Title: Boundaries and Twists in the Color Code

Date: Oct 11, 2017 04:00 PM

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Abstract: <p>We present an in-depth study of the domain walls available in the color code. We begin by presenting new boundaries which gives rise to a new family of color codes. Interestingly, the smallest example of such a code consists of just 4 qubits and weight three parity check measurements, making it an accessible playground for today's experimentalists interested in small scale experiments on topological codes. Secondly, we catalogue the twist defects that are accessible with the color code model. We give lattice representations of these twists and investigate how they interact with one another, and how they interact with the anyons of the system. Our categorisation allows us to explore new approaches for the fault-tolerant storage and manipulation of quantum information in color codes. This research combines and extends recent work with the surface code [1,2] to the color code models, whose continuous domain walls have been studied in generality in [3]. [1] Delfosse, Nicolas, Pavithran Iyer, and David Poulin. "Generalized surface codes and packing of logical qubits." arXiv preprint arXiv:1606.07116 (2016). [2] Brown, Benjamin J., et al. "Poking holes and cutting corners to achieve Clifford gates with the surface code." Physical Review X 7.2 (2017): 021029. [3] Yoshida, Beni. "Topological color code and symmetry-protected topological phases." Physical Review B 91.24 (2015): 245131.</p>

The boundaries and topological defects of the color code[1]

Markus S. Kesselring[2], Ben J. Brown, Fernando Pastawski, Jens Eisert

11 Oct 2017

Freie Universität Berlin

CRC 183
ENTANGLED STATES OF MATTER

[1] arXiv:1710/11:????
[2] markus.kesselring@fu-berlin.de

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Overview

- Motivation
- Introducton to topological error correction codes
- Introduction color codes
  - Stabilizers
  - Anyons
- Boundaries in the color code
  - New small color code!
- Domain walls and twists in the color code
  - Lattice representations
- Putting boundaries and twists to use
  - Encode logical information in holes, twists and combinations
  - Manipulate information by braiding holes and twists

Motivation

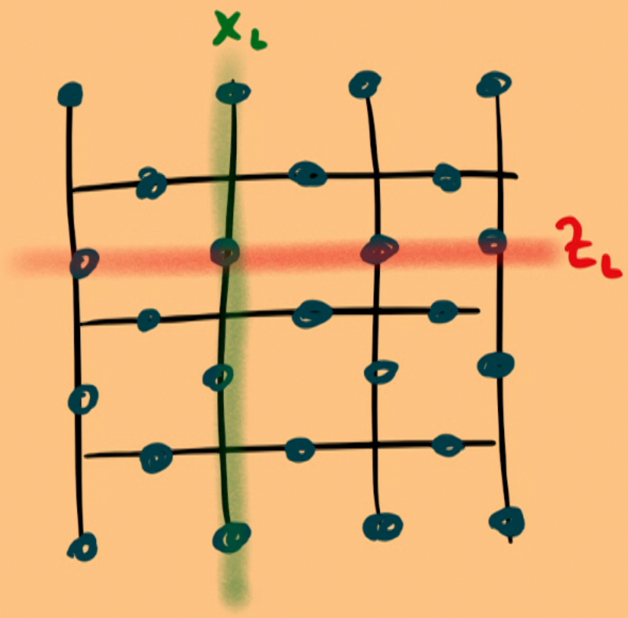
Noise on qubits and gates will mess up quantum computing - thus we need two things:

- **Store information** (robust quantum memories)
  - we want: low encoding rate
- **Manipulate quantum information** (fault tolerance)
  - we want: low space and time overhead

The more ways we know of how to do this, the better!

Robust (topological) quantum memories[3]

Idea: use many physical qubits to encode one logical qubit
Entangle physical qubits with **local** operations



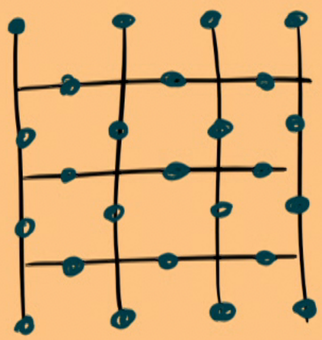
Manipulate logical qubit with **global** operations

[3] Alexei Y. Kitaev: 9707021

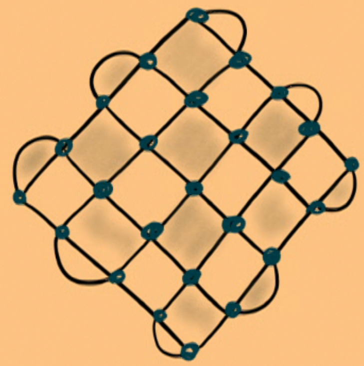
Encoding rates of topological error correction codes: Simple codes

$c = \frac{d^2 \cdot k}{n}$: encoding rate $\rightarrow$ we want this number to be large!

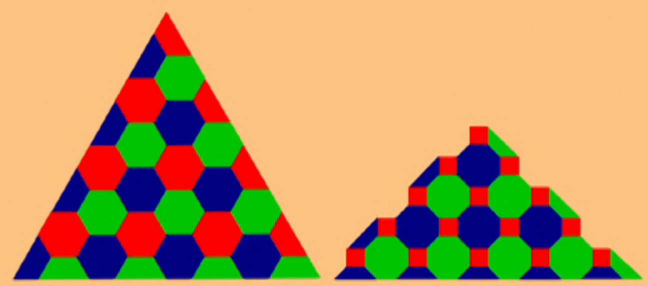
Surface code
$c \simeq 0.5$



Wen plaquette model
$c = 1$



Color codes[4]
6.6.6: $c \simeq 1.33$, 4.8.8 $c \simeq 2$
(figure from[5])



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[4] H. Bombin and M.A. Martin-Delgado: 0605138
[5] Andrew J. Landahl, Jonas T. Anderson, and Patrick R. Rice: 1108.5738

Goal of twist work

Summary: Color codes are better at encoding information.
But surface codes can catch up, if twists and boundaries are used!

Questions: What happens if we introduce twists and boundaries to the color code?

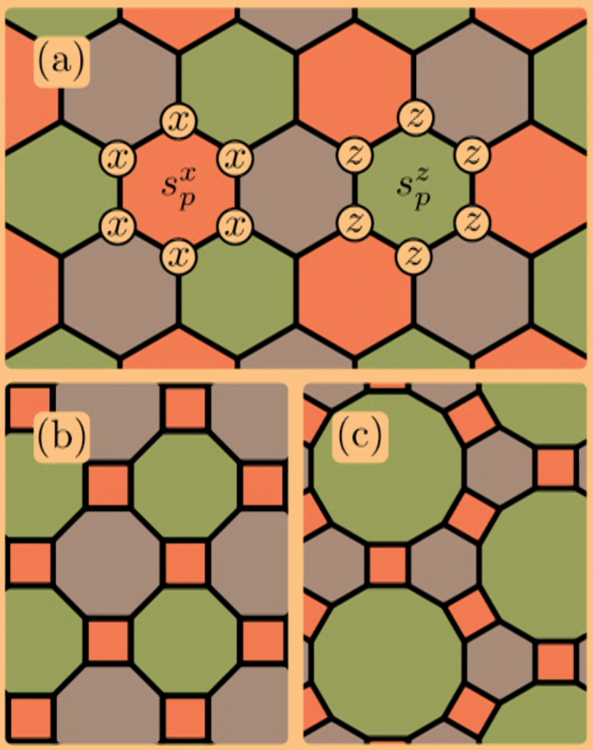
- How do you encode information?
- And how would we perform gates?
- What is the overhead?

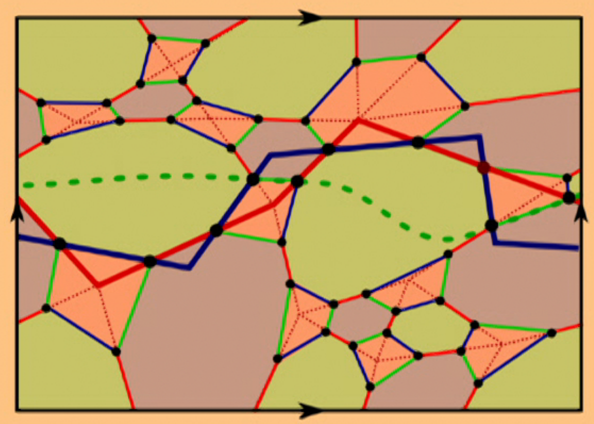
Problem: Color code boundaries and twists are not fully classified and studied. ([9,10,11,12])

Goal: Give lattice representation of boundaries and twists. Find ways to encode logical qubits in them.

[9] Hector Bombin: 1006.5260
[10] Jeffrey C. Y. Teo, Abhishek Roy, and Xiao Chen: 11306.1538, 1511.00912
[11] Beni Yoshida: 1503.07208
[12] Aleksander Kubica, Beni Yoshida, Fernando Pastawski: 1503.02065

Introduction to color codes[13]: Stabilizers





$$H_{CC} = -\sum_p s_p^x - \sum_p s_p^z$$

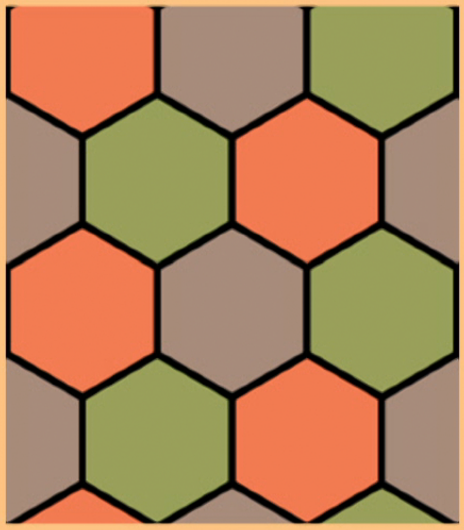
[13] H. Bombin and M.A. Martin-Delgado: 0605138

Introduction to color codes: Anyons

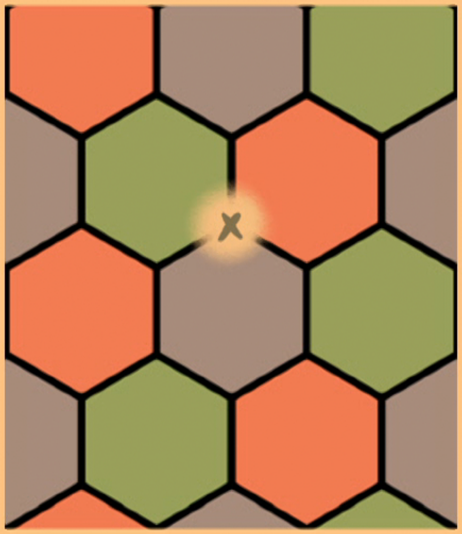
In the color code:

- Anyons are violated stabilizers
- Pauli rotations **create** and **move** anyons
- Anyons follow certain **fusion rules**
- Anyons interact according to **braid statistics**

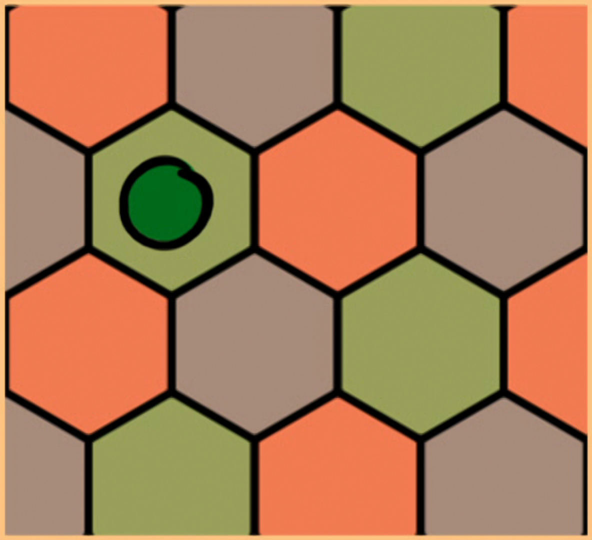
Introduction to color codes:
Creating anyons



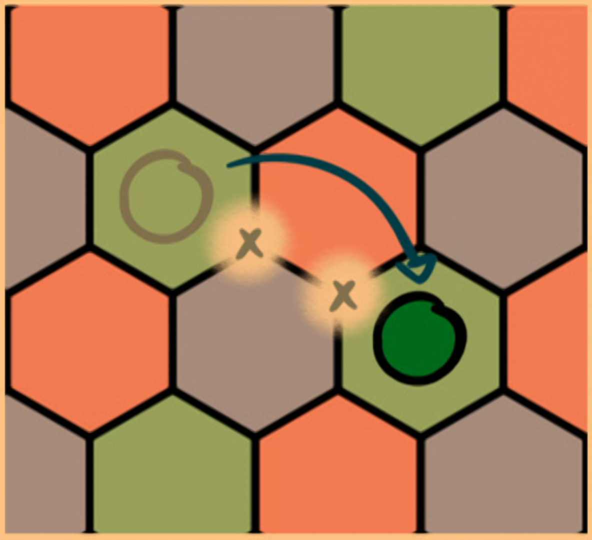
Introduction to color codes: Creating anyons



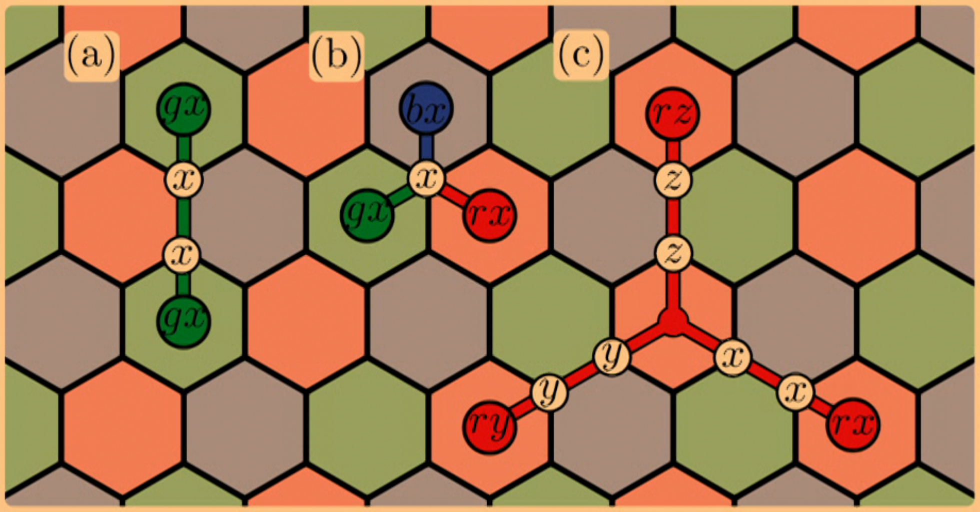
Introduction to color codes:
Moving anyons



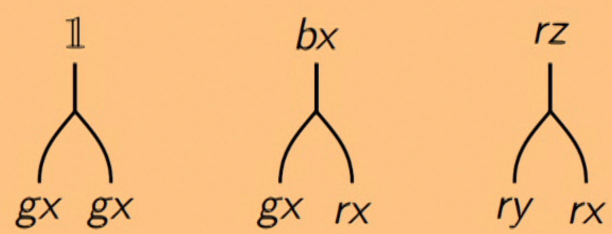
Introduction to color codes:
Moving anyons



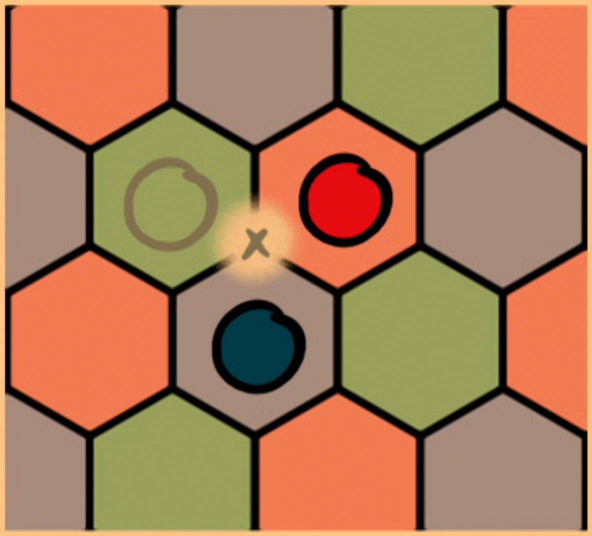
Introduction to color codes:
Fusion of color code anyons



rx	gx	bx
ry	gy	by
rz	gz	bz



Introduction to color codes:
Moving anyons



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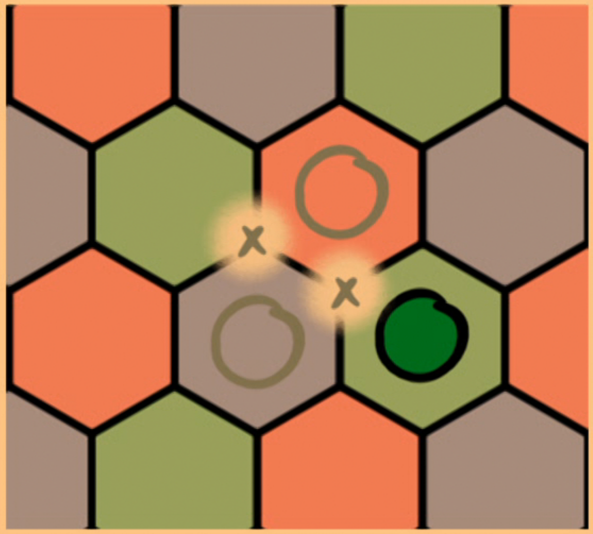
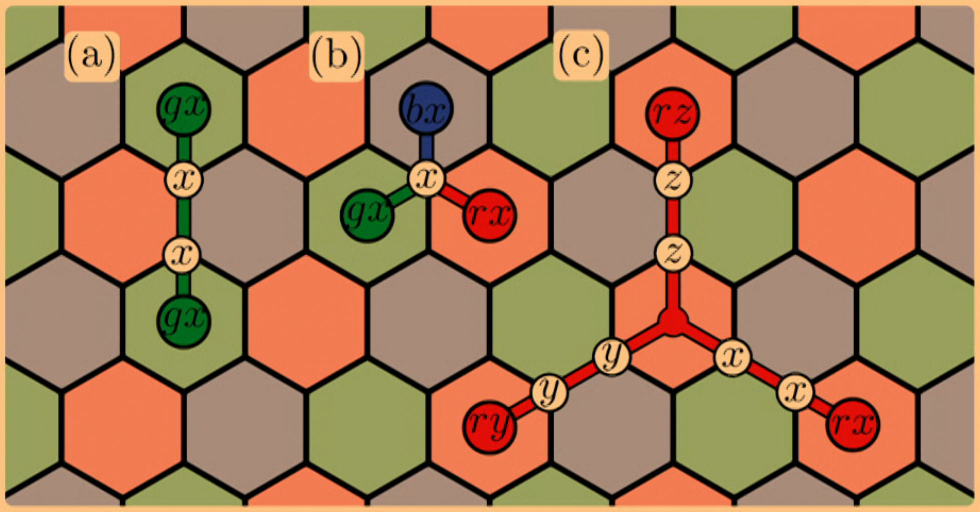
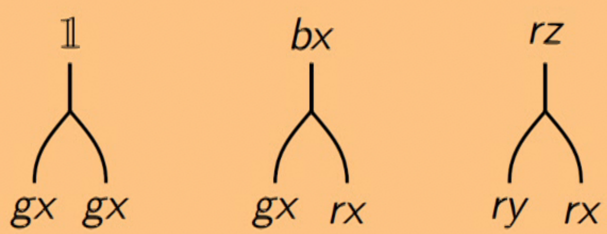
Moving anyons



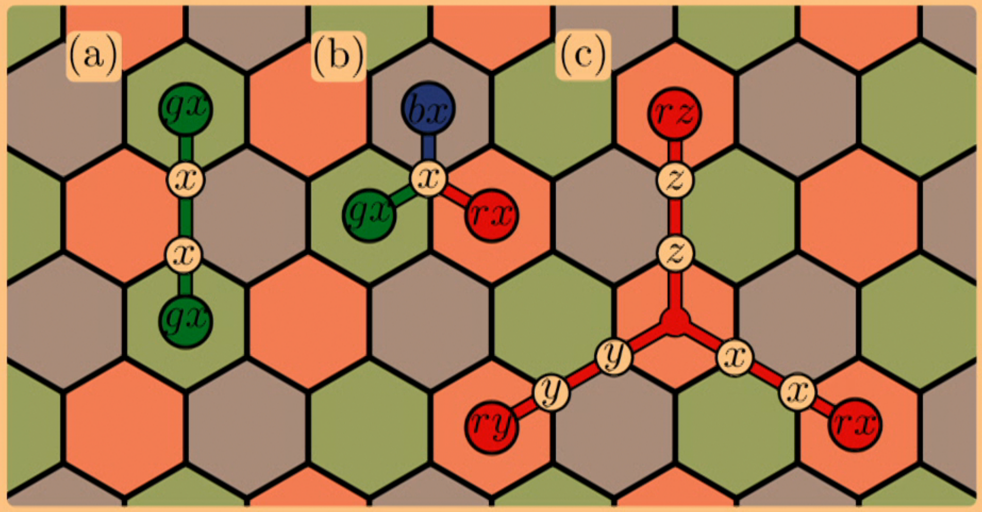
Introduction to color codes:
Fusion of color code anyons



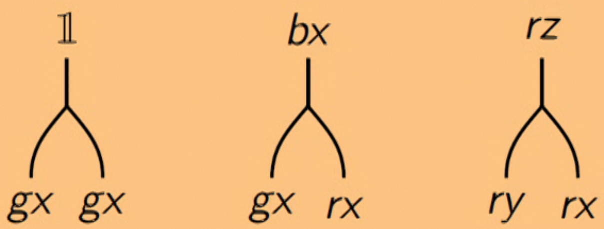
rx	gx	bx
ry	gy	by
rz	gz	bz



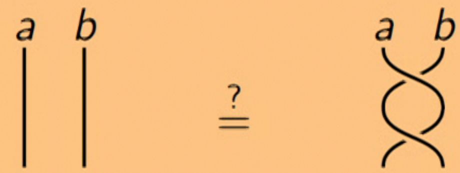
Introduction to color codes: Fusion of color code anyons



rx	*gx*	*bx*
ry	*gy*	*by*
rz	*gz*	*bz*

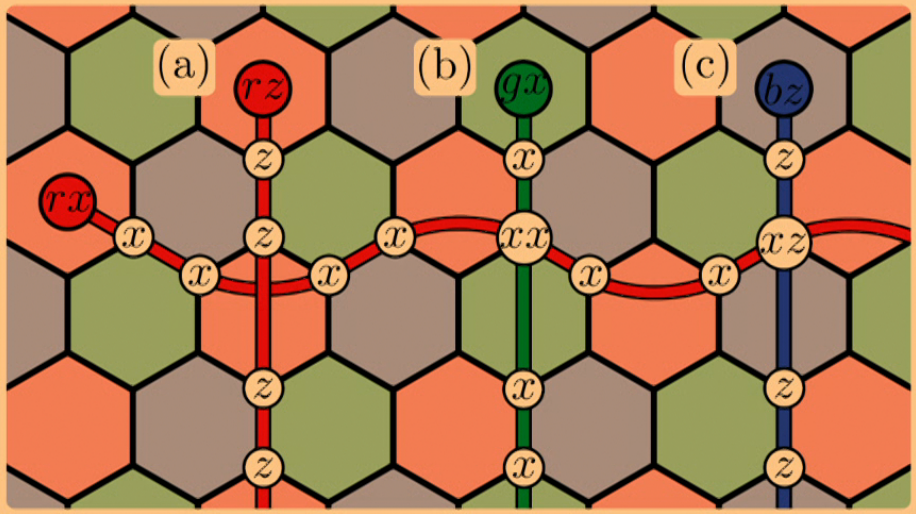


Introduction to color codes: Braiding of anyons



Braid statistics give a way to compare the left and the right side of this equation

Introduction to color codes: Braiding of color code anyons



rx	gx	bx
ry	gy	by
rz	gz	bz

Summary anyons

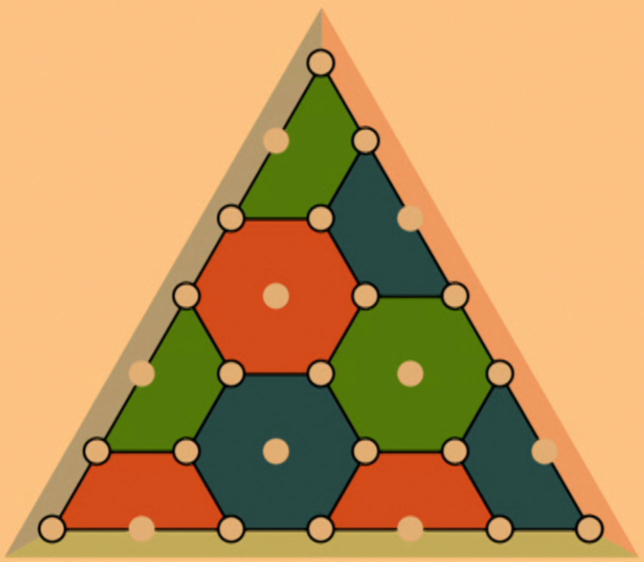
Fusion of two anyons within a row/column results in the third anyon within said row/column.

Braiding of two anyons which are in the same row/column is trivial, otherwise it will result in a phase -1.

rx	gx	bx
ry	gy	by
rz	gz	bz

Boundaries of the color code

Three "known" bounadries of the color code:



14

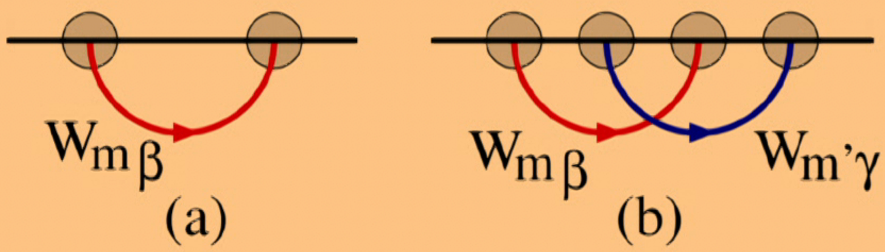
rx	gx	bx
ry	gy	by
rz	gz	bz

[14] Daniel Litinski, MSK, Jens Eisert, and Felix von Oppen: 1704.01589

Gapped bounadries

Anyons that can get absorbed (condense) at a gapped boundary form a *Lagrangian subgroup* $\mathcal{M}$ (see [15])

- bosonic self-exchange statistics
- trivial braiding with other anyons in $\mathcal{M}$

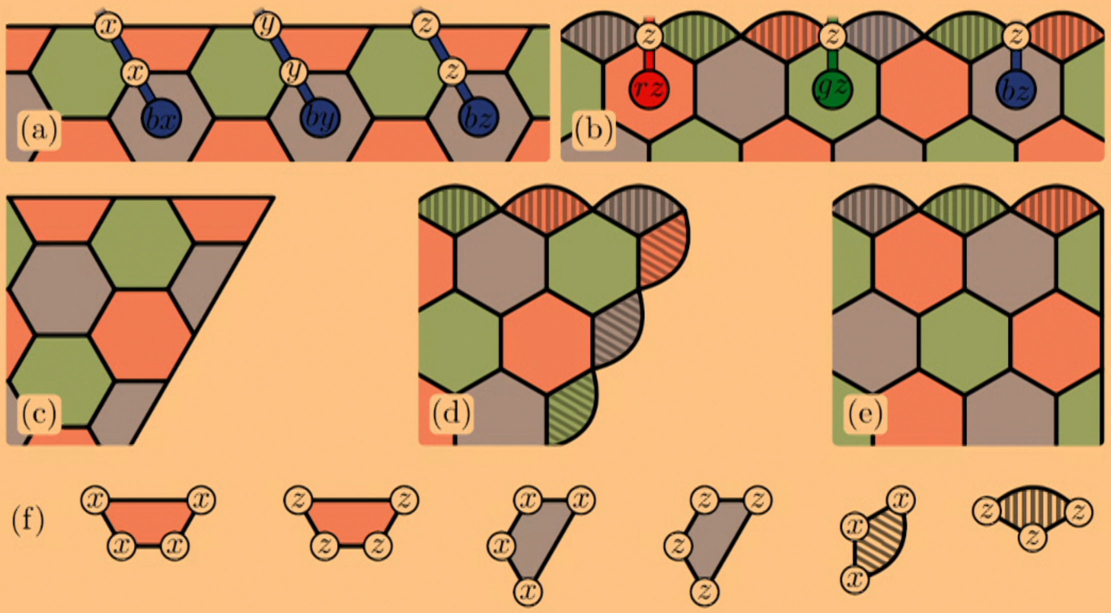


rx	*gx*	*bx*
ry	*gy*	*by*
rz	*gz*	*bz*

[15] Michael Levin: 1301.7355

Color code boundaries

Pauli boundaries \ color boundaries			
	rx	gx	bx
	ry	gy	by
	rz	gz	bz

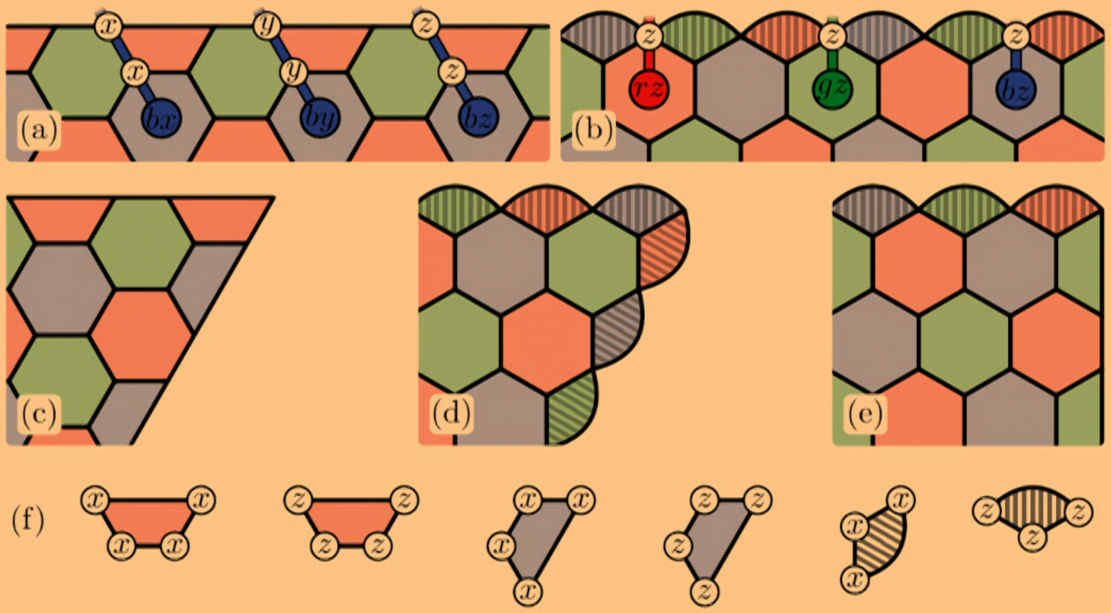


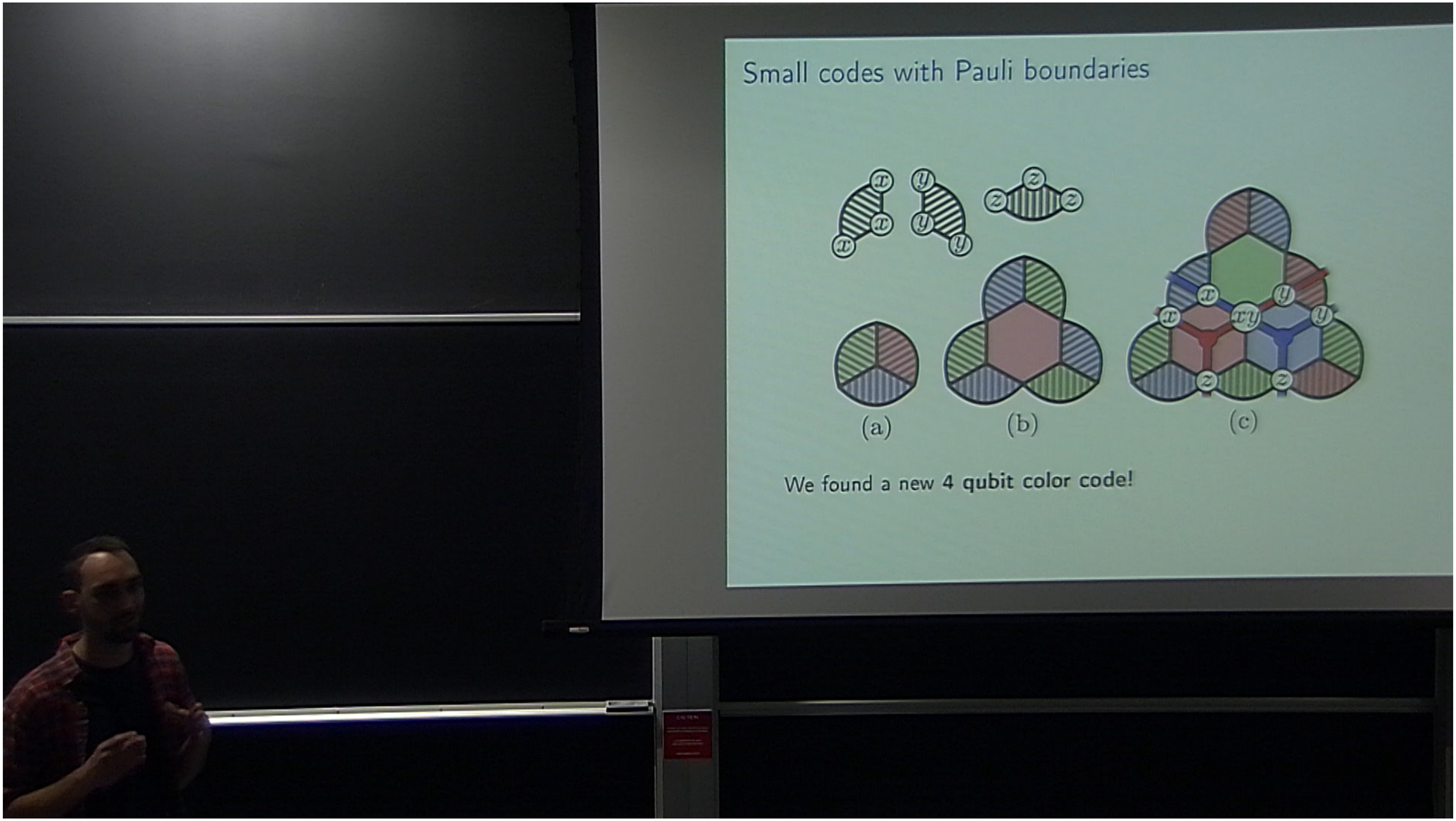
Color code boundaries

color boundaries

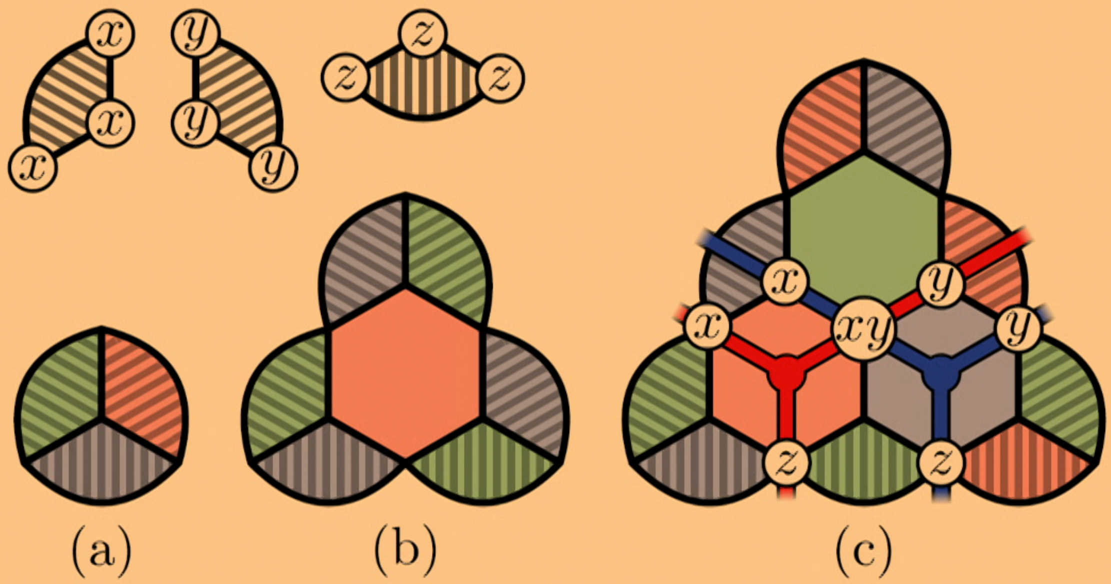
Pauli boundaries

rx	*gx*	*bx*
ry	*gy*	*by*
rz	*gz*	*bz*

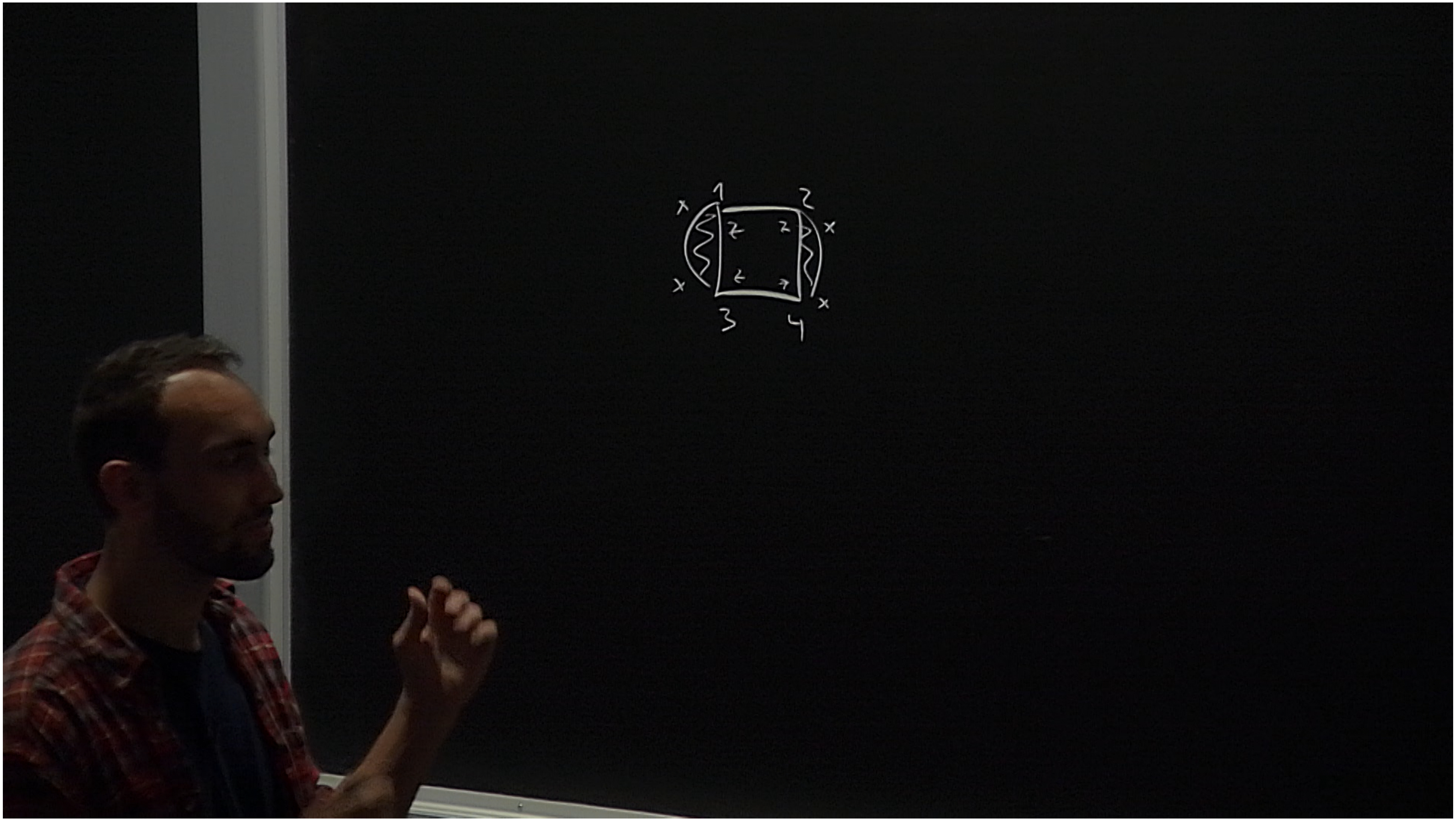



Small codes with Pauli boundaries
(a)
(b)
(c)
We found a new **4 qubit color code!**

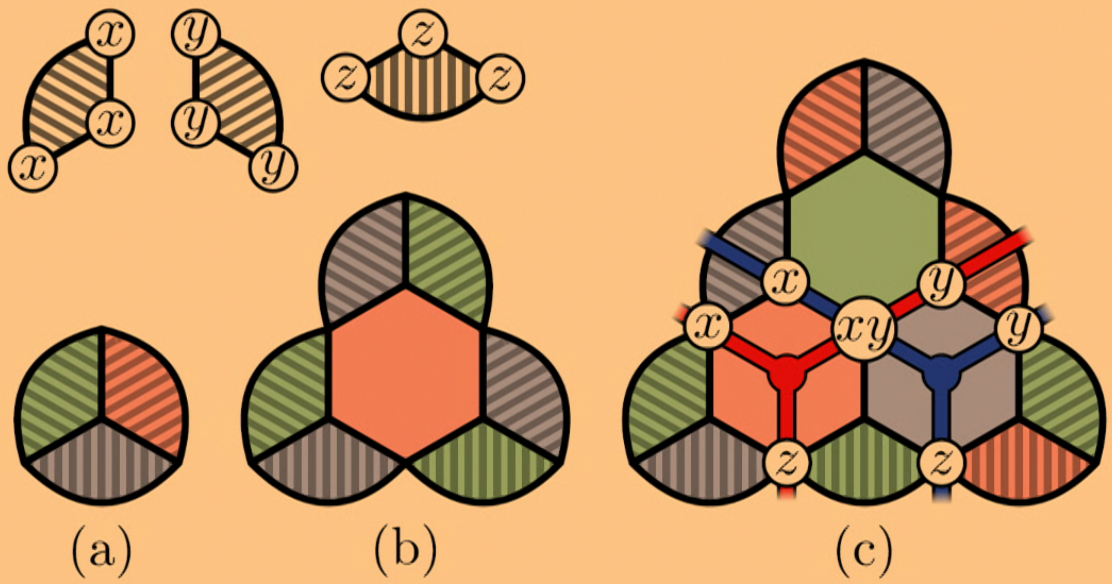
Small codes with Pauli boundaries



We found a new **4 qubit color code!**


1
2
3
4

Small codes with Pauli boundaries



We found a new **4 qubit color code!**

Summary: Boundaries

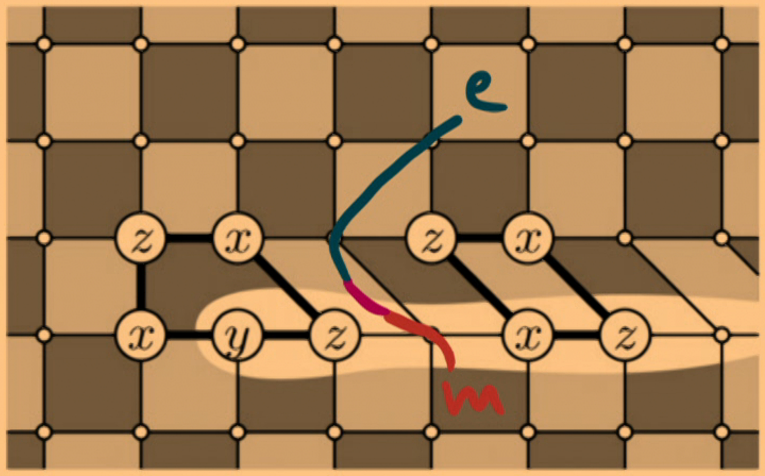
We added **three new boundaries** to the inventory

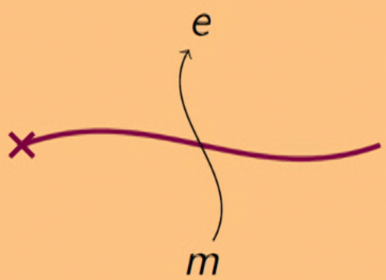
We found lattice representations for all possible **corners** between them

We found a new family of triangular color codes, the smallest example is a **four qubit code**

Domain walls and twists
Example: Twists in the toric code

Twists have first been described for the toric code[16]
They are the **endpoints** of dislocation lines





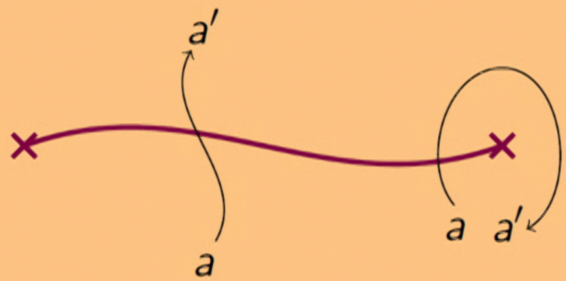
Anyons passing over the dislocation line change label

[16] H. Bombin: 1004.1838 (@ PI)

Domain walls and twists

In general:

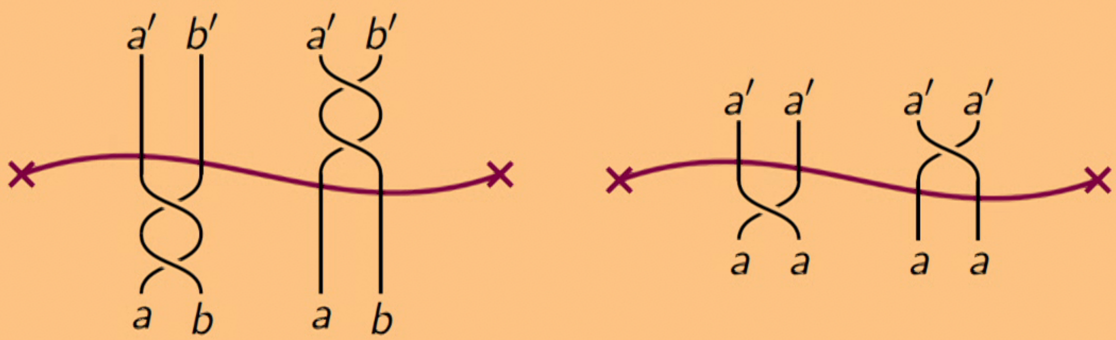
- Domain walls correspond to **maps** $a \mapsto \varphi(a) = a'$
- Anyons crossing domain walls get mapped
- Anyons circling around twist get mapped



There are **consistency conditions** such a map φ has to fulfil

Conditions on Twists
Braiding (and self-statistics)

- **Braiding** (and self-statistics) before and after crossing have to be the same
  - $\varphi(S_{a,b}) = S_{a',b'} = S_{a,b}$
  - $\varphi(T_{a,b}) = T_{a',b'} = T_{a,b}$



Conditions on Twists
Color Code

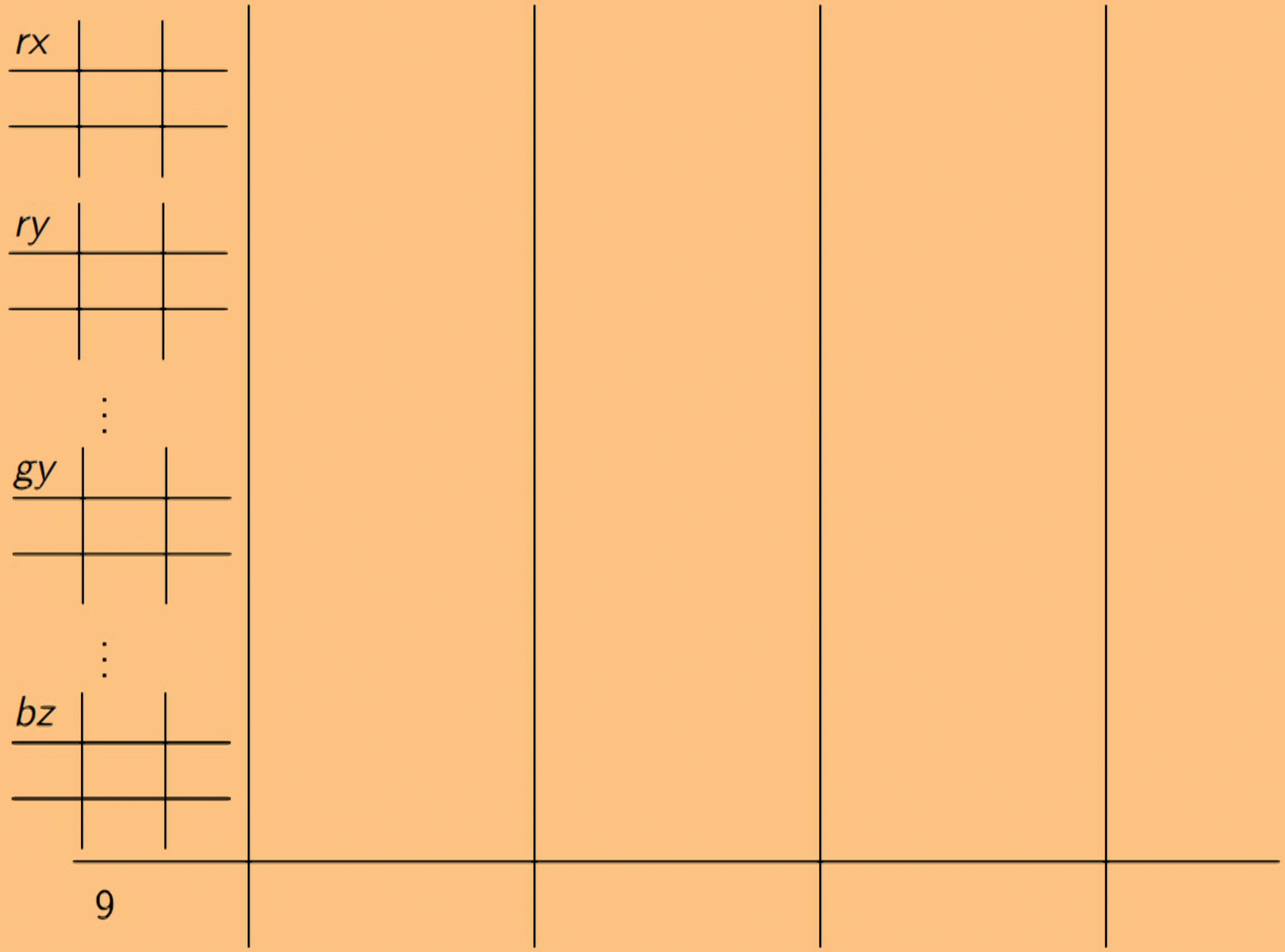
Fusion and braiding must be preserved!
In the color code, this simplifies to:

row $\mapsto$ **row** and **column** $\mapsto$ **column**
or
row $\mapsto$ **column** and **column** $\mapsto$ **row**

$$\begin{array}{c|c|c} rx & gx & bx \\ \hline ry & gy & by \\ \hline rz & gz & bz \end{array} \quad \mapsto \quad \begin{array}{c|c|c} gy & gz & gx \\ \hline by & bz & bx \\ \hline ry & rz & rx \end{array}$$

How many twists are there?
= How many ways can we fill up the 3x3 grid?

Number of domain walls in the color code



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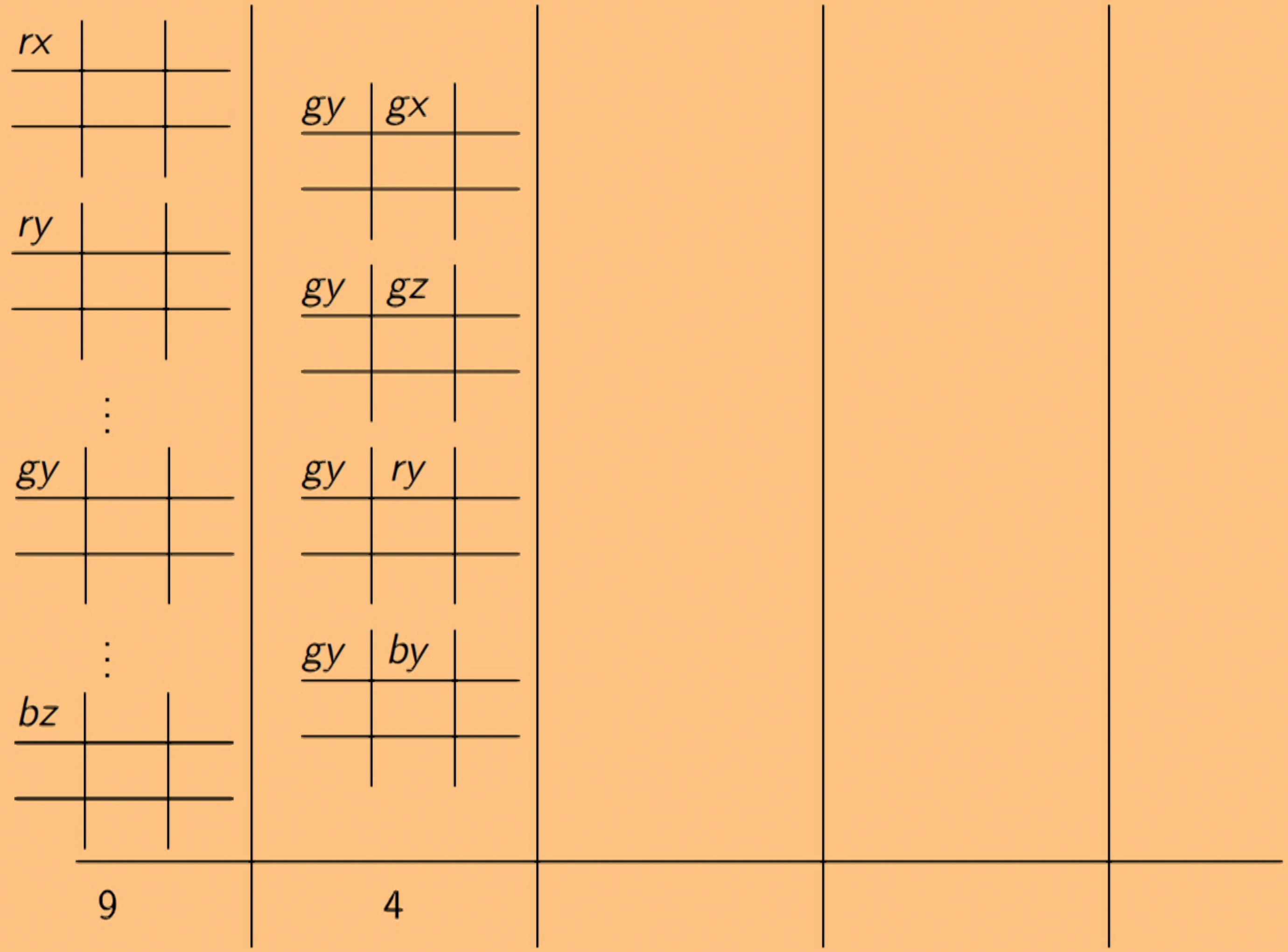

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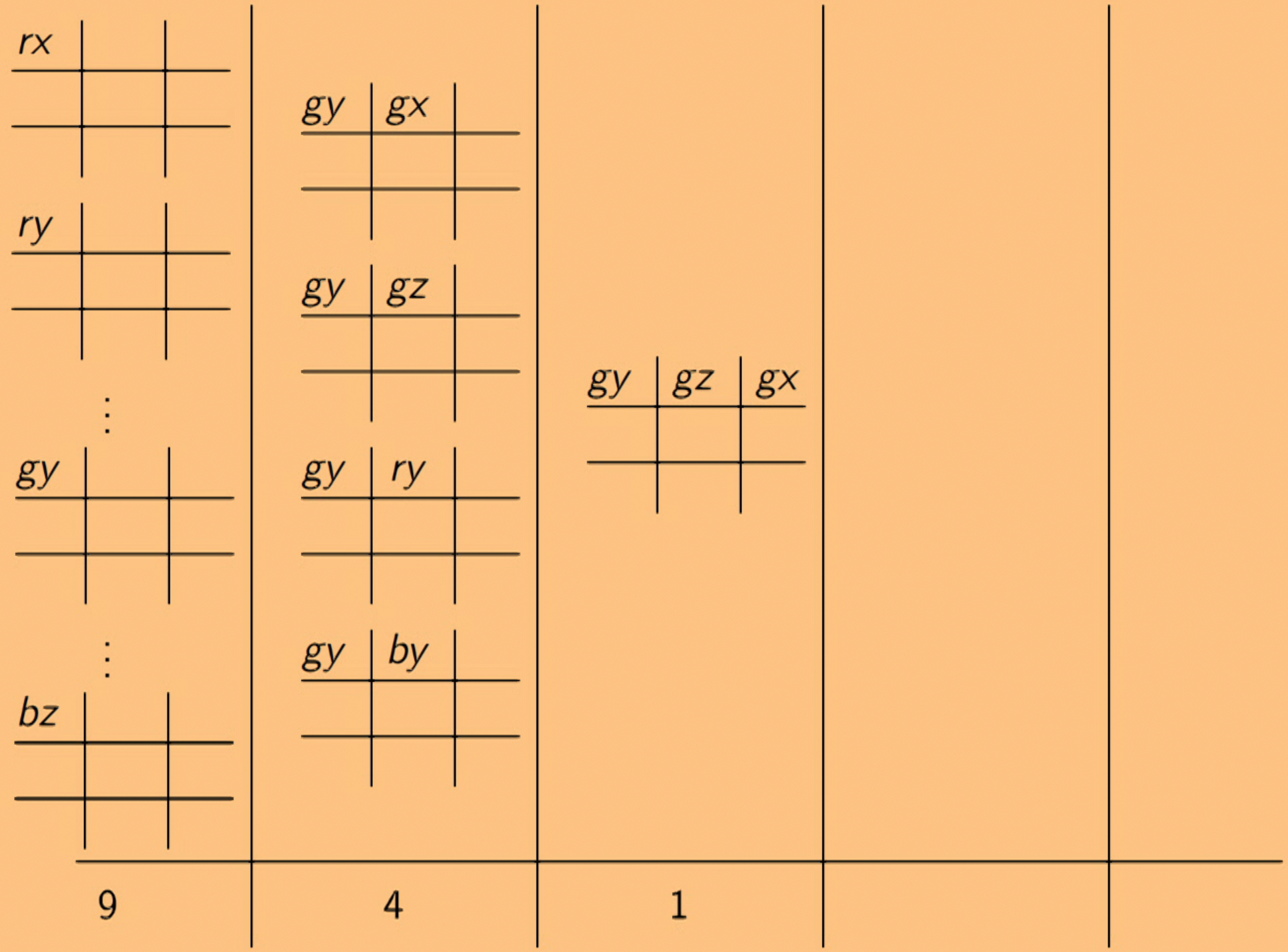

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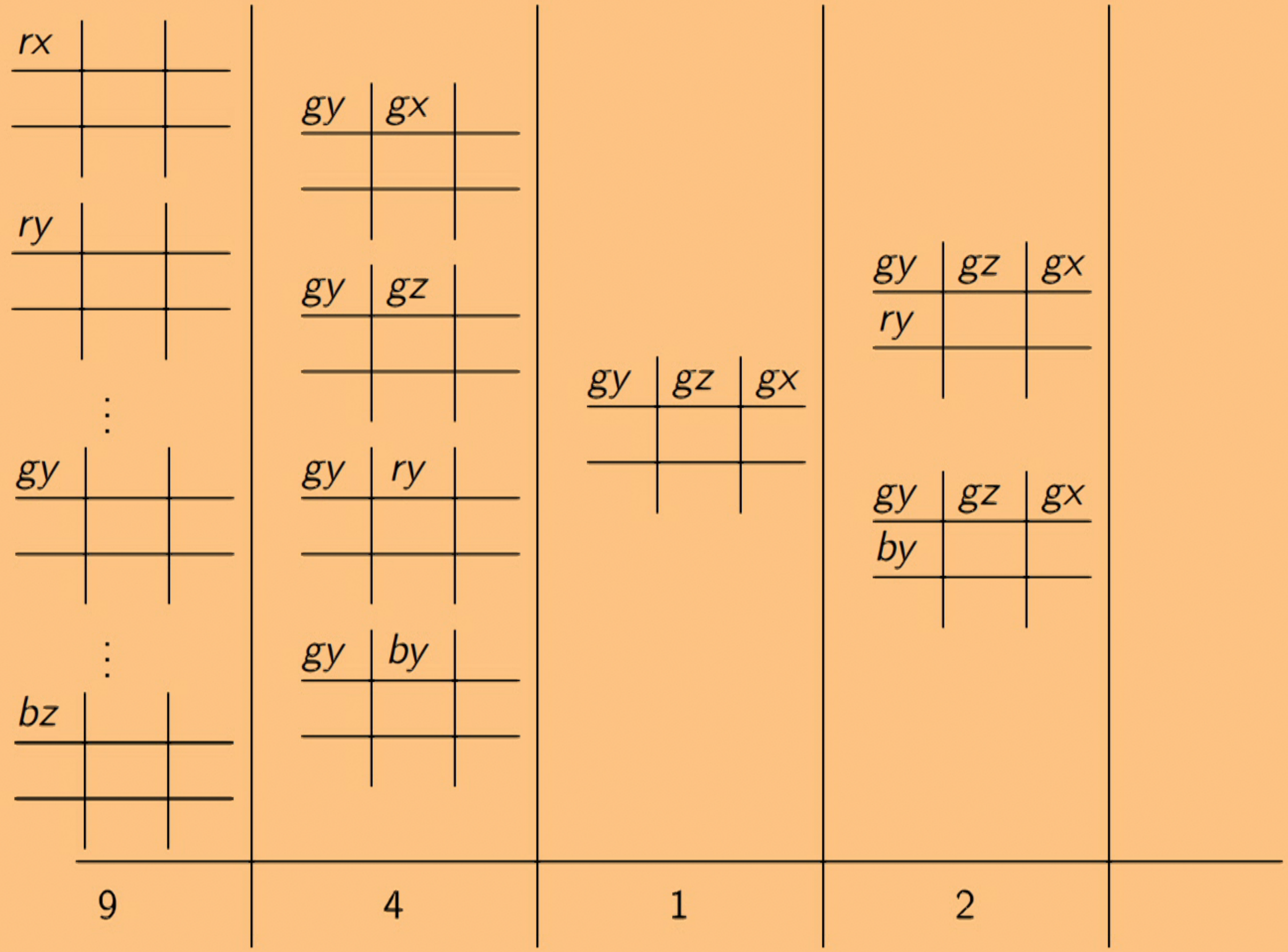

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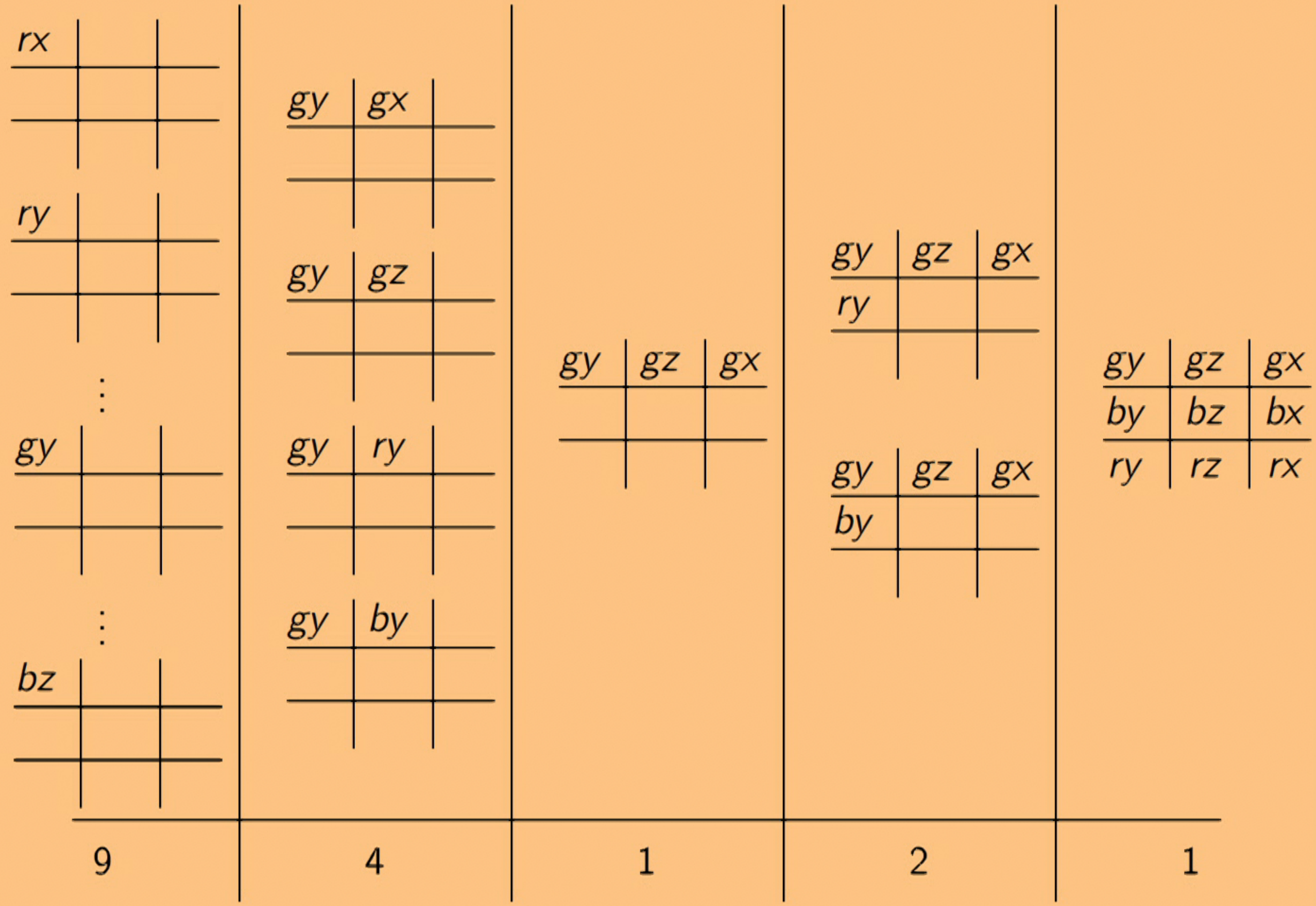


Number of domain walls in the color code

rx	gx	bx
ry	gy	by
rz	gz	bz

$\mapsto$

gy	gz	gx
by	bz	bx
ry	rz	rx

$9 \cdot 4 \cdot 2 = \mathbf{72}$ (including the trivial map)[17]

Compare to toric code: 2 twist (including the trivial twist)

[17] $72 =$ Cardinality of group $(S_3 \times S_3) \rtimes \mathbb{Z}_2$

Families of Domain Walls

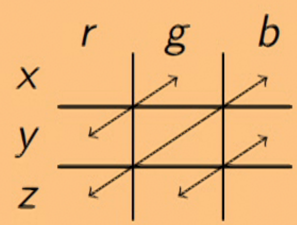
- Color label permuting domain walls

rx	gx	bx
ry	gy	by
rz	gz	bz

- Pauli label permuting domain walls

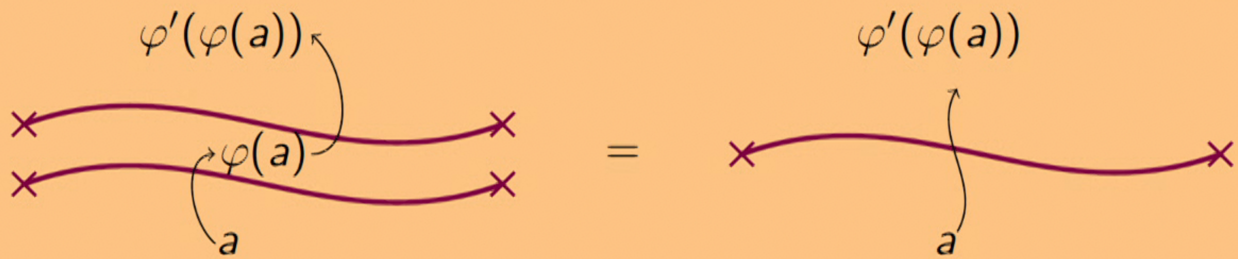
rx	gx	bx
ry	gy	by
rz	gz	bz

- Intertwine domain walls - exchange color and Pauli label

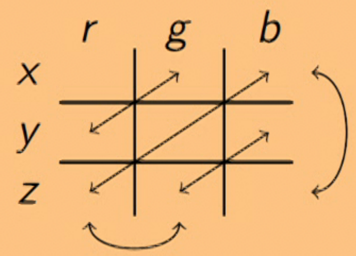


Generating domain walls & twists

Combining domain walls

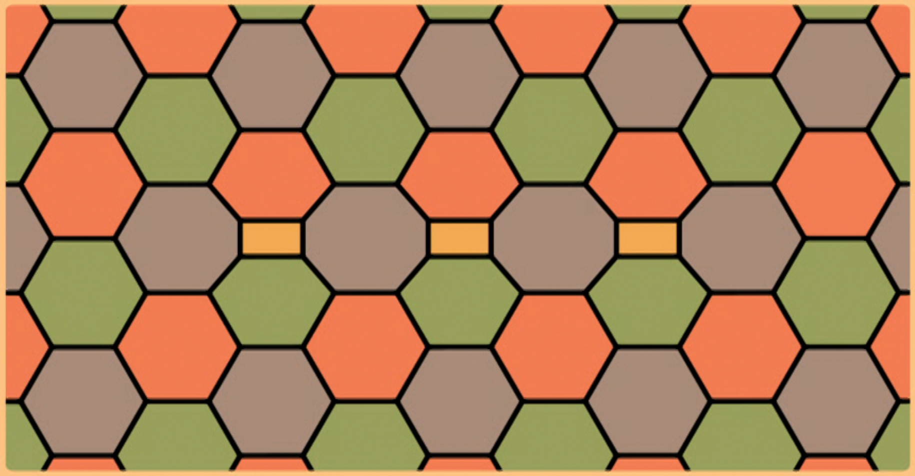


Generating set of domain walls



Lattice representation of domain walls and twists: color twists

Color twists break tricolorability of the lattice



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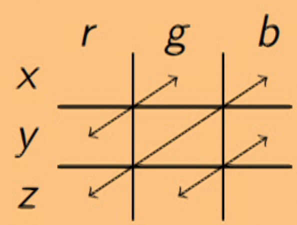
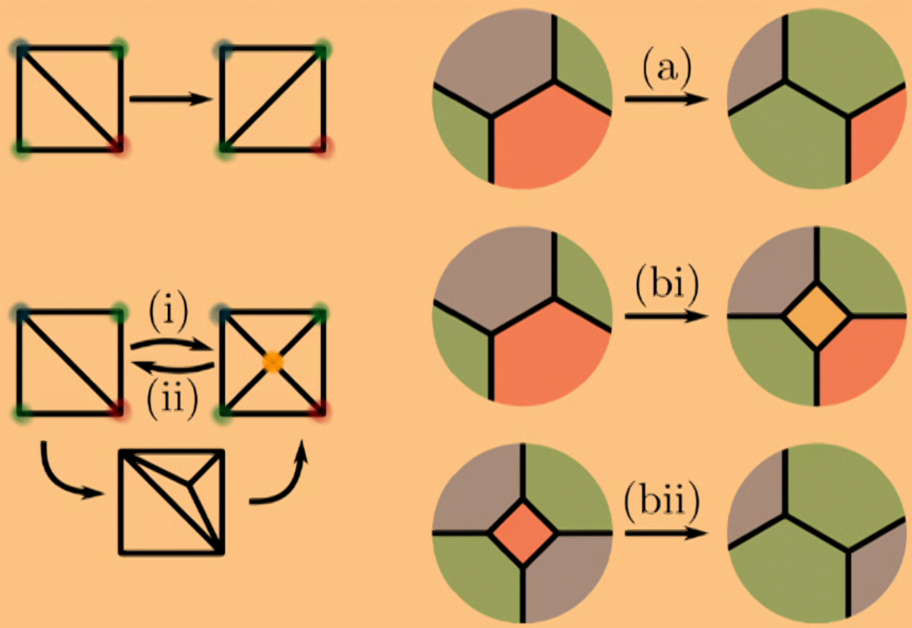


Lattice representation of domain walls and twists: color twists

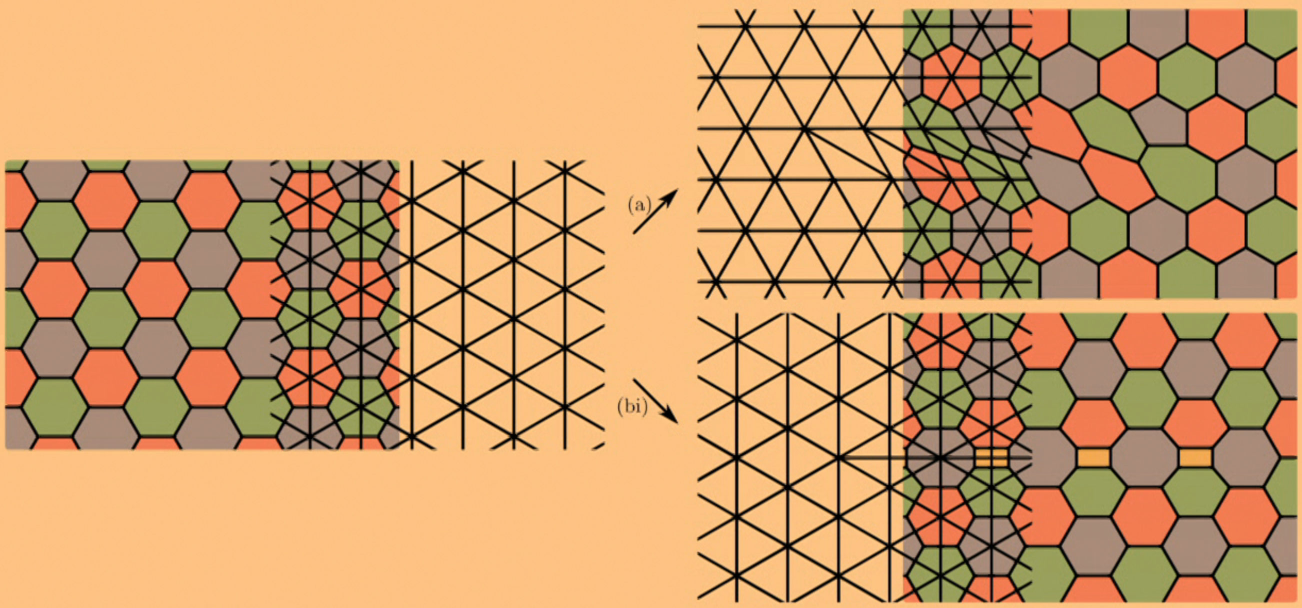
Color twists can generated and moved using **Pachner moves** on the dual lattice



These moves break tricolorability of faces but preserve trivalence of vertices

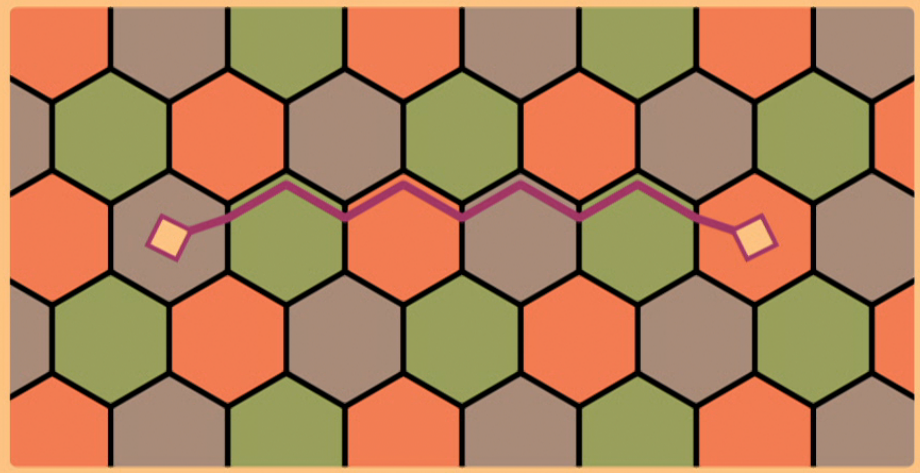
Lattice representation of domain walls and twists: color twists

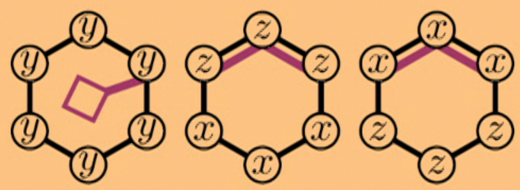
Pachner moves in the 666 color code and the resulting defects



Lattice representation of domain walls and twists: Pauli twists

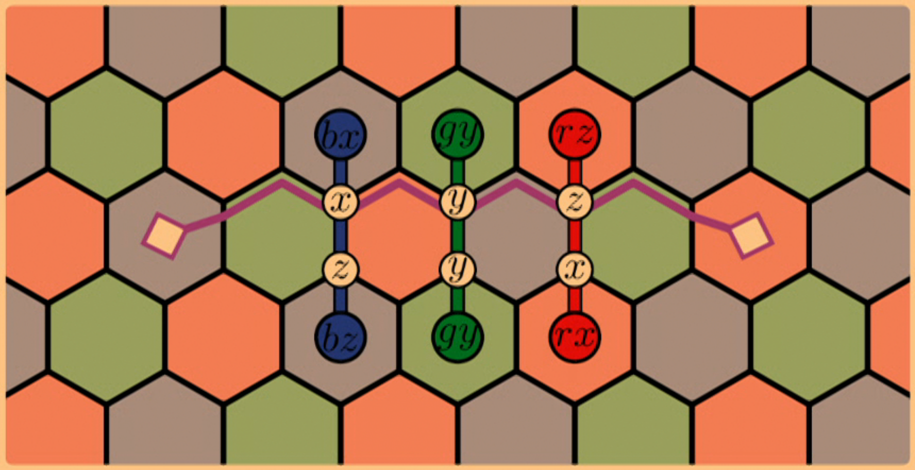
Pauli twists do not change the lattice, only the stabilizers





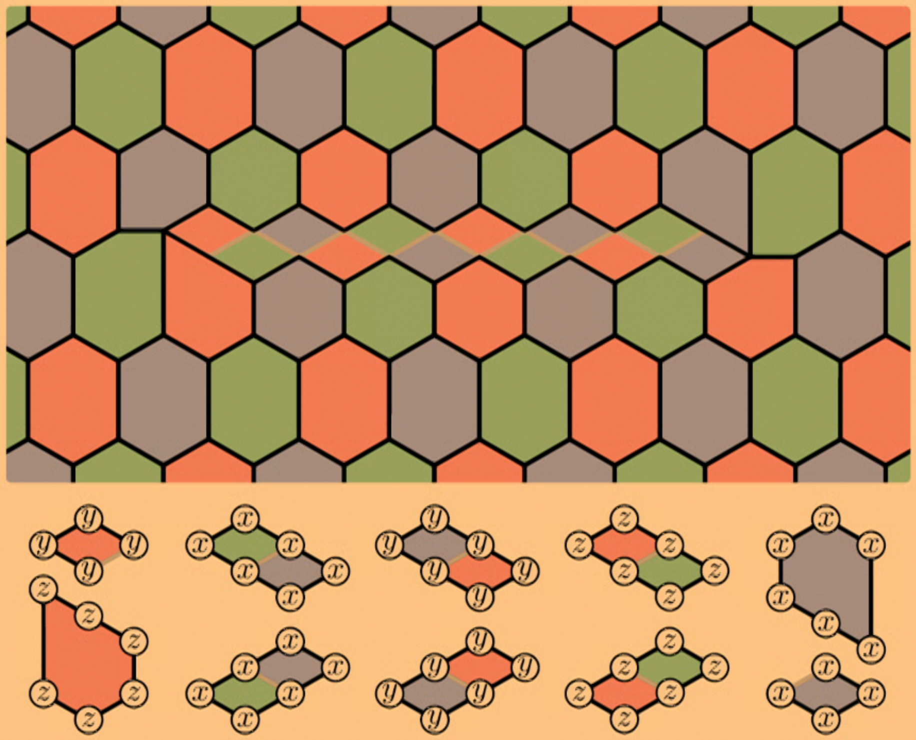
Lattice representation of domain walls and twists: Pauli twists

Pauli twists only change the Pauli label of anyons, not their color

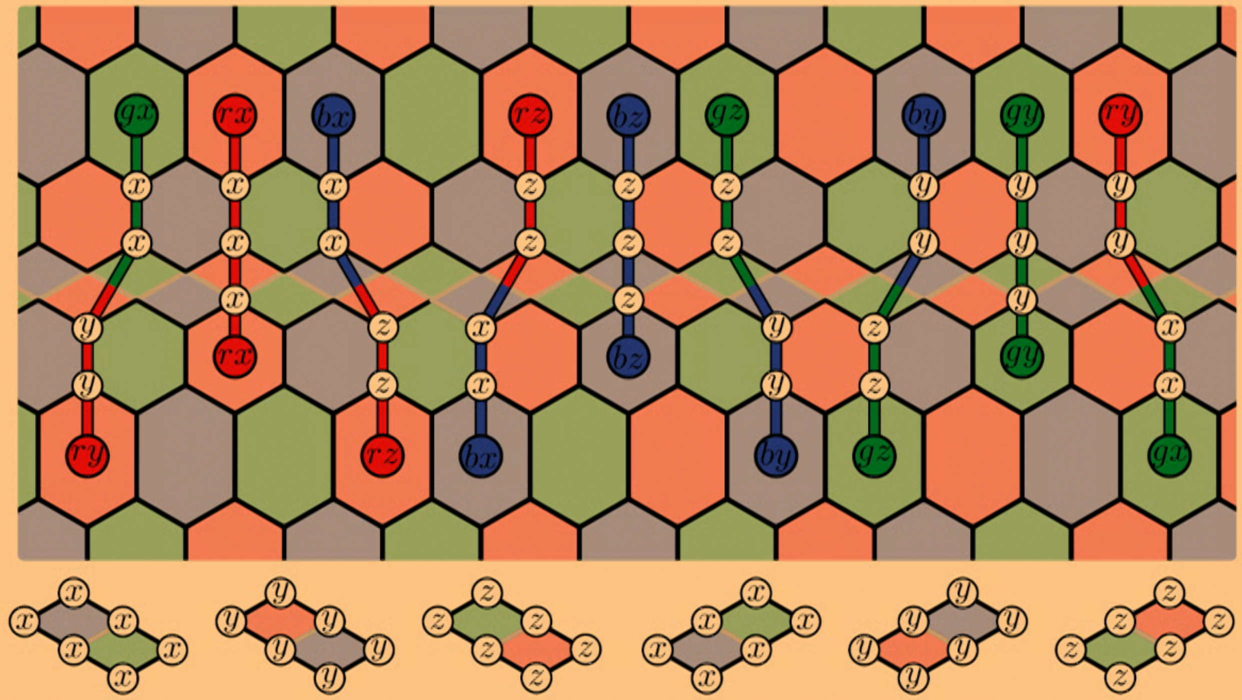


Lattice representation of domain walls and twists: intertwine twists

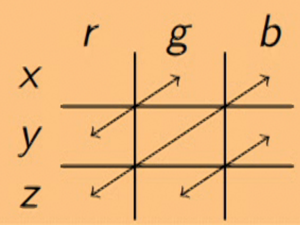
Intertwine domain walls have overlapping domino-brick stabilizers



Lattice representation of domain walls and twists: intertwine twists



This domain wall exchanges Pauli and color labels: $r \leftrightarrow x$, $g \leftrightarrow y$, $b \leftrightarrow z$

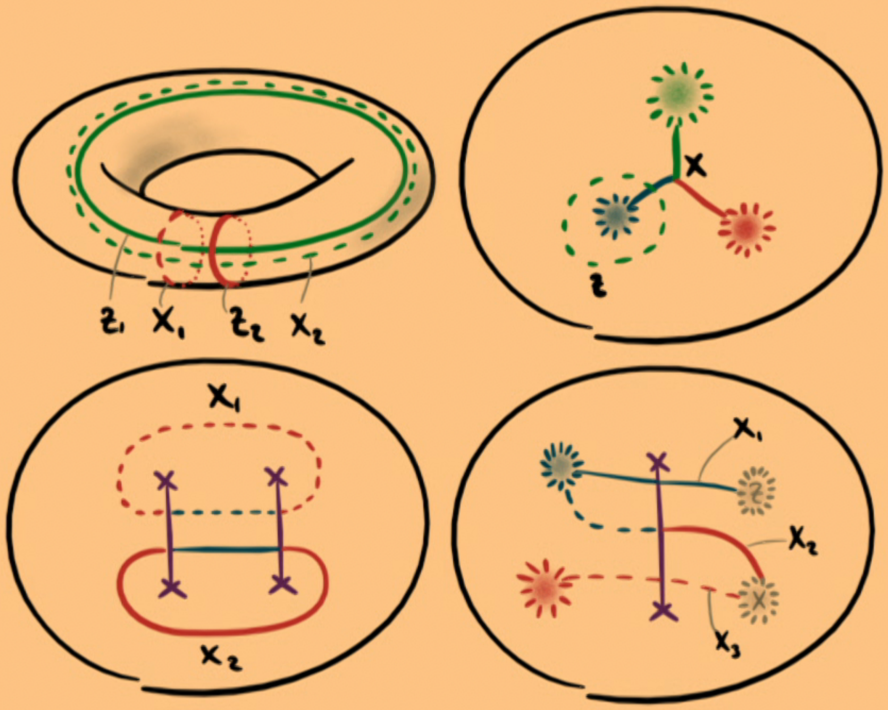


Summary domain walls

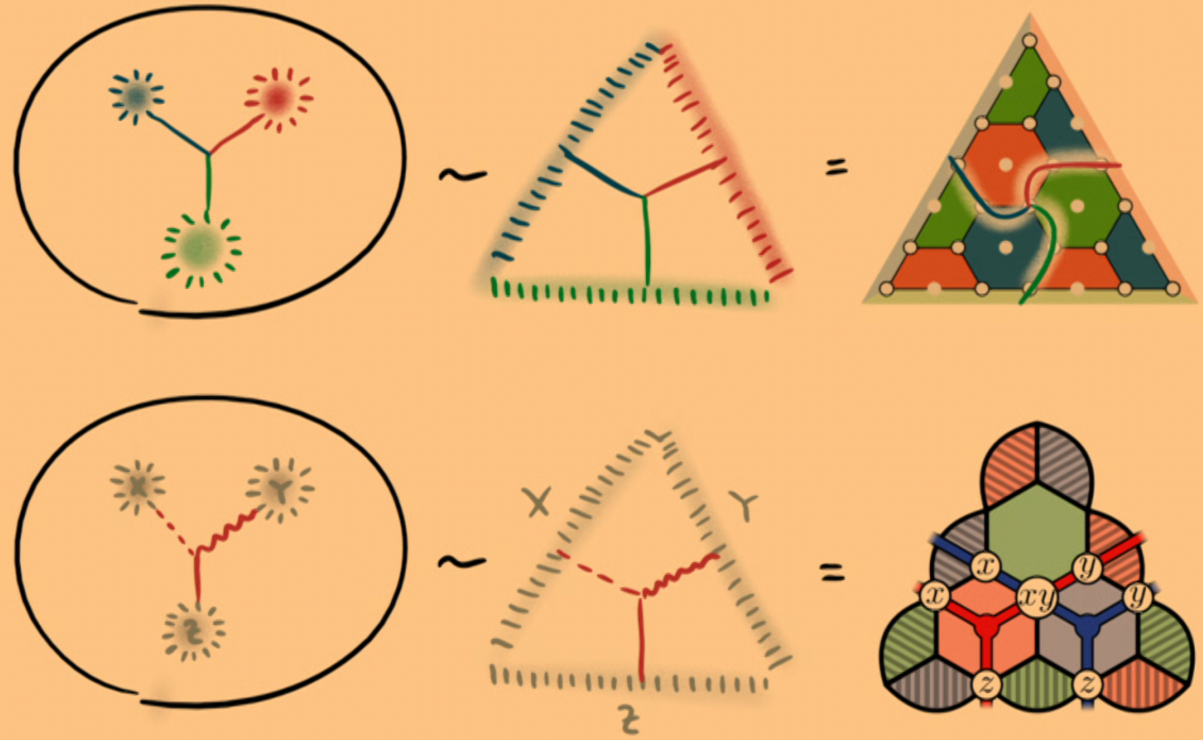
- Domain walls correspond to maps (with some consistency conditions)
- In the color code we find 72 such maps
- They can be generated by just 3
- We found lattice representations of a generating set

How to store quantum information in topological codes?

- (Topologically non-trivial surfaces)
- Holes and Boundaries
- Twists
- Hybrids of the above



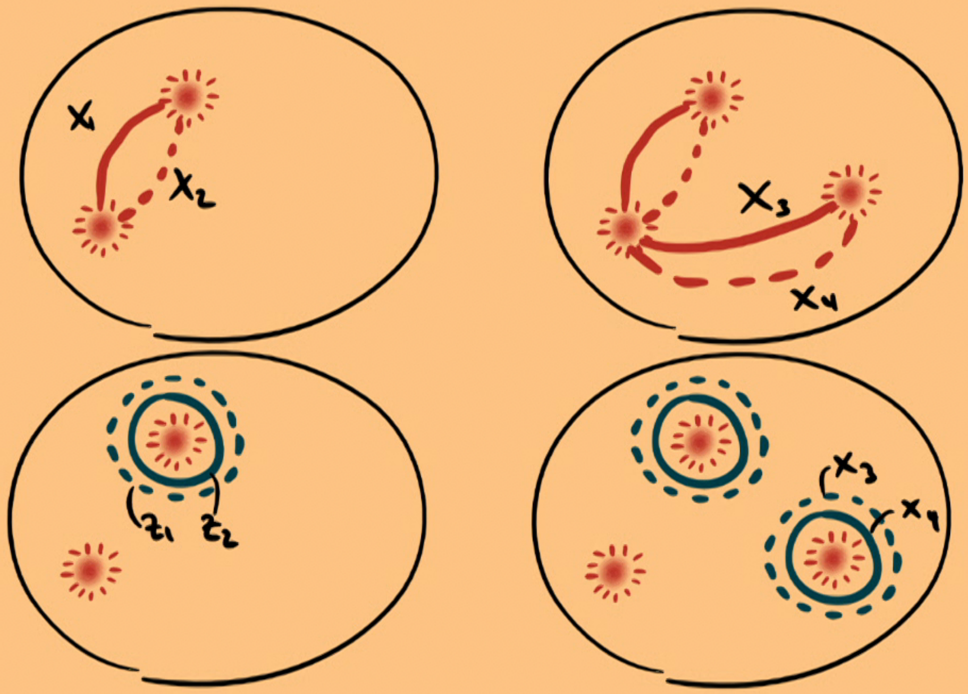
Boundaries: We already know this encoding scheme!



RB

Notation:

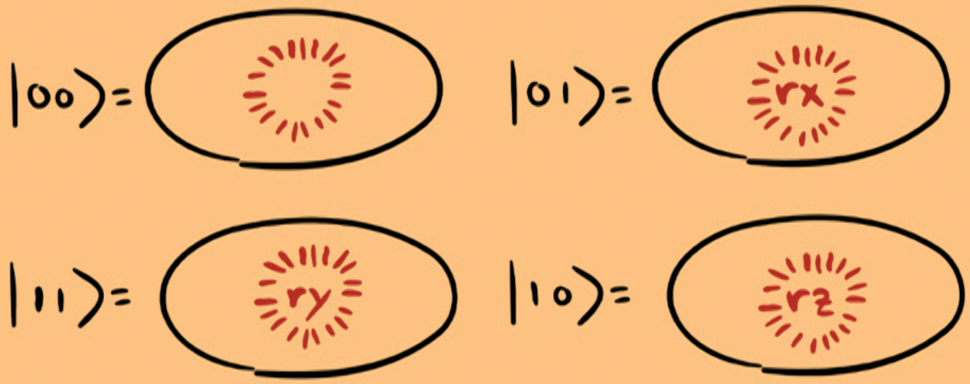
More holes, please!



- An added hole adds **two** logical qubits!

Holes can host several anyons

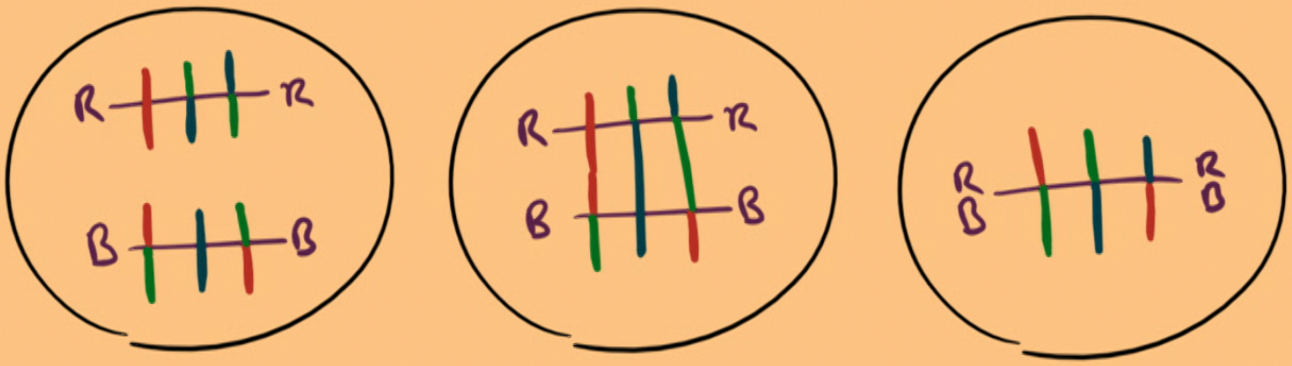
A red hole/boundary can absorb 4 anyons



Logical operators measure (or change) **charge contained in a hole**

4 outcomes of charge measurement $(1, rx, rz$ or $ry)$ correspond to **4 logical states** of two qubits $|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$

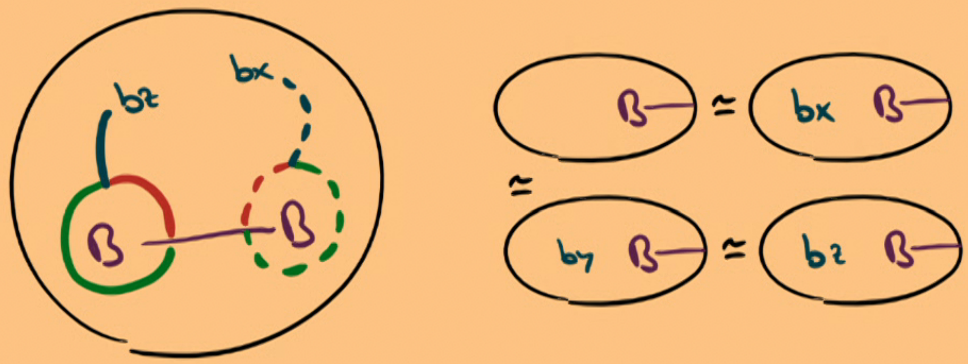
Twist encoded qubits
Notation



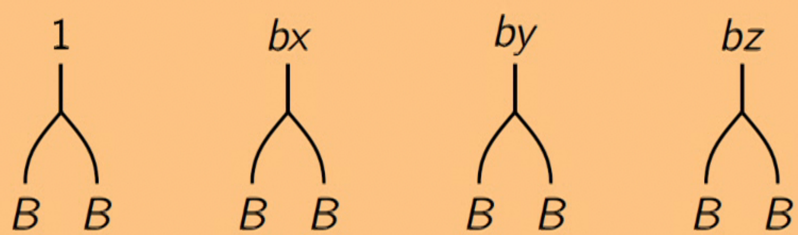
A **red invariant twist** R and a **blue invariant twists** B (left) combine to a **cyclic twist** RB (right)

Quantum Dimension of blue invariant twists

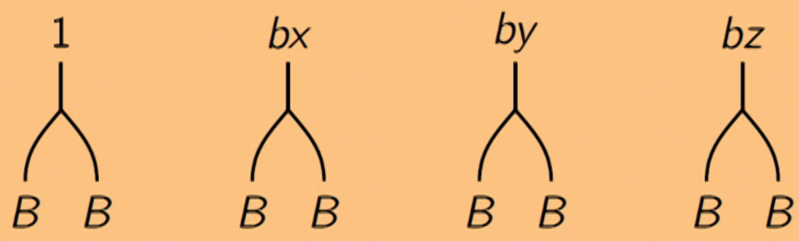
A **blue invariant twists** B can absorb 4 anyons



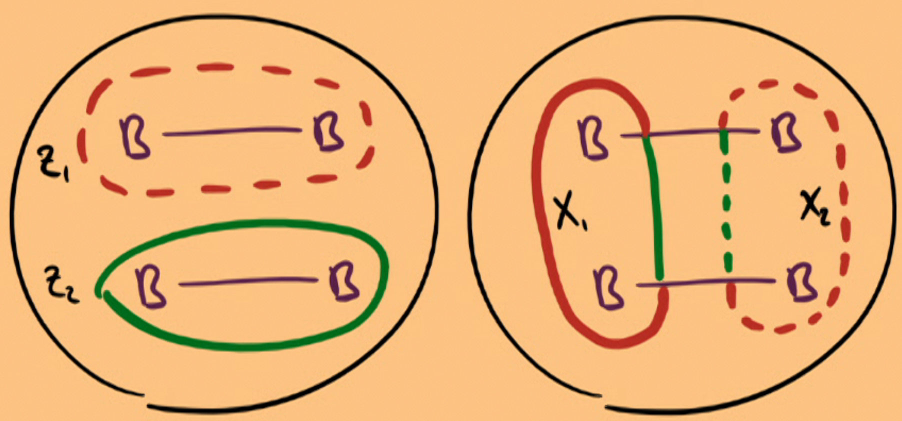
Hence the fusion of two twists can have four outcomes:



Quantum Dimension of blue invariant twists



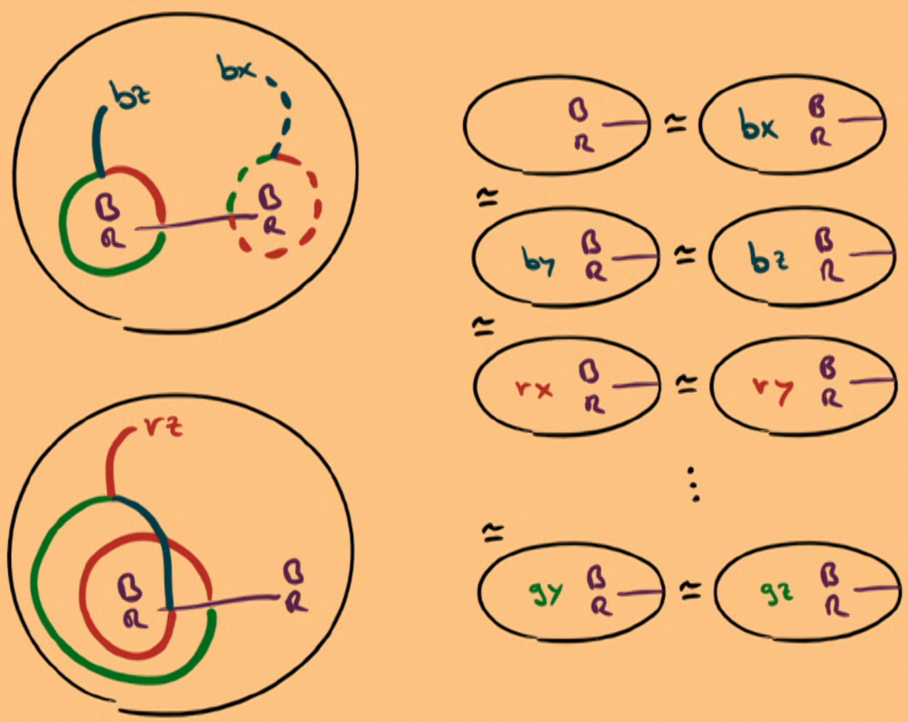
These for outcomes can be written as logical states of two qubits:
$1 \leftrightarrow |00\rangle \quad bx \leftrightarrow |01\rangle \quad bz \leftrightarrow |10\rangle \quad by \leftrightarrow |11\rangle$
Every pair of B-twists adds 2 logical qubits. Quantum dim.: $d_B = 2$



Logical operators have to measure charge of pairs of twists

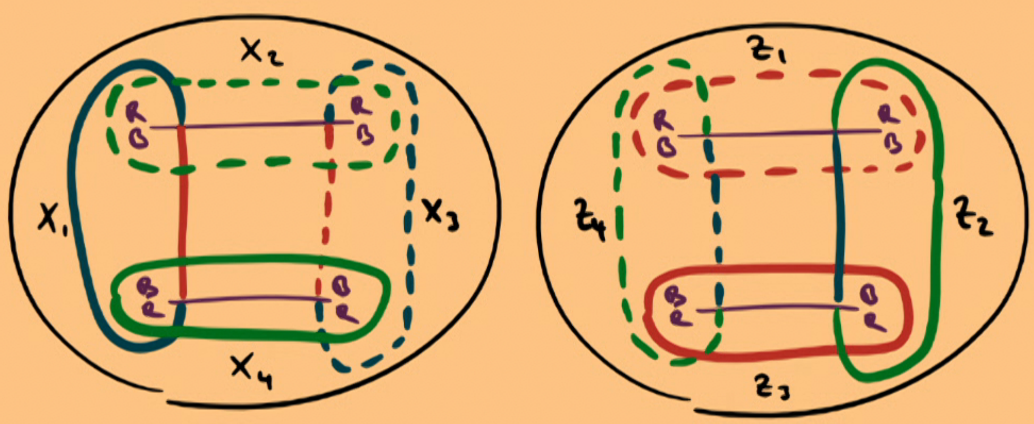
Quantum dimension of cyclic twists

Cyclic twists can absorb all 16 anyons:



Quantum dimension of cyclic twists

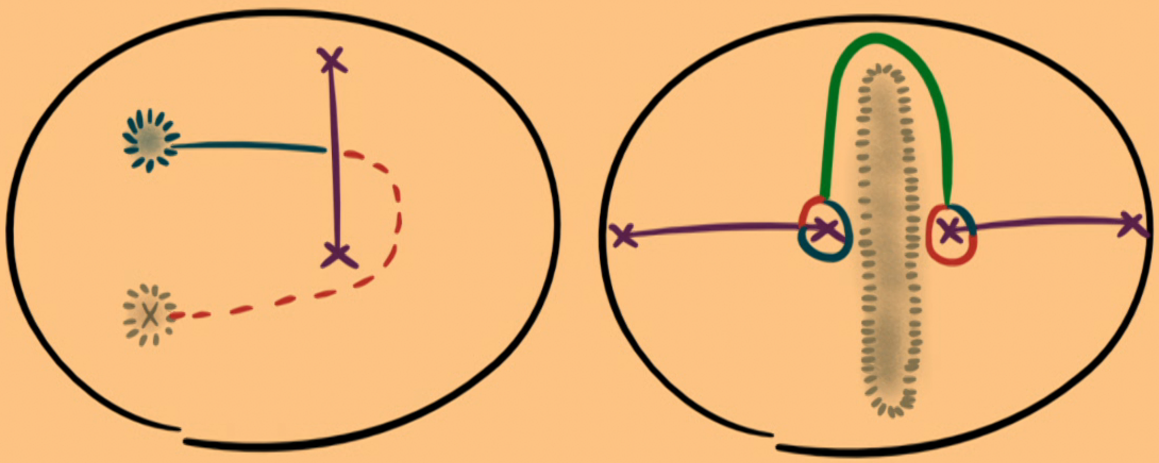
16 distinguishable fusion outcomes mean 4 logical qubits (or $d_{RB} = 4$)



Hybrid qubits

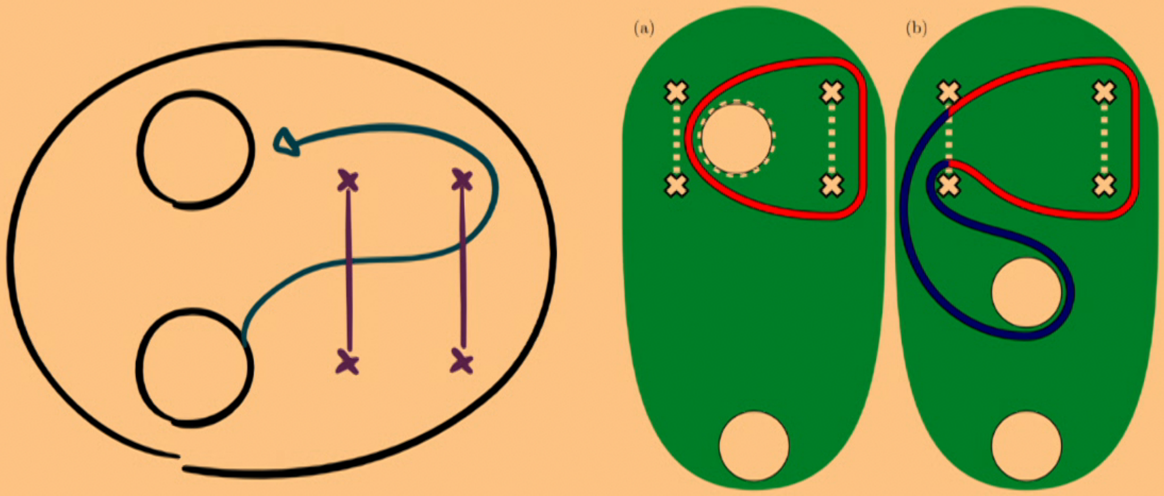
Make code distance larger by

- forcing string to go around far away twist
- forcing string to go around large hole



Manipulate logical information
Braiding of holes and twists

CNOT between hole encoded qubit(s) and twist encoded qubit(s)[18]



[18] Benjamin Brown, Katharina Laubscher, MSK, James Wootton: 1609.04673

Summary: Encoding logical qubits

- Logical qubits can be encoded using:
  - (Non-trivial topologies)
  - Holes or boundaries
  - Twists
  - Combinations of holes and twists
- Braiding of holes and twists implements Clifford gates

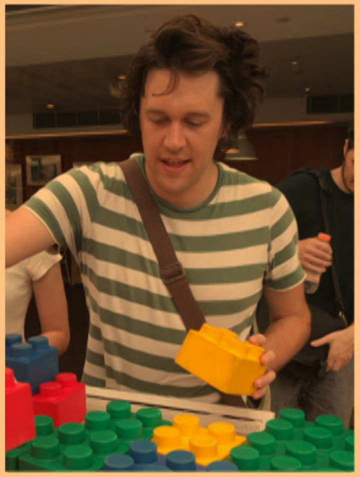
Summary

- Motivation: find new ways to encode and manipulate logical qubits
- We found three new boundaries (and the smallest color code!)
- We found lattice representation of three types of twist
  - Pauli twists
  - Color twists
  - Intertwine twists

  They generate all 72 possible twists!
- Logical qubits can be encoded using
  - Boundaries (holes)
  - Twists
  - Combinations of both
- Braiding twists and holes performs logical gates

Questions?

Thanks to my collaborators:
Ben J. Brown, Fernando Pastawski, Jens Eisert



And thank you for your attention!