Title: Surface Charges to Disentangle Physics from Gauge

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Abstract: <p>Recently it was porposed by Hawking, Perry and Strominger that an infinite number of asymptotic charges may play a role in the decription of black hole entropy. With this context in mind we review the classical definition of surface charges in 3+1 gravity (and electromagnetism) from a slighly different framework by using the tetrad-connection variables. The general derivation follows the canonical covariant symplectic formalism in the language of forms. Applications to 3+1 and 2+1 charged and rotating black hole families are briefly discussed as a check. For exact (global) symmetries it is shown to be explicitly equivalent with the 'invariant' symplectic method. Extension to Lovelock gravity are shwon. As an application the entropy expression found by Myers and Jacobson is recovered. [based on arXiv:1703.10120]</p>

Surface Charges to Disentangle Physics from Gauge

arXiv 1703.10120

Ernesto Frodden

[a collaboration with Diego Hidalgo]

Perimeter Institute, Waterloo, 26 October 2017

CECs
CENTRO
DE ESTUDIOS
CIENTÍFICOS

Main Subject: "Charges in Gauge Theories"

Goal 1 "Compare two explicitly convariant methods to compute charges"

Goal 2 "Charges for GR coupled to EM in the Einstein-Cartan formalism"

Goal 3 "Test the result with known examples"

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Why I care?

Short story of personal failures:

2011 **Quasilocal** energy for BH [Frodden, Gosh, and Pérez (2011): 1110.4055]

Not exactly.. Rather an energy for a family of quasilocal accelerated observers.

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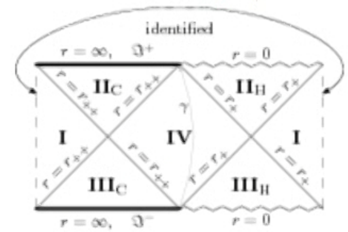
Good enough even for asymptotically de Sitter spacetimes.
But then.. we really need to go to the future to define a mass today?



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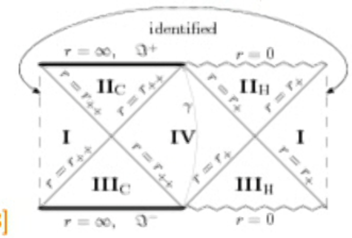

2015 Lets try **quasilocal** again..

Marco and Cédric: *'surface charges you should use'*..
[Barnich and Brandt (2001): 0111246] and [Barnich and Compère (2007): 0708.2378]
Ok.. trying to understand them.. (+ one year).. got it.

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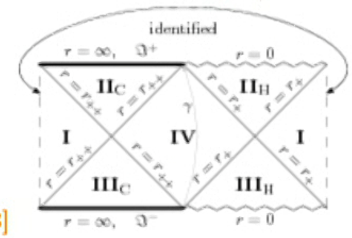

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2016 In the meantime [Strominger, Hawking, and Perry (2016): 1601.00921]

"Infinitely many asymptotic charges may solve the information paradox".. What?

From Strominger 2016 Lectures [1703.05448]

Supertranslated Schwarzschild Solution. In advanced Bondi coordinates the Schwarzschild metric is

$$ds^2 = -Vdv^2 + 2dvdr + r^2\gamma_{AB}d\Theta^A d\Theta^B \ , \quad V \equiv 1 - \frac{2m_B}{r} \ , \tag{7.2.1}$$

where $m_B = GM$. It is not hard to show that, for the Schwarzschild geometry, the supertranslation vector field

$$\zeta_f = f\partial_v + \frac{1}{r}D^A f \partial_A - \frac{1}{2}D^2 f \partial_r \ , \quad f = f(z, \bar{z}) \tag{7.2.2}$$

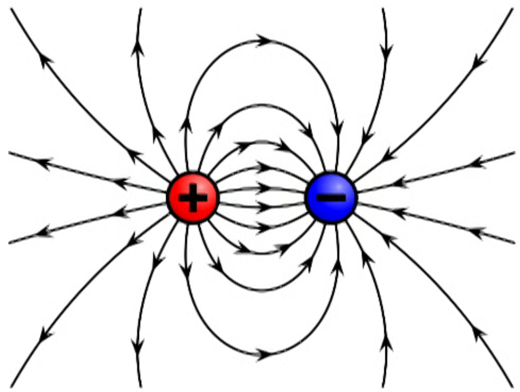
preserves Bondi gauge for all r, not just to leading order. Taking the Lie derivative one finds

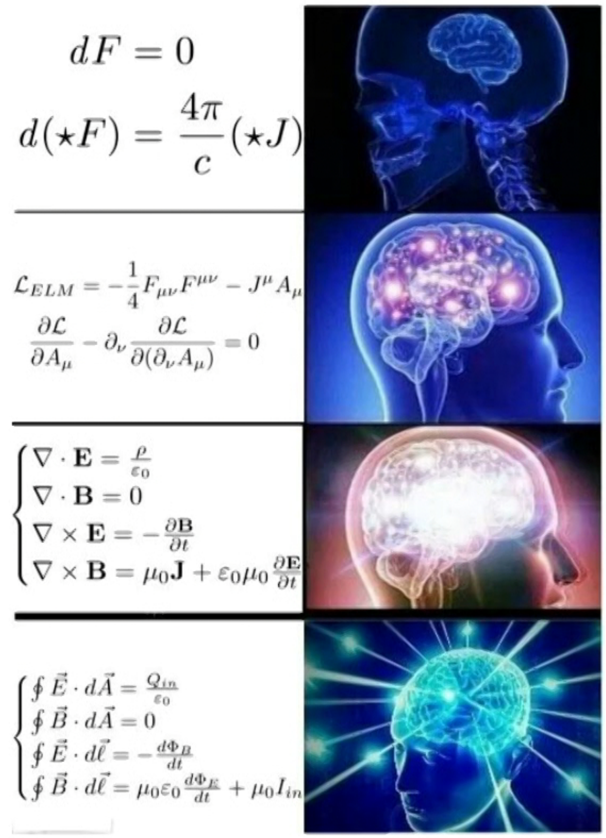
$$\begin{aligned} ds^2 + \mathcal{L}_f ds^2 \ &= -\left(V - \frac{m_B D^2 f}{r^2}\right) dv^2 + 2dvdr - dvd\Theta^A D_A(2Vf + D^2 f) \\ &+ (r^2\gamma_{AB} + 2rD_A D_B f - r\gamma_{AB}D^2 f)d\Theta^A d\Theta^B \ . \end{aligned} \tag{7.2.3}$$

The event horizon is at $r = 2m_B + \frac{1}{2}D^2 f$. This geometry describes a black hole with linearized supertranslation hair. Horizon supertranslations were studied in other gauges in [222–224].

What makes $\delta_f g = \mathscr{L}_f g$ physical?

What is a charge?





1. Physical? in what sense?
2. Local or global?
3. Forget sources, use fields
4. Topological charges?

What is a Noether charge?

...Continuous physical symmetries produce conserved charges...

Field Φ

Infinitesimal symmetry $\delta_{\bar{\epsilon}}\Phi$ (nothing gauge around)

$$
\begin{aligned}
dM_{\bar{\epsilon}} = \delta_{\bar{\epsilon}} L[\Phi] &= \cancel{E_\Phi}\,\delta_{\bar{\epsilon}}\Phi + d\Theta(\delta_{\bar{\epsilon}}\Phi) \quad (E_\Phi \to 0) \\
dM_{\bar{\epsilon}} &= d\Theta(\delta_{\bar{\epsilon}}\Phi)
\end{aligned}
$$

The Noether current is conserved on-shell

$$dJ^N_{\bar{\epsilon}} = 0 \quad \text{with} \quad J^N_{\bar{\epsilon}} \equiv \Theta(\delta_{\bar{\epsilon}}\Phi) - M_{\bar{\epsilon}}$$

Why it does not work with gauge fields?

.. Because Noether identities make the conservation trivial ..

Field Φ
Infinitesimal gauge symmetry $\delta_\epsilon \Phi$

$$\begin{aligned} \delta_\epsilon L[\Phi] &= E_\Phi \delta_\epsilon \Phi + d\Theta(\delta_\epsilon \Phi) \\ dM_\epsilon &= \cancelto{0}{N_\epsilon} + dS_\epsilon + d\Theta(\delta_\epsilon \Phi) \end{aligned}$$

The 'current' is trivially conserved.. off-shell

$$dJ_\epsilon = 0 \quad \text{with} \quad J_\epsilon = \Theta_\epsilon - M_\epsilon + S_\epsilon$$

But $J_\epsilon \sim J_\epsilon^N$ both have the same physical (on-shell) content..

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[WARNING]
.. if you insist, ϵ is arbitrary function and you get infinitely many conserved charges everywhere..

.. for free, but meaningless, right?

$$dJ_\epsilon = 0 \quad \Rightarrow \quad J_\epsilon = d\widetilde{Q}_\epsilon \quad \Rightarrow \quad \text{“}Q_\epsilon = \oint \widetilde{Q}_\epsilon\text{”}$$

the phantom you should protect from

What can we do for gauge symmetries?

Usually more information is directly imposed on Q_ϵ.
It is a delicated strategy: The formalism does not dictate you which information.

Another strategy

- Explore the phase space
- Strengthen the symmetry condition
- Conservation law of a lower level

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Presymplectic structure density to explore the phase space

$$\Omega(\delta_1, \delta_2) = \delta_1 \Theta(\delta_2 \Phi) - \delta_2 \Theta(\delta_1 \Phi)$$

Gauge orbits $\delta_\epsilon \Phi$: The phase space is degenerated

$$\Omega(\delta, \delta_\epsilon) = dk_\epsilon$$

Strengthen the symmetry: Make it **exact** $\delta_{\bar{\epsilon}} \Phi = 0$

$$dk_{\bar{\epsilon}} = 0 \quad \Rightarrow \quad \not\delta Q_{\bar{\epsilon}} = \oint k_{\bar{\epsilon}}$$

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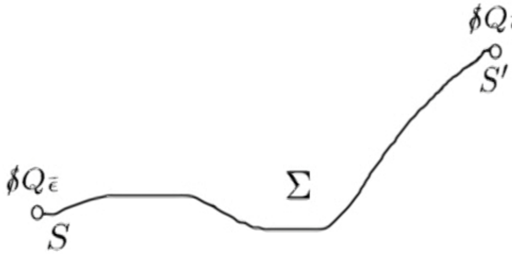

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↑↑↑ **surface charges** ↑↑↑

Two ways for the surface charge potential

- À la Wald and Zoupas [9911095]

$$
\begin{aligned}
\delta J_\epsilon &= d\delta\widetilde{Q}_\epsilon \\
&\vdots \\
\Omega(\delta,\delta_\epsilon) &= dk_\epsilon \\
k_\epsilon &= \delta\widetilde{Q}_\epsilon - \xi \lrcorner \Theta
\end{aligned}
$$

(with $\epsilon = (\xi, \lambda)$, $\delta_\epsilon L = d(\xi \lrcorner L)$, $\delta_\epsilon \Theta = \mathscr{L}_\xi \Theta$, and $\delta\xi = 0$)

- À la Barnich and Brandt [0111246]

$$
k'_\epsilon = I_{\delta\Phi}[S_\epsilon]
$$

(the *homotopy operator* $I_{\delta\Phi}$ satisfies $I d\,\omega + dI\,\omega = \delta\omega$. Remember $E_\Phi \delta_\epsilon \Phi = dS_\epsilon$)

Two questions

- **Which are the conditions on k_ϵ and k'_ϵ to produce a conservation?**

  Both conservations hold

  $$dk_{\bar{\epsilon}} = 0 \quad \text{and} \quad dk'_{\bar{\epsilon}} = 0$$

  if the same three conditions are satisfied

  - On-shell $E_\Phi = 0$
  - Linearized on-shell $\delta E_\Phi = 0$
  - Exact symmetry $\delta_{\bar{\epsilon}} \Phi = 0$

- **Produce k_ϵ and k'_ϵ different charges?** ..**Nope**..

  Again, if the three conditions hold they are in the same class $k_{\bar{\epsilon}} \sim k'_{\bar{\epsilon}}$ because

  $$\oint k_{\bar{\epsilon}} = \oint k'_{\bar{\epsilon}}$$

..the proof in the appendix of [1703.10120] or [Barnich and Compère: 0708.2378]

Two more questions

- **In what sense $\oint Q_{\bar{\epsilon}}$ are quasilocal charges?**

- **What about the integrability of $\oint Q_{\bar{\epsilon}}$?**

$$\delta \oint Q_{\bar{\epsilon}} = 0 \quad \Rightarrow \quad \exists\ Q_{\bar{\epsilon}} \qquad \text{(a finite function of the phase space)}$$

The linearity property may help to solve it (but a general argument is missing):

$$\oint Q_{\alpha_1 \bar{\epsilon}_1 + \alpha_2 \bar{\epsilon}_2} = \alpha_1 \oint Q_{\bar{\epsilon}_1} + \alpha_2 \oint Q_{\bar{\epsilon}_2}$$

with α_1 and α_2 arbitrary functions of the phase space.

Ex0: The old electric charge

- Exact symmetry condition (solvable for any solution)

$$\delta_\lambda A = d\lambda = 0 \quad \Rightarrow \quad \lambda = \lambda_0$$

- Surface charge potential

$$k_\lambda = \lambda \; \delta \star F$$

- Surface charge (trivially integrable if $\delta\lambda_0 = 0$)

$$\not{\delta} Q_{\lambda_0} = \oint k_{\lambda_0} = \oint \lambda_0 \delta \star F = \delta \oint \lambda_0 \star F$$

- The usual Gauss law

$$Q_{\lambda_0} = \lambda_0 \oint \star F$$

Λ-Einstein-Cartan-Maxwell-Euler in $D = 4$

$$L[e, \omega, A] = \frac{\kappa}{2}\varepsilon_{abcd}\bar{R}^{ab}\bar{R}^{cd} + \alpha F * F, \quad \text{with} \quad \bar{R}^{ab} = R^{cd} \pm \frac{1}{\ell^2} e^a \wedge e^b$$

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Exact symmetry ('improved' but not needed) (notation $\xi\lrcorner = i_\xi = \xi\cdot$)

$$\begin{aligned} \delta_{\bar{\epsilon}}\, e^a &= d_\omega \xi\lrcorner e^a + \xi\lrcorner d_\omega e^a + \lambda^a{}_b e^b = 0 \\ \delta_{\bar{\epsilon}}\, \omega^{ab} &= \xi\lrcorner R^{ab} - d_\omega \lambda^{ab} = 0 \\ \delta_{\bar{\epsilon}}\, A &= \xi\lrcorner F - d\lambda = 0 \end{aligned}$$

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Surface charge potential à la Wald

$$k_{\bar{\epsilon}} = -\kappa\, \varepsilon_{abcd}\Big(\lambda^{ab}\delta\bar{R}^{cd} - \delta\omega^{ab}\xi\lrcorner\bar{R}^{cd}\Big) - 2\alpha(\lambda\, \delta * F - \delta A\, \xi\lrcorner * F).$$

Surface charge

$$\not{\delta} Q_{\bar{\epsilon}} = \oint k_{\bar{\epsilon}}$$

Ex1: Gravity coupled to Electromagnetism in $D = 4$

(anti-)de Sitter Kerr-Newman family

A black hole family with

- asymptotically constant curvature
- electrically charged
- rotating

Check: 1703.10120

An alternative version for $I_{\delta\Phi}[\cdot]$?

Surface charge potential à la Barnich $(D=4)$ [Barnich: 0301039]

$$k'_\epsilon = I_{\delta\phi}[S_\epsilon] = \sum_{k=0} \frac{k+1}{k+2} \partial_{\mu_1} \cdots \partial_{\mu_k} \left(\delta\phi^i \frac{\partial}{\partial(\phi^i_{,\mu_1\cdots\mu_k\rho})} \partial_\rho \lrcorner S_\epsilon \right) \underset{\text{first order}}{=} \frac{1}{2} \delta\phi^i \frac{\partial}{\partial \phi^i_{,\rho}} \partial_\rho \lrcorner S_\epsilon$$

For Einstein-Cartan (coordinates hidden, gauge invariance made explicit) [1703.10120]

$$k'^{GR}_\epsilon = I_{\delta e, \delta\omega}[S_\epsilon] = \left(\delta e^a \frac{\partial}{\partial T^a} + \delta\omega^{ab} \frac{\partial}{\partial R^{ab}} \right) S_\epsilon$$

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Λ-Einstein-Cartan in $D = 4$

$$k'^{GR}_\epsilon = -\kappa' \, \varepsilon_{abcd}\Big(\lambda^{ab}\delta(e^c e^d) - \delta\omega^{ab}\xi \lrcorner (e^c e^d)\Big)$$

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Note:

$$\begin{aligned} k^{Euler}_\epsilon &= k^{GR}_\epsilon - k'^{GR}_\epsilon \\ &= -\kappa \, \varepsilon_{abcd} \Big(\lambda^{ab} \delta R^{cd} - \delta\omega^{ab} \xi \lrcorner R^{cd} \Big) = -\kappa \, \varepsilon_{abcd} \Big(d(\lambda^{ab} \delta\omega^{cd}) + \delta_\epsilon \omega^{ab} \delta\omega^{cd} \Big) \end{aligned}$$

The boundary term and the cosmological constant do not contribute..

Lovelock-Cartan: gravity in D

Action [see Hassaine and Zanelli (2016) book on "Chern-Simons (Super)Gravity"]

$$S[e,\omega] = \int_{\mathcal{M}} \sum_{p=0}^{[D/2]} L_p^D \quad \text{with} \quad L_p^D = \alpha_p \varepsilon_{a_1 \cdots a_D} R^{a_1 a_2} \dots R^{a_{2p-1} a_{2p}} e^{a_{2p+1}} \dots e^{a_D}$$

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Each term can be decomposed

$$(D-2p)\varepsilon\{R \cdots R(\lambda \delta e - \delta\omega \xi_\lrcorner e) e \cdots e\} + (p-1)\varepsilon\{d(\lambda \delta\omega R \cdots R e \cdots e) + \delta_\epsilon \omega \delta\omega R \cdots R e \cdots e\},$$

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$$(D-2p)\varepsilon\{R\cdots R(\lambda\delta e - \delta\omega\xi\lrcorner e)e\cdots e\} + (p-1)\varepsilon\{d(\lambda\delta\omega R\cdots Re\cdots e) + \delta_\epsilon\omega\delta\omega R\cdots Re\cdots e\},$$

Equivalent surface charge potential

$$k'_\epsilon = -\sum_{p=1}^{[(D-1)/2]} \alpha'_p \varepsilon_{a_1\cdots a_D} (\lambda^{a_1 a_2}\delta e^{a_3} - \delta\omega^{a_1 a_2}\xi\lrcorner e^{a_3}) R^{a_4 a_5} \cdots R^{a_{2p}a_{2p+1}} e^{a_{2p+2}} \cdots e^{a_D},$$

Lovelock-Cartan: gravity in D

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$$(D-2p)\varepsilon\{R\cdots R(\lambda\delta e - \delta\omega\xi\lrcorner e)e\cdots e\} + (p-1)\varepsilon\{d(\lambda\delta\omega R\cdots Re\cdots e) + \delta_\epsilon\omega\delta\omega R\cdots Re\cdots e\},$$

Equivalent surface charge potential

$$k'_\epsilon = -\sum_{p=1}^{[(D-1)/2]} \alpha'_p \varepsilon_{a_1\cdots a_D} (\lambda^{a_1 a_2} \delta e^{a_3} - \delta\omega^{a_1 a_2} \xi \lrcorner e^{a_3}) R^{a_4 a_5} \dots R^{a_{2p} a_{2p+1}} e^{a_{2p+2}} \dots e^{a_D},$$

Surprise $\qquad k'_\epsilon = I_{\delta e, \delta\omega}[S_\epsilon]$

Note: $\alpha_p = 0$ for $p \geq 1$ recovers [Barnich, Mao, and Ruzziconi: 1611.01777]

Ex2: Wald's entropy for Lovelock [Discussed with Alfredo Guevara]

"as a Noether charge"

[Jacobson and Myers: 9305016]
[Bañados, Teitelboim, and Zanelli: 9309026]

Consider a Killing horizon such that

$$\xi^a \nabla_a \xi_b = \kappa \xi_b$$

The variation on the phase space (fixing $\delta\xi = 0$)

$$\delta\left(\frac{1}{\kappa}\xi^a \nabla_a \xi_b\right) = \xi^a \delta\left(\frac{1}{\kappa}\lambda_{ab}\right) = 0$$

The surface charge $\oint Q_{\frac{1}{\kappa}\xi}$ can be intregrated on a bifurcated horizon: $\xi|_B = 0$

$$S \equiv \frac{1}{\kappa}\oint \sum_{p=0}^{[D/2]} \alpha_p p \varepsilon_{a_1 \cdots a_D} \nabla^{a_1}\xi^{a_2} R^{a_3 a_4} \dots R^{a_{2p-1}a_{2p}} e^{a_{2p+1}} \dots e^{a_D}$$

Lovelock-Cartan: gravity in D

Action [see Hassaine and Zanelli (2016) book on "Chern-Simons (Super)Gravity"]

$$S[e,\omega] = \int_{\mathcal{M}} \sum_{p=0}^{[D/2]} L_p^D \quad \text{with} \quad L_p^D = \alpha_p \varepsilon_{a_1\cdots a_D} R^{a_1 a_2} \ldots R^{a_{2p-1} a_{2p}} e^{a_{2p+1}} \ldots e^{a_D}$$

Surface charge potential

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Ex3: Einstein-Maxwell $D = 3$: BTZ + electric charge + rotation

[Martínez, Teitelboim, and Zanelli: 9912.259]
[see also Pérez, Riquelme, Tempo, and Troncoso: 1509.01750]
[and Adami and Setare: 1511.00527]

$$\begin{aligned} ds^2 &= -N^2 F^2 dt^2 + \frac{1}{F^2} dr^2 + R^2 (N^\phi dt + d\phi)^2 \\ A &= -q \log(r/\ell)[dt - q\omega_r d\phi] \end{aligned}$$

with $F^2 = \frac{r^2}{\ell^2} - \frac{r_p^2}{\ell^2} - \frac{q^2}{2}\left(1 - \omega_r^2\right)\log(r/r_p)$, $\quad N = \ldots$, $N^\phi = \ldots$, and $R = \ldots$

Three phase space parameters q, r_p, and ω_r.

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Three phase space parameters q, r_p, and ω_r.

Choose variables:

- Metric $g_{\mu\nu}$
- Tetrad-connection (e^a, ω^{ab})
- Poincaré CS connection $A = e^a P_a + \omega^{ab} J_{ab}$.

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Find all the exact symmetries parameters $\bar{\epsilon} = (\xi, \lambda)$

$$\begin{aligned} \delta_{\bar{\epsilon}} g_{\mu\nu} &= \mathscr{L}_\xi g_{\mu\nu} = 0 \\ \delta_{\bar{\epsilon}} A_\mu &= \xi^\nu F_{\nu\mu} - \partial_\mu \lambda = 0 \end{aligned}$$

Three exact symmetries

$$\begin{aligned} \epsilon_t &= (\partial_t, \lambda_t) = (\partial_t, q \log(r/\ell)) \\ \epsilon_\phi &= (\partial_\phi, \lambda_\phi) = (\partial_t, -q\omega_r \ell \log(r/\ell)) \\ \epsilon_\lambda &= (0, \lambda_0) \end{aligned}$$

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Compute conserved charges*.. immediatly integrable ($\delta \oint Q_{\bar{\epsilon}} = 0$)

$$\begin{aligned} \oint Q_{\epsilon_t} &= \delta M \quad \text{with} \quad M = \left(-\frac{r_p^2}{\ell^2} \left(\frac{1+\omega_r^2}{1-\omega_r^2} \right) + \frac{q^2}{2} \left(\omega_r^2 + (1+\omega_r^2) \log(r_p/\ell) \right) \right) \\ \oint Q_{\epsilon_\phi} &= \delta J \quad \text{with} \quad J = \frac{\omega_r}{\ell} \left(\frac{2r_p^2}{1-\omega_r^2} - \ell^2 q^2 \left(\frac{1}{2} + \log(r_p/\ell) \right) \right) \\ \oint Q_{\epsilon_\lambda} &= \delta Q \quad \text{with} \quad Q = q\lambda_0 \end{aligned}$$

Surface charges for all

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- Chern-Simons in 2+1, non-abelian (either à la Wald or à la Barnich)

$$k_\lambda = \langle \delta A \lambda \rangle \quad \text{with} \quad \delta_\epsilon A = \xi \lrcorner F - d_A \lambda$$

- Yang-Mills in 3+1.. à la Barnich

$$k'_\lambda = \lambda_I \star \delta F^I$$

- Einstein-Skyrme in 3+1.. à la Wald [Canfora, Frodden, Hidalgo, and Tallarita: *in preparation*]

$$\begin{aligned}
k_\xi^{\mu\nu} &= \delta \widetilde{Q}_\xi^{\mu\nu} - \widetilde{Q}_{\delta\xi}^{\mu\nu} - 2\xi^{[\mu}\Theta^{\nu]} \\
\widetilde{Q}_\xi^{\mu\nu} &= 2\kappa\sqrt{-g}\nabla^{[\mu}\xi^{\nu]} \\
\Theta_{GR}^\mu(\delta g) &= 2\kappa\sqrt{-g}\nabla^{[\alpha}(g^{\mu]\beta}\delta g_{\alpha\beta}) \\
\Theta_{SK}^\mu(\delta U) &= \frac{K}{2}\sqrt{-g}\; Tr\left[(R^\mu + \frac{\lambda}{4}(R_\nu F^{\mu\nu} - F^{\mu\nu}R_\nu))U^{-1}\delta U\right]
\end{aligned}$$

- Gravity with a conformally coupled scalar field in 2+1.. à la Wald

$$\begin{aligned}
\widetilde{Q}_\xi^{\mu\nu} &= \frac{1}{2\kappa}\sqrt{-g}\nabla^{[\mu}\xi^{\nu]}\left(1 - \frac{\kappa}{8}\Phi^2\right) \\
\Theta^\mu &= \frac{1}{2\kappa}\sqrt{-g}\left\{\nabla^{[\alpha}(g^{\mu]\beta}\delta g_{\alpha\beta})\left(1 - \frac{\kappa}{8}\Phi^2\right) - 2\kappa\nabla^\mu\Phi\delta\Phi\right\}
\end{aligned}$$

- Extensions to quantum backreacted solutions? $\quad G_{\mu\nu} = \kappa\langle \hat{T}_{\mu\nu} \rangle$
- Supergravity?

What have we learnt?

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   (a) exact symmetry condition $\delta_{\bar{\epsilon}}\Phi = 0$
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...be careful...
4. Again surface charges require exact symmetries.. or gauge may be spilled on your charge
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   Counterexample in 2+1? [Compère, Mao, Seraj, Sheikh-Jabbari: 1511.06079]
5. How this affects the infinitely many asymptotic 'charges'? .. to be seen..
   Certainly exactness is not usually fulfilled: For the supertranslated BH $\delta_f g \neq 0$

   A little hope: $dk_{\bar{\epsilon}} = 0$ *maybe can be generalized...*
   In a gauge invariant way for perturbed fields everywhere
   "Bring the infinitely many asymptotic charges to the reality of the bulk"

Prospects and comments

- To better understand the BRST origin of the surface charges
- Generalize surface charges to quantum perturbed systems
- More classical applications

A loopy story:

1. Observables are defined at the spacetime boundary
2. Field dependent gauge transformation (*improper*) produce charges at the boundary
3. You get infinitely many boundary charges
4. How do you check if they are physical?
5. .. go to 1

Gracias por la atención