Title: Asymptotic charges from soft factorizations theorems

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Abstract: <p>There is a growing list of examples where soft factorization theorems in scattering amplitudes can be understood as Ward identities of asymptotic charges. I will review some of these, with emphasis on cases that are not associated to usual conservation laws: leading scalar, subleading photon and sub-subleading graviton soft theorems.</p>

Asymptotic charges and soft factorizations theorems

Miguel Campiglia

Universidad de la República, Uruguay

Perimeter Institute, October 2017

Based on works in collaboration with:
Alok Laddha (Chennai Mathematical Institute)
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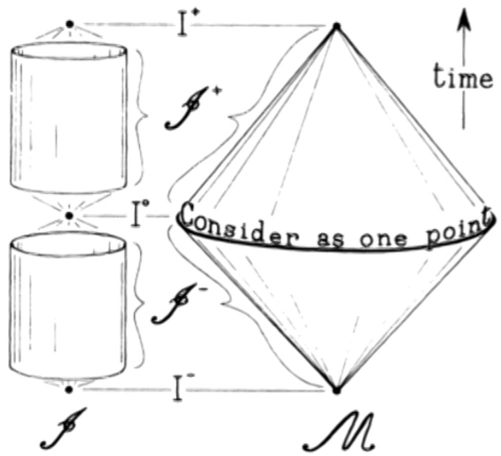
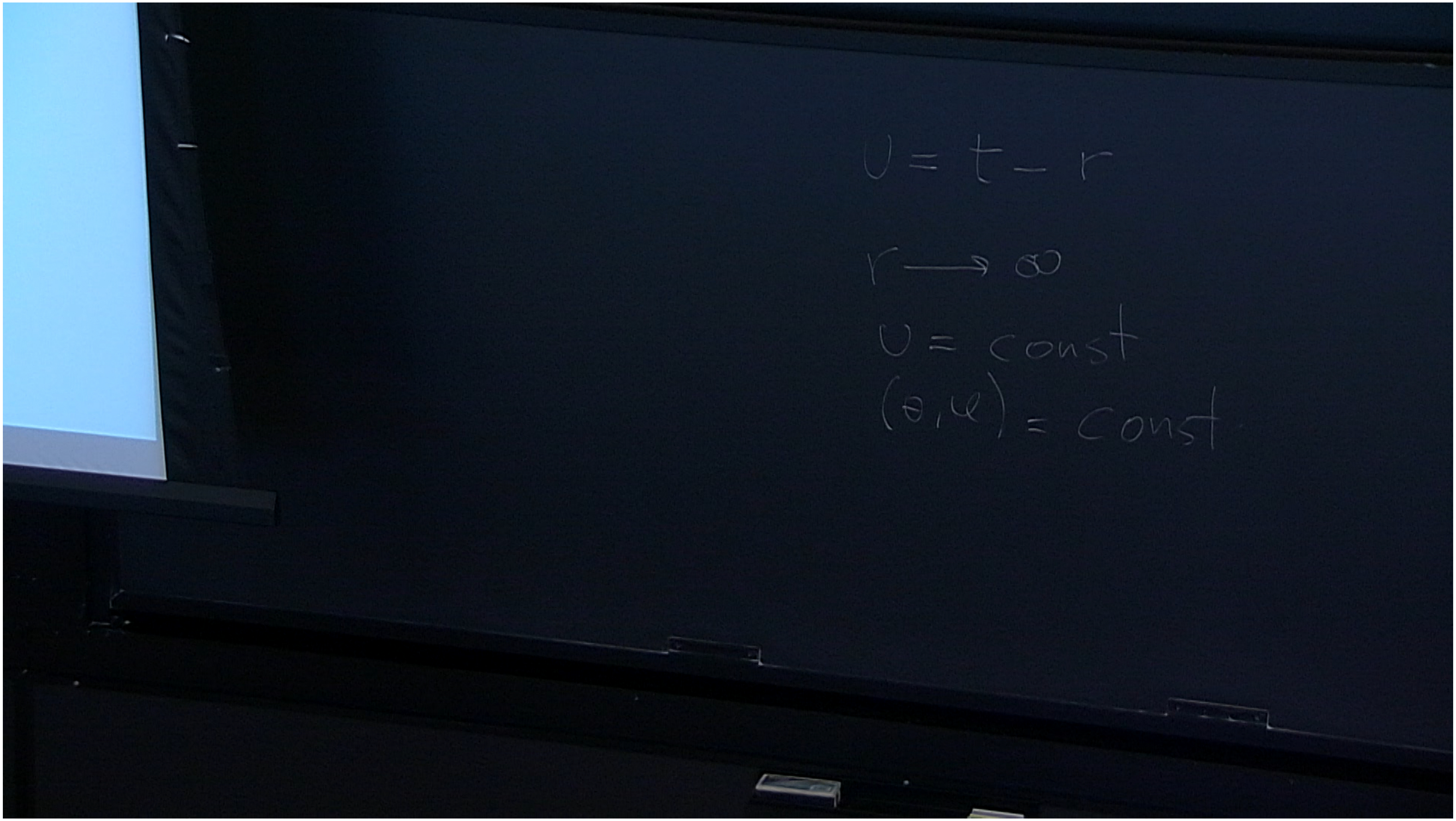
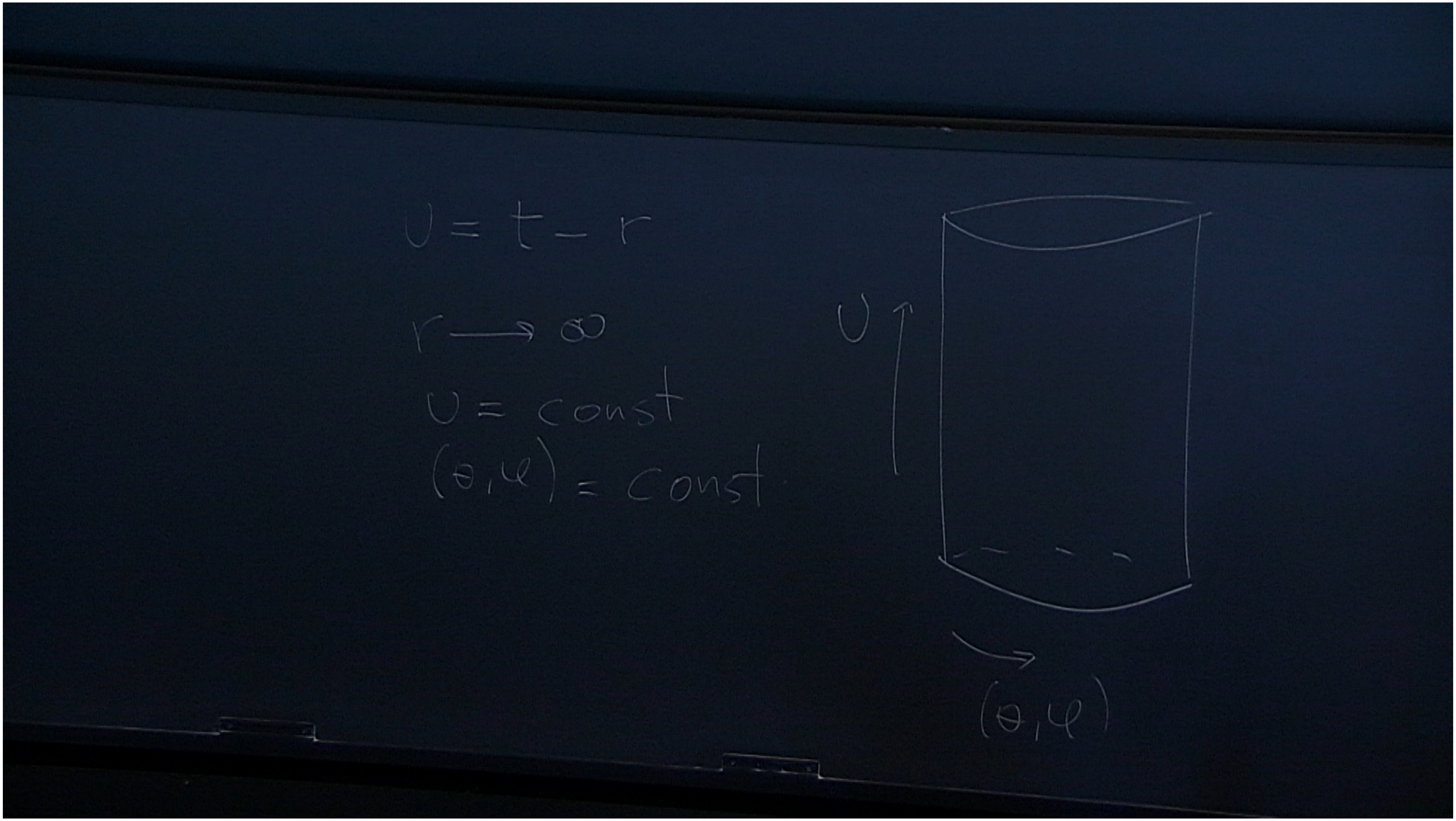
Introduction

- Penrose description of asymptotically flat spacetimes



(Phys Rev Lett 1963)


u = t - r
r ⟶ ∞
u = const
(θ, φ) = const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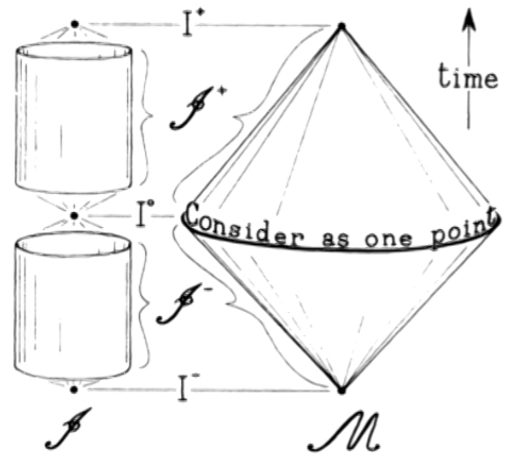


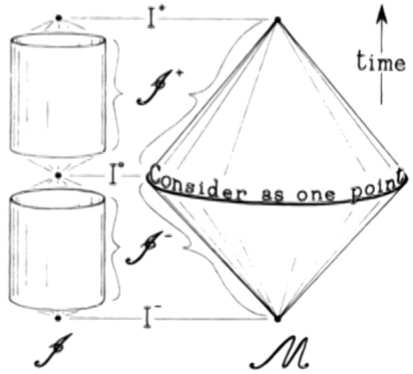
(Phys Rev Lett 1963)

- Penrose's list of motivations include:
  "... (6) geometric derivation of the Bondi-Metzner-Sachs asymptotic symmetry group. A longer term aim of this approach is for a covariant S-matrix theory incorporating gravitation."

- S-matrix for massless fields:

$$\mathcal{S} : \mathcal{H}[\mathscr{I}^-] \to \mathcal{H}[\mathscr{I}^+]$$

- Approach developed in Ashtekar's "Asymptotic Quantization" (1981)



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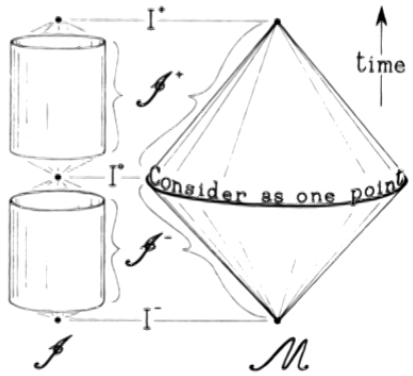

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infinite dimensional generalization of energy-momentum

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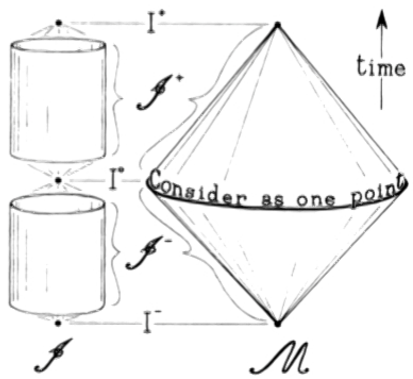

$\Longrightarrow$ BMS supertranslation charges $\mathcal{Q}_{\mathsf{ST}}$

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- Strominger et.al. 2014:

$\mathcal{Q}_{\mathsf{ST}}\mathcal{S} = \mathcal{S}\mathcal{Q}_{\mathsf{ST}} \iff$ Weinberg's soft graviton theorem (1965)

infinite dim generalization of conservation of energy-momentum

- Many works followed establishing similar relations:

$$\text{“}\mathcal{QS} = \mathcal{SQ} \iff \text{soft factorization”}$$

He, Lysov, Mitra, Strominger, Kapec, Pasterski, Porfyriadis, Mohd, MC, Laddha, Pate, Avery, Schwab, Dumitrescu, Low, Mao, Conde, Campoleoni, Francia, Heissenberg, Coito, Mizera, Wu, Hamada, Sugishita, . . .

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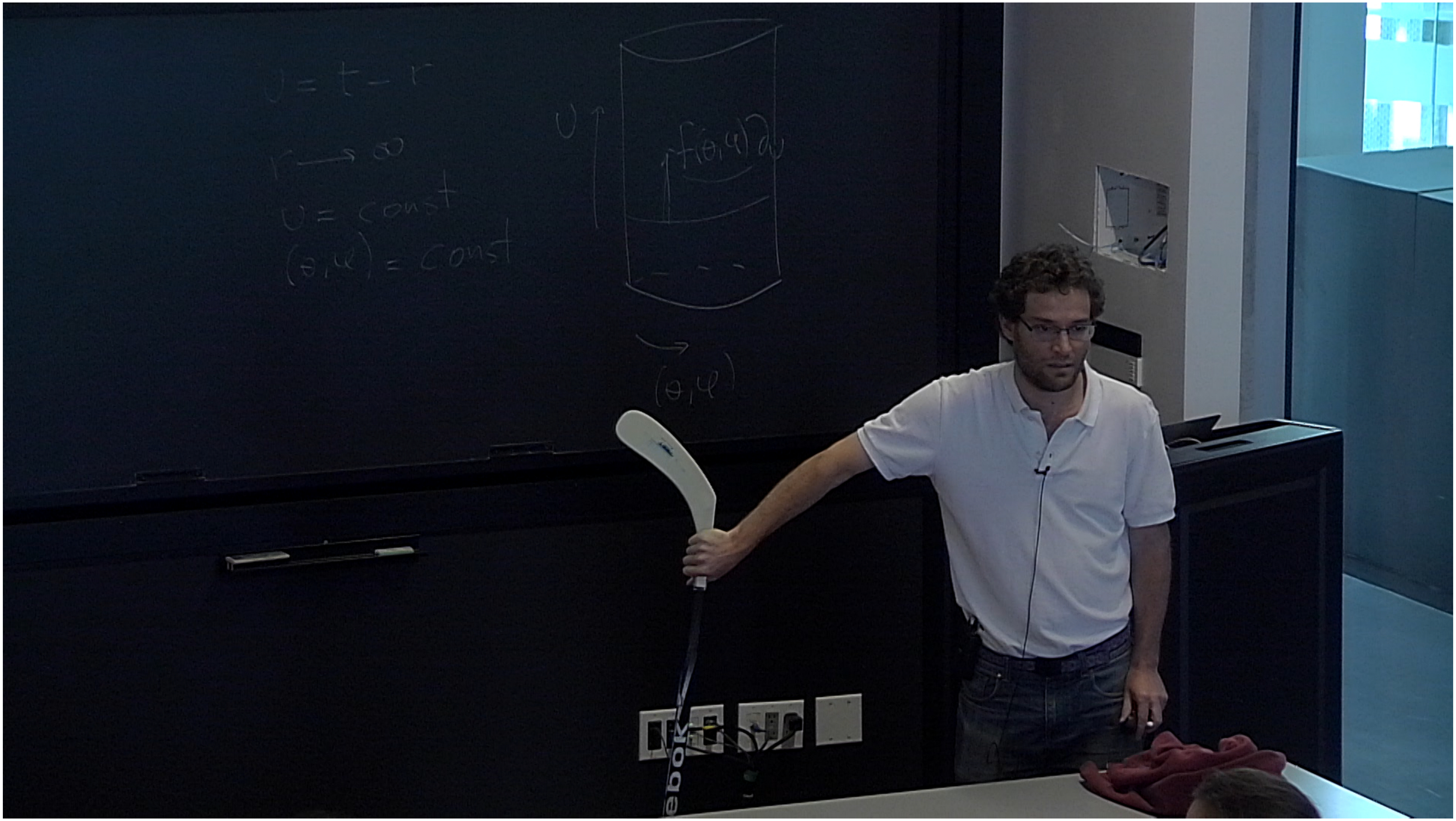
- In this talk I will review some of these examples.

I will restrict attention to the simplest case of 4d tree-level amplitudes of massless particles. But many extensions to this case are known.



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Soft factorizations (Low '58, Weinberg '65, Strominger-Cachazo '14, . . .)

$$\mathcal{A}_{n+1}(\vec{p}_1, \ldots, \vec{p}_n, \vec{q}) \overset{|\vec{q}|\to 0}{=} \ldots$$

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- Terms in blue:
  - Gauge invariance $\Longrightarrow$ conservation law
- Terms in red:
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  - Gauge invariance $\Longrightarrow$ conservation law
  - Universal
- Terms in red:
  - Gauge-invariant $\Longrightarrow$ do not imply conservation laws
  - $S^{(-1)}_{\text{scalar}}$ non-universal. $S^{(0)}_{\text{photon}}$ and $S^{(1)}_{\text{grav}}$ are universal pieces of more general factorizations (Elvang, Jones and Naculich '16; Laddha and Sen '17)

$S^{(-1)}_{\text{photon}}$ and $S^{(-1)}_{\text{grav}}$ factorizations

- Weinberg's soft photon theorem:

$$\mathcal{A}_{n+1}(\vec{p}_1,\ldots,\vec{p}_n;(\vec{q},\varepsilon_\mu)) \overset{|\vec{q}|\to 0}{=} \left(\sum_{i=1}^{n} e_i \frac{\varepsilon_\mu p_i^\mu}{q \cdot p_i}\right)\mathcal{A}_n(\vec{p}_1,\ldots,\vec{p}_n)+O(|\vec{q}|^0)$$

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6 / 15

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Gauge invariance $\implies$ conservation of linear momentum:

$$\mathcal{A}_{n+1}(\varepsilon_{\mu\nu} = c_\mu q_\nu) = 0 \implies \left(\sum_{i=1}^{n} c_\mu p_i^\mu \right) = 0$$

$\mathcal{Q}$'s for $S^{(-1)}_{\text{photon}}$ and $S^{(-1)}_{\text{grav}}$ factorizations

- Strominger et.al:

$$\lim_{\omega\to 0} \omega \mathcal{A}_{n+1}(\omega q) = \left(\omega S^{(-1)}_{\text{photon}}\right) \mathcal{A}_n \iff \mathcal{Q}^{U(1)}[\lambda]\mathcal{S} = \mathcal{S}\mathcal{Q}^{U(1)}[\lambda]$$

$$\lim_{\omega\to 0} \omega \mathcal{A}_{n+1}(\omega q) = \left(\omega S^{(-1)}_{\text{grav}}\right) \mathcal{A}_n \iff \mathcal{Q}^{\text{grav}}[\lambda]\mathcal{S} = \mathcal{S}\mathcal{Q}^{\text{grav}}[\lambda]$$

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- $\mathcal{Q}^{U(1)}[\lambda]$ charge for 'large' $U(1)$ gauge parameter $\lambda(\Omega)$
- $\mathcal{Q}^{\text{grav}}[\lambda] \equiv \mathcal{Q}_{\text{ST}}$ for supertranslation $\lambda(\Omega)\partial_u$

- For both charges

$$\mathcal{Q}[\lambda] = \mathcal{Q}_{\mathsf{hard}}[\lambda] + \mathcal{Q}_{\mathsf{soft}}[\lambda]$$

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- 'Hard' part:

$$\mathcal{Q}_{\text{hard}}[\lambda] = \int_{\mathscr{I}} \lambda(\Omega)\, \rho(u, \Omega)$$

with

$$\begin{aligned} \rho^{U(1)} &= \text{electric charge density flux across } \mathscr{I} \\ \rho^{\text{grav}} &= \text{energy density flux across } \mathscr{I} \end{aligned}$$

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$$\mathcal{Q}^{U(1)}_{\text{soft}}[\lambda] = \int_{\mathscr{I}} D^A \lambda \partial_u A_A, \quad \mathcal{Q}^{\text{grav}}_{\text{soft}}[\lambda] = \int_{\mathscr{I}} D^A D^B \lambda \partial_u C_{AB}$$

8 / 15

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$$\mathcal{Q}^{U(1)}_{\text{soft}}[\lambda] = 0 \iff \lambda = \text{const.} \quad \iff \quad \mathcal{Q}^{U(1)} \propto \text{electric charge}$$

$$\mathcal{Q}^{\text{grav}}_{\text{soft}}[\lambda] = 0 \iff \lambda = Y^m_{\ell=0,1} \quad \iff \quad \mathcal{Q}^{\text{grav}} \propto \text{linear momentum}$$

Other factorizations

- Similarly: $S^{(0)}_{\text{grav}}$ factorization $\iff$ infinite dim generalization of angular momentum conservation. The $\mathcal{Q}$'s are associated to 'superrotations' $\xi = X^A \partial_A$

$$\mathcal{A}_{n+1}(\vec{p}_1, \ldots, \vec{p}_n, \vec{q}) \overset{|\vec{q}| \to 0}{=} \left(S^{(-1)}_{\text{grav}} + S^{(0)}_{\text{grav}} + S^{(1)}_{\text{grav}}\right) \mathcal{A}_n + O(|\vec{q}|^2)$$

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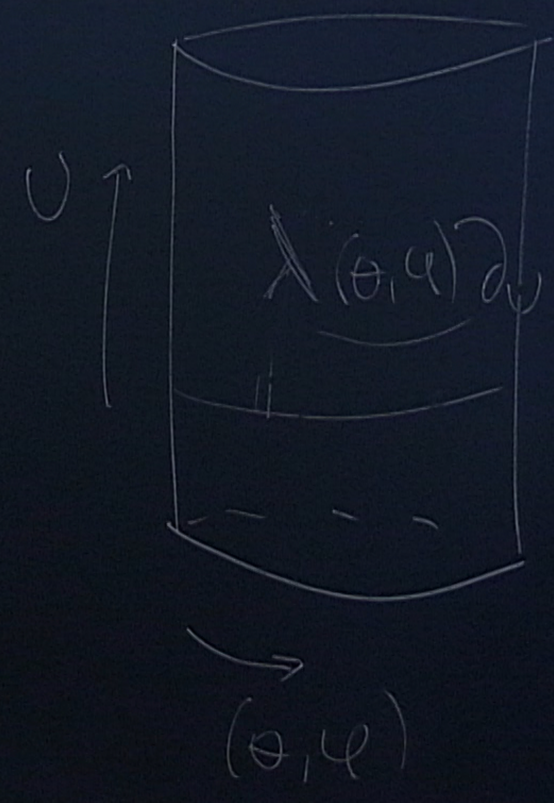

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- One can obtain $\mathcal{Q}$'s by 'reverse engineering' the soft theorems. But this strategy does not guarantee a symmetry interpretation for $\mathcal{Q}$

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Charges for $S^{(-1)}_{\text{scalar}}$ factorization (MC, Coito, Mizera '17)

- A scalar analogue of Weinberg's soft theorem

$$\mathcal{A}_{n+1}(\vec{p}_1, \ldots, \vec{p}_n, \vec{q}) \overset{|\vec{q}|\to 0}{=} \left(\sum_{i=1}^{n} \frac{g}{2} \frac{1}{q \cdot p_i} \right) \mathcal{A}_n(\vec{p}_1, \ldots, \vec{p}_n) + O(|\vec{q}|^0)$$

(for instance in $L = -\frac{1}{2}(\partial\varphi)^2 + \frac{g}{3!}\varphi^3$)

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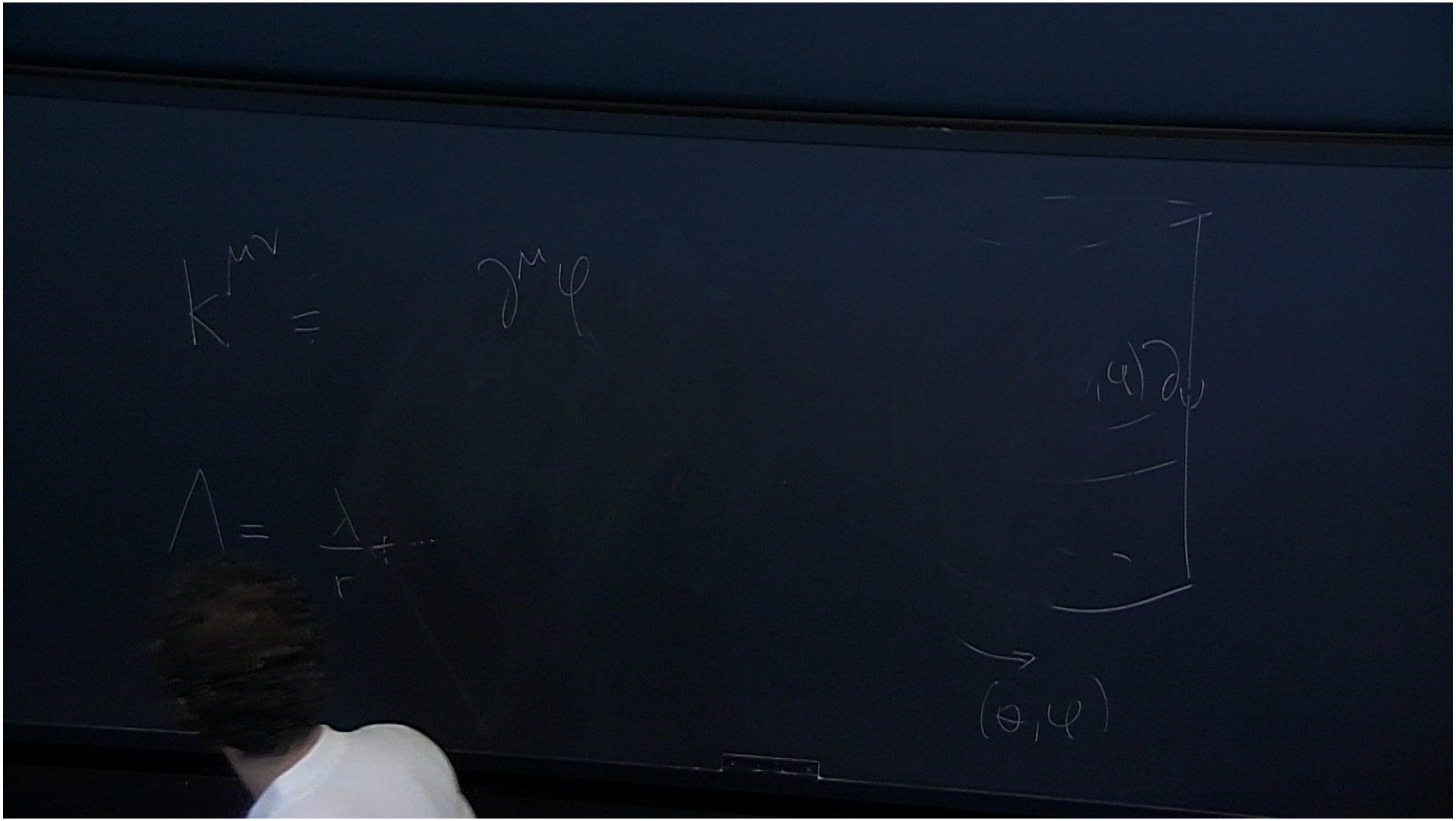
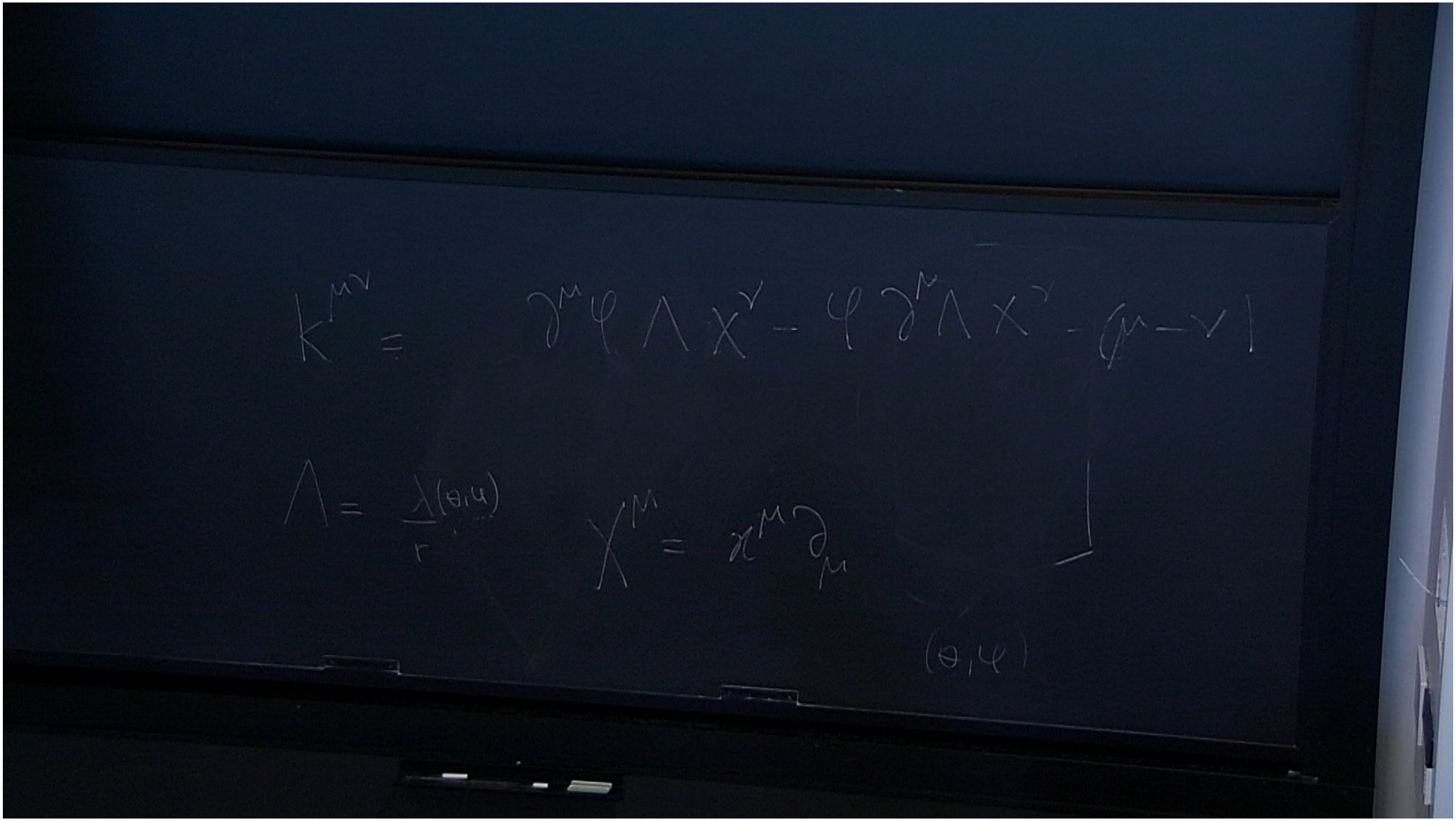

K^μν =
∂^μ φ
Λ = λ/r + ...
(θ, φ)


$K^{\mu\nu} = \partial^\mu \varphi \Lambda X^\nu - \varphi \partial^\mu \Lambda X^\nu - (\mu - \nu)$
$\Lambda = \frac{\lambda(\theta,\varphi)}{r}$
$X^\mu = x^\mu \partial_\mu$
(θ,φ)

Charges for $S^{(0)}_{\text{photon}}$ factorization (Lysov, Pasterski, Strominger '14)

- Subleading soft photon factorization:

$$\lim_{\omega \to 0} \partial_\omega [\omega \mathcal{A}_{n+1}(p_1, \ldots, p_n; \omega q)] = \left(\sum_{i=1}^{n} e_i \frac{\varepsilon_\mu q_\nu J_i^{\mu\nu}}{q \cdot p_i} \right) \mathcal{A}_n(p_1, \ldots, p_n)$$

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- Charges are now parametrized by sphere vector fields $Y^A(\Omega)$:

$$\mathcal{Q}[Y] = \int_{\mathscr{I}} dud^2\Omega((D\cdot Y)(D\cdot A)+(D\times Y)(D\times A))+\mathcal{Q}_{\text{hard}}[Y]$$

$$D\cdot Y \equiv D_A Y^A\ ; \quad D\times Y \equiv \epsilon^{AB} D_A Y_B$$

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- MC, Laddha '16: $\mathcal{Q}[Y]$ can be understood as a large $U(1)$ gauge charge for

$$\Lambda(r, u, \Omega) = r\lambda(\Omega) + \dots$$

$$\implies \mathcal{Q}[\lambda] + \tilde{\mathcal{Q}}[\tilde{\lambda}] = \mathcal{Q}[Y^A = D^A\lambda + \epsilon^{AB} D_B \tilde{\lambda}]$$

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- It also suggest why soft photon theorems stop at subleading order

$S^{(0)}_{\text{photon}}$ charges as large $U(1)$ charges

- Work in Lorenz gauge $\implies \Box\Lambda = 0$
  and take $\Lambda(r, u, \Omega) = r\lambda(\Omega) + \dots$

$$\mathcal{Q}[\Lambda] := \int_{\Sigma_t} dS_a \partial_b(\Lambda F^{ab}) \overset{t\to\infty}{=} t\,\overset{(1)}{\mathcal{Q}}[\lambda] + \overset{(0)}{\mathcal{Q}}[\lambda]$$

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- $\overset{(1)}{\mathcal{Q}}[\lambda]$ coincides with "$S^{(-1)}_{\text{photon}}$ charge" $\implies [\overset{(1)}{\mathcal{Q}}[\lambda], \mathcal{S}] = 0$

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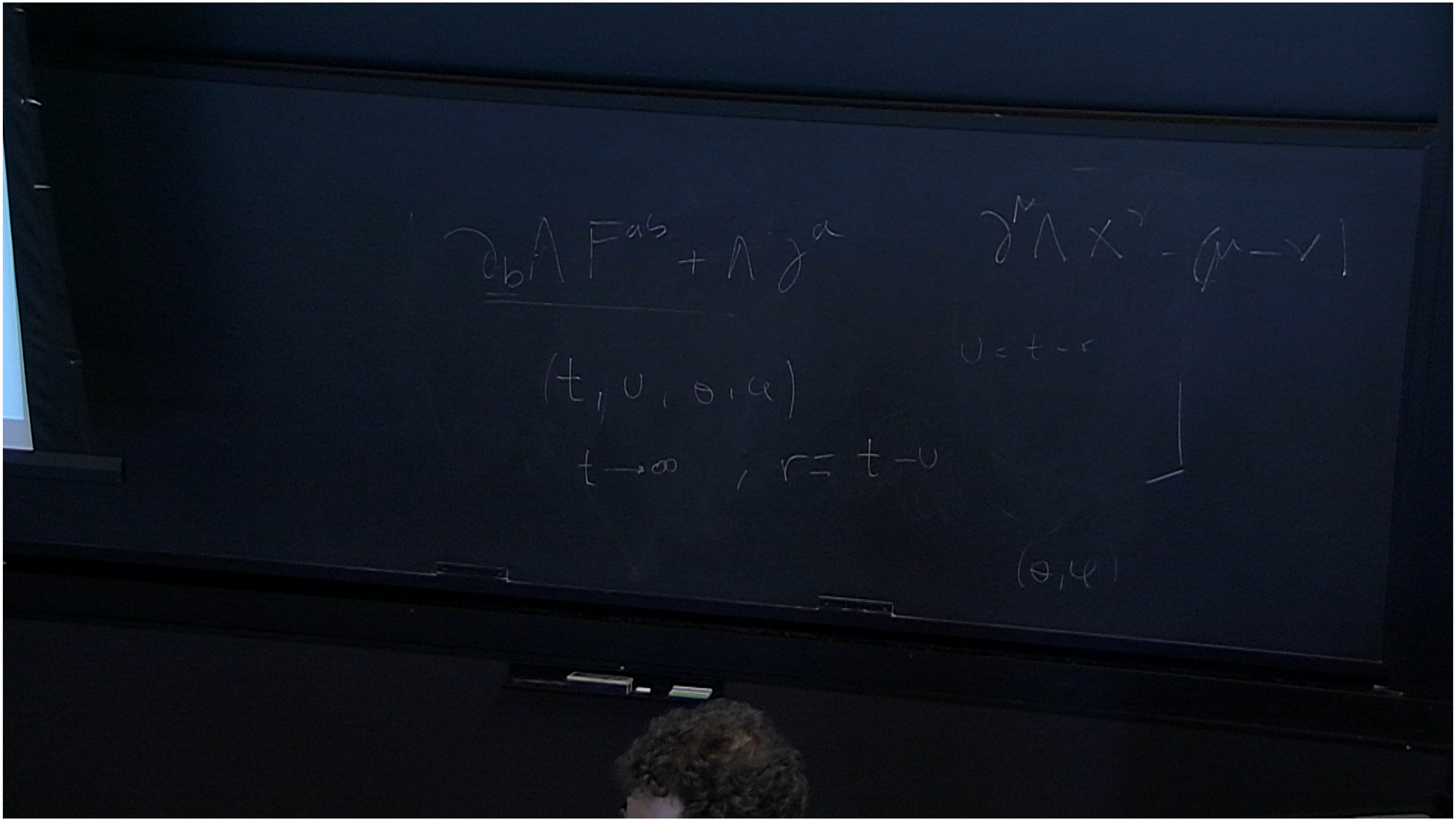
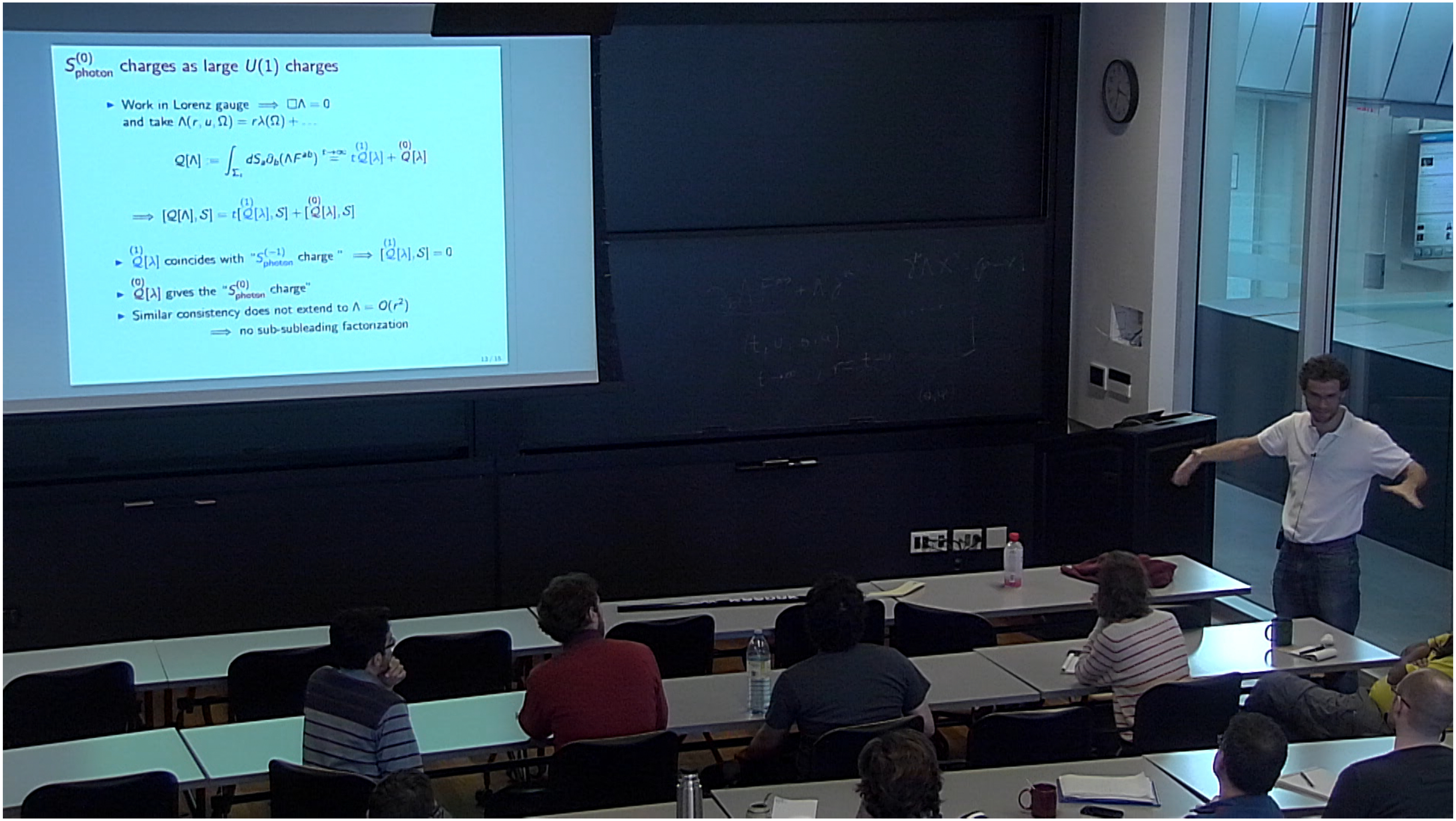

∂bΛ F^ab + Λ j^a
∂^μ Λ x^ν - (μ→ν)
u = t - r
(t, u, θ, φ)
t → ∞ , r = t - u
(θ, φ)



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13 / 15

Discussion

- Things to complete upon:
  - Higher dimensional $\mathcal{Q}$'s for $S^{(0)}_{\text{grav}}$, $S^{(0)}_{\text{photon}}$, $S^{(1)}_{\text{grav}}$
  - Inclusion of massive particles on $\mathcal{Q}$'s for $S^{(0)}_{\text{photon}}$, $S^{(1)}_{\text{grav}}$
  - $S^{(1)}_{\text{grav}}$: correction to $\mathcal{Q}$ from non-minimal coupling

- Some conceptual questions:
  - Large $O(r)$ gauge/diffeos transformations lie outside standard phase space description. Can one improve on this?
  - Is there a symmetry interpretation for $\mathcal{Q}^{\text{scalar}}$?

- Could the $\mathcal{Q}$'s be useful beyond soft theorems?