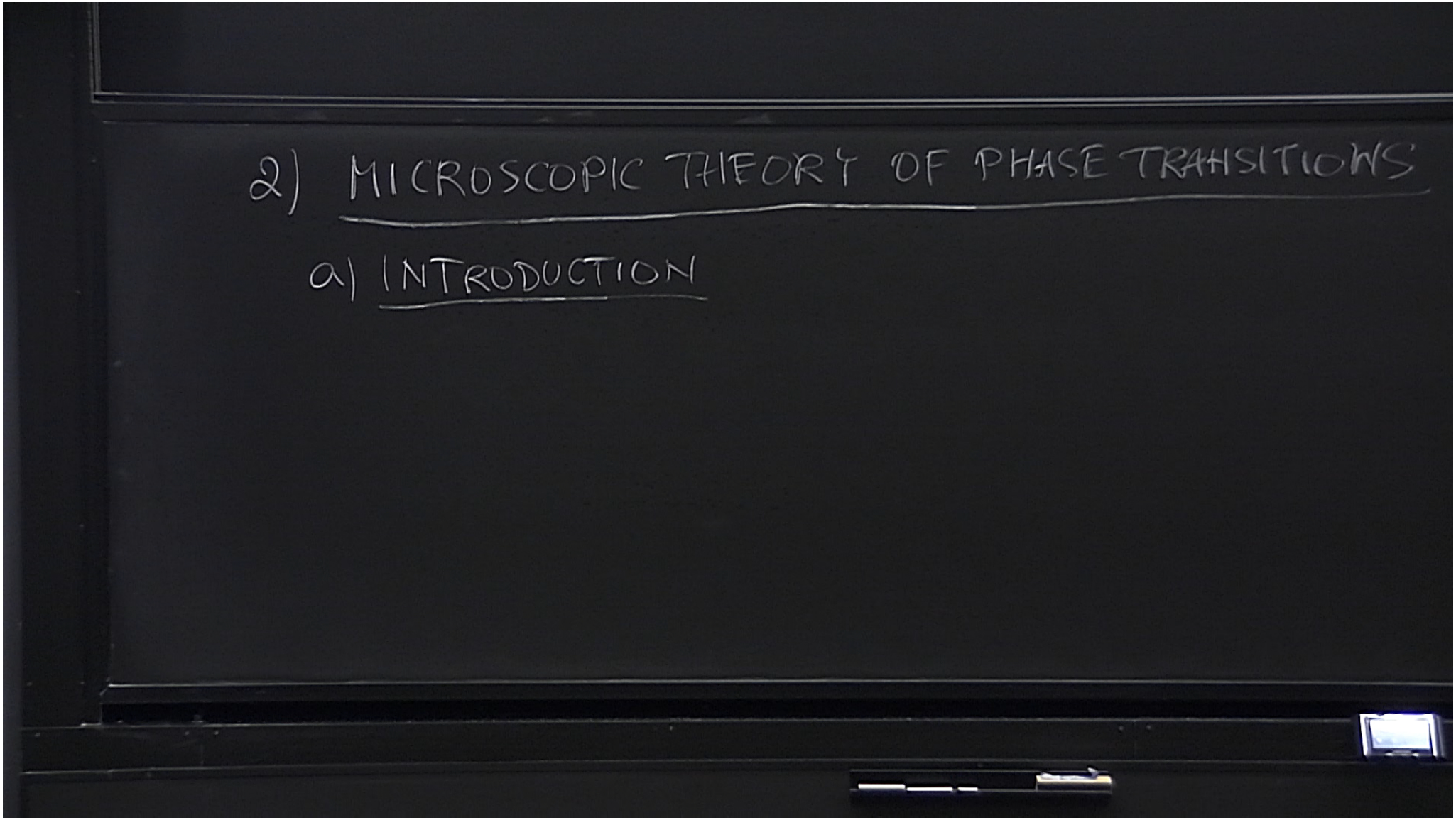
Title: PSI 17/18 - Statistical Mechanics - Lecture 4

Date: Oct 13, 2017 10:45 AM

URL: http://pirsa.org/17100037

Abstract:


2) MICROSCOPIC THEORY OF PHASE TRANSITIONS
a) INTRODUCTION

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- MATTER ORGANIZES INTO PHASES CHARACTERIZED BY DIFFERENT MACROSCOPIC PROPERTIES.

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EX. SOLID/LIQUID/GAS.

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- MATTER ORGANIZES INTO PHASES CHARACTERIZED BY DIFFERENT MACROSCOPIC PROPERTIES.

EX. SOLID/LIQUID/GAS: SOLID... HAS "RIGIDITY TO SHEAR"

2) MICROSCOPIC THEORY OF PHASE TRANSITIONS

a) INTRODUCTION

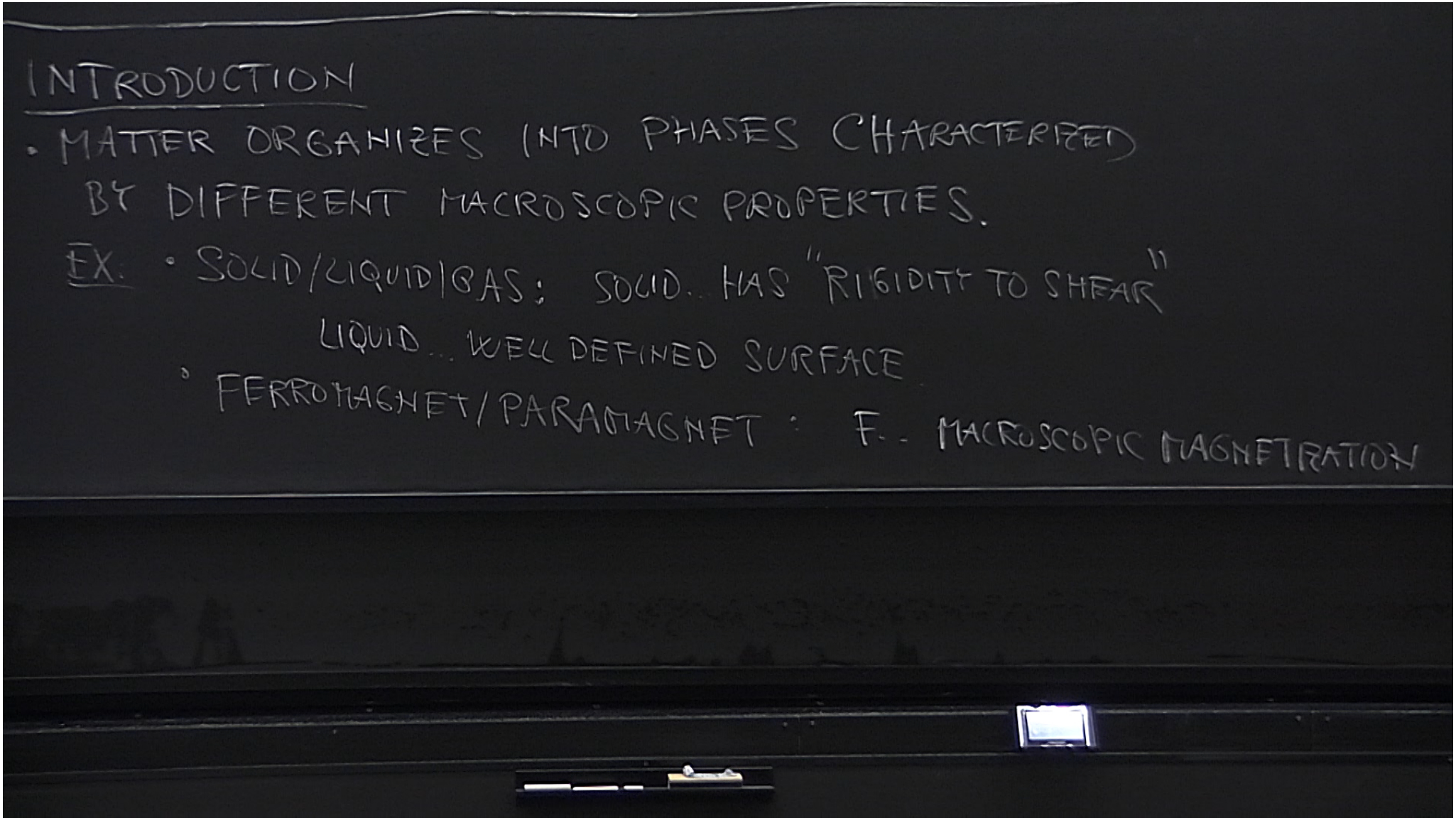
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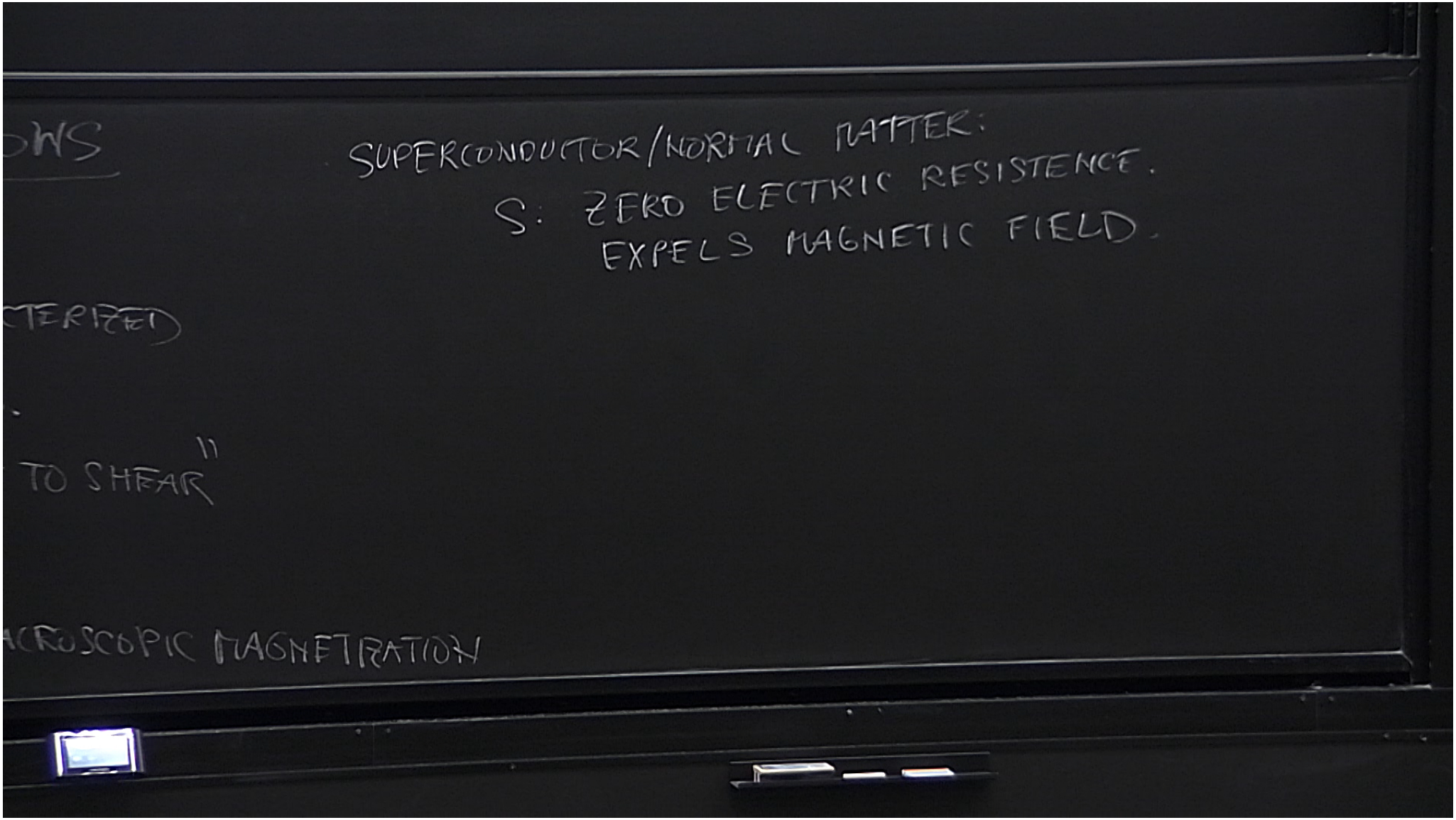
EX: SOLID/LIQUID/GAS: SOLID... HAS "RIGIDITY TO SHEAR"
LIQUID... WELL DEFINED SURFACE

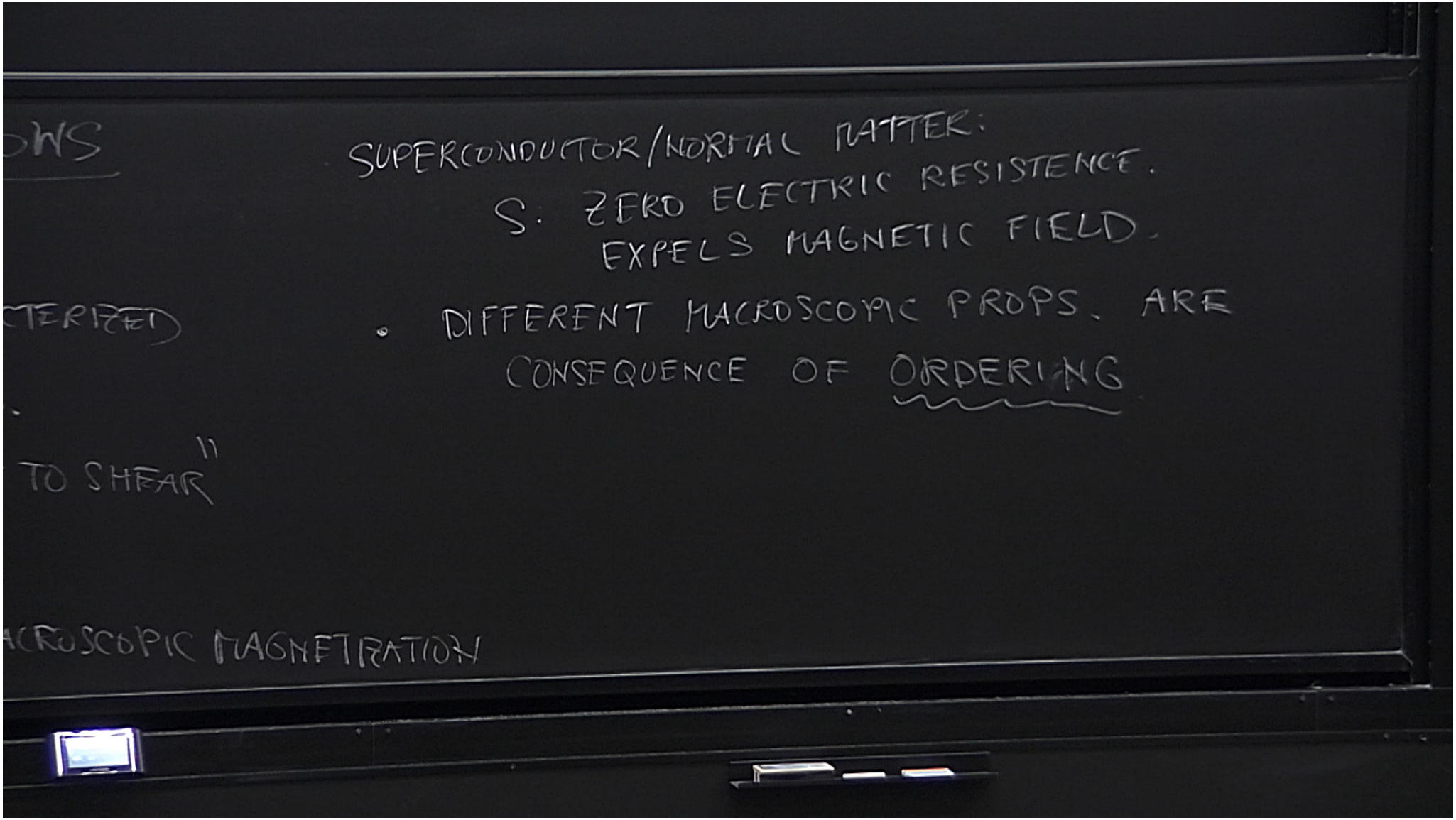
a) INTRODUCTION

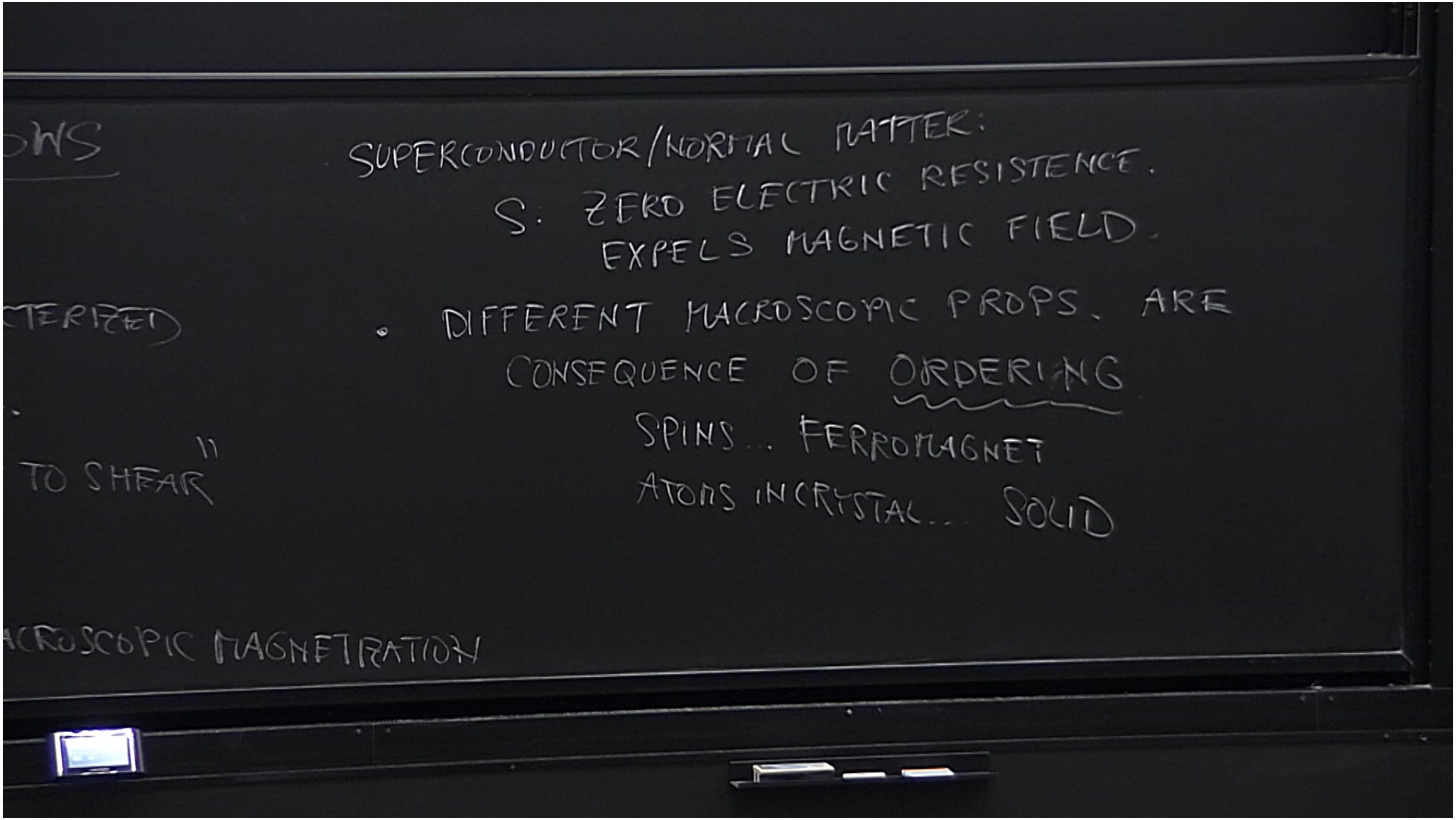
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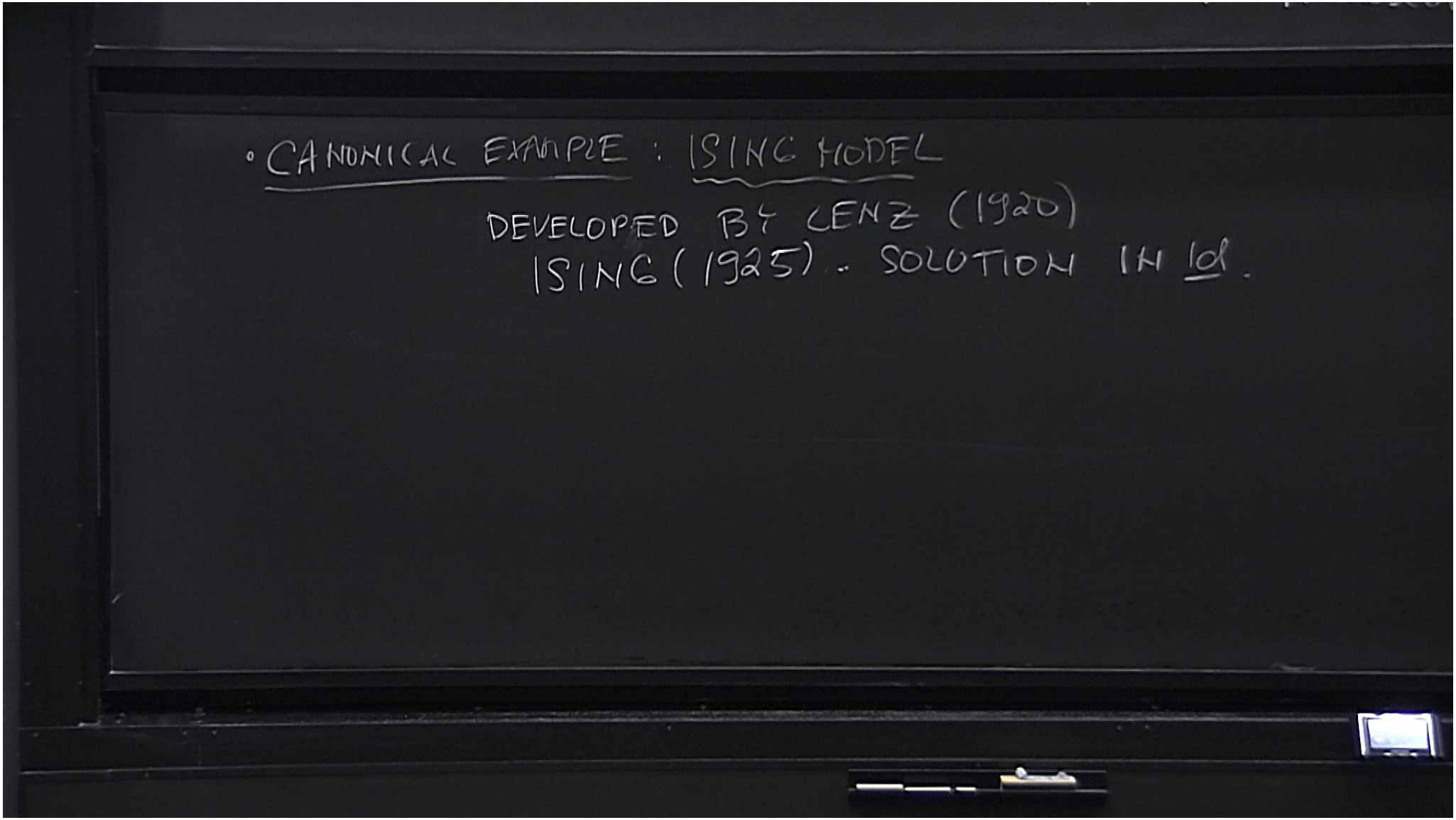
EX: • SOLID/LIQUID/GAS: SOLID... HAS "RIGIDITY TO SHEAR"
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• FERROMAGNET/PARAMAGNET:


INTRODUCTION
• MATTER ORGANIZES INTO PHASES CHARACTERIZED BY DIFFERENT MACROSCOPIC PROPERTIES.
EX: • SOLID/LIQUID/GAS: SOLID.. HAS "RIGIDITY TO SHEAR"
LIQUID... WELL DEFINED SURFACE
• FERROMAGNET/PARAMAGNET: F.. MACROSCOPIC MAGNETIZATION


SUPERCONDUCTOR/NORMAL MATTER:
S: ZERO ELECTRIC RESISTENCE.
EXPELS MAGNETIC FIELD.
TO SHEAR"
MACROSCOPIC MAGNETIZATION


SUPERCONDUCTOR/NORMAL MATTER:
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SUPERCONDUCTOR/NORMAL MATTER:
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CONSEQUENCE OF ORDERING
SPINS.. FERROMAGNET
ATOMS IN CRYSTAL.. SOLID
TO SHEAR"
ACROSCOPIC MAGNETIZATION


• CANONICAL EXAMPLE : ISING MODEL
DEVELOPED BY LENZ (1920)
ISING (1925) . SOLUTION IN 1d .

- CANONICAL EXAMPLE : ISING MODEL

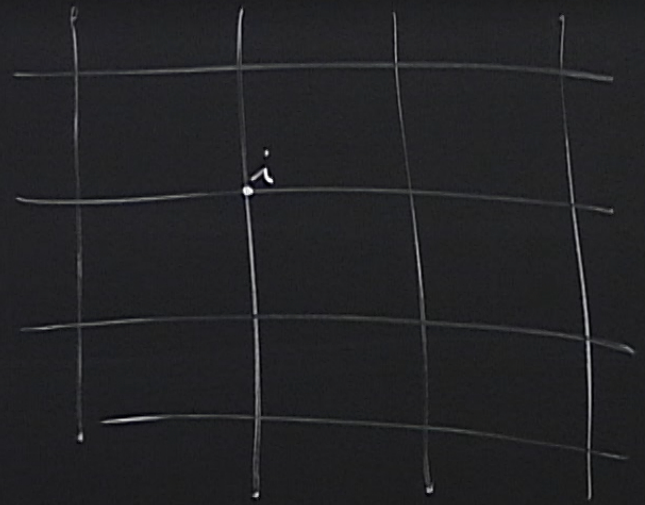
DEVELOPED BY LENZ (1920)
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ONSAGER (1944) . . 2d

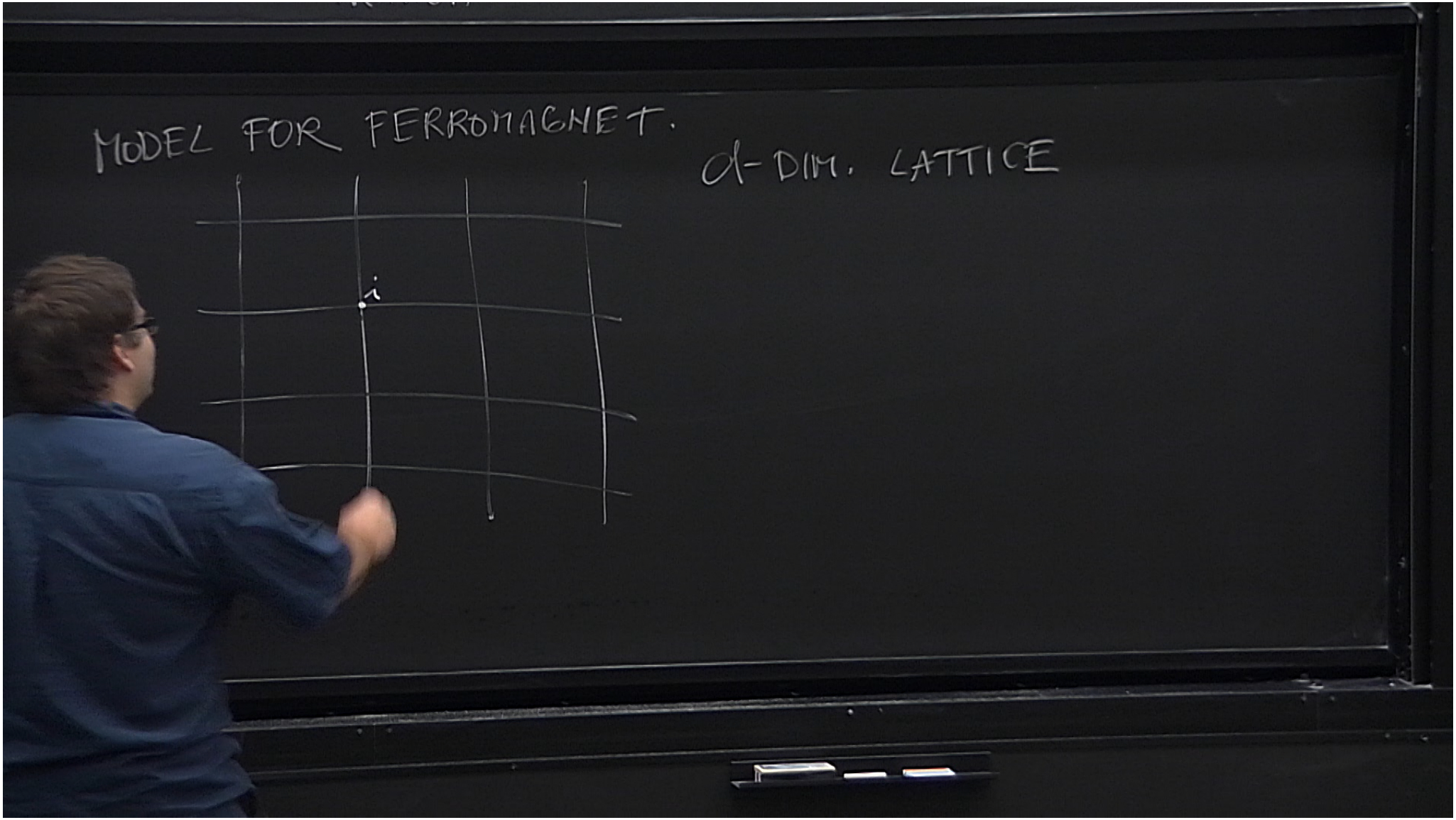
- CANONICAL EXAMPLE : ISING MODEL

DEVELOPED BY LENZ (1920)
ISING (1925) - SOLUTION IN 1d.
ONSAGER (1944) ... 2d.
- 3d ... ONGOING PROBLEM
- 4d AND HIGHER ... SIMPLE
(MFT APPROXIMATION IS EXACT)

MODEL FOR FERROMAGNET.

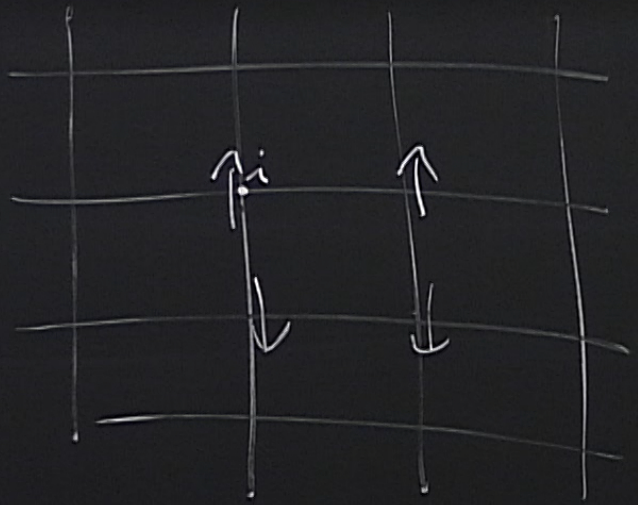
d-DIM.



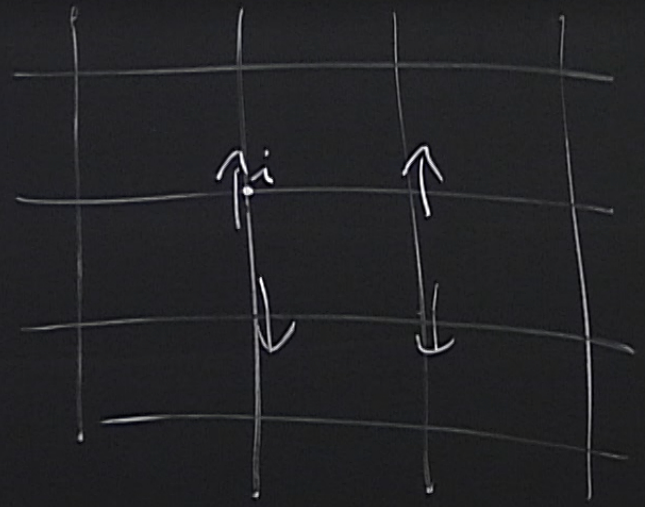

MODEL FOR FERROMAGNET.
d-DIM. LATTICE
i

MODEL FOR FERROMAGNET.

d-DIM. LATTICE



MODEL FOR FERROMAGNET.

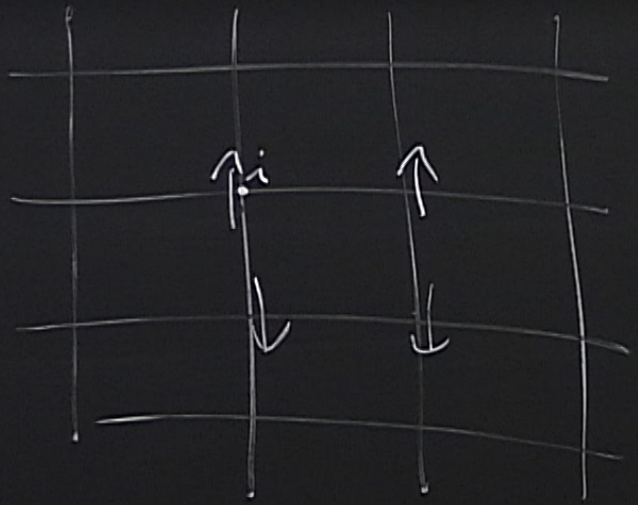


d-DIM. LATTICE

SPINS .. DESCRETE VARIABLES

σ_i

MODEL FOR FERROMAGNET.

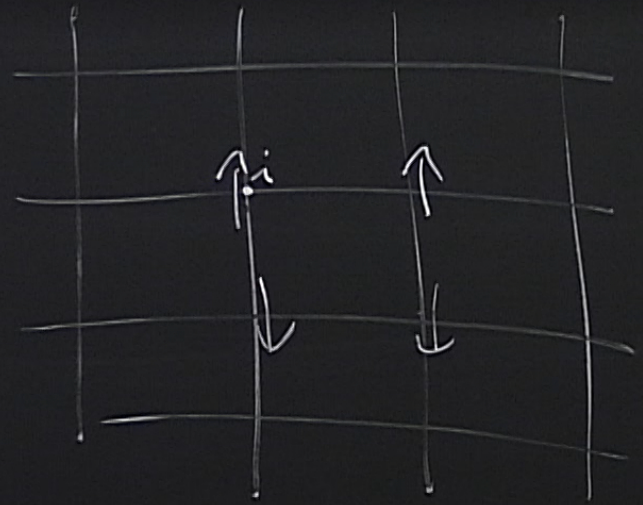


d-DIM. LATTICE

SPINS .. DESCRETE VARIABLES

$\sigma_i = \pm 1$

MODEL FOR FERROMAGNET.

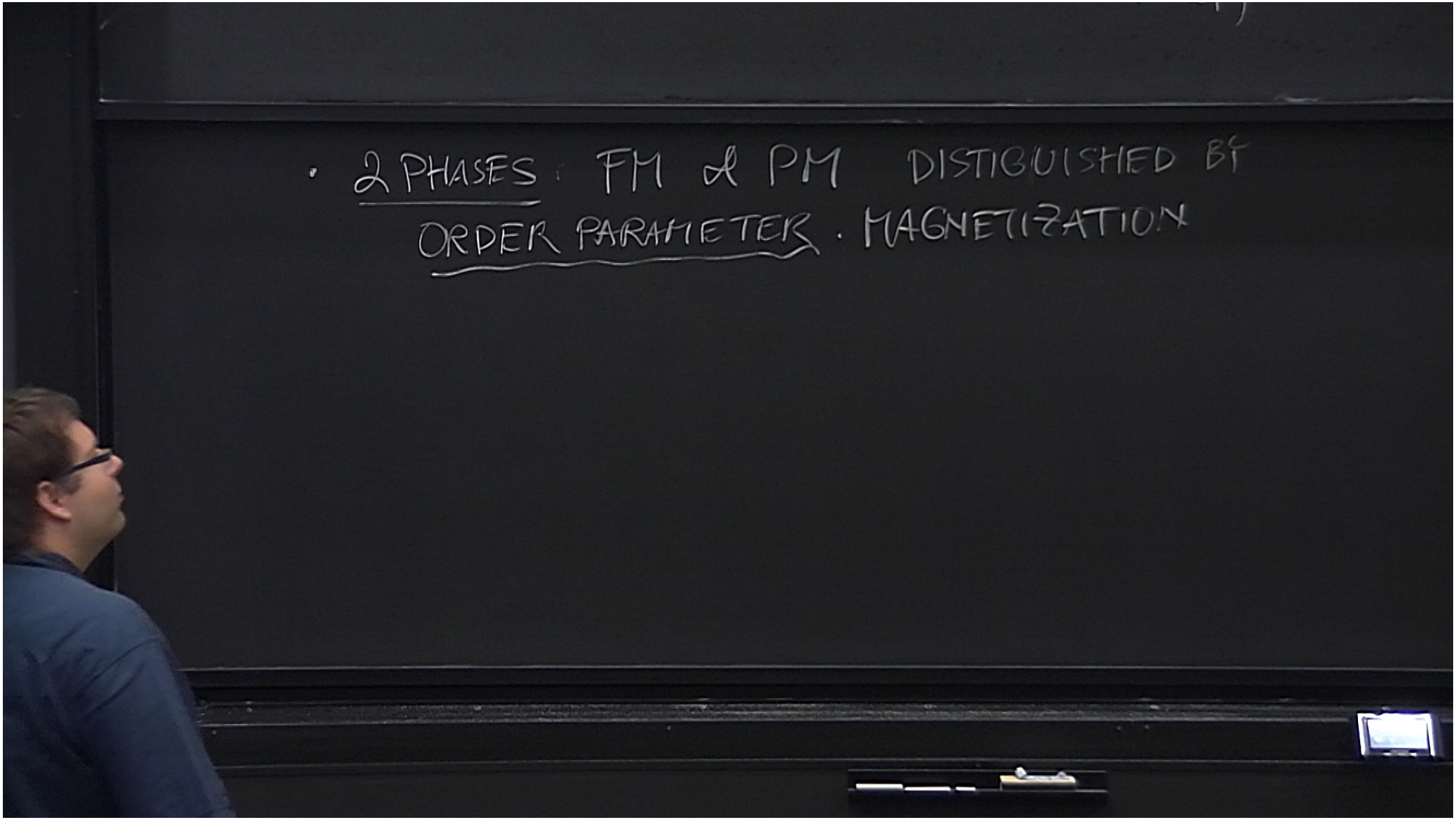


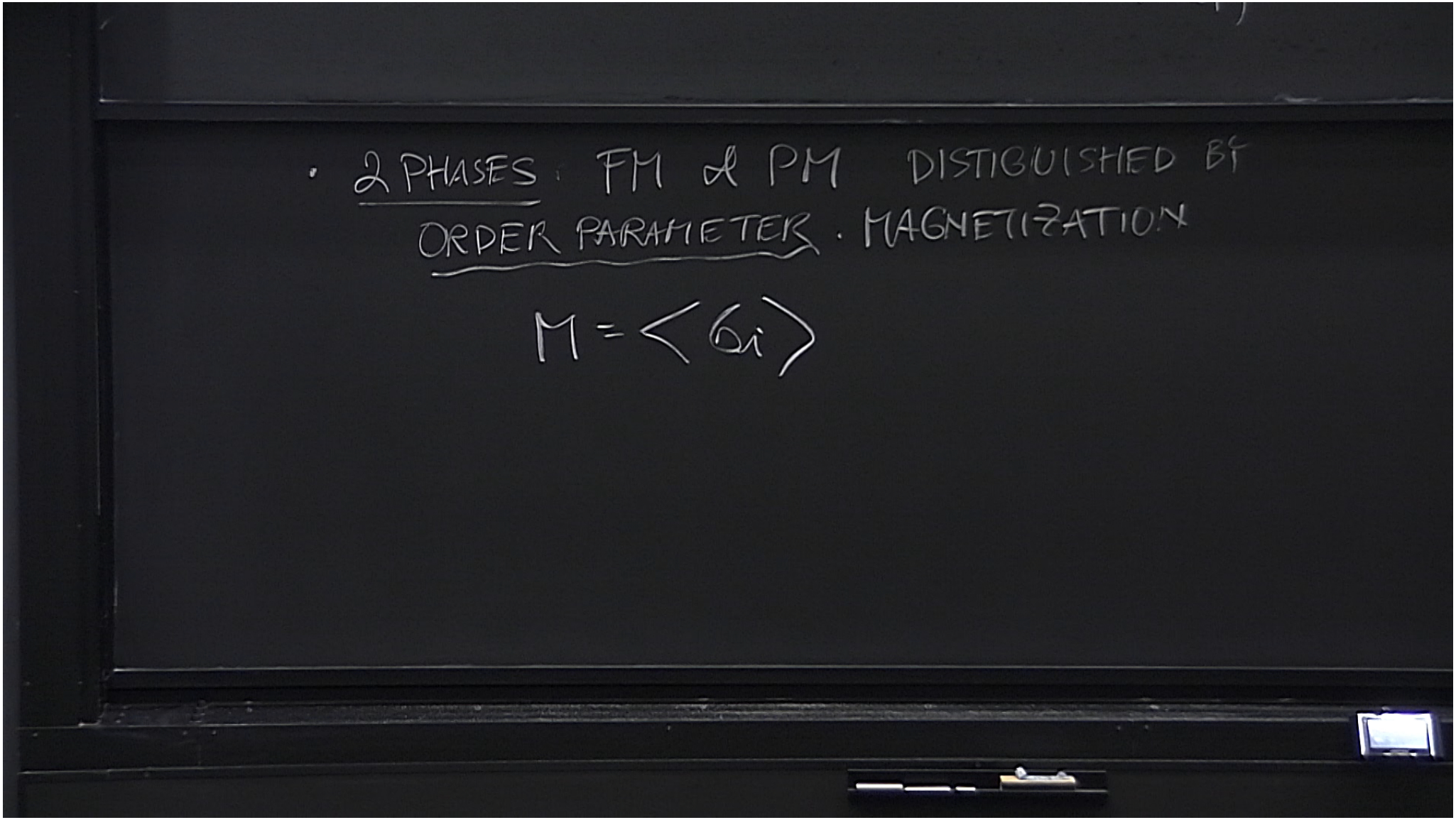
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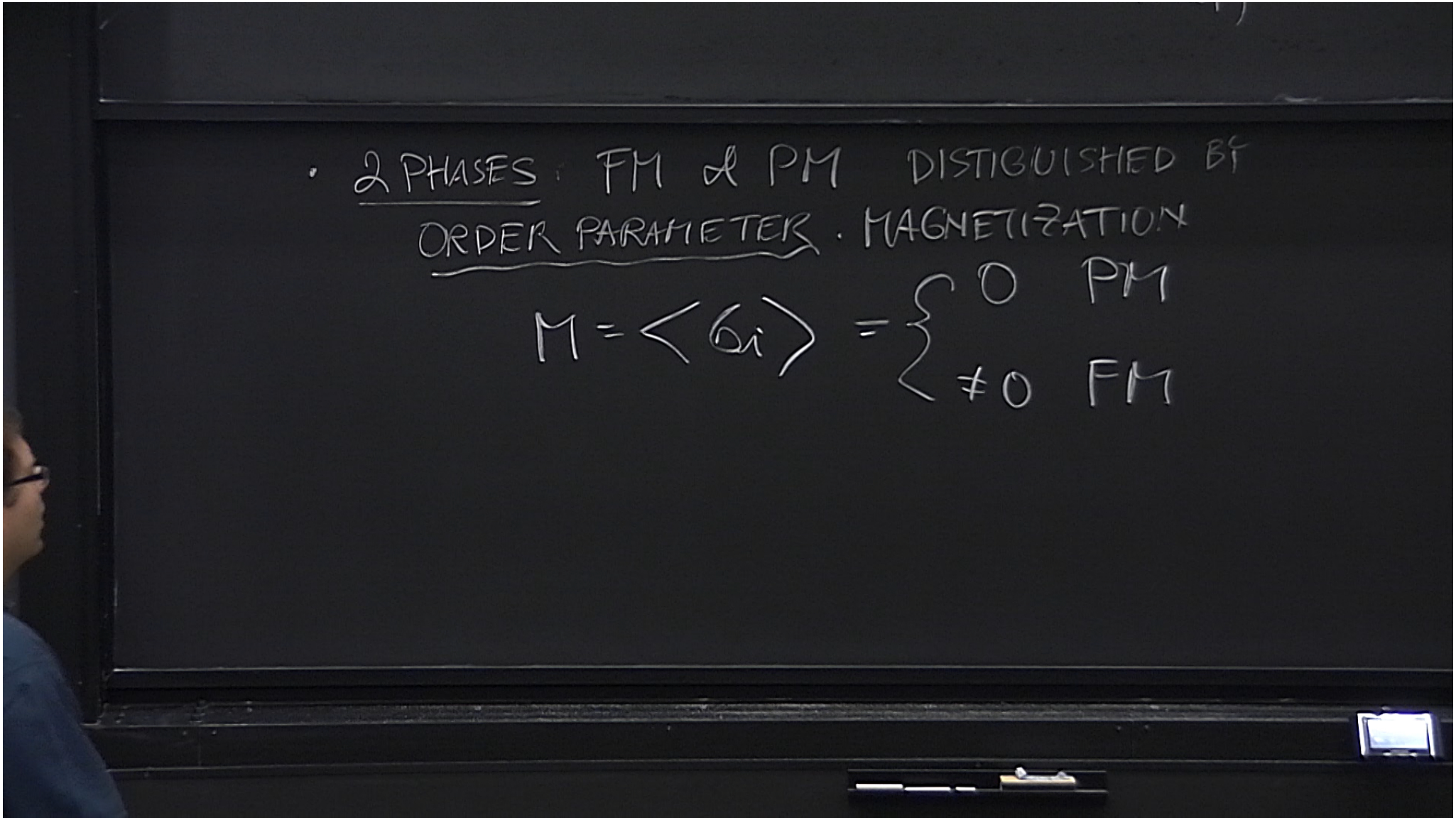
SPINS .. DESCRETE VARIABLES

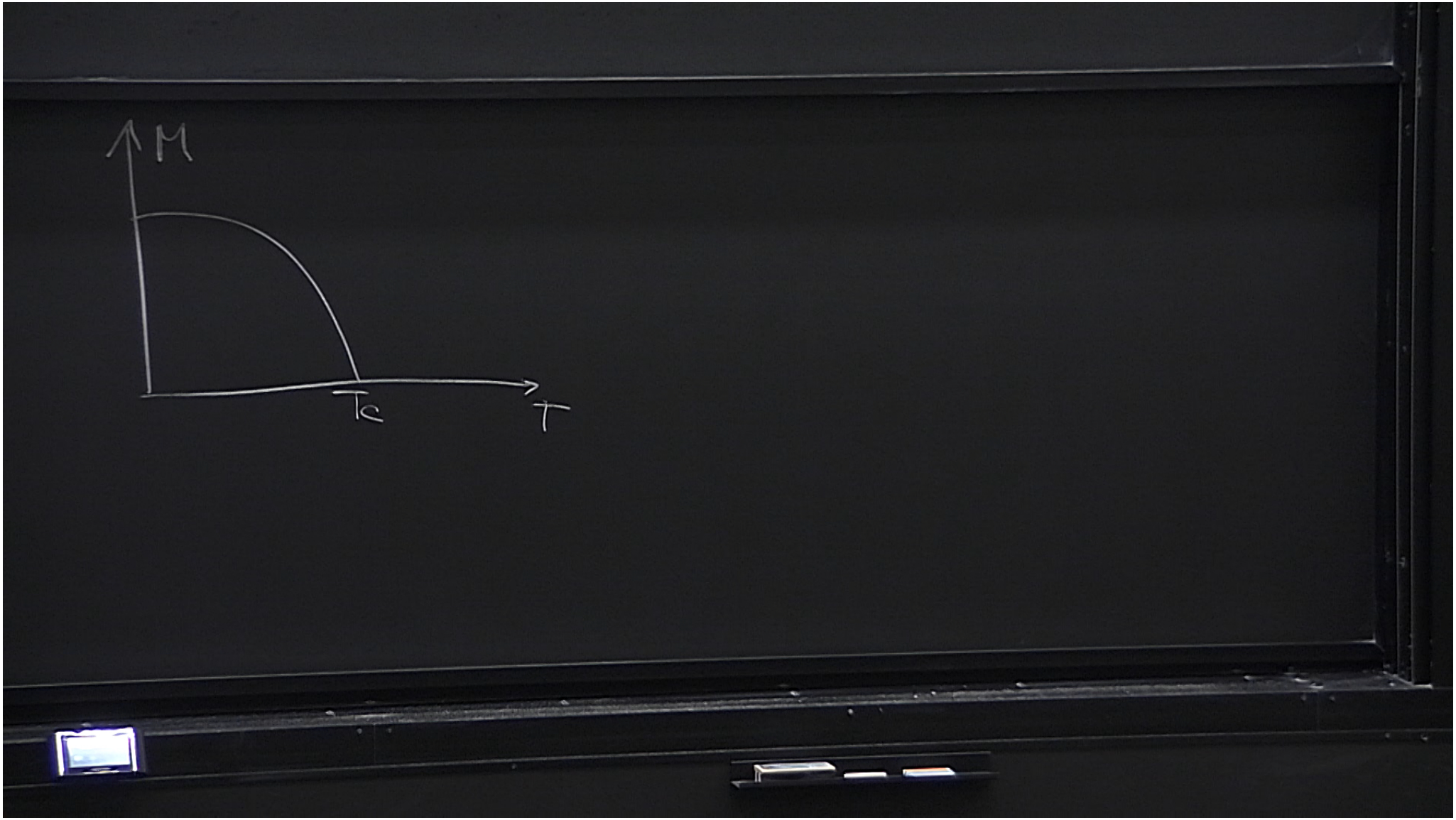
$$\sigma_i = \pm 1$$

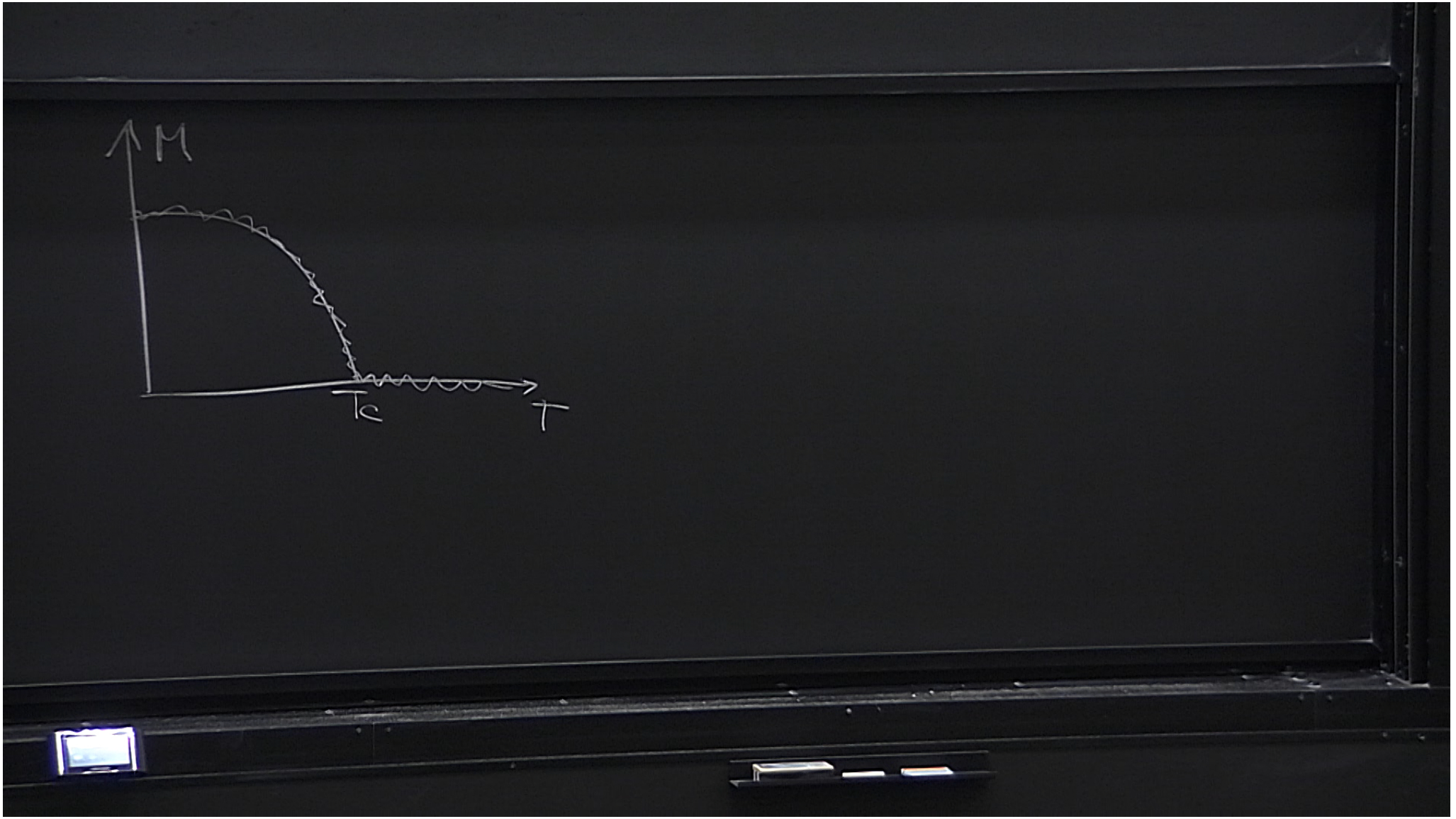
$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j$$

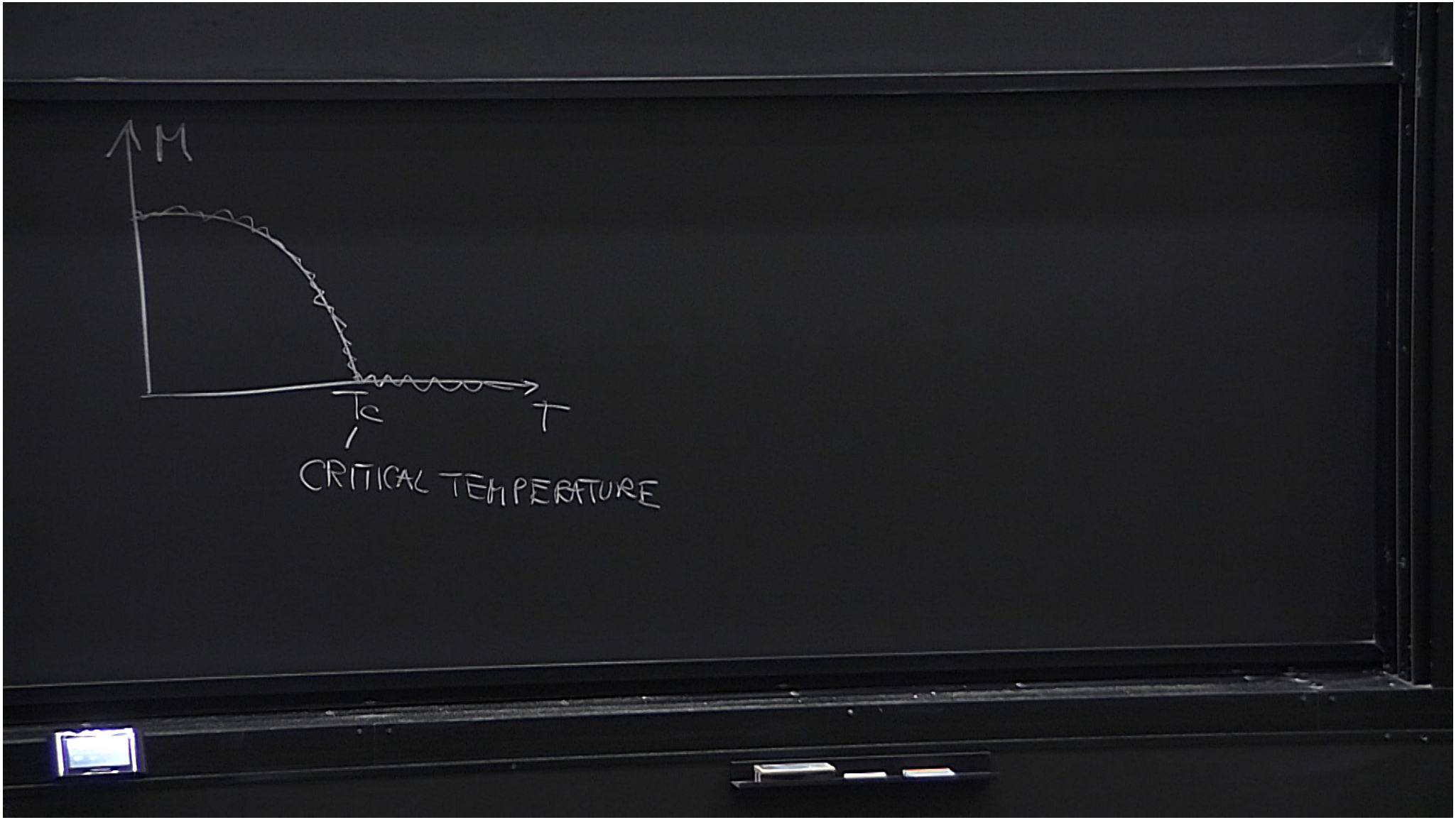

• 2 PHASES : FM & PM DISTIGUISHED BY
ORDER PARAMETER . MAGNETIZATION

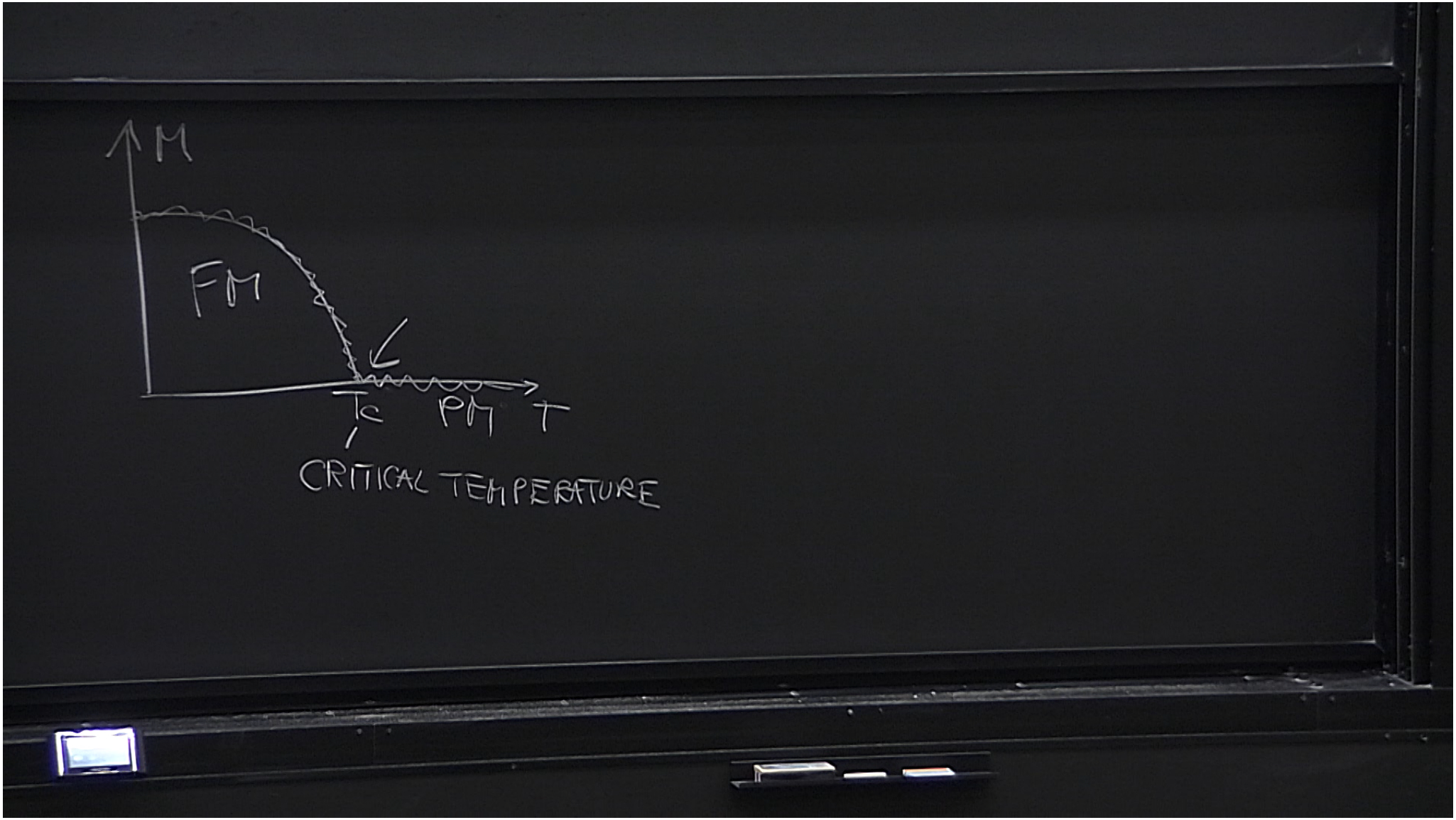

• 2 PHASES: FM & PM DISTIGUISHED BY
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M = < Si >)

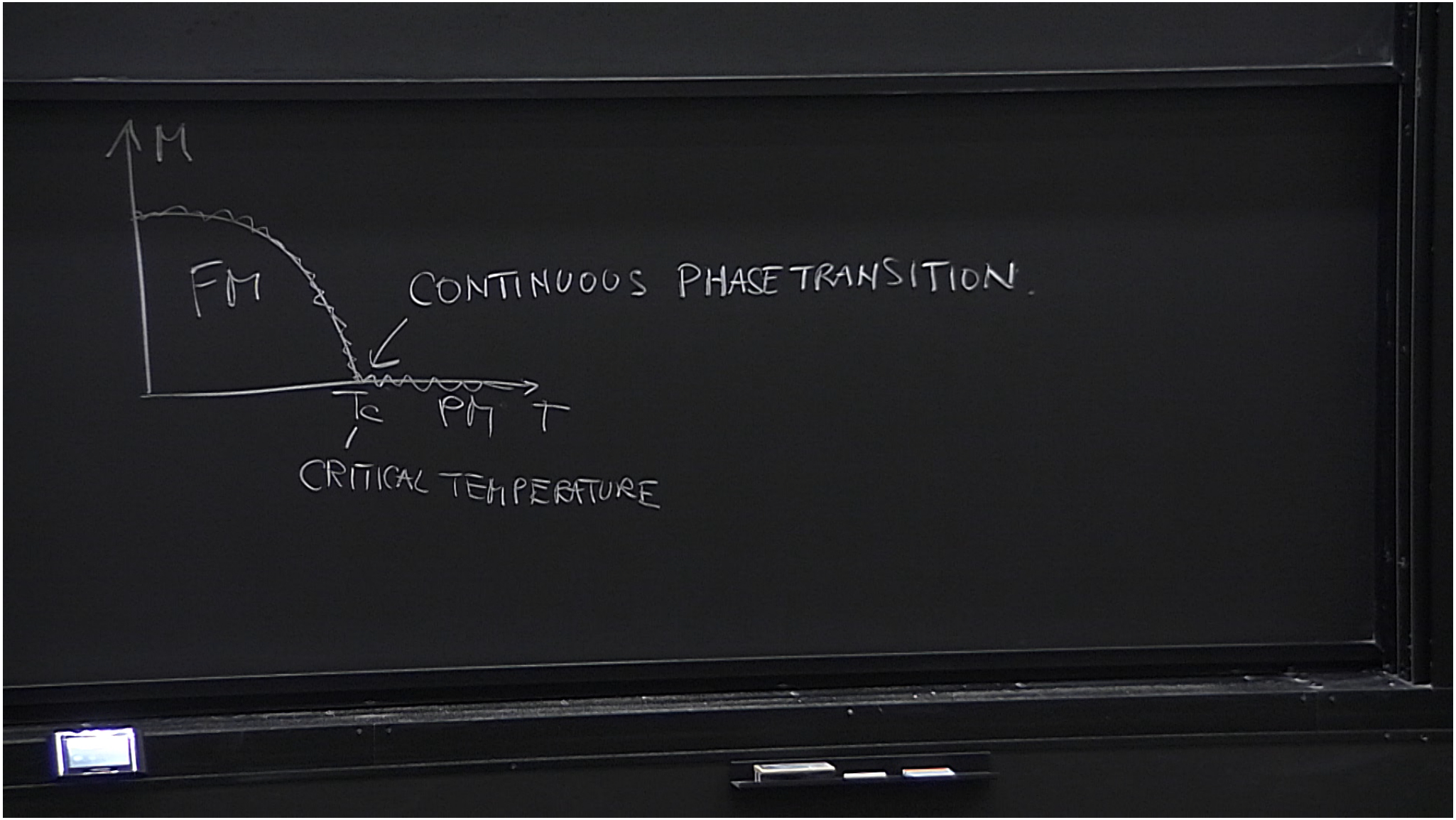

• 2 PHASES: FM & PM DISTIGUISHED BY
ORDER PARAMETER. MAGNETIZATION
$M = \langle \sigma_i \rangle = \begin{cases} 0 & \text{PM} \\ \neq 0 & \text{FM} \end{cases}$

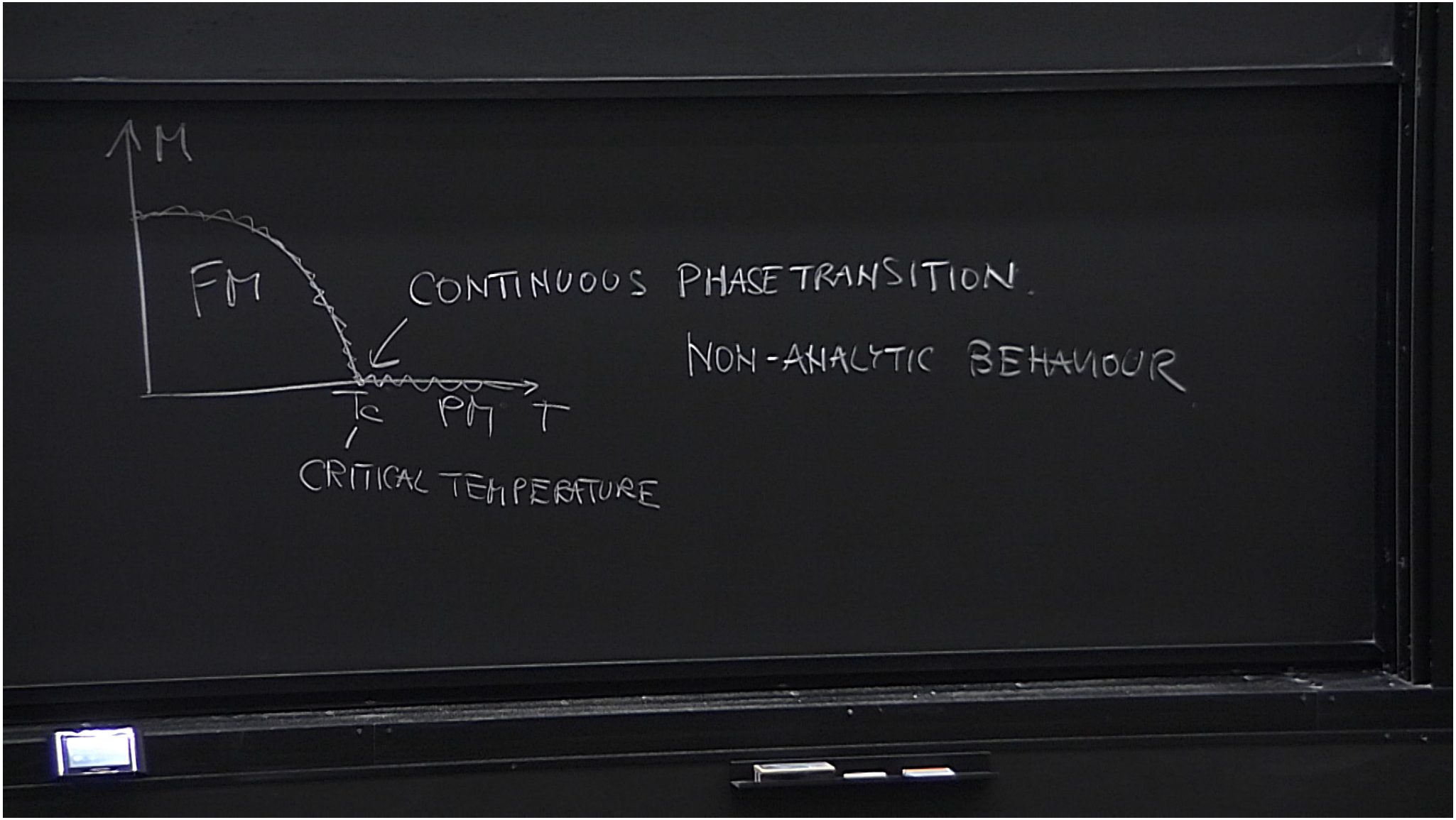

M
Tc
T

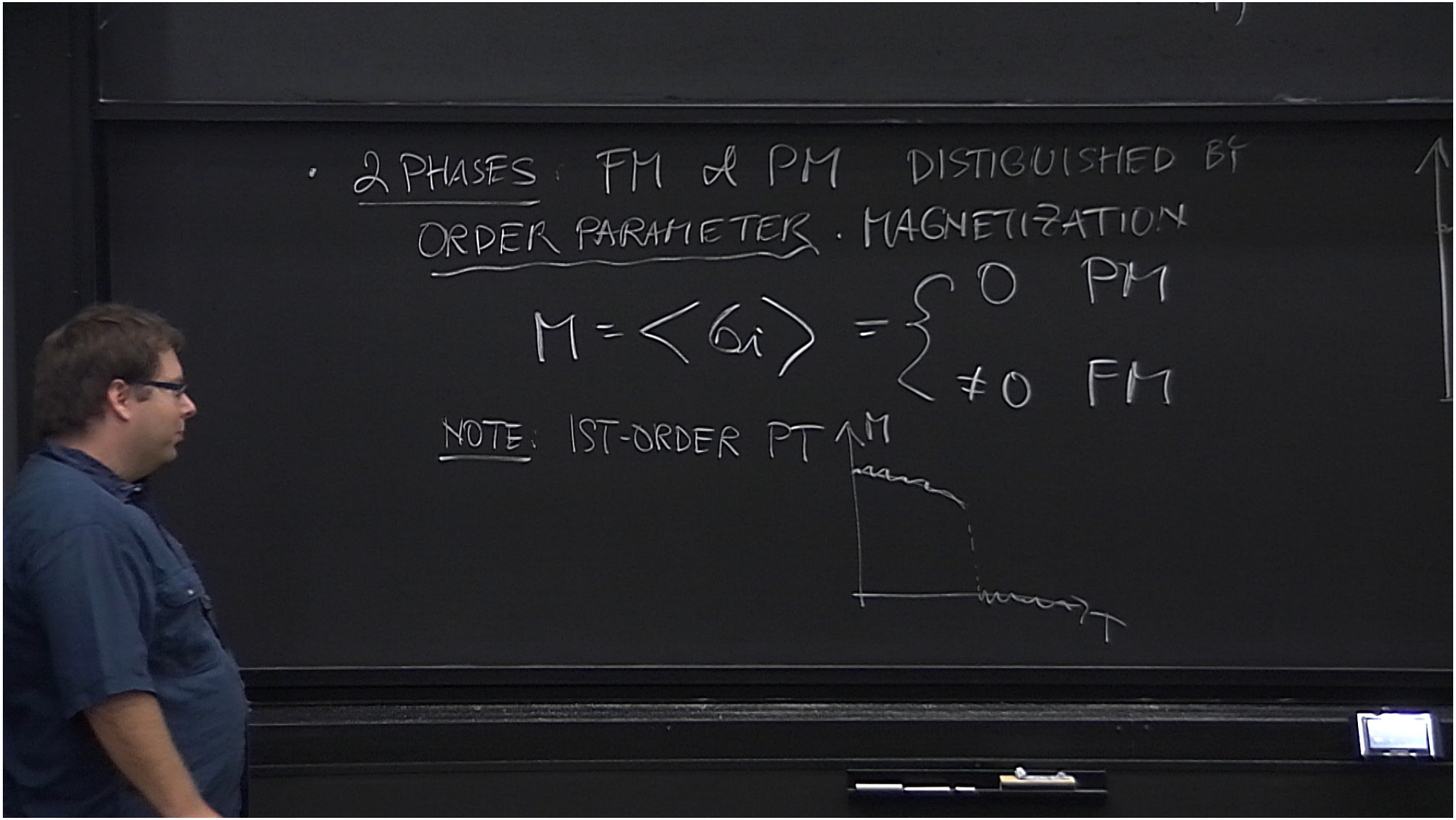

M
Tc
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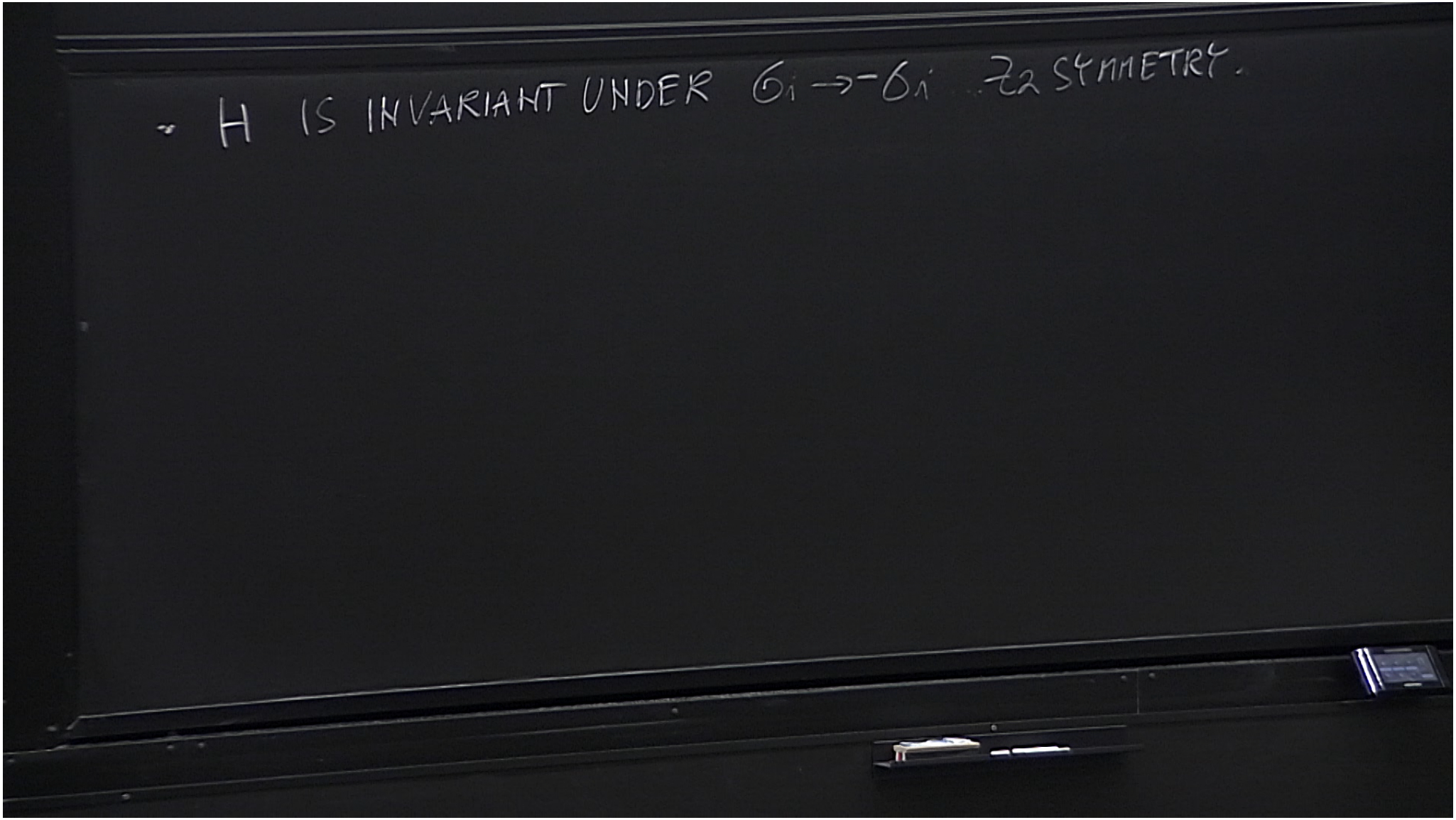

M
Tc
T
CRITICAL TEMPERATURE

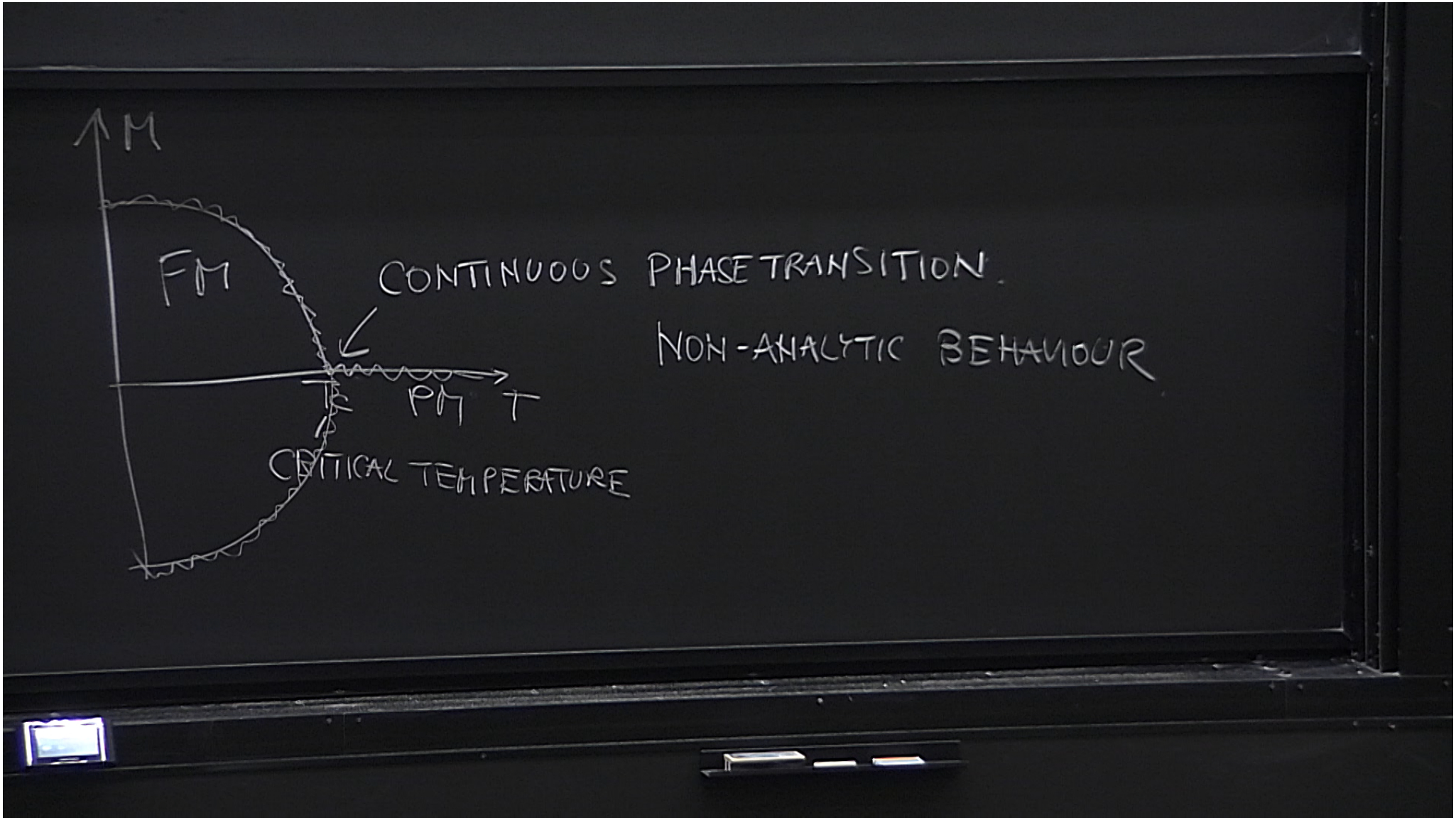

M
FM
Tc
PM
T
CRITICAL TEMPERATURE

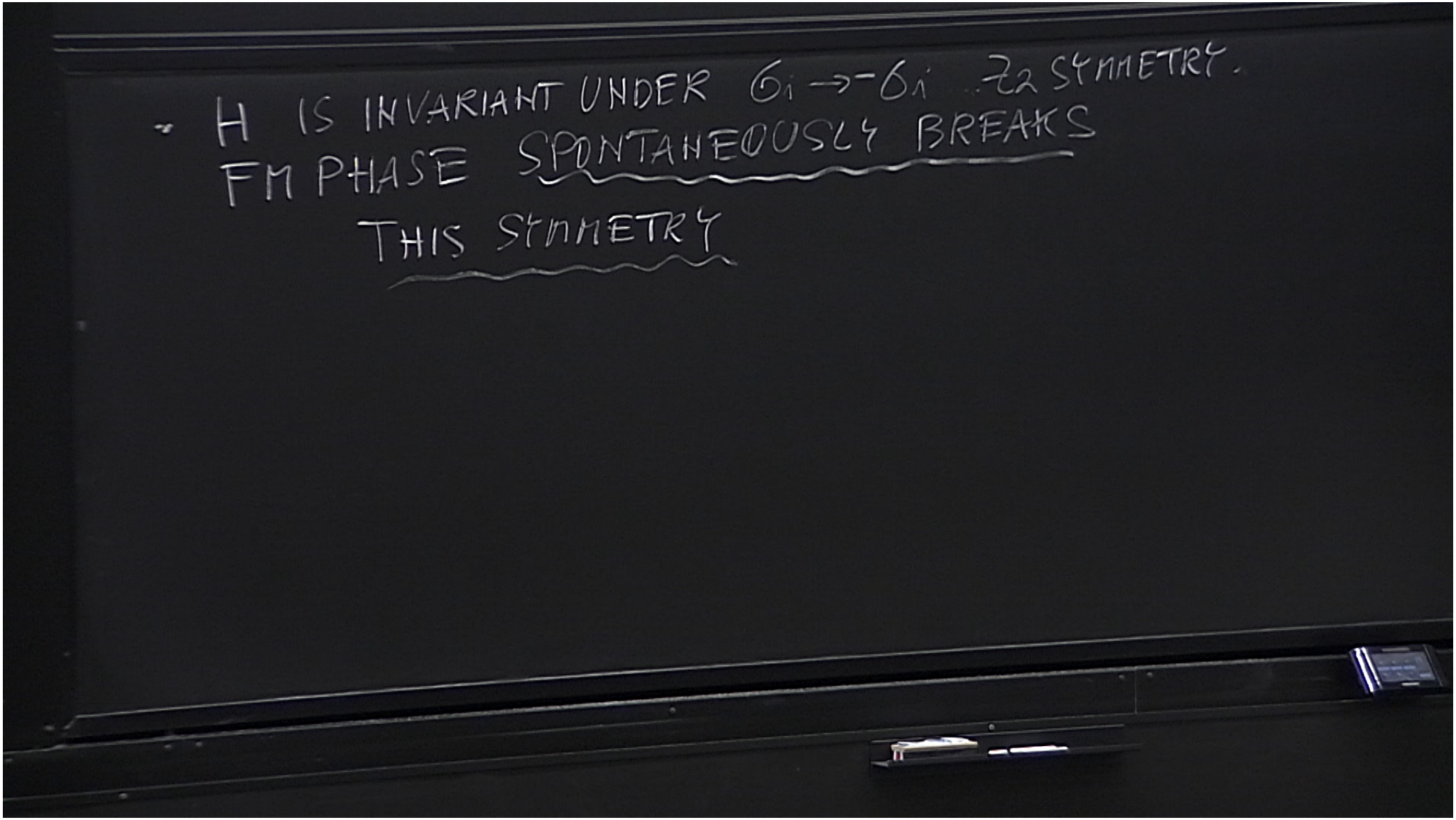

M
FM
CONTINUOUS PHASE TRANSITION.
Tc
PM
T
CRITICAL TEMPERATURE

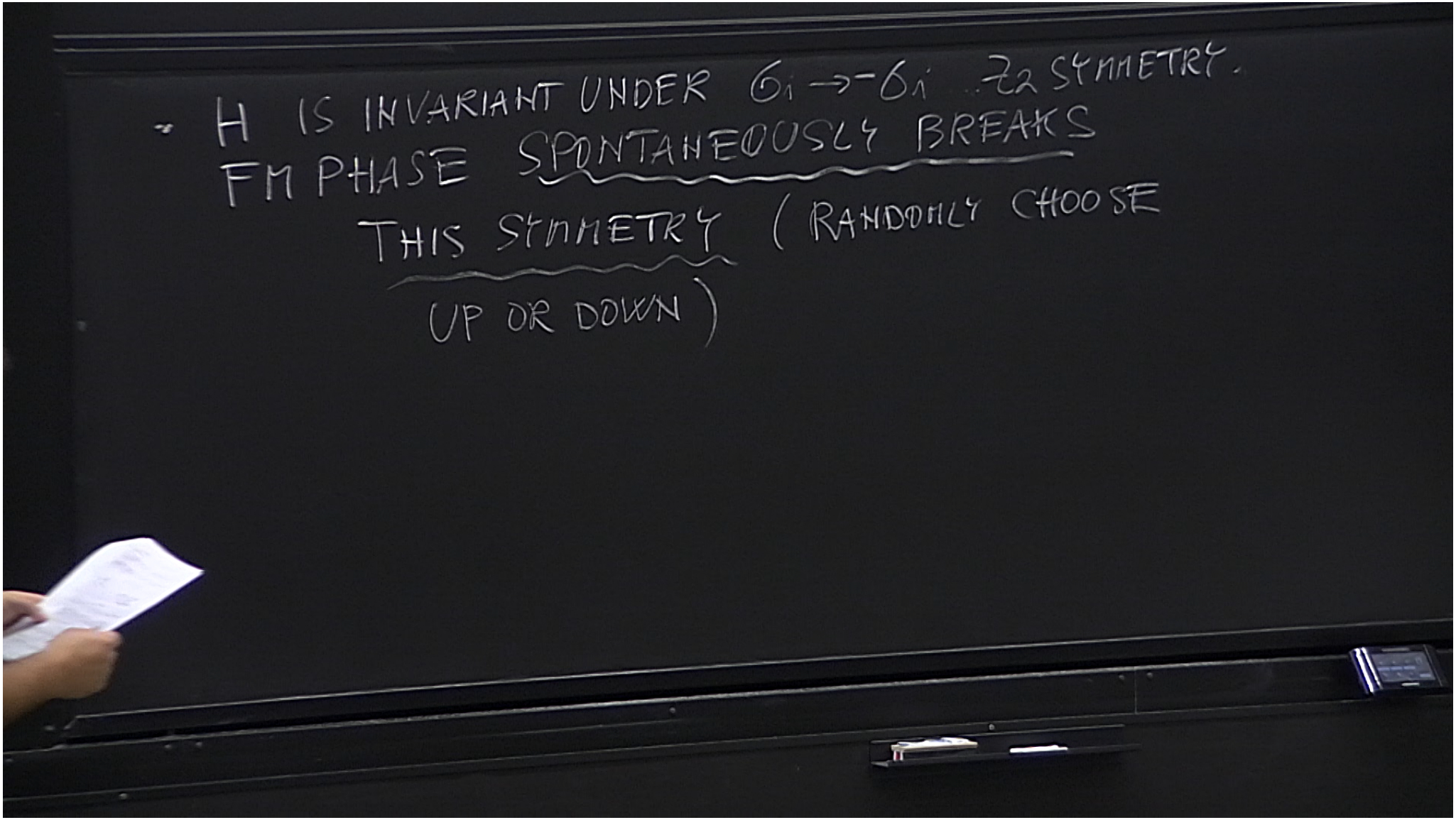

M
FM
CONTINUOUS PHASE TRANSITION.
NON-ANALYTIC BEHAVIOUR
Tc
PM
T
CRITICAL TEMPERATURE


• 2 PHASES: FM & PM DISTIGUISHED BY
ORDER PARAMETER. MAGNETIZATION
$M = \langle \sigma_i \rangle = \begin{cases} 0 & PM \\ \neq 0 & FM \end{cases}$
NOTE: 1ST-ORDER PT
M
T

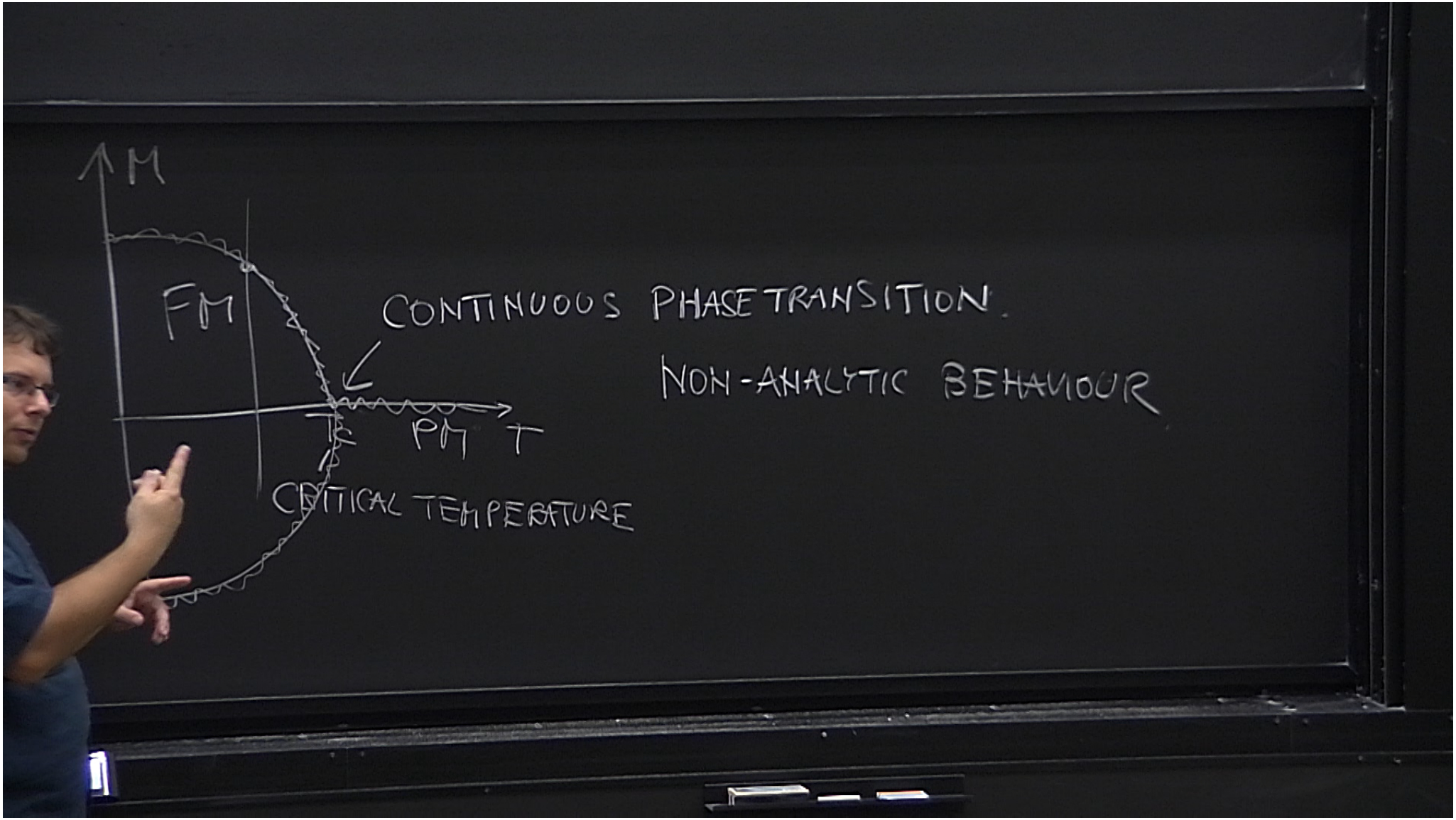

- H IS INVARIANT UNDER $\sigma_i \to -\sigma_i$ Z_2 SYMMETRY.

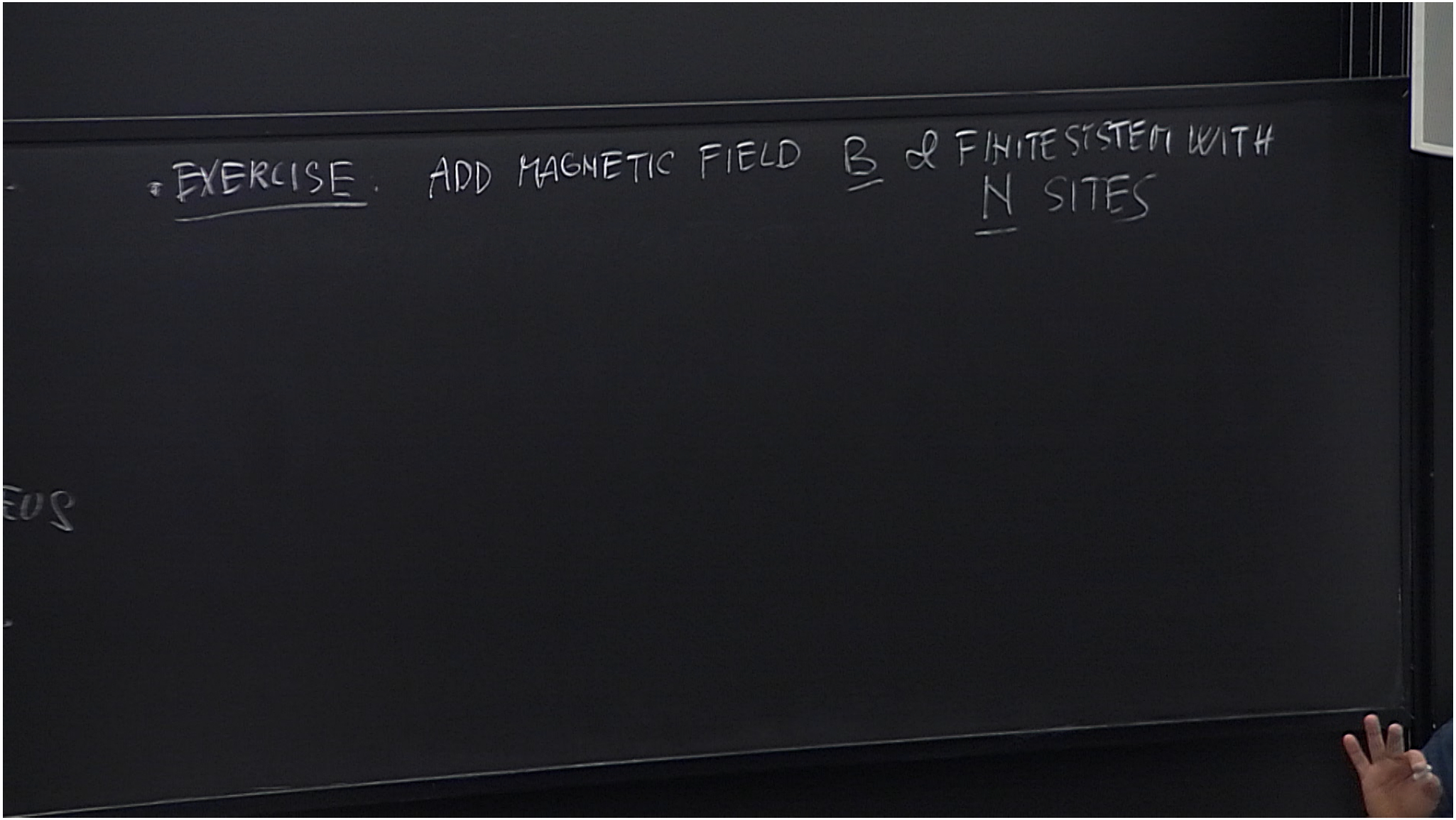

M
FM
CONTINUOUS PHASE TRANSITION.
NON-ANALYTIC BEHAVIOUR
PM
T
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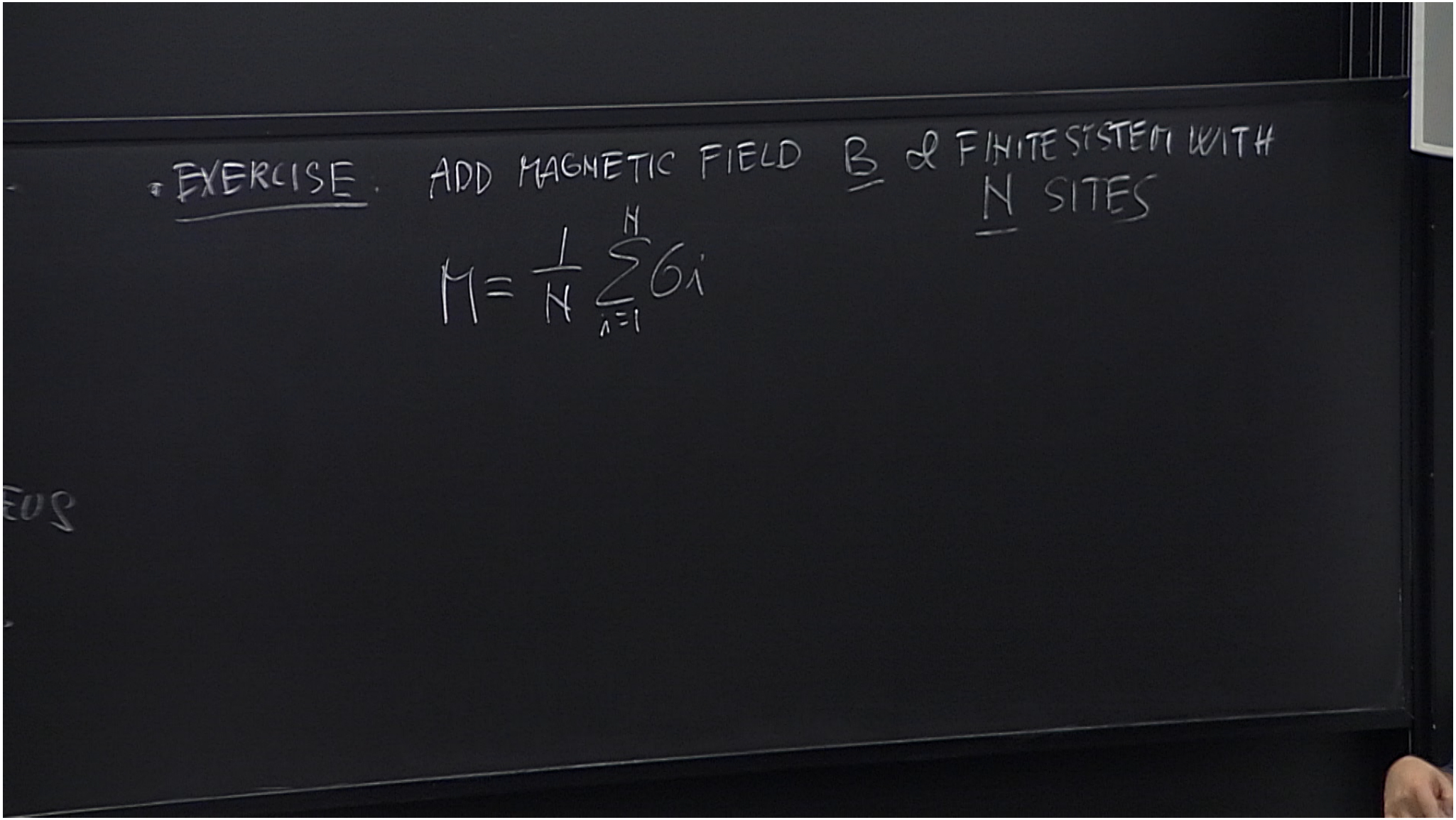

H IS INVARIANT UNDER $\sigma_i \to -\sigma_i$ Z_2 SYMMETRY.
FM PHASE SPONTANEOUSLY BREAKS
THIS SYMMETRY


H IS INVARIANT UNDER $\sigma_i \to -\sigma_i$ Z_2 SYMMETRY.
FM PHASE SPONTANEOUSLY BREAKS
THIS SYMMETRY (RANDOMLY CHOOSE
UP OR DOWN)

- H IS INVARIANT UNDER $\sigma_i \to -\sigma_i$ $\mathbb{Z}_2$ SYMMETRY.
FM PHASE SPONTANEOUSLY BREAKS
THIS SYMMETRY (RANDOMLY CHOOSE
UP OR DOWN)
$\exists$ OF ORDER PARAM $\Leftarrow$ TIED TO SPONTANEOUS
SYMMETRY BREAKING.


M
FM
PM
T
CONTINUOUS PHASE TRANSITION
NON-ANALYTIC BEHAVIOUR
CRITICAL TEMPERATURE


EXERCISE. ADD MAGNETIC FIELD B & FINITE SYSTEM WITH N SITES


EXERCISE. ADD MAGNETIC FIELD B & FINITE SYSTEM WITH N SITES
$M = \frac{1}{N} \sum_{i=1}^{N} G_i$

• EXERCISE. ADD MAGNETIC FIELD B & FINITE SYSTEM WITH N SITES

$$M = \frac{1}{N} \sum_{i=1}^{N} \sigma_i$$

$$\Gamma = \frac{P_{M>0}}{P_{M<0}}$$

• EXERCISE. ADD MAGNETIC FIELD $\underline{B}$ & FINITE SYSTEM WITH $\underline{N}$ SITES

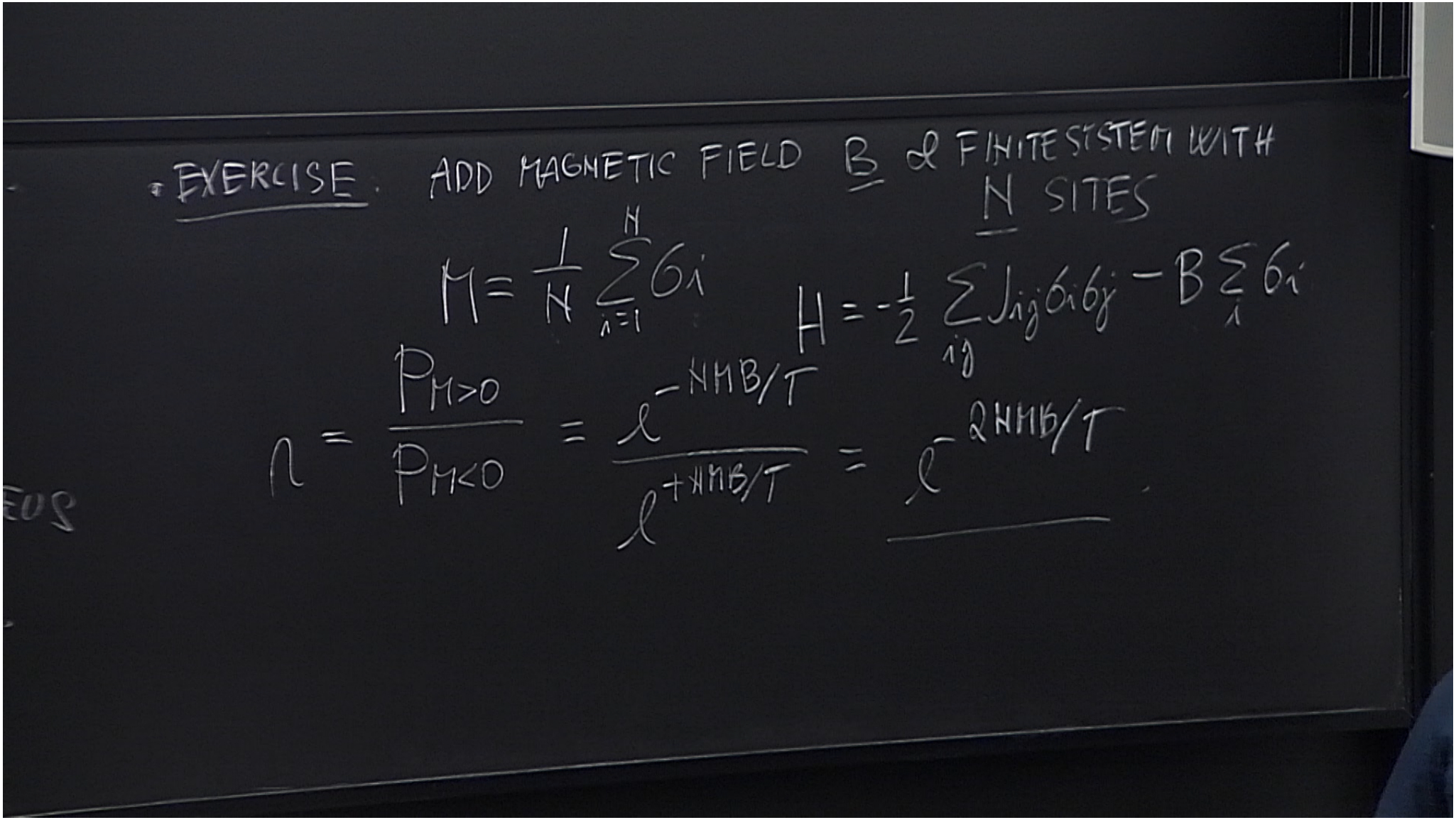
$$M = \frac{1}{N} \sum_{i=1}^{N} \sigma_i \qquad H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j - B \sum_i \sigma_i$$

$$\Omega = \frac{P_{M>0}}{P_{M<0}}$$

• EXERCISE. ADD MAGNETIC FIELD B & FINITE SYSTEM WITH N SITES

$$M = \frac{1}{N}\sum_{i=1}^{N}\sigma_i \qquad H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - B\sum_i \sigma_i$$

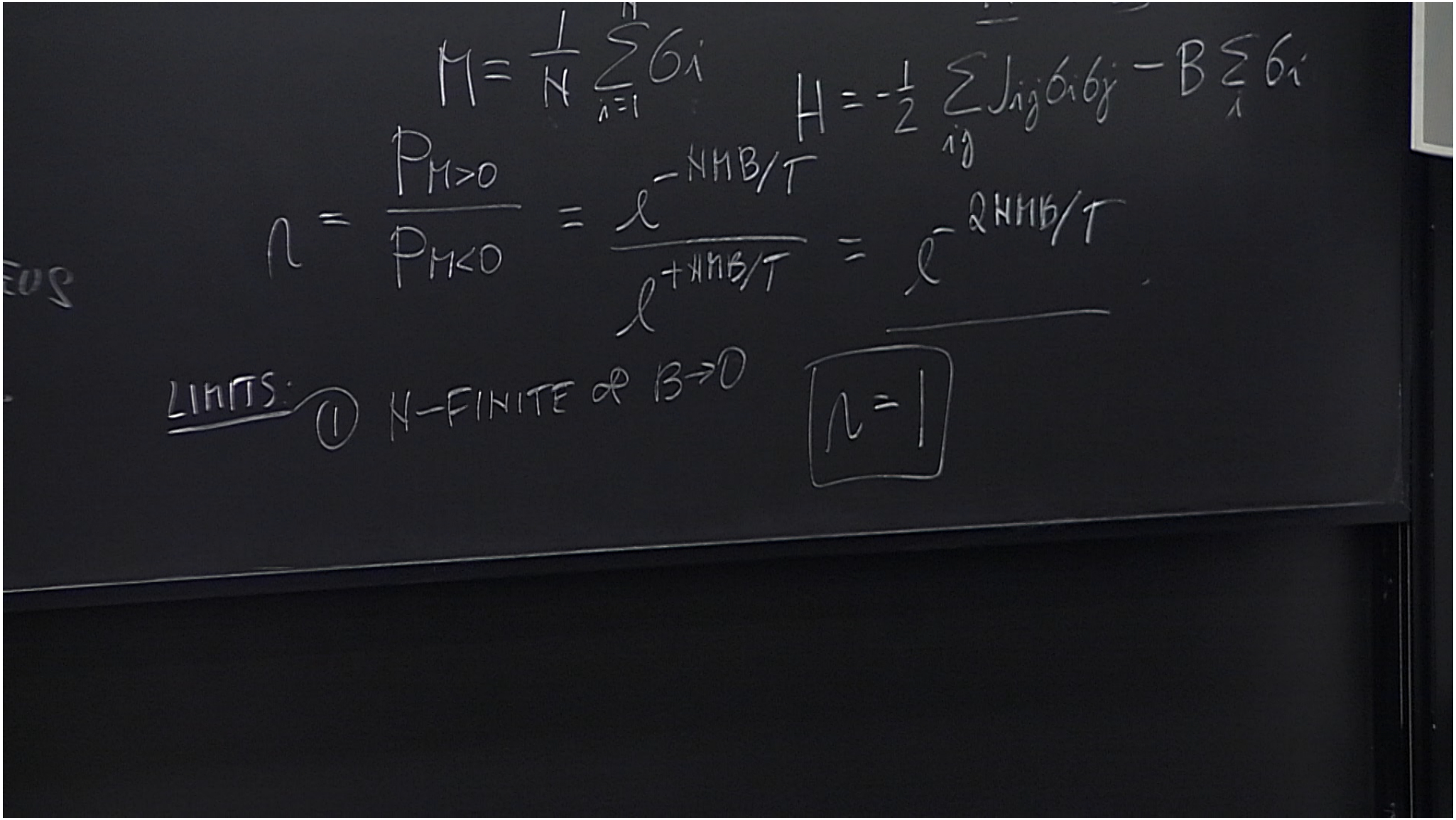
$$r = \frac{P_{M>0}}{P_{M<0}} = \frac{e^{-NMB/T}}{e^{+NMB/T}}$$


EXERCISE. ADD MAGNETIC FIELD B & FINITE SYSTEM WITH N SITES
$M = \frac{1}{N}\sum_{i=1}^{N}\sigma_i$
$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - B\sum_i \sigma_i$
$r = \frac{P_{M>0}}{P_{M<0}} = \frac{e^{-NMB/T}}{e^{+NMB/T}} = e^{-2NMB/T}$

$$M = \frac{1}{N}\sum_{i=1}^{N} \sigma_i \qquad H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - B\sum_i \sigma_i$$

$$R = \frac{P_{M>0}}{P_{M<0}} = \frac{e^{-NMB/T}}{e^{+NMB/T}} = e^{-2NMB/T}$$

LIMITS: ① N-FINITE & $B \to 0$


$M = \frac{1}{N}\sum_{i=1}^{N} \sigma_i$
$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - B\sum_i \sigma_i$
$\Lambda = \frac{P_{M>0}}{P_{M<0}} = \frac{e^{-NMB/T}}{e^{+NMB/T}} = e^{-2NMB/T}$
LIMITS: ① N-FINITE & $B \to 0$
$\Lambda = 1$

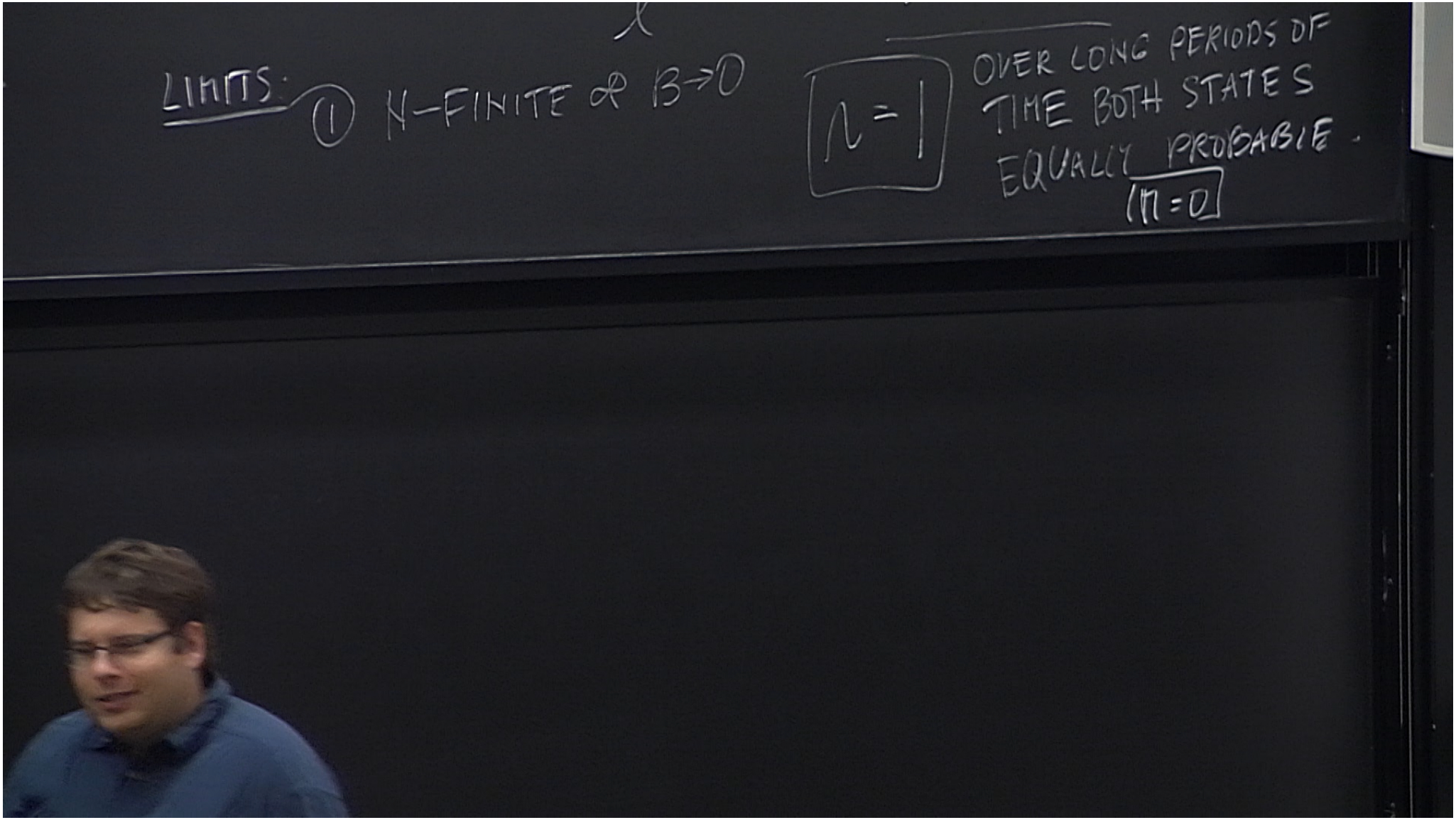
$$M = \frac{1}{N}\sum_{i=1}^{N} \sigma_i \qquad H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - B\sum_i \sigma_i$$

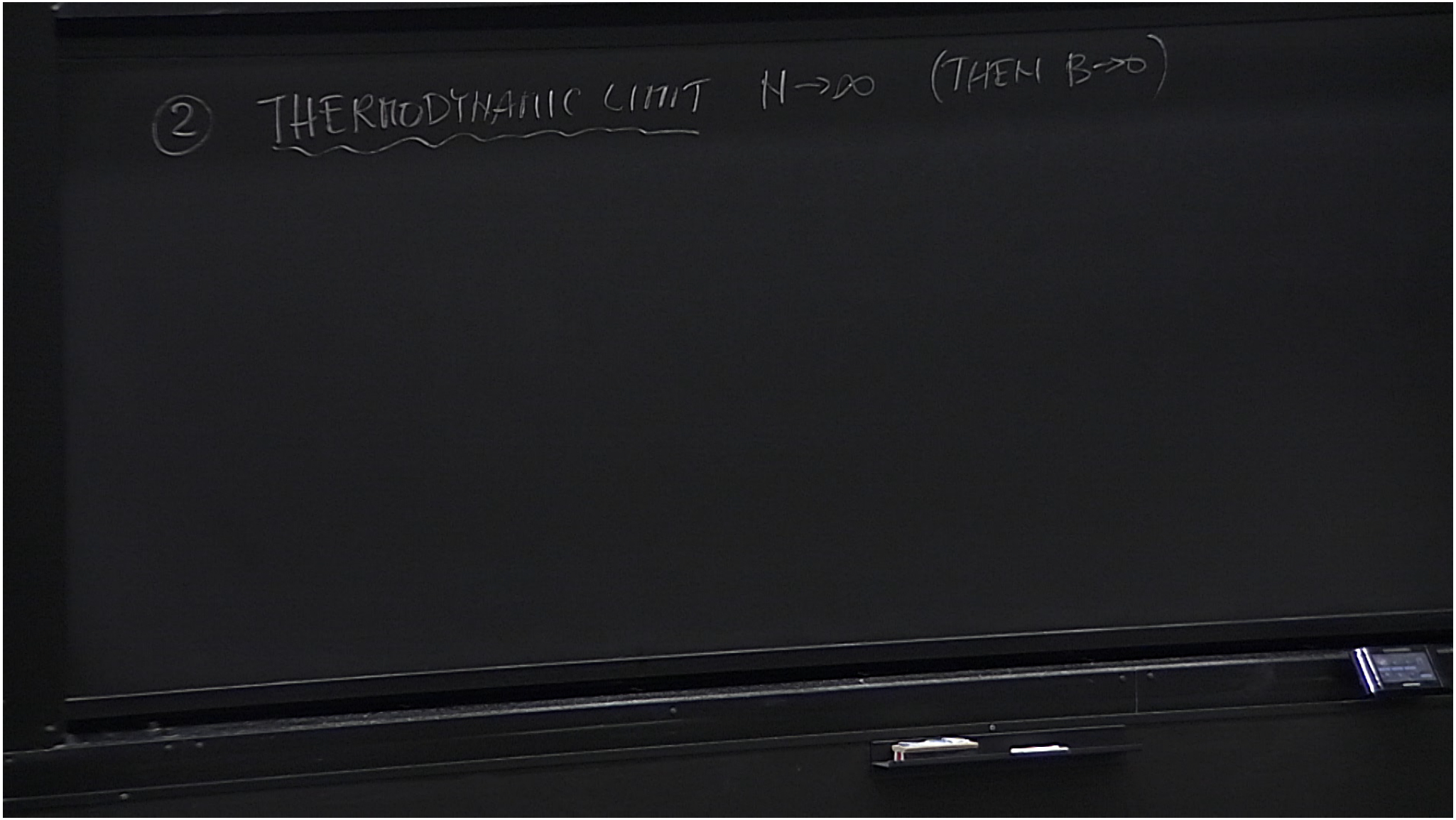
$$R = \frac{P_{M>0}}{P_{M<0}} = \frac{e^{-NMB/T}}{e^{+NMB/T}} = e^{-2NMB/T}$$

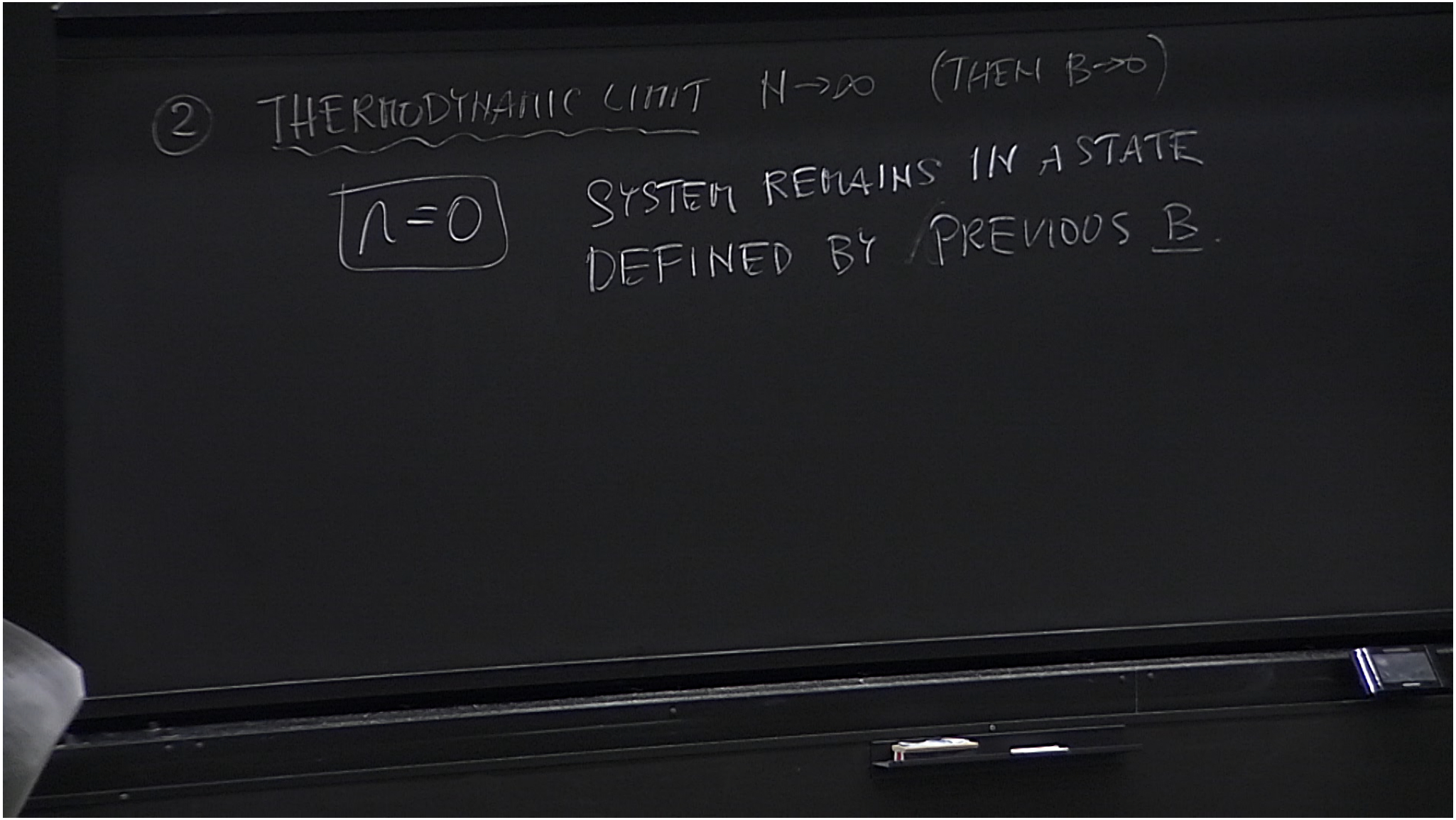
LIMITS: ① N-FINITE & $B \to 0$

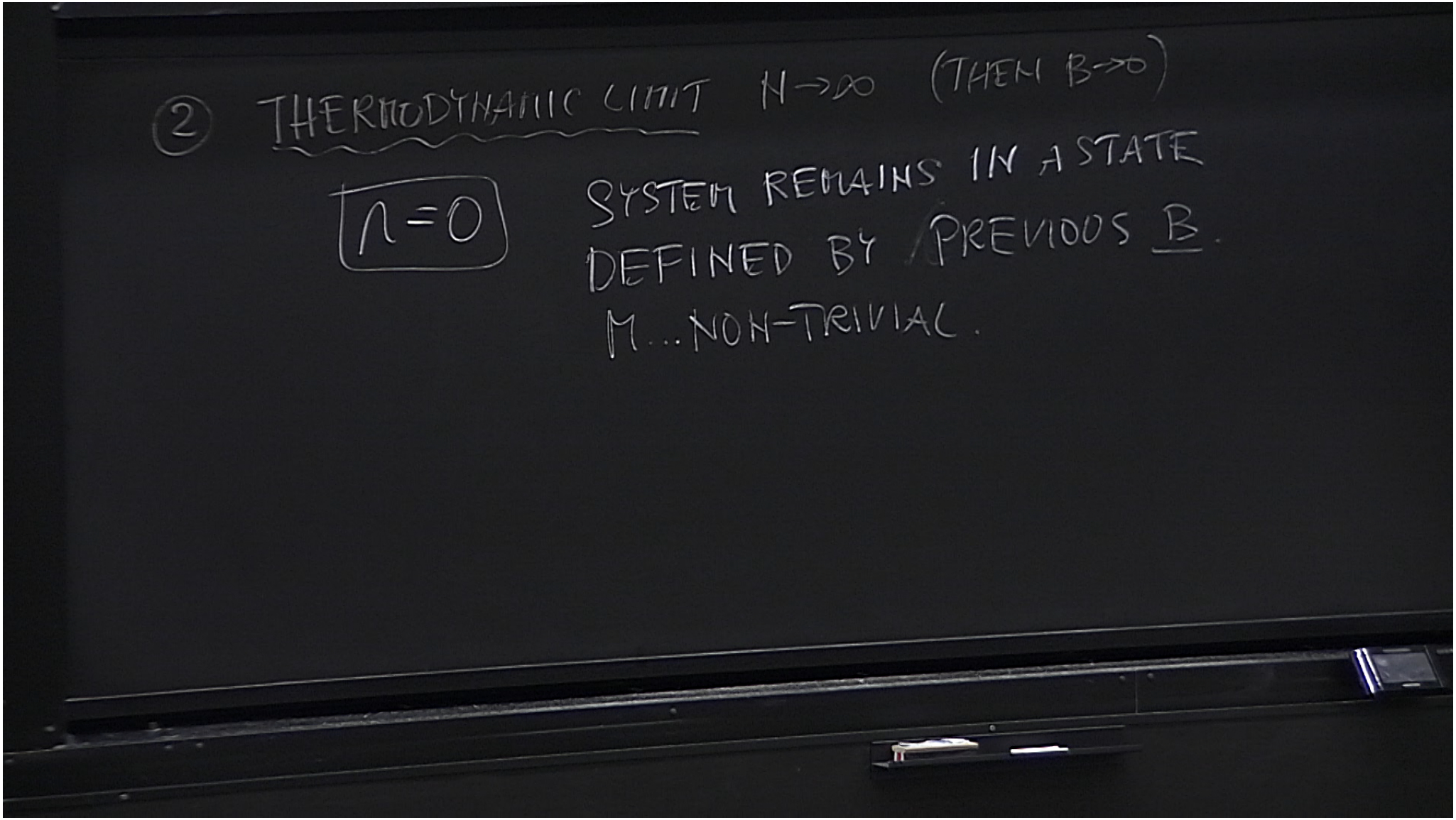
$R = 1$

OVER LONG PERIODS OF TIME BOTH STATES EQUALLY PROBABLE.


LIMITS:
① N-FINITE & β→0
Λ = 1
OVER LONG PERIODS OF TIME BOTH STATES EQUALLY PROBABLE.
(Π = 0)


② THERMODYNAMIC LIMIT N→∞ (THEN B→0)


② THERMODYNAMIC LIMIT $N \to \infty$ (THEN $B \to 0$)
$\Lambda = 0$
SYSTEM REMAINS IN A STATE DEFINED BY PREVIOUS B.


② THERMODYNAMIC LIMIT $N \to \infty$ (THEN $B \to 0$)
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SYSTEM REMAINS IN A STATE DEFINED BY PREVIOUS B.
M...NON-TRIVIAL.

② THERMODYNAMIC LIMIT $N \to \infty$ (THEN $B \to 0$)

$\Lambda = 0$ SYSTEM REMAINS IN A STATE DEFINED BY PREVIOUS B.

M...NON-TRIVIAL.

CONCLUSION. PHASES & PHASE TRANSITIONS ARE DEFINED SHARPLY ONLY FOR INFINITE SYSTEMS

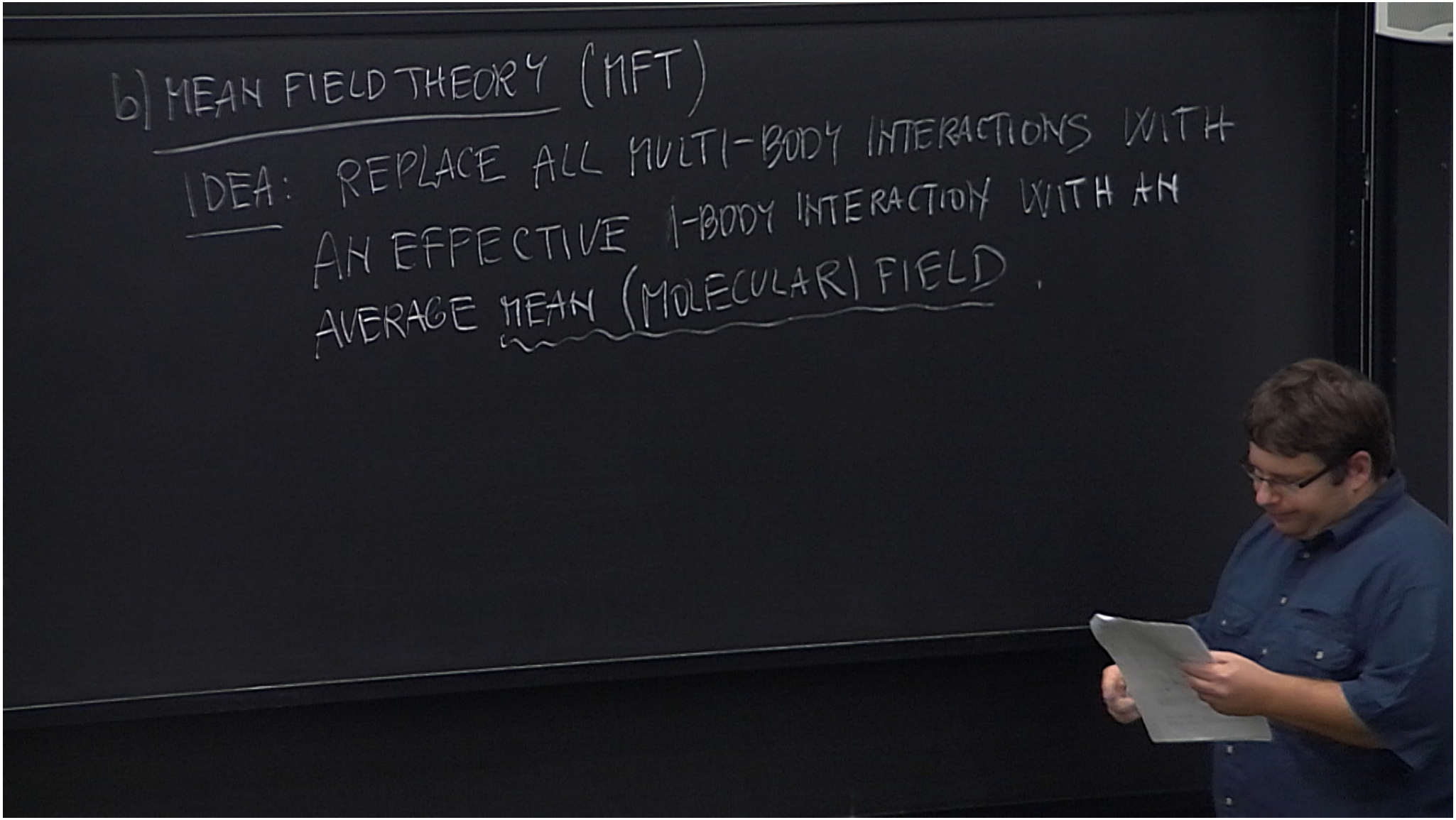
$\Lambda = 0$ SYSTEM REMAINS

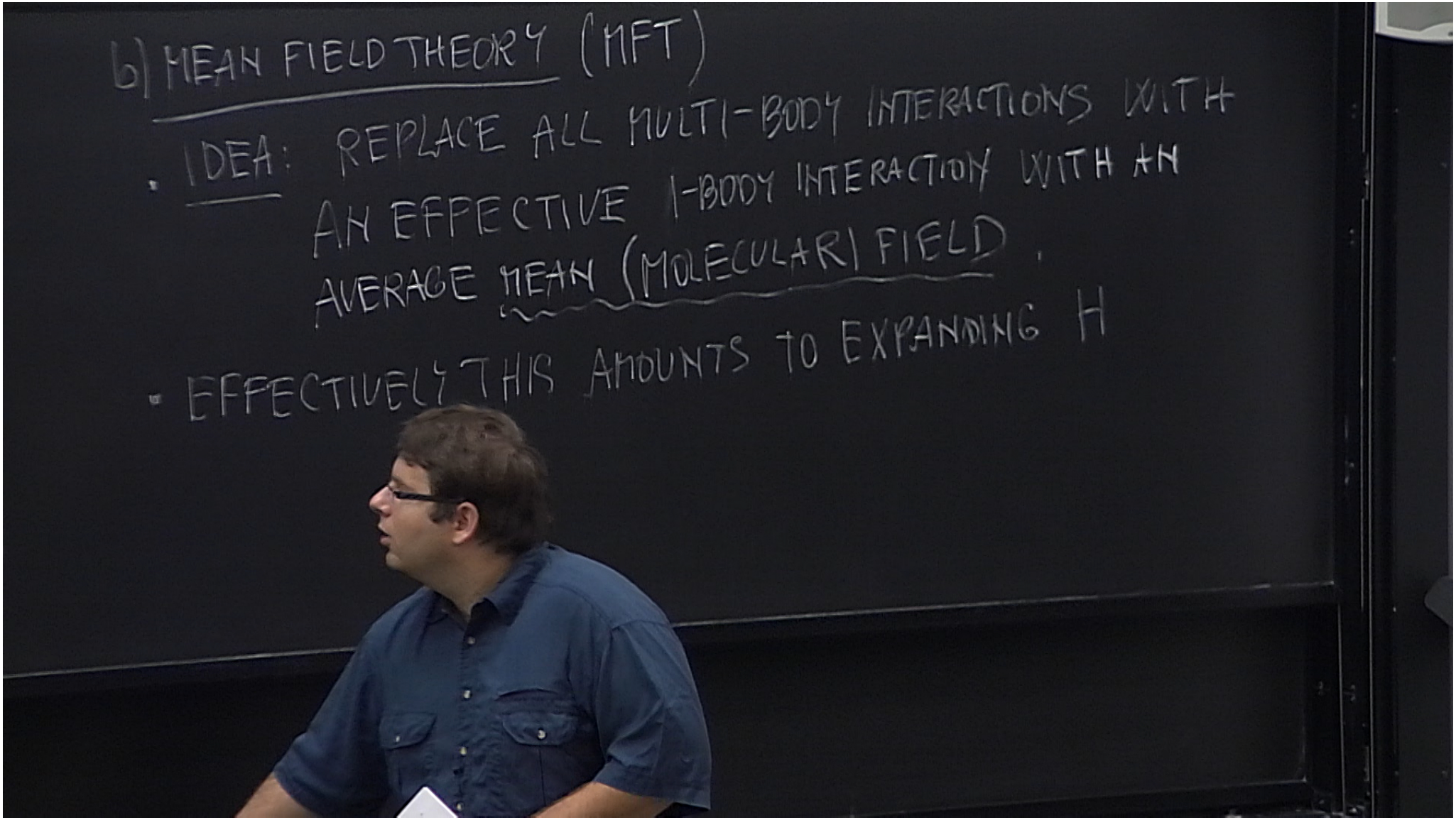
DEFINED BY PREVIOUS B.

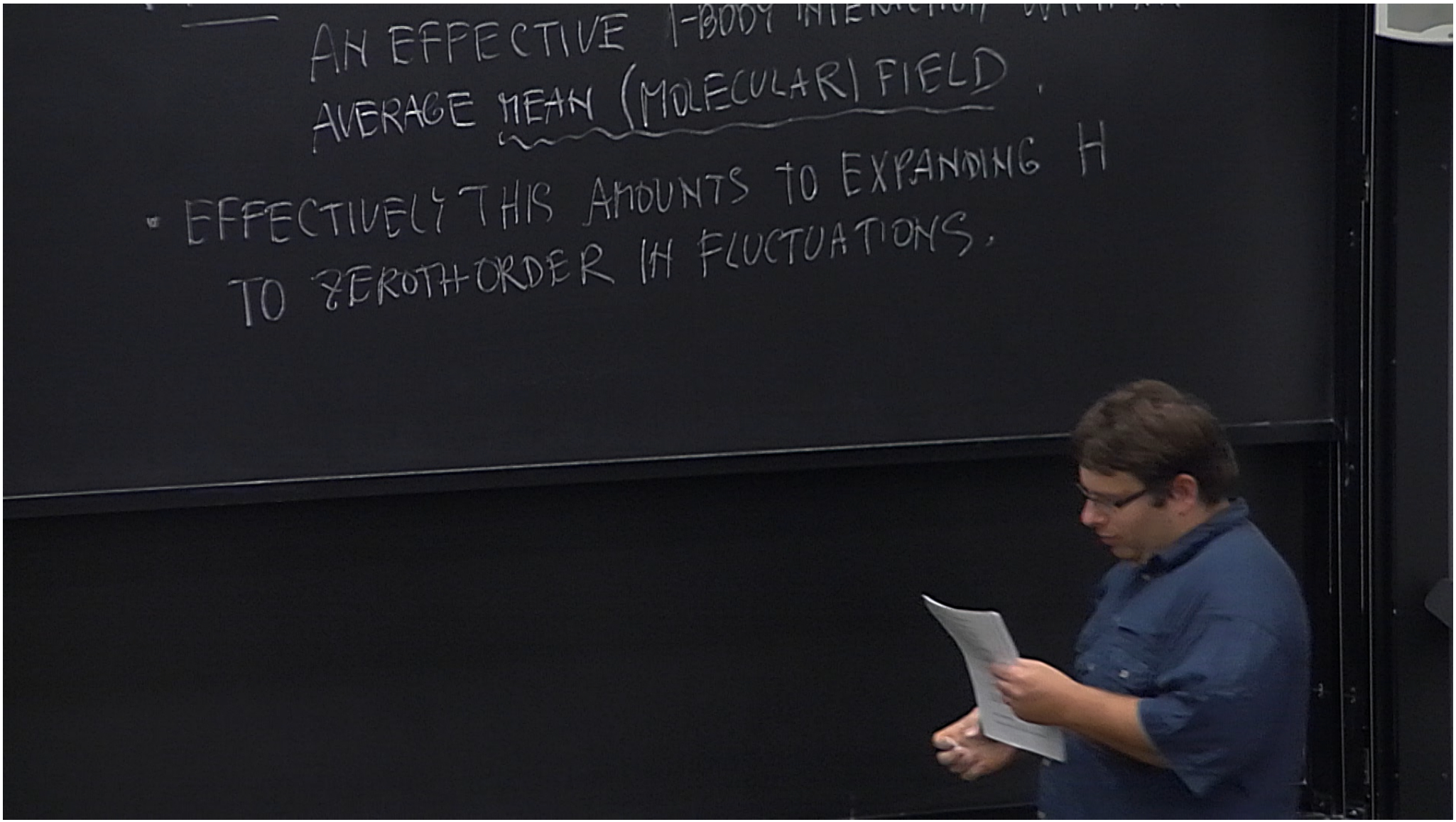
M..NON-TRIVIAL.

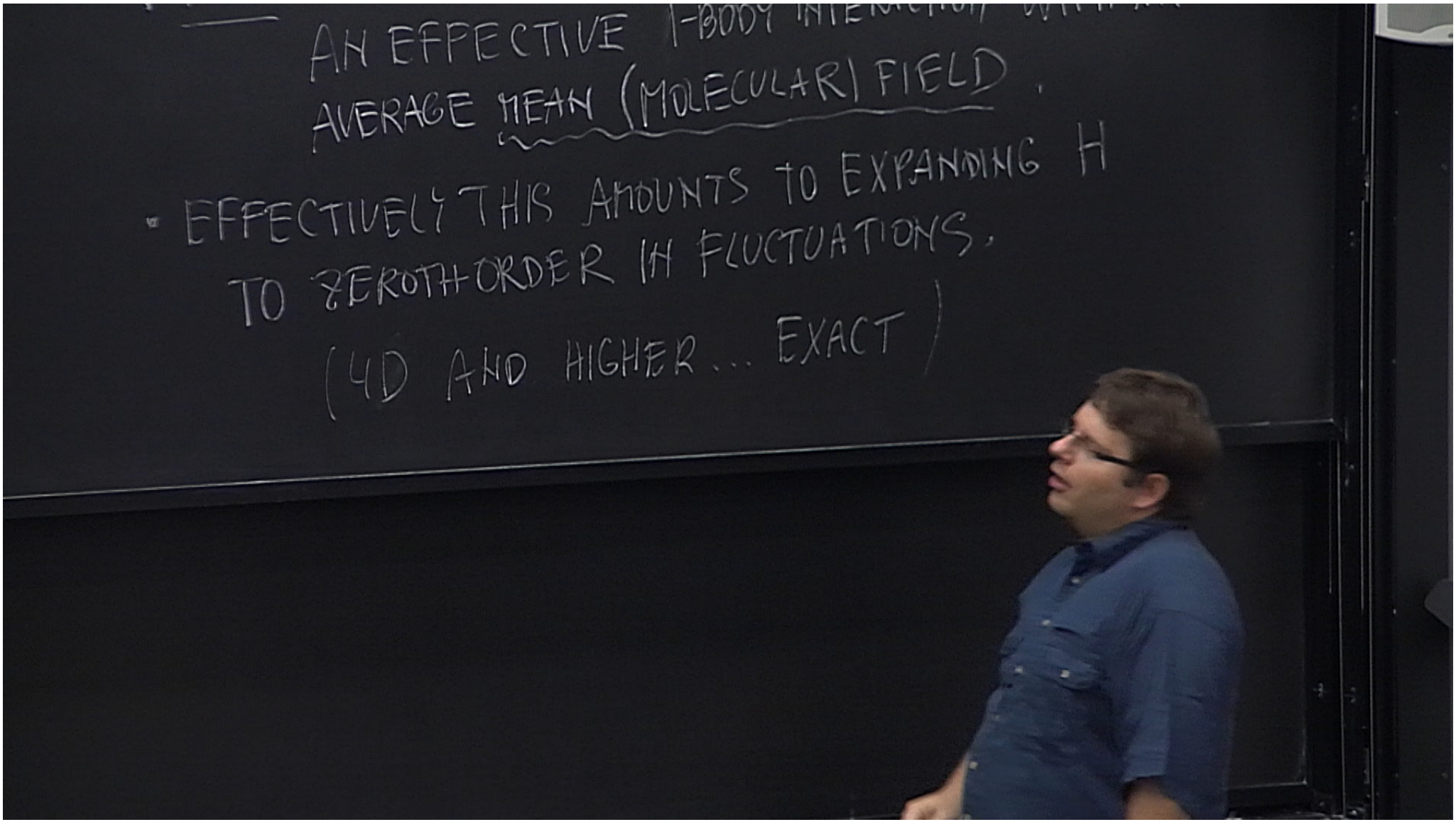
CONCLUSION. PHASES & PHASE TRANSITIONS ARE DEFINED SHARPLY ONLY FOR INFINITE SYSTEMS

(MACROSCOPIC SYSTEMS... LONG TIMESCALE)


b) MEAN FIELD THEORY (MFT)
IDEA: REPLACE ALL MULTI-BODY INTERACTIONS WITH
AN EFFECTIVE 1-BODY INTERACTION WITH AN
AVERAGE MEAN (MOLECULAR) FIELD.


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• IDEA: REPLACE ALL MULTI-BODY INTERACTIONS WITH AN EFFECTIVE 1-BODY INTERACTION WITH AN AVERAGE MEAN (MOLECULAR) FIELD.
• EFFECTIVELY THIS AMOUNTS TO EXPANDING H


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(4D AND HIGHER... EXACT)

APPLICATION TO ISING.

$$\sigma_i = \sigma_i - \langle\sigma_i\rangle + \langle\sigma_i\rangle = \sigma_i - m + m.$$

$$\sigma_i\sigma_j = \left[(\sigma_i - m) + m\right]\left[(\sigma_j - m) + m\right]$$

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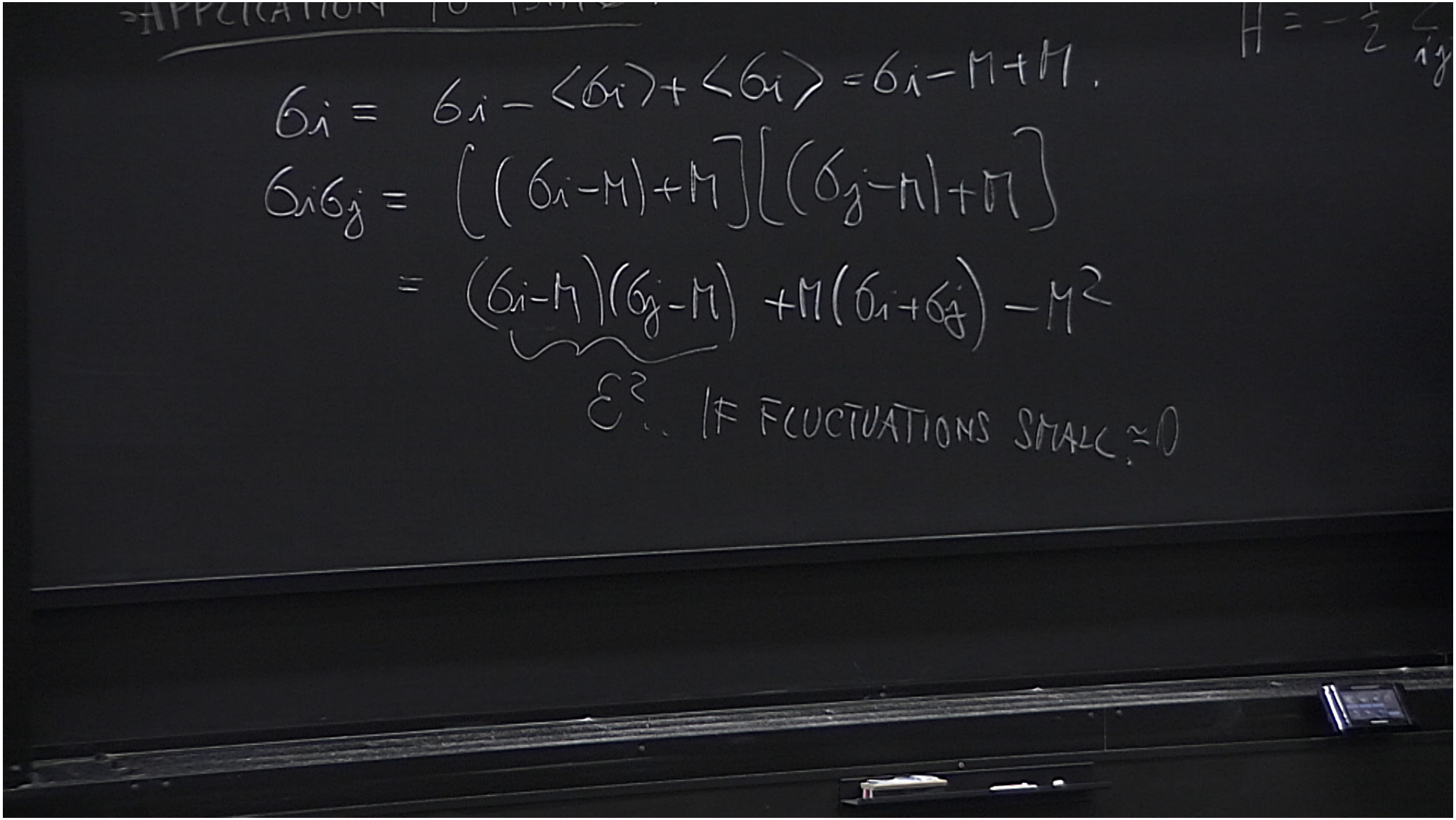
APPLICATION TO ISING.

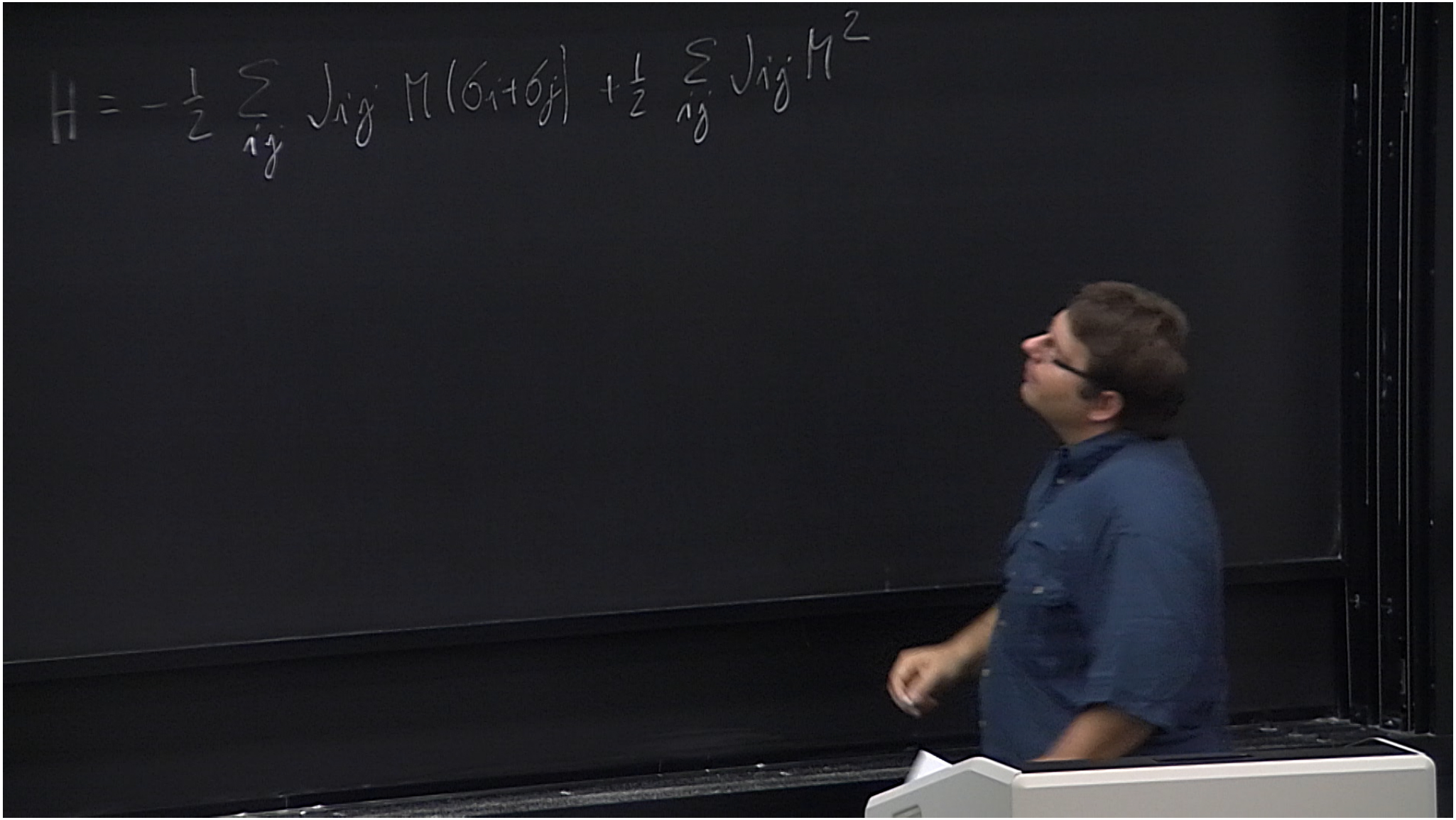
$$\sigma_i = \sigma_i - \langle\sigma_i\rangle + \langle\sigma_i\rangle = \sigma_i - M + M.$$

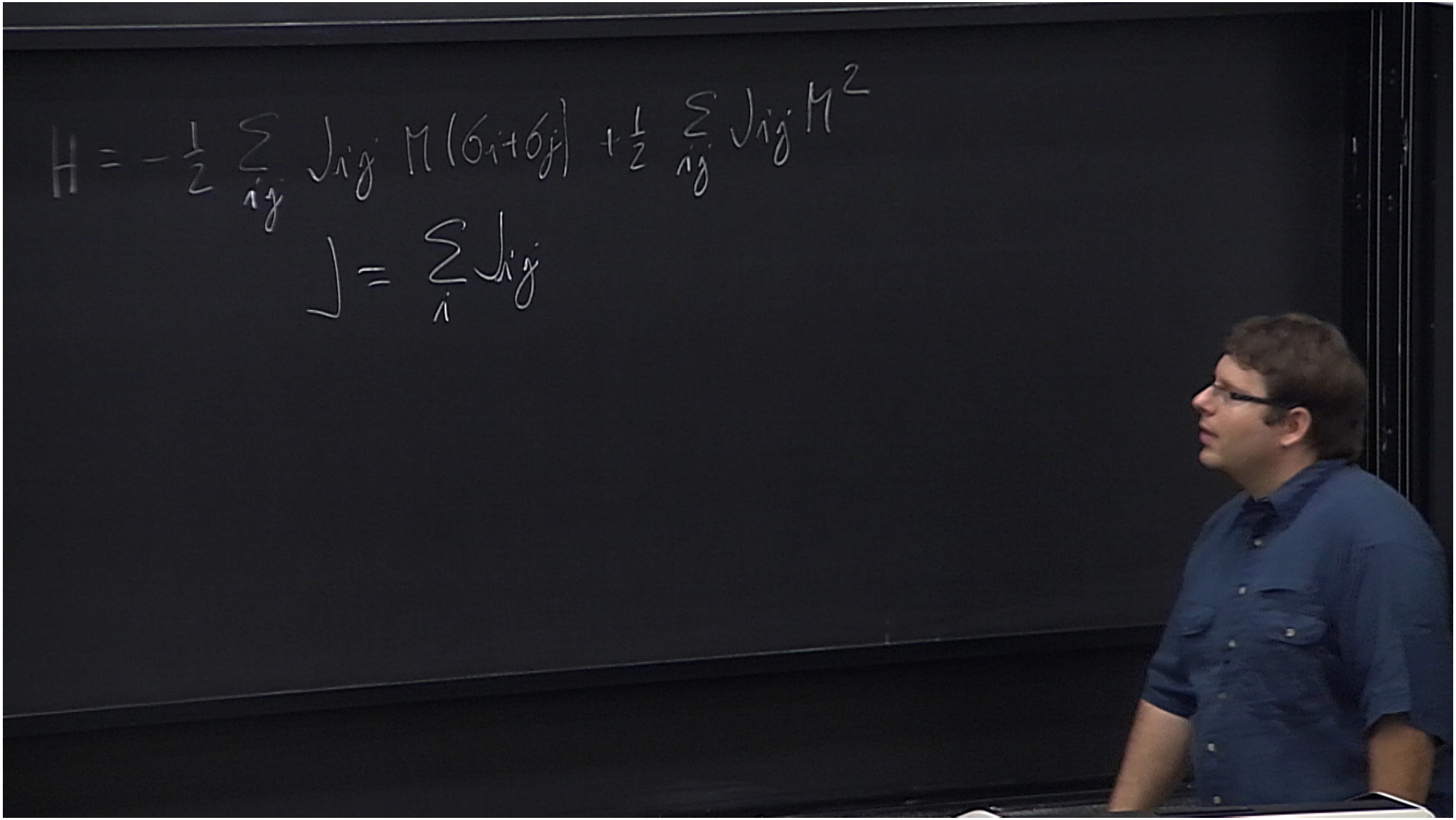
$$\sigma_i\sigma_j = \left[(\sigma_i - M) + M\right]\left[(\sigma_j - M) + M\right]$$

$$= \underbrace{(\sigma_i - M)(\sigma_j - M)}_{\varepsilon^2} + M(\sigma_i + \sigma_j) - M^2$$

ε^2 IF FLUCTUATIONS SMALL.


APPLICATION TO
$H = -\frac{1}{2}\sum_{ij}$
$\sigma_i = \sigma_i - \langle\sigma_i\rangle + \langle\sigma_i\rangle = \sigma_i - M + M$.
$\sigma_i\sigma_j = [(\sigma_i - M) + M][(\sigma_j - M) + M]$
$= (\sigma_i - M)(\sigma_j - M) + M(\sigma_i + \sigma_j) - M^2$
ε^2 IF FLUCTUATIONS SMALL $\simeq 0$


$H = -\frac{1}{2} \sum_{ij} J_{ij} M(\sigma_i + \sigma_j) + \frac{1}{2} \sum_{ij} J_{ij} M^2$


$H = -\frac{1}{2}\sum_{ij} J_{ij} M(\sigma_i + \sigma_j) + \frac{1}{2}\sum_{ij} J_{ij} M^2$
$J = \sum_i J_{ij}$

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\, M(\sigma_i + \sigma_j) + \frac{1}{2}\sum_{ij} J_{ij} M^2$$

$J = \sum_i J_{ij}$ ... INDEPENDENT OF j. LATTICE UNIFORM.

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FINITE RANGE OF INTERACTIONS.

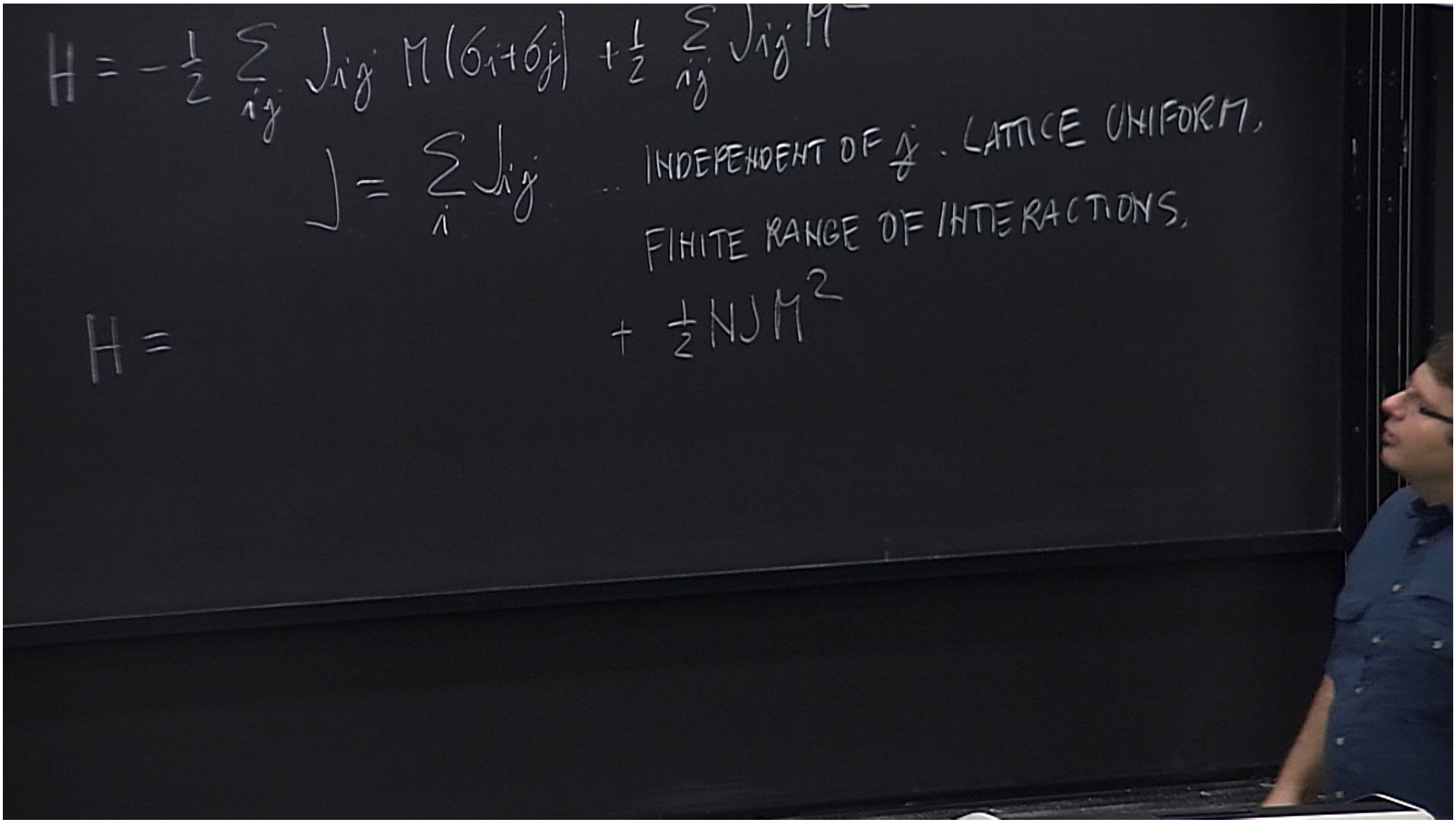
$$H = -\frac{1}{2}\sum_{ij} J_{ij}\, M(\sigma_i + \sigma_j) + \frac{1}{2}\sum_{ij} J_{ij} M^2$$

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FIHITE RANGE OF INTERACTIONS.

$H =$

$$+ \frac{1}{2} N J M^2$$



$$H = -\frac{1}{2}\sum_{ij} J_{ij}\, M(\sigma_i + \sigma_j) + \frac{1}{2}\sum_{ij} J_{ij} M^2$$

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FINITE RANGE OF INTERACTIONS.

$$H = -MJ\sum_i \sigma_i + \frac{1}{2}NJM^2$$

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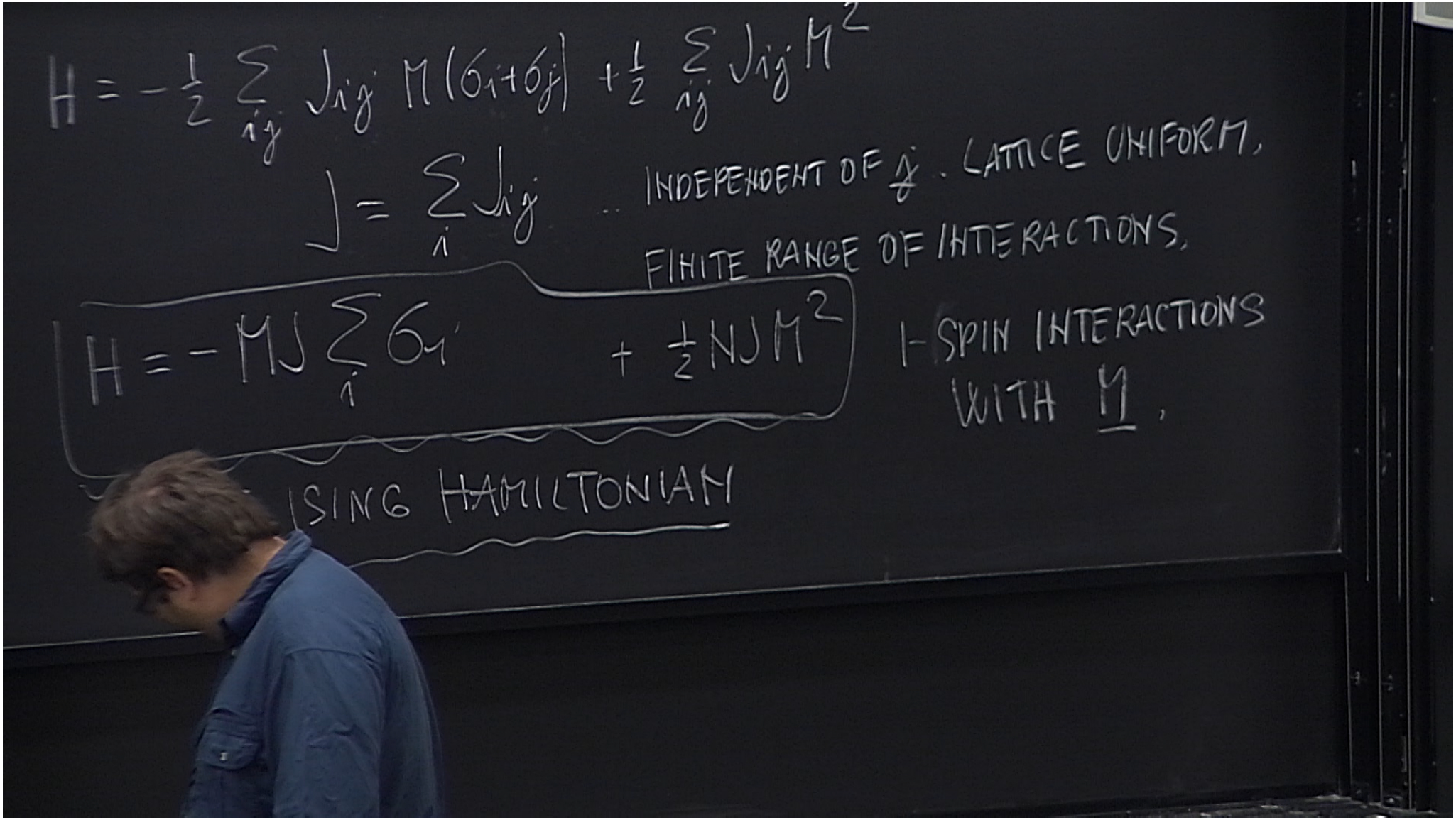
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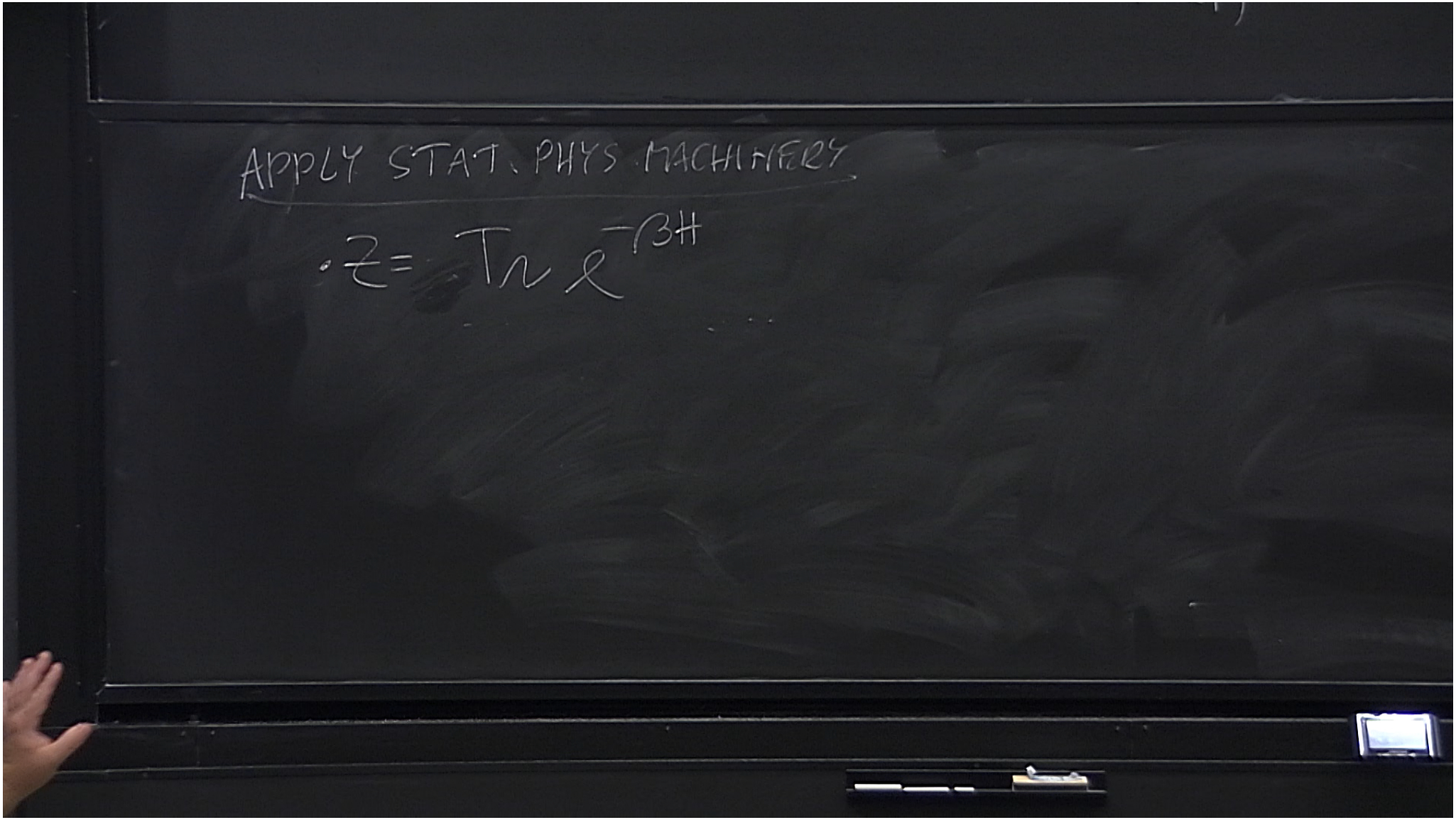
FINITE RANGE OF INTERACTIONS.

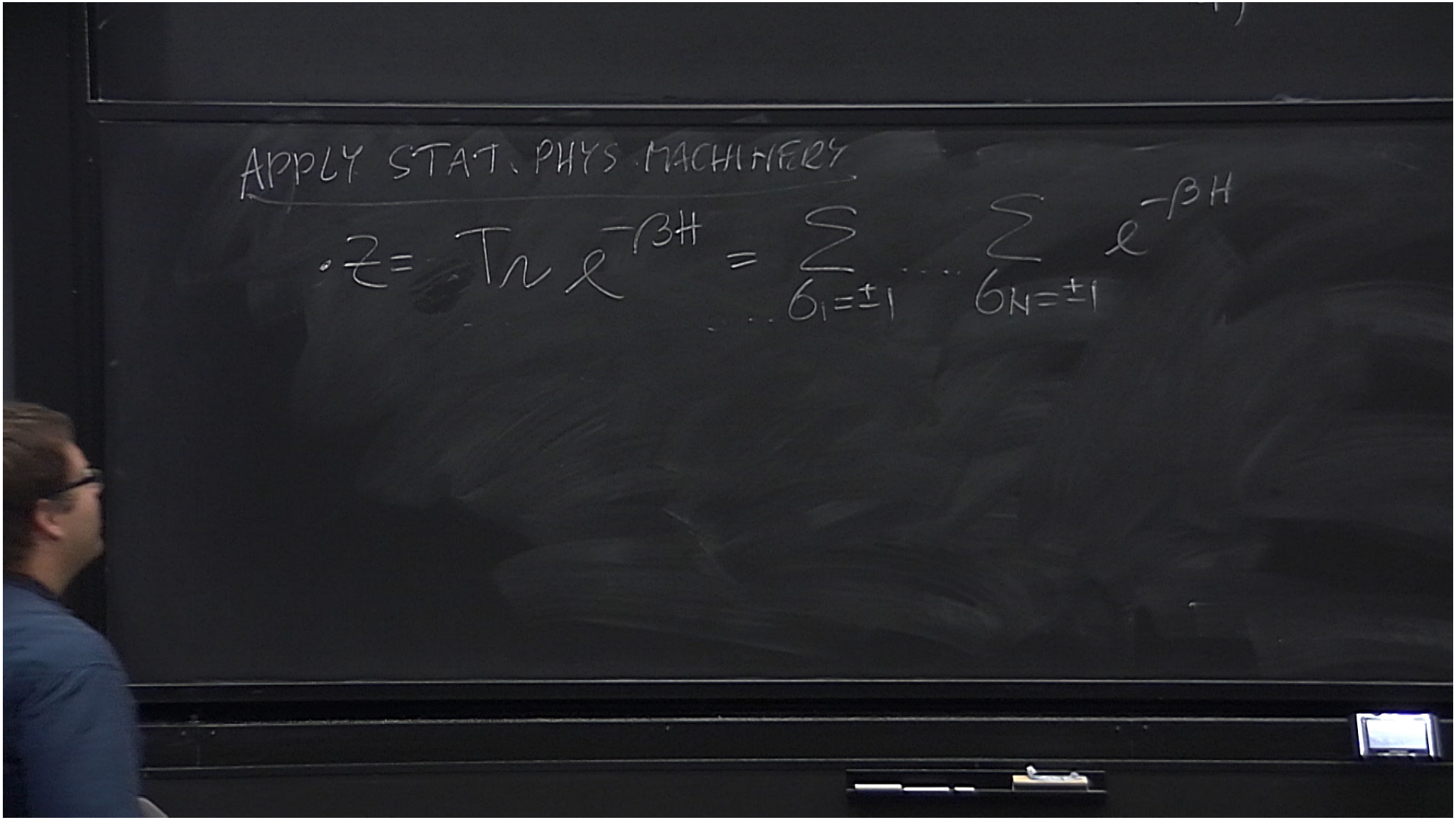
$$H = -MJ\sum_i \sigma_i + \frac{1}{2}NJM^2$$

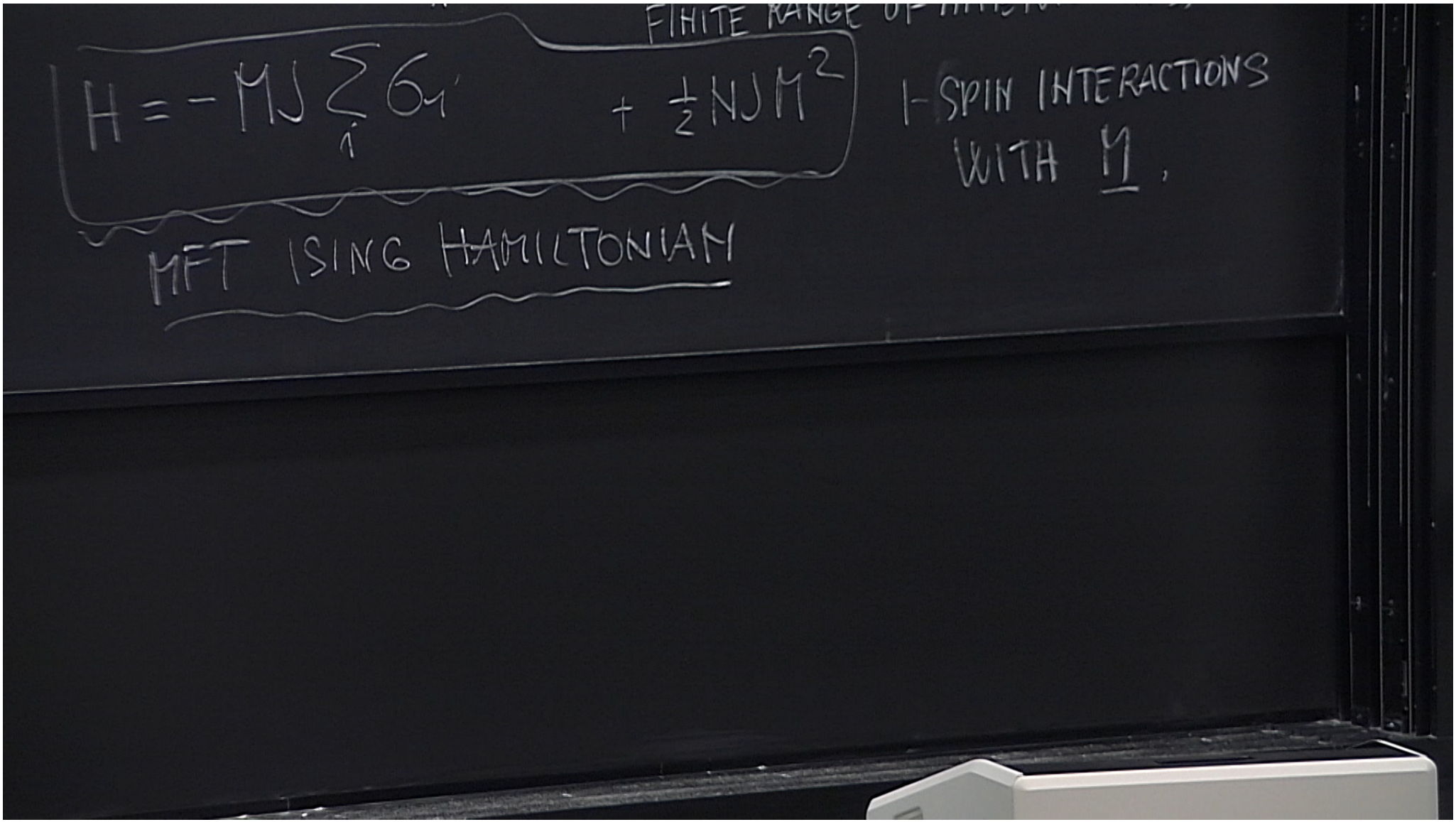
MFT ISING HAMILTONIAN

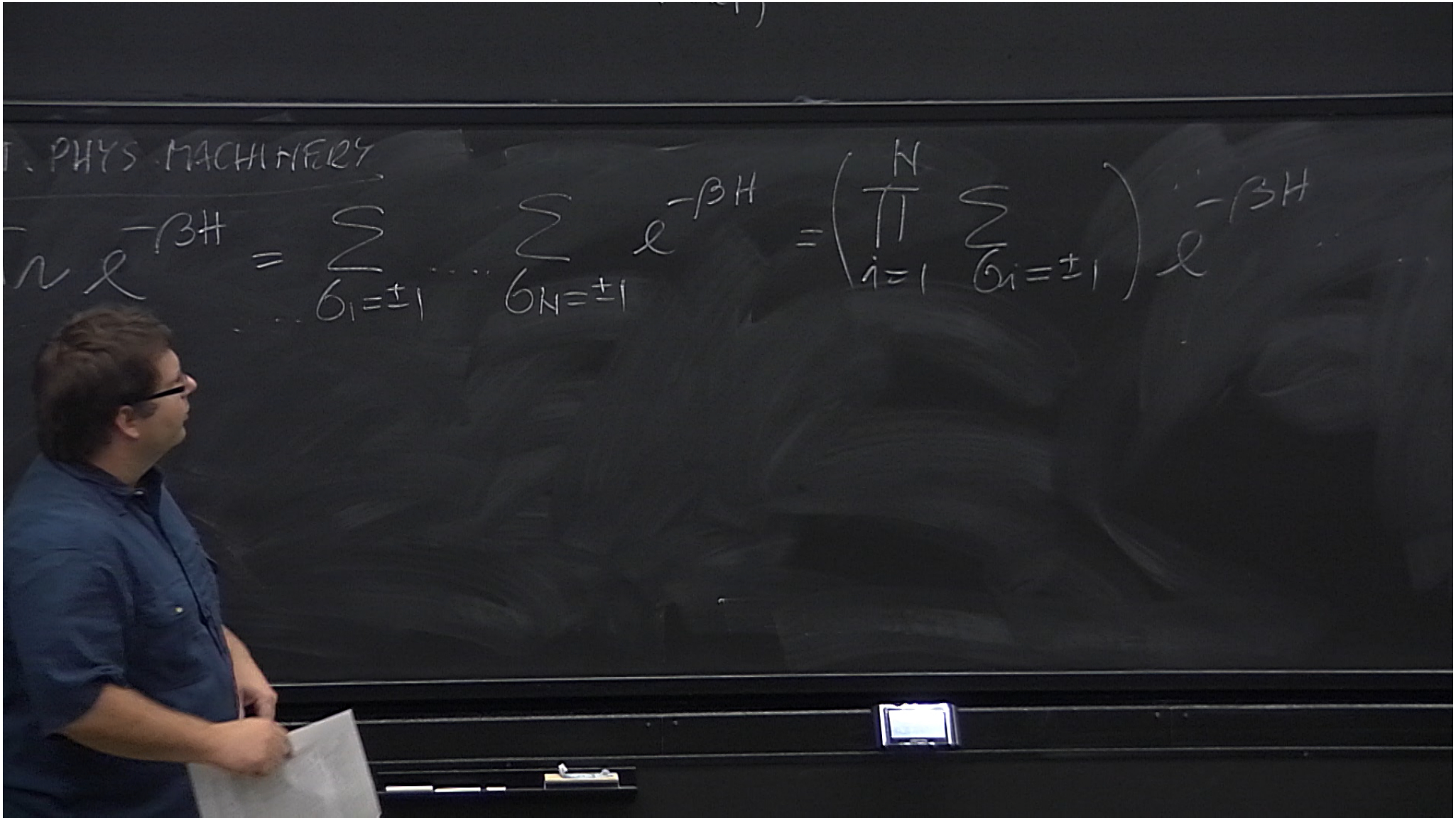


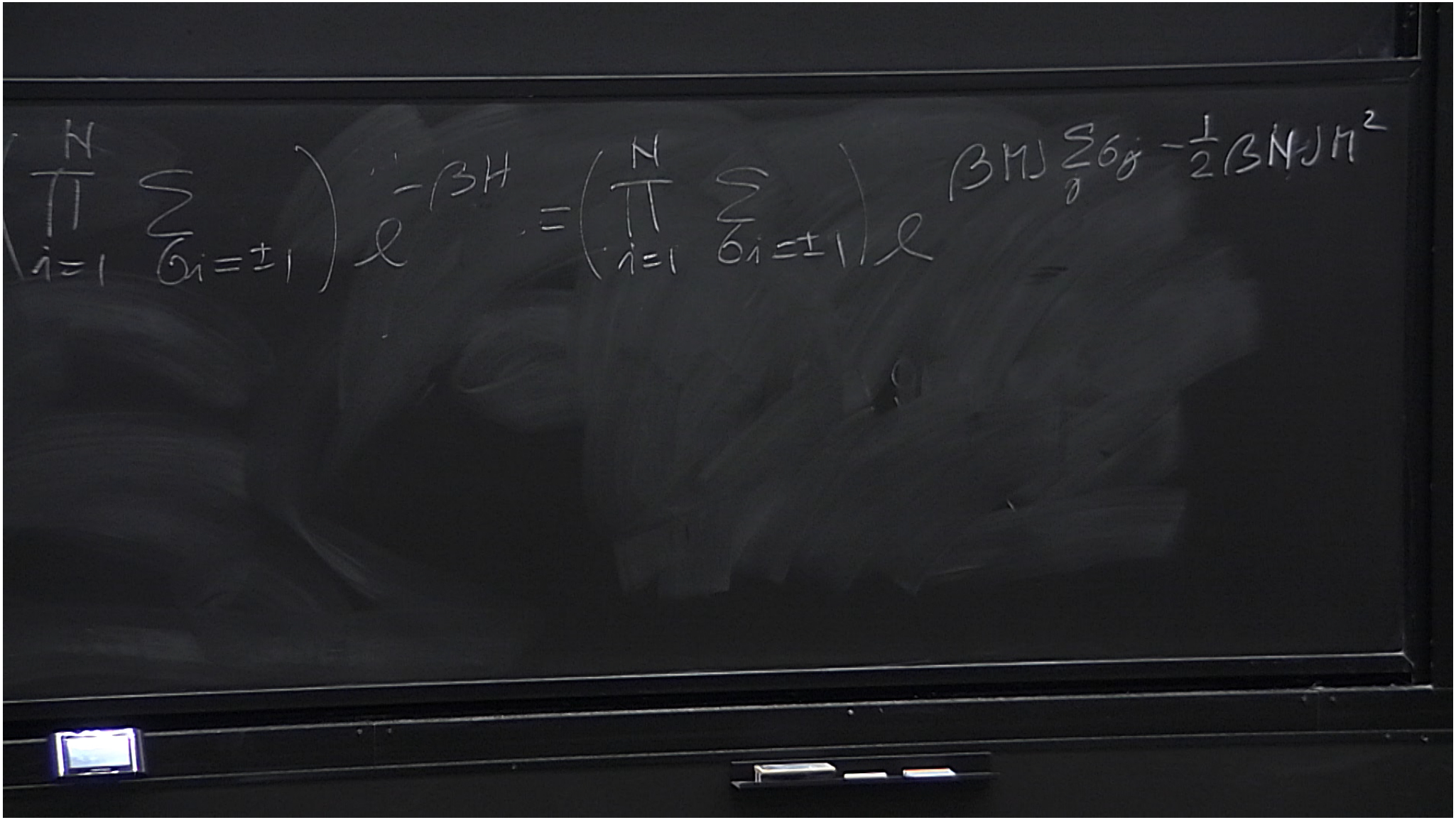

$H = -\frac{1}{2}\sum_{ij} J_{ij} M(\sigma_i + \sigma_j) + \frac{1}{2}\sum_{ij} J_{ij} M^2$
$J = \sum_i J_{ij}$ ... INDEPENDENT OF i. LATTICE UNIFORM.
FINITE RANGE OF INTERACTIONS.
$H = -MJ\sum_i \sigma_i + \frac{1}{2}NJM^2$
1-SPIN INTERACTIONS WITH M.
ISING HAMILTONIAN

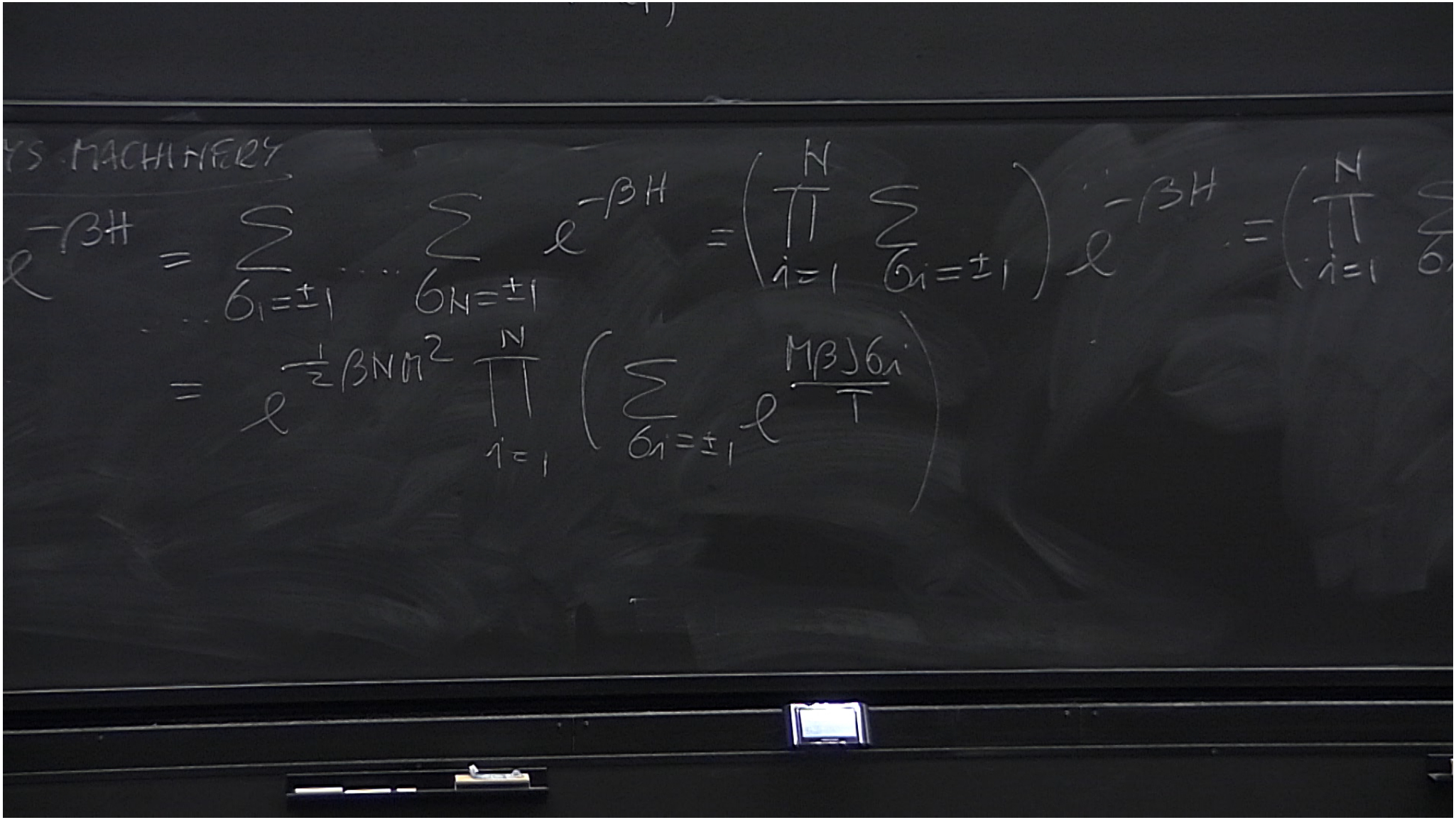

APPLY STAT. PHYS MACHINERY
$Z = \mathrm{Tr}\, e^{-\beta H}$

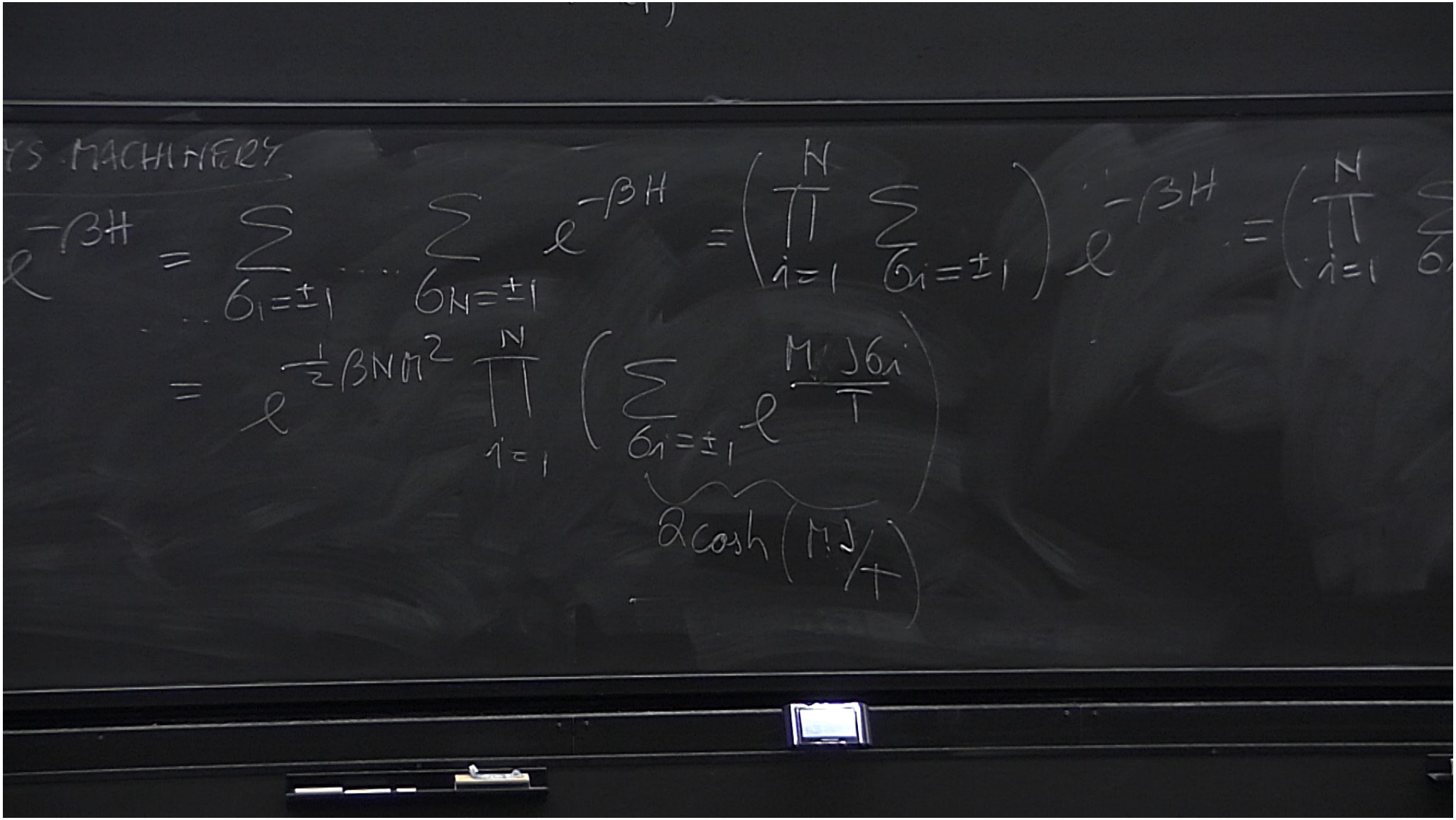

APPLY STAT. PHYS MACHINERY
$Z = \mathrm{Tr}\, e^{-\beta H} = \sum_{\sigma_1=\pm 1} \cdots \sum_{\sigma_N=\pm 1} e^{-\beta H}$

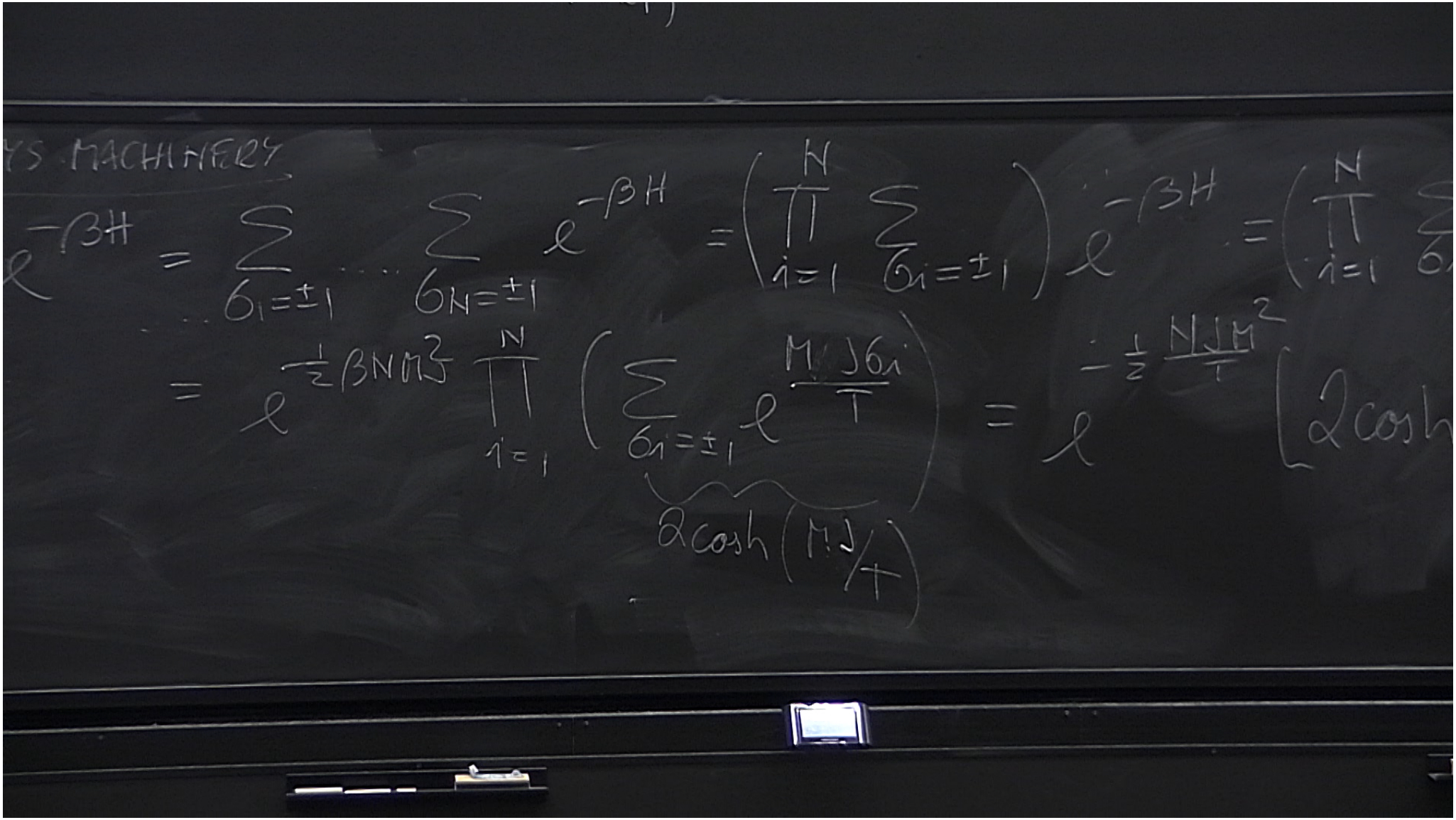

FINITE RANGE
$H = -MJ \sum_i G_i + \frac{1}{2} NJM^2$
MFT ISING HAMILTONIAN
1-SPIN INTERACTIONS
WITH M.

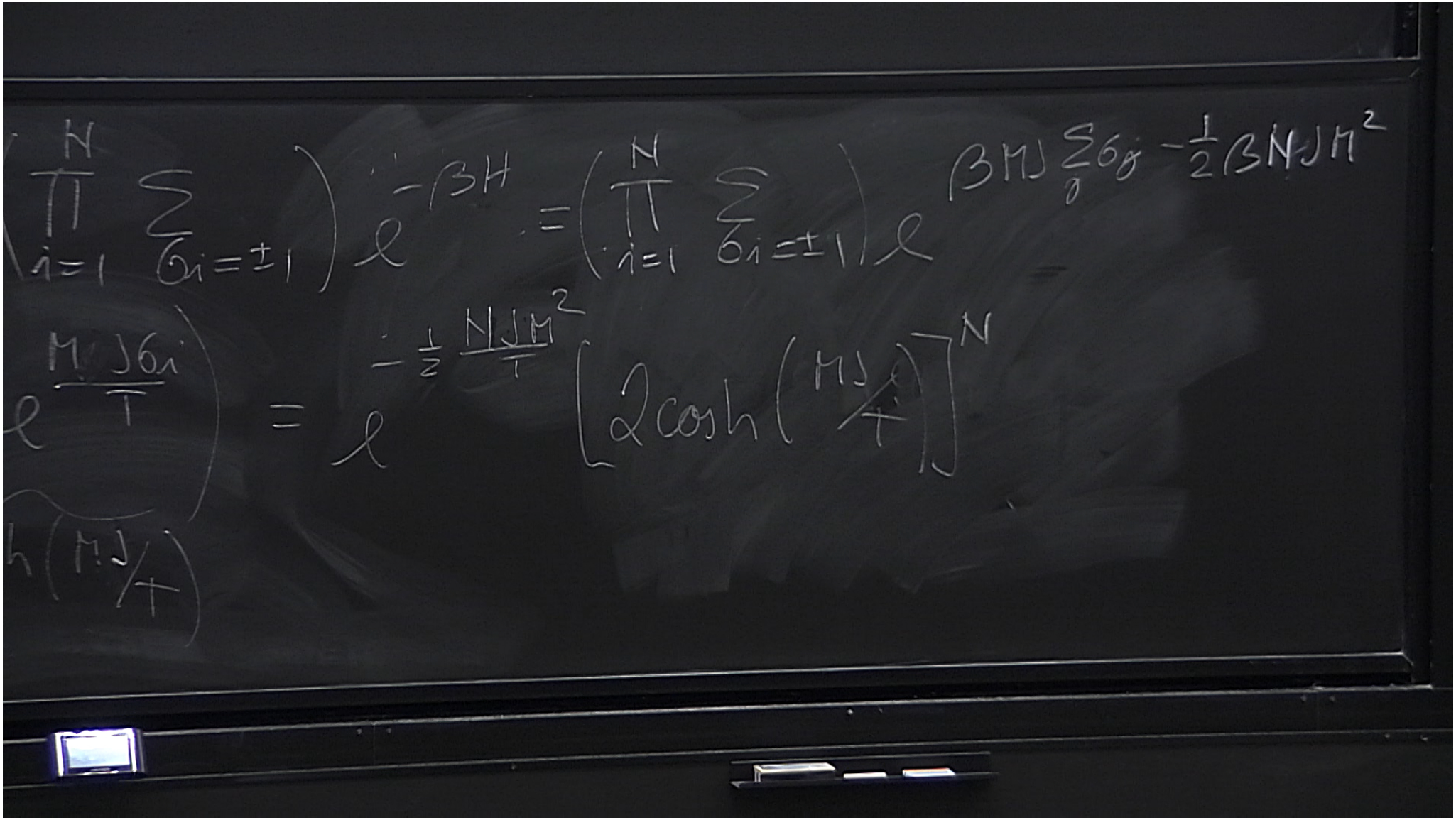

. PHYS MACHINERY
$\mathrm{tr}\, e^{-\beta H} = \sum_{\sigma_1=\pm 1} \cdots \sum_{\sigma_N=\pm 1} e^{-\beta H} = \left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm 1} \right) e^{-\beta H}$

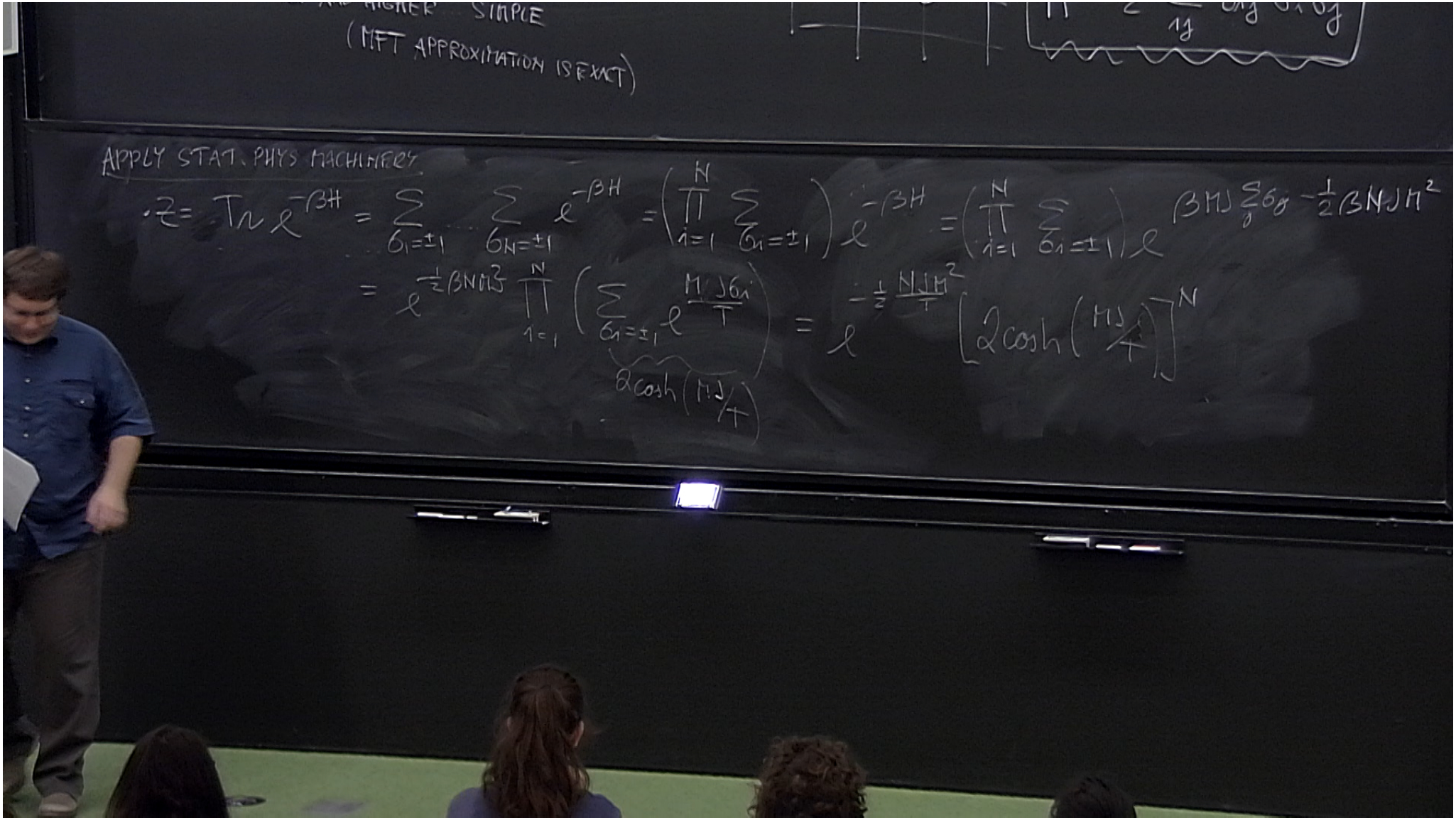

$\left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm 1}\right) e^{-\beta H} = \left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm 1}\right) e^{\beta M J \sum_j \sigma_j - \frac{1}{2}\beta N J M^2}$


S MACHINERY
$e^{-\beta H} = \sum_{\sigma_1=\pm 1} \cdots \sum_{\sigma_N=\pm 1} e^{-\beta H} = \left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm 1} \right) e^{-\beta H}$
$= e^{\frac{1}{2}\beta N M^2} \prod_{i=1}^{N} \left(\sum_{\sigma_i=\pm 1} e^{\frac{M\beta J \sigma_i}{T}} \right)$

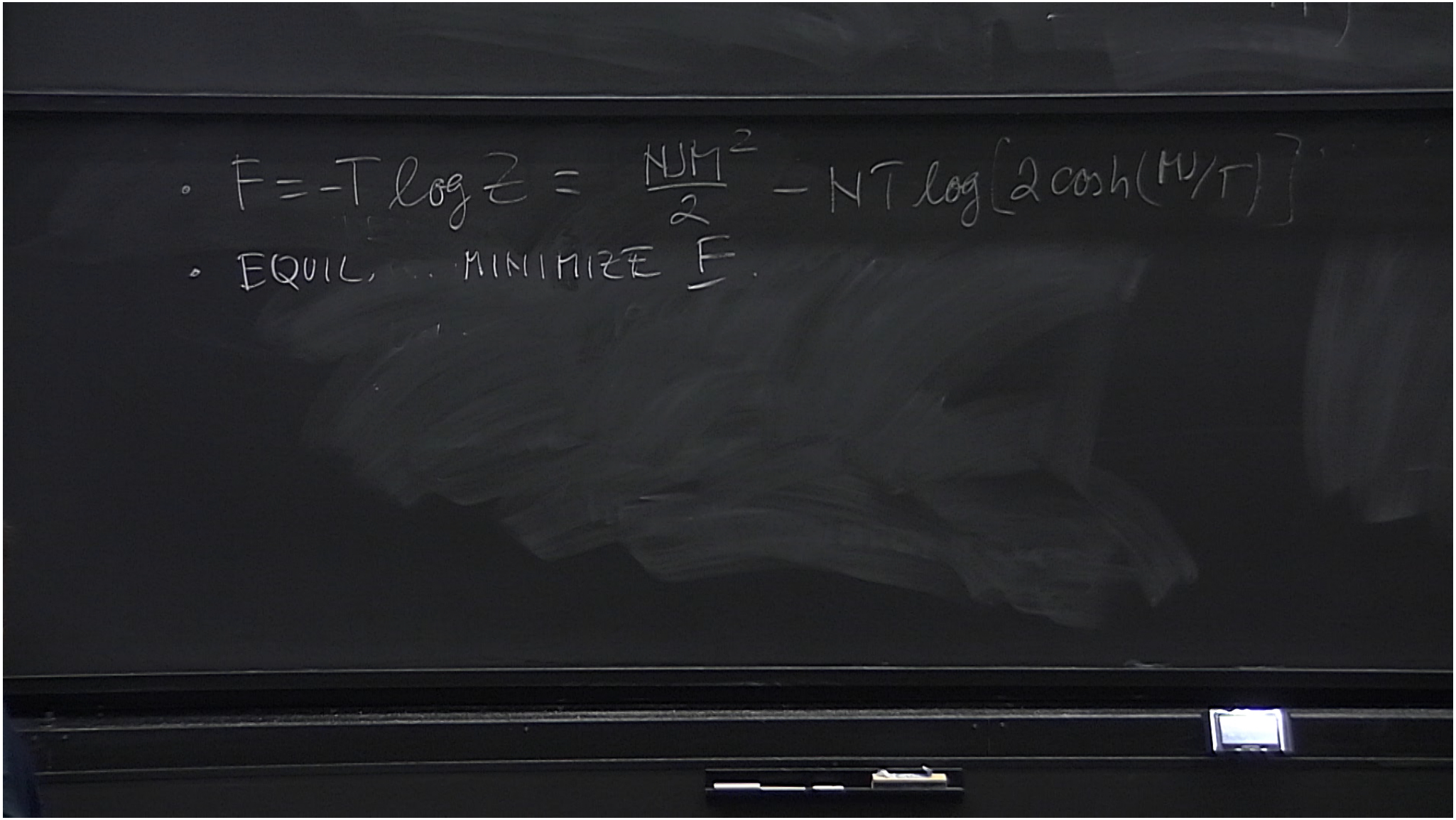

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$2\cosh\left(\frac{MJ}{T}\right)$

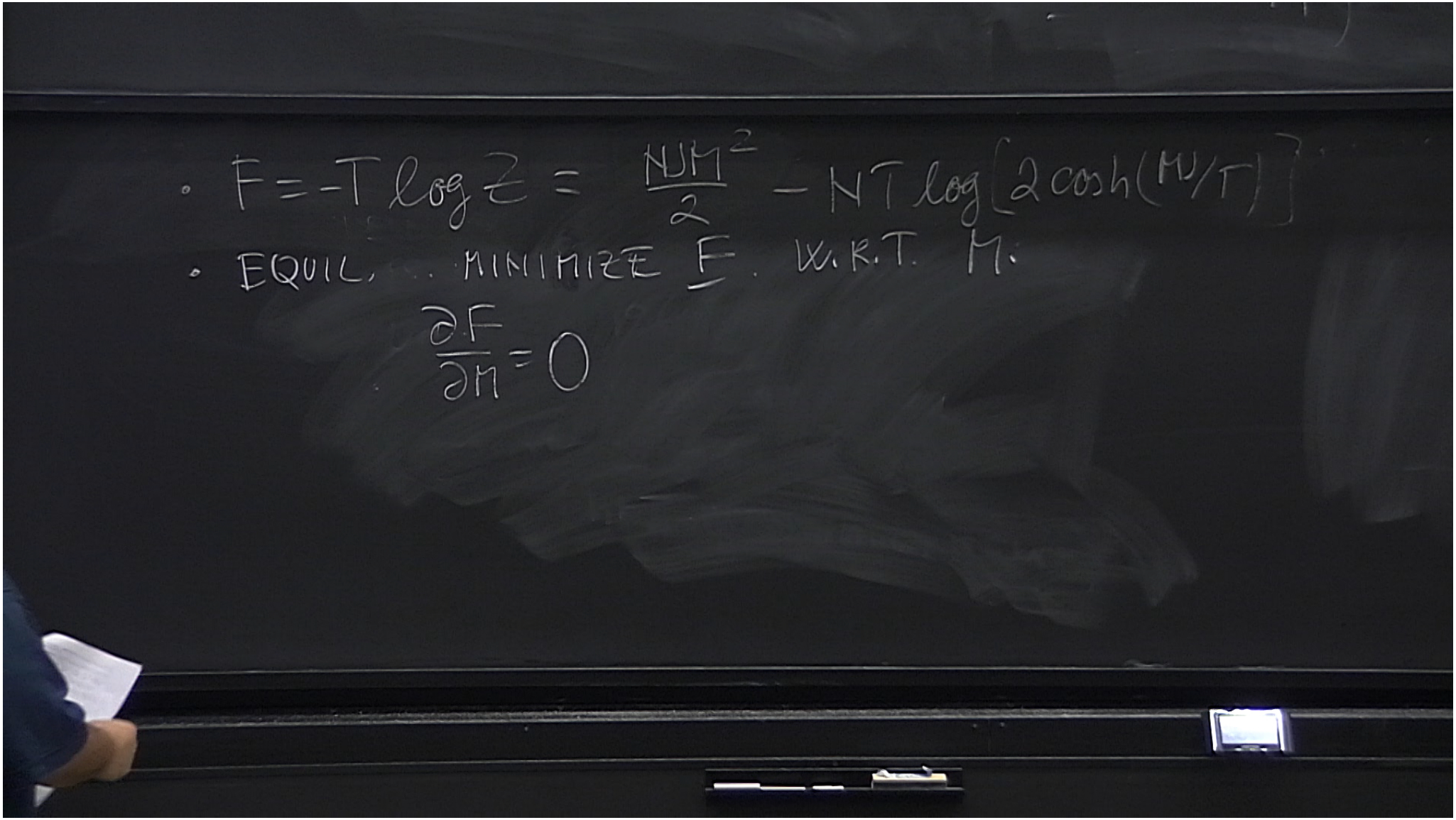

S MACHINERY
$e^{-\beta H} = \sum_{\sigma_1=\pm 1} \cdots \sum_{\sigma_N=\pm 1} e^{-\beta H} = \left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm 1} \right) e^{-\beta H}$
$= e^{\frac{1}{2}\beta N M^2 J} \prod_{i=1}^{N} \left(\sum_{\sigma_i=\pm 1} e^{\frac{M J \sigma_i}{T}} \right) = e^{-\frac{1}{2}\frac{N J M^2}{T}} \left[2\cosh \right.$
$2\cosh(MJ/T)$


$\left(\prod_{i=1}^{N}\sum_{\sigma_i=\pm 1}\right) e^{-\beta H} = \left(\prod_{i=1}^{N}\sum_{\sigma_i=\pm 1}\right) e^{\beta MJ\sum_i \sigma_i - \frac{1}{2}\beta N J M^2}$
$e^{\frac{M J \sigma_i}{T}}\Big) = e^{-\frac{1}{2}\frac{NJM^2}{T}} \left[2\cosh\left(\frac{MJ}{T}\right)\right]^N$
$h\left(\frac{MJ}{T}\right)$


SIMPLE
(MFT APPROXIMATION IS EXACT)
APPLY STAT. PHYS MACHINERY
$Z = \mathrm{Tr}\, e^{-\beta H} = \sum_{\sigma_1=\pm1} \cdots \sum_{\sigma_N=\pm1} e^{-\beta H} = \left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm1}\right) e^{-\beta H} = \left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm1}\right) e^{\beta M J \sum_j \sigma_j - \frac{1}{2}\beta N J M^2}$
$= e^{\frac{1}{2}\beta N J M^2} \prod_{i=1}^{N} \left(\sum_{\sigma_i=\pm1} e^{\frac{M J \sigma_i}{T}} \right) = e^{-\frac{1}{2}\frac{N J M^2}{T}} \left[2\cosh\left(\frac{MJ}{T}\right) \right]^N$
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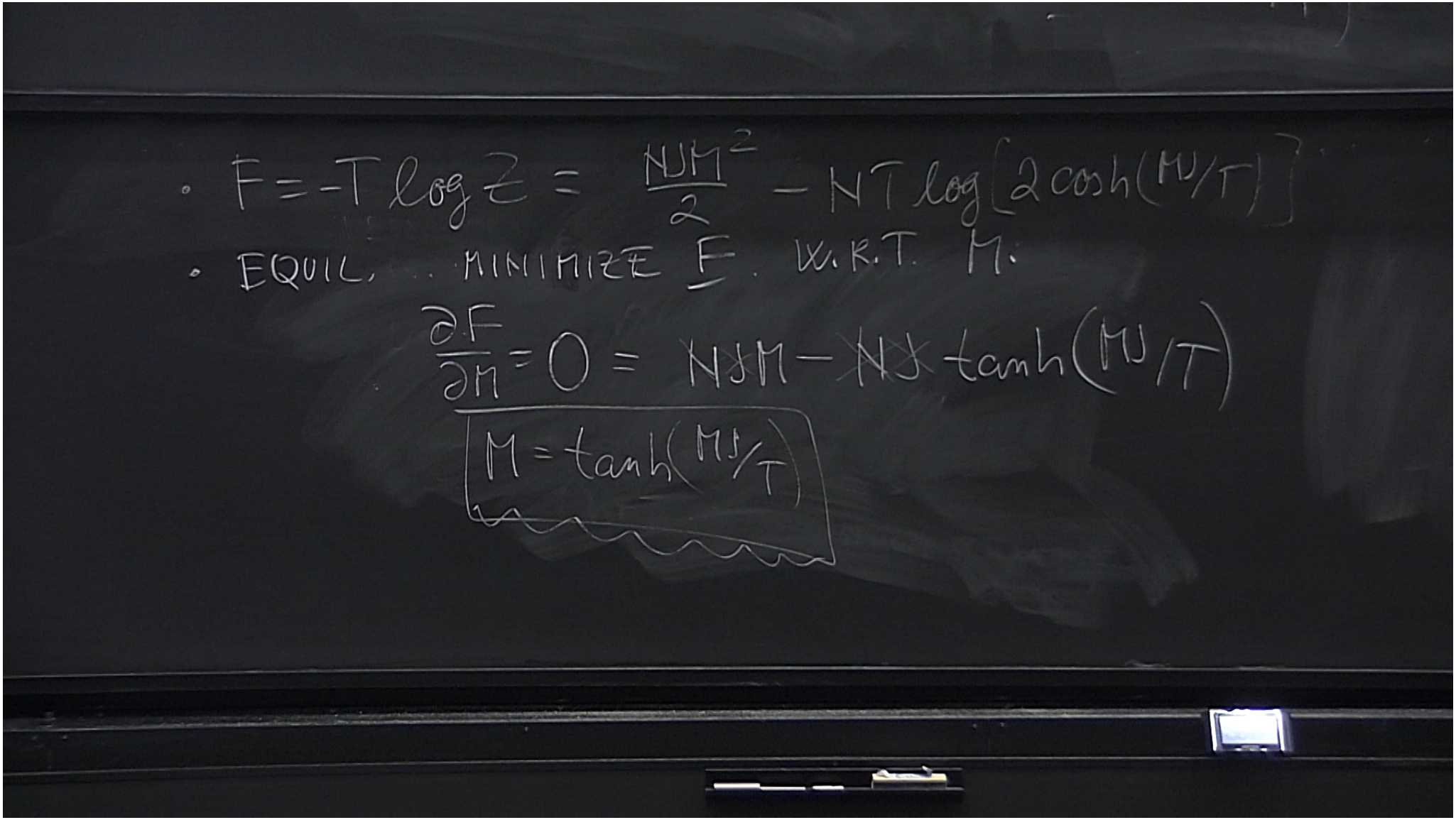
$$F = -T \log Z = \frac{NJM^2}{2} - NT \log\left[2\cosh(MJ/T)\right]$$

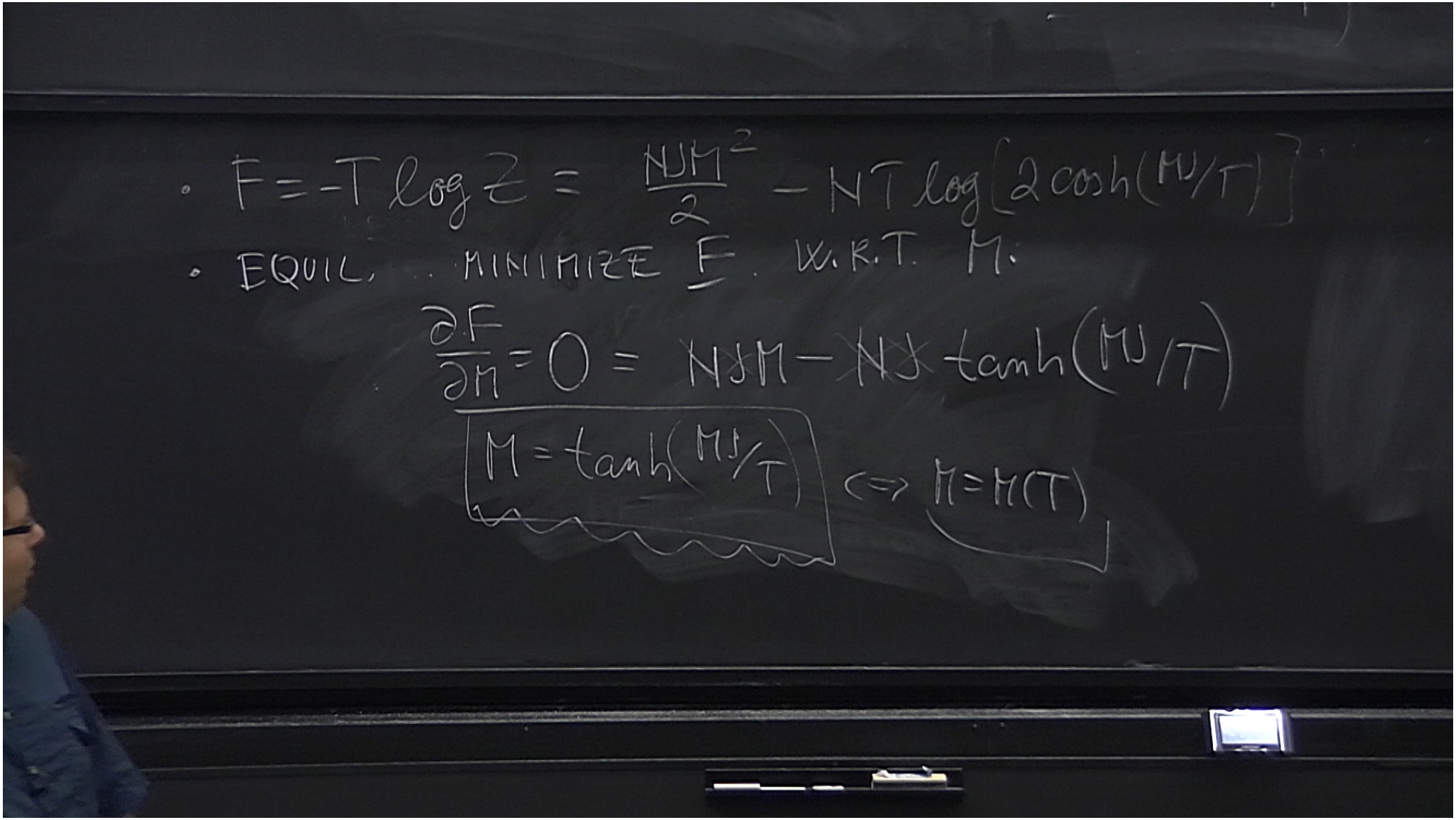

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• EQUIL. ... MINIMIZE F.


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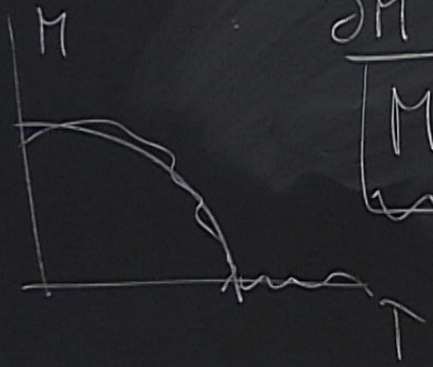

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$M = \tanh(MJ/T)$

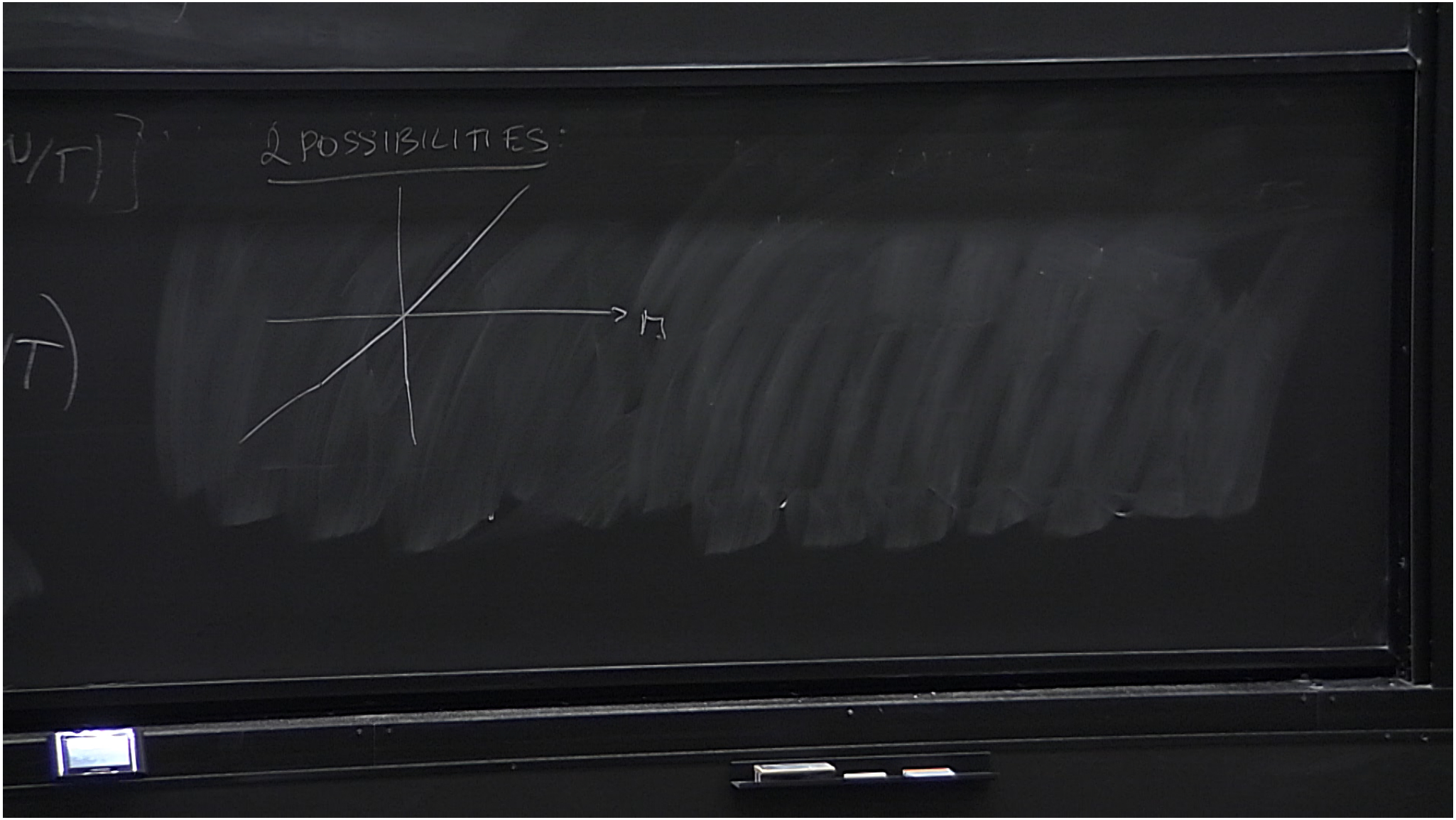

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$\frac{\partial F}{\partial M} = 0 = NJM - NJ \tanh(MJ/T)$
$M = \tanh(MJ/T) \Leftrightarrow M = M(T)$

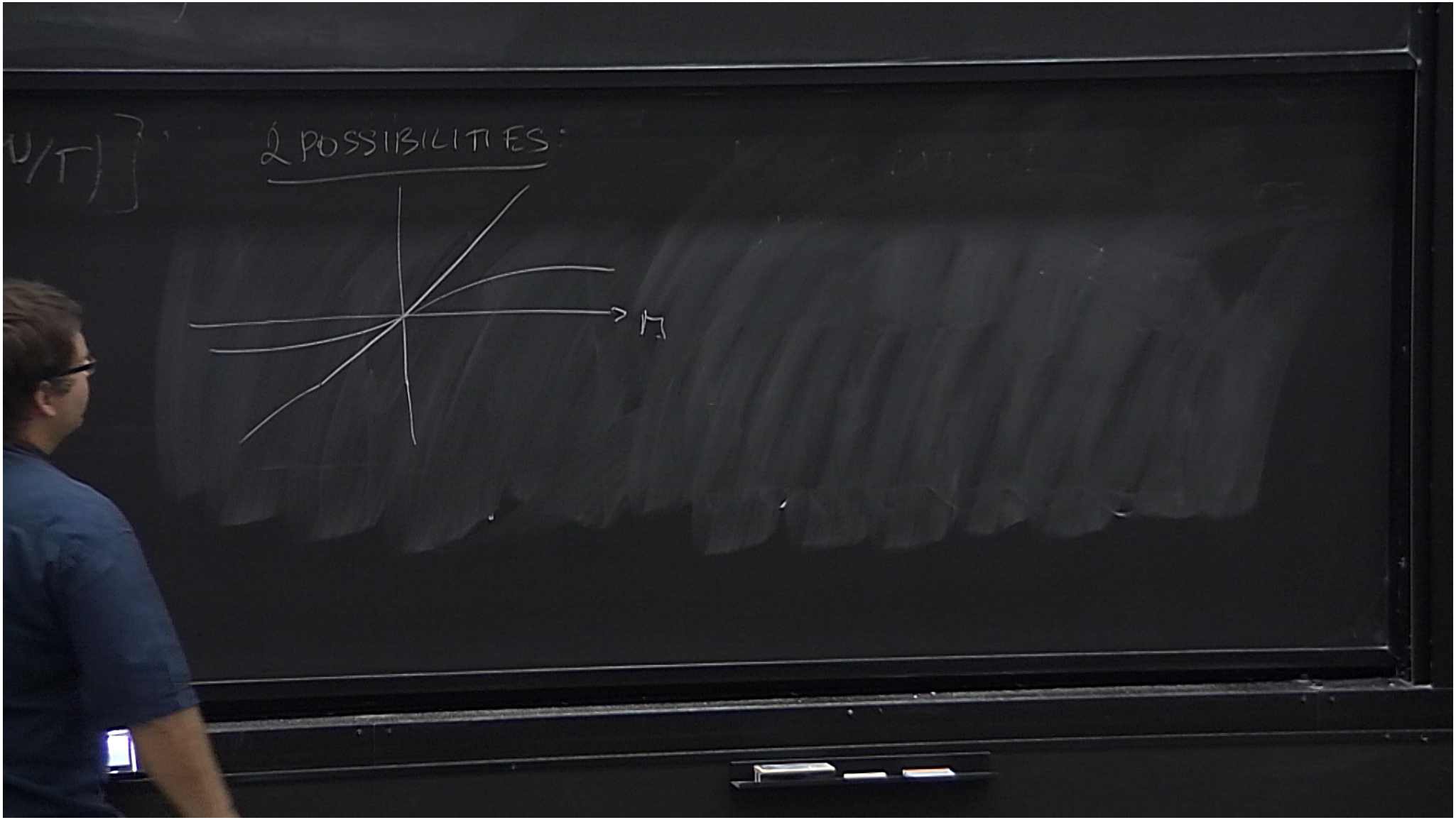
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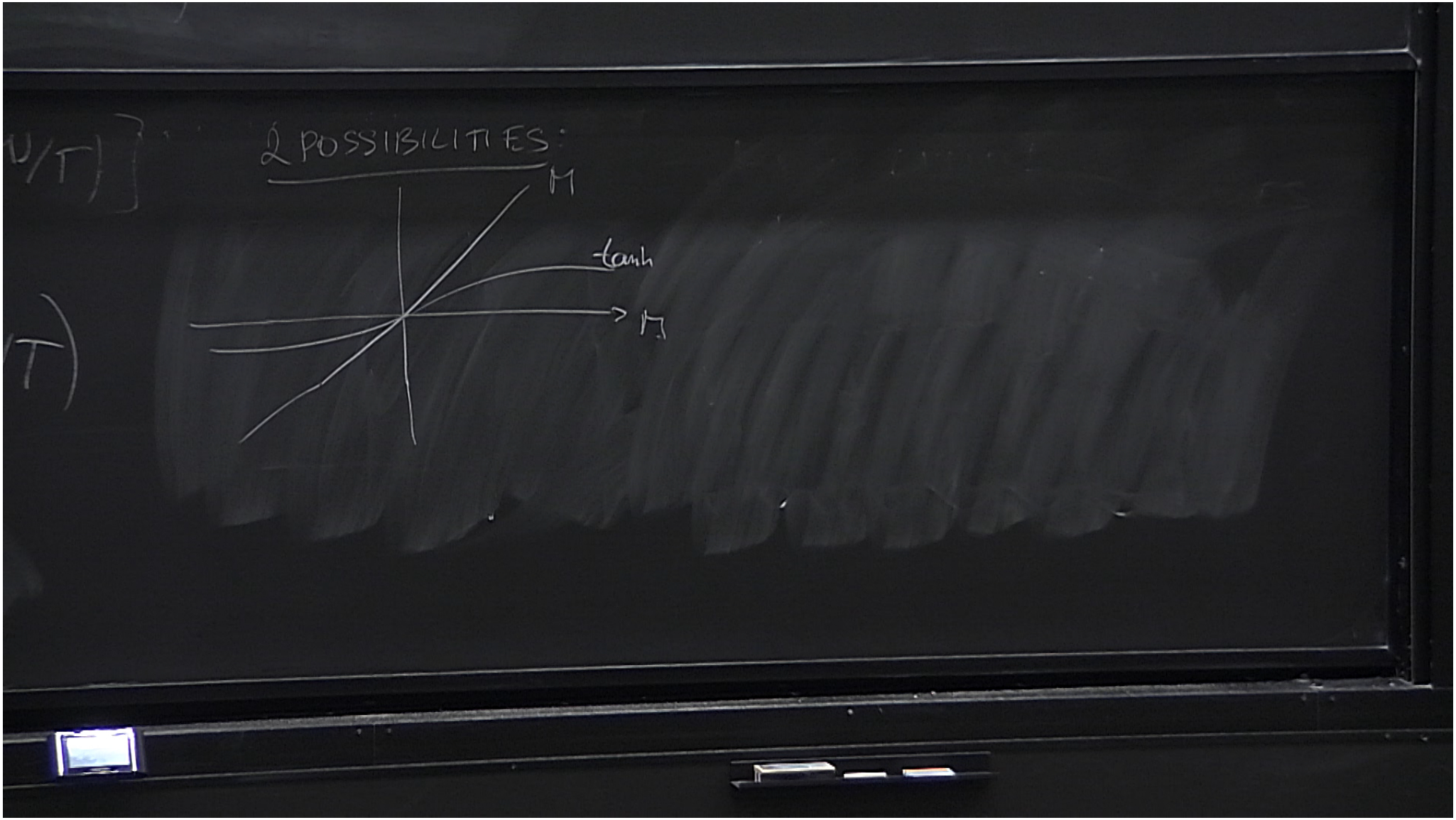
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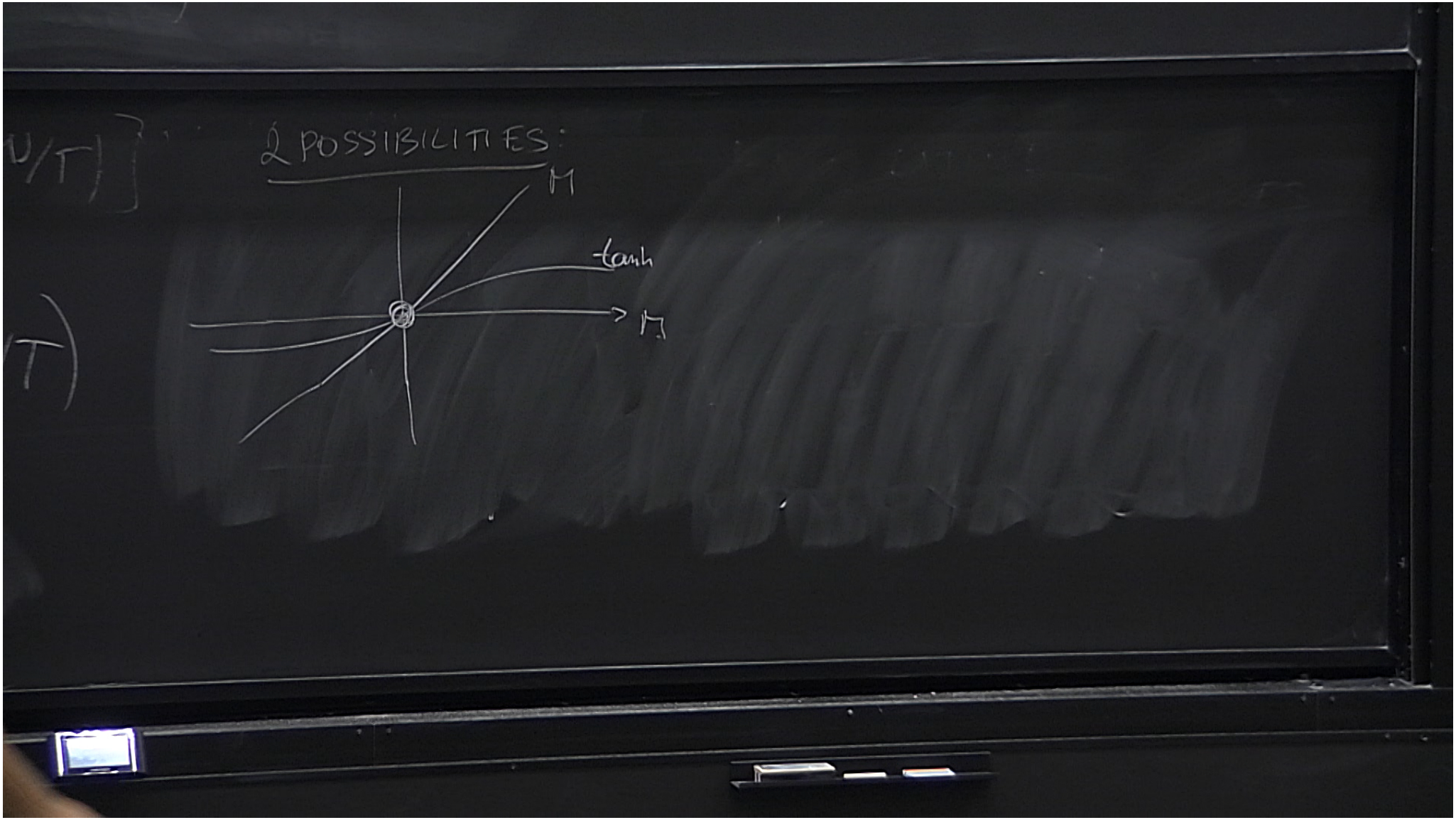
$$\boxed{M = \tanh(MJ/T)} \iff M = M(T)$$

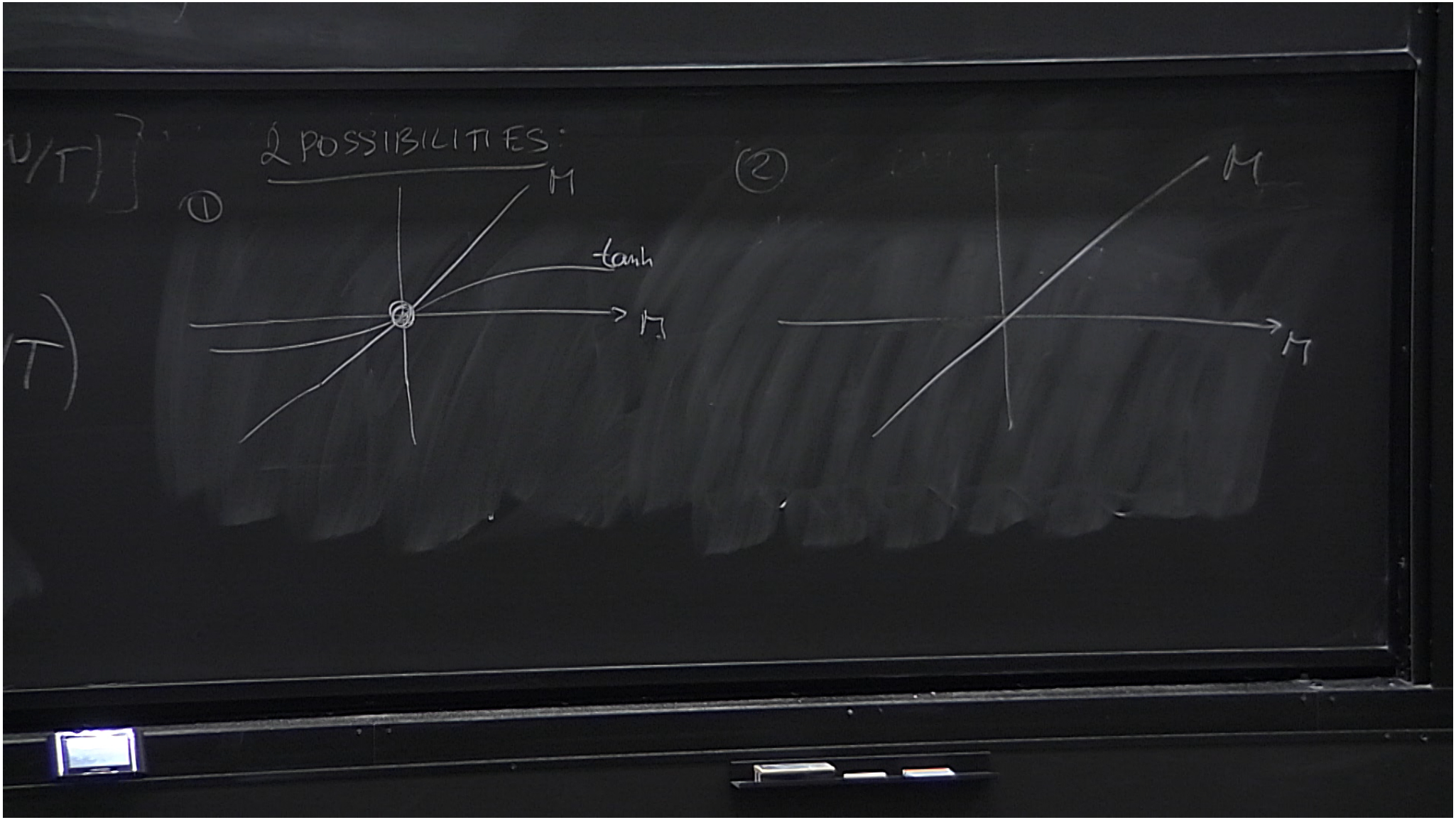


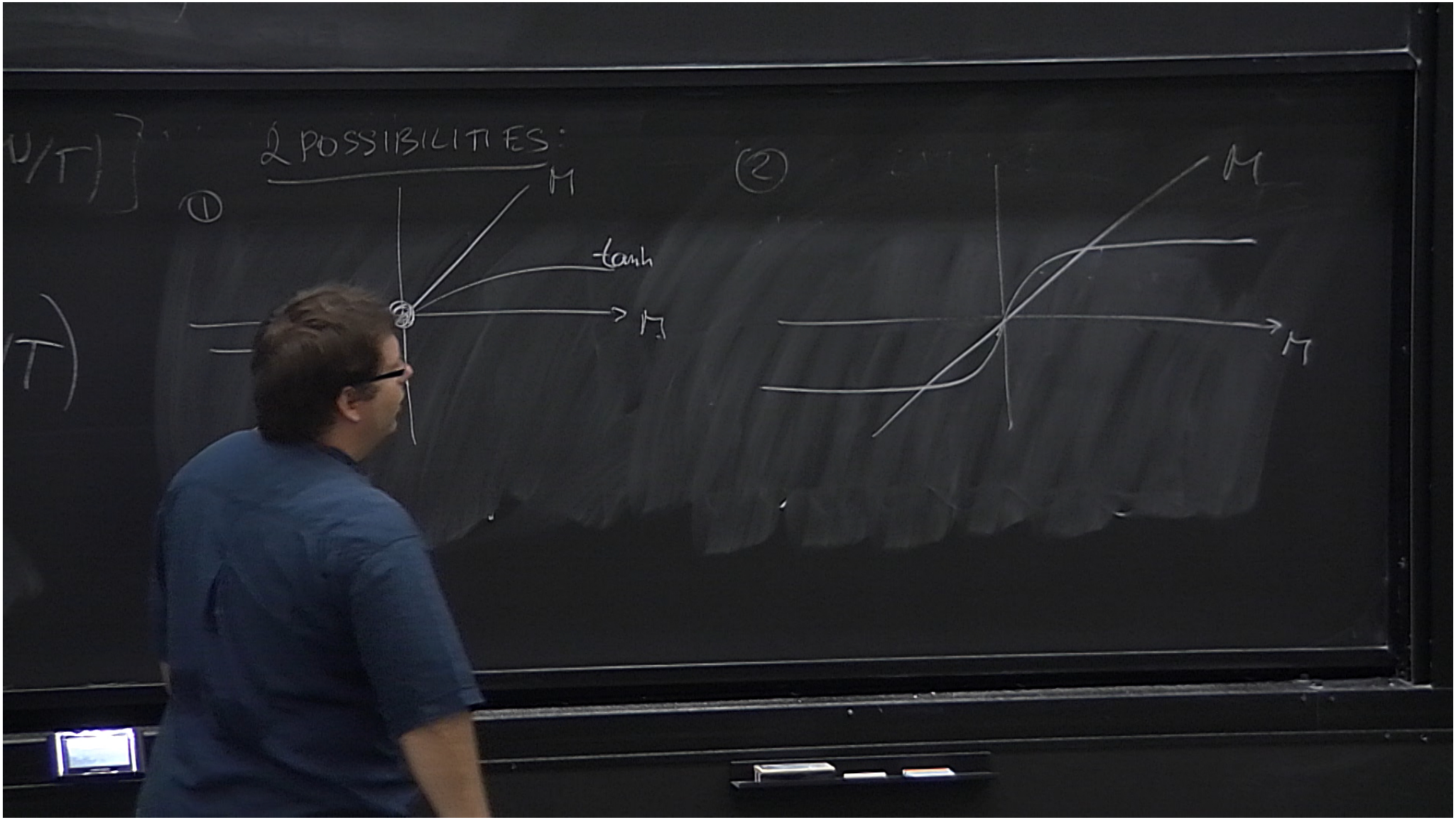

2 POSSIBILITIES:

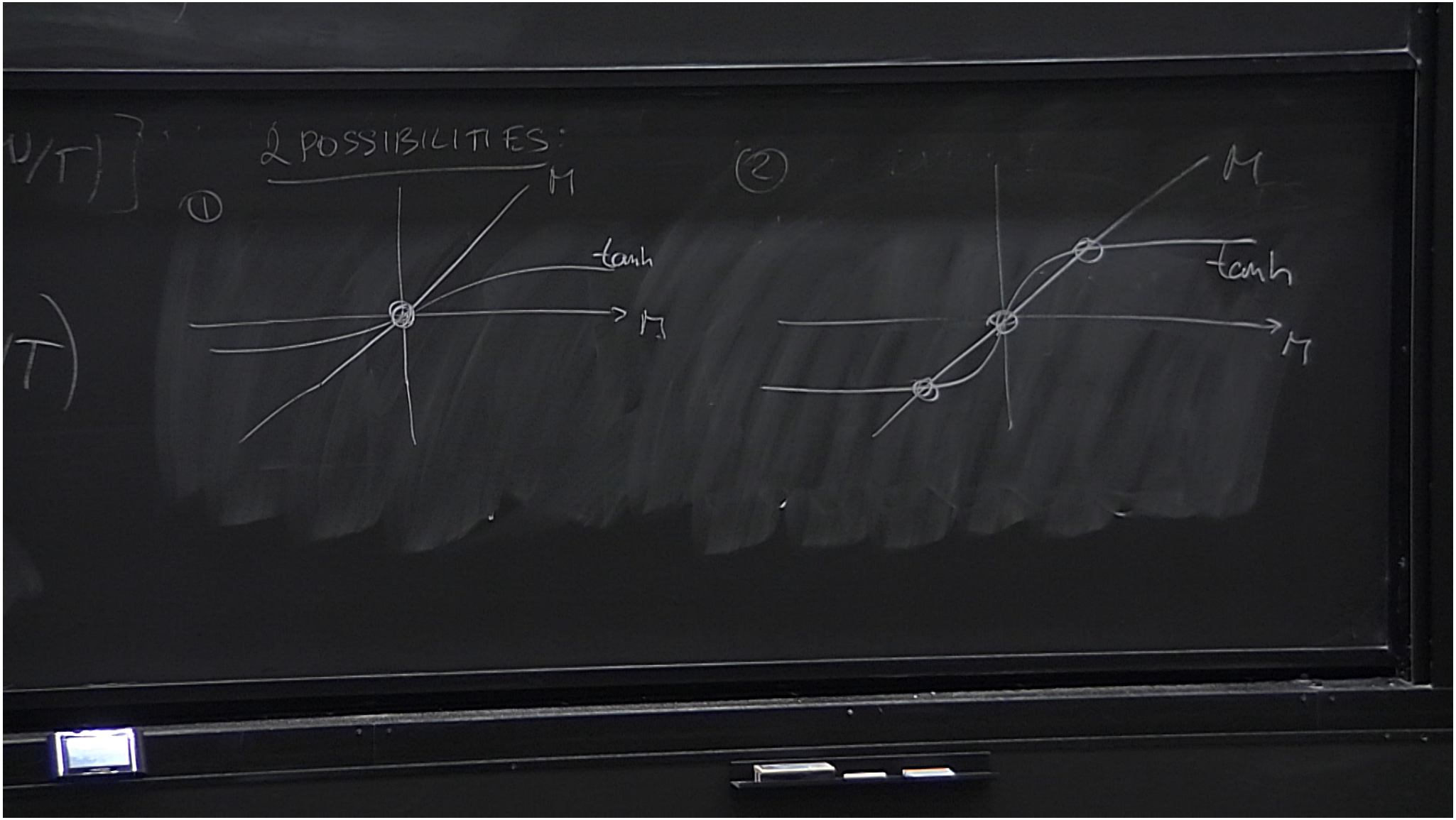

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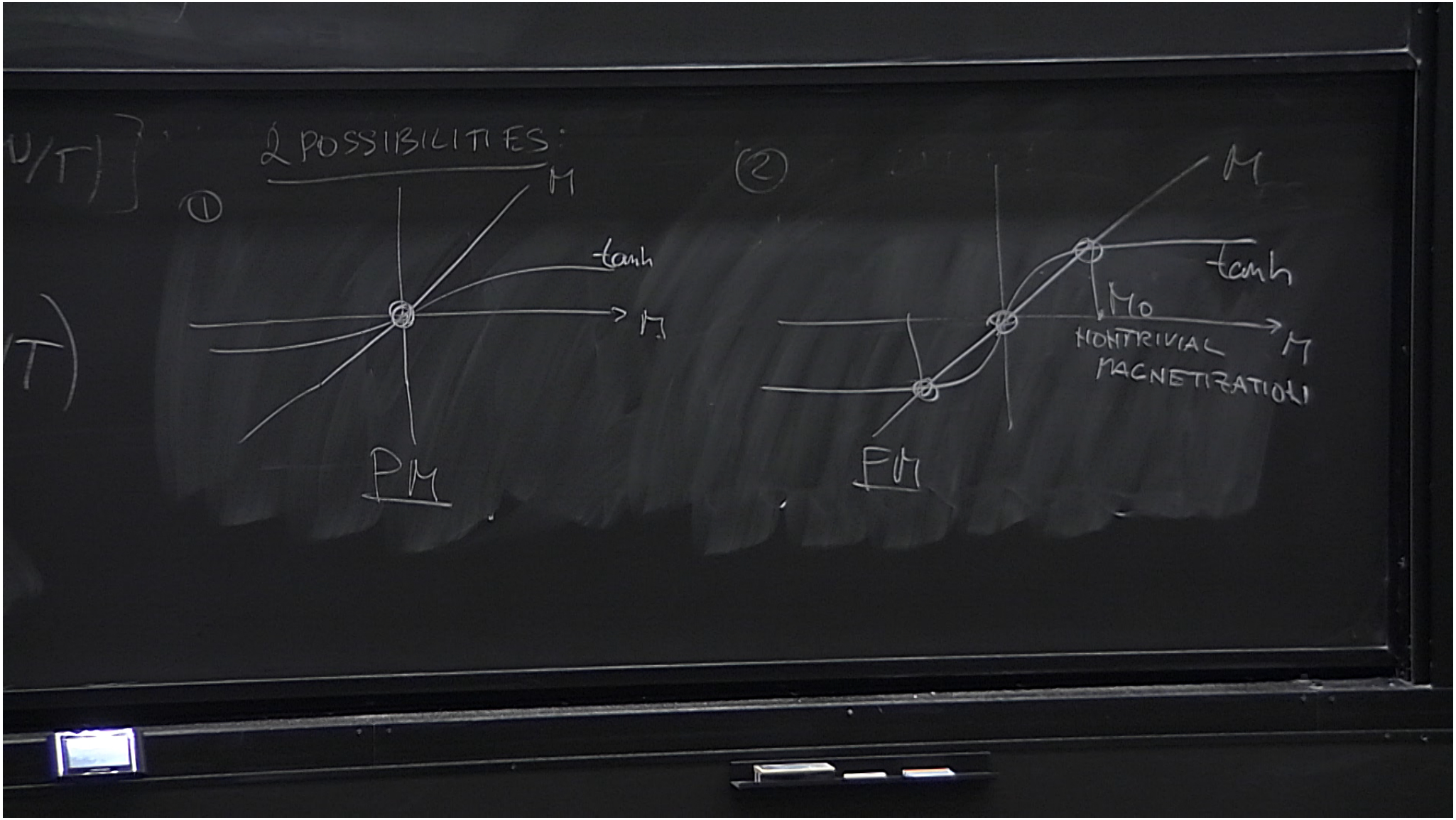

2 POSSIBILITIES:
M
tanh
h


2 POSSIBILITIES:
M
tanh
M


2 POSSIBILITIES:
①
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②
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2 POSSIBILITIES:
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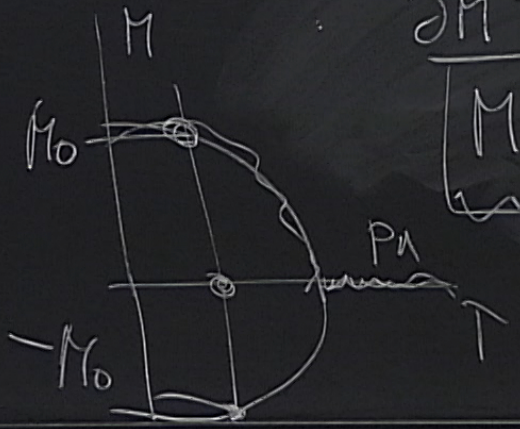

2 POSSIBILITIES:
①
M
tanh
M
②
M
tanh
M

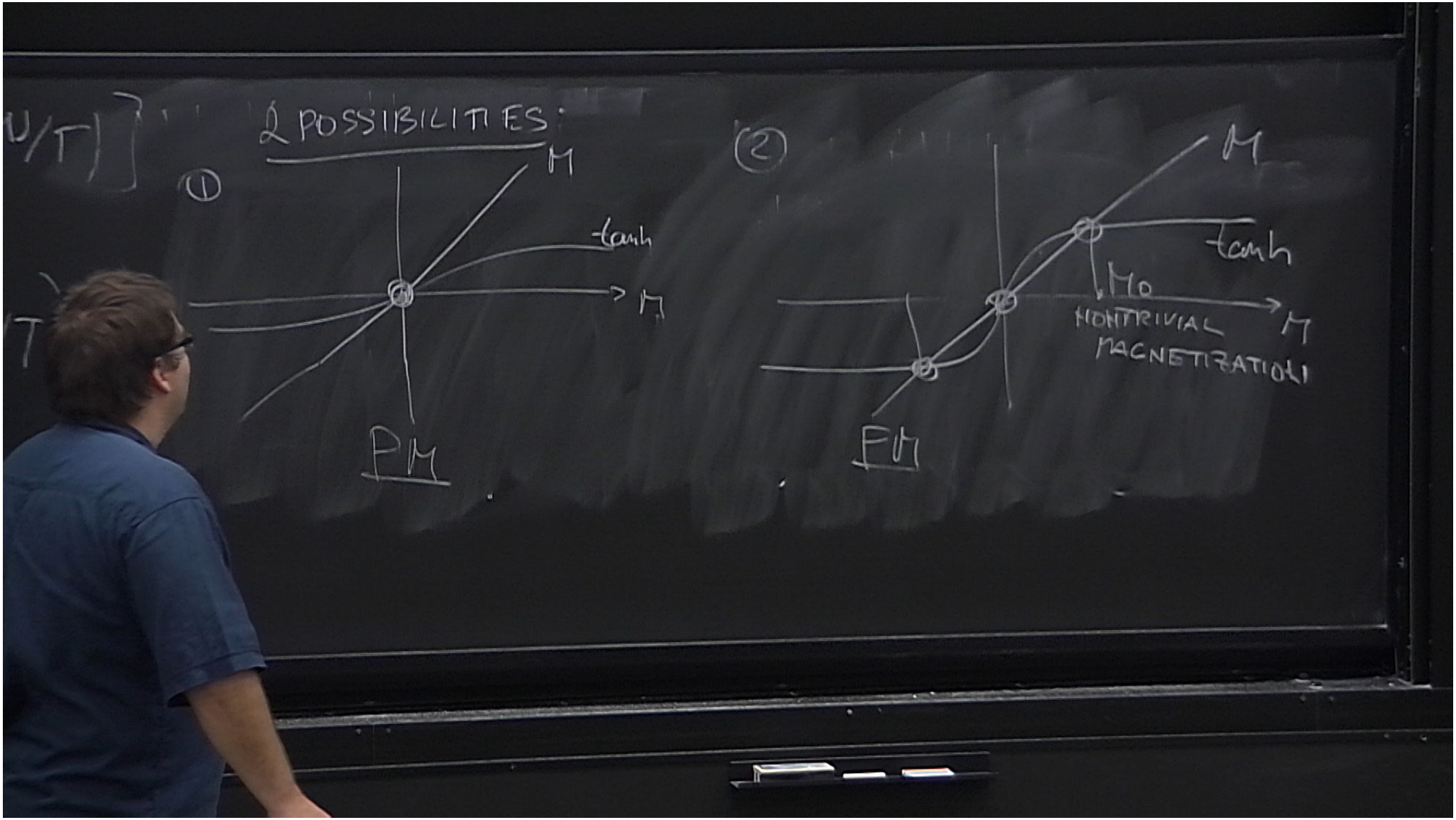

2 POSSIBILITIES:
①
M
tanh
m
PM
②
M
tanh
m_0
NONTRIVIAL MAGNETIZATION
m
PM

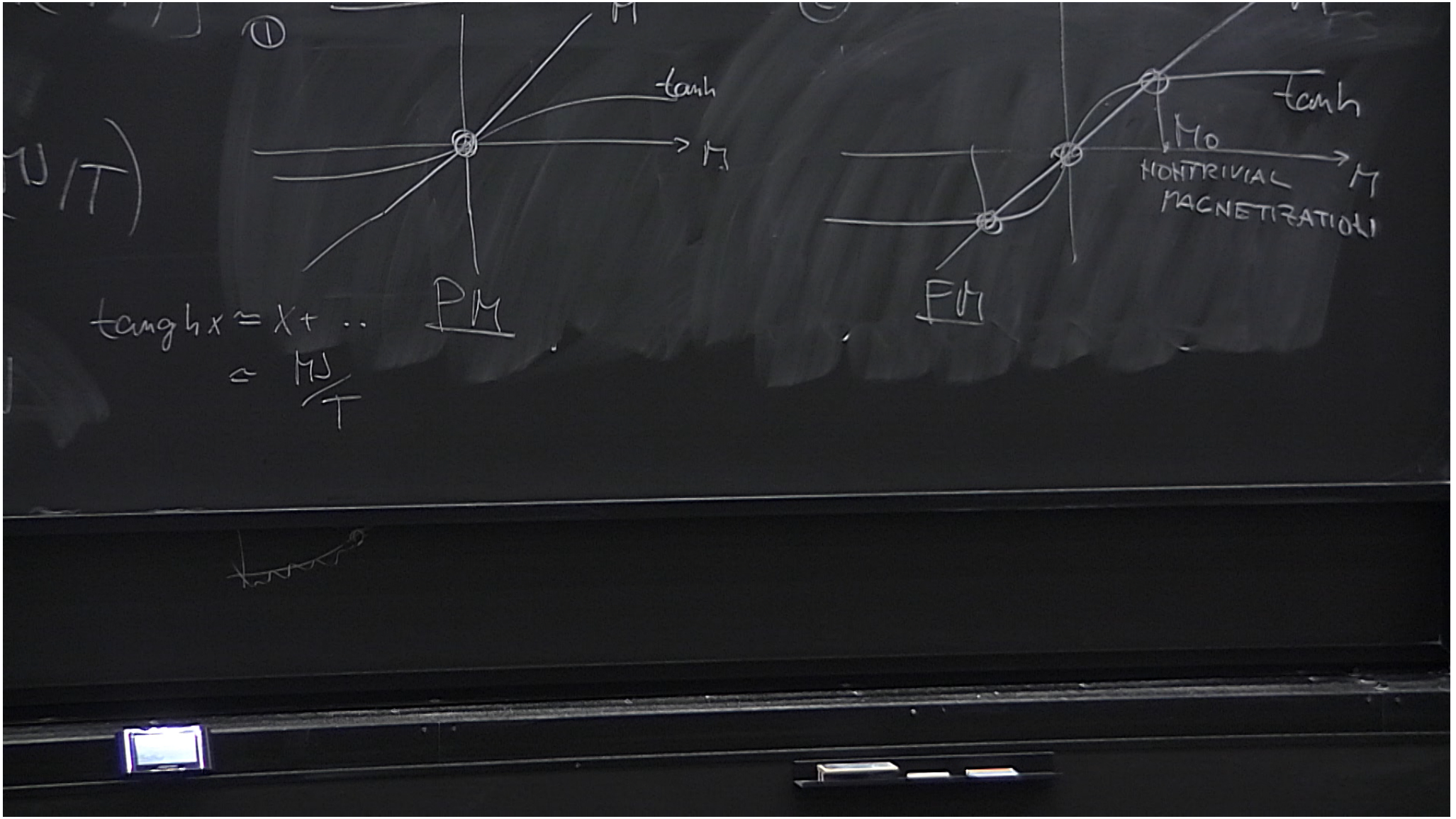
- $F = -T \log Z = \frac{NJM^2}{2} - NT \log\left[2\cosh(MJ/T)\right]$
- EQUIL. ... MINIMIZE F W.R.T. M.

$$\frac{\partial F}{\partial M} = 0 = NJM - NJ \tanh(MJ/T)$$

$$\boxed{M = \tanh(MJ/T)} \iff M = M(T)$$



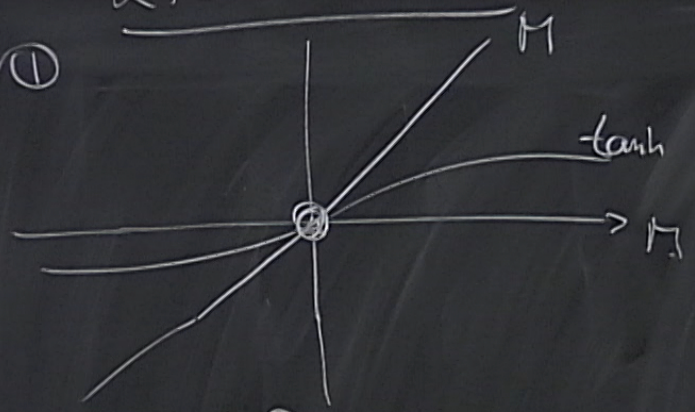

2 POSSIBILITIES:
(1)
M
tanh
PM
(2)
M
tanh
Mo
NONTRIVIAL
MAGNETIZATION
M
FM


tanh
M
PM
tanh
M0
NONTRIVIAL
MAGNETIZATION
M
FM

$\tanh(MJ/T)$

(MJ/T)

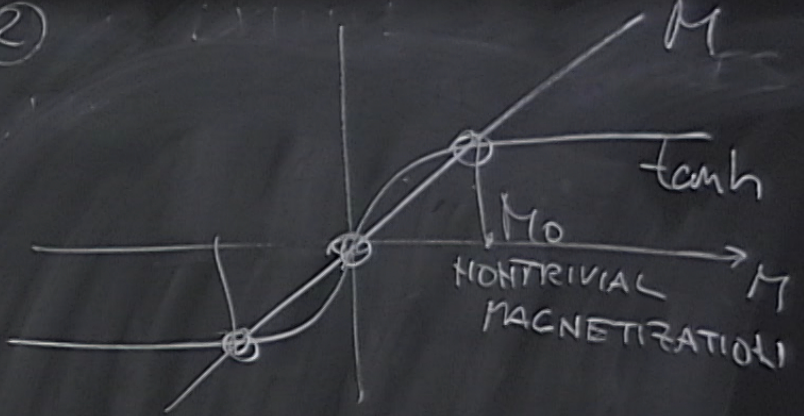
2 POSSIBILITIES:

① 

PM $J<T$

$\tanh x \simeq x + \ldots$

$\simeq \frac{MJ}{T}$

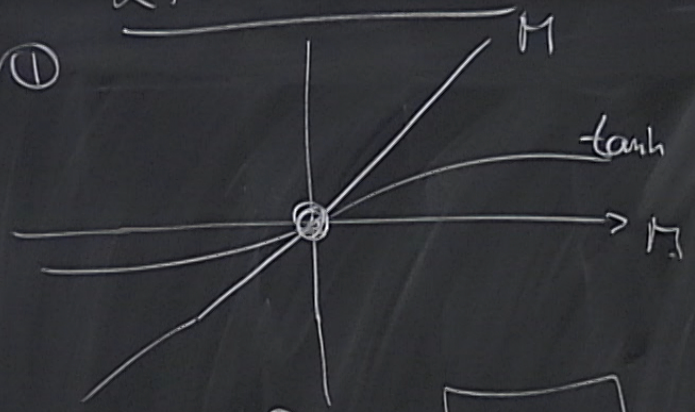
② 

PM $J>T$ $M = \pm M_0$

$h(MJ/T)$]

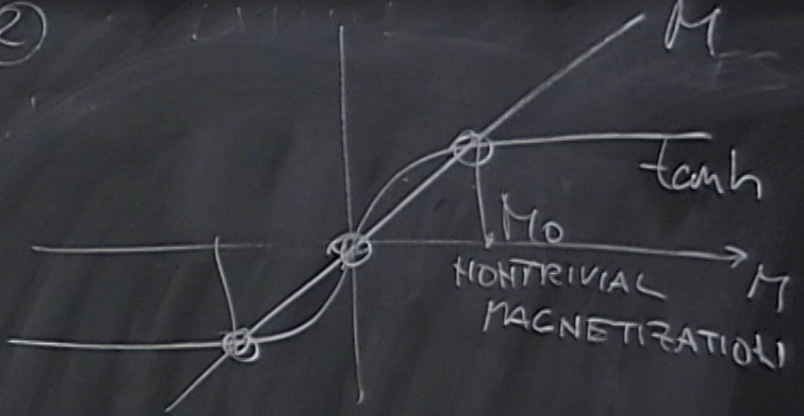
(MJ/T)

2 POSSIBILITIES:

① 

PM

$J<T$

② 

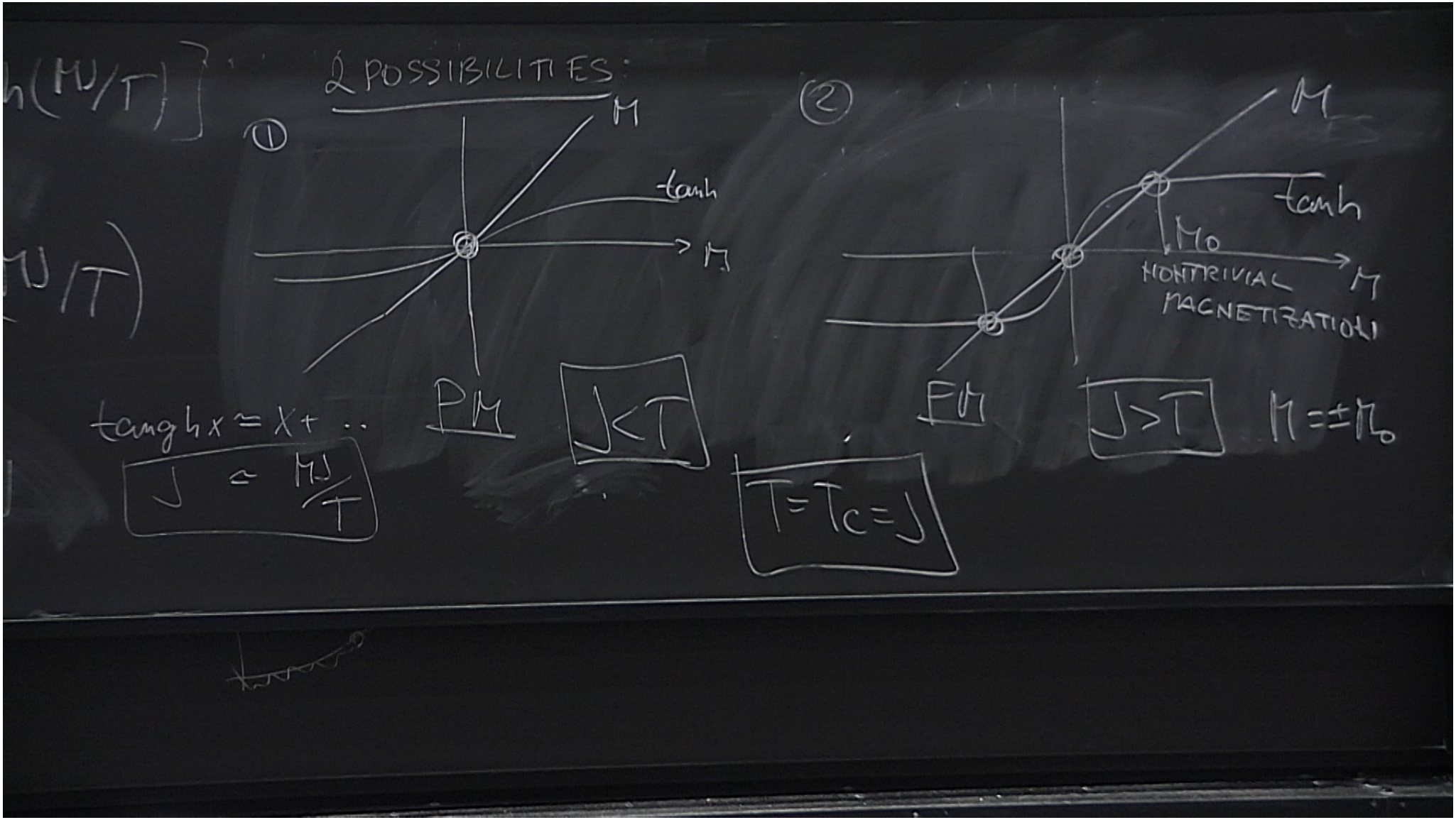
FM

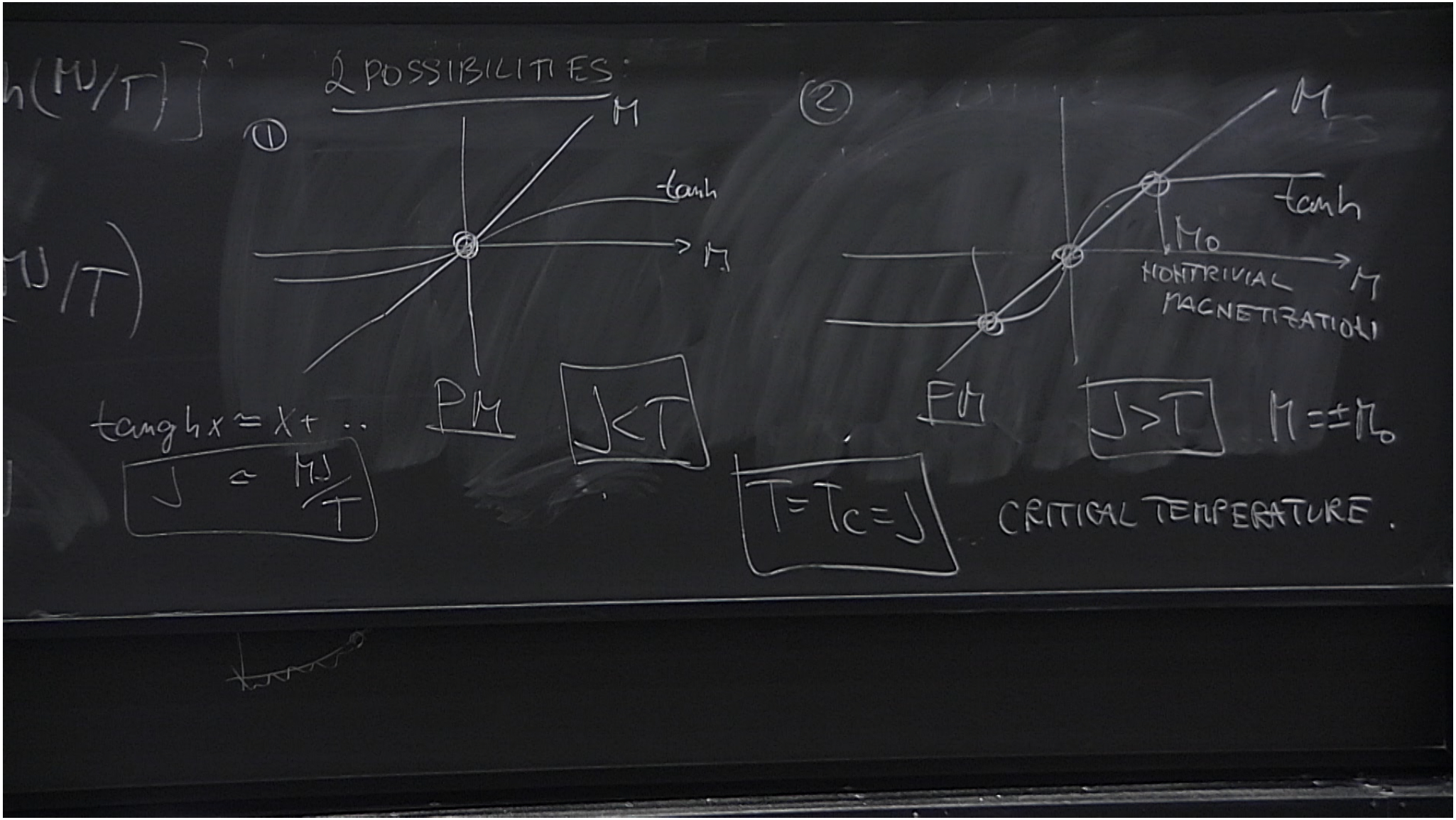
$J>T$

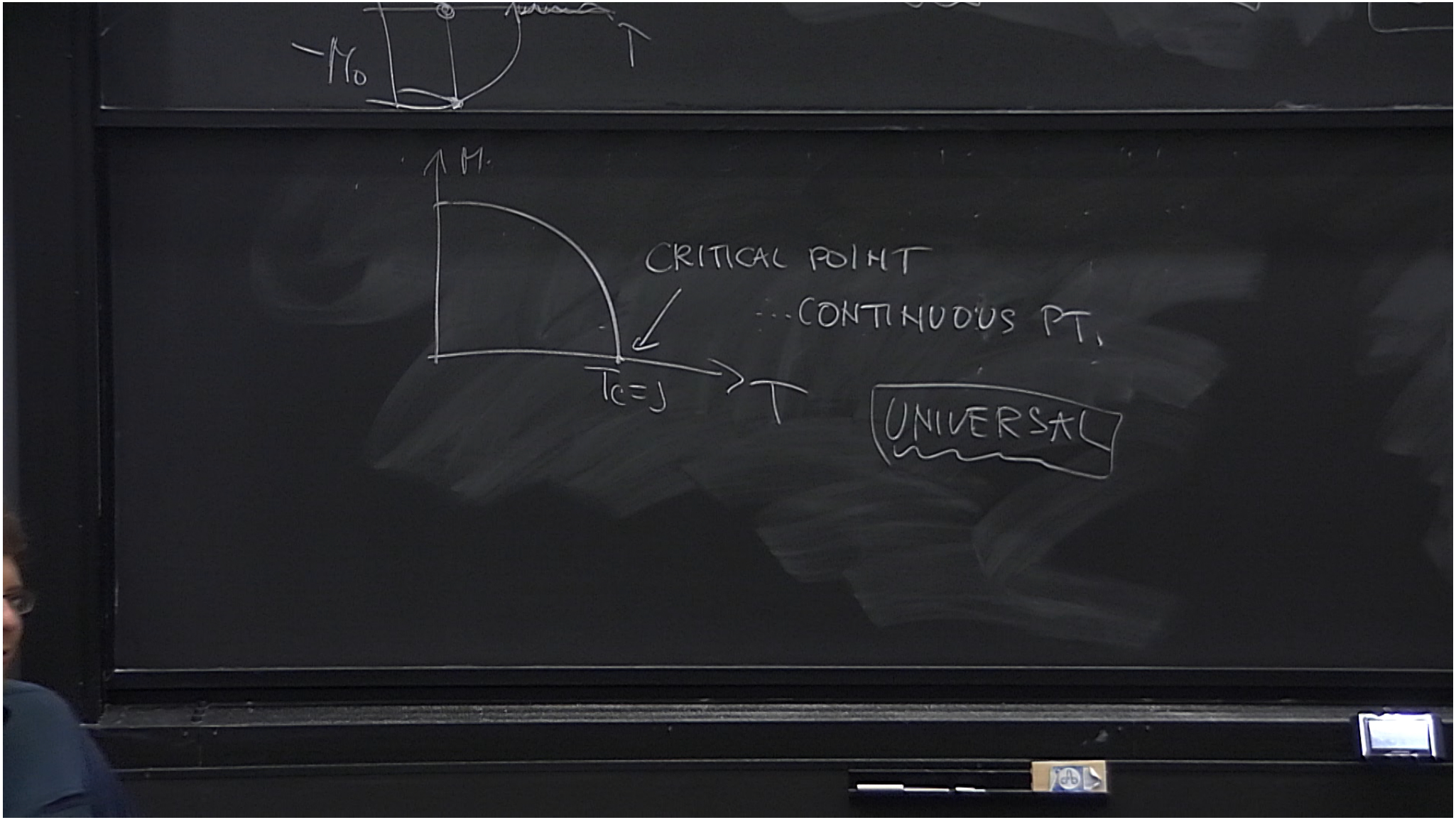
$M = \pm M_0$

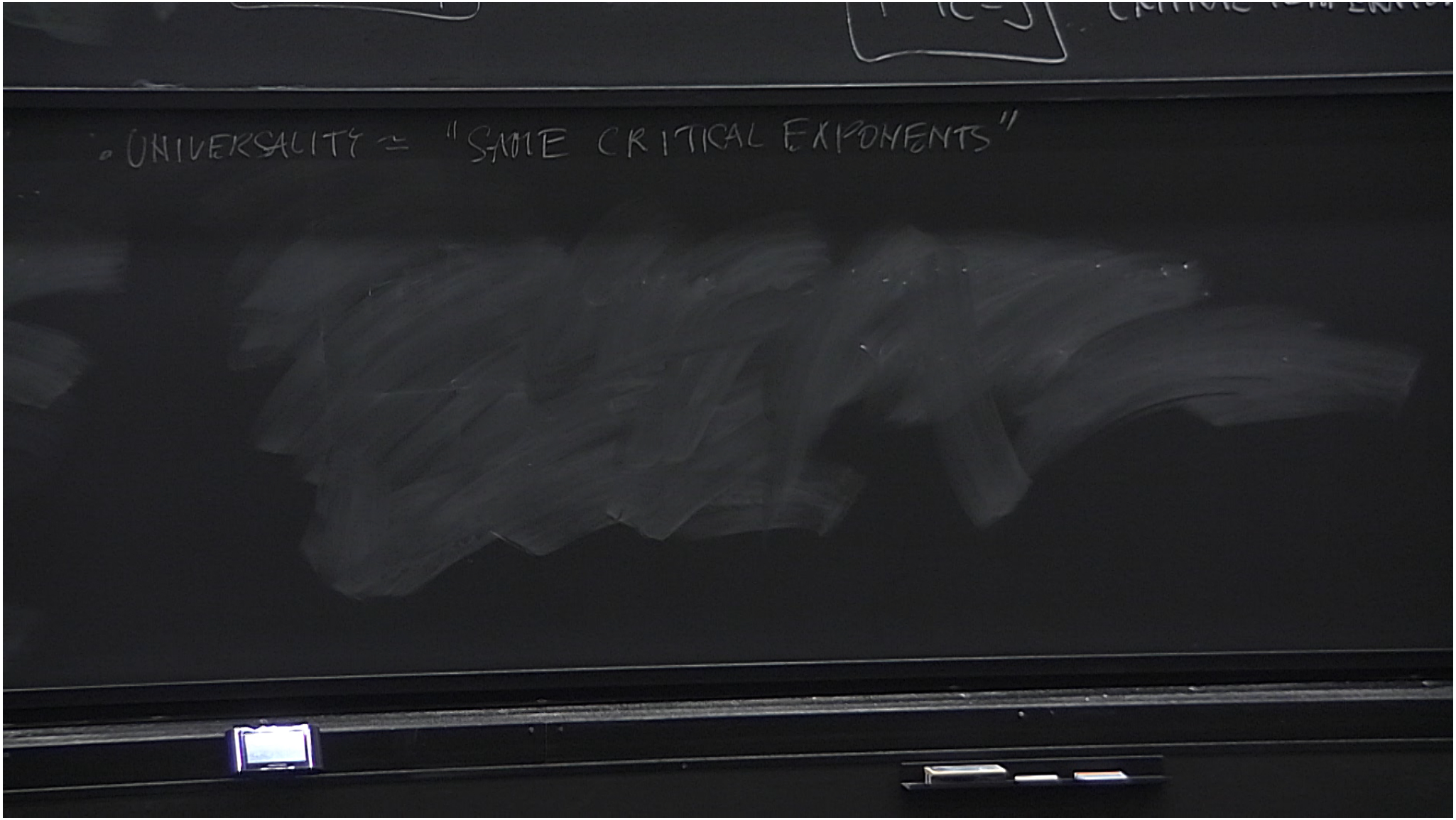
$\tanh x \simeq x + \ldots$

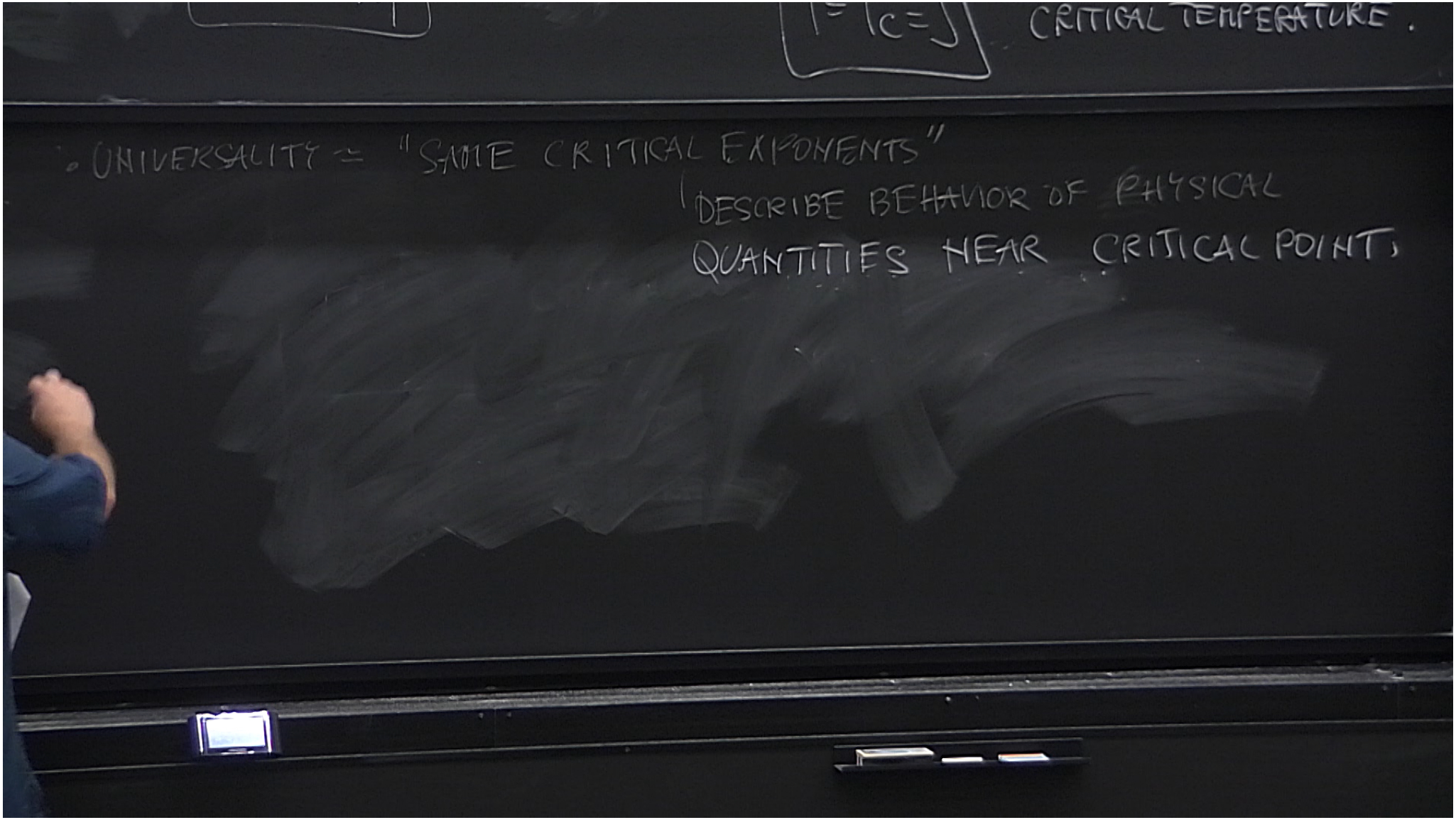
$\simeq MJ/T$

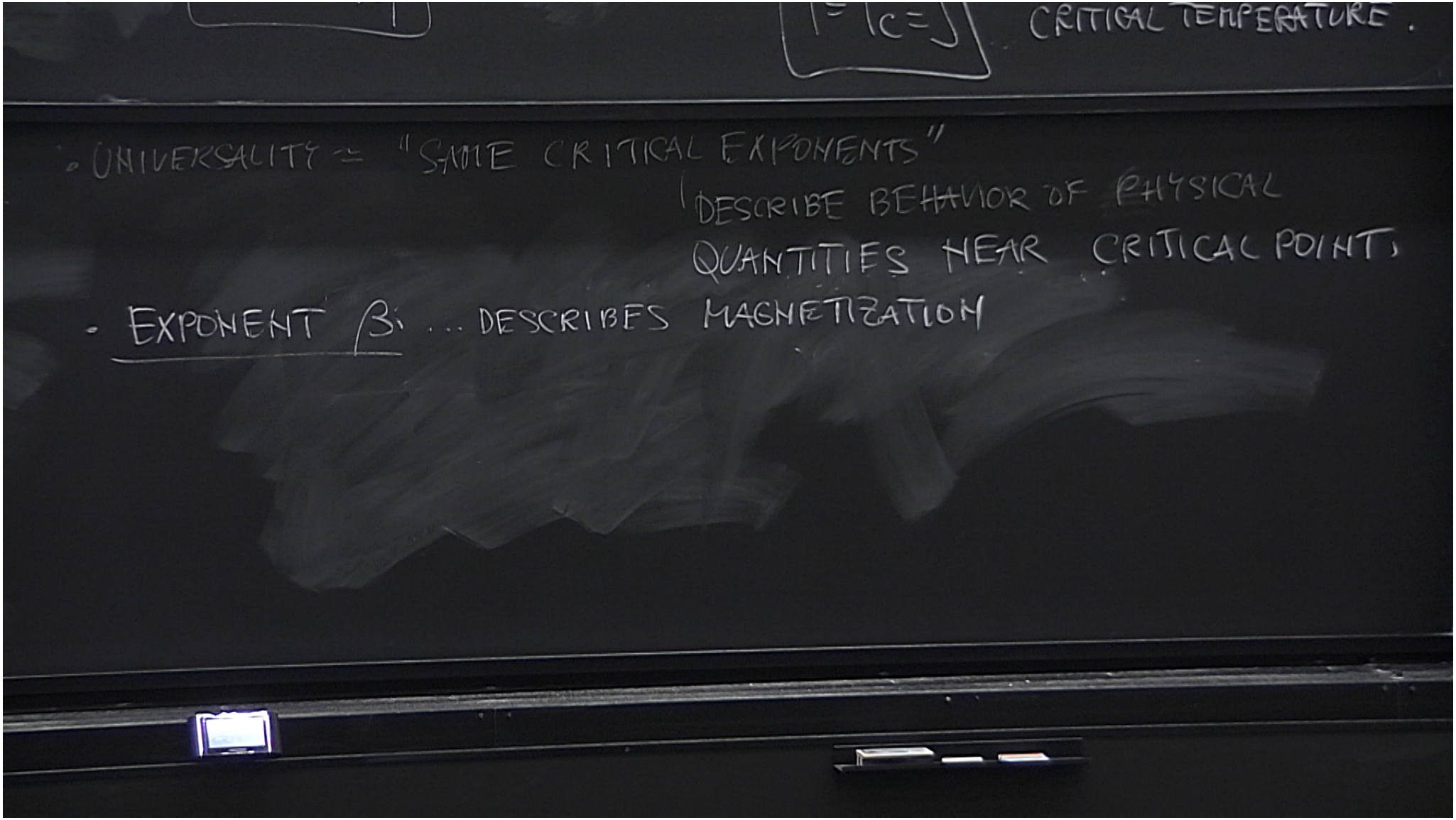

2 POSSIBILITIES:
①
M
tanh
m
PM
J<T
T=Tc=J
②
M
tanh
m₀
NONTRIVIAL MAGNETIZATION
m
FM
J>T
m=±m₀
tanh x ≃ x + ..
J = mJ/T

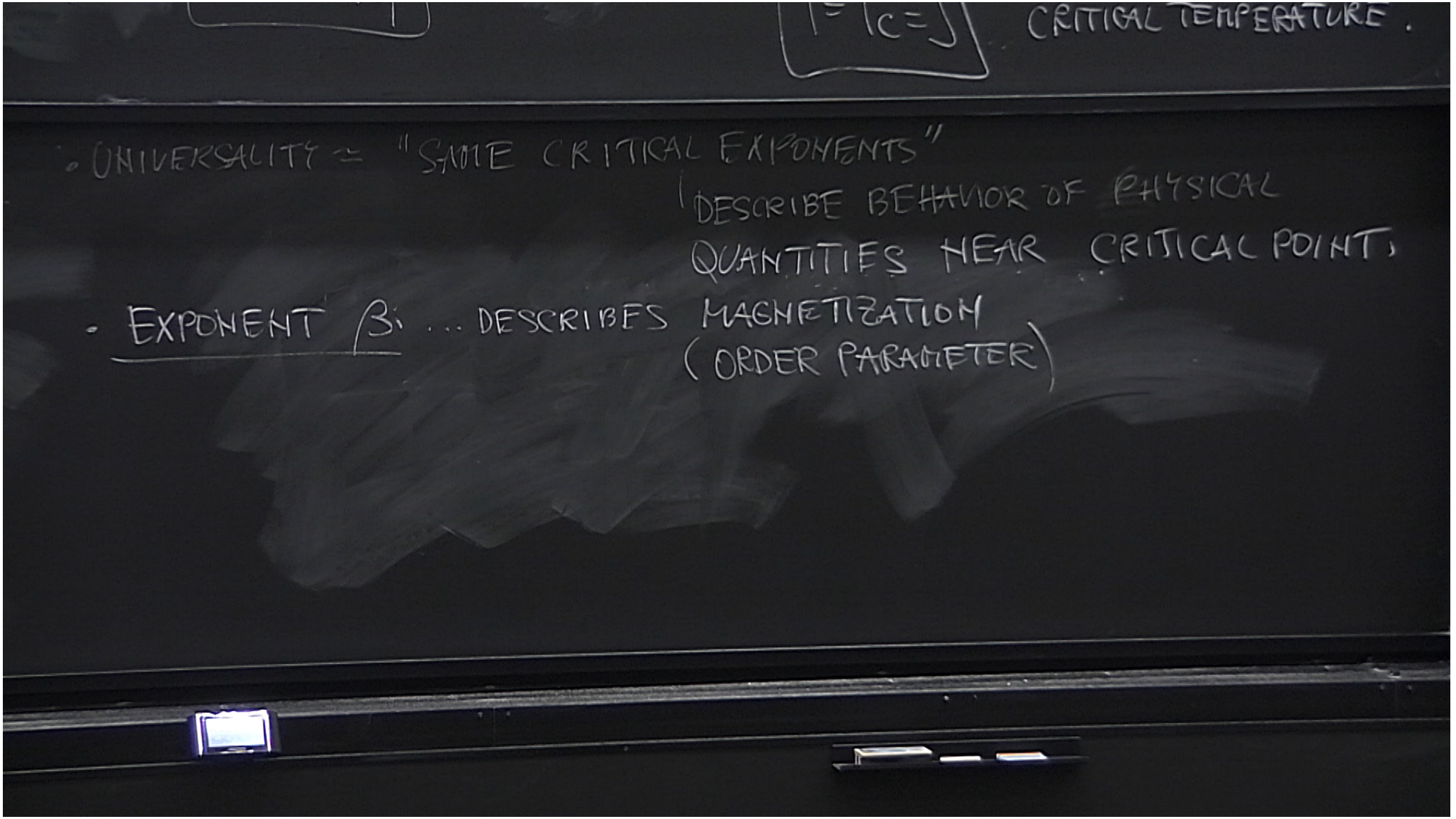

2 POSSIBILITIES:
①
M
tanh
m
PM
J<T
tanh x ≃ x + ..
J = MJ/T
②
M
tanh
M₀
NONTRIVIAL MAGNETIZATION
m
FM
J>T
m = ± m₀
T = T_C = J
CRITICAL TEMPERATURE.


M
CRITICAL POINT
...CONTINUOUS PT.
T
UNIVERSAL


• UNIVERSALITY = "SAME CRITICAL EXPONENTS"

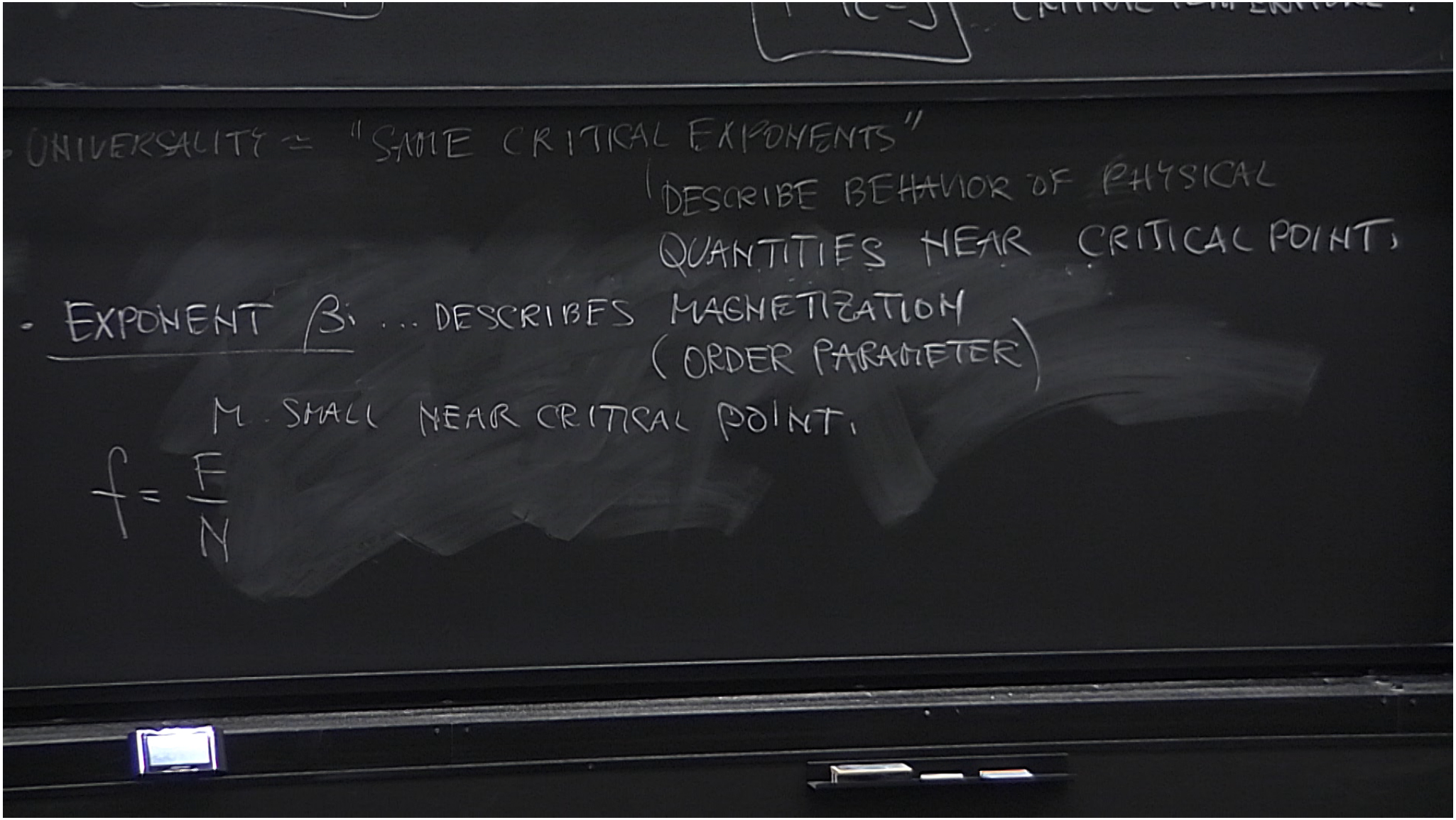

T = T_C = J
CRITICAL TEMPERATURE.
• UNIVERSALITY = "SAME CRITICAL EXPONENTS"
DESCRIBE BEHAVIOR OF PHYSICAL
QUANTITIES NEAR CRITICAL POINT.


CRITICAL TEMPERATURE.
• UNIVERSALITY = "SAME CRITICAL EXPONENTS"
DESCRIBE BEHAVIOR OF PHYSICAL
QUANTITIES NEAR CRITICAL POINT,
• EXPONENT β: ... DESCRIBES MAGNETIZATION


T = T_C = J
CRITICAL TEMPERATURE.
• UNIVERSALITY = "SAME CRITICAL EXPONENTS"
(DESCRIBE BEHAVIOR OF PHYSICAL QUANTITIES NEAR CRITICAL POINT)
• EXPONENT β: ... DESCRIBES MAGNETIZATION (ORDER PARAMETER)

- UNIVERSALITY = "SAME CRITICAL EXPONENTS"
  (DESCRIBE BEHAVIOR OF PHYSICAL QUANTITIES NEAR CRITICAL POINT)
- EXPONENT β: ... DESCRIBES MAGNETIZATION (ORDER PARAMETER)

  M SMALL NEAR CRITICAL POINT.

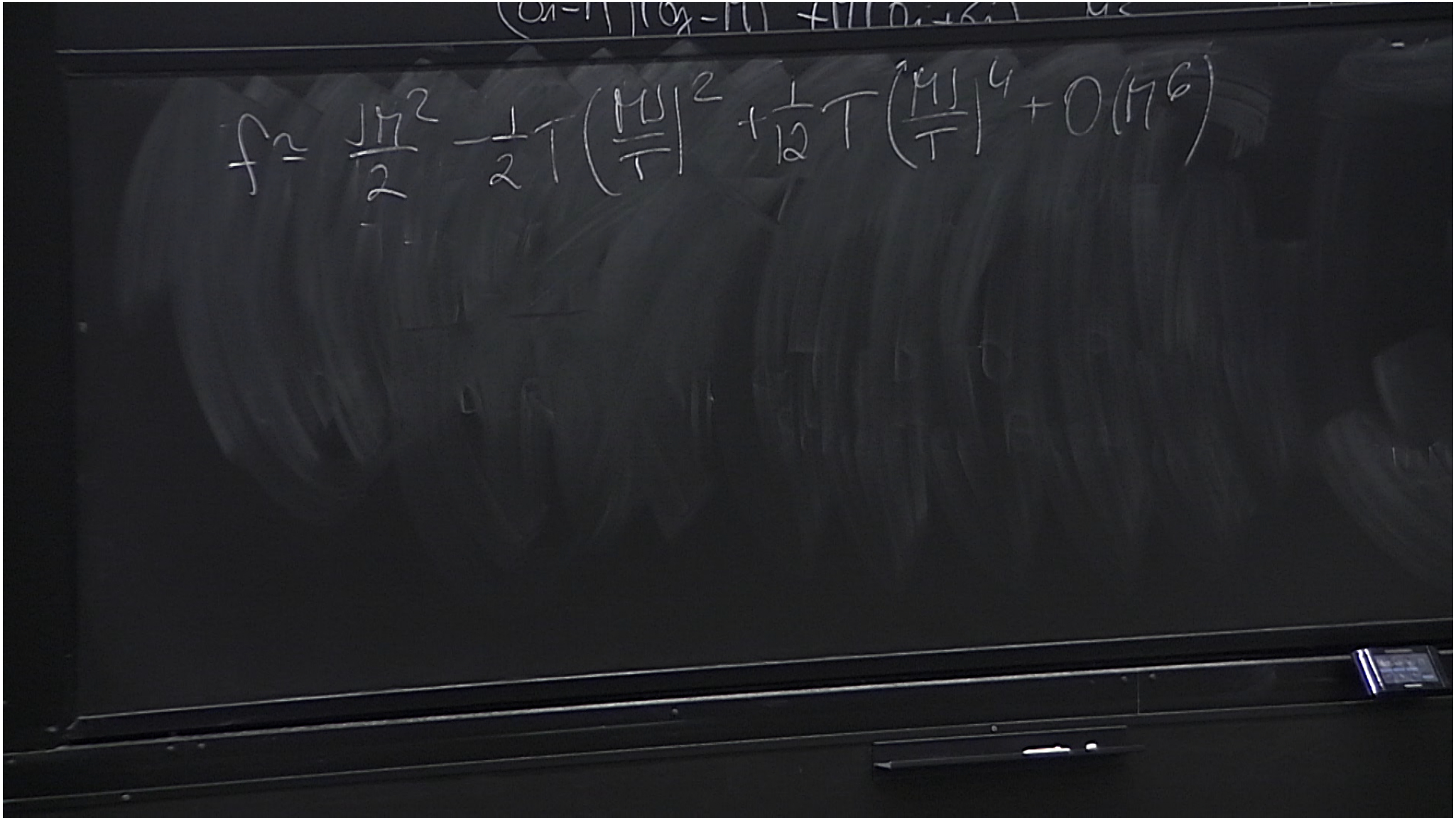

UNIVERSALITY = "SAME CRITICAL EXPONENTS"
DESCRIBE BEHAVIOR OF PHYSICAL QUANTITIES NEAR CRITICAL POINT.
EXPONENT β: ... DESCRIBES MAGNETIZATION (ORDER PARAMETER)
M SMALL NEAR CRITICAL POINT.
f = F/N

- UNIVERSALITY = "SAME CRITICAL EXPONENTS"
  (DESCRIBE BEHAVIOR OF PHYSICAL QUANTITIES NEAR CRITICAL POINT)

- EXPONENT β: ... DESCRIBES MAGNETIZATION (ORDER PARAMETER)

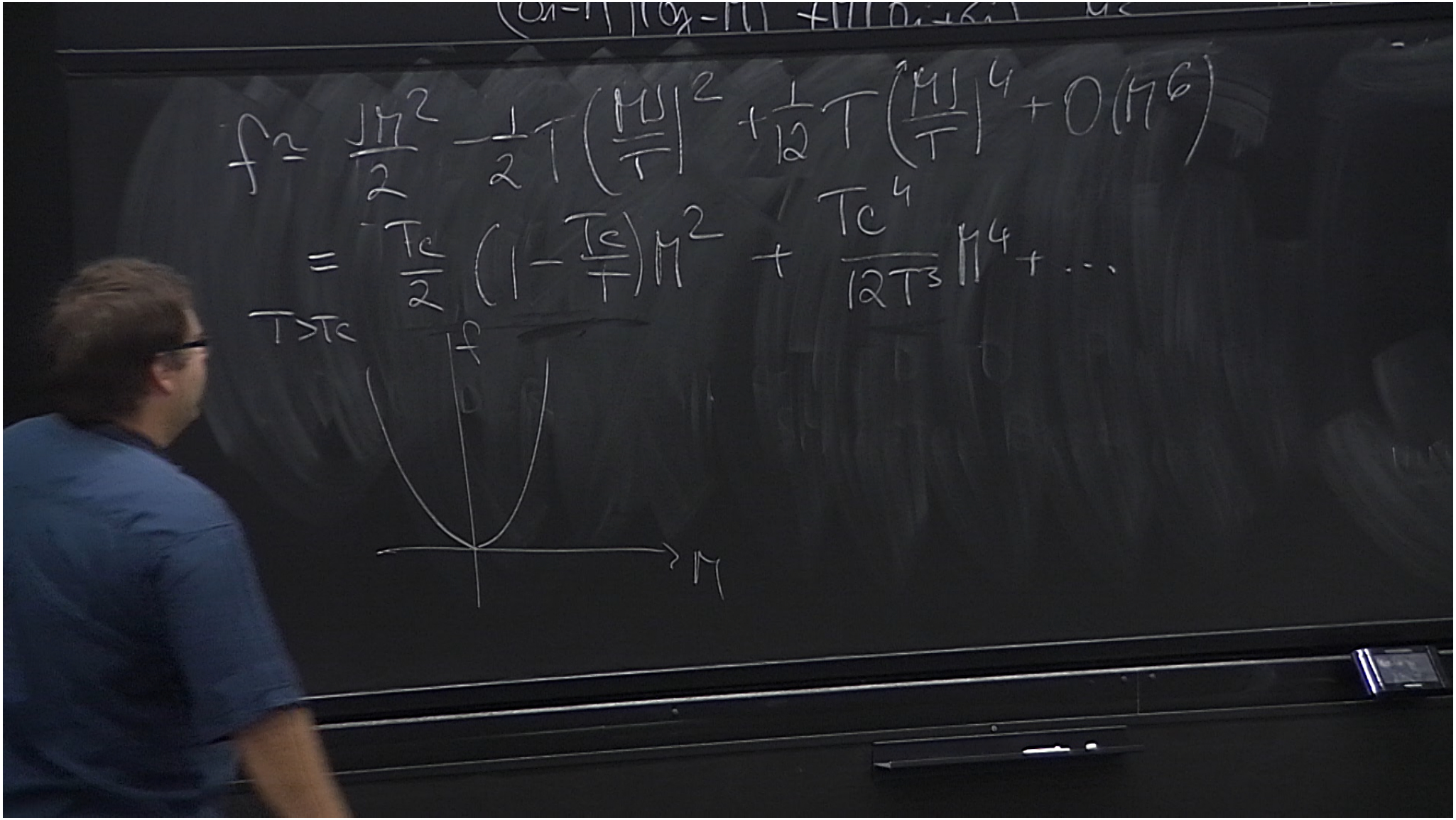
M SMALL NEAR CRITICAL POINT.

$$f = \frac{F}{N} = \frac{JM^2}{2} - T\log\left(2\cosh\frac{MJ}{T}\right)$$


$f \approx \frac{\mu r^2}{2} - \frac{1}{2}T\left(\frac{r\mu}{T}\right)^2 + \frac{1}{12}T\left(\frac{r\mu}{T}\right)^4 + O(r^6)$

$$f = \frac{M^2}{2} - \frac{1}{2}T\left(\frac{M}{T}\right)^2 + \frac{1}{12}T\left(\frac{M}{T}\right)^4 + O(M^6)$$

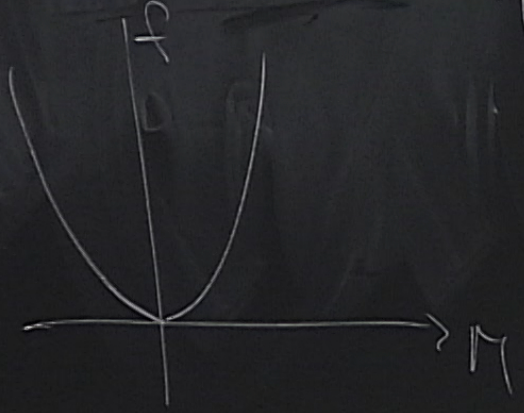
$$= \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4 + \cdots$$


$f \approx \frac{Jm^2}{2} - \frac{1}{2}T\left(\frac{m}{T}\right)^2 + \frac{1}{12}T\left(\frac{m}{T}\right)^4 + O(m^6)$
$= \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right)m^2 + \frac{T_c^4}{12T^3}m^4 + \ldots$
$T > T_c$
f
m

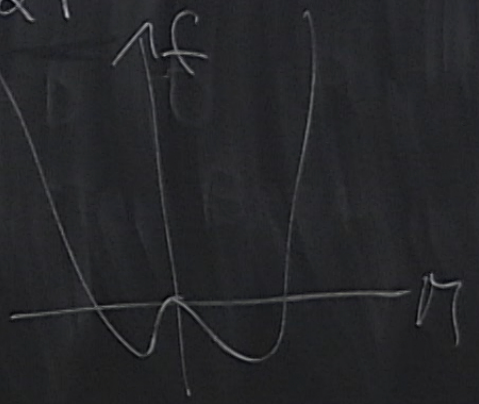
$$f \approx \frac{Jm^2}{2} - \frac{1}{2}T\left(\frac{m}{T}\right)^2 + \frac{1}{12}T\left(\frac{m}{T}\right)^4 + O(m^6)$$

$$= \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right)m^2 + \frac{T_c^4}{12T^3}m^4 + \cdots$$

$T > T_c$

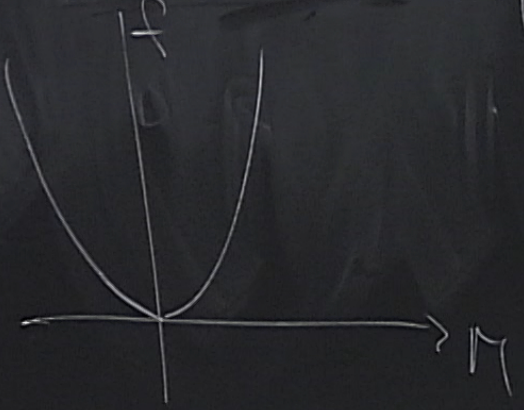


$T < T_c$

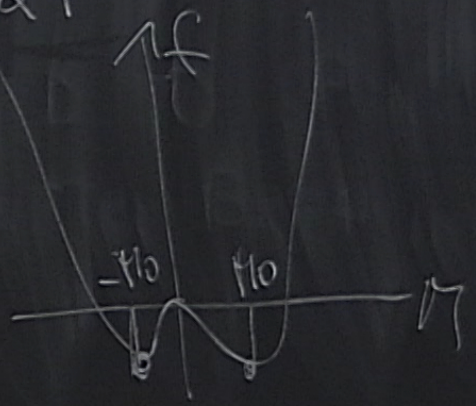


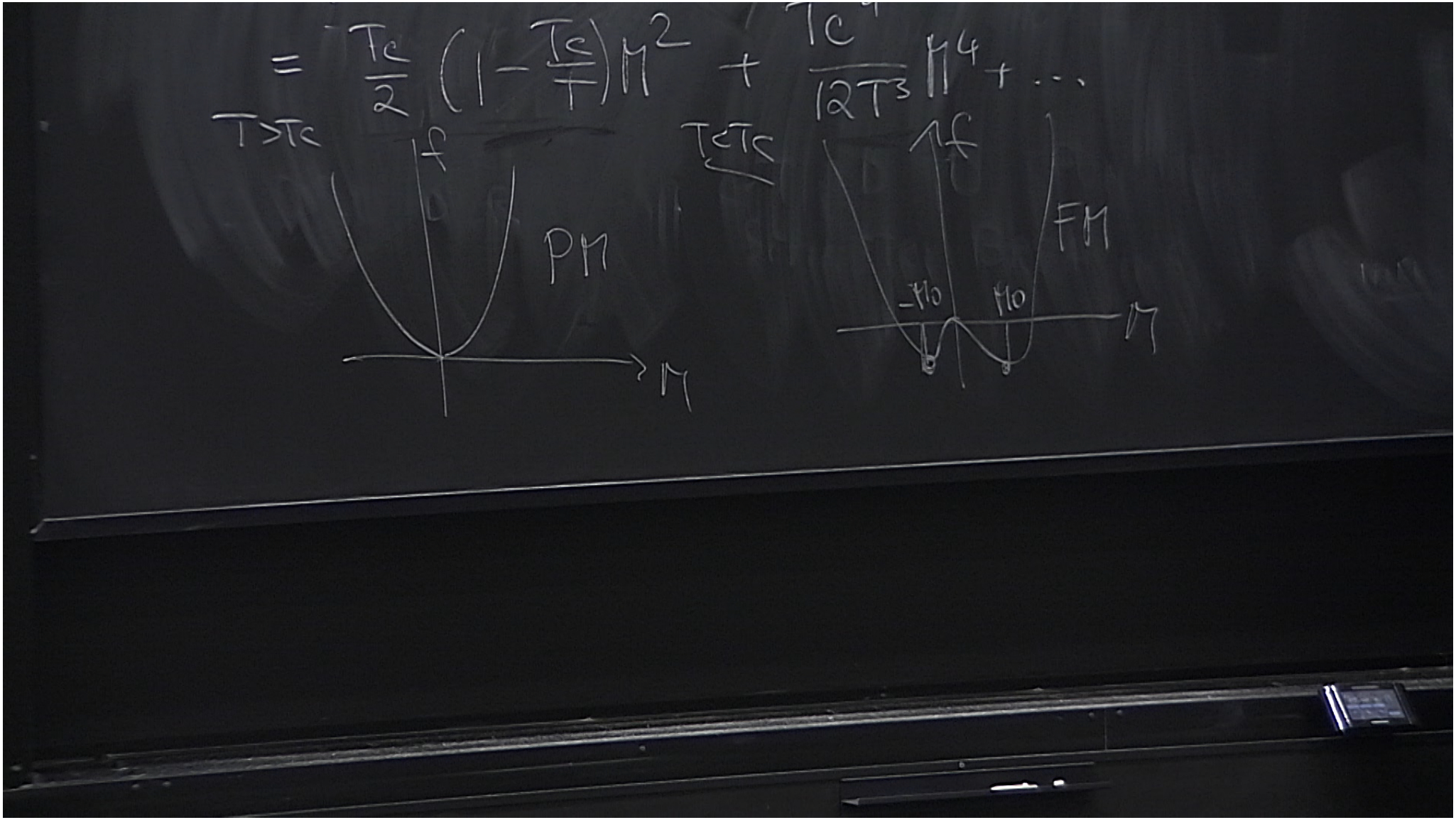
$$= \frac{T_c}{2}\left(1-\frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4 + \cdots$$

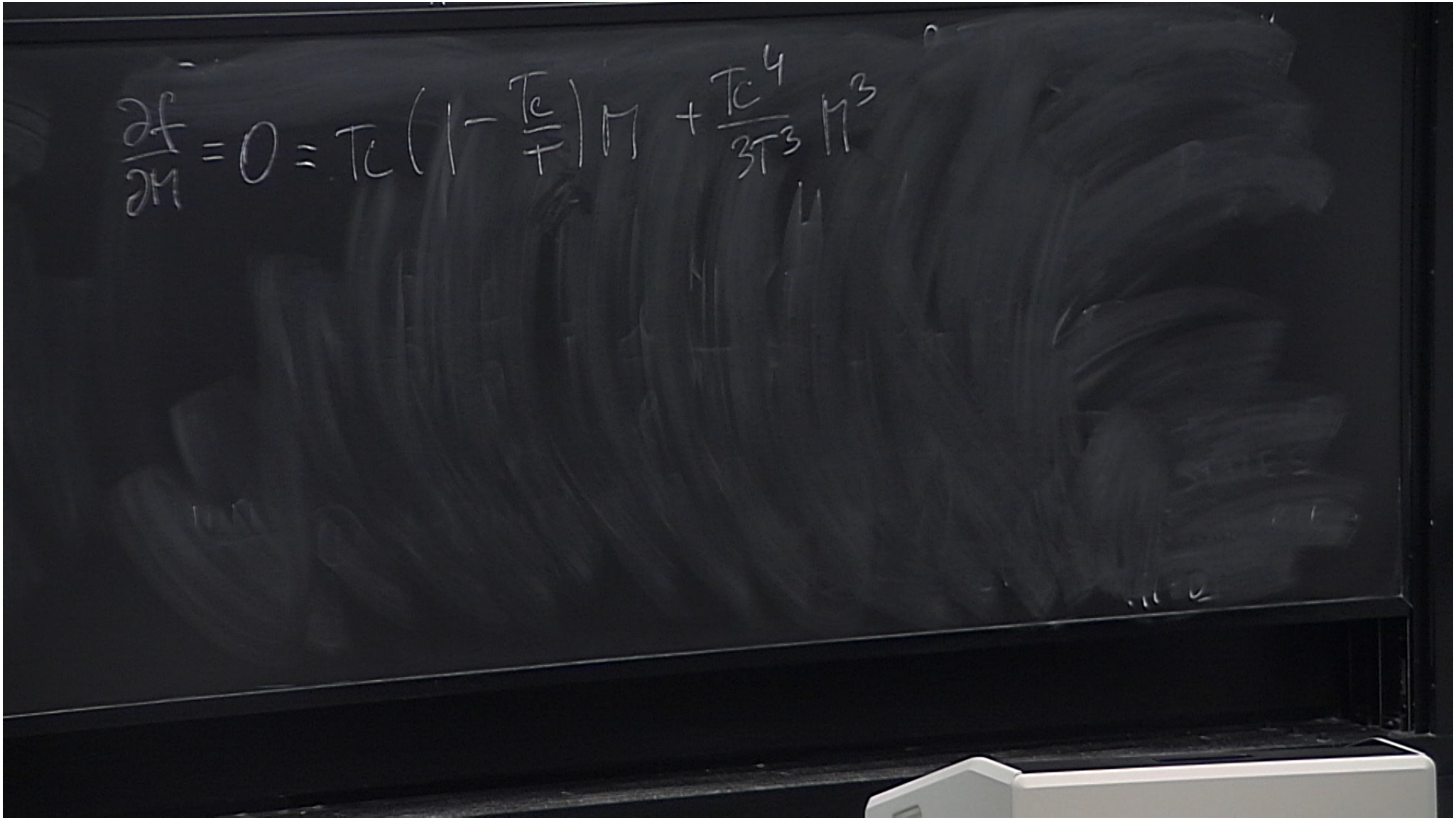
$T > T_c$

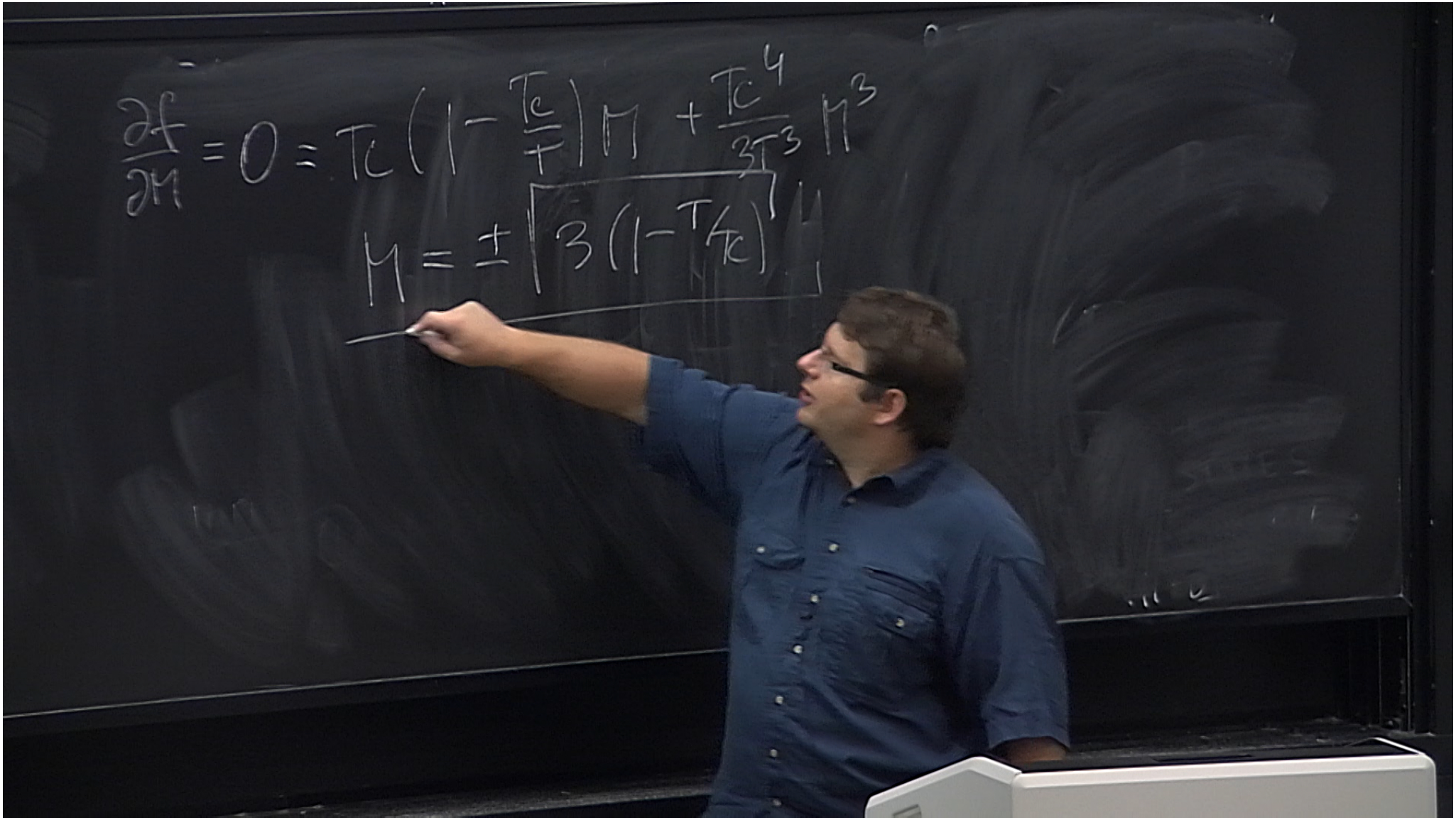


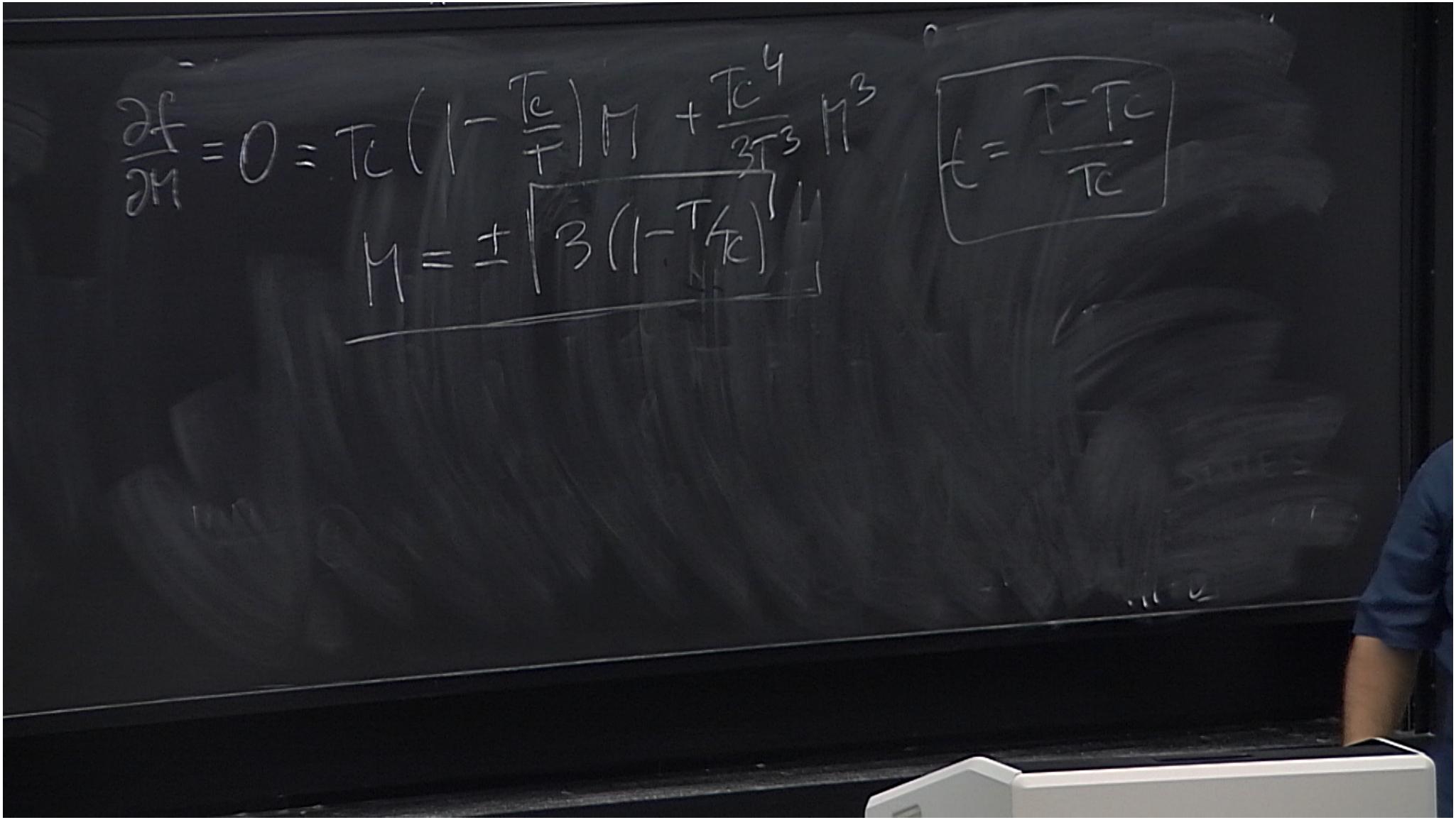
$T < T_c$

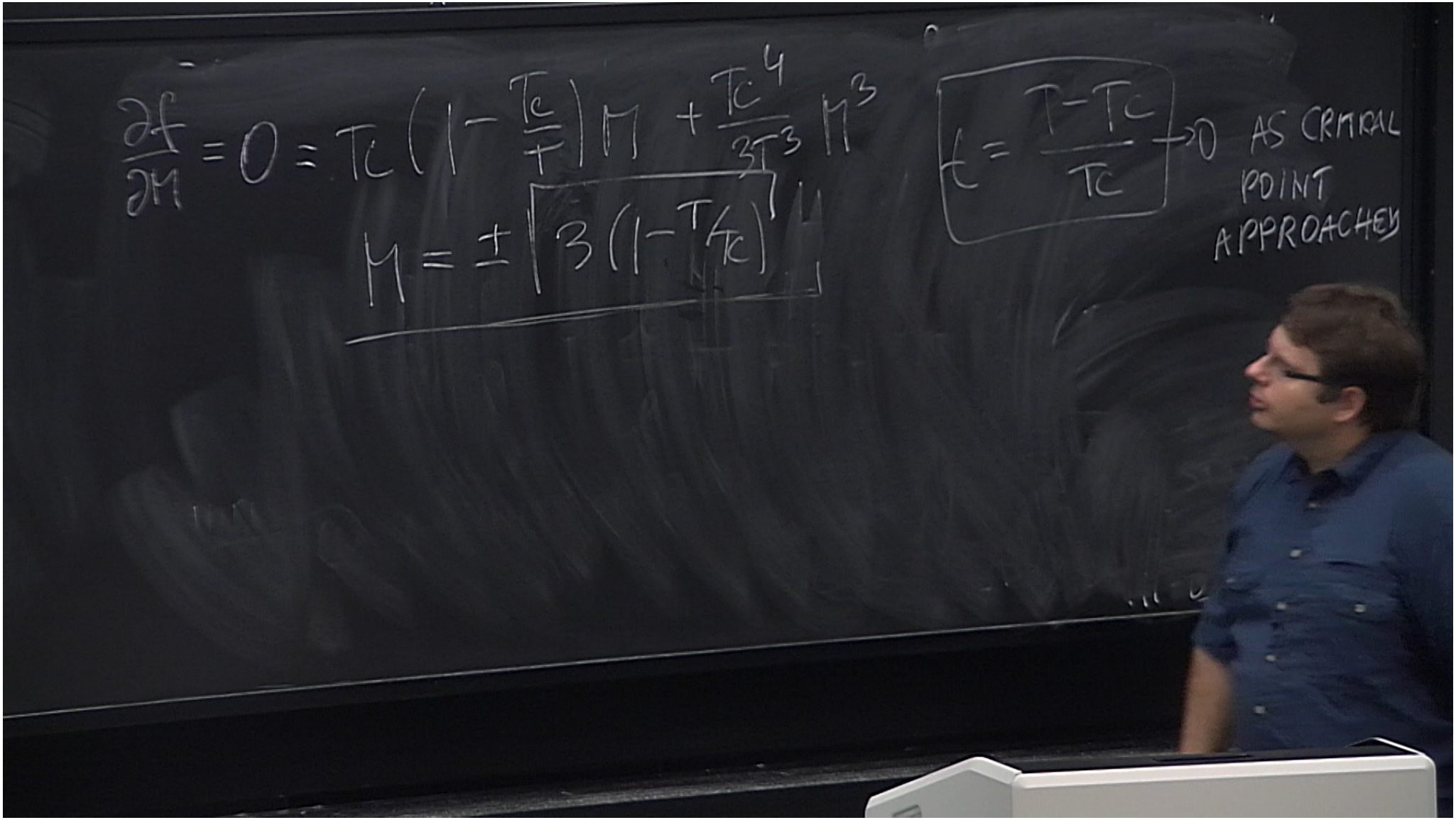


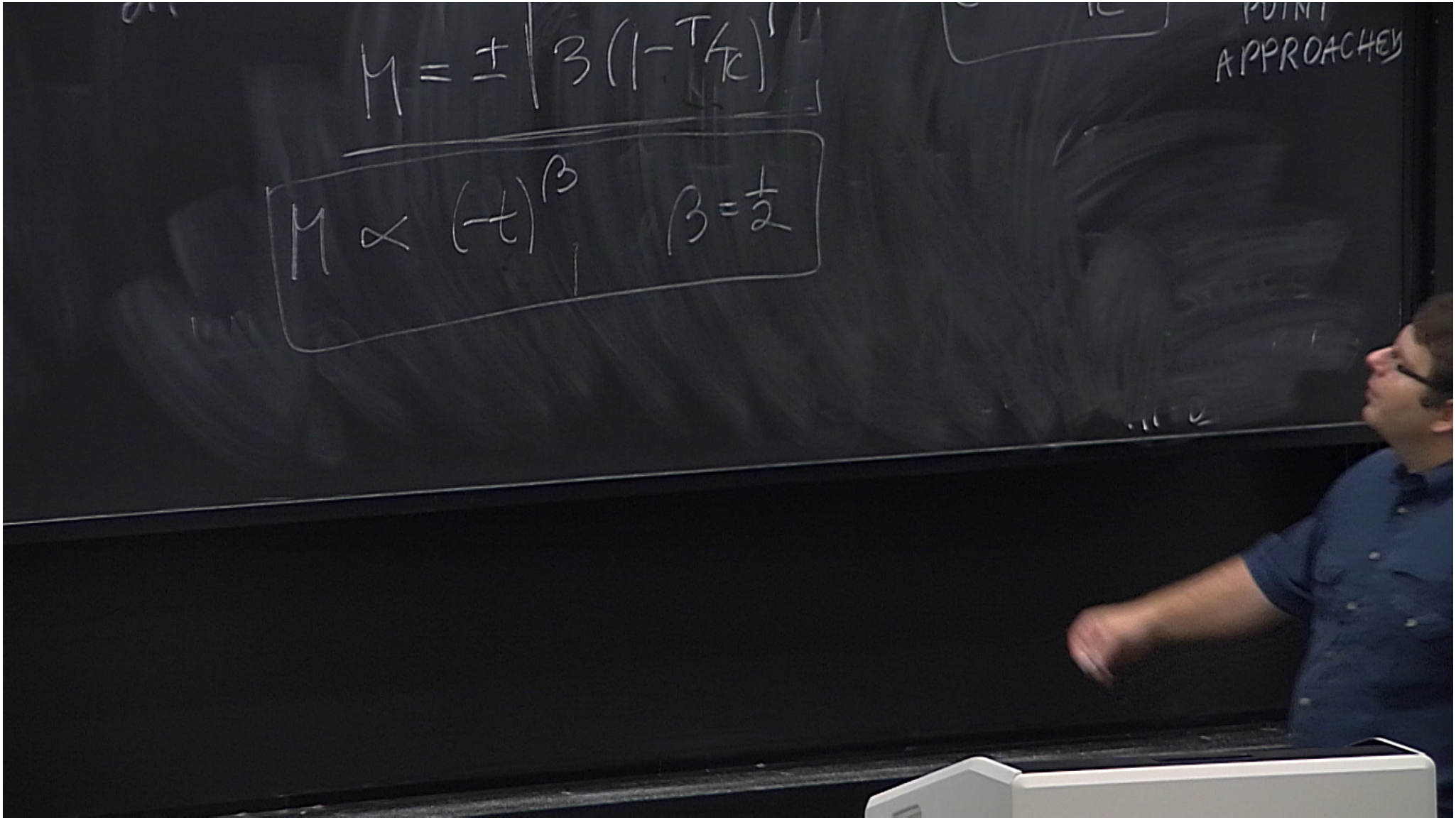

$= \frac{T_c}{2}\left(1-\frac{T_c}{T}\right)M^2 + \frac{T_c}{12T^3}M^4 + \dots$
$T > T_c$
f
PM
M
$T < T_c$
f
FM
$-M_0$
M_0
M

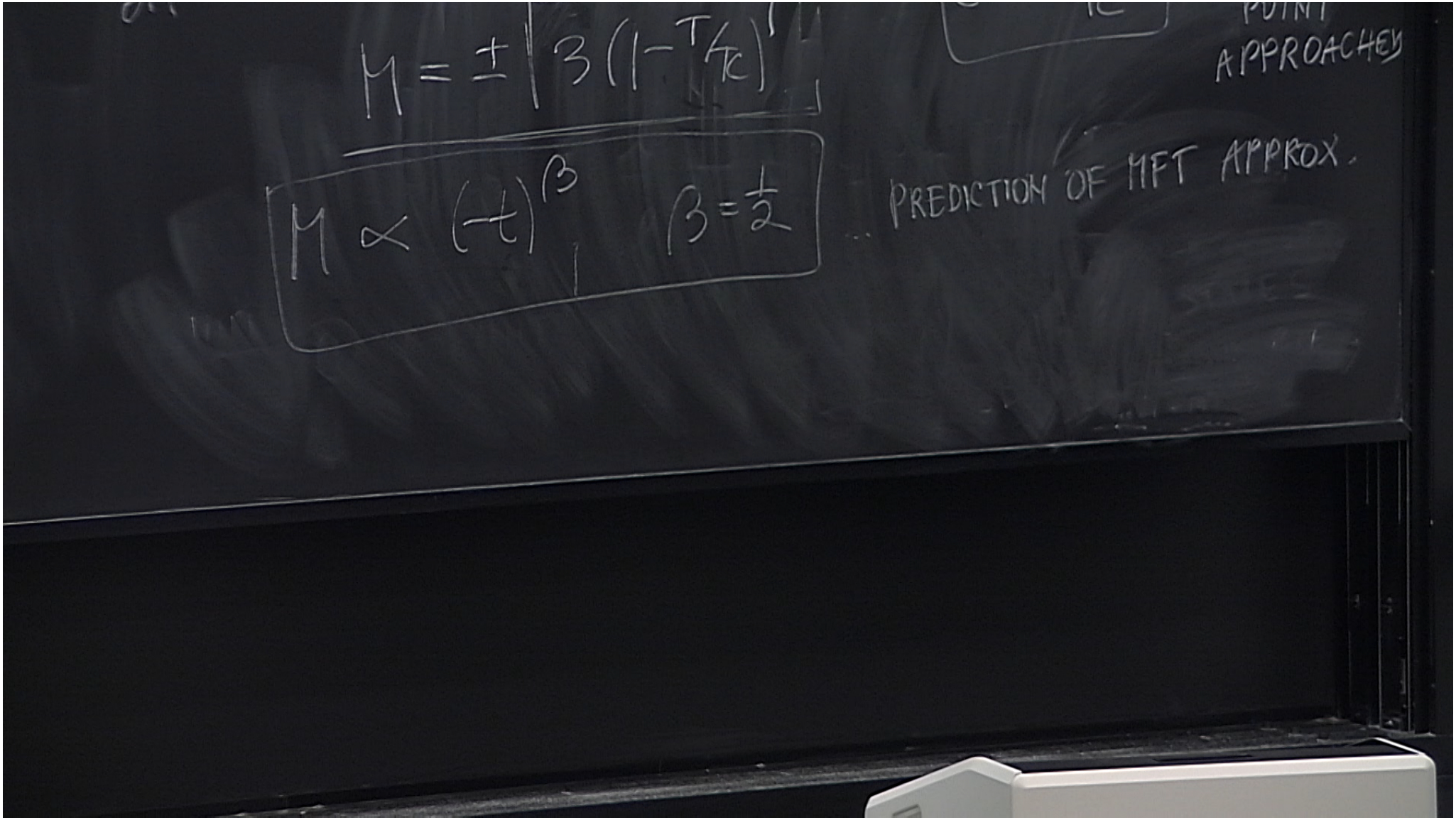

$\frac{\partial f}{\partial M} = 0 = T_C\left(1 - \frac{T_C}{T}\right)M + \frac{T_C^4}{3T^3}M^3$


$\frac{\partial f}{\partial M} = 0 = T_c\left(1 - \frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3$
$M = \pm\sqrt{3\left(1 - \frac{T}{T_c}\right)}$


$\frac{\partial f}{\partial M} = 0 = T_c\left(1 - \frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3$
$M = \pm\sqrt{3\left(1 - \frac{T}{T_c}\right)}$
$\epsilon = \frac{T - T_c}{T_c}$


$\frac{\partial f}{\partial M} = 0 = T_c\left(1 - \frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3$
$M = \pm\sqrt{3\left(1 - \frac{T}{T_c}\right)}$
$t = \frac{T - T_c}{T_c} \to 0$ AS CRITICAL POINT APPROACHED


$M = \pm |3(1 - T/T_c)|^{\frac{1}{2}}$
$M \propto (-t)^{\beta}$ $\beta = \frac{1}{2}$
APPROACHES


$M = \pm |3(1 - T/T_c)|^{1/2}$
$M \propto (-t)^{\beta}$ $\beta = \frac{1}{2}$
PREDICTION OF MFT APPROX.
APPROACHES

$$\frac{\partial f}{\partial M} = 0 = T_c\left(1 - \frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3$$

$$M = \pm\sqrt{3\left(1 - \frac{T}{T_c}\right)}$$

$$M \propto (-t)^{\beta} \qquad \beta = \frac{1}{2}$$

$$t = \frac{T - T_c}{T_c} \to 0$$ AS CRITICAL POINT APPROACHED

... PREDICTION OF MFT APPROX. INDEPENDENT OF d .. DIMENSIONALITY OF ISING MODEL.