Title: Poking holes and cutting corners to achieve Clifford gates with the surface code

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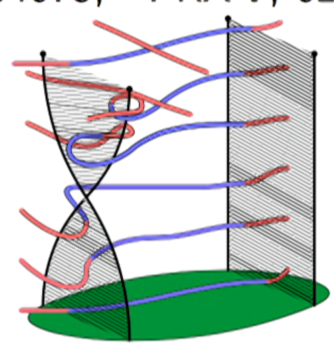
Abstract: <p>The surface code is currently the leading proposal to achieve fault-tolerant quantum computation. Among its strengths are the plethora of known ways in which fault-tolerant Clifford operations can be performed, namely, by deforming the topology of the surface, by the fusion and splitting of codes, and even by braiding engineered Majorana modes using twist defects. Here, we present a unified framework to describe these methods, which can be used to better compare different schemes and to facilitate the design of hybrid schemes. Our unification includes the identification of twist defects with the corners of the planar code. This identification enables us to perform single-qubit Clifford gates by exchanging the corners of the planar code via code deformation. We analyze ways in which different schemes can be combined and propose a new logical encoding. We also show how all of the Clifford gates can be implemented with the planar code, without loss of distance, using code deformations, thus offering an attractive alternative to ancilla-mediated schemes to complete the Clifford group with lattice surgery.</p>

# Poking holes and cutting corners to achieve Clifford gates with the surface code

Benjamin J. Brown

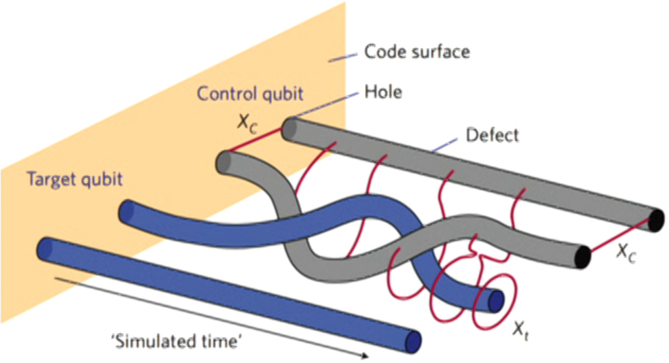
arXiv:1609.04673, PRX **7**, 021029 (2017)



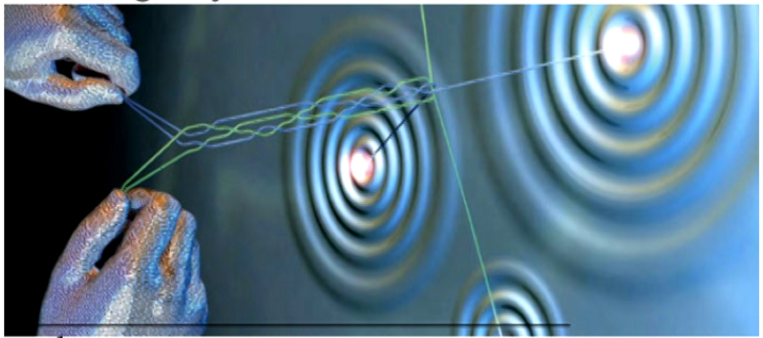
together with K. Laubscher, M. Kesselring and J. Wootton

# Topological quantum computation

Code Deformations by braiding punctures[1]



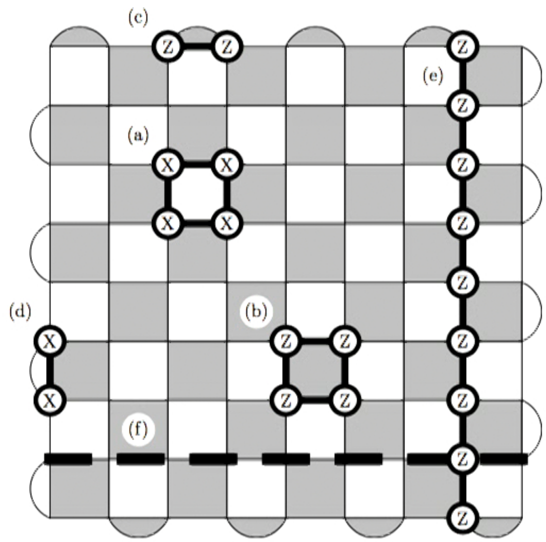

Braiding anyons[2]



[1]Figure from Nat. Phys. **5**, 19 (2009)
[2]Figure from http://www.csee.umbc.edu,

# The planar code

## We first introduce the familiar planar code




- The planar code is a stabilizer code, s.t.

$$S|\psi\rangle = (+1)|\psi\rangle$$

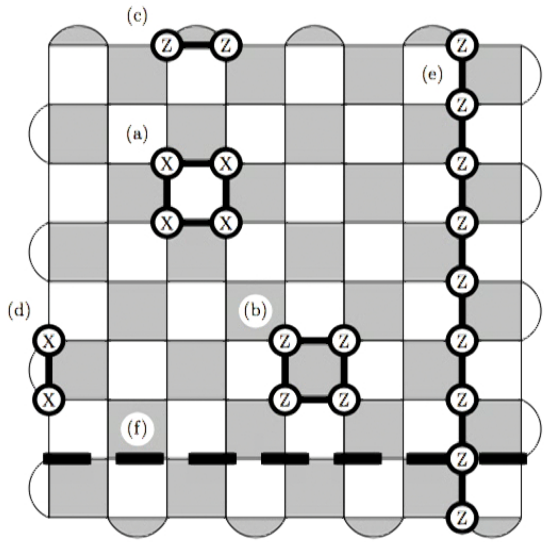
for elements $S \in \mathcal{S}$ of the stabilizer group $\mathcal{S}$ where $|\psi\rangle$ are codewords

- Codewords are manipulated by logical operators $\overline{X}$ and $\overline{Z}$
- (It follows that) logical operators have an unchanged action on the codespace under multiplication by stabilzers

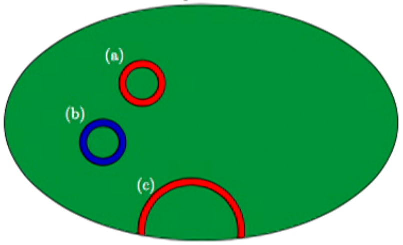
Alexei Kitaev Ann. Phys. (2003), Dennis *et al.* (2002)

# The planar code

## Multiplying (stringlike) logical operators by stabilizers continuously deforms strings

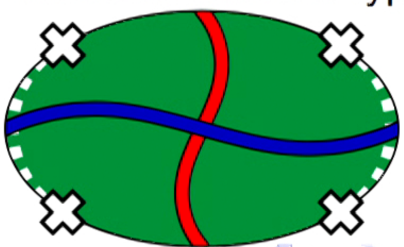



- Stabilizers are represented as closed loops

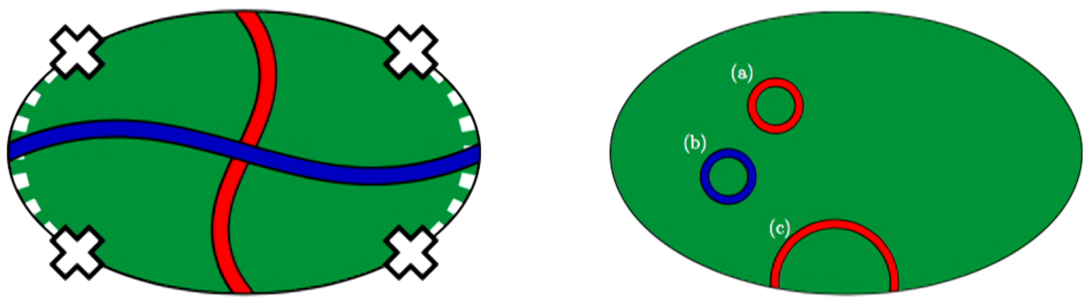



where red(blue) strings indicate strings of Pauli-Zs(Pauli-Xs)

- We also require different (rough and smooth) boundaries to terminate different types of strings

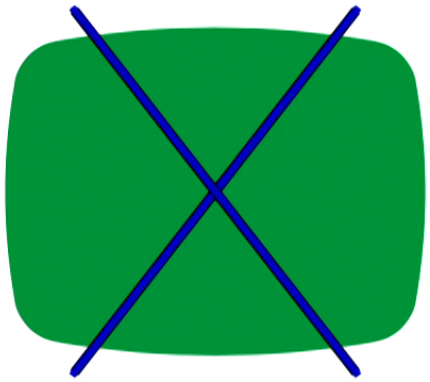


# Strings can be interpreted as world lines of particles

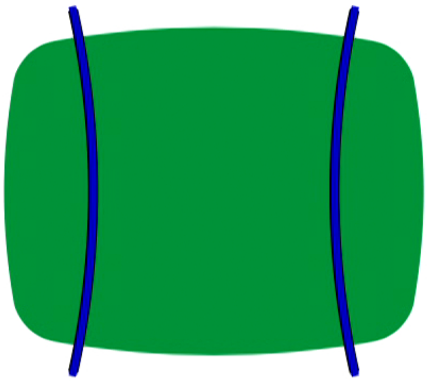


## Strings can be interpreted as world lines of particles

Particles of the same type have bosonic exchange statistics

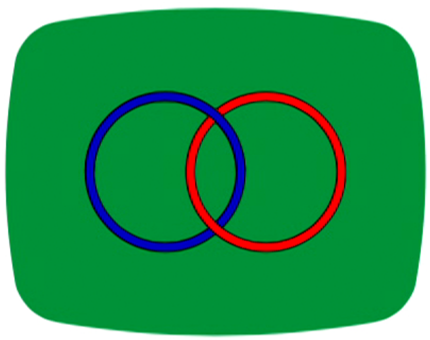


$$= (+1)$$

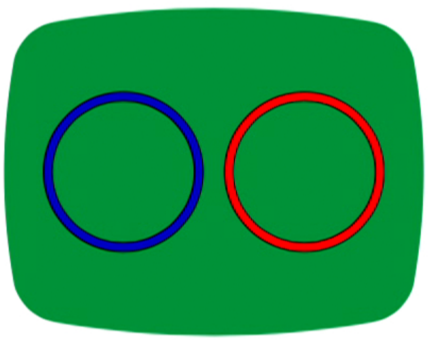


## Strings can be interpreted as world lines of particles

Exchanging two particles of different types give non-trivial exchange statistics ($e$-charges and $m$-charges)

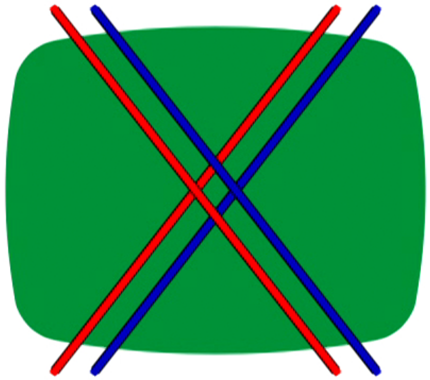


$$= (-1)$$

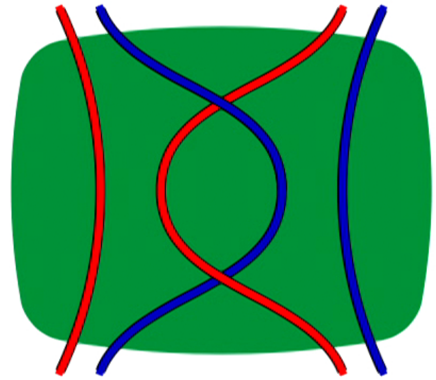


## Strings can be interpreted as world lines of particles

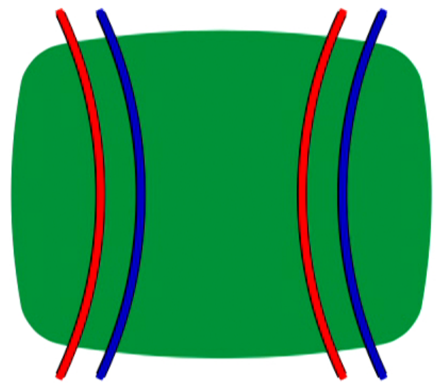
Composite excitations behave like fermions



$$= (+1)$$



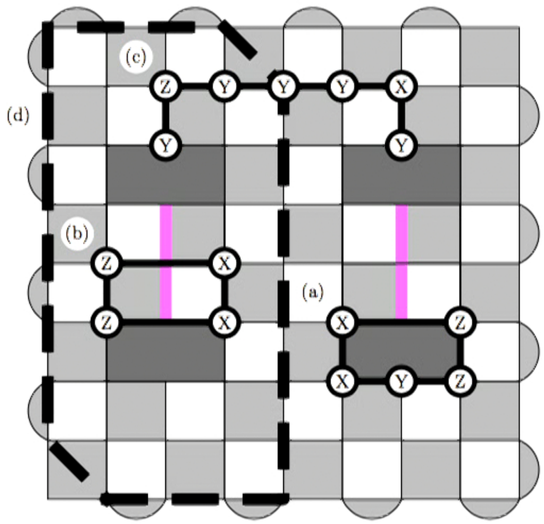
$$= (-1)$$



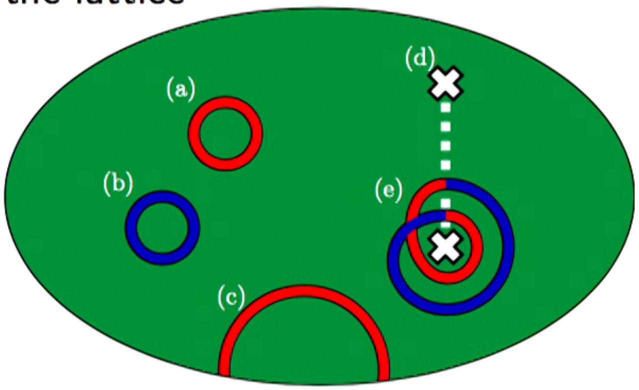
(This follows from facts given in the previous two slides)

# We can also encode qubits using twist defects

## Dislocations change the string type from X to Z, and their end points are Majorana modes



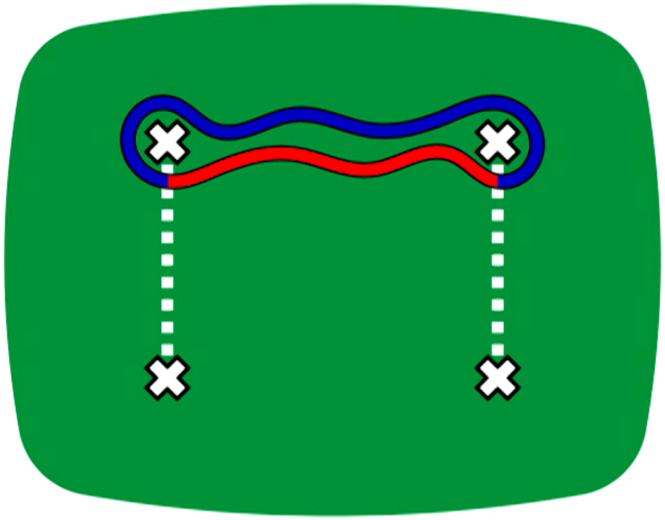

- We will mostly work with this diagrammatic langauge away from the lattice




- Dislocation lines change blue strings to red strings and vice versa.

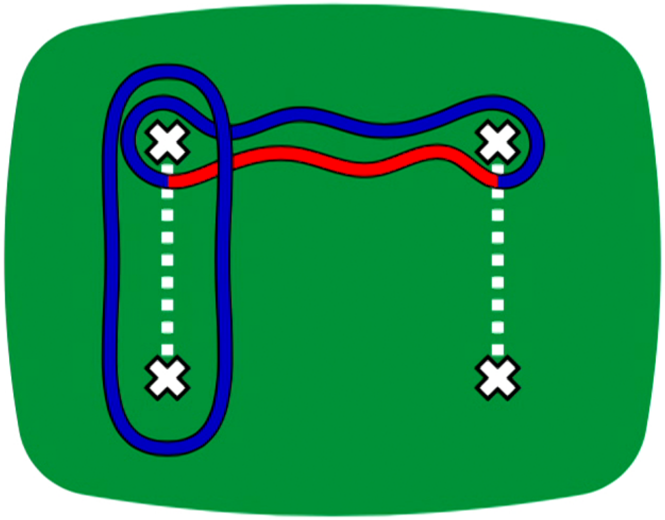
# Interpreting twist defects as Majorana modes

Twist defects can absorb fermions



# Interpreting twist defects as Majorana modes

We can only measure the charge parity of pairs of twists



## Interpreting twist defects as Majorana modes

With these observations we see that twist defects have the fusion rules of Ising anyons (Majorana modes)

With fermionic operators

$$\{a^\dagger, a^\dagger\} = 0, \quad \{a, a^\dagger\} = 1$$

expressed as Majoranas

$$\gamma_j = \gamma_j^\dagger, \quad \{\gamma_j, \gamma_k\} = 2\delta_{j,k}$$

such that

$$a^\dagger = \gamma_1 - i\gamma_2, \quad a = \gamma_1 + i\gamma_2$$

for $n = 0, 1$ we get

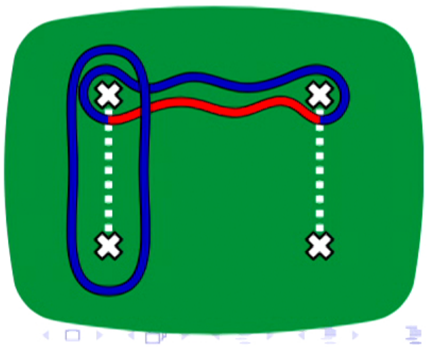
$$a^\dagger a|n\rangle = 2(1 + i\gamma_1\gamma_2)|n\rangle = n|n\rangle$$

With four Majorana modes, $\gamma_1$, $\gamma_2$, $\gamma_3$, $\gamma_4$, we have Pauli operators

$$Z = i\gamma_1\gamma_2$$

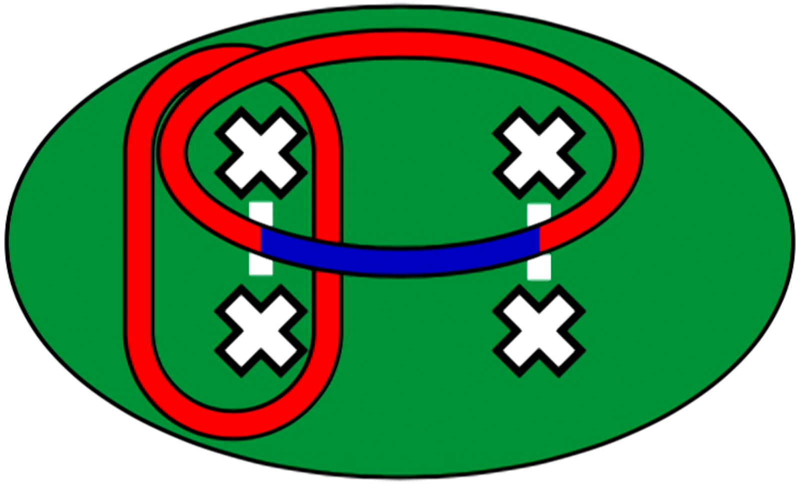
$$Y = i\gamma_2\gamma_3$$

$$X = i\gamma_1\gamma_3$$



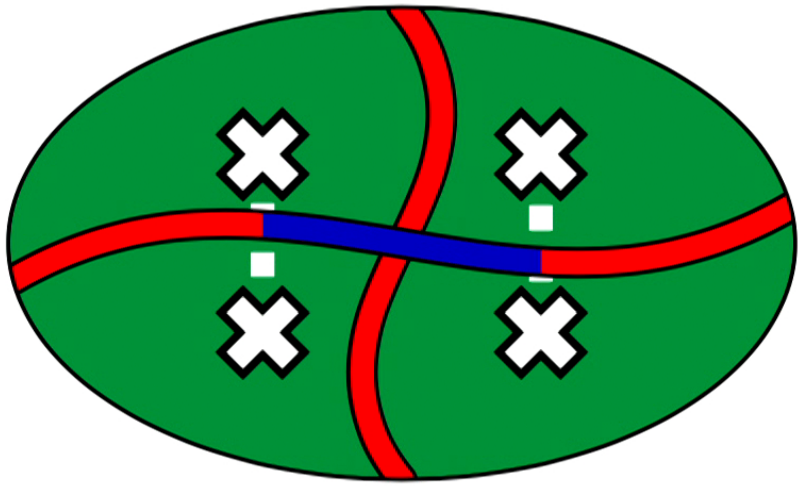
# In the corners of the planar code

We consider four twist defects on the surface code



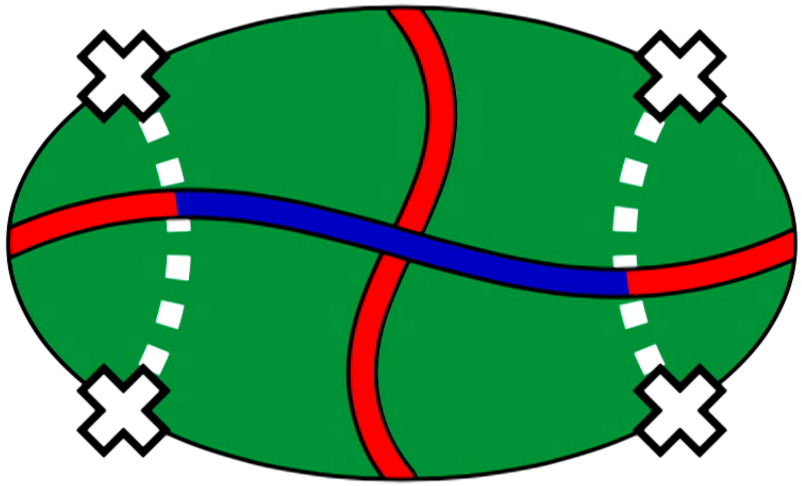
# In the corners of the planar code

We deform the logical operators such that they terminate at the lattice boundary



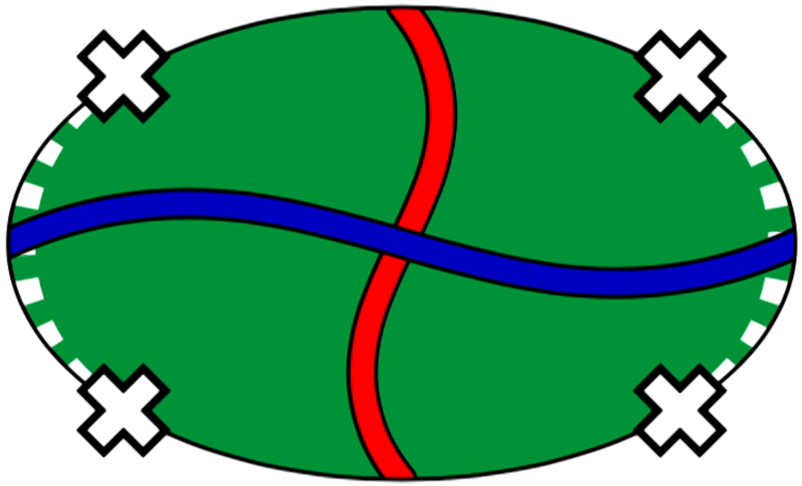
# In the corners of the planar code

The physics of the previous model is unchanged if we move the defects to the boundary



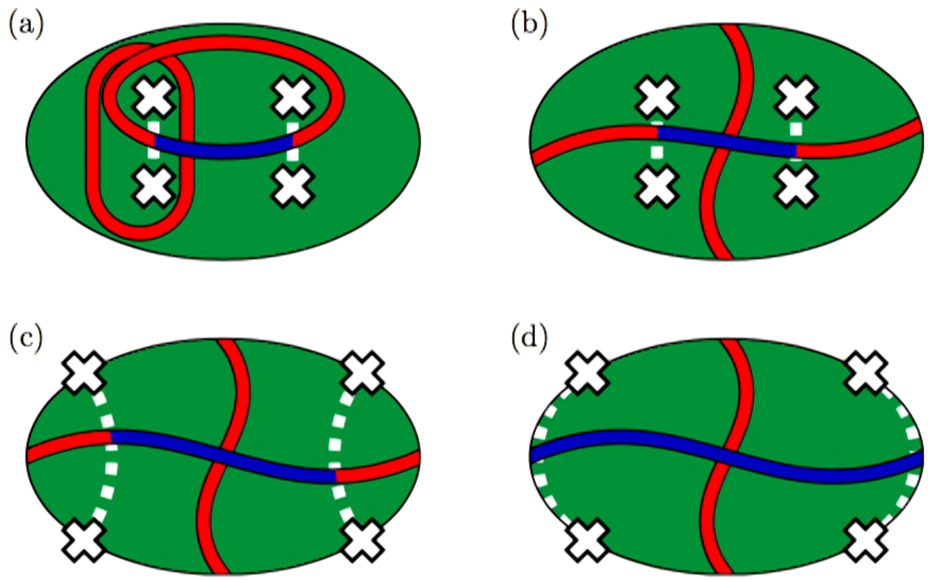
# In the corners of the planar code

Moreover, we can move the dislocation lines to the boundary to recover the planar code



# In the corners of the planar code

planar code corners $\Leftrightarrow$ Majorana modes



# Braiding corners

## We can move holes into the bulk by code deformation