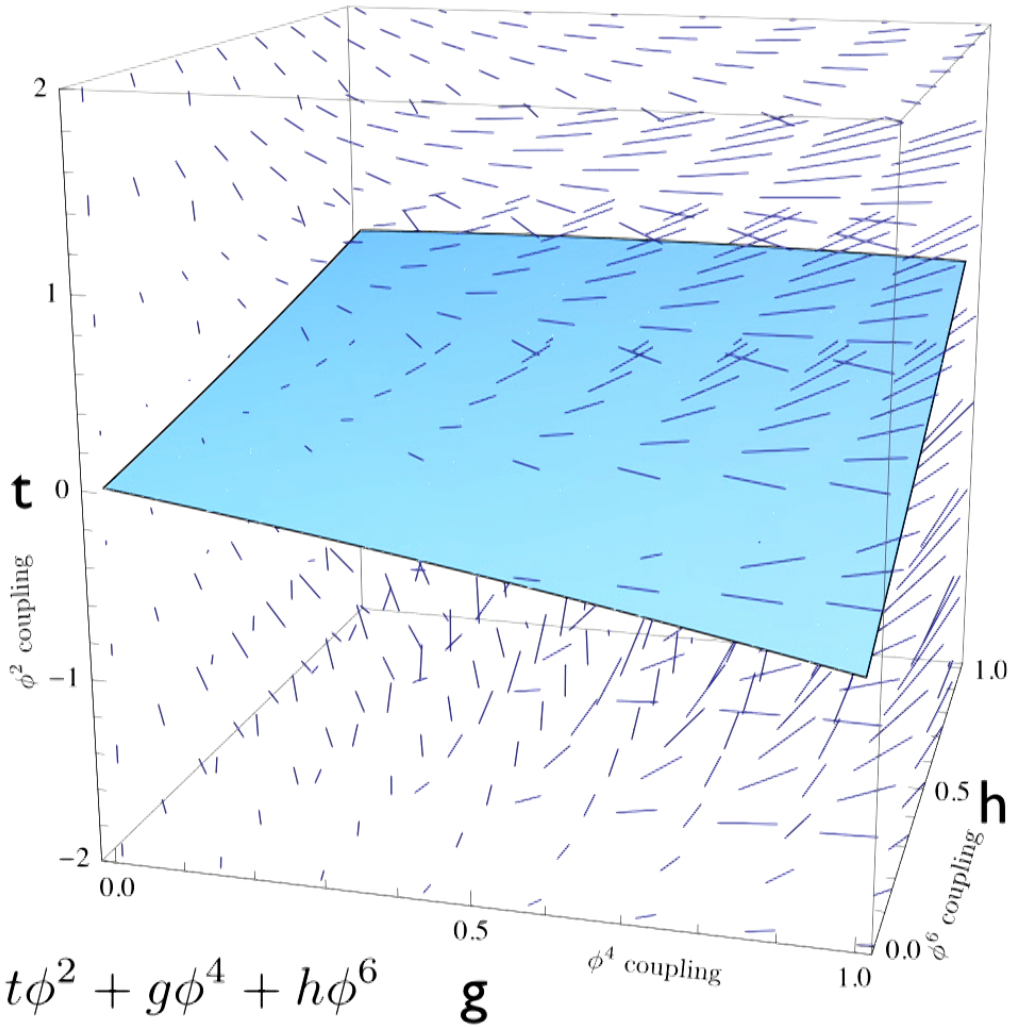
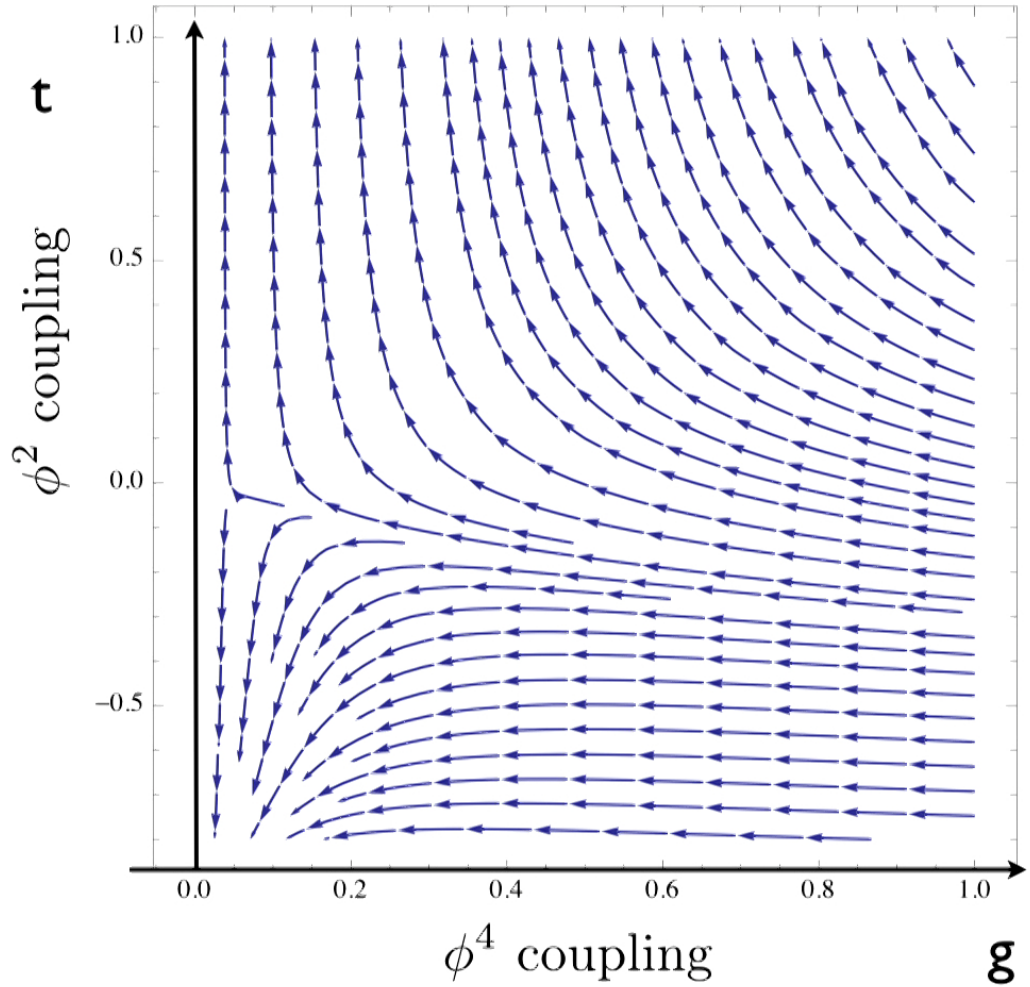
RG flows in the (t,g,h) space

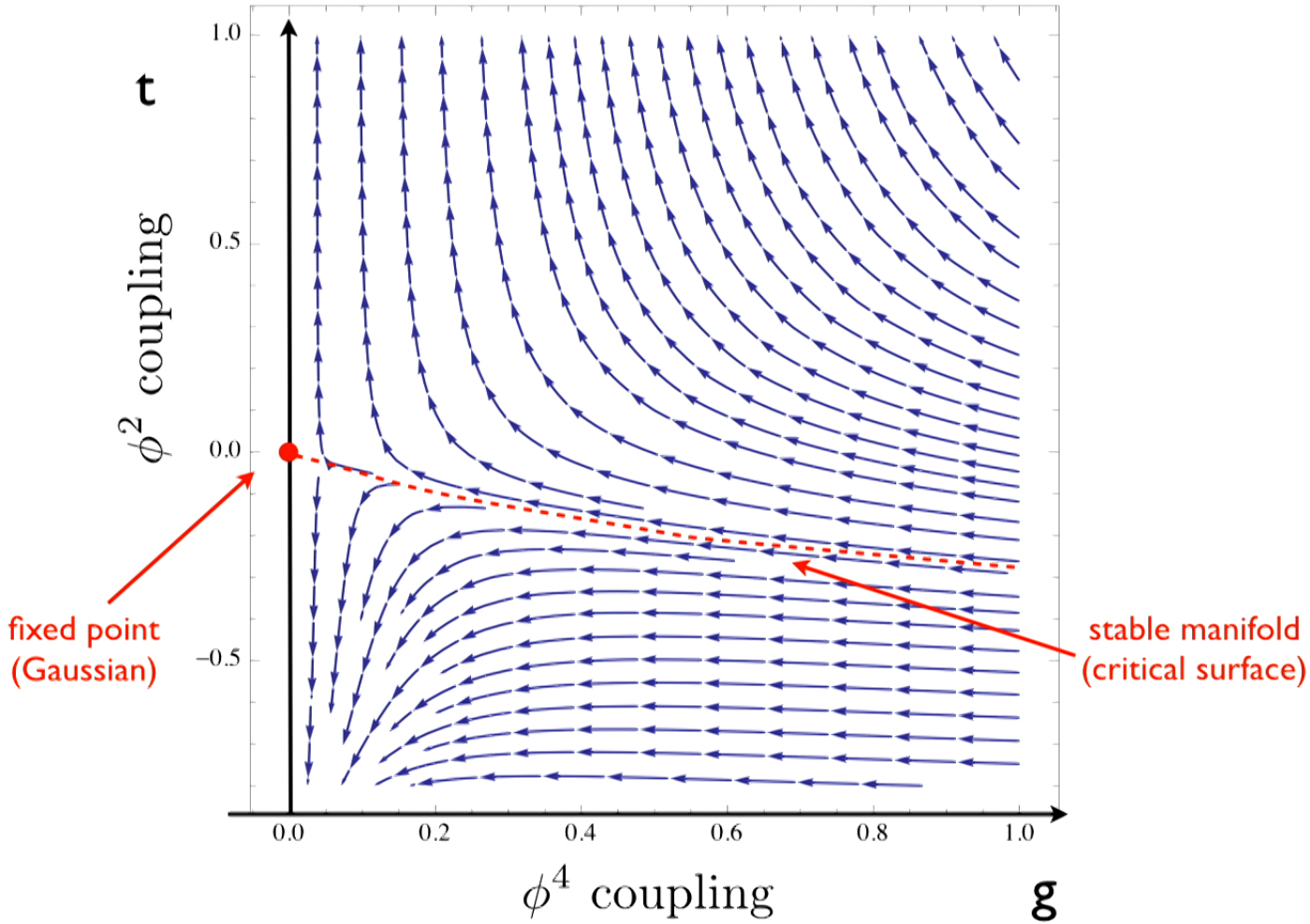
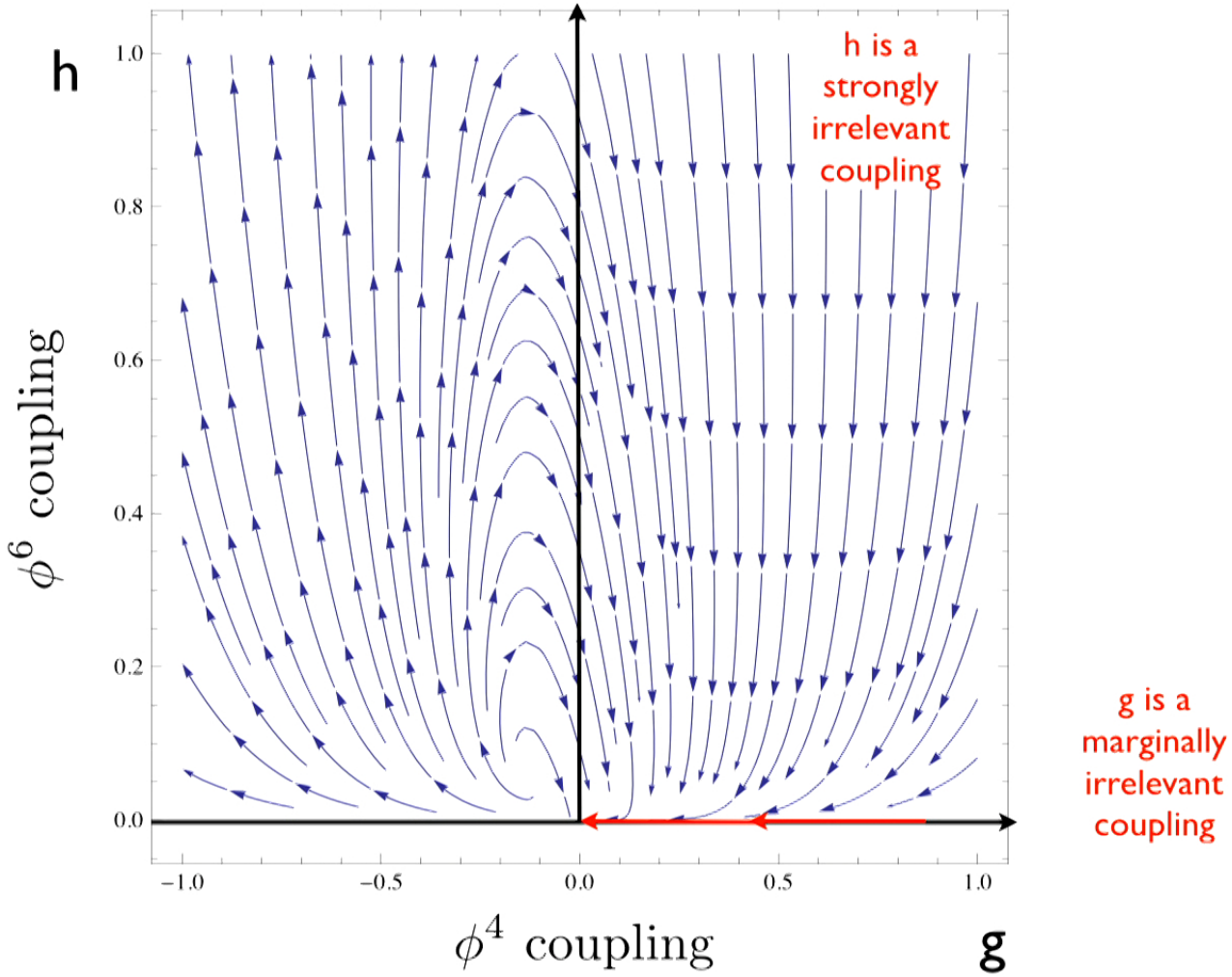
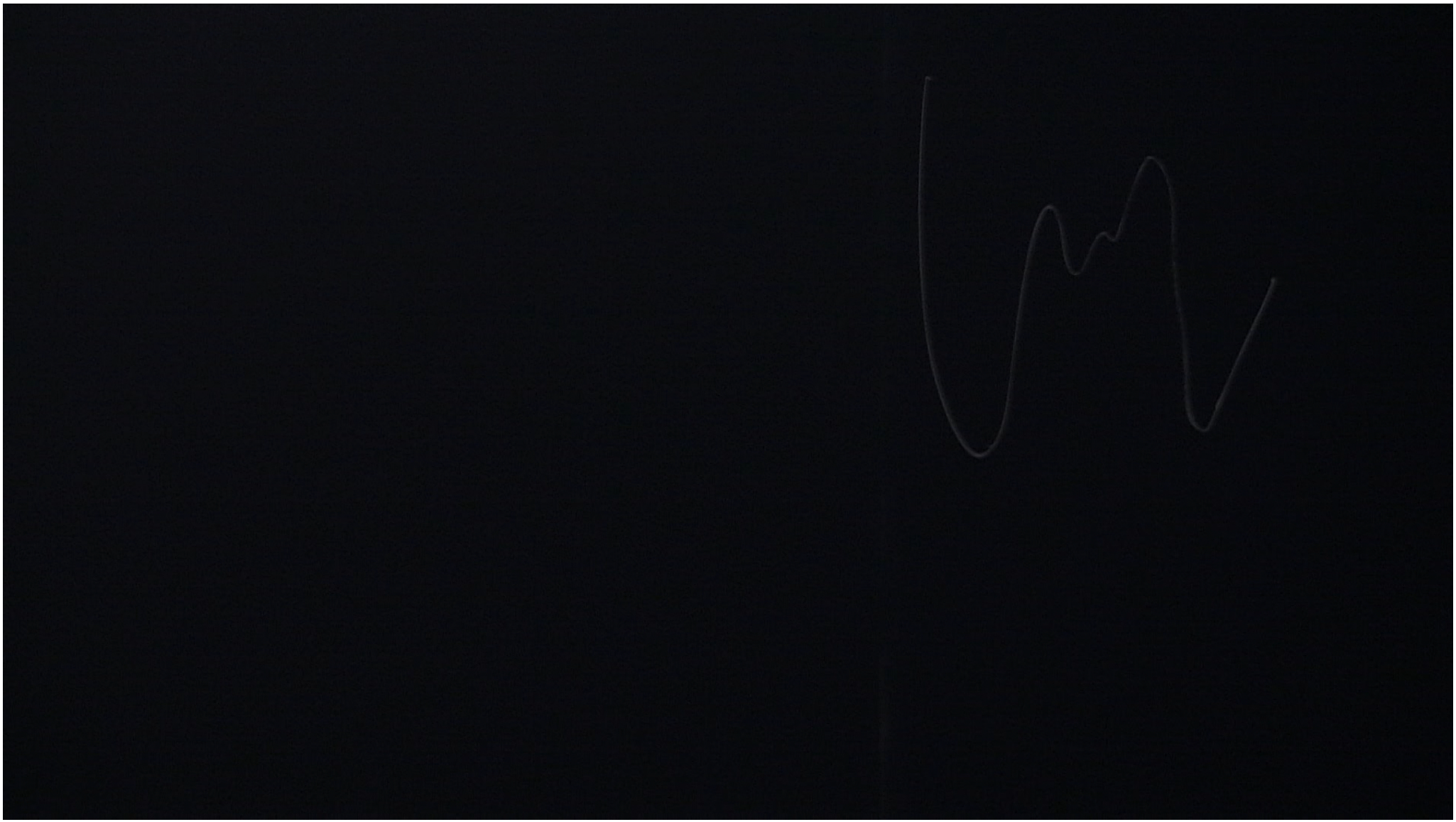


$$V(\phi) = t\phi^2 + g\phi^4 + h\phi^6$$

RG flows in the (t,g) plane



RG flows in the (t,g) plane



RG flows on the critical surface