Title: Order Plus Number ~ Geometry: A Lorentzian Approach to Quantum Gravity

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Abstract: <p>I will give an overview of the causal set approach to quantum gravity, and what makes this "fork in the road" distinct from other approaches. Motivated by deep theorems in Lorentzian geometry, causal set theory (CST) posits that the underlying fabric of spacetime is atomistic and encoded in a locally finite partially ordered set. In the continuum approximation, the partial order corresponds to the causal structure, and the cardinality to the conformal factor. Together, these give the approximate continuum geometry. Lorentz invariance emerges as a consequence, but brings with it a certain "non-localityâ€•, which distinguishes CST from other approaches in an essential way. It also makes the reconstruction of spacetime geometry from the causal set particularly challenging. I will describe some of the progress we have made in this geometric reconstruction program. I will then describe a particular formulation of CST dynamics inspired by the continuum path integral and discuss what we have learnt so far and where it is taking us. </p>

Order and Number ~ Geometry:

A Lorentzian Approach to Quantum Gravity

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Outline

- Order as an Essence of Lorentzian Geometry
- The Causal Set Hypothesis: Order + Number $\sim$ Geometry
  - Geometry from Order
  - Different Routes to Dynamics
  - Phenomenology: (prediction for Λ, non-locality, etc.)
- Directions and Challenges

Nomaan Ahmed, Michel Buck, Will Cunningham, Fay Dowker, Astrid Eichhorn, Lisa Glaser, Joe Henson, Ian Jubb, Abhishek Mathur, Sebastian Mizera, Denjoe O'Connor, David Rideout, Rafael Sorkin, Yasaman Yazdi

Forks on the Road to Quantising Gravity:

– R. Sorkin (1997)

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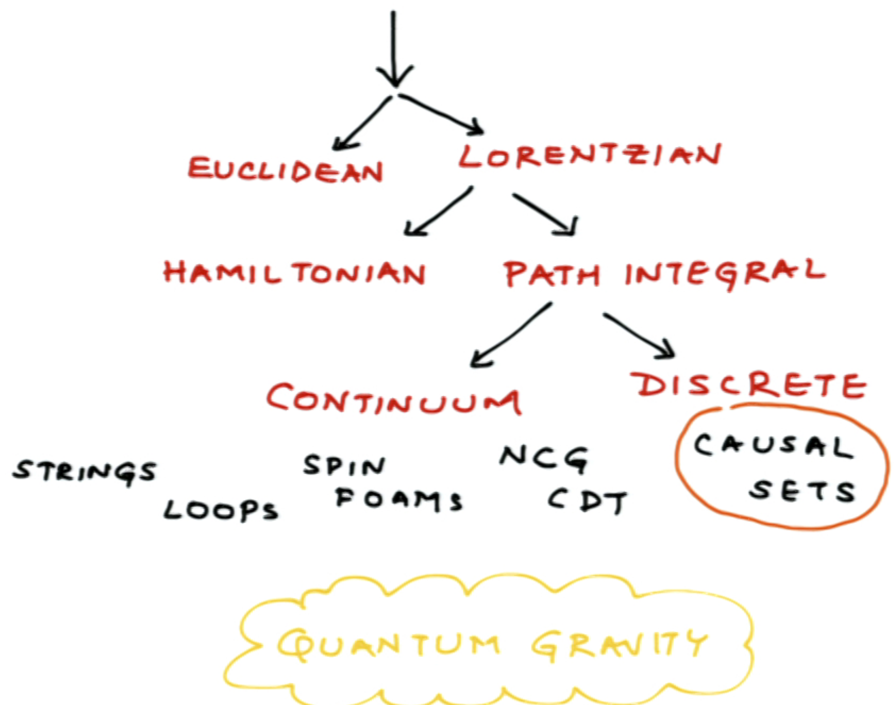


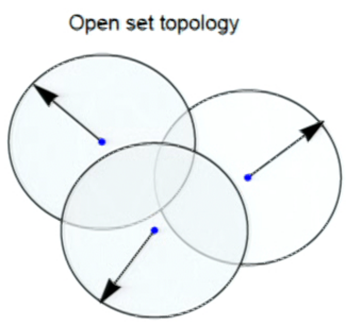
First Fork in the Road: Lorentzian Geometry

What is the essence of Lorentzian geometry?

- Is it merely a generalisation of Riemannian geometry?
  - *"Psuedo-Riemannian"* geometry

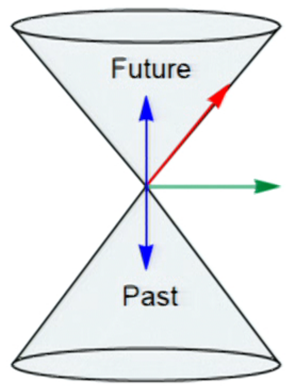
$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 \quad \rightarrow \quad ds^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_3^2$$

  - Topology and Differentiable Structure: same as that in Riemannian geometry



What is the essence of Lorentzian geometry?

- Or is it: $(-, +, +, +)$ essentially different?
  - $ds^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_3^2$ can be positive, zero, or negative.
  - Lightcones



  - Local causality: $x \prec y$ if $\exists$ a future directed causal curve γ from x to y.

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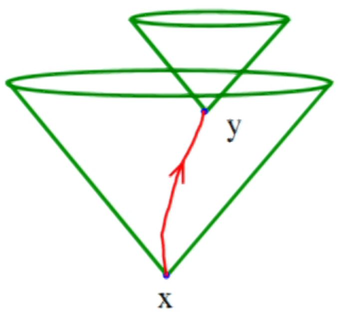


Alexandrov, Seifert, Zeeman, Penrose, Geroch, Kronheimer, Hawking, Malament, Levichev, etc.

Causal Spaces – An Axiomatic Approach
– Penrose and Kronheimer, 1966

Extract from (M, g) its causal essence:

1·2. The *quadruple* $(X, \prec, \ll, \rightarrow)$ will be called a *causal space* if X is a set and $\prec$, $\ll$, $\rightarrow$ are three relations on X satisfying, for each x, y, z in X, the following conditions:

(I) $x \prec x$;

(II) *if* $x \prec y$ *and* $y \prec z$, *then* $x \prec z$;

(III) *if* $x \prec y$ *and* $y \prec x$, *then* $x = y$;

(IV) *not* $x \ll x$;

(V) *if* $x \ll y$ *then* $x \prec y$;

(VI+) *if* $x \prec y$ *and* $y \ll z$, *then* $x \ll z$;

(VI−) *if* $x \ll y$ *and* $y \prec z$, *then* $x \ll z$;

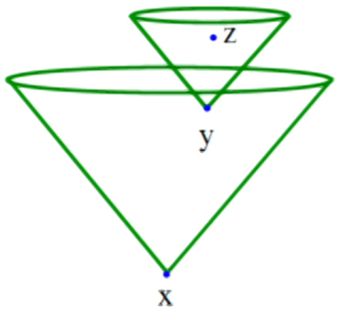
(VII) $x \rightarrow y$ *if and only if* $x \prec y$ *and not* $x \ll y$.

"To admit structures which can be very different from a manifold"

The Causal Structure Poset

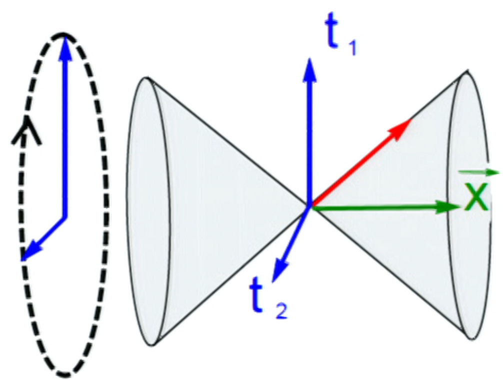
Causal essence: $(M, \prec) \subset (M, g)$

- M: the **set** of events.
- $\prec$:
  - **Reflexive:** $x \prec x$
  - **Acyclic:** $x \prec y$ and $y \prec x \Rightarrow x = y$
  - **Transitive:** $x \prec y$ and $y \prec z \Rightarrow x \prec z$



The Causal Structure Poset

$$ds^2 = -dt_1^2 - dt_2^2 + dx_1^2 + dx_2^2$$



No future or past

There is no causal structure poset for any other signature spacetime

Causal Structure Hierarchy

⋮

- Globally Hyperbolic
- Causally Stable
- Causally Continuous

⋮

- Strong Causality : Alexandrov topology=Manifold topology

⋮

- Future and Past Distinguishable
- Acausal : No causal structure Poset

How primitive is $(M, \prec)$?

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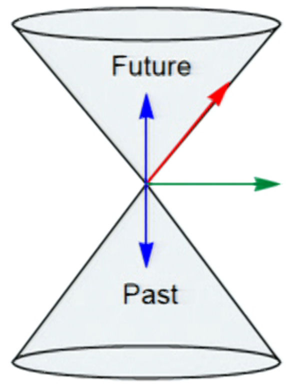
Causal Structure remains invariant under conformal rescaling: $\tilde{g}_{ab} = \Omega^2 g_{ab}$.



$(M, \prec)$ determines the conformal class of the metric.

Let $f : (M_1, g_1) \to (M_2, g_2)$ *be a causal bijection*

$$x_1 \prec_1 y_1 \Leftrightarrow f(x_1) \prec_2 f(x_2)$$

Then f *is a smooth conformal isometry:* f *and* f^{-1} *are smooth and* $f_* g_1 = \Omega^2 g_2$.

S. W. Hawking, A.R. King, P.J. McCarthy (1976); D. Malament (1977)

O. Parrikar, S. Surya (2011)

$(M, \prec)$ determines the conformal class of the metric.

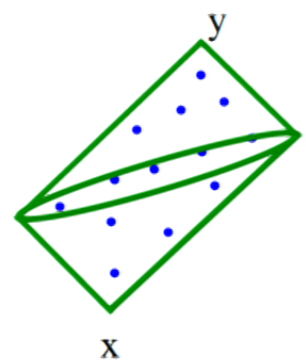
- "Causal structure is $9/10^{\mathrm{th}}$ of the spacetime geometry."
- Remaining $1/10^{\mathrm{th}}$ is the volume element

$$\epsilon = \Omega^n \times \sqrt{g} dx^1 \wedge \ldots \wedge dx^n$$

Spacetime geometry = Causal Structure + Volume

The hypothesis of discreteness

- Cure for infinities: singularities, quantum field theory divergences, entanglement entropy, etc.
- Volume element from discreteness: $N \sim V/V_p$.



Fork in the Road: Discreteness

The hypothesis of discreteness

"To admit structures which can be very different from a manifold. The possibility arises, for example, of a locally countable or discrete event-space equipped with causal relations macroscopically similar to those of a space-time continuum. (Discrete models of space-time have been suggested by a number of authors as a possible way of avoiding the infinities of quantum field theory, etc. See, for instance, (12, 11,3,6,2,1).)"

– Penrose and Kronheimer, 1966

- (1) AHMAVAARA, Y . The structure of space and the formalism of relativistic quantum theory. I. J. Math. Phys. 6 (1965), 87-93.
- (2) BOHM, D. A proposed topological formulation of the quantum theory. The scientist speculates [editor I. J. Good], pp. 302-314. (Heinemann; London, 1962).
- (3) COXETER, H. S. M. and WHITROW, G. J. World-structure and non-Euclidean honeycombs. Proc. Roy. Soc. London Ser. A 201 (1950), 417-437.
- (6) HILL, E.L. Relativistic theory of discrete momentum space and discrete space-time. Phys. Rev. 100 (1955), 1780-1783.
- (11) SCHILD, A. Discrete space-time and integral Lorentz transformations. Canad. J. Math. 1 (1949), 29-47.
- (12) SNYDER, H. S. Quantized space-time. Phys. Rev. 71 (1947), 38-41.

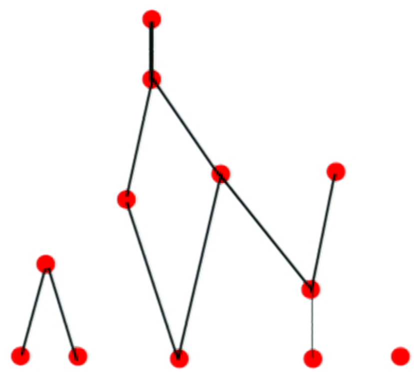
The Causal Set Hypothesis

- Finkelstein(1969),Myrheim(1978), 'tHooft (1979), Hemion(1980)
– L.Bombelli, J.Lee, D. Meyer and R. Sorkin (1987)

Two fundamental building blocks:

- The Causal Structure Poset $(M, \prec) \subset (M, g)$
- Fundamental Spacetime Discreteness

The underlying structure of spacetime is a *causal set* or locally finite poset $(C, \prec)$



Continuum is an Approximation

Spacetime geometry = Causal Structure + Volume

Causal Structure → Partially Ordered Set

Spacetime Volume → Number

Order + Number ∼ Spacetime geometry

The Continuum Approximation

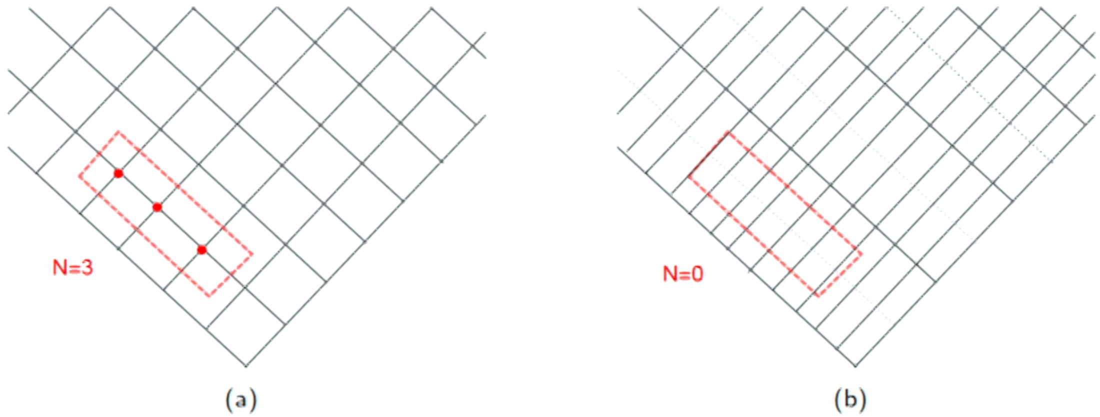
Riemann's dilemma

- A discrete manifold has finite properties, whereas a continuous manifold does not. Natural quantities are to be finite. The world must be discrete.
- A discrete manifold possesses natural internal metrical structure, whereas a continuous manifold must have its metrical structure imposed from without. Natural law is to be unified. The world must be discrete.
- A continuous manifold has continuous symmetries, whereas a discrete manifold does not. Nature possesses continuous symmetries. The world must be continuous.

–from Finkelstein(1969)

The Continuum Approximation

- Regular lattice:

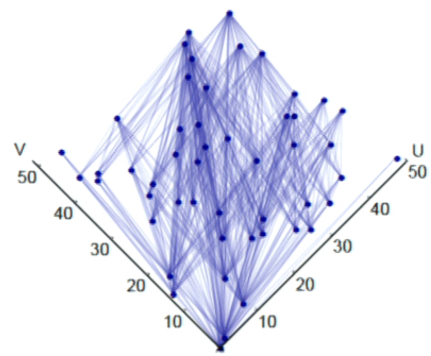


Does not preserve Number-Volume correspondence.

The Continuum Approximation

- "Random lattice" generated via a Poisson process

$$P_V(N) \equiv \frac{1}{N!} \exp^{-\rho V} (\rho V)^N, \quad < N >= \rho V$$



Can this resolve Riemann's dilemma? – Yes!

– Bombelli, Henson and Sorkin (2009)

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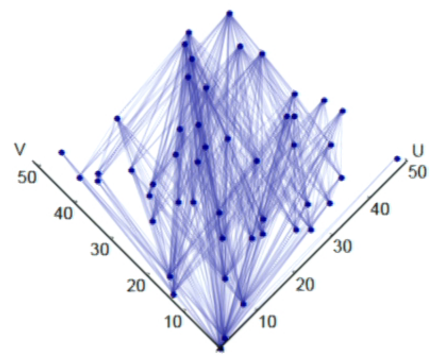

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Lorentz Invariance of a Sprinkling – – L.Bombelli, J.Henson, R. Sorkin (2009)

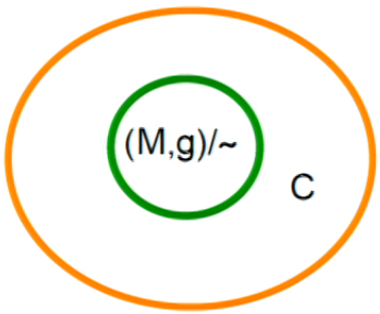
- Ω : space of all sprinklings into $\mathbb{M}^n$
- Poisson process gives a measure μ on Ω which is volume preserving and hence Lorentz invariant.
- Set of all timelike directions forms a unit hyperboloid $H \subset \mathbb{M}^n$
- There is no measurable map $D : \Omega \to H$ which is equivariant, i.e., $D \circ \Lambda = \Lambda \circ D$.

  (Proof: If such a map existed, then $\mu_D \equiv \mu \circ D^{-1}$ is a Lorentz invariant probability measure on H which is not possible since H is non-compact.)

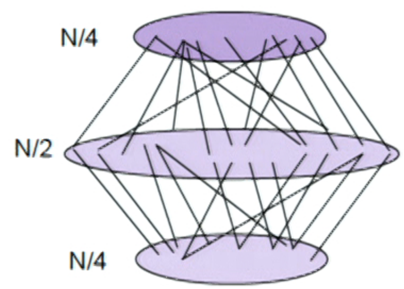
CST: Framework

- (M, g) replaced by locally finite partially ordered sets $C \in \Omega$

$$Z = \sum_M \int D[g]\, e^{\frac{i}{\hbar} S_{EH}[g]} \quad \rightarrow \quad Z = \sum_{C \in \Omega} e^{\frac{i}{\hbar} S[C]}$$



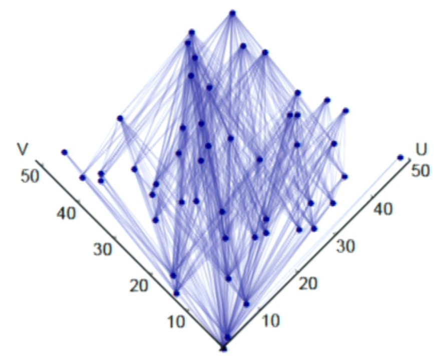
(c)



(d) Typical Causal Set $\sim 2^{\frac{N^2}{4}}$

- Continuum can arise only as an approximation not a limit

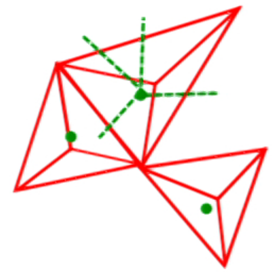
Geometric Reconstruction: Where is Spacetime Hidden in the Order?



Geometric Reconstruction: Where is Spacetime Hidden in the Order?

Simplicial Decomposition of d dimensional spacetime (M, g)

- Triangulate with d-dimensional simplices.
- Fixed valency dual graph

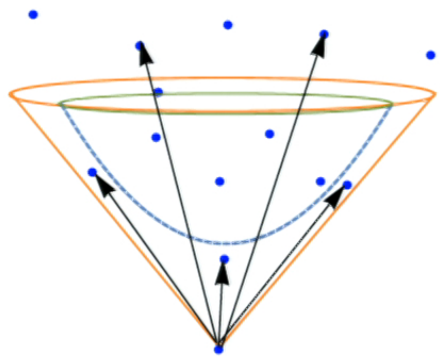


- Dimension, topology easy to extract.
- Geometry: via the Regge Action

Local

Geometric Reconstruction: Where is Spacetime Hidden in the Order?

- A causal set need not be a fixed valency graph.
- There can be an infinite number of nearest neighbours.



Non-Local

Topology and Geometry From Order: (M, g) from C

- Spacetime Dimension: Myrheim (1978), Myer(1988), Sorkin (1987), Glaser and Surya (2013), Eichhorn, Mizera, Surya (2017)

  Myrheim-Myer Estimator: $R = N^2 \frac{\Gamma(d+1)\Gamma(\frac{d}{2})}{4\Gamma(\frac{3d}{2})}$, $R = |\{(x, y)|x \prec y\}|$
- Spatial Homology : Major, Rideout and Surya (2005,2006,2009)
- Timelike Distance: Brightwell and Gregory (1991)
- Spacelike and Spatial Distance: Rideout and Wallden (2009), Eichhorn, Mizera and Surya (2017)
- D'Alembertian, Scalar Curvature and Action: Henson (2006), Sorkin (2007), Benincasa and Dowker (2010), Dowker and Glaser (2013)

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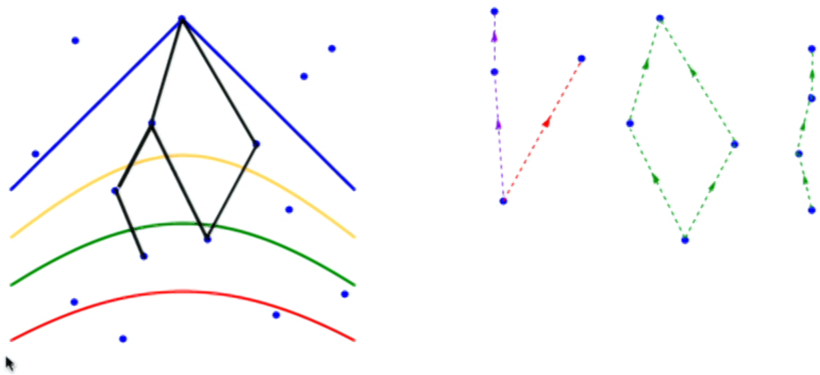

Benincasa-Dowker Action: $\frac{1}{\hbar}S(c) = 4\left(N - 2N_0 + 4N_1 - 2N_2\right)$ – recovers locality!

Topology and Geometry From Order: (M, g) from C

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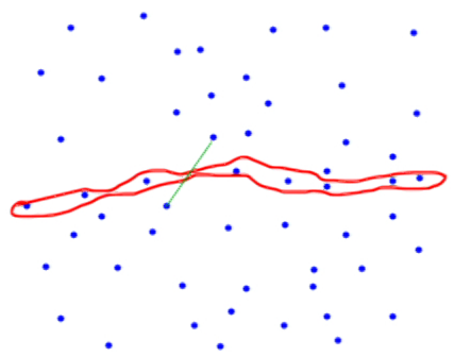
- GHY boundary terms: Buck, Dowker, Jubb and Surya (2015)
- Greens Functions for Scalar fields: Johnston (2008,2009), Ahmed, Dowker and Surya (2017)

Observables or Beables

- Observables are Order Invariants: Label invariance $\sim$ Covariance
- Observables are fundamentally spacetime in character
- No Cauchy hypersurfaces



Fork in the road: the Path Integral

Causal Set Dynamics

- Sequential Growth Dynamics — Rideout and Sorkin (2000,2001)
  - Initial conditions are natural
  - Observables or Beables correspond to properties of causal sets in an event algebra $(\Omega, \mathfrak{A})$: — Brightwell, Dowker, Garcia, Henson and Sorkin (2003), Dowker and Surya (2006)
  - Challenge to formulate a quantum dynamics — Sorkin and Surya (in preparation)

Causal Set Dynamics

- Path Sum Approaches

$$Z = \sum_{c \in \Omega} \exp^{\frac{i}{\hbar} S(c)}$$

  - Suppression of Bilayer orders: Carlip and Loomis (2017)
  - Ω=set of all N-element causal sets Henson, Rideout, Sorkin and Surya (2016)
  - Ω=set of all N element $2d$-causal sets

$$Z_\beta \equiv \sum_{c \in \Omega} \exp^{-\frac{\beta}{\hbar} S(c)}$$

Analytic continuation of a parameter : $i\beta \rightarrow -\beta$

Surya (2011), Glaser and Surya (2015), O'Connor, Glaser and Surya (2017)

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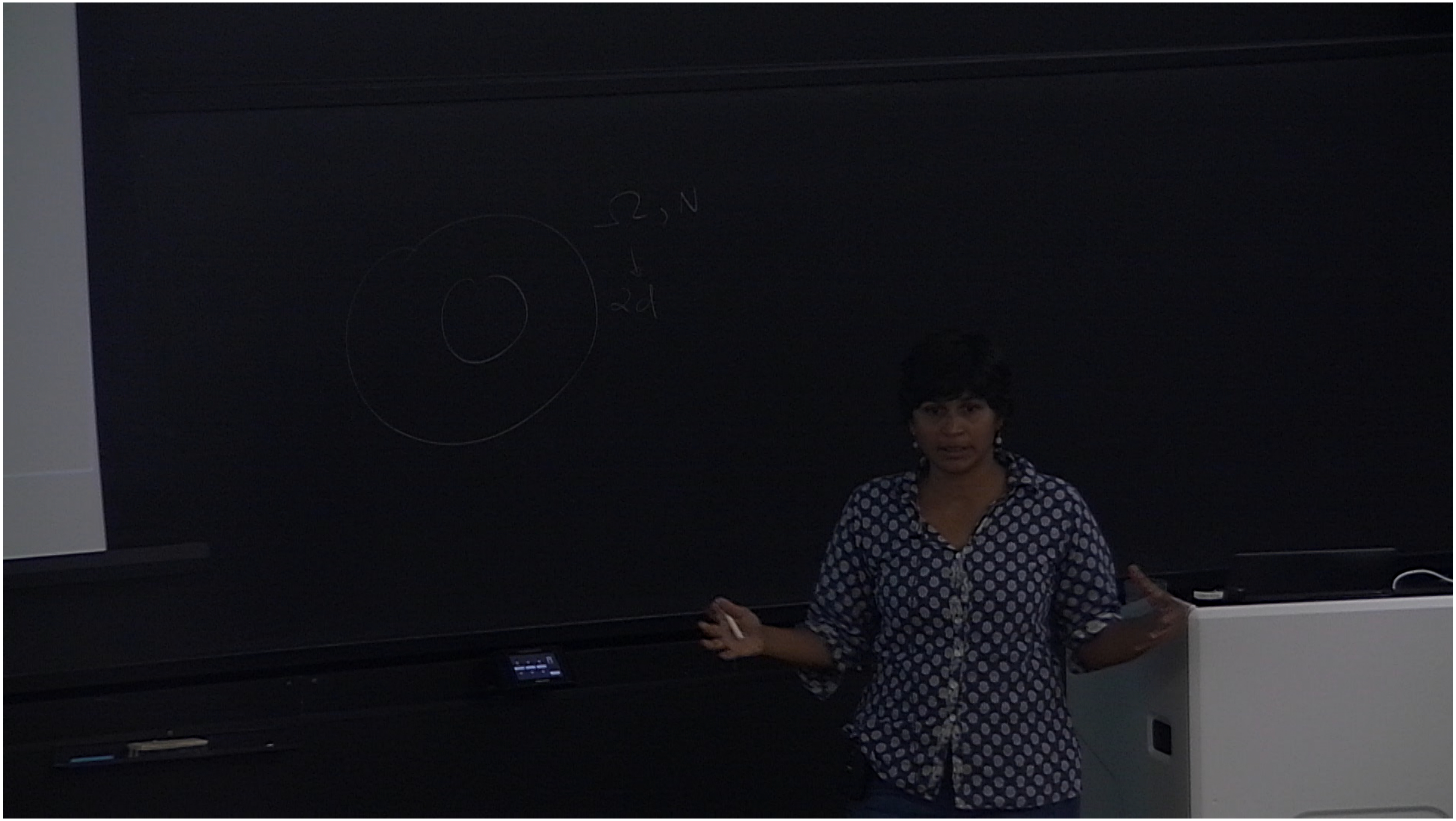
Directions

- Extending Path Sum to Higher dimensions
- Geometric Reconstruction: spacetime topology, etc.
- Quantum Field Theory on Causal Sets
- Quantum Sequential Growth Dynamics
- Phenomenology: Λ, Non-locality, etc.


Ω


Ω, N
2d