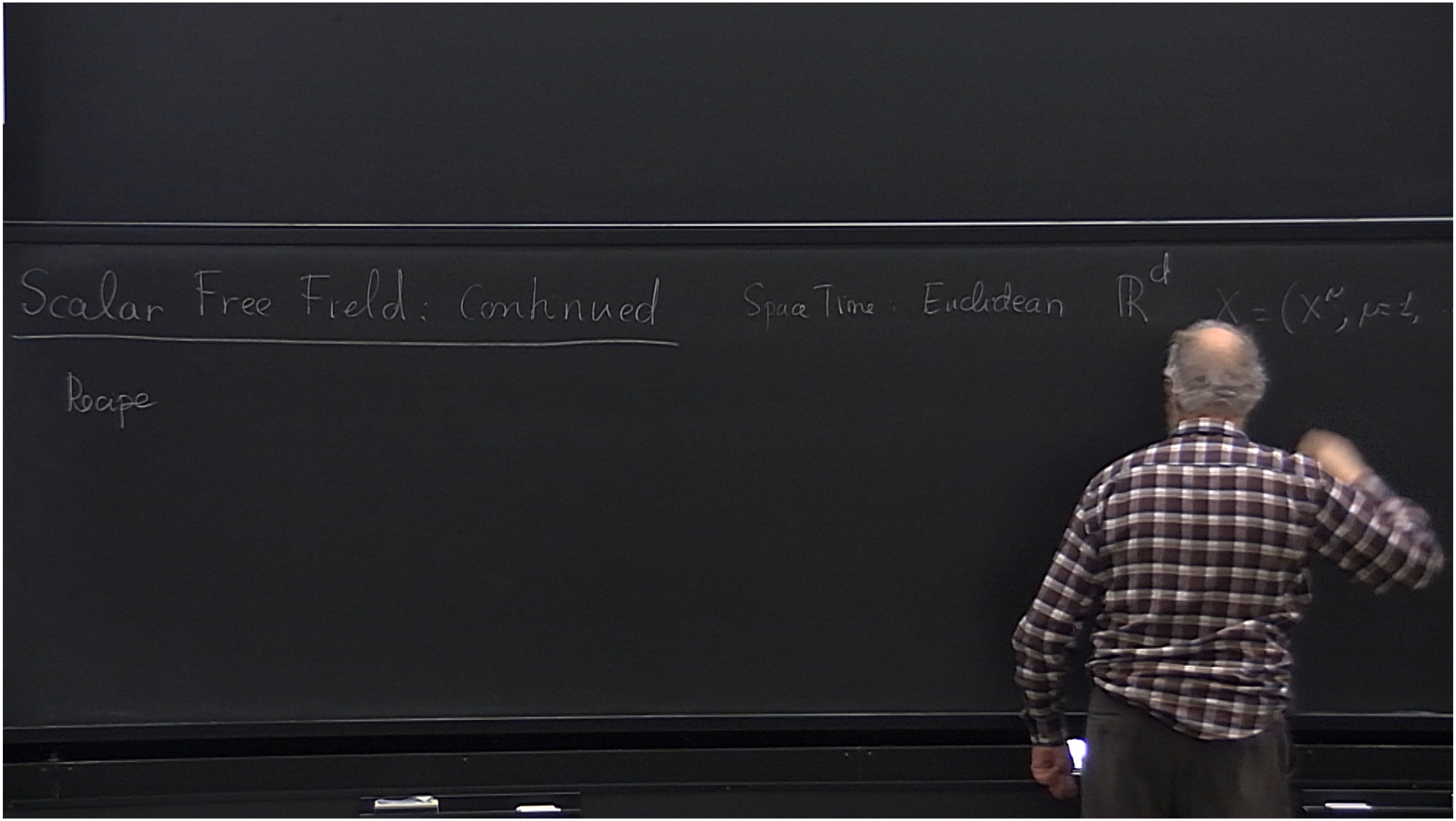
Title: PSI 17/18 - Quantum Field Theory II - Lecture 3

Date: Nov 08, 2017 09:00 AM

URL: http://pirsa.org/17110013

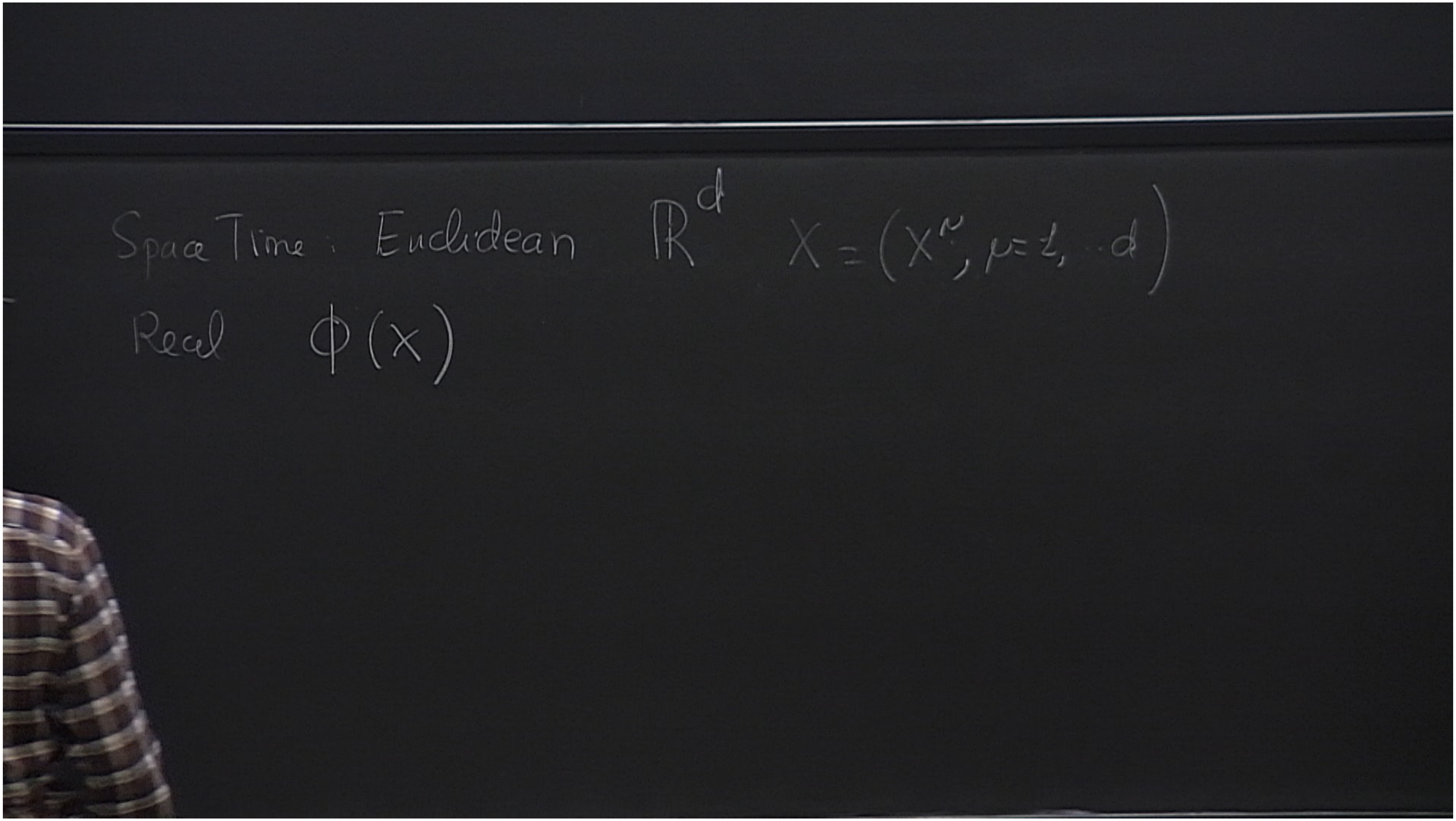
Abstract:



# Scalar Free Field: Continued

Space Time: Euclidean $\mathbb{R}^d$ $X = (X^\mu; \mu = 1,$

Recap



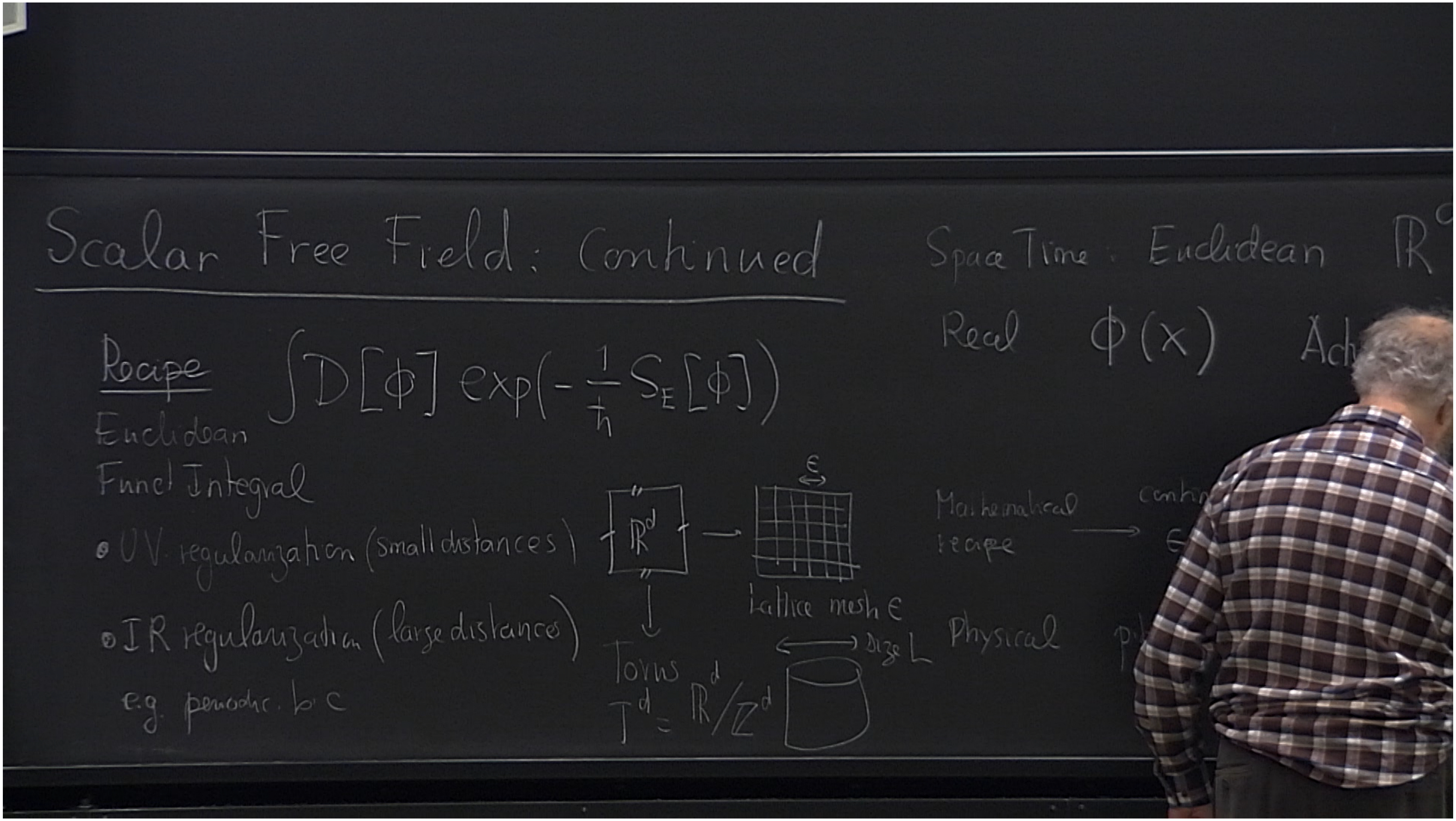
Space Time : Euclidean $\mathbb{R}^d$ $x = (x^\mu, \mu = 1, \dots d)$

Real $\phi(x)$

mass of the quanta
↓

Space Time: Euclidean $\mathbb{R}^d$ $\quad X = (X^\mu, \mu = 1, \dots d)$

Real $\phi(x)$ $\quad$ Action $S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$



# Scalar Free Field: Continued

Space Time: Euclidean $\mathbb{R}^d$

Real $\phi(x)$ Ach

**Recipe** $\int D[\phi] \exp\left(-\frac{1}{\hbar} S_E[\phi]\right)$

Euclidean Funct Integral

- U.V. regularization (small distances)
- IR regularization (large distances)

e.g. periodic b.c.

$\mathbb{R}^d \longrightarrow$ lattice mesh $\epsilon$

$\downarrow$

Torus $T^d = \mathbb{R}^d/\mathbb{Z}^d$ size L

Mathematical recipe $\longrightarrow$ contin

$\epsilon$

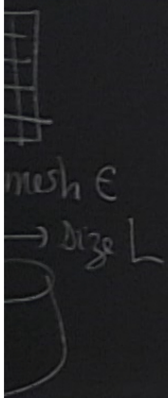
Physical

mass of the quan

Space Time : Euclidean $\mathbb{R}^d$ $X = (X^\mu, \mu = 1, \ldots d)$

Real $\phi(x)$ Action $S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$

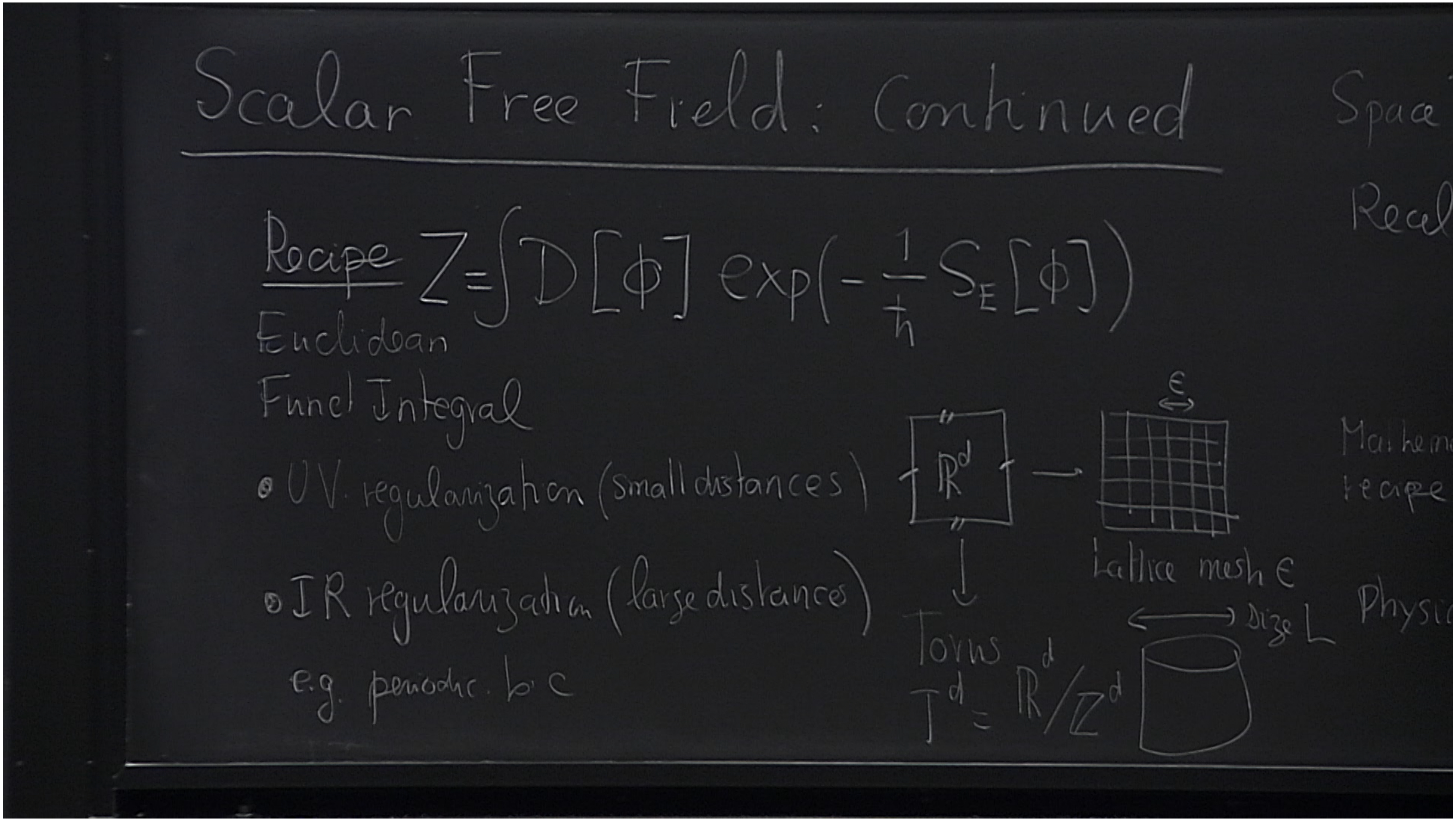
Mathematical recipe $\longrightarrow$ continuum limit $\epsilon \to 0$



Physical

p.b.c in Euclidean Time $\longleftrightarrow$ Finite temperature

p.b.c in Space directions $\longleftrightarrow$ Q Field in a finite Box



# Scalar Free Field: Continued

**Recipe** $Z = \int D[\phi] \exp\left(-\frac{1}{\hbar} S_E[\phi]\right)$

Euclidean

Funct Integral

- UV regularization (small distances)
- IR regularization (large distances)

e.g. periodic b.c

Spac

Real

Mathem

recipe

Physic

mass of the quanta

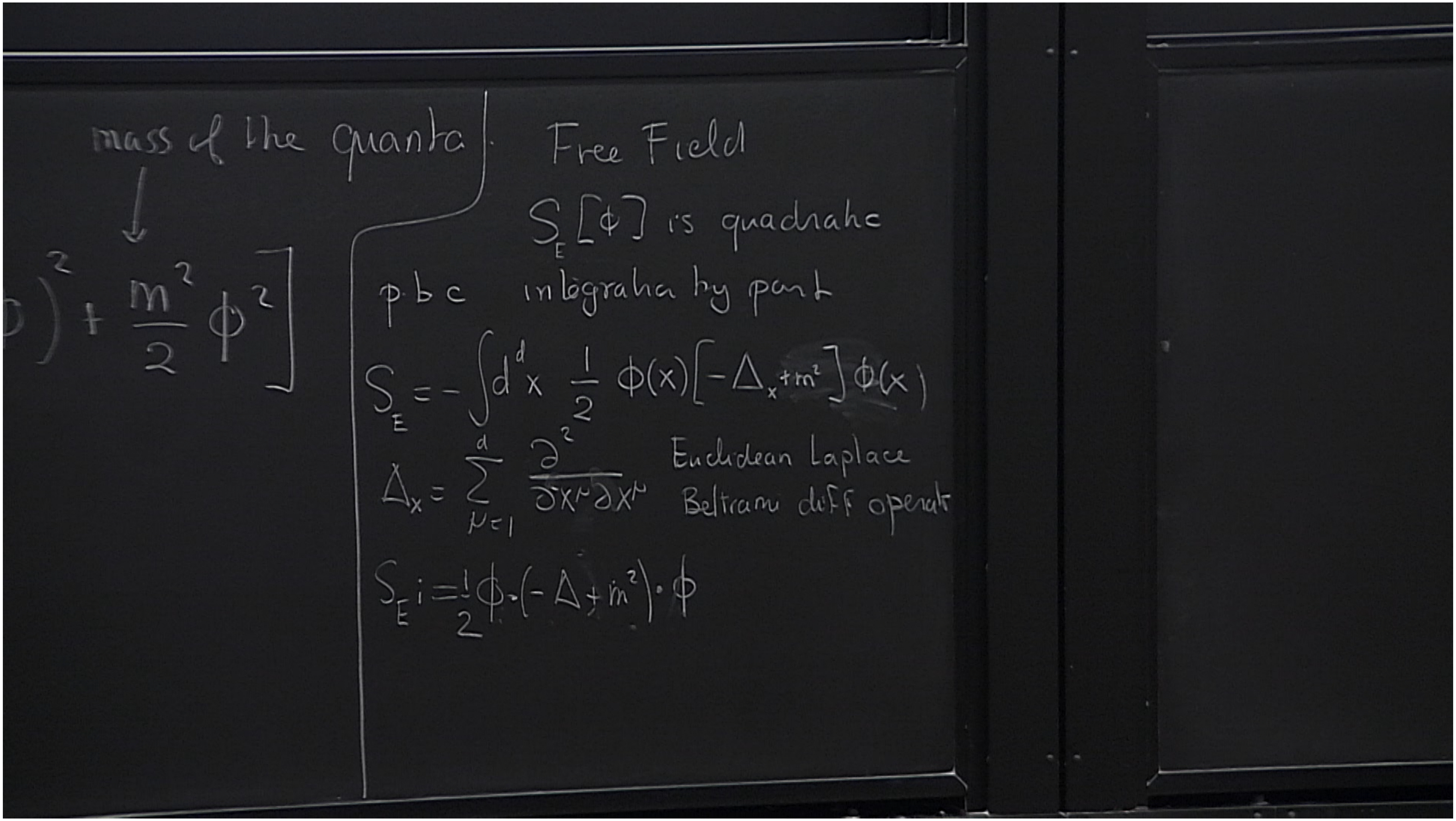
$$)^2 + \frac{m^2}{2}\phi^2\Big]$$

Free Field

$S_E[\phi]$ is quadratic

p.b.c integration by part

$$S_E = -\int d^dx \ \frac{1}{2}\,\phi(x)\Big[-\Delta_x \cdot \phi(x)\Big]$$



mass of the quanta

$$\phi)^2 + \frac{\overset{\downarrow}{m^2}}{2}\phi^2\Big]$$

Free Field

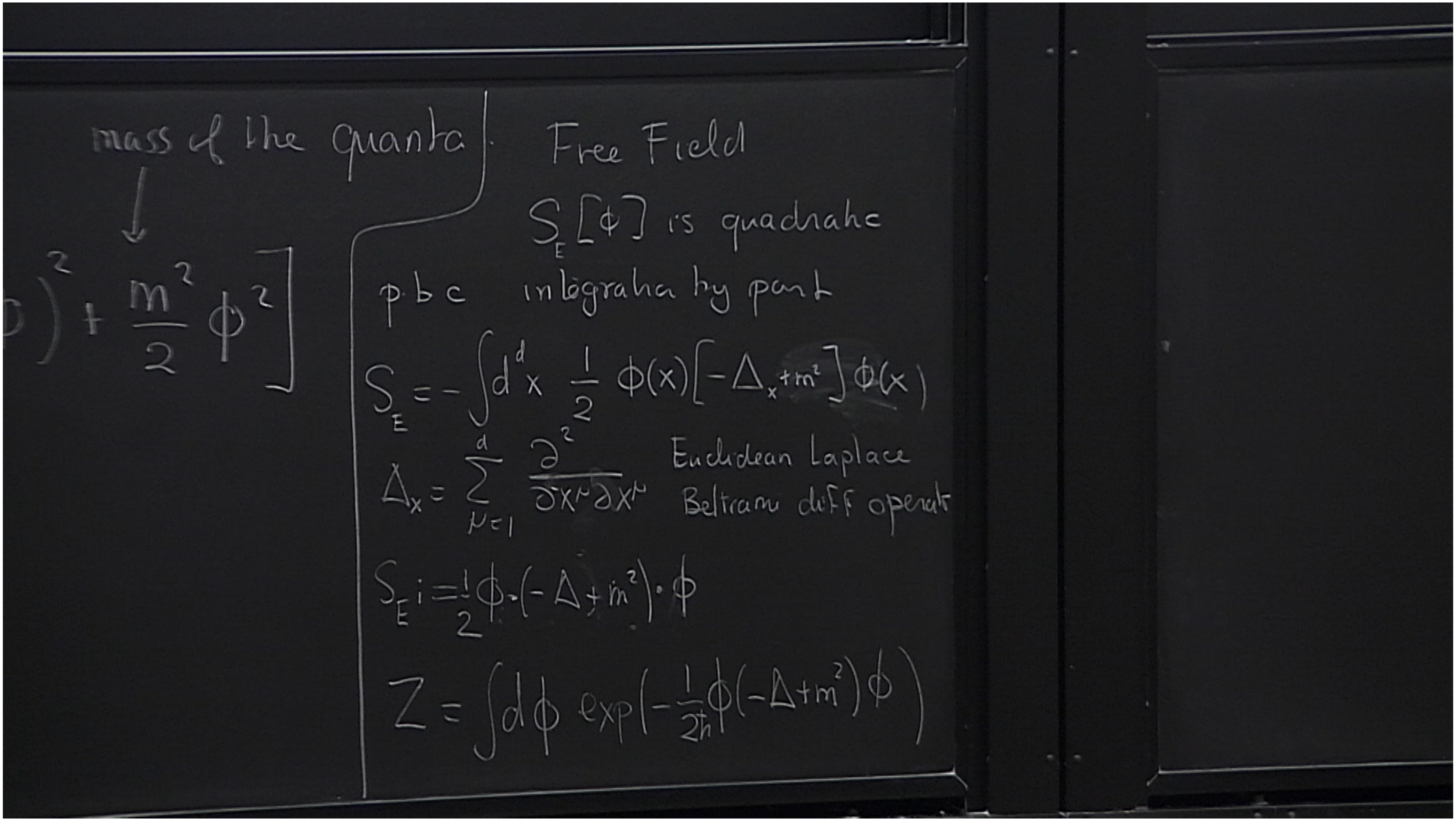
$S_E[\phi]$ is quadratic

p.b.c integration by part

$$S_E = -\int d^dx\ \frac{1}{2}\phi(x)\left[-\Delta_x + m^2\right]\phi(x)$$

$$\Delta_x = \sum_{\mu=1}^{d} \frac{\partial^2}{\partial x^\mu \partial x^\mu}$$ Euclidean Laplace Beltrami diff operat

$$S_E := \frac{1}{2}\phi\cdot(-\Delta + m^2)\cdot\phi$$



mass of the quanta

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Free Field

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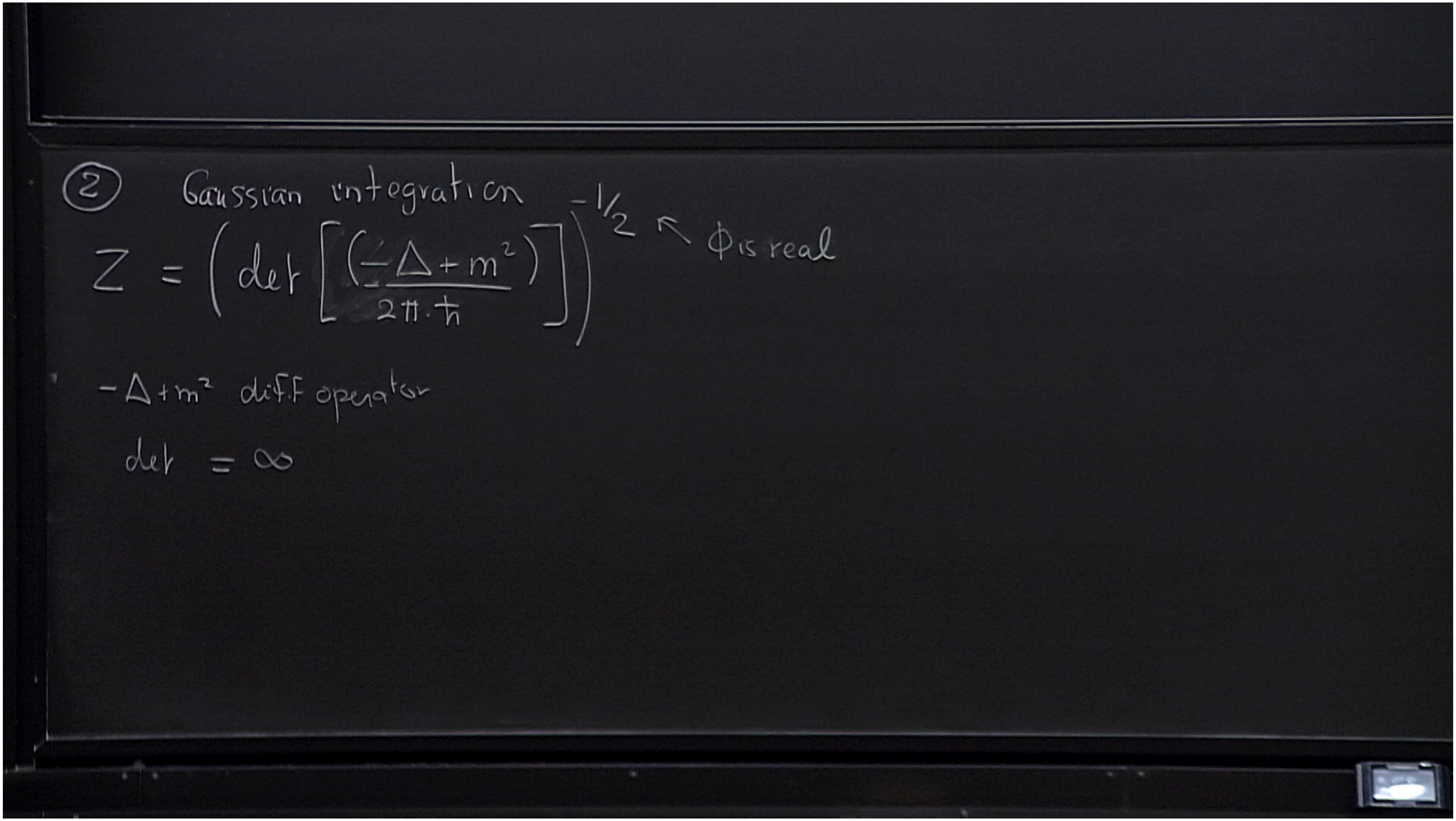
$$S_E := \frac{1}{2}\phi\cdot(-\Delta + m^2)\cdot\phi$$

$$Z = \int d\phi \, \exp\left(-\frac{1}{2\hbar}\phi(-\Delta + m^2)\phi\right)$$

② Gaussian integration

$$Z = \left( \det \left[ \frac{(-\Delta + m^2)}{2\pi \cdot \hbar} \right] \right)^{-1/2} \nwarrow \phi \text{ is real}$$

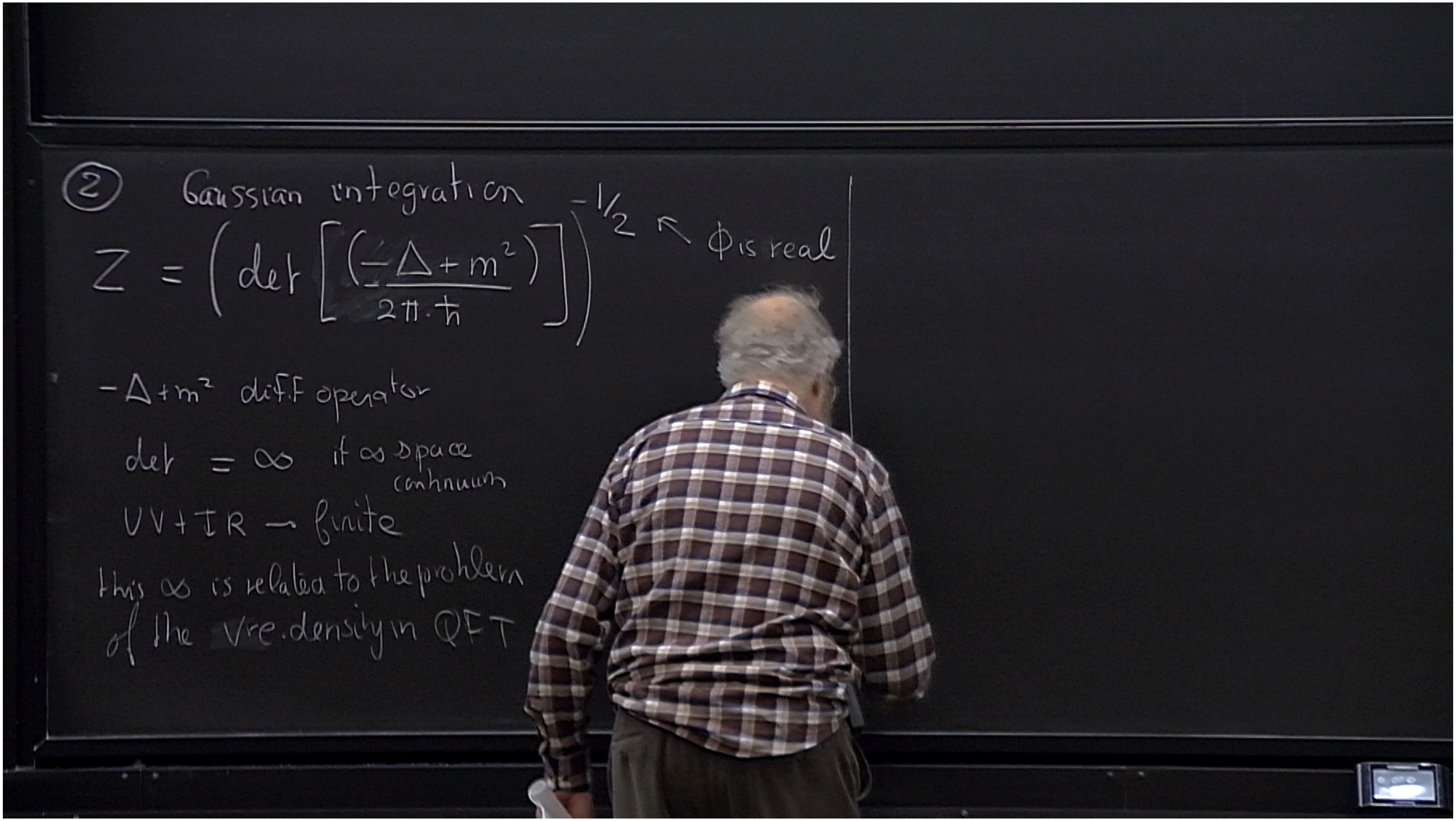


② Gaussian integration

$$Z = \left(\det\left[\left(\frac{-\Delta + m^2}{2\pi\cdot\hbar}\right)\right]\right)^{-1/2} \nwarrow \phi \text{ is real}$$

$-\Delta + m^2$ diff operator

$\det = \infty$

</br>

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$-\Delta + m^2$ diff.f operator

det $= \infty$ if $\infty$ space continuum

UV + IR → finite

this $\infty$ is related to the problem of the vre. density in QFT al

Correlation functions :

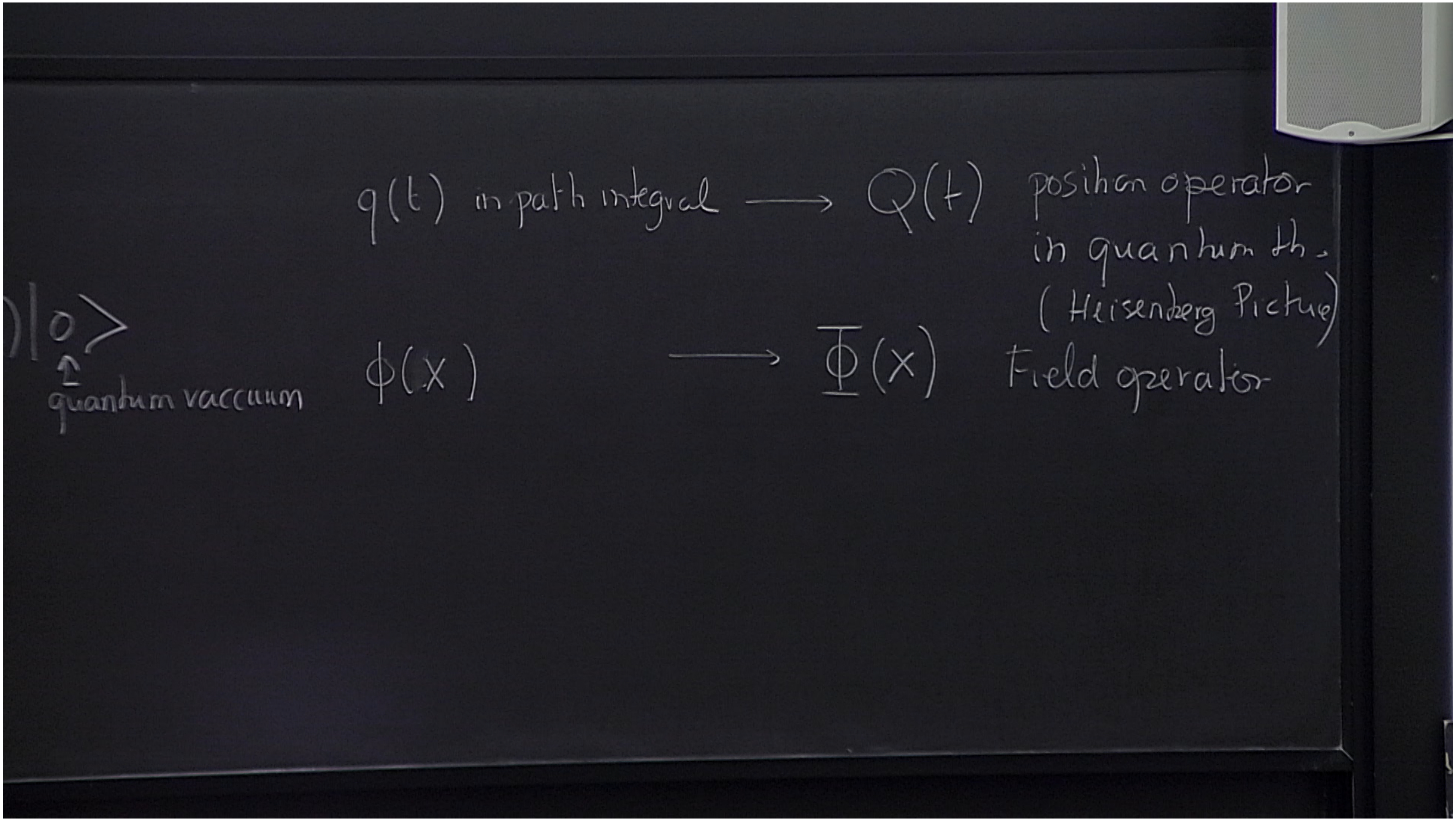
Path Integral $\longrightarrow \langle 0|T(A_1 A_2)|0\rangle$

q(t) in path integral

Functional integral $\longrightarrow \langle 0|\,T(\underset{\uparrow}{\Phi}(x_1)\,\underset{\uparrow}{\Phi}(X_2)|\underset{\uparrow}{0}\rangle$

Field Operator

quantum vaccuum

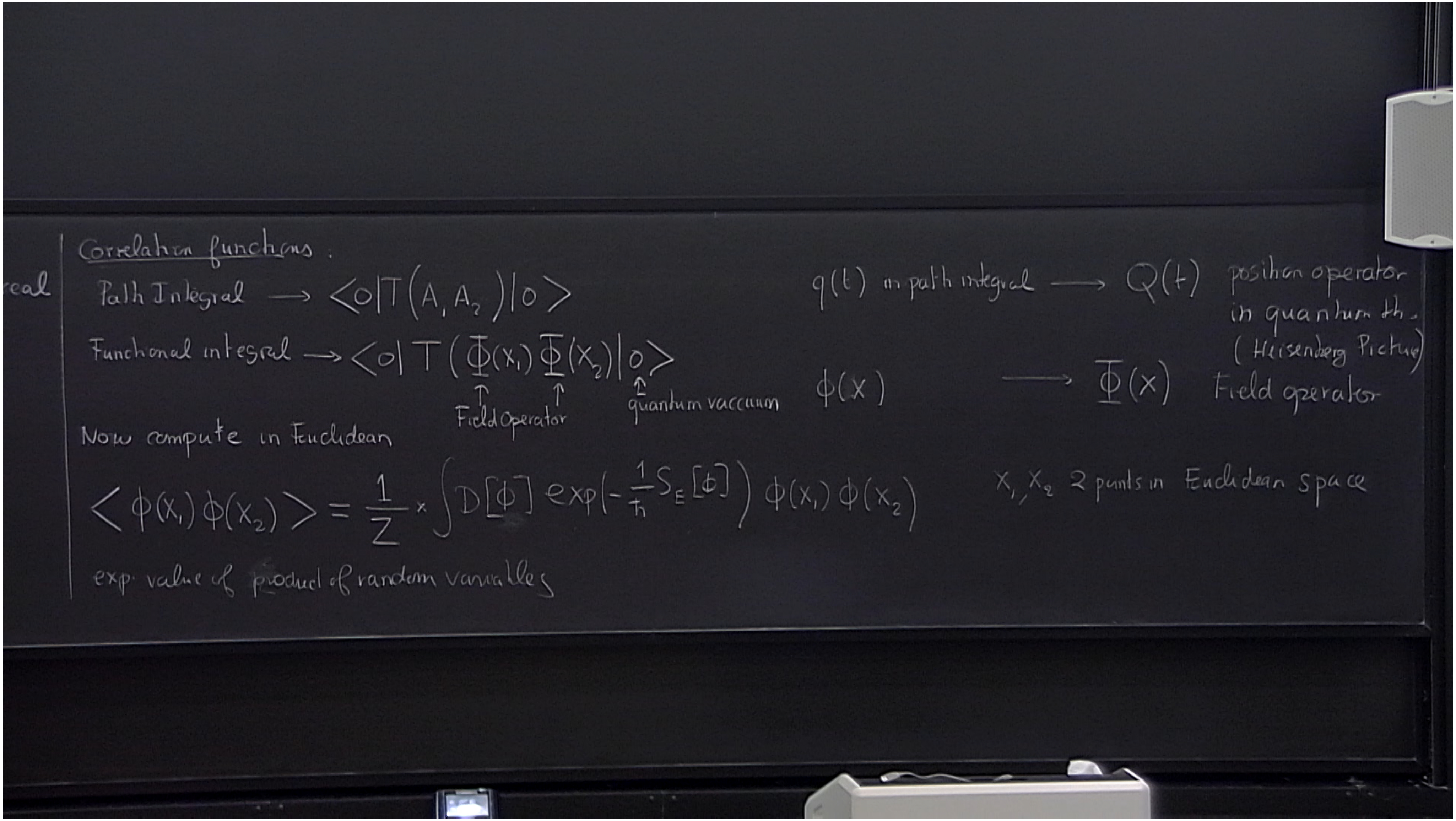


$q(t)$ in path integral $\longrightarrow$ $Q(t)$ position operator in quantum th. (Heisenberg Picture)

$)|0\rangle$

↑ quantum vaccuum

$\phi(x) \longrightarrow \underline{\Phi}(x)$ Field operator



real

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Path Integral $\longrightarrow \langle 0|T(A_1 A_2)|0\rangle$

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Field Operator

quantum vaccuum

$q(t)$ in path integral $\longrightarrow Q(t)$ position operator in quantum th. (Heisenberg Picture)

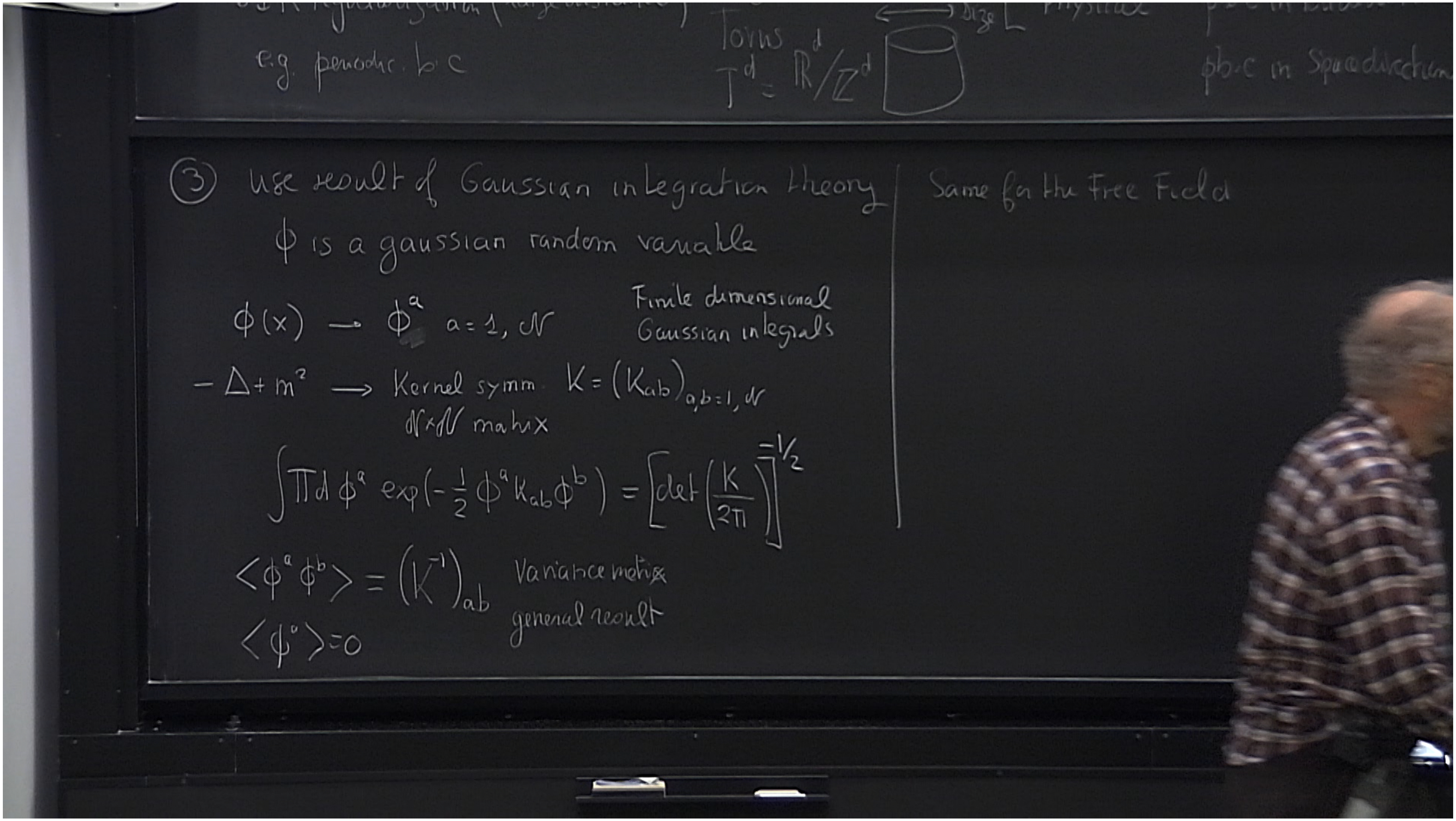
$\phi(x) \longrightarrow \Phi(x)$ Field operator

Now compute in Euclidean

$$\langle \phi(x_1)\,\phi(x_2)\rangle = \frac{1}{Z}\times\int D[\phi]\,\exp\left(-\frac{1}{\hbar}S_E[\phi]\right)\phi(x_1)\,\phi(x_2)$$

$x_1, x_2$ 2 points in Euclidean space

exp. value of product of random variables



e.g. periodic b.c

Torus $T^d = \mathbb{R}^d/\mathbb{Z}^d$

size L

physical

pb.c in space direction

③ use result of Gaussian integration theory

$\phi$ is a gaussian random variable

$\phi(x) \longrightarrow \phi^a \quad a = 1, \mathcal{N}$

Finite dimensional Gaussian integrals

$-\Delta + m^2 \longrightarrow$ Kernel symm. $K = (K_{ab})_{a,b=1,\mathcal{N}}$

$\mathcal{N}\times\mathcal{N}$ matrix

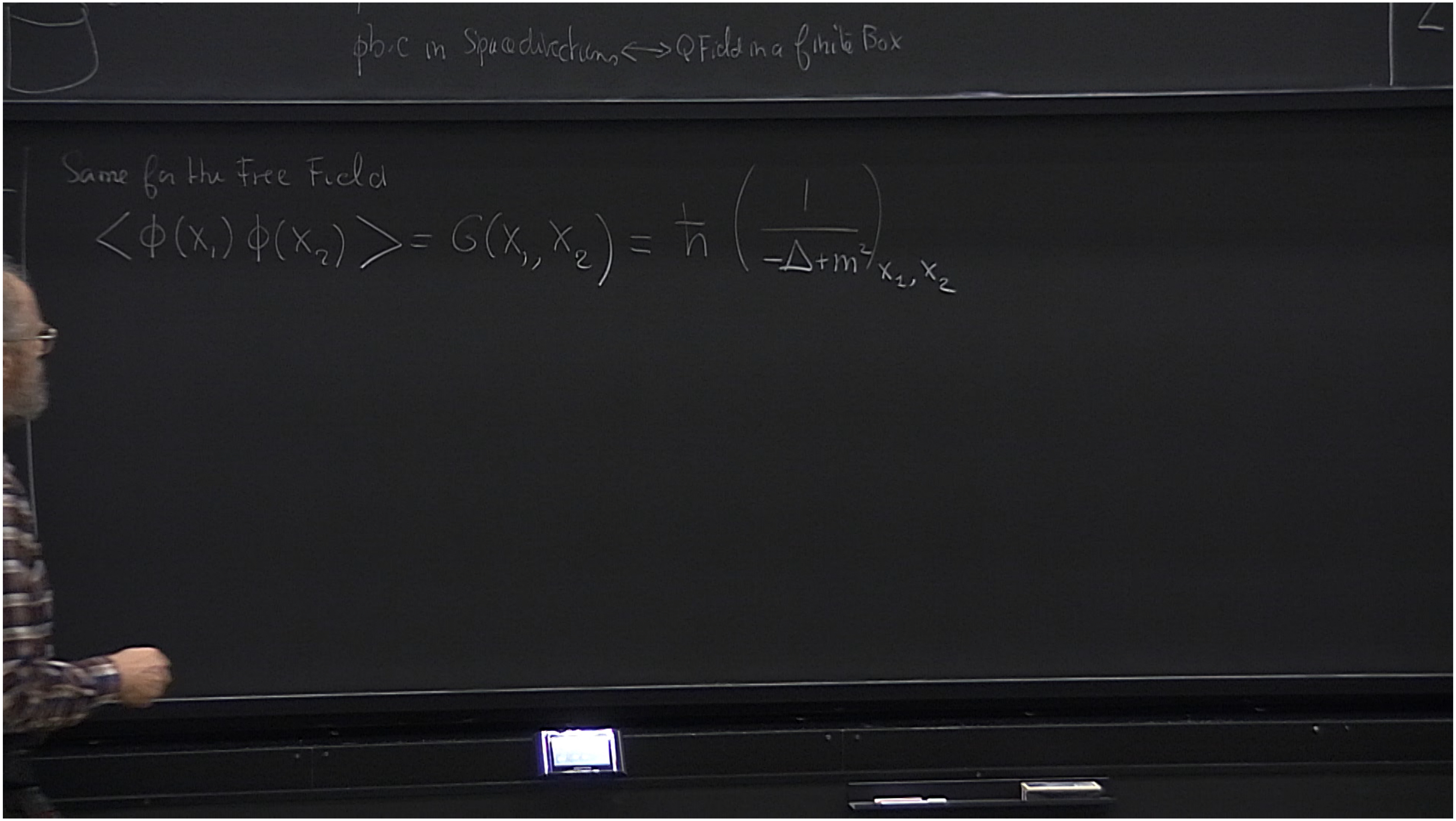
$$\int \prod d\phi^a \exp\left(-\frac{1}{2}\phi^a K_{ab}\phi^b\right) = \left[\det\left(\frac{K}{2\pi}\right)\right]^{-1/2}$$

$\langle \phi^a \phi^b \rangle = \left(K^{-1}\right)_{ab}$ Variance matrix

general result

$\langle \phi^a \rangle = 0$

Same for the Free Field



p.b.c in Spacedirections $\longleftrightarrow$ Q Field in a finite Box

Same for the Free Field

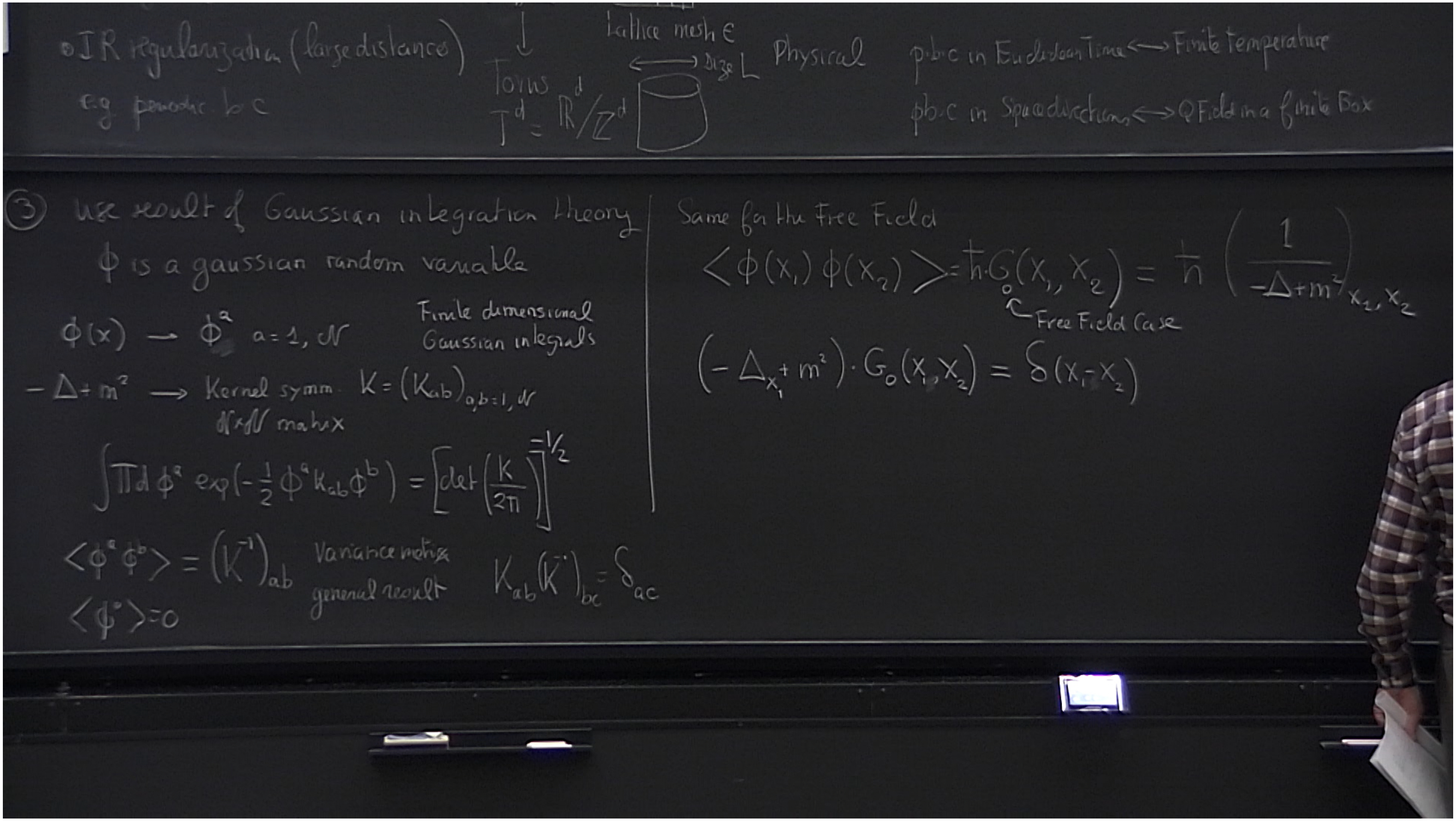
$$\langle \phi(x_1)\,\phi(x_2)\rangle = G(x_1, x_2) = \hbar \left(\frac{1}{-\Delta+m^2}\right)_{x_1,x_2}$$

pb.c in Spacedirections ⟷ QField in a finite Box

Same for the Free Field

$$\langle \phi(x_1)\,\phi(x_2) \rangle = \hbar \cdot G_0(x_1, x_2) = \hbar \left( \frac{1}{-\Delta + m^2} \right)_{x_1, x_2}$$

⤷ Free Field Case



- IR regularization (large distances)

e.g. periodic b.c

$\downarrow$

Torus

$T^d = \mathbb{R}^d/\mathbb{Z}^d$

Lattice mesh $\epsilon$

$\longleftrightarrow$ size L

Physical

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$\langle \phi^a \phi^b \rangle = (K^{-1})_{ab}$ Variance matrix general result $\quad K_{ab}(K^{-1})_{bc} = \delta_{ac}$

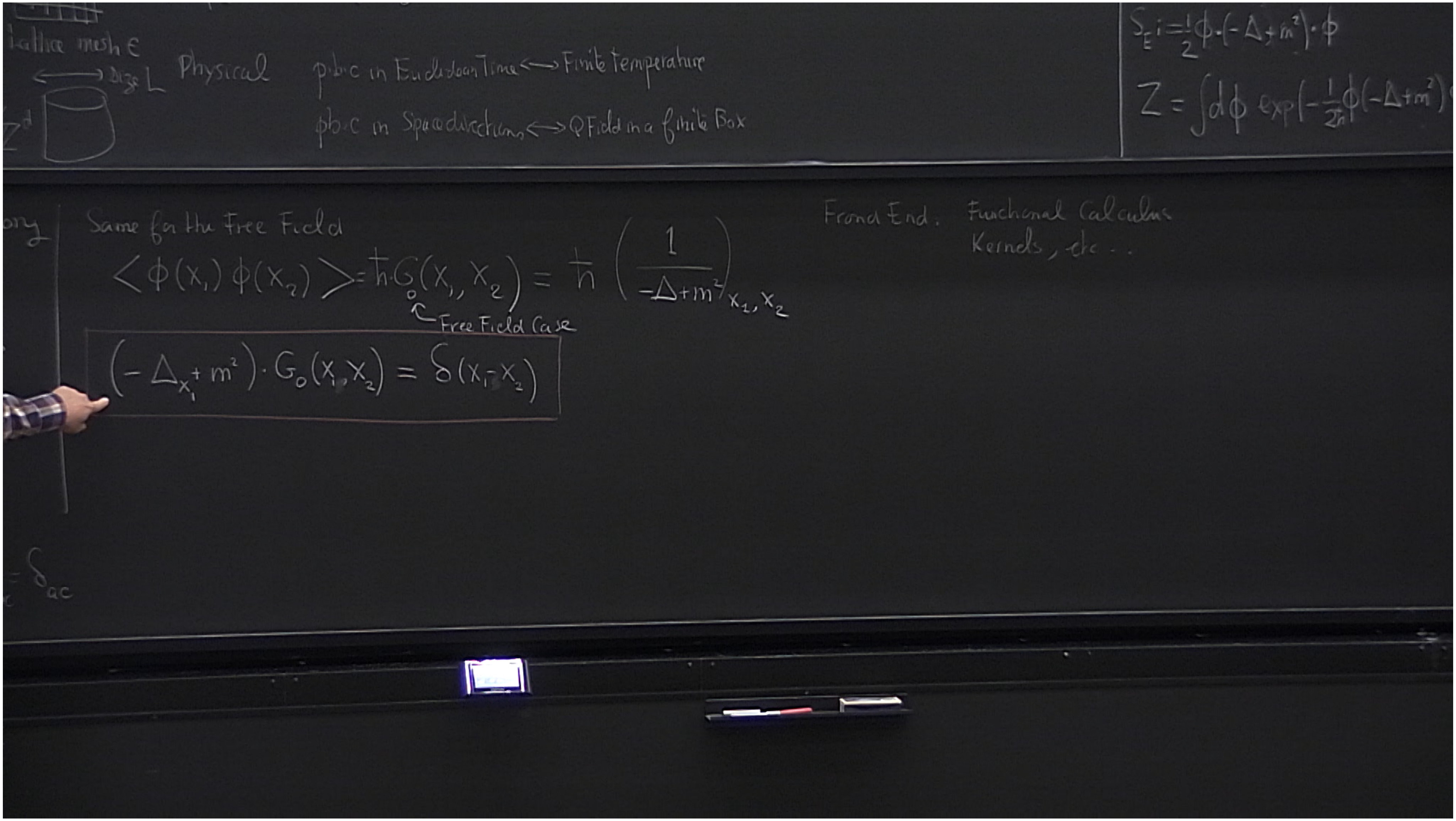
$\langle \phi^a \rangle = 0$

Same for the Free Field

$$\langle \phi(x_1)\,\phi(x_2) \rangle = \hbar \cdot G_0(x_1, x_2) = \hbar \left(\frac{1}{-\Delta + m^2}\right)_{x_1, x_2}$$

Free Field Case

$$(-\Delta_{x_1} + m^2)\cdot G_0(x_1, x_2) = \delta(x_1 - x_2)$$



lattice mesh $\epsilon$

$\longleftrightarrow$ Size L

$\mathbb{Z}^d$

Physical

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$S_E := \frac{1}{2}\phi\cdot(-\Delta+m^2)\cdot\phi$

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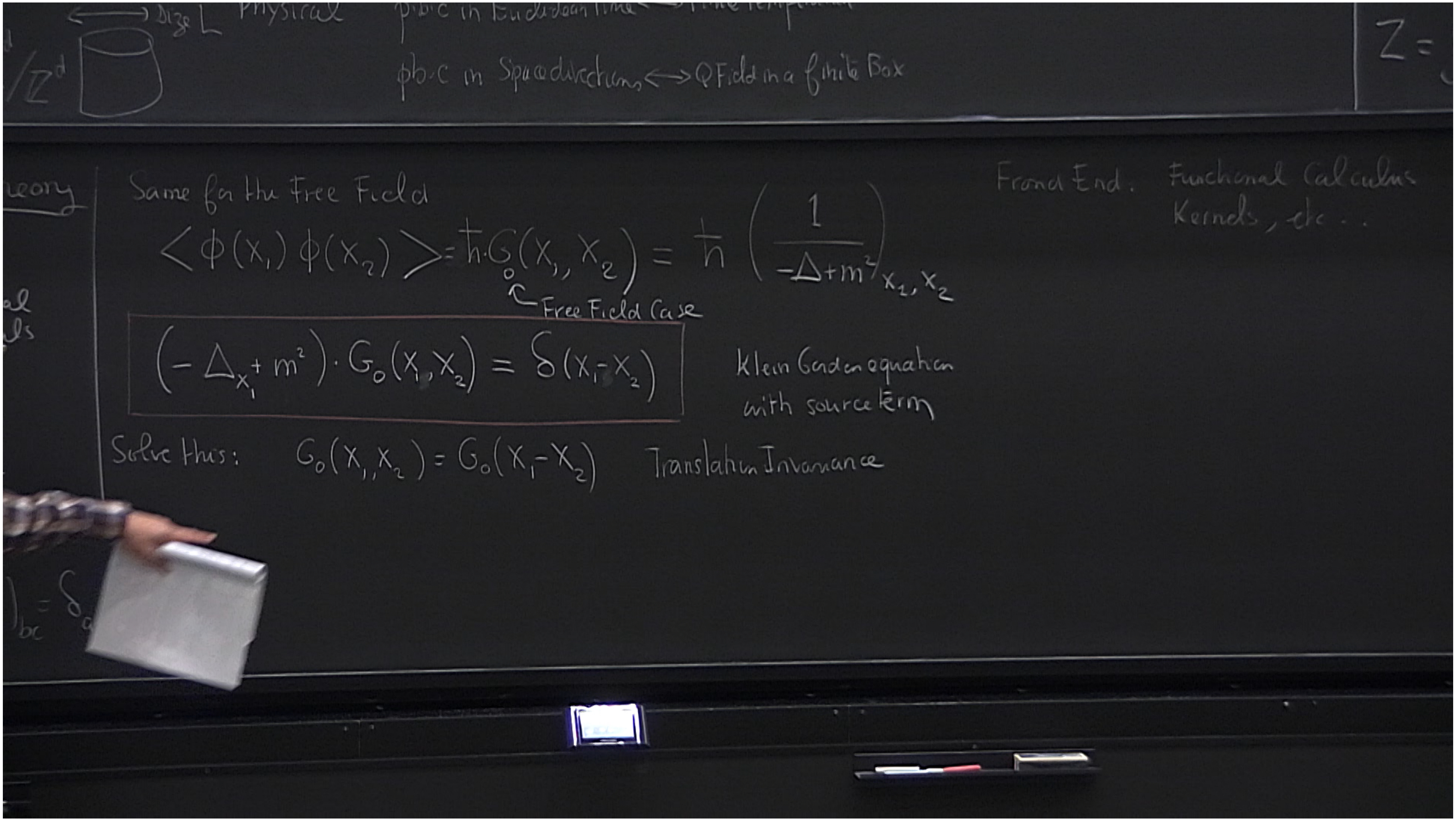
$$\boxed{\left(-\Delta_{x_1} + m^2\right)\cdot G_0(x_1,x_2) = \delta(x_1 - x_2)}$$

Front End: Functional Calculus

Kernels, etc . .

$= \delta_{ac}$



size L Physical p.b.c in Euclidean Time ⟷

$/\mathbb{Z}^d$

p.b.c in Space directions ⟷ Q Field in a finite Box

$Z =$

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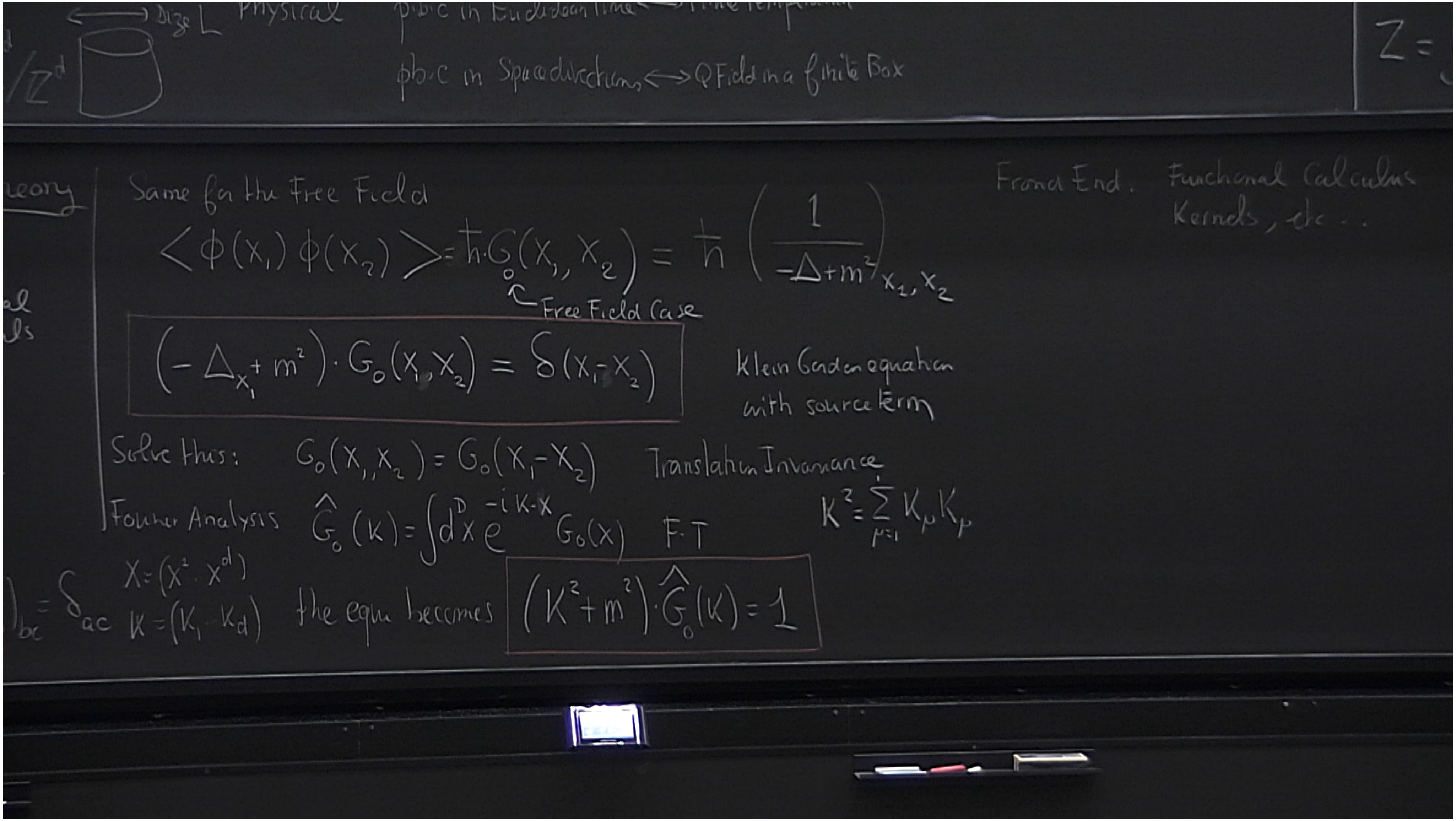
$$\boxed{\left(-\Delta_{x_1} + m^2\right) \cdot G_0(x_1, x_2) = \delta(x_1 - x_2)}$$

Klein Gordon equation with source term

Solve this: $G_0(x_1, x_2) = G_0(x_1 - x_2)$ Translation Invariance

Front End: Functional Calculus Kernels, etc . .



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p.b.c in Euclidean Time ⟷ Finite temperature

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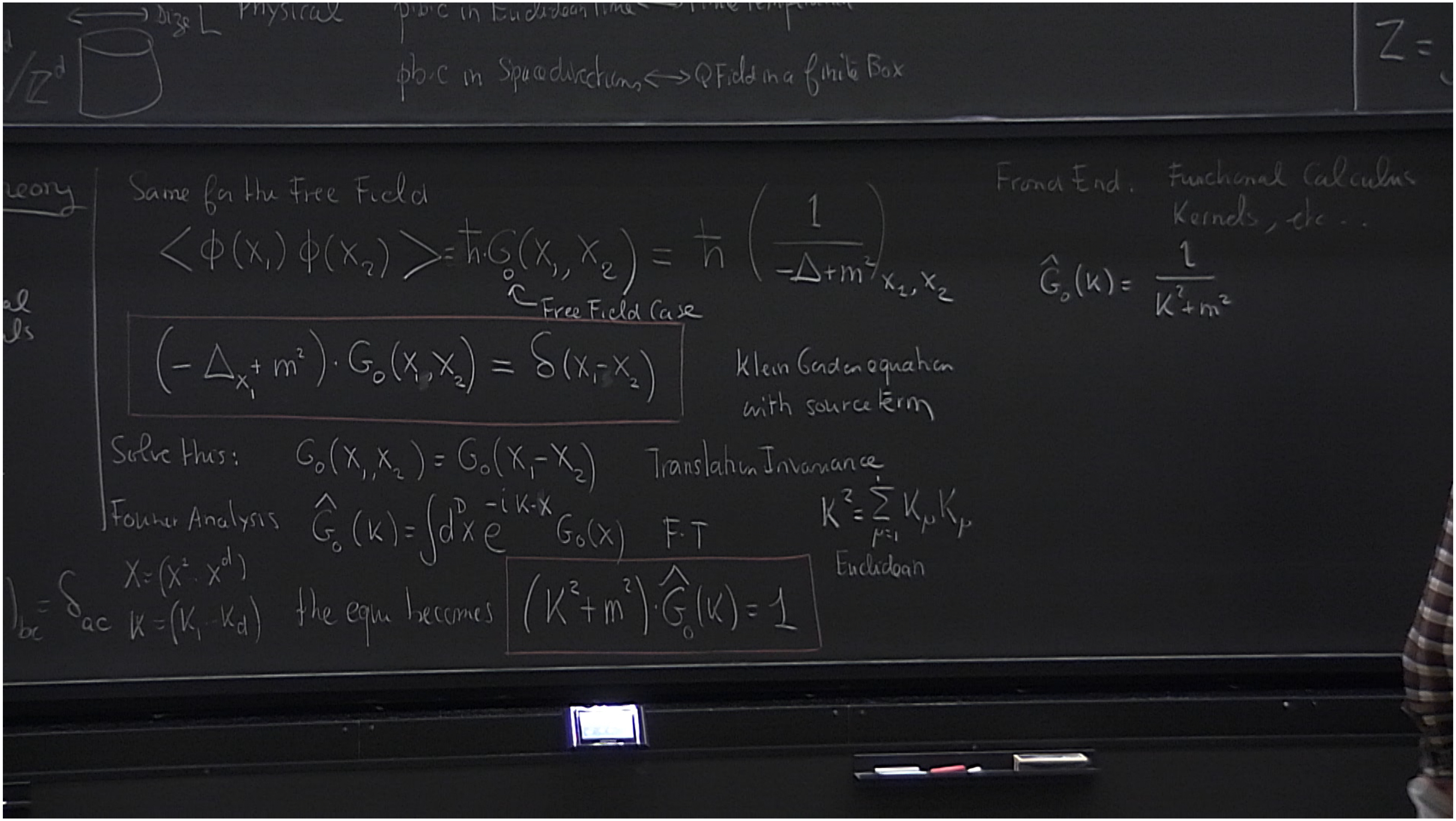
Fourier Analysis $\hat{G}_0(k) = \int d^D x\, e^{-i k \cdot x}\, G_0(x)$  F.T

$K^2 = \sum_{\mu=1}^{d} K_\mu K_\mu$

$X = (X^1 \cdot X^d)$

$)_{bc} = \delta_{ac}$  $K = (K_1 \cdot K_d)$  the eqn becomes $\boxed{(K^2 + m^2) \cdot \hat{G}_0(K) = 1}$

Front End. Functional Calculus Kernels, etc . .



$\longleftrightarrow$ Size L Physical

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$k^2 = \sum_{\mu=1} k_\mu k_\mu$

Euclidean

$)_{bc} = \delta_{ac}$

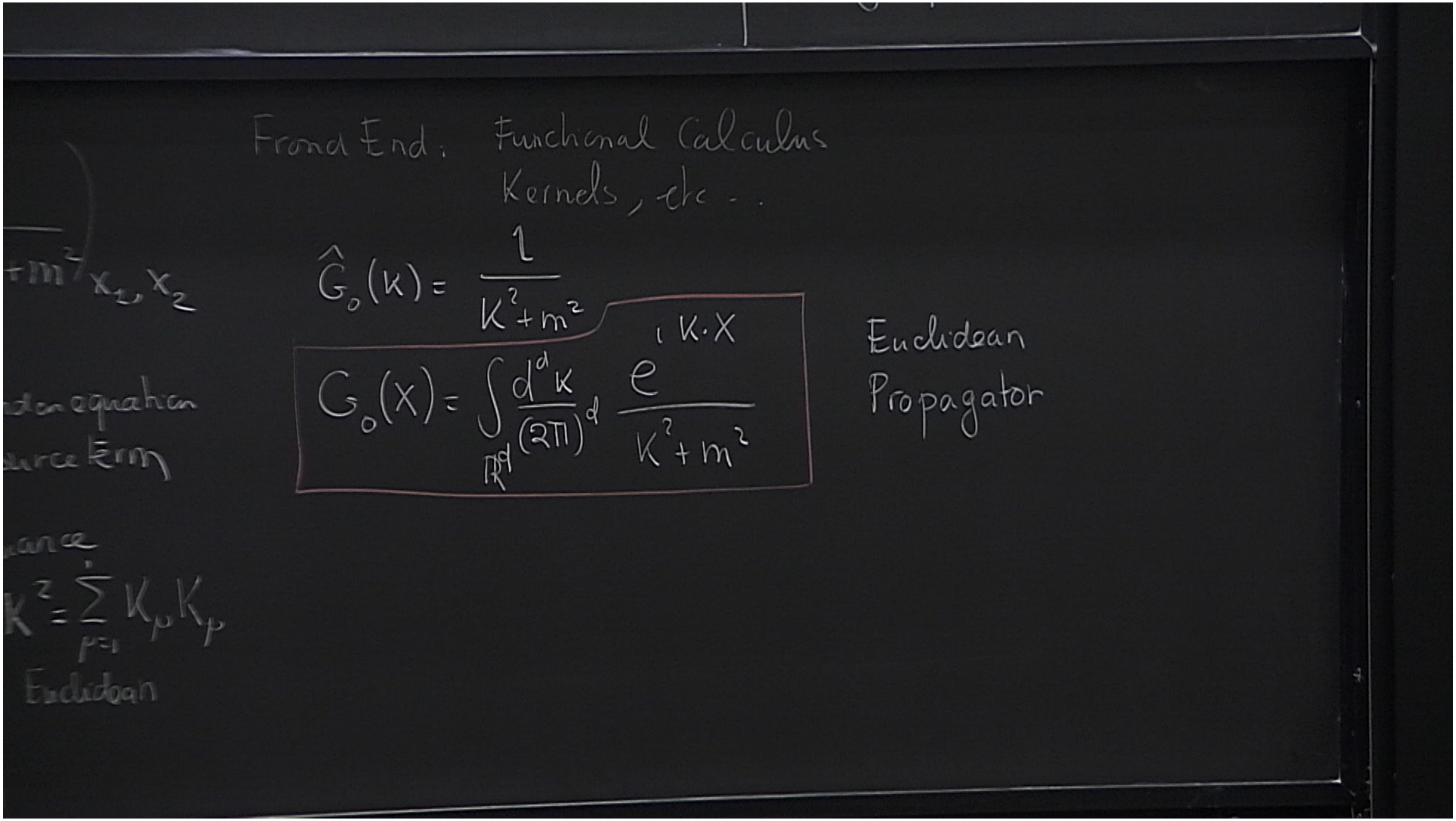
$x = (x^1 \cdot x^d)$

$k = (k_1 \cdot k_d)$

the eqn becomes $\boxed{(k^2+m^2)\cdot\hat{G}_0(k) = 1}$

Front End. Functional Calculus Kernels, etc..

$$\hat{G}_0(k) = \frac{1}{k^2+m^2}$$



$+m^2) x_2, x_2$

den equation

urce term

ance

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Euclidean

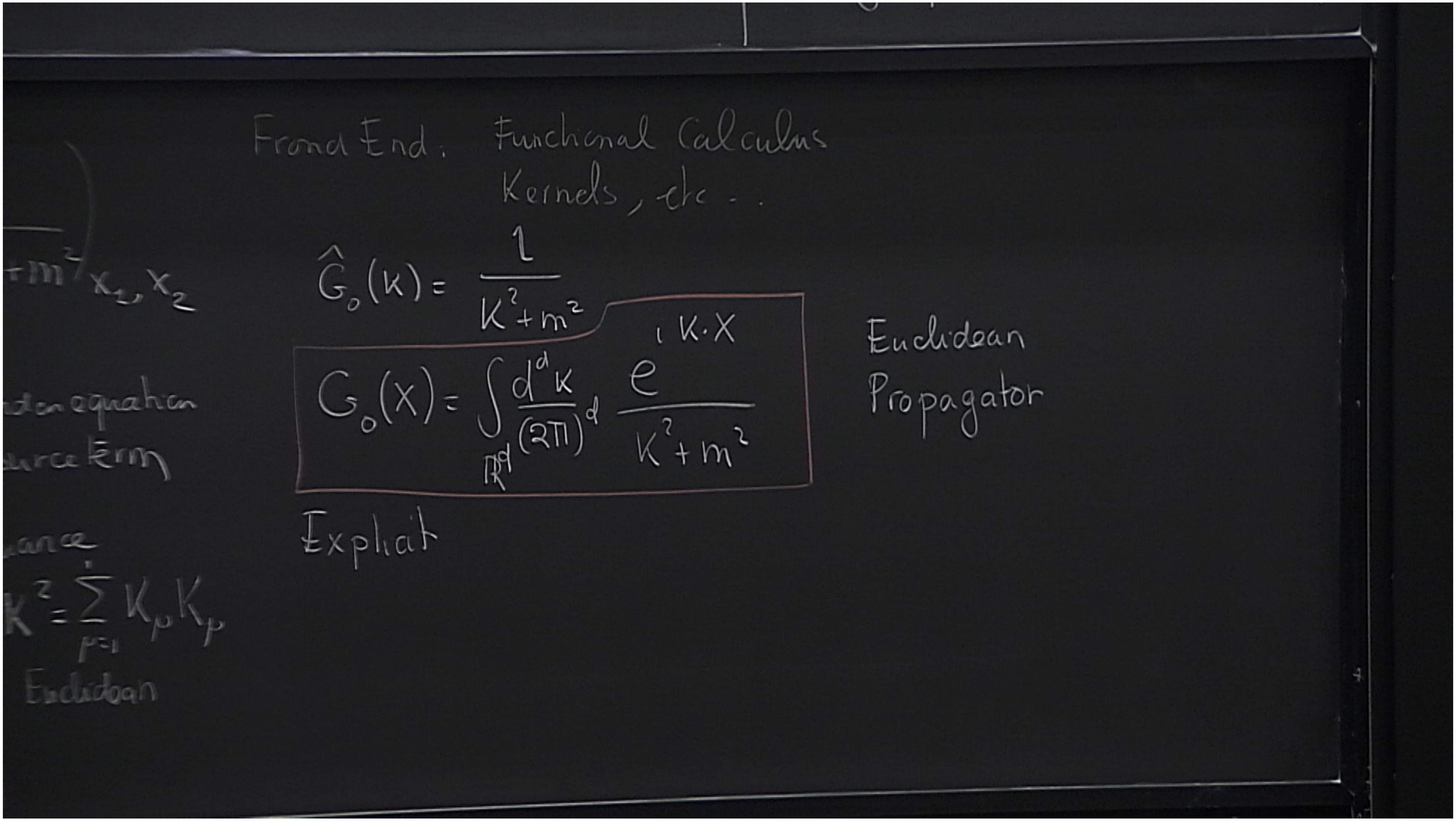
Front End: Functional Calculus

Kernels, etc . .

$$\hat{G}_0(k) = \frac{1}{k^2+m^2}$$

$$\boxed{G_0(x) = \int_{\mathbb{R}^d} \frac{d^d k}{(2\pi)^d} \frac{e^{ik\cdot x}}{k^2+m^2}}$$

Euclidean Propagator



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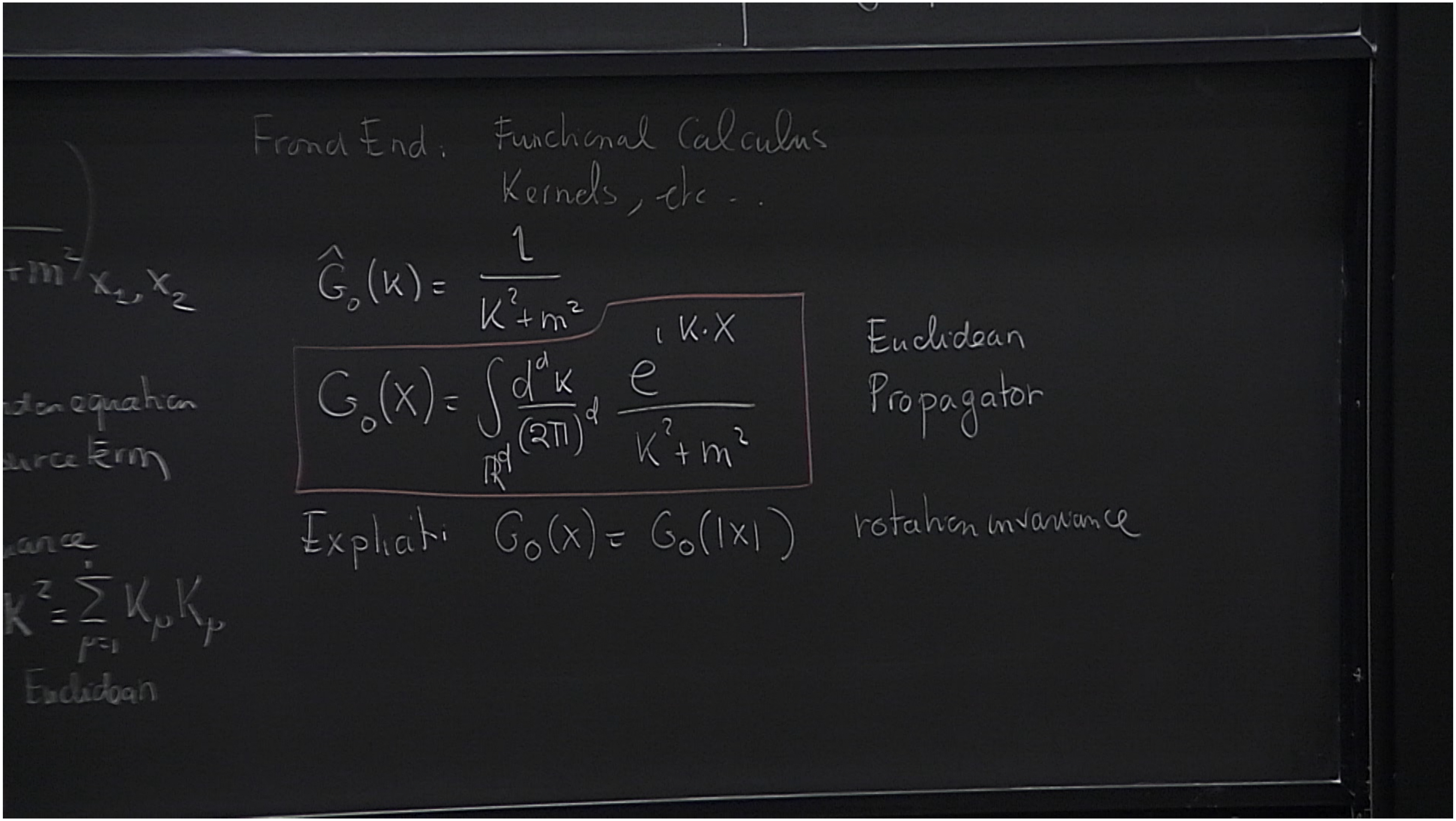
Euclidean Propagator

Explicit



$+m^2)_{x_2, x_2}$

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Euclidean

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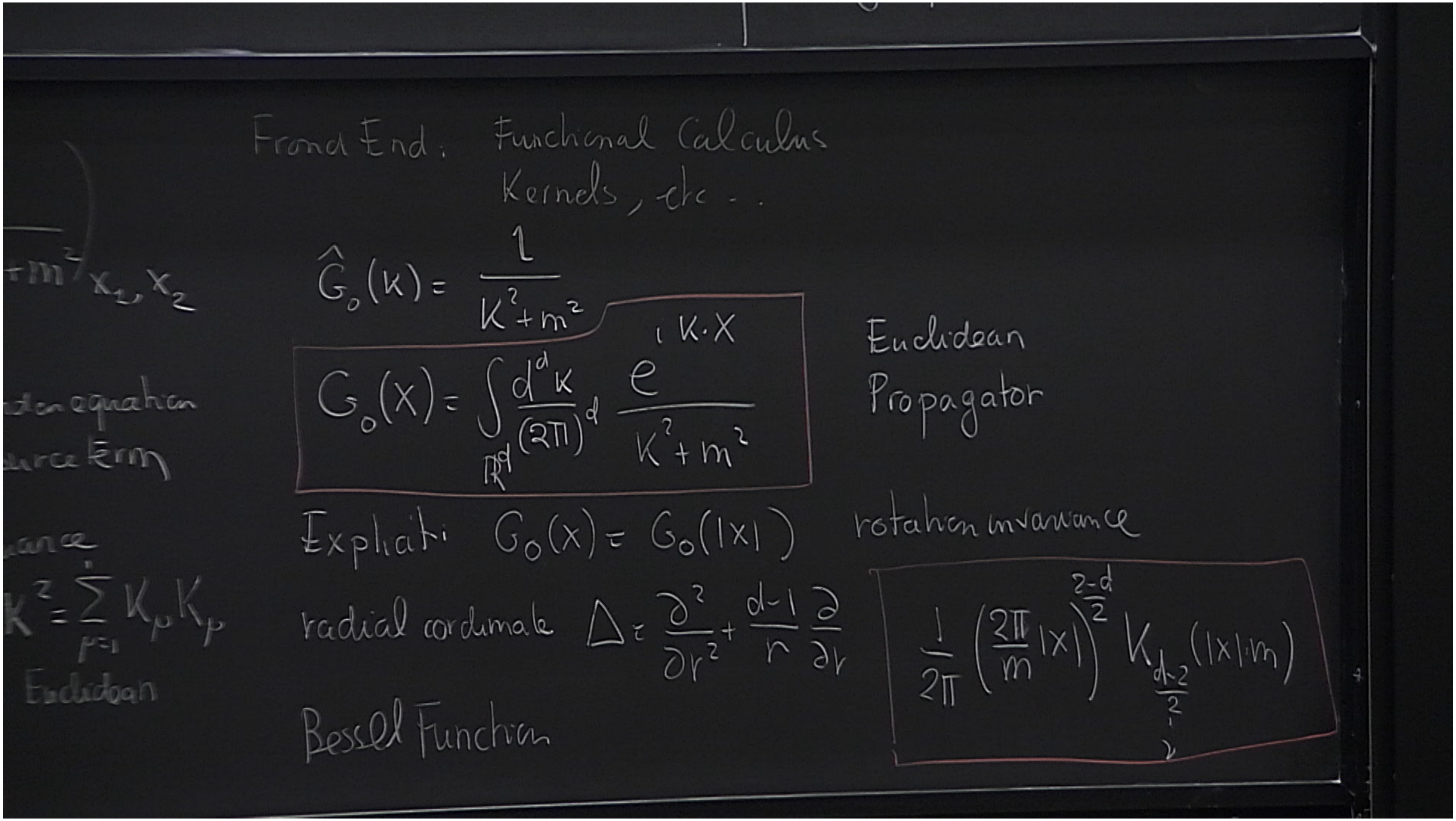
Euclidean Propagator

Explicit: $G_0(X) = G_0(|X|)$ rotation invariance



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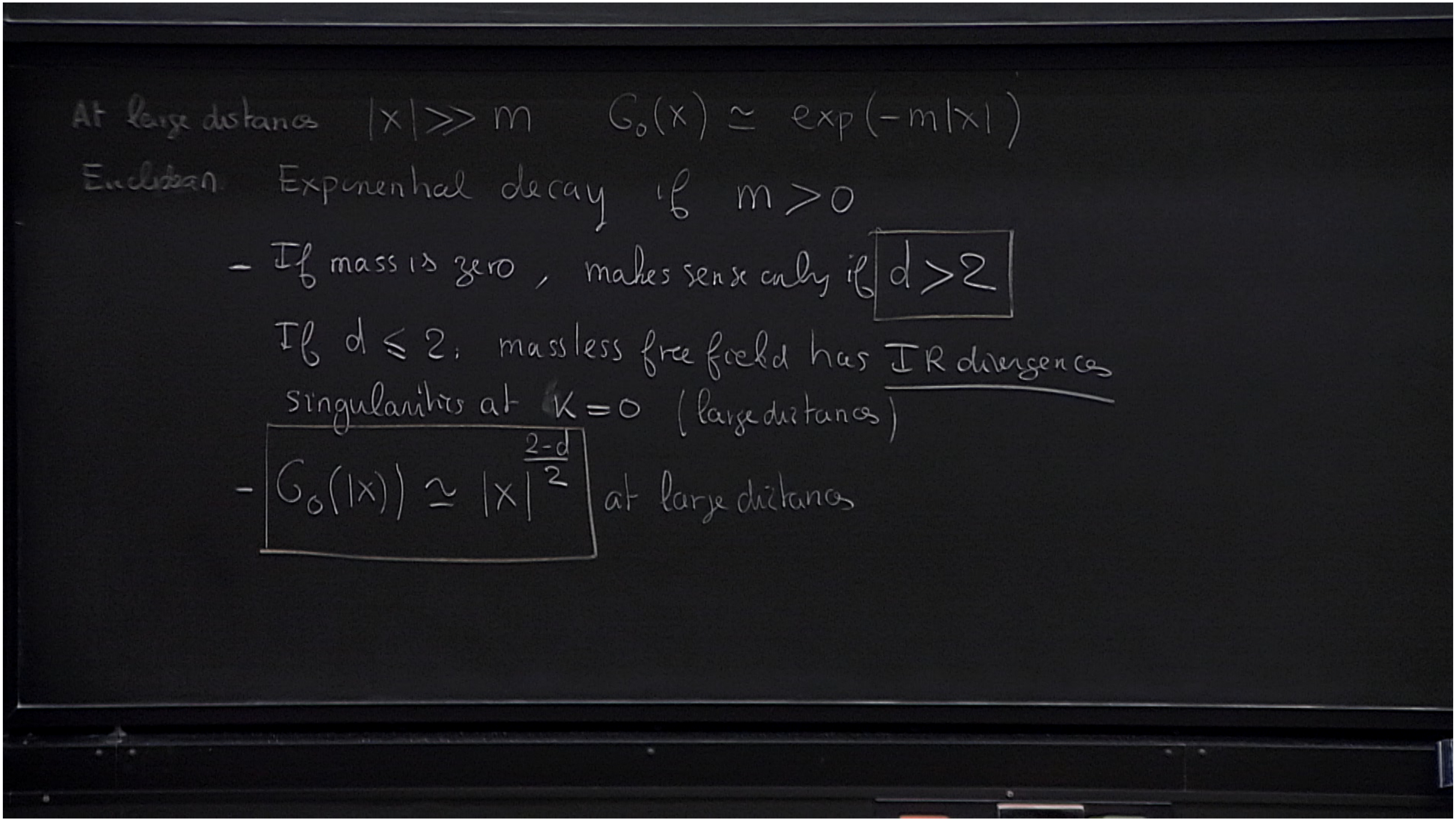
Euclidean
Propagator

Explicit: $G_0(x) = G_0(|x|)$ rotation invariance

radial coordinate $\Delta = \frac{\partial^2}{\partial r^2} + \frac{d-1}{r}\frac{\partial}{\partial r}$

$$\boxed{\frac{1}{2\pi}\left(\frac{2\pi}{m}|x|\right)^{\frac{2-d}{2}} K_{\underset{\nu}{\frac{d-2}{2}}}(|x|m)}$$

Bessel Function



At large distances $|x| \gg m$ $G_0(x) \simeq \exp(-m|x|)$

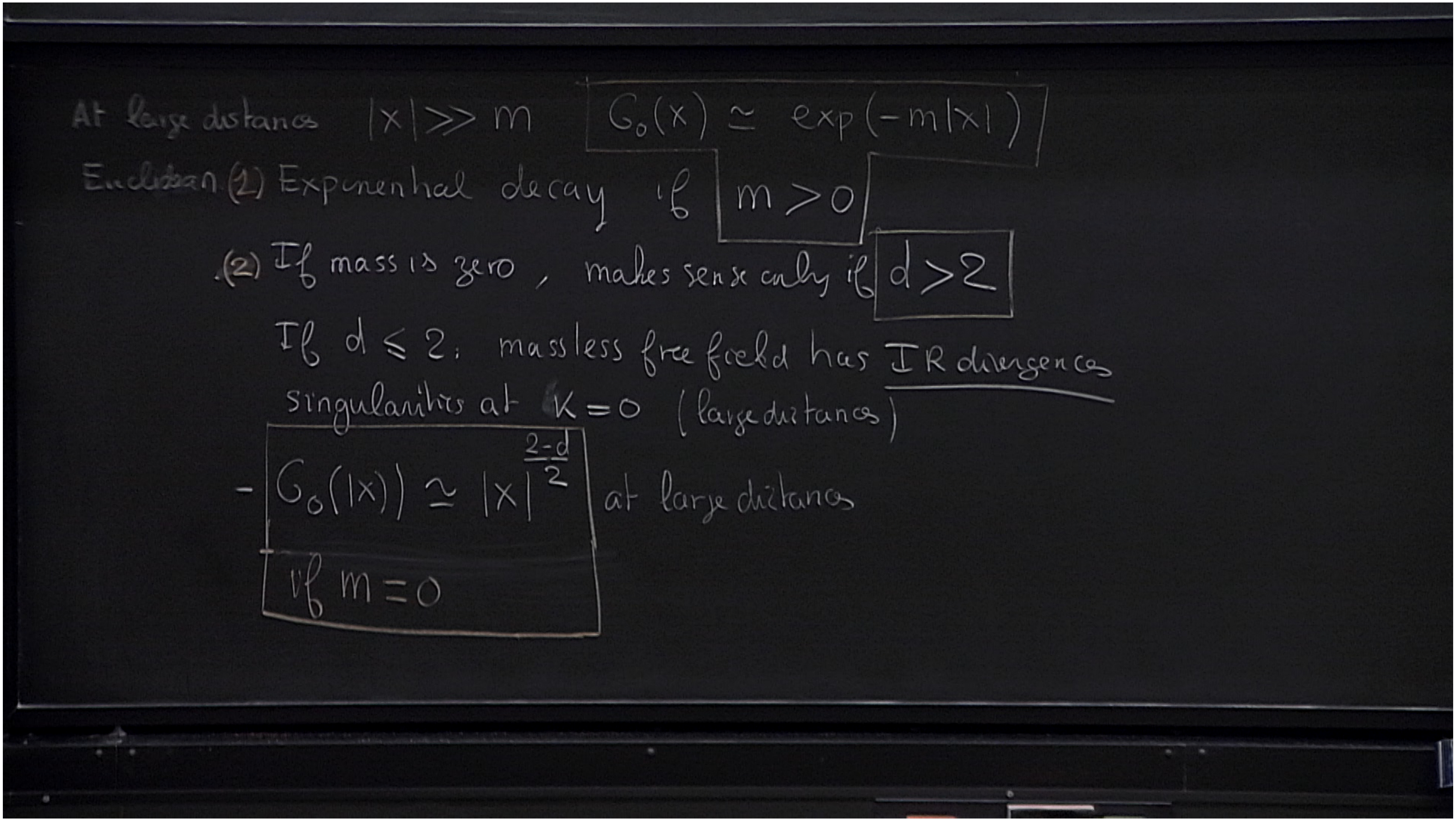
Euclidean Exponential decay if $m > 0$

- If mass is zero, makes sense only if $\boxed{d > 2}$

If $d \leq 2$: massless free field has IR divergences

singularities at $k = 0$ (large distances)

- $\boxed{G_0(|x|) \simeq |x|^{\frac{2-d}{2}}}$ at large distances



At large distances $|x| \gg m$ $\quad \boxed{G_0(x) \simeq \exp(-m|x|)}$

Euclidean (1) Exponential decay if $\boxed{m > 0}$

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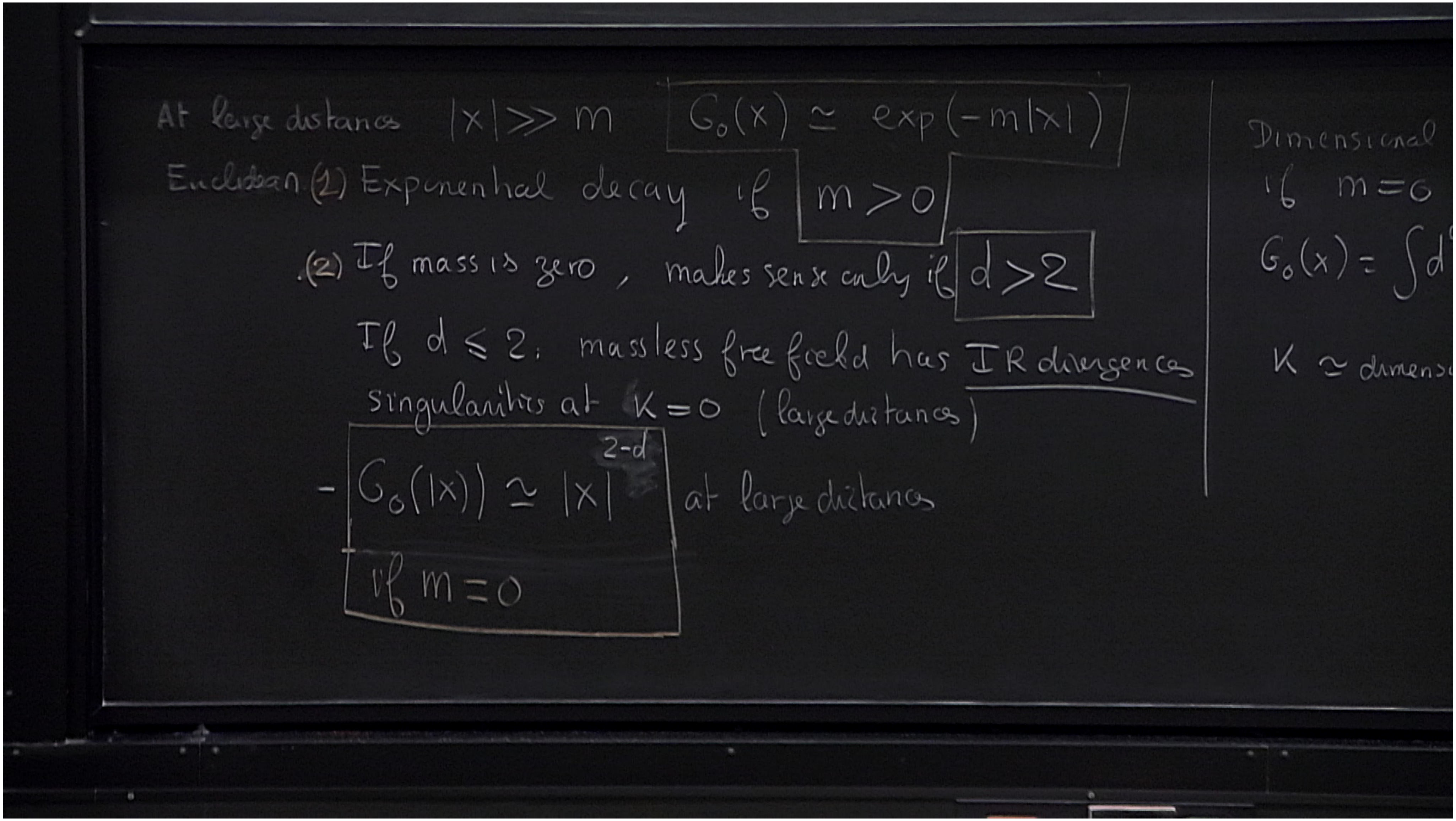

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If $d \leqslant 2$: massless free field has IR divergences

singularities at $k=0$ (large distances)

$- \boxed{G_0(|x|) \simeq |x|^{2-d}}$ at large distances

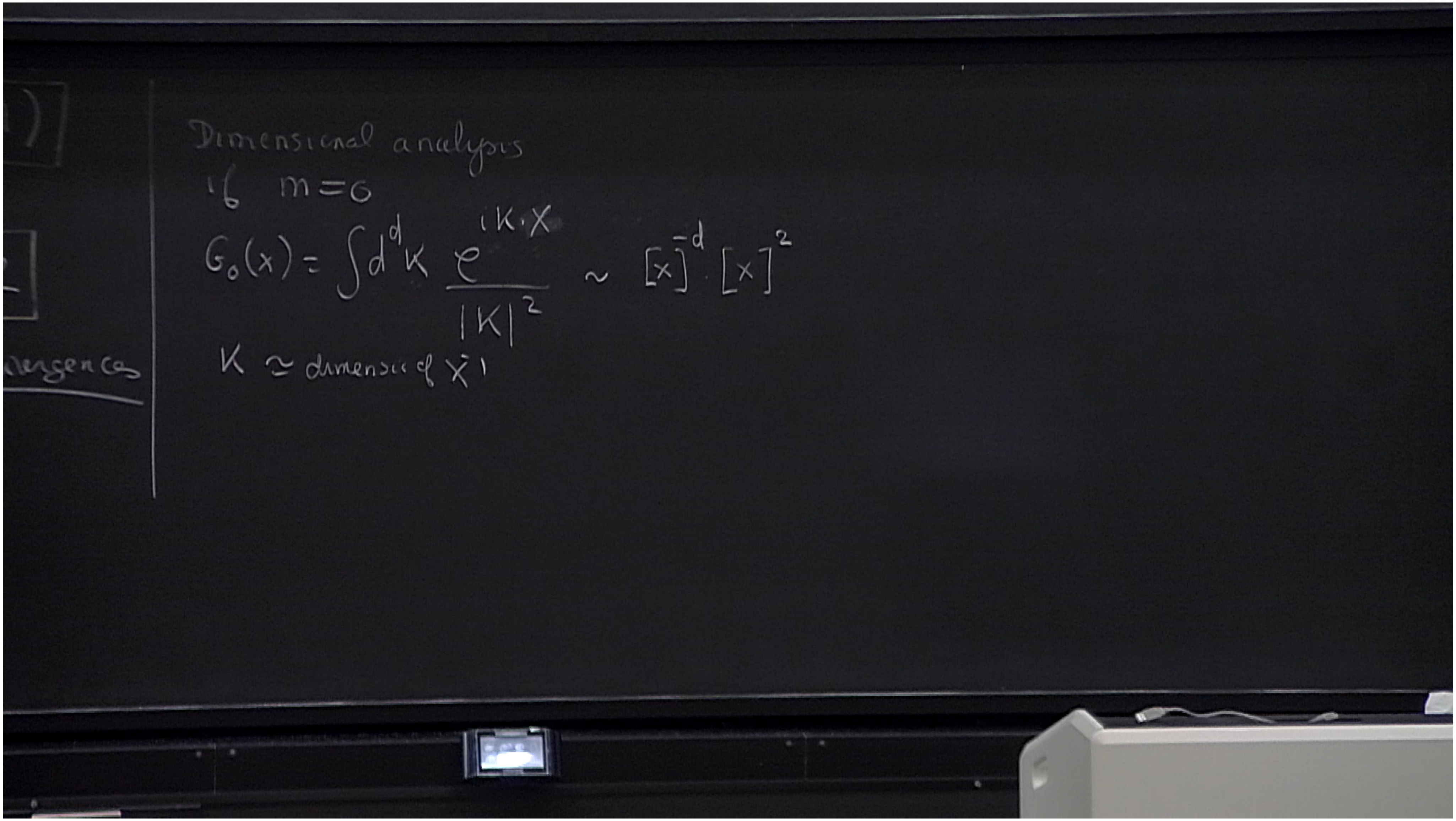
$\boxed{\text{if } m=0}$

Dimensional

if $m=0$

$G_0(x) = \int d$

$K \simeq$ dimens



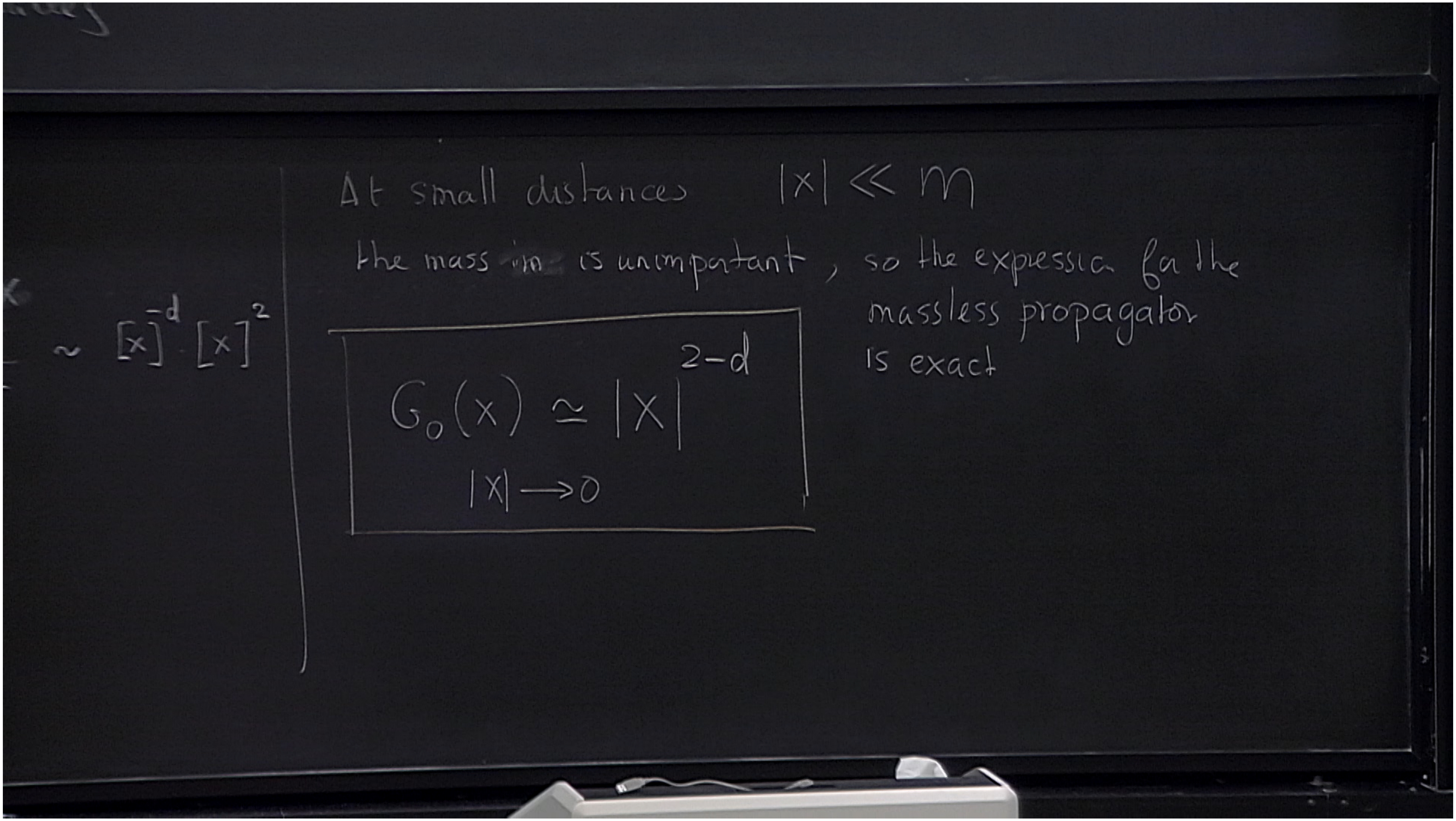
vergences

Dimensional analysis

if $m=0$

$$G_0(x) = \int d^d k \, \frac{e^{ik\cdot x}}{|k|^2} \sim [x]^{-d} \cdot [x]^2$$

$k \simeq$ dimension of $x^{-1}$

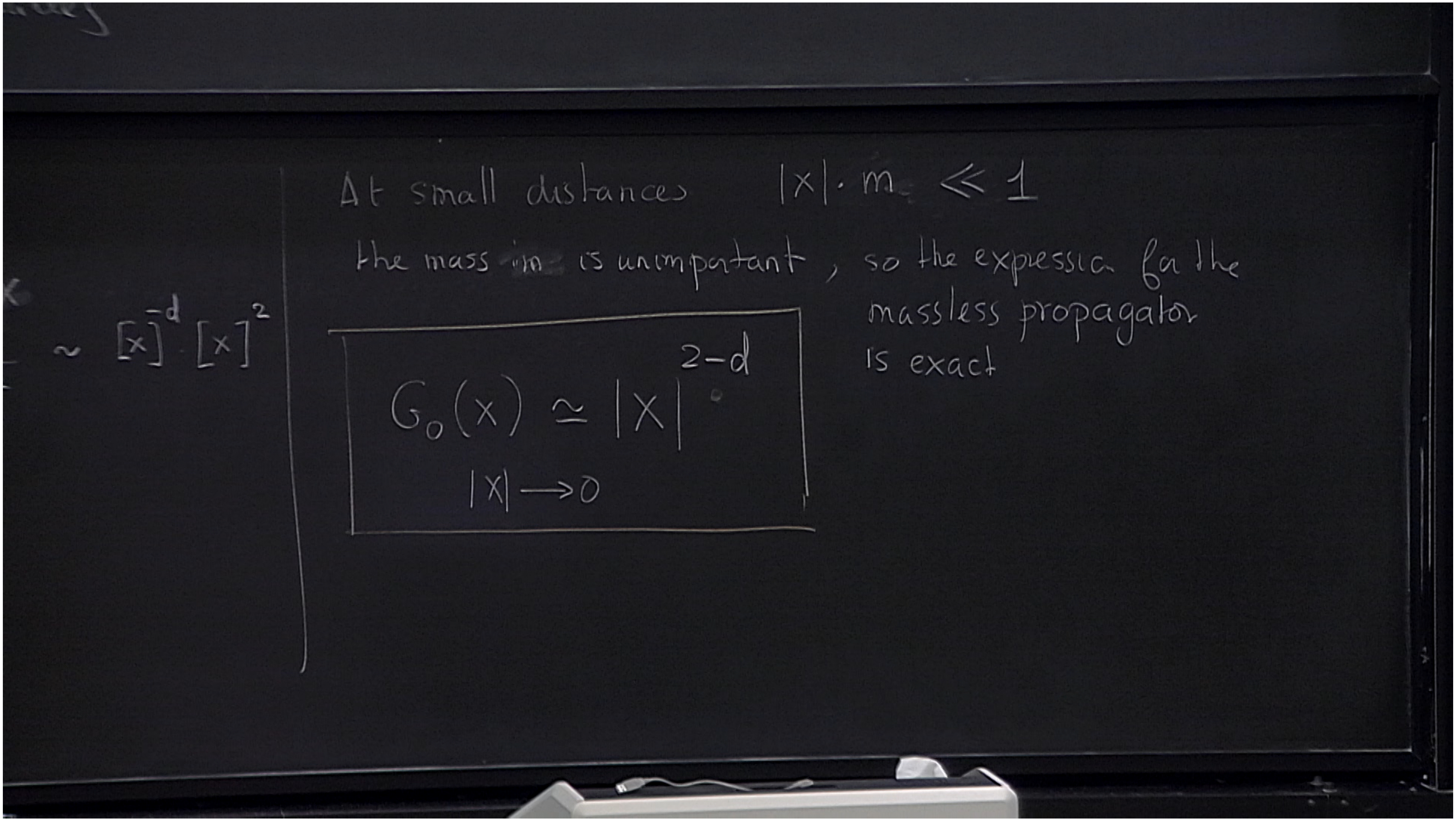


$\sim [x]^{-d} \cdot [x]^{2}$

At small distances $|x| \ll m$

the mass is unimportant, so the expression for the massless propagator is exact

$$\boxed{\begin{gathered} G_0(x) \simeq |x|^{2-d} \\ |x| \to 0 \end{gathered}}$$

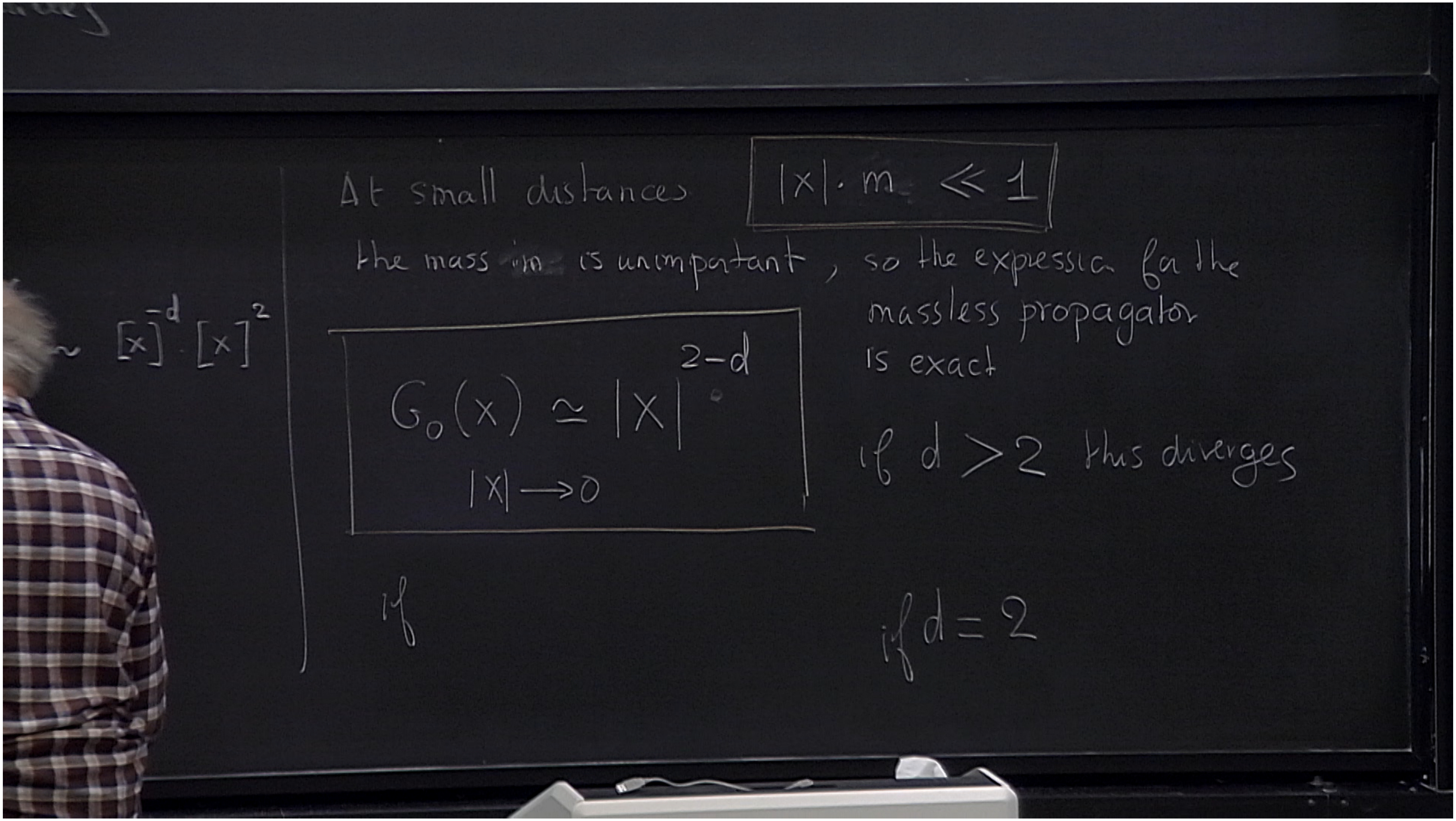


$\sim [x]^{-d}\cdot[x]^{2}$

At small distances $|x|\cdot m \ll 1$

the mass m is unimportant, so the expression for the massless propagator is exact

$$\boxed{\begin{array}{c} G_0(x) \simeq |x|^{2-d} \\ |x|\longrightarrow 0 \end{array}}$$



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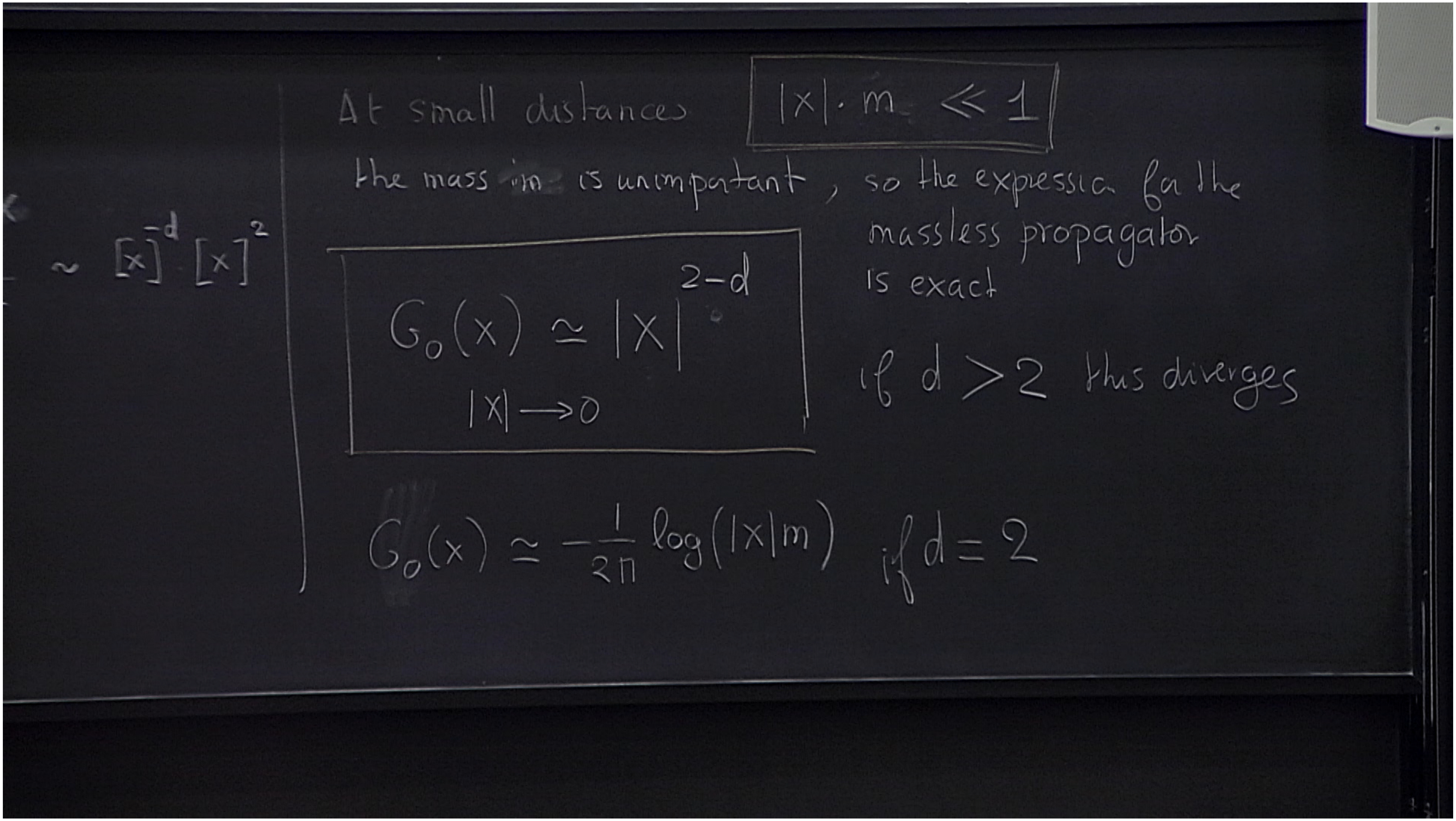
$$\boxed{G_0(x) \simeq |X|^{2-d} \quad |X| \to 0}$$

if $d > 2$ this diverges

if

if $d = 2$



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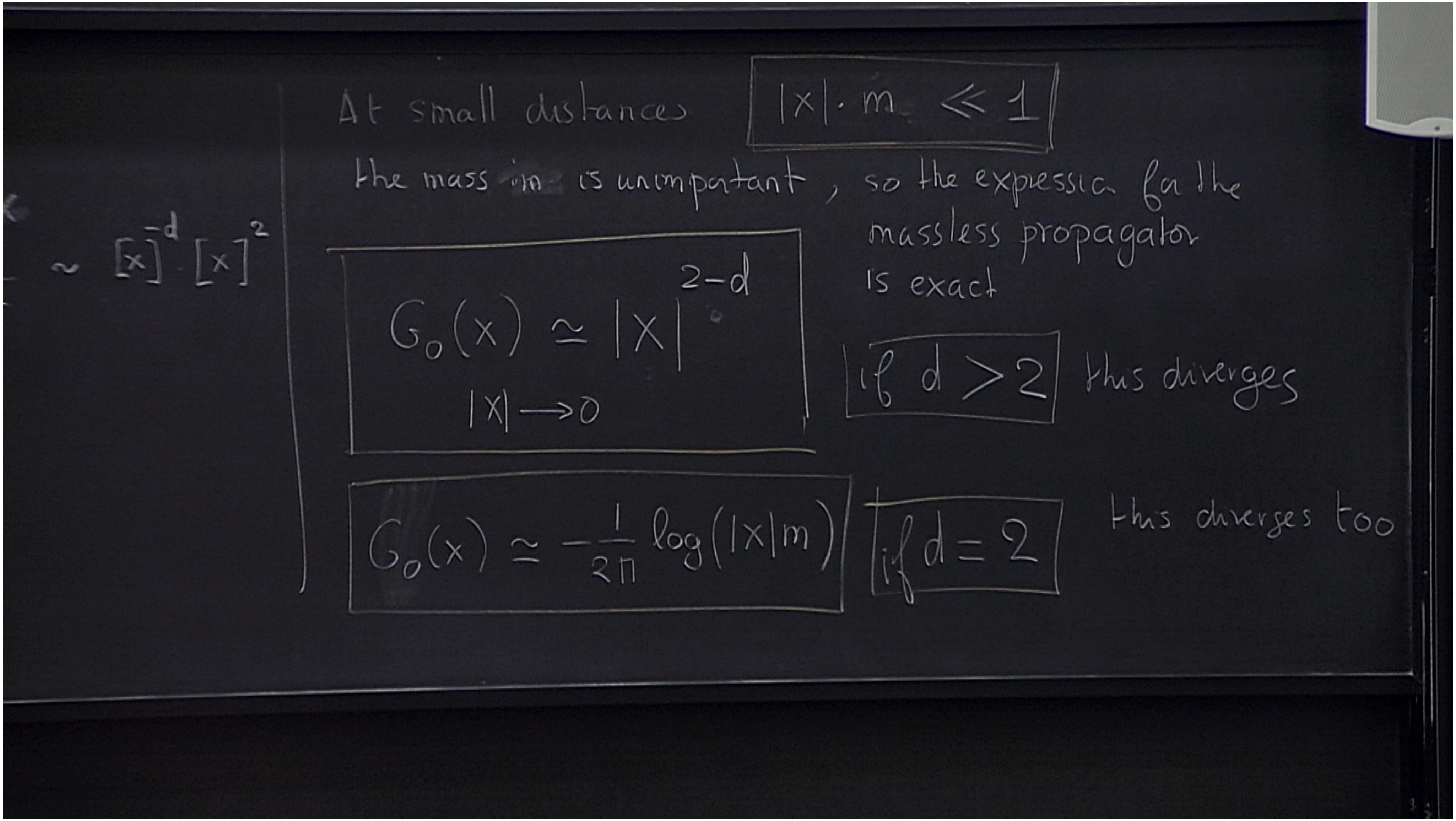

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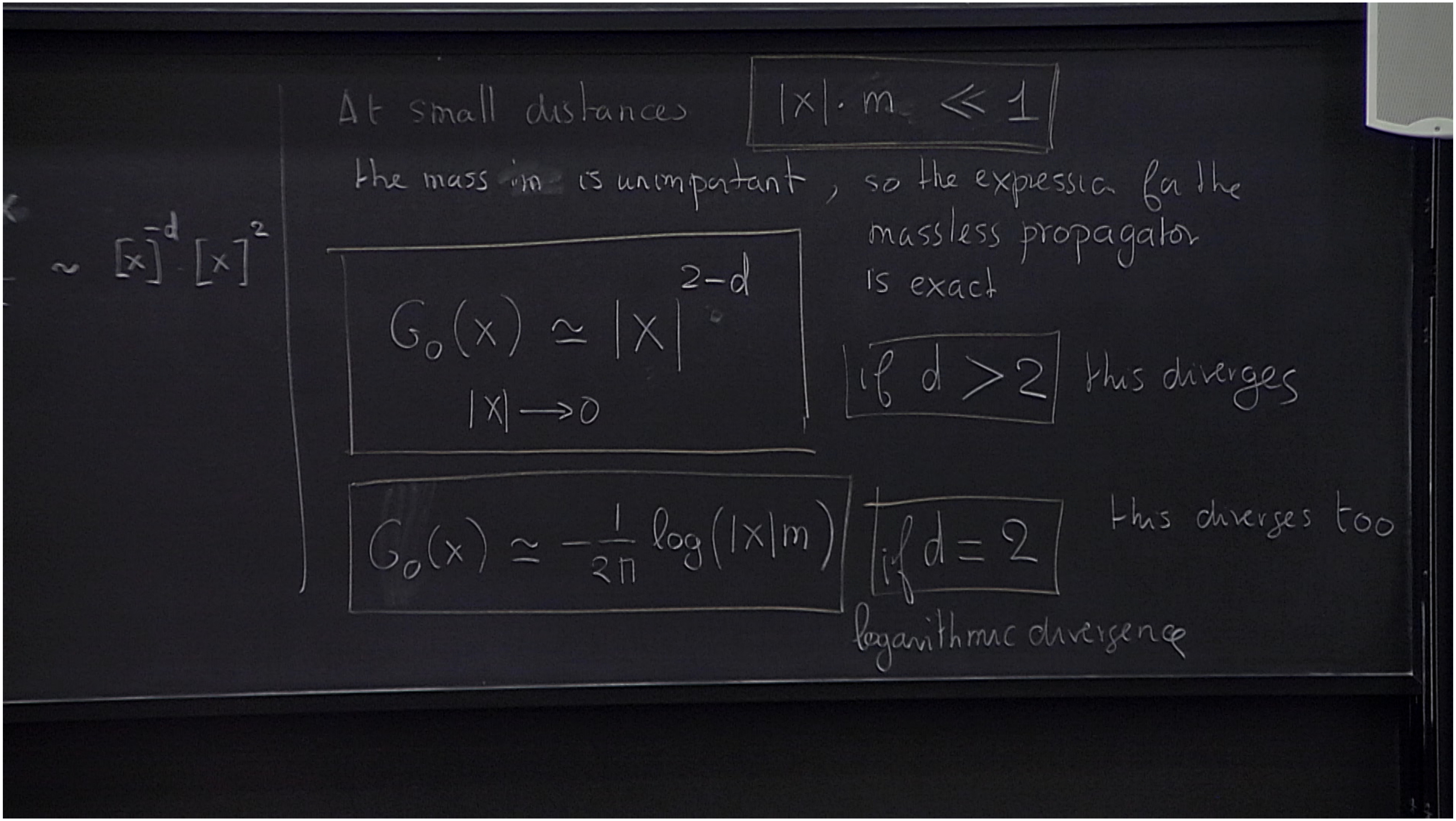

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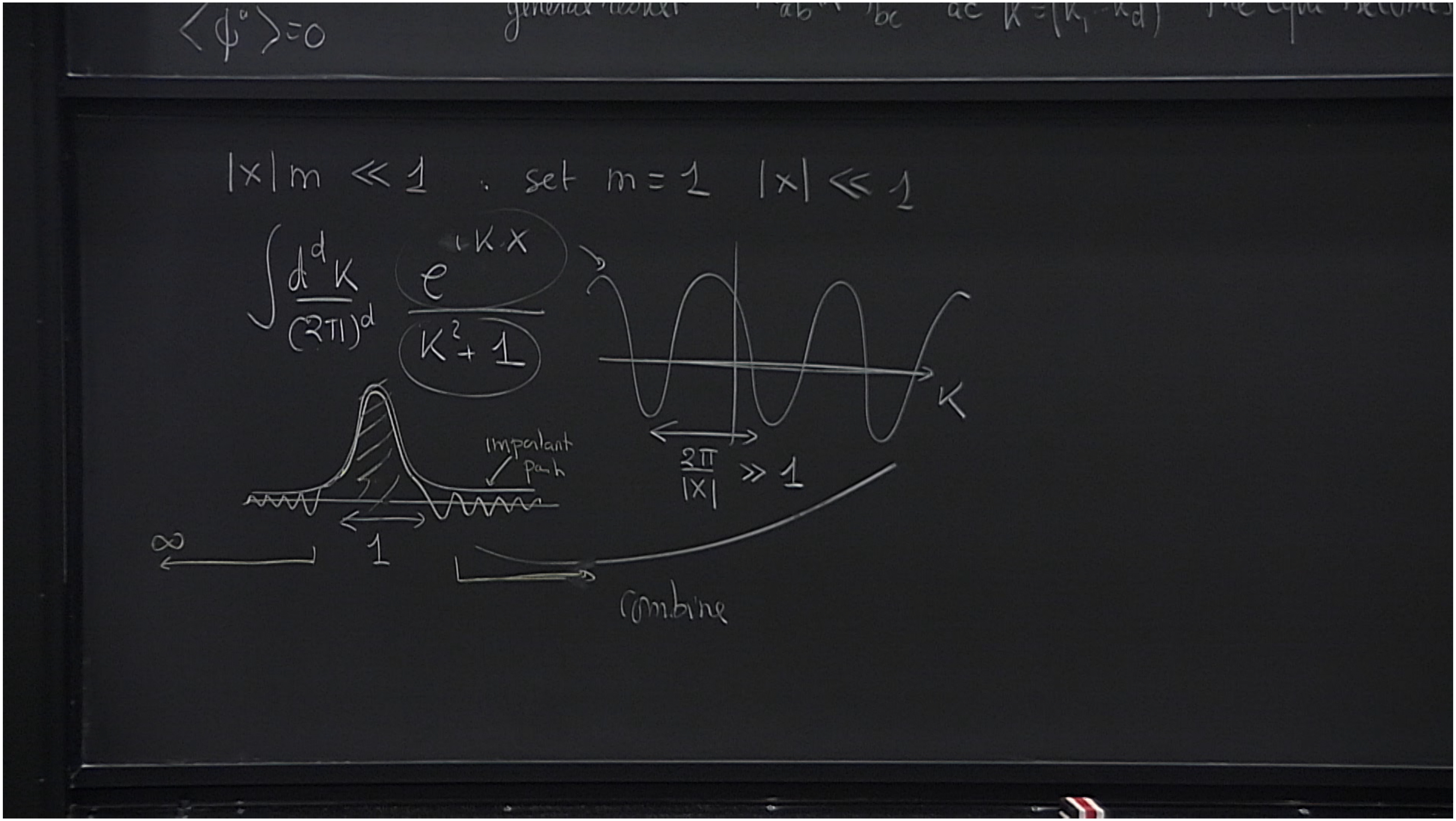
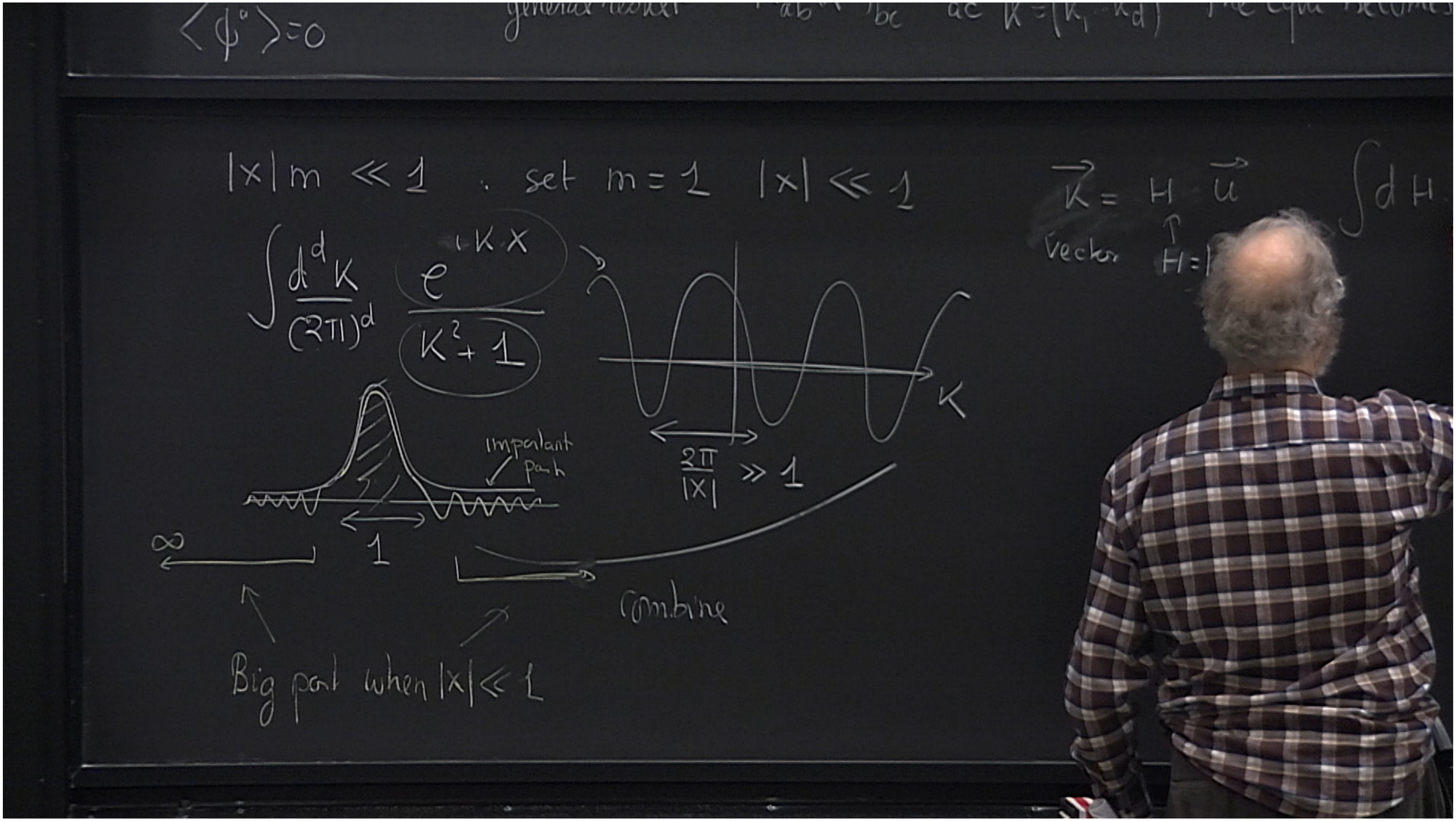
logarithmic divergence



$\langle \phi^a \rangle = 0$ general vector $\ldots_{ab} \ldots_{bc} \ldots_{ac}$ $k = (k_1 \ldots k_d)$ The eqn becomes

$|x|\, m \ll 1$ . set $m = 1$ $|x| \ll 1$

$$\int \frac{d^d k}{(2\pi)^d} \frac{\boxed{e^{ik\cdot x}}}{\boxed{k^2 + 1}}$$



$\langle \phi^a \rangle = 0$ general result ab bc ac $K = (k_1 \dots k_d)$ The eqn becomes

$|x| m \ll 1$ . set $m = 1$ $|x| \ll 1$

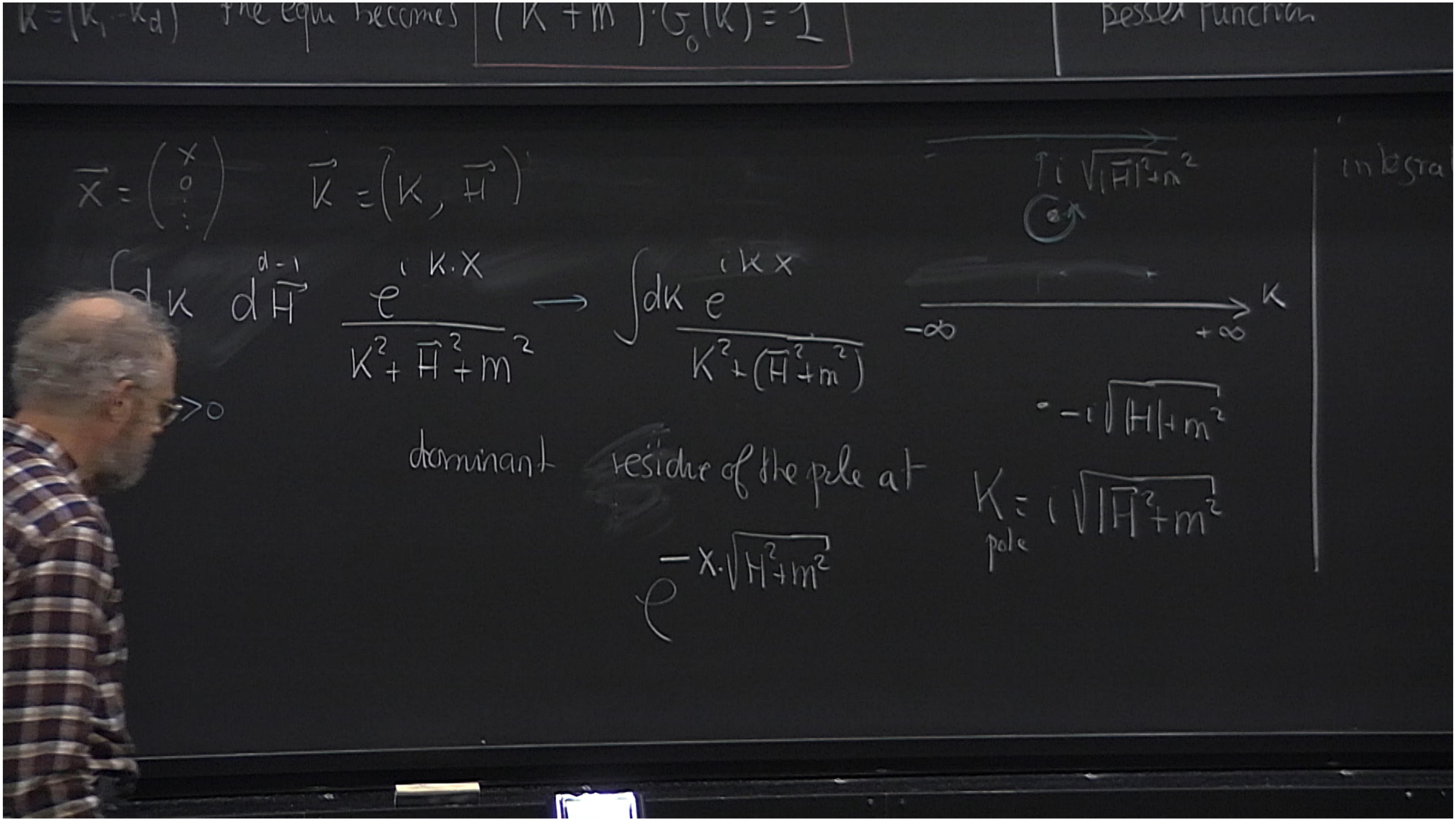
$$\int \frac{d^d k}{(2\pi)^d} \frac{e^{ik\cdot x}}{k^2 + 1}$$

Big part when $|x| \ll 1$

$\vec{K} = H \vec{u}$

vector $H = |$

$\int dH$



$k = (k_1 \cdots k_d)$ the equ becomes $\boxed{(k^2+m^2)\cdot G_0(k) = 1}$

Bessel function

$$\vec{x} = \begin{pmatrix} x \\ 0 \\ \vdots \end{pmatrix} \qquad \vec{k} = (k, \vec{H})$$

$$\int dk\, d^{d-1}\vec{H}\, \frac{e^{ik\cdot x}}{k^2+\vec{H}^2+m^2} \longrightarrow \int dk\, \frac{e^{ikx}}{k^2+(\vec{H}^2+m^2)}$$

$>0$

$i\sqrt{|\vec{H}|^2+m^2}$

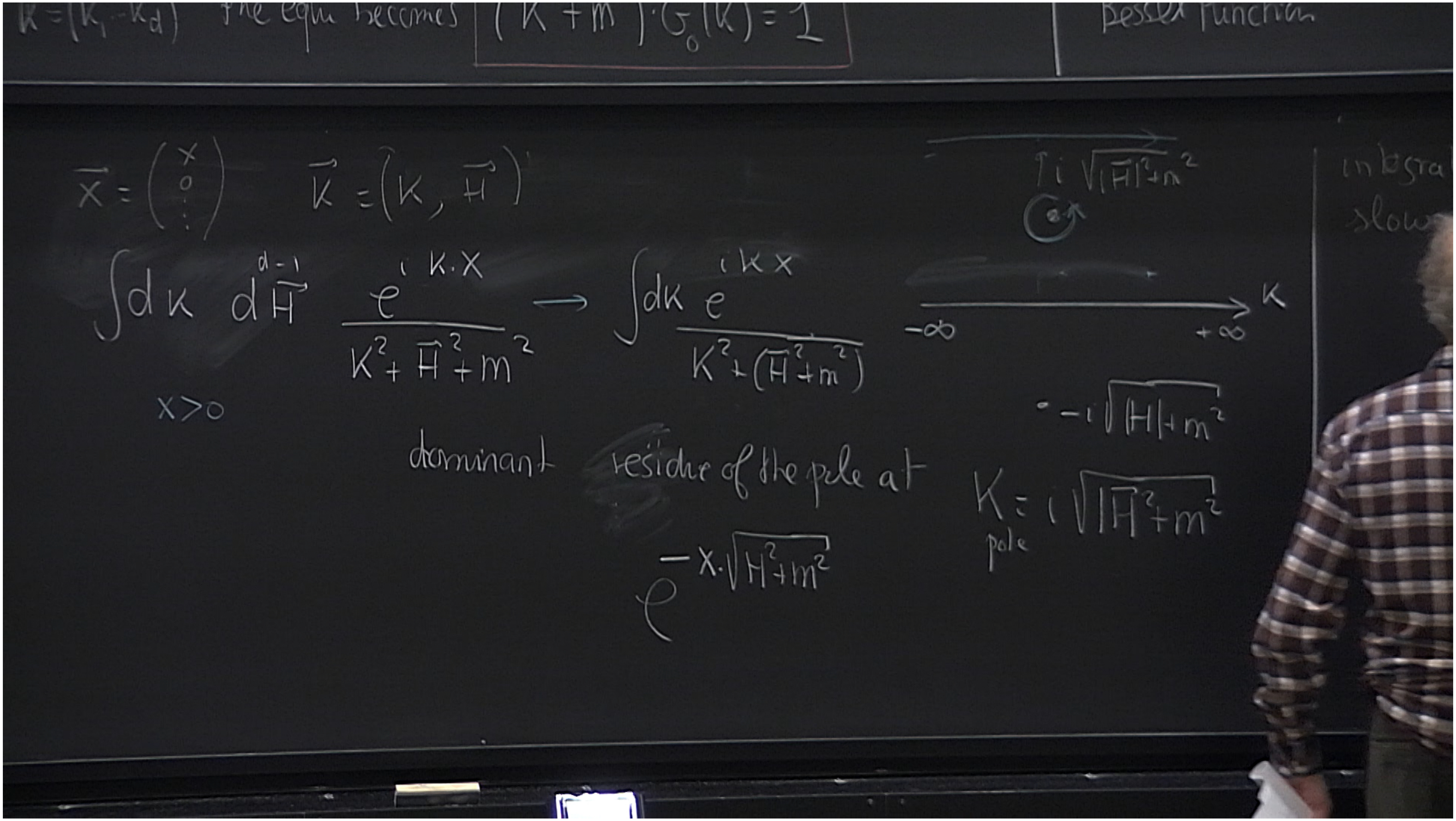
$-\infty$ $+\infty$ $k$

$-i\sqrt{|H|+m^2}$

dominant residue of the pole at $k_{pole} = i\sqrt{|\vec{H}|^2+m^2}$

$$e^{-x\cdot\sqrt{H^2+m^2}}$$

integra



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$x > 0$

$i\sqrt{|\vec{H}|^2 + m^2}$

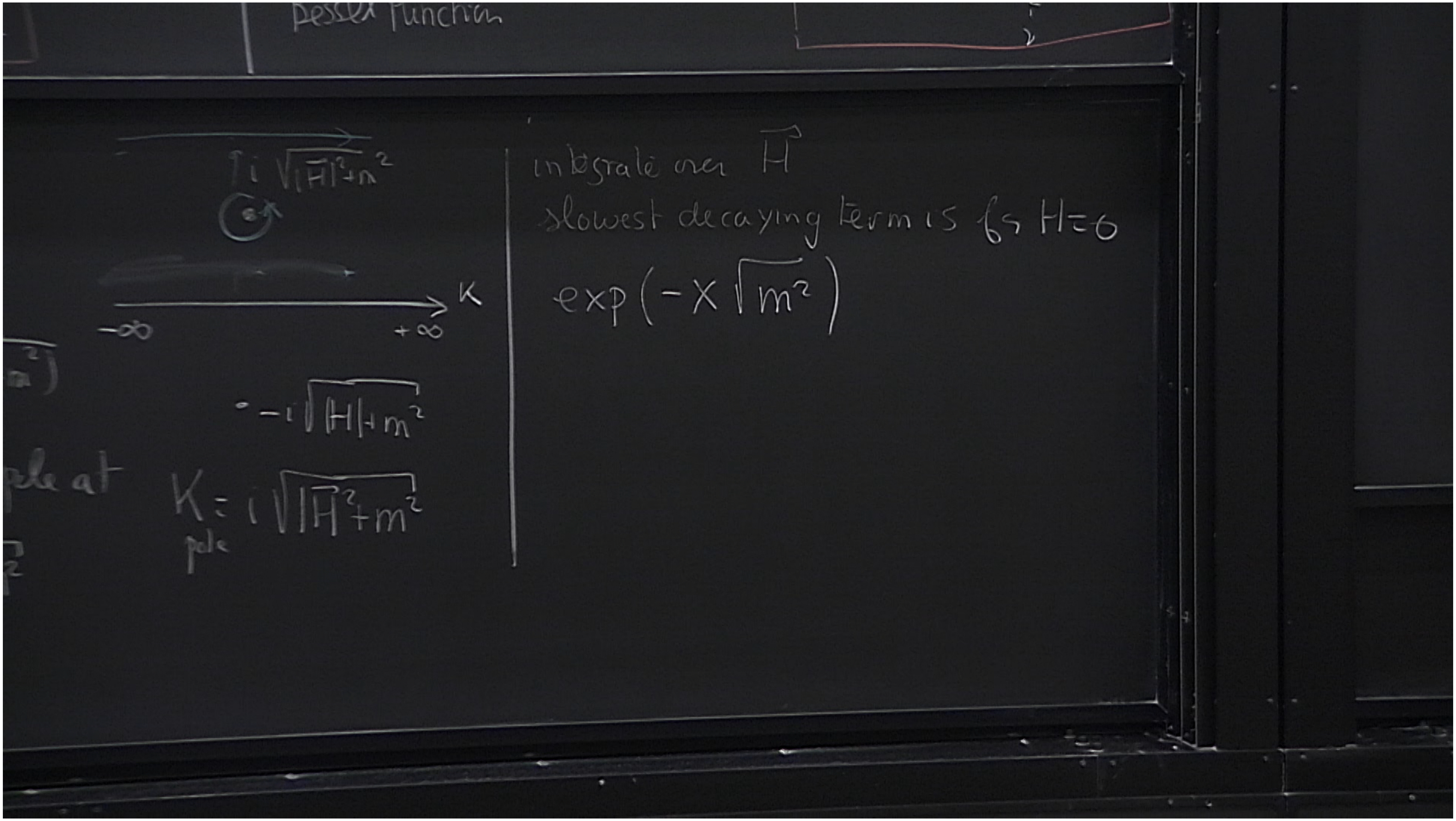
$-\infty$ $+\infty$ $k$

$-i\sqrt{|H| + m^2}$

dominant residue of the pole at $k_{pole} = i\sqrt{|\vec{H}|^2 + m^2}$

$$e^{-x\cdot\sqrt{H^2+m^2}}$$

integral slow



Bessel function

$i\sqrt{|\vec{H}|^2+m^2}$

$K$

$-\infty$ $+\infty$

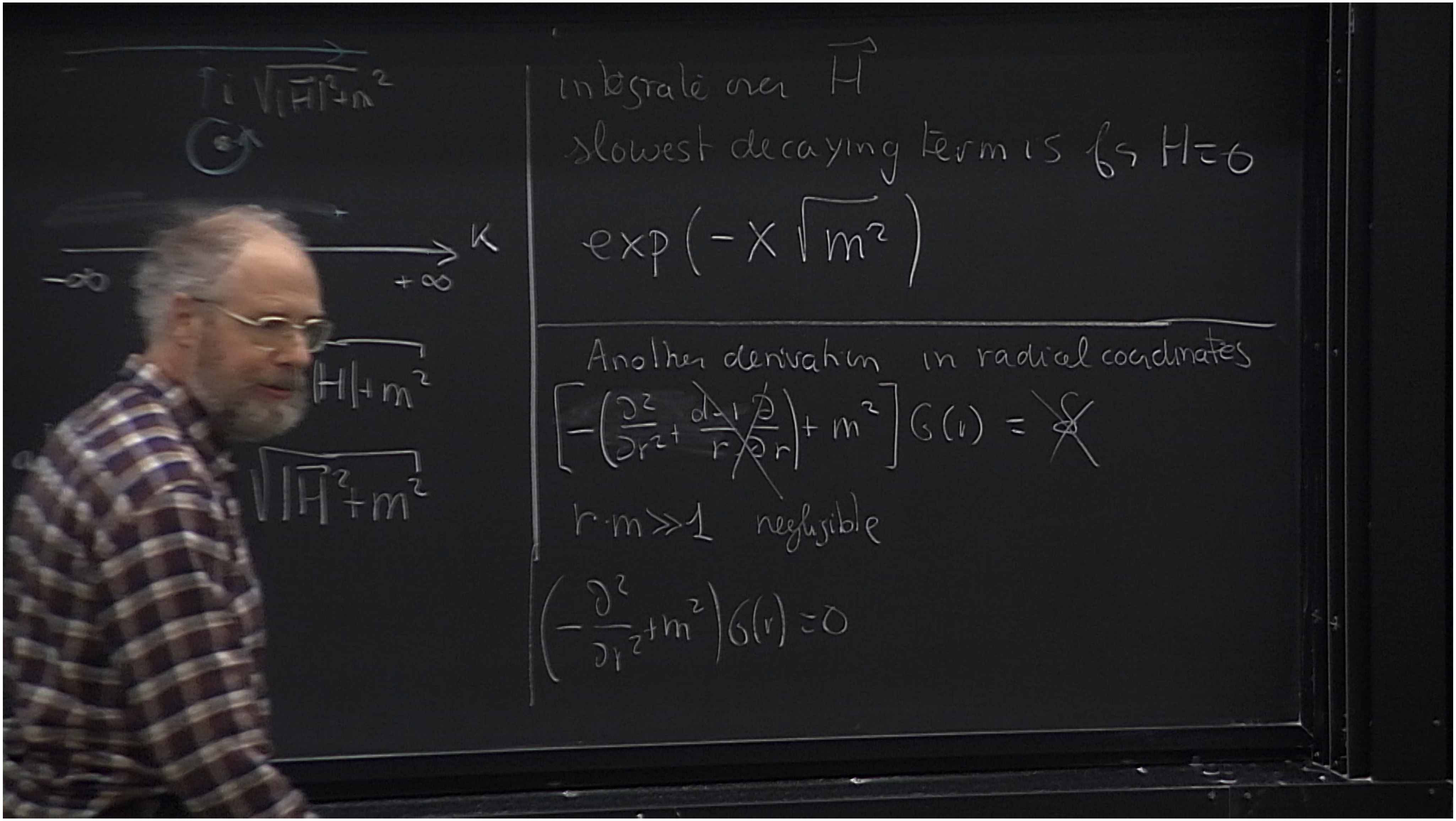
$-i\sqrt{|H|+m^2}$

pole at $K_{pole} = i\sqrt{|\vec{H}|^2+m^2}$

integrate over $\vec{H}$

slowest decaying term is for $H=0$

$\exp(-X\sqrt{m^2})$



$i\sqrt{|\vec{H}|^2+m^2}$

$-\infty$ $\xrightarrow{\phantom{xxxxxxxx}}$ $+\infty$ $K$

$|H|+m^2$

$\sqrt{|\vec{H}|^2+m^2}$

integrate over $\vec{H}$

slowest decaying term is for $H=0$

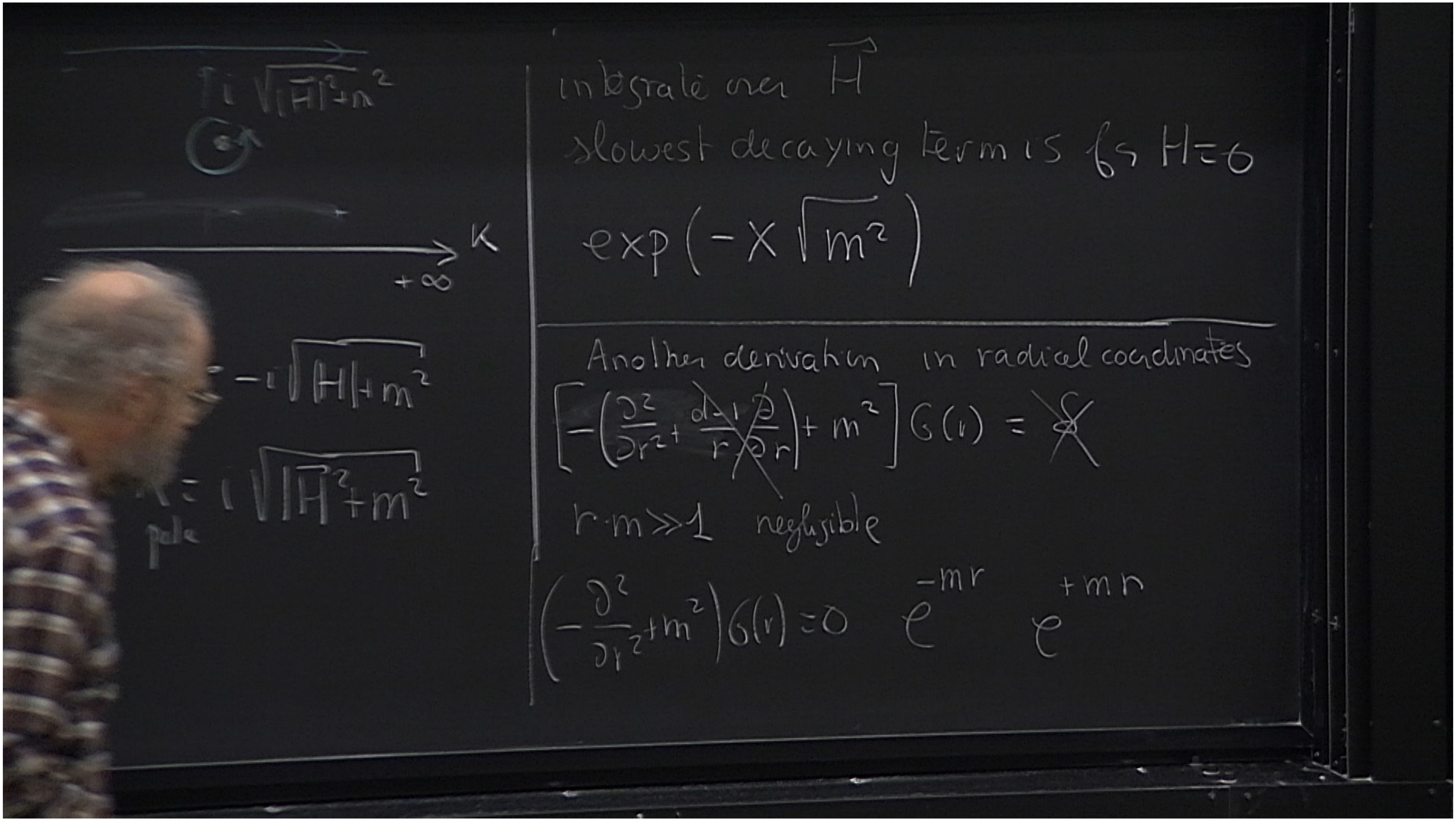
$$\exp\left(-X\sqrt{m^2}\right)$$

Another derivation in radial coordinates

$$\left[-\left(\frac{\partial^2}{\partial r^2}+\cancel{\frac{d-1}{r}\frac{\partial}{\partial r}}\right)+m^2\right]G(r) = \cancel{\delta}$$

$r\cdot m \gg 1$ negligible

$$\left(-\frac{\partial^2}{\partial r^2}+m^2\right)G(r)=0$$



$\pm i\sqrt{|\vec{H}|^2+m^2}$

$\xrightarrow{\quad} K$

$+\infty$

$= -i\sqrt{|H|+m^2}$

$k = i\sqrt{|\vec{H}|^2+m^2}$

pole

integrate over $\vec{H}$

slowest decaying term is for $H=0$

$$\exp\left(-X\sqrt{m^2}\right)$$

Another derivation in radial coordinates

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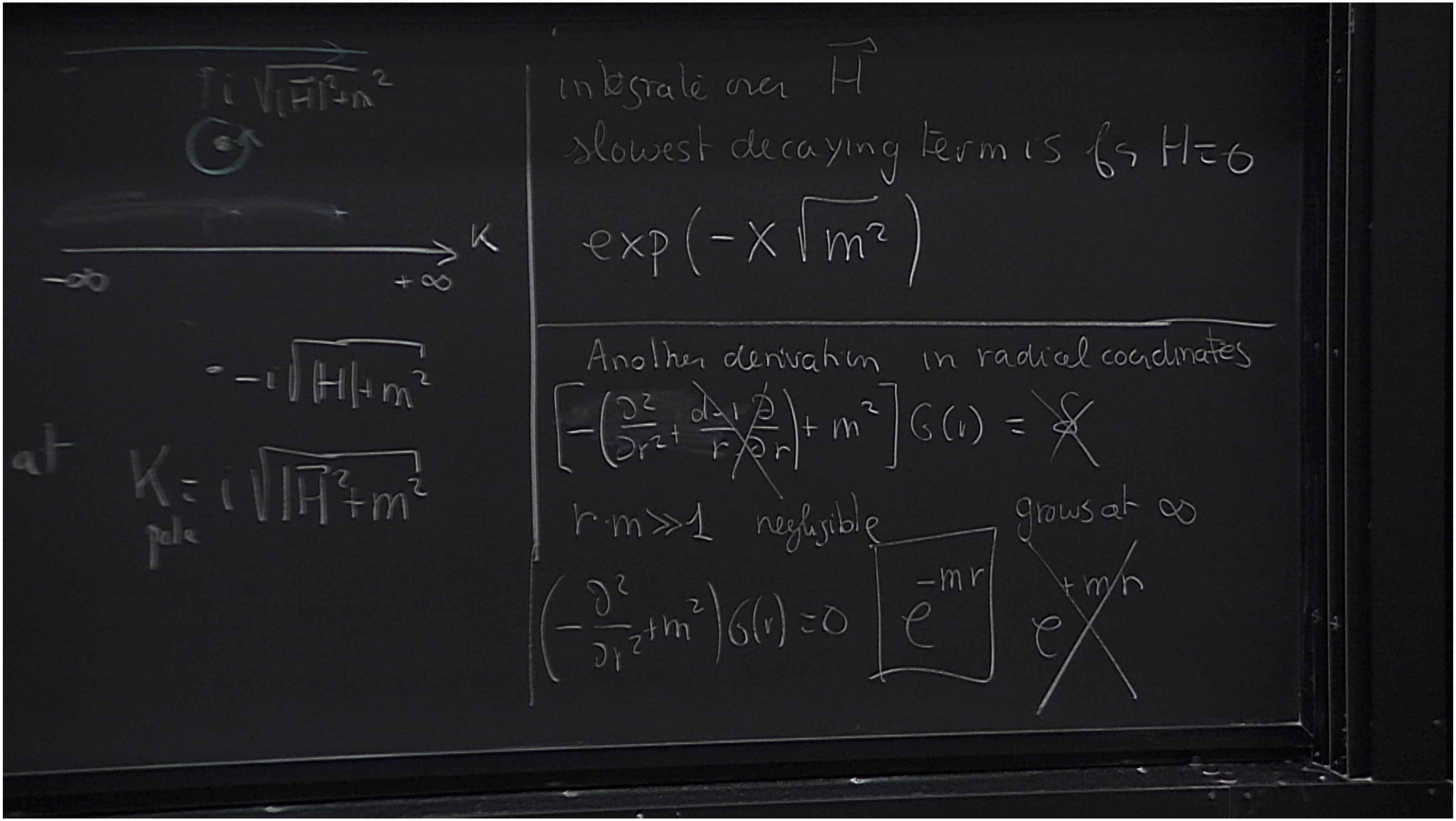


$\pm i\sqrt{|\vec{H}|^2+m^2}$

$K$

$-\infty$ $+\infty$

$-i\sqrt{|H|+m^2}$

at $K = i\sqrt{|\vec{H}|^2+m^2}$

pole

integrate over $\vec{H}$

slowest decaying term is for $H=0$

$$\exp\left(-X\sqrt{m^2}\right)$$

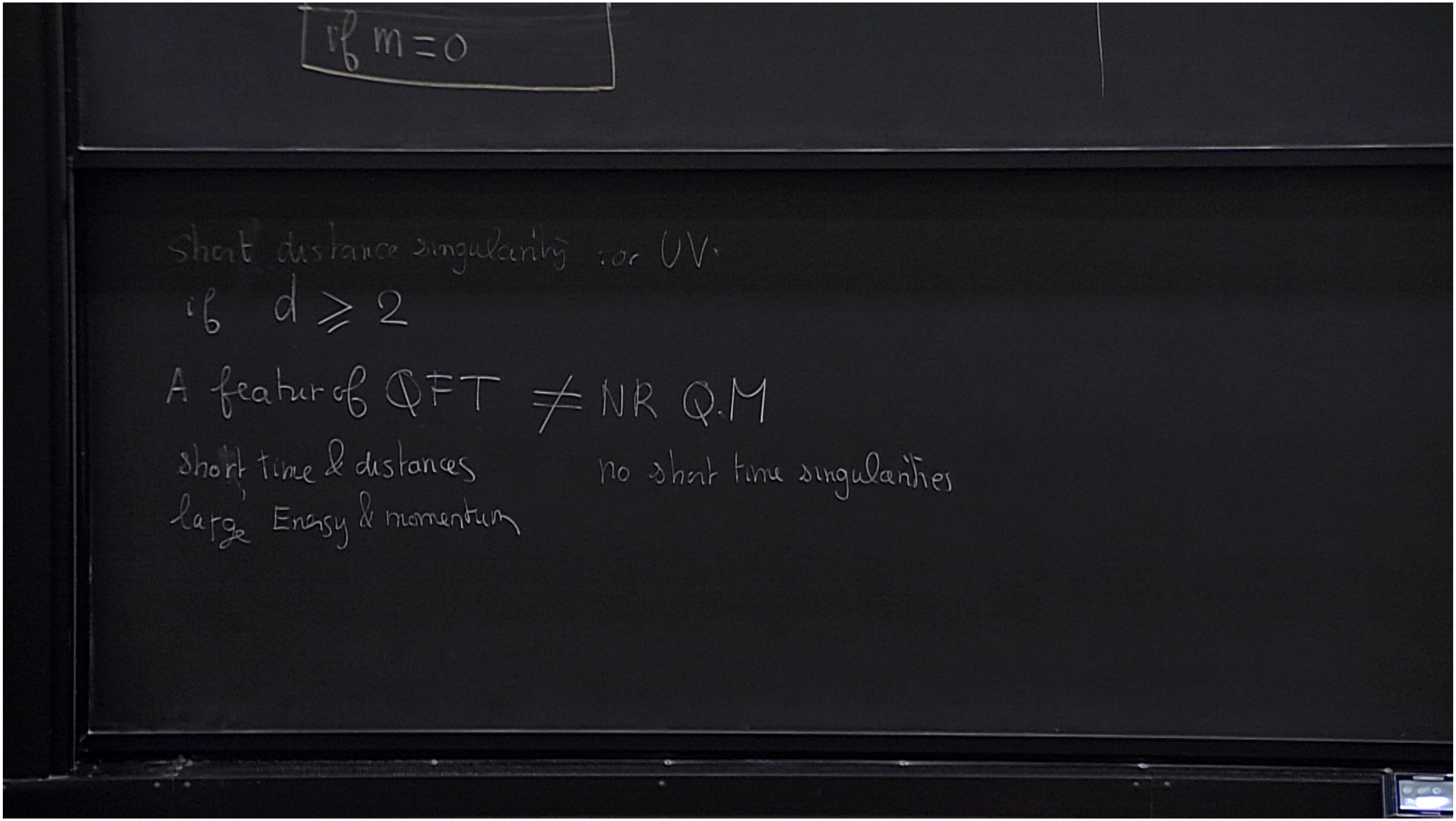
Another derivation in radial coordinates

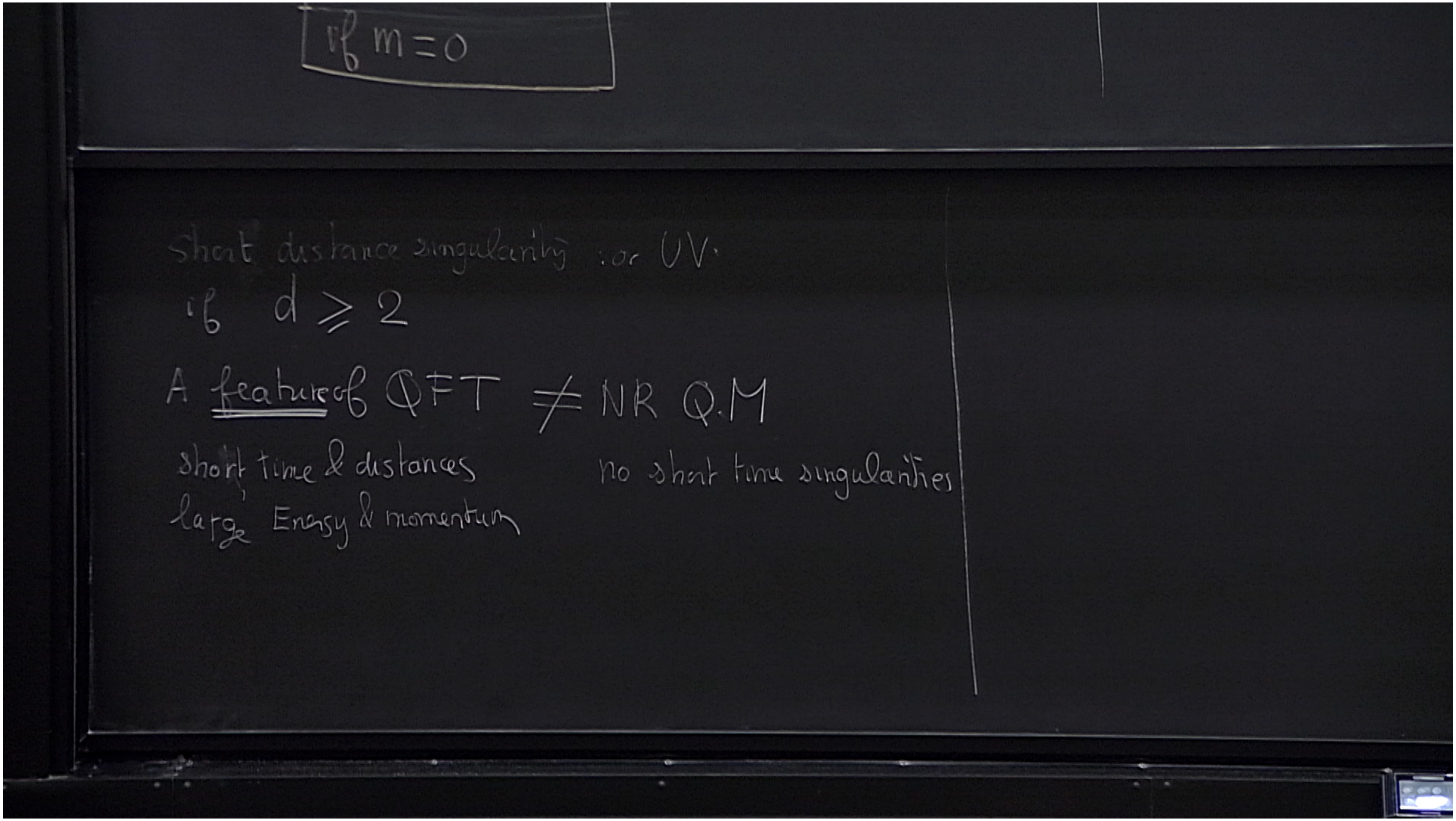
$$\left[-\left(\frac{\partial^2}{\partial r^2}+\cancel{\frac{d-1}{r}\frac{\partial}{\partial r}}\right)+m^2\right]G(r) = \cancel{\delta}$$

$r\cdot m \gg 1$ negligible

grows at $\infty$

$$\left(-\frac{\partial^2}{\partial r^2}+m^2\right)G(r)=0 \quad \boxed{e^{-mr}} \quad \cancel{e^{+mr}}$$





$\boxed{\text{if } m=0}$

Short distance singularity : or UV.

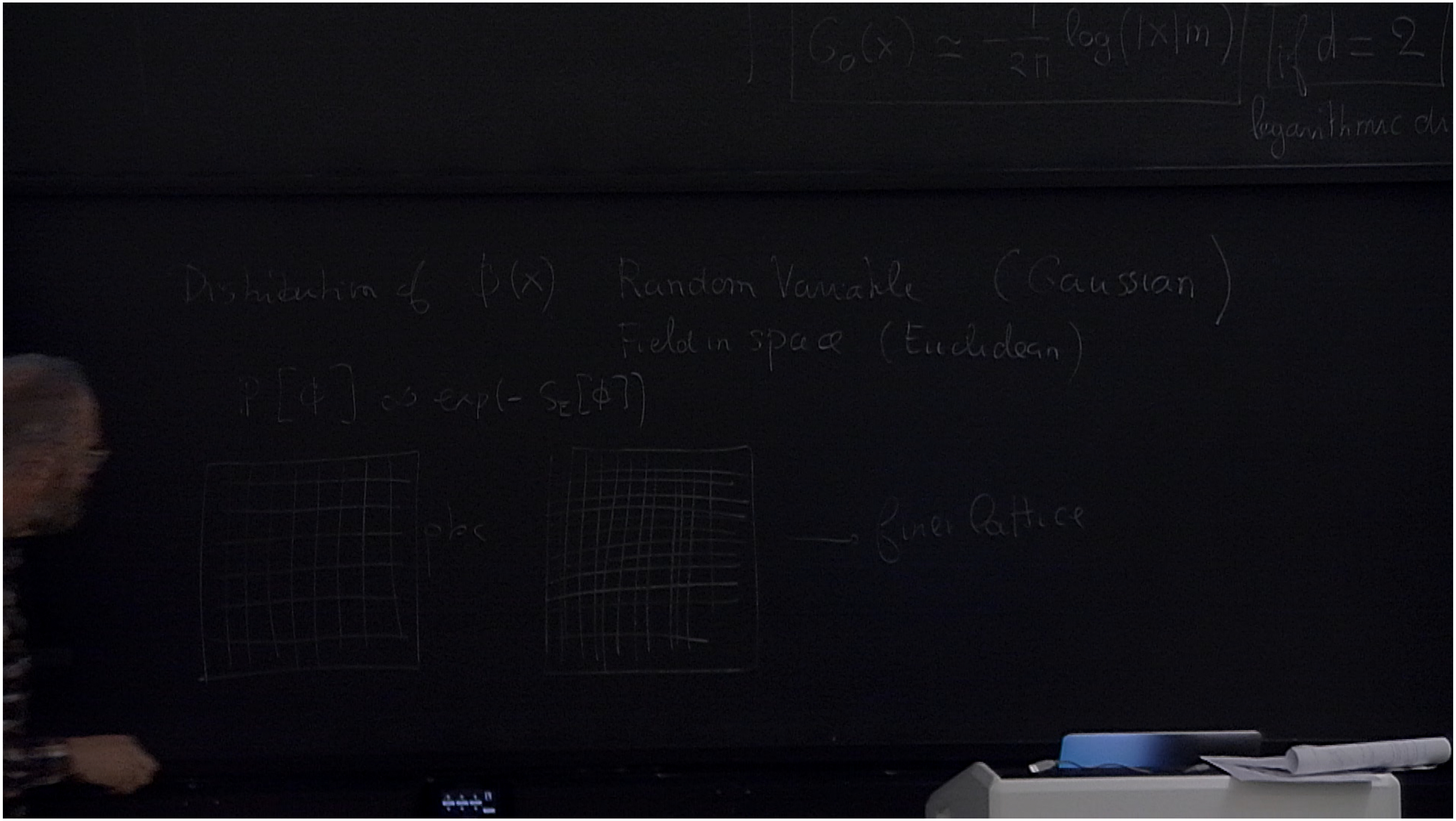
if $d \geq 2$

A feature of QFT $\neq$ NR Q.M

short time & distances

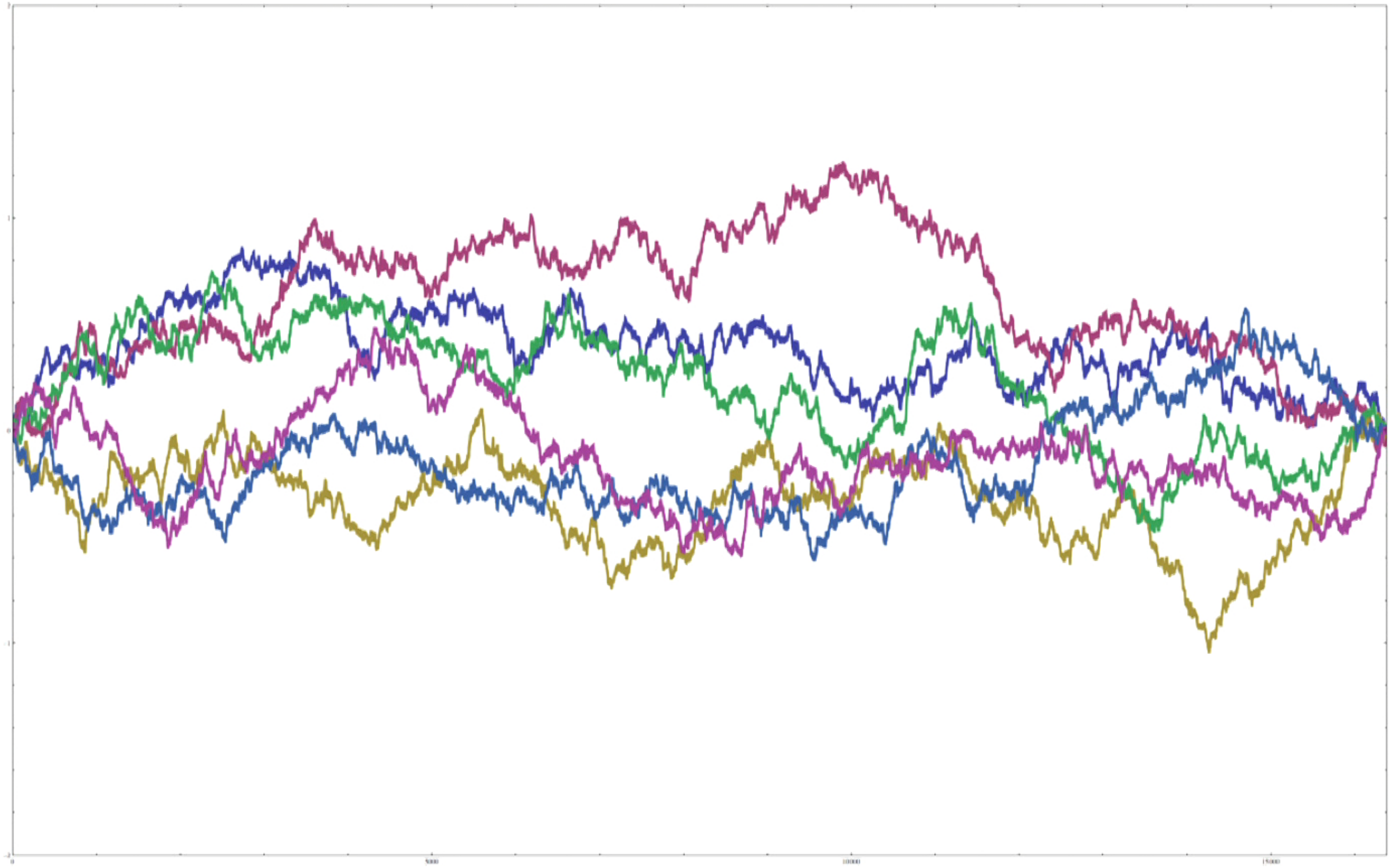
large Energy & momentum

no short time singularities

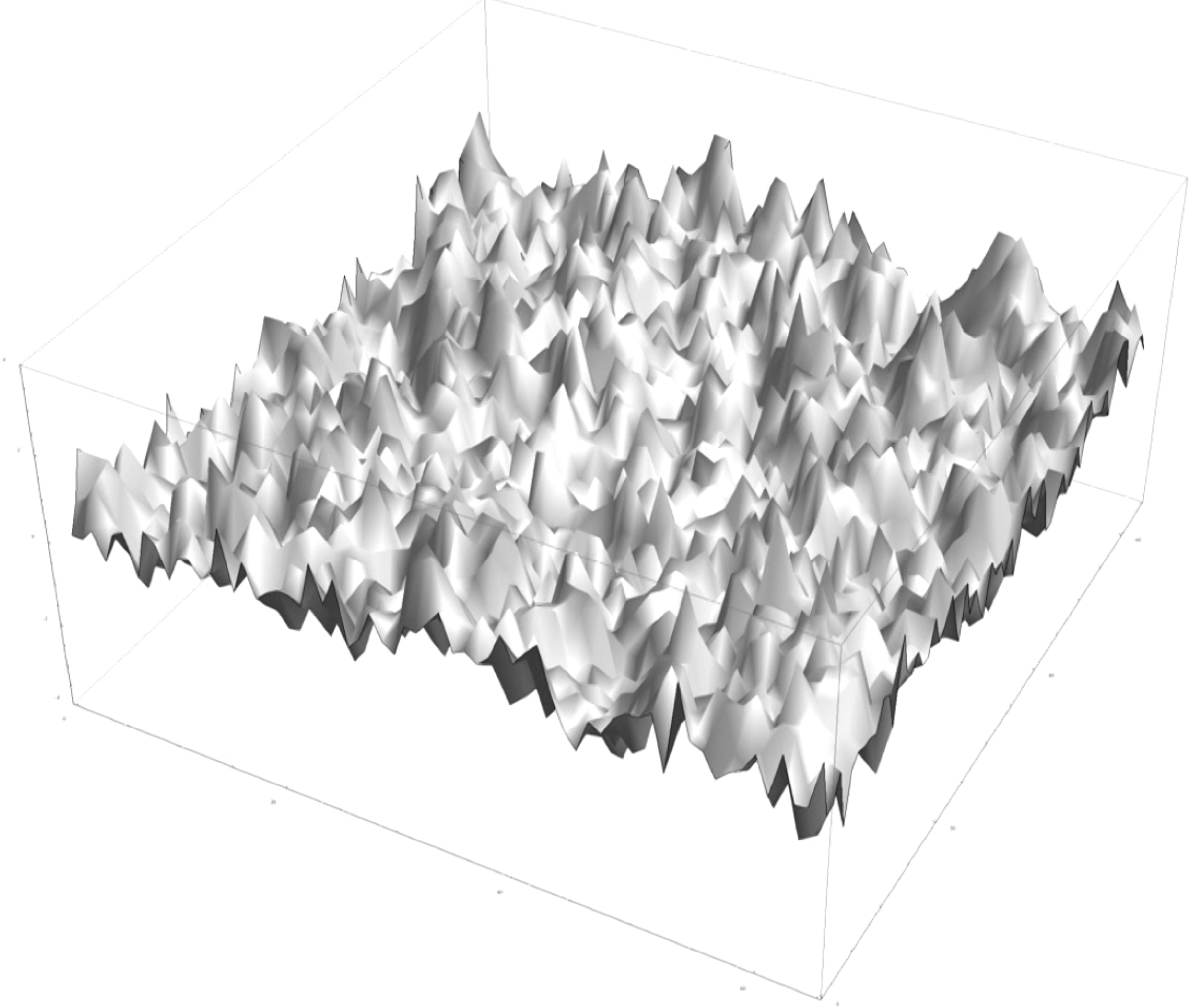
$\boxed{G_0(x) \simeq -\frac{1}{2\pi}\log(|x|m)}$
if $d = 2$
logarithmic di
Distribution of $\phi(x)$ Random Variable (Gaussian)
Field in space (Euclidean)
$P[\phi] \propto \exp(-S_E[\phi])$
pbc
$\longrightarrow$ finer lattice

# Samples of Gaussian free field configurations (Euclidean, periodic b.c.)

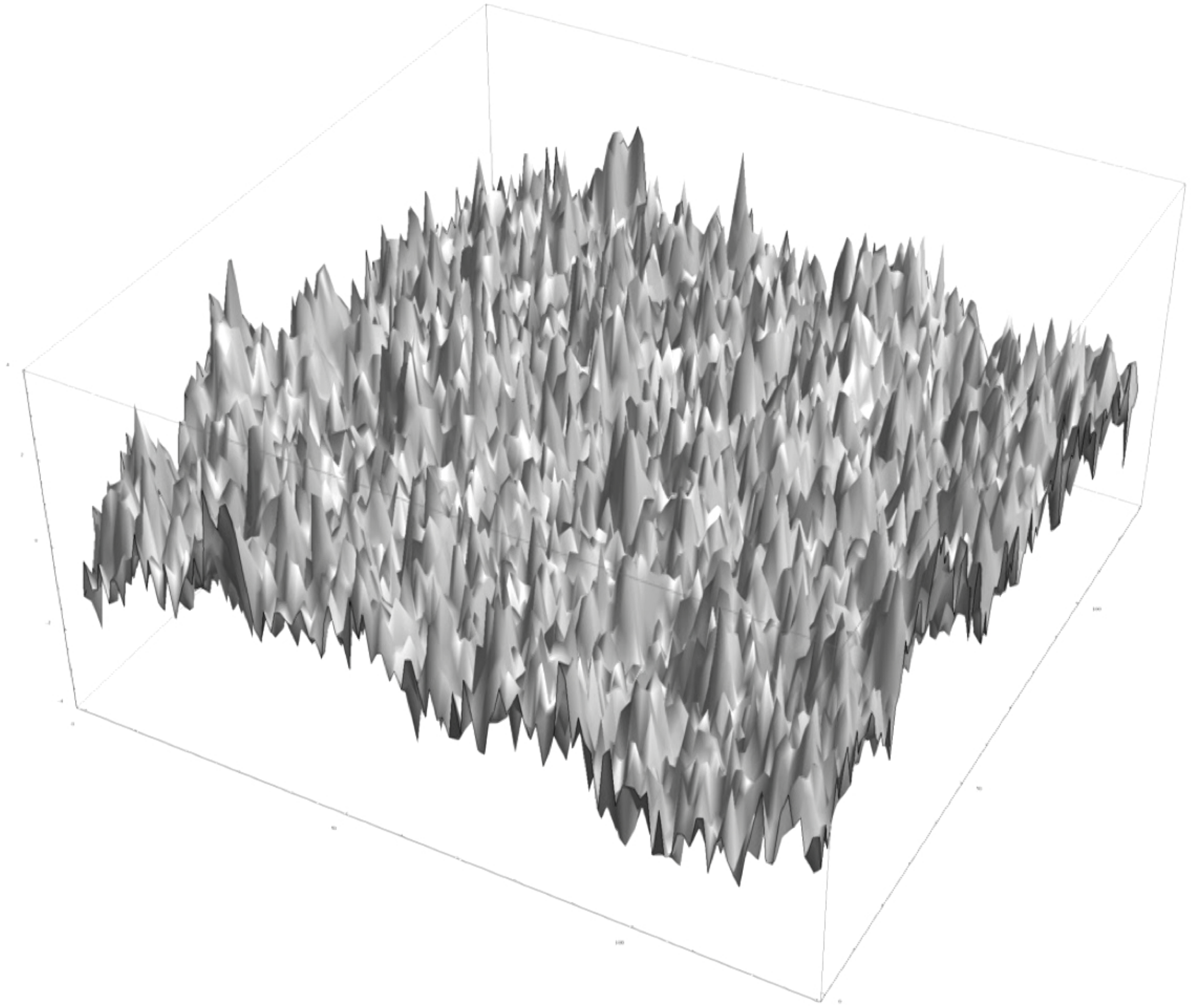
# D=1 : Gaussian Free Fields = Random Walk (i.e. Brownian or Wiener Process)



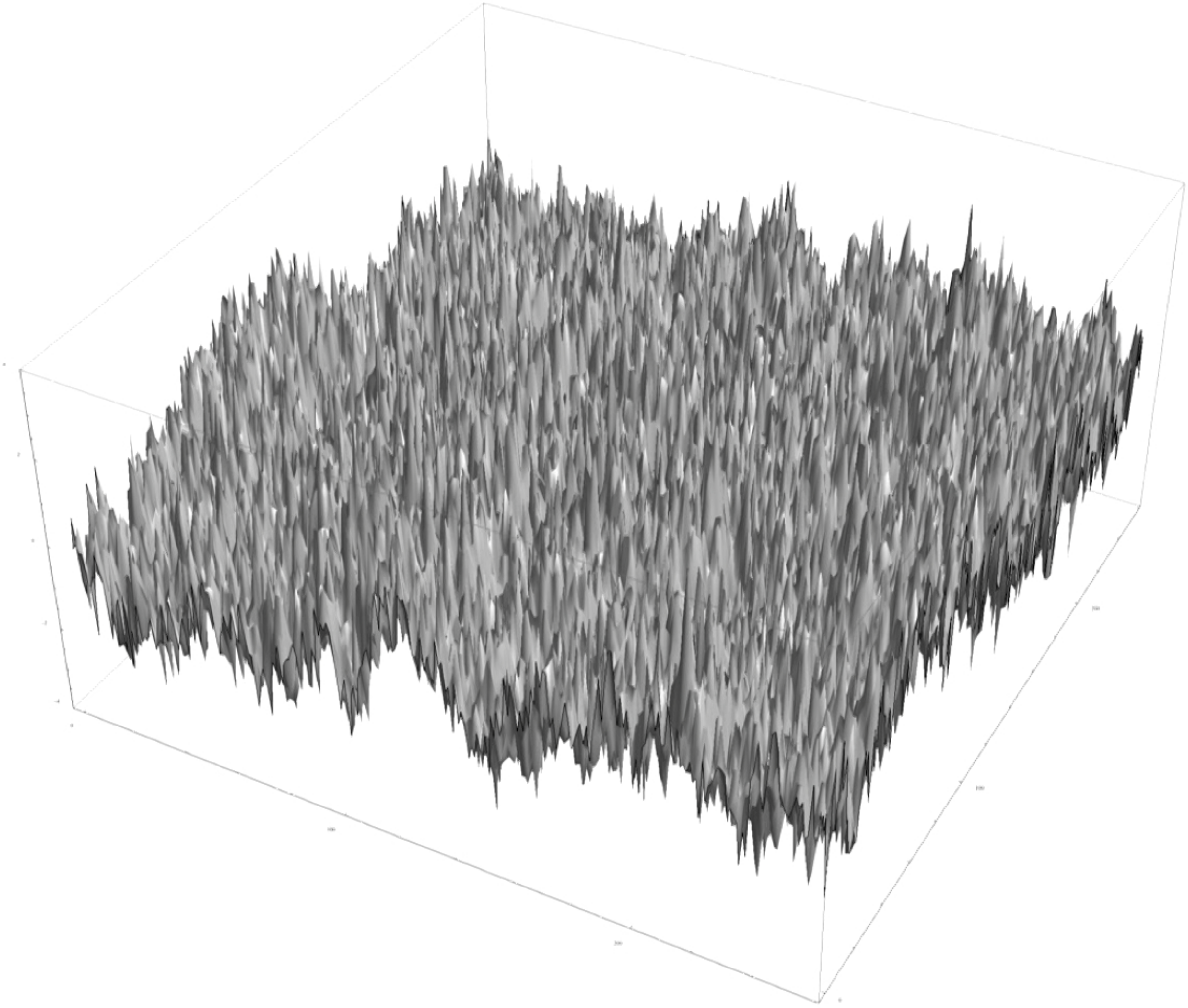
# Gaussian Free Field  D=2  64x64 lattice



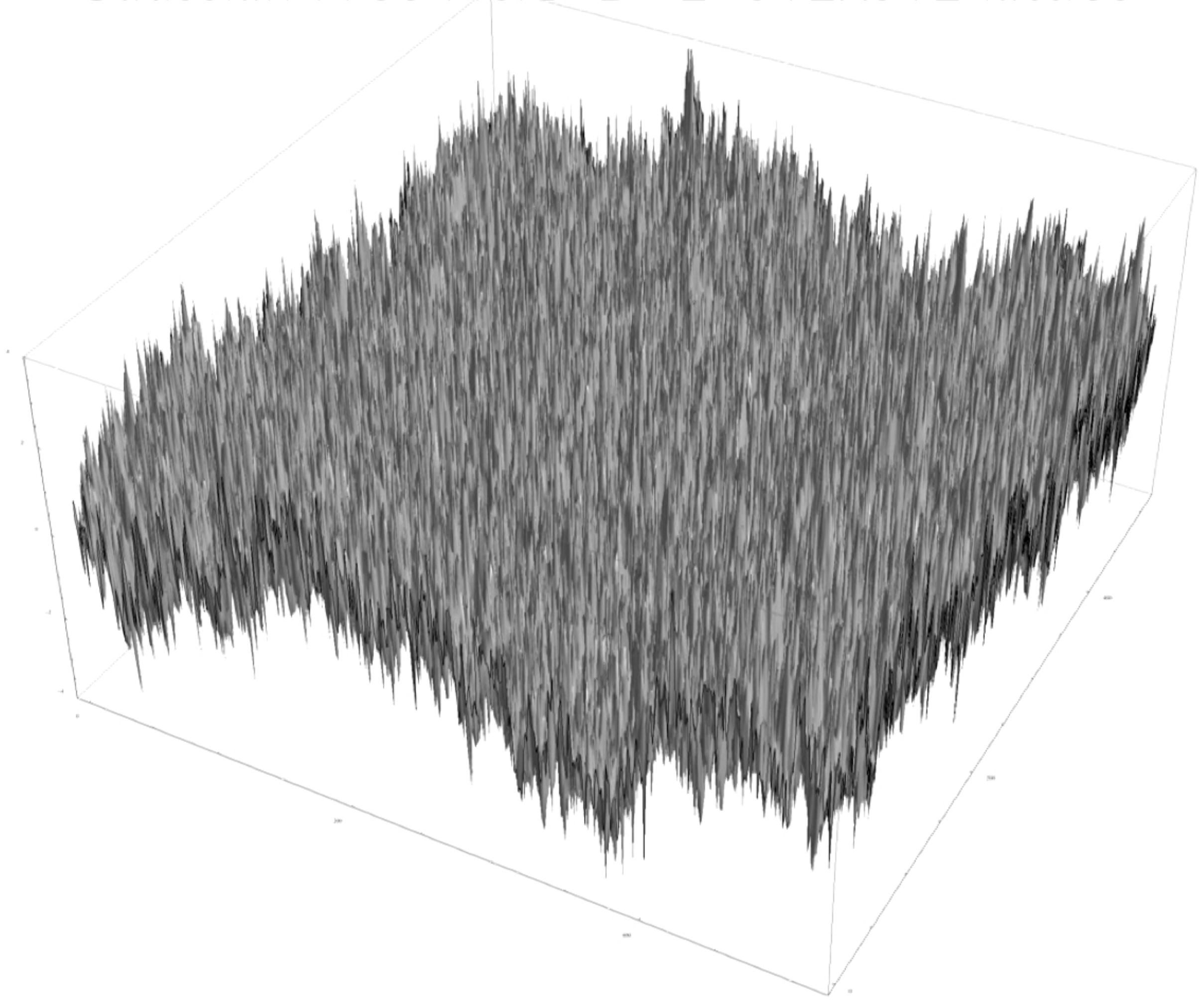
# Gaussian Free Field  D=2  128x128 lattice



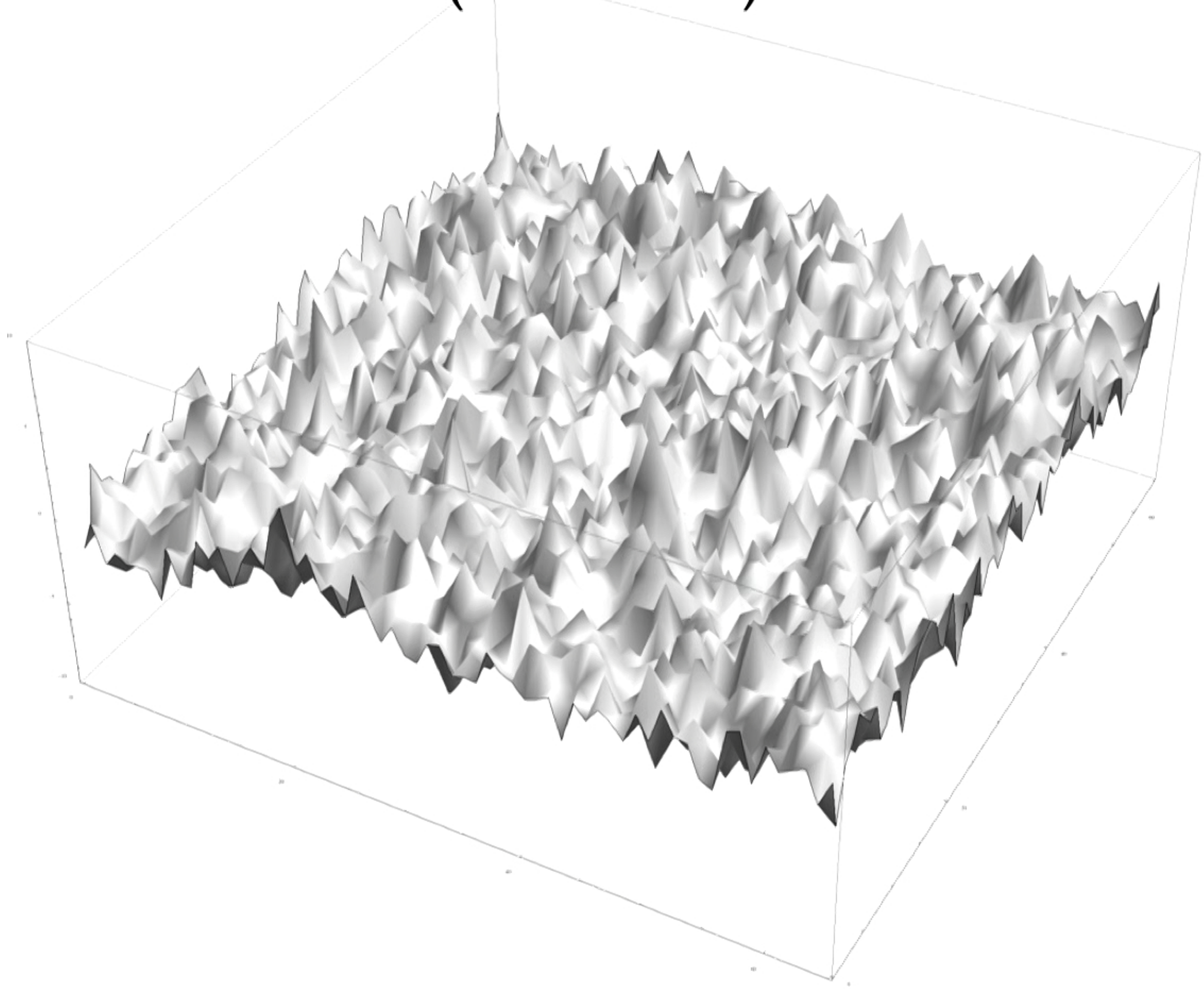
# Gaussian Free Field  D=2  256x256 lattice



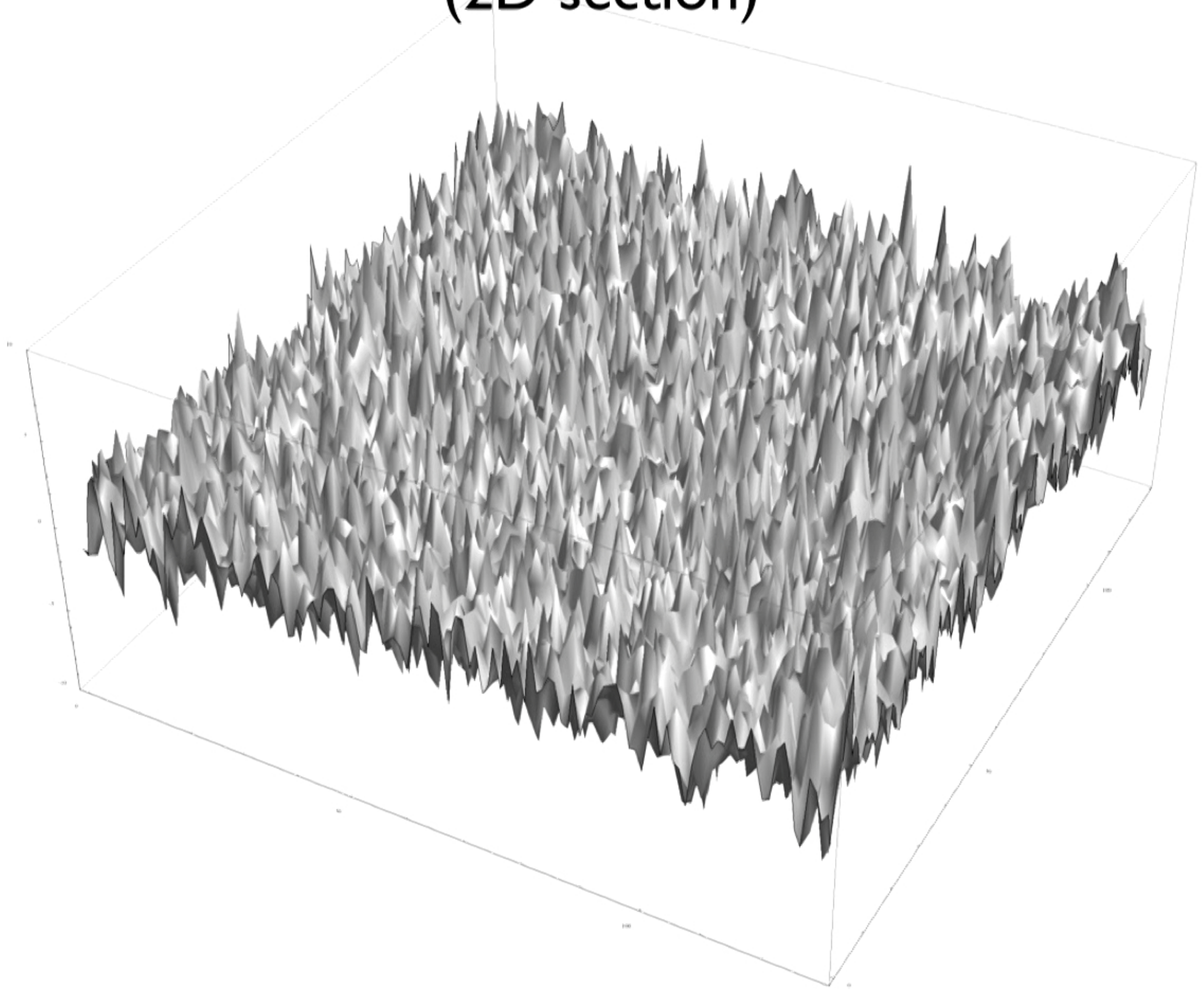
# Gaussian Free Field  D=2  512x512 lattice



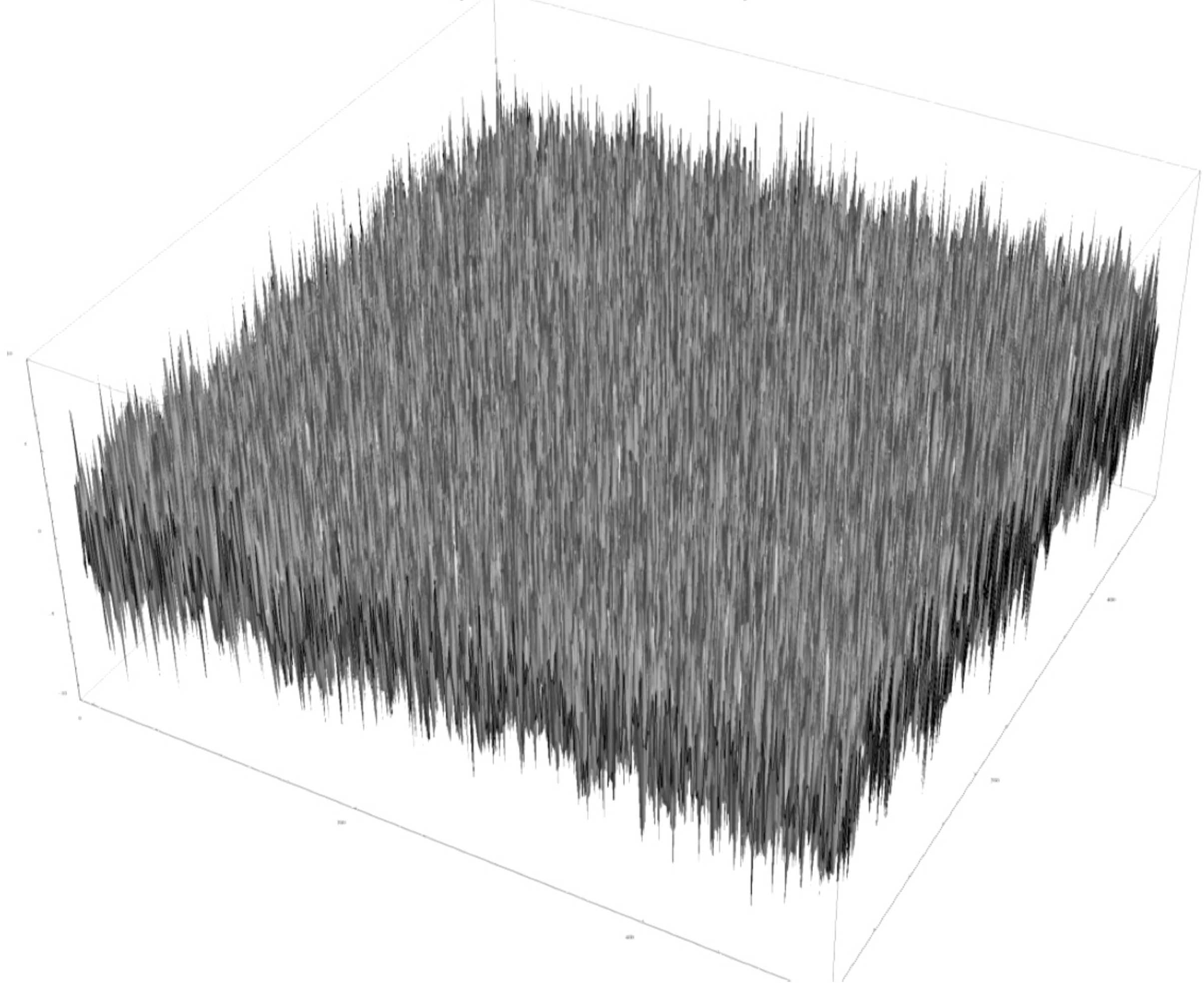
# Gaussian Free Field D=3 64x64x64 lattice (2D section)



# Gaussian Free Field  D=3  128x128x128 lattice (2D section)



# Gaussian Free Field  D=3  512x512x512 lattice
(2D section)



# Free field mass m , dimension D=2 propagator in Euclidean space