Title: PSI 17/18 - Quantum Field Theory I - Lecture 14

Date: Oct 27, 2017 09:00 AM

URL: http://pirsa.org/17100033

Abstract:

Quantum Electrodynamics (QED)

Free-level

$$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{e.m.} + \mathcal{L}_{int}$$

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{L}_{int} = -e \bar{\psi} \gamma^{\mu} \psi A_{\mu}$$

Quantum Electrodynamics (QED)

Tree-level

$$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{e.m.} + \mathcal{L}_{int}$$

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{L}_{int} = -e \bar{\psi} \gamma^{\mu} \psi A_{\mu}$$

Dirac – global symmetry:

$$\psi(x) \to e^{i\alpha}\psi(x)$$

$$\bar{\psi}(x) \to e^{-i\alpha}\bar{\psi}(x)$$

$\mathcal{L}_{e.m.}$ – Gauge symmetry

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

Quantum Electrodynamics (QED)

- Free-level

$$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{e.m.} + \mathcal{L}_{int}$$

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{L}_{int} = -e \bar{\psi} \gamma^{\mu} \psi A_{\mu}$$

$\mathcal{L}_{Dirac}$

$\bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi$

$\mathcal{L}_{e.m.}$

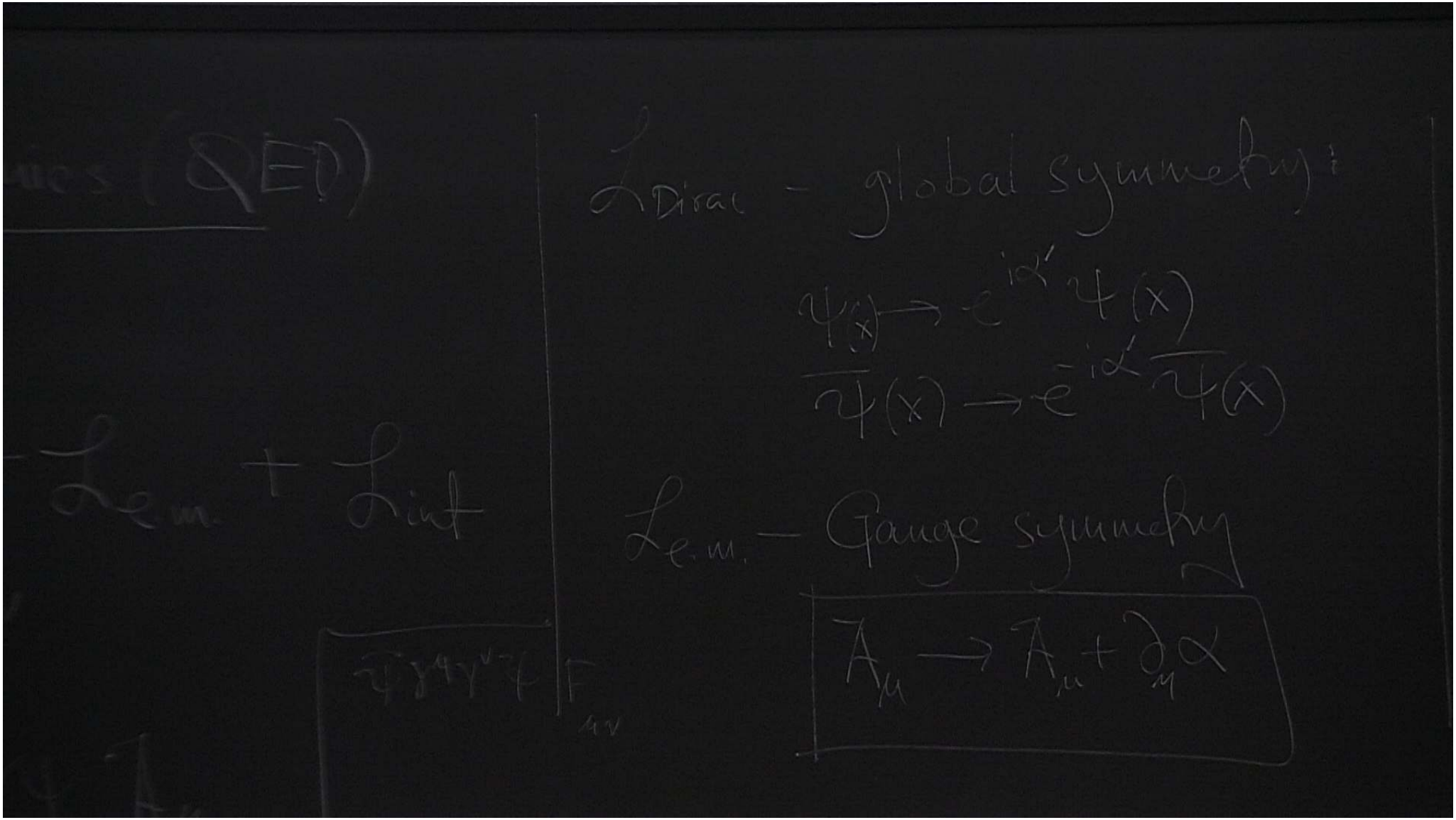
$F_{\mu\nu}$

Doesn't seem to be gauge invariant.

$$\psi(x) \longrightarrow \psi'(x) = e^{-ie\alpha(x)}\,\psi(x)$$

$$i\,\bar{\psi}\,\partial\!\!\!/\,\psi \longrightarrow i\,\bar{\psi}\,\partial\!\!\!/\,\psi + \bar{\psi}\,e(\partial\!\!\!/\,\alpha)\,\psi$$

$$\mathcal{L}_{int} \longrightarrow \mathcal{L}_{int} - e\,\bar{\psi}(\partial\!\!\!/\,\alpha)\psi$$


(QED)
$\mathcal{L}_{em} + \mathcal{L}_{int}$
$\mathcal{L}_{Dirac}$ – global symmetry:
$\psi(x) \to e^{i\alpha}\psi(x)$
$\bar{\psi}(x) \to e^{-i\alpha}\bar{\psi}(x)$
$\mathcal{L}_{e.m.}$ – Gauge symmetry
$\bar{\psi}\gamma^{\mu}\psi F_{\mu\nu}$
$A_\mu \to A_\mu + \partial_\mu \alpha$

Gauge transformation

$$A_\mu \longrightarrow A_\mu + \frac{1}{e}\partial_\mu \alpha$$

$$\psi(x) \rightarrow e^{-i\alpha(x)}\psi(x)$$

$$\bar{\psi}(x) \quad e^{+i\alpha(x)}\bar{\psi}(x)$$

gauging a symmetry.

$$\psi^a \qquad a = 1,2$$

$$\mathcal{L} = i\bar{\psi}^a \partial\!\!\!/ \psi^a - m\bar{\psi}^a\psi^a$$

$$\Psi = \begin{pmatrix}\psi^1 \\ \psi^2\end{pmatrix} \qquad \Psi \longrightarrow g\Psi$$

$$g \in SU(2)$$

$$\mathcal{L}_{\text{scalar QED}} = \mathcal{L}_\varphi + \mathcal{L}_{e.m} + \mathcal{L}_{int}$$

$$\mathcal{L}_{int} = -ie\,(\partial_\mu \varphi^\dagger \varphi - \varphi^\dagger \partial_\mu \varphi)\,\vec{A}^\mu$$

$$\varphi \longrightarrow e^{-i\alpha(x)}\,\varphi$$

$$\varphi^\dagger \longrightarrow e^{+i\alpha(x)}\,\varphi^\dagger$$

$$\vec{A}_\mu \longrightarrow \vec{A}_\mu + \frac{1}{e}\,\partial_\mu \alpha$$

$+ \mathcal{L}_{int}$

$R - \varphi^\dagger \partial_\mu \varphi) \bar{A}^\mu$

Covariant derivatives

$$D_\mu \psi = \partial_\mu \psi + ie \bar{A}_\mu$$

$$D_\mu \psi \rightarrow e^{-i\alpha(x)} D_\mu \psi$$

$$D_\mu \varphi = \partial_\mu \varphi + ie \bar{A}_\mu \varphi$$

$$D_\mu \varphi \rightarrow e^{-i\alpha(x)} D_\mu \varphi$$

$+ \mathcal{L}_{int}$

$R - \varphi^\dagger \partial_\mu \varphi) \vec{A}^\mu$

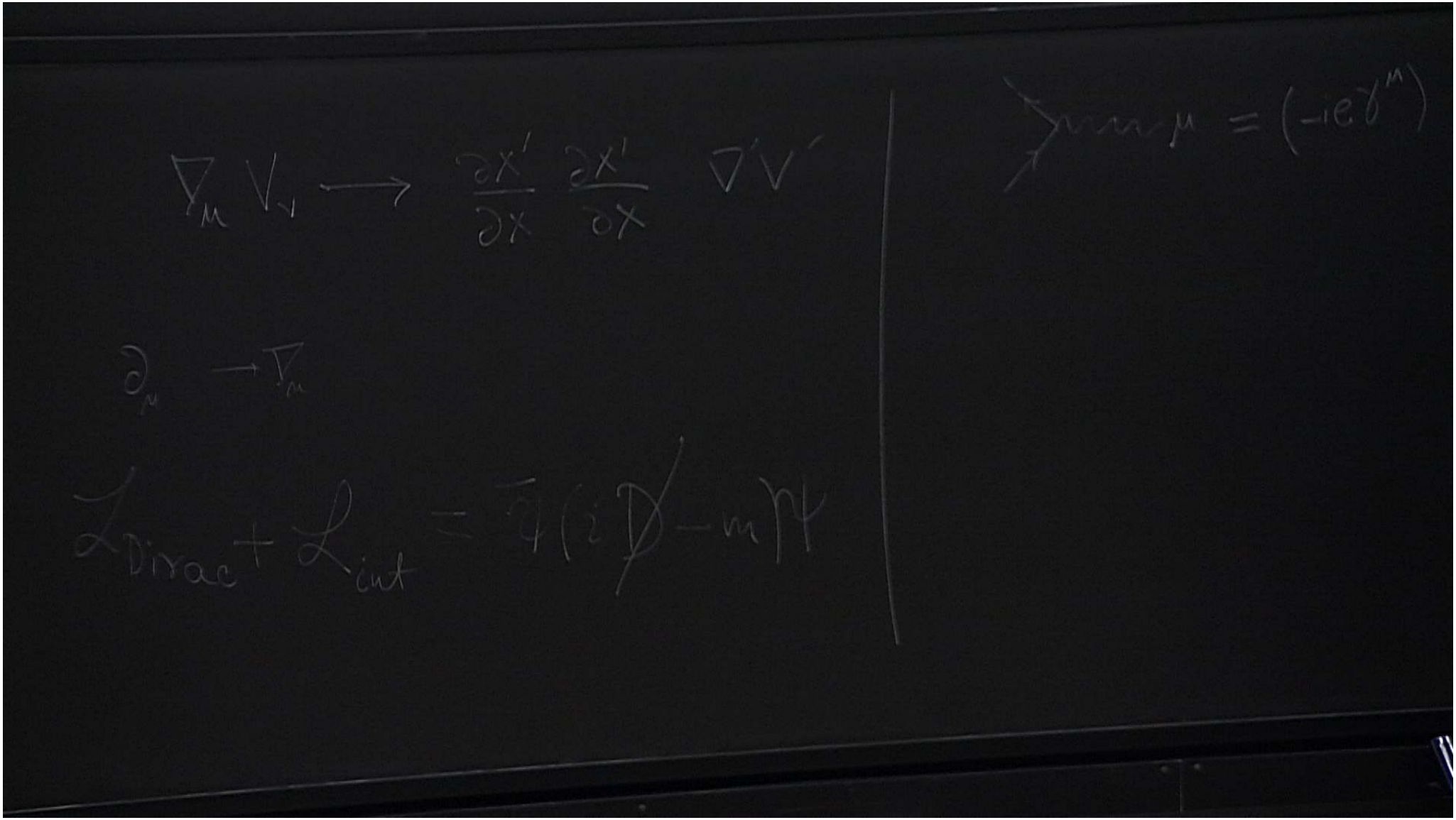
Covariant derivatives

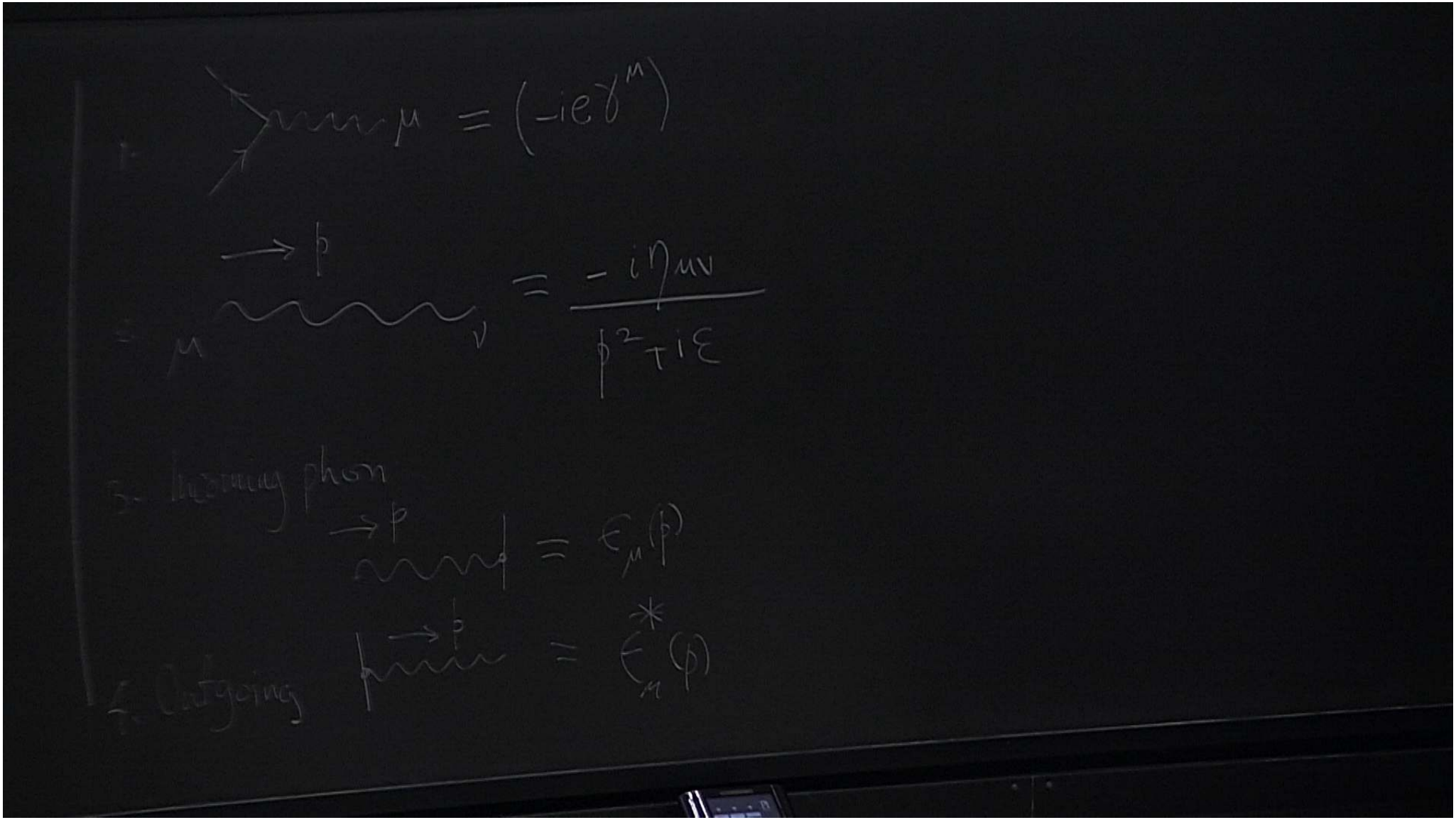
$$D_\mu \psi = \partial_\mu \psi + ie \vec{A}_\mu$$

$$D_\mu \psi \rightarrow e^{-i\alpha(x)} D_\mu \psi$$

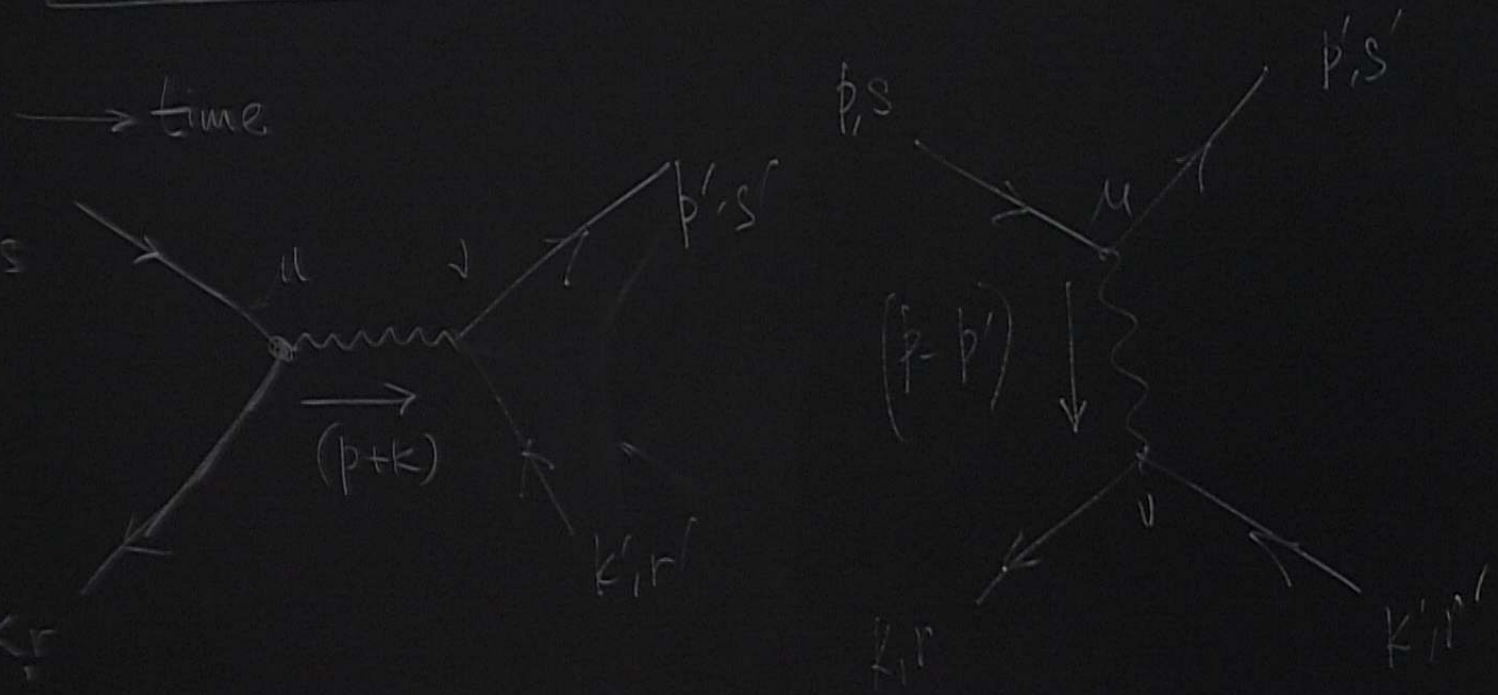
$$D_\mu \varphi = \partial_\mu \varphi + ie \vec{A}_\mu \varphi$$

$$D_\mu \varphi \rightarrow e^{-i\alpha(x)} D_\mu \varphi$$


$\nabla_\mu V_\nu \longrightarrow \frac{\partial x'}{\partial x}\frac{\partial x'}{\partial x}\nabla' V'$
$\partial_\nu \rightarrow \nabla_\mu$
$\mathcal{L}_{Dirac} + \mathcal{L}_{int} = \bar{\psi}(i\not{D} - m)\psi$
$\gamma^\mu \quad \mu = (-ie\gamma^\mu)$


$= (-ie\gamma^\mu)$
p
μ
ν
$= \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$
$= \epsilon_\mu(p)$
$= \epsilon^*_\mu(p)$

Electron - Positron Scattering



$$i\mathcal{M} = \left(\bar{v}_r(k)(-ie\gamma^\mu)u_s(p)\right)\frac{-i\eta_{\mu\nu}}{(p+k)^2+i\varepsilon}\bar{u}_{s'}(p')(-ie\gamma^\nu)u$$

$$-\bar{u}_{s'}(p')(-ie\gamma^\mu)\frac{-i\eta_{\mu\nu}}{(p-p')^2+i\varepsilon}\bar{v}_{r'}(k')(-ie\gamma^\nu)u_{r'}(k')$$

$$\frac{-i\eta_{\mu\nu}}{(p+k)^2+i\varepsilon}\,\bar{u}_{s'}(p')(-ie\gamma^\nu)v_{r'}(k')$$

$$\frac{-i\eta_{\mu\nu}}{(p-p')^2+i\varepsilon}\,\bar{v}_{r'}(k)(-ie\gamma^\nu)u_{r'}(k')$$

$$\mathcal{M} = \left(\bar{v}_r(k)(-ie\gamma^\mu)u_s(p)\right)\frac{-i\eta_{\mu\nu}}{(p+k)^2+i\varepsilon}\left(\bar{u}_{s'}(p')\right.$$

$$\left.-\bar{u}_{s'}(p')(-ie\gamma^\mu)u_s(p)\right)\frac{-i\eta_{\mu\nu}}{(p-p')^2+i\varepsilon}\bar{v}_{r'}(k)(-ie\gamma^\nu)u_{r'}(k$$

$$i\mathcal{M} = \left(\bar{v}_r(k)(-ie\gamma^\mu)u_s(p)\right)\frac{-i\eta_{\mu\nu}}{(p+k)^2+i\varepsilon}\left(\bar{u}_{s'}(p')(-ie\right.$$

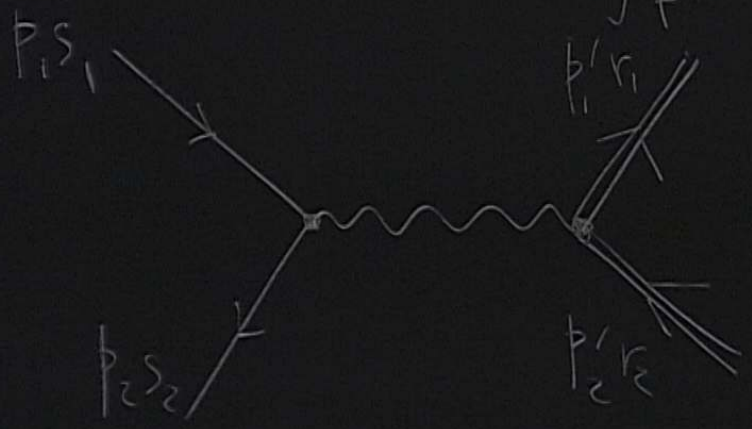
$$= \bar{u}_{s'}(p')(-ie\gamma^\mu)u_s(p)\frac{-i\eta_{\mu\nu}}{(p-p')^2+i\varepsilon}\bar{v}_{r'}(k)(-ie\gamma^\nu)v_{r'}(k')$$

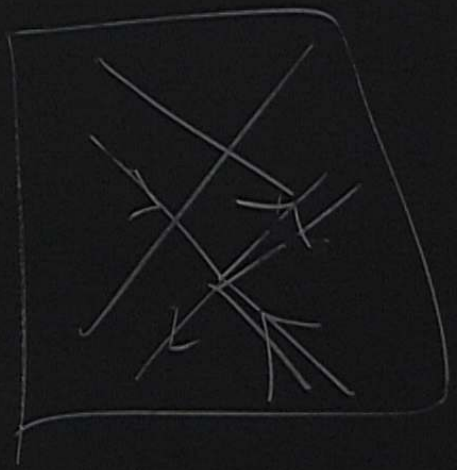
$e^+e^- \longrightarrow \mu^+\mu^-$

$$\mathcal{L} = \bar{\psi}_e(i\not{\partial} - m_e)\psi_e + \bar{\psi}_\mu(i\not{\partial} - m_\mu)\psi_\mu$$

$$+ \mathcal{L}_{e.m.} + \mathcal{L}_{g.f.}$$

$i\mathcal{M} =$



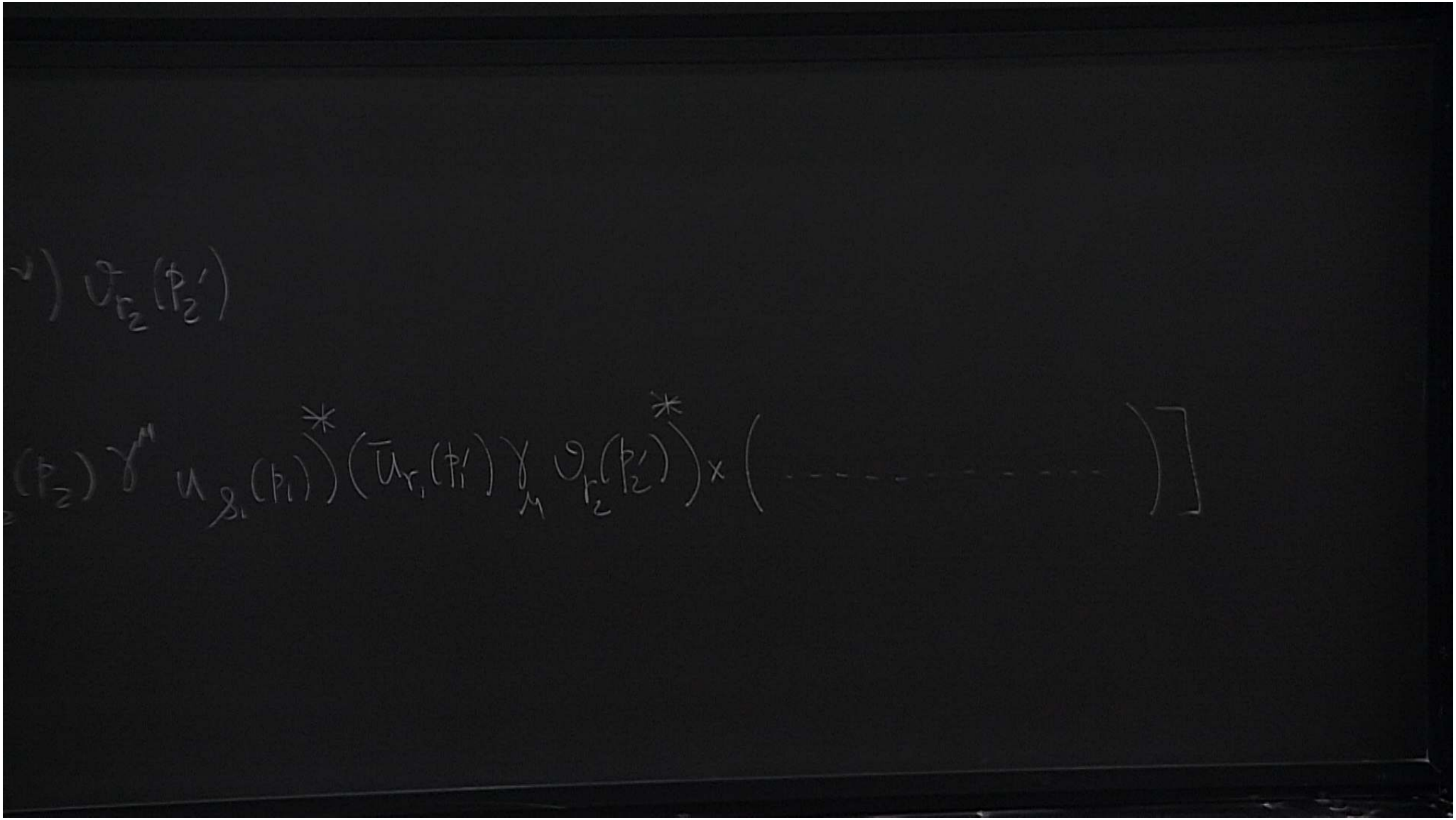


$$i\mathcal{M} = \bar{v}_{s_2}(p_2)(-ie\gamma^\mu)u_{s_1}(p_1)$$

$$\frac{-i\eta_{\mu\nu}}{(p_1+p_2)^2}\,\bar{u}_{r_1}(p_1')(-ie\gamma^\nu)v_{r_2}(p_2')$$

$$iM = \bar{v}_{s_2}(p_2)(-ie\gamma^\mu)u_{s_1}(p_1)\frac{-i\eta_{\mu\nu}}{(p_1+p_2)^2}\bar{u}_{r_1}(p_1')(-ie\gamma^\nu)v_{r_2}(p_2')$$

$$\frac{1}{4}\sum_{\substack{s_1,r_1\\ s_2,r_2}}|M|^2 = \frac{1}{4}\sum\frac{e^4}{(p_1+p_2)^4}\left[\left(\bar{v}_{s_2}(p_2)\gamma^\mu u_{s_1}(p_1)\right)\left(\bar{u}_{r_1}(p_1')\gamma_\mu v_{r_2}(p_2')\right)^*\times\right.$$



$\gamma^0 \gamma^{\mu\dagger} \gamma^0 = \gamma^\mu$

$(\gamma^0)^2 = 1$

$\gamma^\mu) \, v_{r_2}(p_2')$

$\left(\bar{u}_{s_2}(p_2)\gamma^\mu u_{s_1}(p_1)\right)^* \left(\bar{u}_{r_1}(p_1')\gamma_\mu v_{r_2}(p_2')\right)^* \times (\cdots\cdots\cdots\cdots\cdots)\Big]$

$\left(\bar{u}_{s_2}(p_2)\gamma^\mu u_{s_1}(p_1)\right)^* = u^\dagger_{s_1}(p_1) \, (\gamma^0)^2 \, \gamma^{\mu\dagger} \gamma^0 \, v_{s_2}(p_2) = \bar{u}_{s_1}(p_1)\gamma^\mu v_{s_2}(p_2)$

$$\frac{1}{4}\sum|\mathcal{M}|^2 = \frac{e^4}{4(p+k)^4}\sum\Big[\underbrace{\left(\bar{u}_{s_1}(p_1)\gamma^\mu v_{s_2}(p_2)\right)\left(\bar{v}_{s_2}(p_2)\gamma^\nu u_{s_1}(p_1)\right)}\times\left(\bar{u}_{r_1}(p_1')\right.$$

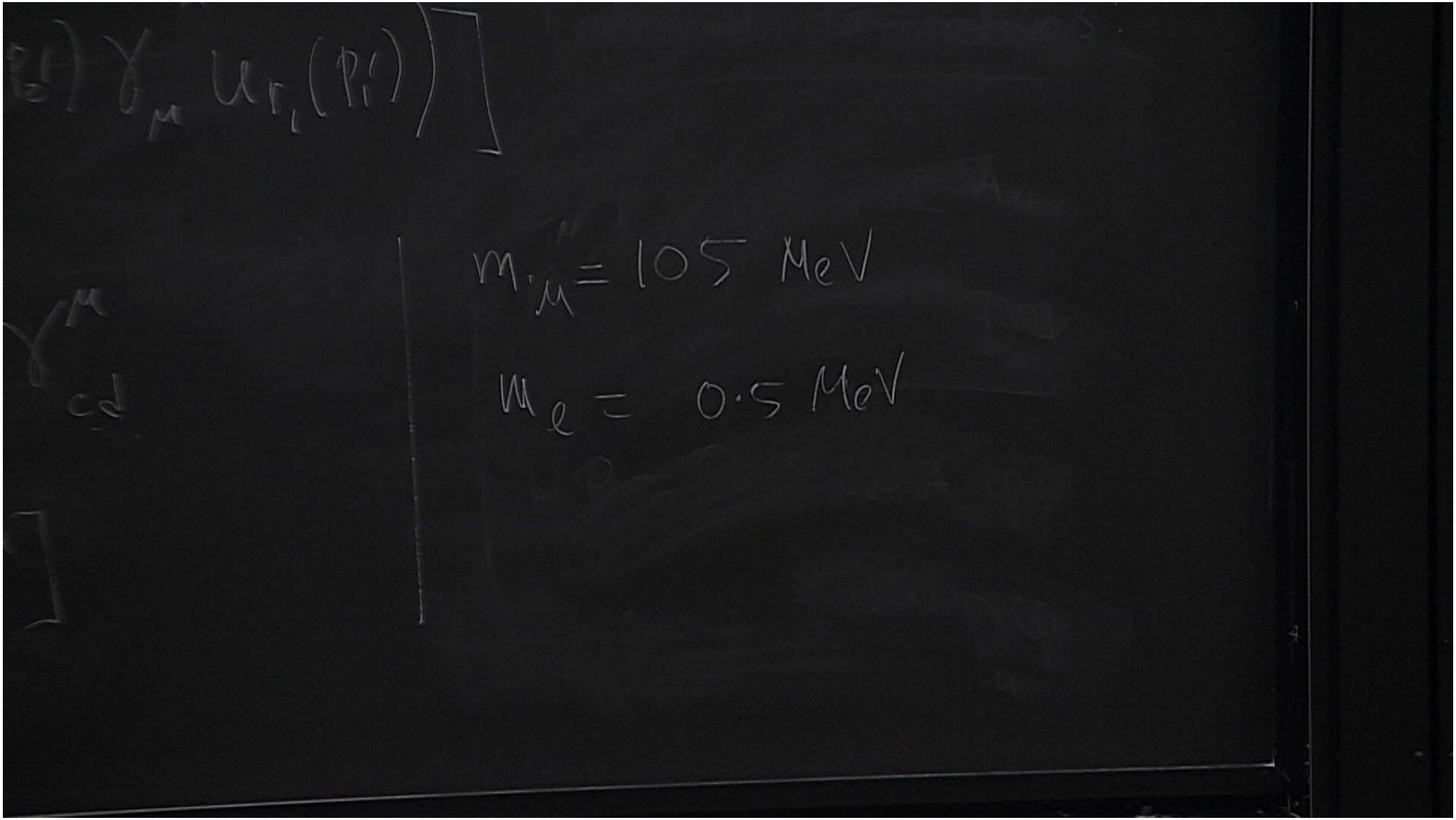
$$\sum_{s_1 s_2}\bar{v}_{s_2}(p_2)_a\gamma^\nu_{ab}u_{s_1}(p_1)_b\;\bar{u}_{s_1}(p_1)_c\gamma^\mu_{cd}v_{s_2}(p_2)_d = (\not{p}_2 - m_e)_{da}$$

$$\begin{cases}\sum_r v_r(p)\bar{v}_r(p) = \not{p} - m_e\\ \sum_r u_r(p)\bar{u}_r(p) = \not{p} + m_e\end{cases}$$

$$\gamma^\nu u_{s_1}(p_1)\Big) \times \Big(\bar{u}_{r_1}(p_1')\,\gamma_\nu\, v_{r_2}(p_2')\;\bar{v}_{r_2}(p_2')\,\gamma_\mu\, u_{r_1}(p_1')\Big)\Big]$$

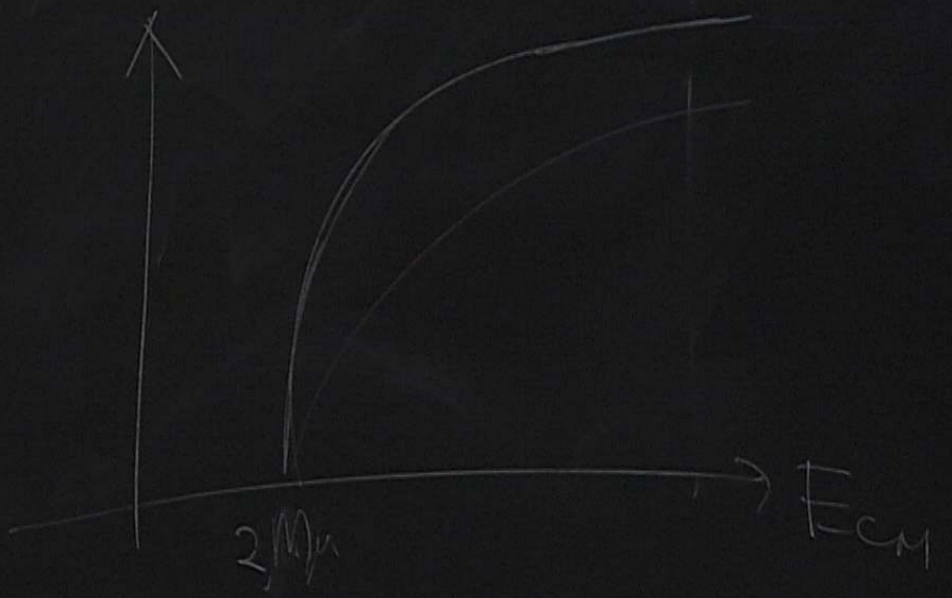
$$= (\not{p}_2 - m_e)_{da}\,\gamma^\nu_{ab}\,(\not{p}_1 + m_e)_{bc}\,\gamma^\mu_{cd}$$

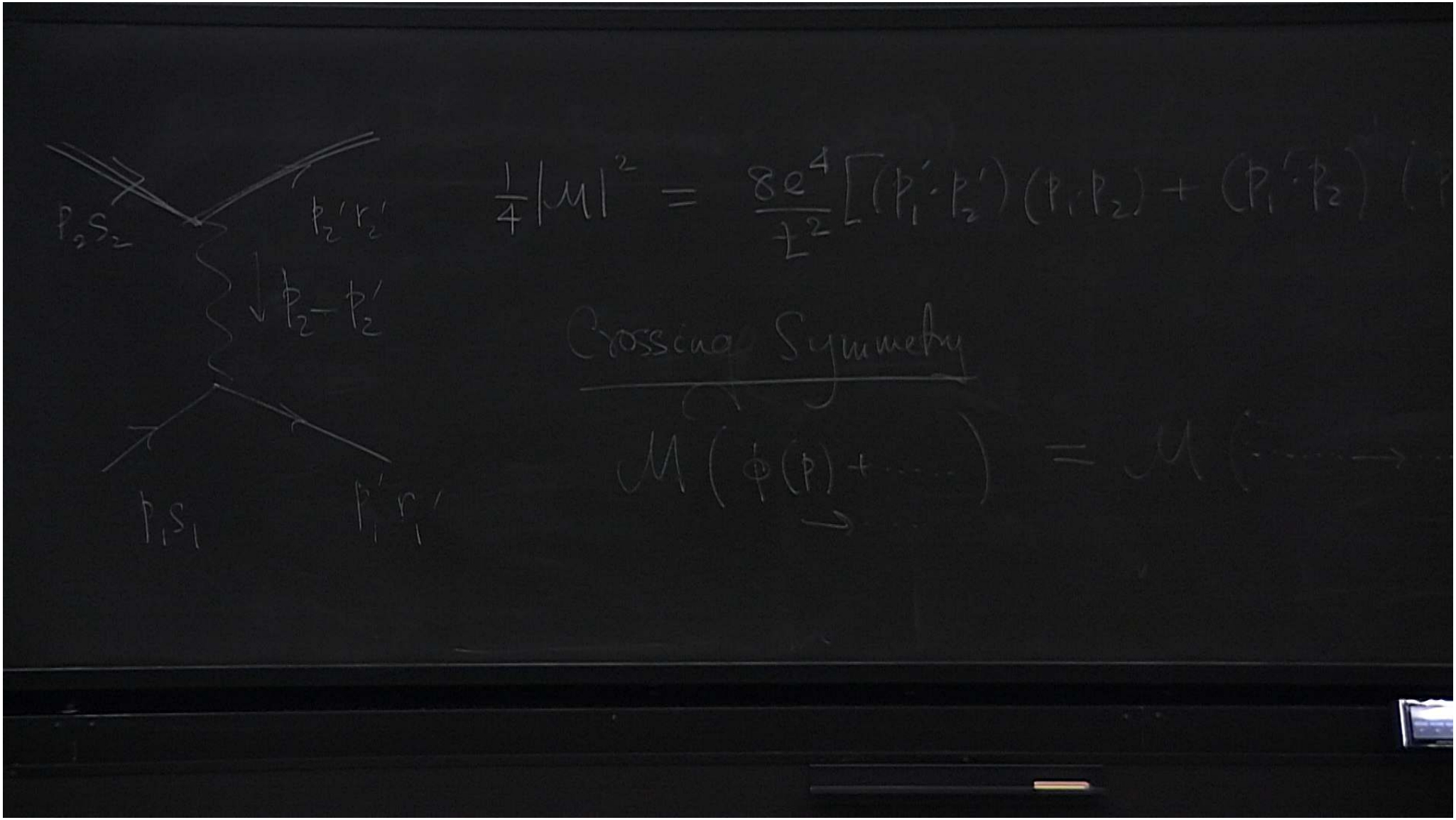
$$= \mathrm{Tr}\left[(\not{p}_2 - m_e)\gamma^\nu(\not{p}_1 + m_e)\gamma^\mu\right]$$

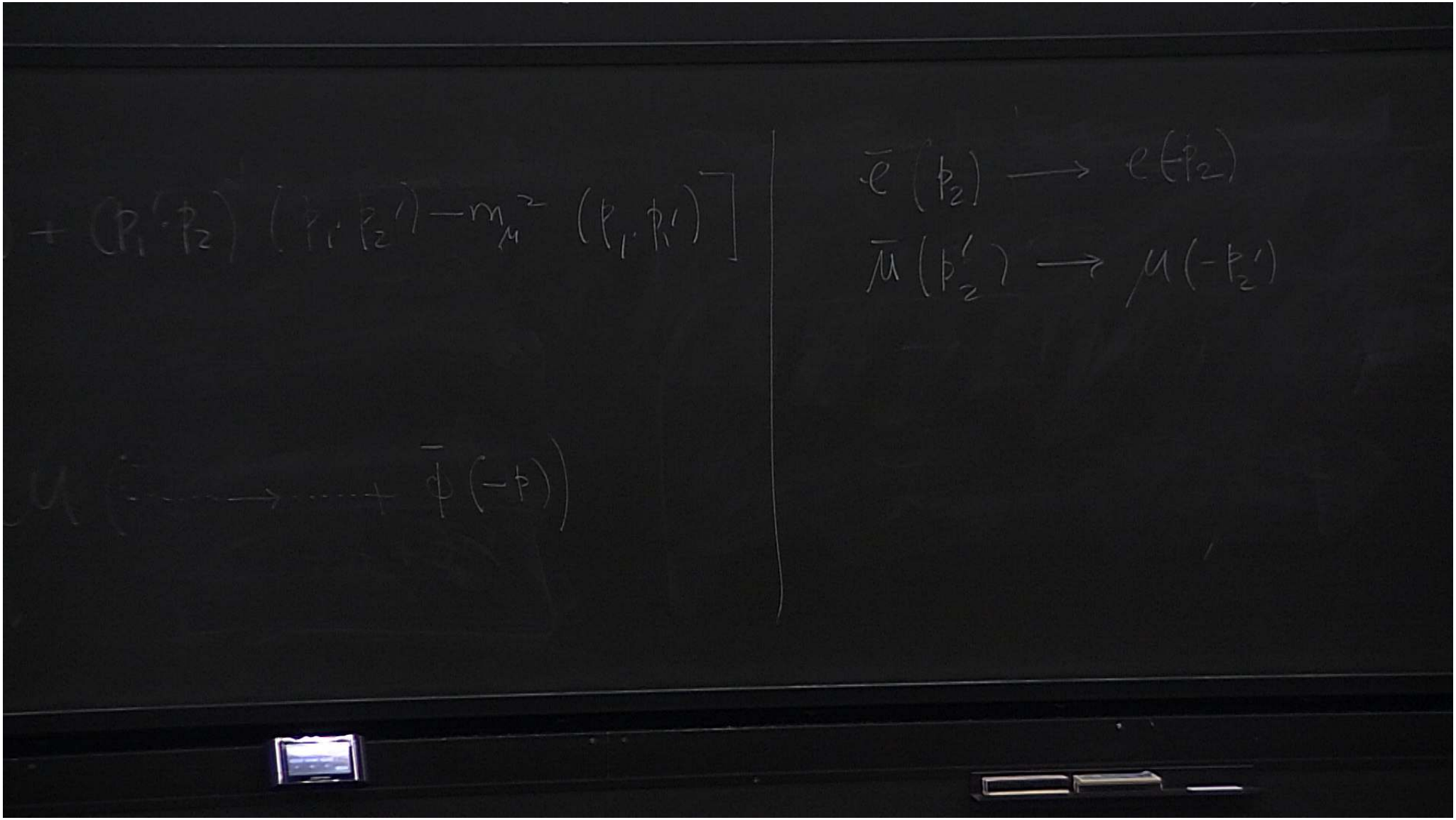

$m_\mu = 105$ MeV
$m_e = 0.5$ MeV

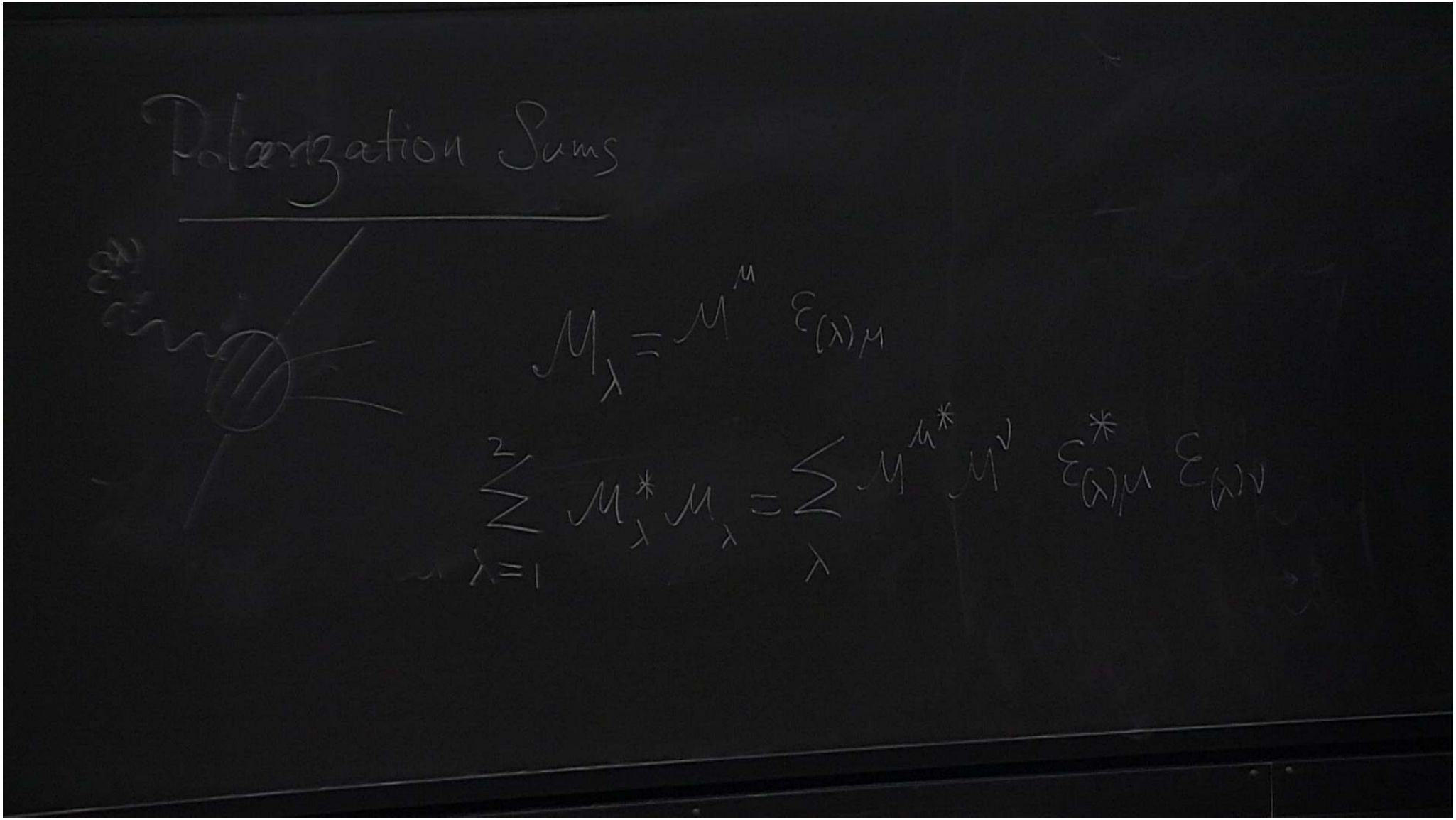
$$\sigma_{tot} = \frac{4\pi\alpha^2}{3E_{CoM}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left(1+\frac{1}{2}\frac{m^2}{E^2}\right)$$

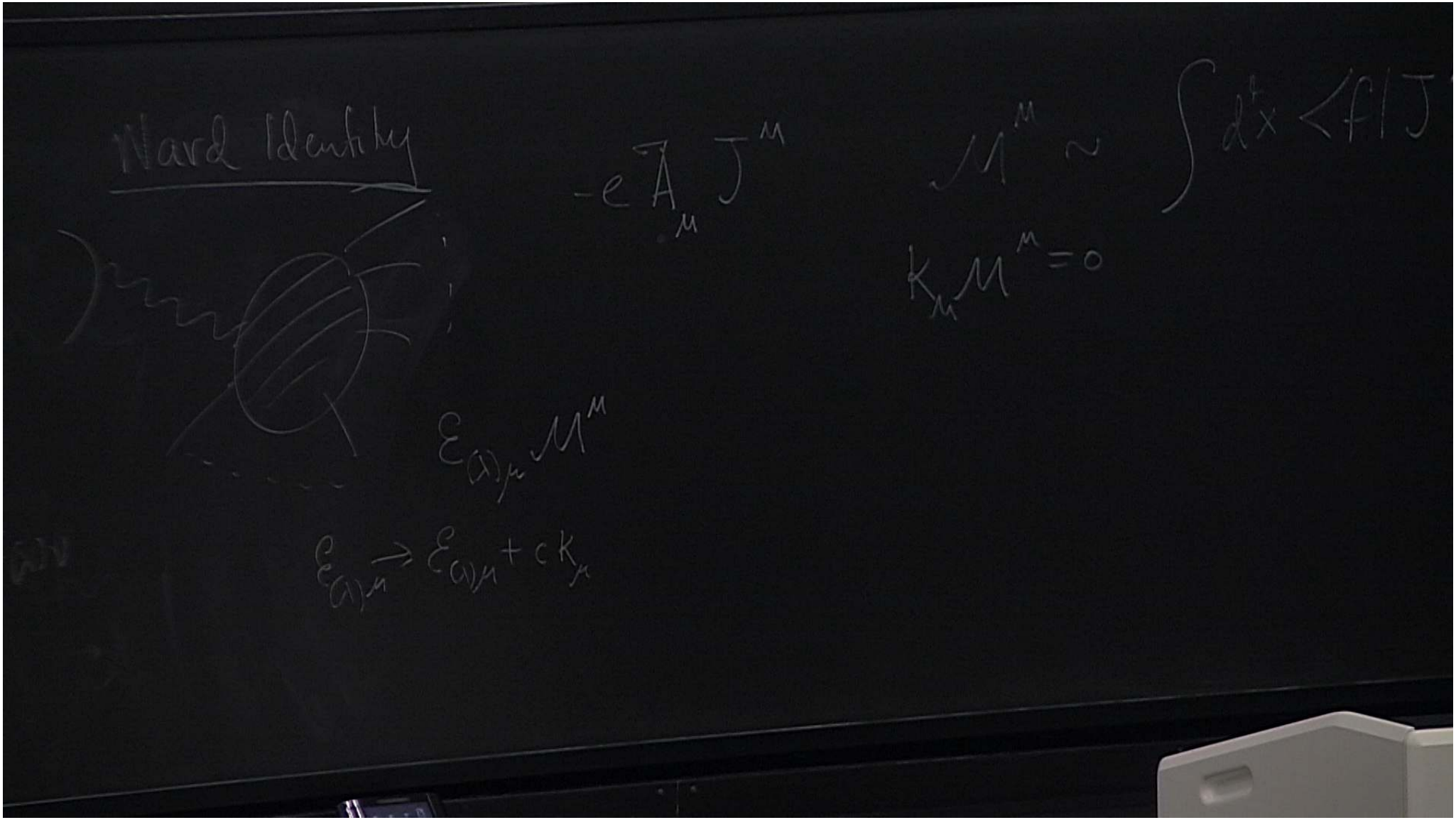
$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137.04}$$

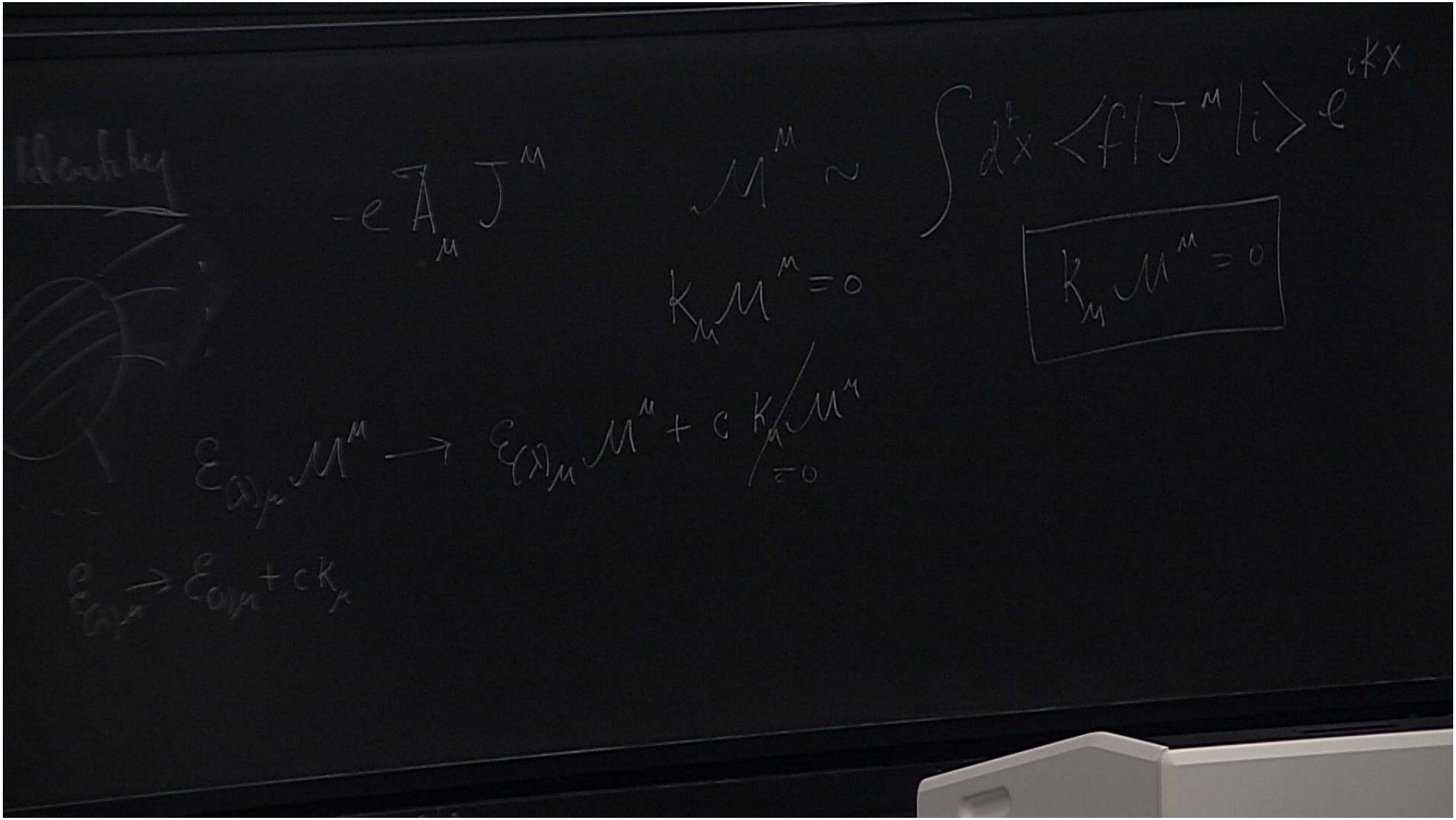


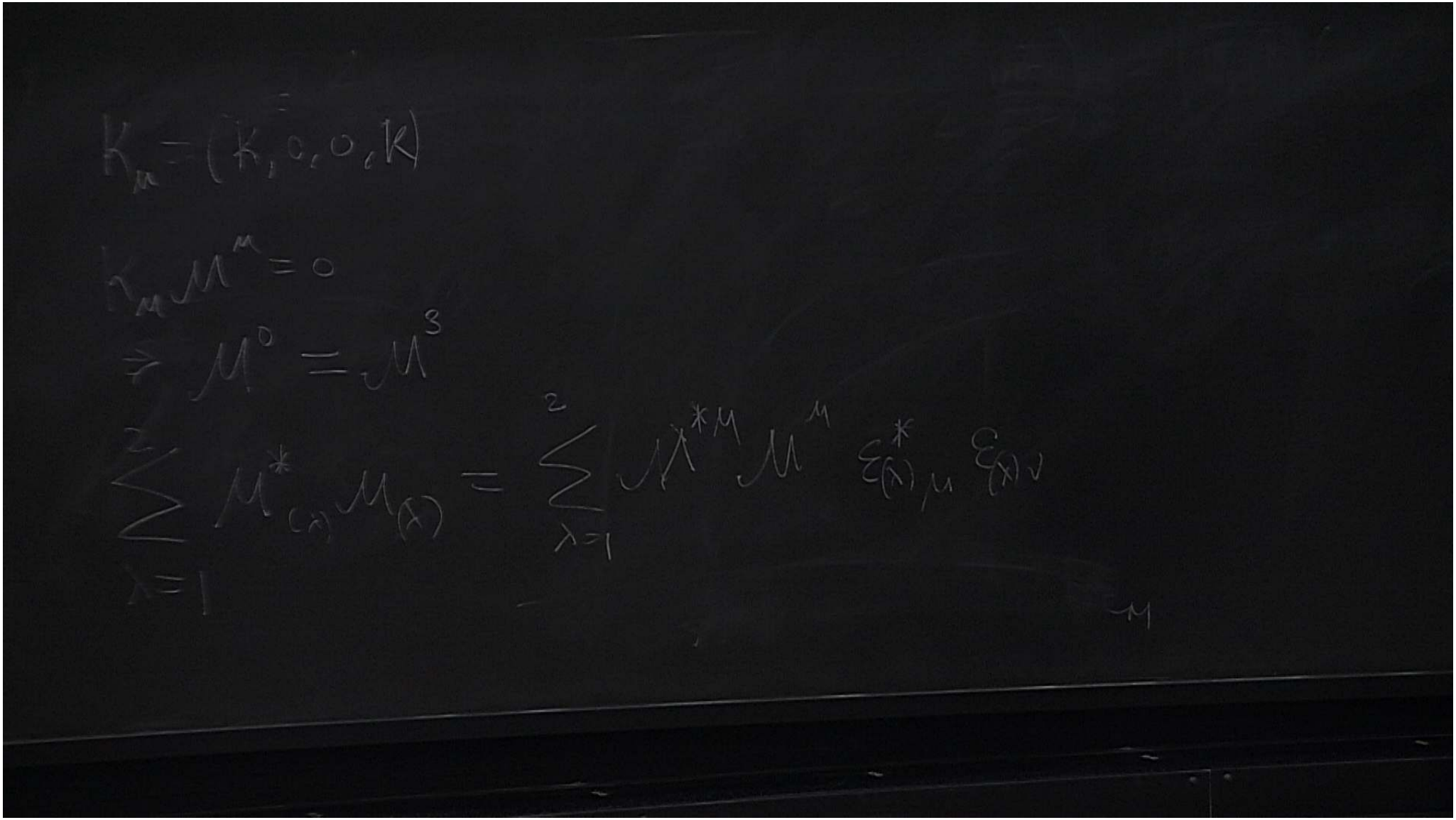

$p_2 s_2$
$p_2' r_2'$
$p_2 - p_2'$
$p_1 s_1$
$p_1' r_1'$
$\frac{1}{4}|\mathcal{M}|^2 = \frac{8e^4}{t^2}[(p_1' \cdot p_2')(p_1 \cdot p_2) + (p_1' \cdot p_2$
Crossing Symmetry
$\mathcal{M}(\phi(p) + \ldots) = \mathcal{M}($

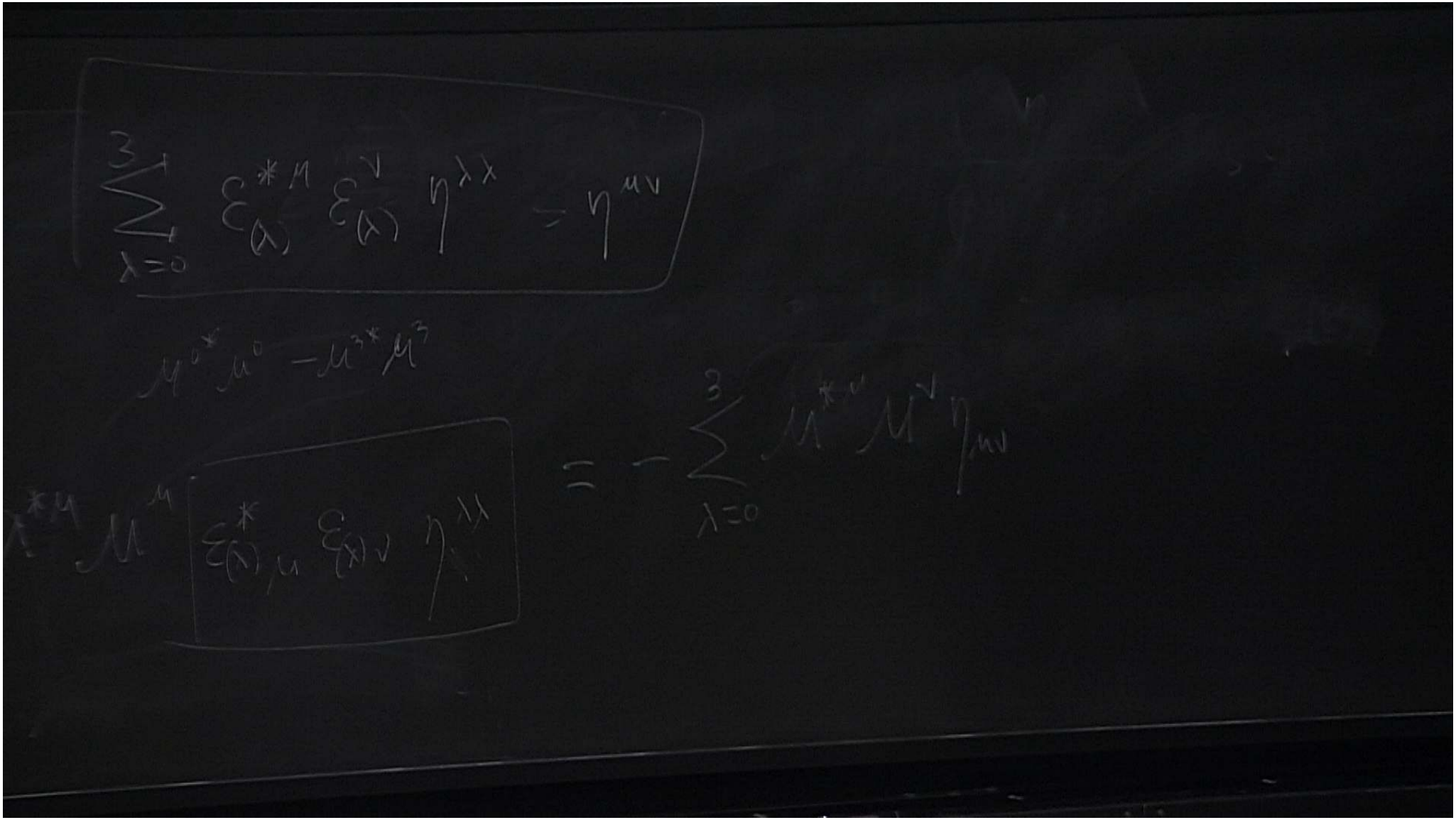

$+ (p_1 \cdot p_2)(p_1 \cdot p_2') - m_\mu^2 (p_1 \cdot p_1')]$
$\bar{\psi}(-p))$
$\bar{e}(p_2) \longrightarrow e(-p_2)$
$\bar{\mu}(p_2') \longrightarrow \mu(-p_2')$

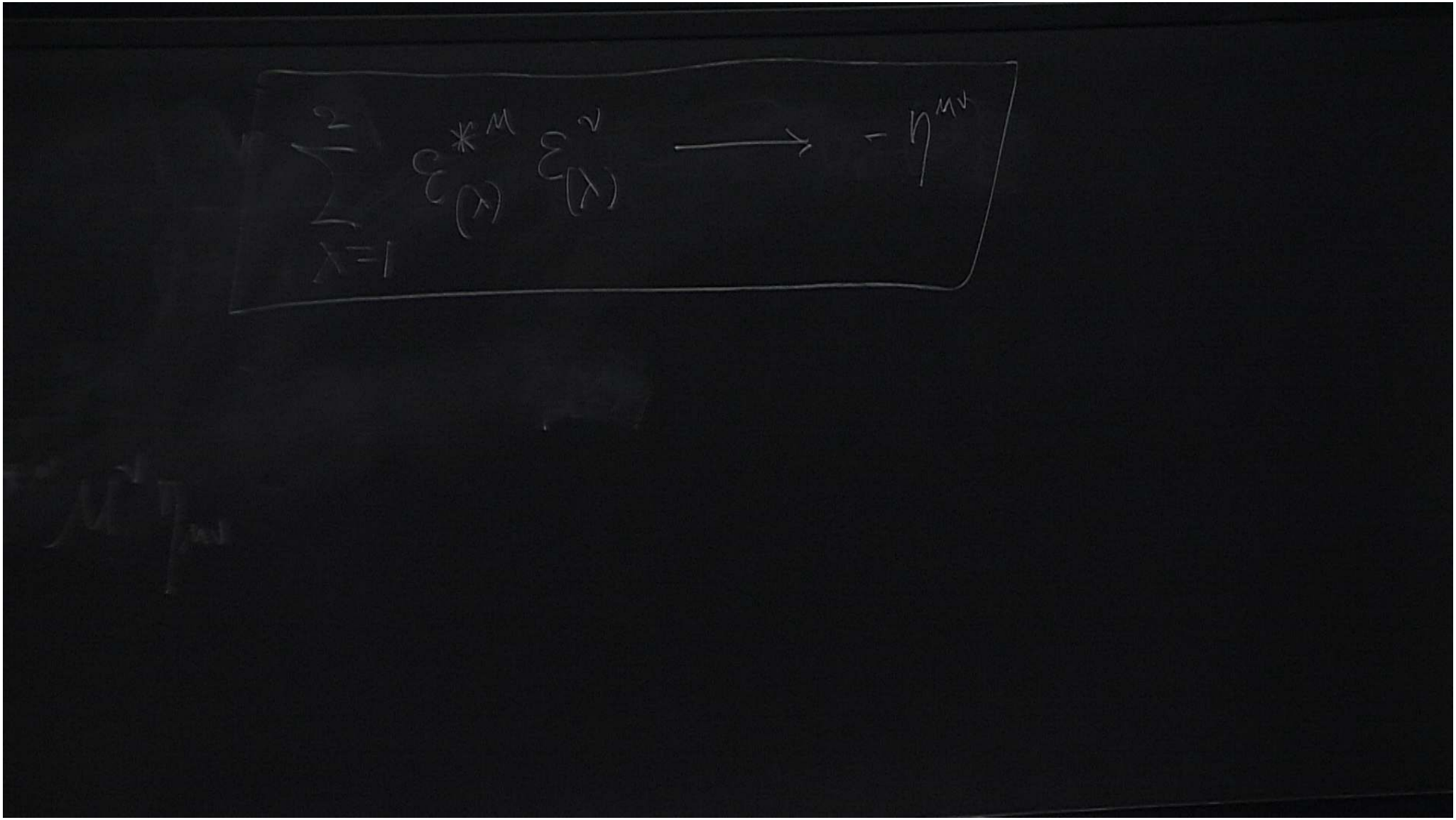

Polarization Sums
$\mathcal{M}_\lambda = \mathcal{M}^\mu \varepsilon(\lambda)_\mu$
$\sum_{\lambda=1}^{2} \mathcal{M}_\lambda^* \mathcal{M}_\lambda = \sum_\lambda \mathcal{M}^{\mu *} \mathcal{M}^\nu \varepsilon^*_{(\lambda)\mu} \varepsilon_{(\lambda)\nu}$


Ward Identity
$-e\tilde{A}_\mu J^\mu$
$\mathcal{M}^\mu \sim \int d^4x \langle f|J$
$k_\mu \mathcal{M}^\mu = 0$
$\varepsilon_{(1)\mu} \mathcal{M}^\mu$
$\varepsilon_{(1)\mu} \to \varepsilon_{(1)\mu} + c k_\mu$


Identity
$-e\vec{A}_\mu J^\mu$
$\mathcal{M}^\mu \sim \int d^4x \langle f|J^\mu|i\rangle e^{ikx}$
$k_\mu \mathcal{M}^\mu = 0$
$k_\mu \mathcal{M}^\mu = 0$
$\varepsilon_{(\lambda)\mu}\mathcal{M}^\mu \to \varepsilon_{(\lambda)\mu}\mathcal{M}^\mu + c\,k_\mu \mathcal{M}^\mu$
$\varepsilon_{(\lambda)\mu} \to \varepsilon_{(\lambda)\mu} + c k_\mu$

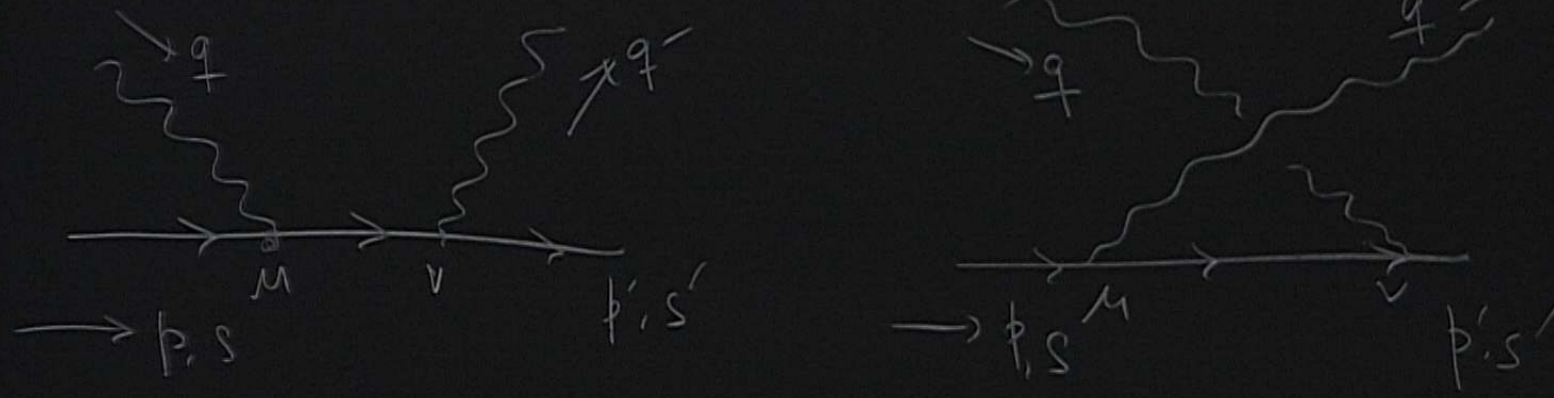

$K_\mu = (k, 0, 0, k)$
$K_\mu u^\mu = 0$
$\Rightarrow u^0 = u^3$
$\sum_{\lambda=1}^{2} u^*_{(\lambda)} u_{(\lambda)} = \sum_{\lambda=1}^{2} u^{*\mu} u^\nu \varepsilon^*_{(\lambda)\mu} \varepsilon_{(\lambda)\nu}$


$\sum_{\lambda=0}^{3} \varepsilon^{*\mu}_{(\lambda)} \varepsilon^{\nu}_{(\lambda)} \eta^{\lambda\lambda} = \eta^{\mu\nu}$
$u^{0*} u^{0} - u^{3*} u^{3}$
$= - \sum_{\lambda=0}^{3} u^{*\mu} u^{\nu} \eta_{\mu\nu}$
$\varepsilon^{*}_{(\lambda)\mu} \varepsilon_{(\lambda)\nu} \eta^{\mu\nu}$


$\sum_{\lambda=1}^{2} \varepsilon^{*\mu}_{(\lambda)} \varepsilon^{\nu}_{(\lambda)} \longrightarrow -\eta^{\mu\nu}$

$$i\mathcal{M} = \bar{u}_{s'}(p')\, \varepsilon^{*}_{\nu}(q')(-ie\gamma^{\nu}) \left(\frac{i(\not{p} + \not{q} + m)}{(p+q)^2 - m^2} \right.$$

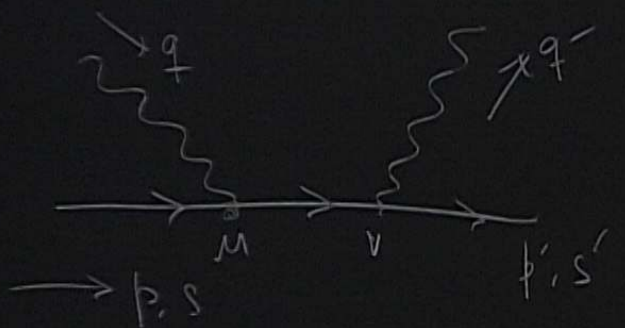
Compton Scattering

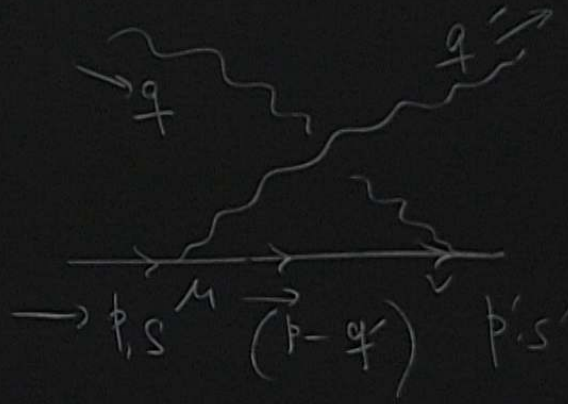


Compton Scattering

$i\mathcal{M} = \bar{u}$

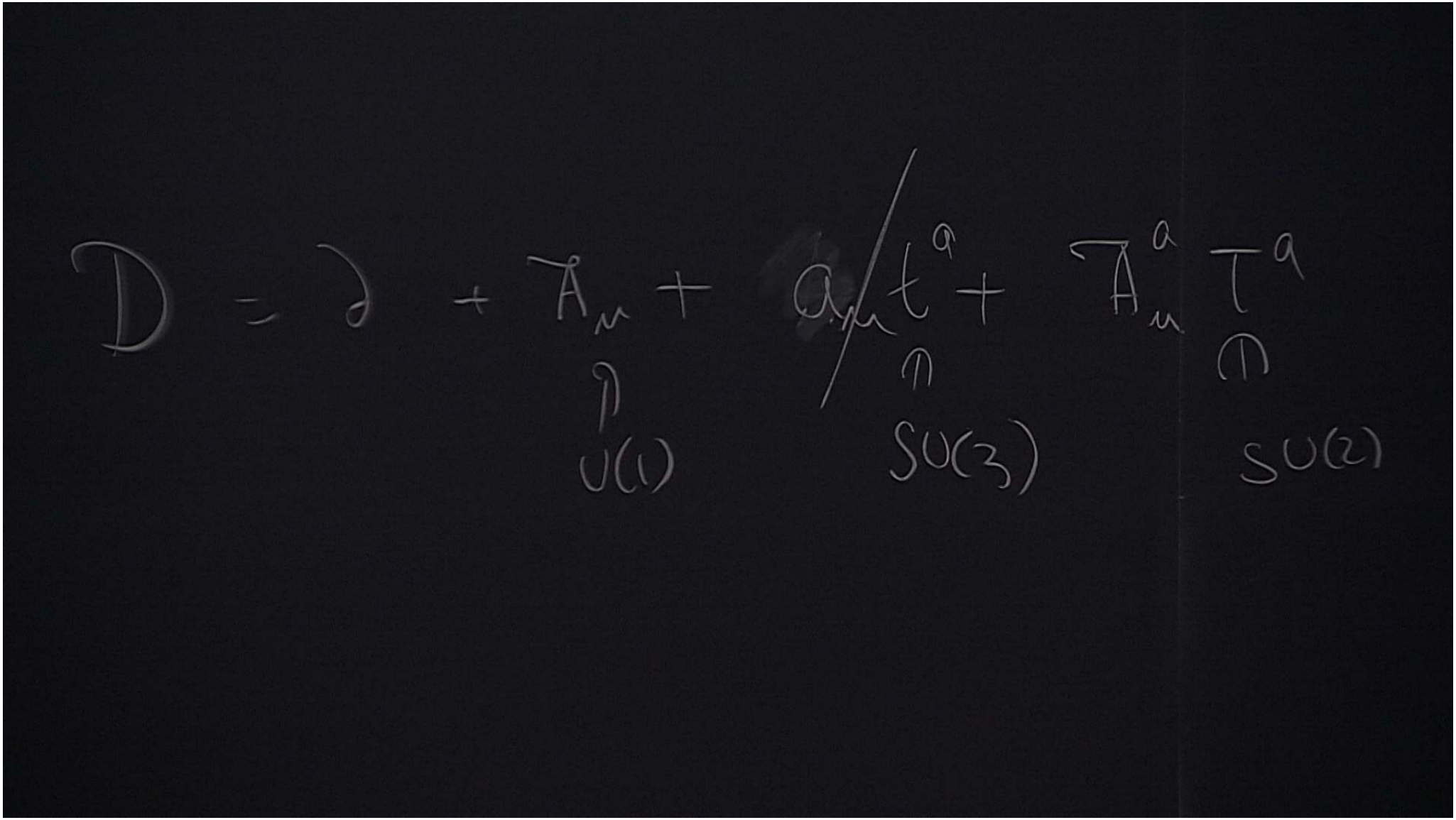
$+ \bar{u}_s$





$$(-ie\gamma^\nu)\, i\,\frac{(\not{p}+\not{q}+m)}{(p+q)^2-m^2}\,(-ie\gamma^\mu)\,\varepsilon_\mu(q)\,u_s(p)$$

$$(ie\gamma^\nu)\, i\,\frac{(\not{p}-\not{q}'+m)}{(p-q')^2-m^2}\,\varepsilon^{*}_{\mu}(q')\,(-ie\gamma^\mu)\,u_s(p)$$


$D = \partial + A_\mu + a_\mu t^a + T^a_\mu T^a$
$\mathbb{R}$
$U(1)$
$\cap$
$SU(3)$
$\cap$
$SU(2)$