Title: Quantum Mechanics Without Wavefunctions

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Abstract: Seven years ago, the first paper was published [1] on what has come to be known as the â€œMany Interacting Worldsâ€• (MIW) interpretation of quantum mechanics (QM) [2,3,4]. MIW is based on a new formulation of QM [1,5,6], in which the wavefunction Î¨(t, x) is discarded entirely. Instead, the quantum state is represented as an ensemble, x(t, C), of quantum trajectories or â€œworlds.â€• Each of these worlds has well-defined real-valued particle positions and momenta, and is thereby classical-like. Unlike a classical ensemble, however, nearby trajectories/worlds can interact with each other dynamically, giving rise to quantum effects. In this respect, MIW is very different from the Everett Many-Worlds Interpretation (MWI); another key difference is that no world branching occurs.

The MIW approach offers a direct â€œrealistâ€• description of nature that may be beneficial in interpreting quantum phenomena such as entanglement, measurement, spontaneous decay, etc. It provides a useful analysis of MWI, explaining how the illusion of world branching emerges in that context. Moreover, x(t, C) satisfies a trajectory-based action principle, which allows quantum theory (via the Euler-Lagrange equation and Noetherâ€™s theorem) to be placed on the same footing as classical theories. In this manner, a straightforward relativistic generalization can also be obtained [7,8], which offers a notion of global simultaneity even for accelerating observers. Whereas the original MIW theory is fully consistent with Schroedinger wave mechanics, the more recently developed flavors offer the promise of new experimental predictions. These and other developments, e.g. for many dimensions, multiple particles, and spin, may also be discussed.

[1] B. Poirier, Chem. Phys. 370, 4 (2010).
[2] M. J. W. Hall, D.-A. Deckert, and H. Wiseman, Phys. Rev. X 4, 041013 (2014)
[3] B. Poirier, Phys. Rev. X, 4, 040002 (2014).
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[5] P. Holland, Ann. Phys. 315, 505 (2005).
[6] J. Schiff and B. Poirier, J. Chem. Phys. 136, 031102 (2012).
[7] B. Poirier, arXiv:1208.6260 [quant-ph], (2012).
[8] H.-M. Tsai and B. Poirier, EmQM15: Emergent Quantum Mechanics 2015, ed. G. GrÃ¶ssing, (J. Physics, IOP, 2016) 701, 012013.
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T E X A S T E C H U N I V E R S I T Y

October 24, 2017

Quantum Mechanics Without Wavefunctions

aka *"Many Interacting Worlds"*

Bill Poirier

Texas Tech University

Quantum Foundations Seminar

Perimeter Institute

Waterloo, Ontario, CANADA

TEXAS TECH UNIVERSITY

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Quantum Mechanics Without Wavefunctions aka *"Many Interacting Worlds"*

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J.-M. Rost (MPI Dresden), Christoph Meier (Toulouse)

H. Wiseman, D.-A. Deckert, M. Hall (discrete MIW)

Charles Sebens, Lee Smolin (max variety)

Chemical Physics

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15. Bohmian mechanics without pilot waves

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Bill Poirier

In David Bohm's causal/trajectory interpretation of quantum mechanics, a physical system is regarded as consisting of both a particle and a wavefunction, where the latter "pilots" the trajectory evolution...

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Bohmian mechanics without pilot waves

Bill Poirier

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ABSTRACT

In David Bohm's causal/trajectory interpretation of quantum mechanics, a physical system is regarded as consisting of *both* a particle *and* a wavefunction, where the latter "pilots" the trajectory evolution of the former. In this paper, we show that it is possible to discard the pilot wave concept altogether, thus developing a complete mathematical formulation of time-dependent quantum mechanics directly in terms of real-valued trajectories alone. Moreover, by introducing a kinematic definition of the quantum potential,

damental than Eq. (11). Basically, this implies that no quantum effects can be attributed to the behavior of a single trajectory alone. Rather, all quantum behavior in nature is due to an interaction amongst the different trajectories within a given ensemble, with

We conclude with a brief discussion of some of the potential *interpretive* ramifications of the new formulation. In Bohmian mechanics, there is only one system trajectory, whereas the present approach offers an entire ensemble of trajectories. If one presumes objective existence for a single trajectory only, then the remaining trajectories in the ensemble must be regarded as "virtual," in some sense. On the other hand, one might prefer to regard *all* trajectories in the quantum ensemble as equally valid and real. It is hard to imagine how this could be achieved, without positing that each trajectory inhabits a separate world. It must be emphasized, however, that this version of the many worlds interpretation would be *very* different from the standard form [19–21]. In a nutshell, the latter associates

Communication: Quantum mechanics without wavefunctions

Jeremy Schiff[1,a] and Bill Poirier[2,b]

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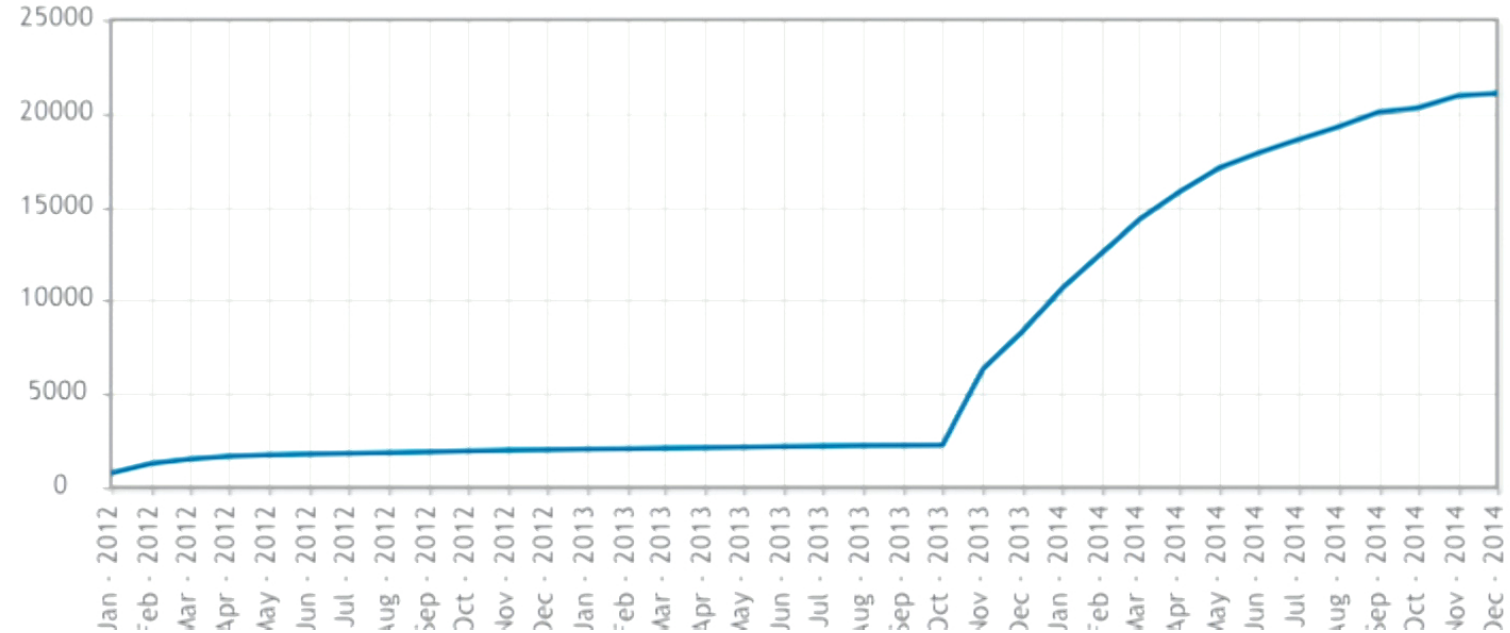

Editorial: Does Research on Foundations of Quantum Mechanics Fit into PRX's Scope?

And we have invited a Commentary by Bill Poirier from Texas Tech University that we hope will enhance your understanding of the paper and of our decision to publish it.

The Editors

The Many Interacting Worlds Approach to Quantum Mechanics

Bill Poirier*, Department of Chemistry and Biochemistry, and Department of Physics, Texas Tech University, Box 41061, Lubbock, Texas 79409-1061

A Commentary on:
Quantum Phenomena Modeled by Interactions between Many Classical Worlds
Michael J. W. Hall, Dirk-André Deckert, and Howard M. Wiseman
Phys. Rev. X, 4, 041013 (2014)

About the Commentary author:

Bill Poirier is Chancellor's Council Distinguished Research Professor and also Barnie E. Rushing Jr. Distinguished Faculty Member at Texas Tech University, in the Department of Chemistry and Biochemistry and also the Department of Physics. He received his Ph.D. in theoretical physics from the University of California, Berkeley, followed by a chemistry research associateship at the University of Chicago. His research interest is in understanding and solving the Schrödinger equation, from both foundational and practical perspectives.

Brief Outline for Remainder of Talk

1. Brief description of what this approach is.
2. First principles derivation: 1D time-independent case
3. Noether's theorem / symplectic structure / numerics
4. Time dependent case
5. Entanglement / measurement / collapse
6. Relativistic generalization.

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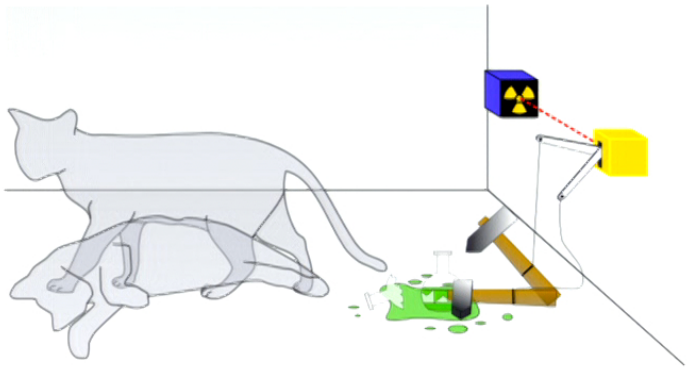
Getting Rid of Ψ Altogether:

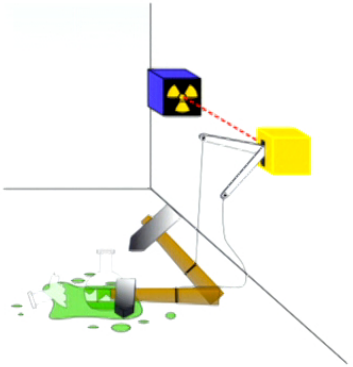
How can that even be possible?

Not , **But**



Ψ

$\not{h}$



Is Ψ Alive or Dead ?

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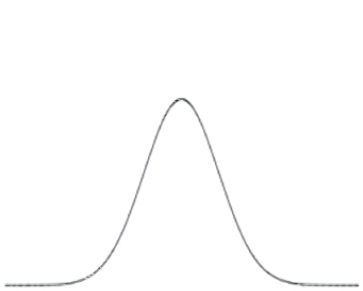
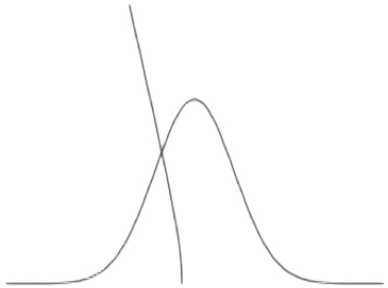
But if not Ψ, then what?

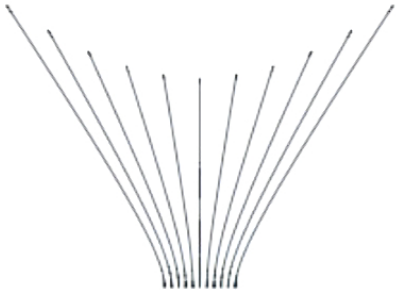
Answer: Trajectories only

- The wavefunction $\Psi(x,t)$ is replaced with an *ensemble* (family) of trajectories, $x(C,t)$.
  - parameter C labels individual trajectories within the ensemble.
  - resembles classical statistical mechanics/trajectory simulations.
- The individual trajectories turn out to be the quantum trajectories of David Bohm. *However…*
- *This is* **NOT** *Bohmian Mechanics!*
  - Bohm uses a *single* trajectory, $x(t)$.
  - Bohm *also* uses the wavefunction, $\Psi(x,t)$.

Copenhagen quantum mechanics	Bohmian mechanics	Quantum trajectory-based formulation (non-relativistic)
Ψ represents the state of the system. TDSE drives evolution of $\Psi(x,t)$.	Ψ and $x(t)$ together represent the state of the system. Ψ leads to quantum potential Q, driving trajectory dynamics via: $m\ddot{x} + \frac{\partial V(x)}{\partial x} + \frac{\partial Q(x,t)}{\partial x} = 0$	There is no Ψ. $x(t, C)$ (trajectory ensemble) alone represents the state of the system, and leads to Q. $x(t, C)$ satisfies its own PDE that replaces the TDSE (with $'$ denoting partial derivative w/ respect to C.) $m\ddot{x} + \frac{\partial V(x)}{\partial x} + \frac{\hbar^2}{4m}\left(\frac{x''''}{x'^4} - 8\frac{x'''x''}{x'^5} + 10\frac{x''^3}{x'^6}\right) = 0$







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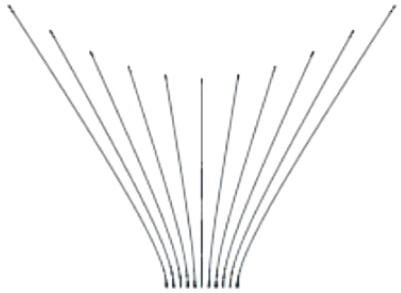
Key Features (especially for relatiistic case):

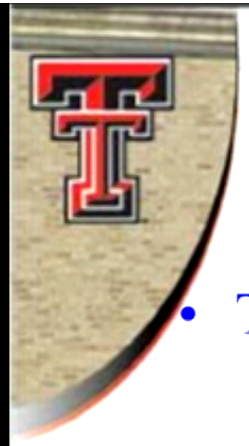
1. x was an *independent* variable, but is now the *dependent* field quantity.
2. PDE 4th order in "space" (C), 2nd order in time (t). C and t not treated on equal footing; *are x and t ?*
3. "Spatial" derivatives = interworld interaction = quantum "weirdness."
4. Quantum potential shows no explicit dependence on C and t (and x if V = const) unlike in Bohm.
5. Ensemble of quantum trajectories foliate spacetime (no crossing trajectories).



Combining Both Constraints

- Either physical constraint by itself leads to a unique set of solution trajectories
  - In general, i.e. for arbitrary choice of $f[x]$ and $g[\dot{x}]$,

    Action extremizing trajectories are not the same as Hamiltonian conserving trajectories
- Satisfying *both* conditions simultaneously is very special:
  - Noether's theorem: explicit t invariance of L implies existence of a conserved energy quantity, denoted E.
  - Our condition: that Noether E be equal to the Hamiltonian H.
  - imposes *severe* restrictions on allowed forms for $f[x]$ and $g[\dot{x}]$.



Functional Form of Q

- Technical Note:
  - action extremization via "generalized" Euler-Lagrange eqn:

$$\left[\frac{\partial L}{\partial x}\right] - \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}}\right] + \frac{d^2}{dt^2}\left[\frac{\partial L}{\partial \ddot{x}}\right] - \cdots = 0$$

- Allowed meromorphic solutions (dynamical laws):

$V[x]$ = completely unconstrained.

$$T[\dot{x}] = (m/2)\dot{x}^2$$

$$Q[\dot{x},\ddot{x},\ldots] = \begin{cases} \Delta E = \text{constant} & \text{order 0 (classical mechanics)} \\ \text{no solutions} & \text{order 1} \\ \text{no solutions} & \text{order 2} \\ -\dfrac{B}{2m}\left(\dfrac{5}{4}\dfrac{\ddot{x}^2}{\dot{x}^4} - \dfrac{1}{2}\dfrac{\dddot{x}}{\dot{x}^3}\right) & \text{order 3 (quantum mechanics, } B = \hbar^2) \end{cases}$$

Reactive Scattering Calculations: Cross Sections & Rates

Time independent

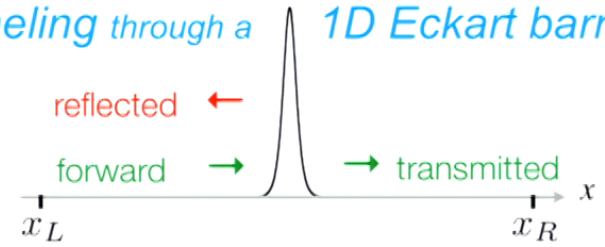
quantum trajectories (#5)

tunneling through a 1D Eckart barrier

Eckart potential:

$V(x) = V_0 \, \mathrm{sech}^2(\alpha x)$

$\alpha = 3.0\, a.u.$
$m = 2000\, a.u.$
$V_0 = 400\, cm^{-1}$



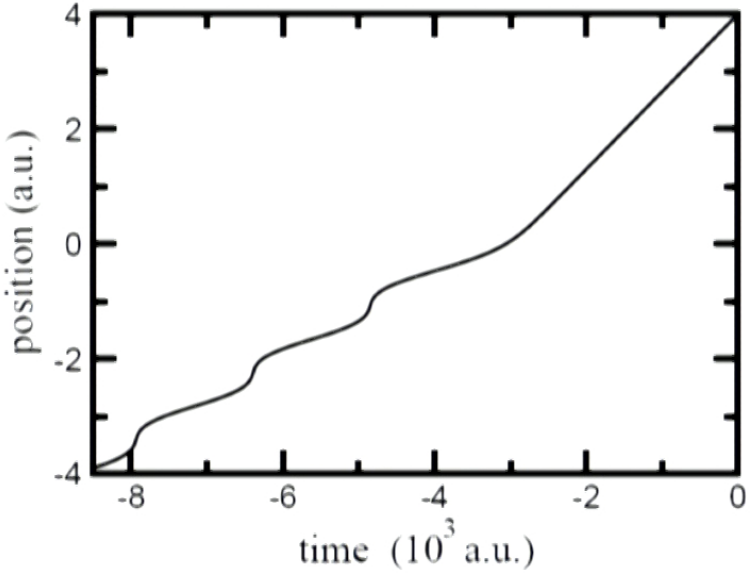
$\Psi(x \to +\infty) = k_R^{-1/2} T \exp[ik_R x]$

$\Psi(x \to -\infty) = k_L^{-1/2} (\exp[ik_L x] + R \exp[-ik_L x])$

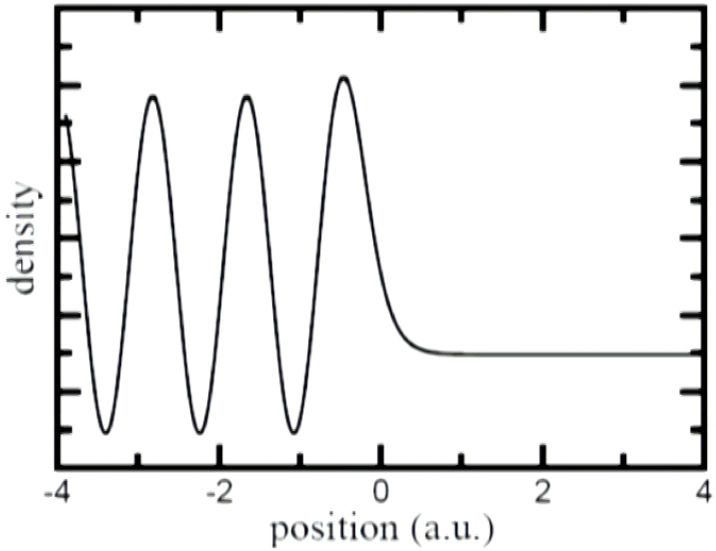
- Challenges in the continuum:
  - Two linearly independent eigenstate solutions for each energy *E*, requiring imposition of special boundary conditions.
  - Energy eigenstates extend infinitely far in both directions, necessitating use of optical potentlals to absorb outgoing flux.
  - Exact quantum dynamics calculation in the “deep” tunneling regime nearly impossible, even in 1D.

Numerical Solution of the 1D TISE:

Eckart Barrier



Trajectory, $x(t)$



Wavefunction density, $\rho(x)$

Solve 4th order real-valued ODE in t, to obtain $x(t)$.

- similar to Newton's second law, w/ extra terms.
- two initial conditions specify E and x_0.
- remaining two specify boundary conditions of solution ψ

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Noether's Theorem

"There is…a minority…who recognize…that the most beautiful and satisfying experiences open to humankind are not derived from the outside, but…with the development of the individual's own feeling, thinking and acting. The genuine artists, investigators and thinkers have always been persons of this kind. However inconspicuously the life of these individuals runs its course, none the less the fruits of their endeavors are the most valuable contributions which one generation can make to its successors. Within the past few days a distinguished mathematician, Professor Emmy Noether…died in her fifty-third year. In the judgment of the most competent living mathematicians, Fräulein Noether was ***the most significant creative mathematical genius thus far produced*** since the higher education of women began.

Emmy Noether Einstein's Obituary (excerpts)

"Any differentiable symmetry (invariance) of the action of a physical system has a corresponding conservation law."

scattering through a 1D Eckart barrier

- Main difference with classical trajectories on the left (reactant) asymptote:
  - $p = m\dot{x}$ is ***not*** conserved.
  - one can show that ***Noether momentum***

$$p_{Noether} = m\dot{x} + \frac{\hbar^2}{4m}\left(\frac{\dddot{x}}{\dot{x}^4} - \frac{2\ddot{x}^2}{\dot{x}^5}\right)$$

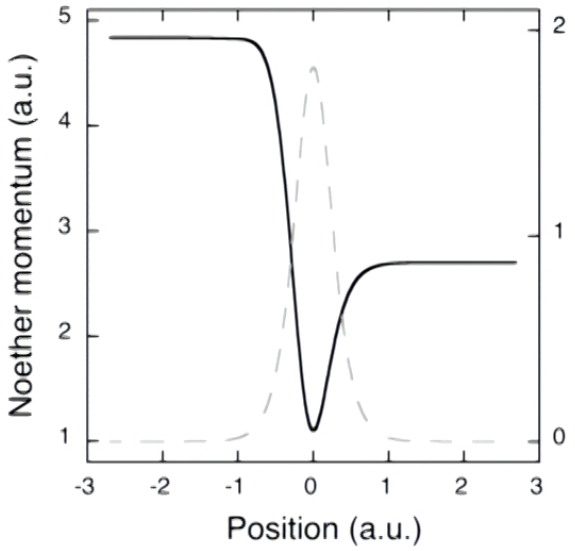
  is conserved

- Asymptotic $p_{Noether}$:

$$p_L = p_{Noether}(t \to -\infty) = \hbar k_L \left(\frac{1+|R|^2}{1-|R|^2}\right)$$

- Transmission probability:

$$P_T = \frac{2\hbar k_L}{\hbar k_L + p_L}$$



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1. Brief description of what this approach is.
2. First principles derivation: 1D time-independent case
3. Noether's theorem / symplectic structure / numerics
4. Time dependent case: basic 1D derivation
5. Entanglement / measurement / collapse
6. Relativistic generalization.

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Quantum Trajectories Derivation

1D Time-dependent Wavepackets

- Individual trajectories no longer able to represent $\psi(x,t)$.
  - the wavefunction $\psi(x,t)$ is replaced with an *ensemble* of trajectories, $x(C,t)$.
  - parameter C labels individual trajectories within the ensemble.
  - resembles classical statistical mechanics.
- Variables x and t no longer related via coordinate transformation.
  - trajectory field description provided by $x=x(C,t)$.
  - C is a parameter used to distinguish a given trajectory for *all* t.
  - e.g., $C = x_0 = x(x_0, t=0)$ is the initial value of a given trajectory in the ensemble.
  - other choices of the C parameter also exist.

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Quantum Trajectories Derivation

1D Time-dependent Wavepackets

- Trajectories governed by their own self-contained PDE.
  - we now have "spatial" derivatives in terms of C, (i.e., across trajectories), in addition to time derivatives.
  - allowed forms of T[], V[], and Q[] turn out to be identical to time-independent case, except with C rather than t derivatives for Q[].
  - all quantum effects/quantum forces arise from C derivatives, i.e. stem from interaction across nearby worlds.

- Goal: Derive a PDE to describe time evolution of $x(C,t)$ field.
  - depends on partial derivatives in both time *and* space (really C):

$$\dot{x} = \left.\frac{\partial x(C,t)}{\partial t}\right|_C \quad \text{and} \quad x' = \left.\frac{\partial x(C,t)}{\partial C}\right|_t, \quad \text{etc.}$$

Bohmian trajectories

Substituting $\psi(x,t) = R(x,t)e^{iS(x,t)/\hbar}$ (R, S real) into

$$i\hbar\psi_t = -\frac{\hbar^2}{2m}\psi_{xx} + V(x,t)\psi$$

gives

$$\begin{aligned} R_t + \frac{S_x R_x}{m} + \frac{RS_{xx}}{2m} &= 0\,, \\ S_t + \frac{S_x^2}{2m} + V(x,t) - \frac{\hbar^2}{2m}\frac{R_{xx}}{R} &= 0\,. \end{aligned}$$

Bohmian trajectories are obtained by solving

$$m\frac{dx}{dt} = S_x\,.$$

This gives us a one parameter family of trajectories $x(t,C)$ where C is some trajectory label.

1D Time-dependent Wavepackets

Ensemble of trajectories, labeled by independent variable C, leading to "spatial" derivative, $' = (d/dC)|t$ and following Euler-Lagrange eqn:

$$m\frac{\partial^2 x}{\partial t^2} + V_x(x,t) + \frac{\hbar^2}{4m}\left(\frac{x''''}{x'^4} - 8\frac{x''x'''}{x'^5} + 10\frac{x''^3}{x'^6}\right) = 0\,.$$

Noether's theorem also leads to following Conservation Laws:

Invariant under translation of t (if V indep of t) and of C.
Associated conservation laws:

$$\partial_t\left(\frac{m}{2}\dot{x}^2 + V(x) + \frac{\hbar^2}{8m}\frac{x''^2}{x'^4}\right) + \frac{\hbar^2\partial_C}{4m}\left(\left(\frac{x'''}{x'^4} - \frac{2x''^2}{x'^5}\right)\dot{x} - \frac{x''\dot{x}'}{x'^4}\right) = 0$$

$$\partial_t\left(m\dot{x}x'\right) + \partial_C\left(-\frac{1}{2}m\dot{x}^2 + V(x) + \frac{\hbar^2}{8m}\left(\frac{2x'''}{x'^3} - \frac{5x''^2}{x'^4}\right)\right) = 0$$

In the free particle case $V = 0$ there is a further conservation law arising from x-translation symmetry:

$$\partial_t\left(m\dot{x}\right) + \frac{\hbar^2}{4m}\partial_C\left(\frac{x'''}{x'^4} - \frac{2x''^2}{x'^5}\right) = 0\,.$$

Experimental Validation of *Many Interacting Worlds ?*

Three potential avenues by which MIW might lead to experimental outcomes that differ from those predicted by standard quantum theory:

1. Higher order contributions (i.e., beyond 4^{th}) to the trajectory-based dynamical law (continuous MIW).
2. Single particle relativistic quantum trajectory predictions (continuous MIW).
3. "Aliasing" effects due to discretization (discrete MIW).

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Continuous vs. Discrete MIW

Continuous MIW	Discrete MIW
continuous ensemble, $x(C,t)$	discrete ensemble, $x_i(t)$
exact solution of PDE	approximate discretization
unique dynamical law	dynamical law unspecified
action extremization principle	unclear at present
invariance/symmetry principle	unclear at present
relativistic generalization	unclear at present
Heisenberg/many-D/spin	under development
probability measure required	probability arises naturally
natural classical limit	natural classical limit
no trajectory crossing	no trajectory crossing

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Relativistic Generalization

- Usual approach with Ψ-based Lagrangian leads to Klein-Gordon wave equation, which fails to give a meaningful single-particle interpretation.
  - The free-particle Klein-Gordon equation is:

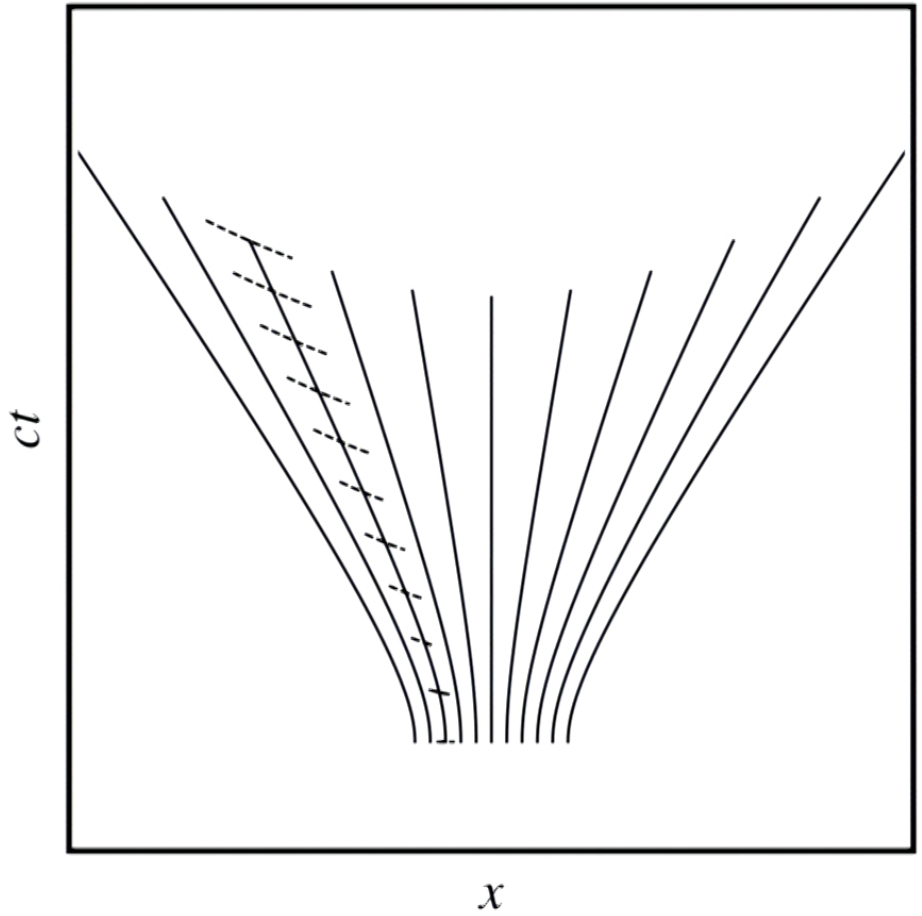
$$\left(-\frac{\partial^2}{\partial(ct)^2}+\nabla^2+\left(\frac{mc}{\hbar}\right)^2\right)\Phi(t,x)=0$$

  - Non-physical negative-energy solutions.
  - The temporal part of the four-current density is:

$$j^0=\frac{i\hbar}{2mc}\left(\Phi^*\frac{\partial}{\partial t}\Phi-\Phi\frac{\partial}{\partial t}\Phi^*\right)$$

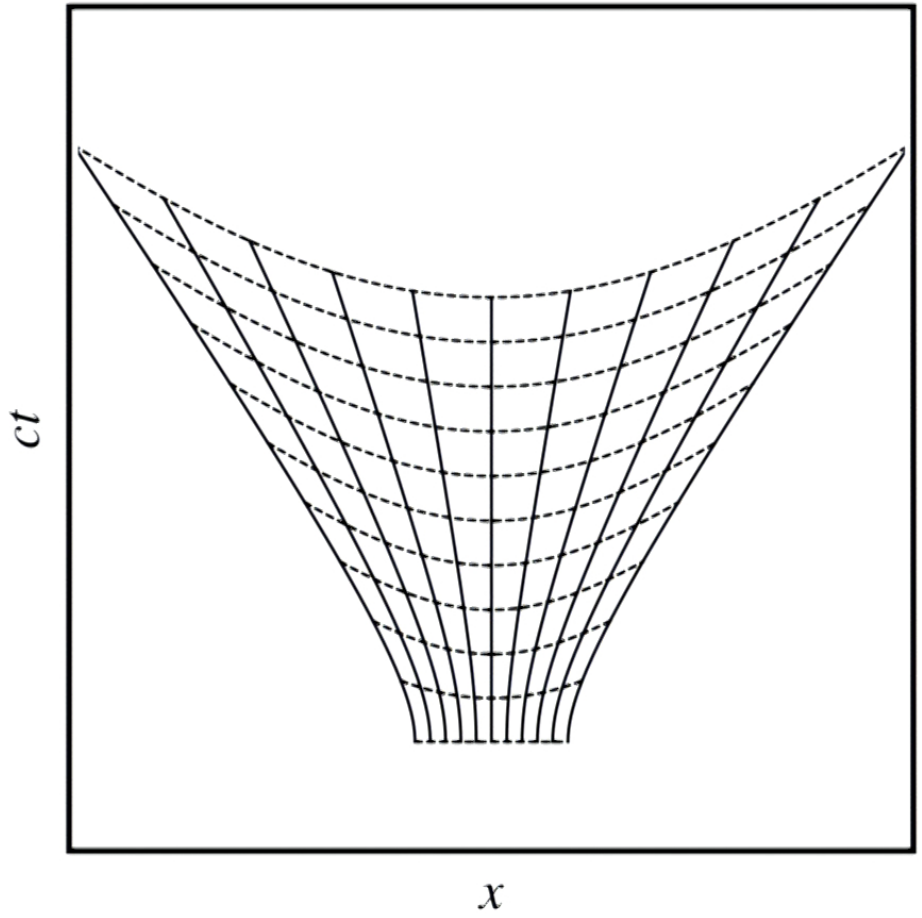
  - j^0 is ***not*** positive-definite in general.
  - The four-current density j^α is ***not*** time-like in general.
- All of above issues seem to be avoided in our relativistic trajectory-based approach.
- Our approach is "natural," because it involves action-extremizing trajectories.

Simultaneity for Accelerating *Quantum* Particles (1+1)



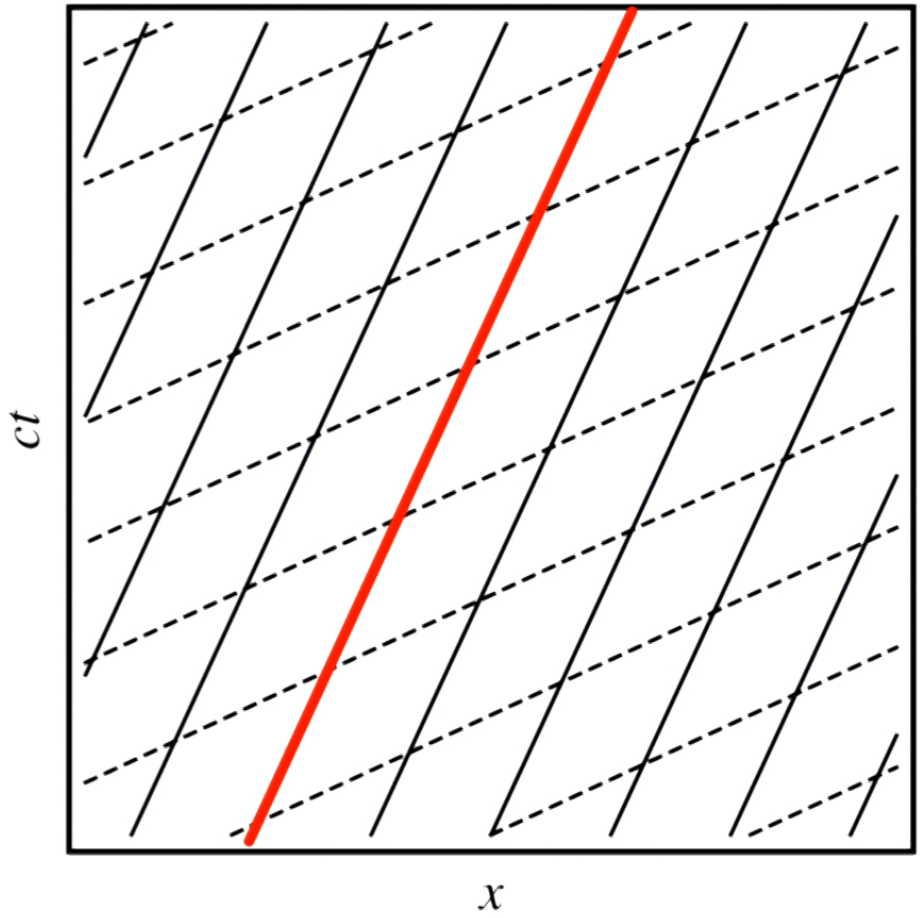
- The system now consists of an *ensemble* of quantum trajectories.
- Each trajectory has its own local simultaneity segments.

Simultaneity for Accelerating *Quantum* Particles (1+1)



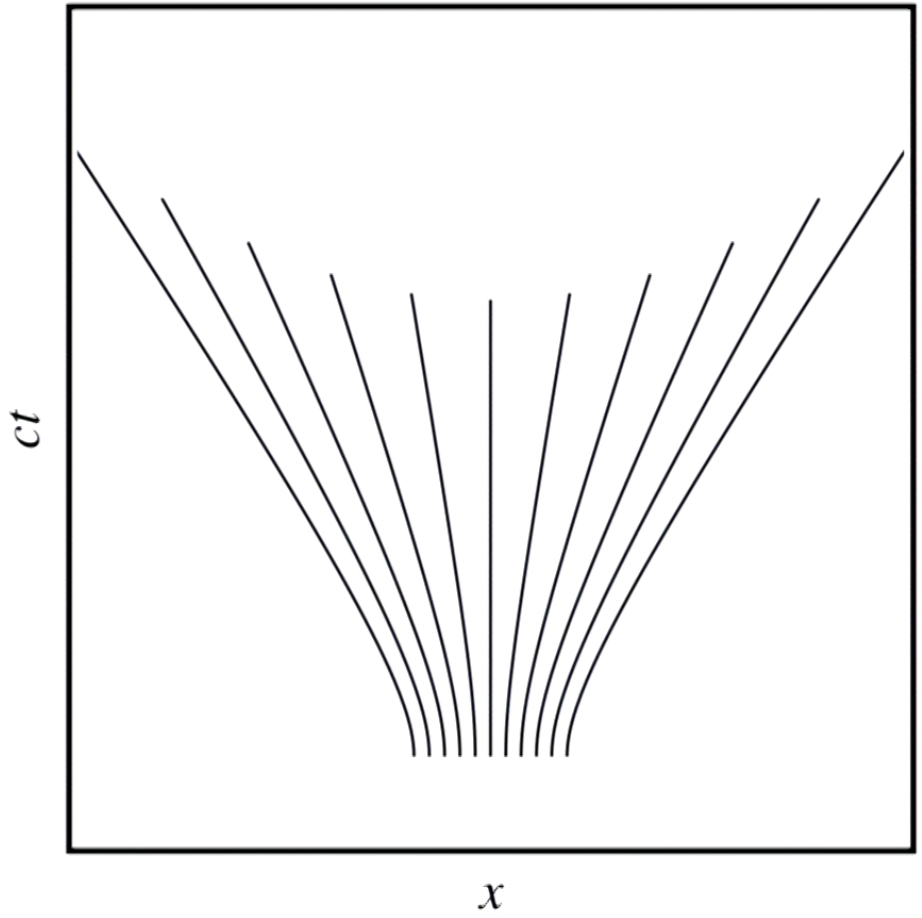
- The system now consists of an *ensemble* of quantum trajectories.
- Each trajectory has its own local simultaneity segments.
- Gluing all of these together, we can construct *global simultaneity submanifolds.*

Einsteinian Relativity (1+1)



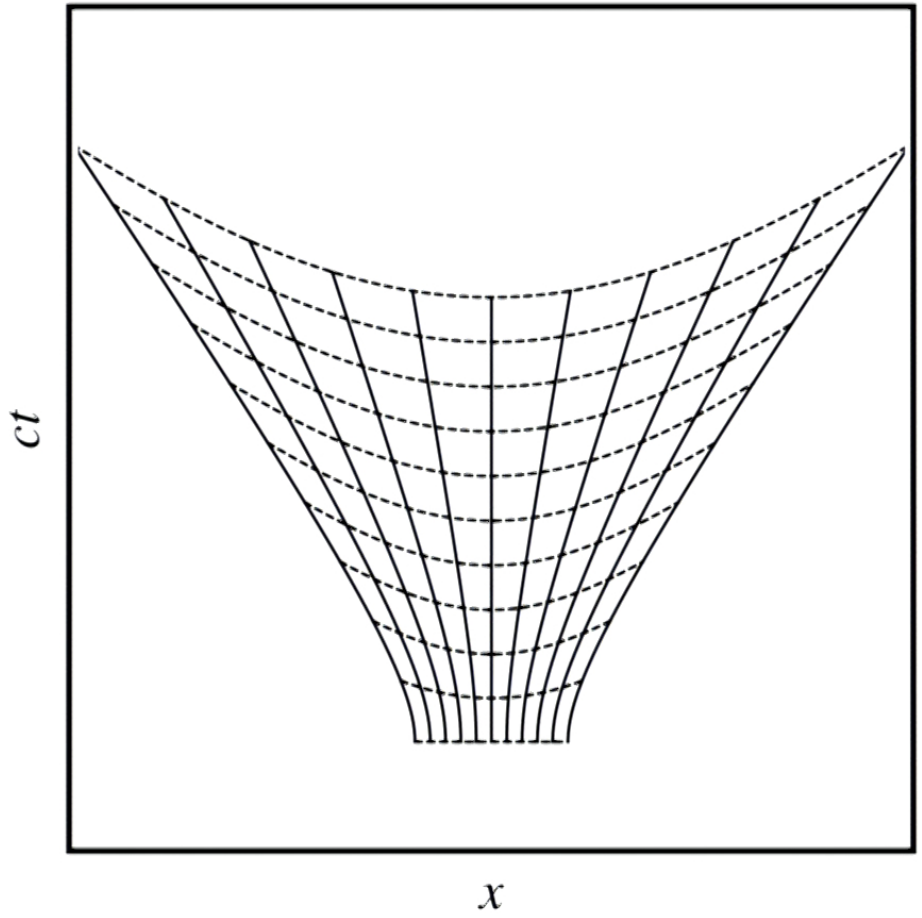
- Simultaneity well-defined for a given inertial observer, but, *depends* on observer.
- A single inertial particle (red curve) suffices to define an entire (ct', x') inertial frame (whose contours are the dashed and solid lines, respectively.)

Simultaneity for Accelerating *Quantum* Particles (1+1)



• The system now consists of an *ensemble* of quantum trajectories.

Simultaneity for Accelerating *Quantum* Particles (1+1)



- The system now consists of an *ensemble* of quantum trajectories.
- Each trajectory has its own local simultaneity segments.
- Gluing all of these together, we can construct *global simultaneity submanifolds.*

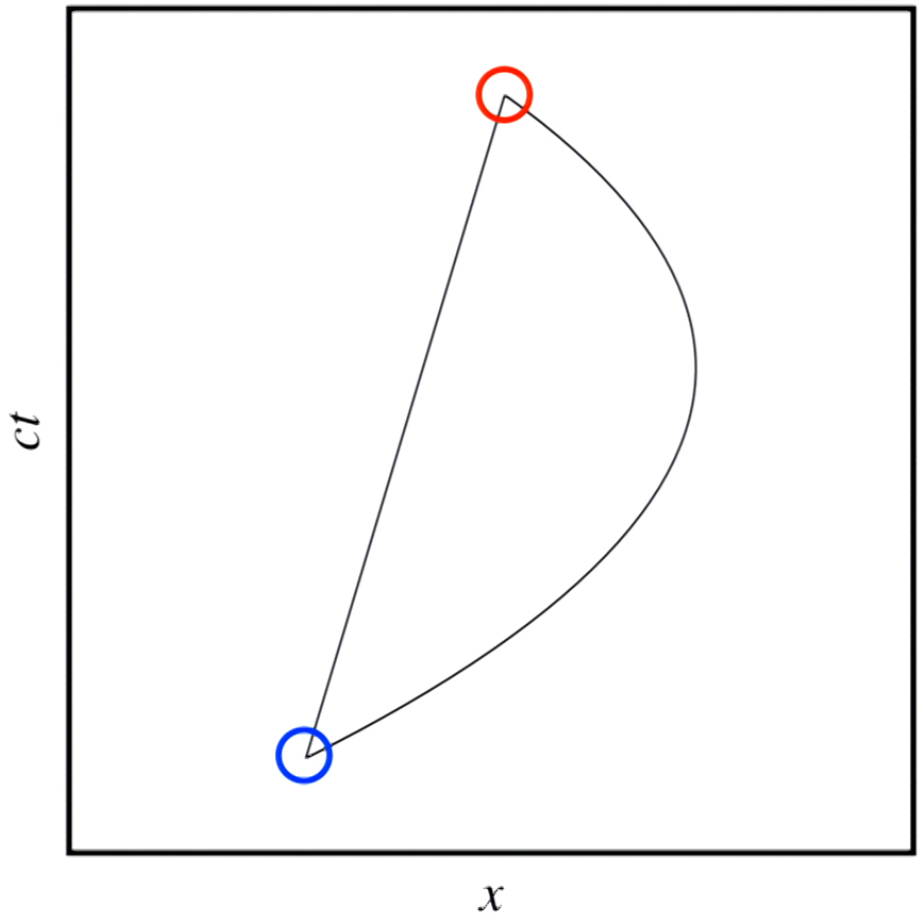
Ensemble Time and the Generalized Twin "Paradox"

- Simultaneity submanifolds are contours of a scalar time-like function, called the *ensemble time,* λ
- *Is it possible to take* proper time τ *to be an ensemble time,* λ ?
  - In general, *NO*, this is not possible.
  - The relation between τ and λ can be found from the metric tensor:

$$g_{00} = -\left(\frac{d\tau}{d\lambda}\right)^2$$

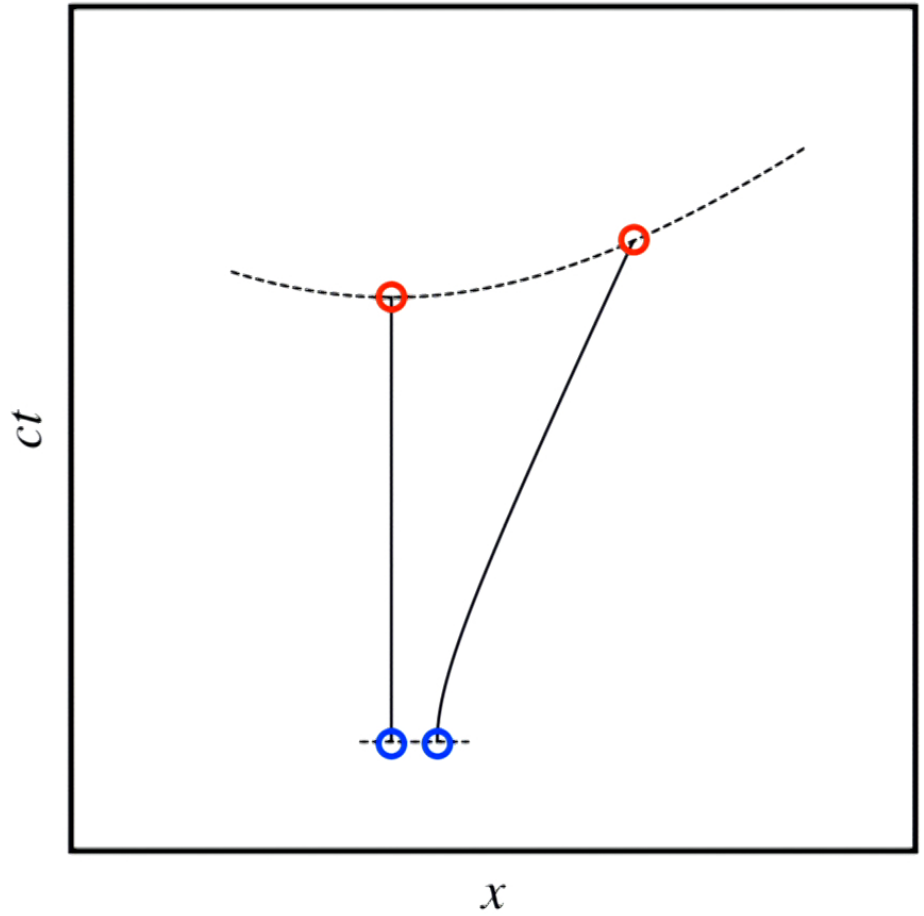
  - Note: g_{00} is *negative*, in keeping with the -+++ metric signature.
- The difference between τ and λ gives rise to the *generalized (quantum) twin paradox.*

Regular Twin "Paradox"



- Two "twin" observers cross paths at the blue circle event.
- Left twin: inertial motion; right twin: accelerated motion
- Right twin is *younger* when paths recross at the red circle event.

Generalized (Quantum) Twin "Paradox"



- Two "copies" of the same observer follow two, non-crossing paths.
- Both agree that the two blue circle events occur simultaneously.
- Both also agree that the two red circle events occur simultaneously.
- One trajectory has experienced less elapsed proper time than the other.

Ensemble Proper Time, & the Relativistic Quantum Potential

- Gravitational potential vs. quantum potential (weak-field limit)

$$-\left(1+2\frac{m\Phi}{mc^2}\right) \approx g_{00} \approx -\left(1-2\frac{Q}{mc^2}\right)$$

- Note: Q can be either positive *OR* negative!
- When $Q > 0$ (classically allowed), $d\tau < d\mathcal{T}$
  - The passage of the proper time for a given trajectory is **slower** than that of an inertial trajectory (time *dilation*).
- When $Q < 0$ (classically forbidden), $d\tau > d\mathcal{T}$
  - The passage of the proper time for a given trajectory is **faster** than that of an inertial trajectory (time *compression*).

Some "Have You Tried?" Questions:

- Bound eigenstate calculations: yes
- 1D wavepacket calculations: yes
- Multidimensional generalization: yes
- Mixed quantum classical methods: yes, but need help
- Quantum capture probabilities: yes, but need help
- Spin generalization: yes
- Relativistic conservation laws: working on it
- Relativistic "Gaussian" wavepackets: yes
- Lorentz-transformed wavepackets: yes
- Single-particle Dirac equation: working on it
- Multiple-particle Dirac equation: definitely need help
- Others working on cosmology, but still could use some help

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TEXAS TECH UNIVERSITY

Relativistic Derivation

Dynamical PDE (eqs. of motion)

- By extremizing the action, we obtain the equation of motion for the trajectory ensemble.

$$\frac{\partial^2 x^\alpha}{\partial \mathcal{T}'^2} = \exp\left[-\frac{2Q}{mc^2}\right]\frac{f^\alpha}{m} - \frac{1}{mc^2}\frac{\partial Q}{\partial \mathcal{T}'}\frac{\partial x^\alpha}{\partial \mathcal{T}'}$$

- PDE is fourth order in **C**, second order in $\mathcal{T}'$, *but* treats all inertial coordinates x^α on equal footing.
- Choosing uniformizing coordinates: $f(\mathbf{C} = \mathbf{P}) = 1$

$$Q = -\frac{\hbar^2}{2m}\gamma^{-1/4}\frac{\partial}{\partial C^i}\left[\gamma^{1/2}\gamma^{ij}\frac{\partial}{\partial C^j}\gamma^{-1/4}\right], \qquad f^\alpha = -\frac{\partial x^\alpha}{\partial C^i}\gamma^{ij}\frac{\partial}{\partial C^j}Q$$