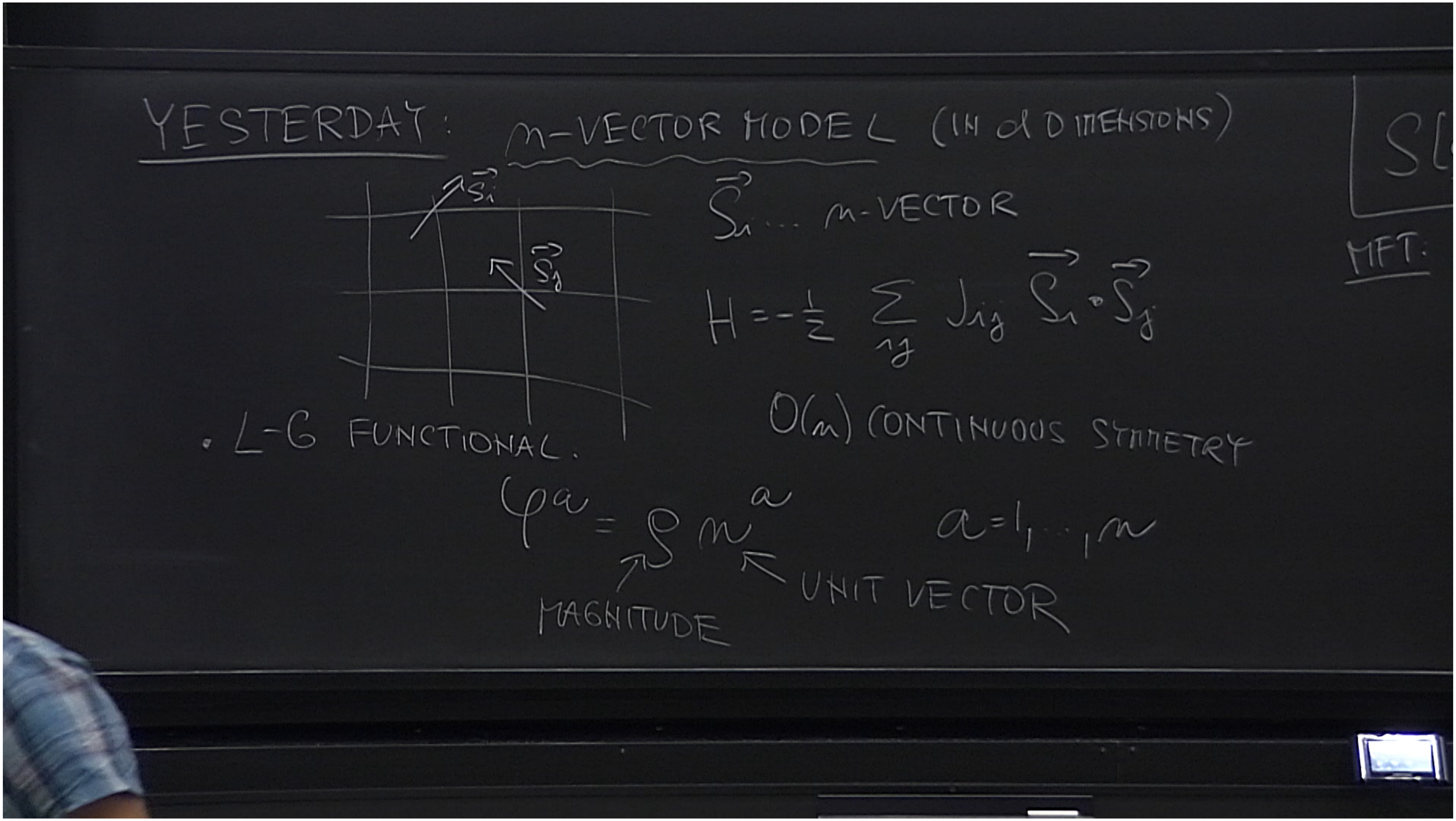
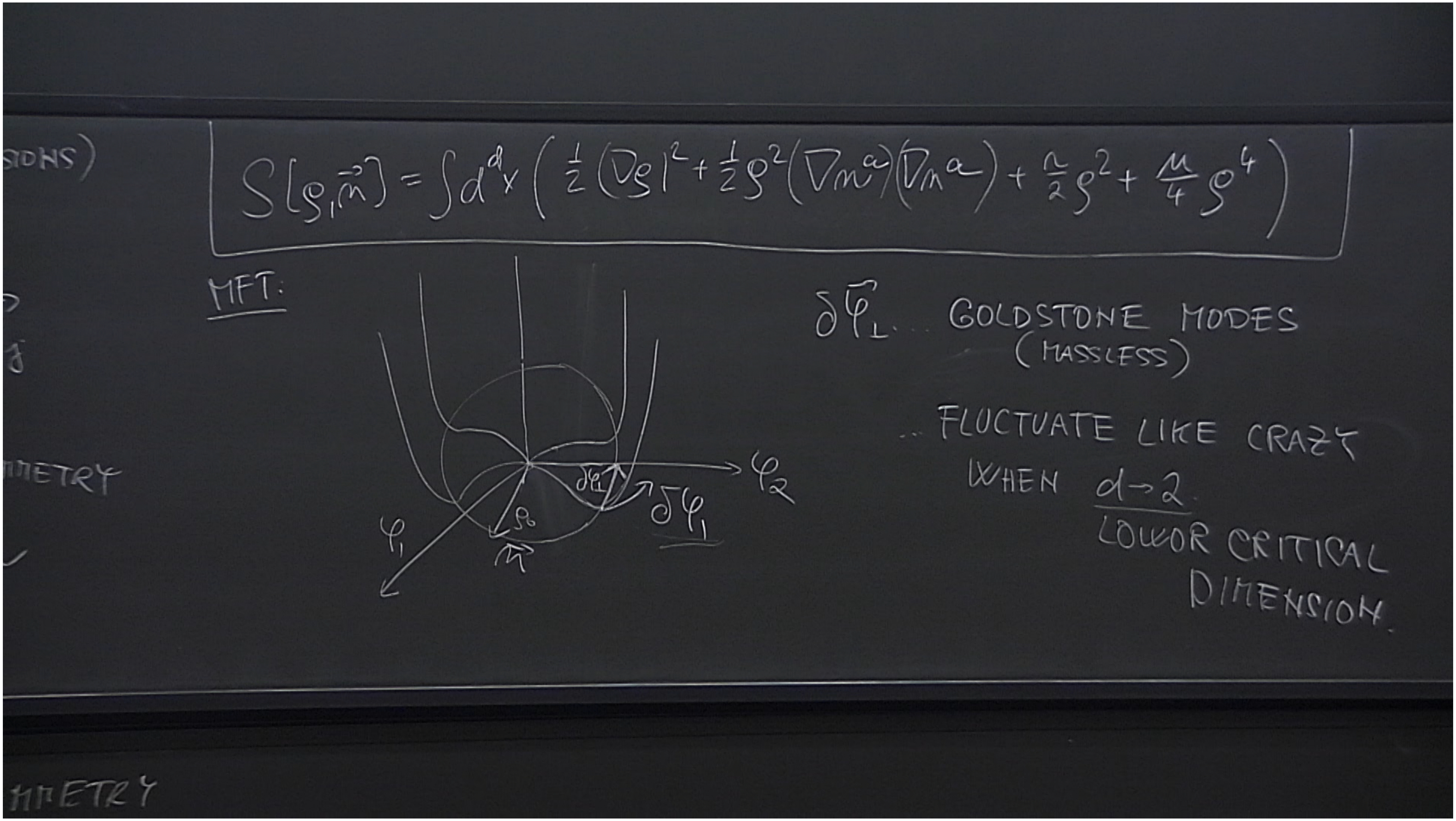
Title: PSI 17/18 - Statistical Mechanics - Lecture 11

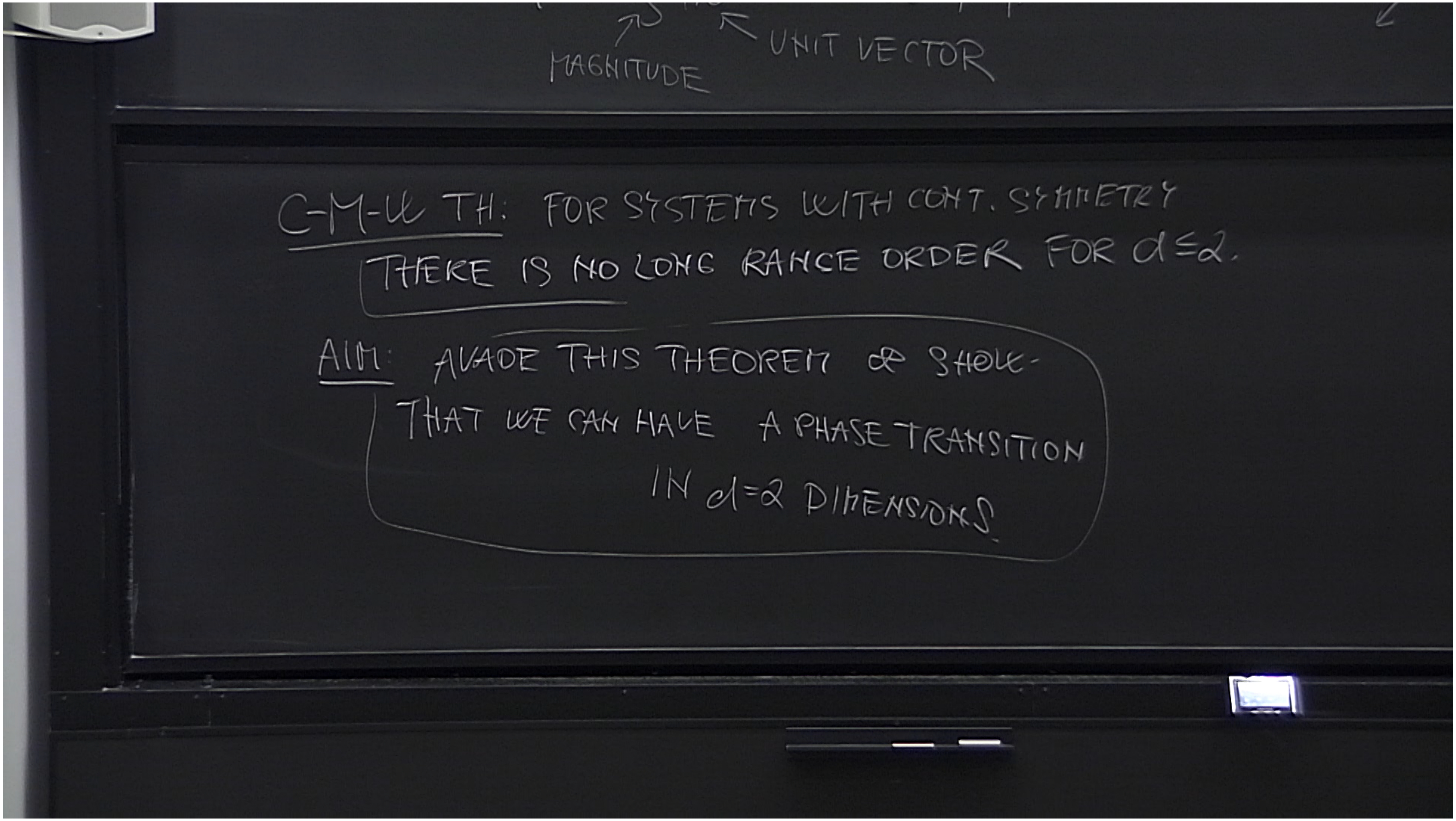
Date: Oct 24, 2017 10:45 AM

URL: http://pirsa.org/17100044

Abstract:


YESTERDAY:
n-VECTOR MODEL (IN d DIMENSIONS)
$\vec{S}_i$
$\vec{S}_j$
$\vec{S}_i$ ... n-VECTOR
$H = -\frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$
O(n) CONTINUOUS SYMMETRY
• L-G FUNCTIONAL.
$\varphi^a = \rho\, n^a$
$a = 1, \ldots, n$
MAGNITUDE
UNIT VECTOR
MFT:


$S[\rho, \vec{n}] = \int d^d x \left(\frac{1}{2} (\nabla \rho)^2 + \frac{1}{2} \rho^2 (\nabla n^a)(\nabla n^a) + \frac{r}{2} \rho^2 + \frac{u}{4} \rho^4 \right)$
MFT:
φ_1
φ_2
ρ_0
$\vec{n}$
$\delta\varphi_\perp$
$\delta\varphi_1$
$\delta\vec{\varphi}_\perp$ ... GOLDSTONE MODES
(MASSLESS)
... FLUCTUATE LIKE CRAZY
WHEN $d \to 2$.
LOWER CRITICAL
DIMENSION.


MAGNITUDE
UNIT VECTOR
C-M-W TH: FOR SYSTEMS WITH CONT. SYMMETRY
THERE IS NO LONG RANGE ORDER FOR d≤2.
AIM: AVADE THIS THEOREM & SHOW-
THAT WE CAN HAVE A PHASE TRANSITION
IN d=2 DIMENSIONS

n^a $a = 1, \ldots, n$

← UNIT VECTOR

UDE

LOWER CRITICAL DIMENSION

YSTEMS WITH CONT. SYMMETRY

ONG RANGE ORDER FOR $d \le 2$.

THEOREM OF SHOLE-

HAVE A PHASE TRANSITION

IN $d=2$ DIMENSIONS

NEAR $d=2$

FLUCTUATIONS OF ρ NOT IMPORTANT, BUT

— " — OF n^a ARE. $\Rightarrow \rho = \rho_0$.

CHANGE UNITS

$$S[\vec{n}] = \frac{\rho^2}{2} \int d^d x \; D n^a D n^a = \frac{1}{2T} \int d^d x \; D n^a D n^a$$

$a = 1, \ldots, n$

...CTOR

LOWER CRITICAL DIMENSION

... CONT. SYMMETRY

...ORDER FOR $d \leq 2$.

... SHOLE-

... TRANSITION

...NSIONS

NEAR $d=2$

FLUCTUATIONS OF ρ NOT IMPORTANT, BUT

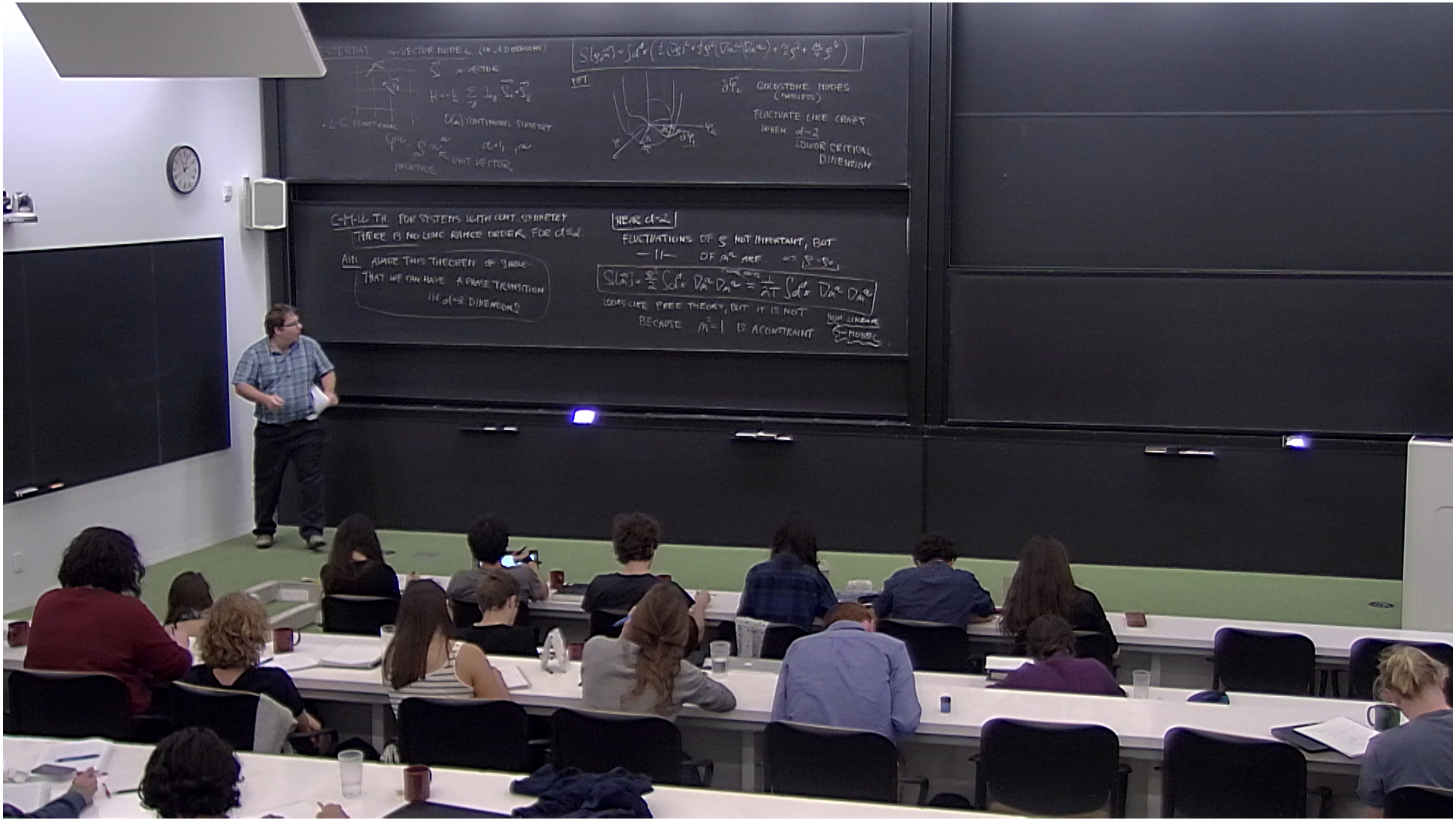
— " — OF n^a ARE. $\Rightarrow$ $\rho = \rho_0$.

CHANGE UNITS

$$S[\vec{n}] = \frac{\rho^2}{2} \int d^dx \, \partial_\mu n^a \partial_\mu n^a = \frac{1}{2T} \int d^dx \, \partial_\mu n^a \partial_\mu n^a$$

LOOKS LIKE FREE THEORY, BUT IT IS NOT

BECAUSE $n^2 = 1$ IS A CONSTRAINT


C-M-W TH. FOR SYSTEMS WITH CONT. SYMMETRY
THERE IS NO LONG RANGE ORDER FOR d≤2
AIM: EVADE THIS THEOREM & SHOW THAT WE CAN HAVE A PHASE TRANSITION IN d=2 DIMENSIONS
NEAR d=2
FLUCTUATIONS OF ρ NOT IMPORTANT, BUT
OF n ARE
LOOKS LIKE FREE THEORY, BUT IT IS NOT
BECAUSE n²=1 IS A CONSTRAINT
NON LINEAR σ-MODEL
GOLDSTONE MODES
FLUCTUATE LIKE CRAZY
LOWER CRITICAL DIMENSION
UNIT VECTOR

NEAR $d=2$

FLUCTUATIONS OF ρ NOT IMPORTANT, BUT

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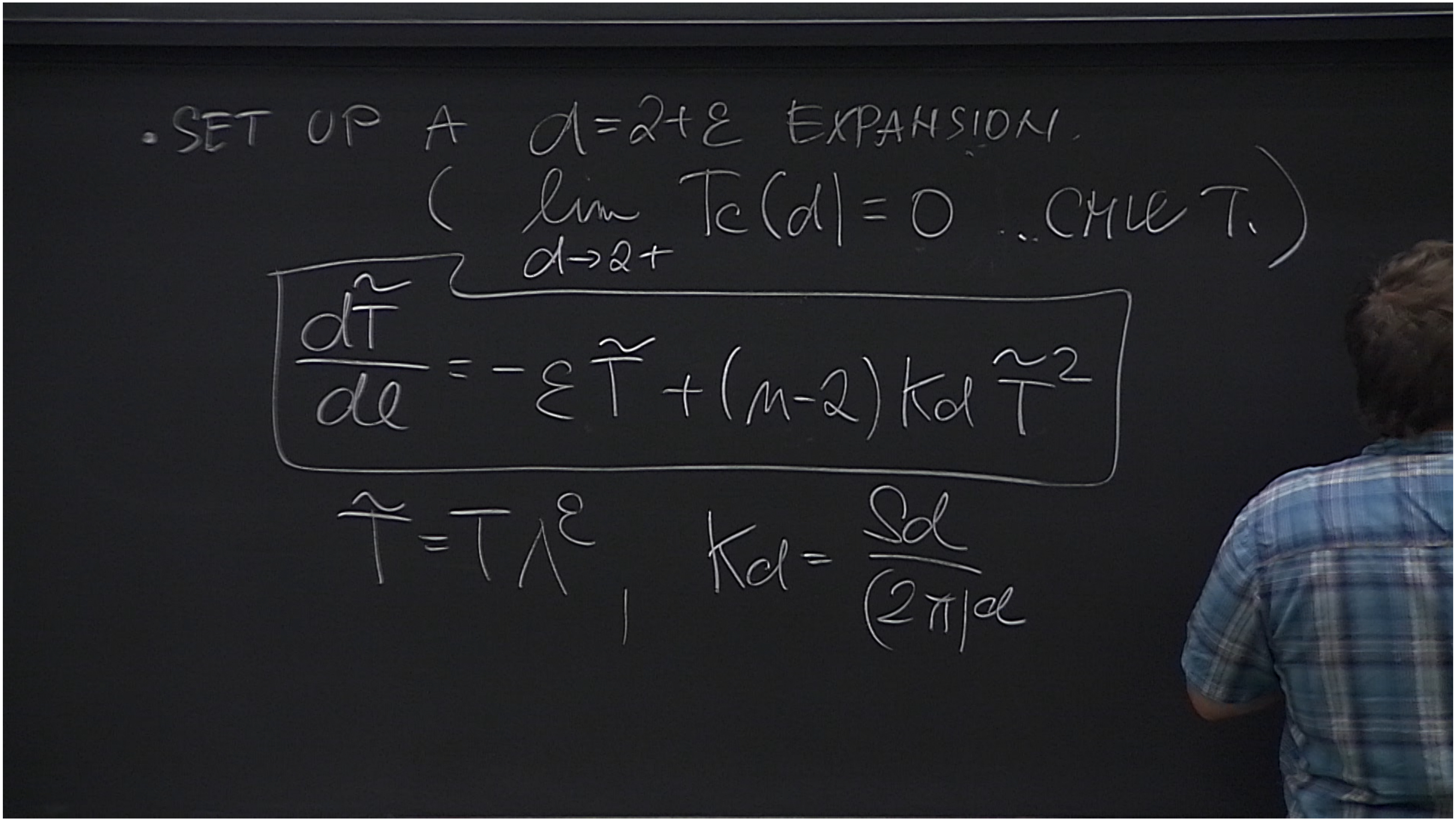
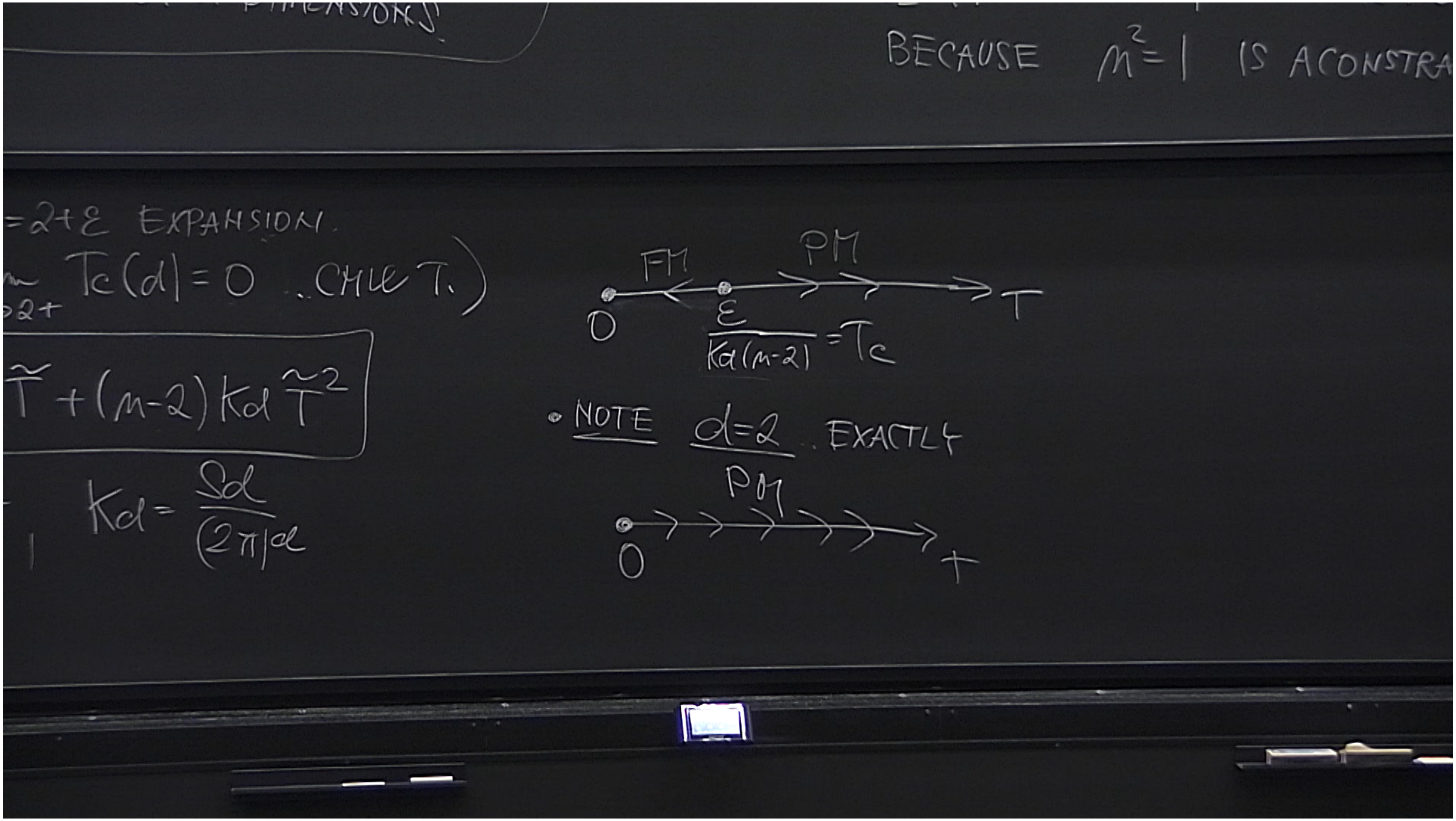
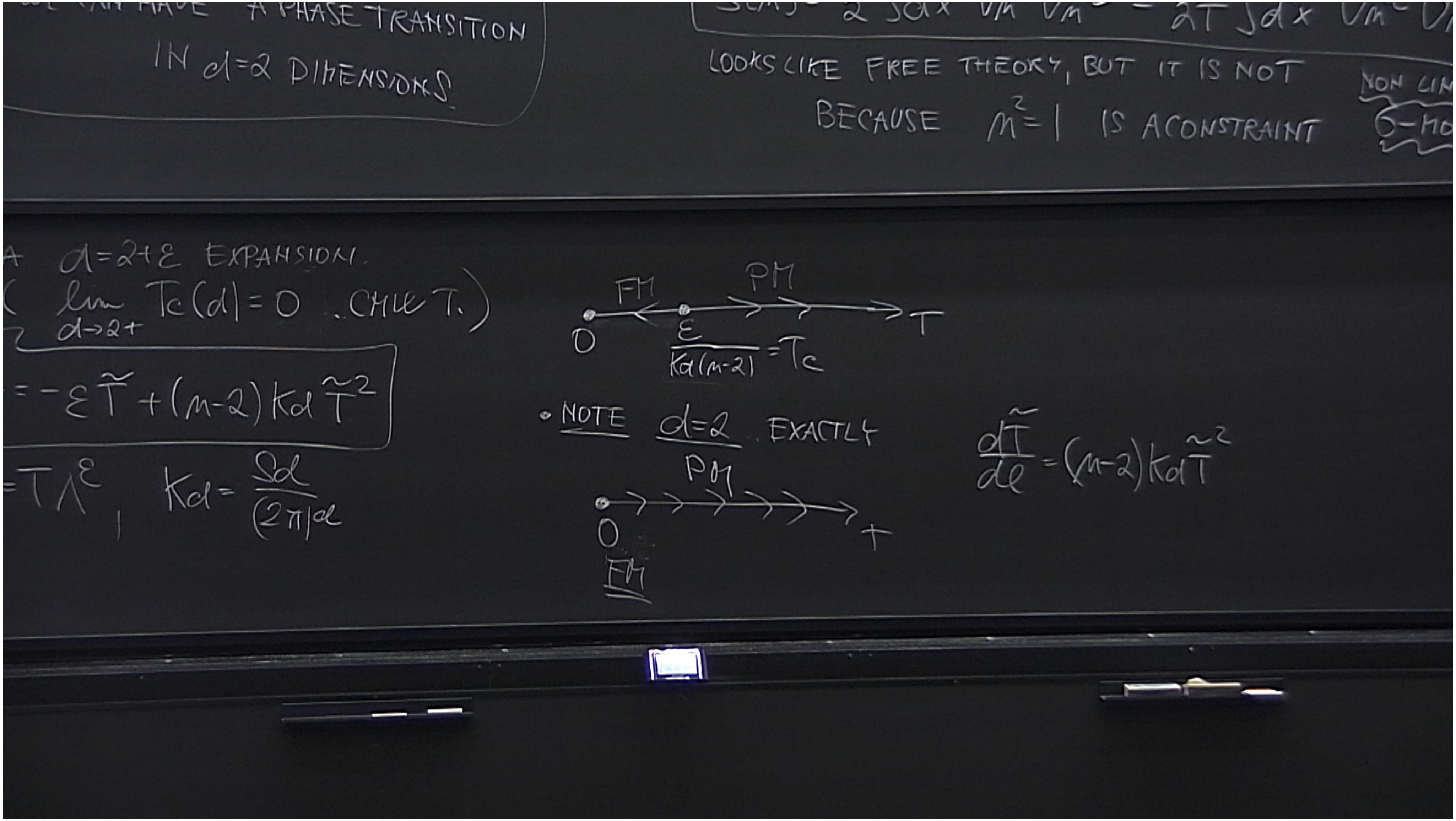
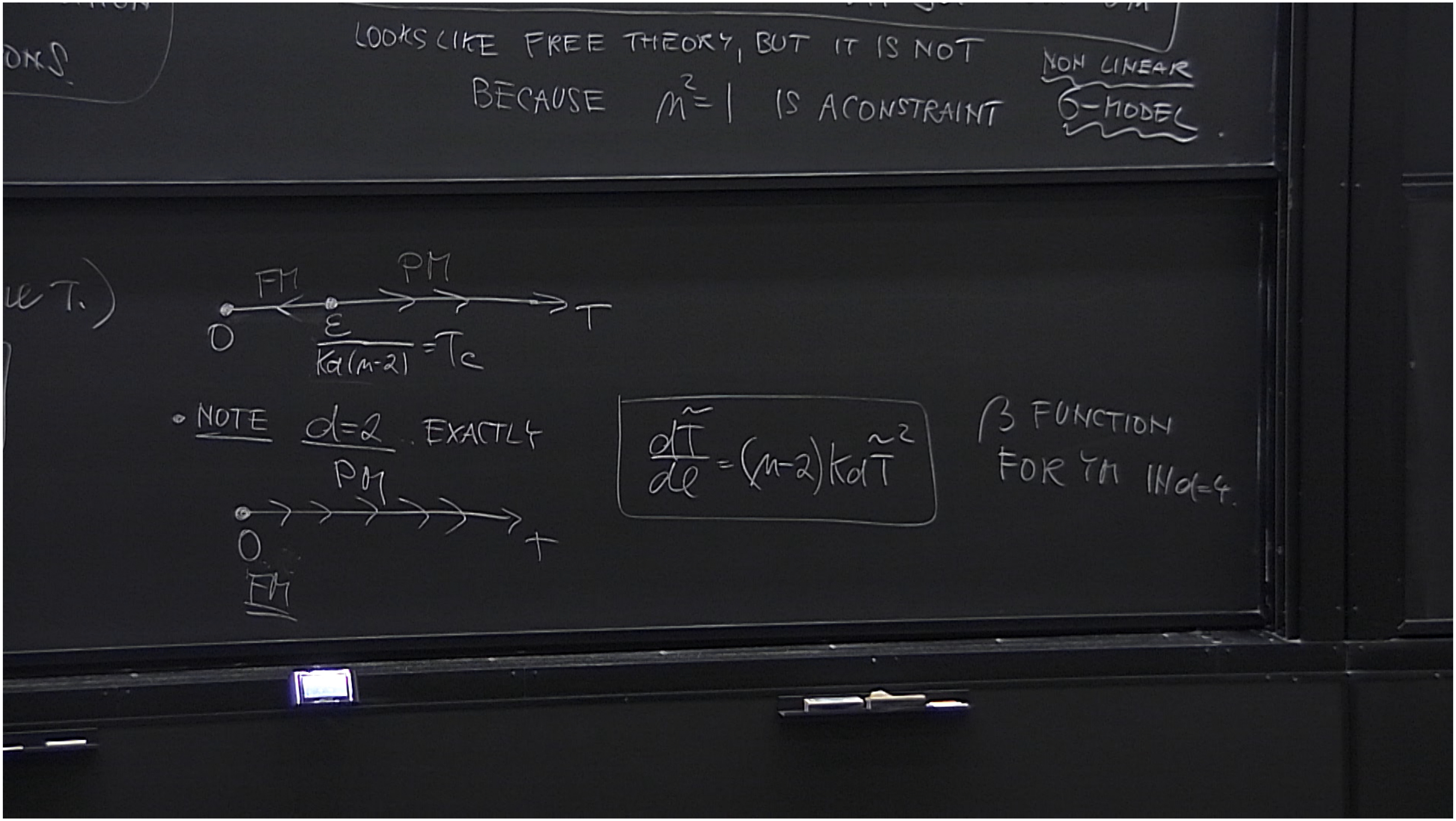
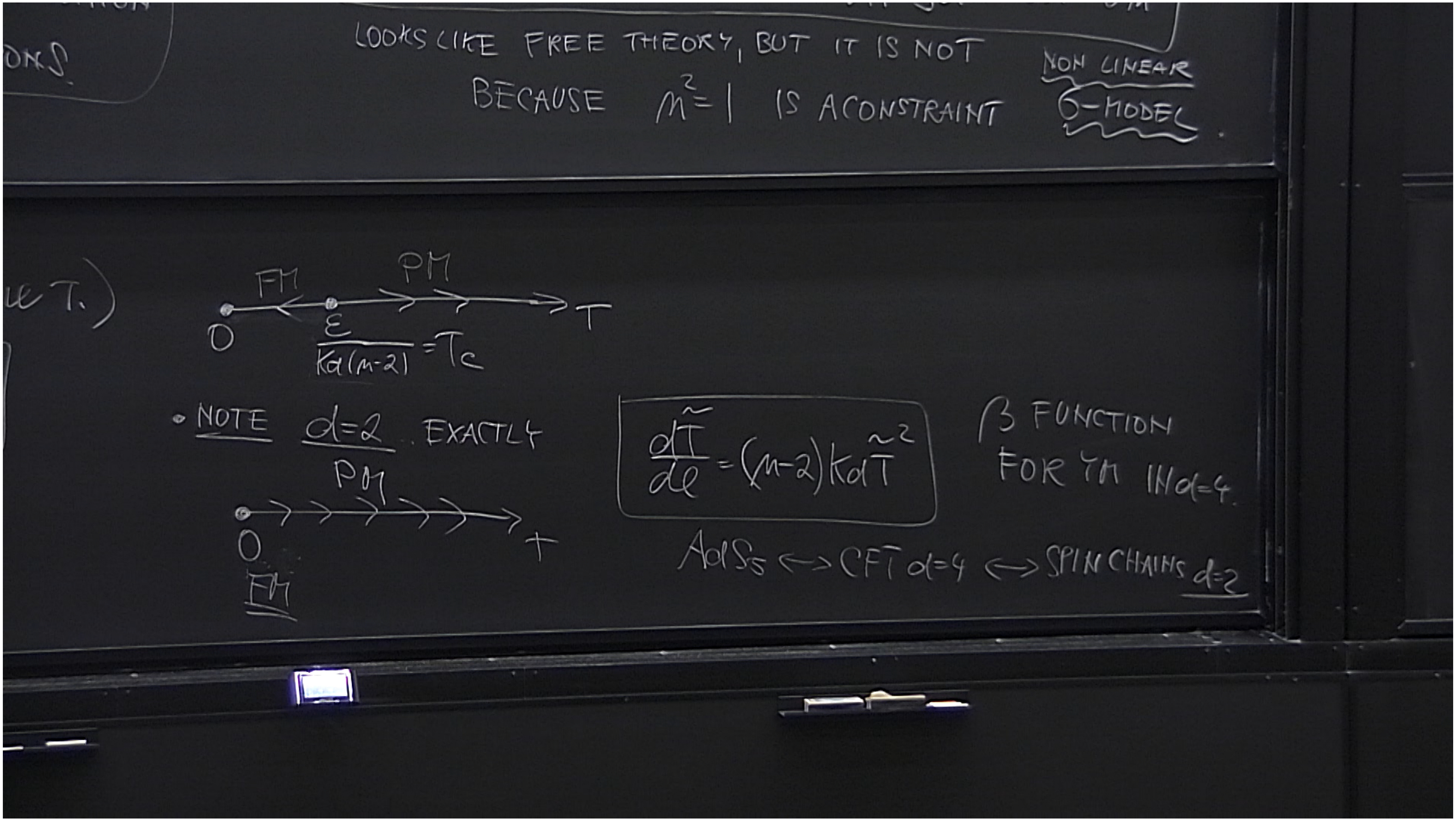
NON LINEAR σ-MODEL.

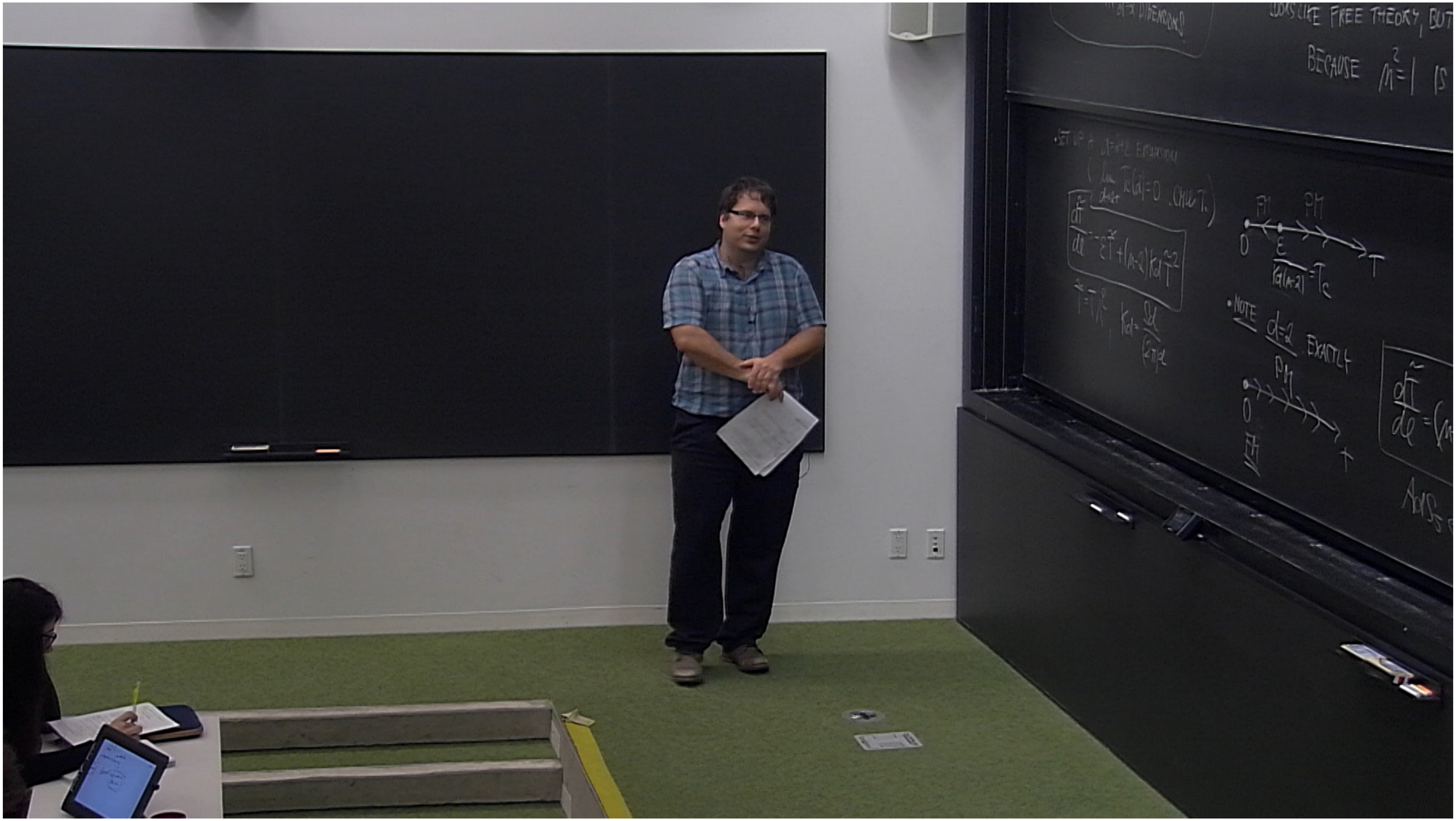

• SET UP A d=2+ε EXPANSION.
(lim d→2+ Tc(d) = 0 ...CRIT T.)
dT̃/dl = −εT̃ + (n−2) Kd T̃²
T̃ = TΛ^ε , Kd = Sd/(2π)^d

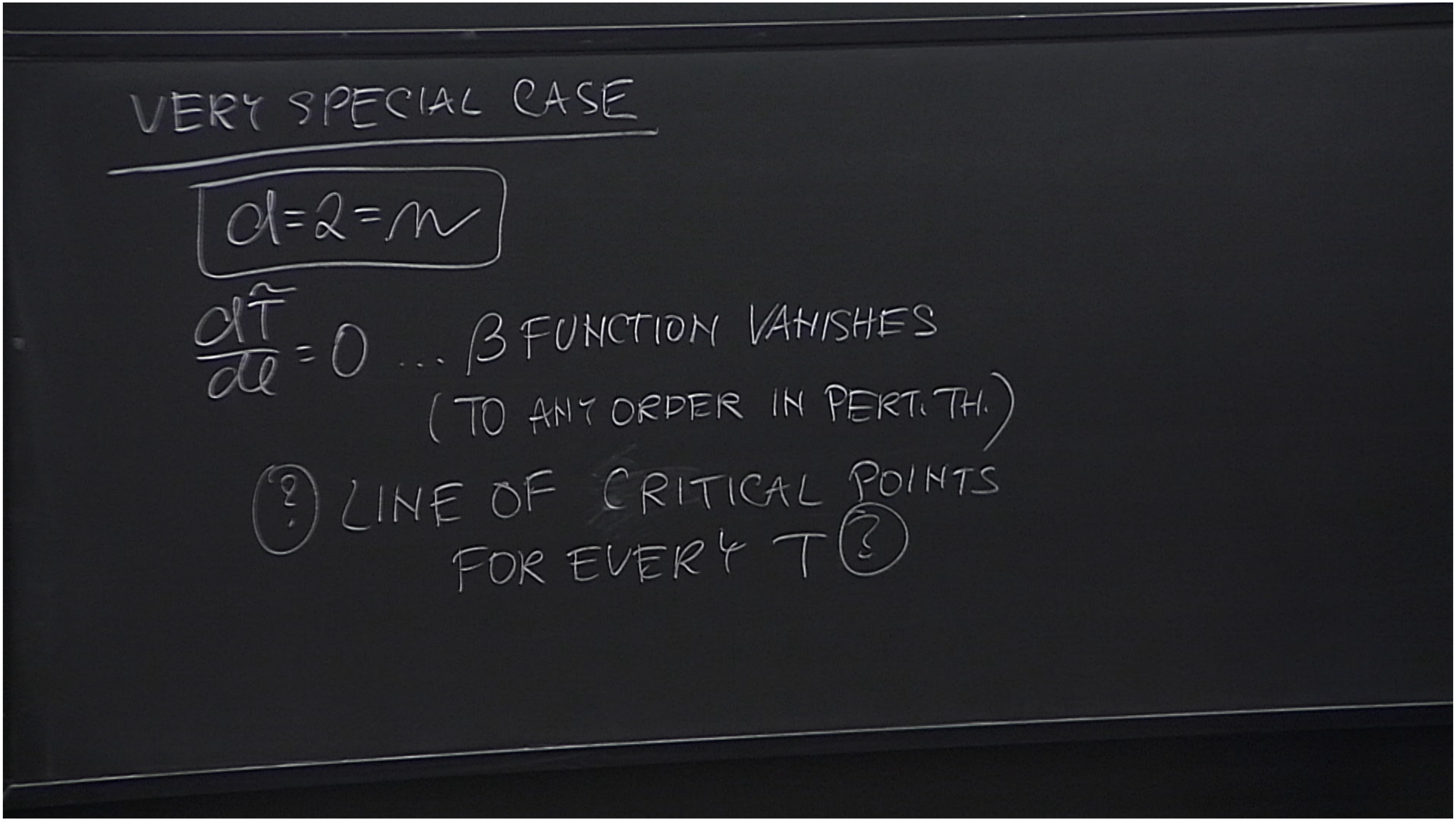

BECAUSE m² = 1 IS ACONSTRA
=2+ε EXPANSION.
Tc(d) = 0
T̃ + (m-2) Kd T̃²
Kd = Sd/(2π)^d
FM
PM
0
T
ε/(Kd(m-2)) = Tc
• NOTE d=2 EXACTLY
PM
0
T


A PHASE TRANSITION
IN d=2 DIMENSIONS
LOOKS LIKE FREE THEORY, BUT IT IS NOT
BECAUSE n²=1 IS A CONSTRAINT
d=2+ε EXPANSION
FM
PM
T
O
ε / Kd(n-2) = Tc
NOTE d=2 EXACTLY
PM
O
T
FM
dT̃/dℓ = (n-2) Kd T̃²


LOOKS LIKE FREE THEORY, BUT IT IS NOT
BECAUSE $n^2 = 1$ IS A CONSTRAINT
NON LINEAR σ-MODEL.
FM
PM
0
T
$\frac{\varepsilon}{K_d(n-2)} = T_c$
• NOTE d=2 EXACTLY
PM
0
FM
T
$\frac{d\tilde{T}}{d\ell} = (n-2) K_d \tilde{T}^2$
β FUNCTION FOR YM IN d=4.


LOOKS LIKE FREE THEORY, BUT IT IS NOT
BECAUSE n² = 1 IS A CONSTRAINT
NON LINEAR σ-MODEL
FM
PM
T
0
ε / Kd(n-2) = Tc
NOTE d=2 EXACTLY
dT̃/dl = (n-2) Kd T̃²
β FUNCTION FOR YM IN d=4
AdS5 ⟷ CFT d=4 ⟷ SPIN CHAINS d=2


LOOKS LIKE FREE THEORY, BUT
BECAUSE
NOTE d=2 EXACTLY


VERY SPECIAL CASE
$d=2=n$
$\frac{d\tilde{T}}{dl}=0$ ... β FUNCTION VANISHES
(TO ANY ORDER IN PERT. TH.)
? LINE OF CRITICAL POINTS
FOR EVERY T ?

VANISHES

IN PERT. TH.)

CAL POINTS

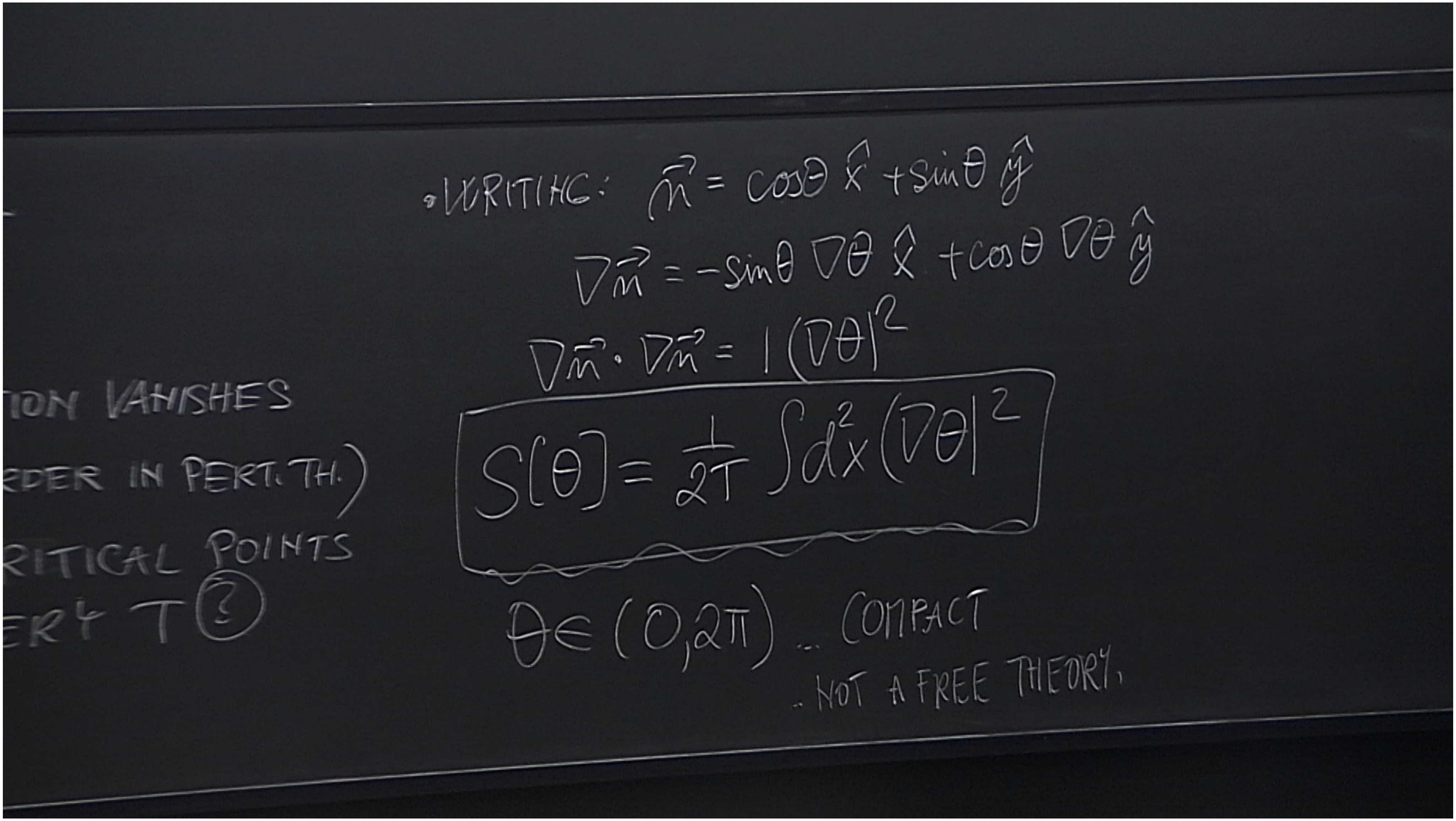
T ?

• WRITING: $\vec{n} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$

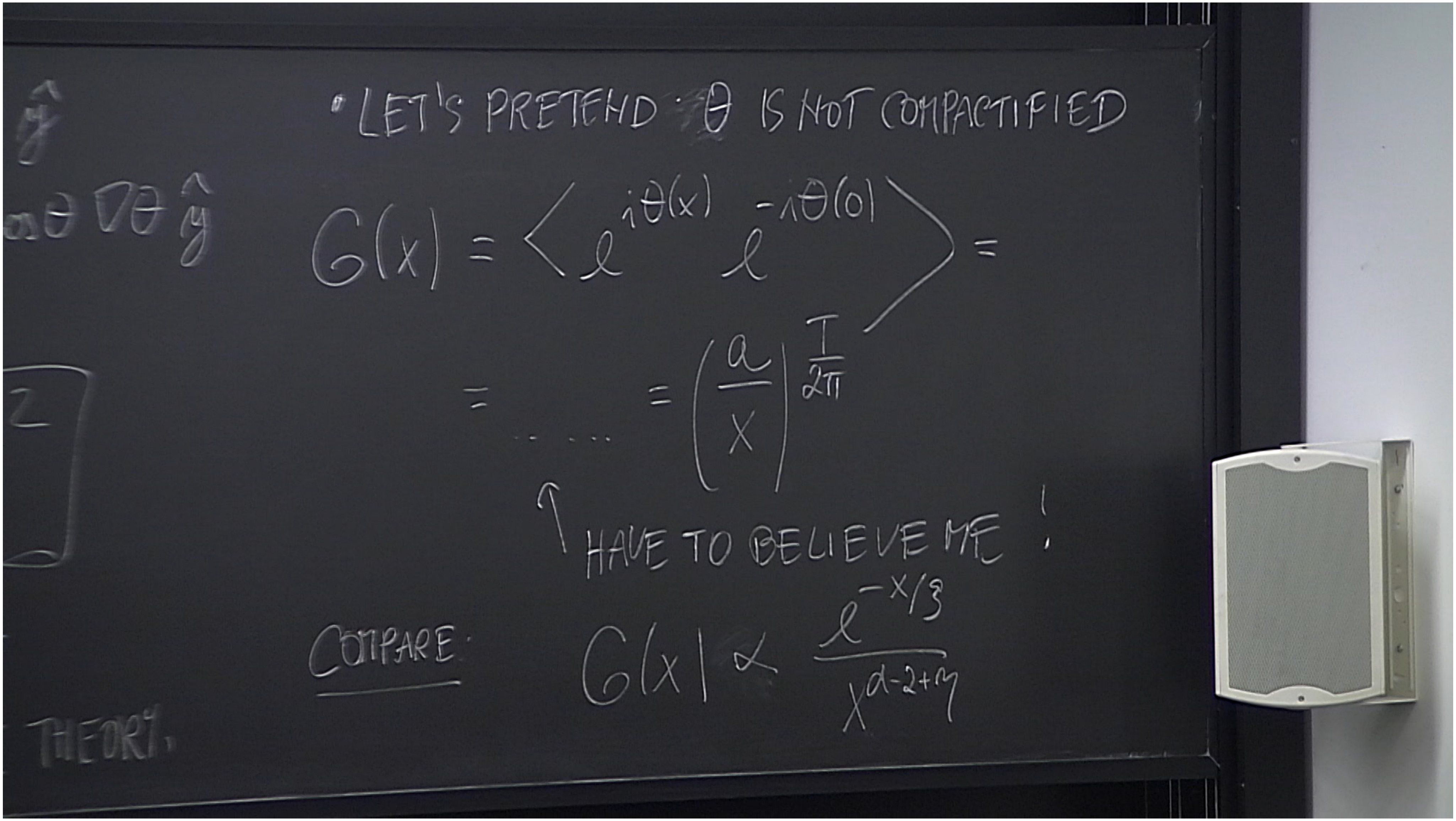
$$\nabla\vec{n} = -\sin\theta\,\nabla\theta\,\hat{x} + \cos\theta\,\nabla\theta\,\hat{y}$$

$$\nabla\vec{n}\cdot\nabla\vec{n} = 1\,(\nabla\theta)^2$$

$$S[\theta] = \frac{1}{2T}\int d^2x\,(\nabla\theta)^2$$



• WRITING: $\vec{n} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$
$\nabla\vec{n} = -\sin\theta\,\nabla\theta\,\hat{x} + \cos\theta\,\nabla\theta\,\hat{y}$
$\nabla\vec{n}\cdot\nabla\vec{n} = 1\,(\nabla\theta)^2$
$S[\theta] = \frac{1}{2T}\int d^2x\,(\nabla\theta)^2$
$\theta \in (0, 2\pi)$ ... COMPACT
.. NOT A FREE THEORY.
ION VANISHES
RDER IN PERT. TH.)
RITICAL POINTS
ERY T


• LET'S PRETEND: θ IS NOT COMPACTIFIED
$G(x) = \langle e^{i\theta(x)} e^{-i\theta(0)} \rangle =$
$= \ldots\ldots = \left(\frac{a}{x}\right)^{\frac{T}{2\pi}}$
HAVE TO BELIEVE ME!
COMPARE:
$G(x) \propto \frac{e^{-x/\xi}}{x^{d-2+\eta}}$
THEORY,

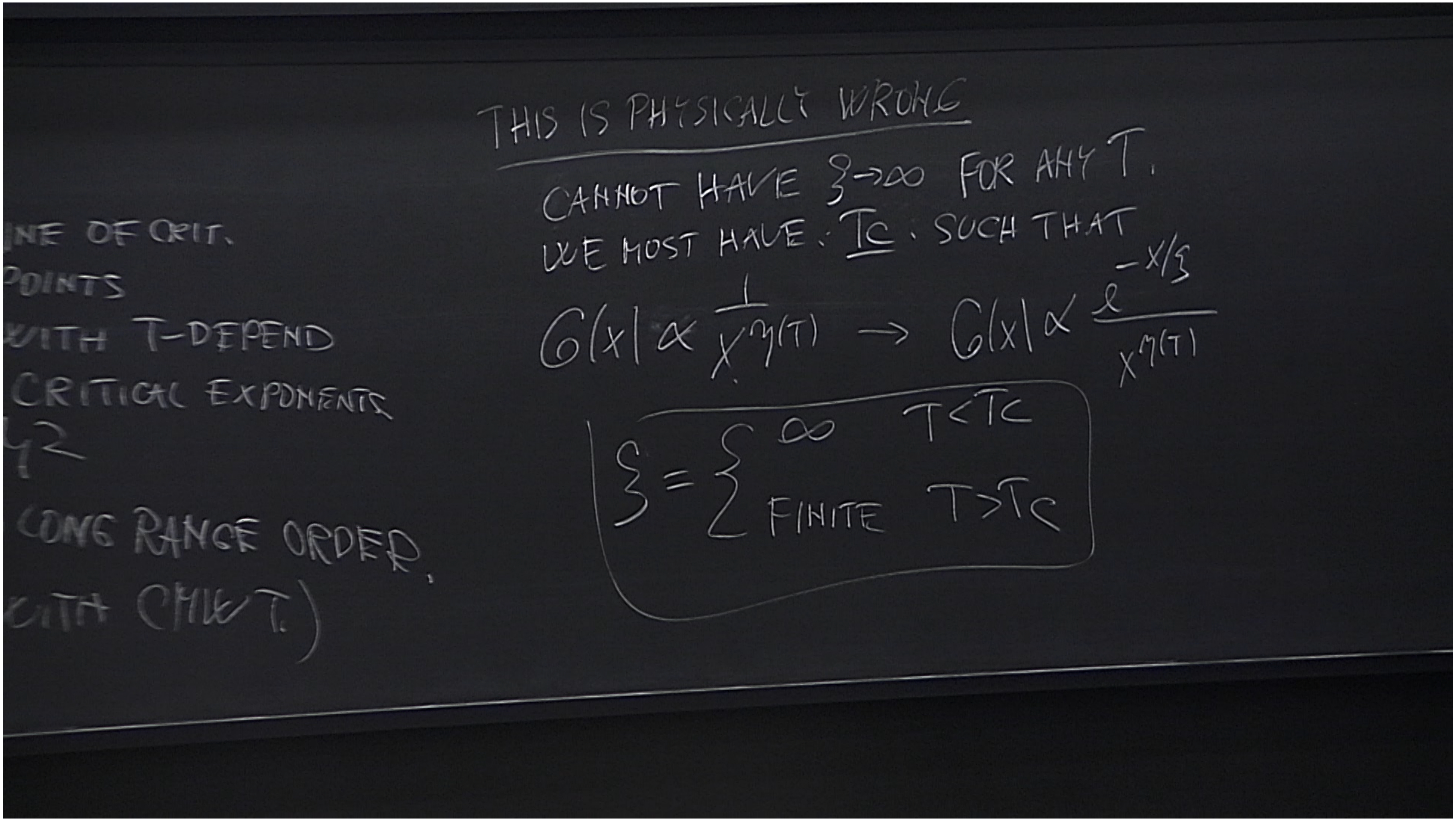
THIS IS PHYS

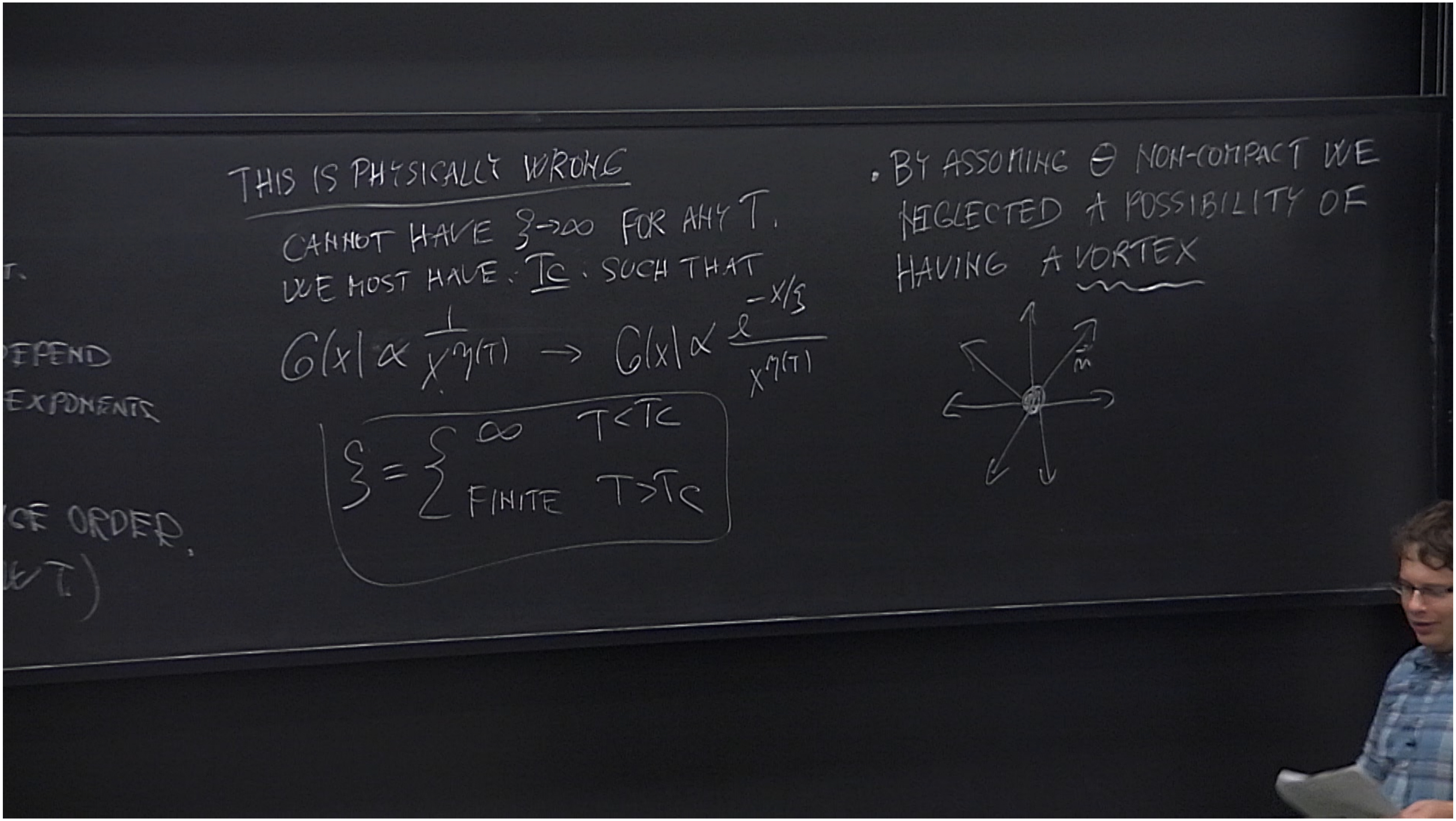
WOULD CONCLUDE:

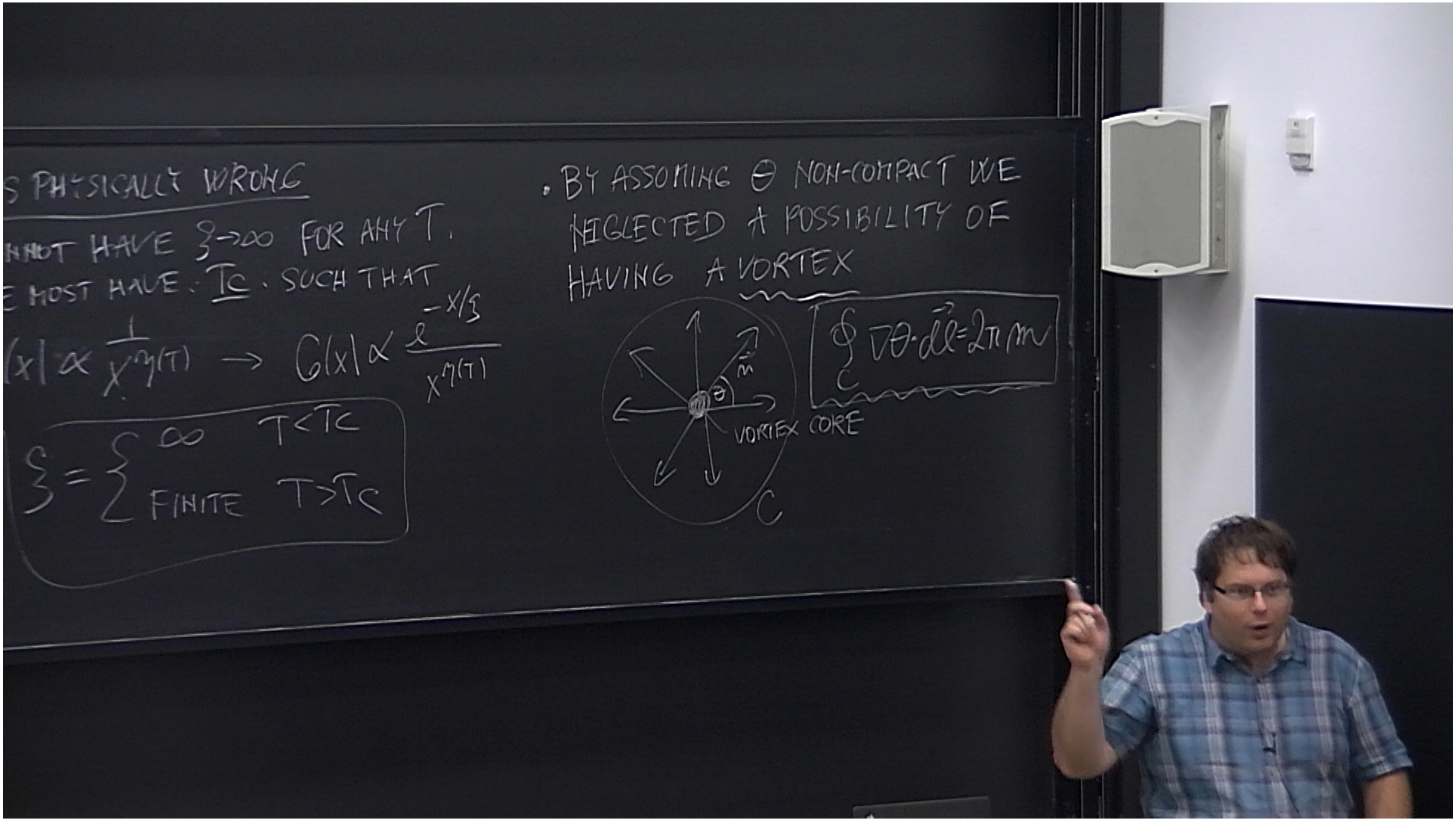
- $\xi \to \infty$ LINE OF CRIT. POINTS
  $\eta = \frac{T}{2\pi} \ldots$ WITH T-DEPEND CRITICAL EXPONENTS
- $G(x \to \infty) \to 0 \neq M^2$

THERE IS NO LONG RANGE ORDER.

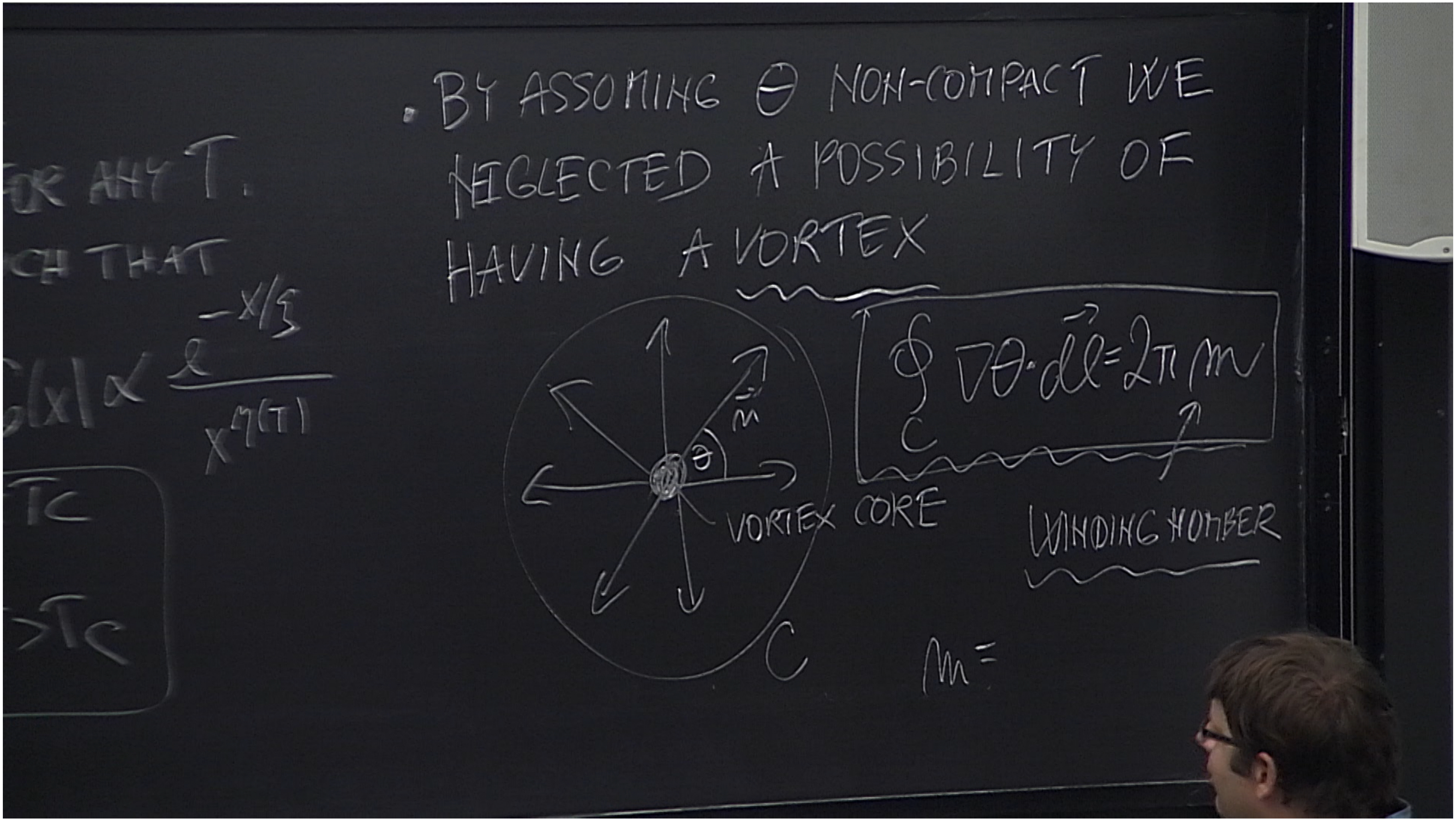
(AGREES WITH CMWT.)


THIS IS PHYSICALLY WRONG
CANNOT HAVE ξ→∞ FOR ANY T.
WE MUST HAVE Tc SUCH THAT
G(x) ∝ 1/x^η(T) → G(x) ∝ e^(-x/ξ)/x^η(T)
ξ = { ∞ T<Tc ; FINITE T>Tc }
POINTS
WITH T-DEPEND
CRITICAL EXPONENTS
LONG RANGE ORDER.

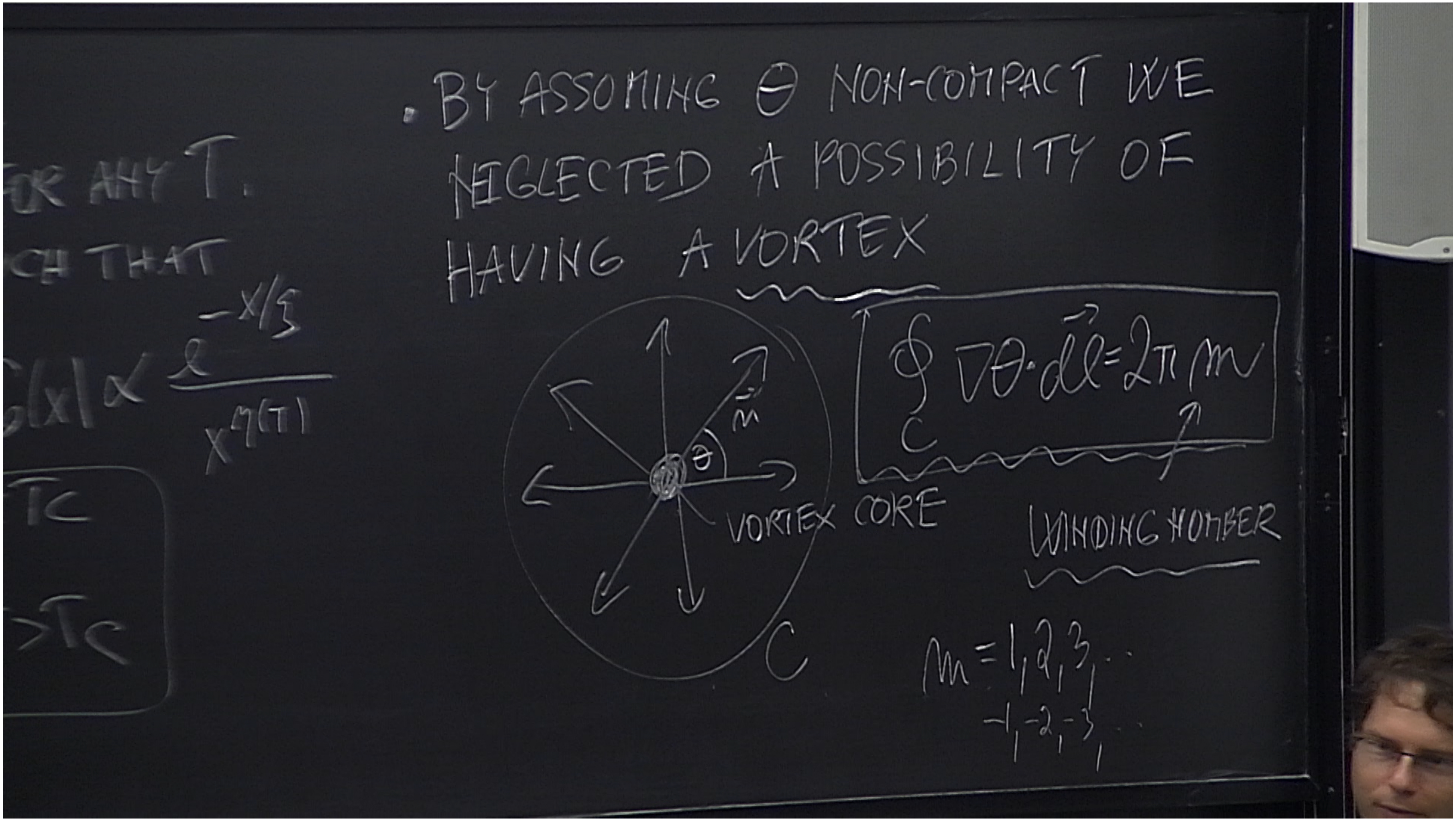

EPEND
EXPONENTS
ORDER.
THIS IS PHYSICALLY WRONG
CANNOT HAVE $\xi \to \infty$ FOR ANY T.
WE MUST HAVE T_c SUCH THAT
$G(x) \propto \frac{1}{x^{\eta(T)}} \to G(x) \propto \frac{e^{-x/\xi}}{x^{\eta(T)}}$
$\xi = \begin{cases} \infty & T<T_c \\ \text{FINITE} & T>T_c \end{cases}$
• BY ASSUMING θ NON-COMPACT WE NEGLECTED A POSSIBILITY OF HAVING A VORTEX
$\vec{m}$

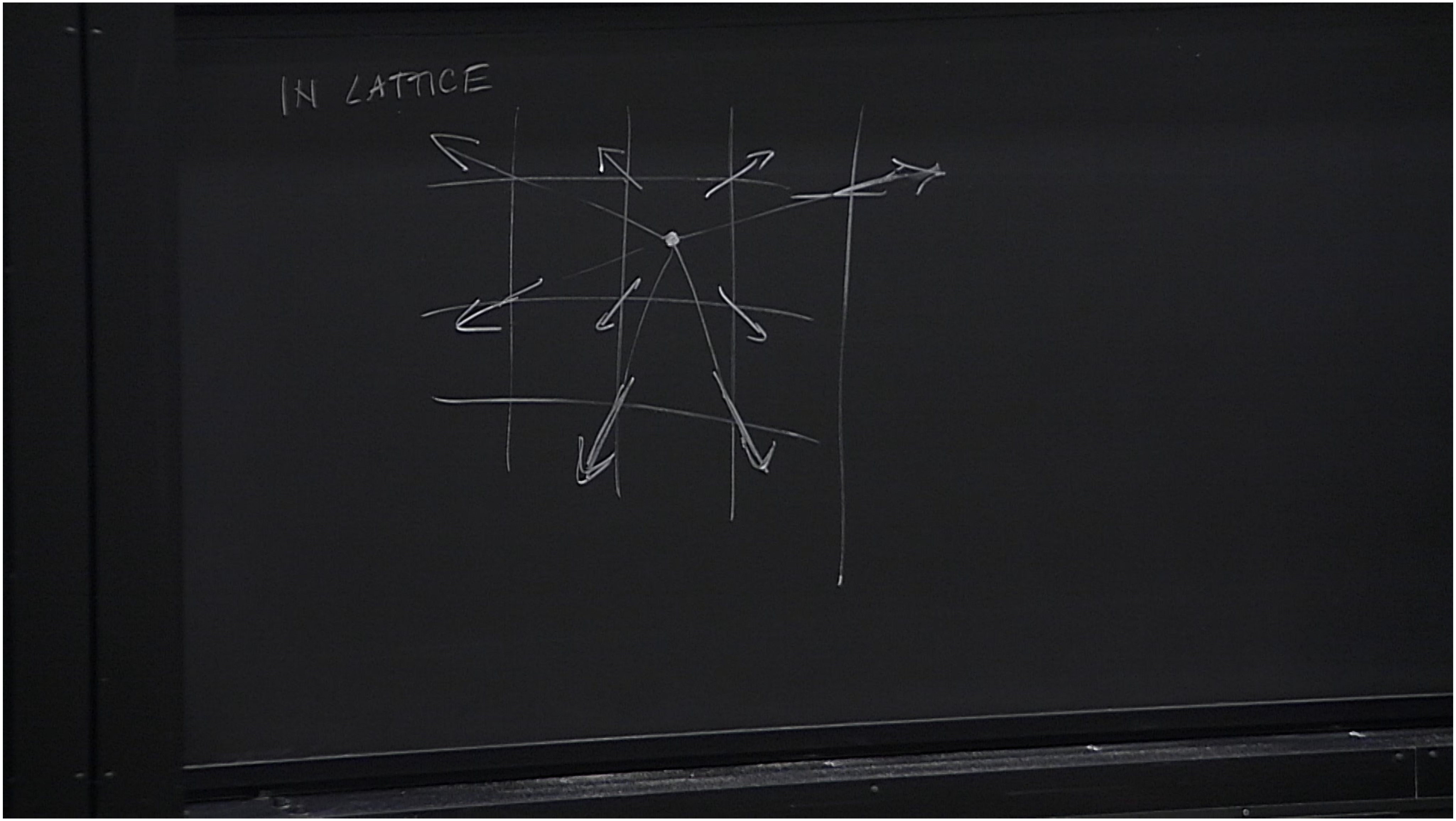


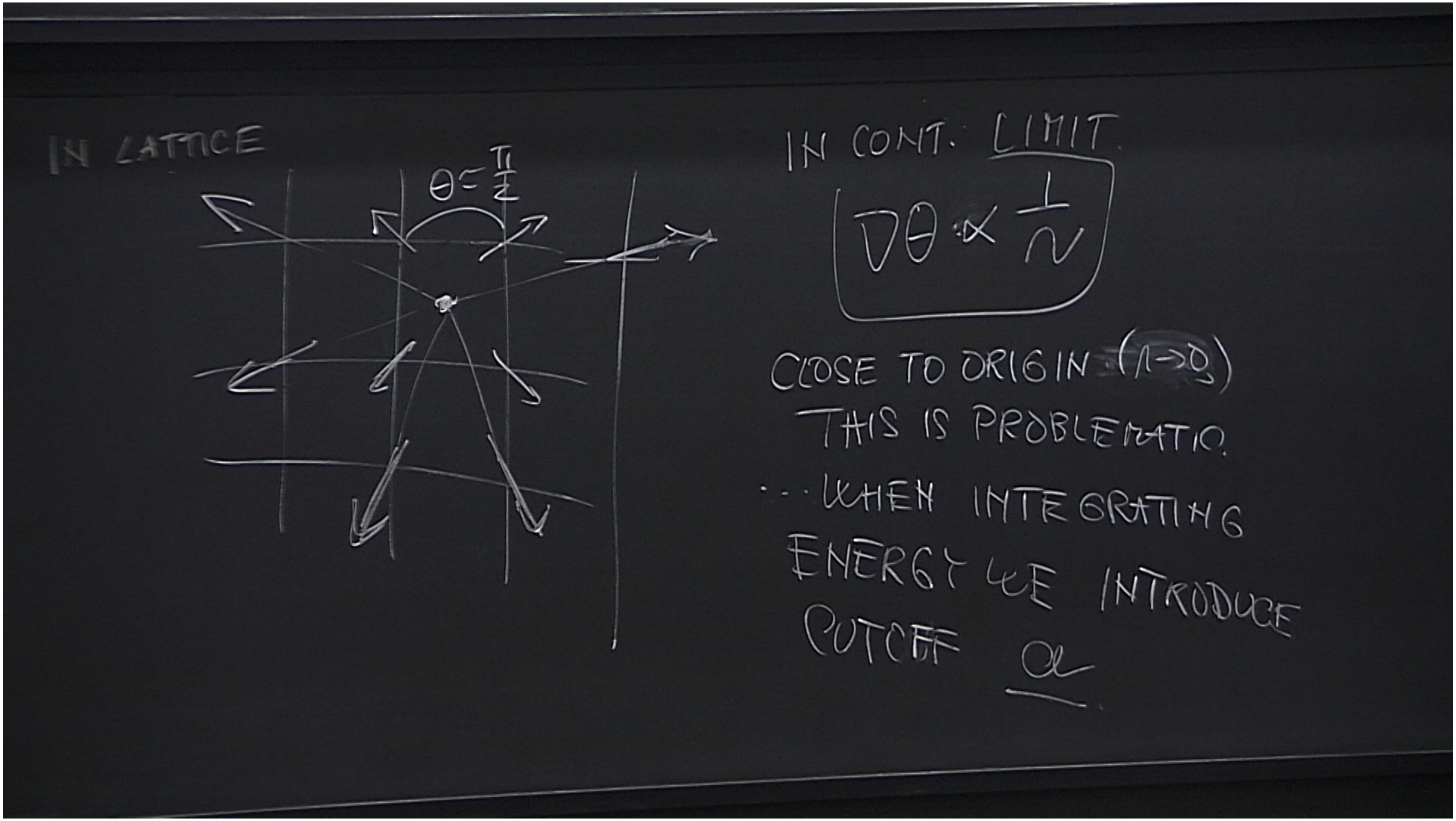
S PHYSICALLY WRONG
NNOT HAVE $\xi \to \infty$ FOR ANY T.
E MOST HAVE: Tc: SUCH THAT
$(x) \propto \frac{1}{x^{\eta(T)}} \to G(x) \propto \frac{e^{-x/\xi}}{x^{\eta(T)}}$
$\xi = \begin{cases} \infty & T<T_c \\ \text{FINITE} & T>T_c \end{cases}$
• BY ASSUMING θ NON-COMPACT WE NEGLECTED A POSSIBILITY OF HAVING A VORTEX
$\oint_C \nabla\theta \cdot d\vec{l} = 2\pi m$
VORTEX CORE
C



• BY ASSUMING θ NON-COMPACT WE NEGLECTED A POSSIBILITY OF HAVING A VORTEX
∮_C ∇θ · dl = 2π m
WINDING NUMBER
VORTEX CORE
C
m =
FOR ANY T.
SUCH THAT
G(x) ∝ e^{-x/ξ} / x^{η(T)}


FOR ANY T.
CH THAT
G(x) ∝ e^{-x/ξ} / x^{η(T)}
T < Tc
T > Tc
• BY ASSUMING Θ NON-COMPACT WE
NEGLECTED A POSSIBILITY OF
HAVING A VORTEX
∮_C ∇Θ · dl = 2π m
C
VORTEX CORE
WINDING NUMBER
m = 1, 2, 3, …
−1, −2, −3, …


IN LATTICE

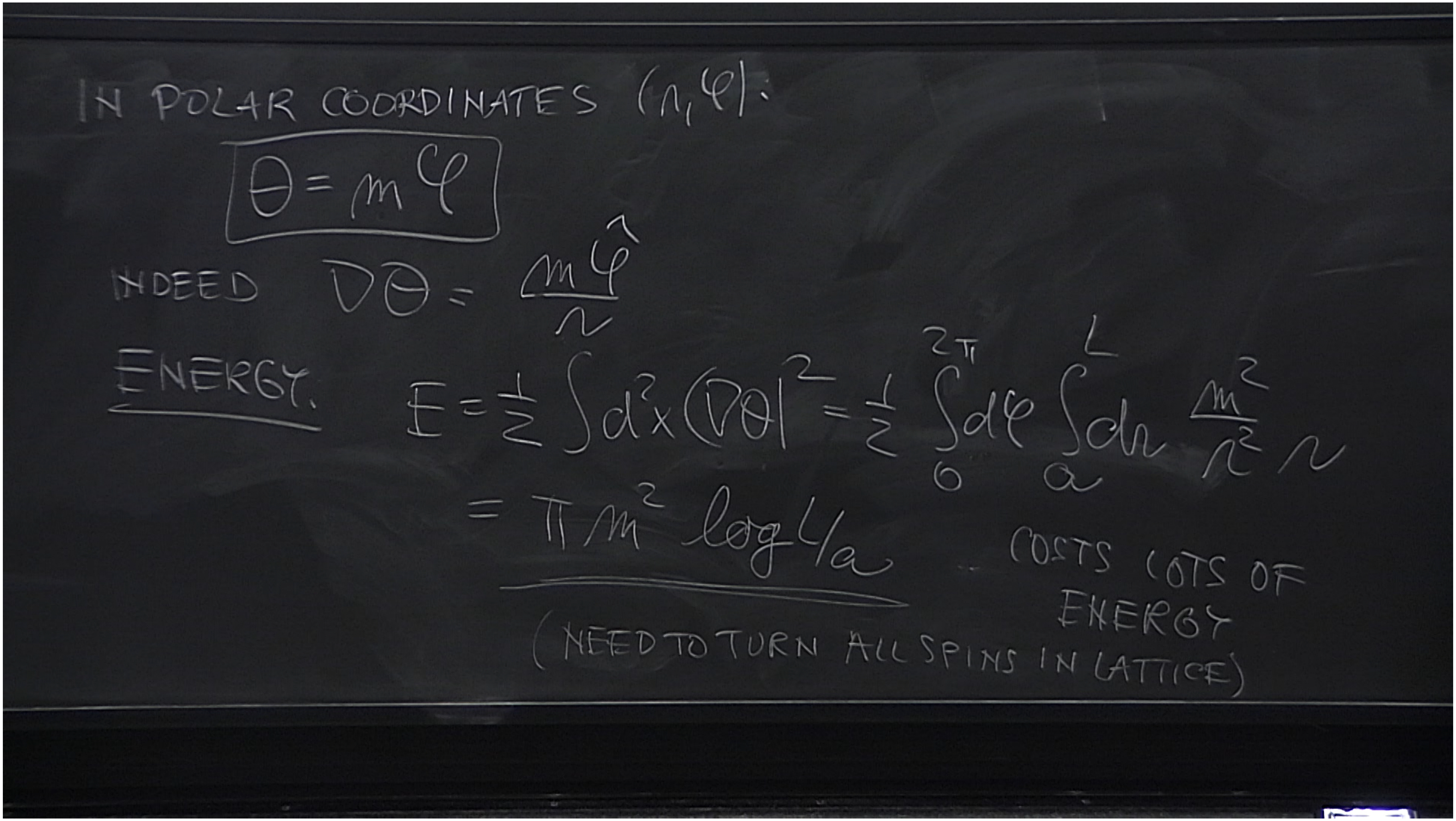

IN LATTICE
θ = π/2
IN CONT. LIMIT
∇θ ∝ 1/N
CLOSE TO ORIGIN (r→0)
THIS IS PROBLEMATIC
... WHEN INTEGRATING
ENERGY WE INTRODUCE
CUTOFF a

• LET'S ESTIMATE THE ENERGY OF A VORTEX:

$$H = \frac{1}{2} \int d^2x \, (\nabla \theta)^2$$

$$\delta H = \frac{1}{2} \int d^2x \, (\nabla \delta\theta) \nabla \theta = -\frac{1}{2} \int d^2x \, (\nabla^2 \theta) \delta\theta = 0$$

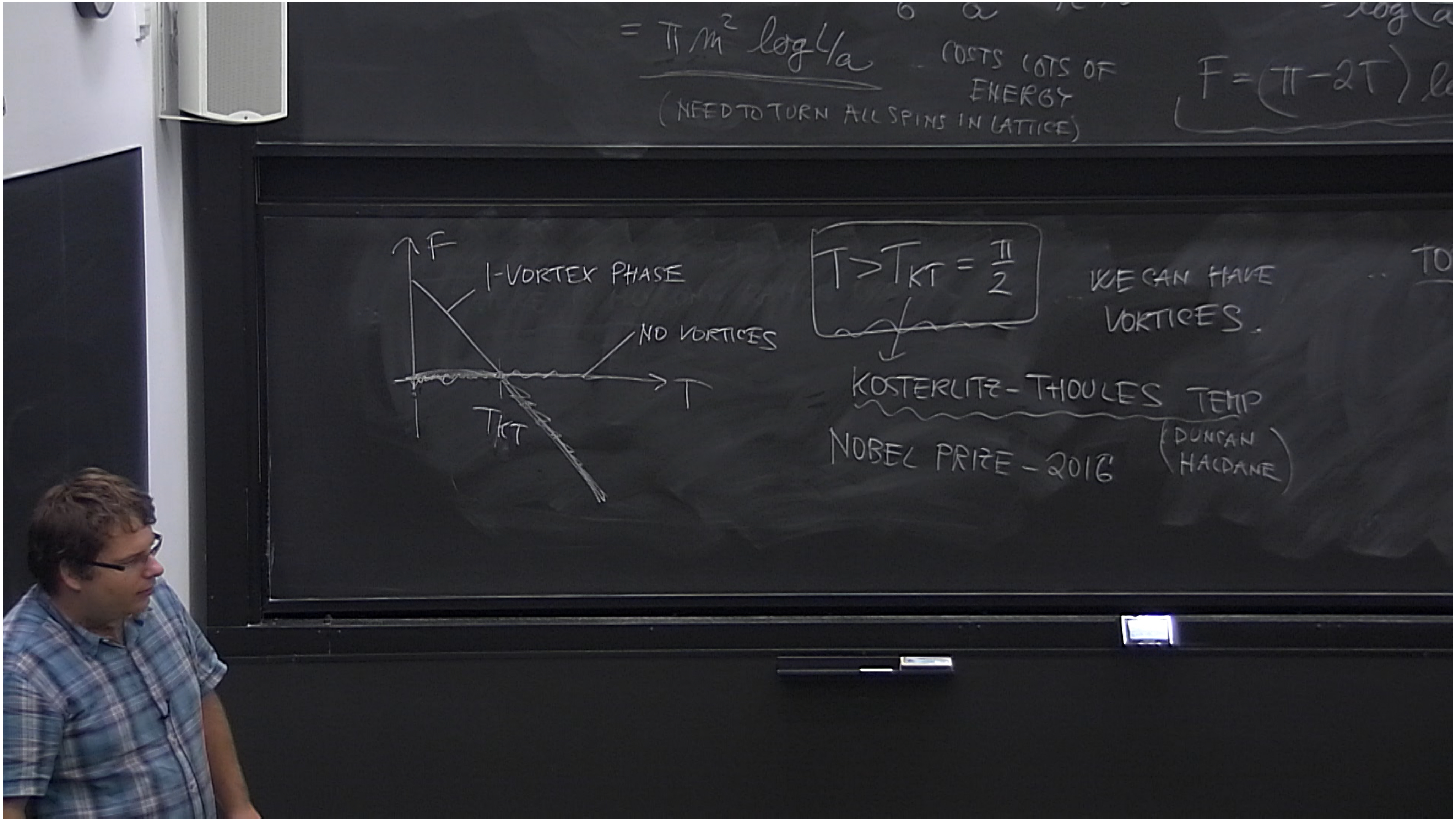
$$\boxed{\nabla^2 \theta = 0} \quad \Big| \quad \boxed{\oint_C \nabla\theta \cdot d\vec{l} = 2\pi m}$$


IN POLAR COORDINATES (r, φ):
$\Theta = m\varphi$
INDEED $\nabla\Theta = \frac{m\hat{\varphi}}{r}$
ENERGY.
$E = \frac{1}{2}\int d^2x\,(\nabla\Theta)^2 = \frac{1}{2}\int_0^{2\pi} d\varphi \int_a^L dr\,\frac{m^2}{r^2}\,r$
$= \pi m^2 \log L/a$
.. COSTS LOTS OF ENERGY
(NEED TO TURN ALL SPINS IN LATTICE)

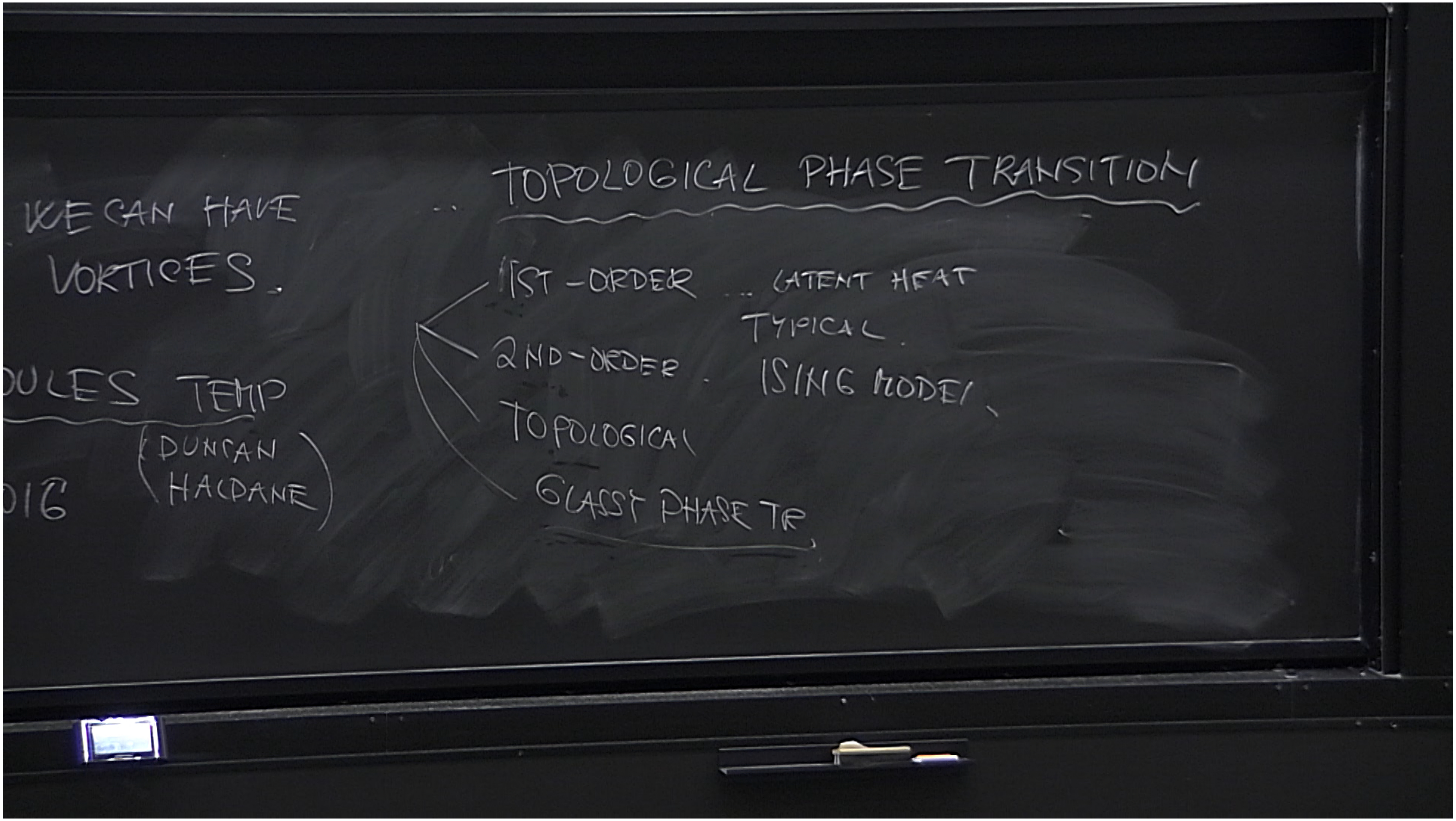
- AT FINITE T . .. TO FIND IF VORTICES ARE POSSIBLE CONSIDER FREE ENERGY

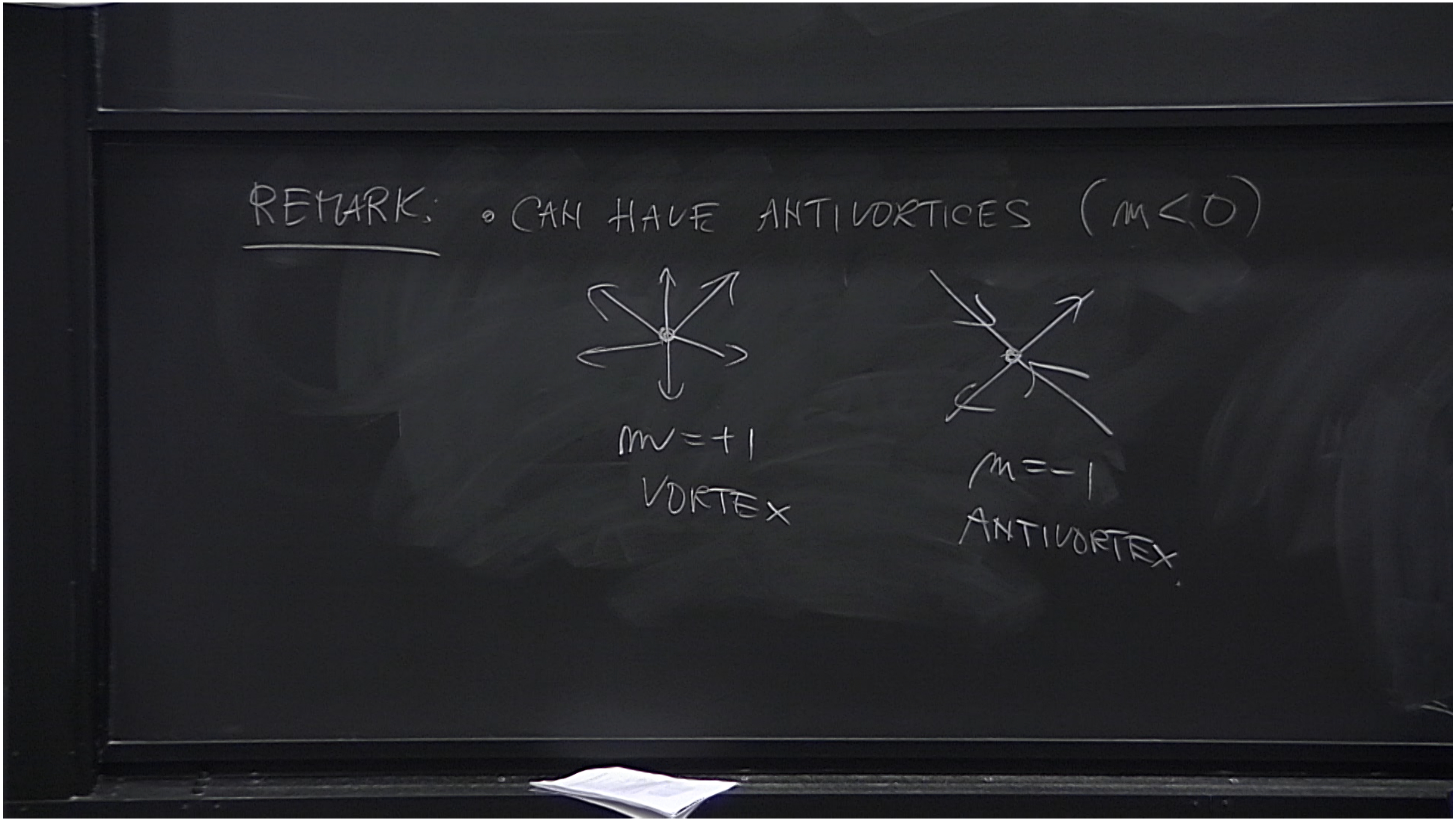
$$F = E - TS$$

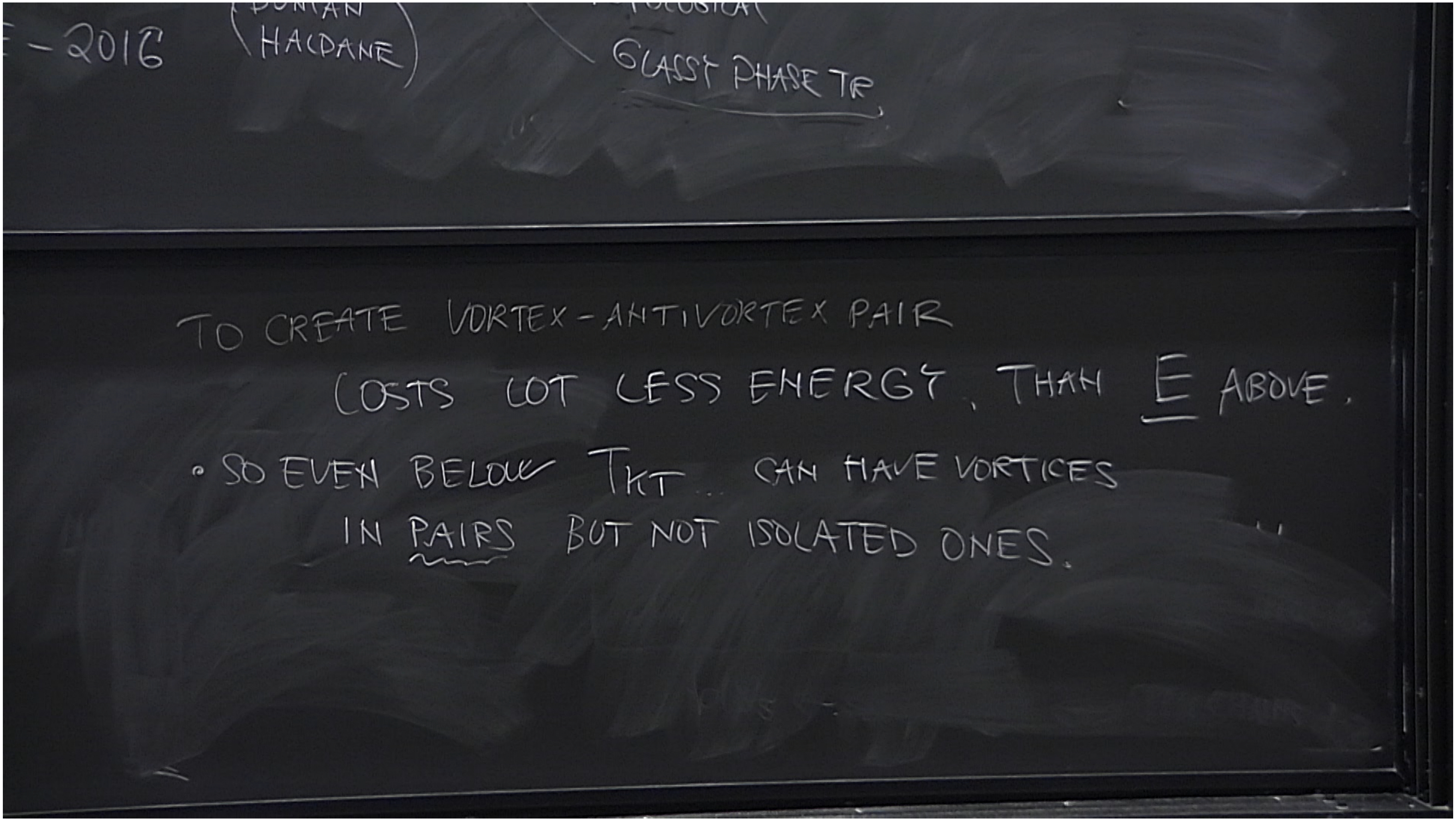
$$S = \log(\#\,\mathrm{STATES}) \simeq$$

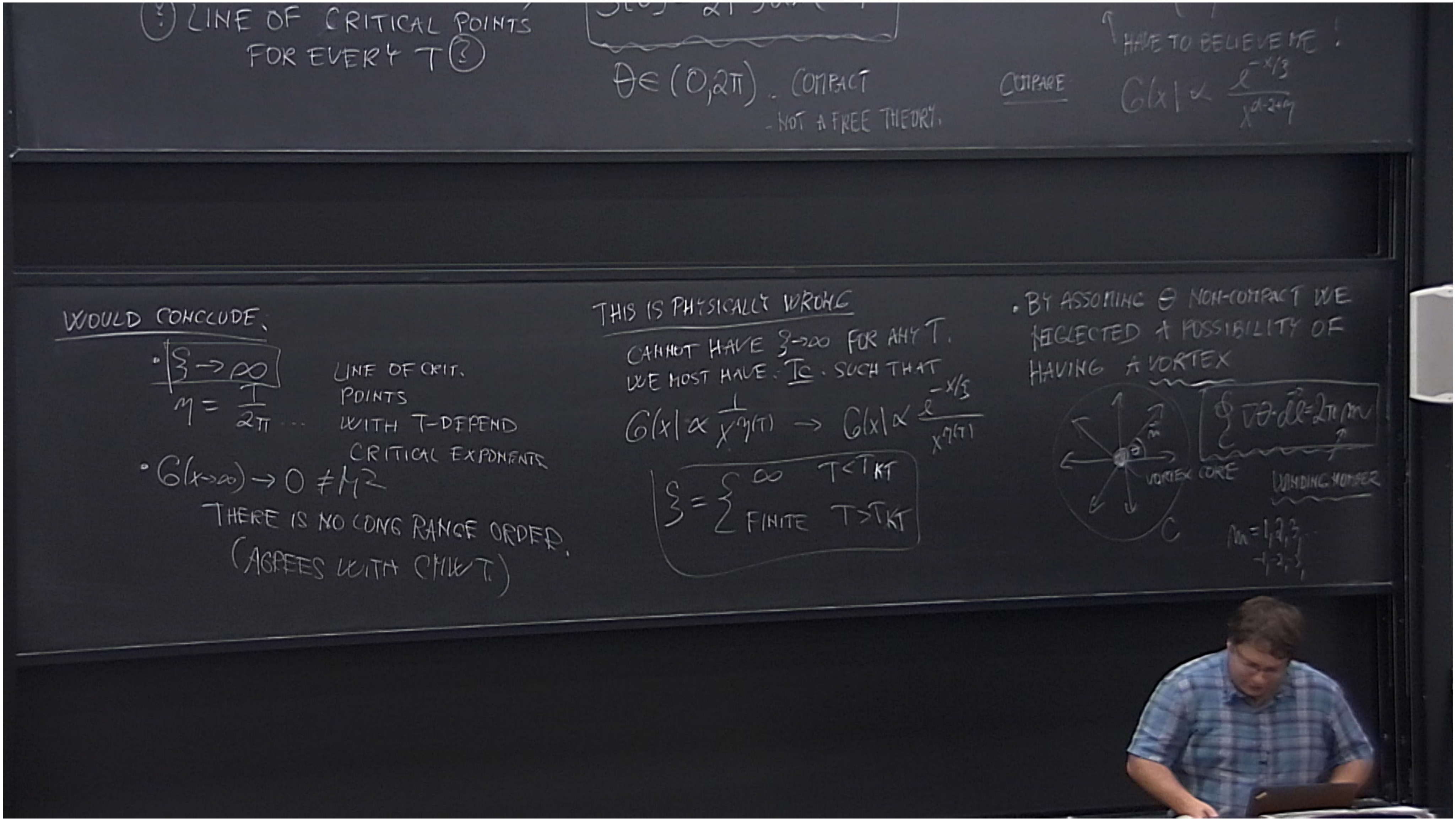


= π m² log L/a
COSTS LOTS OF ENERGY
(NEED TO TURN ALL SPINS IN LATTICE)
F = (π − 2T)
F
1-VORTEX PHASE
NO VORTICES
T
T_KT
T > T_KT = π/2
WE CAN HAVE VORTICES.
KOSTERLITZ-THOULES TEMP
NOBEL PRIZE - 2016
(DUNCAN HALDANE)


WE CAN HAVE
VORTICES.
TOPOLOGICAL PHASE TRANSITION
1ST-ORDER ... LATENT HEAT
TYPICAL.
2ND-ORDER . ISING MODEL.
TOPOLOGICAL
GLASSY PHASE TR
ULES TEMP
(DUNCAN
HALDANE)
016


REMARK: • CAN HAVE ANTIVORTICES (m<0)
m=+1
VORTEX
m=-1
ANTIVORTEX


- 2016
HALDANE
GLASSY PHASE TR
TO CREATE VORTEX - ANTIVORTEX PAIR
COSTS LOT LESS ENERGY, THAN E ABOVE.
• SO EVEN BELOW T_{KT} .. CAN HAVE VORTICES
IN PAIRS BUT NOT ISOLATED ONES.

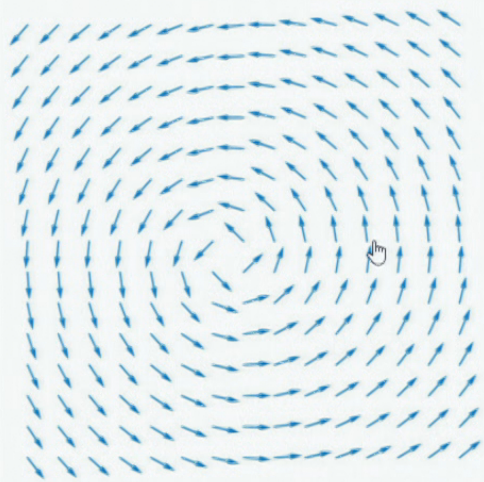

LINE OF CRITICAL POINTS
FOR EVERY T
θ ∈ (0,2π). COMPACT
- NOT A FREE THEORY.
HAVE TO BELIEVE ME!
COMPARE
G(x) ∝ e^{-x/ξ} / x^{d-2+η}
WOULD CONCLUDE:
• ξ → ∞
η = T/2π ...
LINE OF CRIT. POINTS
WITH T-DEPEND CRITICAL EXPONENTS
• G(x→∞) → 0 ≠ M²
THERE IS NO LONG RANGE ORDER.
(AGREES WITH CMWT)
THIS IS PHYSICALLY WRONG
CANNOT HAVE ξ→∞ FOR ANY T.
WE MUST HAVE: Tc. SUCH THAT
G(x) ∝ 1/x^{η(T)} → G(x) ∝ e^{-x/ξ}/x^{η(T)}
ξ = ∞ T<T_KT
FINITE T>T_KT
• BY ASSUMING θ NON-COMPACT WE NEGLECTED A POSSIBILITY OF HAVING A VORTEX
∮ ∇θ·dl = 2πm
VORTEX CORE
WINDING NUMBER
C
m = 1, 2, 3, ...
-1, -2, -3,

John Baez | Azimuth

file:///C:/Data/Fyzika/PSI/2018/Courses/StatPhys/Old2/Scripta/KT/John Baez _ Azimuth.htm

When you heat up your thin film, it gets a bit more energy, so the spins can do more interesting things.

Here's one interesting possibility, called a 'vortex':



The spins swirl around like the flow of water in a whirlpool. Each spin is *fairly* close to being lined up to its neighbors, except near the middle where they're doing a terrible job.

The total energy of a vortex is enormous. The reason is not the problem at the middle, which certainly contributes some energy. The reason is that 'fairly' close is not good enough. The spins fail to perfectly line up with their neighbors even far away from the middle of this picture. This problem is bad enough to make the energy huge. (In fact, the energy would be *infinite* if our thin film of material went on forever.)

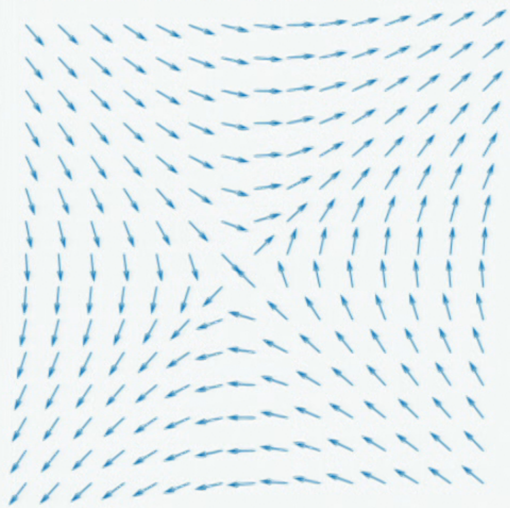
So, even if you heat up your substance, there won't be enough energy to make many vortices. This made people think vortices were irrelevant.

Azimuth

Blog at WordPress.com.

So, even if you heat up your substance, there won't be enough energy to make many vortices. This made people think vortices were irrelevant.

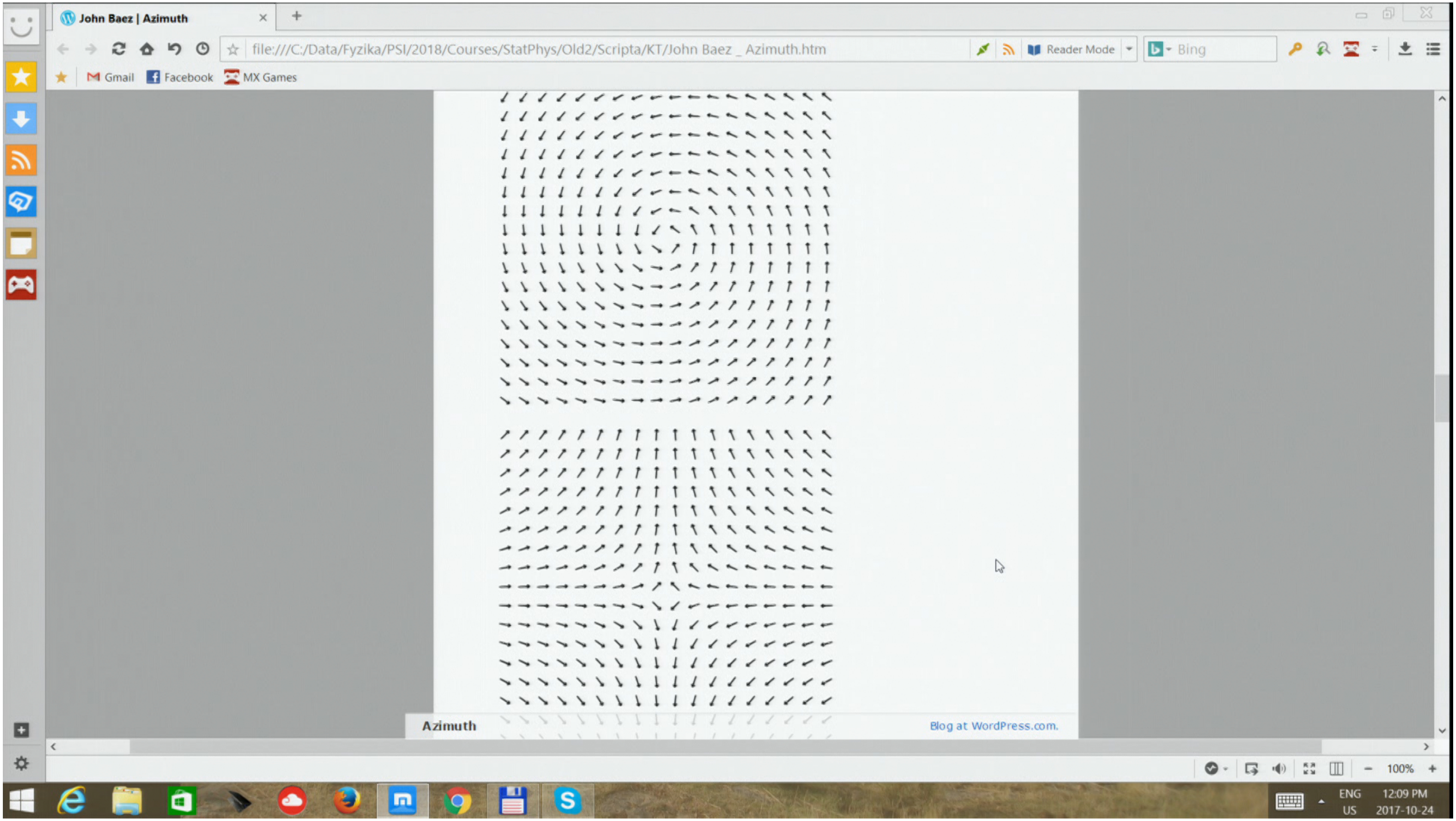
But there's another possibility, called an 'antivortex':



A single antivortex has a huge energy, just like a vortex. So again, it might seem antivortices are irrelevant if you're wondering what your stuff will do when it has just a little energy.

But here's what Kosterlitz and Thouless noticed: the combination of *a vortex together with an antivortex* has much less energy than either one alone! So, when your thin film of stuff is hot enough, the spins will form 'vortex-antivortex pairs'.

Brian Skinner has made a beautiful animation showing how this happens. A vortex-antivortex pair can appear out of nothing:


John Baez | Azimuth
file:///C:/Data/Fyzika/PSI/2018/Courses/StatPhys/Old2/Scripta/KT/John Baez _ Azimuth.htm
Reader Mode
Bing
Gmail
Facebook
MX Games
Azimuth
Blog at WordPress.com.
100%
ENG
US
12:09 PM
2017-10-24

A single antivortex has a huge energy, just like a vortex. So again, it might seem antivortices are irrelevant if you're wondering what your stuff will do when it has just a little energy.

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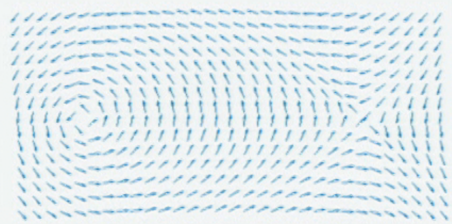

... and then disappear again!

Thanks to this process, at low temperatures our thin film will contain a dilute 'gas' of vortex-antivortex pairs. Each vortex will stick to an antivortex, since it takes a lot of energy to separate them. These vortex-antivortex pairs act a bit like particles: they move around, bump into each other, and so on. But unlike most ordinary particles, they can appear out of nothing, or disappear, in the process shown above!

As you heat up the thin film, you get more and more vortex-antivortex pairs, since there's more energy available to create them. But here's the really surprising thing. Kosterlitz and Thouless showed that as you turn up the heat, there's a certain temperature at which the vortex-antivortex pairs suddenly 'unbind' and break apart!

Why? Because at this point, the density of vortex-antivortex pairs is so high, and they're bumping into each other so much, that we can't tell which vortex is the partner of which antivortex. All we've got is a thick soup of vortices and antivortices!

What's interesting is that this happens *suddenly* at some particular temperature. It's a bit like how ice *suddenly* turns into liquid water

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