

Title: Moore-Tachikawa conjecture, affine Grassmannian and Coulomb branches of star-shaped quivers

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Abstract: 

Moore and Tachikawa conjecture that there exists a functor from the category of 2-bordisms to a certain category whose objects are algebraic groups and morphisms between  $G$  and  $H$  are given by affine symplectic varieties with an action of  $G \times H$ . I will explain a proof of this conjecture due to Ginsburg and Kazhdan, and its relation to Coulomb branches of certain quiver gauge theories which allows to make interesting calculations.

# Moore - Tachikawa conj

Category  $\mathcal{C}$

Objects = affine alg. groups

Morphisms  $\text{Hom}_e(G_1, G_2)$

Isom classes of "symplectic varieties with Hamiltonian  $G_1 \times G_2$ -action"

↓  
affine, Poisson, generically symplectic  
(normal)

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Hamiltonian — class moment map

$G_1 \times G_2$ -action

Composition

$$G_1 \xrightarrow{X_1} G_2 \xrightarrow{X_3} G_3$$

Composition:

$$\frac{X_1 \times X_2 // G_2}{(G_1 \times G_2) \times (G_2 \times G_3) \cup G_2}$$

$$= \text{Spec } \mathbb{C} \left[ \mathcal{M}_{X_1 \times X_2}^{-1}(0) \right]^{G_2}$$

What is  $\text{id} : G \rightarrow G$ ?

Answer  $T^*G$

Exercise

Show that indeed

$T^*G$  is identity

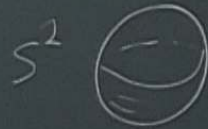
Moore - Tachikawa

Given a reductive (connected)  $G$

$\mathbb{J}$  functor

oriented  
2-bordisms  $\longrightarrow$

$\mathcal{L}$



$$\mathfrak{g} \cong \mathfrak{g}^*$$

Given  $G \mapsto \mathcal{Z}_G =$  "the universal centralizer".

"moduli space" of pairs  $(x, g)$   $x \in \mathfrak{g} = \text{Lie } G$  regular  
 $g \in G$

$K \subset \mathfrak{g}$   $(e, h, f)$ -principal  $\mathfrak{sl}_2$ -triple in  $\mathfrak{g}$   $\text{ad}_g(x) = x$

Kostant slice

$K = e + \mathcal{Z}(f)$   $\mathcal{Z}(f) = \text{centralizer of } f \text{ in } \mathfrak{g}$

$\mathcal{Z}_G = \left\{ (g, x) \mid \begin{array}{l} g \in G \\ x \in K \\ \text{ad}_g(x) = x \end{array} \right\}$  - smooth and carries canonical symplectic structure.

What is  $\text{id} : G \rightarrow G$ ?

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Show that indeed

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$\int_{x_1, x_2}^{-1} (0) \int_{G_2}$

Moore - Tachikawa

Given a reductive (connected)  $G$

$\mathcal{J}$  functor

oriented  
2-bordisms

$\longrightarrow \mathcal{Z}$

$\bigcirc \longmapsto G$

$S^2 \longmapsto \mathcal{Z}G$

What is  $\text{id} : G \rightarrow G$ ?

Answer  $T^*G$

Exercise

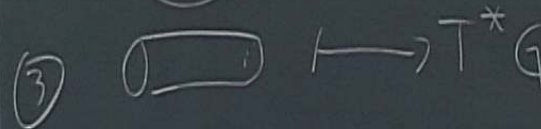
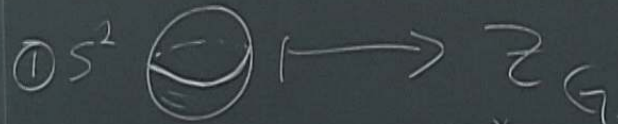
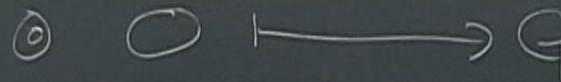
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Moore - Tachikawa

Given a reductive (connected)  $G$

$\mathcal{J}$  functor  $\mathcal{Z}\text{-bordisms} \rightarrow \mathcal{Z}$



② Explicit answer for 

$G \times K$

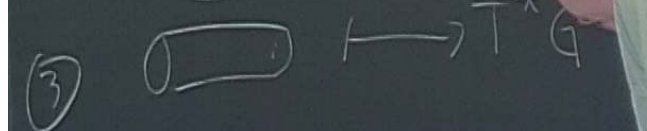
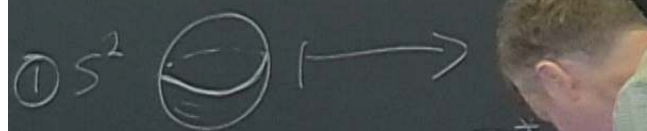
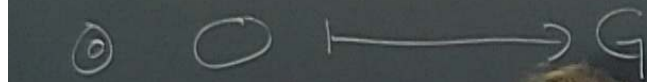
$K \subset$

Kostant slice

Moore - Tachikawa

reductive (connected)  $G$

for oriented 2-bordisms  $\rightarrow \mathcal{Z}$



Explicit answer

Given  $G \mapsto \mathcal{Z}_G =$  "the universal moduli space" of pairs  $(x, \dots)$

$\dim G =$   
 $= \text{rank } G + 2 \cdot \# \text{positive roots}$

$K \subset \mathfrak{g}$  of  $(e, h, f)$ -principal  $\mathfrak{sl}_2$ -triple in  $\mathfrak{g}$

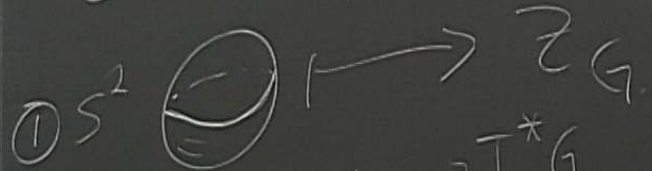
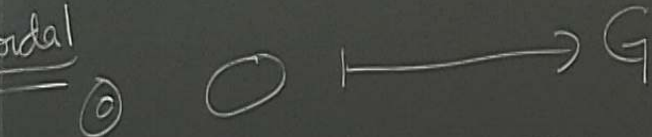
Kostant slice  $K = e + \mathcal{Z}(f)$   $\mathcal{Z}(f) = \text{centralizer of } f$

$\dim K = \text{rank } G$ .  $\mathcal{Z}_G = \{ (g, x) \mid g \in G, x \in K, \text{ad}_g(x) = x, g x g^{-1} = x \}$

Moore - Tachikawa

Given a reductive (connected)  $G$

$\exists$  functor  $\mathcal{Z}$ -bordisms  $\rightarrow \mathcal{Z}$   
monoidal



Explicit answer for  $G \times K$

Given  $G \mapsto \mathcal{Z}_G = \dots$

$\dim G = \text{rank } G + 2 \times \# \text{ positive roots}$

"moduli space"  $\neq$

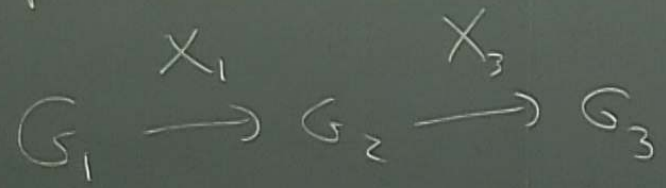
$K \subset \text{of } (e, h, f)$ -principal

Kostant slice  $K = e + \mathcal{Z}(f)$

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$G_2$ -action

Composition



Composition:

$$\frac{X_1 \times X_2 // G_2}{(G_1 \times G_2) \times (G_2 \times G_3) \downarrow \Delta G_2}$$

$$= \text{Spec } \mathbb{C} \left[ \mathcal{O}_{X_1 \times X_2}^{-1}(0) \right]^{G_2}$$

Monoidal category

$$G_1 \otimes G_2 = G_1 \times G_2$$

What  
 Answer  
 Exercise  
 Show  
 $T^*$

$\Sigma$  - Riemann surface whose boundary  
has  $n$  components



Symplectic variety  $X$   
with an action of  $G^n$

" symplectic varieties with Hamiltonian  $G_1 \times G_2$ -action

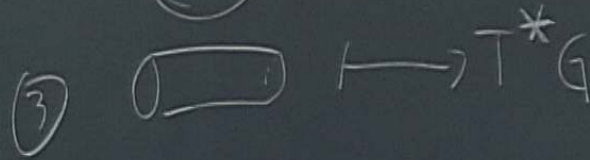
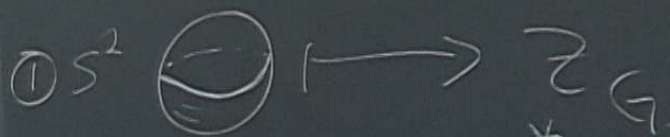
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(normal)

Hamiltonian - chosen moment map

Moore - Tachikawa

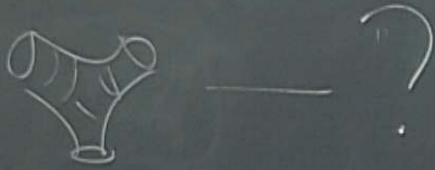
Given a reductive (connected)  $G$

$\mathcal{J}$  functor  
monoidal  
 $\text{oriented } 2\text{-bordisms} \longrightarrow \mathcal{Z}$



② Explicit answer  
 for  $\downarrow$   
 $G \times K$

(4)



Essentially determines everything.

Explicit answer

for 

↓

$G \times K$

( $G$  simple, ADE).

Higgs branch of  
certain 3d QFT

d (0,2)-theory  
compactify on  $S^1 \times \Sigma$   
3d theory

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Hamiltonian - class

(connected)  $G$   
 central  
 analysis  $\rightarrow \mathcal{Y}$   
 $\mathbb{C} \rightarrow G$   
 $\mathbb{C} \rightarrow \mathbb{Z}G$   
 $\mathbb{C} \rightarrow T^*G$   


filling in a hole  
 corresponds  
 to  $KW_G$

(4)



Essentially determines everything.

(2) Explicit answer  
 for  $G \times K$

$$\begin{aligned}
 \mathbb{Z}_G &\simeq KW_{G \times G}(T^*G) \\
 G \times K &= KW_{G \times G}(T^*G) \\
 KW_G(G \times K) &= \mathbb{Z}_G.
 \end{aligned}$$

<sup>simp</sup>  
 $X$ -variety with  $G$ -action

Let  $N \subset G$  be a maximal unip  
 Let  $\chi: N \rightarrow \mathbb{C}$  be a non-deg homomorphism

$$N/(N, N) \simeq \bigoplus_{\alpha_i} \mathbb{C}_{\alpha_i}$$

simple roots

$$KW_G(X) = \mu_N^{-1}(\chi)/N$$

$$\chi: \mathfrak{n} \rightarrow \mathbb{C}$$

(4)



Essentially determines everything.

$$\mathbb{Z}_G \simeq \text{KW}_{G \times G}(T^*G)$$

$$G \times K = \text{KW}_G(T^*G)$$

$$\text{KW}_G(G \times K) = \mathbb{Z}_G.$$

<sup>simp</sup>  
X-variety with G-action.

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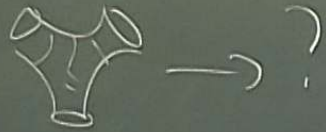
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$$\chi: \mathfrak{n} \rightarrow \mathbb{C}$$



$\times G_2$ -action

$$G = SL(2) \quad ? = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$G = SL(3) \quad ? = \text{the closure of the minimal nilpotent orbit in } E_6$$

EXERCISE Invent a Hamiltonian action of  $SL(3)^3$  on this



What is  $\text{id}: G \rightarrow G$ ?

Answer  $T^*G$

indeed

tity



$\times G_{\mathbb{Z}}$ -action

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$G = SL(4) \quad ? \text{ don't know}$



What is  $\text{id}: G \rightarrow G$ ?

Answer  $T^*G$

indeed  
fity

Construction

Ginzburg-Kazhdan + Bapat

$$G = SL(N) \text{ or } GL(N)$$

there is a different way of looking at it.

3 on this

Change of notation.

Fix a reductive group  $G$

{

Construct the sought-for functor for  $G^\vee$   
(Langlands dual group).

functor for  $G^V$   
 (Langlands dual group).

$$GR_G = G((+)) / G[[+]] = \mathcal{O} = \mathbb{C}[[t]]$$

= moduli space of principal

$G$ -bundles on  $D = \text{Spec } \mathbb{C}[[t]]$

+ triv at  $D^* = \text{Spec } \mathbb{C}((t))$

Going to study  $D_{G(\mathcal{O})}(GR_G)$ .

<sup>simp</sup>  
 $X$ -variety with  $\mathcal{O}$

let  $N \subset G$  be a  
 let  $\mathcal{O} \subset \mathcal{O}$

$$GR_G = G(\mathbb{A}) / G(\mathbb{Z}) = \mathcal{O} = \mathbb{C}[[t]]$$

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Going to study  $D_{G(\mathcal{O})}(GR_G)$

carries a canonical monoidal symmetric structure

functor for  $G^\vee$   
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Going to study  $D_{G(\mathcal{O})} (GR_G)$

carries a canonical monoidal structure  
 symmetric convolution of sheaves

functor for  $G^v$   
 (Langlands dual group)

Given an object  $\mathcal{A} \in \mathcal{D}(Gr_G)$

it makes sense to say that  $\mathcal{A}$  is Ring object.

$$\mathcal{A} * \mathcal{A} \xrightarrow{m} \mathcal{A}$$

structure  
evolution of sheaves

Given an object  $\mathcal{A} \in \mathcal{D}(\Gamma_{\mathbb{A}^1})$

it makes sense to say that  $\mathcal{A}$  is Ring object.

$$\mathcal{A} * \mathcal{A} \xrightarrow{m} \mathcal{A}$$

(Commutative Ring objects)

Structure  
convolution of sheaves

Let  $X_n$  be

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it makes sense to say that  $\mathcal{A}$  is Ring object.

$$\mathcal{A} * \mathcal{A} \xrightarrow{m} \mathcal{A}$$

(Commutative Ring objects)

Given  $n \geq 0 \rightarrow \mathcal{A}^n$  Ring object endowed with an action of  $(G^V)^n$ .

of sheaves  $\mathbb{C}[$

$$= G(\mathbb{A}^1) / G[\mathbb{A}^1] = \mathcal{O} = \mathbb{C}[\mathbb{A}^1]$$

moduli space of principal

$G$ -bundles on  $D = \text{Spec } \mathbb{C}[\mathbb{A}^1]$

+ trivial on  $D^* = \text{Spec } \mathbb{C}^*$

to study  $D$   $(GR_G)$

carries a canonical monoidal symmetric

structure convolution of sheaves

Given an object  $\mathcal{A} \in \mathcal{D}(GR_G)$

it makes sense to say that  $\mathcal{A}$  is ring object.

Let  $X_n$  be the image of  $S^2$  with  $n$  punctures under RT functor.

$$\mathcal{A} * \mathcal{A} \xrightarrow{m} \mathcal{A}$$

(commutative ring objects)

Given  $n \geq 0 \rightarrow \mathcal{A}_n$  ring object endowed with an action of  $(G^V)^n$ .

$$\mathbb{C}[X_n] = H^*_{G(\theta)}(\mathcal{A}_n)$$

$$D(G_R) \supset \text{Per}_{G(0)}(G_R)$$

$\hookrightarrow \mathbb{R}$

Rep  $G^\vee$

$$G_R = \bigsqcup_{\lambda \in \Lambda^+} G_R^\lambda$$

$\uparrow$   
G(0)-orbits

$\Lambda =$  weight lattice of  $G^\vee$

$\Lambda^+ =$  dominant weights

$$\Lambda^+ \ni \lambda$$

$$V(\lambda) \cong IC^\lambda = IC(\overline{G_R^\lambda})$$

$$\chi_i =$$

$$D(GR)_{G(0)} \supset \text{Per}_{G(0)}(GR_G)$$

$\hookrightarrow \mathbb{R}$   
 Rep  $G^\vee$  equiv of monoidal categories

$$GR_G = \bigsqcup_{\lambda \in \Lambda^+} GR^\lambda$$

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 $G(0)$ -orbits

$\Lambda =$  weight lattice of  $G^\vee$

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$$\lambda \in \Lambda^+ \quad V(\lambda) \text{ mod } \mathfrak{m} \quad IC^\lambda = IC(\overline{GR}^\lambda)$$

$$\mathcal{A}_1 = \mathcal{S}^{-1} \text{ (Regular representation)}$$

$\parallel$

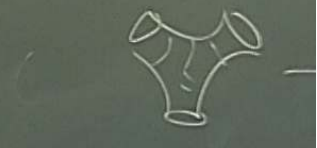
$$\bigoplus_{\lambda \in \Lambda^+} IC^\lambda \otimes V(\lambda)^*$$

of  
 idal  
 egories  
 lattice of  $G^v$   
 int nts

$$A_1 = \mathcal{S}^{-1} \text{ (Regular representation)}$$

||

$$\bigoplus_{\lambda \in \Lambda^+} \mathbb{C}^{\lambda} \otimes V(\lambda)^*$$



$$G = SL(2)$$

$$G = SL(3)$$

Exercise Inve

$$G = SL(4) \quad \text{don}$$

$$\mathcal{F} \in \text{Perv}_{G(0)}(GR_G)$$

$\lambda(\mathcal{F})$  as a vector space

$$\text{it is } H^*(\overline{GR}_G, \mathbb{C})$$

$$\Lambda^+ \ni \lambda$$

$$D(GR) \supset \text{Perv}_{G(0)}(GR_G)$$

$\lambda$  ?

Rep  $G^V$

equiv of monoidal categories

$\Lambda =$  weight lattice

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$$GR_G = \bigsqcup_{\lambda \in \Lambda^+} GR^\lambda$$

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$(\overline{GR}_G)$

equiv of  
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categories

weight lattice of  $G^\vee$

dominant weights

$(\overline{GR}^\lambda)$

$G \times K$

$$\mathcal{A}_i = \mathcal{S}^{-1} \text{ (Regular representation)}$$

$\parallel$

$$\bigoplus_{\lambda \in \Lambda^+} IC^\lambda \otimes V(\lambda)^*$$

$$K = \mathfrak{t}/\mathfrak{w} \quad \mathfrak{t} \text{ cog Cartan}$$

$(GR_G)$

equiv of monoidal categories

$\Lambda =$  weight lattice of  $G^\vee$

$\Lambda^+ =$  dominant weights

$(\overline{GR}^+)$

$G \times K$

$$\mathcal{A}_1 = \mathcal{S}^{-1} \text{ (Regular representation)}$$

$\parallel$

$$\bigoplus_{\lambda \in \Lambda^+} IC^\lambda \otimes V(\lambda)^*$$

$K = t/w$   $t =$  coc of Cartan

ANSWER for  $\mathcal{A}_n$

$$\mathcal{A}_n = \underbrace{\mathcal{A}_1 \overset{!}{\otimes} \mathcal{A}_1 \overset{!}{\otimes} \dots \overset{!}{\otimes} \mathcal{A}_1}_{n \text{ times}}$$

$Y$  - any var. over  $\mathbb{C}$   
 $\mathcal{F}, \mathcal{G}$  - constructible complexes  
 can form  $\mathcal{F} \overset{!}{\otimes} \mathcal{G}$   
 also  $\mathcal{F} \overset{!}{\otimes} \mathcal{G}$   
 $\mathcal{F} \boxtimes \mathcal{G}$  - sheaf on  $Y \times Y$   
 $\Delta: Y \rightarrow Y \times Y$   
 $\mathcal{F} \overset{!}{\otimes} \mathcal{G} = \Delta^* (\mathcal{F} \boxtimes \mathcal{G})$

$$\mathcal{F} \overset{!}{\otimes} \mathcal{G} = \Delta^* (\mathcal{F} \boxtimes \mathcal{G})$$

(Also the natural tensor product for D-modules)

ANSWER for  $\mathcal{A}_n$

$$\mathcal{A}_n = \underbrace{\mathcal{A}_1 \overset{!}{\otimes} \mathcal{A}_1 \overset{!}{\otimes} \dots \overset{!}{\otimes} \mathcal{A}_1}_{n \text{ times}}$$

$$\mathcal{A}_0 = \omega_{\mathbb{A}^1}$$

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(also the natural  
tensor product  
for D-modules)

$H_c^*(Y, \omega_Y) = \text{homology of } Y$

$H^*(Y, \omega_Y) = \text{Borel-Moore hom. of } Y$

$G^\vee$   
(Langlands dual group)

$$\text{Gr}_G = G((t))$$

= moduli space

$G$ -bundles

+ triv.

Going to study

Cart

$H_c^*(Y, \omega_Y) = \text{homology of } Y$

$H^*(Y, \omega_Y) = \text{Borel-Moore hom. of } Y$

$\mathbb{C}[X_0] = G(\mathbb{C})$ -equivariant

homology of  $GTR_G = \mathbb{C}[Z_G]$

(Bezrukavnikov - Finkelberg - Mirkovic)

$n=2$

of  $\gamma$ .

$$n=2$$

$$A_1 \otimes A_1$$

$$H_{G(\mathcal{O})}^*(A_1 \otimes A_1) = \mathbb{C}[T^* \mathbb{C}^2]$$

Finkelberg-Mirkovic)

Given an

it mak

sheave)

ANSWER for  $\mathcal{A}_n$

$$\mathcal{A}_n = \underbrace{\mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \dots \otimes \mathcal{A}_1}_{n \text{ times}}$$

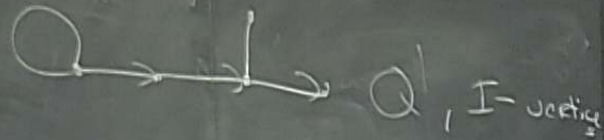
$$\mathcal{A}_0 = \omega_{\mathbb{R}^6}$$

$\mathbb{C}[X_n]$  has a canonical  
quantization

$$\hbar^* \omega_{(0) \times \mathbb{R}^6}(\mathcal{A}_n)$$

a condition

$(A_n)$   
 $x \in x^*$



Given  $G_{\text{gauge}}$  - reductive

$N$  - rep of  $G_{\text{gauge}}$

$M_c$  - affine singular sym.  
variety

$$\vec{v}_i, \vec{w}_i \in \mathbb{Z}_{\geq 0}$$

$$\dim V_i = v_i$$

$$\dim W_i = w_i$$

$$G_{\text{gauge}} = \prod_{i \in I} \text{GL}(V_i)$$

$$N = \bigoplus_{i \rightarrow j} \text{Hom}(V_i, V_j) \oplus \left( \bigoplus_i \text{Hom}(V_i, W_i) \right)$$

Theorem

$$G = GL(N)$$

Then  $X_n$  is the Coulomb branch

of the quiver theory for  $*$ -shaped quiver.

$$n = \# \text{ legs}$$

$$N = \text{length of each leg}$$



$$\oplus \text{Hom}(V_i, W_i)$$

sheaves

Theorem

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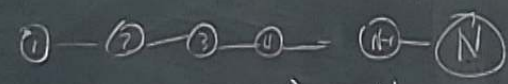
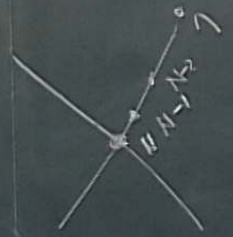
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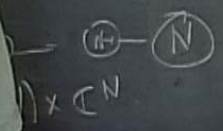
$N = \text{length of each leg}$

$\oplus_i \text{Hom}(V_i, W_i)$



$GL(N) \times \mathbb{C}^N$

pedi quiver  
egs  
th of each leg



$$\mathcal{R}_{G_{\text{gauge}}, \mathbb{N}}$$



$$\mathcal{S}P_{G_{\text{gauge}}, \mathbb{N}}$$

$$G_{\text{gauge}} = \prod_{i=1}^N GL(i)$$

$$\mathbb{N} = \bigoplus_{i=0}^{N-1} \text{Hom}(e^i, e^{i+1})$$

$\mathcal{P}$ -principal  $G$ -bundle on  $\mathcal{D}$   
 $\kappa$ :- triv. off  $\mathcal{P}$  on  $\mathcal{D}^*$   
 $\rho$ -section of  $\mathcal{P}_{\mathbb{N}}$  on  $\mathcal{D}$   
 which has no pole w.r. to  $\kappa$

let  $X_n$  be the image of  $S$  with  $n$  poles under

$$\mathcal{P}[\mathcal{M}_c] = H_{G_{\text{gauge}}(0)}^{BM}(\mathcal{R}_{G_{\text{gauge}}, \mathbb{N}})$$

Theorem

$G = GL(N)$

Then  $X_n$  is the Coulomb branch

of the quiver theory for  $*$ -shaped quiver

$(V_i, W_i)$



$n = \# \text{ legs}$

$N = \text{legs on each leg}$



$$\pi: \mathcal{R}_{G_{\text{gauge}}, N} \rightarrow \mathcal{G}R_{GL(N)}$$

$$\pi_* \omega = A_{\Delta} \supset GL(N)$$

$$\mathcal{R}_{G_{\text{gauge}}, N}$$



$$\mathcal{G}R_{G_{\text{gauge}}, N}$$

$$G_{\text{gauge}} = \prod_{i=1}^n GL(N_i)$$

$$N = \bigoplus_{i=0}^{N-1} \text{Hom}(C^i, C^{i+1})$$

$\mathcal{P}$ -principal  $G$ -b  
 $K$  - triv. off  $\mathcal{P}$   
 $\rho$ -section of  
 which has no

Theorem

$G = GL(N)$

Then  $X_n$  is the Coulomb branch

of the quiver theory for  $n$ -shaped quiver

$n = \# \text{ legs}$

$N = \text{length of each leg}$



$GL(N) \times \mathbb{C}^N$

$\pi: \mathcal{R}_{G_{\text{gauge}}, N} \rightarrow \mathcal{G}R_{GL(N)}$

$\pi_* \omega = \mathcal{A}_1 \supset GL(N)$

$\mathcal{R}_{G_{\text{gauge}}, N}$



$\mathcal{G}R_{G_{\text{gauge}}, N}$

$G_{\text{gauge}} = \prod_{i=1}^n GL(i)$

$N = \bigoplus_{i=0}^{N-1} \text{Hom}(E^i, E^{i+1})$

$\mathcal{P}$ -principal  $G$   
 $\kappa$ :- triv. off  
 $\rho$ -section of  
 which has  $v$

ANSWER for  $\mathcal{A}_n$

$$\mathcal{A}_n = \underbrace{\mathcal{A}_1 \otimes \mathcal{A}_1 \otimes \dots \otimes \mathcal{A}_1}_{n \text{ times}}$$

$$= \omega_{\mathbb{G}_G}$$

$$\mathbb{C}[\mathcal{X}_n] = H_{\mathbb{G}(0)}^*(\mathbb{G}_G, \mathcal{A}_n)$$

$\mathbb{C}[\mathcal{X}_n]$  has a canonical quantization

$$H_{\mathbb{G}(0)}^*(\mathcal{A}_n)$$



Given  $G_{\text{gauge}}$ -red

$N$ -rep of

$\mathcal{M}_C$  - affine singular

$\dim V_2$

$\dim W_2$

$G_{\text{gauge}}$

$N =$