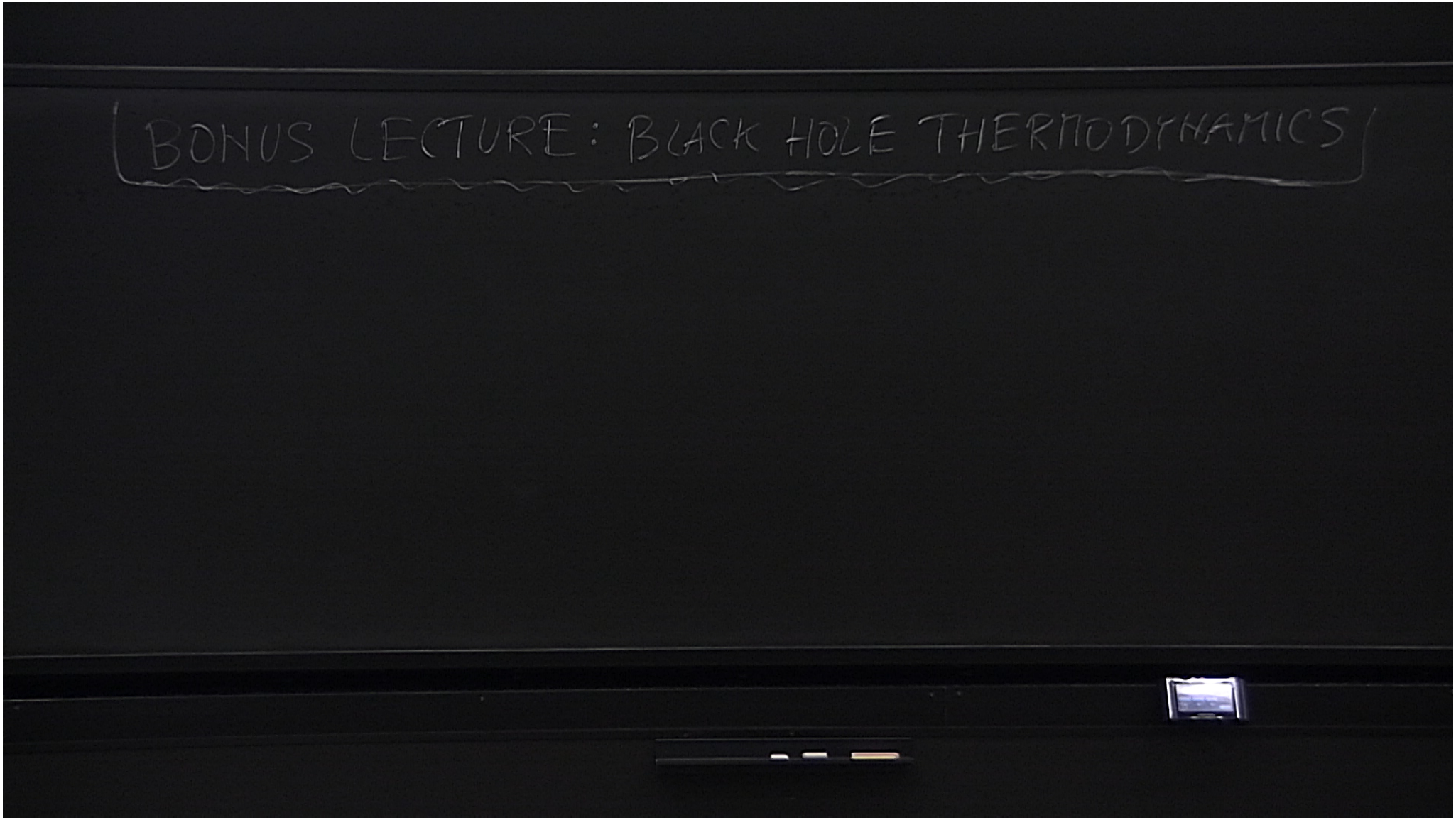
Title: PSI 17/18 - Statistical Mechanics - Lecture 14

Date: Oct 27, 2017 10:45 AM

URL: http://pirsa.org/17100047

Abstract:


BONUS LECTURE: BLACK HOLE THERMODYNAMICS

BONUS LECTURE: BLACK HOLE THERMODYNAMICS

a) MOTIVATION

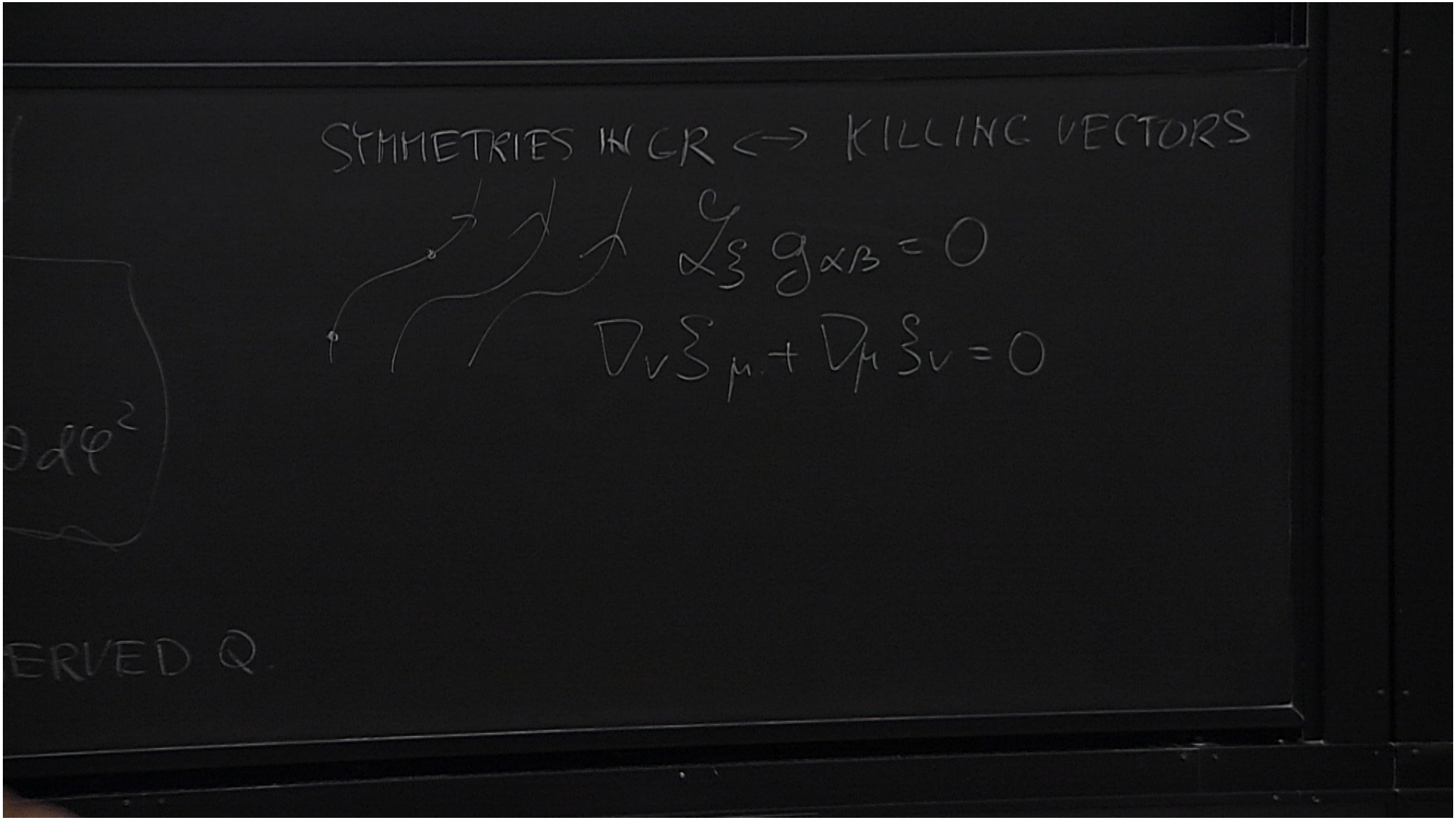
SCHWARZSCHILD

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$f = 1 - \frac{2M}{r}, \quad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$

- ASYMPTOTIC MASS (TOTAL ENERGY)

NOETHER'S THEOREM: SYMMETRY <-> CONSERVED Q


SYMMETRIES IN GR ↔ KILLING VECTORS
$\mathcal{L}_\xi g_{\alpha\beta} = 0$
$D_\nu \xi_\mu + D_\mu \xi_\nu = 0$
$d\varphi^2$
ERIVED Q.

SYMMETRIES IN GR $\leftrightarrow$ KILLING VECTORS

$\mathcal{L}_\xi g_{\alpha\beta} = 0$

$D_\nu \xi_\mu + D_\mu \xi_\nu = 0$

SPEC. SCH IS STATIC

$d\varphi^2$

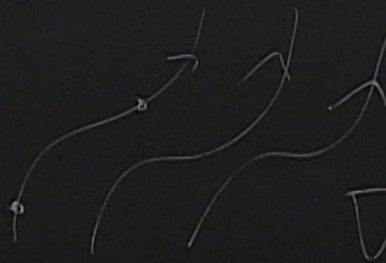
ERIVED Q.

ICS

$+\sin^2\theta\, d\varphi^2$

ONSERVED Q.

SYMMETRIES IN GR $\leftrightarrow$ KILLING VECTORS



$\mathcal{L}_\xi g_{\alpha\beta} = 0$

$D_\nu \xi_\mu + D_\mu \xi_\nu = 0$

SPEC. SCH IS STATIC $k = \frac{\partial}{\partial t}$

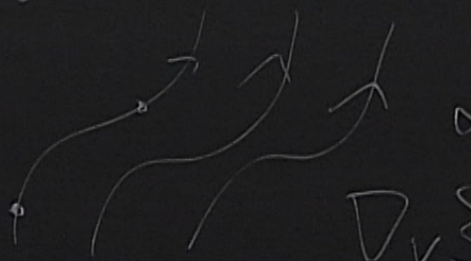
$M = -\frac{1}{8\pi} \int_{S^2_\infty} *dk$ (2-FORM) $=$

ICS

$+\sin^2\theta\, d\varphi^2$

ONSERVED Q.

SYMMETRIES IN GR $\longleftrightarrow$ KILLING VECTORS



$$\mathcal{L}_\xi g_{\alpha\beta} = 0$$

$$D_\nu \xi_\mu + D_\mu \xi_\nu = 0$$

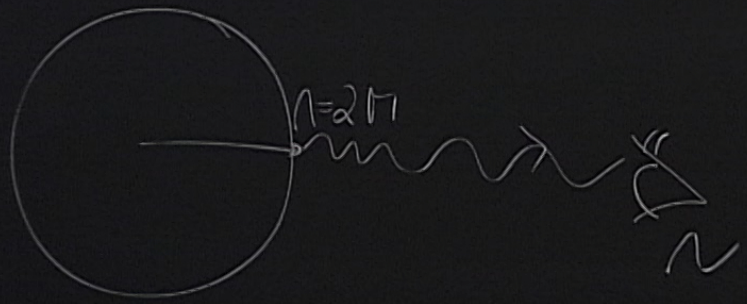
SPEC. SCH IS STATIC $\quad k = \frac{\partial}{\partial t}$

2-FORM

$$M = -\frac{1}{8\pi} \int_{S^2_\infty} *dk = M$$

- BLACK HOLE HORIZON

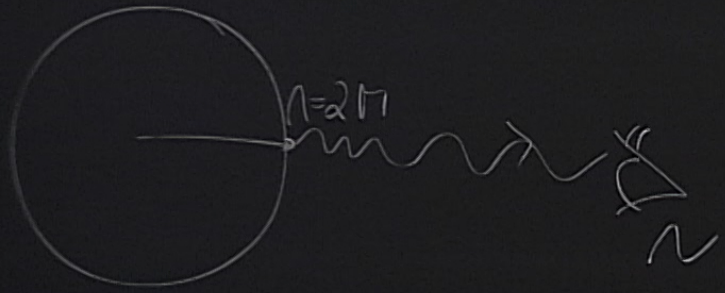
$r_+ = 2M$ ... "BOUNDARY OF BH"



$$\frac{V_r}{V_{r+}} = \frac{\Delta\tau_{r+}}{\Delta\tau_r}$$

- BLACK HOLE HORIZON

$r_+ = 2M$ "BOUNDARY OF BH"



$$\frac{V_r}{V_{r_+}} = \frac{\Delta\tau_{r_+}}{\Delta\tau_r} = \frac{\sqrt{f(r_+)}\,\Delta t}{\sqrt{f(r)}\,\Delta t} = 0$$

r_+ SURFACE OF INFINITE REDSHIFT.

THIS IS WHY BLACK HOLE

SUR

A

SURFACE AREA.

$$A = \pi r_+^2$$ (AREA GAUGE FOR Ω)

$dt = 0 = dr$

$ds^2 = r_+^2 d\Omega^2$. INDUCED METRIC ON HORIZON

$$A = \int \sqrt{g}\, d\theta d\varphi =$$

REDSHIFT.

HOLE

SURFACE AREA.

$$\boxed{A = 4\pi r_+^2}$$ (AREA GAUGE FOR $\underline{\Omega}$)

$= 0$

$dt = 0 = dr$

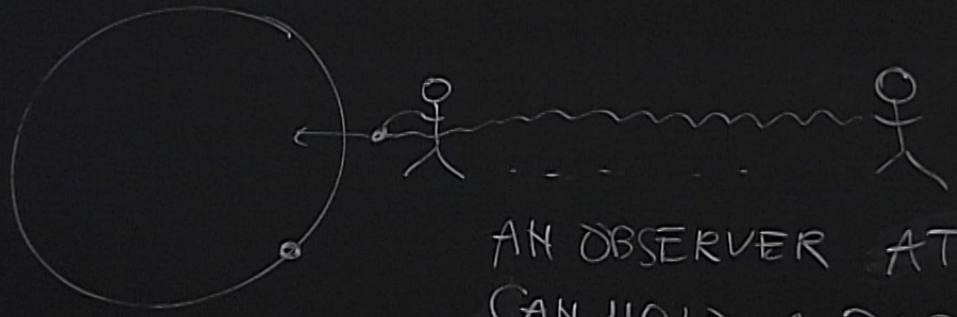
$ds^2 = r_+^2 d\Omega^2$ .. INDUCED METRIC ON HORIZON

REDSHIFT.

$$A = \int \sqrt{\gamma}\, d\theta d\varphi = \int_{\partial\varphi} r_+^2 \sin\theta \, d\theta d\varphi = 4\pi r_+^2$$

THIS IS WHY BLACK HOLE

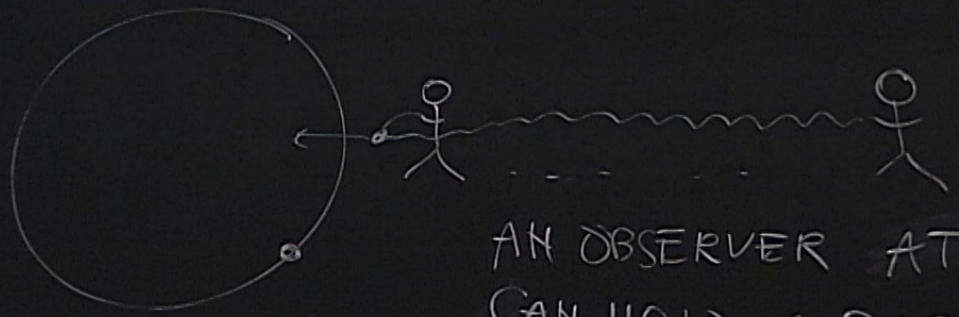
• SURFACE GRAVITY



AN OBSERVER AT INFINITY CAN HOLD A PARTICLE ON THE HORIZON USING A MASSLESS STRING WITH FINITE FORCE

THIS IS WHY .. BLACK HOLE

• SURFACE GRAVITY



AN OBSERVER AT INFINITY CAN HOLD A PARTICLE ON THE HORIZON USING A MASSLESS STRING WITH FINITE FORCE

BLACK HOLE

$$A = \int \sqrt{g}\, d\theta d\varphi = \int_0^{2\pi} r_+^2 \sin\theta \, d\theta d\varphi = 4\pi r_+^2$$

κ .. SURFACE GRAVITY . ACCELERATION ($m=1$)

$$\kappa = \frac{f'(r_+)}{2} = \frac{\frac{d}{dr}\left(1-\frac{2m}{r}\right)\Big|_{r=r_+}}{2} = \frac{1}{4m} = \frac{1}{2r_+}$$

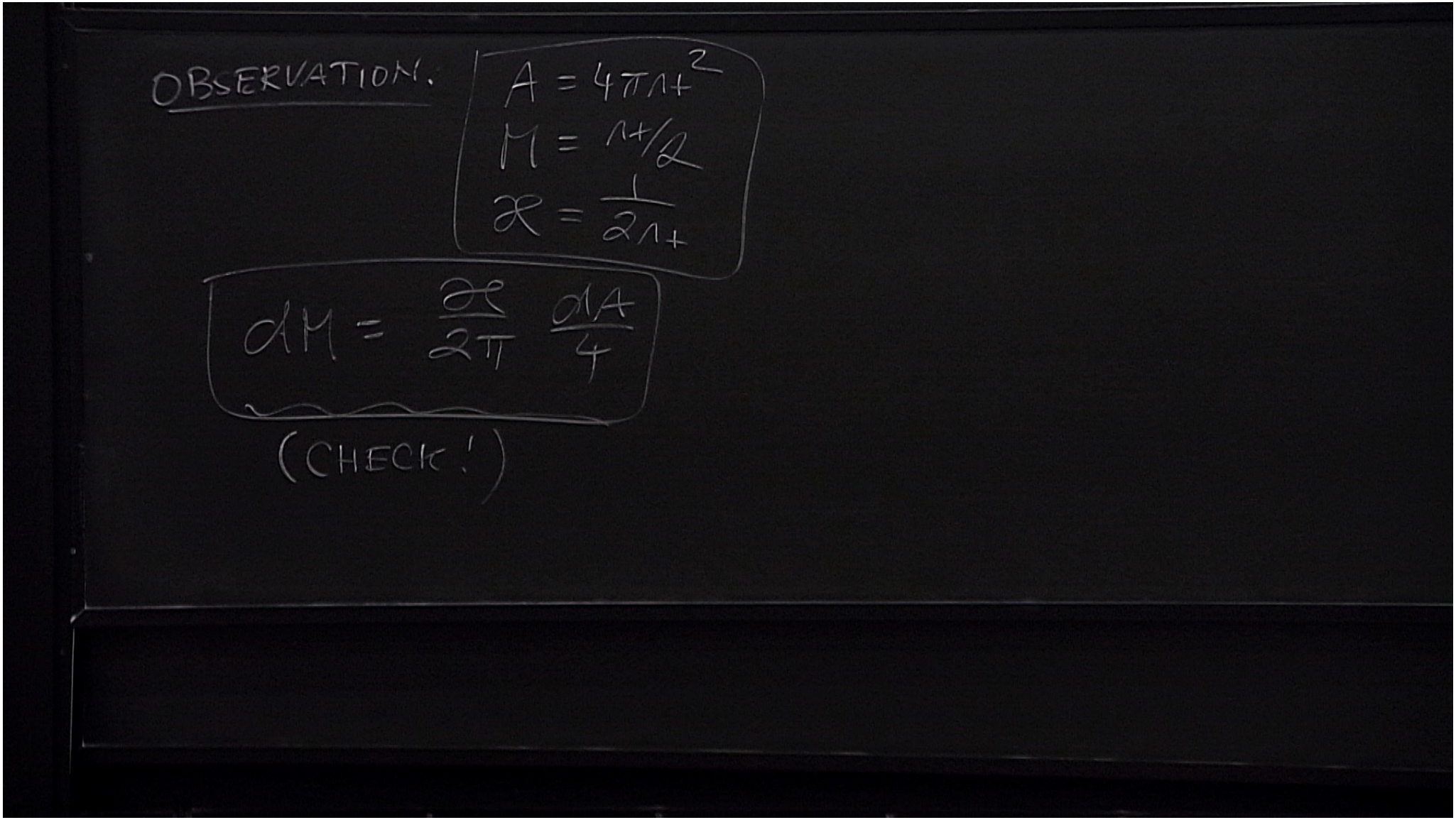
OBSERVATION:

$$g = \frac{m}{r^2}\Big|_{r=r_+} = \frac{m}{r_+^2} = \frac{1}{4m}$$

g_{NEWTON}

COINCIDENCE

FORCE


OBSERVATION.
$A = 4\pi r_+^2$
$M = r_+/2$
$\varkappa = \frac{1}{2r_+}$
$dM = \frac{\varkappa}{2\pi}\frac{dA}{4}$
(CHECK!)

b) BLACK HOLE MECHANICS (73)

BARDEEN, CARTER, HAWKING

FOR THE MOST GENERAL (MASS M, ROTATION J, CHARGE Q)

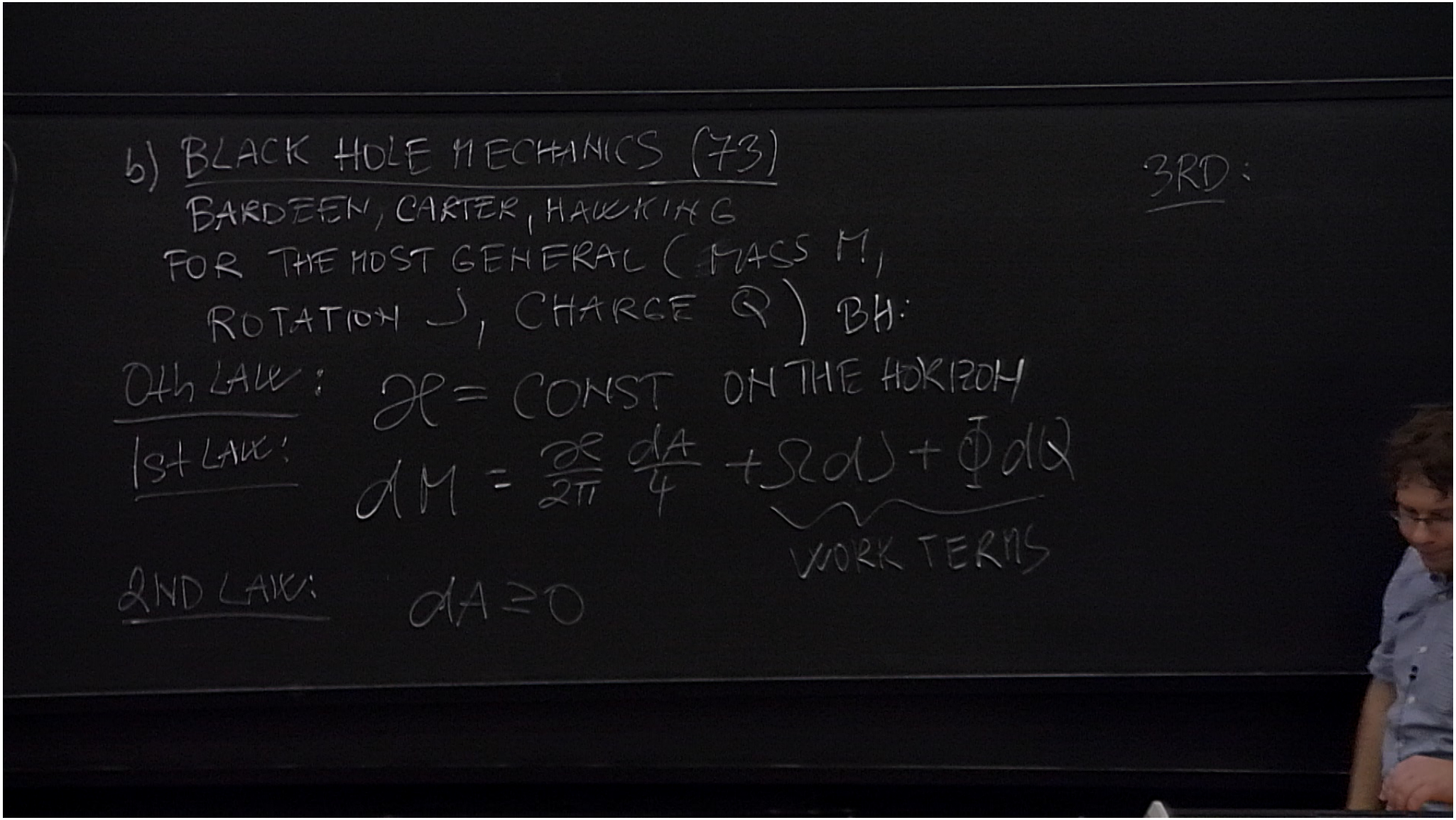
b) BLACK HOLE MECHANICS (73)

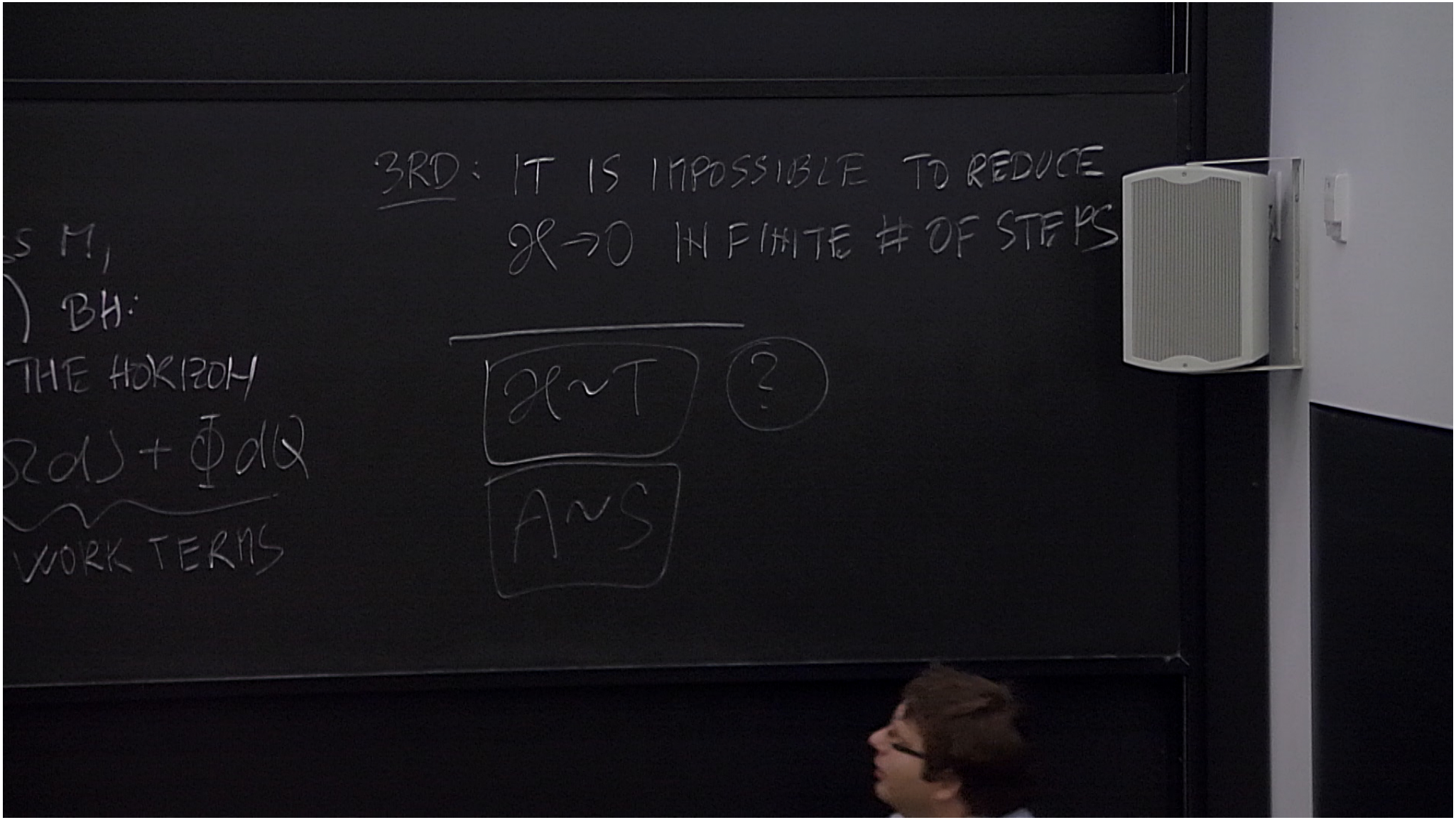
BARDEEN, CARTER, HAWKING

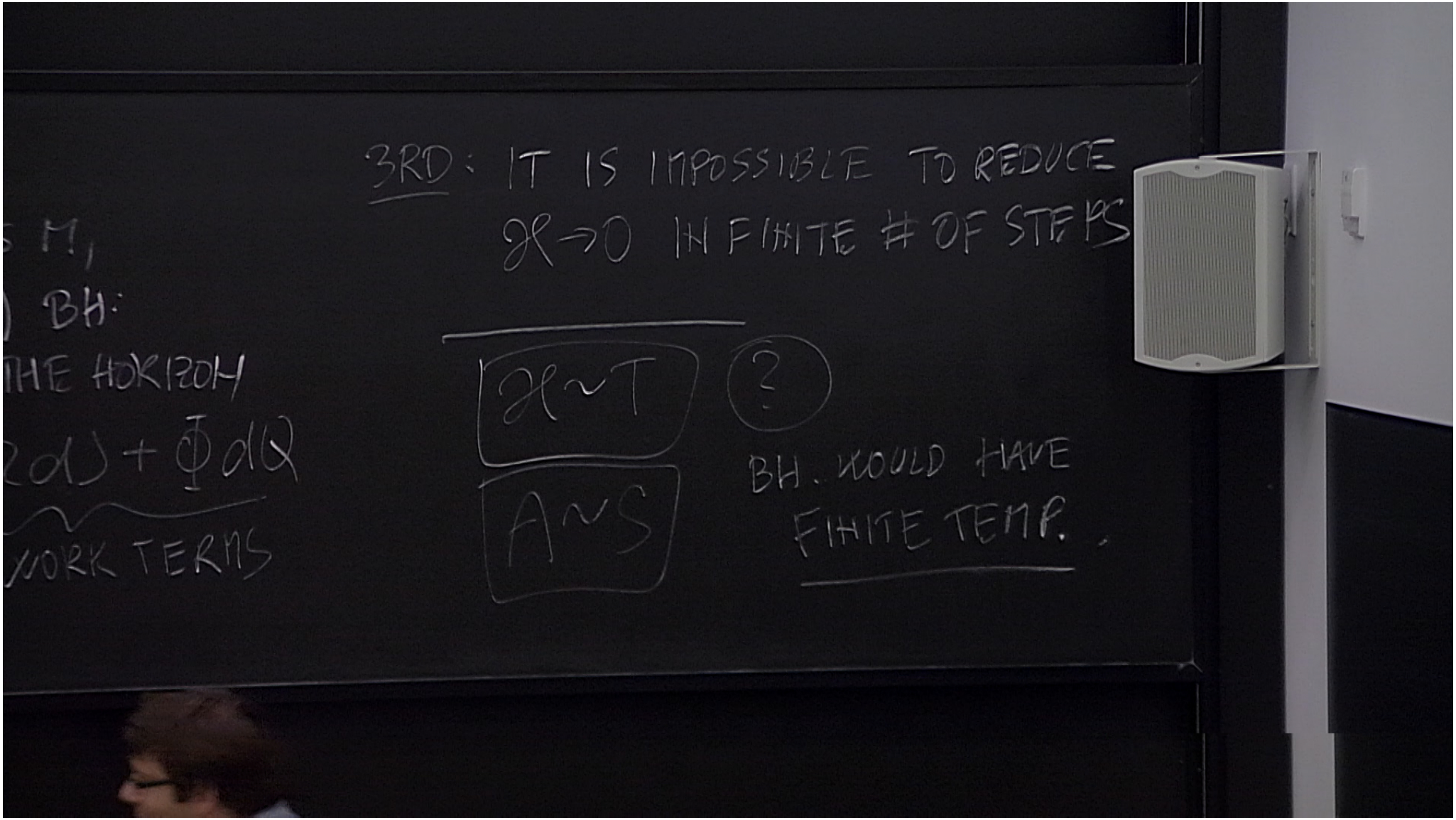
FOR THE MOST GENERAL (MASS M, ROTATION J, CHARGE Q) BH:

0th LAW: $\mathcal{K}$ = CONST ON THE HORIZON

1st LAW: $dM = \frac{\mathcal{K}}{2\pi}\frac{dA}{4} + \underbrace{\Omega dJ + \Phi dQ}_{\text{WORK TERMS}}$


b) BLACK HOLE MECHANICS (73)
BARDEEN, CARTER, HAWKING
FOR THE MOST GENERAL (MASS M, ROTATION J, CHARGE Q) BH:
3RD:
0th LAW: $\mathcal{K}$ = CONST ON THE HORIZON
1st LAW: $dM = \frac{\mathcal{K}}{2\pi}\frac{dA}{4} + \Omega dJ + \Phi dQ$
WORK TERMS
2ND LAW: $dA \geq 0$


3RD: IT IS IMPOSSIBLE TO REDUCE
$\mathcal{K} \to 0$ IN FINITE # OF STEPS
BH:
THE HORIZON
$\delta Q + \oint dQ$
WORK TERMS
$\mathcal{K} \sim T$
?
$A \sim S$


3RD: IT IS IMPOSSIBLE TO REDUCE
$\mathcal{R} \to 0$ IN FINITE # OF STEPS
M,
BH:
THE HORIZON
$\int dQ$
WORK TERMS
$\mathcal{R} \sim T$
?
A ~ S
BH. WOULD HAVE
FINITE TEMP.

CLASSICALLY .. IMPOSSIBLE (BH .. ULTIMATE SPONGE)

.. T=0

C) BLACK HOLE THERMODYNAMICS

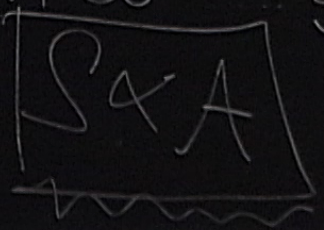
- WHEELER'S CUP OF TEA: WHERE DOES THE ENTROPY GO? .. BEKENSTEIN

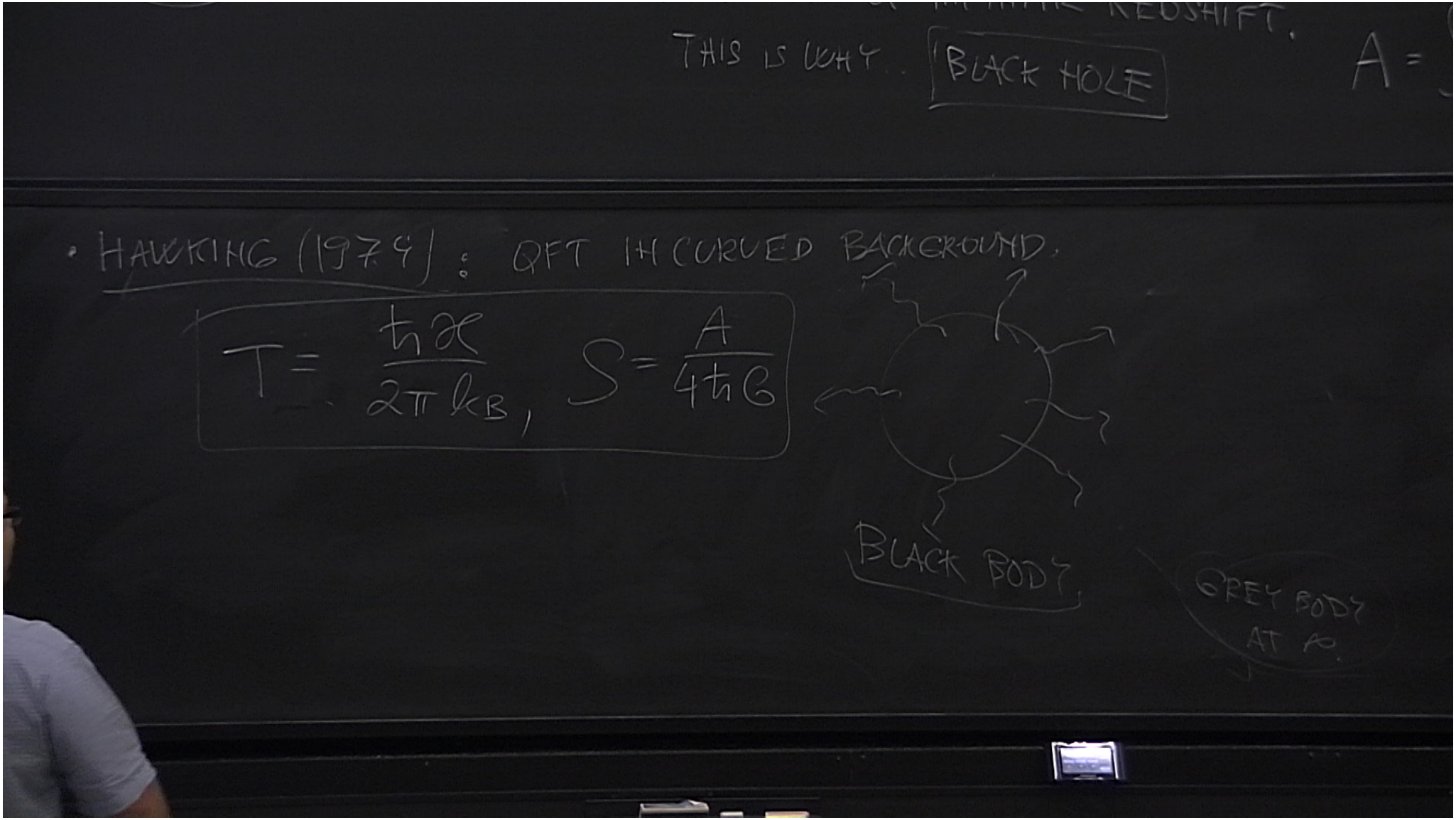
CLASSICALLY .. IMPOSSIBLE (BH .. ULTIMATE SPONGE)

.. T=0

C) BLACK HOLE THERMODYNAMICS

- WHEELER'S CUP OF TEA: WHERE DOES THE ENTROPY GO? BEKENSTEIN

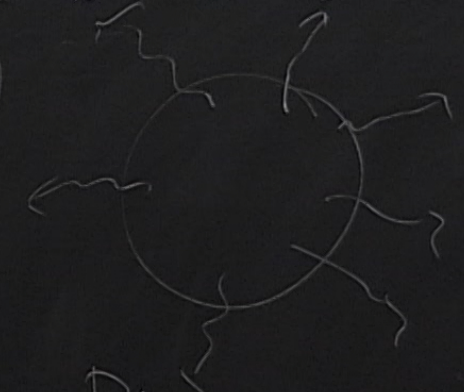



THIS IS WHY
BLACK HOLE
A =
• HAWKING (1974): QFT IN CURVED BACKGROUND.
$T = \frac{\hbar \kappa}{2\pi k_B}$, $S = \frac{A}{4\hbar G}$
BLACK BODY
GREY BODY AT A.

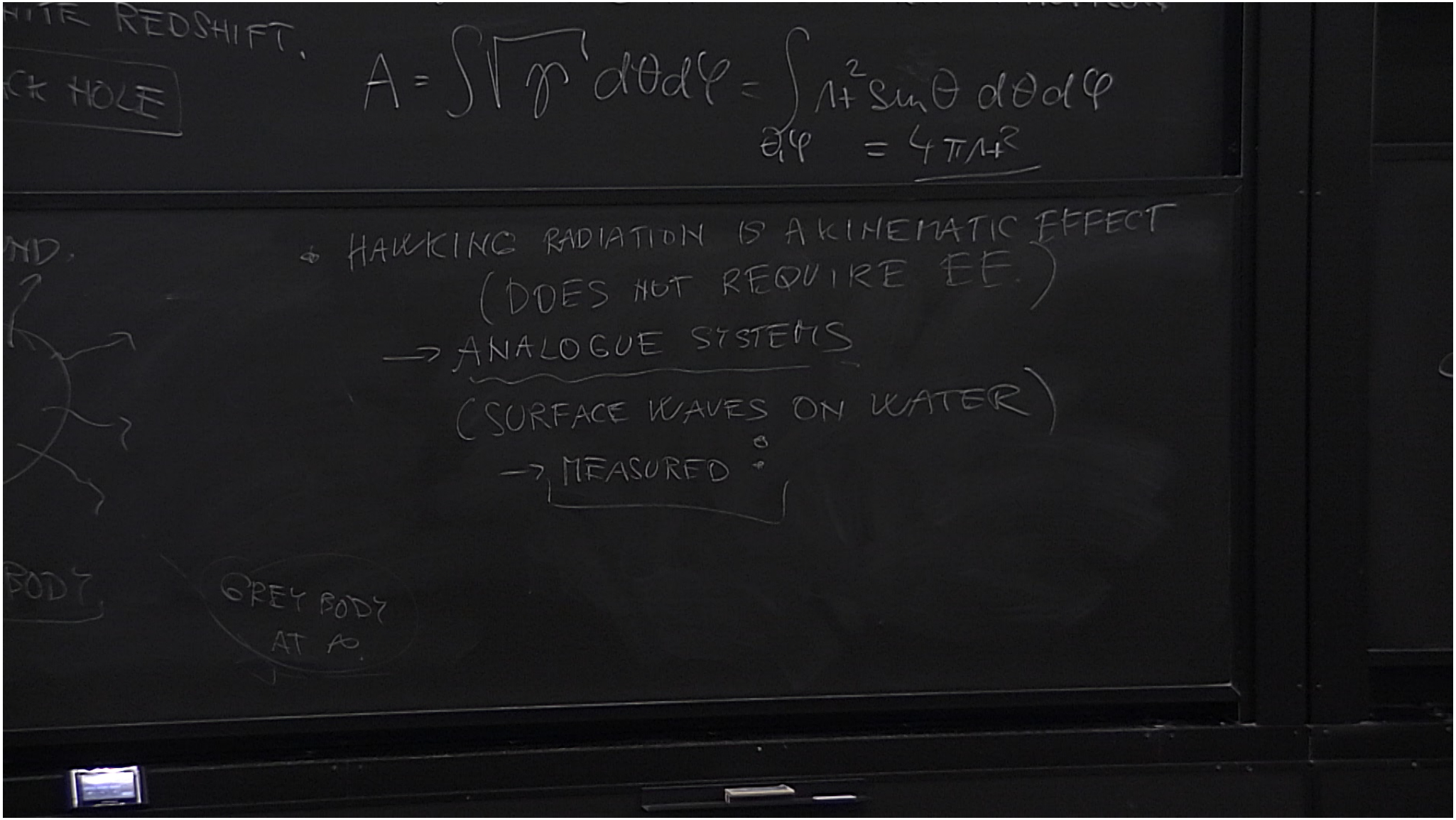
THIS IS WHY BLACK HOLE

• HAWKING (1974): QFT IN CURVED BACKGROUND.

$$T = \frac{\hbar \kappa}{2\pi k_B}, \quad S = \frac{A}{4\hbar G}$$



- DERIVATION CRITICIZED.
- BUT EUCLIDEAN PI ✓
  LQG ✓
  ST ✓


ITE REDSHIFT.
CK HOLE
$A = \int \sqrt{\gamma}\, d\theta d\varphi = \int_0^{2\pi} \int_0^{\pi} r_+^2 \sin\theta \, d\theta d\varphi$
$= 4\pi r_+^2$
ND.
• HAWKING RADIATION IS A KINEMATIC EFFECT
(DOES NOT REQUIRE EE.)
→ ANALOGUE SYSTEMS
(SURFACE WAVES ON WATER)
→ MEASURED
BODY
GREY BODY
AT A_0.

d) EUCLIDEAN TRICK

- THERMAL GREEN FUNCTIONS HAVE PERIODICITY IN IMAGINARY TIME: $\tau = it$

$$G(\tau) = G(\tau + \beta) \qquad \beta = \frac{1}{T}$$

- IN PARTICULAR CONSIDE GRAV. FIELD. → EUCLIDEANIZED SCH. METRIC.

$$ds_E^2 = f d\tau^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

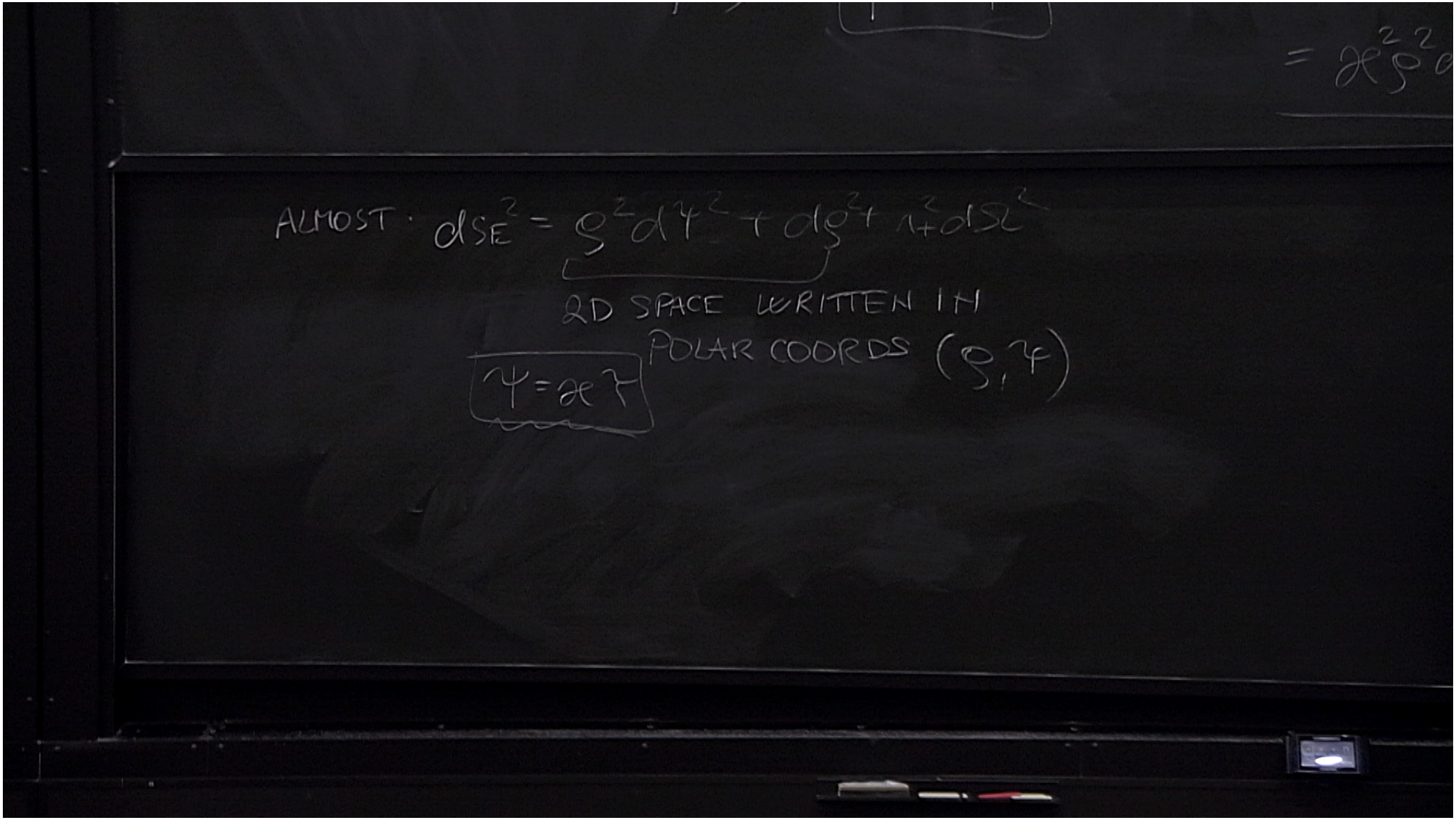
- NEAR HORIZON

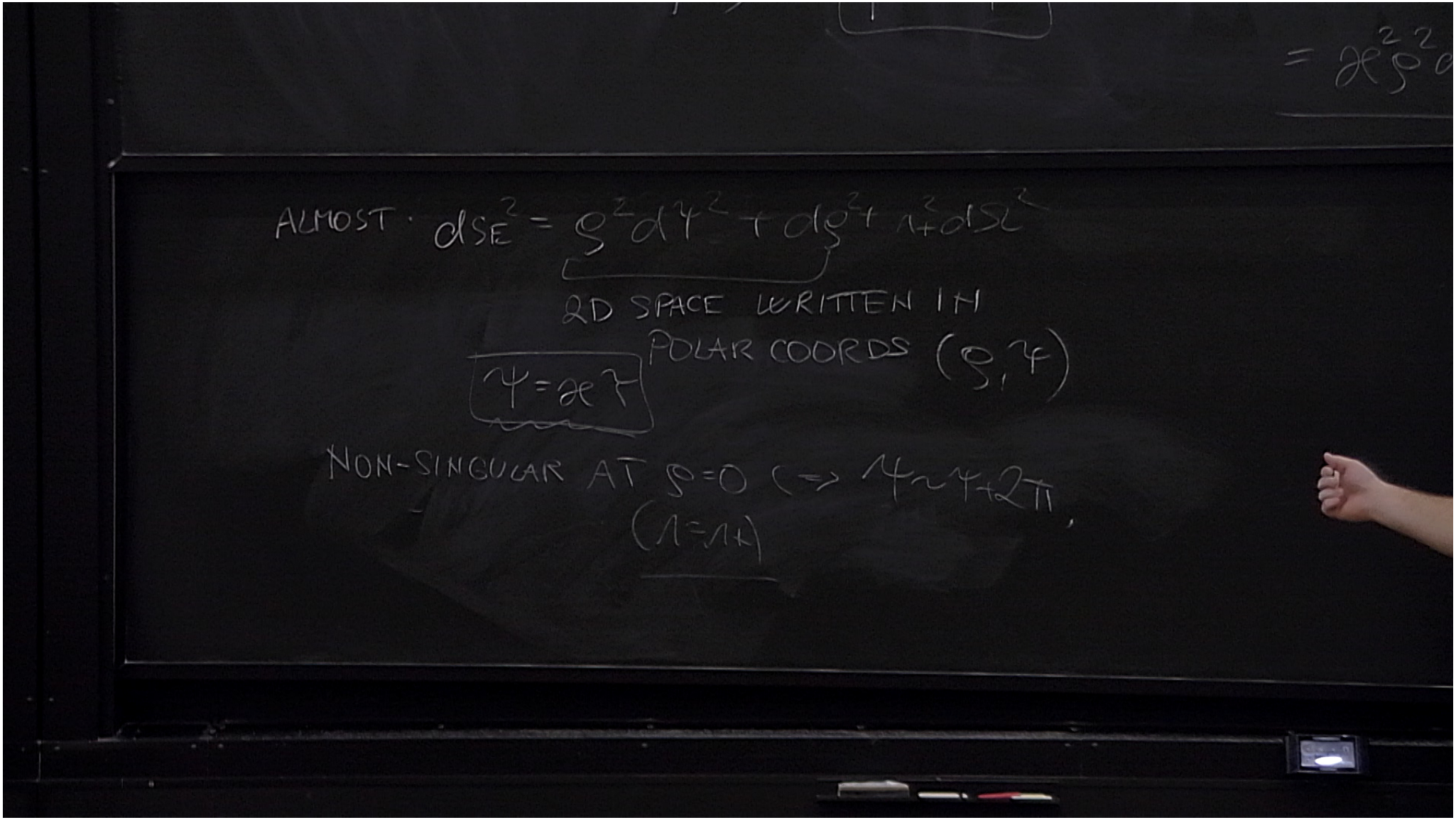
$$f(r) = \underbrace{f(r_+)}_{0} + \underbrace{(r - r_+)}_{\Delta r} f'(r_+) + \dots = \Delta r\, f'(r_+) = \underline{2\Delta r\, \kappa}.$$

$$ds_E^2 = 2\Delta r\, \kappa\, d\tau^2 + \frac{dr^2}{2\kappa\Delta r} + r_+^2 d\Omega^2 \qquad d\rho = \frac{dr}{\sqrt{2\kappa\Delta r}} \Leftrightarrow \Delta r = \frac{\kappa}{2}\rho^2$$

$$= \kappa^2\rho^2 d\tau^2 + d\rho^2 + r_+^2 d\Omega^2$$

SPACE)


ALMOST · $dSE^2 = \rho^2 d\gamma^2 + d\rho^2 + \rho^2 d\Omega^2$
2D SPACE WRITTEN IN POLAR COORDS (ρ, γ)
$\gamma = \alpha\gamma$


ALMOST · $ds_E^2 = \rho^2 d\psi^2 + d\rho^2 + \dots$
2D SPACE WRITTEN IN POLAR COORDS (ρ, ψ)
$\psi = \alpha\varphi$
NON-SINGULAR AT $\rho = 0$ $\Leftrightarrow$ $\psi \sim \psi + 2\pi$,
$(\alpha = \pm 1)$

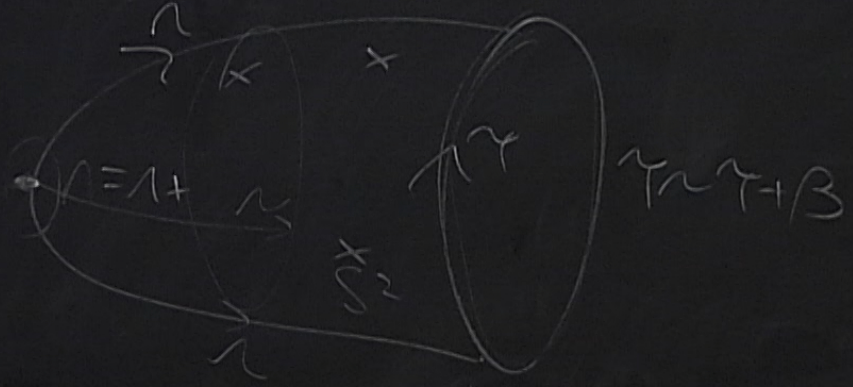
BLACK HOLE

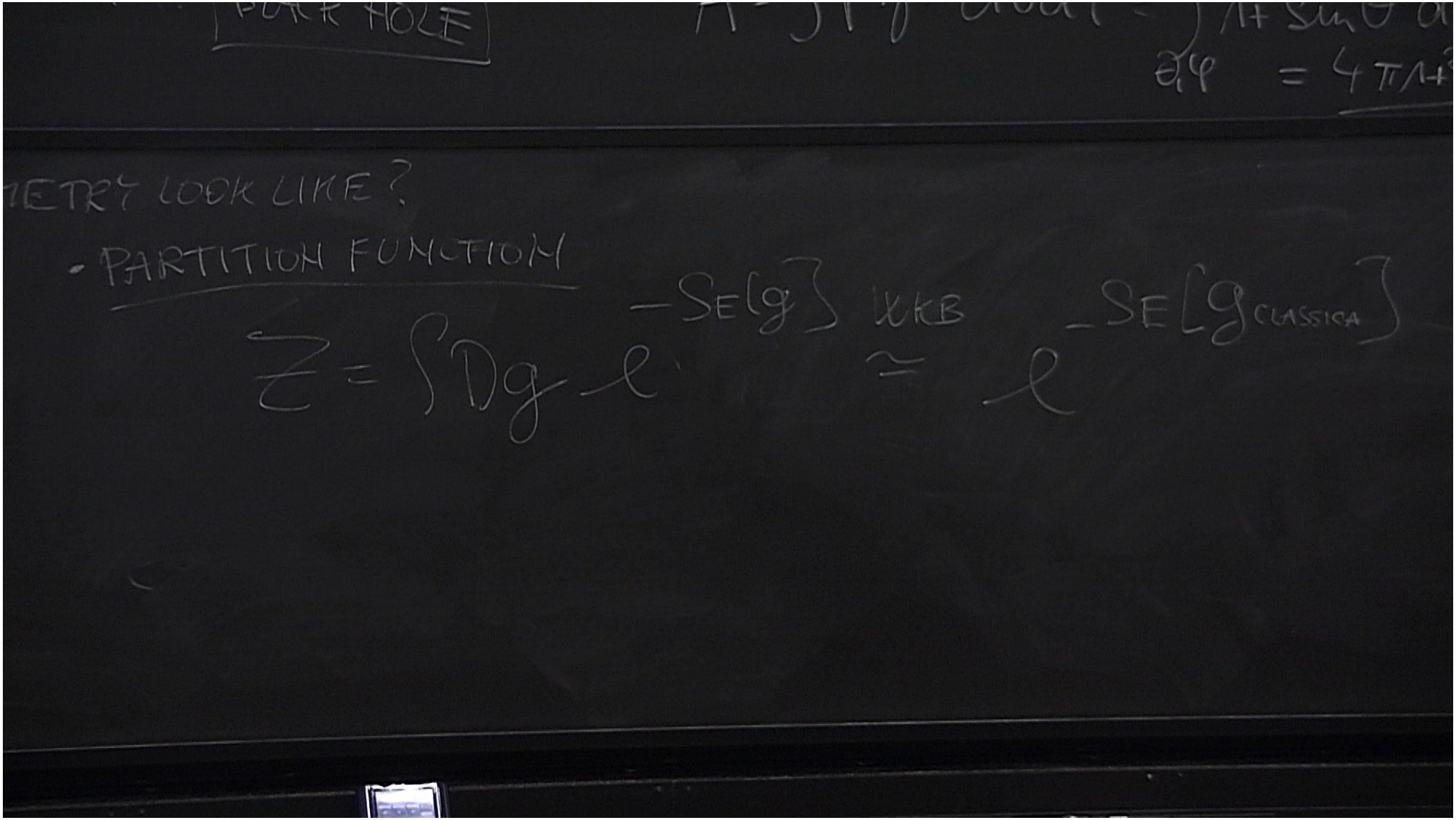
HOW DOES THIS EUCLIDEAN GEOMETRY LOOK LIKE?

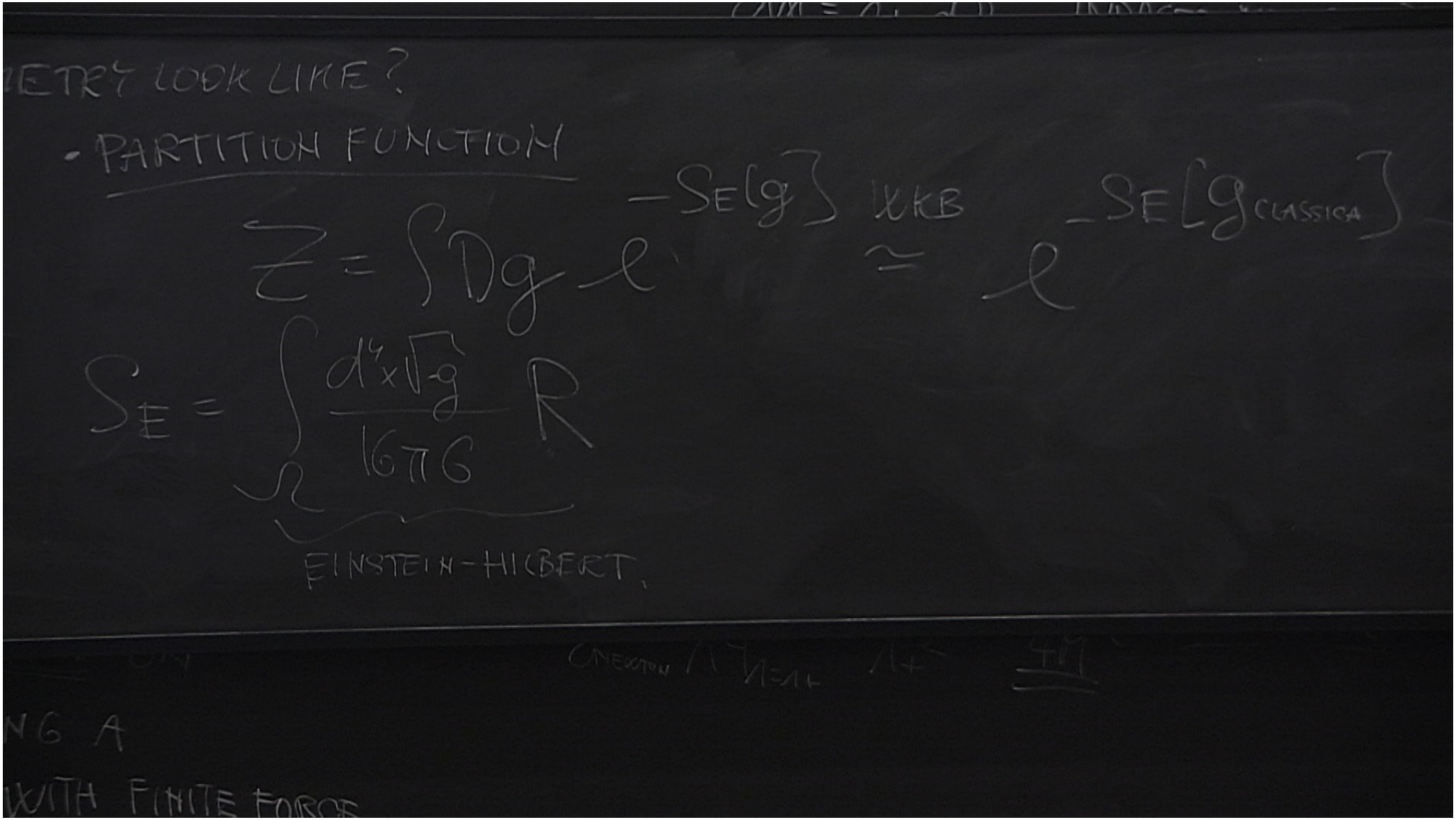
≃ CIGAR

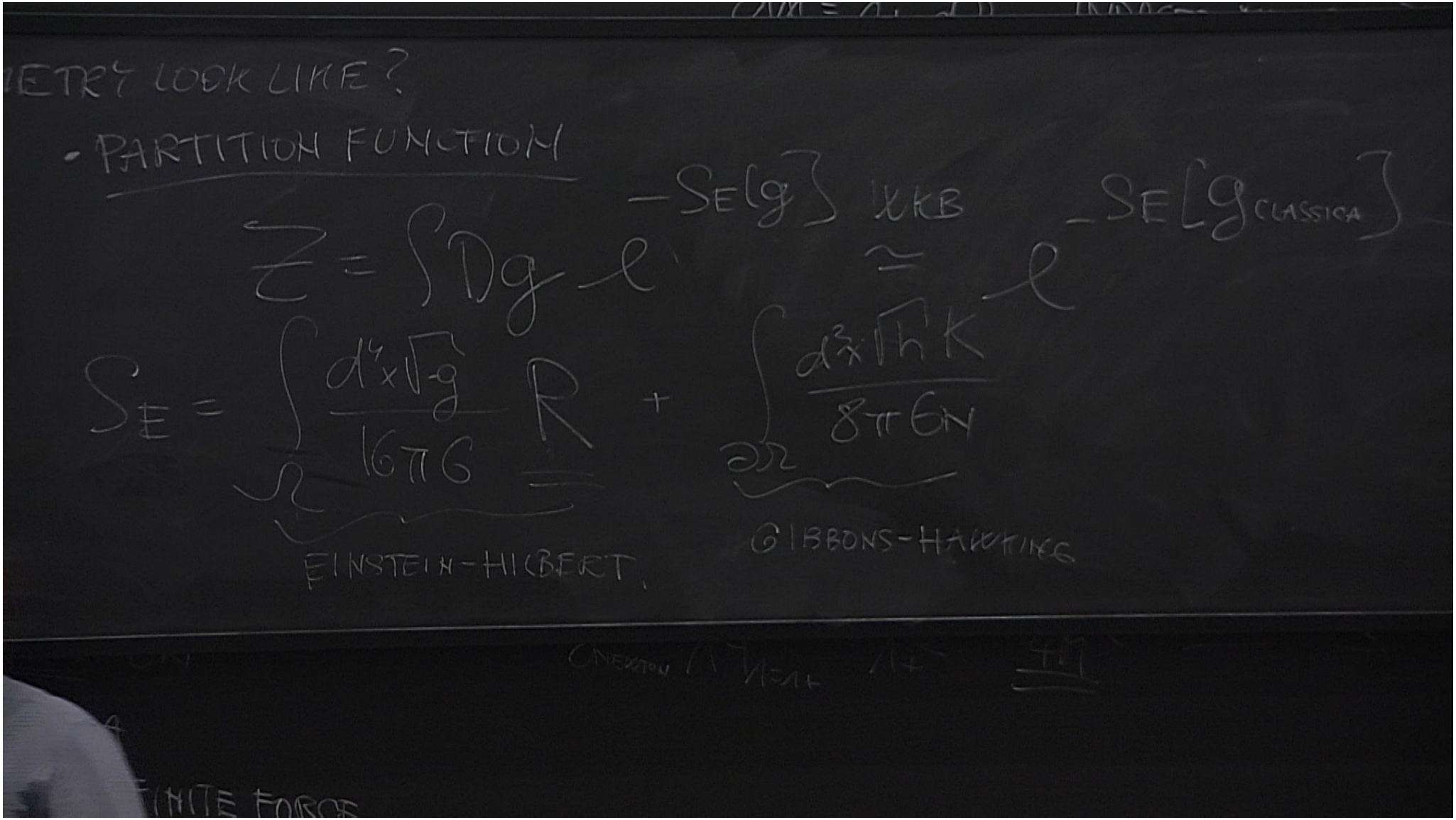
- PARTITION FUNCTION

$Z = \int$




BLACK HOLE
METRY LOOK LIKE?
• PARTITION FUNCTION
$Z = \int Dg\, e^{-S_E[g]}$
WKB
$\simeq e^{-S_E[g_{CLASSICA}]}$


ETRY LOOK LIKE?
• PARTITION FUNCTION
$Z = \int Dg\, e^{-S_E[g]}$ WKB $\simeq e^{-S_E[g_{CLASSICA}]}$
$S_E = \int \frac{d^4x\sqrt{g}}{16\pi G} R$
EINSTEIN-HILBERT.
NG A
WITH FINITE FORCE


ETRY LOOK LIKE?
• PARTITION FUNCTION
$Z = \int Dg\, e^{-S_E[g]}$
$\hbar \to 0$
$\simeq e^{-S_E[g_{CLASSICA}]}$
$S_E = \int_{\Omega} \frac{d^4x\sqrt{g}}{16\pi G} R + \int_{\partial\Omega} \frac{d^3x\sqrt{h}K}{8\pi G_N}$
EINSTEIN-HILBERT
GIBBONS-HAWKING

$f(n)$ at 0 $\quad dt = 0 = dn$

$dn^2 = n^2 d\Omega^2$

ETRY LOOK LIKE?

- PARTITION FUNCTION

$$Z = \int Dg\, e^{-S_E[g]} \overset{WKB}{\simeq} e^{-S_E[g_{CLASSICA}]}$$

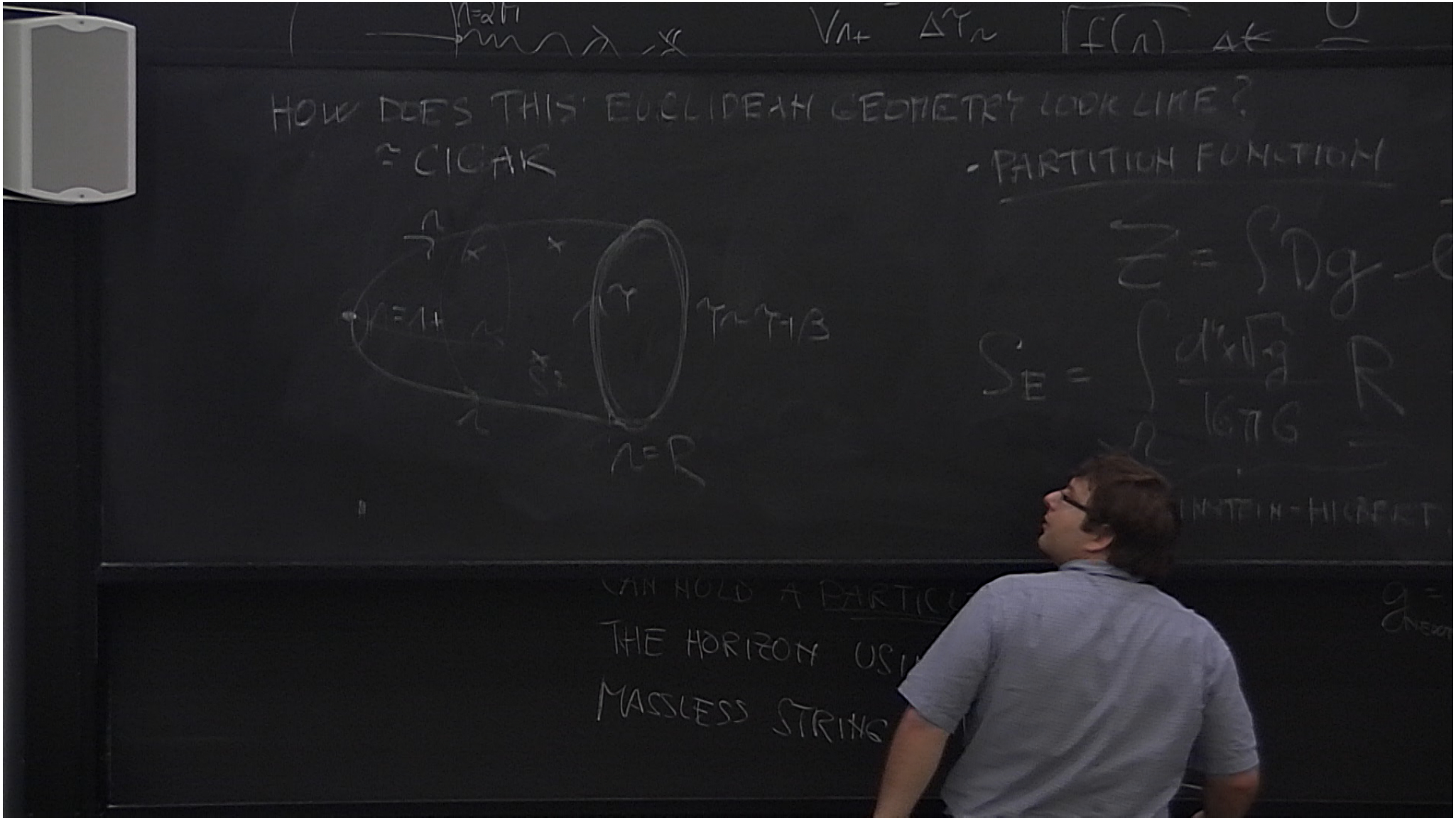
$$S_E = \int_{\Omega} \frac{d^4x\sqrt{g}}{16\pi G} R + \int_{\partial\Omega} \frac{d^3x\sqrt{h}K}{8\pi G_N} - \int_{\partial\Omega} \frac{d^3x\sqrt{h}K_0}{8\pi G_N}$$

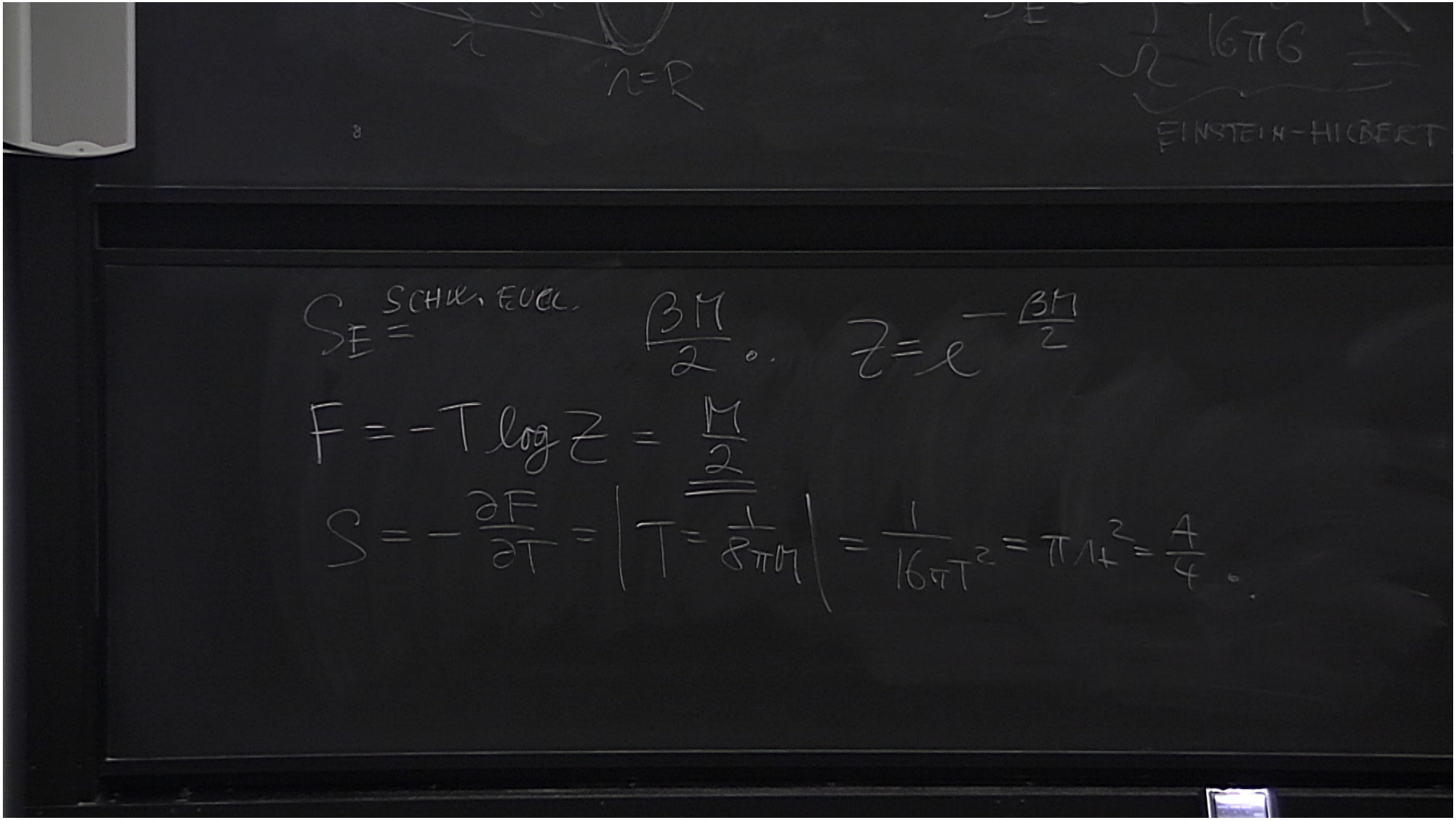
EINSTEIN-HILBERT. GIBBONS-HAWKING TO TUNE VALUE OF S_E.

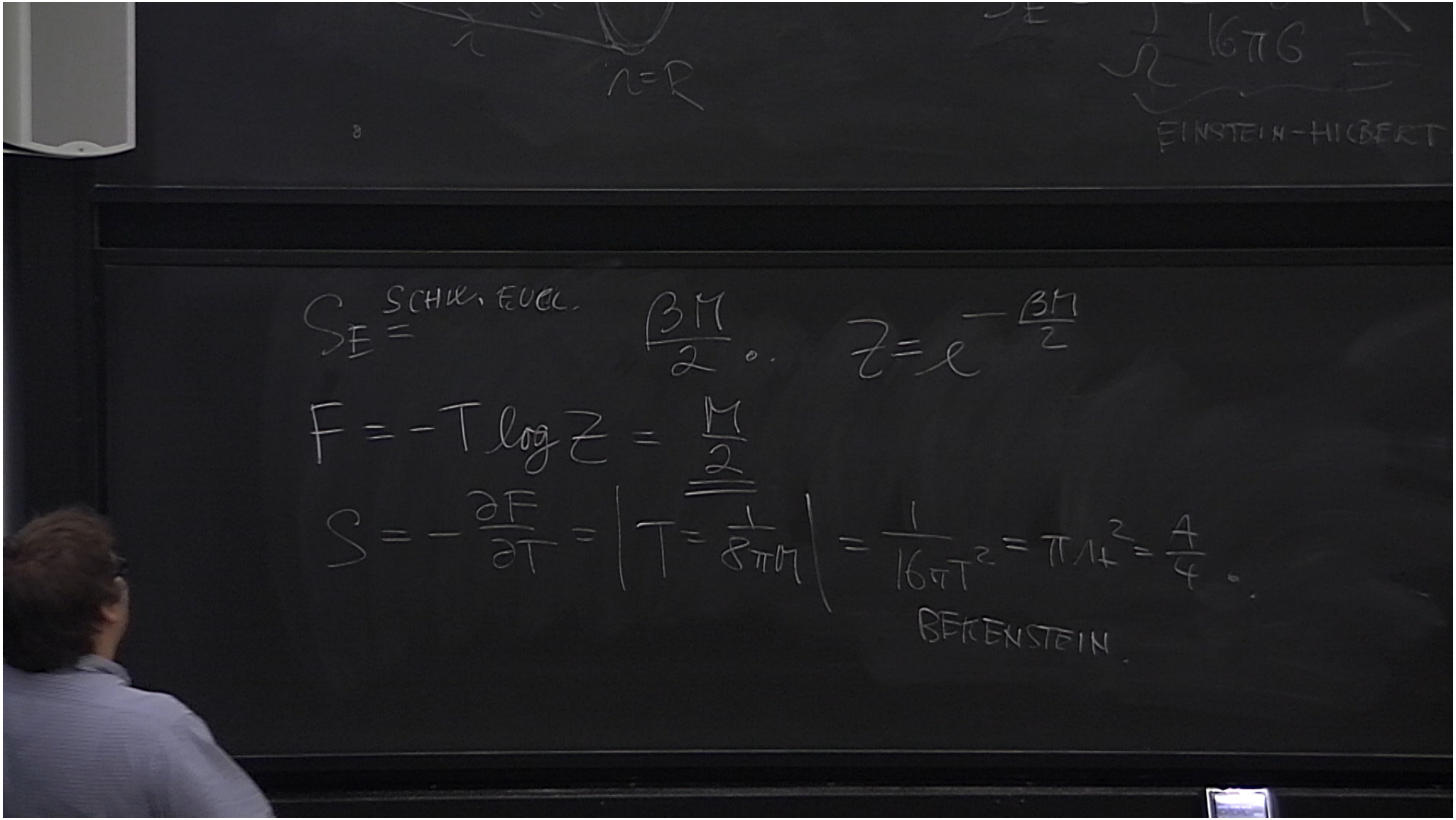
CANCELLATION

NG A

WITH FINITE FORCE


HOW DOES THIS EUCLIDEAN GEOMETRY LOOK LIKE?
= CIGAR
- PARTITION FUNCTION
T=T+1/β
Λ=R
THE HORIZON
MASSLESS


r=R
16πG
EINSTEIN-HILBERT
$S_E = \overset{SCHW. EUCL.}{\frac{\beta M}{2}}$ .. $Z = e^{-\frac{\beta M}{2}}$
$F = -T \log Z = \frac{M}{2}$
$S = -\frac{\partial F}{\partial T} = \left| T = \frac{1}{8\pi M} \right| = \frac{1}{16\pi T^2} = \pi A_+^2 = \frac{A}{4}$ ..

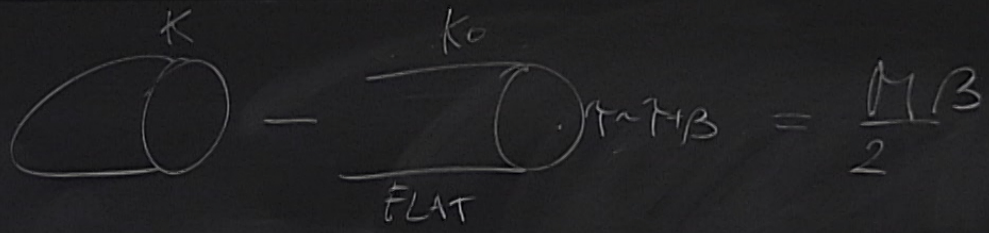

$\lambda = R$
$\frac{1}{16\pi G}$
EINSTEIN-HILBERT
$S_E = \frac{\beta M}{2}$ SCHW. EUCL.
$Z = e^{-\frac{\beta M}{2}}$
$F = -T \log Z = \frac{M}{2}$
$S = -\frac{\partial F}{\partial T} = \left| T = \frac{1}{8\pi M} \right| = \frac{1}{16\pi T^2} = \pi A^2 = \frac{A}{4}$
BEKENSTEIN

$\frac{1}{16\pi G}$ EINSTEIN-HILBERT.

$\frac{1}{8\pi G_N}$ GIBBONS-HAWKING

$\frac{1}{8\pi G_N}$ TO TUNE VALUE OF S_E.

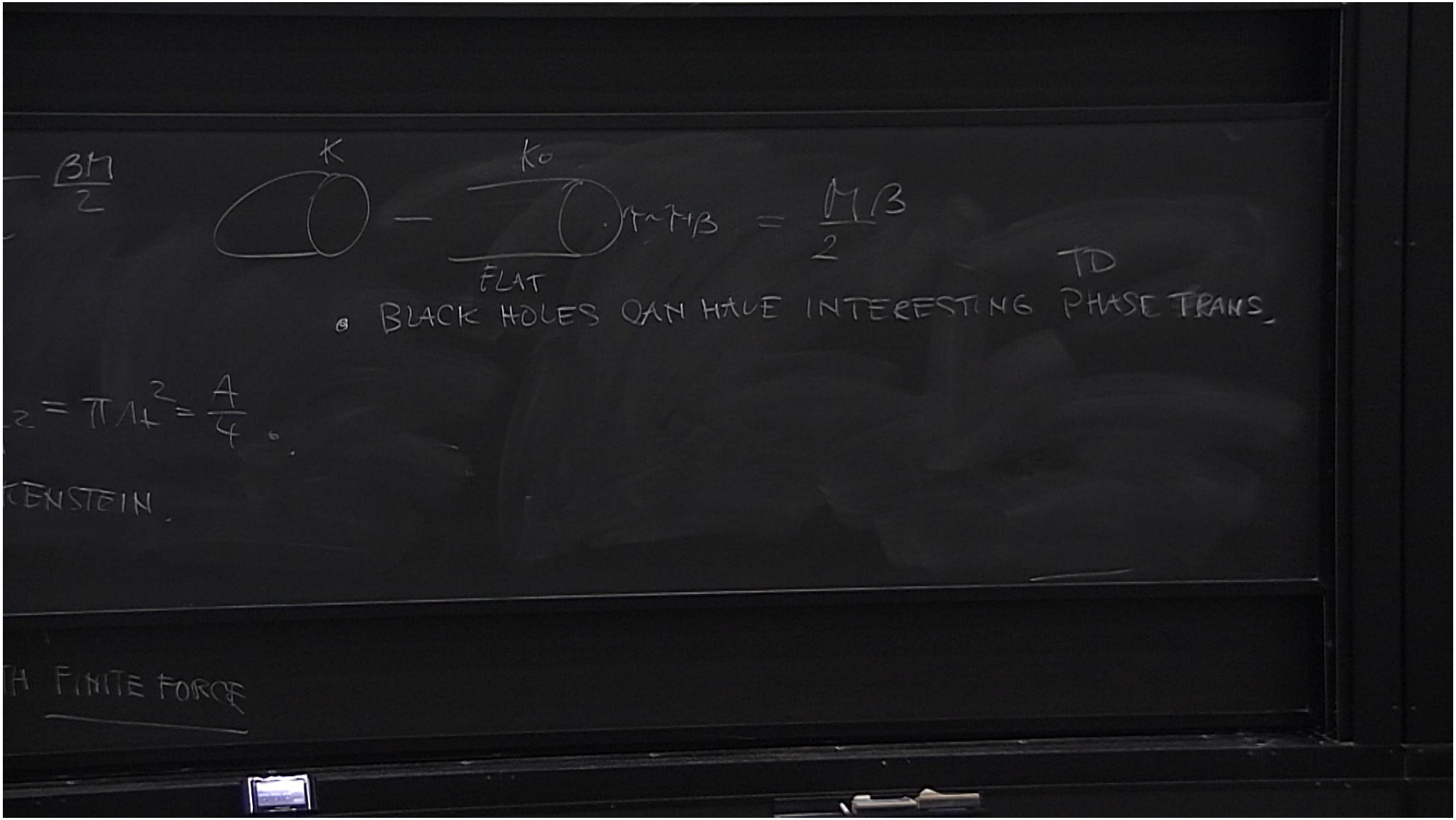
$-\frac{\beta H}{2}$



$= \frac{M\beta}{2}$

$= \pi r_+^2 = \frac{A}{4}$.

KENSTEIN.


K
k₀
FLAT
TD
BLACK HOLES CAN HAVE INTERESTING PHASE TRANS
FINITE FORCE

$\frac{1}{2\kappa}\int R\sqrt{g}$ $\frac{1}{8\pi G}$ = $\frac{1}{2\kappa}\int$ $\frac{1}{8\pi G}$ + $\frac{1}{2\kappa}\int$ $\frac{1}{8\pi G}$

EINSTEIN-HILBERT GIBBONS-HAWKING TO TUNE VALUE OF SE.

$\frac{\beta\Lambda}{2}$



$T \sim \frac{\Lambda\beta}{2}$

TD

- BLACK HOLES CAN HAVE INTERESTING PHASE TRANS.

$$F = M - TS$$

MASS

$r_+ = \pi\Lambda r_+^2 = \frac{A}{4}$.

EINSTEIN.

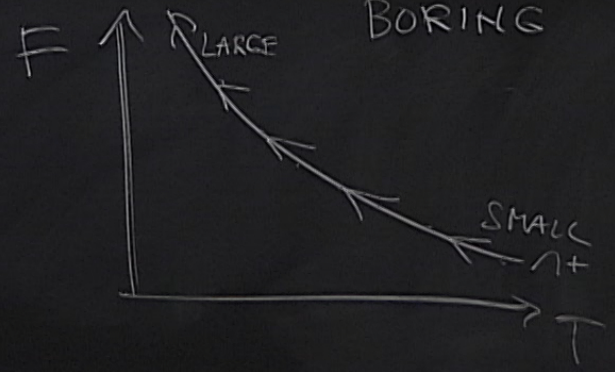
$\mathcal{H}$. SURFACE GRAVITY. ACCELERATION ($m=1$)

$\mathcal{H} = \frac{f'(r_+)}{2} = \frac{d}{dr}\left(1 - \frac{2\Lambda}{r}\right)\Big|_{r=r_+} = \frac{1}{4M} = \frac{1}{2r_+}$

$$S = -\frac{\partial F}{\partial T} = \left| T = \frac{1}{8\pi M} \right| = \frac{1}{16\pi T^2} = \pi/4^2 = \frac{A}{4}$$

BEKENSTEIN.

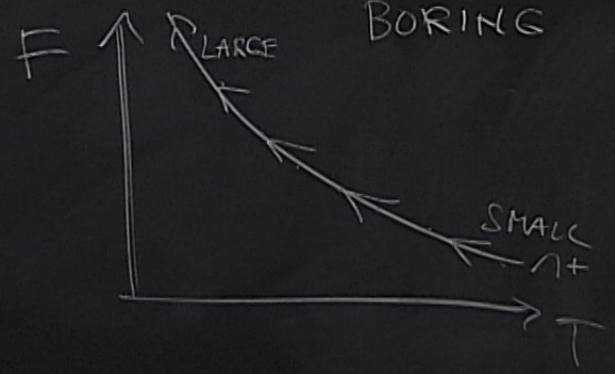
- SCHW BH: $F = F(T, \ldots)$ BORING



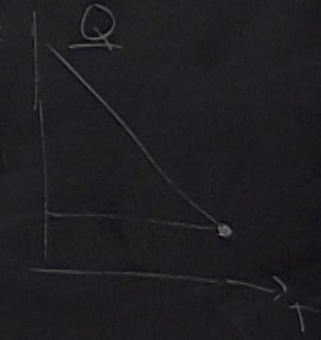
$$S = -\frac{\partial F}{\partial T} = \left| T = \frac{1}{8\pi M} \right| = \frac{1}{16\pi T^2} = \pi A^2 = \frac{A}{4}$$

BEKENSTEIN.

- SCHW BH: $F = F(T, \ldots)$ BORING



- SCHW + $\Lambda < 0$:



$r_+^2 = \pi \Lambda_+^2 = \frac{A}{4}$.

$F = M - TS$

MASS

TENSTEIN.

• SCHW + $\Lambda < 0$:

$f = 1 - \frac{2m}{r} + \frac{r^2}{\ell^2}$.

$\Lambda r - \frac{1}{\ell^2}$

T

$r_+^2 = \pi \Lambda_+^2 = \frac{A}{4}$.

$F = M - TS$

MASS

KENSTEIN.

. SCHW + $\Lambda < 0$:

$f = 1 - \frac{2m}{r} + \frac{r^2}{\ell^2}$

SYSTEM = BH + RADIATION

RADIATION → LARGE BH

P.T.

ADS/CFT. DECONFINMENT OF QUARK.

