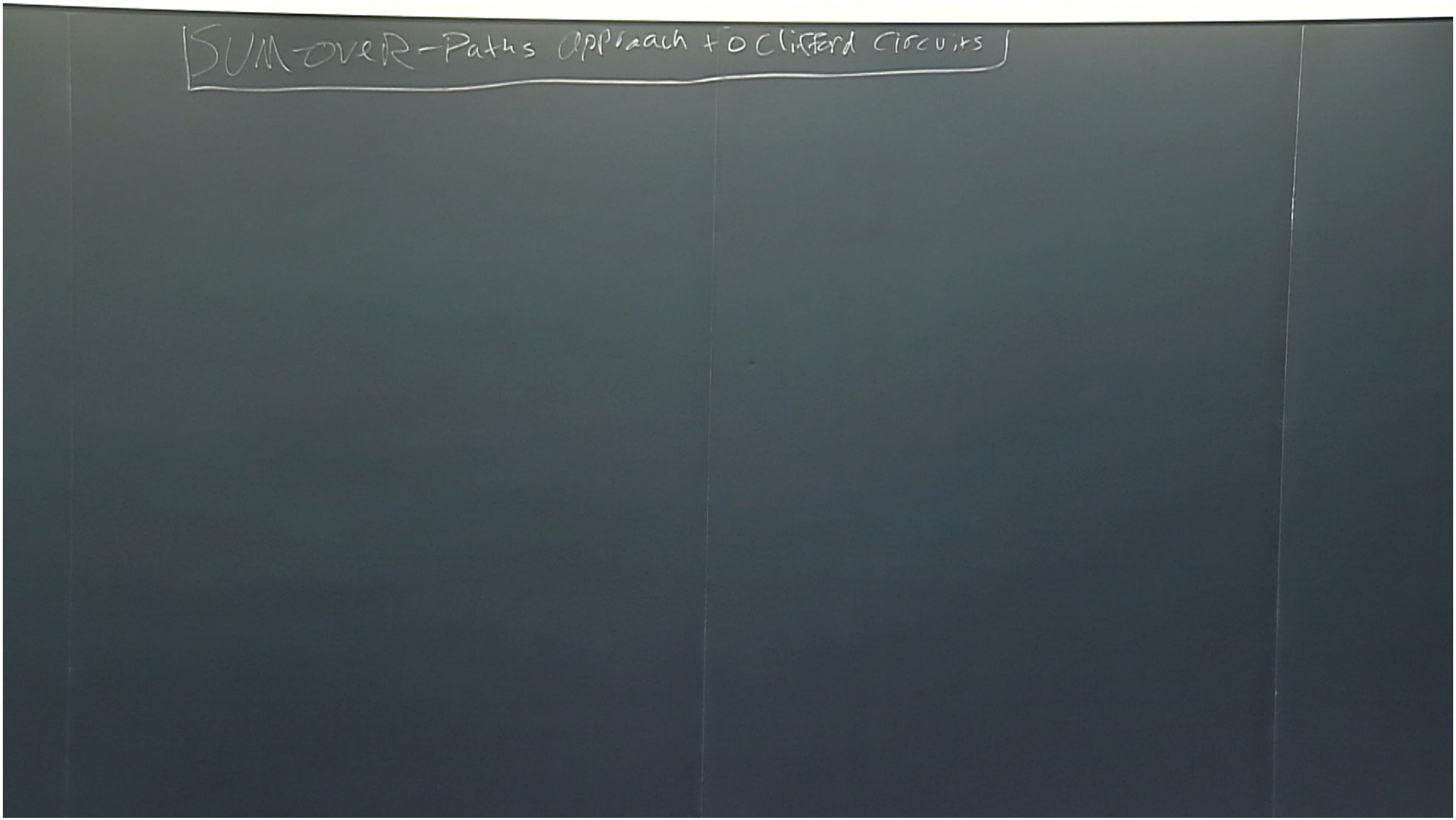
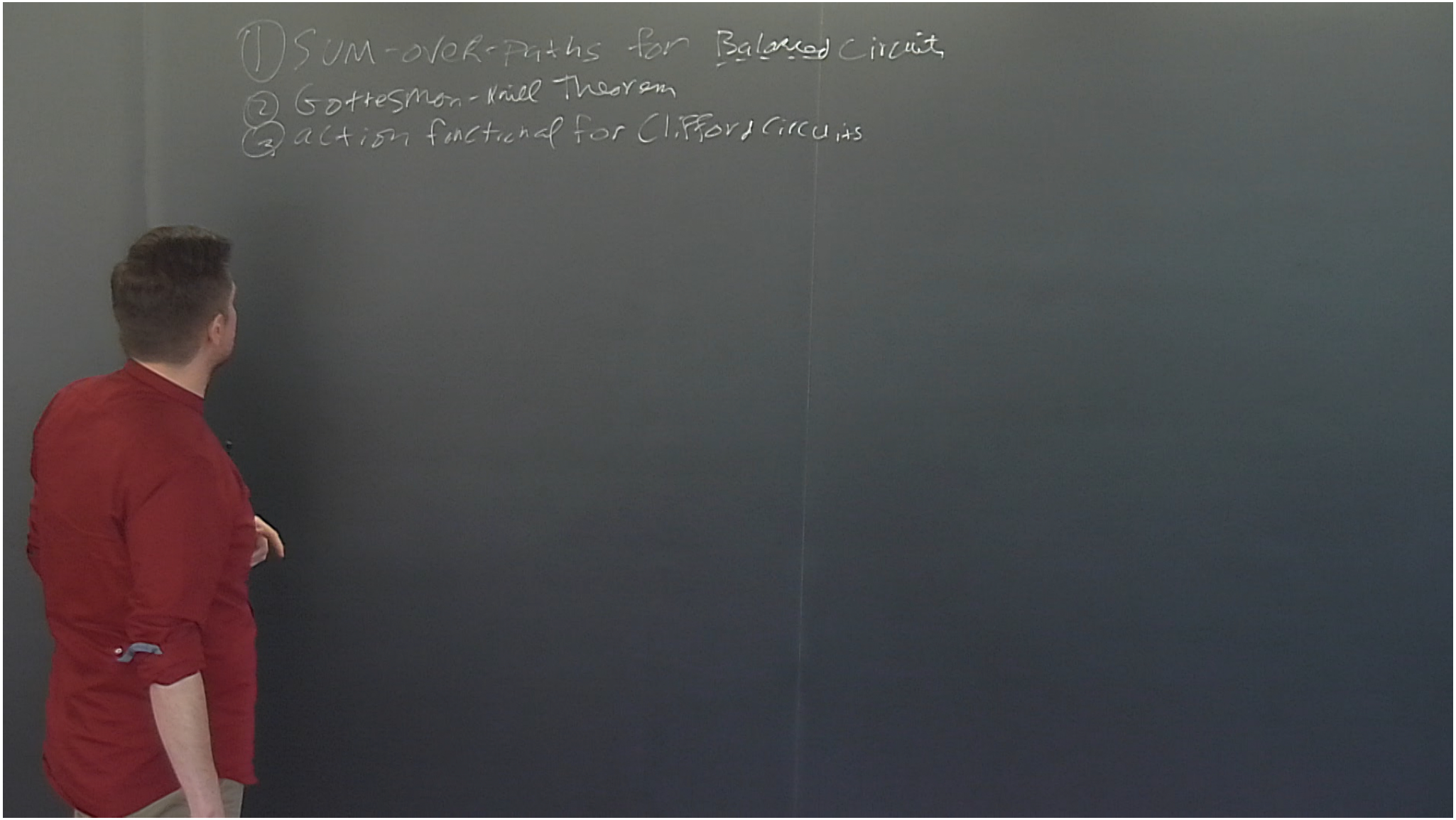
Title: The sum-over-paths technique for Clifford circuits

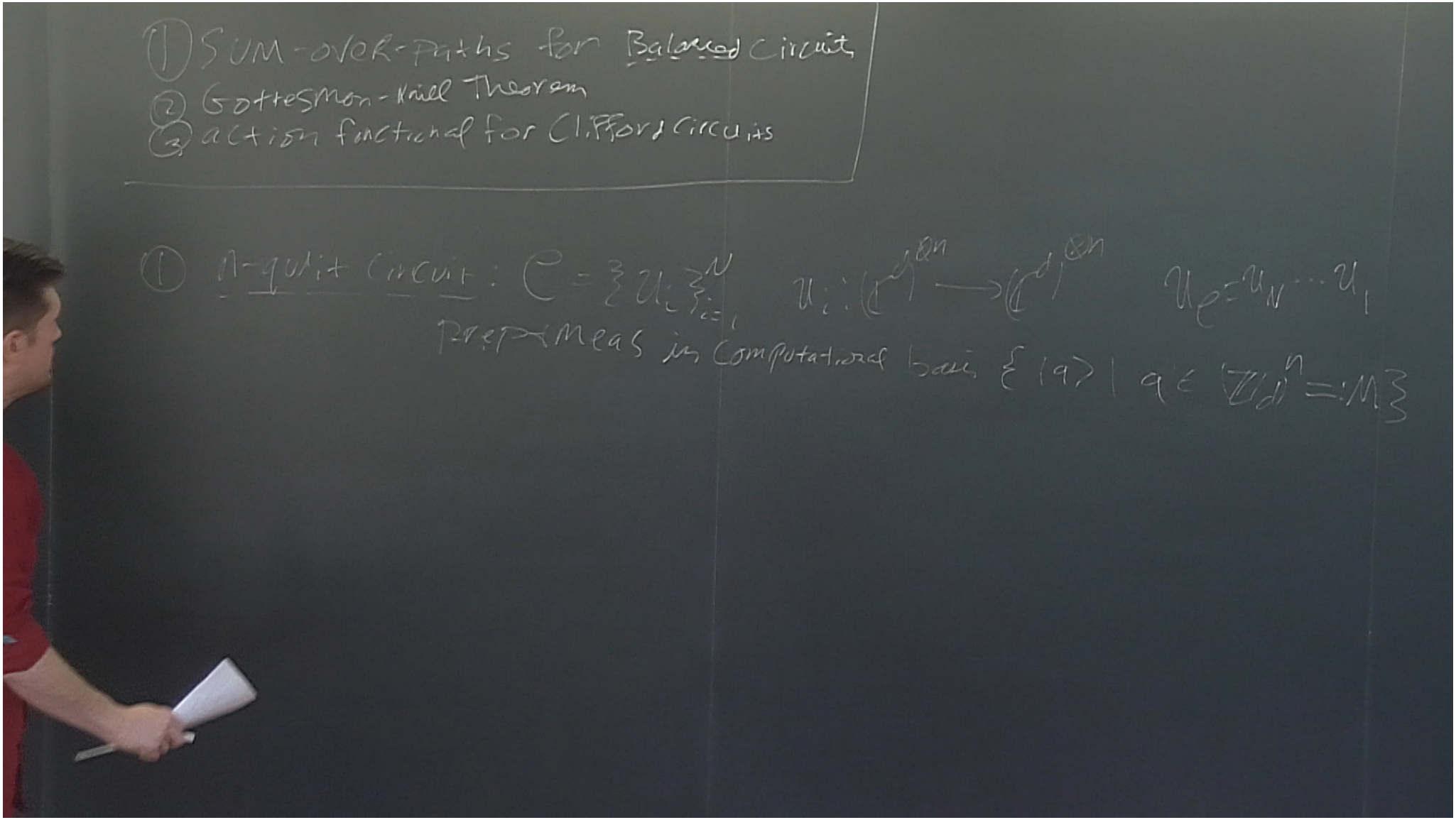
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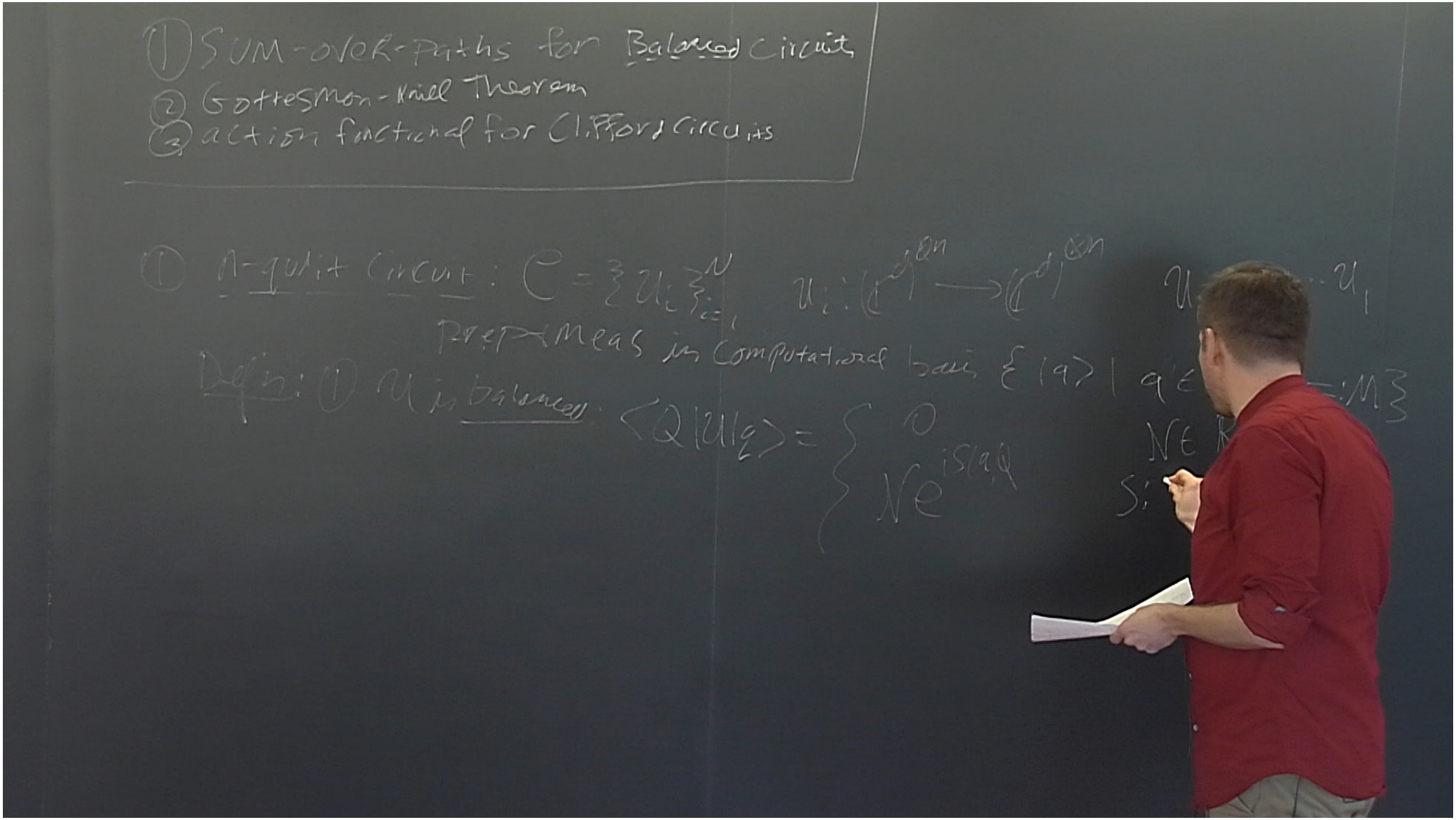
URL: http://pirsa.org/17100062

Abstract: The path integral formulation of quantum mechanics has been immensely influential, particularly in high energy physics. However, its applications to quantum circuits has so far been more limited. In this talk I will discuss the sum-over-paths approach to computing transition amplitudes in Clifford circuits. In such a formulation, the relative phases of different discrete-time paths through the configuration space can be defined in terms of a classical action which is provided by the discrete Wigner representation. As an application of the sum-over-paths technique I will show how to recover a version of the Gottesman-Knill theorem, namely that the transition amplitudes in Clifford circuits can be computed efficiently.


SUM-OVER-Paths approach to Clifford circuits


1) SUM-over-Paths for Balanced Circuits
2) Gottesman-Knill Theorem
3) action functional for Clifford circuits


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2) Gottesman-Knill Theorem
3) action functional for Clifford circuits
1) n-qudit circuit: $\mathcal{C} = \{U_i\}_{i=1}^{N}$, $U_i : (\mathbb{C}^d)^{\otimes n} \to (\mathbb{C}^d)^{\otimes n}$, $U_{\mathcal{C}} = U_N \cdots U_1$
Prep-Meas in Computational basis $\{|q\rangle \mid q \in \mathbb{Z}_d^n =: M\}$


1) SUM-over-Paths for Balanced Circuits
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Prep → Meas in Computational basis $\{|q\rangle \mid q \in$
Defn: 1) U_f is balanced: $\langle Q|U|q\rangle = \begin{cases} 0 \\ Ne^{is(q)} \end{cases}$
$U = U_N \cdots U_1$
$N \in \mathbb{R}$
$S:$

1. SUM-over-Paths for Balanced Circuits
2. Gottesman-Knill Theorem
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Prep: $|0\rangle$, Meas in computational basis $\{|q\rangle \mid q \in \mathbb{Z}_d^n =: M\}$

Def'n: U is balanced: $\langle Q|U|0\rangle = \begin{cases} 0 \\ Ne^{iS(q)} \end{cases}$ $N \in \mathbb{R}_{>0}$, $S : M \to \mathbb{R}$

1. SUM-over-Paths for Balanced Circuits
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Prep-Meas in computational basis $\{|q\rangle \mid q \in (\mathbb{Z}_d)^n =: M\}$

Defn: 0. $U_{\mathcal{C}}$ is balanced: $\langle Q | U_{\mathcal{C}} | q \rangle = \begin{cases} 0 \\ N e^{i S(q,Q)} \end{cases}$ $N \in \mathbb{R}_{>0}$, $S : M^2 \to \mathbb{R}$

e.g. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

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---

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2. C is balanced: U_i balanced

SUM-over-Paths approach to Clifford circuits

Sum-over-Paths: $\langle q_N | u_\ell | q_0 \rangle = \sum_{(q_1, \ldots, q_{N-1}) \in M}$

$= u_N \cdots u_1$

$\int_{\mathbb{R}^n} \cdots =: M \}$

$\to R$

SUM-OVER-Paths approach to Clifford circuits

SUM-over-Paths: $\langle q_N | U_e | q_0 \rangle = \sum_{(q_1, \ldots, q_{N-1}) \in M} \langle q_N | u_N | q_{N-1} \rangle \langle q_{N-1} | u_{N-1} | q_{N-2} \rangle \cdots \langle q_1 | u_1 | q_0 \rangle$

$U = u_N \cdots u_1$

$\mathbb{C}^{d^n} =: M$

$R > 0$

$\to R$

SUM-over-Paths approach to Clifford circuits

SUM-over-Paths: $\langle q_N | U_e | q_0 \rangle = \sum_{(q_1, \ldots, q_{N-1}) \in M} \langle q_N | u_N | q_{N-1} \rangle \langle q_{N-1} | u_{N-1} | q_{N-2} \rangle \cdots \langle q_1 | u_1 | q_0 \rangle$

$P_N(q_0, q_N) = \{q_0\} \times M^{N-1} \times \{q_N\}$

$= \sum_{\gamma \in P_N(q_0, q_N)} A(\gamma)$

SUM-over-Paths approach to Clifford circuits

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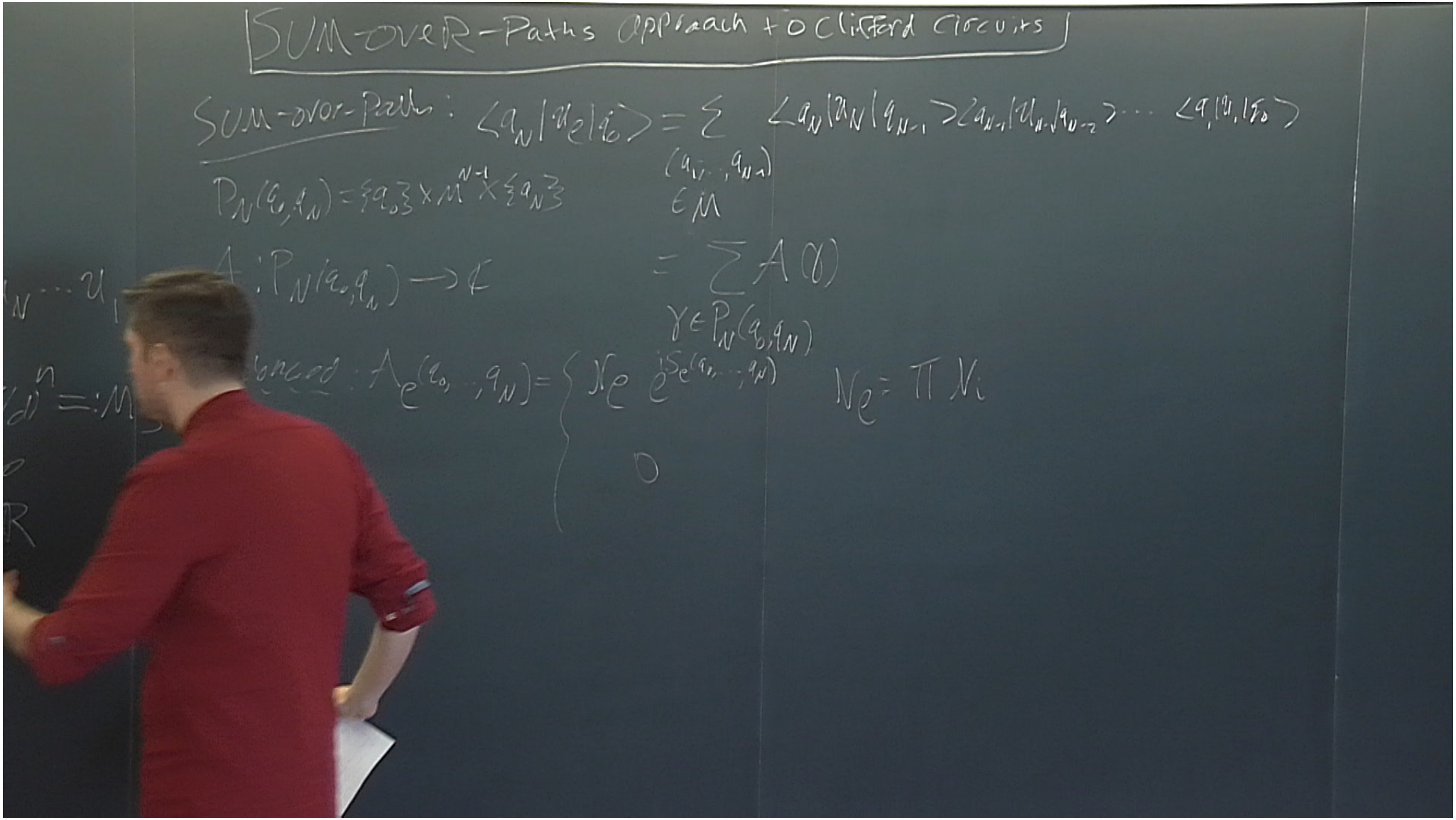
$A : P_N(q_0, q_N) \to \mathbb{C}$

$= \sum_{\gamma \in P_N(q_0, q_N)} A(\gamma)$

$\ldots u_1$

$d^n =: M\}$

R


SUM-OVER-Paths approach to Clifford circuits
SUM-over-Paths: $\langle q_N|U_C|q_0\rangle = \sum_{(q_1,\dots,q_{N-1})\in M} \langle q_N|u_N|q_{N-1}\rangle\langle q_{N-1}|U_{N-1}|q_{N-2}\rangle\cdots\langle q_1|u_1|q_0\rangle$
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$A: P_N(q_0,q_N)\to\mathbb{C}$
$= \sum_{\gamma\in P_N(q_0,q_N)} A(\gamma)$
Lemma: $A_C(q_0,\dots,q_N) = \begin{cases} N_C\, e^{iS_C(q_0,\dots,q_N)} \\ 0 \end{cases}$
$N_C = \prod N_i$

SUM-over-Paths approach to Clifford circuits

SUM-over-Paths: $\langle q_N | U_C | q_0 \rangle = \sum_{(q_1, \ldots, q_{N-1}) \in M} \langle q_N | U_N | q_{N-1} \rangle \langle q_{N-1} | U_{N-1} | q_{N-2} \rangle \cdots \langle q_1 | U_1 | q_0 \rangle$

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Balanced: $A_C(q_0, \ldots, q_N) = \begin{cases} N_C \, e^{i S_C(q_0, \ldots, q_N)} \\ 0 \end{cases}$

$N_C = \prod N_i$

$S_C(q_0, \ldots, q_N) = \sum S_i(q_{i-1}, q_i)$

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$S_C(q_0, \ldots, q_N) = \sum_i S_i(q_{i-1}, q_i)$

$\langle q_N | U_C | q_0 \rangle = N_C \sum_{\gamma \in P_C(q_0, q_N)} e^{i \xi(\gamma)}$

$P_C(q_0, q_N) = \{\gamma \in P_N(q_0, q_N) \mid A(\gamma) \neq 0\}$

SUM-OVER-Paths approach to Clifford circuits

SUM-over-Paths: $\langle q_N | U_C | q_0 \rangle = \sum_{(q_1, \ldots, q_{N-1}) \in M} \langle q_N | U_N | q_{N-1} \rangle \langle q_{N-1} | U_{N-1} | q_{N-2} \rangle \cdots \langle q_1 | U_1 | q_0 \rangle$

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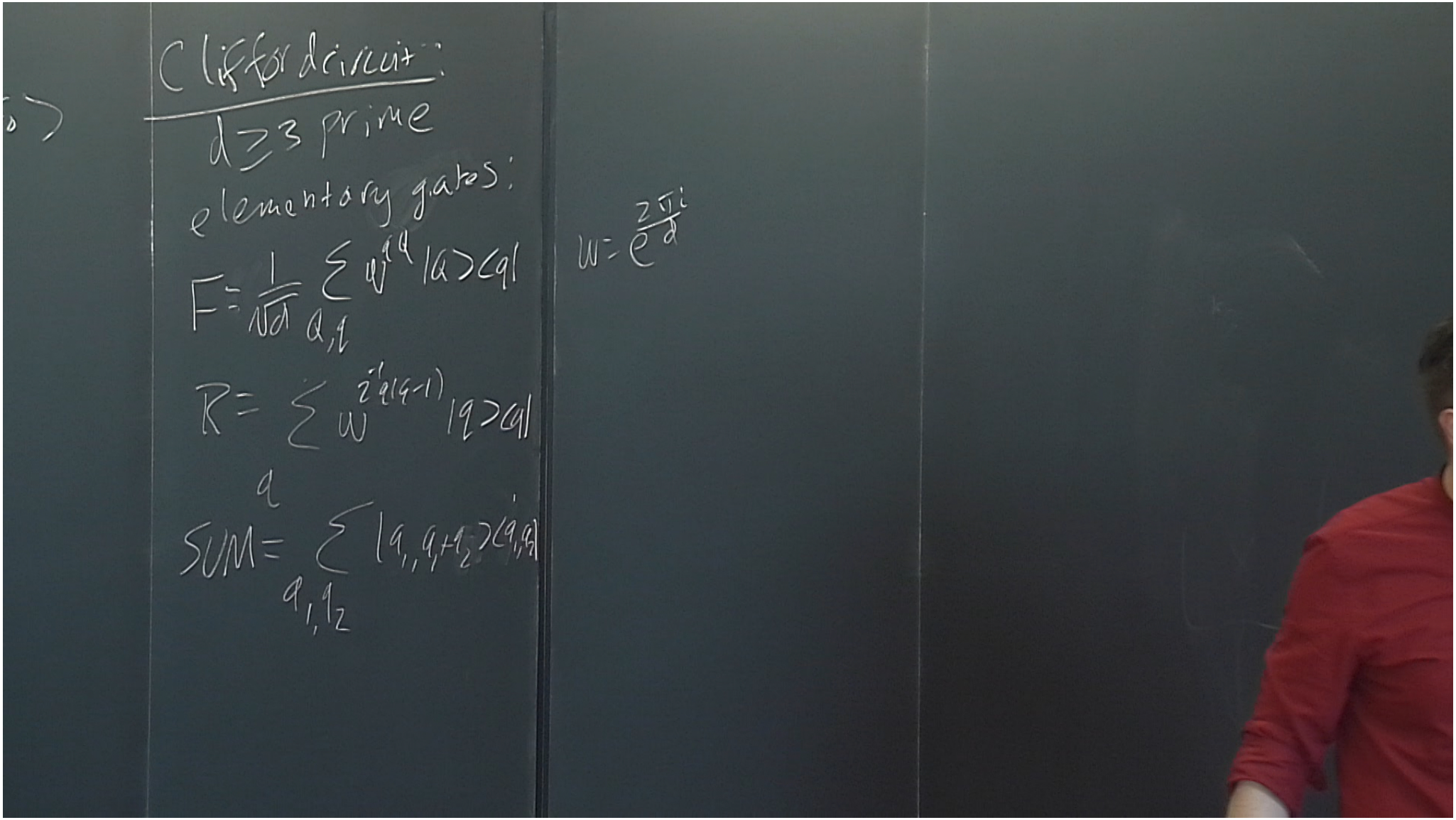
$N_C = \prod N_i$

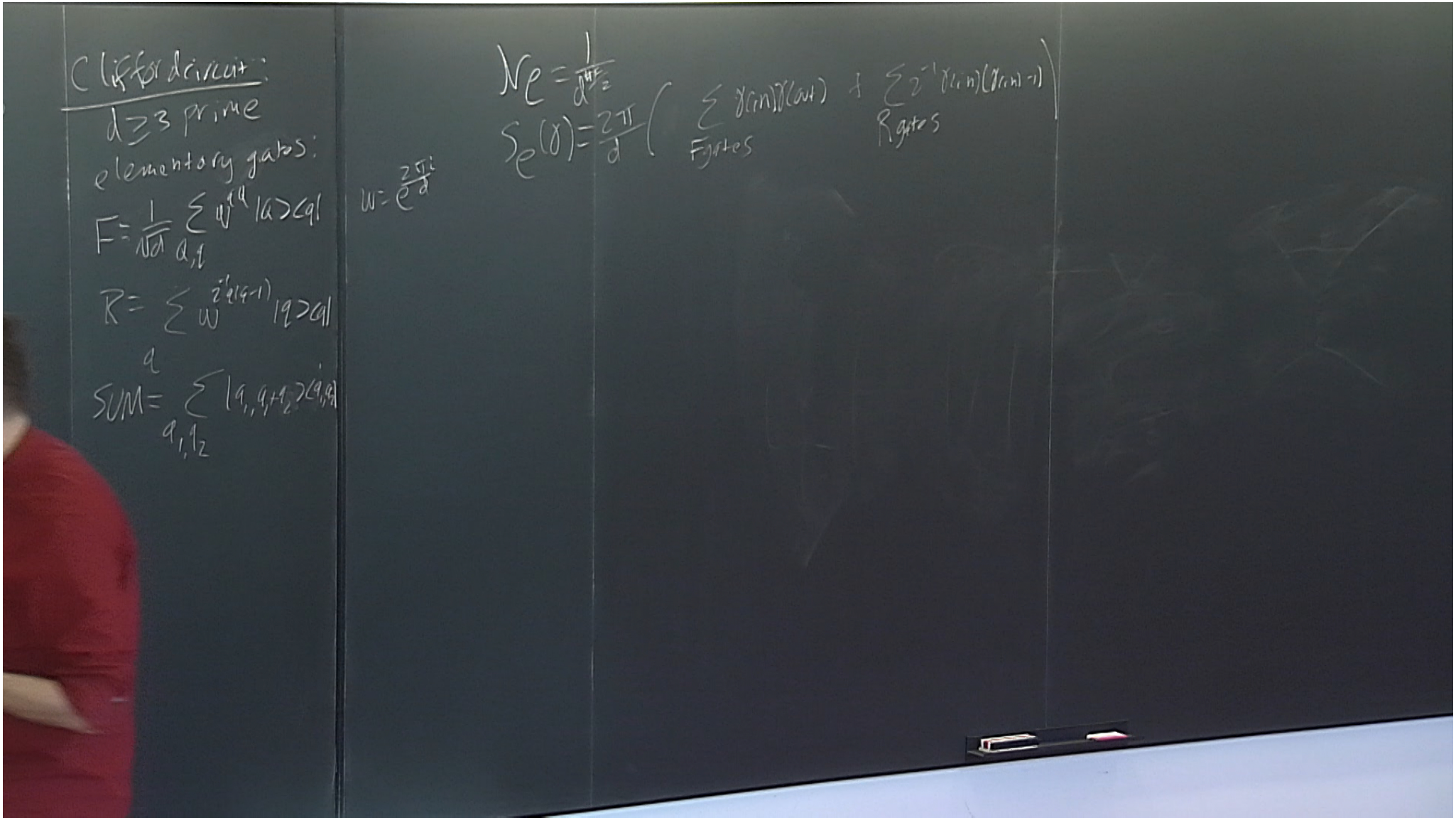
$S_C : P_N(q_0, q_N) \to \mathbb{R}$

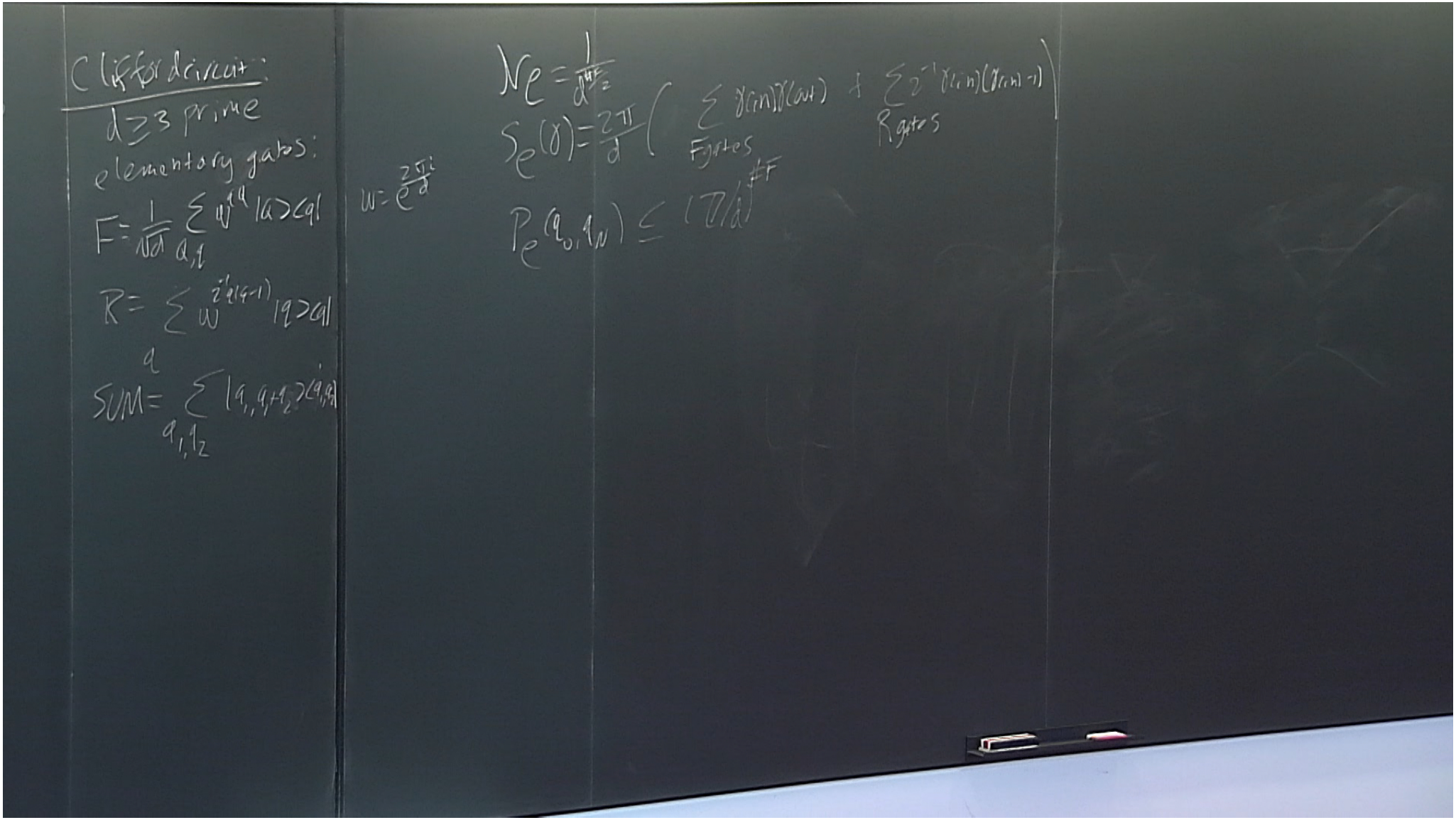
$S_C(q_0, \ldots, q_N) = \sum S_i(q_{i-1}, q_i)$

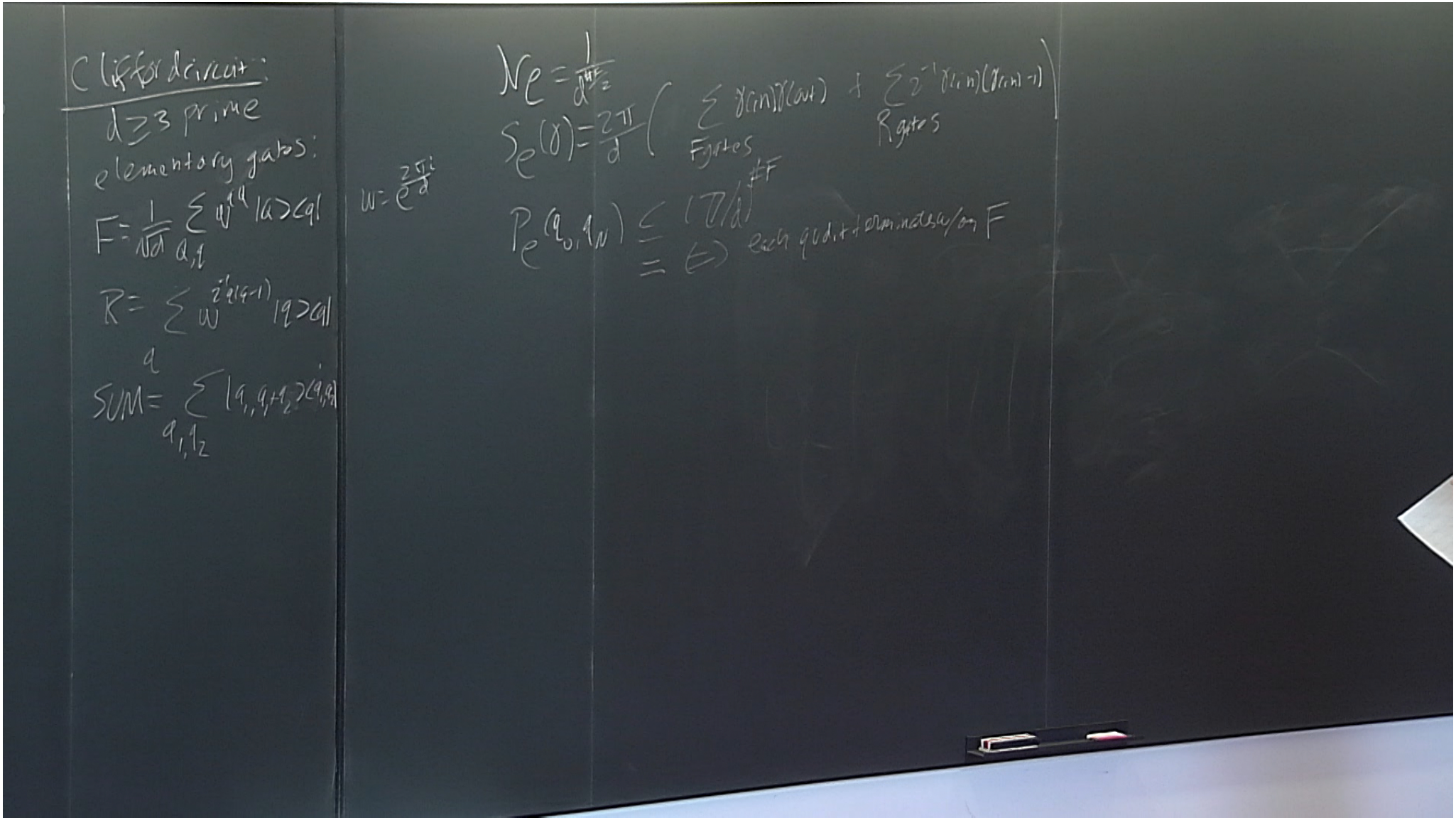
$\langle q_N | U_C | q_0 \rangle = N_C \sum_{\gamma \in P_C(q_0, q_N)} e^{i \xi(\gamma)}$

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Clifford circuit:
$d \geq 3$ prime
elementary gates:
$F = \frac{1}{\sqrt{d}} \sum_{q,t} w^{qt} |q\rangle\langle q|$
$w = e^{\frac{2\pi i}{d}}$
$R = \sum_q w^{z^{-1} q(q-1)} |q\rangle\langle q|$
$SUM = \sum_{q_1,q_2} |q_1, q_1+q_2\rangle\langle q_1, q_2|$


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$F = \frac{1}{\sqrt{d}} \sum_{q,l} w^{ql} |q\rangle\langle l|$
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$SUM = \sum_{q_1,q_2} |q_1, q_1+q_2\rangle\langle q_1, q_2|$
$w = e^{\frac{2\pi i}{d}}$
$N_e = \frac{1}{d^{n_F/2}}$
$S_e(\gamma) = \frac{2\pi}{d}\left(\sum_{F\text{gates}} \gamma(in)\gamma(out) + \sum_{R\text{gates}} 2^{-1}\gamma(in)(\gamma(in)-1) \right)$


Clifford circuit:
$d \geq 3$ prime
elementary gates:
$F = \frac{1}{\sqrt{d}} \sum_{q,l} w^{lq} |q\rangle\langle l|$
$R = \sum_q w^{\frac{1}{2}q(q-1)} |q\rangle\langle q|$
$SUM = \sum_{q_1,q_2} |q_1, q_1+q_2\rangle\langle q_1, q_2|$
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$N_\varepsilon = \frac{1}{d^{\#F}}$
$S_\varepsilon(\gamma) = \frac{2\pi}{d}\left(\sum_{\text{F gates}} \gamma(\text{in})\gamma(\text{out}) + \sum_{R \text{ gates}} 2^{-1}\gamma(\text{in})(\gamma(\text{in})-1) \right)$
$P_\varepsilon(q_0, q_N) \leq (\pi/d)^{\#F}$


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$P_\epsilon(q_0, q_N) \leq (\pi/d)^{\#F}$
$\Leftrightarrow$ each qudit terminates on an F

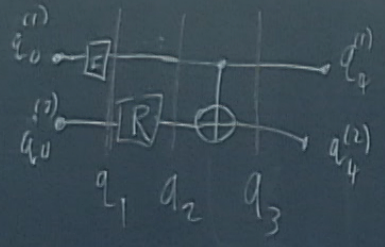
$$N_e = \frac{1}{d^{\#_F}}$$

$$S_e(\gamma) = \frac{2\pi}{d}\left(\sum_{F\text{ gates}} \gamma(in)\gamma(out) + \sum_{R\text{ gates}} 2^{-1}\gamma(in)(\gamma(in)-1) \right)$$

$$P_e(q_0, q_N) \leq (\pi/d)^{\#F}$$

$\Leftrightarrow$ each qudit is diminished on F

e.g.



$$N_e = \frac{1}{d^{\#F/2}}$$

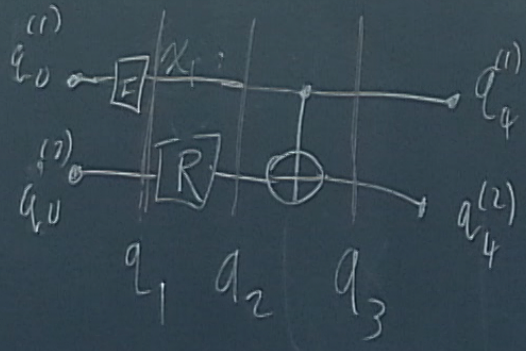
$$S_e(\gamma) = \frac{2\pi}{d}\left(\sum_{F\text{ gates}} \gamma(in)\gamma(out) + \sum_{R\text{ gates}} 2^{-1}\gamma(in)(\gamma(in)-1) \right)$$

$$w = e^{\frac{2\pi i}{d}}$$

$$P_e(q_0, q_N) \leq (\pi/d)^{\#F}$$

$\Leftrightarrow$ each qudit terminates w/ on F

e.g.



$x_1 = q_4^{(1)}$

$x_1 + q_0^{(2)} = q_4^{(2)}$

(2)

$\sum 2^{-1} \gamma(i,n)(\gamma(i,n)-1)$
R gates

ominates w/ on F

(1) $\chi_4 = q_4^{(1)}$

4

(2) $\chi_4 + q_0^{(2)} = q_4^{(2)}$

(2) Gottesman-Knill theorem

$N_e = \frac{1}{d^{\#F/2}}$

$S_e(\gamma) = \frac{2\pi}{d}\left(\sum \gamma(in)\gamma(out) + \sum 2^{-1}\gamma(in)(\gamma(in)-1) \right)$

F gates R gates

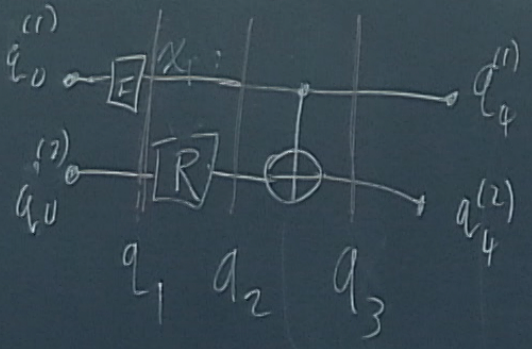
② Gottesman-Π

polynomial

$P_e(q_0, q_N) \leq (\pi/d)^{\#F}$

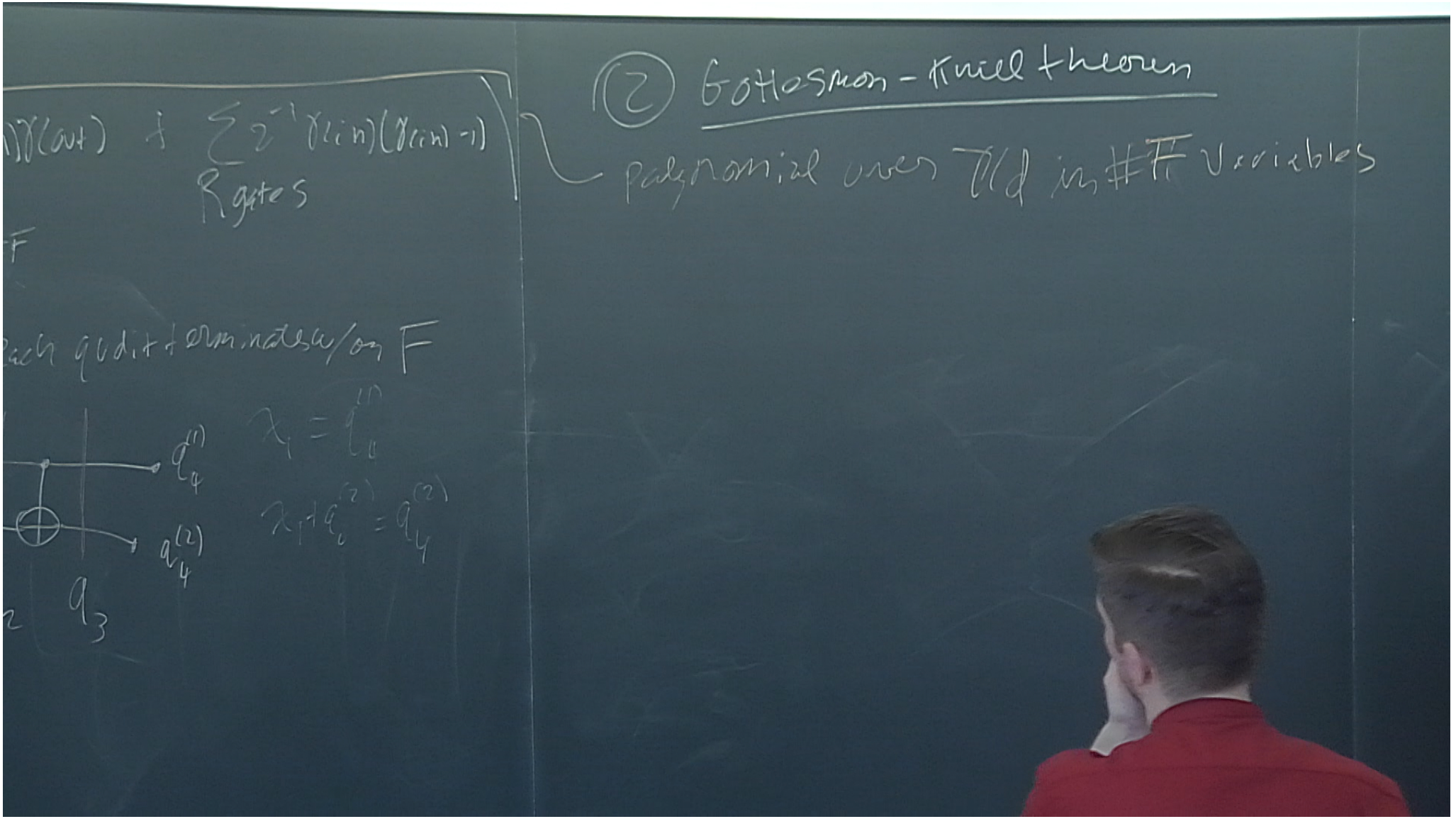
$= \Leftrightarrow$ each qudit terminates on F

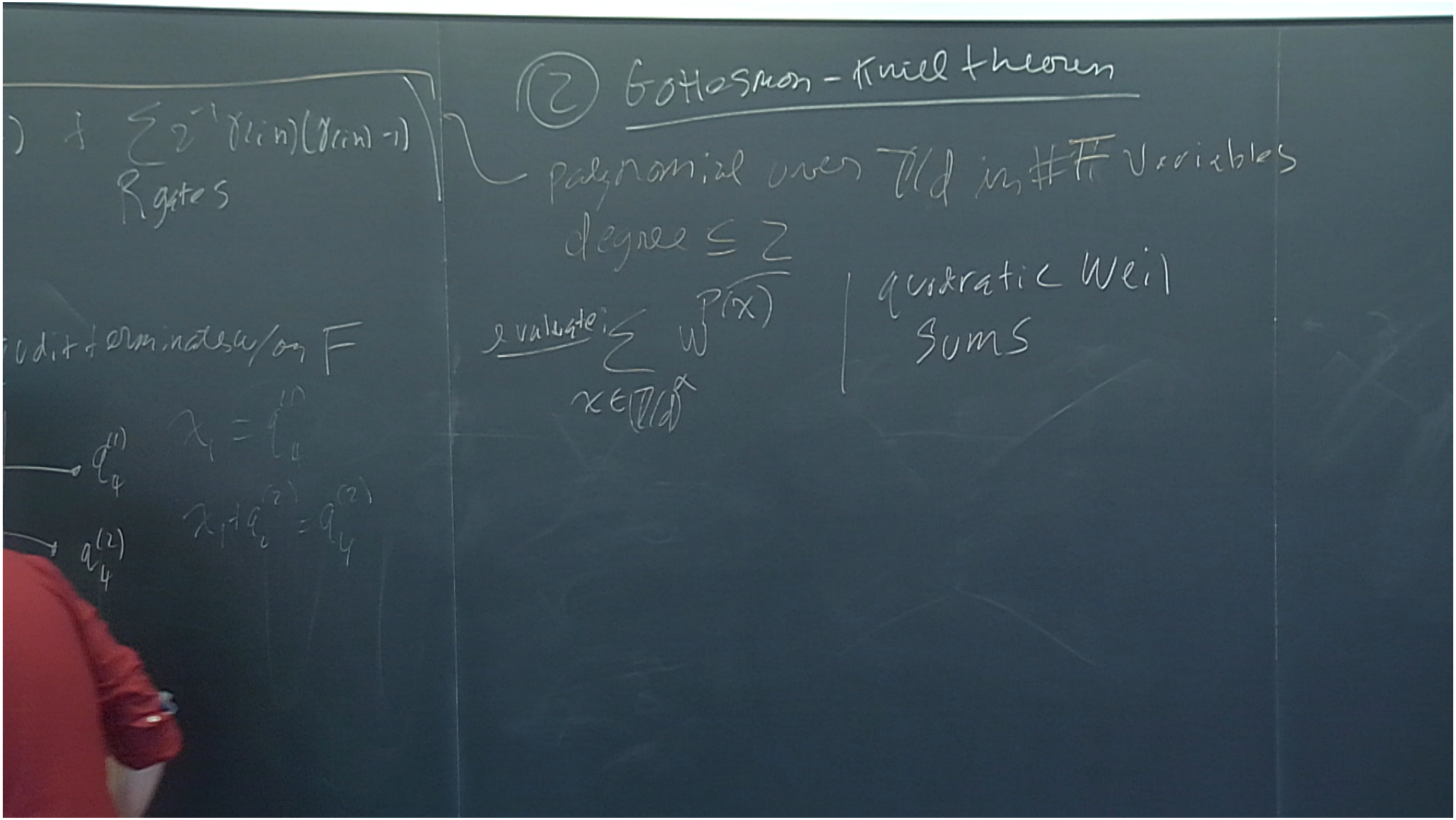
e.g.

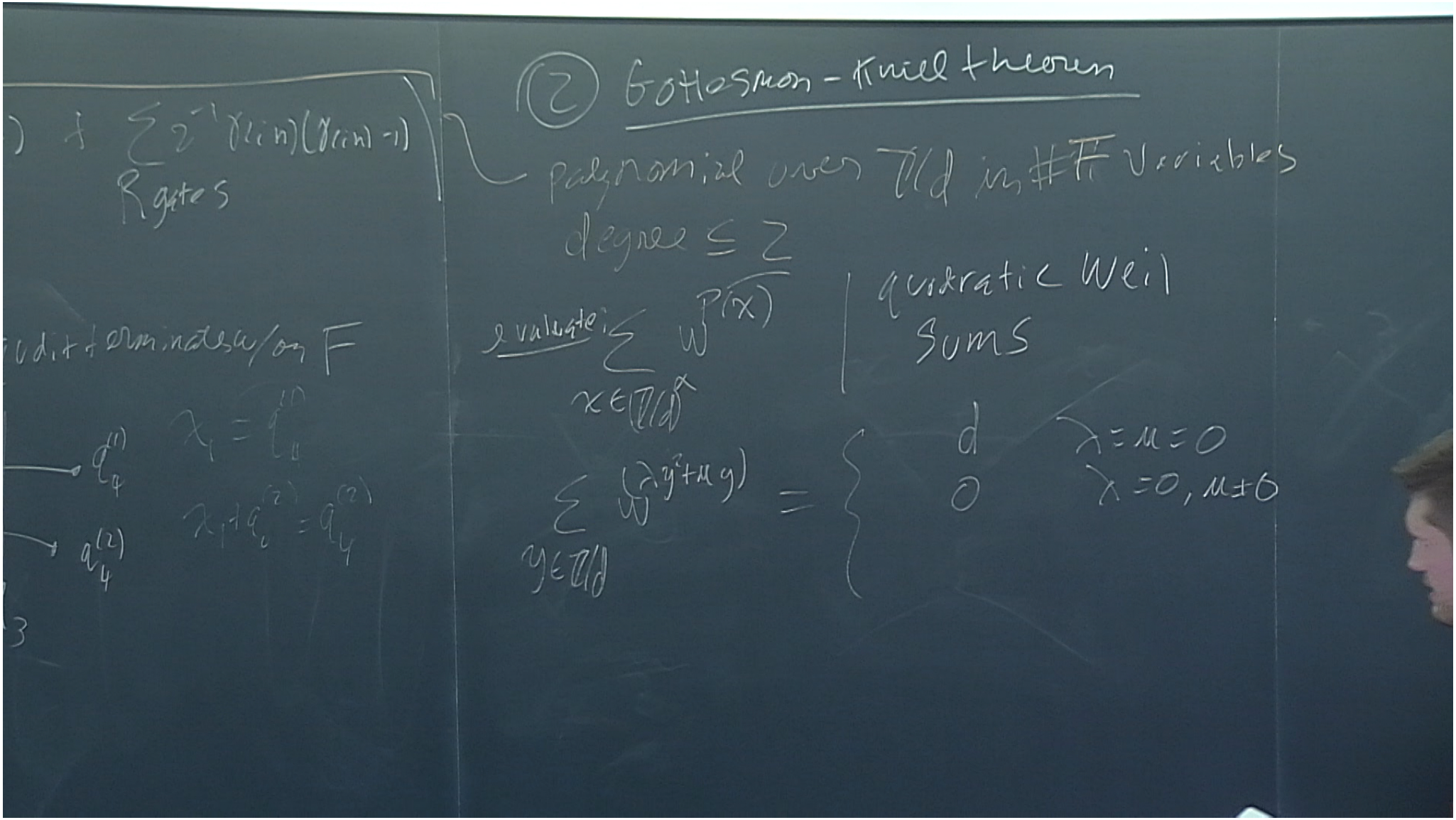


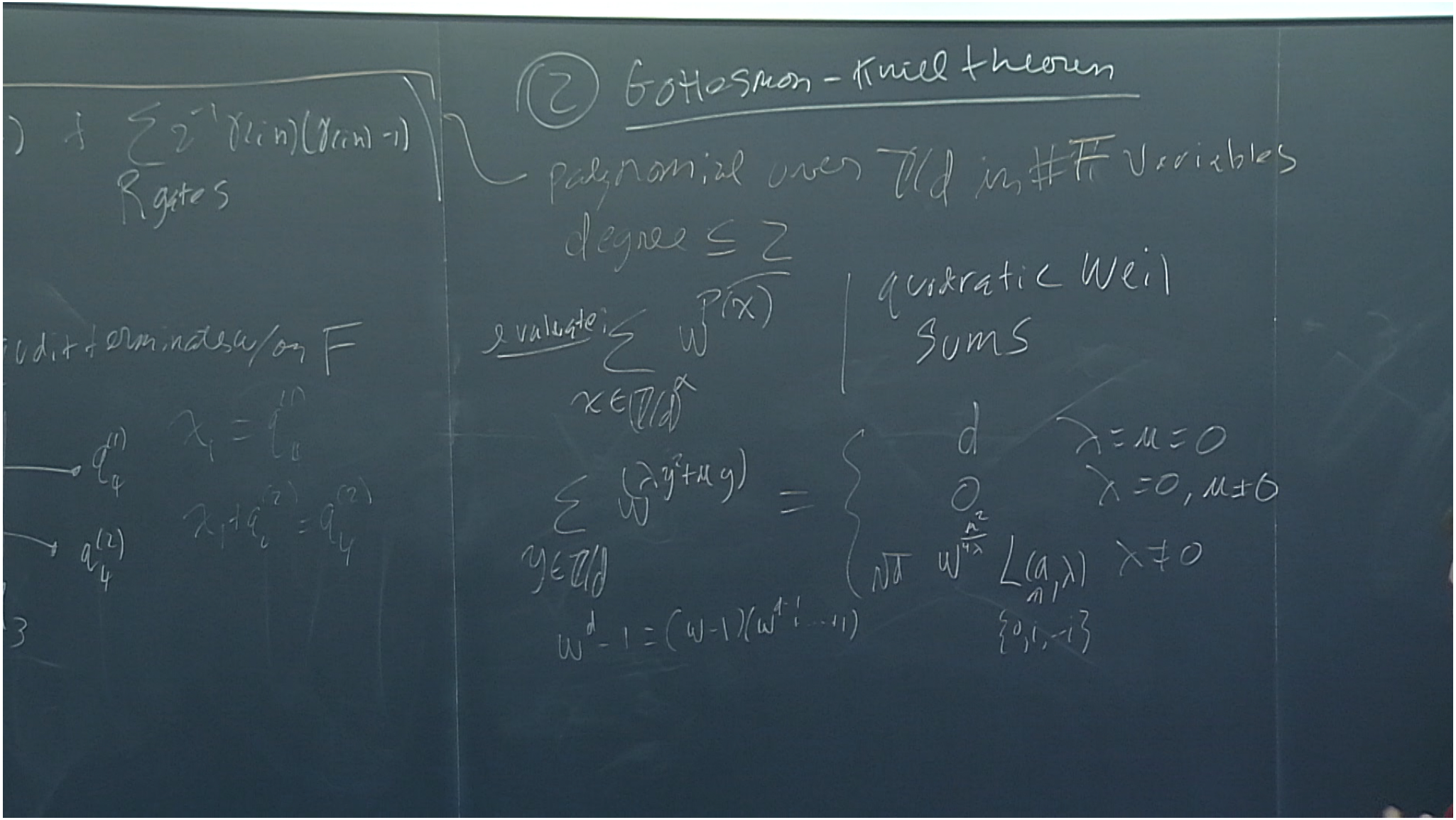
$x_1 = q_4^{(1)}$

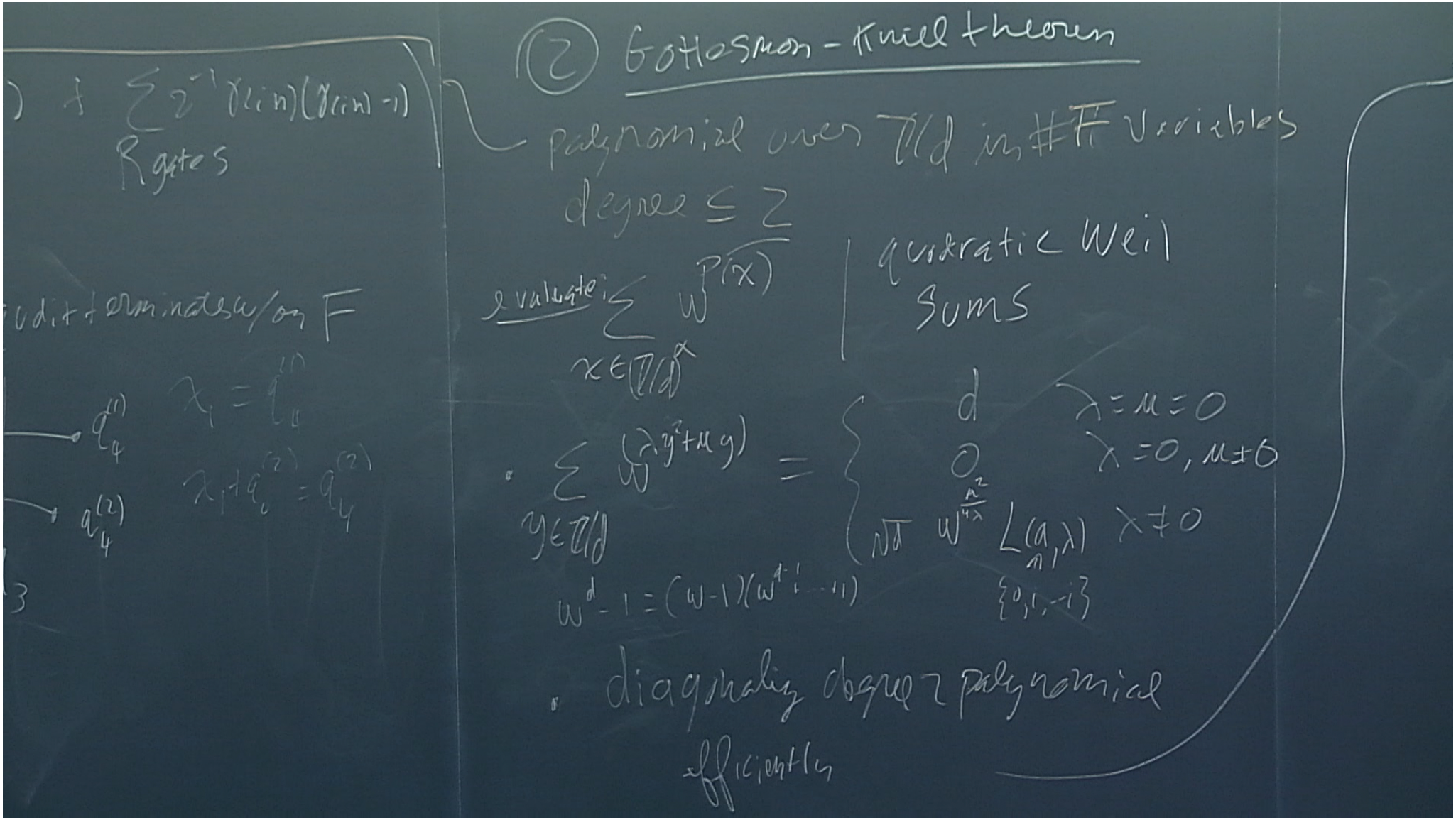
$x_1 + q_0^{(2)} = q_4^{(2)}$

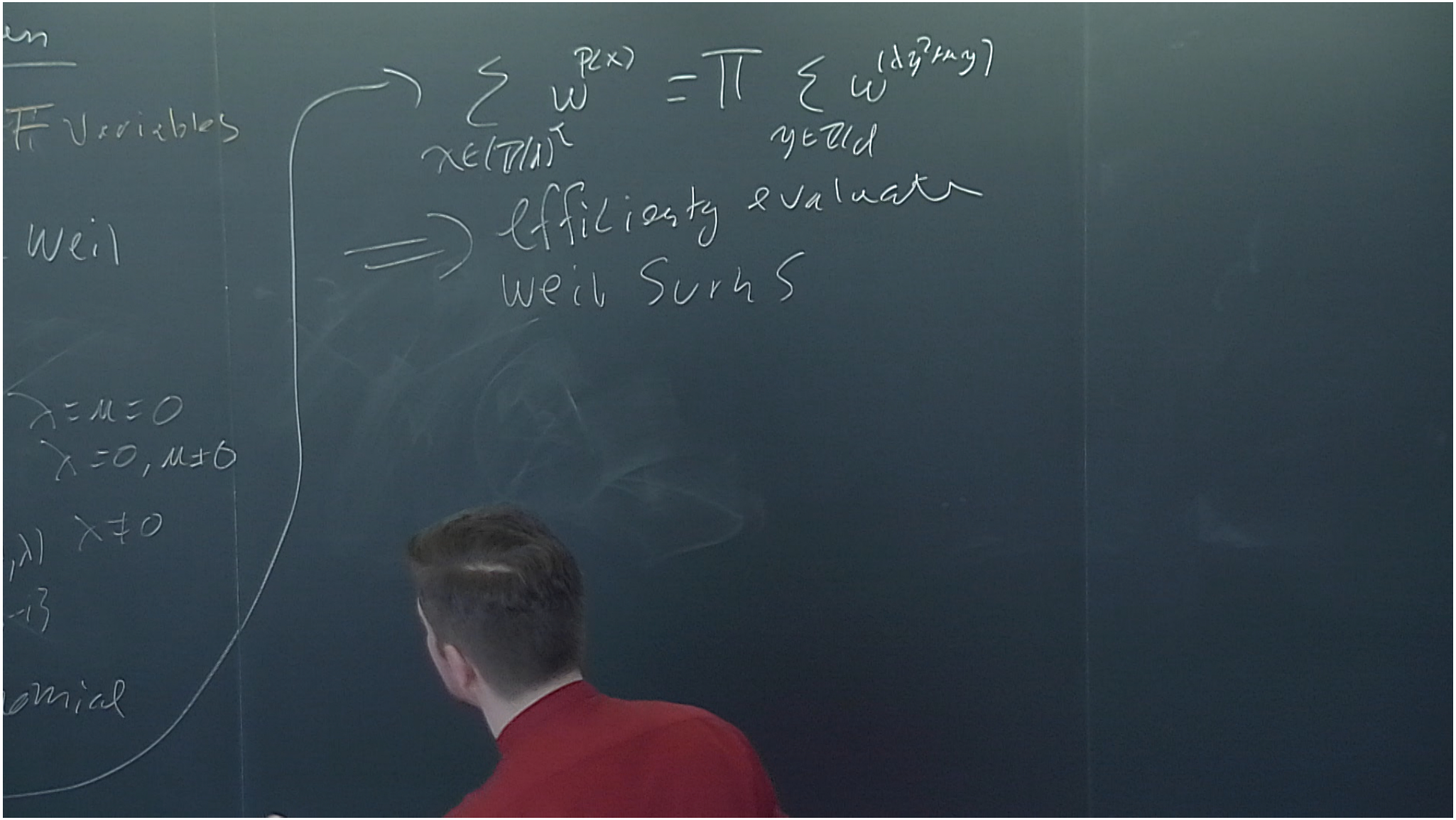

(2) Gottesman-Knill theorem
polynomial over ℤd in #F variables
R gates
F
each qudit terminates on F
$q_4^{(1)}$
$q_4^{(2)}$
q_3
$x_1 = q_0^{(1)}$
$x_1 + q_0^{(2)} = q_4^{(2)}$

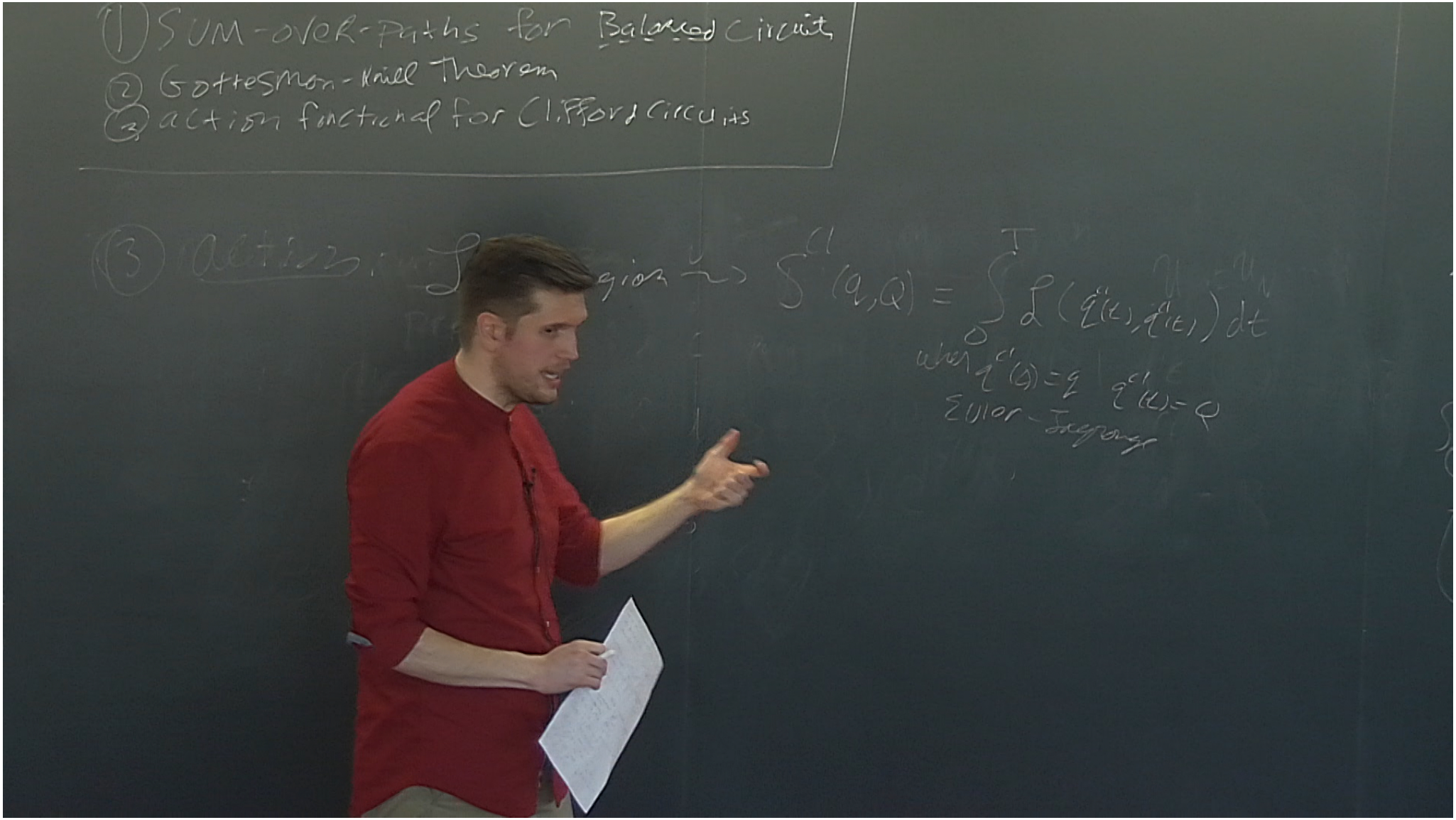

R gates
② Gottesman-Knill theorem
polynomial over ℤ/d in #F variables
degree ≤ 2
evaluate
quadratic Weil sums


R gates
② Gottesman-Knill theorem
polynomial over ℤ/d in #F variables
degree ≤ 2
quadratic Weil sums
evaluate
Σ_{x∈(ℤ/d)^n} ω^{P(x)}
Σ_{y∈ℤ/d} ω^{(λy²+μy)} = { d λ=μ=0 ; 0 λ=0, μ≠0


R gates
(2) Gottesman-Knill theorem
polynomial over $\mathbb{Z}/d$ in #F variables
degree ≤ 2
evaluate: $\sum_{x \in (\mathbb{Z}/d)^n} w^{P(x)}$
quadratic Weil sums
$\sum_{y \in \mathbb{Z}/d} w^{(\lambda y^2 + \mu y)} = \begin{cases} d & \lambda = \mu = 0 \\ 0 & \lambda = 0, \mu \neq 0 \\ \sqrt{d}\, w^{\frac{\mu^2}{4\lambda}} L(a, \lambda) & \lambda \neq 0 \end{cases}$
$w^d - 1 = (w-1)(w^{d-1} + \cdots + 1)$
$\{0, 1, -i\}$


Gottesman-Knill theorem
polynomial over Z/d in #F variables
degree ≤ 2
evaluate
quadratic Weil sums
R gates
audit terminates w/ on F
diagonalizing degree 2 polynomial
efficiently


Variables
Weil
$\chi=\mu=0$
$\chi=0, \mu\neq 0$
$\chi\neq 0$
$\sum_{x\in(\mathbb{F}_q)^\times} w^{P(x)} = \prod_{y\in\mathbb{F}_q} \xi w^{(\deg y)\mu_y}$
$\Longrightarrow$ efficiently evaluate
Weil Sums


1) SUM-over-Paths for Balanced Circuits
2) Gottesman-Knill Theorem
3) action functional for Clifford circuits
3) action:
when q^cl(0) = q q^cl(t) = Q
Euler-Lagrange

1. SUM-over-Paths for Balanced Circuits
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"Total action"

(3) action: $\int$ lagrangian $\rightsquigarrow$ $S^{C}(q,Q) = \int_0^T \mathcal{L}(\dot{q}^C(t), q^C(t))\,dt$

where $q^C(0) = q$ $q^C(t) = Q$

Euler-Lagrange

1. SUM-over-Paths for Balanced Circuits
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"Total action"

(3) action: $\int$ Lagrangian $\rightsquigarrow$ $S^{C'}(q,Q) = \int_0^T L(\dot{q}^{C'}(t), q^{C'}(t))\,dt$

$p = -\frac{\partial S^{C'}}{\partial q}$ $P = \frac{\partial S^{C'}}{\partial Q}$

when $q^{C'}(0) = q$ $q^{C'}(t) = Q$

Euler-Lagrange

1. SUM-over-Paths for Balanced Circuits
2. Gottesman-Knill Theorem
3. action functional for Clifford circuits

(3) action: $\int$ lagrangian $\rightsquigarrow$ $S^{C_1}(q,Q) = \int_0^T \mathcal{L}(\dot{q}^{C_1}(t), q^{C_1}(t))\,dt$

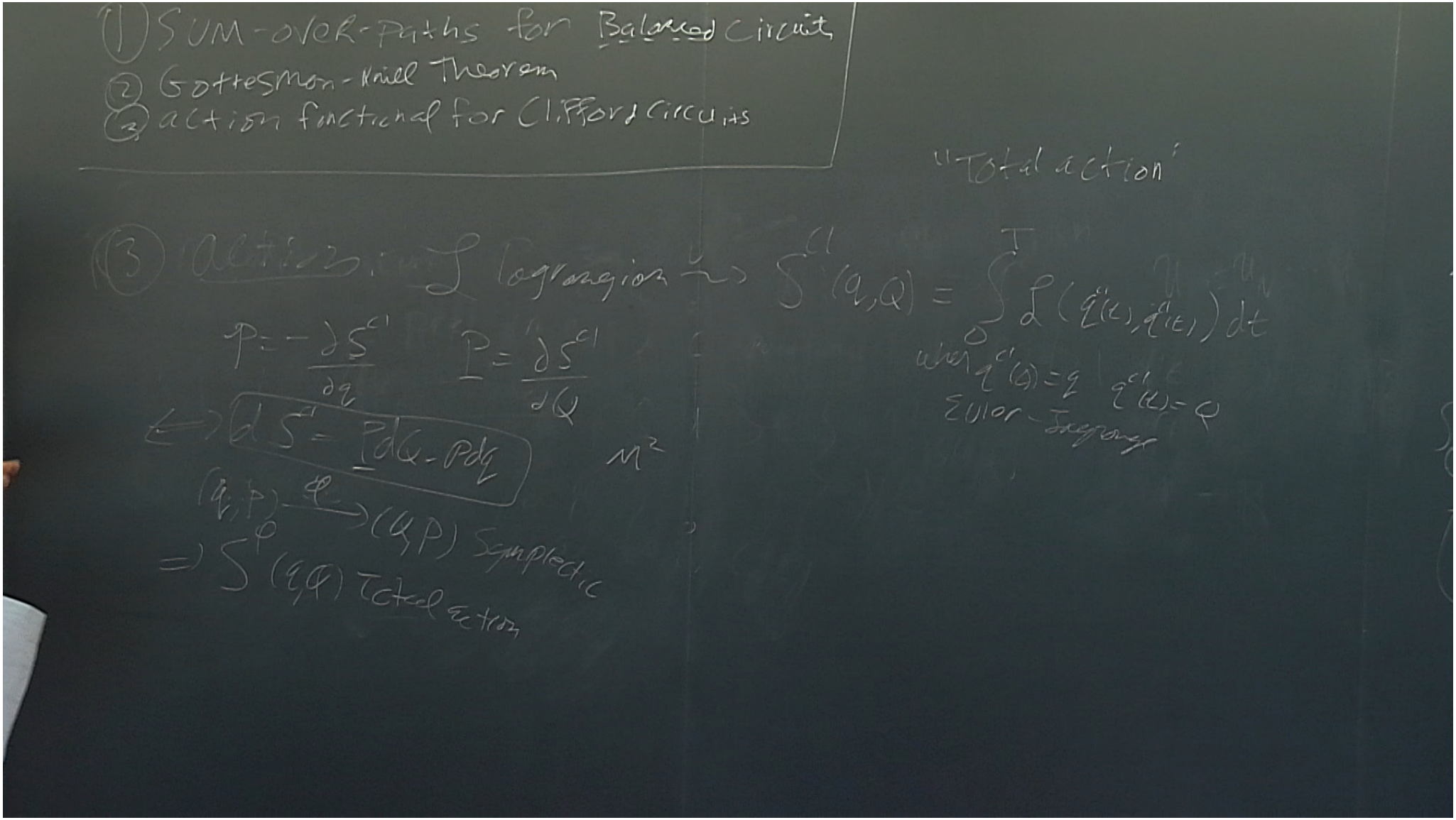
"Total action"

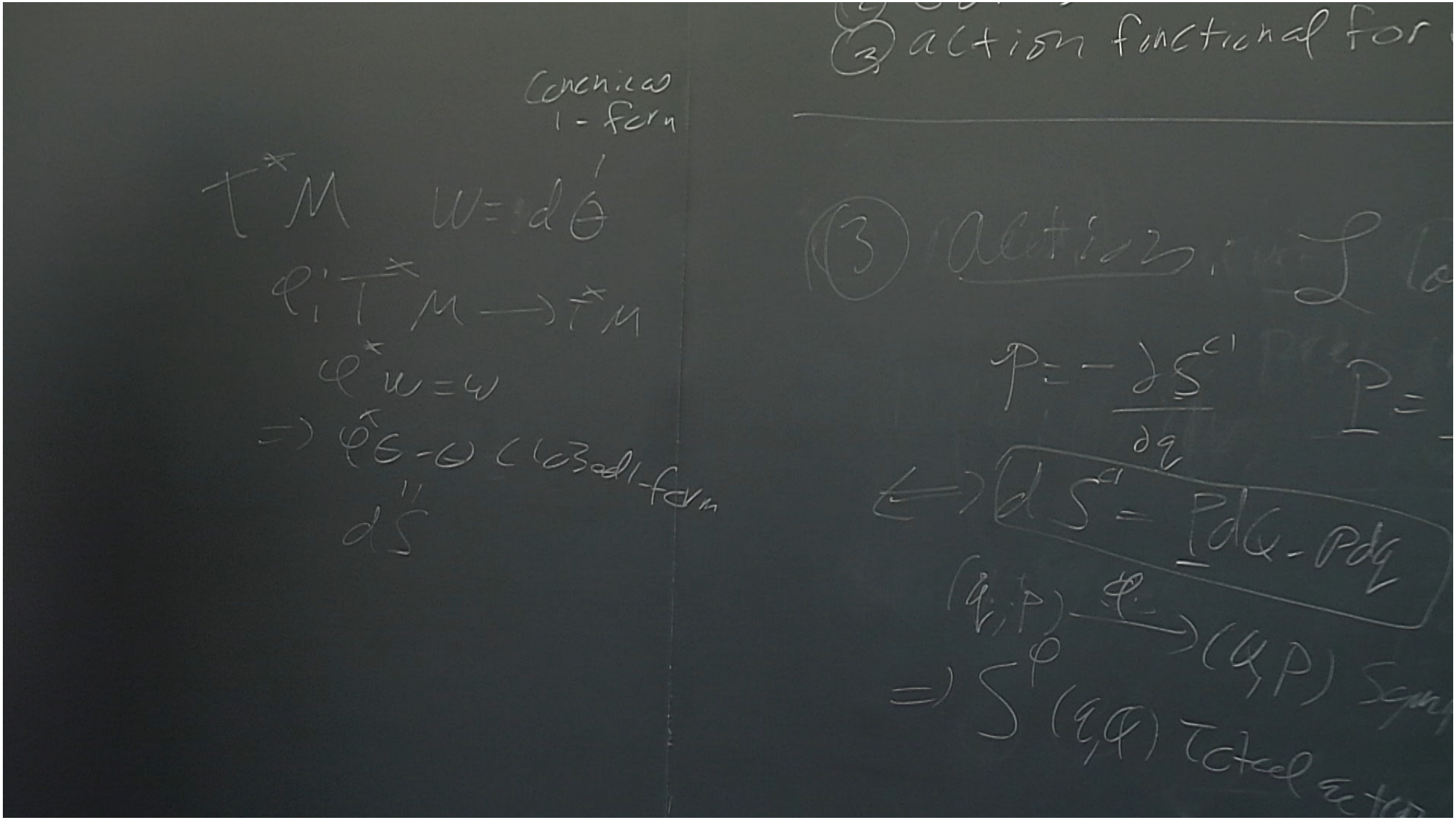
when $q^{C_1}(0) = q$ $q^{C_1}(t) = Q$

Euler-Lagrange

$$p = -\frac{\partial S^{C_1}}{\partial q} \qquad P = \frac{\partial S^{C_1}}{\partial Q}$$

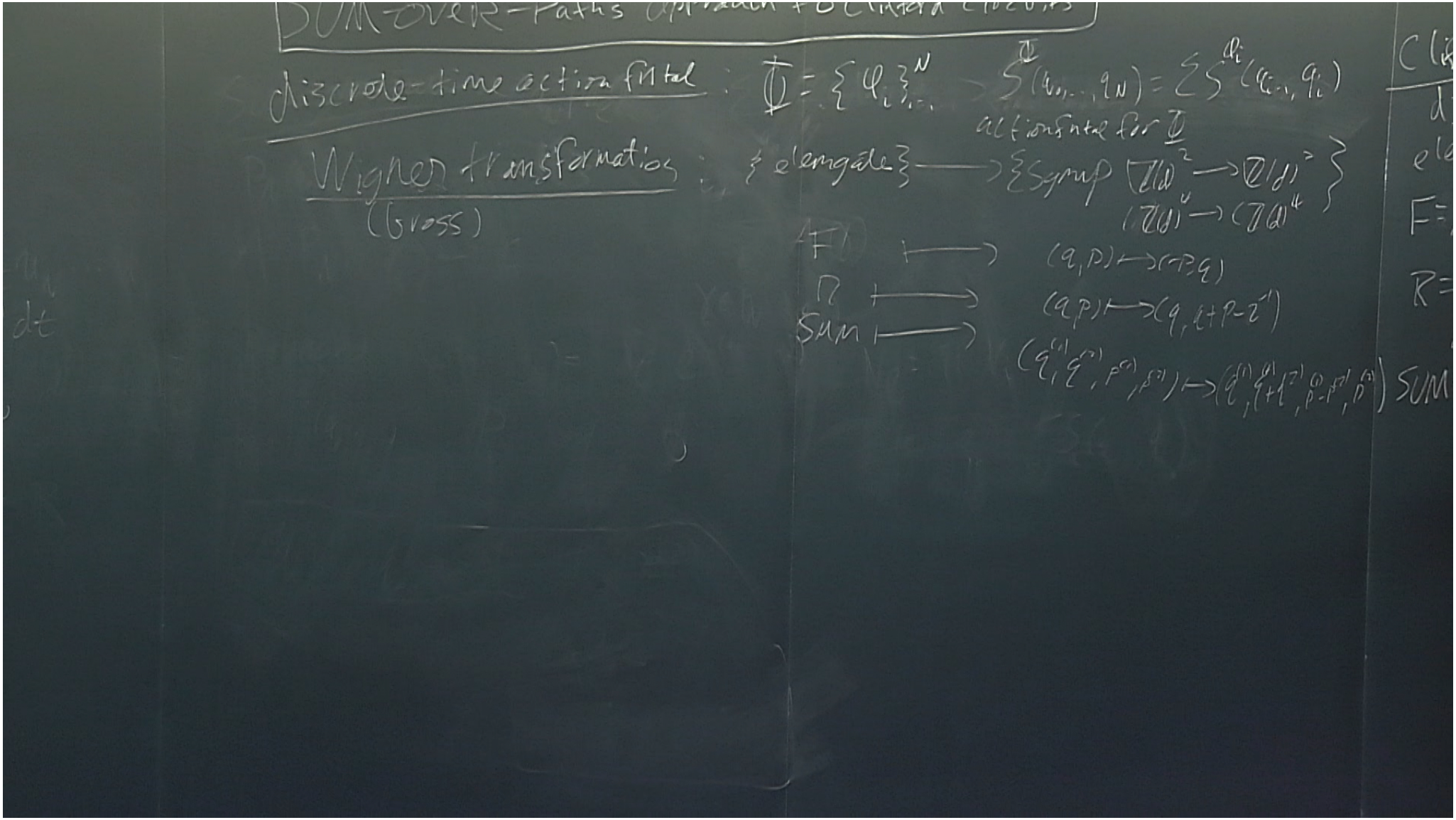
$$\leftrightarrow dS^{C_1} = P\,dQ - p\,dq$$

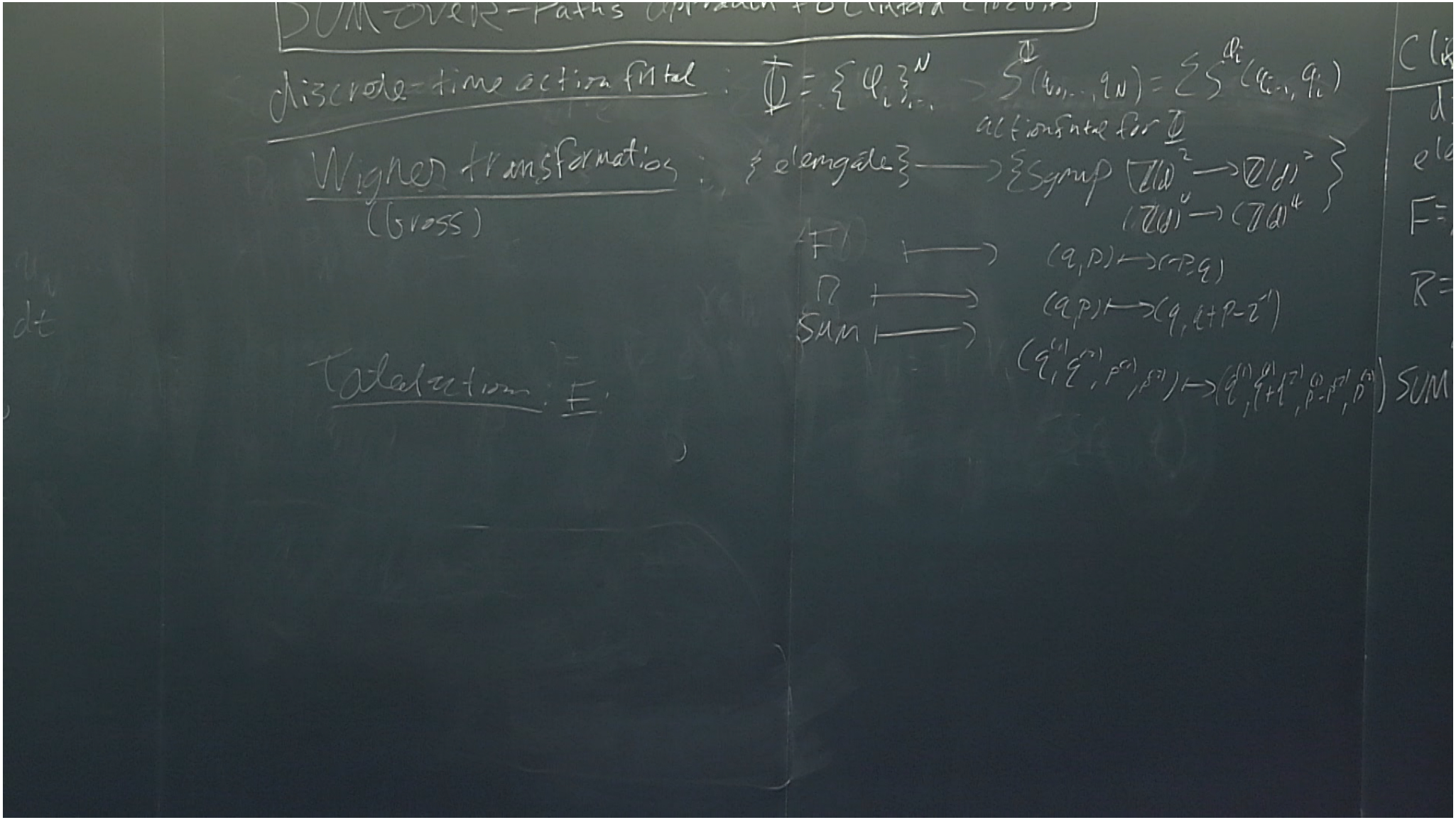

(1) SUM-over-Paths for Balanced Circuits
(2) Gottesman-Knill Theorem
(3) action functional for Clifford circuits
(3) action: ∫ lagrangian ⇝ S^cl(q,Q) = ∫_0^T L(q̇^cl(t), q^cl(t)) dt
"Total action"
when q^cl(0) = q q^cl(t) = Q
Euler-Lagrange
p = −∂S^cl/∂q   P = ∂S^cl/∂Q
⟷ dS^cl = P dQ − p dq
(q,p) —Φ→ (Q,P) Symplectic
⇒ S^Φ(q,Q) Total action


canonical 1-form
T^*M $\omega = d\theta$
$\varphi: T^*M \to T^*M$
$\varphi^*\omega = \omega$
$\Rightarrow \tilde{\varphi}\theta - \theta$ closed 1-form
dS
(3) action functional for
(3) action
$p = -\frac{\partial S}{\partial q}$
$\Leftrightarrow dS = pdq - Pdq$
$(q,p) \xrightarrow{\varphi} (Q,P)$ symp
$\Rightarrow S(q,Q)$ Tdel act

SUM-OVER-Paths

discrete-time action functional: $\Phi = \{q_i\}_{i=0}^{N} \to S^{\Phi}(q_0, \ldots, q_N) = \sum_i S^{q_i}(q_{i-1}, q_i)$


SUM-OVER-Paths
discrete-time action fctal
Wigner transformation
(Gross)
elementale
symp
action fctal for
SUM


SUM-OVER-Paths
discrete-time action functional
Wigner transformation
(Gross)
elementale
symp
action functional for
Calculation
SUM

SUM-OVER-Paths

discrete-time action fields : $\underline{\Phi} = \{q_i\}_{i=1}^{N}$ $\quad S^{\underline{\Phi}}(q_1, \ldots, q_N) = \sum S^{q_i}(q_1, \ldots, q_i)$

action fields for $\underline{\Phi}$

Wigner transformation : $\{\text{elementals}\} \longrightarrow \{\text{symp } (\mathbb{Z}/d)^2 \to (\mathbb{Z}/d)^2,\ (\mathbb{Z}/d)^4 \to (\mathbb{Z}/d)^4\}$

(Gross)

$F \longmapsto (q,p) \longmapsto (-p,q)$

$\Pi \longmapsto (q,p) \longmapsto (q, q+p-z^{-1})$

$\text{SUM} \longmapsto (q^{(1)}, q^{(2)}, p^{(1)}, p^{(2)}) \longmapsto (q^{(1)}, q^{(2)}+q^{(1)}, p^{(1)}-p^{(2)}, p^{(2)})$ SUM

Total action : $E \cdot \underline{P}\,dQ - p\,dq = q\,d(-p) - p\,dq$

$= -$

SUM-OVER-Paths

discrete-time action functional: $\underline{\Phi} = \{q_i\}_{i=\dots}^{N}$ $\quad S^{\underline{\Phi}}(q_0, \dots, q_N) = \sum S^{q_i}(q_{i-1}, q_i)$

action functional for $\underline{\Phi}$

Wigner transformations (gross): $\{\text{elementals}\} \longrightarrow \{\text{symp } (\mathbb{R}(d)^2 \to \mathbb{R}(d)^2, (\mathbb{R}(d)^\vee \to (\mathbb{R}(d)^\vee\}$

$F \longmapsto (q,p) \longmapsto (-p,q)$

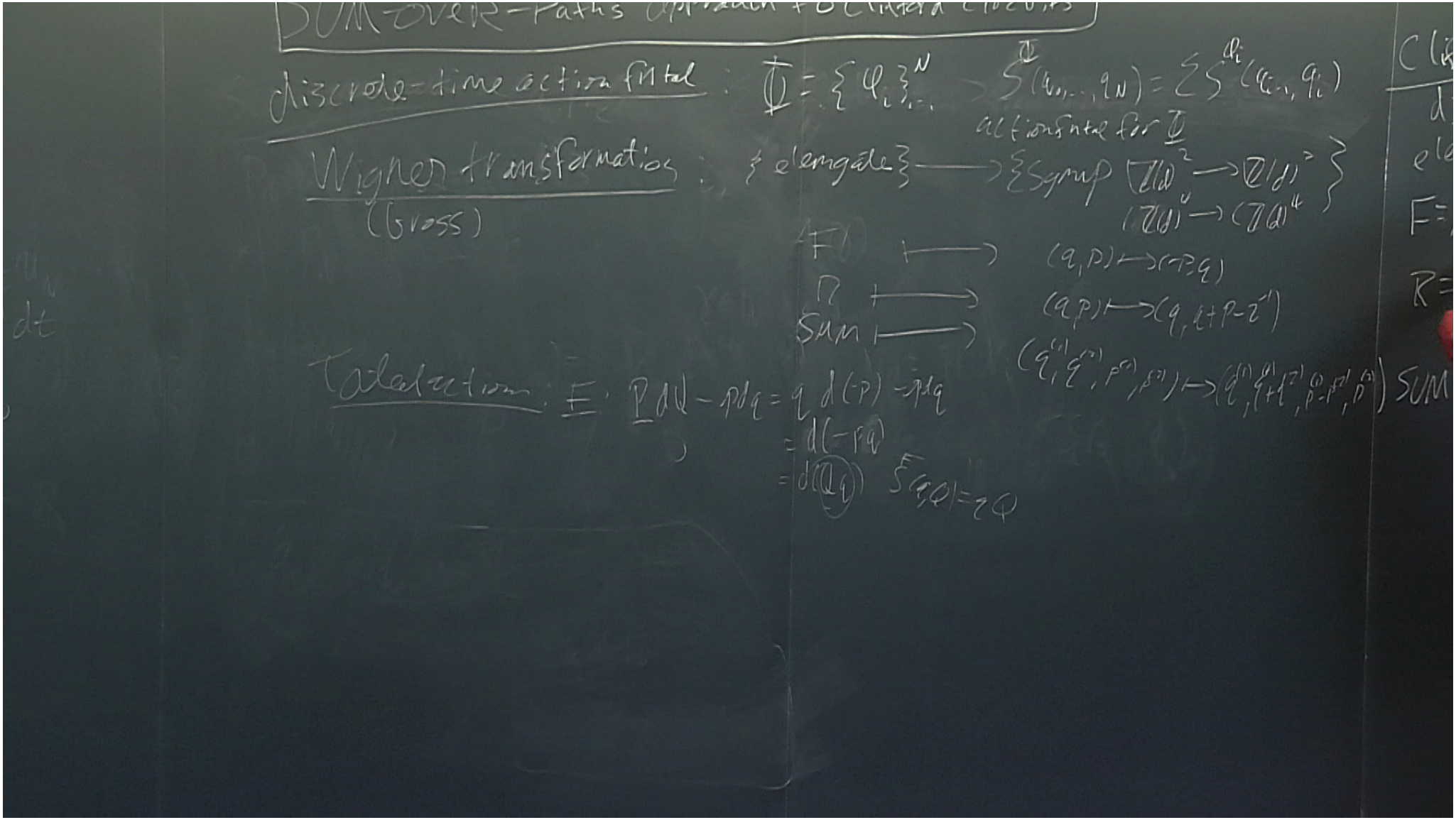
$\Pi \longmapsto (q,p) \longmapsto (q, q+p-z^{-1})$

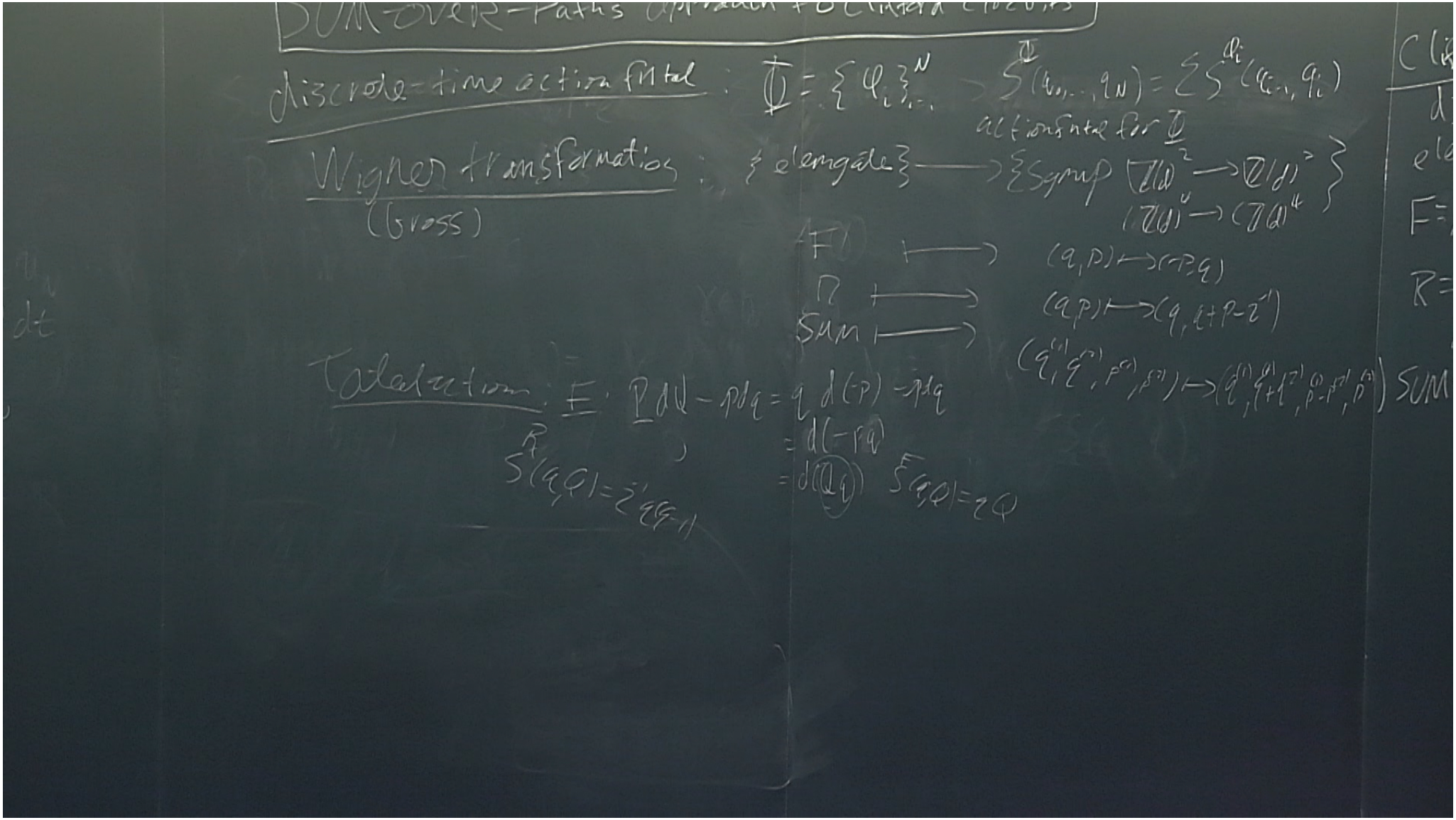
$\text{Sum} \longmapsto (q^{(1)}, q^{(2)}, p^{(1)}, p^{(2)}) \longmapsto (q^{(1)}, q^{(1)}+q^{(2)}, p^{(1)}-p^{(2)}, p^{(2)})$ SUM

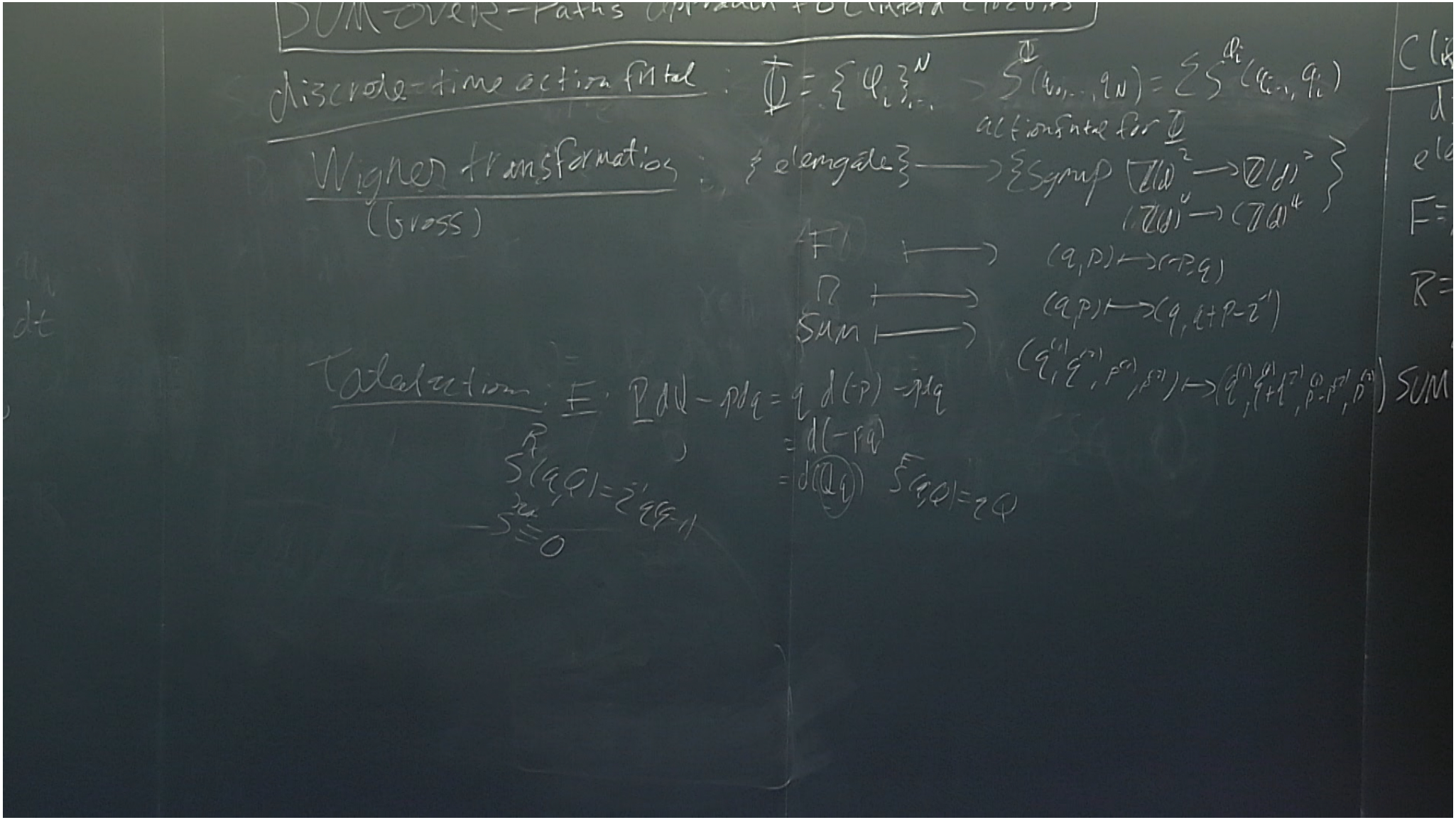
Total action: $\underline{E}: \underline{p}\,dq - p\,dq = q\,d(-p) - p\,dq$

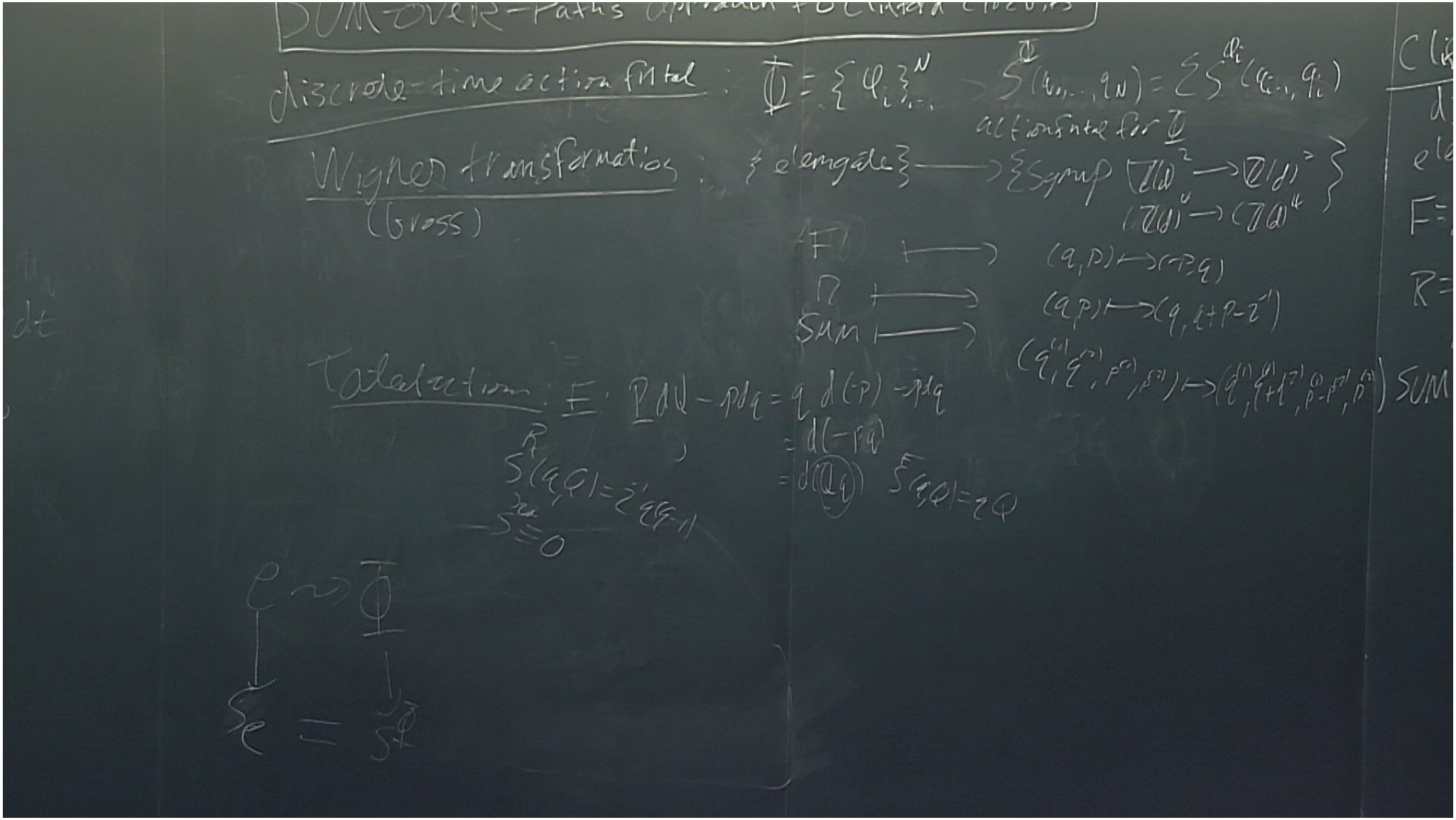
$= d(-pq)$

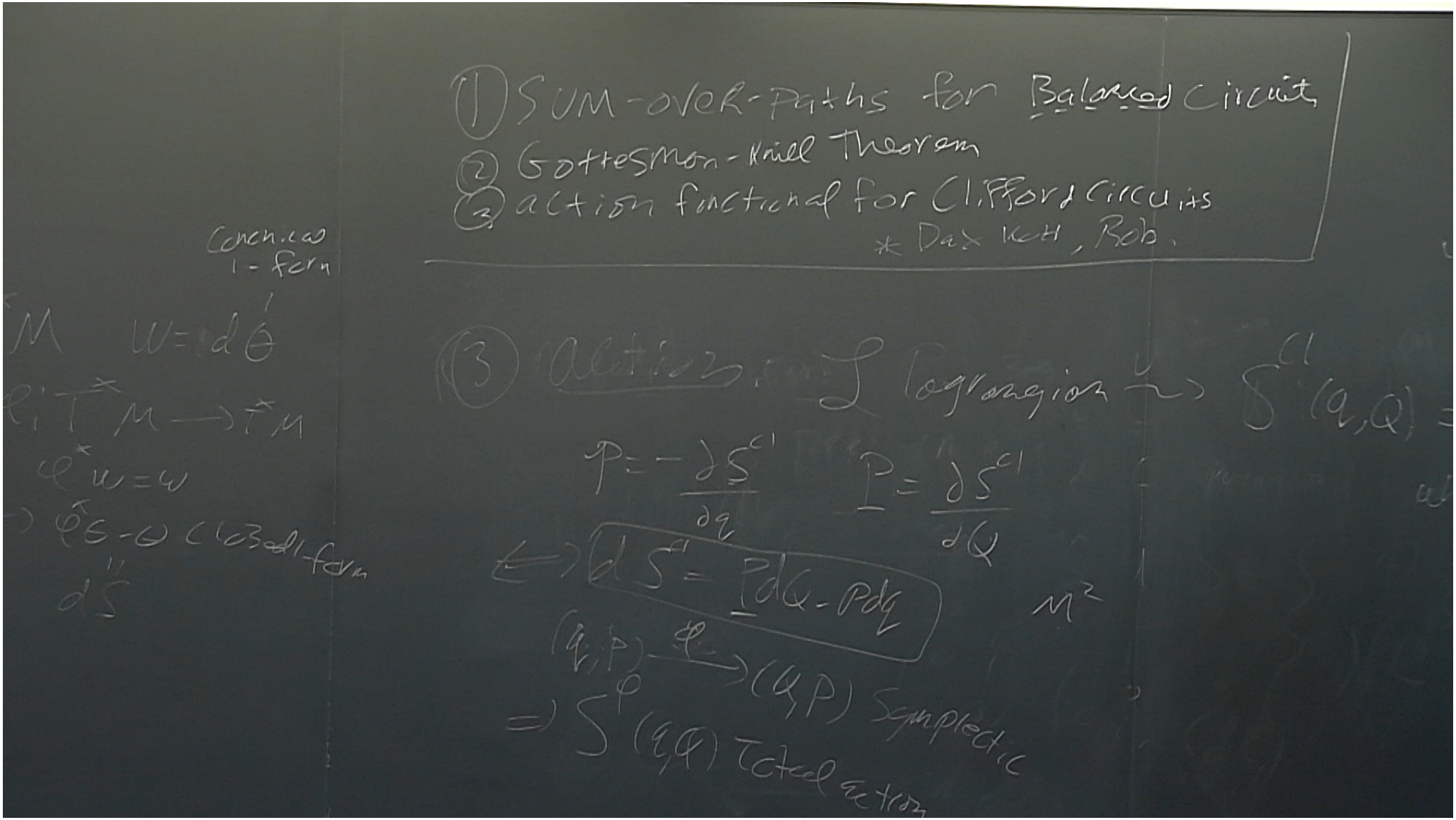
$= d(Q_q)$


SUM-OVER-Paths
discrete-time action functional
Wigner transformations
(Gross)
Calculation:
SUM

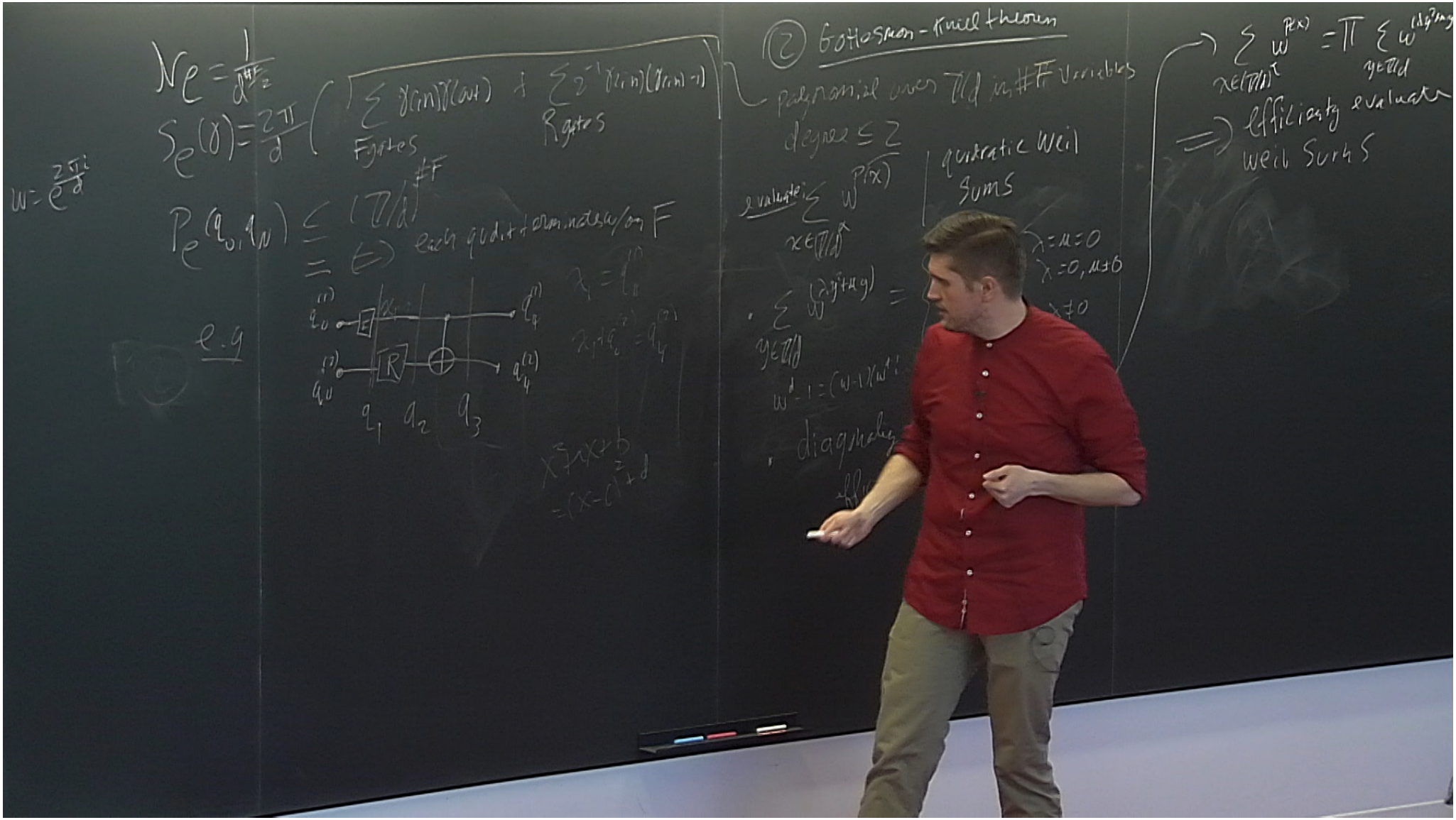

SUM-OVER-Paths
discrete-time action functional
Wigner transformation
(Gross)
{elementals}
{symp
action functional for
Calculation:
SUM


SUM-OVER-Paths
discrete-time action fntal
Wigner transformation
(Gross)
{elementale}
{symp
action fntal for
Tdual
SUM
Total action:
R
S(q,q1)=2'qq-1
=0


SUM-OVER-Paths
discrete-time action functional
Wigner transformation
(Gross)
elementals
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Translation
SUM


1) SUM-over-Paths for Balanced Circuits
2) Gottesman-Knill Theorem
3) action functional for Clifford Circuits
* Daw ICH, Rob.
canonical 1-form
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(3) action: $\mathcal{L}$ Lagrangian $\rightsquigarrow$ $S^{cl}(q,Q)$
$p = -\frac{\partial S^{cl}}{\partial q}$ $P = \frac{\partial S^{cl}}{\partial Q}$
$dS = Pdq - pdq$
M^2
$(q,p) \xrightarrow{\varphi} (Q,P)$ Symplectic
$S^{\varphi}(q,q)$ Total action


Gottesman-Knill theorem
quadratic Weil sums
e.g.


Gottesman-Knill theorem
quadratic Weil sums
e.g.