Title: PSI 17/18 - Quantum Field Theory II - Lecture 6

Date: Nov 13, 2017 09:00 AM

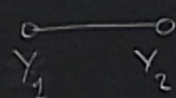
URL: http://pirsa.org/17110016

Abstract:

ϕ^4 perturbation theory: diagrammatics (continued)

Euclidean $\quad S[\phi] = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right)$

Feynman diagrams



propagator (free) $\quad G_0(y_1 - y_2) \xleftarrow{F.T} \frac{1}{k^2+m^2}$

vertices: interaction $\quad (-g)$

$$\sum_k \frac{1}{k!}\left(\frac{-g}{4!}\right)^k \int_{x's} \langle \phi(z_1)\cdots\phi(z_N)\,\phi^4(x_1)\cdots\phi^4(x_k)\rangle_0 = Z(z_1 \cdots z_N) = \sum_{\text{diagrams } G} (-g)^k \underbrace{C_G}_{\text{sym. factor}} \underbrace{I_G(z_1\cdots z_N)}_{\text{integral}}$$

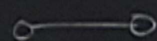
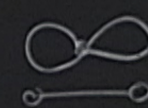
Correlation functions (Green Funct)

$$\langle \phi(z_1) \cdots \phi(z_N) \rangle = \frac{Z(z_1 \cdots z_N)}{Z}$$

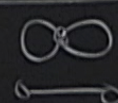
N pt function

0 pt. function

$N = 2$

 $- g\left[\left(\frac{1}{2}\right)\right.$  $+ \frac{1}{8}$  $\left.\right] + O(g^2)$

divided by

$1 - g\frac{1}{8}$  $+ O(g^2)$

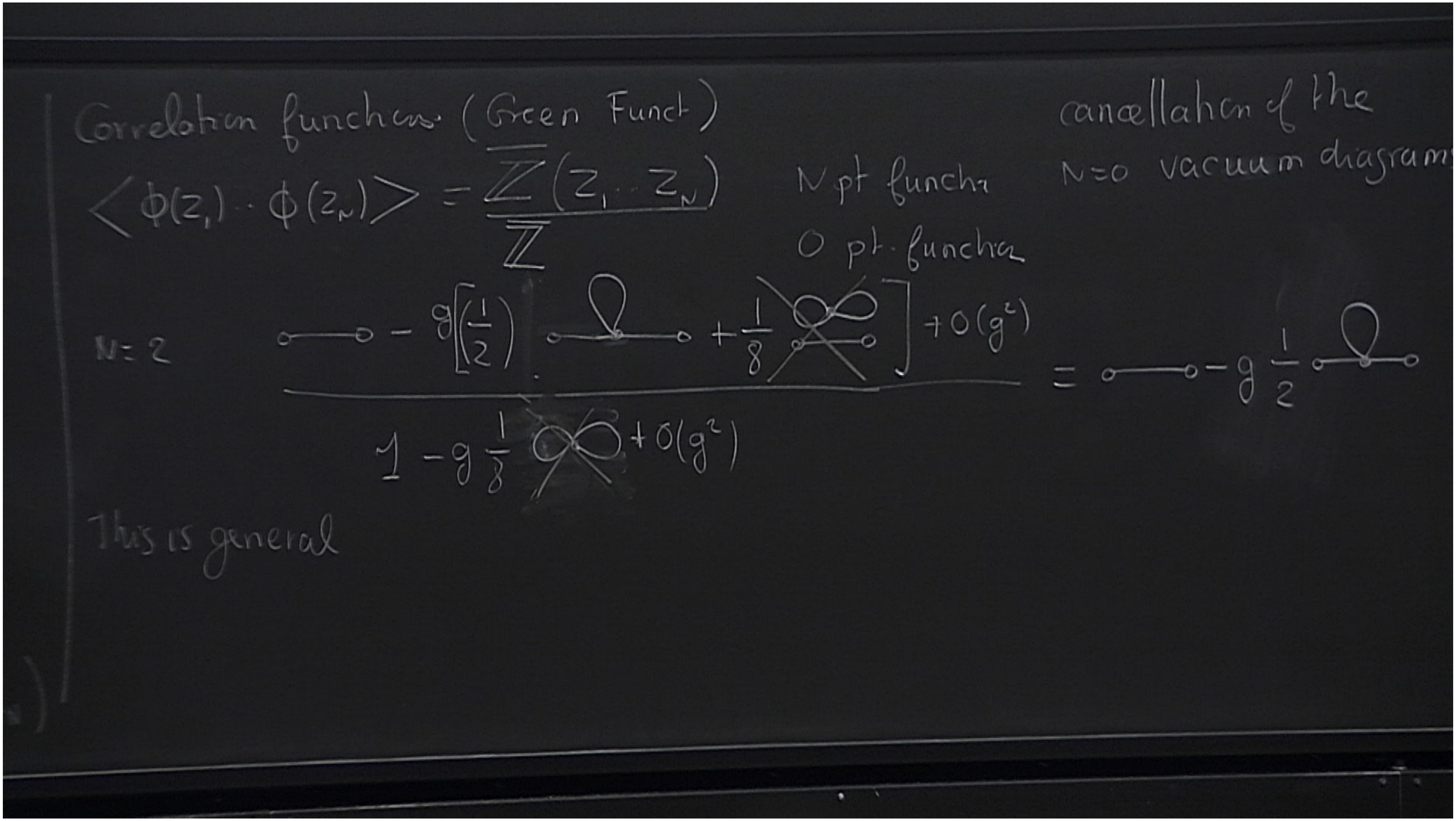
Correlation functions (Green Funct)

$$\langle \phi(z_1)\cdots\phi(z_N)\rangle = \frac{\underline{\mathbb{Z}}(z_1 \cdots z_N)}{\mathbb{Z}}$$

N pt function

0 pt. function

$$N=2 \qquad \frac{\;-\; g\left[\left(\frac{1}{2}\right) \;+\; \frac{1}{8}\;\right] + O(g^2)}{1 - g\,\frac{1}{8}\; + O(g^2)}$$


Correlation functions (Green Funct)
$\langle \phi(z_1) \cdots \phi(z_N) \rangle = \frac{Z(z_1 \ldots z_N)}{Z}$
N pt function
0 pt. function
cancellation of the
N=0 vacuum diagrams
N = 2
$+O(g^2)$
This is general

2 points diagrams

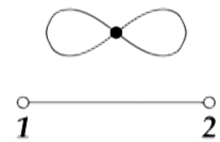
$N = 2,\ K = 0$

$$\langle \Phi(z_1)\Phi(z_2)\rangle_0 \ =$$

$N = 2,\ K = 1$

$$\frac{1}{4!}\int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi^4(x_1)\rangle_0 \ = \ \frac{1}{2}$$

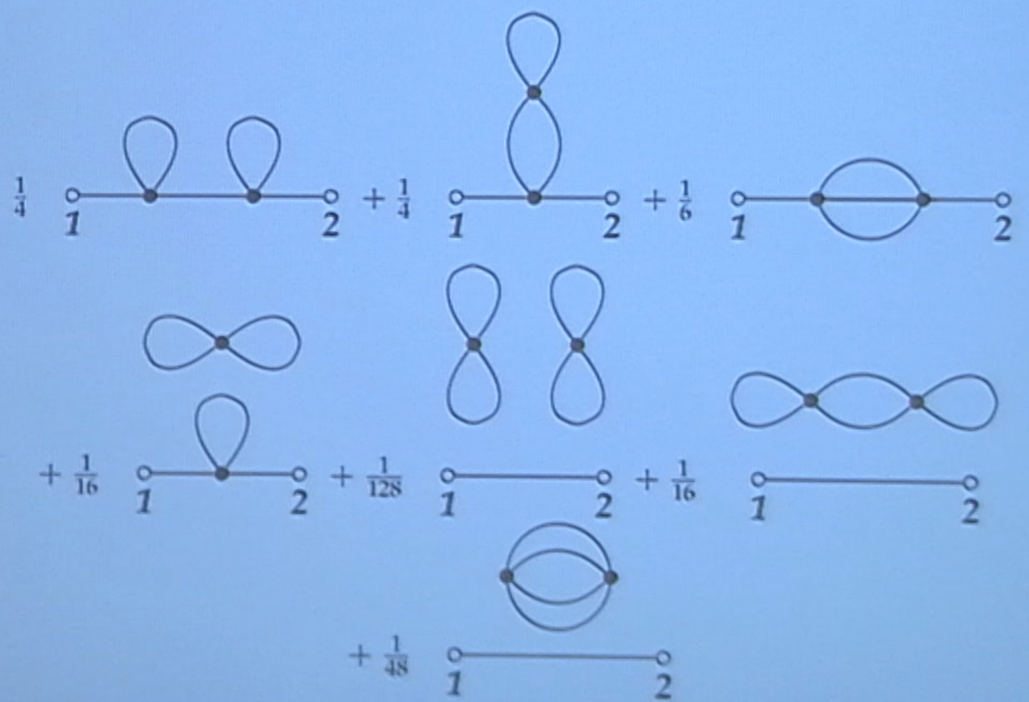
$$+ \ \frac{1}{8}$$



2 points diagrams (continued)

$N = 2,\ K = 2$

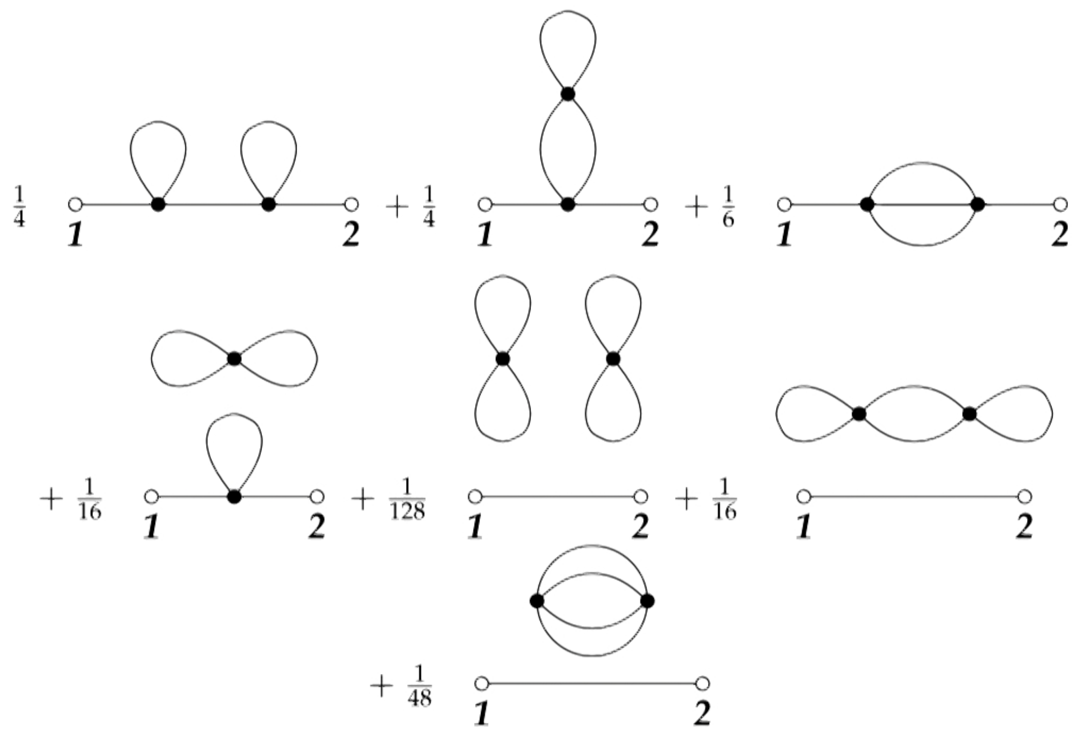
$$\frac{1}{2!\,(4!)^2}\int_{x_1}\int_{x_2}\langle\Phi(z_1)\Phi(z_2)\Phi^4(x_1)\Phi^4(x_2)\rangle_0 \;=$$



2 points diagrams (continued)

$N = 2,\ K = 2$

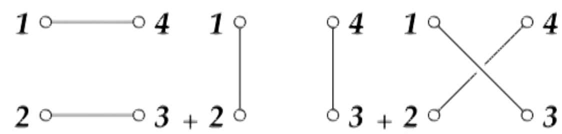
$$\frac{1}{2!\,(4!)^2}\int_{x_1}\int_{x_2}\langle\Phi(z_1)\Phi(z_2)\Phi^4(x_1)\Phi^4(x_2)\rangle_0 \ =$$



4 points diagrams

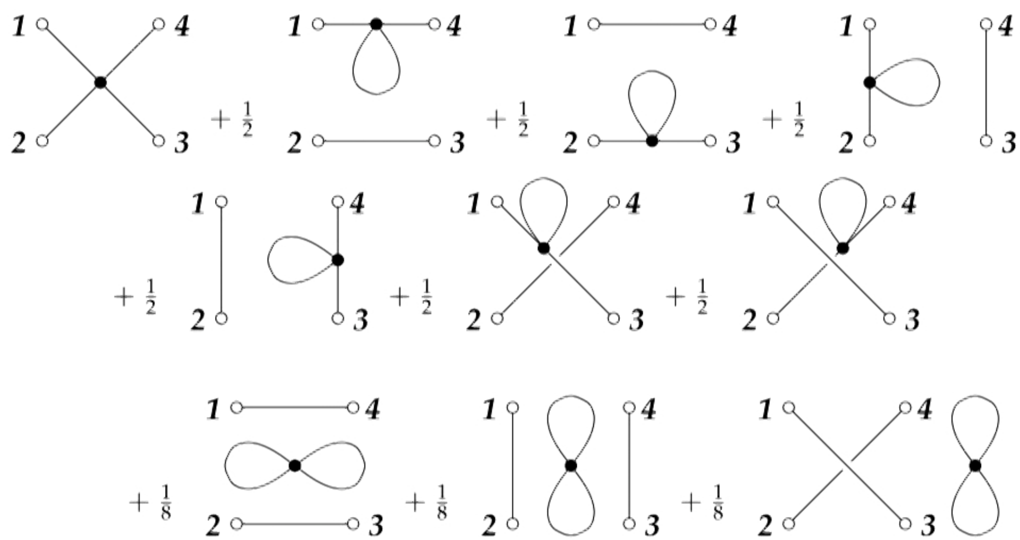
$N = 4,\ K = 0$

$$\langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4)\rangle_0 \ =$$

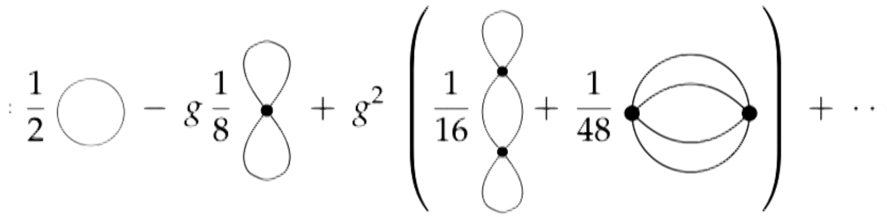


$N = 4,\ K = 1$

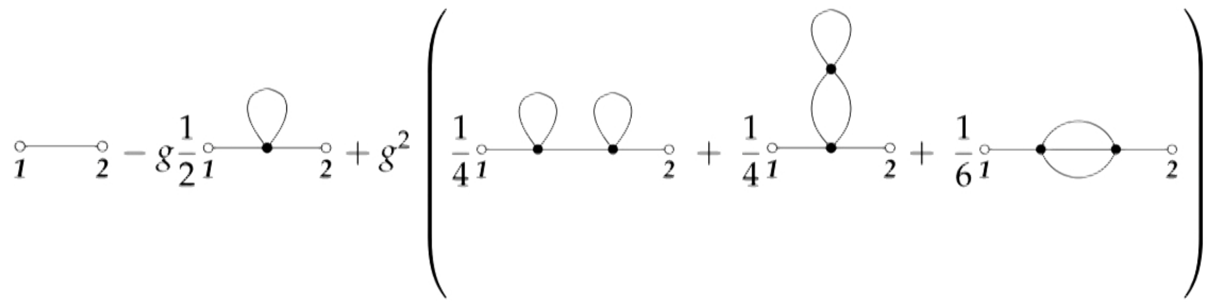
$$\frac{1}{4!}\int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4)\Phi^4(x_1)\rangle_0 \ =$$



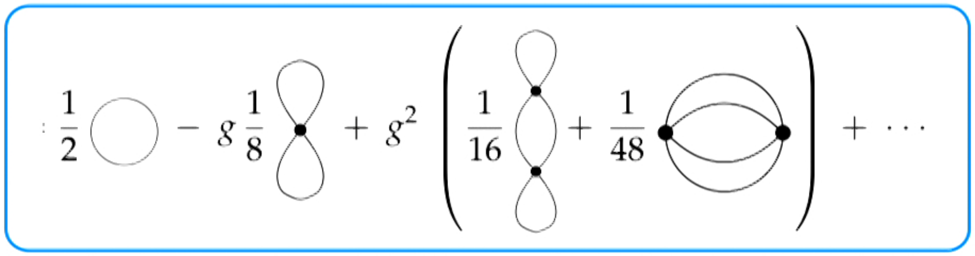
Connected vacuum diagrams



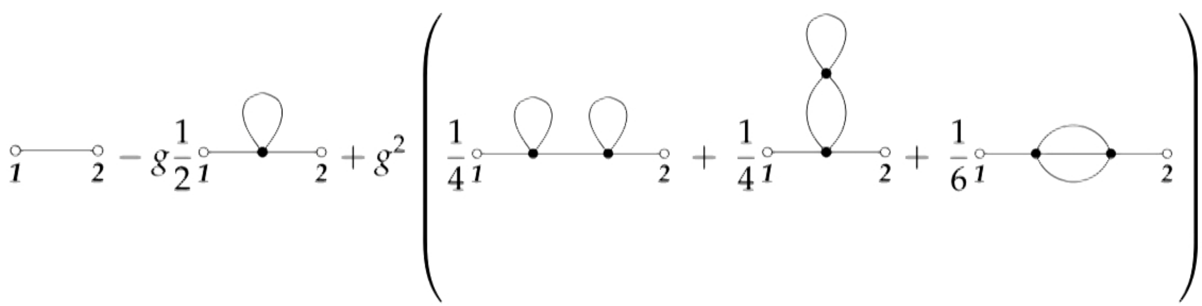
2 points function = connected 2 points function



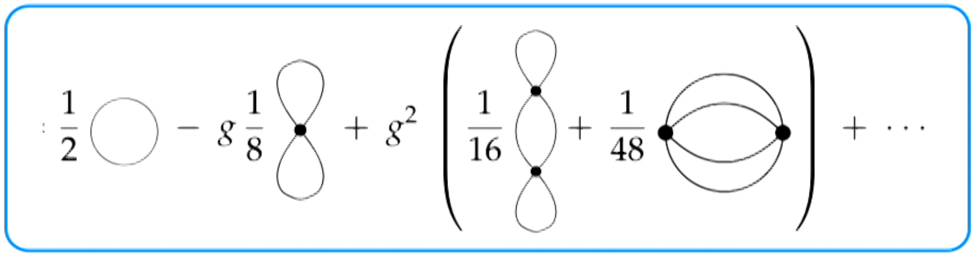
Connected vacuum diagrams



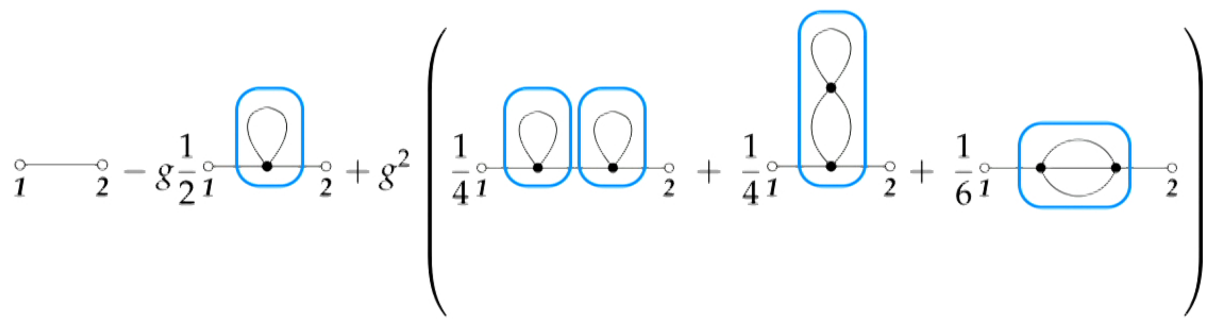
2 points function = connected 2 points function



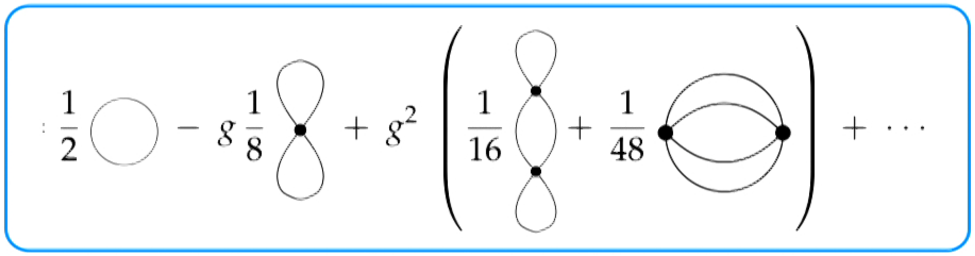
Connected vacuum diagrams



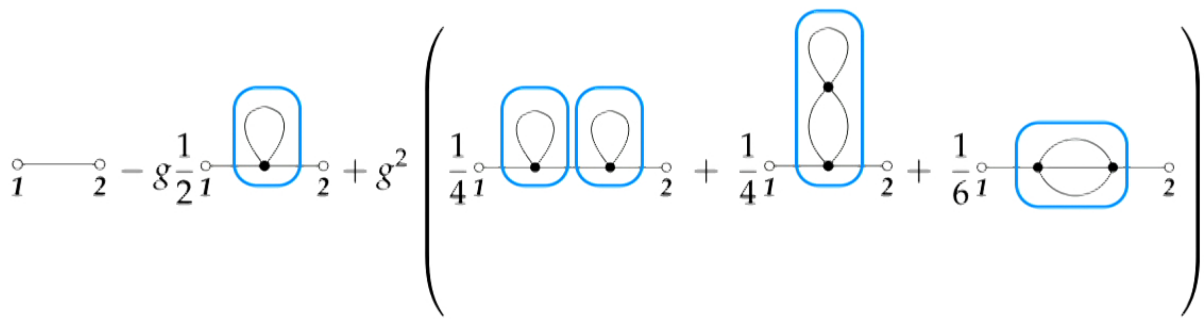
2 points function = connected 2 points function



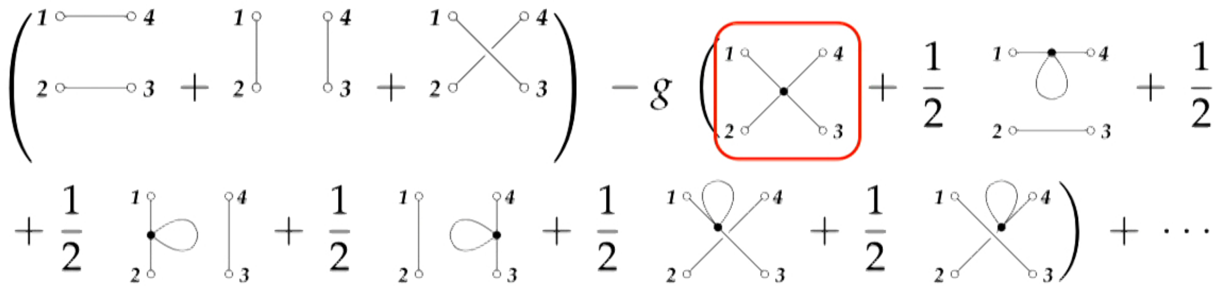
Connected vacuum diagrams



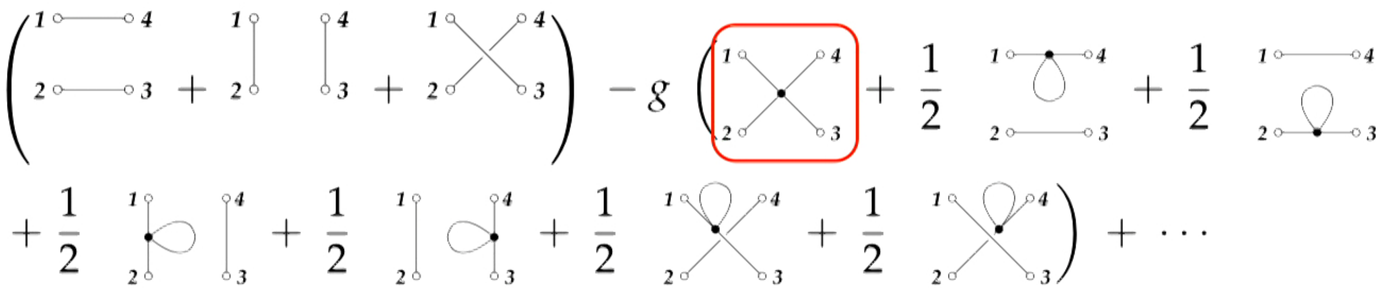
2 points function = connected 2 points function



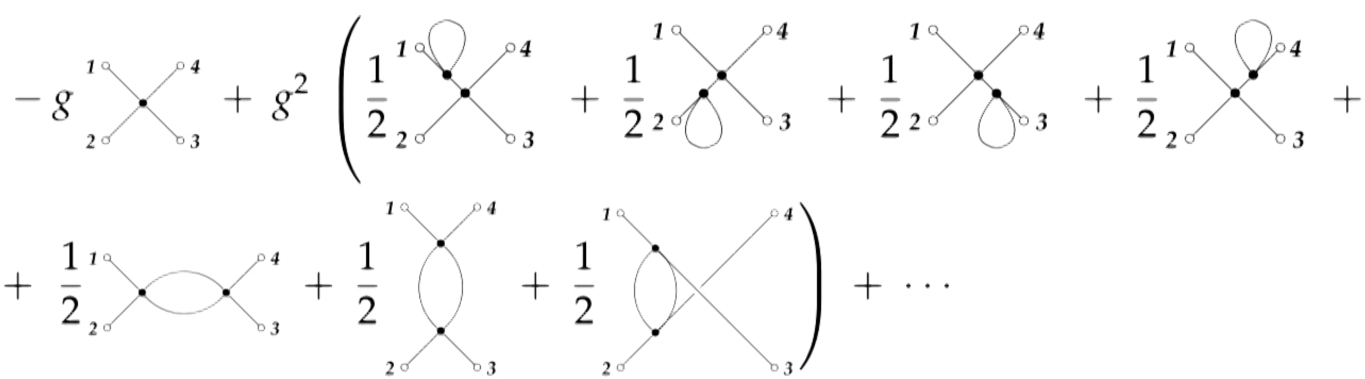
4 points function (up to order 1)



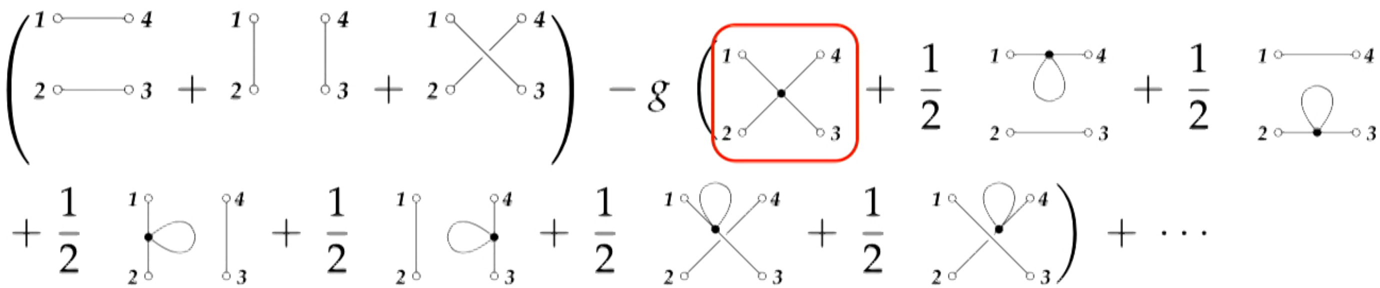
4 points function (up to order 1)



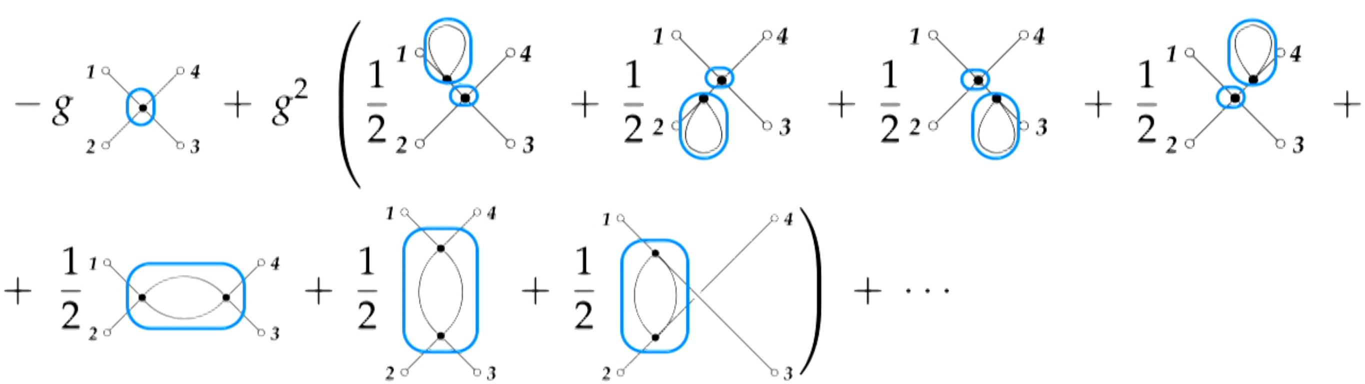
Connected 4 points function (up to order 2)



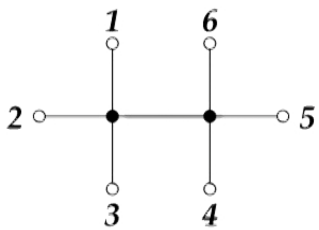
4 points function (up to order 1)



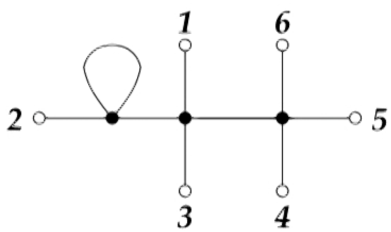
Connected 4 points function (up to order 2)

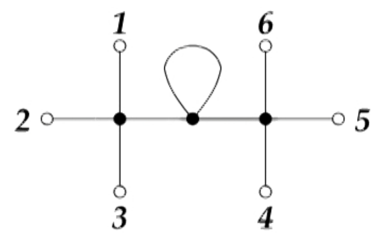


Connected 6 points function (up to order 3)

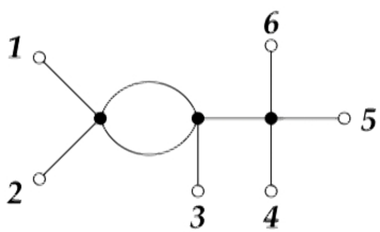
g^2 

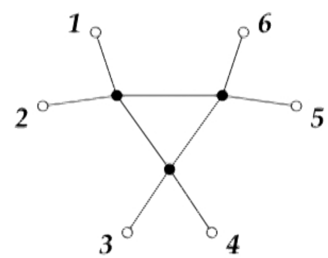
$+\cdots$

$-g^3\frac{1}{2}$ 

$+\cdots$ $-g^3\frac{1}{2}$ 

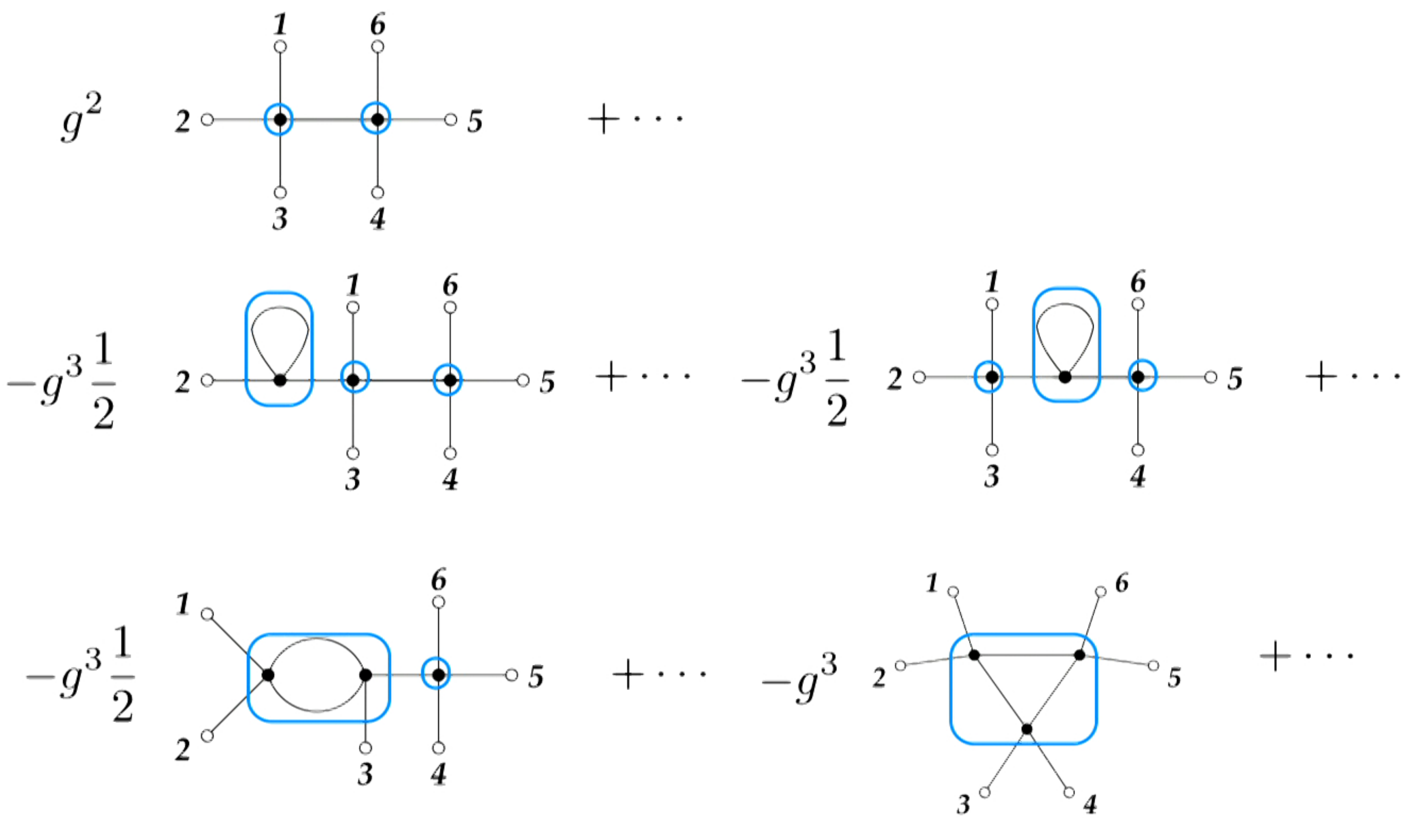
$+\cdots$

$-g^3\frac{1}{2}$ 

$+\cdots$ $-g^3$ 

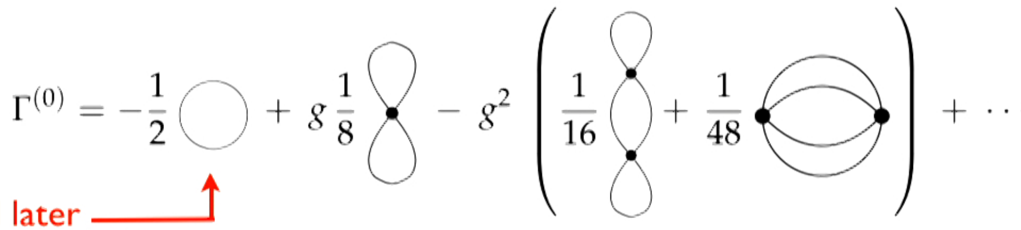
$+\cdots$

Connected 6 points function (up to order 3)



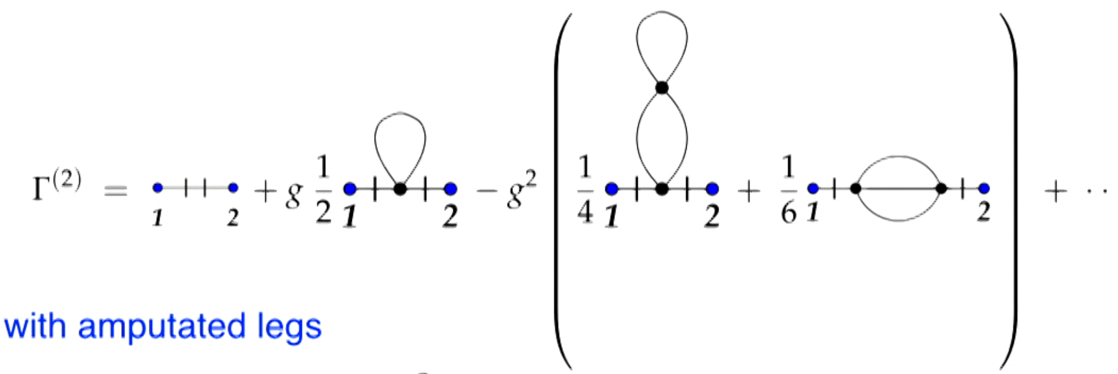
One Particle Irreducible (1PI) functions

Irreducible vacuum diagrams = vacuum self energy
(= connected vacuum diagrams for Φ^4)

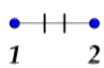


to be explained later

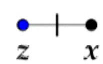
Irreducible 2 points function = inverse of the 2 points function



with amputated legs

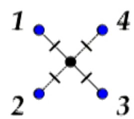
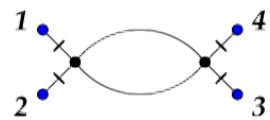


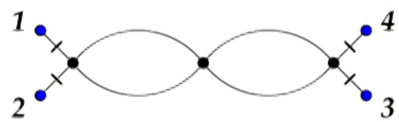
$$= (-\Delta + m^2)_{z_1 z_2}$$
$$= p^2 + m^2$$

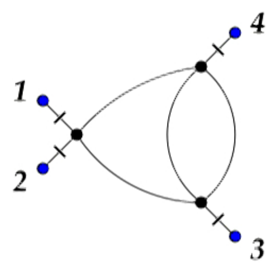


$$= \delta(z - x)$$
$$= 1$$

Irreducible 4 points function

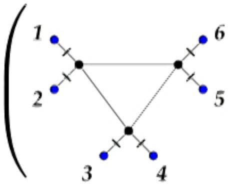
$\Gamma(z_1, \cdots z_4) = g$  $- \, g^2 \Big(\frac{1}{2}$  $+ \; 2 \text{ permutations} \Big)$

$+ \; g^3 \Big(\frac{1}{4}$  $+ \; 2 \text{ permutations} \Big)$

$+ \, g^3 \Bigg(\frac{1}{2}$  $+ \; 5 \text{ permutations} \Bigg) + \cdots$

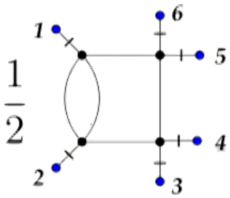
Irreducible 6 points function

$$\Gamma(z_1, \cdots z_6) \; = \; g^3 \; \Bigg($$



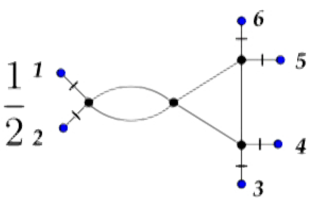
$$+ \; 14 \text{ permutations} \Bigg)$$

$$- \, g^4 \; \Bigg(\frac{1}{2}$$



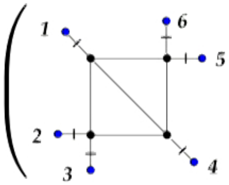
$$+ \; 89 \text{ permutations} \Bigg)$$

$$- \, g^4 \; \Bigg(\frac{1}{2}$$



$$+ \; 44 \text{ permutations} \Bigg)$$

$$- \, g^4 \; \Bigg($$



$$+ \; 44 \text{ permutations} \Bigg) \; + \; \cdots$$

Generating functions for perturbation theory and correlation functions

$$\sum_{N=0}^{\infty} \iint_{z's} j(z_1)\cdots j(z_N)\,\overline{\mathbb{Z}}\,(z_1, \cdots z_N)$$

source term $j(z)$ function of $z \in \mathbb{R}^d \longrightarrow$ real

Generating functions for perturbation theory and correlation functions

$$Z[j] = \sum_{N=0}^{\infty} \frac{1}{N!} \iint_{z's} j(z_1) \dots j(z_N) \overline{Z}(z_1, \dots z_N)$$

source term $j(z)$ function of $z \in \mathbb{R}^d \longrightarrow$ real

Generating functions for perturbation theory and correlation functions

$$\boxed{Z[j] = \sum_{N=0}^{\infty} \frac{1}{N!} \iint_{z's} j(z_1) \cdots j(z_N)\, \overline{\mathbb{Z}}(z_1, \cdots z_N)}$$

Functional of the source j

Source term $j(z)$ function of $z \in \mathbb{R}^d \longrightarrow$ real

Generating functions for perturbation theory and correlation functions

$$\boxed{Z[j] = \sum_{N=0}^{\infty} \frac{1}{N!} \iint_{z's} j(z_1) \cdots j(z_N)\, \overline{Z}(z_1, \dots z_N)}$$

Functional of the source j

<u>source term</u> $j(z)$ function of $z \in \mathbb{R}^d \longrightarrow$ real

⚠ do not forget $\hbar$

$$= \int d^d x \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right)$$

y_1 y_2 propagator (free) $G_0(y_1 - y_2) \xleftarrow{F.T} \dfrac{1}{K^2+m^2}$

vertices: interaction $(-g)\hbar^{-1}$ sym. factor integral

$$\ldots \phi^4(x_K)\rangle_0 = Z(z_1 \ldots z_N) = \sum_{\text{diagrams } G} (-g)^K \, C_G \, I_G(z_1 \ldots z_N)$$

$$= \int d^d x \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right)$$

y_1 y_2 propagator (free) $G_0(y_1 - y_2) \xleftarrow{F.T} \dfrac{\hbar}{k^2+m^2}$

vertices: interaction $(-g)\hbar^{-1}$ sym. factor integral

$$\langle \cdots \phi^4(x_k) \rangle_0 = Z(z_1 \cdot z_N) = \sum_{\text{diagrams } G} (-g)^k \, C_G \, I_G(z_1 \cdot\cdot z_N)$$

2

A bit of counting

Diagram with K internal vertices

L propagators

$\hbar^{L-K}$

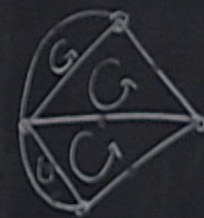
Euler. Any graph: connected

$$L - V = B - 1$$

vertices   # of internal loops

②

A bit of counting



$7-4=4-1$

Diagram with K internal vertices ✕

L propagators ——

$$\left(\frac{1}{\hbar}\right)^{L-K}$$

Euler: Any graph: connected

$$L - V = B - 1$$

V: # vertices

B: # of internal loops (1st Betti number)

if not connected

$$L-V=B-C$$

C: # connected components

for perturbation theory and correlation functions

$(z_1)\cdots j(z_N)\, Z(z_1, \ldots z_N)$

's

function of $z \in \mathbb{R}^d \longrightarrow$ real

Functional of the source j

For us

$V = K + N$

internal external

A bit of coun

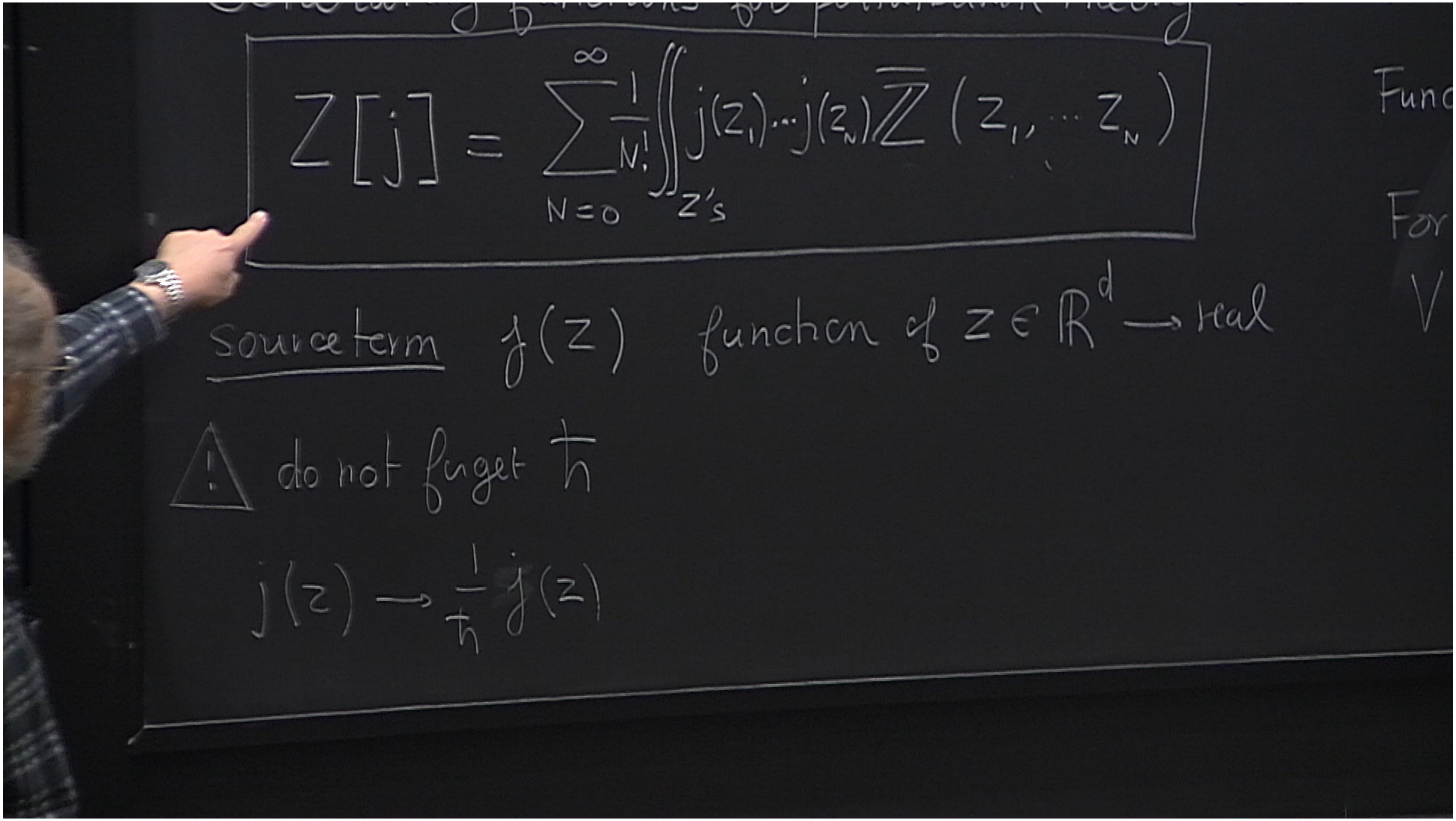
Diagram wi

$\hbar^{L-K}$

Euler. Any

$L - V$

ver



$Z[j] = \sum_{N=0}^{\infty} \frac{1}{N!} \iint_{z's} j(z_1) \dots j(z_N) \overline{Z}(z_1, \dots z_N)$
Source term $j(z)$ function of $z \in \mathbb{R}^d \longrightarrow$ real
do not forget $\hbar$
$j(z) \longrightarrow \frac{1}{\hbar} j(z)$
Func
For

Generating functions for perturbation theory and correlatio

$$\boxed{Z[j] = \sum_{N=0}^{\infty} \frac{1}{N!} \iint_{z's} \frac{j(z_1)\cdots j(z_N)}{\hbar^N} \bar{Z}(z_1, \ldots z_N)}$$

Function

For us

$V =$

interi

Source term $j(z)$ function of $z \in \mathbb{R}^d \longrightarrow$ real

⚠ do not forget $\hbar$

$$j(z) \longrightarrow \frac{1}{\hbar} j(z)$$

Generating functions for perturbation theory and correlatio

$$\boxed{Z[j] = \sum_{N=0}^{\infty} \frac{1}{N!} \iint_{z's} \frac{j(z_1)\cdots j(z_N)}{\hbar^N} \overline{Z}(z_1, \cdots z_N)}$$

Functiona

For us

$V =$

interi

Source term $j(z)$ function of $z \in \mathbb{R}^d \longrightarrow$ real

⚠ do not forget $\hbar$

$j(z) \longrightarrow \frac{1}{\hbar} j(z)$

A Diagram $G \longrightarrow \hbar^{L-V}$

Generating functions for perturbation theory and correlatio

$$\boxed{Z[j] = \sum_{N=0}^{\infty} \frac{1}{N!} \iint_{z's} \frac{j(z_1) \cdots j(z_N)}{\hbar^N} \bar{Z}(z_1, \dots z_N)}$$

Function a

For us

$V = $

inter

Source term $j(z)$ function of $z \in \mathbb{R}^d \longrightarrow$ real

⚠ do not forget $\hbar$

$j(z) \longrightarrow \frac{1}{\hbar} j(z)$ A Diagram $G \longrightarrow \hbar^{L-V} = \hbar^{B-1}$

for perturbation theory and correlation functions

$$\frac{(z_1)\cdots j(z_N)}{\hbar^N} \overline{Z}(z_1, \cdots z_N)$$

Functional of the source j

function of $z \in \mathbb{R}^d \longrightarrow$ real

For us

$$V = K + N$$

internal external

A Diagram $G \longrightarrow \hbar^{L-V} = \hbar^{B-1}$ $B = \#$ loops of the diagram

A bit of com

Diagram wi

$\hbar^{L-K}$

Euler. Any

$L - V$

#ve

$$\langle \ldots (z_N) \rangle = \frac{Z(z_1 \ldots z_N)}{Z}$$

N pt function   N=0 vacuum diagrams

0 pt. function

$$\frac{\circ\!\!-\!\!\circ - g\left[\left(\frac{1}{2}\right) + \frac{1}{8}\right] + O(g^2)}{1 - g\,\frac{1}{8} + O(g^2)} = \circ\!\!-\!\!\circ - g\,\frac{1}{2}$$

perturbation theory is a topological expansion $= \sum \hbar^B$

$B = \#$ loops

diagrams G

Functional Integral definition : general

$$Z[j] = \int D[\phi] \exp\left(-\frac{1}{\hbar}\left(S[\phi] - j\cdot\phi\right)\right)$$ where $j \cdot \phi$

All diagrams (not connected, vacuum diagrams included)

diagrams G

1. Functional Integral definition : general

$$Z[j] = \int D[\phi] \, \exp\left(-\frac{1}{\hbar}\left(S[\phi] - j\cdot\phi\right)\right)$$

where

All diagrams (not connected, vacuum diagrams included)

x_s diagrams G

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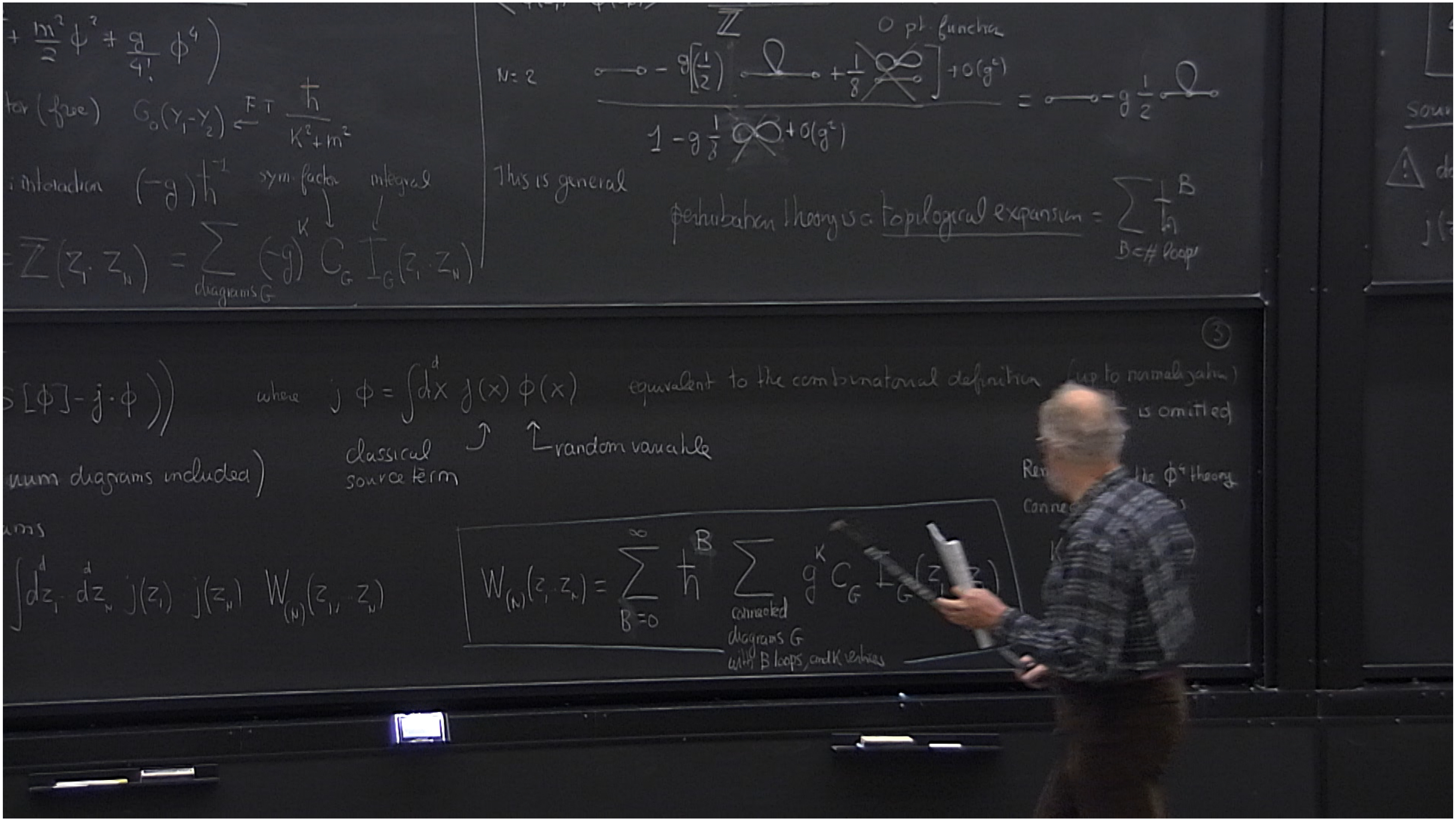
0 pt. function
N=2
This is general
perturbation theory is a topological expansion
B = # loops
tor (free)
F.T.
interaction
sym. factor
integral
diagrams G
where
classical source term
random variable
equivalent to the combinatorial definition (up to normalization)
is omitted
num diagrams included)
connected diagrams G with B loops, and K vertices

$G_0(y_1-y_2) \xleftarrow{F.T} \frac{\hbar}{K^2+m^2}$

$(-g)\,\hbar^{-1}$ sym. factor integral

$) = \sum_{\text{diagrams } G} (-g)^K C_G \, I_G(z_1 \cdots z_N))$

This is general

$1 - g\frac{1}{8} + O(g^2)$

$= \circ\!-\!\circ - g\frac{1}{2}$

perturbation theory is a topological expansion $= \sum_{B=\#\text{ loops}} \hbar^B$

(5)

$))$ where $j\cdot\phi = \int d^dx \; j(x)\,\phi(x)$ equivalent to the combinatorial definition (up to normalization)

$\frac{1}{Z_0}$ is omitted

classical source term

random variable

ms included)

Remark. For the ϕ^4 theory connected diagrams

$K = B + (N-2)$

$(z_1) \cdots j(z_N) \; W_{(N)}(z_1, \cdots z_N)$

$$\boxed{W_{(N)}(z_1 \cdots z_N) = \sum_{B=0}^{\infty} \hbar^B \sum_{\substack{\text{connected}\\ \text{diagrams } G\\ \text{with } B \text{ loops, and } K \text{ vertices}}} g^K C_G \, I_G(z_1 \cdots z_N)}$$

$G(z_1 \ldots z_N)$

$B = \#$ loops

③

$= \int d^dx \, J(x) \, \phi(x)$ equivalent to the combinatorial definition (up to normalization)

classical force term

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Remark. For the ϕ^4 theory connected diagrams

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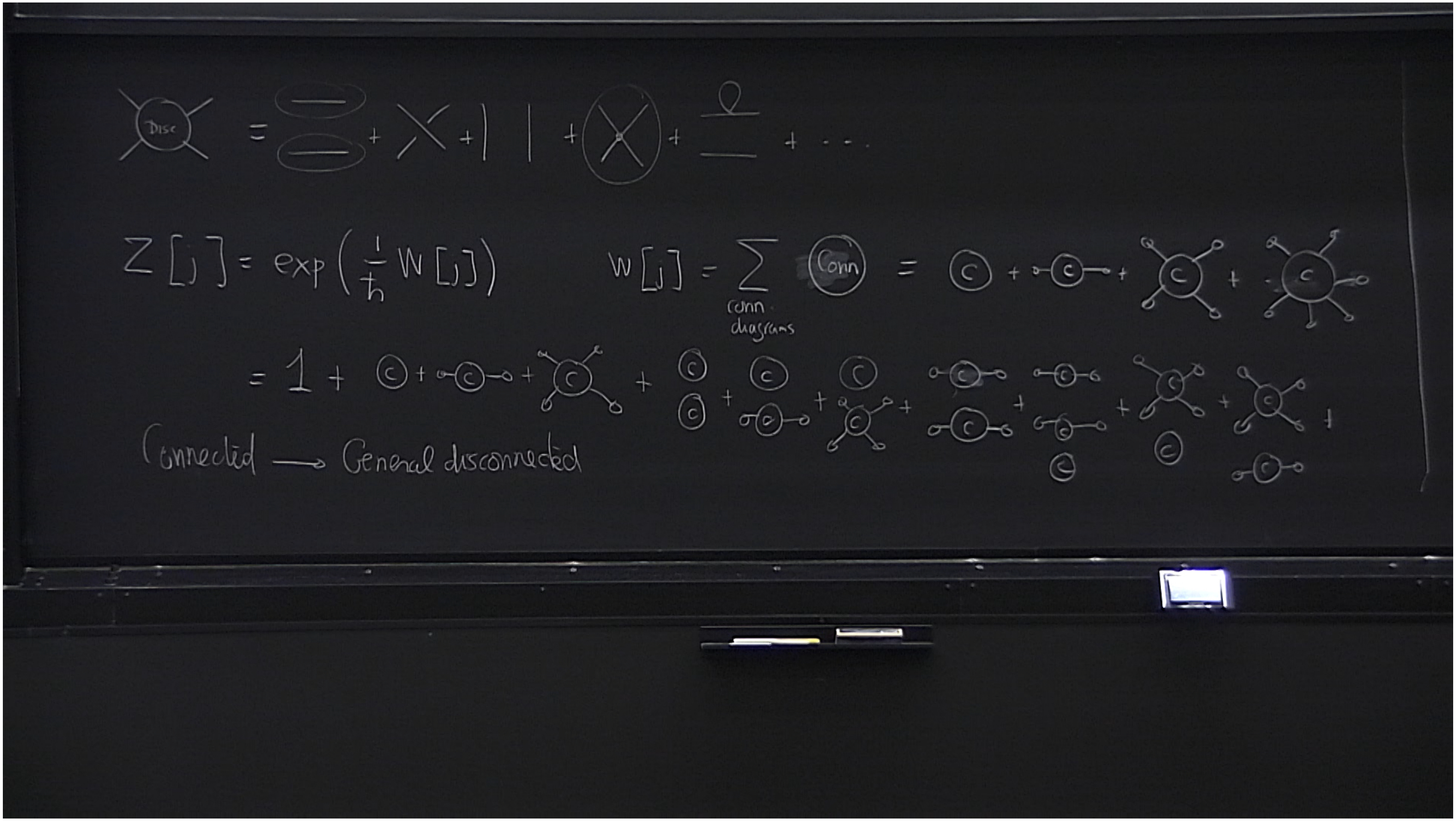
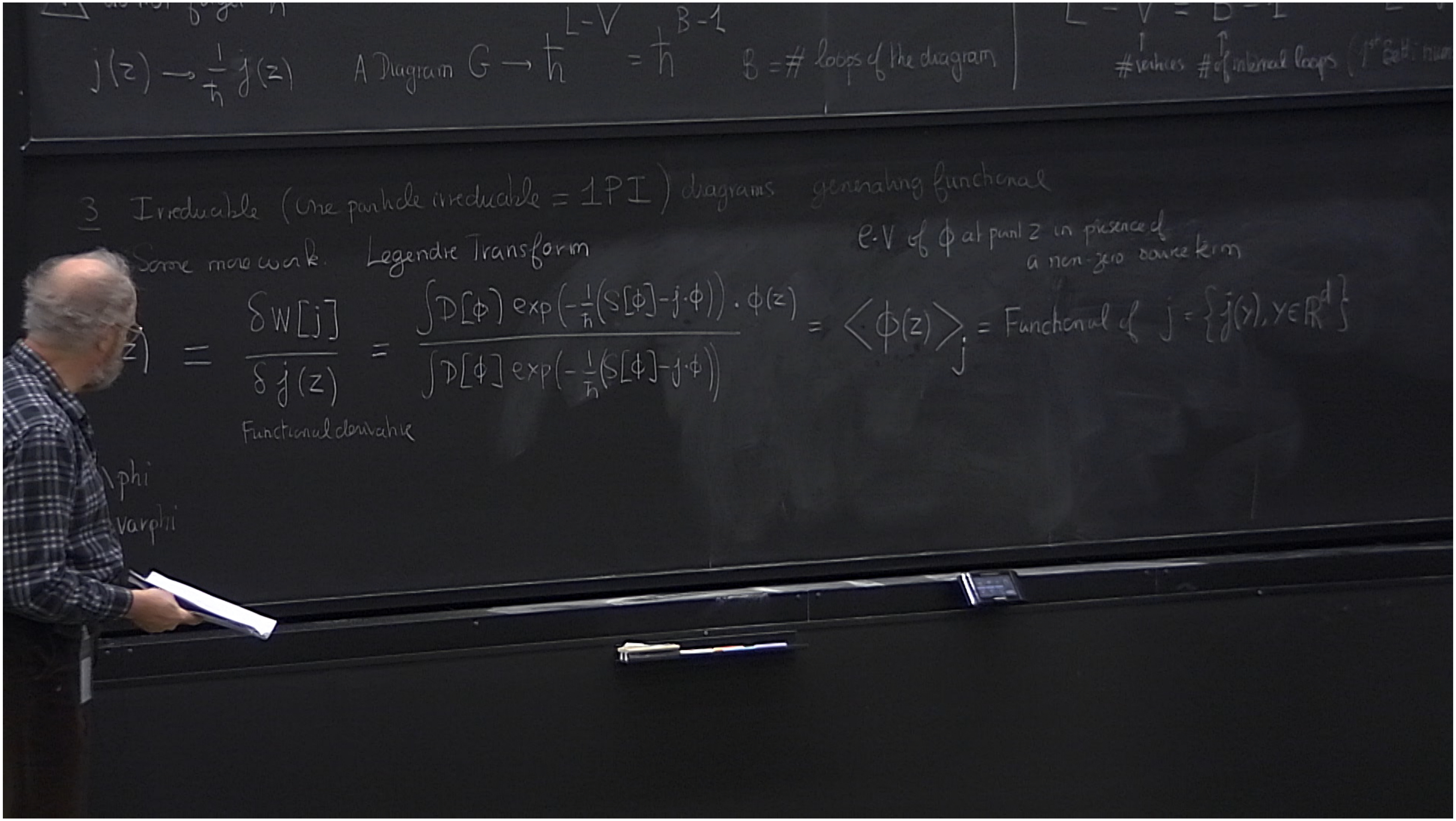
$K = B + \left(\frac{N-2}{2}\right)$

Ex: Check this identity

Ex. what about ϕ^3


Disc
Z[j] = exp(1/ħ W[j])
W[j] = Σ Conn
conn diagrams
Connected → General disconnected


j(z) → 1/ħ j(z)
A Diagram G → ħ^{L-V} = ħ^{B-1}
B = # loops of the diagram
L - V = B - 1
#vertices
# of internal loops
(1st Betti num
3 Irreducible (one particle irreducible = 1PI) diagrams generating functional
Some more work. Legendre Transform
e.V of φ at point z in presence of
a non-zero source term
δW[j] / δj(z) = ∫D[φ] exp(-1/ħ(S[φ]-j·φ)) · φ(z) / ∫D[φ] exp(-1/ħ(S[φ]-j·φ)) = ⟨φ(z)⟩_j = Functional of j = {j(y), y ∈ R^d}
Functional derivative
\phi
\varphi

$j(z) \to \frac{1}{\hbar} j(z)$ A Diagram $G \to \hbar^{L-V} = \hbar^{B-1}$ B = # loops of the diagram

$L - V = B - 1$

vertices # of internal loops (1st Betti num

3 Irreducible (one particle irreducible = 1PI) diagrams generating functional

Some more work. Legendre Transform

e.V of ϕ at point z in presence of a non-zero source term

$$\varphi(z) = \frac{\delta W[j]}{\delta j(z)} = \frac{\int D[\phi] \exp\left(-\frac{1}{\hbar}(S[\phi] - j\cdot\phi)\right) \cdot \phi(z)}{\int D[\phi] \exp\left(-\frac{1}{\hbar}(S[\phi] - j\cdot\phi)\right)} = \langle \phi(z) \rangle_j = \text{Functional of } j = \{j(y), y \in \mathbb{R}^d\}$$

Functional derivative

ϕ = \phi

φ = \varphi

#vertices #of internal loops

3 Irreducible (One particle irreducible = 1PI) diagrams generating functional

Some more work. Legendre Transform

e.v of ϕ at point z in presence of a non-zero source term

$$\boxed{\underset{\text{classical}}{\varphi(z)} = \underset{\text{Functional derivative}}{\frac{\delta W[j]}{\delta j(z)}}} = \frac{\int D[\phi]\, \exp\left(-\frac{1}{\hbar}\left(S[\phi]-j\cdot\phi\right)\right)\cdot\phi(z)}{\int D[\phi]\, \exp\left(-\frac{1}{\hbar}\left(S[\phi]-j\cdot\phi\right)\right)} = \boxed{\langle \phi(z) \rangle_j} = \text{Functional of } j = \{j(y), y\in\mathbb{R}^d\}$$

random variable

ϕ = \phi

φ = \varphi

$\varphi(z)$ Background field: $\simeq$ e.v of quantum field ϕ when $j \neq 0$

Legendre transform.

φ is the conjugate variable to j

$$W[j] \xrightarrow[L.Tr.]{} \boxed{\Gamma[\varphi] = j\cdot\varphi - W[j]} \;, \quad j\cdot\varphi = \int d^dz\, j(z)\,\varphi(z)$$

$\varphi = \langle \Phi \rangle_j$ functional of j $\longrightarrow$ j is the inverse functional of φ

$$j = j_\varphi$$

Legendre transform

φ is the conjugate variable to j

$$W[j] \xrightarrow[\text{L. Tr}]{} \boxed{\Gamma[\varphi] = j\cdot\varphi - W[j]} \quad , \quad j\cdot\varphi = \int d^d z\, j(z)\,\varphi(z)$$

$\varphi = \langle \phi \rangle_j$ functional of j $\longrightarrow$ j is the inverse functional of φ : which one ?

$$\varphi = \frac{\delta W}{\delta j} \qquad j = j_\varphi$$

Legendre transform

φ is the conjugate variable to j

$$W[j] \xrightarrow[\text{L.Tr}]{} \boxed{\Gamma[\varphi] = j\cdot\varphi - W[j]}\ , \quad j\cdot\varphi = \int d^dz\, j(z)\varphi(z)$$

$\varphi = \langle\phi\rangle_j$ functional of j $\longrightarrow$ j is the inverse functional of φ . which one ?

$$\boxed{\varphi(z) = \frac{\delta W[j]}{\delta j(z)}} \qquad j = j_\varphi \Rightarrow \boxed{j(z) = \frac{\delta \Gamma[\varphi]}{\delta\varphi(z)}}$$

consequence of $x_i \to y_i$ $\quad J_{ij} = \frac{\partial y_i}{\partial x_j}$, $\quad J'_{ij} = \frac{\partial x_i}{\partial y_j} = (J^{-1})_{ij}$

with B loops, and K vertices

Ex:

(5)

$\Gamma[\varphi]$ is the (quantum) effective action

z)

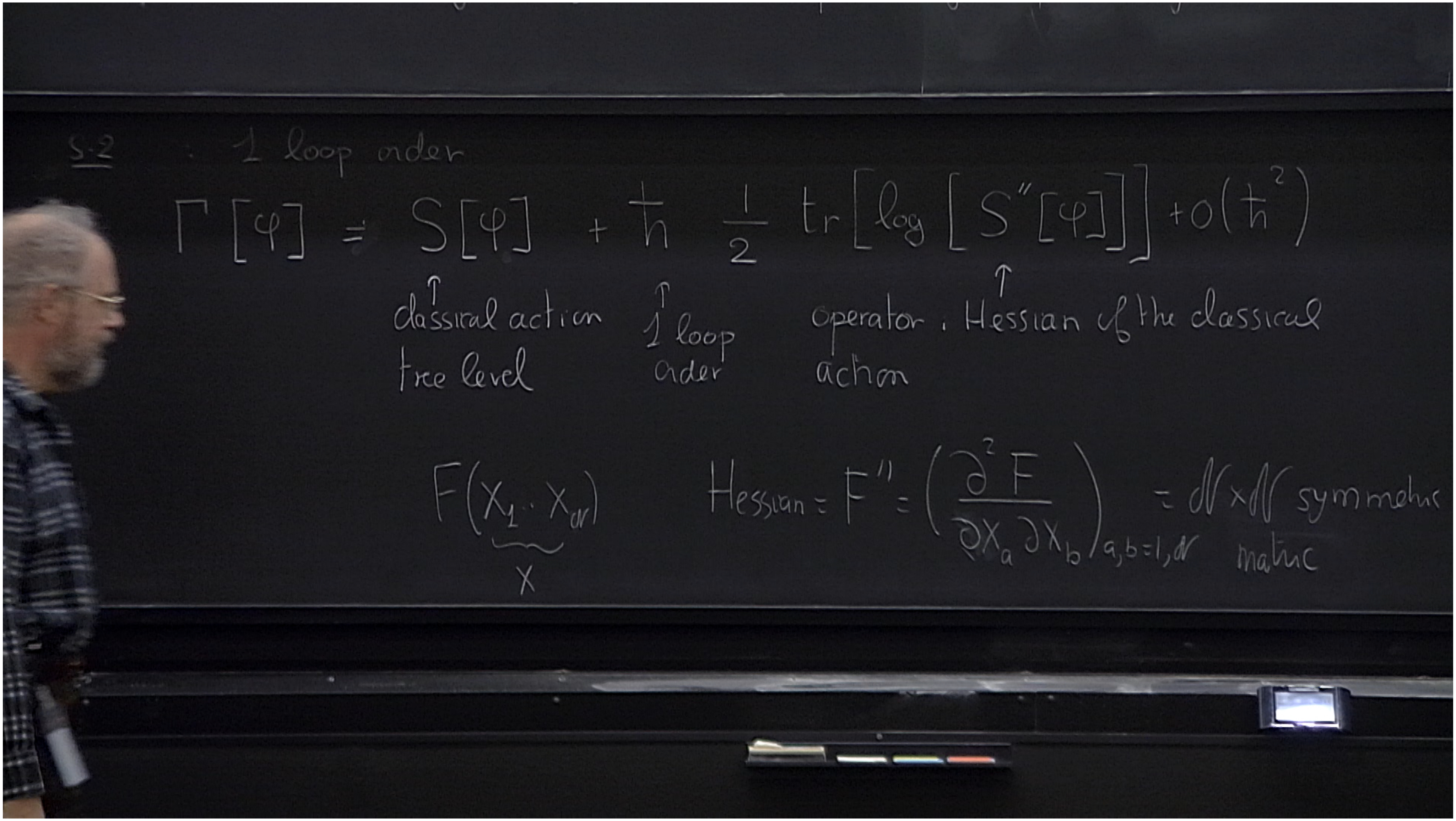
2 statements

$\Gamma[\varphi]$ is the generating functional of irreducible (1PI) diagrams

$$\Gamma[\varphi] = \sum_N \frac{1}{N!} \int d^d z_1 \cdots d^d z_N \, \varphi(z_1) \cdots \varphi(z_N) \cdot \Gamma_{(N)}(z_1 \cdots z_N)$$

$$\Gamma_{(N)}(z_1 \cdots N_N) = \sum_{\text{1PI diagrams}} \hbar^B \, C_G \, I_G(z_1 \cdots z_N)$$

amplitude of 1PI graph with N legs amputated


S.2 : 1 loop order
$\Gamma[\varphi] = S[\varphi] + \hbar \frac{1}{2} \mathrm{tr}\left[\log\left[S''[\varphi]\right]\right] + O(\hbar^2)$
classical action
tree level
1 loop
order
operator : Hessian of the classical
action
$F(X_1 .. X_d)$
X
Hessian $= F'' = \left(\frac{\partial^2 F}{\partial X_a \partial X_b}\right)_{a,b=1,d}$ $= d \times d$ symmetric
matrix

§2 . 1 loop order

$$\Gamma[\varphi] \doteq S[\varphi] + \hbar \, \frac{1}{2} \, \mathrm{tr}\left[\log\left[S''[\varphi]\right]\right] + O(\hbar^2)$$

$S[\varphi]$: classical action, tree level

$\hbar$: 1 loop order

$S''[\varphi]$: operator, Hessian of the classical action

$$F(\underbrace{x_1 .. x_d}_{X}) \qquad \text{Hessian} = F'' = \left(\frac{\partial^2 F}{\partial x_a \partial x_b}\right)_{a,b=1,d} = d \times d \text{ symmetric matrix}$$

$S'' =$

S''

oper

S'' = Linear operator whose Kernel $S''(x_1, x_2)$ is

$$S''(x_1, x_2) = \frac{\delta^2 S[\varphi]}{\delta\varphi(x_1)\,\delta\varphi(x_2)}$$

function of x_1, x_2 (in fact it is a distribution)

functional of φ

Operator

ψ

$$S'' \cdot \psi = \tilde{\psi} \qquad \tilde{\psi}(x_1) = \int d^d x_2 \; S''(x_1, x_2)\, \psi(x_2)$$

integral Kernel

4

$S'' =$ Linear operator whose Kernel $S''(x_1, x_2)$ is

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$\psi \qquad S'' \cdot \psi = \tilde{\psi} \qquad \tilde{\psi}(x_1) = \int d^d x_2 \; S''(x_1, x_2) \cdot \psi(x_2)$

integral Kernel

...ahir $\operatorname{tr} \log S'' = \log \det S''$: functional of φ

$\phi = \backslash\text{phi}$

$\varphi = \backslash\text{varphi}$ $\varphi(z)$ Background field: $\simeq$ e.v of quantum field ϕ when $j \neq 0$

S.2 . 1 loop order

$$\boxed{\Gamma[\varphi] = S[\varphi] + \hbar \, \frac{1}{2} \, \mathrm{tr}\left[\log\left[S''[\varphi]\right]\right] + O(\hbar^2)}$$

classical action, tree level

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