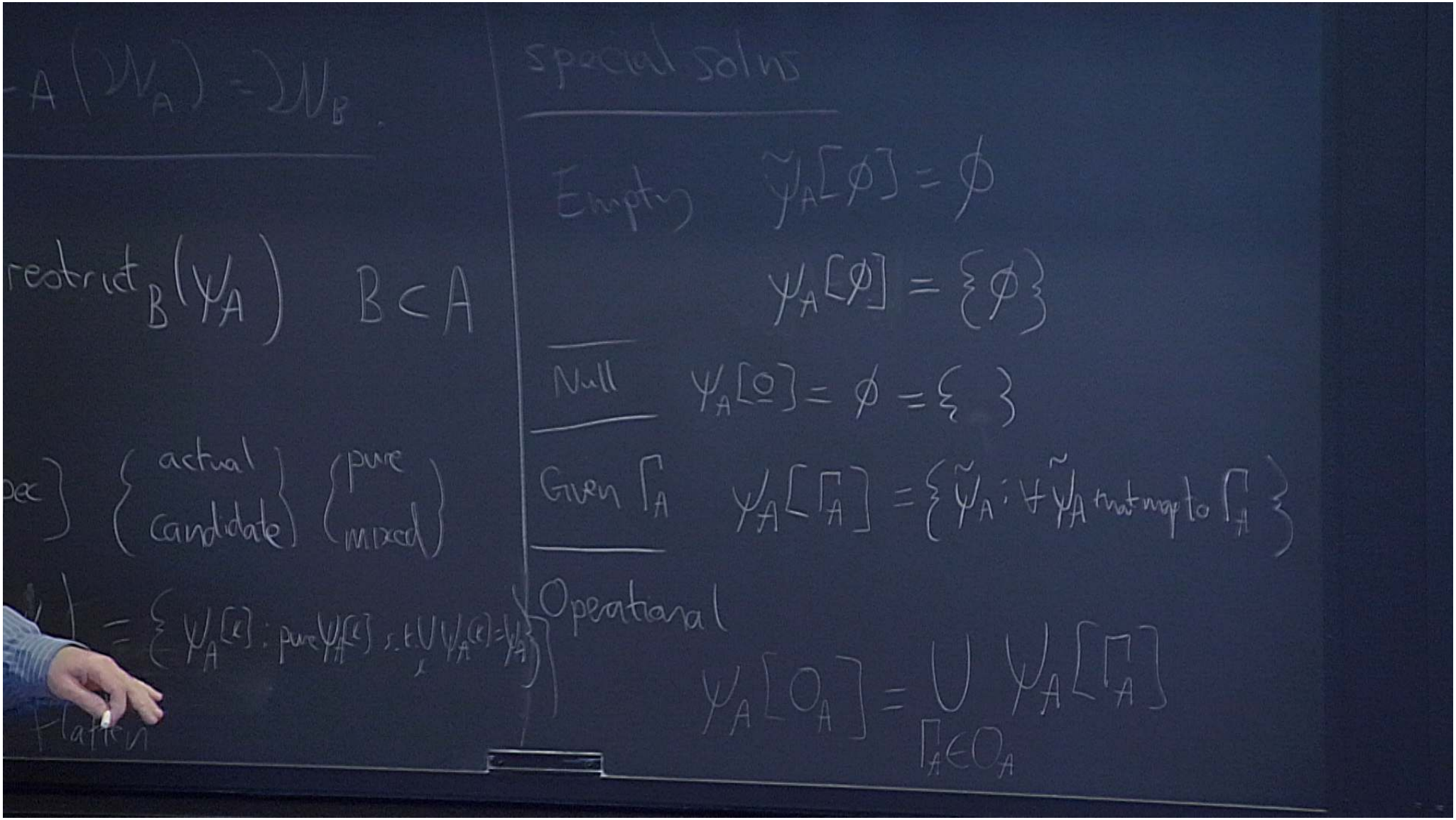
Title: Operational General Relativity - Lecture 3

Date: Oct 30, 2017 02:00 PM

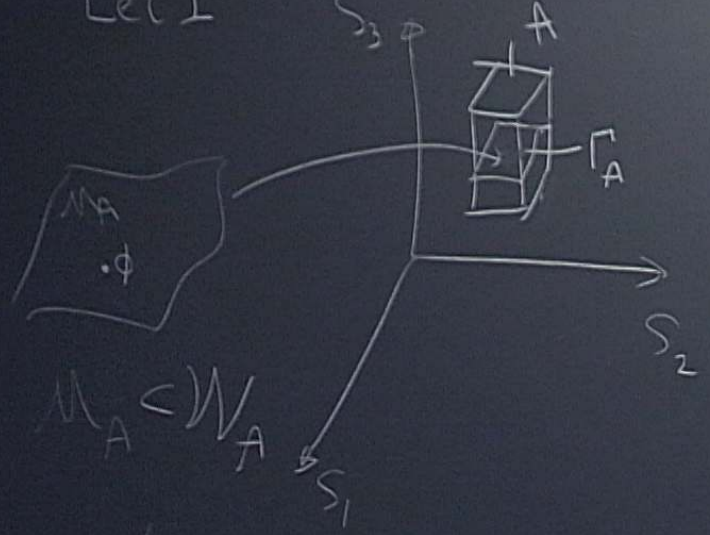
URL: http://pirsa.org/17100076

Abstract:


special solns
Empty $\tilde{\psi}_A[\emptyset] = \emptyset$
$\psi_A[\emptyset] = \{\emptyset\}$
Null $\psi_A[\underline{0}] = \emptyset = \{\ \}$
Given Γ_A $\psi_A[\Gamma_A] = \{\tilde{\psi}_A : \forall \tilde{\psi}_A$ that map to $\Gamma_A\}$
Operational
$\psi_A[O_A] = \bigcup_{\Gamma_A \in O_A} \psi_A[\Gamma_A]$
$\mathrm{restrict}_B(\psi_A)$ $B \subset A$
actual
candidate
pure
mixed
$= \{\psi_A[x] : \text{pure } \psi_A[x] \text{ s.t. } \bigcup_x \psi_A[x] = \psi_A\}$
Flatten

Summary

Lec 1



$\mathcal{M}_A \subset \mathcal{W}_A$

$$\tilde{\psi}_A = \{ (p, \phi) : \forall p \in \mathcal{M}_A \}$$

$$\psi_A = \{ \varphi_A^* \tilde{\psi}_A : \forall \varphi_A \}$$

$$\varphi^I_{B \leftarrow A}(\mathcal{W}_A) = \mathcal{W}_B .$$

Lec 2

$$\psi_B = \text{restrict}_B(\psi_A) \qquad B \subset A$$

Ω

$$\Omega[\text{spec}] \quad \begin{Bmatrix} \text{actual} \\ \text{candidate} \end{Bmatrix} \quad \begin{Bmatrix} \text{pure} \\ \text{mixed} \end{Bmatrix}$$

$$\text{sort}(\psi_A) = \{ \psi_A[k] : \text{pure } \psi_A[k] \text{ s.t. } \bigcup_k \psi_A[k] = \psi_A \}$$

flatten

$\leftarrow_A(\mathcal{W}_A) = \mathcal{W}_B$.

$\text{restrict}_B(\Psi_A)$ $B \subset A$

spec } { actual / candidate } { pure / mixed }

$\Psi_A) = \{\Psi_A[\Gamma] : \text{pure } \Psi_A[\Gamma] \text{ s.t. } \bigcup_\Gamma \Psi_A[\Gamma] = \Psi_A\}$ Operational

flatten

special solns

Empty $\tilde{\Psi}_A[\emptyset] = \emptyset$

$\Psi_A[\emptyset] = \{\emptyset\}$

Null $\Psi_A[0] = \emptyset = \{\}$

Given Γ_A $\Psi_A[\Gamma_A] = \{\tilde{\Psi}_A : \forall \tilde{\Psi}_A \text{ that map to } \Gamma_A\}$

$\Psi_A[O_A] = \bigcup_{\Gamma_A \in O_A} \Psi_A[\Gamma_A]$

$\Omega_A[\text{actual/mixed}]$

$\Omega_B[\text{actual/mixed}]$

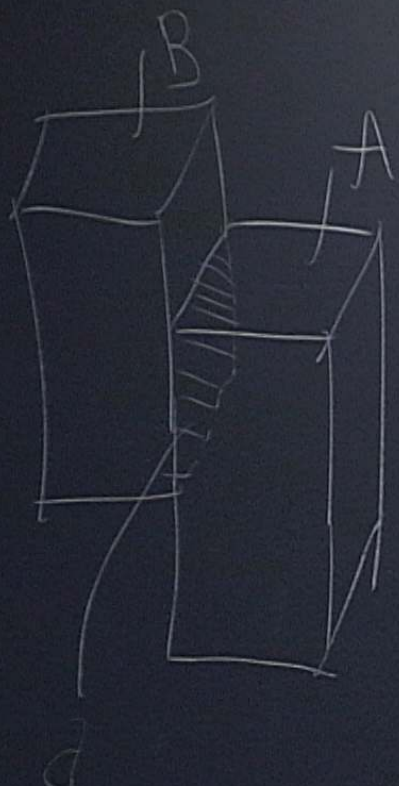
where

$$\tilde{\psi}_A \cup_a \tilde{\psi}_B = \begin{cases} \text{"}\psi_A \cup \psi_B\text{"} & \text{when match} \\ - & \text{else.} \end{cases}$$

$\{-\} = \{\}$

$$\psi_A \cup_a \psi_B = \left\{ \tilde{\psi}_A \cup_a \tilde{\psi}_B : \forall \tilde{\psi}_A \in \psi_A, \tilde{\psi}_B \in \psi_B \right\}$$

Composition



$\psi_A \in \Omega_A$ [actual / mixed]

$\psi_B \in \Omega_B$ [actual / mixed]

where

$\tilde{\psi}_A \, V_d \, \tilde{\psi}_B = \{$

$\{-\} = \{\}$

Define

$\psi_A \, V_d \, \psi_B = \{ \tilde{\psi}_A \, V_d \, \tilde{\psi}_B : \forall \tilde{\psi}_A \in \psi_A ,$

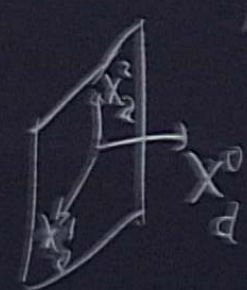
Vmathsf

Typing surfaces $\mathfrak{a}, \mathfrak{b}, \ldots$

$$\mathfrak{a} = \left(\mathrm{set}(\mathfrak{a}), \mathrm{coord}(\mathfrak{a}) \right)$$

$$\mathrm{coord}(\mathfrak{a}) = \left(X^{\ell}_{\mathfrak{a}} : \ell = 0 \text{ to } K-1 \right)$$

scalars



Composition

$\psi_A \in \Omega_A$[actual / mixed]

$\psi_B \in \Omega_B$[actual / mixed]

where

$\tilde{\psi}_A V_a \tilde{\psi}$

$\xi - 3 = \xi$

Define

$\psi_A V_a \psi_B = \{ \tilde{\psi}_A V_a \tilde{\psi}_B : \forall$

$A_a B^a$

Set(cl)

Mathsf

$X^\circ_a(\Sigma) = 0$ is an eqn for set(a)

dir(a) indicates side of set(a) in which X°_a increases.

$a^R = (\text{set}(a), \text{coord}^R(a))$ replace X°_a by $-X^\circ_a$

$\tilde{\gamma}_A = \{(p, \phi) : \forall p \in \mathcal{M}_A\}$ $\tilde{\gamma}_B = \{(p, \phi) : \forall$

$\mathcal{M}_A$

First (architecture) see if $\mathcal{M}_A$ matches $\mathcal{M}_B$ (under

Then (wallpaper) see if fields ϕ and some derivatives, Π, mat

$\in \mathcal{M}_A \}$ $\quad \widetilde{\mathcal{V}}_B = \{(p, \phi) : \forall p \in \mathcal{M}_B \}$

$\mathcal{M}_A$ $\qquad$ $\mathcal{M}_B$

ee if $\mathcal{M}_A$ matches $\mathcal{M}_B$ (under $\overset{I}{\varphi}_{B \subset A}$) to form new manifold.

ields ϕ and some derivatives, π, match.

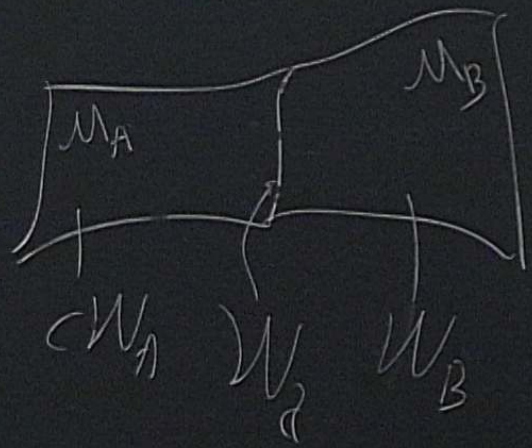
$\mathcal{W}_A$

$:= \{ p : \forall p \in \mathcal{M}_A \text{ s.t. } \underline{S}(p) \in \text{set}(a) \} \subset \mathcal{W}_A$

$\mathcal{W}_a$

$= \varphi^{I}_{a \in A}(\mathcal{M}_{a|A})$

$A|a$

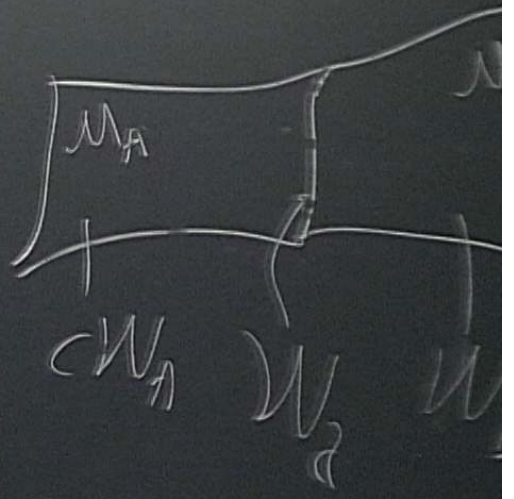


Define for $\tilde{\mathcal{V}}_A$

$$\mathcal{M}_{a|A} := \{ p : \forall p \in \mathcal{M}_A \text{ s.t. } \underline{S}(p) \in \text{set}(a) \} \subset$$

map into $\mathcal{W}_a$

$$\mathcal{M}_{a|A|a} = \varphi^{I}_{a \leftarrow A}\left(\mathcal{M}_{a|A}\right)$$



$\tilde{\varphi}_B$ $\mathcal{M}_{a|B|a}$

matching cond

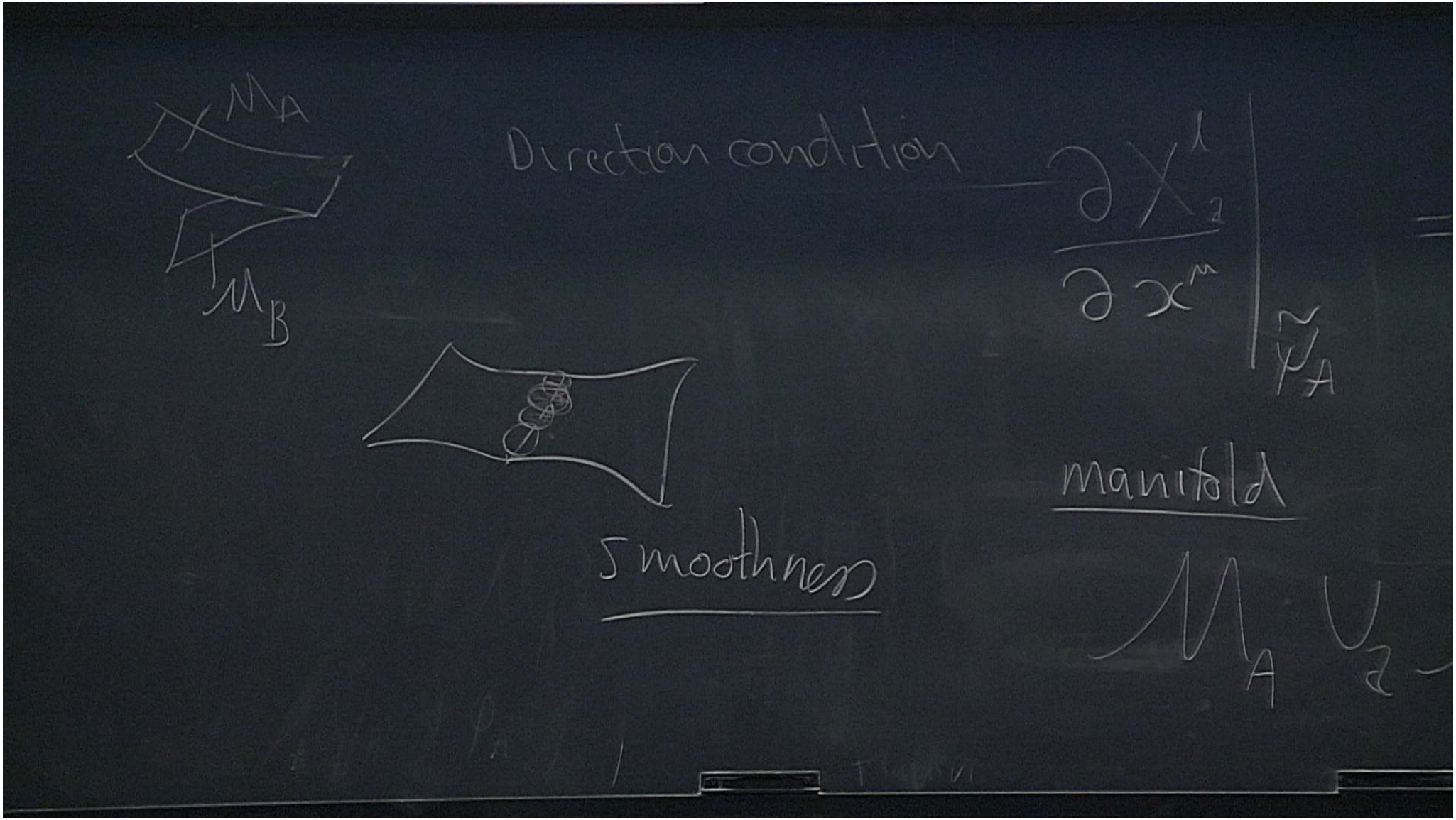
$$\mathcal{M}_{a|A|a}\Big|_{\tilde{\psi}_A} = \mathcal{M}_{a|B|a}\Big|_{\tilde{\psi}_B}$$

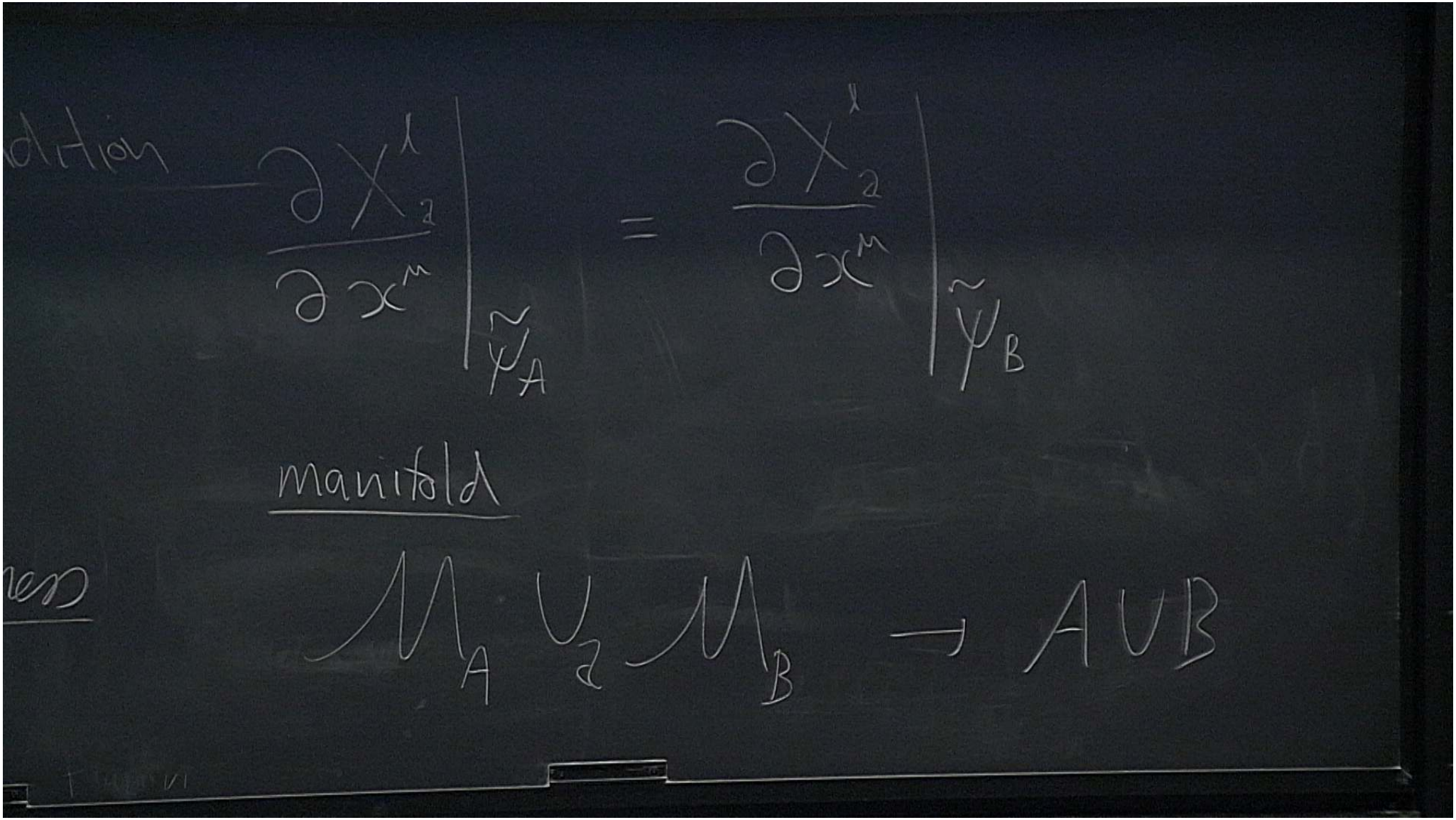
Identity $p \in \mathcal{M}_{a|A}$ with $\mathcal{O}^{I}_{B\leftarrow A}(p) \in \mathcal{M}_{a|B}$

$\tilde{\psi}_A$ ψ_B

manifold

$M_A \cup_2 M_B \to A \cup B$


M_A
M_B
Direction condition
$\frac{\partial x'^{\mu}}{\partial x^{\nu}}\Big|_{\tilde{\gamma}_A}$
smoothness
manifold
M_A
U_2



dition

$$\left.\frac{\partial X^l_a}{\partial x^\mu}\right|_{\tilde{\psi}_A} = \left.\frac{\partial X^l_a}{\partial x^\mu}\right|_{\tilde{\psi}_B}$$

manifold

ness

$$\mathcal{M}_A \cup_a \mathcal{M}_B \to A \cup B$$