Title: Boundary Trace Anomalies and Boundary Conformal Field Theory

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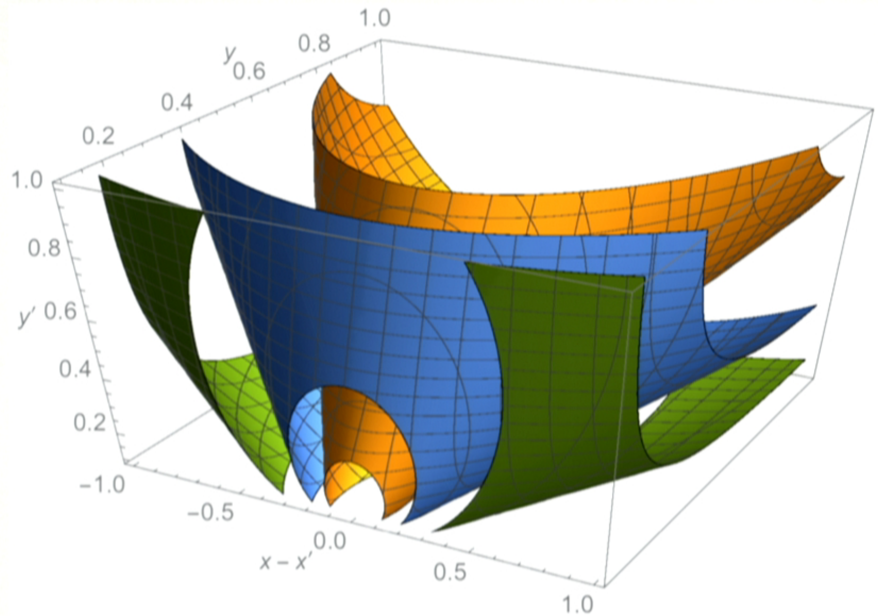
Abstract: <p>I discuss some aspects of boundary conformal field theories (bCFTs). I will demonstrate that free bCFTs have a universal way of satisfying crossing symmetry constraints. I will introduce a simple class of interacting bCFTs where the interaction is restricted to the boundary. Finally, I will discuss relationships between boundary trace anomalies and boundary limits of stress-tensor correlation functions.</p>



$$v = \frac{(x-x')^2 + (y-y')^2}{(x-x')^2 + (y+y')^2}$$

Boundary Trace Anomalies and Boundary Conformal Field Theory

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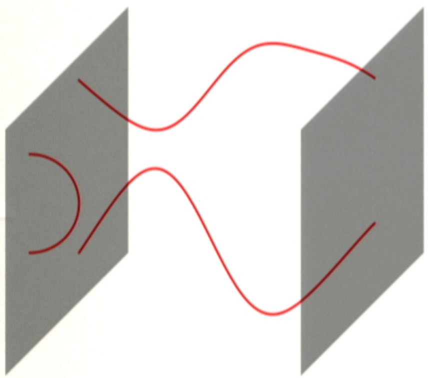
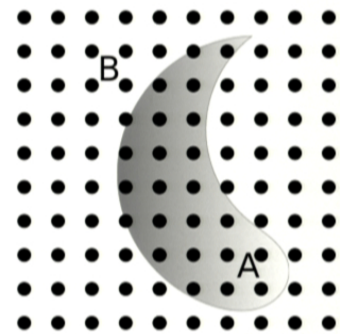
K.-W. Huang and CPH arXiv:1707.06224
Huang, Jensen, and CPH arXiv:1709.07431

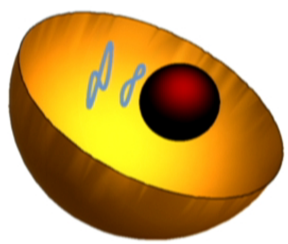
Outline

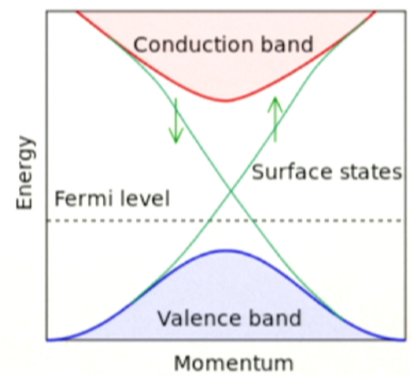
- Why bCFT?
- Review of trace anomalies and especially boundary contributions.
- New results: relations between boundary trace anomalies and two and three point functions of the displacement operator.
- Demonstration that the b_2 charge can depend on marginal couplings.

What do D-branes, AdS/CFT, topological insulators, and entanglement entropy for field theories have in common?





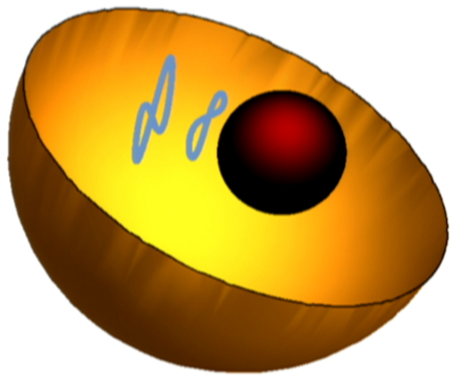




AdS/CFT

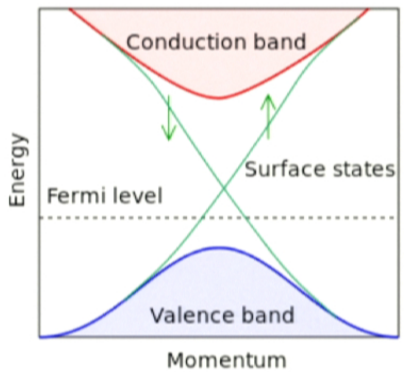
- Conformal boundary of anti-de Sitter space is where the conformal field theory "lives".

- Myriad developments — quantum gravity, strongly coupled field theories, black holes, … — perhaps best summarized by the fact that Maldacena's original paper now has over 15,000 citations.



Topological Insulators

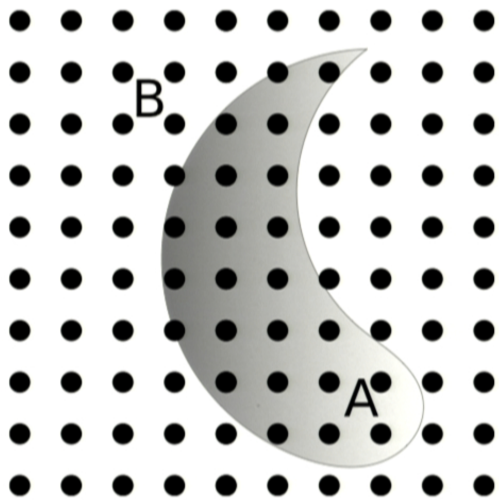
- The insulating bulk material has conducting surface states that are protected by symmetry
- Predicted in the late 80s for time reversal symmetry (Pankratov, Pakhomov, Volkov), the effects were first observed in real materials in 2007 (Konig, Wiedmann, et al.) and have now been studied and seen in many other shapes and forms as well.



Entanglement Entropy

- In field theory, entanglement is often measured with respect to spatial regions, leading to the importance of the "entangling surface"

- Led to new proofs of the "a" and "c" theorems for renormalization group flow of field theories in 2 and 4 dimensions. Also given a new perspective to black hole physics.



Would all of these developments have been "obvious" if we just understood quantum field theory in the presence of a boundary a little better to begin with?

Boundary Conformal Field Theory

- Surprisingly unexplored. McAvity and Osborn '93 and '95 papers on two point functions have fewer than 100 citations each.
- Flat space: A planar boundary breaks the SO(d,2) symmetry to SO(d-1,2).
- Curved space: Require that the boundary and boundary terms in the action preserved Weyl invariance.

Stress Tensor Trace

Today's talk: To understand new terms in the trace anomaly associated with the presence of a boundary.

A quick review:

classically, Weyl invariance implies $J^\mu = x_\nu T^{\nu\mu}; \quad \nabla_\mu J^\mu = 0$

$$\Rightarrow T^\mu{}_\mu = 0 \quad \text{given} \quad \nabla_\mu T^{\mu\nu} = 0$$

quantum mechanically, there are anomalies…

Trace Anomaly in 4d (no boundary)

$$\langle T^{\mu}{}_{\mu} \rangle = \frac{1}{16\pi^2}(cW^2 - aE.D. + d\Box R)$$

Weyl curvature — Euler density — scheme dependent, ignore

- c is also the coefficient of the $T_{\mu\nu}$ two-point function.
- Cardy ('88) *a*-conjecture and later Komargodski-Schwimmer proof ('11) $a_{\rm UV} > a_{\rm IR}$
- Casini-Teste-Torroba ('17) entanglement entropy proof of *a*-theorem

—towards a map of 4d QFTs

Trace Anomaly in 6d (no boundary)

$$\langle T^{\mu}{}_{\mu} \rangle \sim a\, E.D. + c_1 W^3 + c_2 W^3 + c_3 W \Box W$$

hints from supersymmetry and AdS/CFT of a 6d *a*-theorem

Trace Anomaly with a Codimension One Boundary

K_{AB} extrinsic curvature
hat on K removes trace

2D $\langle T^\mu{}_\mu \rangle = \frac{c}{24\pi}(R + 2K\delta(x^n))$

Jensen-O'Bannon ('15) b-theorem
$a_{\rm UV} > a_{\rm IR}$

3D $\langle T^\mu{}_\mu \rangle = \frac{1}{4\pi}(-aR + b\,\mathrm{tr}\,\hat{K}^2)\delta(x^n)$

4D $\langle T^\mu{}_\mu \rangle = \frac{1}{16\pi^2}(cW^2 - aE.D. + (-b_1\,\mathrm{tr}\,\hat{K}^3 + b_2 K^{AB} W_{nAnB})\delta(x^n))$

Solodukhin-Fursaev ('16) conjecture
$b_2 = 8c$

5D $\langle T^\mu{}_\mu \rangle \sim \delta(x_\perp)(b_1 W^2 + b_2 K^4 + \ldots)$

6D $\langle T^\mu{}_\mu \rangle \sim a\,E.D. + c_1 W^3 + c_2 W^3 + c_3 W \Box W + \delta(x_\perp)(b_1 K W^2 + \ldots)$

Today's Talk

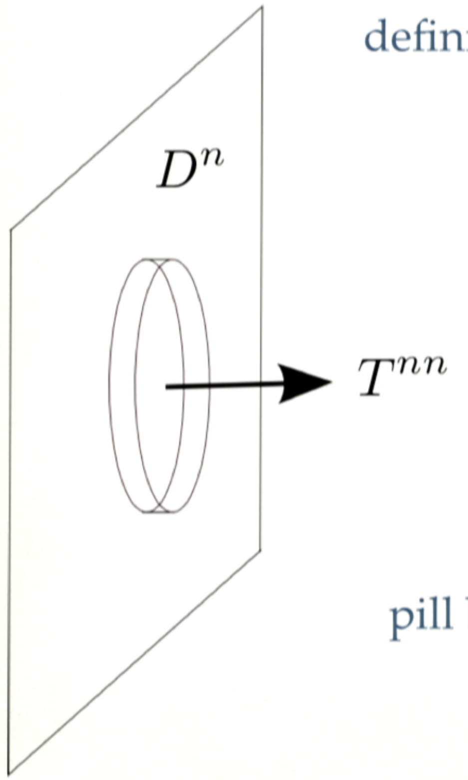
Can we say anything more about

3d:
- aR — conjecture
- $b\,\mathrm{tr}\,\hat{K}^2$ — yes

4d:
- $b_1\,\mathrm{tr}\,\hat{K}^3$ — yes
- $b_2 K^{AB} W_{AnBn}$ — yes

Related to displacement operator two and three point functions

Displacement Operator



definition: operator sourced by small changes in the embedding $\quad \frac{\delta I}{\delta x^n} \equiv D^n$

diffeomorphism Ward identity:

$$\partial_\mu T^{\mu n} = D^n \delta(x^n)$$

$$\partial_\mu T^{\mu A} = 0$$

tangential components still conserved

pill box argument implies

$$T^{nn}(\vec{x}, x^n)|_{x^n=0} = D^n(\vec{x})$$

Results

$$\langle D^n(\vec{x}) D^n(0)\rangle = \frac{c_{nn}}{|\vec{x}|^{2d}}$$

$$\langle D^n(\vec{x}) D^n(\vec{x}') D^n(0)\rangle = \frac{c_{nnn}}{|\vec{x}|^d |\vec{x}'|^d |\vec{x}-\vec{x}'|^d}$$

3d

- aR <TT> (?)
- $b\,\mathrm{tr}\,\hat{K}^2$ $\quad b = \frac{\pi^2}{8} c_{nn}$

Argument analogous to Osborn and Petkou's result for *c* in 4d. (Fails for *a* in 3d because R is topological.)

4d

- $b_1\,\mathrm{tr}\,\hat{K}^3$ $\quad b_1 = \frac{2\pi^3}{35} c_{nnn}$
- $b_2 K^{AB} W_{AnBn}$ $\quad b_2 = \frac{2\pi^4}{15} c_{nn}$

plan:

- where do these results come from?
- what about $b_2 = 8c$?
- conjecture for *a* in 3d

$b_2 K^{AB} W_{nAnB}$ in 4d

Could extract from $\langle D^n \rangle_{W \neq 0}$

We'll do something more roundabout.

$$I^{(b_2)} = \frac{b_2}{16\pi^2} \frac{\mu^\epsilon}{\epsilon} \int_{\partial M} KW$$

We will cancel $\dfrac{\delta^2 I^{(b_2)}}{\delta g_{\mu\nu} \delta g_{\lambda\rho}}$

$$I^{(c)} = \frac{c}{16\pi^2} \frac{\mu^\epsilon}{\epsilon} \int_M W^2$$

against a boundary term in $\dfrac{\delta^2 I^{(c)}}{\delta g_{\mu\nu} \delta g_{\lambda\rho}}$

An order of limits issue spoils the naive relation between b_2 and c.

Aside on Stress Tensor 2-point function with boundary

Depends on a function $\alpha(v)$ of a cross ratio $\quad v = \dfrac{(x-x')^2}{(x-x')^2 + 4x^n x'^n}$

$v \to 1$: boundary limit

$v \to 0$: coincident limit

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(x') \rangle = \frac{1}{(x-x')^8} \left(\frac{4\alpha}{3} I_{\mu\nu,\rho\sigma} + (v^2 \partial_v^2 \alpha) \hat{I}_{\mu\nu,\rho\sigma} - \frac{1}{6}(v\partial_v \alpha)(\hat{\beta}_{\mu\nu,\rho\sigma} + 7\hat{I}_{\mu\nu,\rho\sigma}) \right)$$

same tensor structure that appears in absence of a boundary

precise form of $\hat{\beta}_{\mu\nu,\rho\sigma}$ and $\hat{I}_{\mu\nu,\rho\sigma}$ not so important

essential point is that they can't cancel the b_2 boundary term

$b_2 K^{AB} W_{nAnB}$ in 4d (part 2)

$$I^{(c)} = \frac{c}{16\pi^2}\frac{\mu^\epsilon}{\epsilon}\int_M W^2$$ is valid in the coincident limit.

To reproduce the scale dependence of the 2-pt function in the boundary limit, $v \to 1$, we should use this effective action with a different value of c, in order to match to $\alpha(1)$.

Canceling $\frac{\delta^2 I^{(b_2)}}{\delta g_{\mu\nu}\delta g_{\lambda\rho}}$ against $\frac{\delta^2 I^{(c)}}{\delta g_{\mu\nu}\delta g_{\lambda\rho}}$

then leads to a relation between $\alpha(1)$, i.e. c_{nn}, and b_2.

Why is $b_2 = 8c$ for free fields?

$v \to 1$: boundary limit

$v \to 0$: coincident limit

For free theories $\alpha(v) \sim 1 + v^{2d}$

theory without boundary (the 1)

effect of image points on other side of the boundary (the v^{2d})

$$\Longrightarrow \quad 2\alpha(0) = \alpha(1)$$

$\alpha(0) \sim c$ by the old Osborn-Petkou ('93) argument

Mixed Dimensional QED

$$S = -\frac{1}{4}\int_{\mathcal{M}} \mathrm{d}^4x\, F^{\mu\nu}F_{\mu\nu} + \int_{\partial\mathcal{M}} \mathrm{d}^3x (i\bar{\psi}\not{D}\psi)$$

Gorbar, Gusynin, Miransky '01
S.-J. Rey '07
Kaplan, Lee, Son, Stephanov '09
S. Teber '12

where $D_\mu = \nabla_\mu - igA_\mu$

boundary conditions: $F_{nA} = g\bar{\psi}\gamma_A\psi$

- relation to graphene
- β-function for g vanishes perturbatively
- behavior under electric-magnetic duality (Son '17)
- relation to large N_f QED_3 (Kotikov-Teber '13)

Perturbative corrections to α(1)

In standard Feynman gauge, for Maxwell theory with a boundary

$$\langle A_A(x) A_B(x') \rangle = \delta_{AB} \left(\frac{1}{(x-x')^2} + \frac{1}{(\vec{x}-\vec{x}')^2 + (y+y')^2} \right)$$

(Neumann boundary condition for tangential components.)

The coupling to the boundary produces a small change

$$\langle A_A(x) A_B(x') \rangle = \delta_{AB} \left(\frac{1}{(x-x')^2} + \frac{1 - O(g^2)}{(\vec{x}-\vec{x}')^2 + (y+y')^2} \right)$$

For the stress tensor two point function

$$\implies \alpha(1) = \alpha(0)(2 - O(g^2))$$

Marginal Directions

$\alpha(1)$ and hence b_2 depended on the exactly marginal coupling.

Unlike the situation for the bulk charges a and c in 4d.

Wess-Zumino consistency forces a to be constant along marginal directions.

No such argument for c. However, SUSY fixes c to be a constant, and it's unknown how to construct 4d CFTs with marginal directions but without SUSY.

Conjecture for a in 3d

The coefficients can be written in terms of the stress tensor two point function

$$a_{(3d)} = \frac{\pi^2}{9}\left(\epsilon(1) - \frac{3}{4}\alpha(1) + 3C\right)$$

$$\langle T_{\mu\nu}(\mathbf{x}, y) T_{\rho\sigma}(\mathbf{0}, y')\rangle = A_{\mu\nu,\rho\sigma}(\mathbf{x}, y, y') \frac{1}{|\mathbf{x}|^{2d}} \, ,$$

$$A_{AB,CD}(\mathbf{x}, y, y') = \alpha(v) \frac{d}{d-1} I^{(d)}_{AB,CD} + \left(2\epsilon(v) - \frac{d}{d-1}\alpha(v)\right) I^{(d-1)}_{AB,CD}$$

gives the right answer for free fields

leads to a possible bound $\quad \dfrac{a_{(3d)}}{b} \geq -\dfrac{2}{3}$

Summary of results

$$b \,\mathrm{tr}\, \hat{K}^2$$

$$b_1 \,\mathrm{tr}\, \hat{K}^3$$

$$b_2 K^{AB} W_{AnBn}$$

- Related boundary central charges in three and four dimensional bCFTs to two and three point functions of the displacement operator.
- Argued that in 4d, the relation $b_2 = 8c$ is special to free theories
- Discussed mixed dimensional QED as an example where $b_2 \neq 8c$ and where b_2 depends on a marginal coupling.

Future Projects

- Verify our proposal for b_2 by computing it directly, for mixed QED in a curved space-time.
- Higher codimension defects. (Billo, Goncalves et al. '16)
- Find bounds on these boundary central charges. (Hofman-Maldacena '08)
- Computation of these central charges in AdS/CFT, for example in Janus solutions. (Takayanagi '11, Miao et al., Astaneh et al. '17)
- Models with only boundary interactions, like mixed QED.
- Boundary bootstrap. (Liendo et al. '12)

Larger Vision: Structure of QFT

- Constrain QFT by constraining CFTs
- Provide a more local view of QFT by figuring out how to deal with boundaries.