

Title: Aspects of field theory with higher derivatives

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Abstract: <p>I will discuss related aspects of field theories with higher-derivative Lagrangians but second-order equations of motion, with a focus on the Lovelock and Horndeski classes that have found use in modifications to general relativity. In the first half I will investigate when restricting to such terms is and is not well-justified from an effective field theory perspective. In the second half I will discuss how non-perturbative effects, like domain walls and quantum tunneling, are modified in the presence of these kinetic terms</p>

Aspects of Field Theory with Higher Derivatives



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Based on:
Carrillo-González, Masoumi, ARS, & Trodden 1607.05260 (PRD)
and 1703.00909 (PRD), ARS & Trodden 1709.09695

Outline

- ❖ Motivation
- ❖ Higher derivatives, modified gravity, and EFTs
- ❖ Galileon tunneling
- ❖ Galileon solitons (if time)

Why non-canonical kinetic terms?

- ❖ Scalars with **non-canonical kinetic terms** are ubiquitous in cosmology, e.g.,
 - ❖ Dark energy/inflation (K-essence, P(X), ...)
 - ❖ Modified gravity (Horndeski, galileons, Lovelock, ...)
- ❖ What do particle physics considerations tell us about these theories?
- ❖ Are they **consistent**? Are there **new observable effects**?

Galileons

- ❖ Higher-derivative scalar field theories on flat space — discovered in response to DE problem, starting with DGP
- ❖ Properties of the galileons:
 - ❖ Second-order equations of motion
 - ❖ Galilean invariance $\phi \rightarrow \phi + c + b_\mu x^\mu$
 - ❖ Vainshtein mechanism: nonlinearities kill “fifth force”
 - ❖ Non-renormalization theorem

Galileon Lagrangians

- ❖ The four Lagrangians in D=4 consistent with galilean symmetry and with second-order eoms on flat space:

$$\begin{aligned}
 \mathcal{L}_2 &\sim (\partial\phi)^2 && \sim \varepsilon\varepsilon\partial\phi\partial\phi \\
 \mathcal{L}_3 &\sim (\partial\phi)^2\Box\phi && \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi \\
 \mathcal{L}_4 &\sim (\partial\phi)^2 [(\Box\phi)^2 - \phi_{\mu\nu}^2] && \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi\partial^2\phi \\
 \mathcal{L}_5 &\sim (\partial\phi)^2 [(\Box\phi)^3 - 3\phi_{\mu\nu}^2\Box\phi + 2\phi_{\mu\nu}^3] && \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi\partial^2\phi\partial^2\phi
 \end{aligned}$$

with

$$\begin{aligned}
 \phi_\mu &\equiv \partial_\mu\phi \\
 \phi_{\mu\nu} &\equiv \partial_\mu\partial_\nu\phi
 \end{aligned}$$

Generalizing galileons: without gravity

- ❖ Most general flat-space scalars with second-order eoms:

$$f_n(\phi, (\partial\phi)^2) \times \mathcal{L}_n$$

- ❖ Lose galilean invariance (though may have other interesting symmetries)
- ❖ Some special cases—DBI, conformal, and (A)dS galileons—have interesting origins in higher-dimensional physics
- ❖ **Vainshtein mechanism!** How does this affect nonperturbative solutions?

Generalizing galileons: with gravity

Horndeski

- ❖ Generalizing to include gravity yields **Horndeski gravity**, the most general scalar-tensor theory with second-order equations of motion ($X = (\partial\phi)^2$):

$$\mathcal{L}_2 = G_2(\phi, X),$$

$$\mathcal{L}_3 = G_3(\phi, X)\square\phi,$$

$$\mathcal{L}_4 = G_4(\phi, X)R - 2G_{4,X} [(\square\phi)^2 - \phi_{\mu\nu}^2],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X} [(\square\phi)^3 - 3\phi_{\mu\nu}^2\square\phi + 2\phi_{\mu\nu}^3]$$

- ❖ These Lagrangians are now ubiquitous in cosmology!

Effective field theory and modified gravity: All* theories are effective theories

- ❖ **Decoupling:** high-energy physics not needed to understand phenomena at low energies
- ❖ EFT approach underlies our ability to use nonrenormalizable theories, including GR
- ❖ The lens through which modern physical theories are viewed!
- ❖ **Unless you have a theory of everything at hand, new theories should be seen as EFTs**

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What does (and doesn't) go into an EFT

- ❖ Write down most general Lagrangian consistent with particle content and symmetries, expanded in powers of E/M
 - ❖ M : some (high) energy scale signaling breakdown of EFT
- ❖ Requirements: locality, analyticity, etc.
- ❖ Notably **not** required: second-order equations of motion

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Modified gravity as an EFT

- ❖ Modifications of GR: useful for dark energy, dark matter, inflation, etc.
- ❖ These theories are nonrenormalizable, hence should be seen as effective theories
 - ❖ Construct MG analogously to beyond-Standard Model physics
- ❖ Most MG theories are constructed to be **ghost-free**: Horndeski, Lovelock, generalized galileons, etc.
- ❖ Higher derivatives are generic in EFTs — healthy UV does **not** imply ghost-free EFT

Questions

- ❖ When should (and shouldn't) we restrict to ghost-free theories like Horndeski and Lovelock?
- ❖ What new higher-derivative terms can one write down?
- ❖ Are these phenomenologically interesting?

Healthy UV can have unhealthy EFT

I. Is quantum gravity unstable?

- ❖ GR is lowest-energy term in EFT expansion:

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_{\text{Pl}}^2}{2} R - 2\Lambda + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{d_1}{M^2} R^3 + \dots$$

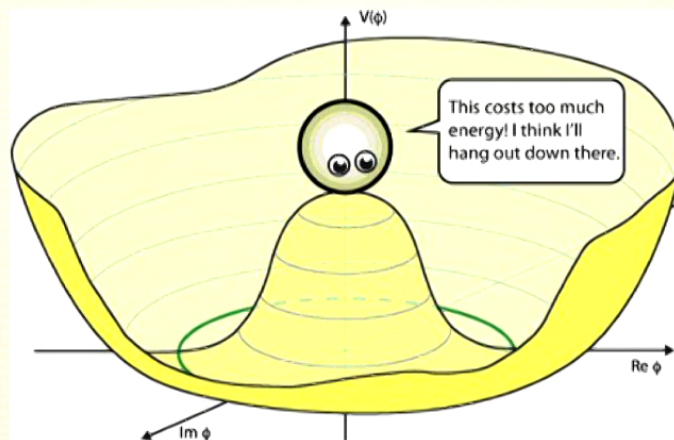
- ❖ All terms (besides R^n) lead to ghosts
- ❖ **Is flat space unstable in quantum gravity?**
- ❖ Answer (fortunately) is no

Healthy UV can have unhealthy EFT

II. U(1)-breaking scalar

- ❖ Illustrative example (Burgess and Williams 1404.2236):

$$S = \int d^4x \left[-\frac{1}{2} \partial_\mu \Phi^\star \partial^\mu \Phi - V(\Phi^\star \Phi) \right]$$



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II. U(1)-breaking scalar

❖ Writing

$$\Phi(x) = \frac{v}{\sqrt{2}} (1 + \rho(x)) e^{i\theta(x)}$$

we have a massless Goldstone θ and a massive ρ
with $M^2 = \lambda v^2$

❖ The action becomes

$$\mathcal{L} = \int d^4x \left[-\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(1 + \rho)^2(\partial\theta)^2 - V(\rho) \right]$$

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II. U(1)-breaking scalar

$$\mathcal{L} = \int d^4x \left[-\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(1+\rho)^2(\partial\theta)^2 - V(\rho) \right]$$

- ❖ For energies $\ll M$, we can **integrate out** ρ to obtain an effective action for θ
- ❖ Solution to ρ eom is highly nonlocal, but can localize by writing perturbatively in $1/M$

$$\square\rho - (1+\rho)(\partial\theta)^2 - V' = 0 \quad \Longrightarrow \quad \rho = -\frac{(\partial\theta)^2}{M^2} - \frac{(\partial\theta)^4 + 2\square(\partial\theta)^2}{2M^4} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

Healthy UV can have unhealthy EFT

II. U(1)-breaking scalar

- ❖ At low energies, EFT for angular mode θ is

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 - \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu\rho}\theta^{,\nu}{}_{,\rho} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

- ❖ $\mathcal{O}(M_4)$ term is **ghostly!**
- ❖ Original theory is healthy — what happened?
- ❖ Resumming full $1/M$ expansion (nonlocal) would cure ghost — higher derivatives are an artifact of truncation
- ❖ EFT point of view: ghost is of mass $\sim M$, cannot be produced within EFT

Ghostbusting

- ❖ EFTs with higher derivatives require additional initial conditions — **spurious** solutions
- ❖ Only a subset of these are **physical** insofar as they reflect solutions of the full theory
- ❖ Ghost instabilities associated to higher derivatives are **not present** in physical solutions



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Ghostbusting

- ❖ Given a higher-derivative EFT, how do we identify these physical solutions?
- ❖ Simply solving the equations of motion will lead to **disaster**
- ❖ Are they (exact) solutions to some other, ghost-free theory?
 - ❖ If yes: justifies use of ghost-free theories
 - ❖ If no: opens up theory space



How do we identify physical solutions?

- ❖ Consider in 1D particle mechanics

$$L = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\epsilon\ddot{x}^2 + \mathcal{O}(\epsilon^2)$$

the eom is

$$\ddot{x} - \epsilon x^{(4)} = 0$$

and has solutions

$$x = A + Bt + Ce^{t/\epsilon} + De^{-t/\epsilon}$$

- ❖ Exponential solutions are bad but not physical—not consistent with perturbative expansion in ϵ
- ❖ Only the $C=D=0$ solutions are **physical** insofar as they perturbatively reflect solutions of the full (UV) theory

Reduction of order method

- ❖ Instead of setting initial conditions to remove runaways (numerically unstable?), can **reduce order** of EFT eom.
- ❖ In this example, defining $E \equiv \ddot{x} - \epsilon x^{(4)} = \mathcal{O}(\epsilon^2)$ we see that, to order in which we're working,

$$E + \epsilon \ddot{E} = \ddot{x} + \mathcal{O}(\epsilon^2) \quad \implies \quad \ddot{x} = \mathcal{O}(\epsilon^2)$$

so that the physical C=D=0 solutions emerge naturally

- ❖ Allows to straightforwardly deal with higher derivatives in EFTs, e.g., numerically

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When do the physical solutions correspond to an action?

❖ Key: **field redefinitions**. Recall the U(1) scalar:

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 - \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu\rho}\theta^{,\nu}_{,\rho} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

and send

$$\theta \rightarrow \theta + \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu}\theta^{,\nu}$$

Leaves us with a second-order theory, the quartic galileon!

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 + \frac{1}{M^4}(\partial\theta)^2 [\theta_{,\mu\nu}\theta^{,\mu\nu} - (\square\theta)^2] + \mathcal{O}\left(\frac{1}{M^6}\right)$$

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- ❖ The problematic higher derivatives have been shunted off to $\mathcal{O}(M^{-6})$, which we can safely ignore
- ❖ Physical solutions to this EFT could be obtained by exactly solving a quartic galileon (up to $\mathcal{O}(M^{-4})$)

Prescription for identifying genuine higher derivatives

- ❖ Construct EFT operator basis up to two redundancies:
Integrations by parts and **field redefinitions (equations of motion)**
 - ❖ This is very well understood in beyond-SM physics
- ❖ Extra ingredient for modified gravity: construct operator bases with **as few higher derivatives as possible**
- ❖ Those higher-derivative terms that are left **should** be included!
- ❖ Deal with these by solving equations of motion **perturbatively**

Shift-symmetric scalar EFT basis: no ghosts until dimension 12!

Dimension	Operators
4	$X = (\partial\phi)^2$
5	<i>None</i>
6	<i>None</i>
7	Cubic galileon*
8	X^2
9	<i>None</i>
10	Quartic galileon
11	<i>None</i>
12	$X^3, (\phi_{\mu\nu}^2)^2$

Modified gravity EFT: scalar-tensor operator basis

- ❖ Field redefinitions enforce lowest-order equations of motion. Scalar-tensor operators which are unaffected should involve **Riemann couplings**
- ❖ Consider scalar-tensor EFT in derivative expansion (e.g., Weinberg's EFT of inflation)

Derivatives	Operators
4	X^2 , Gauss-Bonnet [Weinberg]
6	X^3 , quartic Horndeski, <i>five new higher-derivative operators</i>

Six-derivative scalar tensor operators generated through Riemann couplings

$$R_{\mu\nu\alpha\beta}\phi^{\mu\alpha}\phi^{\nu\beta}, \quad R_{\mu\nu\alpha\beta}\phi^\mu\phi^\alpha\phi^{\nu\beta}, \quad XR_{\mu\nu\alpha\beta}^2, \\ R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu}, \quad (\nabla R_{\mu\nu\alpha\beta})^2$$

- ❖ Should be considered alongside comparable-size Horndeski terms in, e.g., inflation, dark energy. *No reason* a priori to ignore them
- ❖ Must deal with higher derivatives in one of two ways:
 - ❖ Solve equations of motion order by order, or
 - ❖ Reduce order of equations of motion using perturbative nature

Take-home message

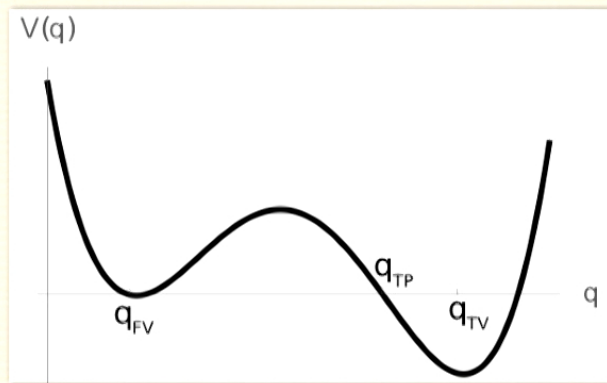
- ❖ **Advocate for consideration of modified gravity as an EFT, including allowing higher derivatives**
- ❖ Healthy UV does not imply ghost-free low-energy EFT
- ❖ Additional justification required to restrict to theories with second-order equations of motion
- ❖ Both Horndeski and non-Horndeski terms should be treated in EFT expansion
 - ❖ Not impossible — e.g., DBI

Such theories have interesting non-perturbative physics

- ❖ Long known that canonical scalars have a zoo of **interesting nonlinear phenomena**
- ❖ This talk:
 - ❖ Quantum tunneling
 - ❖ Solitons
- ❖ Both arise when potentials have **non-degenerate minima**
- ❖ These have been well-understood for decades. What changes when we introduce newly-discovered kinetic structures?

Tunneling

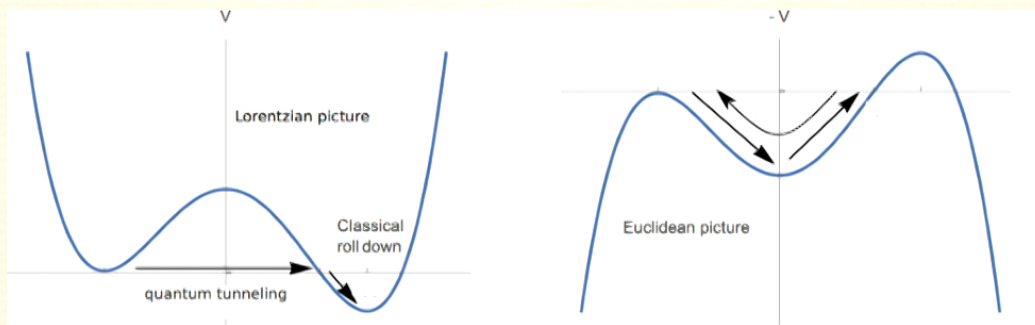
- ❖ Consider a potential with two minima at different heights:



- ❖ Classically both minima are stable, but quantum mechanics induces **decay** of false vacuum via tunneling

Tunneling: Lorentzian and Euclidean pictures

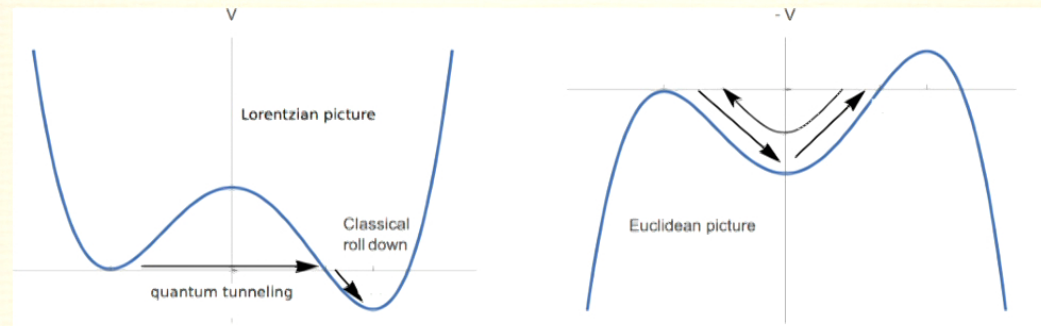
- ❖ Prescription for determining decay rate (Coleman 1977):
- ❖ Transform to **Euclidean time**, i.e., invert potential



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Tunneling: Computing the decay rate



- ❖ The **action** of the Euclidean “bounce” solution determines the decay rate:

$$\frac{\Gamma}{V} \sim e^{-B}$$

$$B = S_E(\text{bounce}) - S_E(\text{FV})$$

Does WKB hold with non-canonical kinetic terms?

Details: arXiv:1703.00909

- ❖ The result $\Gamma/V \sim e_{\cdot B}$ was proven for the canonical scalar field using WKB approximation
- ❖ By solving for the wavefunctional $\psi[\phi]$ in the semi-classical limit, we show that for general

$$L = L(\phi, \dot{\phi}, \partial_i \phi, \partial_i \partial_j \phi)$$

the dominant contribution to the decay rate comes from the solution to the Euclidean equation of motion

- ❖ **Explicitly see: non-canonical kinetic terms do not change Coleman prescription for decay rate***

- ❖ *Assumption: second-order eoms

Decay rates

- ❖ Problem of finding decay rate Γ amounts to solving Euclidean eqms with $O(4)$ symmetry
- ❖ Warm up: $L = P(X) + V(\phi)$ with $X=(\partial\phi)^2$. Euclidean action:

$$S_E = 2\pi^2 \int \rho^3 (P + V) d\rho$$

- ❖ Define **non-standard Lagrangian** with volume factor removed:

$$S_E \equiv 2\pi^2 \int \rho^3 L d\rho$$

and similarly non-standard canonical momentum:

$$\pi_\phi \equiv \frac{\partial L}{\partial \dot{\phi}}$$

P(X) decay rates in the thin-wall limit

- ❖ Consider **thin-wall limit** (small potential difference between the two minima):

$$\epsilon \equiv V_{\text{FV}} - V_{\text{TV}} \ll V$$

- ❖ The bounce factor in this limit is

$$B = \frac{27\pi^2 S_1^4}{2\epsilon^3}$$

- ❖ with S_1 the tension of the bubble wall,

$$S_1 \equiv \int_{\text{wall}} \pi_\phi d\phi$$

- ❖ In the canonical case $P(X) = X/2$ this reduces to Coleman's famous result ✓
- ❖ *However*, $\Gamma/V \sim e_{\text{B}}$ depends extremely sensitively on even small changes in S_1

Galileon decay rates

- ✦ Finally consider **galileons** (incl. non-gravitational generalizations). For concreteness consider cubic galileon,

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi - V(\phi)$$

- ✦ Two regimes:

- ✦ **Standard:** Canonical kinetic term dominates, standard decay rate:

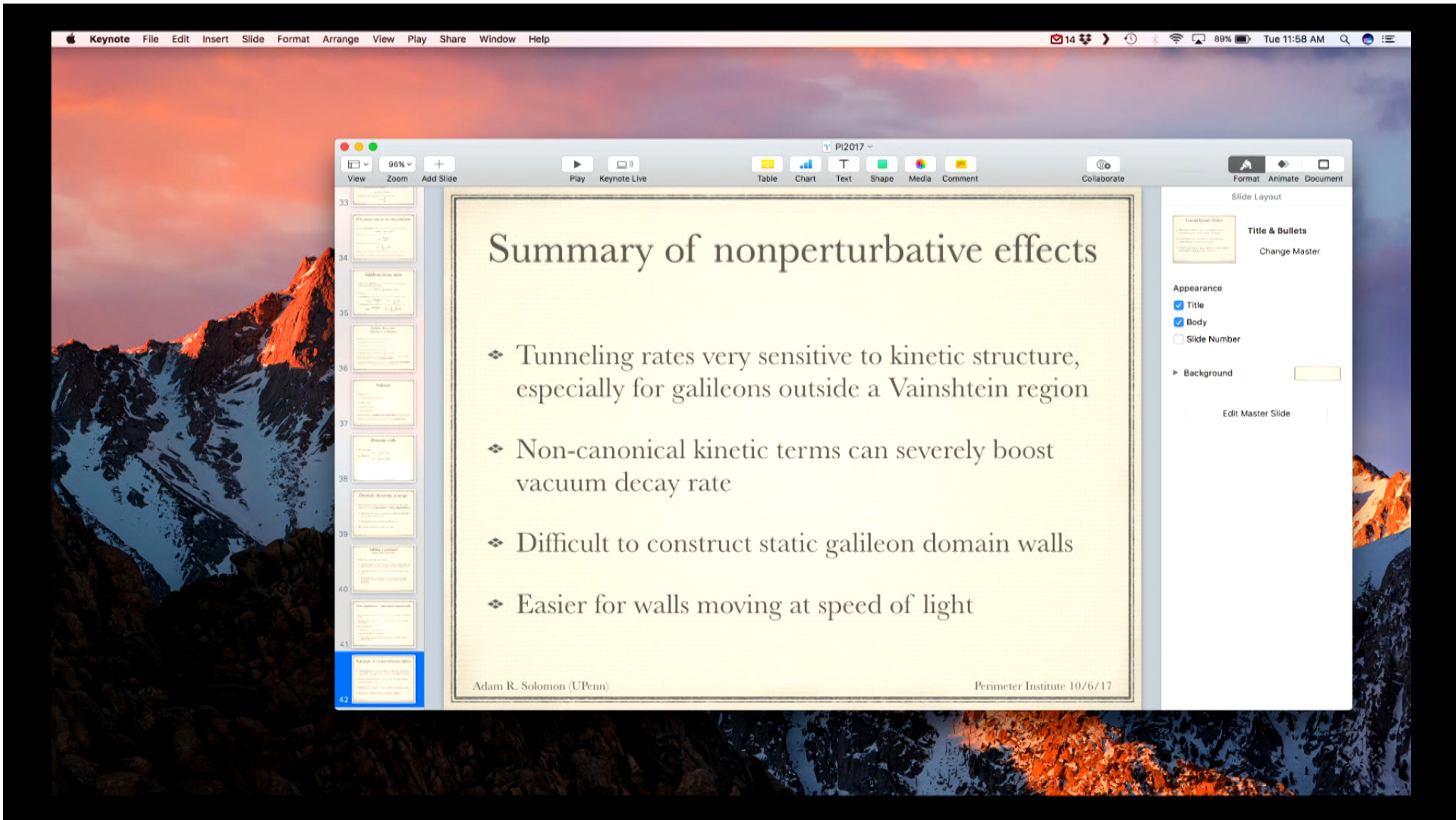
$$B_{\text{can}} = \frac{27\pi^2(S_1^{\text{can}})^4}{2\epsilon^3}, \quad S_1^{\text{can}} = \int_{\text{wall}} \dot{\phi} d\phi$$

- ✦ **Vainshtein:** Galileons dominate, qualitatively different decay rate:

$$B_{\text{gal}} = \frac{2\pi^2(S_1^{\text{gal}})^2}{\epsilon} \quad S_1^{\text{gal}} = \int_{\text{wall}} \frac{6}{\Lambda^3} \dot{\phi}^2 d\phi$$

Galileon decay rates: Vainshtein mechanism

- ❖ Which regime we're in depends on free parameters:
 - ❖ ϵ : difference between potential heights
 - ❖ $\Delta\phi$: difference between location of the two minima
 - ❖ Λ : energy scale associated to the galileon
- ❖ Canonical(/galileon) term dominates if $\frac{\epsilon}{\Delta\phi} \gg (\ll) \Lambda^3$
- ❖ Equivalently: depends on whether Euclidean bubble size ρ is larger or smaller than a threshold value, akin to a **Vainshtein radius**
- ❖ If the galileon dominates, the decay rate can be **many orders of magnitude** larger than for a canonical scalar with the same potential



Summary of nonperturbative effects

- ❖ Tunneling rates very sensitive to kinetic structure, especially for galileons outside a Vainshtein region
- ❖ Non-canonical kinetic terms can severely boost vacuum decay rate
- ❖ Difficult to construct static galileon domain walls
- ❖ Easier for walls moving at speed of light

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