Title: PSI 17/18 - Quantum Field Theory I - Lecture 12

Date: Oct 25, 2017 09:00 AM

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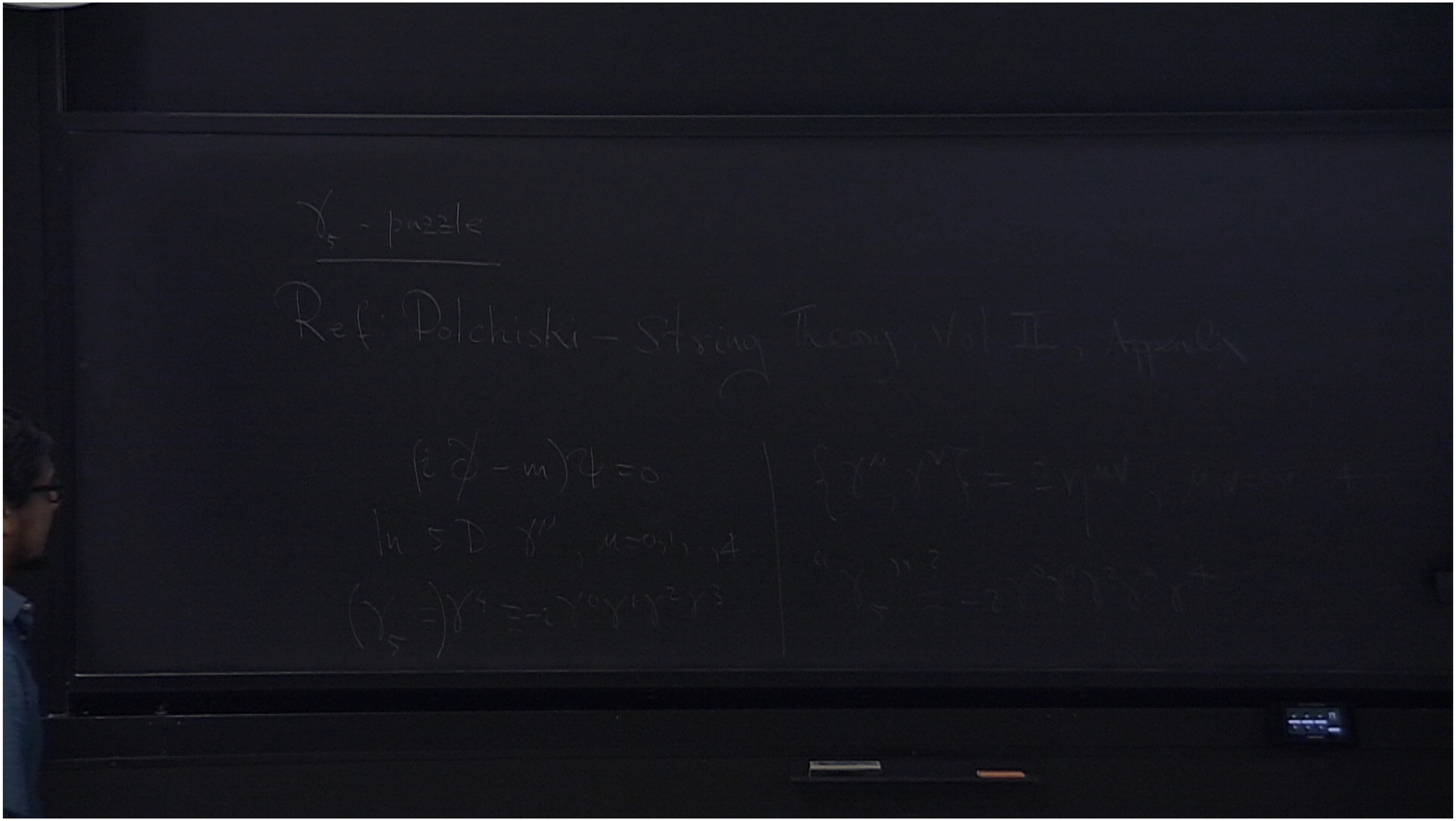
Abstract:



γ5 - puzzle
Ref: Polchiski - String Theory, Vol II, Appendix
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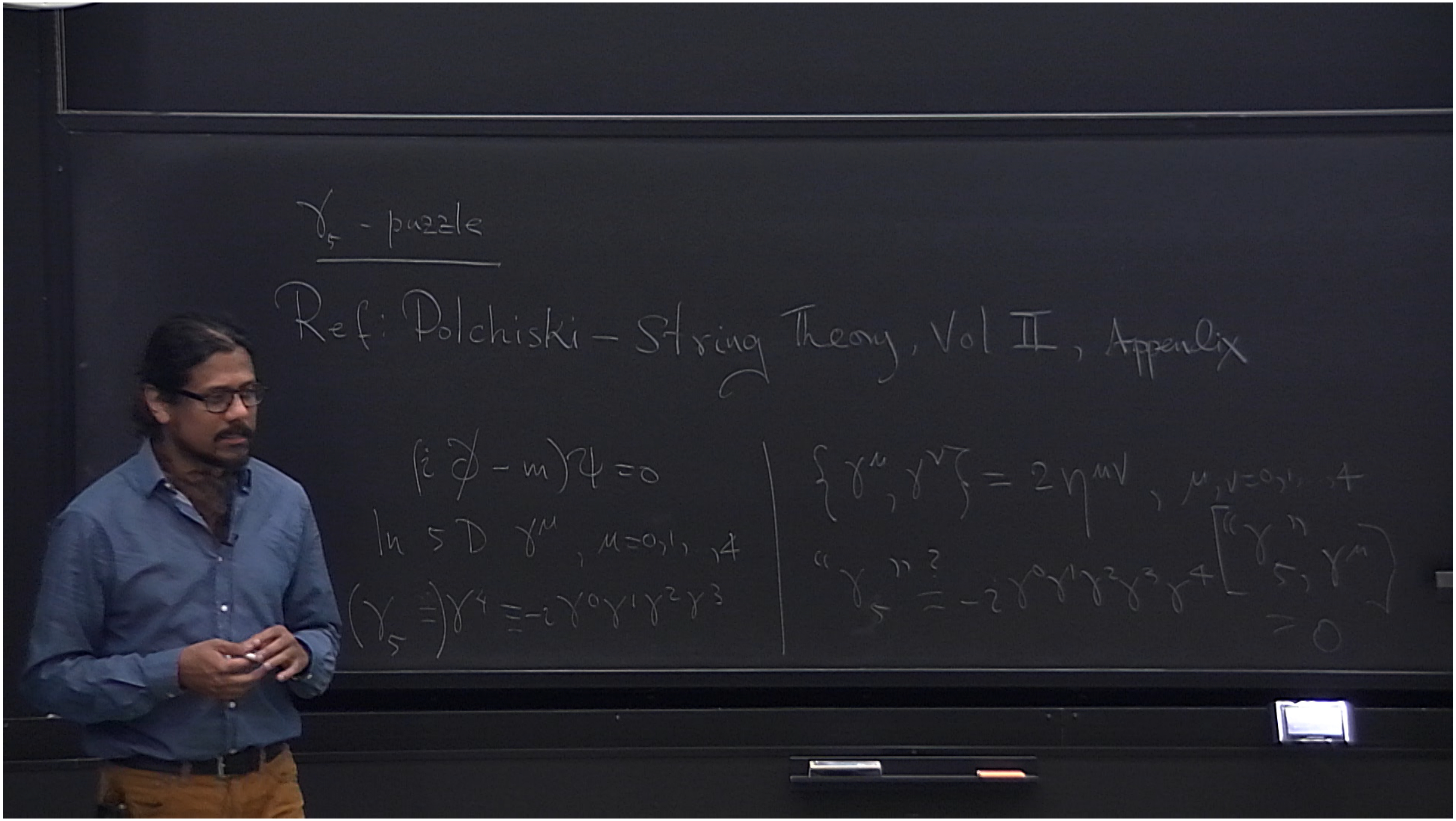
In 5D γ^μ, μ = 0, 1, , 4
(γ5 =) γ^4 = -i γ^0 γ^1 γ^2 γ^3
{γ^μ, γ^ν} = 2η^μν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"γ5" ≟ -i γ^0 γ^1 γ^2 γ^3 γ^4
["γ5", γ^μ] = 0

In 1934, Yukawa proposed that nucleons interact by exchanging a heavy spin-0 pseudo-scalar massive particle called meson. This accounted for the extremely short range of the strong interaction of nucleons. This theory has remained very interesting beyond its original scope because it is a renormalizable theory and its generalization provides the interaction terms between fermions of the standard model.

The version of the Yukawa theory that we consider in this lecture is a theory of nucleons and antinucleons, described by a spinor field $\psi(x)$ and its conjugate $\overline{\psi}(x)$, interacting via exchanging a massive spin-0 particle, described by a real scalar field $\varphi(x)$. The Lagrangian for the theory is given by

$$\mathcal{L}_{\text{Nucl.}} = \mathcal{L}_f + \mathcal{L}_\varphi + \mathcal{L}_{\text{Yuk.}} \tag{9}$$

where,

$$\mathcal{L}_f = \overline{\psi}(i\partial\!\!\!/ - m)\psi \tag{10}$$

$$\mathcal{L}_\varphi = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}M^2\varphi^2 \tag{11}$$

$$\mathcal{L}_{\text{Yuk}} = -\lambda\varphi\overline{\psi}\psi, \tag{12}$$

where $\lambda > 0$ and $[\lambda] = 0$. Notice that $\mathcal{L}_{\text{Nucl.}}$ is invariant under a global U(1) phase rotation:

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where $\lambda > 0$ and $[\lambda] = 0$. Notice that $\mathcal{L}_{\text{Nucl.}}$ is invariant under a global U(1) phase rotation:

$$\begin{aligned} \psi(x) &\to e^{i\alpha}\psi(x) \\ \overline{\psi}(x) &\to e^{-i\alpha}\overline{\psi}(x) \end{aligned} \tag{13}$$

where the states on the right-hand-side are in the Heisenberg picture and they are time-independent. We can write the S operator as

$$S = \mathbb{1} + iT \tag{15}$$

Since we are only interested in that part of the amplitude which describes some intermediate interaction we want to compute the part that only involves the T operator. Associated to this amplitude we define a matrix element $\mathcal{M}$ by

$$\langle(\vec{p}_1, s_1), (\vec{p}_2, s_2)| T |(\vec{k}_1, r_1), (\vec{k}_2, r_2)\rangle = (2\pi)^4 \delta^4(k_1 + k_2 - p_1 - p_2) \times i\mathcal{M}(k_1 k_2 \to p_1 p_2). \tag{16}$$

Using the LSZ reduction formula and the Dyson series expansion of the interacting vacuum in terms of the free vacuum[1], we are interested in the computation below:

$$\langle(\vec{p}_1, s_1), (\vec{p}_2, s_2)| iT |(\vec{k}_1, r_1), (\vec{k}_2, r_2)\rangle$$
$$= \lim_{T \to \infty(1-i\epsilon)} \left\{ {}_0\langle(\vec{p}_1, s_1), (\vec{p}_2, s_2)| \hat{T} \exp\left[-i \int_{-T}^{T} dt H_I(t)\right] |(\vec{k}_1, r_1), (\vec{k}_2, r_2)\rangle_0 \right\}_{\text{connected, amputated}}$$

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Wick's Theorem

Before we can compute these matrix elements we shall need Wick's theorem for fermions. For two fermions fields, it is eas to show Wick's theorem

$$\hat{T}[\psi_a(x)\overline{\psi}_b(y)]=:\psi_a(x)\overline{\psi}_b(y):+\psi_a(x)\overline{\psi}_b(y). \qquad (18)$$

where the contraction is defined by

$$\psi_a(x)\overline{\psi}_b(y)=\begin{cases}\{\psi_a^+(x),\overline{\psi}_b^-(y)\}\text{ if }x^0>y^0\\ \ldots\end{cases} \qquad (19)$$

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where the contraction is defined by

$$\wick{\c{\psi}_a(x)\c{\overline{\psi}}_b(y)} = \begin{cases} \{\psi_a^+(x), \overline{\psi}_b^-(y)\} \text{ if } x^0 > y^0 \\ -\{\overline{\psi}_b^+(y), \psi_a^-(x)\} \text{ if } x^0 > y^0 \end{cases} \tag{19}$$

This can be shown to be the Feynman propagator:

$$\wick{\c{\psi}_a(x)\c{\overline{\psi}}_b(y)} = \langle 0|\, \hat{T}(\psi_a(x)\overline{\psi}_b(y))\, |0\rangle \,. \tag{20}$$

We also define the following contractions which are computed to be zero:

$$\begin{aligned} \wick{\c{\psi}\c{\psi}} &= \{\psi, \psi\} = 0 \\ \wick{\c{\overline{\psi}}\c{\overline{\psi}}} &= \{\overline{\psi}, \overline{\psi}\} = 0 \end{aligned} \tag{21}$$

[1]For details see chapter 4 of [2].

In-Class Group Exercise

Assume $x^0 > y^0$ and prove Wick's theorem for two fermion fields. It is helpful to decompose the field in terms of its positive- and negative-frequency parts:

$$\begin{aligned} \psi &= \psi^+ + \psi^- \\ \overline{\psi} &= \overline{\psi}^- + \overline{\psi}^+ \end{aligned} \tag{22}$$

where ψ^+ (ψ^-) contains the annihilation (creation) operator. Similarly, $\overline{\psi}^+$ ($\overline{\psi}^-$) also contains the annihilation (creation) operator.

Explanation

Since we assume $x^0 > y^0$, we can write

$$\begin{aligned} \hat{T}(\psi(x)\overline{\psi}(y)] &= \psi(x)\overline{\psi}(y) \\ &= (\psi^+ + \psi^-)(\overline{\psi}^- + \overline{\psi}^+) \\ &= \psi^+\overline{\psi}^- + \psi^+\overline{\psi}^+ + \psi^-\overline{\psi}^- + \psi^-\overline{\psi}^+ \\ &= (\overline{\psi}^-\psi^+ - \overline{\psi}^-\psi^+) + \psi^+\overline{\psi}^- + \psi^+\overline{\psi}^+ + \psi^-\overline{\psi}^- + \psi^-\overline{\psi}^+ \end{aligned} \tag{23}$$

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\hat{T}(\psi(x)\overline{\psi}(y)] &= \psi(x)\overline{\psi}(y) \\
&= (\psi^{+} + \psi^{-})(\overline{\psi}^{-} + \overline{\psi}^{+}) \\
&= \psi^{+}\overline{\psi}^{-} + \psi^{+}\overline{\psi}^{+} + \psi^{-}\overline{\psi}^{-} + \psi^{-}\overline{\psi}^{+} \\
&= (\overline{\psi}^{-}\psi^{+} - \overline{\psi}^{-}\psi^{+}) + \psi^{+}\overline{\psi}^{-} + \psi^{+}\overline{\psi}^{+} + \psi^{-}\overline{\psi}^{-} + \psi^{-}\overline{\psi}^{+} \\
&= \{\psi^{+}, \overline{\psi}^{-}\} - \overline{\psi}^{-}\psi^{+} + \psi^{+}\overline{\psi}^{+} + \psi^{-}\overline{\psi}^{-} + \psi^{-}\overline{\psi}^{+} \\
&= S_F(x-y) + :\psi(x)\overline{\psi}(y):
\end{aligned} \tag{23}$$

One can extend the Wick theorem to multi-fermion fields to obtain

$$= \{\psi^+, \overline{\psi}^-\} - \overline{\psi}^- \psi^+ + \psi^+ \overline{\psi}^+ + \psi^- \overline{\psi}^- + \psi^- \overline{\psi}^+$$

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One can extend the Wick theorem to multi-fermion fields to obtain

$$\hat{T}[\psi_1 \overline{\psi}_2 \psi_3 \ldots] =: \psi_1 \overline{\psi}_2 \psi_3 \cdots : + \text{ all possible contractions } \times : \text{remaining fields} : \tag{24}$$

To figure out the sign of the last term we work out the example of one contraction. If the field and the conjugate field are not adjacent to each other, one needs to permute the field inside the normal ordering with the appropriate number of minus signs until they are next to each other. Then we can bring out the contraction outside the normal ordering:

$$: \overbrace{\psi_1 \psi_2 \overline{\psi}_3} \cdots : = - : \overbrace{\psi_1 \overline{\psi}_3} \psi_2 \cdots := - \overbrace{\psi_1 \overline{\psi}_3} (: \psi_2 \cdots :) \tag{25}$$

the contracted part outside the normal ordering is a c-number function.

5 Feynman Rules from Scattering Processes

We want to extract the Feynman rules for fermions from the elementary processes of Yukawa theory.

5.1 Nucleon-Nucleon Scattering

To this end, let us consider a nucleon-nucleon scattering process. First let us write the field operators in terms of creation/annihilation operators:

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_r \left[u_r(\vec{p}) b_r(p) e^{-ip\cdot x} + v_r(p) c_r^\dagger(p) e^{ip\cdot x}\right] \tag{26}$$

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$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left[a(p) e^{-ip\cdot x} + a^\dagger(p) e^{ip\cdot x}\right]. \tag{28}$$

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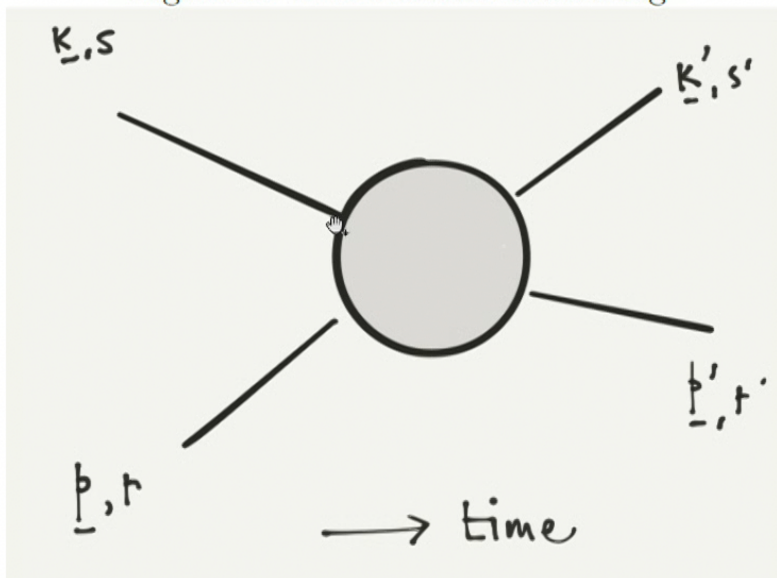
Graphically we represent this process as in figure 3. The interaction Hamiltonian for the Yukawa theory is

Figure 3: Two-Particle Scattering



$$H_I = \lambda \int d^3x \varphi(x) \overline{\psi}(x) \psi(x). \tag{32}$$

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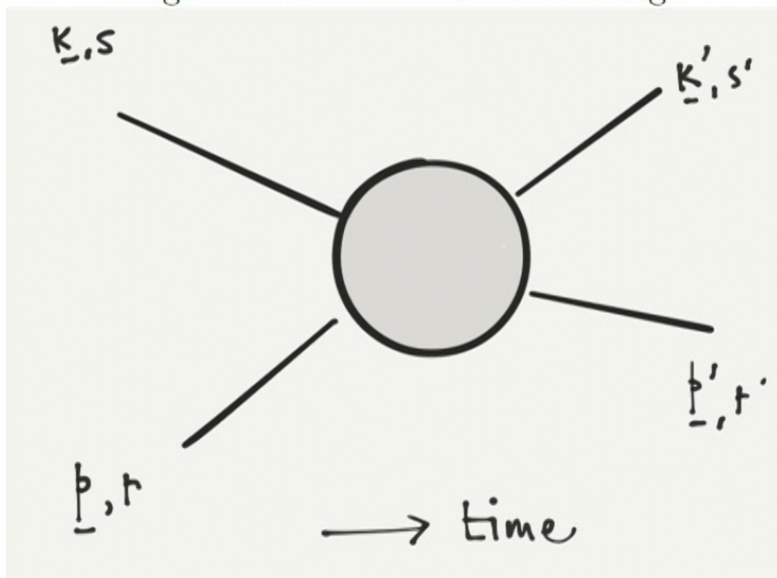

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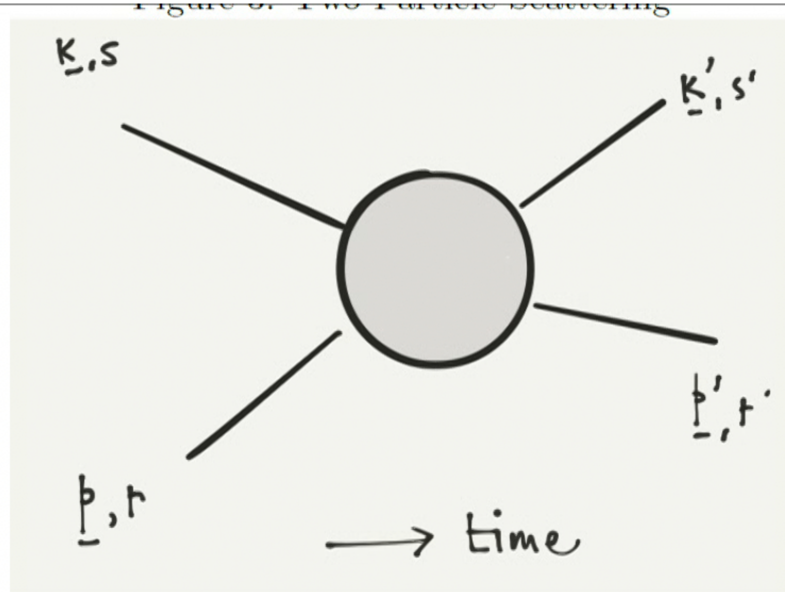

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We can denote the part of $\varphi(x)$ containing the annihilation and creation operators by $\varphi^+(x)$ and $\varphi^-(x)$, respectively. Since neither our in- or out-states contain any mesons, we have

$$\varphi^+(x)\,|\text{in}\rangle = 0 \tag{33}$$

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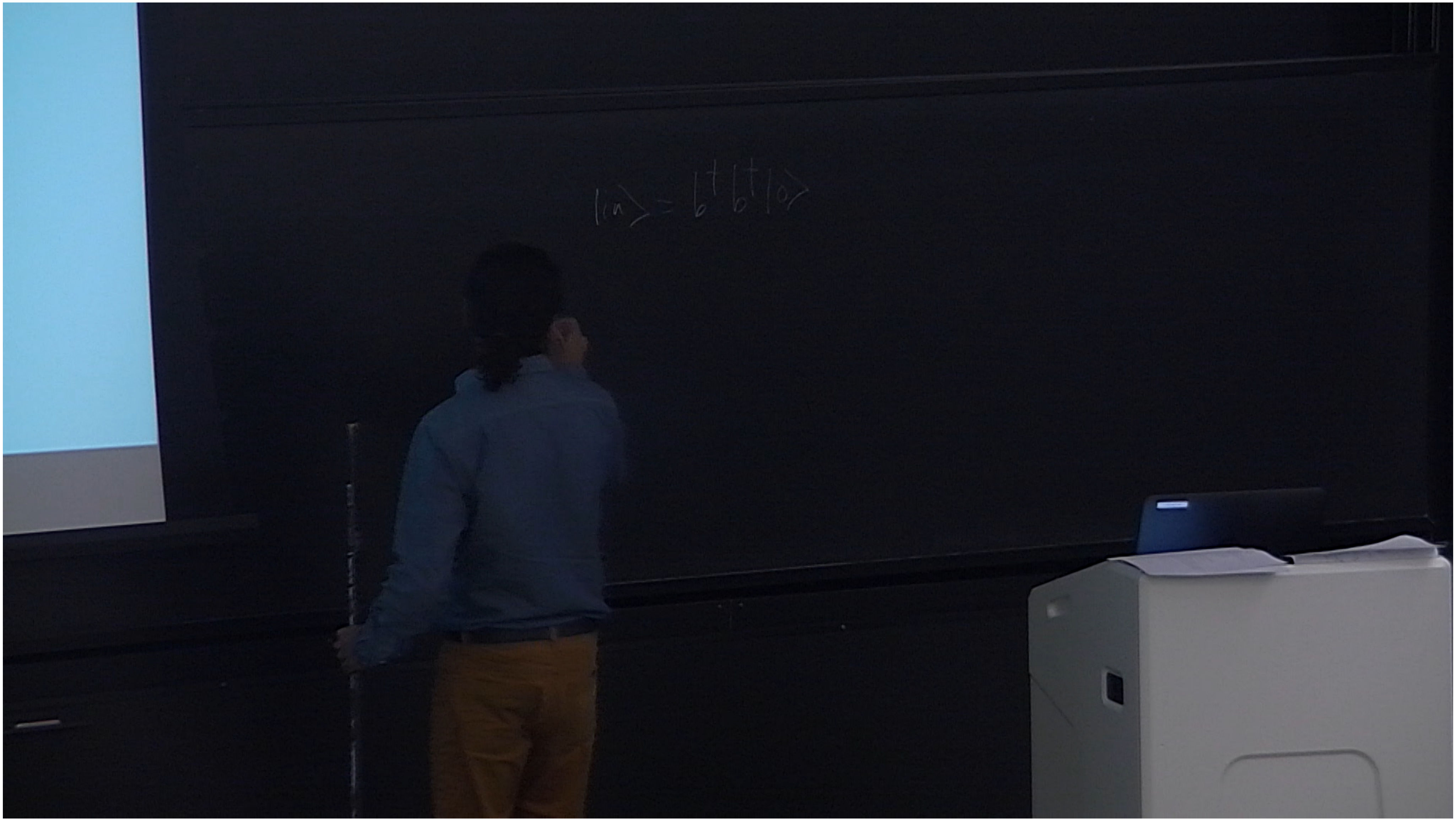
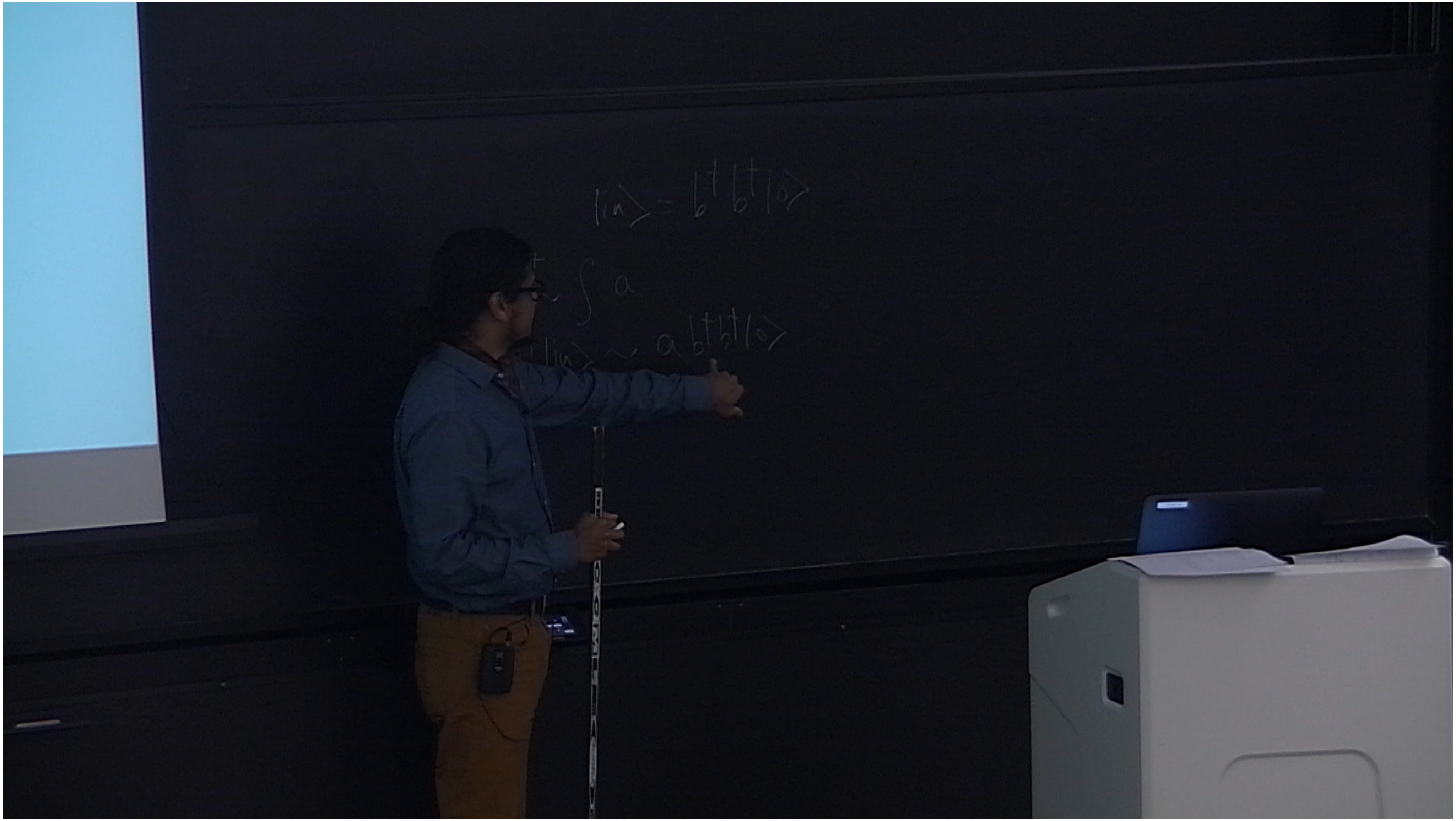
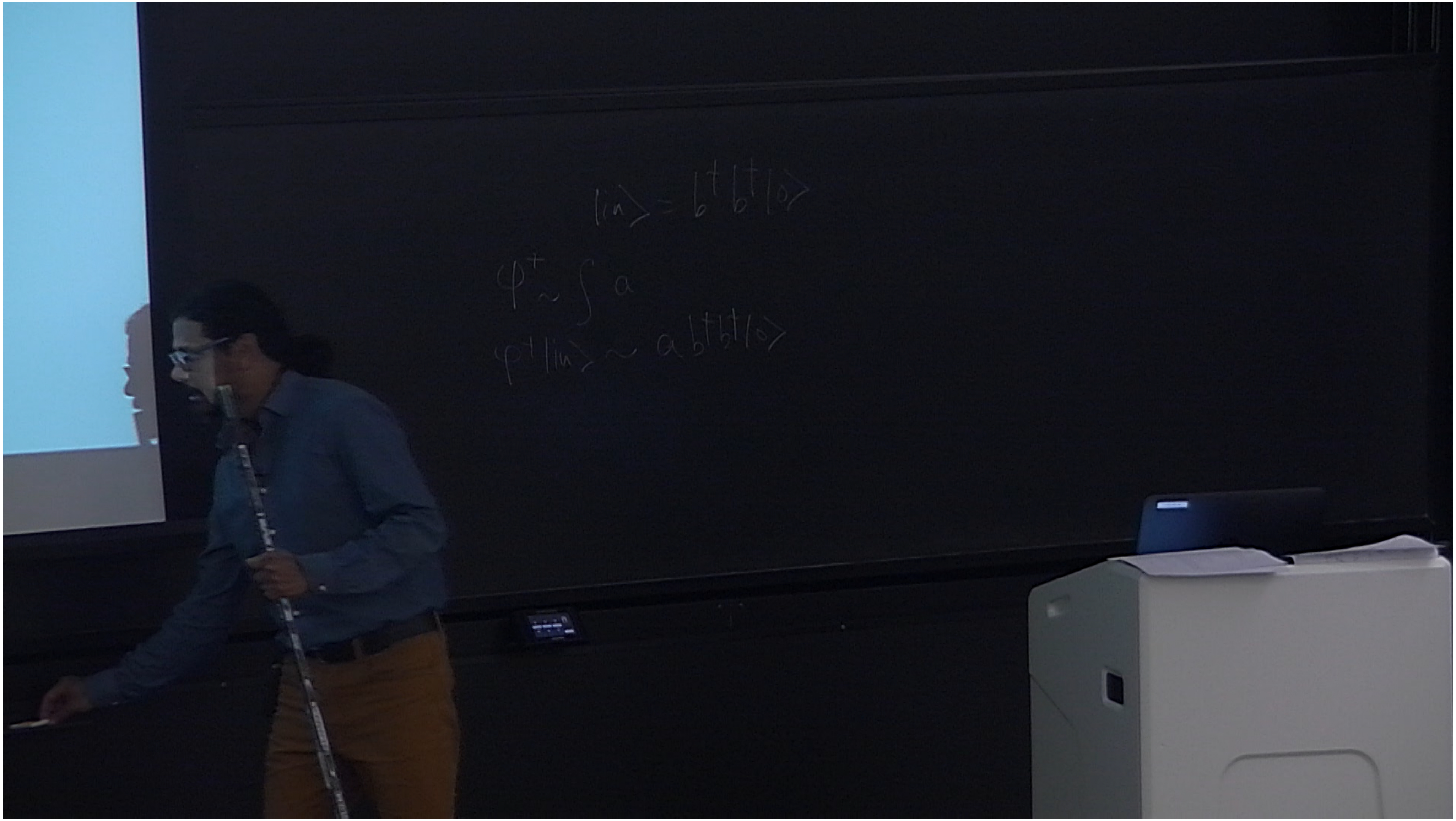

$|in\rangle = b^\dagger b^\dagger |0\rangle$




|in> = b† b† |0>
φ† ~ ∫ a
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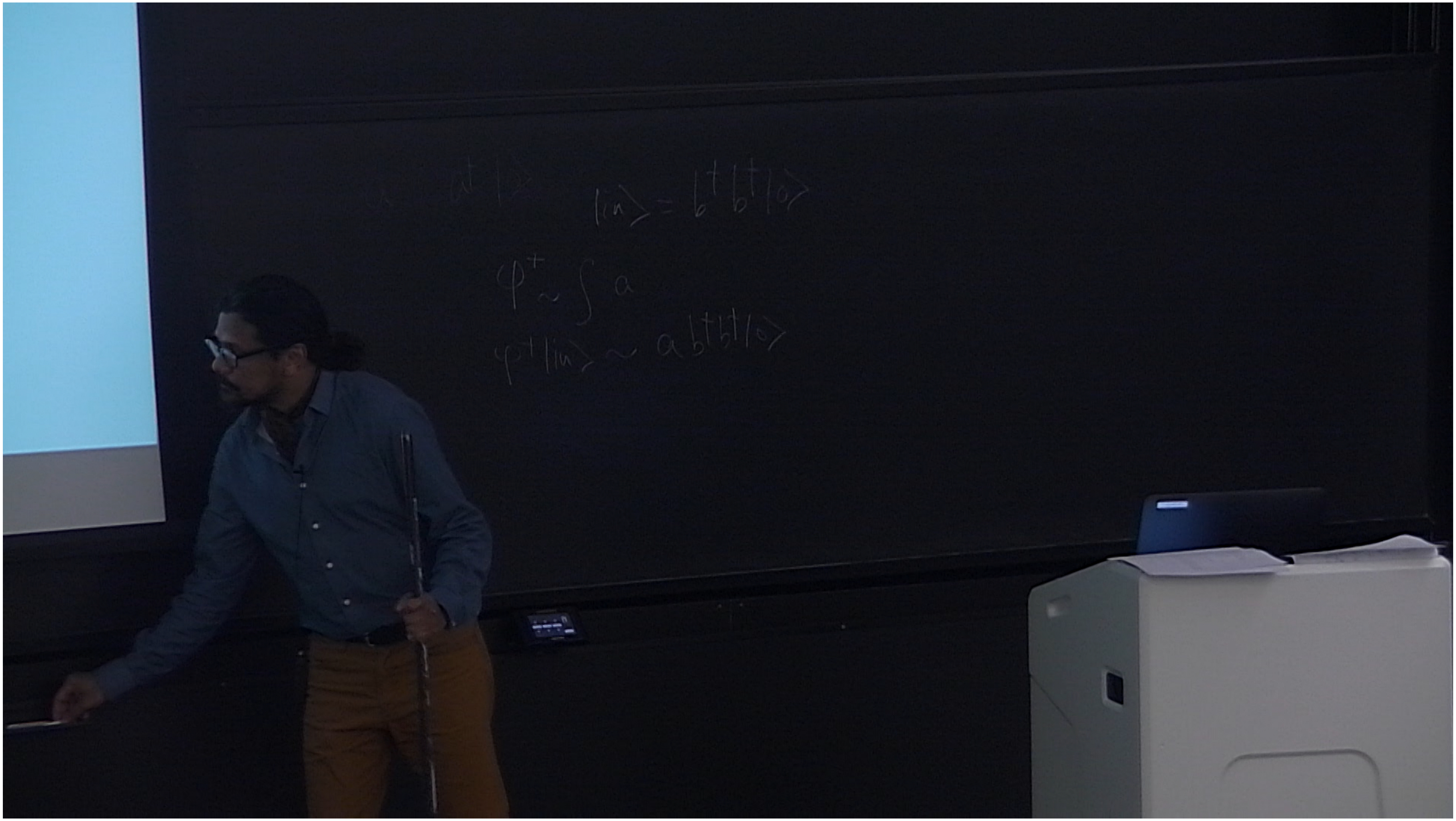
|in> = b† b† |0>
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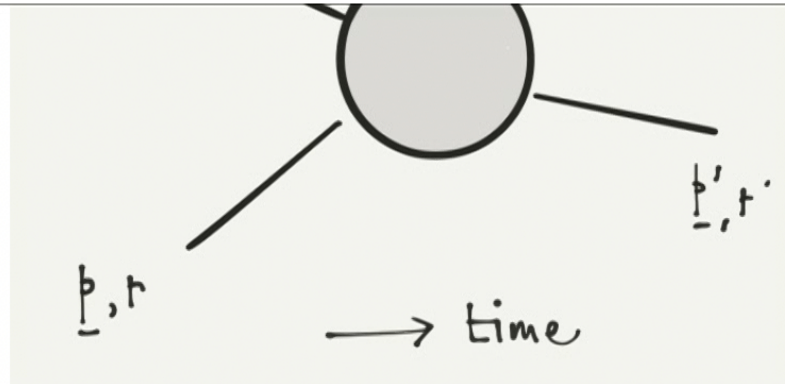

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$\mathcal{O}(\lambda^2)$ contribution:

On the other hand, in the $\mathcal{O}(\lambda^2)$ term we expect there to be a Wick contraction of the form $\varphi\overbracket{(x_1)\varphi}(x_2)$ and so that term won't be annihilated. Explicitly,

$$iT = \frac{(-i\lambda)^2}{2!}\,\langle\text{out}|\,\hat{T}\int d^4x_1 d^4x_2(\varphi\overline{\psi}\psi)_1(\varphi\overline{\psi}\psi)_2\,|\text{in}\rangle$$

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$$\text{[Using Wick's theorem:]} \tag{36}$$
$$= \frac{(-i\lambda)^2}{2!}\langle\text{out}|\int d^4x_1 d^4x_2 : (\overline{\psi}\psi)_1(\overline{\psi}\psi)_2 : |\text{in}\rangle\,\varphi_1\varphi_2$$

Since out initial and final states only contain particles, we are only interested in those terms in $:(\overline{\psi}\psi)_1(\overline{\psi}\psi)_2:$ that contains b and $b^\dagger$ (i.e., no c or $c^\dagger$):

$$i\hat{T} = -\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2 \frac{d^3k_1 d^3k_2}{(2\pi)^6 4E_{k_1}E_{k_2}} \langle \text{out}| \sum_{t,t'} \overline{\psi}_1^{(-)} \cdot u_t(k_1)\overline{\psi}_2^{(-)} \cdot u_{t'}(k_2) e^{-ik_1\cdot x_1 - ik_2\cdot x_2} \times \\ b_t(k_1)b_{t'}(k_2) \underbrace{\left[b_s^\dagger(k) b_r^\dagger(p)\,|0\rangle\right]}_{|\text{in}\rangle} \tag{37}$$

where the overall minus sign comes from pushing $b_t(k_1)$ through $\overline{\psi}_2^{(-)}$. In the last line we have a string of four creation and annihilation operators acting on the vacuum. We can use the anticommutation relationships,

$$\begin{aligned} &b_t(k_1)b_{t'}(k_2)b_s^\dagger(k)b_r^\dagger(p)\,|0\rangle \\ &= -(2\pi)^6 4E_k E_p \left[\delta_{ts}\delta_{t'r}\delta^3(\vec{k}_1 - \vec{k})\delta^3(\vec{k}_2 - \vec{p}) - \delta_{t's}\delta_{tr}\delta^3(\vec{k}_1 - \vec{p})\delta^3(\vec{k}_2 - \vec{k})\right]|0\rangle \end{aligned} \tag{38}$$

Using the delta-functions to do the $\vec{k}_1$ and $\vec{k}_2$ integrals and using the Kronecker deltas to do the spin sums, we find

$:(\overline{\psi}\psi)_1(\overline{\psi}\psi)_2:$ that contains b and $b^\dagger$ (i.e., no c or $c^\dagger$):

$$
\begin{aligned}
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b_t(k_1)b_{t'}(k_2)\underbrace{\left[b_s^\dagger(k)b_r^\dagger(p)\,|0\rangle\right]}_{|\text{in}\rangle}
\end{aligned}
\tag{37}
$$

where the overall minus sign comes from pushing $b_t(k_1)$ through $\overline{\psi}_2^{(-)}$. In the last line we have a string of four creation and annihilation operators acting on the vacuum. We can use the anticommutation relationships,

$$
\begin{aligned}
& b_t(k_1)b_{t'}(k_2)b_s^\dagger(k)b_r^\dagger(p)\,|0\rangle \\
& = -(2\pi)^6 4E_kE_p\left[\delta_{ts}\delta_{t'r}\delta^3(\vec{k}_1-\vec{k})\delta^3(\vec{k}_2-\vec{p}) - \delta_{t's}\delta_{tr}\delta^3(\vec{k}_1-\vec{p})\delta^3(\vec{k}_2-\vec{k})\right]|0\rangle
\end{aligned}
\tag{38}
$$

Using the delta-functions to do the $\vec{k}_1$ and $\vec{k}_2$ integrals and using the Kronecker deltas to do the spin sums, we find

$$
\begin{aligned}
iT = -\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2\, \underbrace{\langle 0|\, b_{r'}(p')b_{s'}(k')}_{\langle\text{out}|}\Big\{-\overline{\psi}_1^{(-)} \cdot u_s(k)\overline{\psi}_2^{(-)} \cdot u_r(p)e^{-ik\cdot x_1 - ip\cdot x_2} \\
+\overline{\psi}_1^{(-)} \cdot u_r(p)\overline{\psi}_2^{(-)} \cdot u_s(k)e^{-ip\cdot x_1 - ik\cdot x_2}\Big\}\,|0\rangle\, \overbrace{\varphi_1\varphi_2}.
\end{aligned}
\tag{39}
$$

$$\underbrace{\qquad\qquad}_{|\text{in}\rangle}$$

where the overall minus sign comes from pushing $b_t(k_1)$ through $\overline{\psi}_2^{(-)}$. In the last line we have a string of four creation and annihilation operators acting on the vacuum. We can use the anticommutation relationships,

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Using the delta-functions to do the $\vec{k}_1$ and $\vec{k}_2$ integrals and using the Kronecker deltas to do the spin sums, we find

$$\begin{aligned} iT = & -\frac{(-i\lambda)^2}{2!}\int d^4x_1d^4x_2\,\underbrace{\langle 0|\,b_{r'}(p')b_{s'}(k')}_{\langle\text{out}|}\Big\{-\overline{\psi}_1^{(-)}\cdot u_s(k)\overline{\psi}_2^{(-)}\cdot u_r(p)e^{-ik\cdot x_1-ip\cdot x_2} \\ & +\overline{\psi}_1^{(-)}\cdot u_r(p)\overline{\psi}_2^{(-)}\cdot u_s(k)e^{-ip\cdot x_1-ik\cdot x_2}\Big\}\,|0\rangle\,\overline{\varphi_1\varphi_2}. \end{aligned} \tag{39}$$

We can now do a similar expansion for the $\overline{\psi}^{(-)}$ fields and doing a similar computation (but involving $\langle\text{out}|$) we find for the *first* term above:

$$\begin{aligned} & -\frac{(-i\lambda)^2}{2!}\int d^4x_1d^4x_2\Big[(-\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))e^{ip'\cdot x_1+ik'\cdot x_2-ik\cdot x_1-ip\cdot x_2} \\ & +(\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))e^{ik'\cdot x_1+ip'\cdot x_2-ik\cdot x_1-ip\cdot x_2}\Big]\,\Delta_F(x_1-x_2)\,\langle 0|0\rangle\,. \end{aligned} \tag{40}$$

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$$
\begin{aligned}
&b_t(k_1)b_{t'}(k_2)b_s^\dagger(k)b_r^\dagger(p)\,|0\rangle \\
&= -(2\pi)^6 4E_k E_p \left[\delta_{ts}\delta_{t'r}\delta^3(\vec{k}_1-\vec{k})\delta^3(\vec{k}_2-\vec{p}) - \delta_{t's}\delta_{tr}\delta^3(\vec{k}_1-\vec{p})\delta^3(\vec{k}_2-\vec{k})\right]|0\rangle
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$$
\begin{aligned}
iT = -\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2\, \underbrace{\langle 0|\, b_{r'}(p')b_{s'}(k')}_{\langle \mathrm{out}|} \Big\{ -\overline{\psi}_1^{(-)}\cdot u_s(k)\overline{\psi}_2^{(-)}\cdot u_r(p)e^{-ik\cdot x_1 - ip\cdot x_2} \\
+\overline{\psi}_1^{(-)}\cdot u_r(p)\overline{\psi}_2^{(-)}\cdot u_s(k)e^{-ip\cdot x_1 - ik\cdot x_2}\Big\}\,|0\rangle\, \varphi_1\varphi_2 .
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$$
\begin{aligned}
&-\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2 \Big[(-\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))e^{ip'\cdot x_1 + ik'\cdot x_2 - ik\cdot x_1 - ip\cdot x_2} \\
&+ (\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))e^{ik'\cdot x_1 + ip'\cdot x_2 - ik\cdot x_1 - ip\cdot x_2}\Big]\,\Delta_F(x_1-x_2)\,\langle 0|0\rangle\,.
\end{aligned}
\tag{40}
$$

By relabelling you can show that both terms in iT are the same and so, using the covariant expression for the scalar propagator for Δ_F:

$$= -(2\pi)^6 4E_k E_p \left[\delta_{ts}\delta_{t'r}\delta^3(\vec{k}_1 - \vec{k})\delta^3(\vec{k}_2 - \vec{p}) - \delta_{t's}\delta_{tr}\delta^3(\vec{k}_1 - \vec{p})\delta^3(\vec{k}_2 - \vec{k})\right] |0\rangle$$

Using the delta-functions to do the $\vec{k}_1$ and $\vec{k}_2$ integrals and using the Kronecker deltas to do the spin sums, we find

$$iT = -\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2 \underbrace{\langle 0| b_{r'}(p')b_{s'}(k')}_{\langle \text{out}|}\Big\{-\overline{\psi}_1^{(-)}\cdot u_s(k)\overline{\psi}_2^{(-)}\cdot u_r(p)e^{-ik\cdot x_1 - ip\cdot x_2} + \overline{\psi}_1^{(-)}\cdot u_r(p)\overline{\psi}_2^{(-)}\cdot u_s(k)e^{-ip\cdot x_1 - ik\cdot x_2}\Big\} |0\rangle\, \varphi_1\varphi_2. \tag{39}$$

We can now do a similar expansion for the $\overline{\psi}^{(-)}$ fields and doing a similar computation (but involving $\langle \text{out}|$) we find for the *first* term above:

$$-\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2 \Big[(-\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))e^{ip'\cdot x_1 + ik'\cdot x_2 - ik\cdot x_1 - ip\cdot x_2} + (\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))e^{ik'\cdot x_1 + ip'\cdot x_2 - ik\cdot x_1 - ip\cdot x_2}\Big]\Delta_F(x_1 - x_2)\langle 0|0\rangle\,. \tag{40}$$

By relabelling you can show that both terms in iT are the same and so, using the covariant expression for the scalar propagator for Δ_F:

$$\Delta_F(x-y) = i\int \frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot(x-y)}}{q^2 - M^2 + i\epsilon} \tag{41}$$

and doing the d^4x_1 and d^4x_2 integrals we get:

$$+\psi_1^{(+)} \cdot u_r(p)\psi_2^{(+)} \cdot u_s(k)e^{-ip\cdot x_1 - ik\cdot x_2}\} |0\rangle \varphi_1\varphi_2.$$

We can now do a similar expansion for the $\overline{\psi}^{(-)}$ fields and doing a similar computation (but involving $\langle \text{out}|$) we find for the *first* term above:

$$-\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2 \Big[(-\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))e^{ip'\cdot x_1+ik'\cdot x_2-ik\cdot x_1-ip\cdot x_2} + (\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))e^{ik'\cdot x_1+ip'\cdot x_2-ik\cdot x_1-ip\cdot x_2}\Big]\Delta_F(x_1-x_2)\langle 0|0\rangle . \quad (40)$$

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and doing the d^4x_1 and d^4x_2 integrals we get:

$$\begin{aligned} iT &= (-i\lambda)^2\int d^4q \frac{(2\pi)^4}{q^2-M^2+i\epsilon}\left[\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))\delta^4(p'-k+q)\delta^4(k'-p-q)\right. \\ &\quad \left. -(\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))\delta^4(k'-k+q)\delta^4(p'-p-q))\right] \\ &= (2\pi)^4\delta(p+k-p'-k')(-i\lambda)^2\left[\frac{\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))}{(k'-p)^2-M^2+i\epsilon} - \frac{(\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))}{(p'-p)^2-M^2+i\epsilon}\right] \end{aligned} \quad (42)$$

⟨out|) we find for the *first* term above:

$$-\frac{(-i\lambda)^2}{2!}\int d^4x_1 d^4x_2 \Big[(-\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))e^{ip'\cdot x_1+ik'\cdot x_2-ik\cdot x_1-ip\cdot x_2}$$
$$+(\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))e^{ik'\cdot x_1+ip'\cdot x_2-ik\cdot x_1-ip\cdot x_2}\Big]\,\Delta_F(x_1-x_2)\,\langle 0|0\rangle\,. \tag{40}$$

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and doing the d^4x_1 and d^4x_2 integrals we get:

$$iT = (-i\lambda)^2\int d^4q\frac{(2\pi)^4}{q^2-M^2+i\epsilon}\left[\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))\delta^4(p'-k+q)\delta^4(k'-p-q)\right.$$
$$\left.-(\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))\delta^4(k'-k+q)\delta^4(p'-p-q))\right]$$
$$=(2\pi)^4\delta(p+k-p'-k')(-i\lambda)^2\left[\frac{\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))}{(k'-p)^2-M^2+i\epsilon}-\frac{(\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))}{(p'-p)^2-M^2+i\epsilon}\right] \tag{42}$$

Figure 4: The Feynman Diagrams for nucleon-nucleon scattering

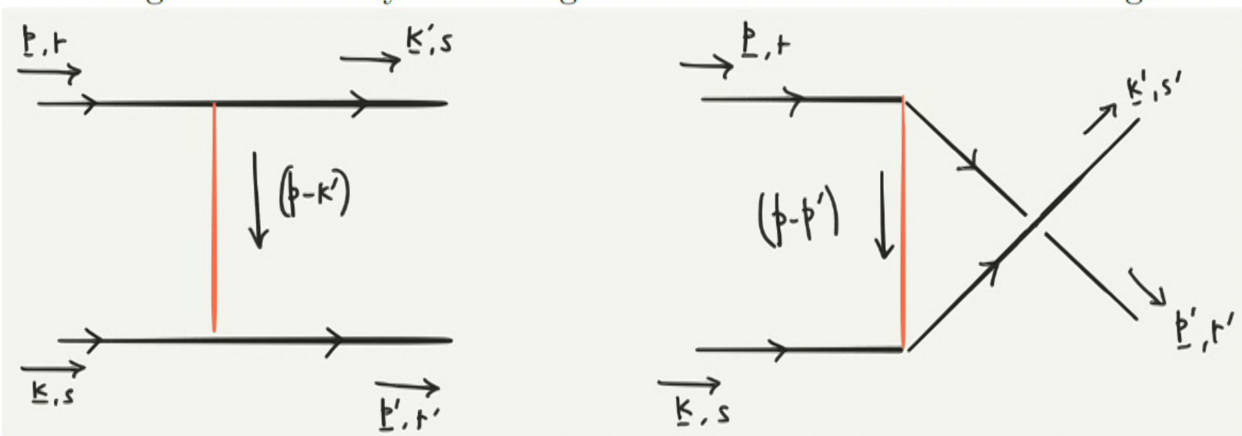
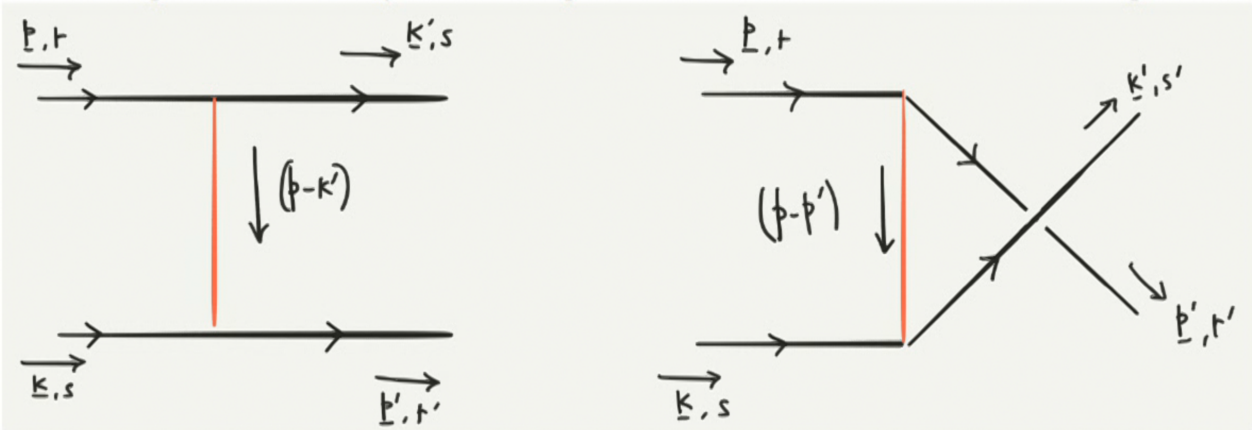
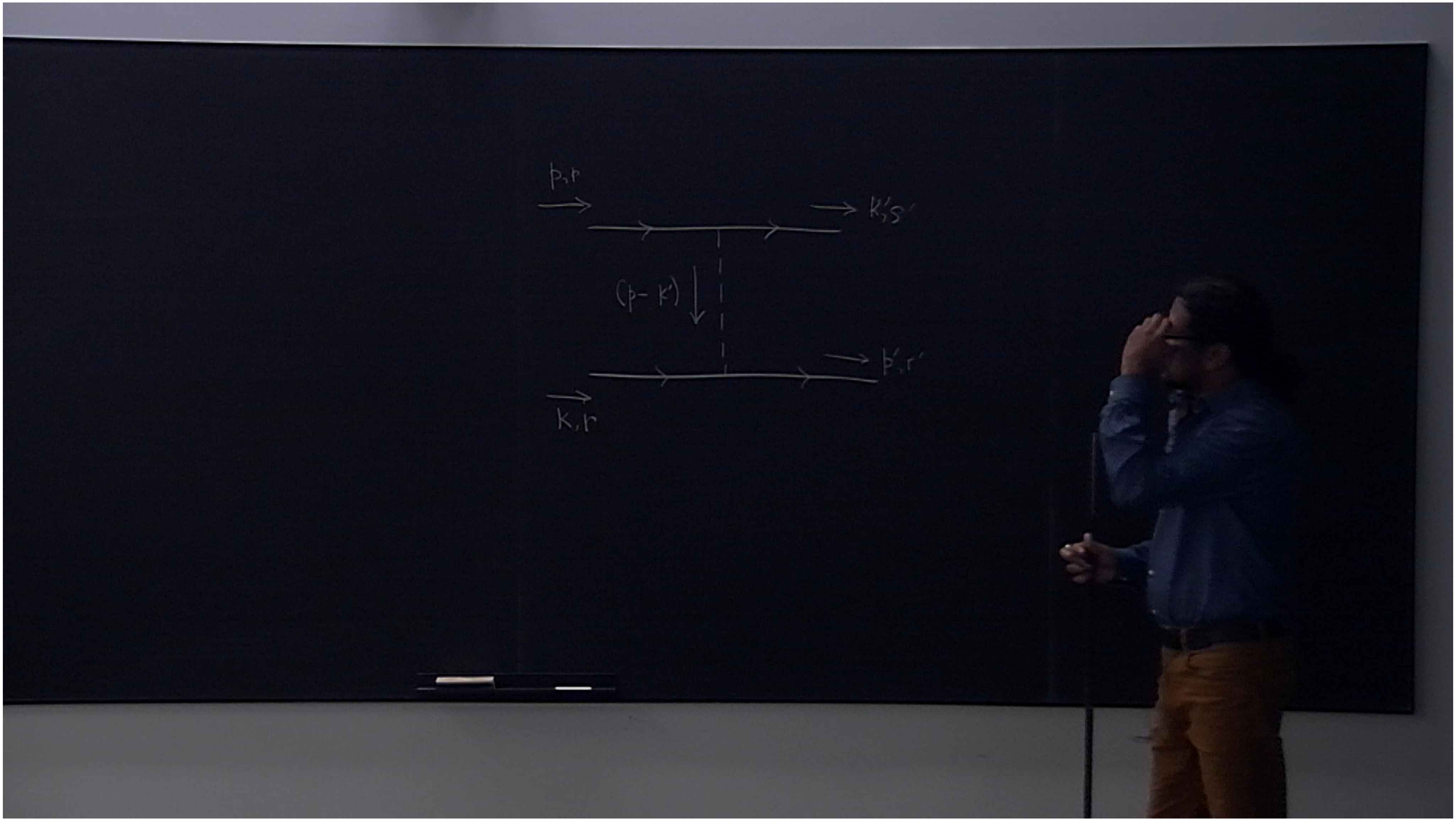
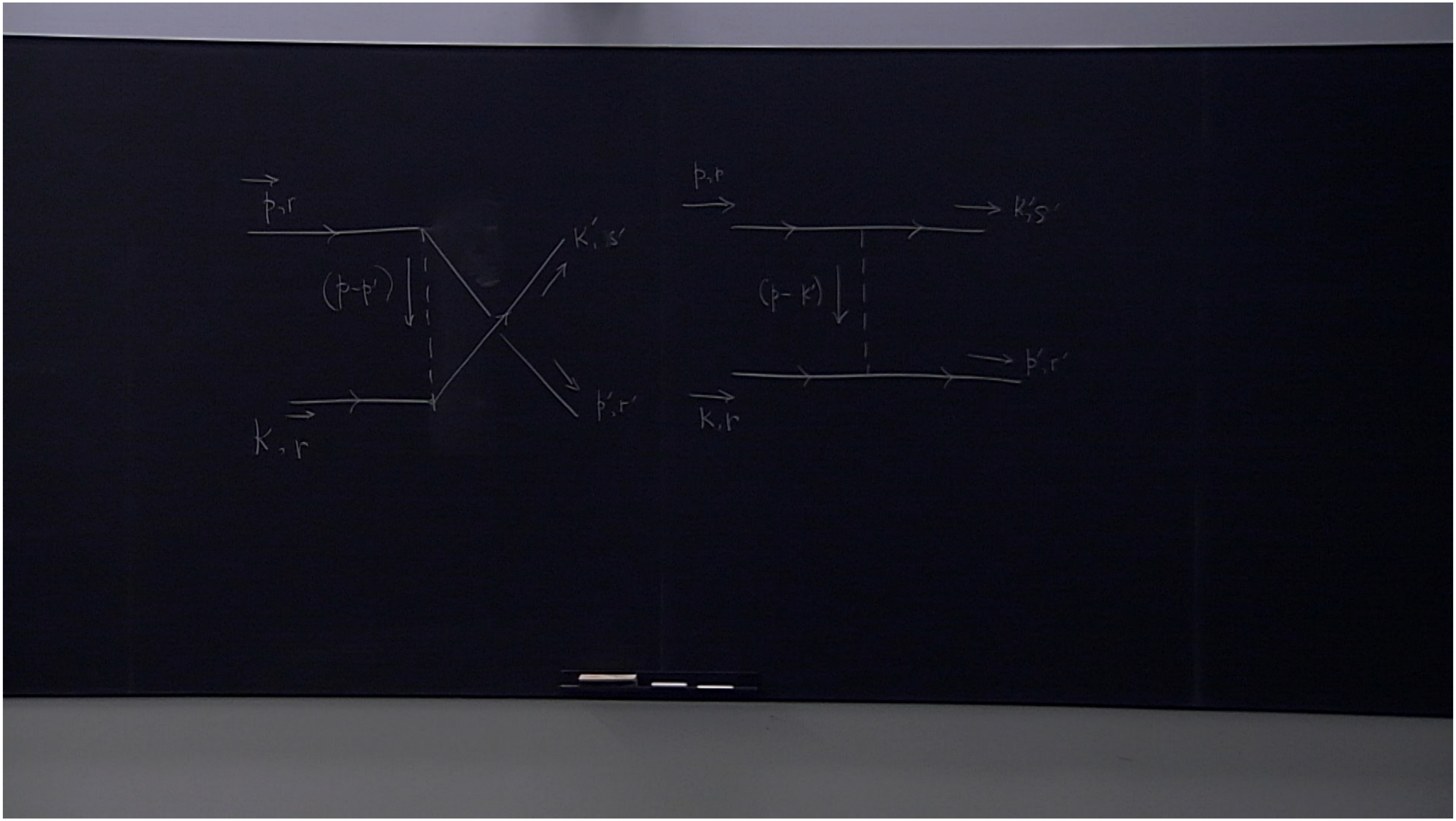


Figure 4: The Feynman Diagrams for nucleon-nucleon scattering




p, r
k', s'
(p - k')
p', r'
k, r


p, r
(p−p')
k', s'
p', r'
k, r
p, r
k', s'
(p−k')
p', r'
k, r

Thus we get for our matrix element:

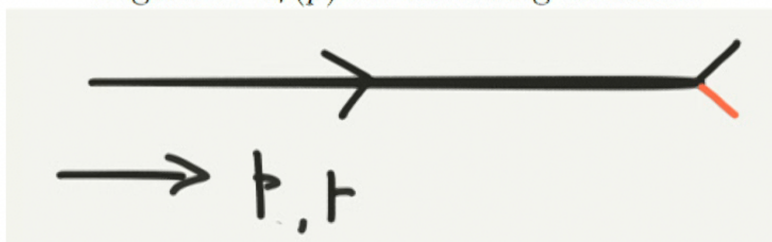
$$i\mathcal{M} = (-i\lambda)^2 \left[\frac{\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))}{(k'-p)^2 - M^2 + i\epsilon} - \frac{(\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))}{(p'-p)^2 - M^2 + i\epsilon} \right]. \tag{43}$$

We can interpret these two terms in terms of the following diagrams, respectively:

And extract from these the following Feynman rules:

1. For each incoming fermion of momentum p and spin polarization r, we associate $u(p)$ and draw the follwing diagram:

Figure 5: $u_r(p)$ for incoming fermions



2. For each outgoing fermion of momentum p and spin polarization r, we associate $\overline{u}(p)$ and draw the follwing diagram:

Figure 6: $\overline{u}_r(p)$ for outgoing fermions

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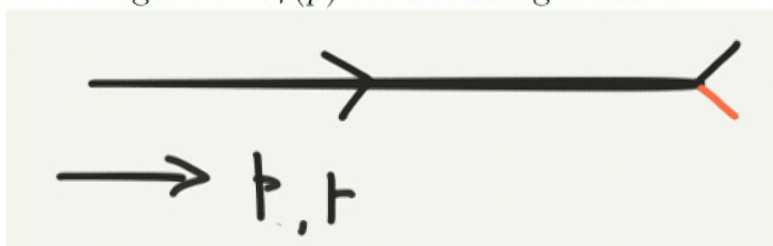

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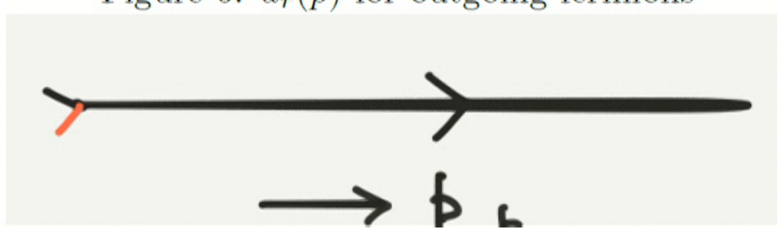
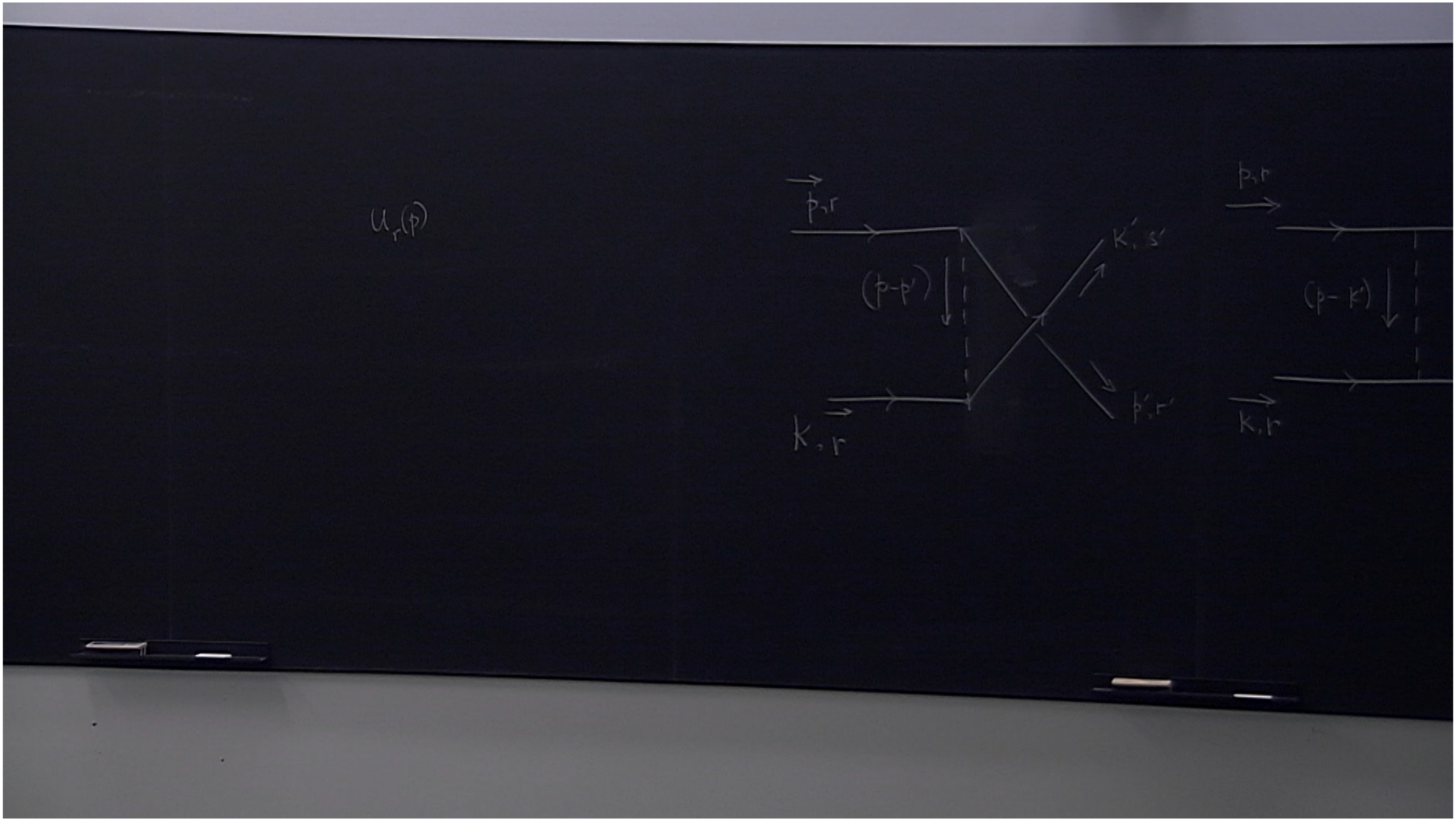
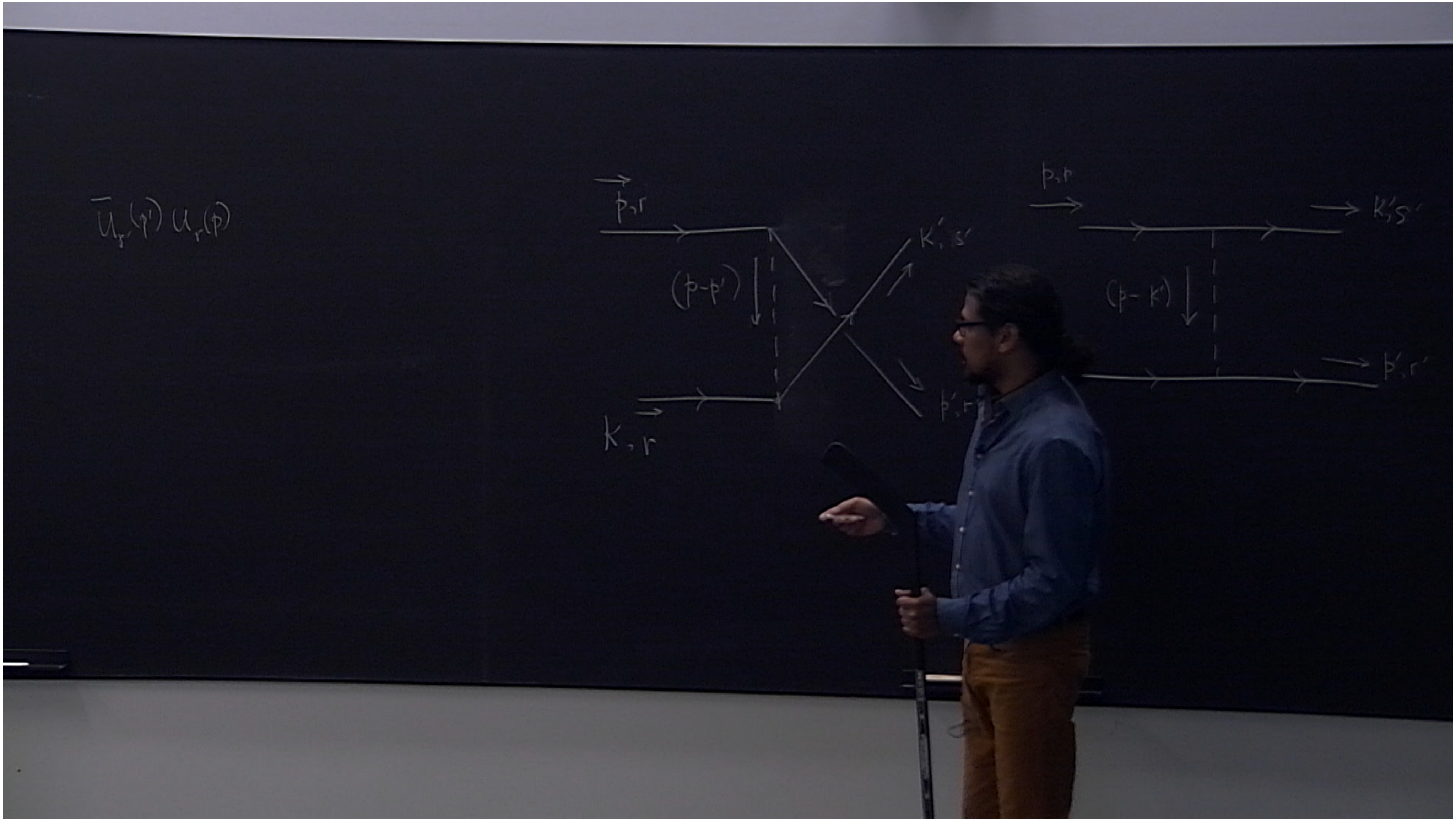
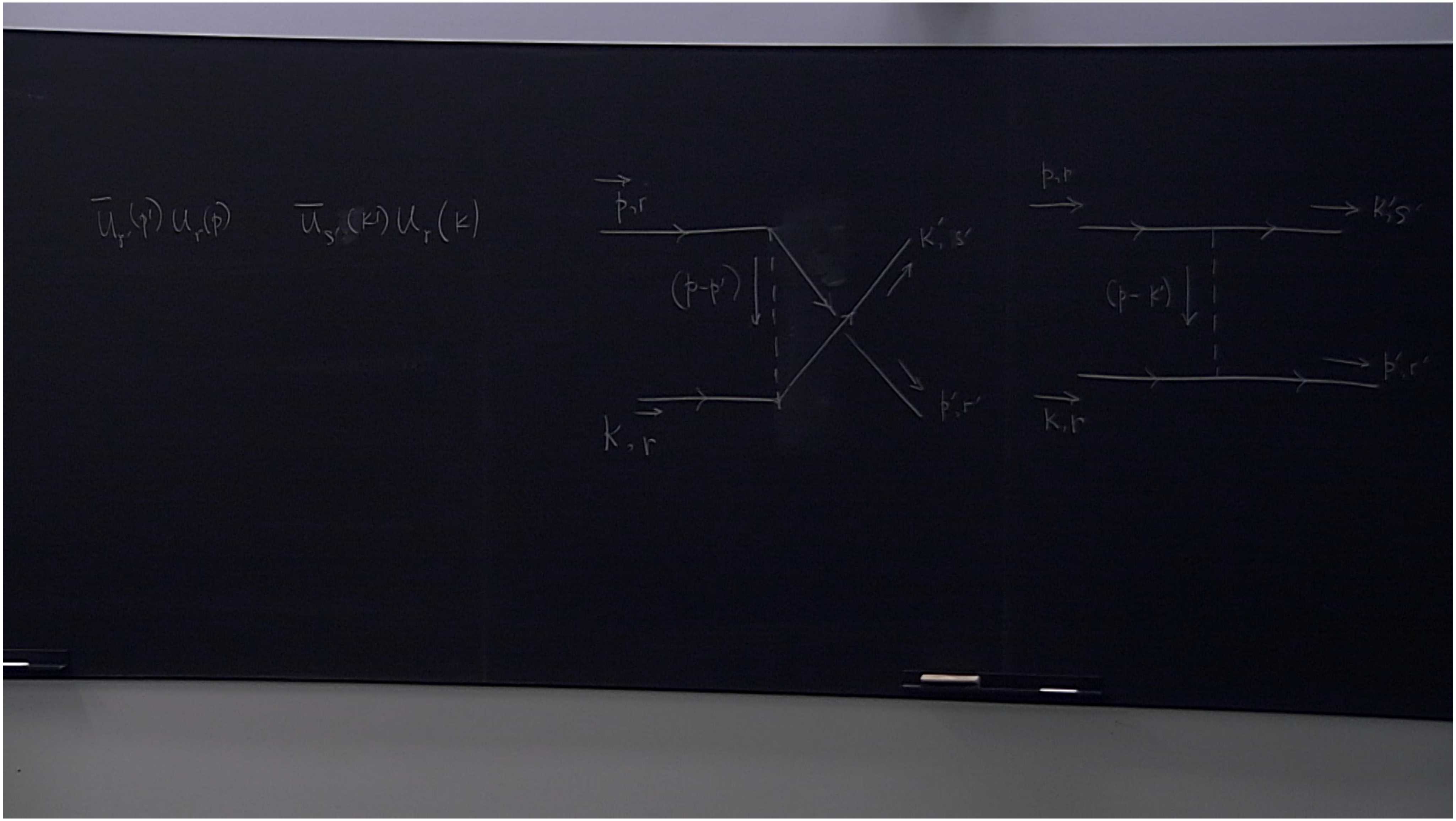
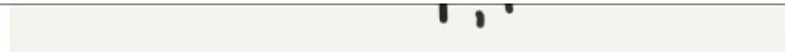



u_r(p)
p, r
(p-p')
K', s'
K, r
p', r'
p, r
(p- K')
K, r


$\bar{u}_{r'}(p')\, u_r(p)$
p, r
(p-p')
k', s'
p', r'
k, r
p, r
(p - k')
k', s'
p', r'



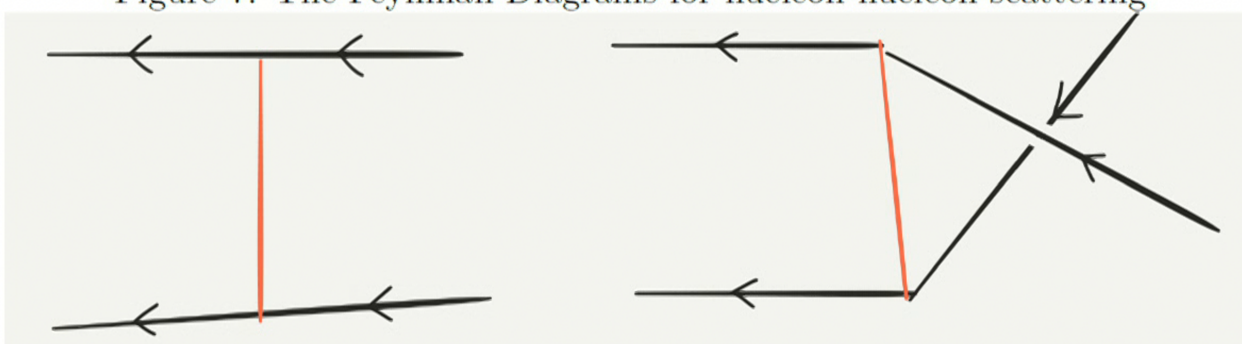
ū_{r'}(p') u_r(p)
ū_{s'}(k') u_r(k)
p, r
(p-p')
k, r
k', s'
p', r'
p, r
(p- k')
k', s'
p', r'
k, r

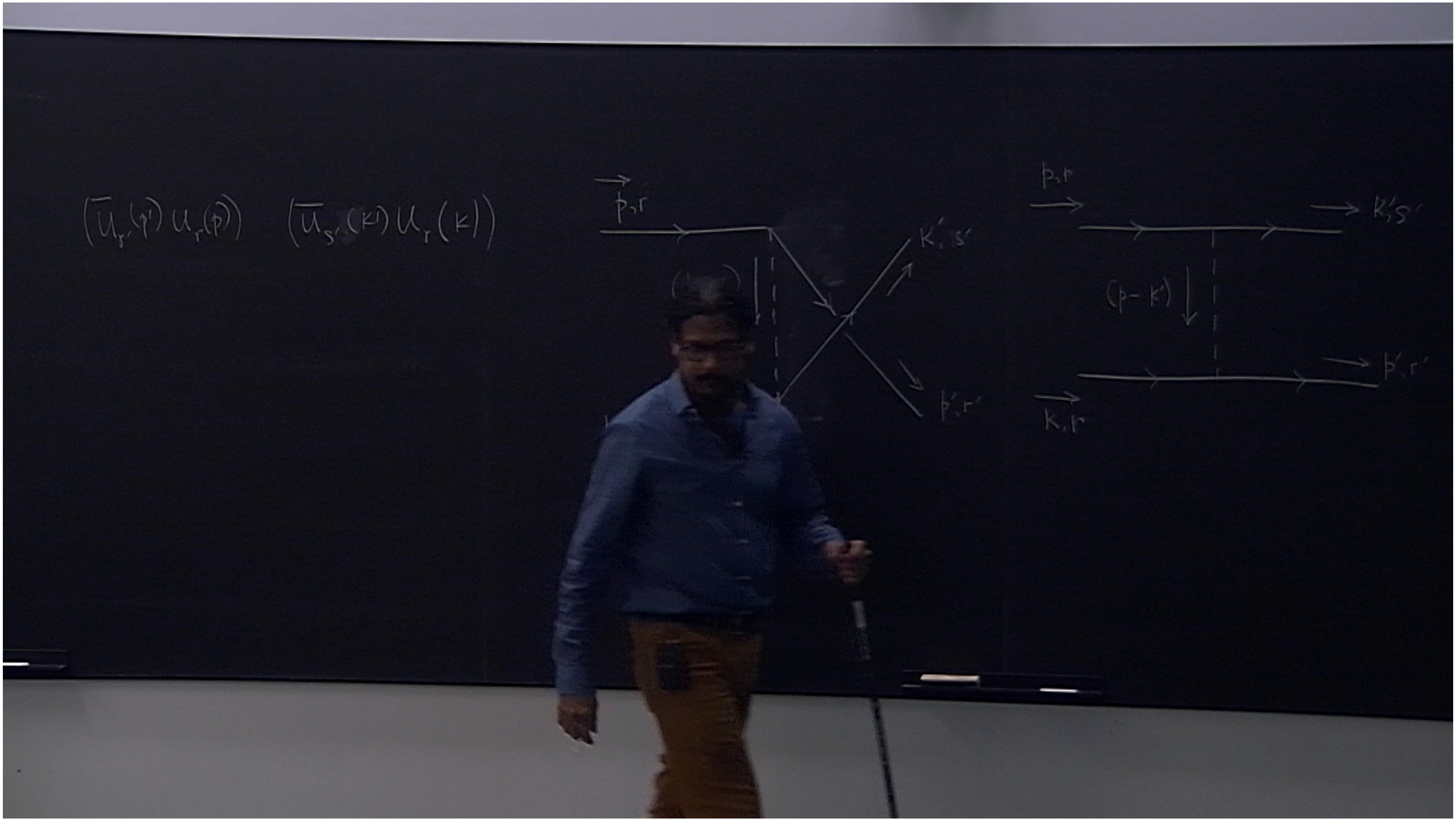


3. We label the fermion lines with an arrow so that the arrow flows in the direction of time. We write down the spinor factors above from left to right as we follow the fermion line opposite to the direction of the arrow. It doesn't matter which fermion line we start with first.

By considering the antinucleon-antinucleon scattering,

Figure 7: The Feynman Diagrams for nucleon-nucleon scattering

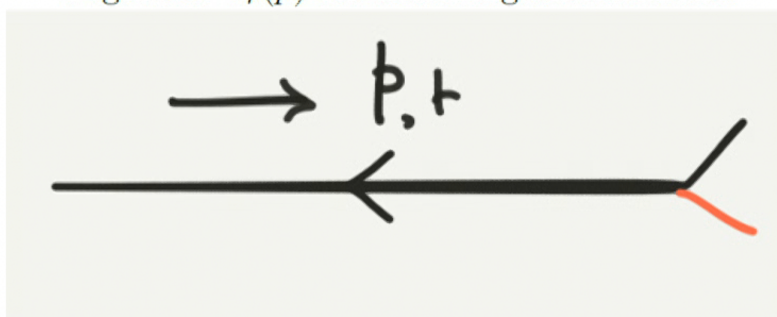



(ū_{r'}(p') u_r(p))
(ū_{s'}(k') u_r(k))
p, r
k', s'
p', r'
p, r
(p − k')
k', s'
p', r'
k, r

we arrive at the following Feynman rules:

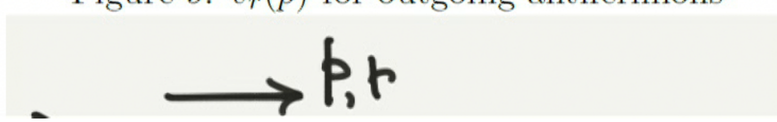
4. Incoming antifermions have associated with them $\overline{v}_r(p)$ and the diagram:

Figure 8: $\overline{v}_r(p)$ for incoming antifermions



5. Outgoing antifermions have associated with them $v_r(p)$ and the diagram:

Figure 9: $v_r(p)$ for outgoing antifermions



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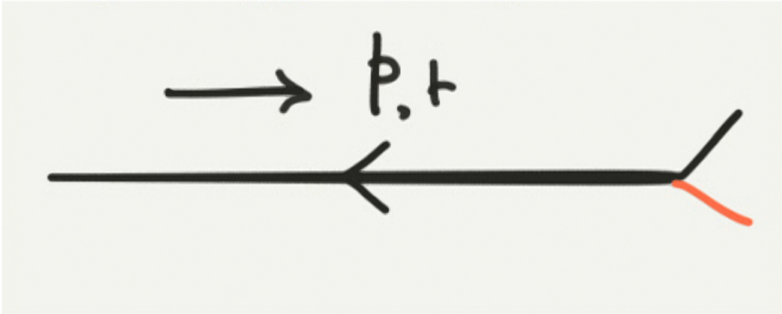

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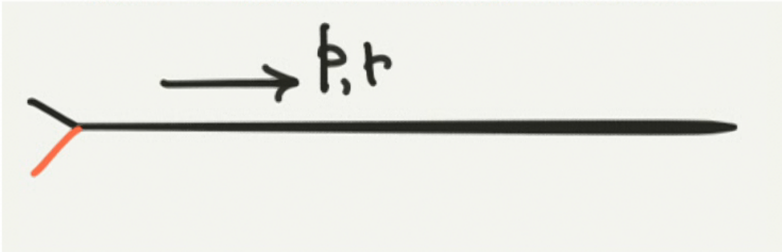
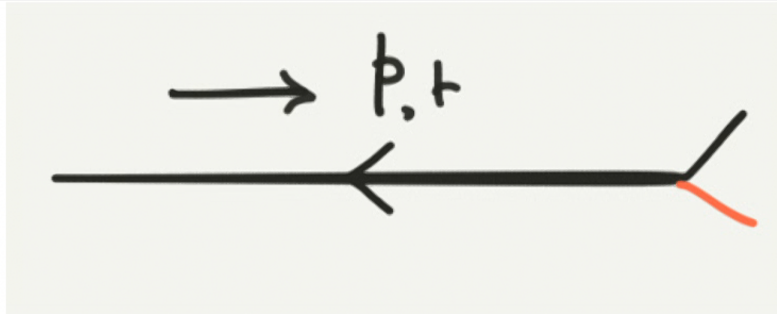
Figure 9: $v_r(p)$ for outgoing antifermions



6. Associate with each vertex a factor $(-i\lambda)$ and the diagram:



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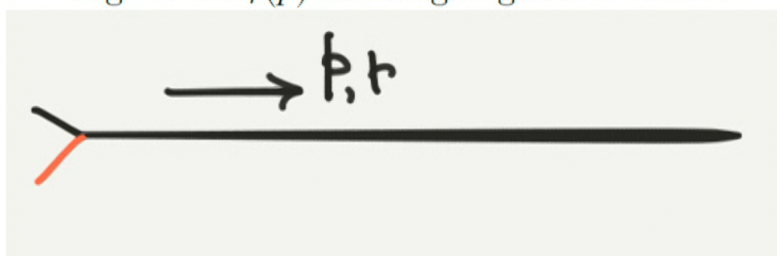
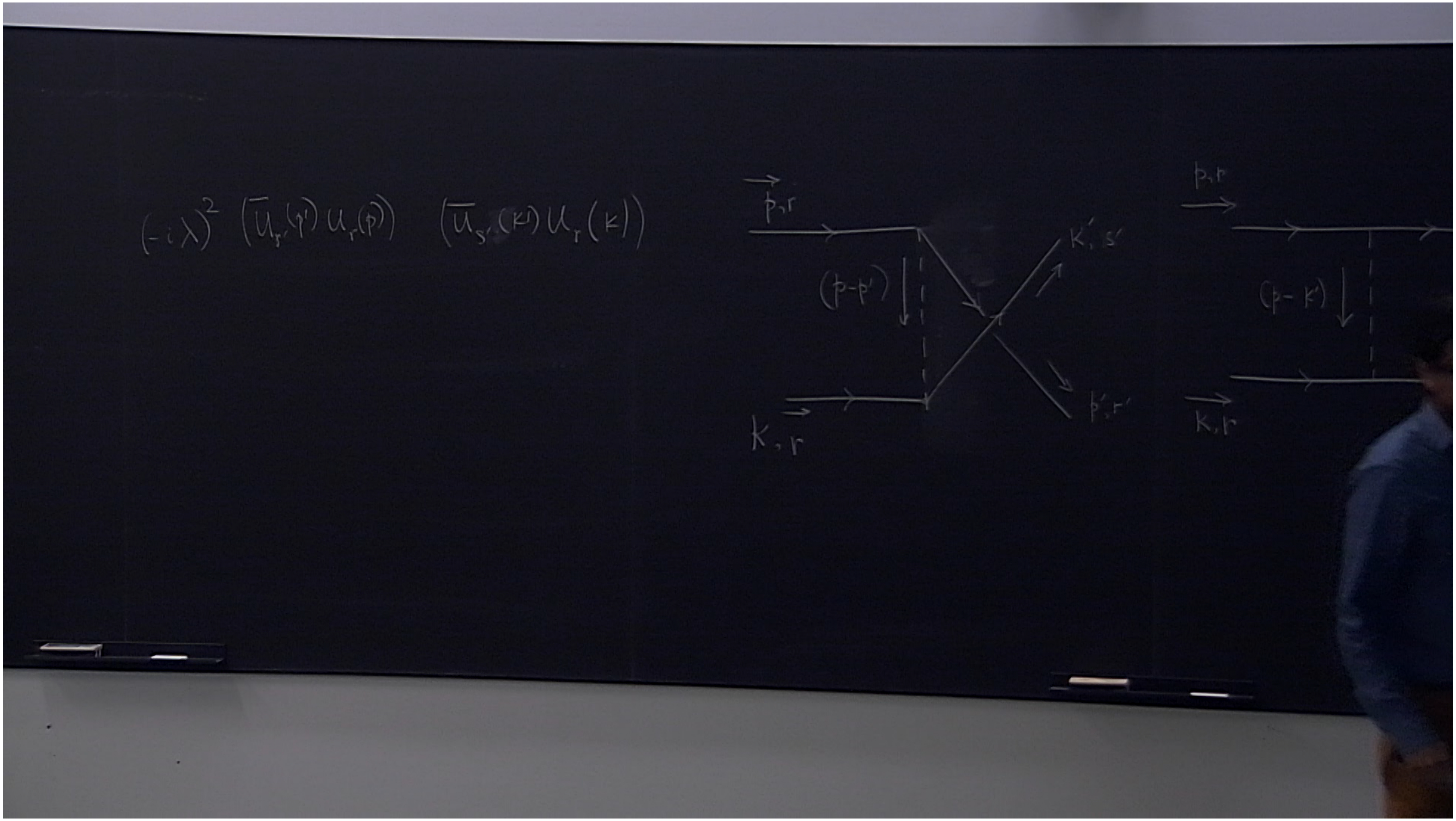


6. Associate with each vertex a factor $(-i\lambda)$ and the diagram:

Figure 10: $-i\lambda$ with any of these vertices



$(-i\lambda)^2 \left(\bar{u}_{r'}(p')\, u_r(p)\right) \left(\bar{u}_{s'}(k')\, u_r(k)\right)$
p, r
(p-p')
k', s'
p', r'
k, r
p, r
(p- k')
k, r

Thus we get for our matrix element:

$$i\mathcal{M} = (-i\lambda)^2 \left[\frac{\overline{u}_{r'}(p')u_s(k))(\overline{u}_{s'}(k')u_r(p))}{(k'-p)^2 - M^2 + i\epsilon} - \frac{(\overline{u}_{s'}(k')u_s(k))(\overline{u}_{r'}(p')u_r(p))}{(p'-p)^2 - M^2 + i\epsilon} \right]. \tag{43}$$

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Figure 4: The Feynman Diagrams for nucleon-nucleon scattering

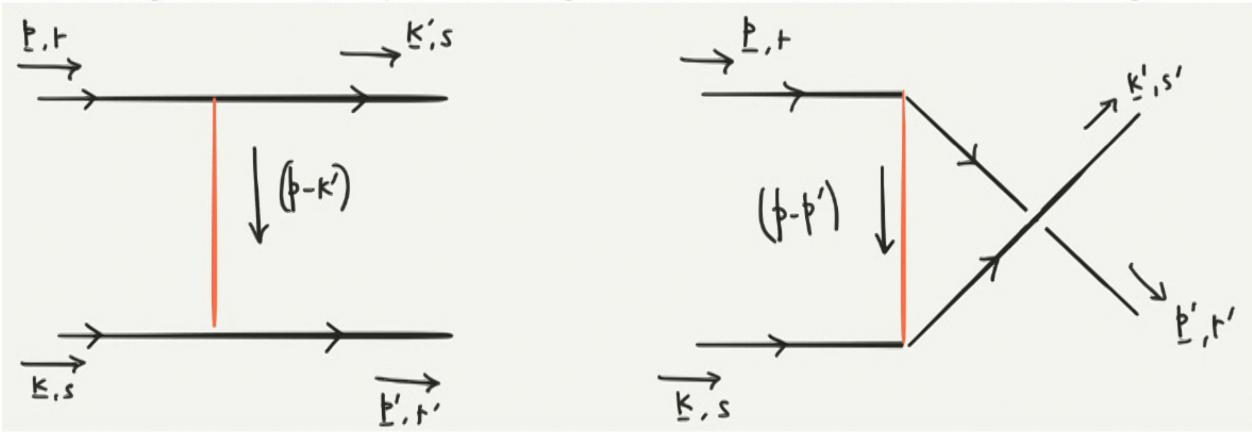
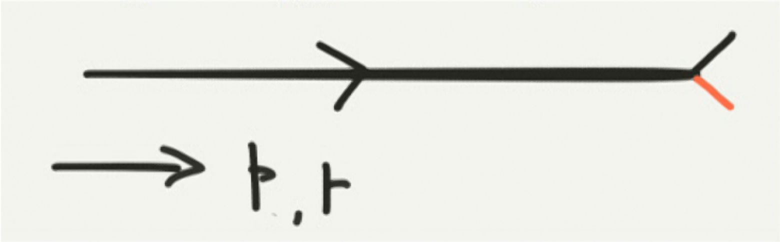
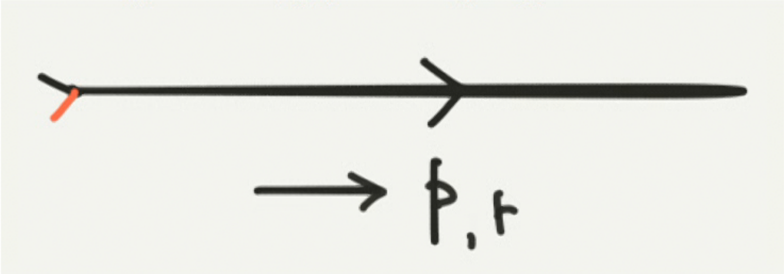


Figure 5: $u_r(p)$ for incoming fermions



2. For each outgoing fermion of momentum p and spin polarization r, we associate $\overline{u}(p)$ and draw the follwing diagram:

Figure 6: $\overline{u}_r(p)$ for outgoing fermions



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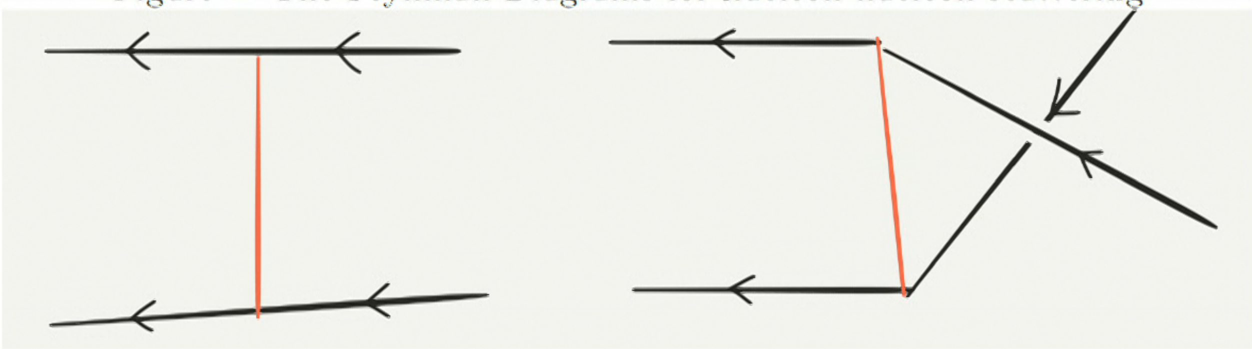
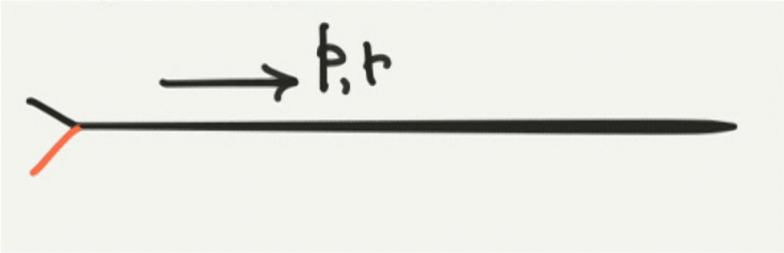


Figure 9: $v_r(p)$ for outgoing antifermions



6. Associate with each vertex a factor $(-i\lambda)$ and the diagram:

Figure 10: $-i\lambda$ with any of these vertices

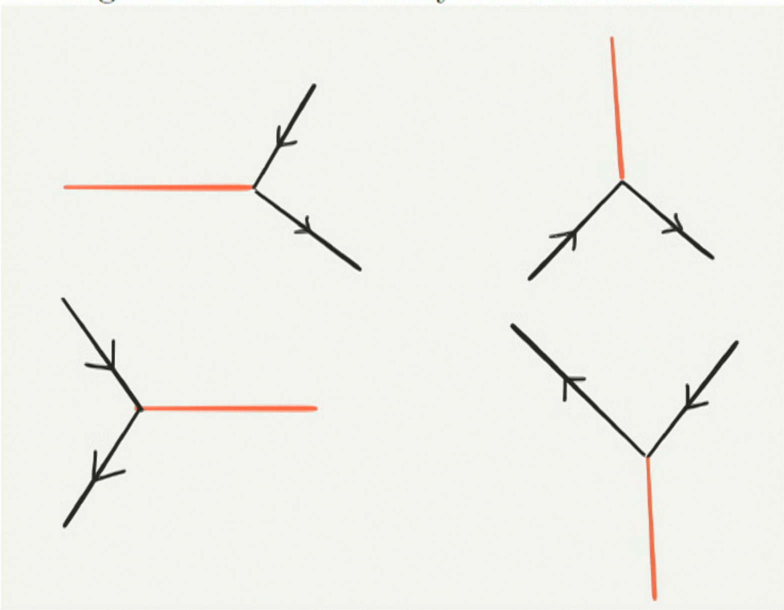
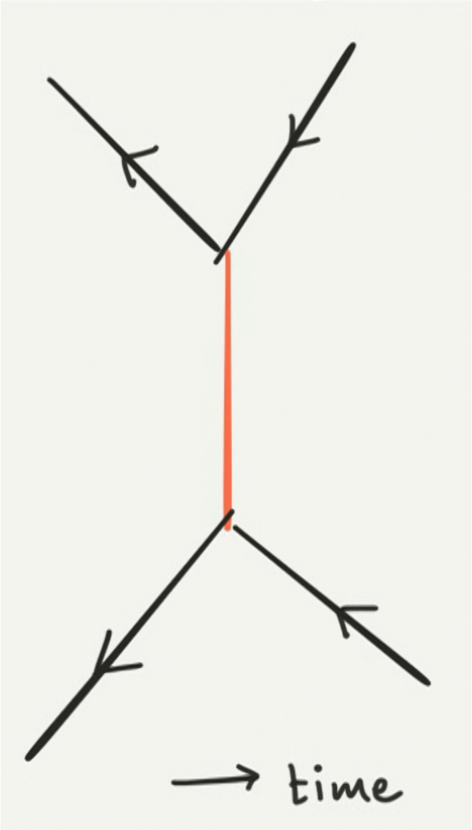
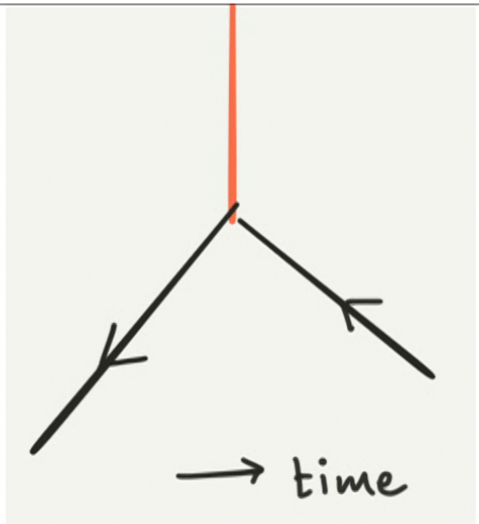


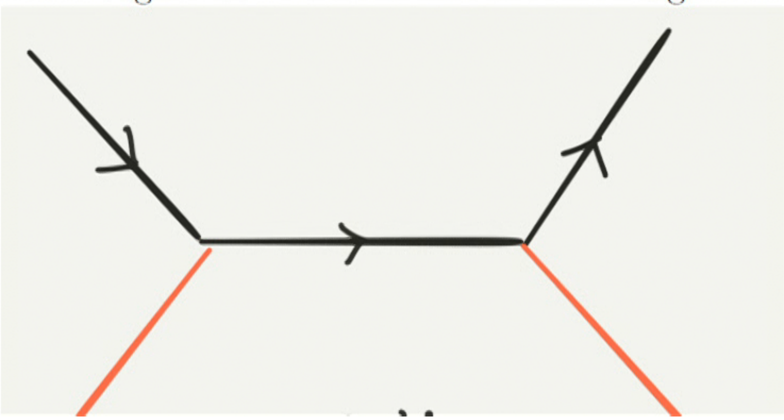
Figure 11: Follow arrow against their flow.

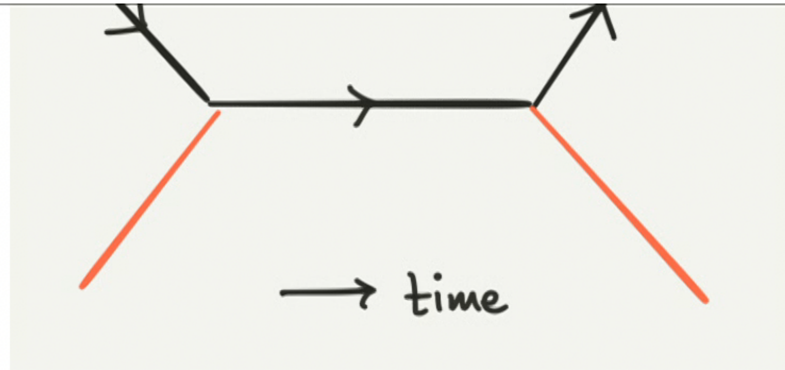




To derive the Feynman rule for internal fermions we scatter a fermion with a meson

Figure 12: Nucleon-meson scattering



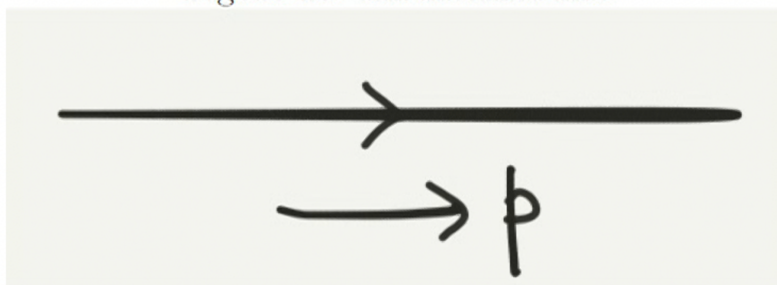


8. and obtain

$$\frac{i(\not{p} - m)}{p^2 - m^2 + i\epsilon}. \tag{44}$$

We represent the internal line by:

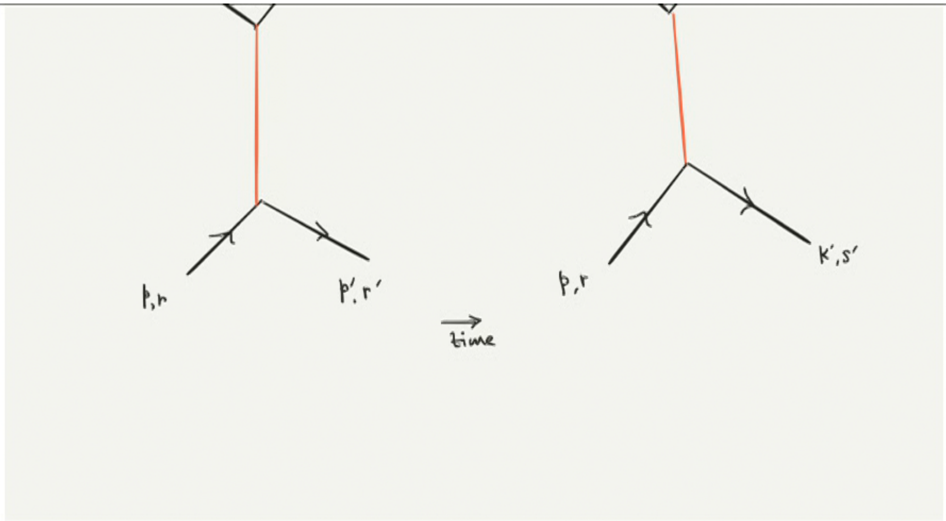
Figure 13: An internal line



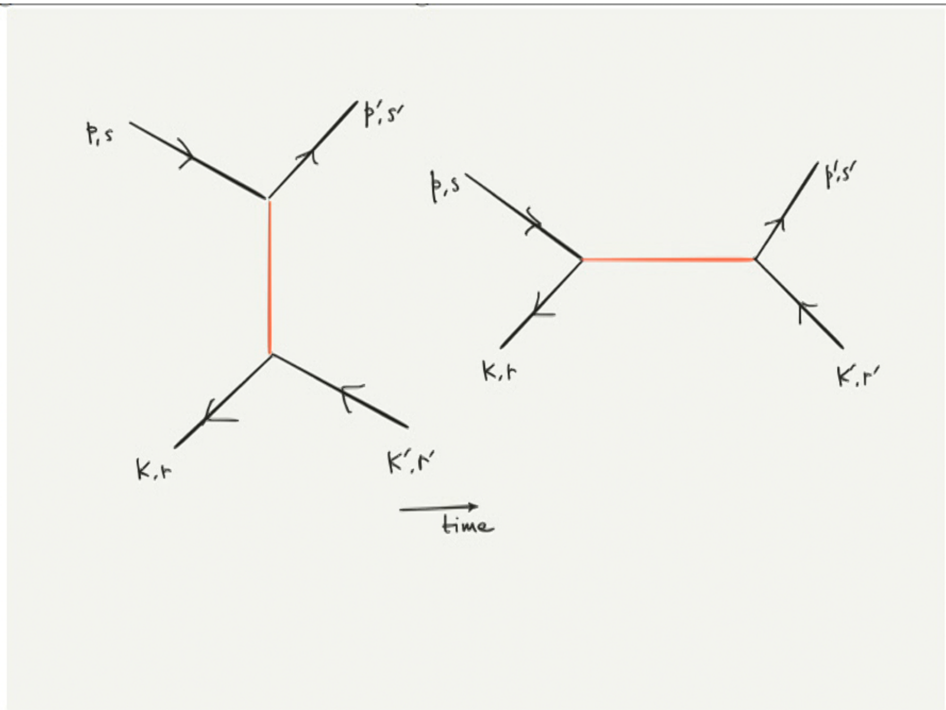
9. We impose four-momentum conservation at each vertex.

10. For fermionic final states with indistinguishable particles we introduce a relative minus sign if a diagram can be converted into another one by an odd number of permutations of the momenta and spin labels of the final fermionic particles:

Figure 14: Relative minus sign in nucleon-nucleon scattering



For final fermionic particles which are distinguishable we can still have a relative minus sign. This comes from examining how the operators in the normal ordered term of the Dyson series expansion interacts with the external states.



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$$:(\overline{\psi}^{(+)}\psi^{(-)})_1(\overline{\psi}^{(-)}\psi^{(+)})_2 : \overbrace{\phi_1\phi_2} \sim -\overline{\psi}_2^{(-)}\psi_1^{(-)}\overline{\psi}_1^{(+)}\psi_2^{(+)}\overbrace{\phi_1\phi_2} \tag{45}$$

whereas for the second diagram we have

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Thus we see that there's a relative minus sign between the two terms. We shall explore more of this minus sign in the next tutorial.

11. When there is an internal loop and there is an undetermined momentum in that loop, then we integrate over the loop momentum:

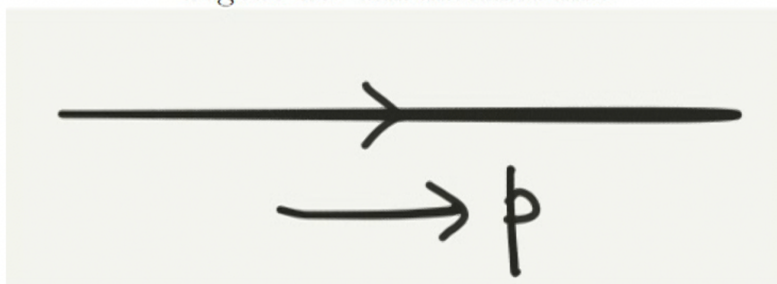
$$\int \frac{d^4q}{(2\pi)^4} \tag{47}$$

12. We introduce -1 for each fermion loop and trace over the fermionic loop spinor indices.

6 Possible Interview Questions

1. How does causality demand that time ordering of fermions involve a minus sign?

Figure 13: An internal line



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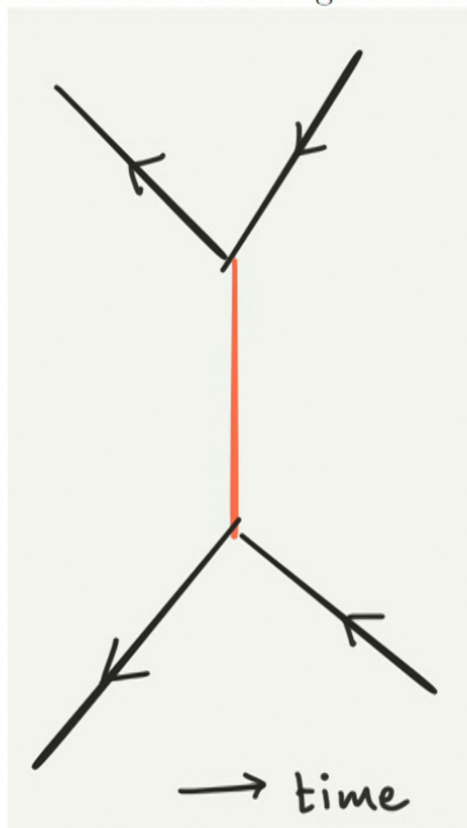


$$\frac{i}{p^2 - M^2}$$

$$i \; \frac{(-i\lambda)^2 \left(\bar{u}_{r'}(p')\, u_r(p)\right) \; \left(\bar{u}_{s'}(k')\, u_r(k)\right)}{\left((p-p')^2 - M^2 + i\epsilon \right)}$$

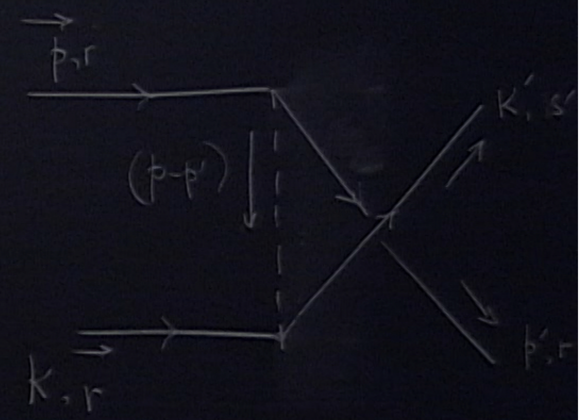
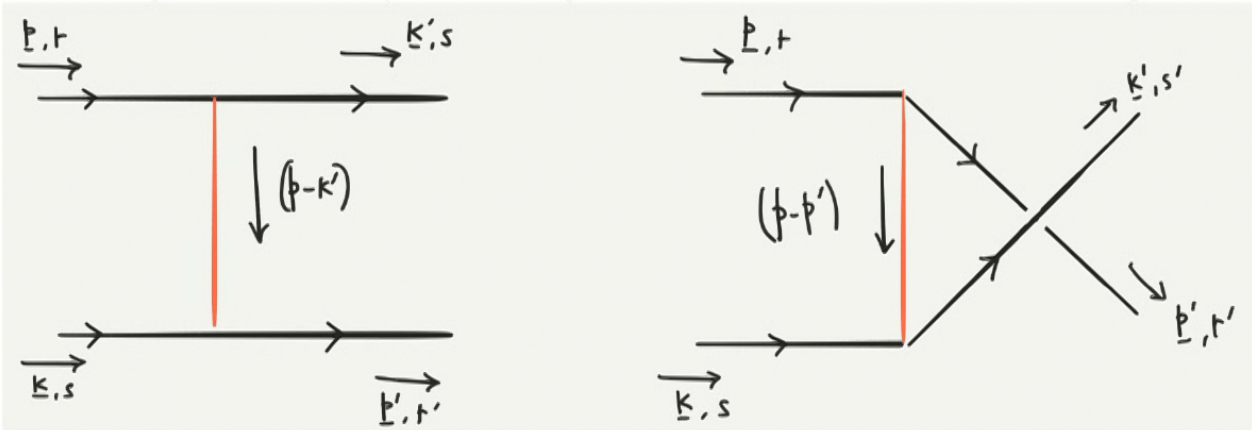


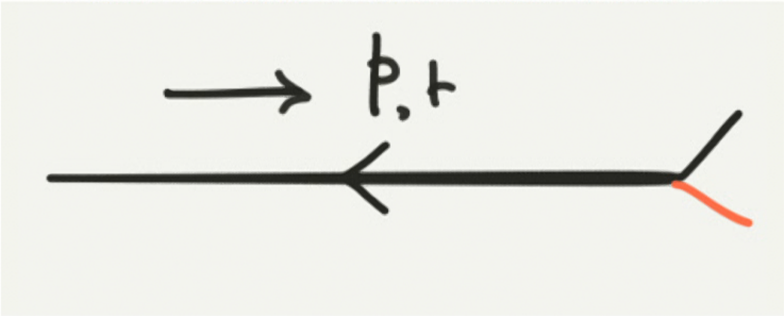
Figure 4: The Feynman Diagrams for nucleon-nucleon scattering



we arrive at the following Feynman rules:

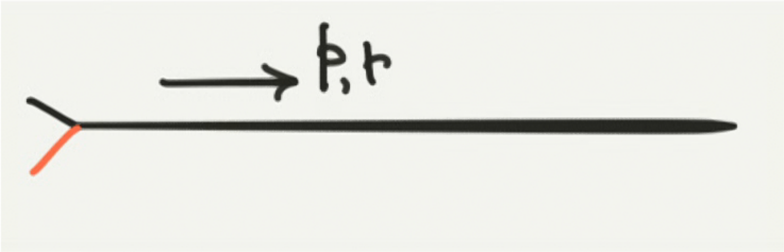
4. Incoming antifermions have associated with them $\overline{v}_r(p)$ and the diagram:

Figure 8: $\overline{v}_r(p)$ for incoming antifermions



5. Outgoing antifermions have associated with them $v_r(p)$ and the diagram:

Figure 9: $v_r(p)$ for outgoing antifermions



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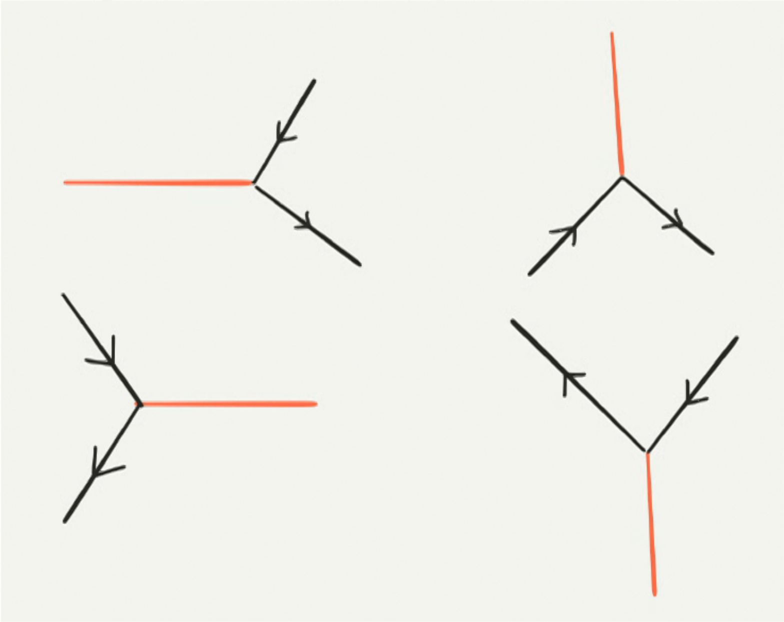
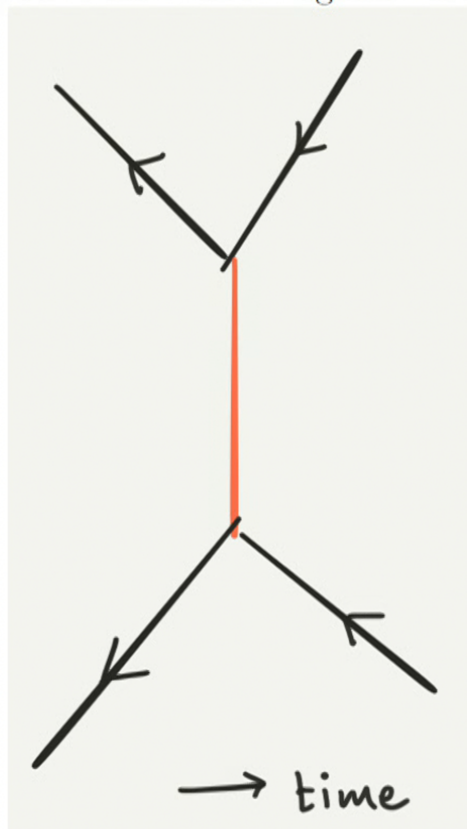


7. Label antifermion lines with arrows which flows opposite the arrow of time. This arrow represents the flow of charge. Again, write down the spinor factors from left to right as you follow the line against the arrow.

Figure 11: Follow arrow against their flow.



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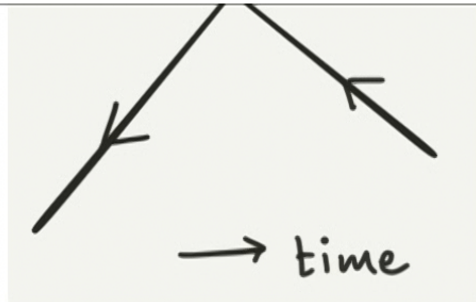

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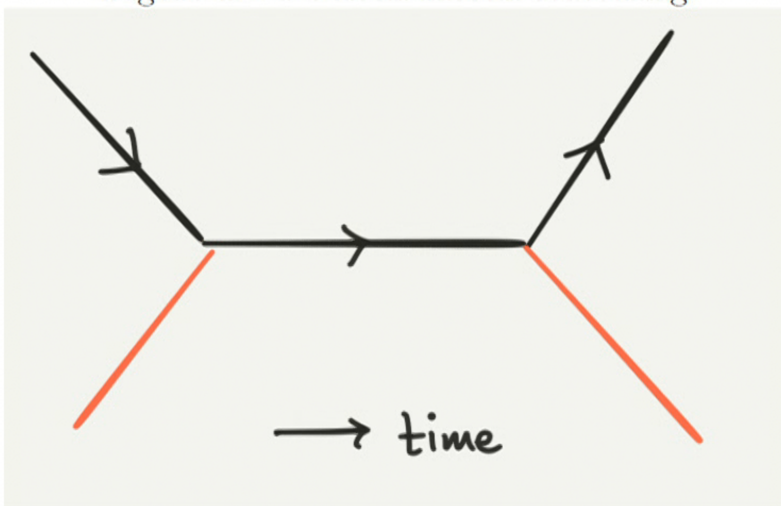

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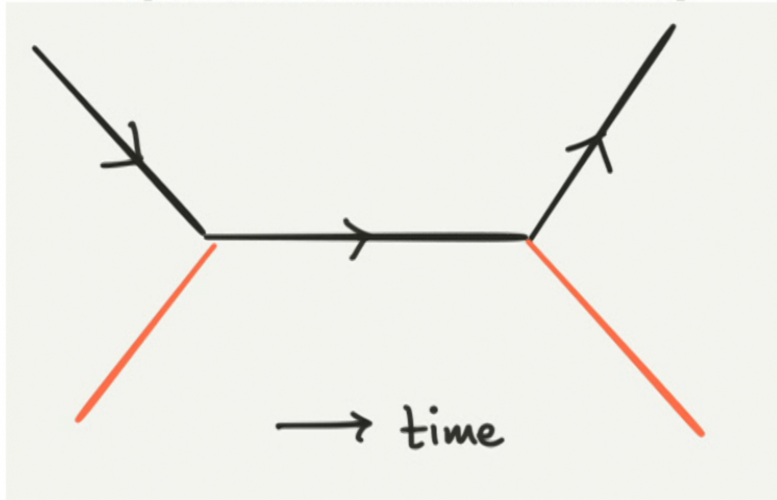

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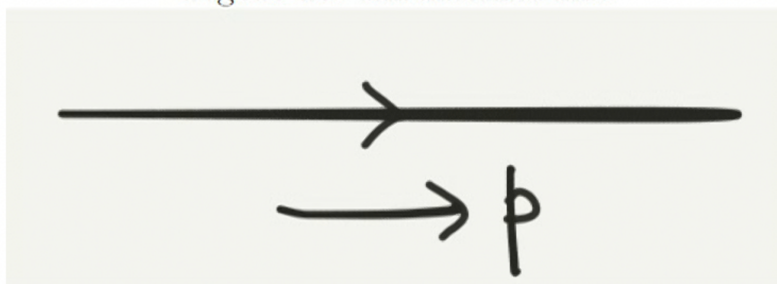

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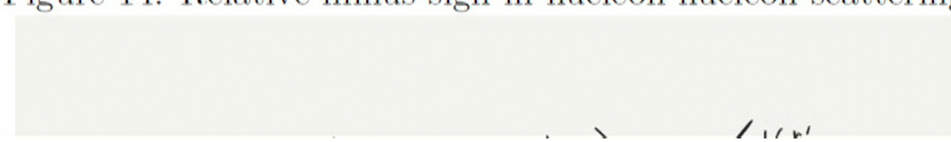

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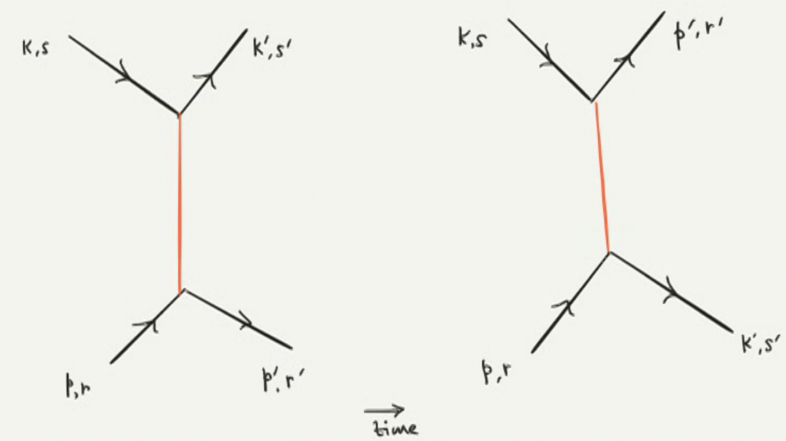
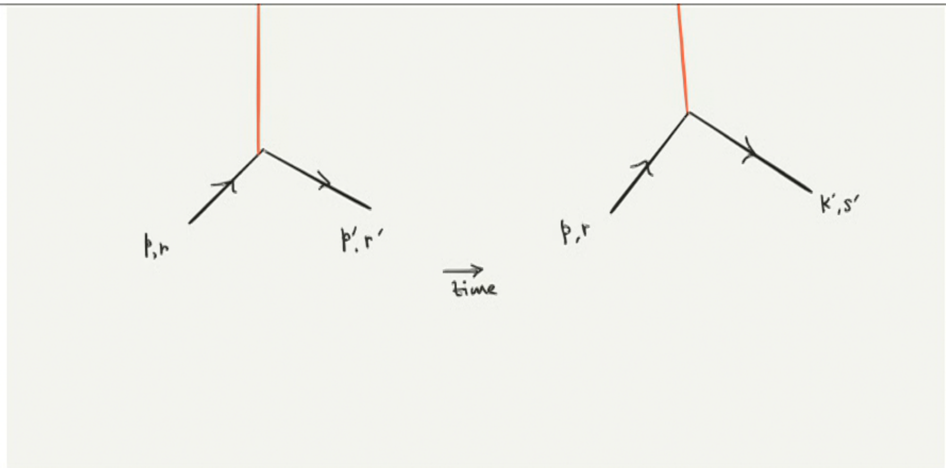
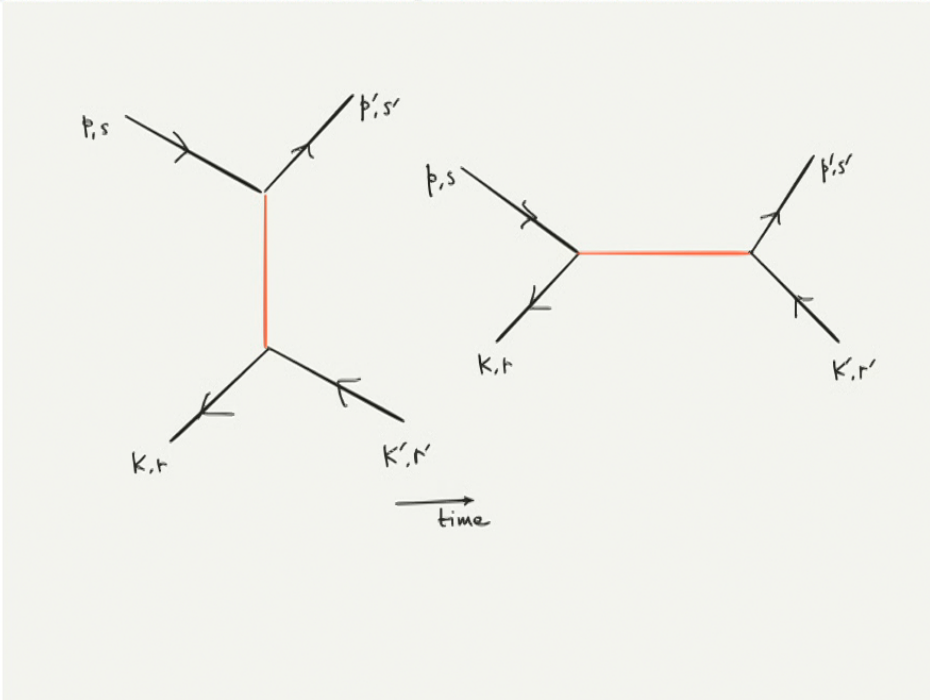




For final fermionic particles which are distinguishable we can still have a relative minus sign. This comes from examining how the operators in the normal ordered term of the Dyson series expansion interacts with the external states.

Figure 15: Relative minus sign in nucleon-antinucleon scattering



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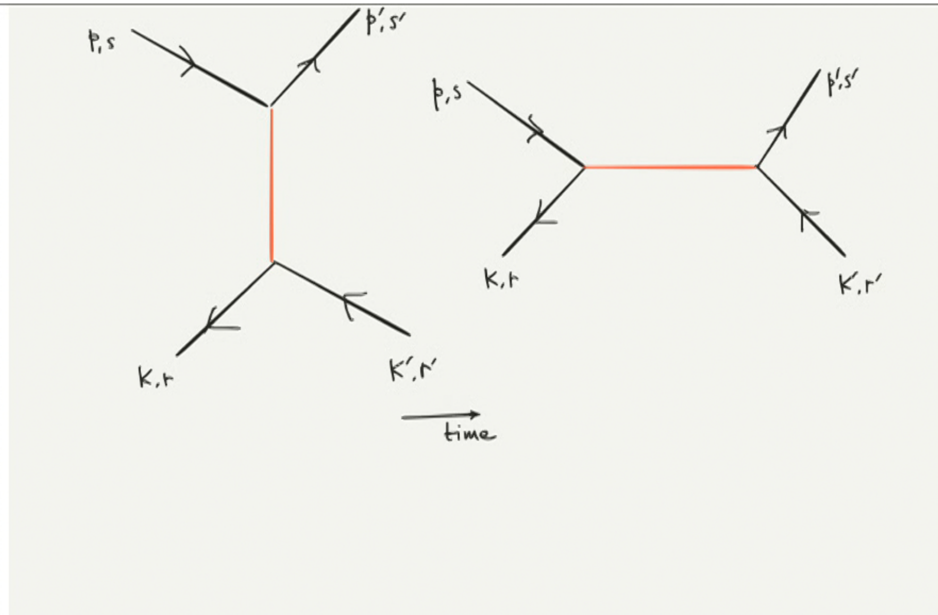

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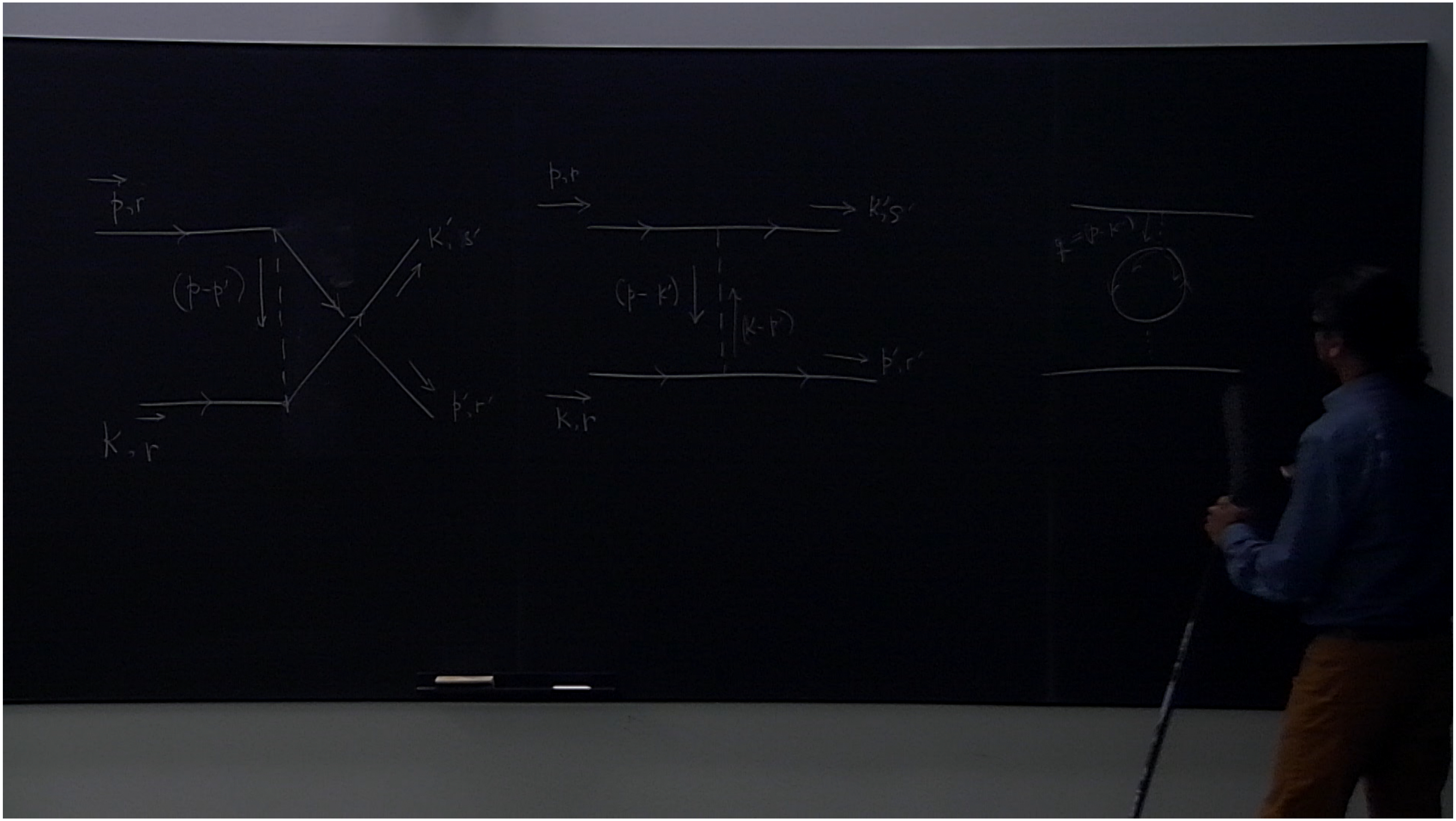


p, r
(p-p')
k, r
k', s'
p', r'
p, r
k', s'
(p - k')
(k - p')
p', r'
k, r

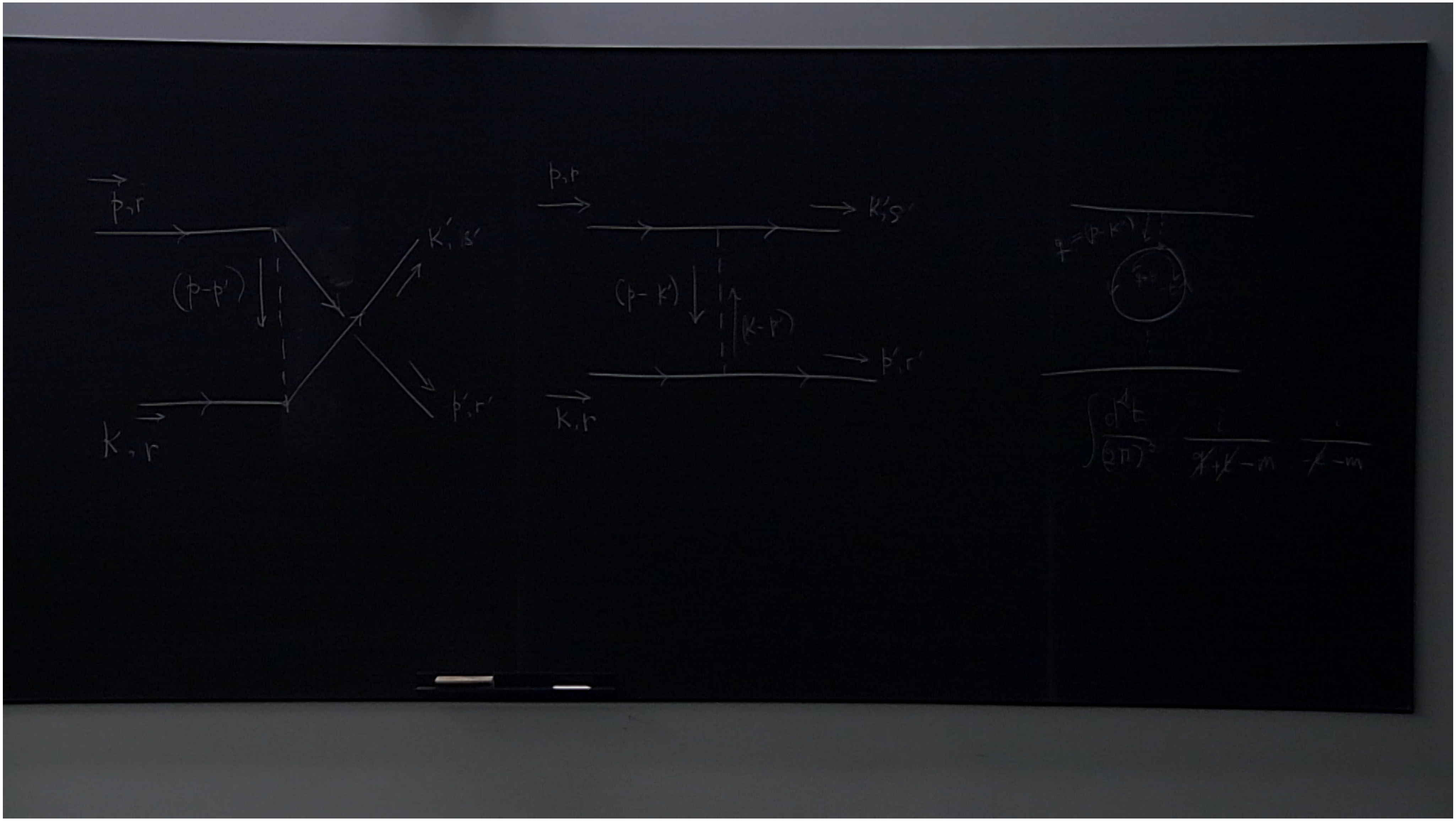
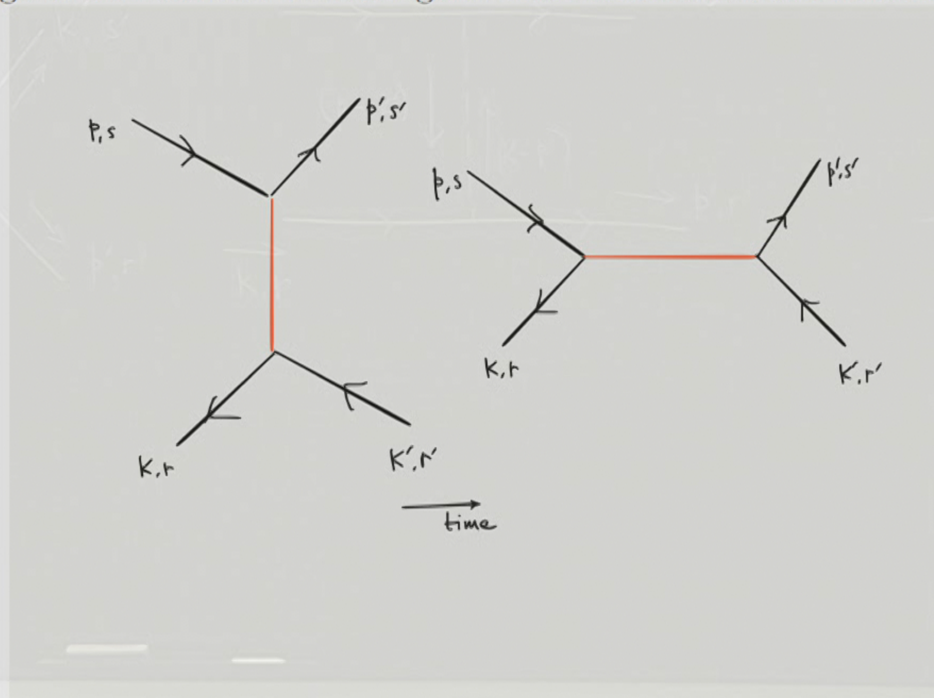

p, r
k, r
k', s'
p', r'
(p-p')
(p-k')
(k-p')
q = (p-k')
$\int \frac{d^4 k}{(2\pi)^4}$

Figure 15: Relative minus sign in nucleon-antinucleon scattering



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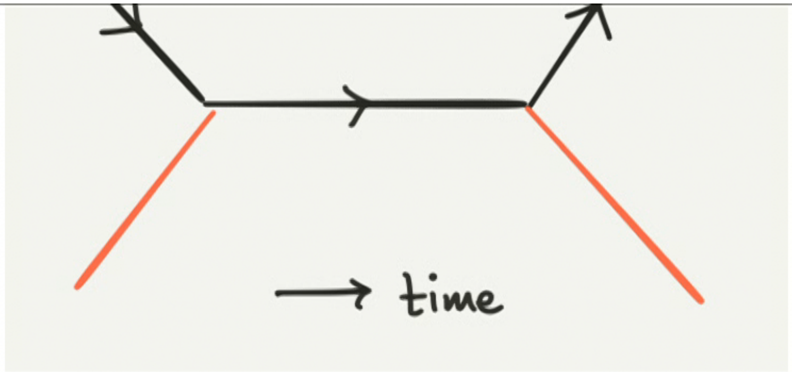

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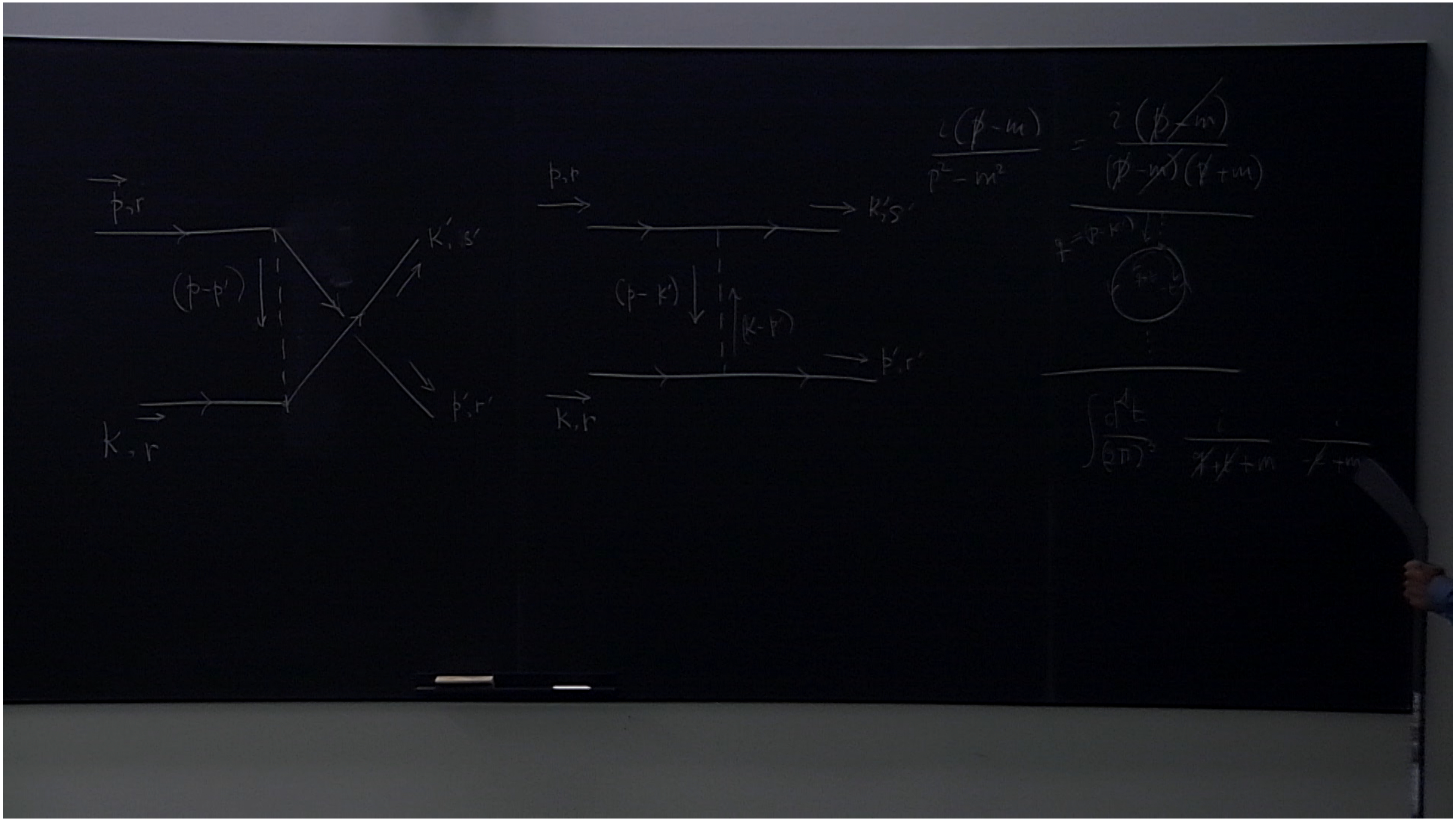
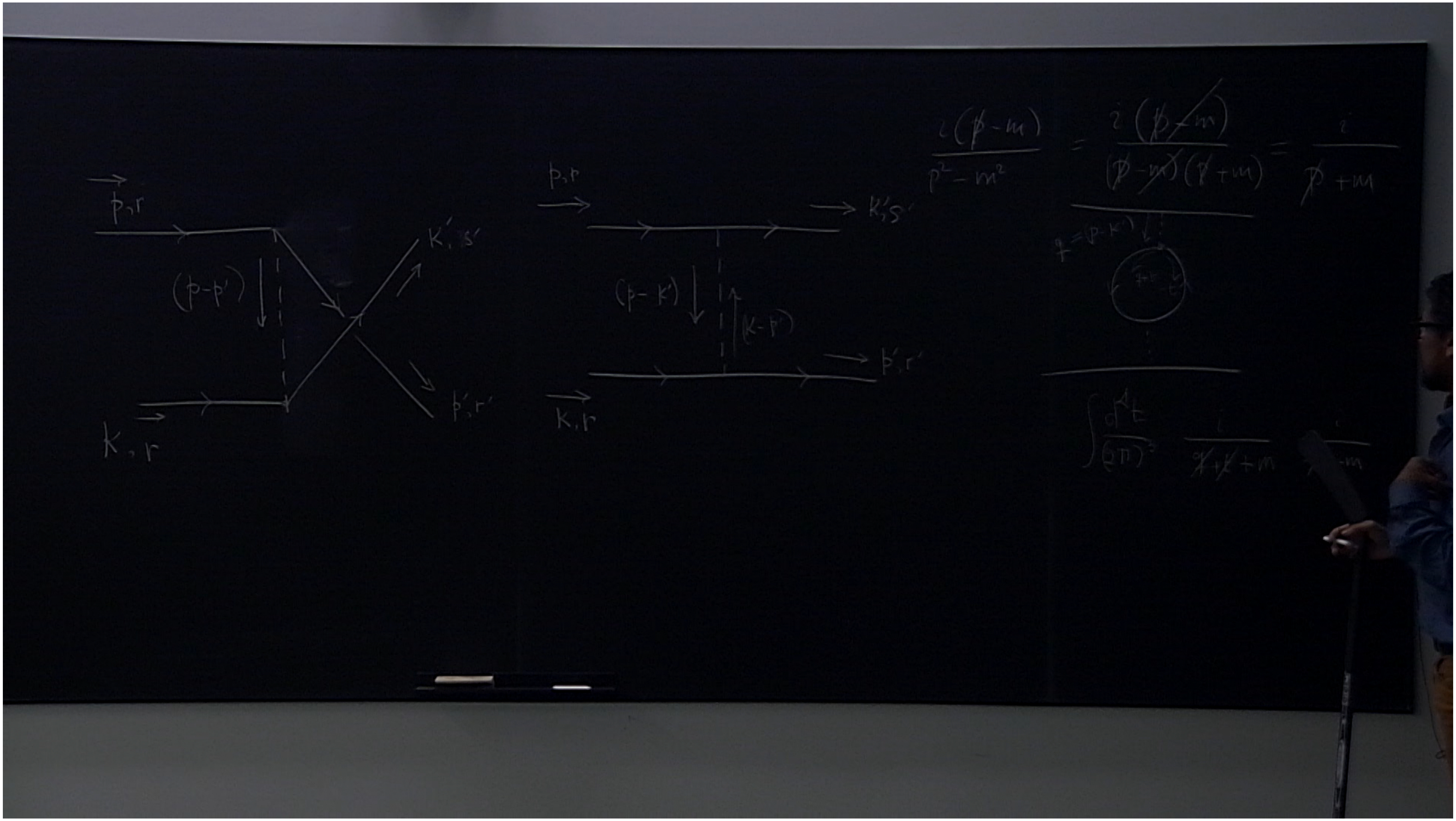
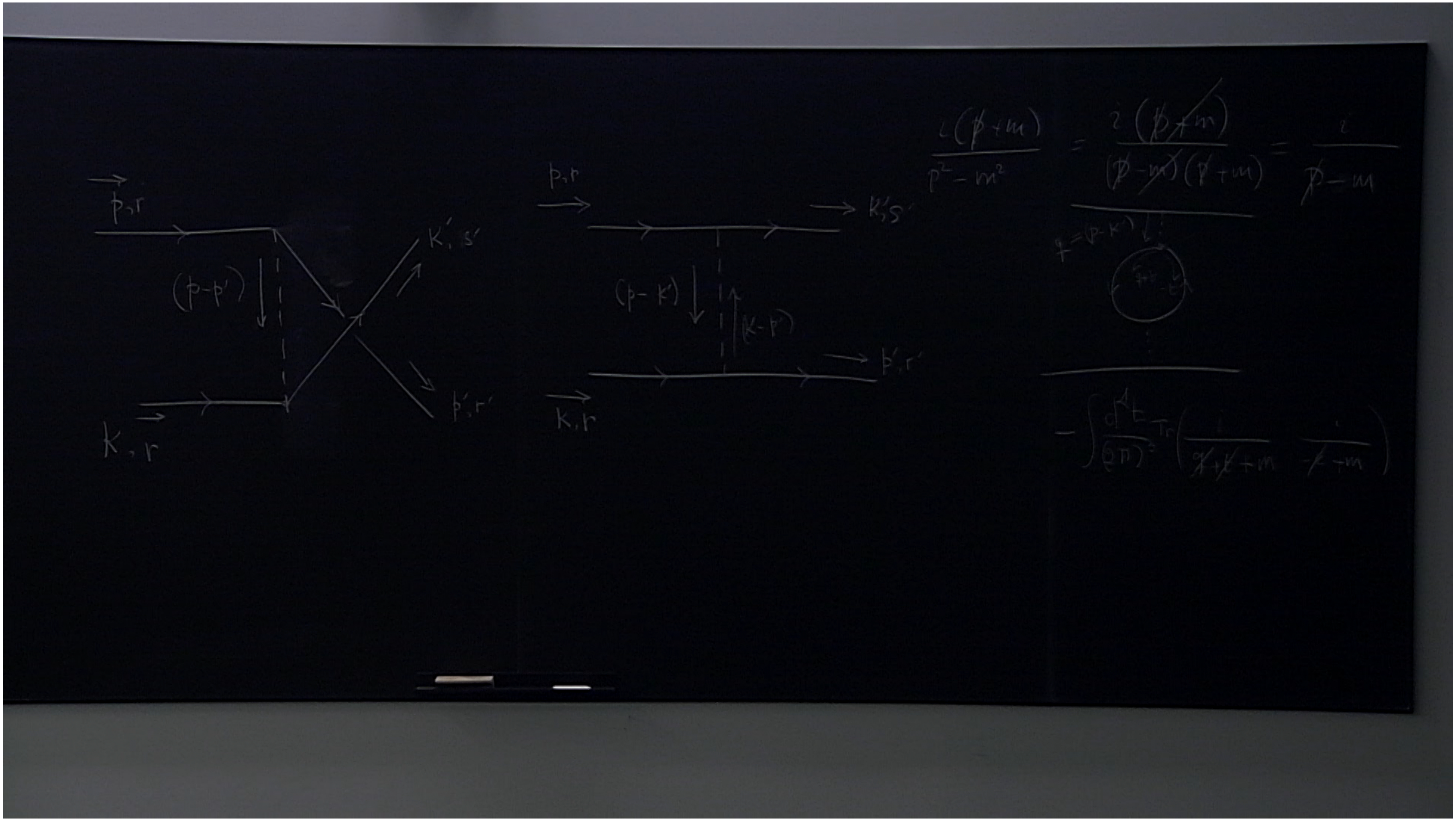

p, r
k, r
K', s'
p', r'
(p-p')
(p-k')
(k-p')




p, r
K, r
K', s'
p', r'
(p−p')
(p−K')
(K−p')

3. Draw various tree-level processes in Yukawa theory and write down their amplitudes according to the Feynman rules.

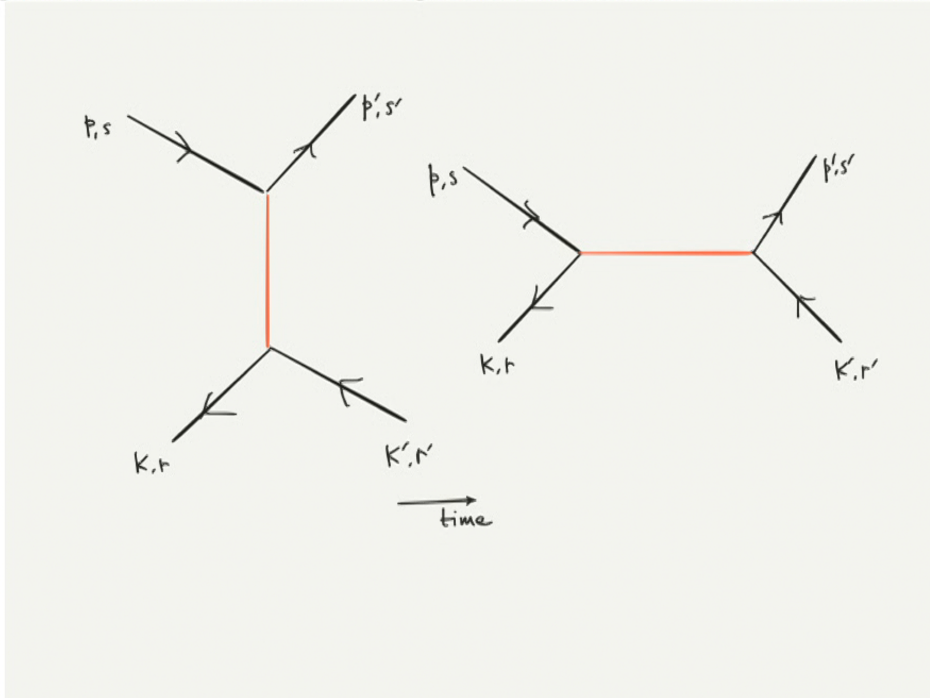
References

[1] M. D. Schwartz, 'Quantum Field Theory and the Standard Model.'

[2] M. E. Peskin and D. V. Schroeder, 'An Introduction to quantum field theory,'

[3] David Tong, 'Lectures on Quantum Field Theory.'

Figure 15: Relative minus sign in nucleon-antinucleon scattering



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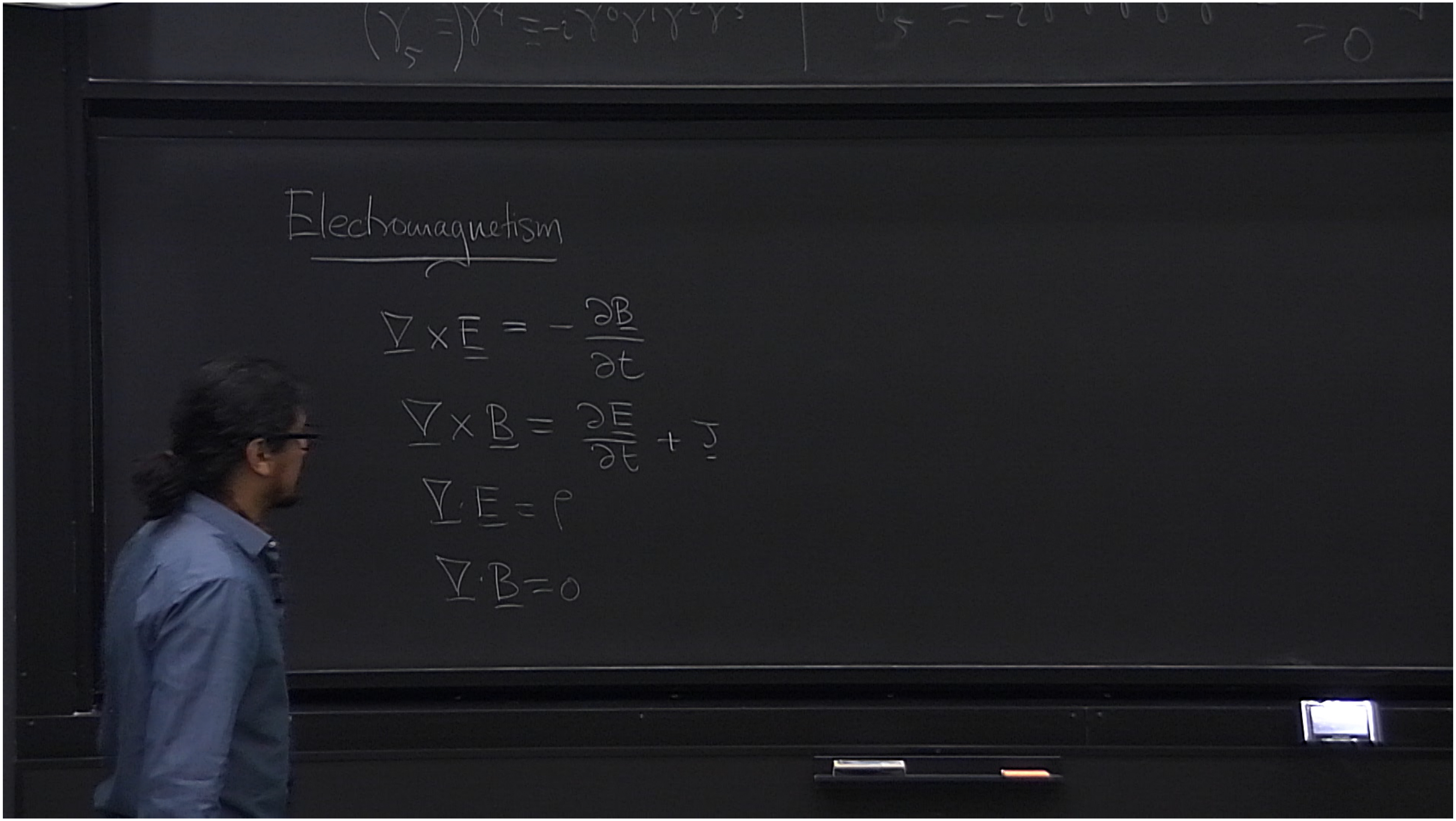
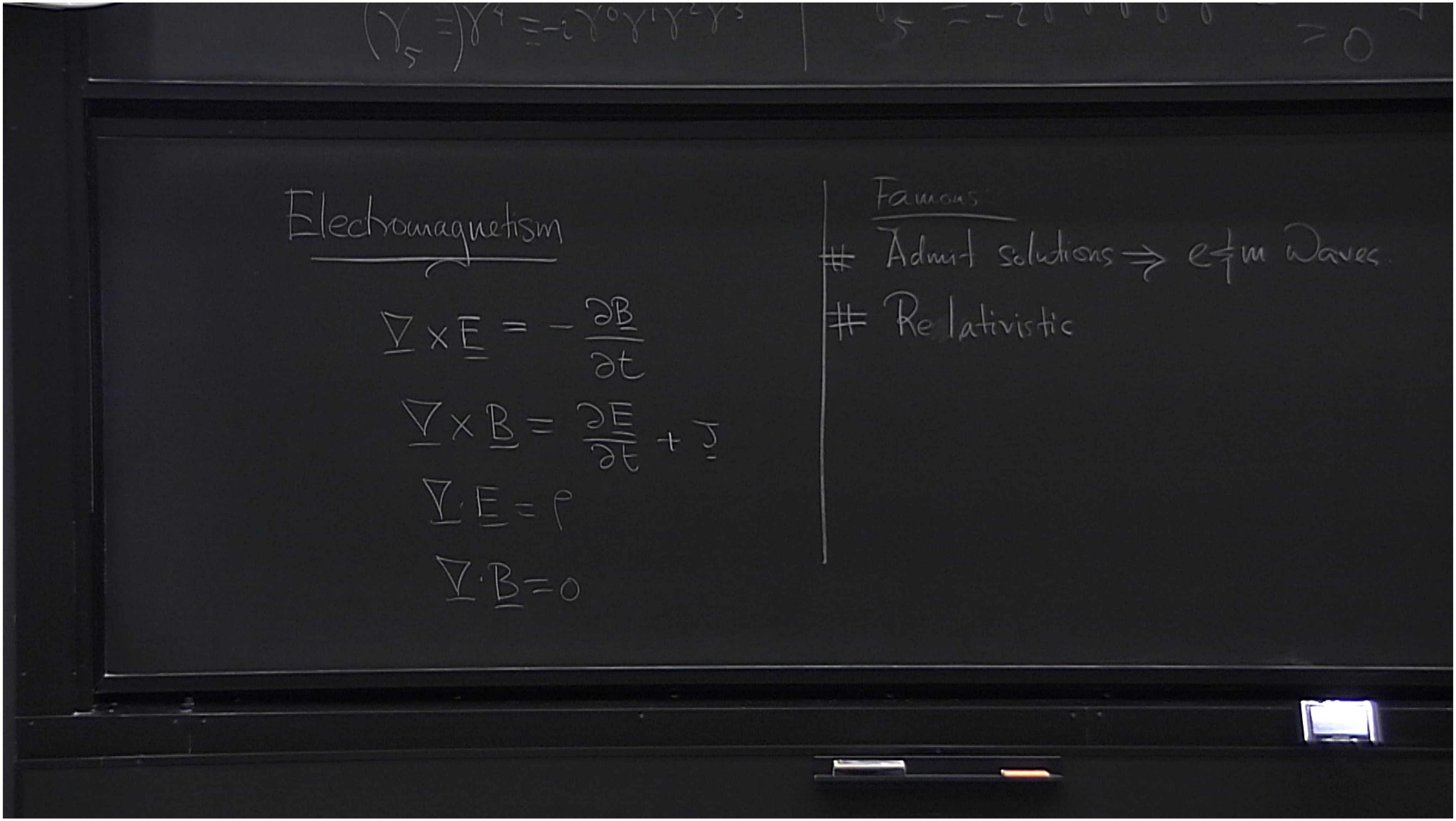

Electromagnetism
$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$
$\nabla \times \underline{B} = \frac{\partial \underline{E}}{\partial t} + \underline{J}$
$\nabla \cdot \underline{E} = \rho$
$\nabla \cdot \underline{B} = 0$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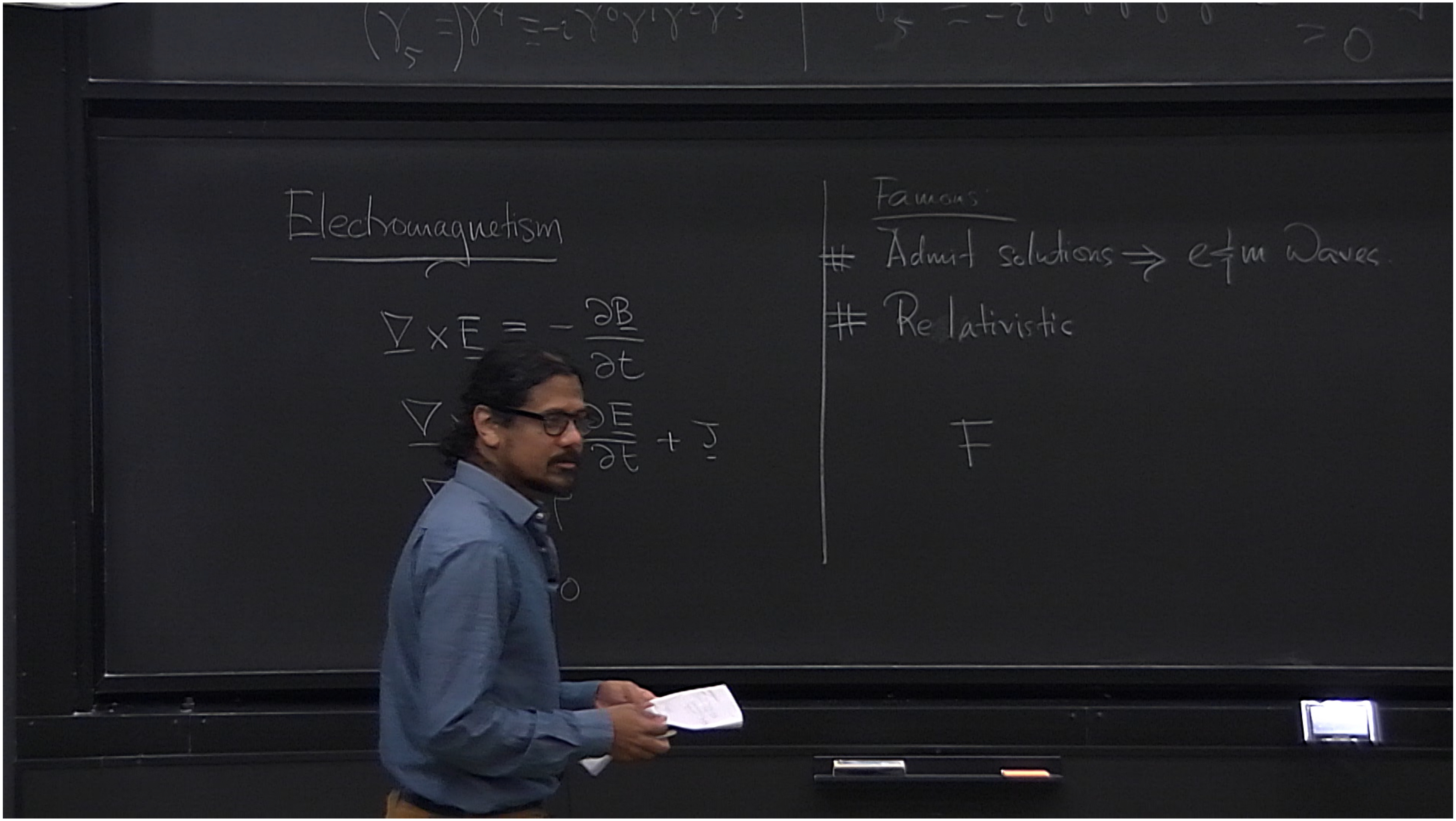
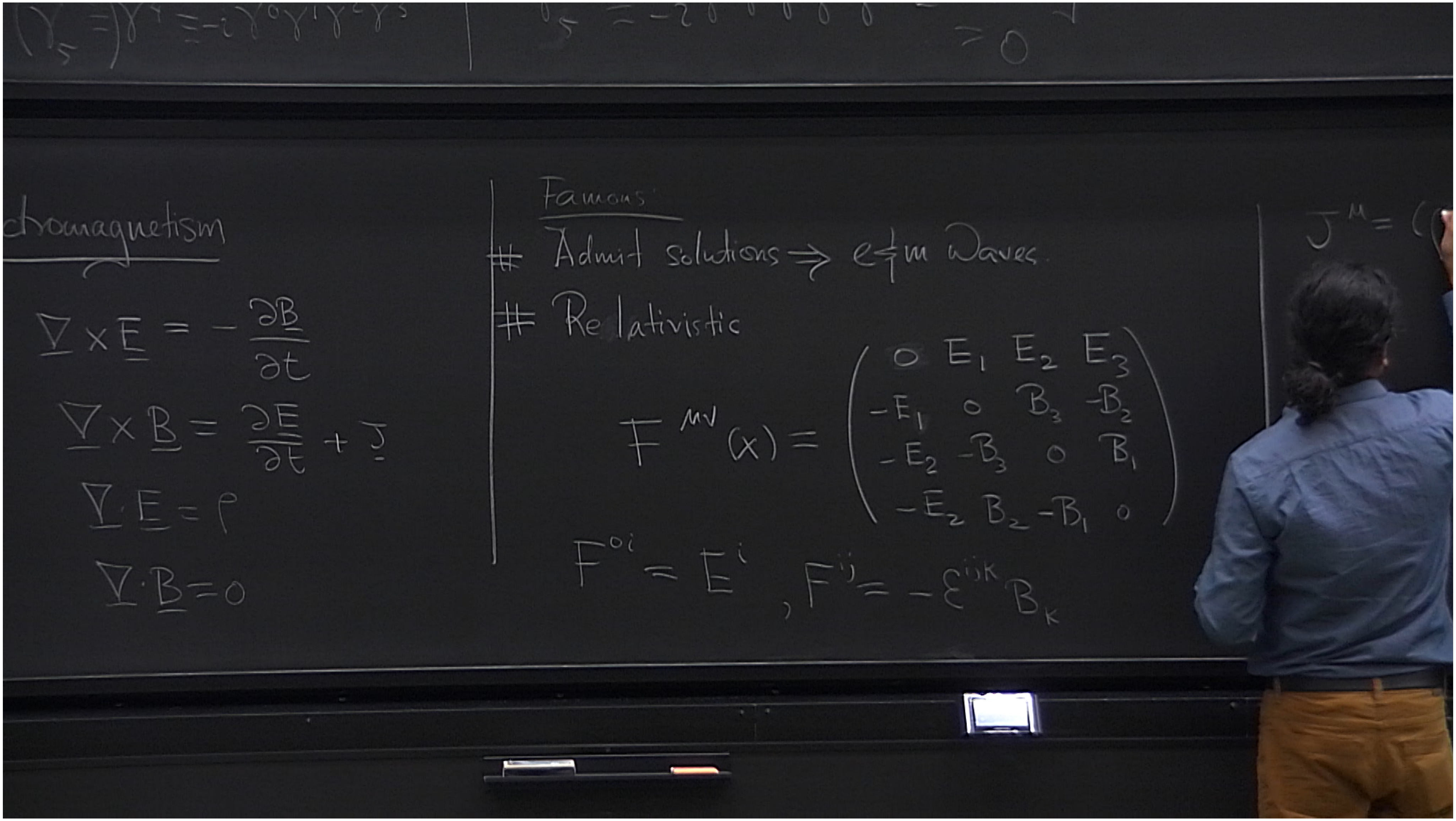
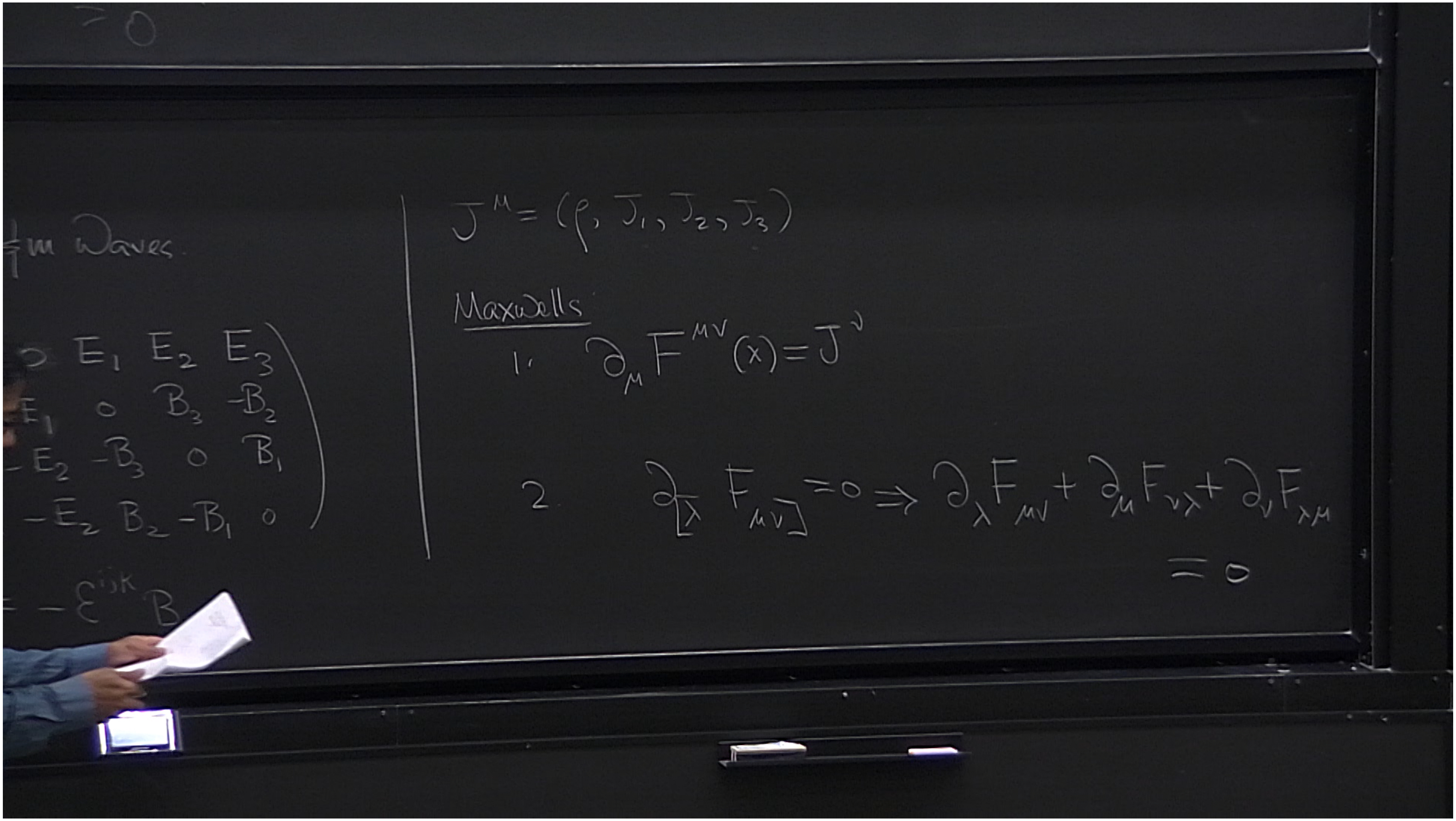
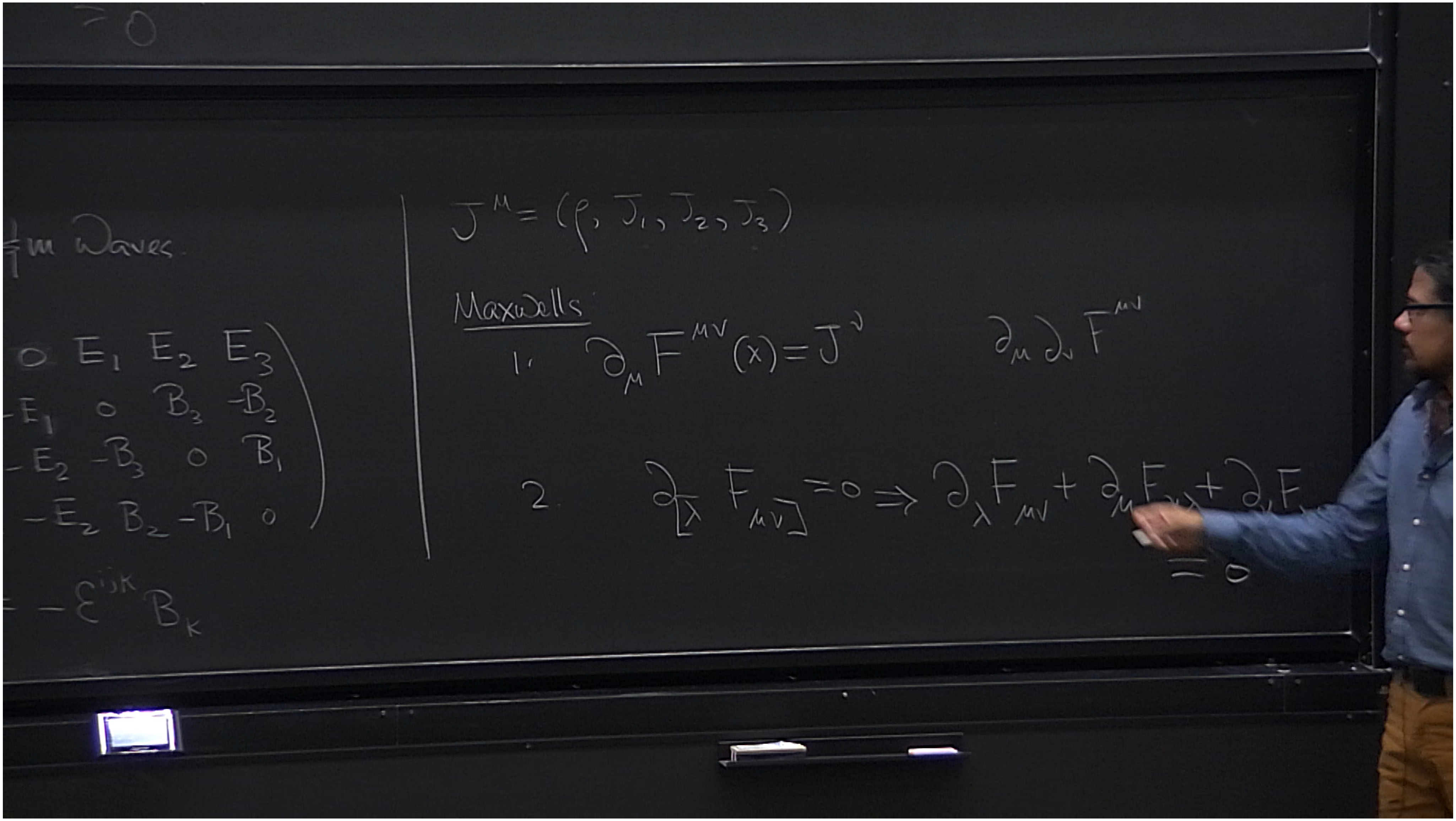
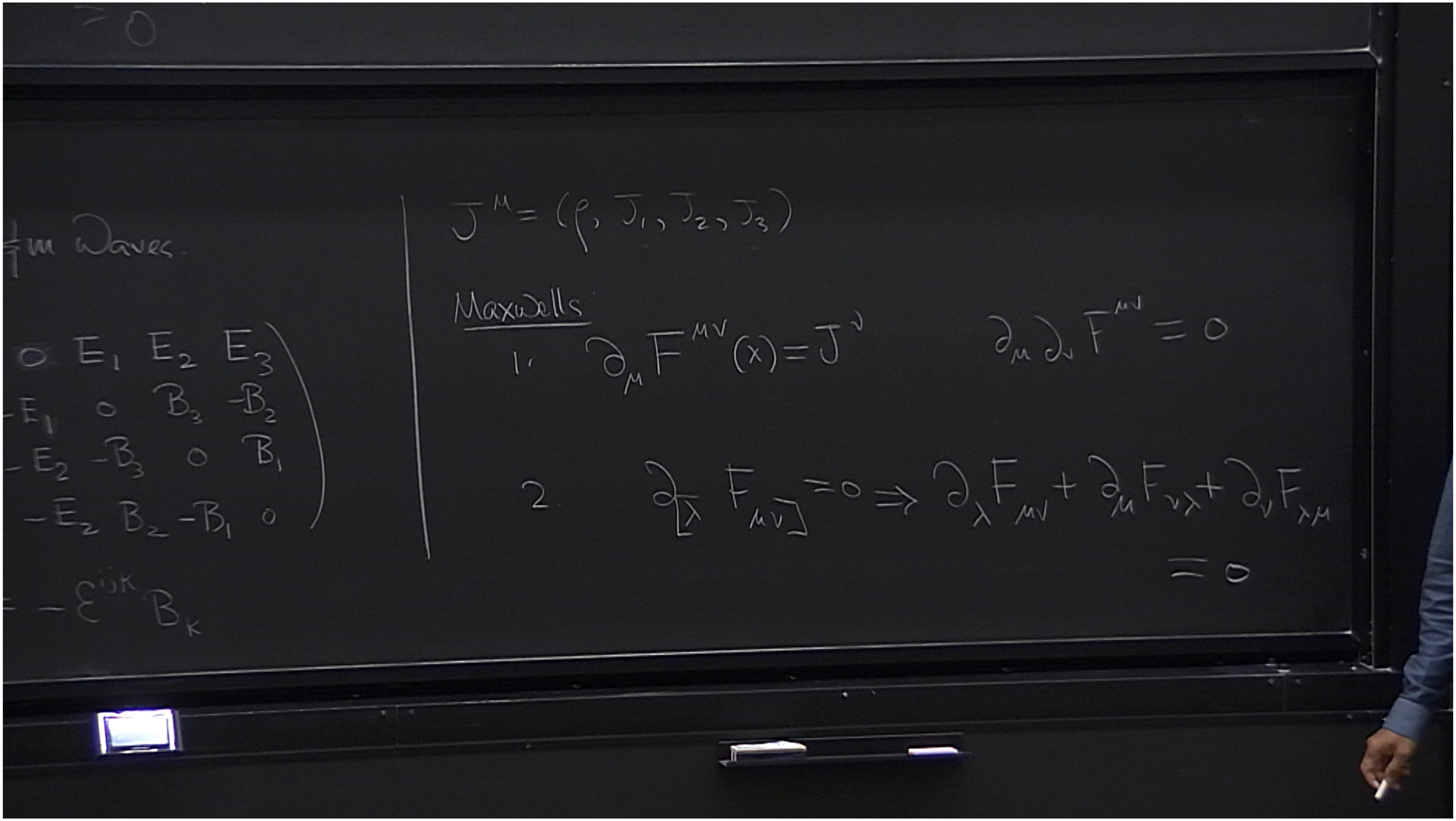
Famous:
Admit solutions ⇒ e&m Waves
Relativistic


Electromagnetism
∇ × E = − ∂B/∂t
Famous
Admit solutions ⇒ e&m Waves
Relativistic
F


ctromagnetism
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times B = \frac{\partial E}{\partial t} + J$
$\nabla \cdot E = \rho$
$\nabla \cdot B = 0$
Famous:
Admit solutions ⇒ e&m Waves.
Relativistic
$F^{\mu\nu}(x) = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_2 & B_2 & -B_1 & 0 \end{pmatrix}$
$F^{0i} = E^i \;,\; F^{ij} = -\varepsilon^{ijk} B_k$
$J^\mu = ($


Em Waves
J^μ = (ρ, J_1, J_2, J_3)
Maxwells
1. ∂_μ F^{μν}(x) = J^ν
2. ∂_[λ F_μν] = 0 ⇒ ∂_λ F_μν + ∂_μ F_νλ + ∂_ν F_λμ = 0


Em Waves.
$J^{\mu} = (\rho, J_1, J_2, J_3)$
Maxwells
1. $\partial_{\mu} F^{\mu\nu}(x) = J^{\nu}$
$\partial_{\mu} \partial_{\nu} F^{\mu\nu}$
2. $\partial_{[\lambda} F_{\mu\nu]} = 0 \Rightarrow \partial_{\lambda} F_{\mu\nu} + \partial_{\mu} F_{\nu\lambda} + \partial_{\nu} F_{\lambda\mu} = 0$
$= -\varepsilon^{ijk} B_k$


Em Waves.
$J^\mu = (\rho, J_1, J_2, J_3)$
Maxwells
1. $\partial_\mu F^{\mu\nu}(x) = J^\nu$
$\partial_\mu \partial_\nu F^{\mu\nu} = 0$
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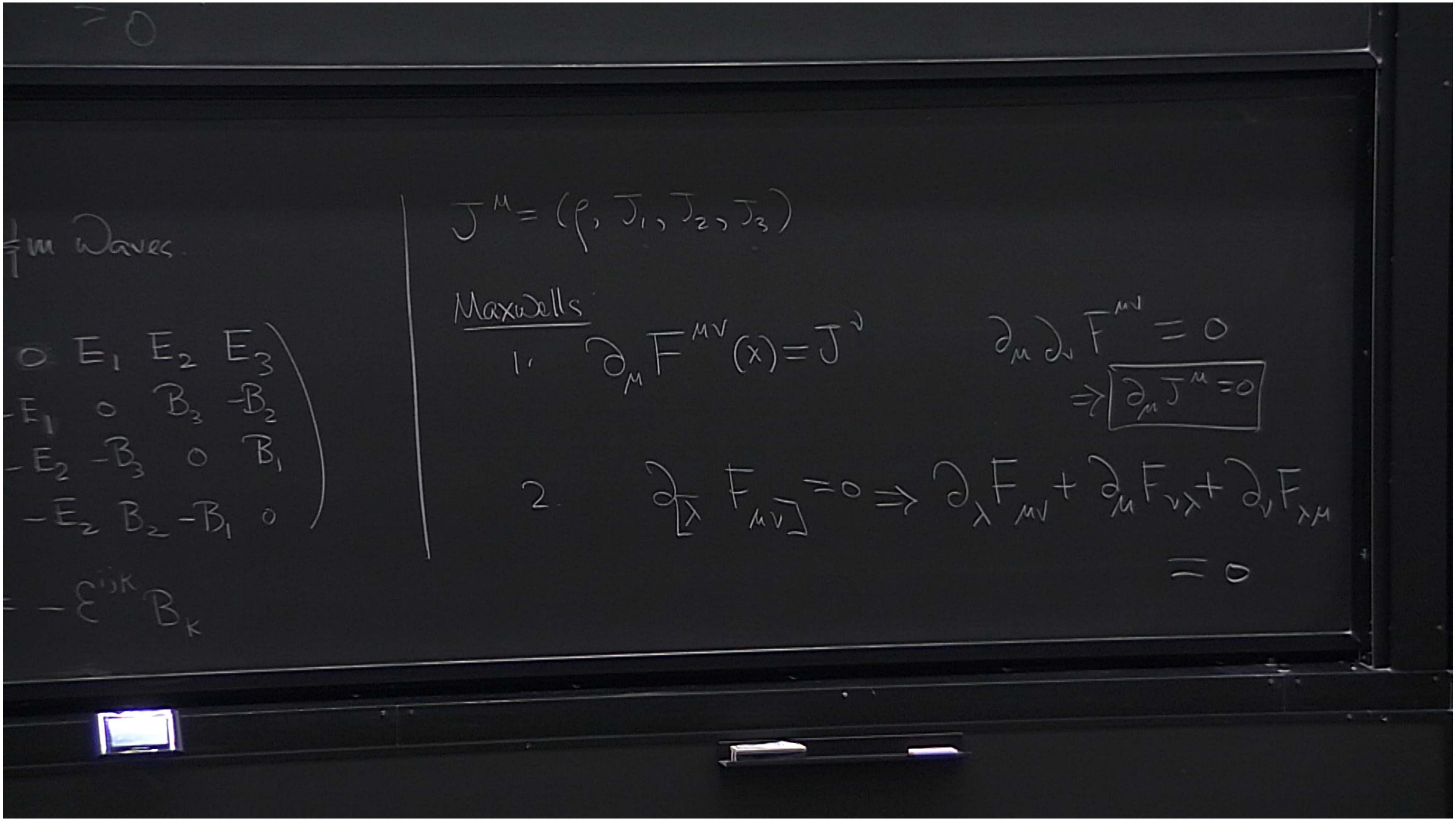
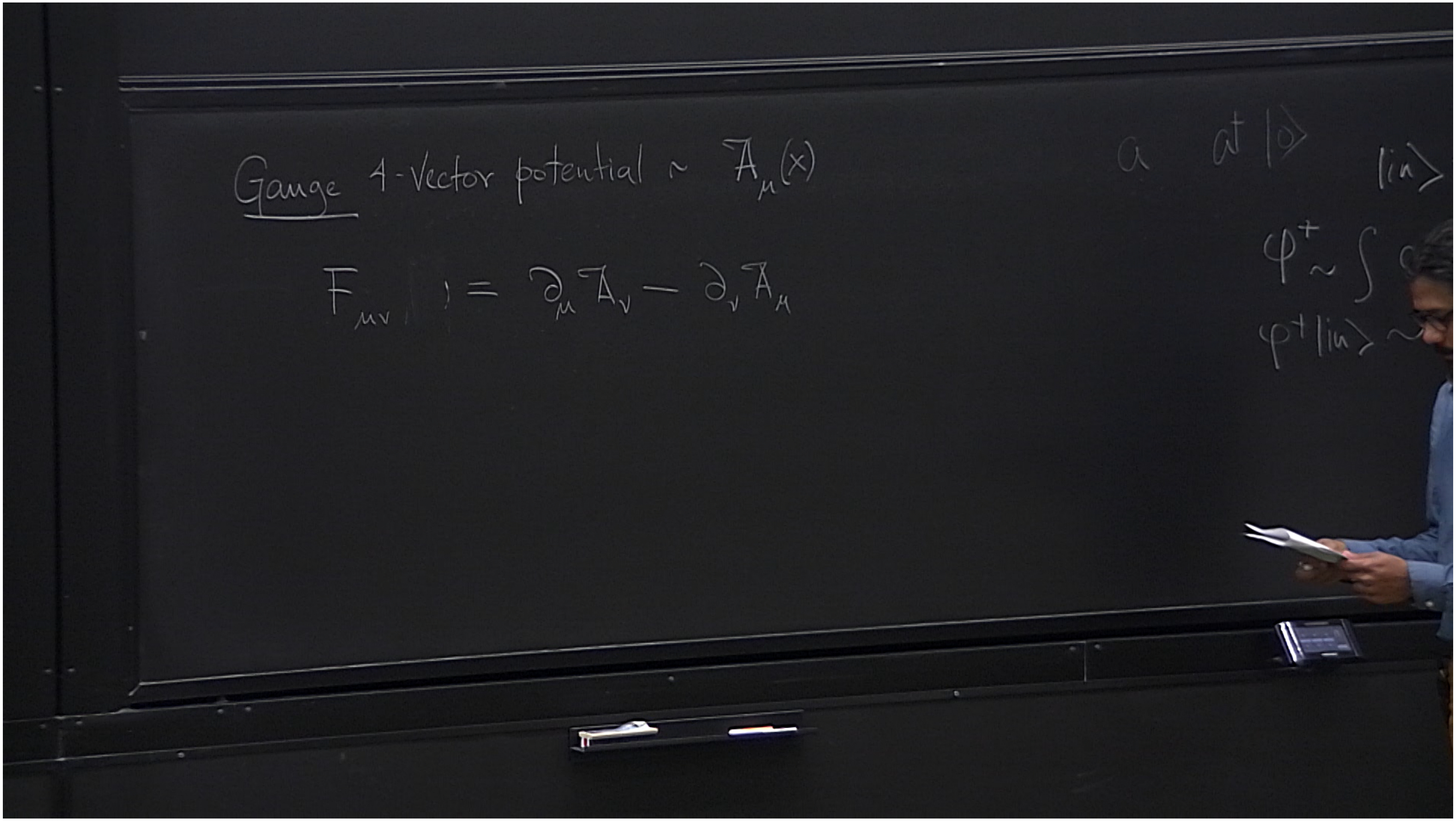
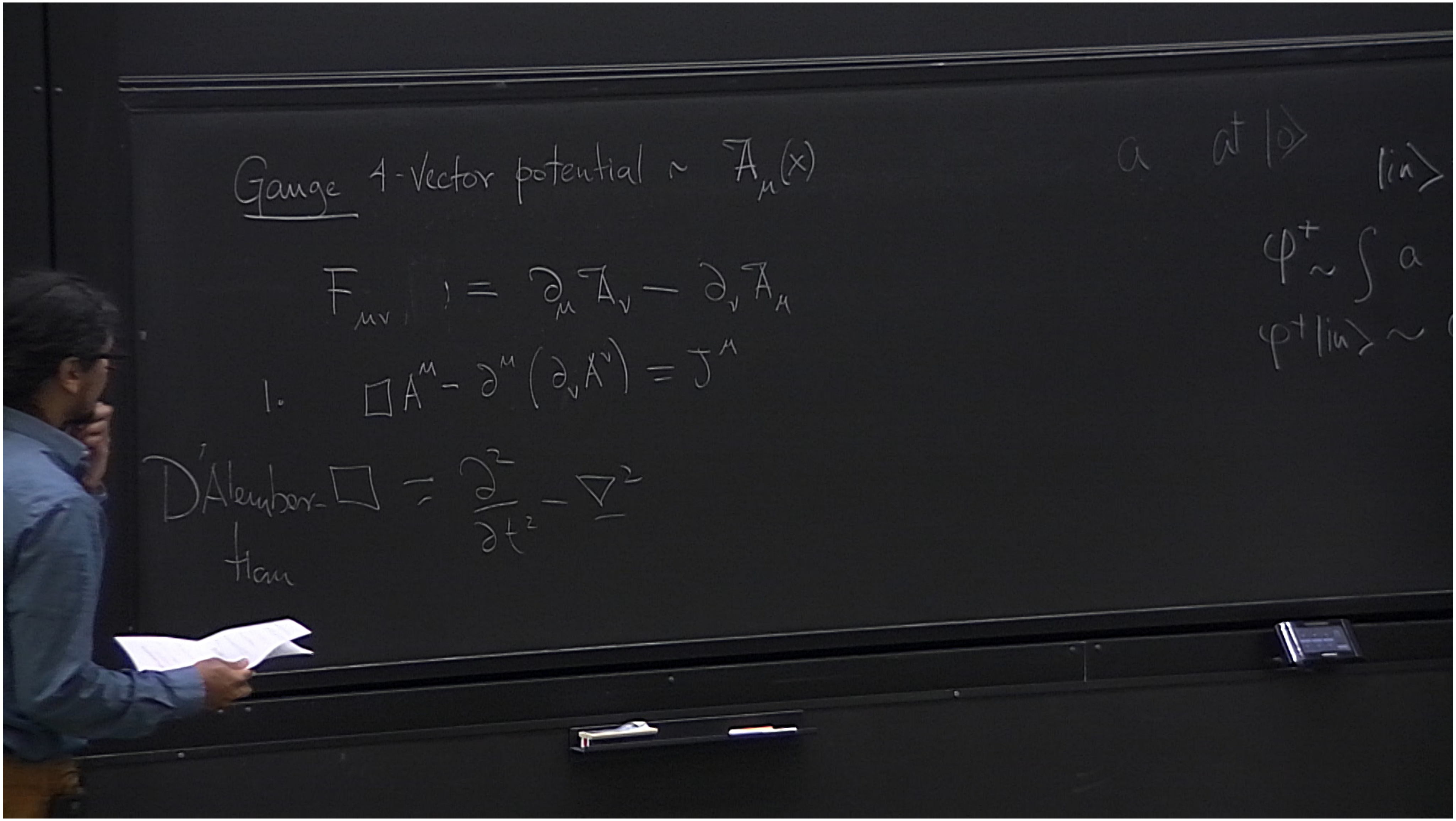
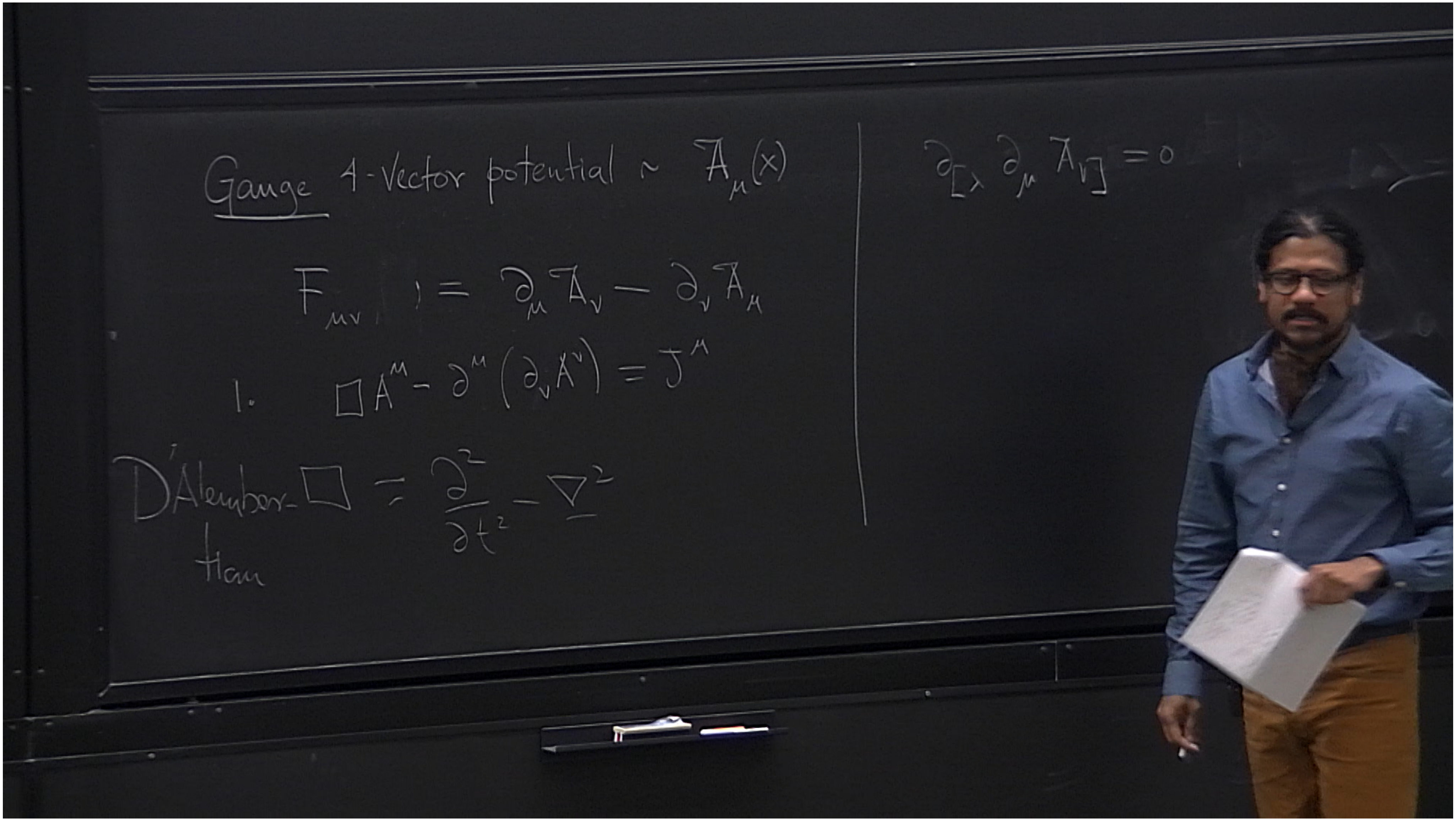
$= 0$
$-\varepsilon^{ijk} B_k$


Em Waves
$\begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$
$= -\varepsilon^{ijk} B_k$
$J^\mu = (\rho, J_1, J_2, J_3)$
Maxwells
1. $\partial_\mu F^{\mu\nu}(x) = J^\nu$
$\partial_\mu \partial_\nu F^{\mu\nu} = 0$
$\Rightarrow \partial_\mu J^\mu = 0$
2. $\partial_{[\lambda} F_{\mu\nu]} = 0 \Rightarrow \partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0$


Gauge 4-Vector potential ~ Aμ(x)
Fμν = ∂μ Aν − ∂ν Aμ
a a† |0⟩
|in⟩
φ+ ~ ∫
φ+|in⟩ ~


Gauge 4-Vector potential ~ $A_\mu(x)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
1. $\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = J^\mu$
D'Alembertian $\Box = \frac{\partial^2}{\partial t^2} - \nabla^2$
a $a^\dagger|0\rangle$
$|in\rangle$
$\varphi^+ \sim \int a$
$\varphi^+|in\rangle \sim$


Gauge 4-Vector potential ~ $A_\mu(x)$
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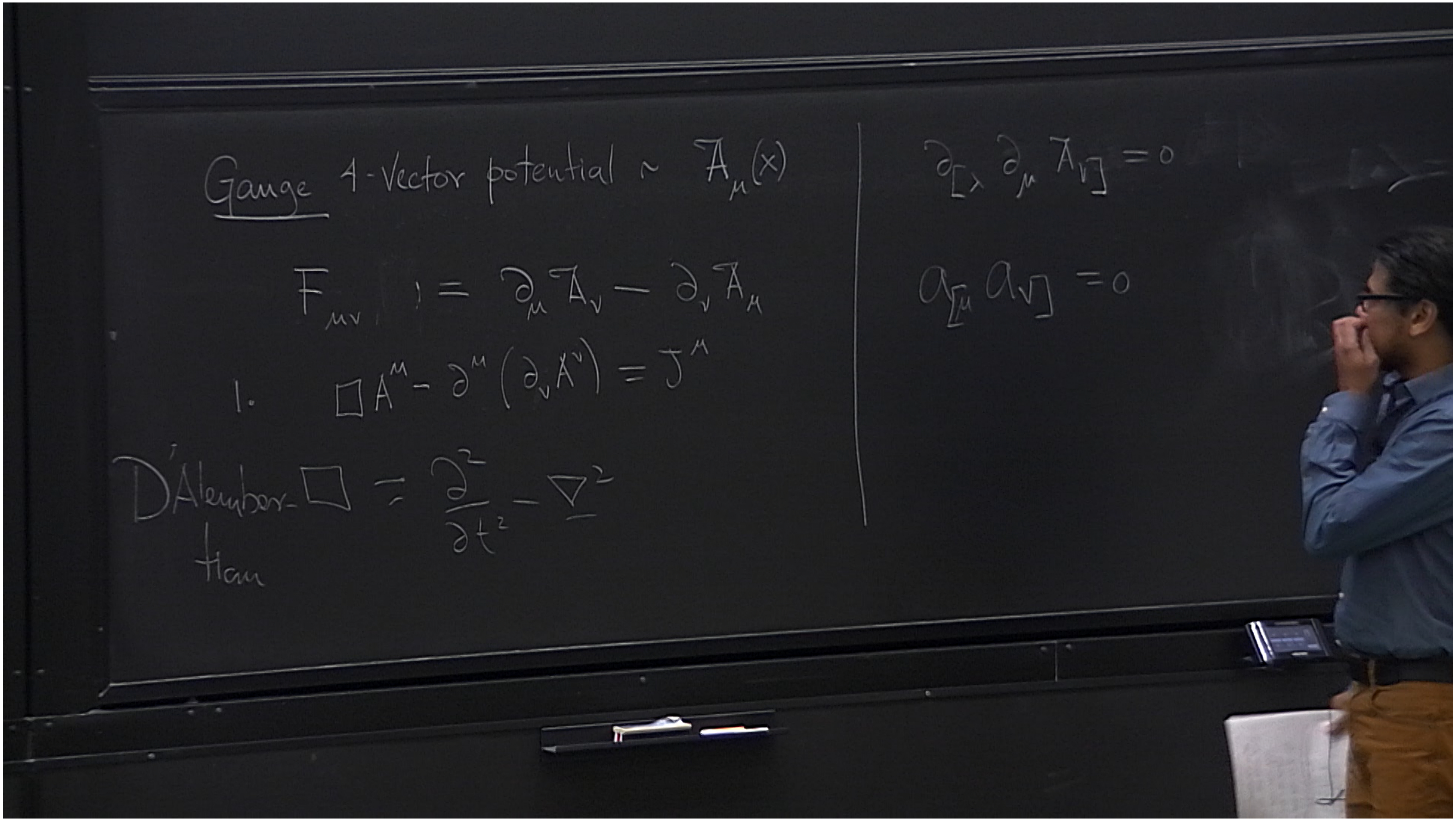
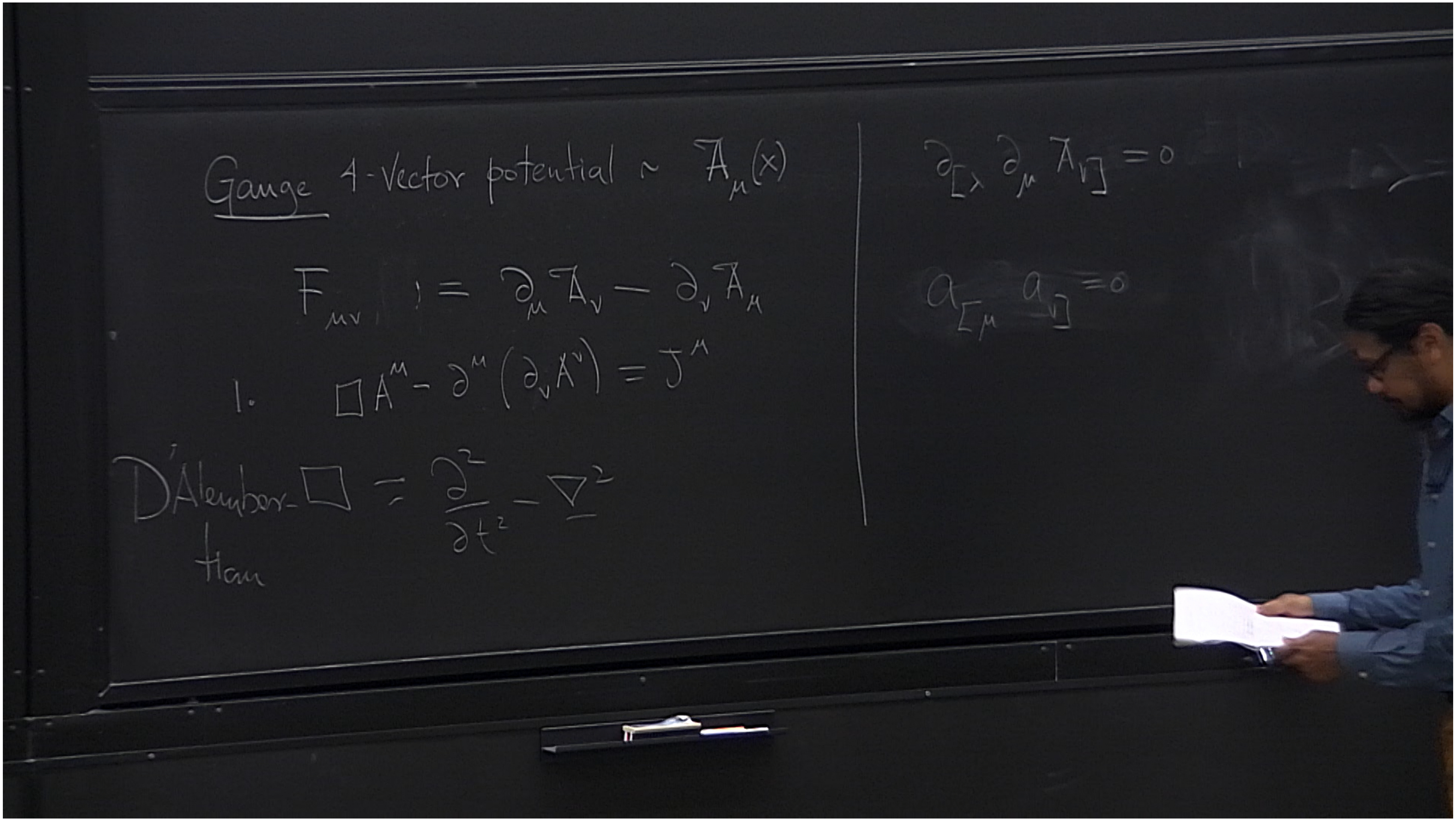
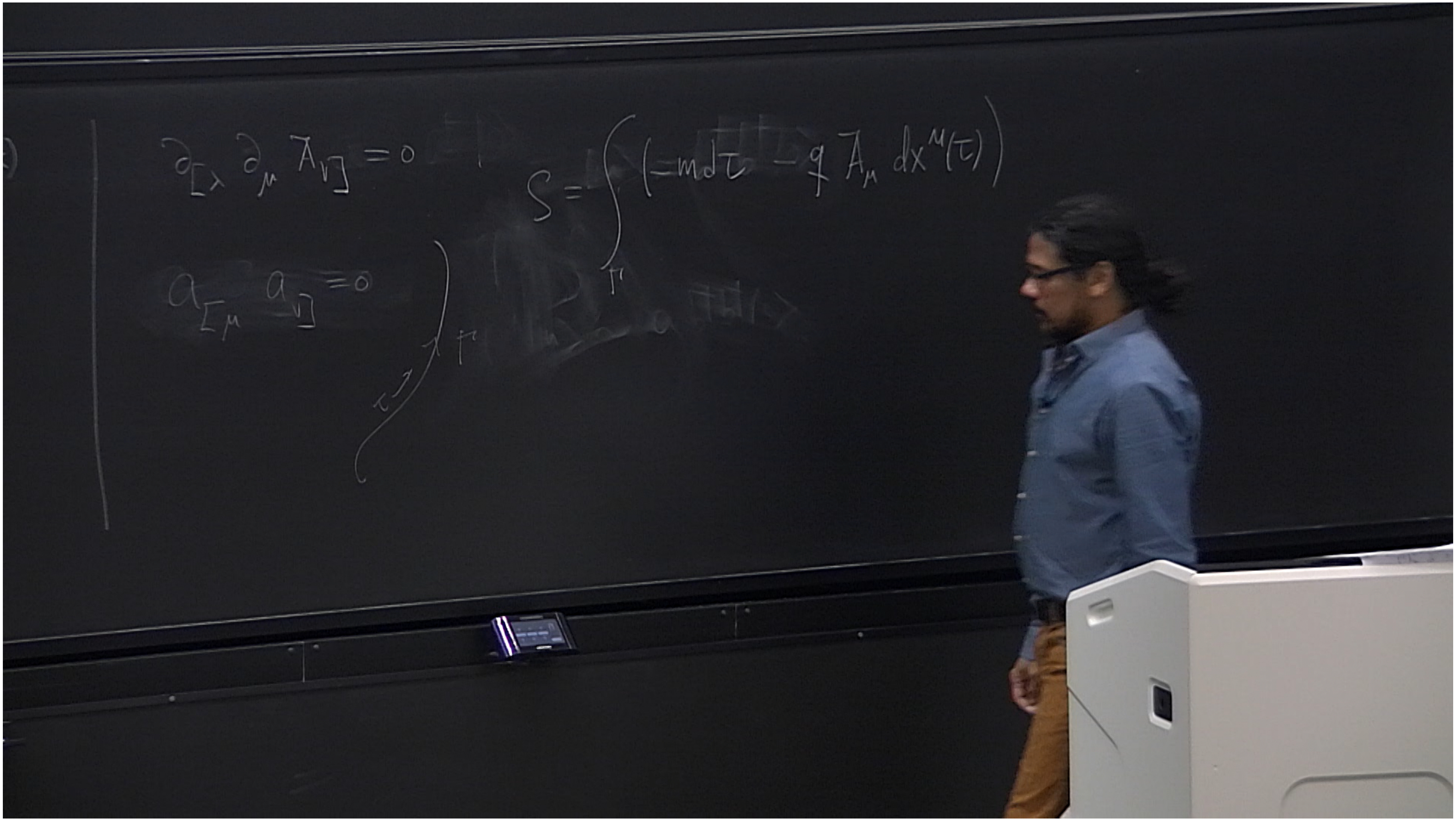
1. $\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = J^\mu$
D'Alembertian $\Box = \frac{\partial^2}{\partial t^2} - \nabla^2$
$\partial_{[\lambda}\partial_\mu A_{\nu]} = 0$

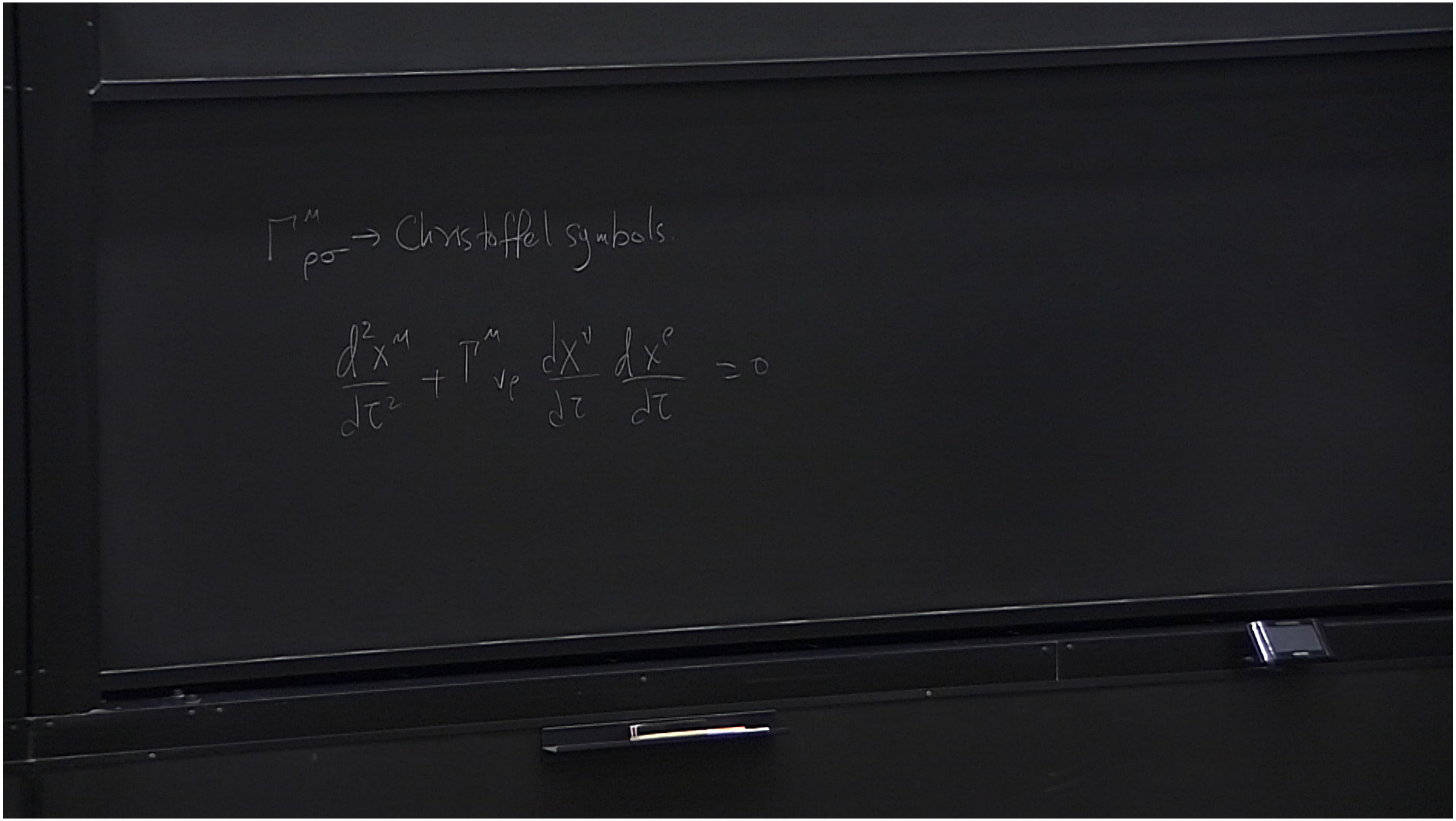

Gauge 4-Vector potential ~ $A_\mu(x)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
1. $\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = J^\mu$
D'Alembertian $\Box = \frac{\partial^2}{\partial t^2} - \nabla^2$
$\partial_{[\lambda}\partial_\mu A_{\nu]} = 0$
$a_{[\mu} a_{\nu]} = 0$

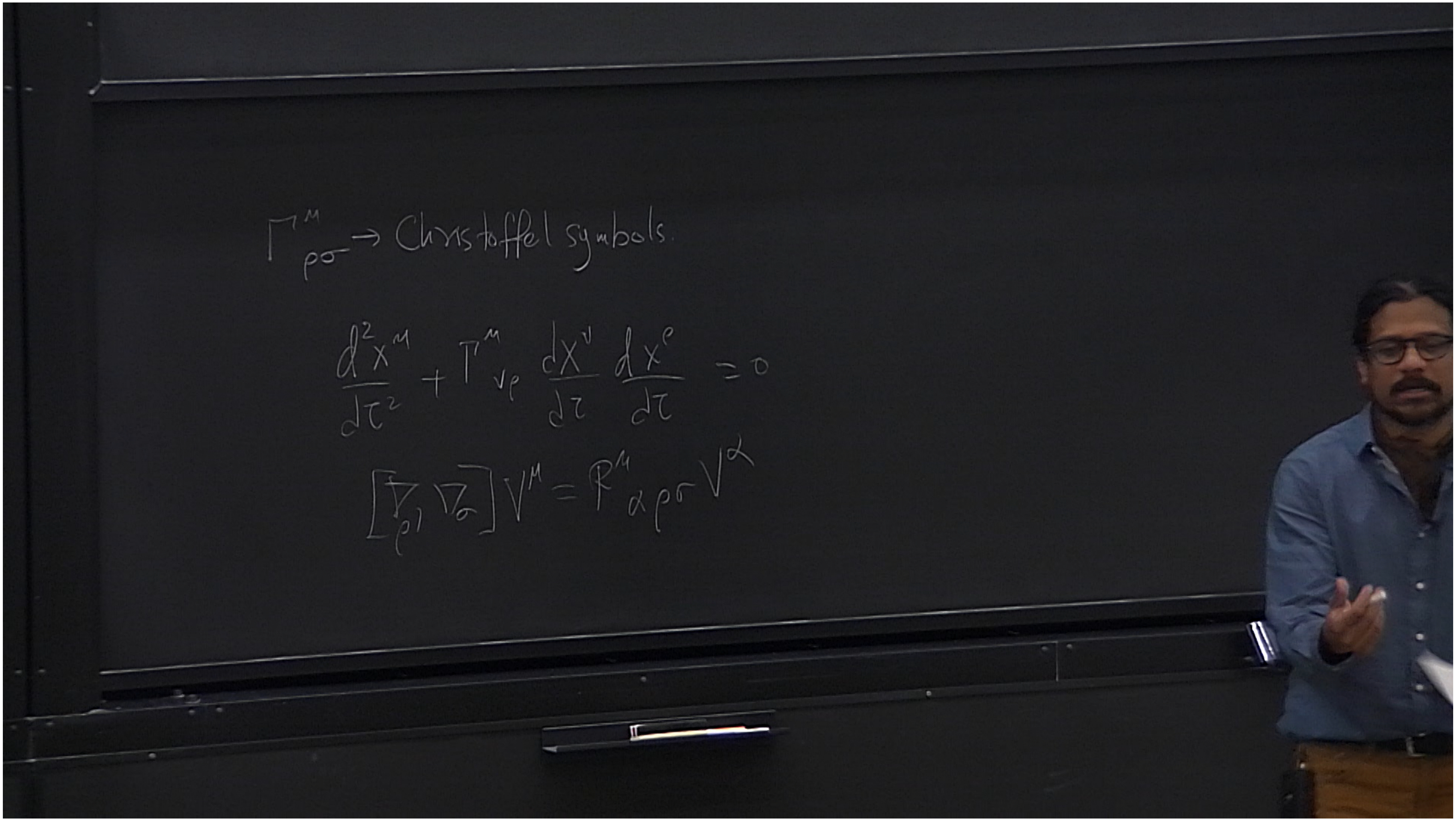

Gauge 4-Vector potential ~ A_μ(x)
F_μν := ∂_μ A_ν − ∂_ν A_μ
1. □A^μ − ∂^μ(∂_ν A^ν) = J^μ
D'Alembertian □ = ∂²/∂t² − ∇²
∂_[λ ∂_μ A_ν] = 0
a_[μ a_ν] = 0

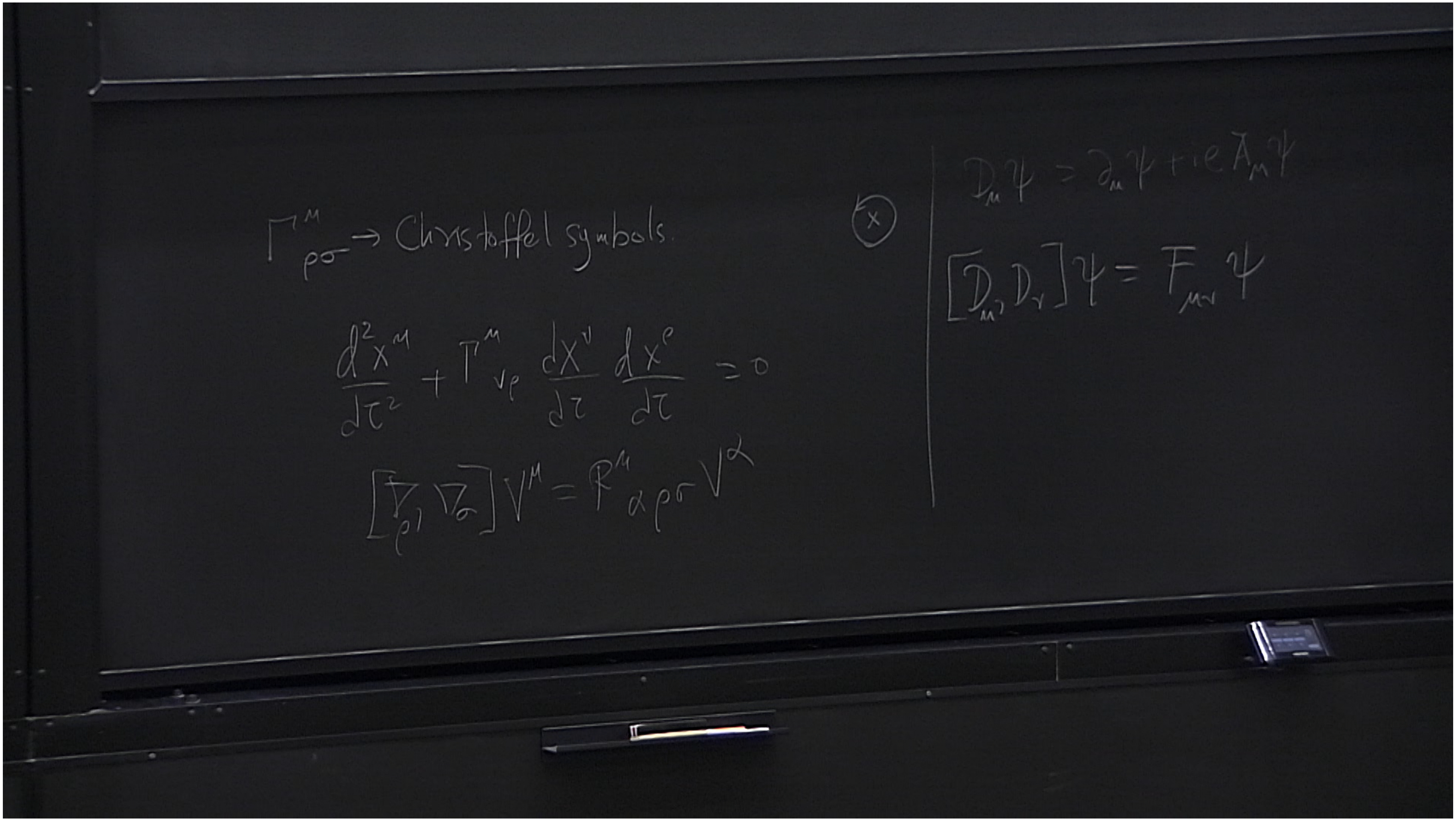


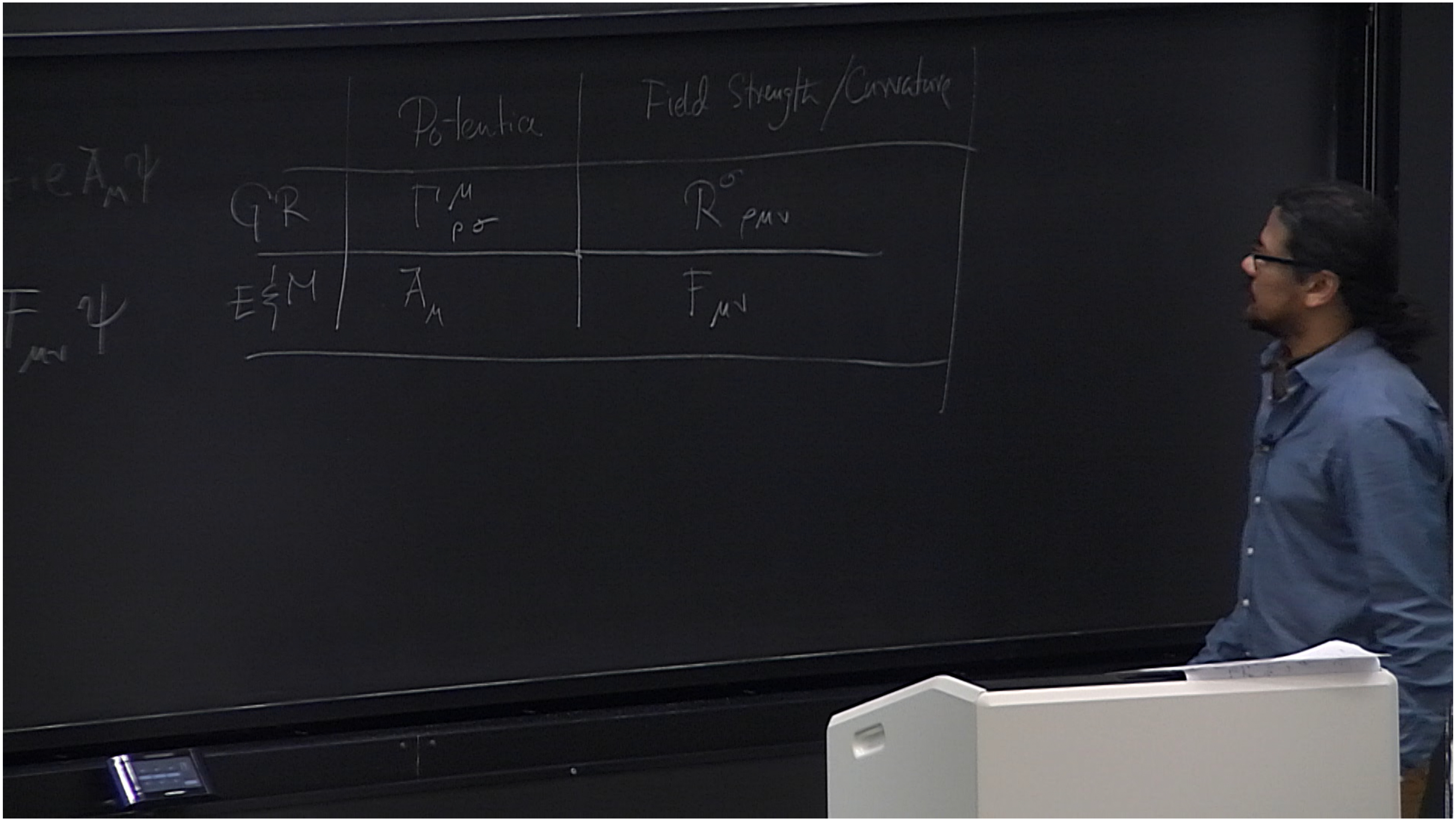
$\partial_{[\lambda} \partial_{\mu} A_{\nu]} = 0$
$S = \int_{\Gamma} \left(-m\, d\tau - q A_{\mu}\, dx^{\mu}(\tau) \right)$
$a_{[\mu} a_{\nu]} = 0$

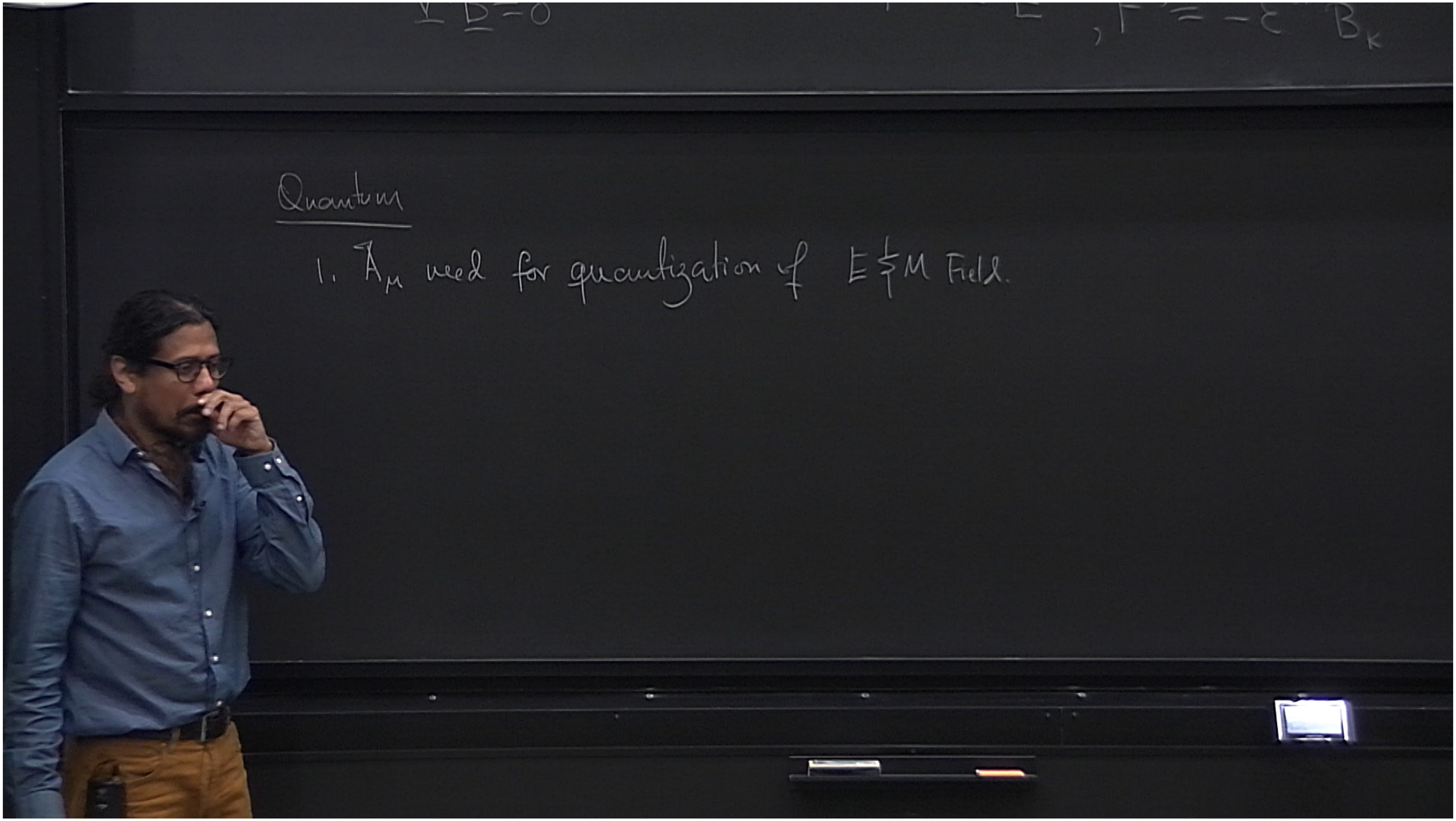
$\Gamma^{\mu}_{\rho\sigma} \rightarrow$ Christoffel symbols.

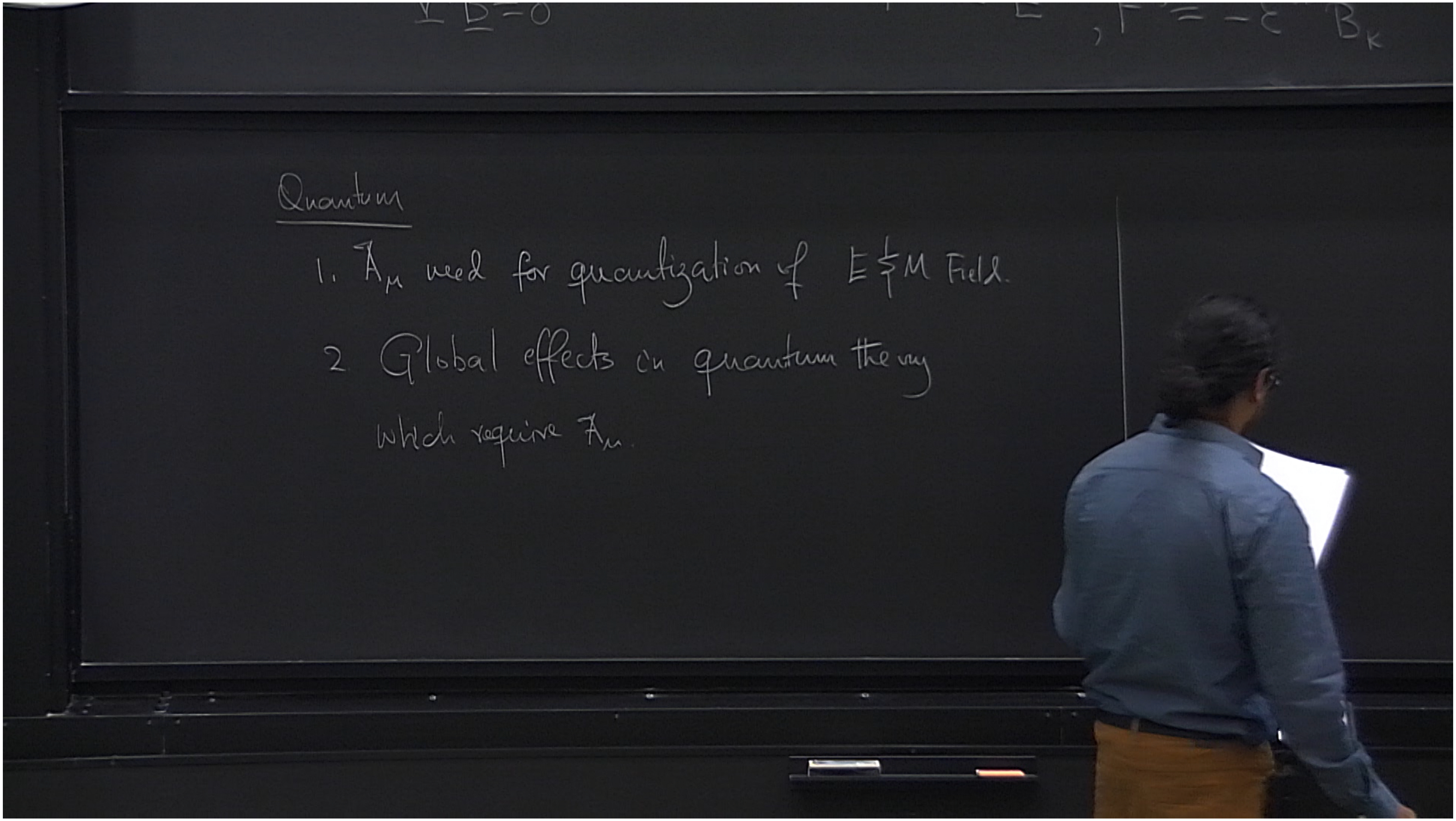

$\Gamma^{\mu}_{\rho\sigma} \rightarrow$ Christoffel symbols.
$\frac{d^2x^\mu}{d\tau^2} + \Gamma^{\mu}_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$

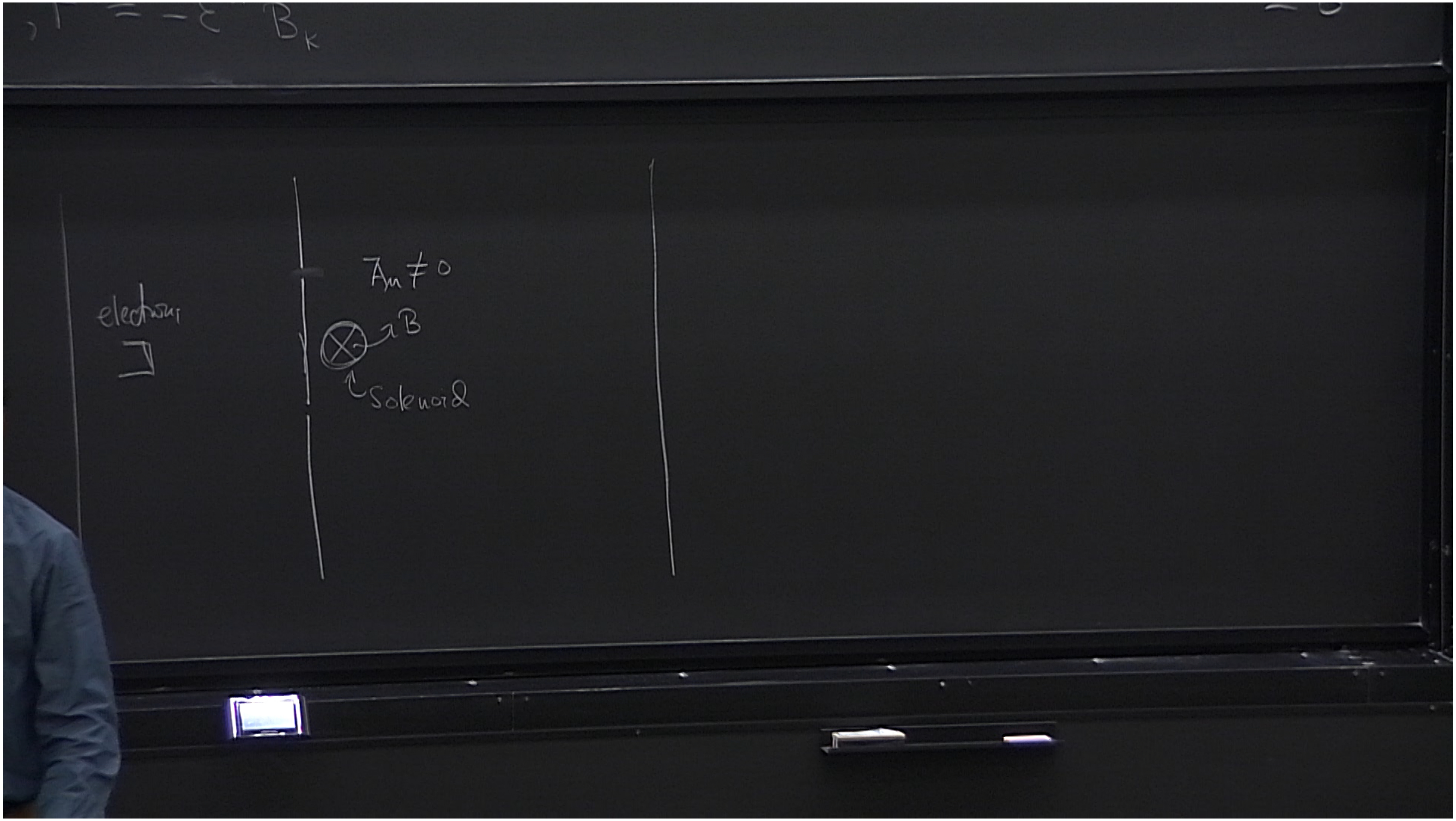

$\Gamma^{\mu}_{\rho\sigma} \rightarrow$ Christoffel symbols.
$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^{\mu}_{\nu\rho} \frac{dx^\nu}{d\tau} \frac{dx^\rho}{d\tau} = 0$
$[\nabla_\rho, \nabla_\sigma] V^\mu = R^{\mu}{}_{\alpha\rho\sigma} V^\alpha$

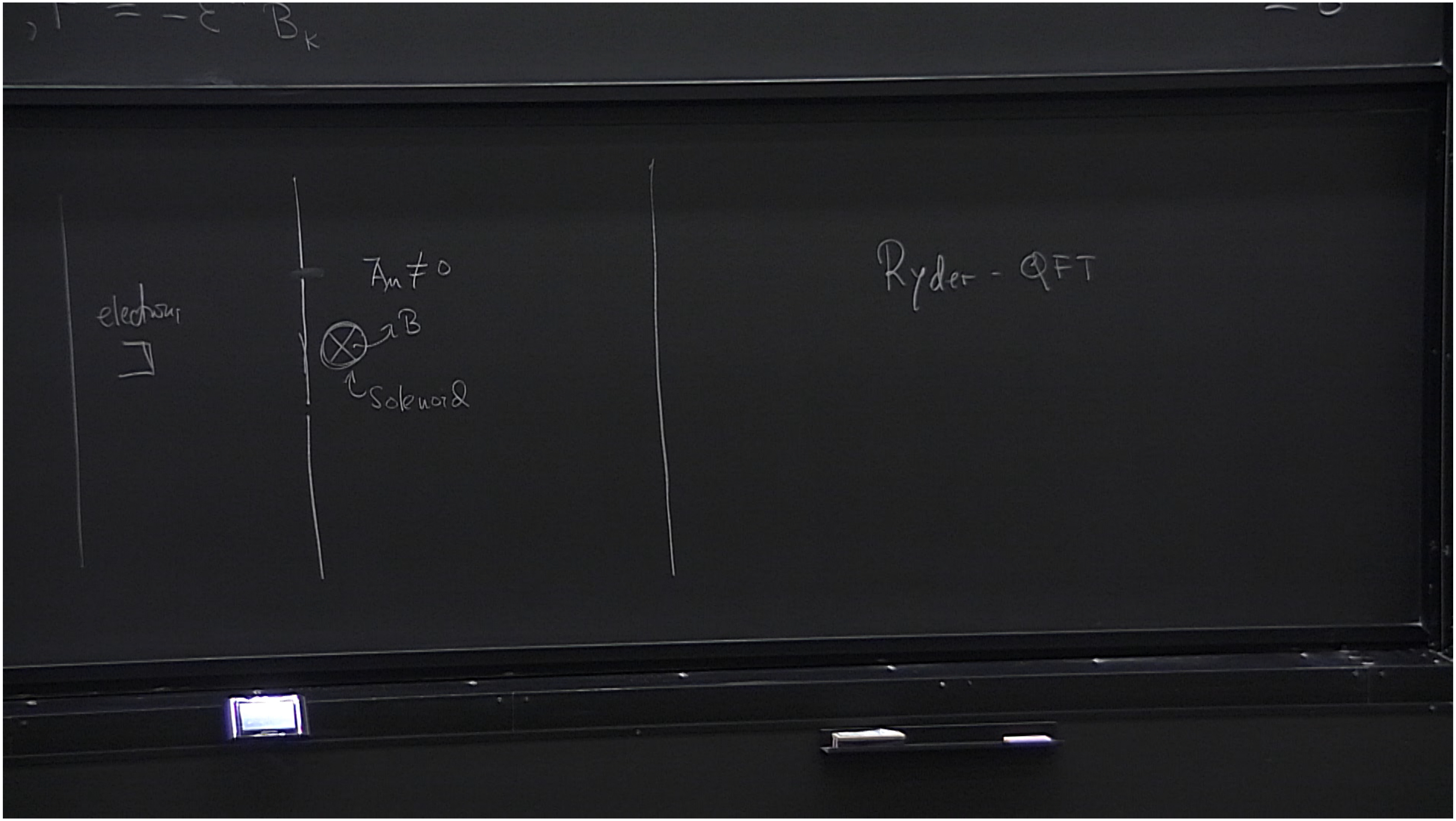

$\Gamma^{\mu}_{\rho\sigma} \rightarrow$ Christoffel symbols.
$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$
$[\nabla_\rho, \nabla_\sigma] V^\mu = R^\mu{}_{\alpha\rho\sigma} V^\alpha$
$D_\mu \psi = \partial_\mu \psi + ie A_\mu \psi$
$[D_\mu, D_\nu]\psi = F_{\mu\nu}\psi$

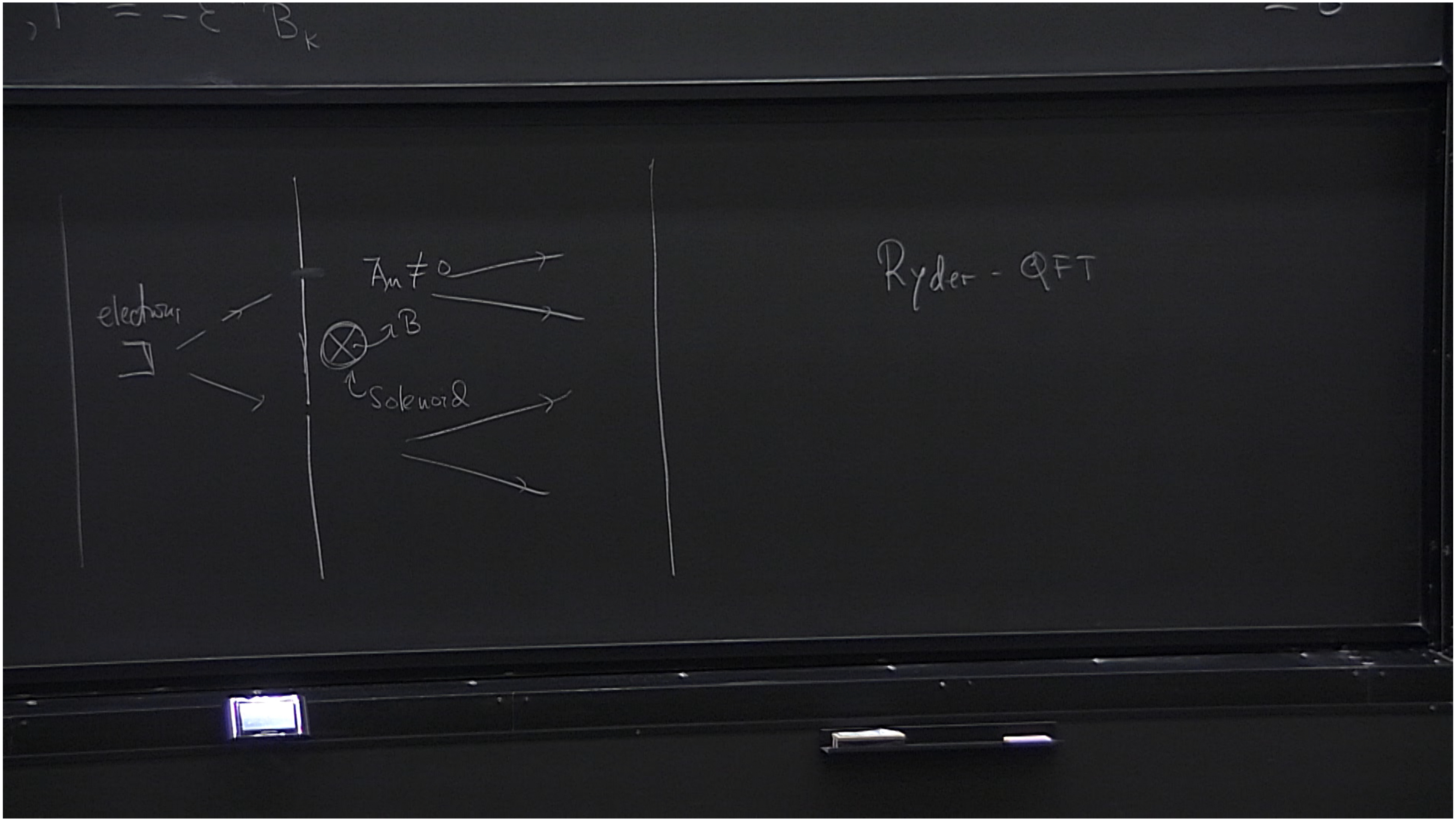

Potential
Field Strength / Curvature
GR
$\Gamma^{\mu}_{\rho\sigma}$
$R^{\sigma}_{\rho\mu\nu}$
E&M
A_μ
$F_{\mu\nu}$
$ieA_\mu\psi$
$F_{\mu\nu}\psi$


Quantum
1. A_μ need for quantization of E&M Field.

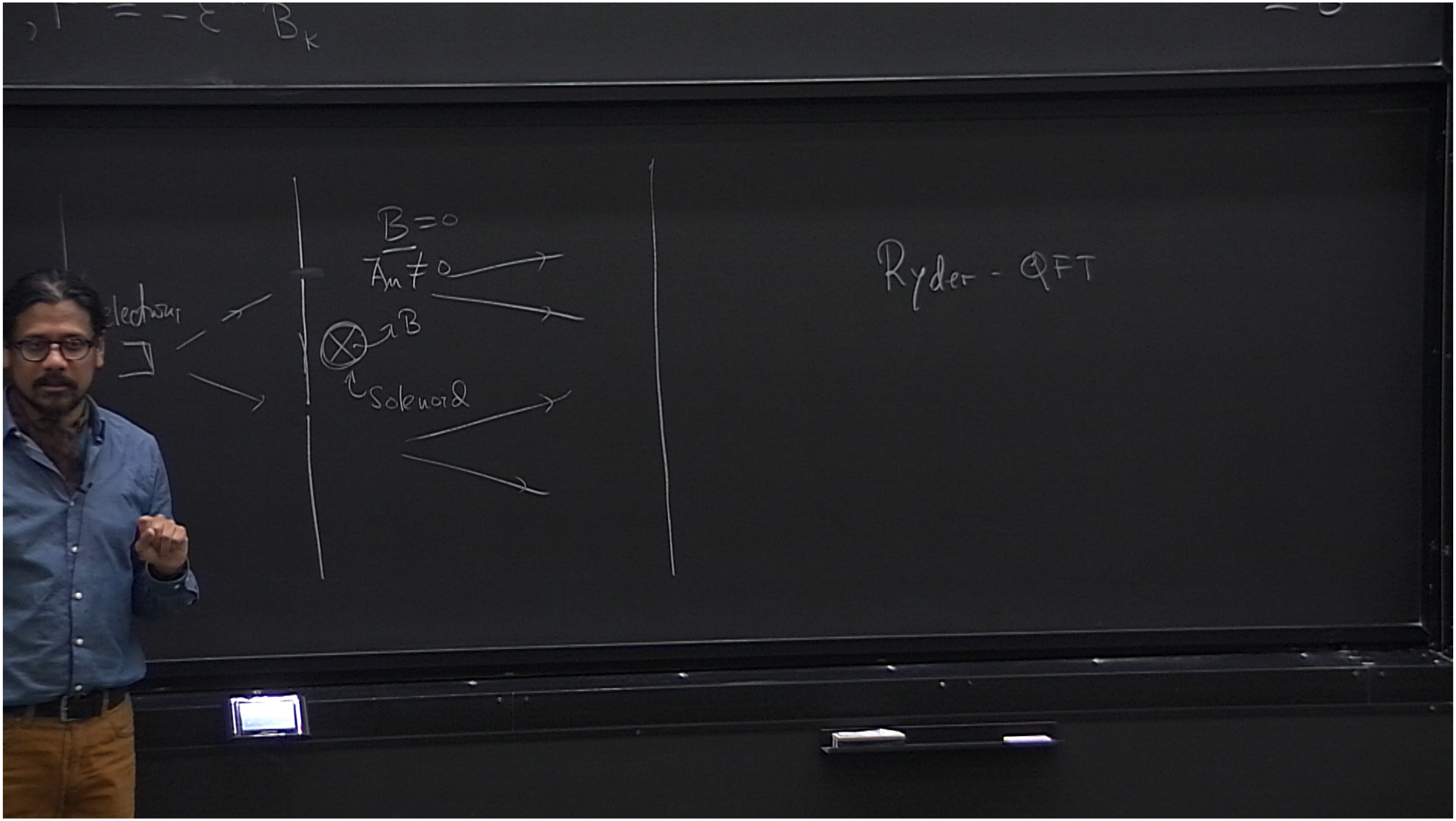

Quantum
1. A_μ need for quantization of E & M Field.
2 Global effects in quantum theory
which require A_μ.

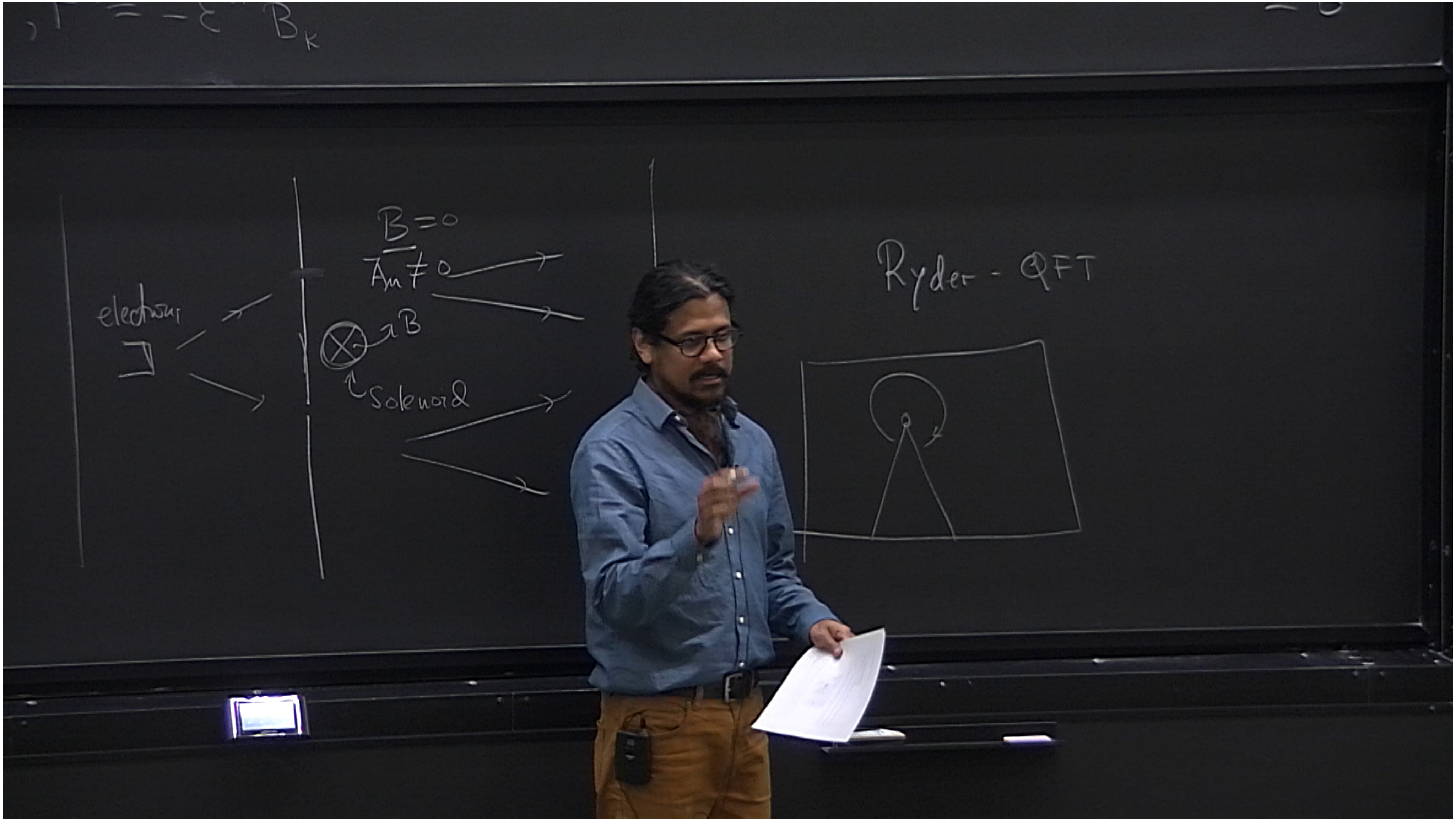

electron
Am ≠ 0
B
Solenoid

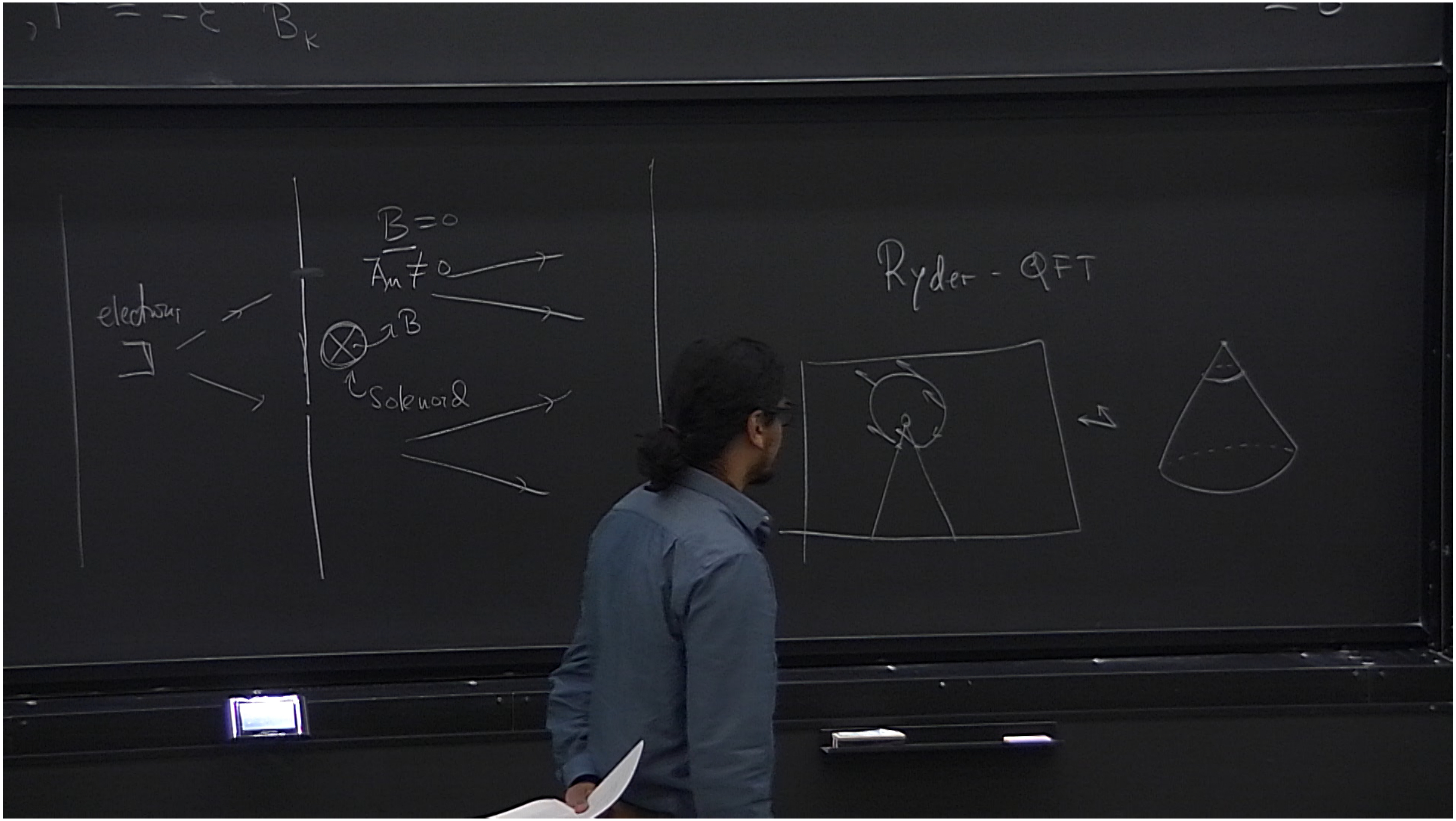

electrons
Aμ ≠ 0
B
Solenoid
Ryder - QFT



electrons
A_μ ≠ 0
B
Solenoid
Ryder - QFT


B=0
$A_\mu \neq 0$
electron
B
Solenoid
Ryder - QFT


B=0
electrons
Solenoid
Ryder - QFT


B=0
$A_\mu \neq 0$
electrons
B
Solenoid
Ryder - QFT