Title: General Relativity for Cosmology - Lecture 16

Date: Nov 03, 2017 04:00 PM

URL: http://pirsa.org/17110000

Abstract:

Most likely timeline:

$\Lambda \approx 0.7\, \rho_{critical}$ ("dark energy")

$\rho_{matter} \approx 0.3\, \rho_{critical}$

$\rho_{dark\ matter} \approx 0.9\, \rho_{matter}$

$\rho_{visible\ matter} \approx 0.1\, \rho_{matter}$

short period of matter domination by "inflaton" particles which then decay leaving a hot soup of all sorts of particles

A field, ϕ, inducing "dark energy" dominates ; radiation dominates ; matter dominates ; A Λ, inducing a "dark energy" dominates

.... ? 10^{-43}s — 10^{-32}s — 10^{-11}s — 10^{0}s — 10^{13}s — 10^{19}s $\approx \log(t)$

Planck era

Inflation

$w = -1 + \varepsilon$ ← very small

10^{0}s: nuclei form

10^{13}s: atoms form & CMB released (10^5 yrs)

10^{19}s: today

Why inflation?

at this time, the temperature was so high that particle collisions occurred at a typical energy of $1\,\mathrm{TeV} = 1.6 \cdot 10^{-19} \cdot 10^{12}\,\mathrm{J}$ which is about the maximal energy that accelerator experiments

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Best fit today: $K = 0$

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Evolution of Friedmann-Lemaître spacetimes

- Depending on what is the major contributor to $T_{\mu\nu}$, there is an effective "Equation of State": $\boxed{p = p(\rho)}$

- Periods of time in which the eqn. of state can be approximated as:

$$p(\rho) = w\rho \quad \text{with} \quad w = \text{const.}$$

are called Cosmic Epochs. For example:

Radiation-dominated epoch:	$w = 1/3$
Matter ("dust")-dominated epoch:	$w =$
Dark energy-dominated epoch:	$w =$

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Radiation-dominated epoch:	$w = 1/3$
Matter ("dust")-dominated epoch:	$w = 0$
Dark energy-dominated epoch:	$w = -1$

For any given epoch, use its $p(\rho)$ to solve (from previous lecture)

Continuity equation $\longrightarrow$ $\frac{d}{da}(\rho a^3) = -3p(\rho)a^2$, i.e.: $\frac{d}{da}(\rho(a)a^3) = -3a^2 w\, \rho(a)$

to obtain $\rho(a)$, which shows how energy is diluting:

Solution: $\boxed{\rho(a) = \rho_0\, a^{-3(w+1)}}$ Exercise: verify

Key special cases:

dilution of matter, i.e., energy proportional to $\frac{1}{\text{Volume}} \sim \frac{1}{a^3}$

dilution of energy $\sim \frac{1}{\text{Volume}}$ and energy loss due to wavelength stretching $\sim \frac{1}{a}$

vacuum energy due to cosmological constant is of course constant.

$$\rho(a) = \begin{cases} \rho_m\, a^{-3} & \text{in matter-dominated epoch } (w=0) \\ \rho_r\, a^{-4} & \text{in radiation-dominated epoch } (w = 1/3) \\ \rho_\nu\, a^{0} & \text{in dark energy-dominated epoch } (w=-1) \end{cases}$$

We know of no physical mechanism that could cause $w<-1$. Yet, some evidence suggests

$\varepsilon > 0$

- Now use $\rho(a)$ to turn the Friedmann eqn. into an ordinary differential equation for $a(t)$:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho(a)$$

(we omit the Λ term by agreeing to incorporate Λ in the definition of ρ, p.)

- Observational evidence: the universe is spatially flat, i.e., $K=0$, in a good approximation.

- Solution for $K=0$ and $w \neq -1$:

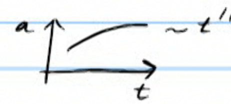
$$\boxed{a(t) = \left(\frac{\pm t}{t_0}\right)^{\frac{2}{3(1+w)}}}$$

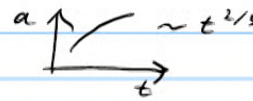
Note that, because $\dot{a}$ is squared in the Friedmann equation,

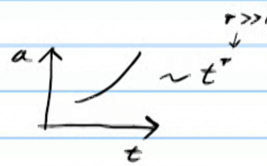
$$a(t) = \left(\frac{\pm t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

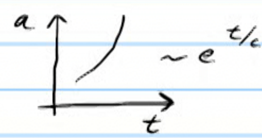
Key epochs: (Exercise/project: what if $K>0$ or $K<0$?)

$$a(t) = \begin{cases} (t/t_r)^{1/2} & \text{in a radiation-dominated epoch } \quad w = 1/3 \\ (t/t_d)^{2/3} & \text{in a matter-dominated epoch: } \quad w = 0 \\ (t/t_p)^{r} & \text{with } r \gg 1 \text{ in a so-called "power law epoch": } w = -1 + \frac{2}{3r} \text{ (Exercise: verify)} \\ e^{t/t_v} & \text{in a totally dark energy dominated epoch: } w = -1. \text{ Exercise: Show this.} \end{cases}$$









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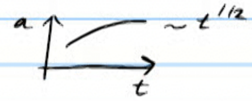

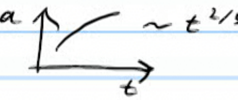

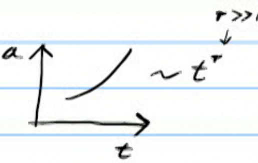

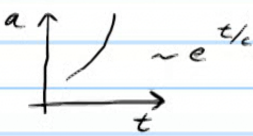

Definition: Any epoch in which $\ddot{a} > 0$, i.e., in which $w < -1/3$ (exercise: verify), is called an "inflationary epoch".

Most likely timeline:

see below for precise definition of $\rho_{critical}$

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at this time, the temperature was so high that particle collisions occurred at a typical energy of $1\,TeV = 1.6 \cdot 10^{-19} \cdot 10^{12}\,J$ which is about the maximal energy that accelerator experiments e.g. at CERN can currently impart on particles.

The flatness problem:

Reconsider the experimental finding of spatial flatness, $K=0$:

- Rewrite the Friedmann equation

$$3\left(\frac{\dot{a}}{a}\right)^2 + 3\,\frac{K}{a^2} = 8\pi G\,\rho + \Lambda$$

Hubble parameter (const. in space but not in time)

by incorporating Λ in $\rho_{tot} = \rho + \frac{\Lambda}{8\pi G}$ and setting $H := \frac{\dot{a}}{a}$:

$$H(t)^2 + \frac{K}{a(t)^2} = \frac{8\pi G}{3}\,\rho_{tot}(t)$$

$\Rightarrow$ At any given time, the critical energy density for $K=0$, i.e., for space to be flat, is:

How close to critical are we now, and at other times?

Definition:

how close to one is/was it?

$$\Omega(t) := \frac{\rho_{tot}(t)}{\rho_{crit}(t)} \quad \text{, i.e.: } \rho_{tot}(t) = \Omega(t) \frac{3}{8\pi G} H(t)^2$$

Thus, the Friedmann equation becomes:

$$H(t)^2 + \frac{K}{a(t)^2} = \Omega(t)\, H(t)^2$$

i.e.

$$\boxed{\Omega(t) - 1 = \frac{K}{\dot{a}(t)^2}}$$

Exercise: check

Calculate backwards through the matter-dominated epoch, $a \sim t^{2/3}$ $\dot{a} \sim t^{-1/3}$:

(radiation-dominated epoch)

▫ Given that $\Omega(t_1)-1 = \mathcal{O}(1)$ today, at time t_1, much earlier, say at $t_0 = 10^{-6} t_1$, we had

$$\Omega(t_0)-1 = \mathcal{O}(10^{-4})$$

accelerator physics goes so far

At $t_n = 10^{-30} t_1$ (i.e. at $t = 10^{-11}s$) we had

in radiation-dominated epoch the effect is even greater since $\Omega - 1 \sim t$

$$\Omega(t_n)-1 \approx \mathcal{O}(10^{-24})$$

⇒ Flatness is not stable! The universe must have started out flat with tremendous accuracy to be still as flat as we see it today.

Experimental constraints?

In order to account for the degree of flatness observed today (and cross-checked in the CMB), a period of near-exponential inflation should have expanded the universe by a factor of at least

$$\frac{a(t_{before})}{a(t_{after})} \approx e^{60}$$

The conjecture of an early inflationary epoch also explains

- The absence of exotic high mass particles that would likely have been produced close to Planck time (and only then).

But also: At the end of the inflationary epoch, how did it happen that the universe was filled with a high density of matter?

Currently favored solution:

The inflationary epoch occurred when a scalar field ϕ had a large potential:

temporarily large

Recall:
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

so that, because of the large $V(\phi)$ we had:

$$w = \frac{p_\phi}{\rho_\phi} \simeq -1 \quad \text{i.e. power law inflation}$$

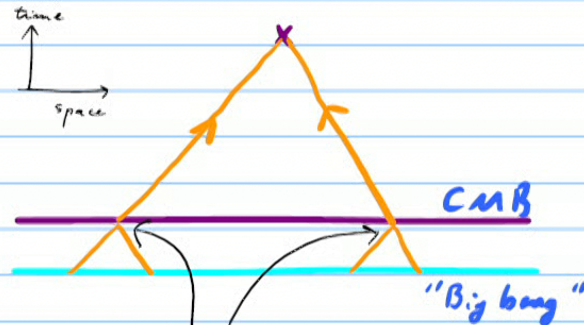
After inflation, $V(\phi)$ becomes the kinetic and mass energy of all sorts of particles, thus making a hot primordial soup.

Speculation:
How did inflation start?
In any spacetime a single quantum fluctuation of ϕ might elevate $V(\phi)$ locally so as to spawn a new universe!

The conjecture of an early inflationary epoch also solves:

- The "horizon problem":

Why does the CMB have the same properties even when checking in opposite directions in the sky?



these two areas of the surface that emitted CMB photons do not have a common past. How come they are so similar?

Concretely: Only patches on the CMB sky of angular extent < 1 degree have a common past, if there was no inflation.

- Answer: If the inflationary epoch expanded spacetime sufficiently.

- the occurrence and precise statistics of inhomogeneities in the universe!

- How? The quantum fluctuations of scalar fields (unlike those of spinor fields of, e.g., electrons and vector fields of, e.g., photons) are being amplified in an inflationary epoch, along with those of g.

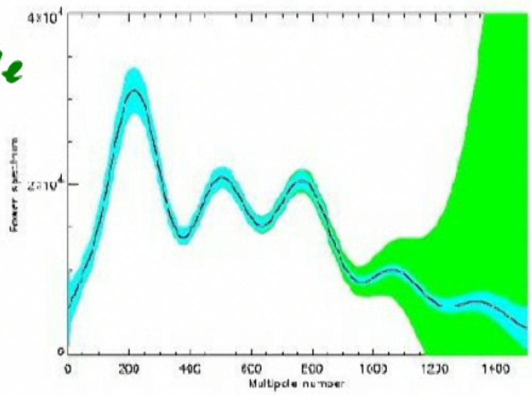
  $\Rightarrow$ They are thought to have seeded the inhomogeneities in the CMB and therefore ultimately the condensation of hydrogen into galaxies and stars.

- Experimental check: Statistics from quantum fluctuations matches data with very good precision:

If caused by quantum fluctuations of ϕ and the metric g, then the predicted statistics was (1980s):

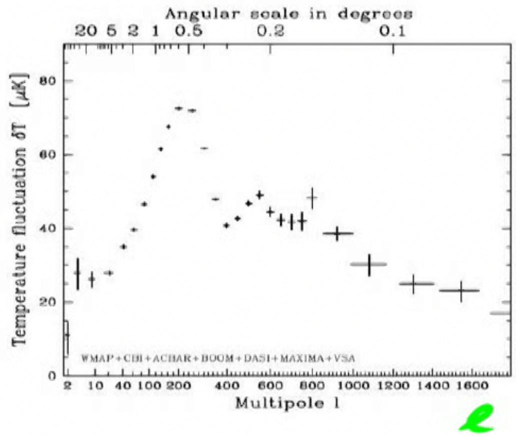
Theory

Amplitude



"angular wavelength^{-1}" ℓ

Experiment



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Remark: A competing theory held that phase transitions, as the universe cooled, left behind "topological defects" in the vacuum, much like

These small inhomogeneities (presumably caused by quantum fluctuations) then explain the statistical distribution of galaxies:

time →

Visualization:

The fluctuations soon grow quickly (Jeans instability)