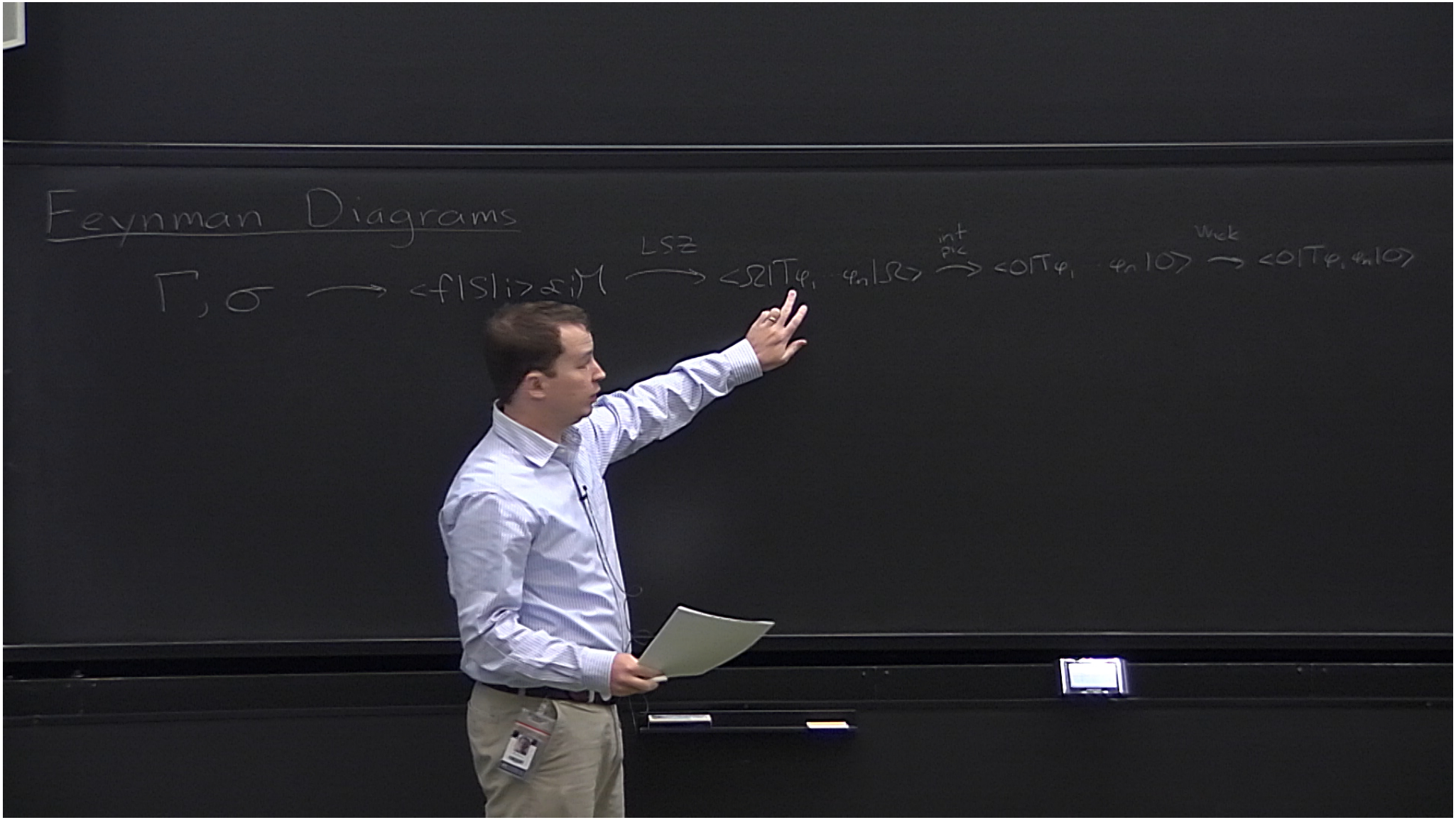
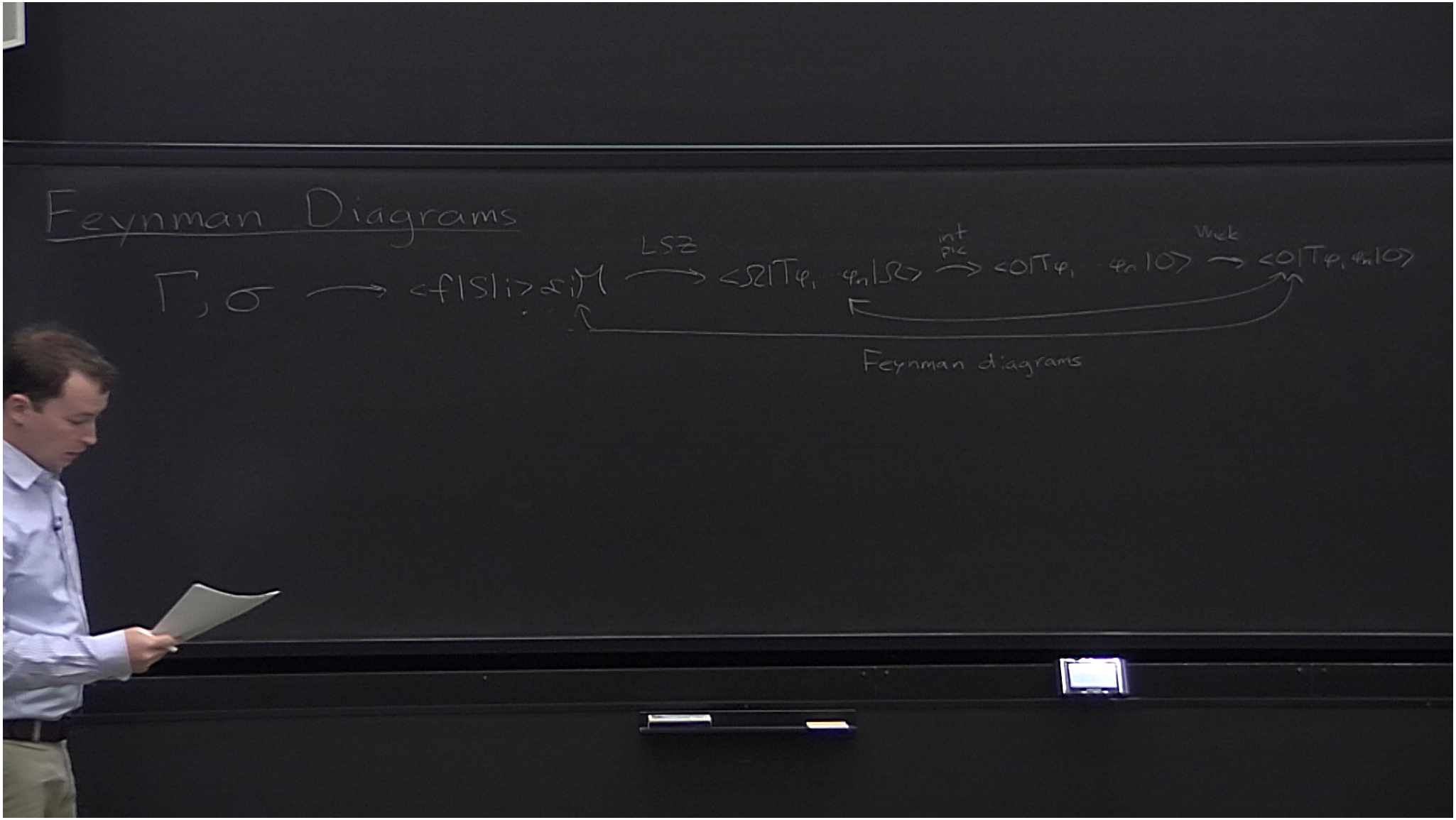
Title: PSI 17/18 - Quantum Field Theory I - Lecture 6

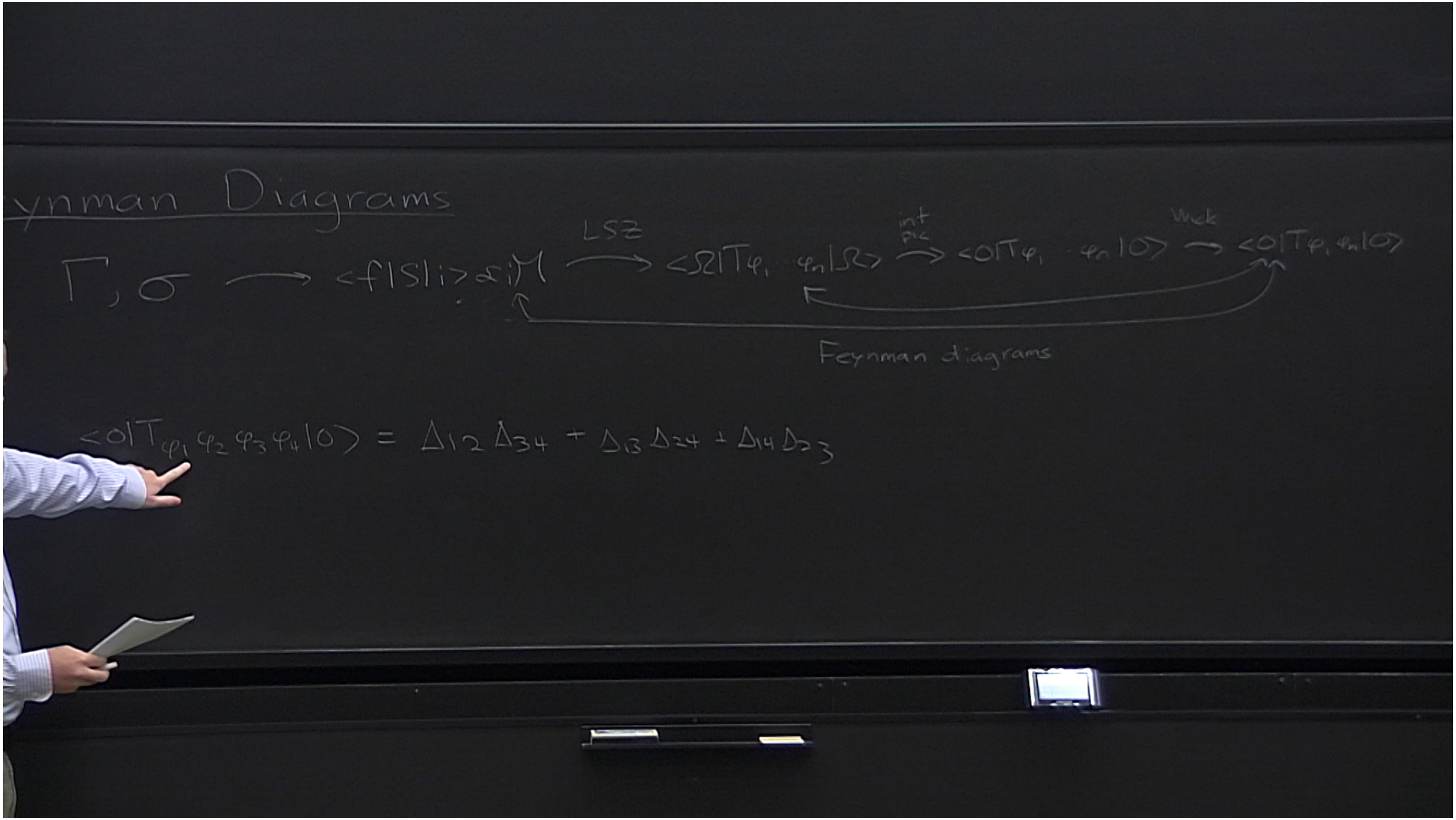
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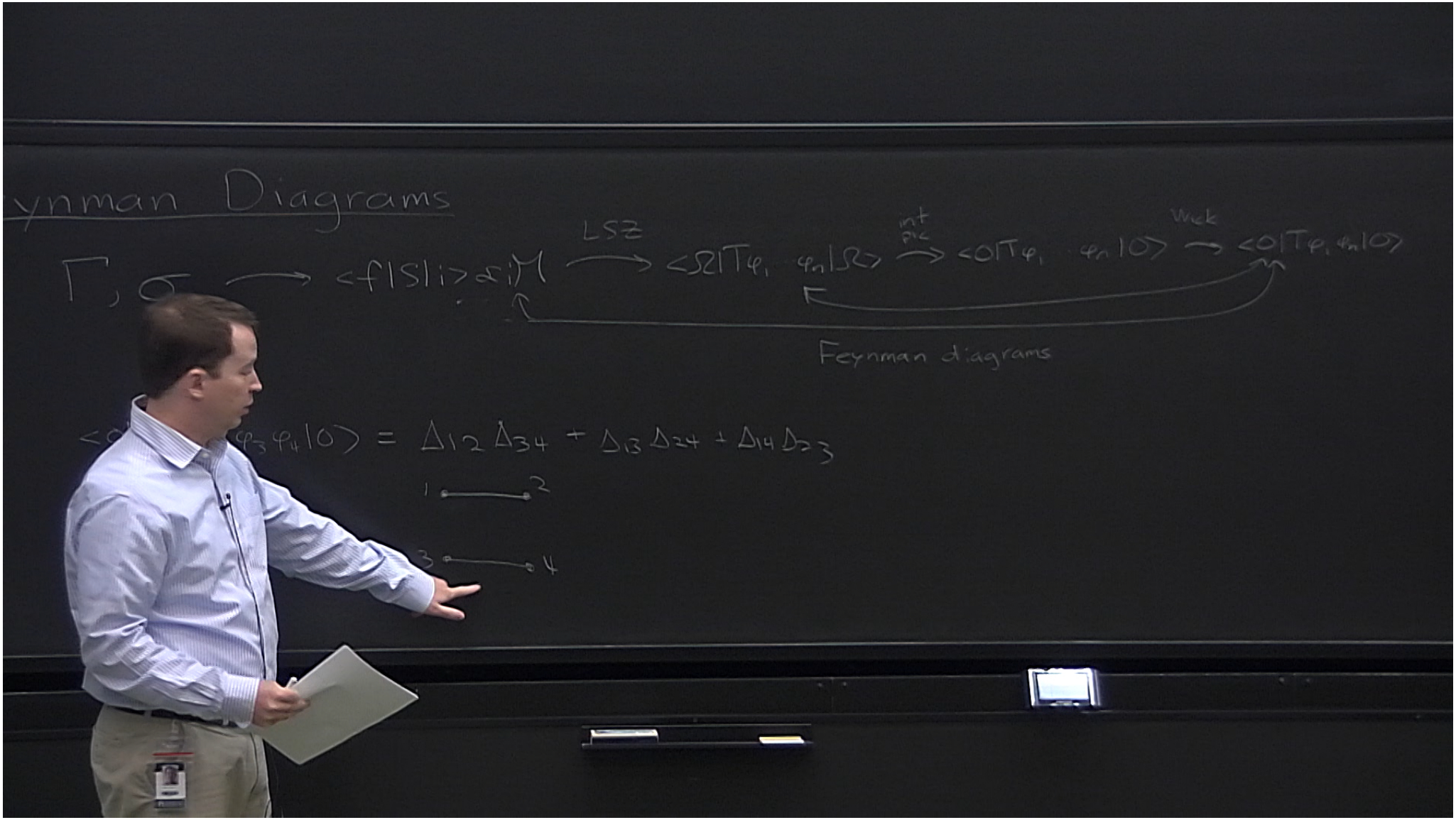
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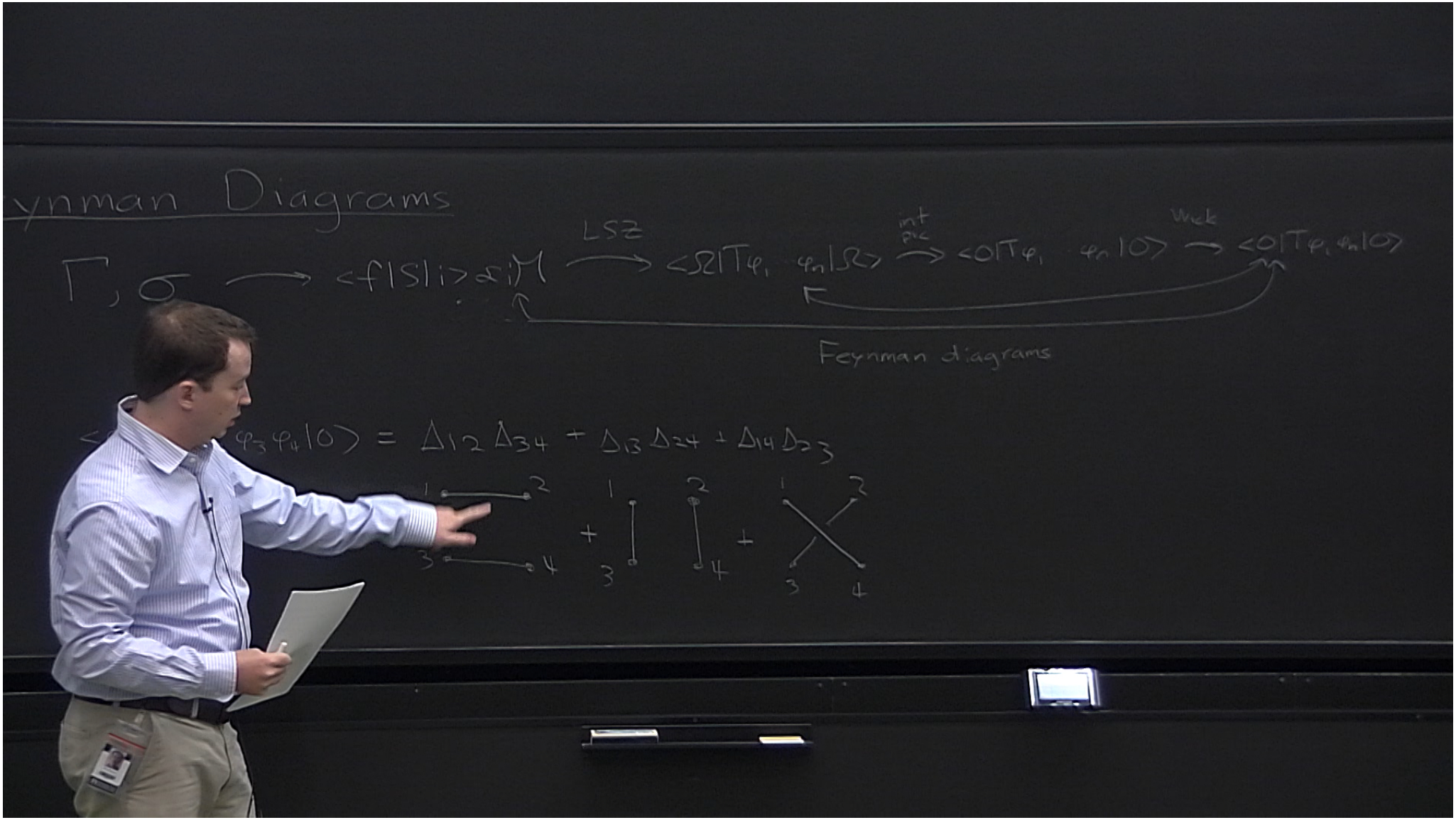
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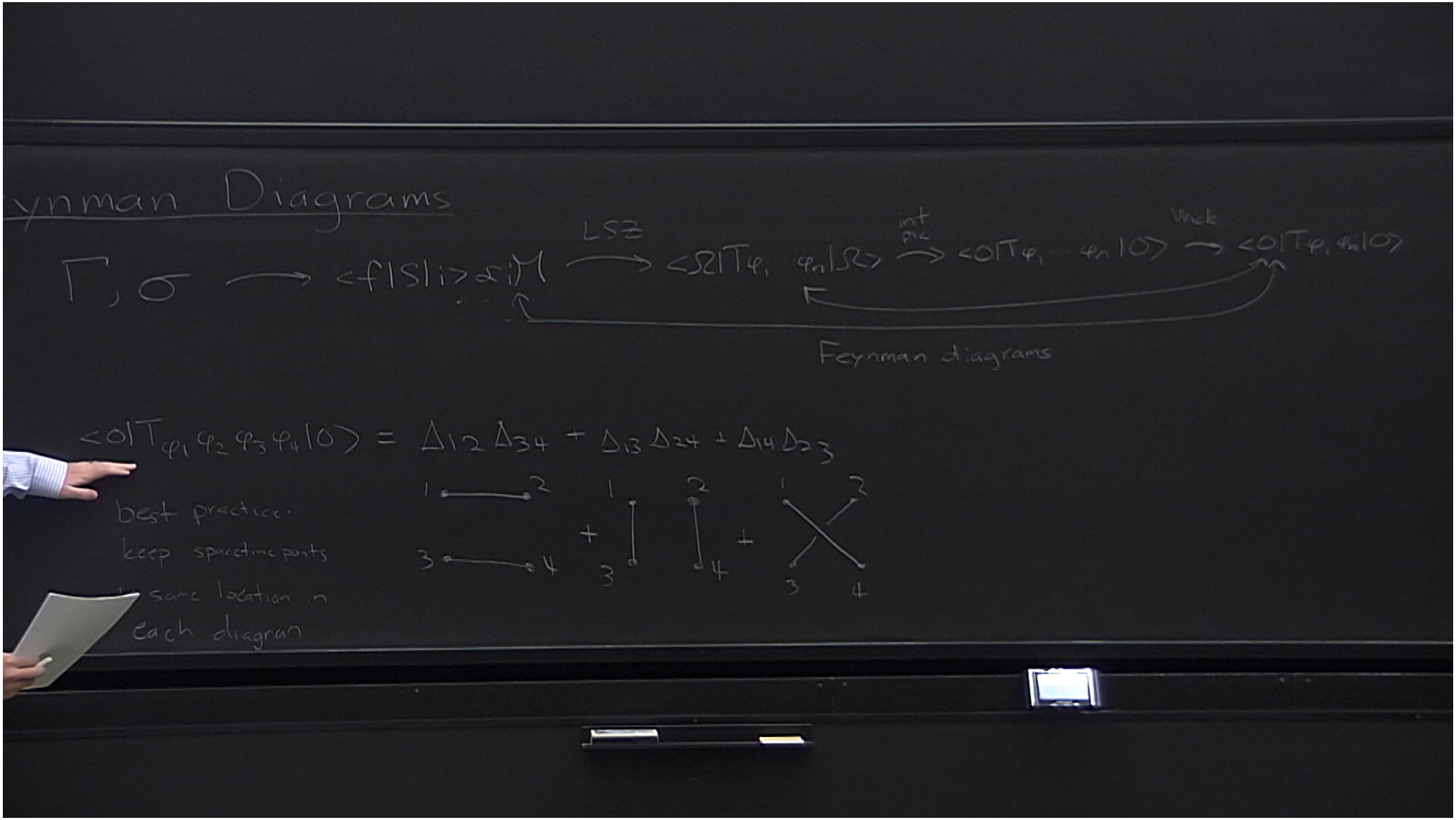

Feynman Diagrams
LSZ
int pic
Wick

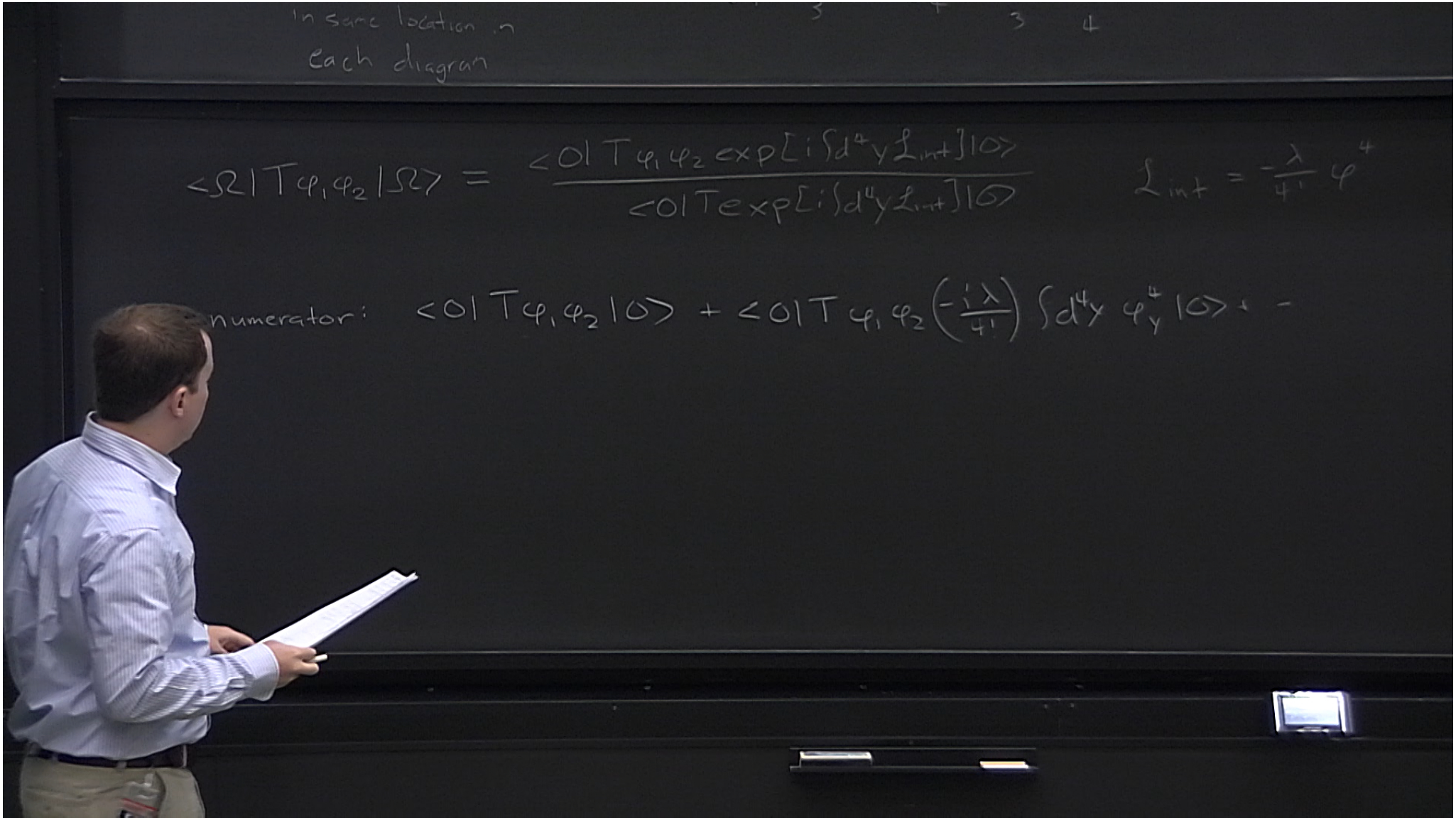

Feynman Diagrams
LSZ
int pic
Wick
Feynman diagrams

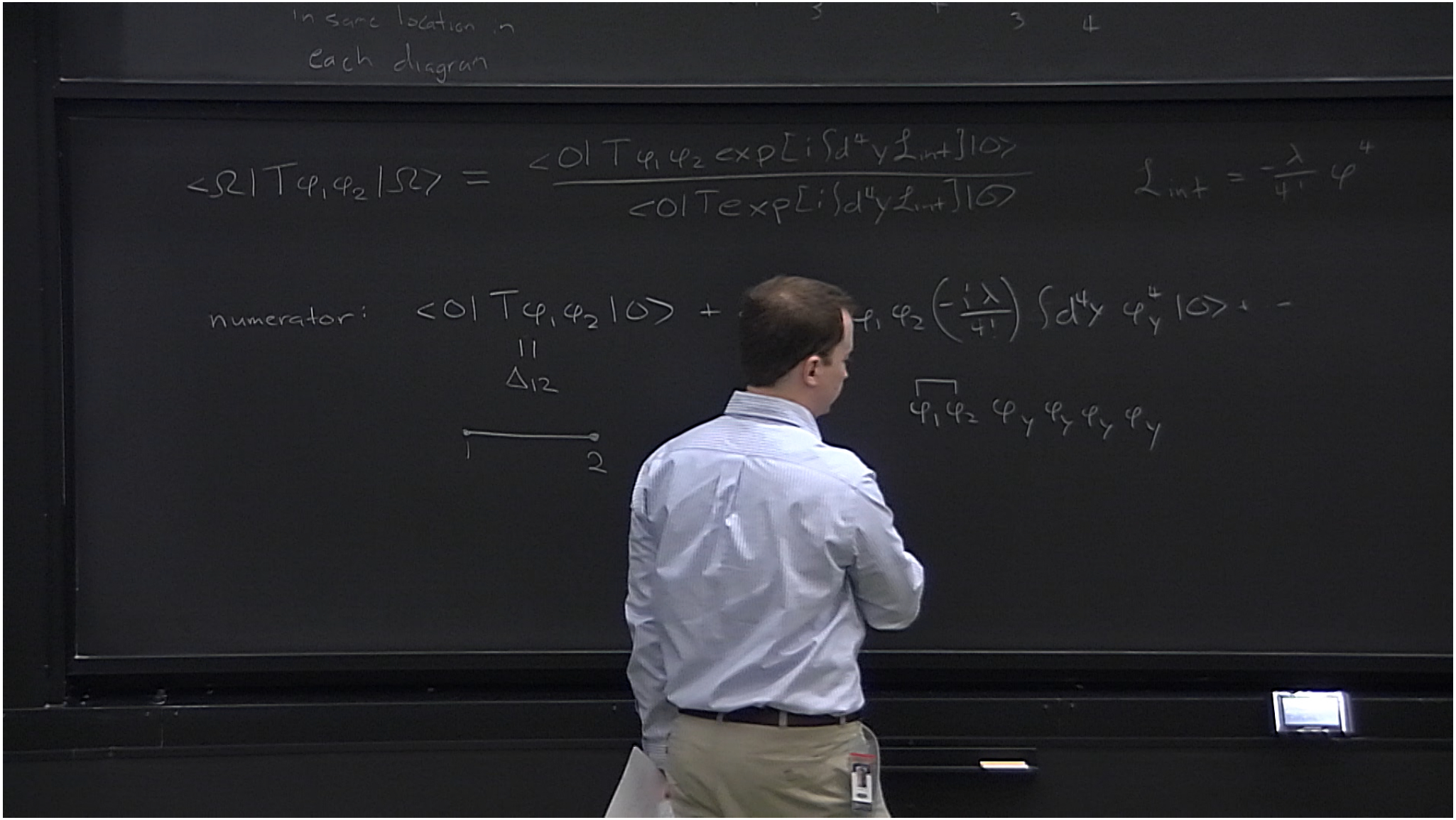

ynman Diagrams
Γ, σ
LSZ
int pic
Wick
Feynman diagrams
<0|T φ1 φ2 φ3 φ4 |0> = Δ12 Δ34 + Δ13 Δ24 + Δ14 Δ23

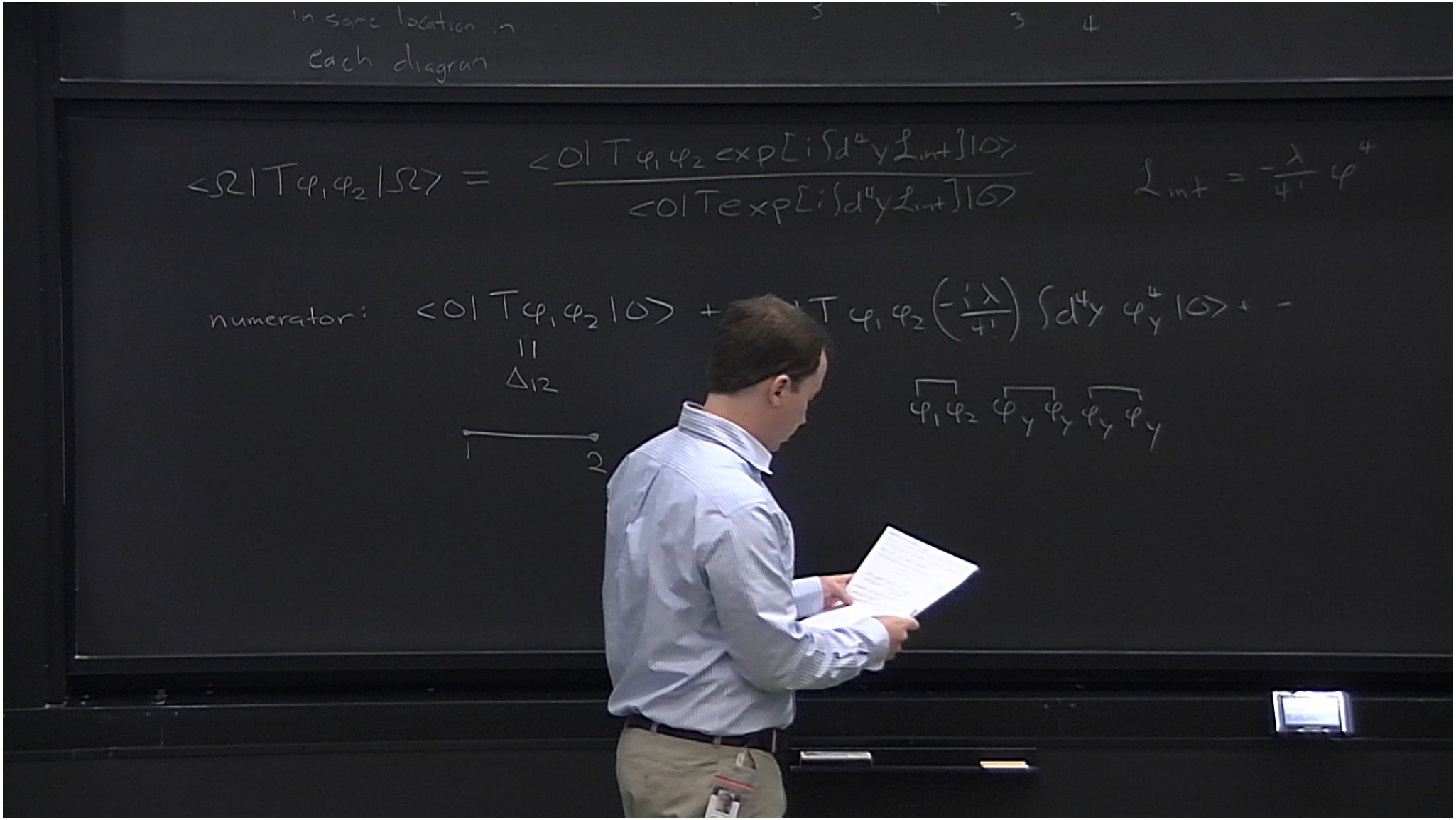

ynman Diagrams
LSZ
int pic
Wick
Feynman diagrams
= Δ12 Δ34 + Δ13 Δ24 + Δ14 Δ23
1
2
3
4

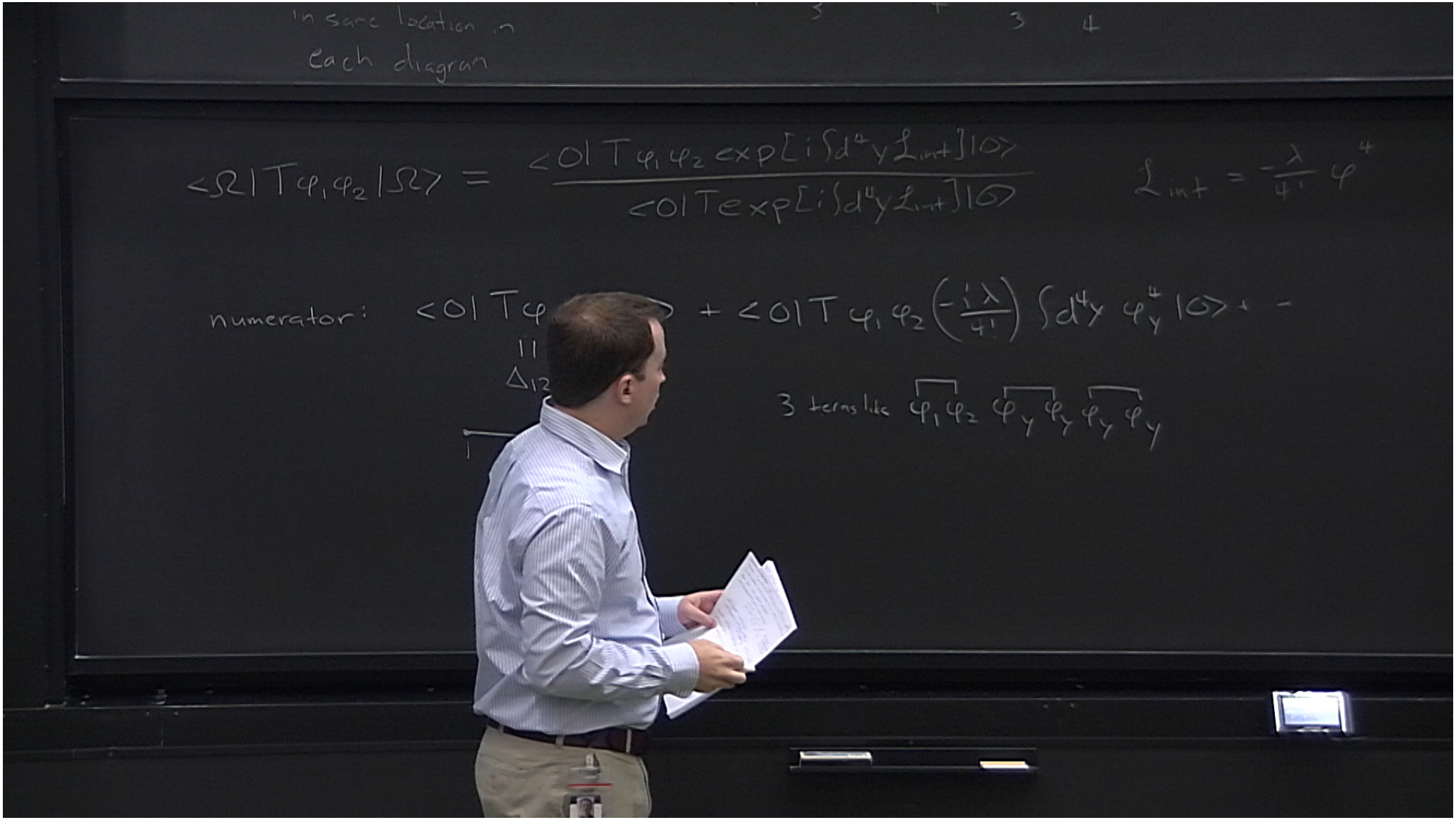

ynman Diagrams
LSZ
int pic
Wick
Feynman diagrams

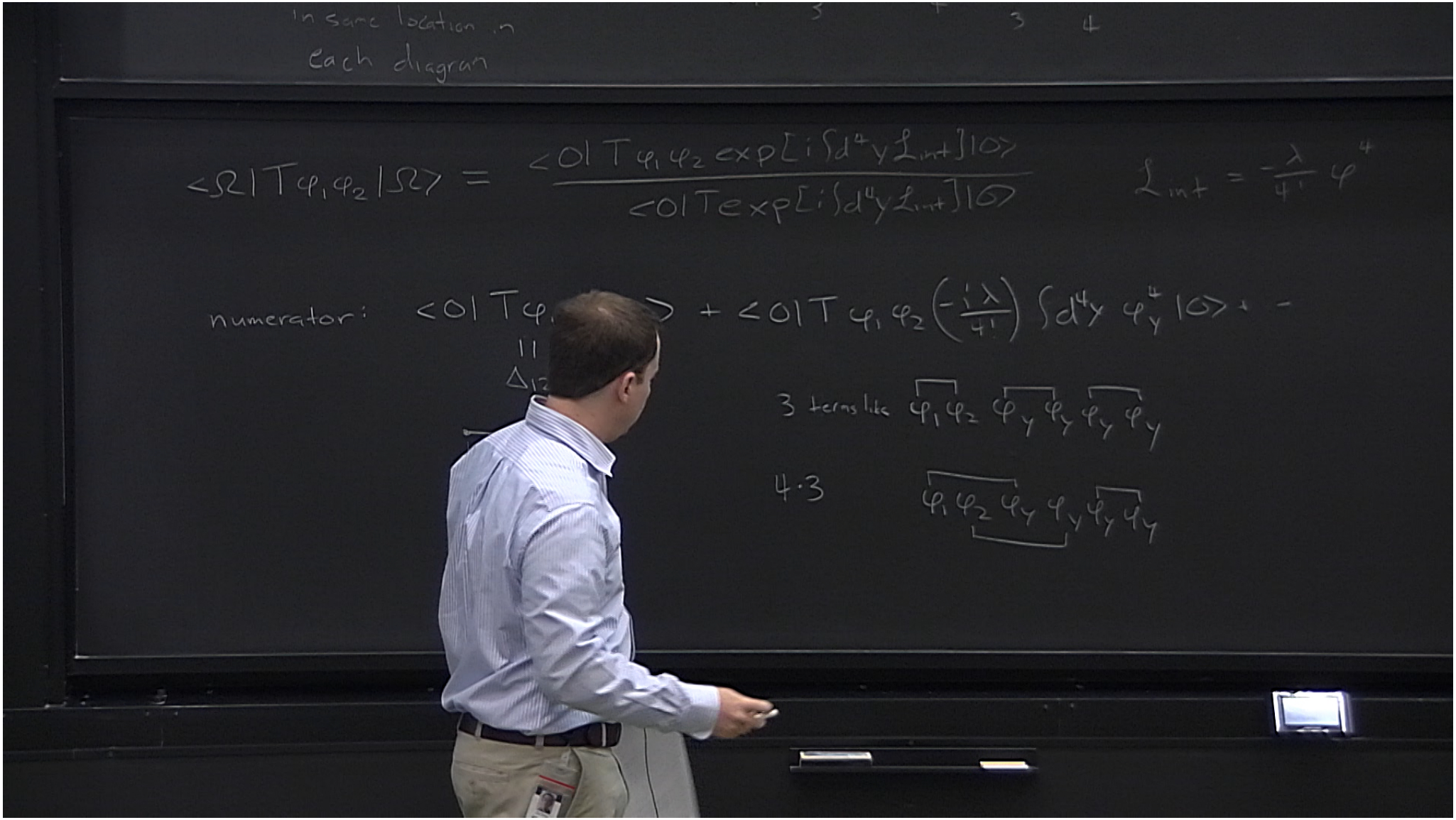

ynman Diagrams
Γ, σ
<f|S|i>
LSZ
int
Wick
Feynman diagrams
best practice:
keep spacetime points
same location in
each diagram

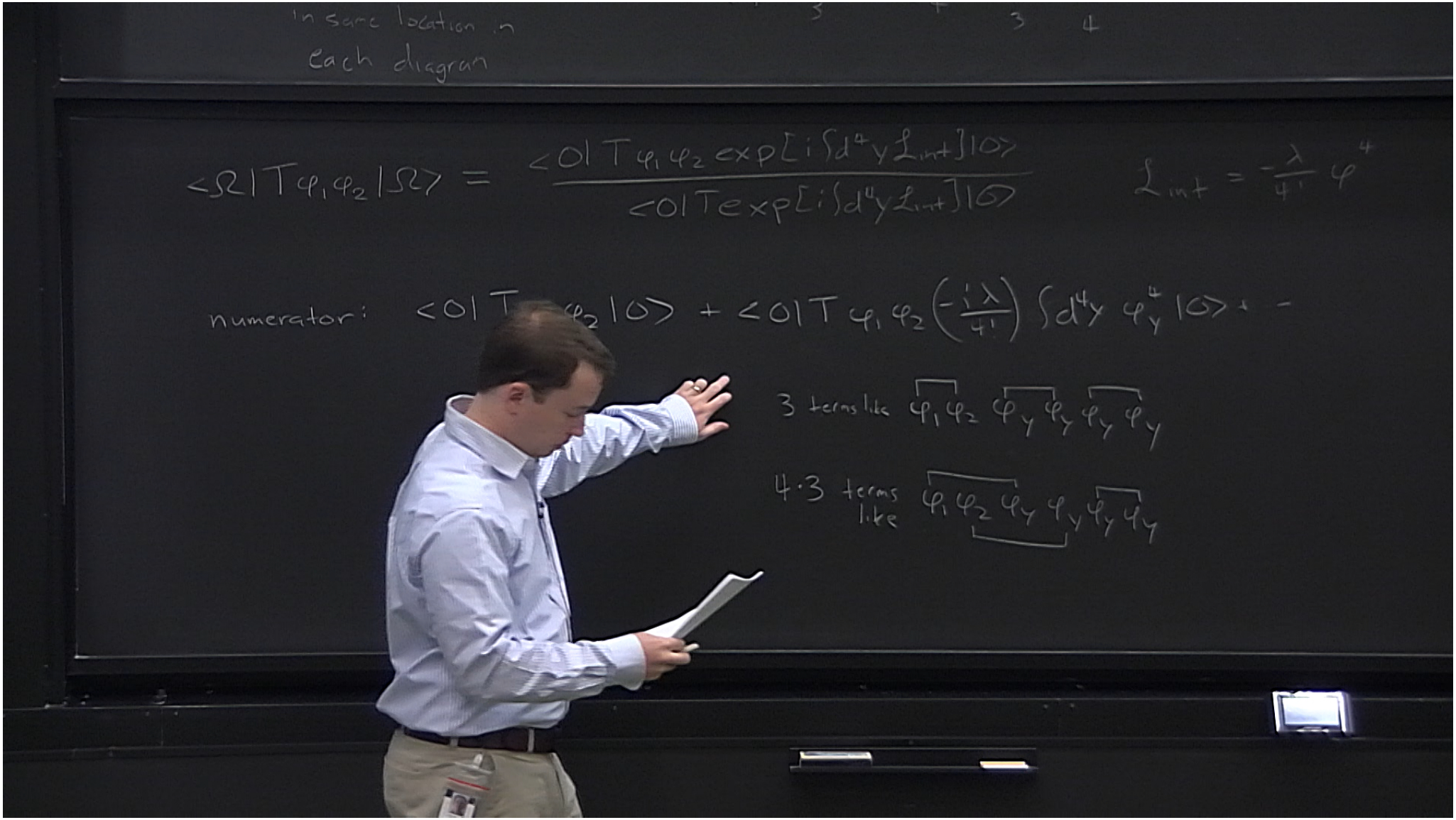

in same location in each diagram
$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2 \exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}$
$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$
numerator: $\langle 0|T\varphi_1\varphi_2|0\rangle + \langle 0|T\varphi_1\varphi_2\left(-\frac{i\lambda}{4!}\right)\int d^4y\, \varphi_y^4|0\rangle + \cdots$

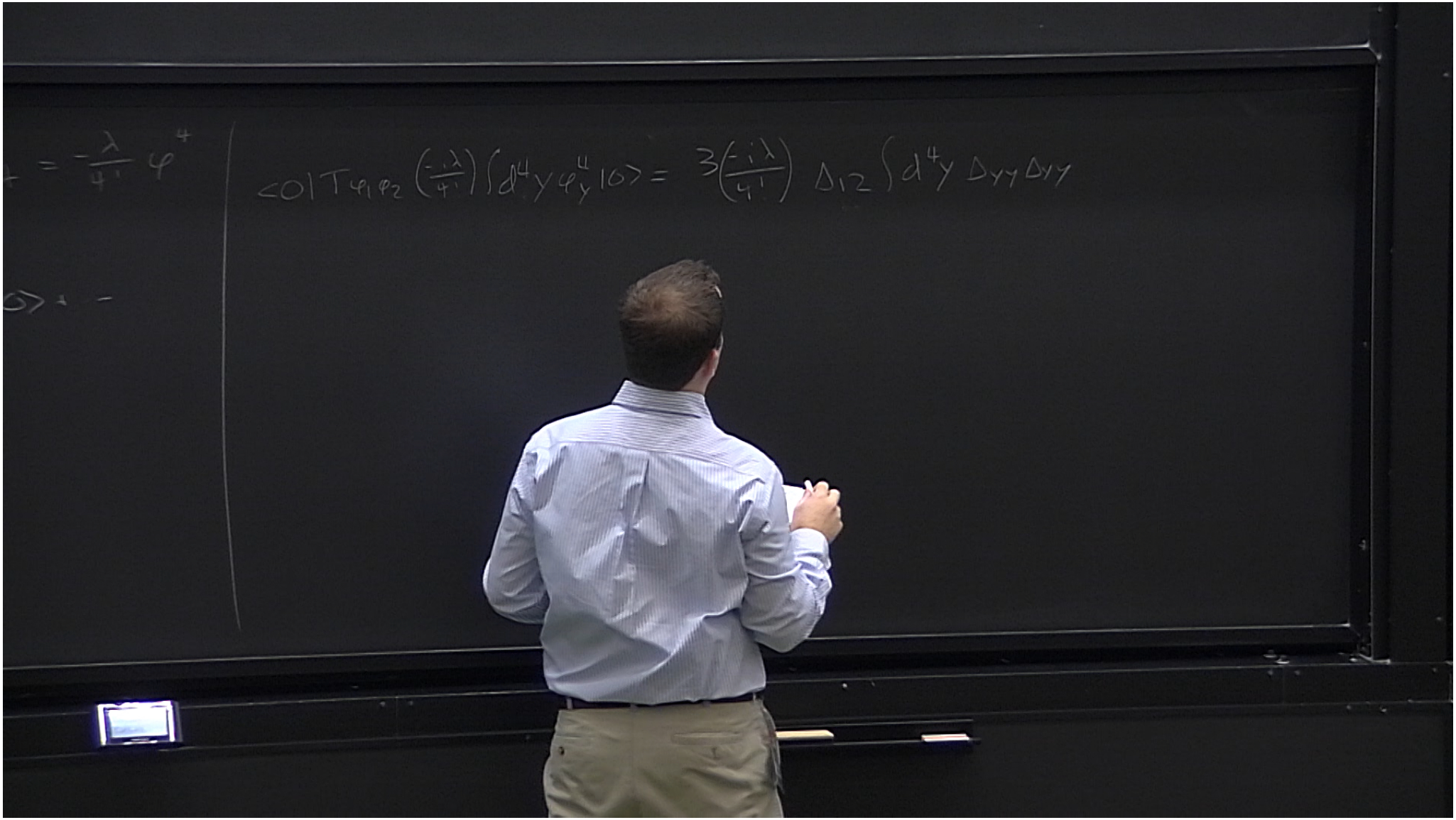

in same location in each diagram
$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2 \exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}$
$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$
numerator: $\langle 0|T\varphi_1\varphi_2|0\rangle + \ldots \varphi_1\varphi_2\left(-\frac{i\lambda}{4!}\right)\int d^4y\, \varphi_y^4|0\rangle + \ldots$
$= \Delta_{12}$
1
2
$\varphi_1\varphi_2\varphi_y\varphi_y\varphi_y\varphi_y$

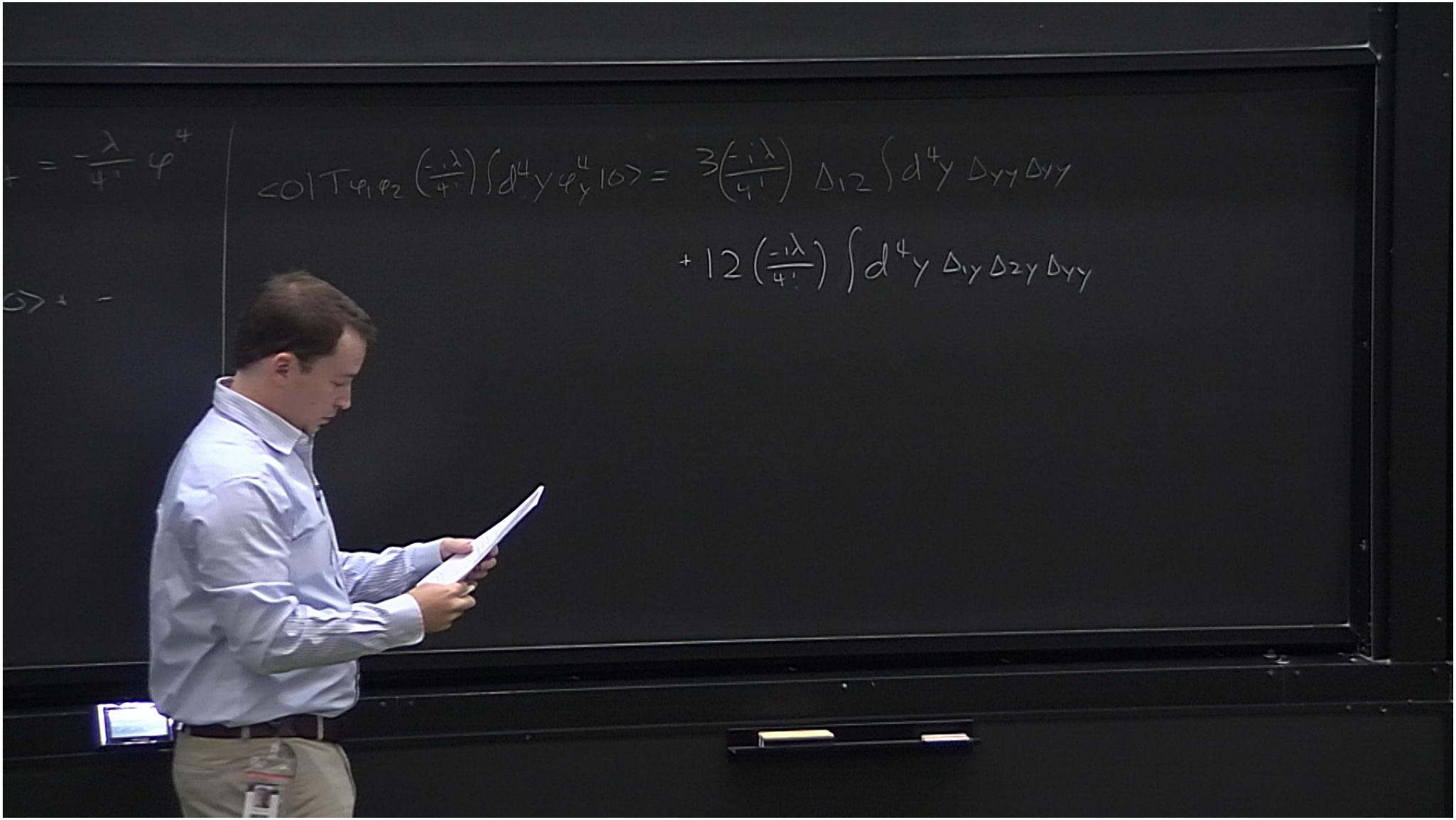

in same location in
each diagram
$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2 \exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}$
$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$
numerator: $\langle 0|T\varphi_1\varphi_2|0\rangle + \ldots T\varphi_1\varphi_2\left(-\frac{i\lambda}{4!}\right)\int d^4y\, \varphi_y^4|0\rangle + \ldots$
$= \Delta_{12}$
1
2
$\varphi_1\varphi_2\,\varphi_y\varphi_y\,\varphi_y\varphi_y$

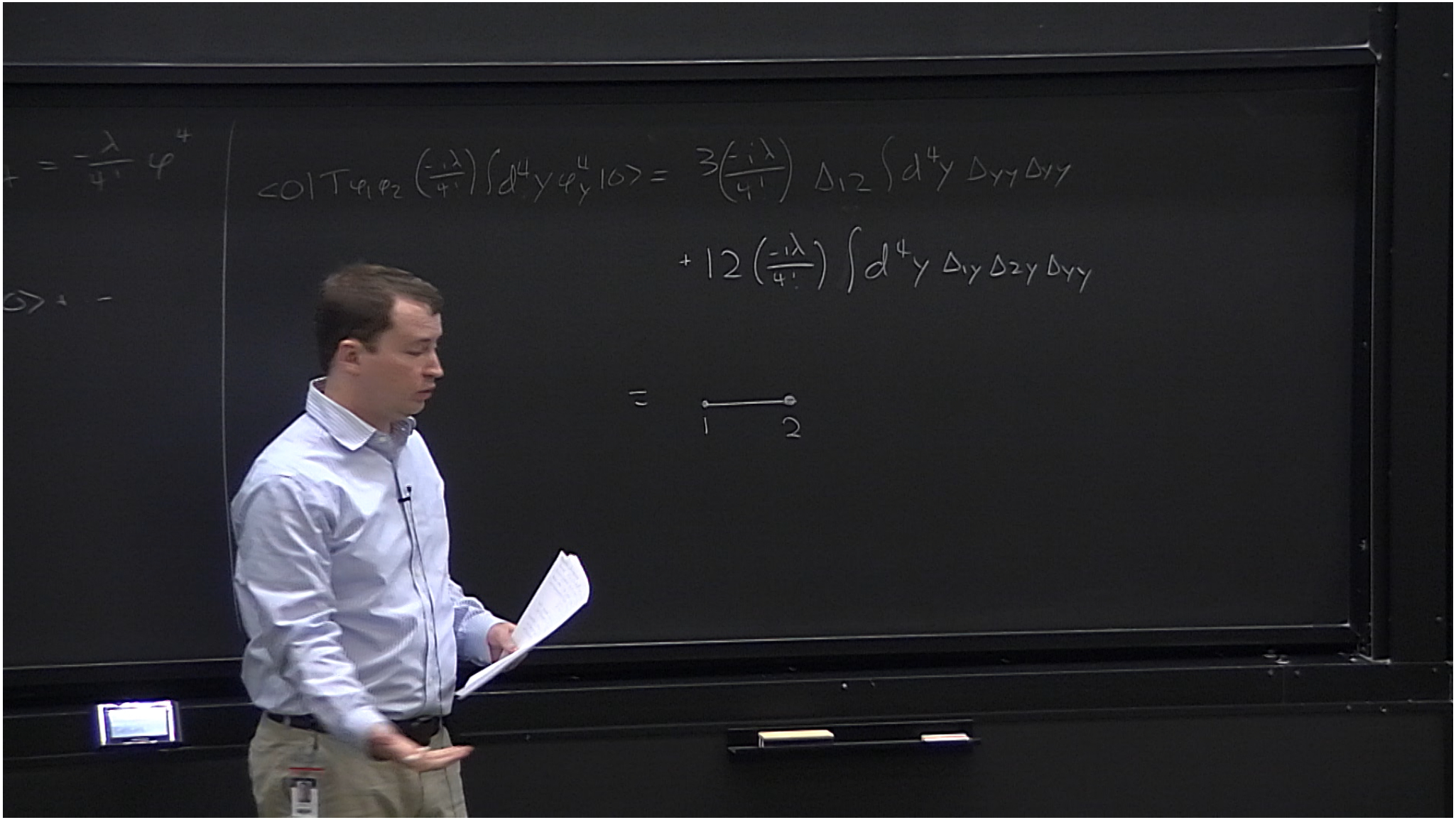

in same location in
each diagram
$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2 \exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}$
$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$
numerator: $\langle 0|T\varphi$ $+ \langle 0|T\varphi_1\varphi_2\left(-\frac{i\lambda}{4!}\right)\int d^4y\, \varphi_y^4|0\rangle + \cdots$
$= \Delta_{12}$
3 terms like $\overline{\varphi_1\varphi_2}\,\overline{\varphi_y\varphi_y}\,\overline{\varphi_y\varphi_y}$

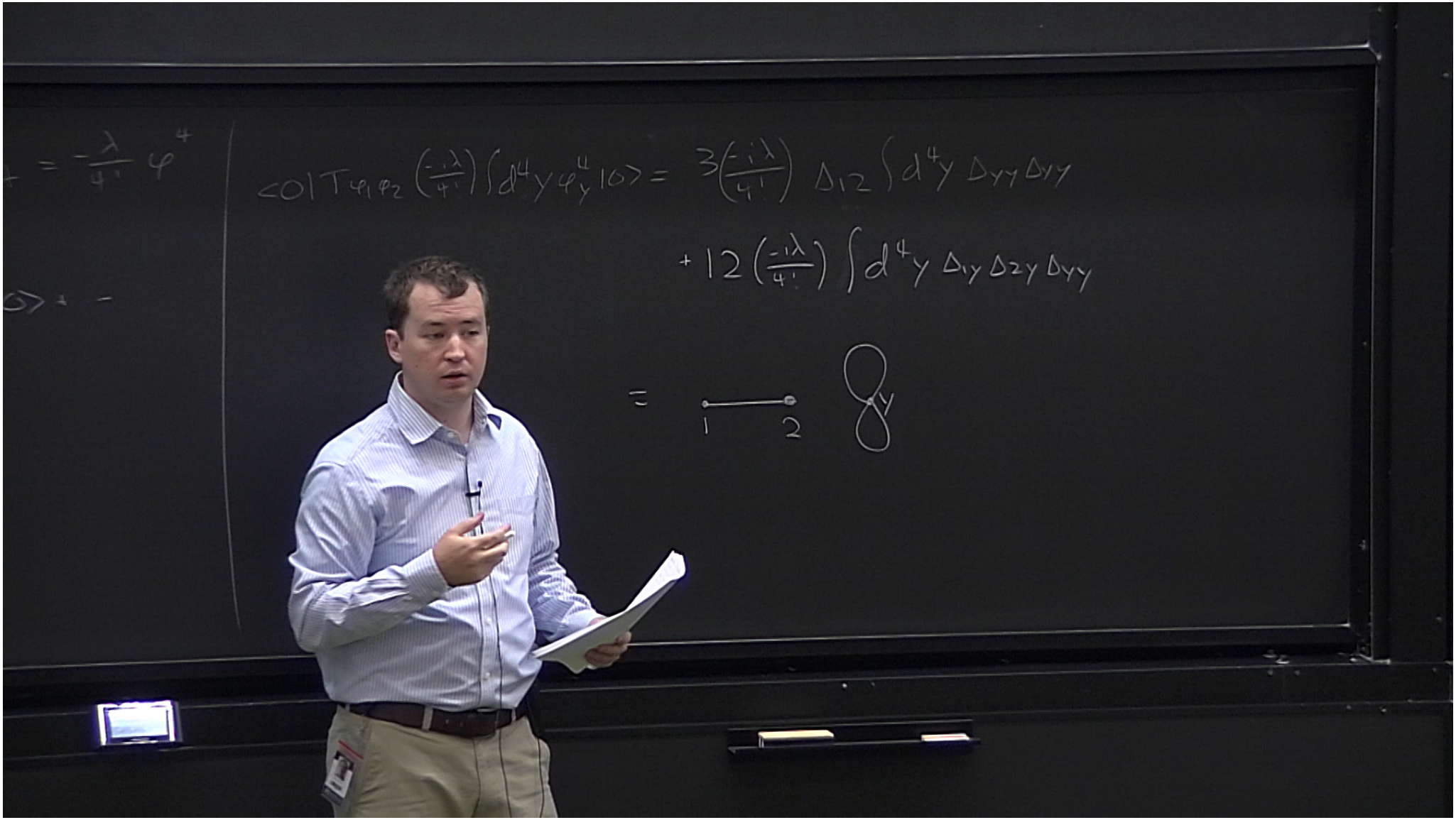

in same location in each diagram
$\langle \Omega | T\varphi_1\varphi_2 | \Omega \rangle = \frac{\langle 0 | T\varphi_1\varphi_2 \exp[i\int d^4y \mathcal{L}_{int}] | 0 \rangle}{\langle 0 | T\exp[i\int d^4y \mathcal{L}_{int}] | 0 \rangle}$
$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$
numerator: $\langle 0 | T\varphi_1\varphi_2 \left(-\frac{i\lambda}{4!}\right)\int d^4y\, \varphi_y^4 | 0 \rangle + \cdots$
$= \Delta_{12}$
3 terms like $\varphi_1\varphi_2\,\varphi_y\varphi_y\,\varphi_y\varphi_y$
4·3 $\varphi_1\varphi_2\,\varphi_y\varphi_y\,\varphi_y\varphi_y$


in same location in each diagram
$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2 \exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4y \mathcal{L}_{int}]|0\rangle}$
$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$
numerator: $\langle 0|T\varphi_1\varphi_2|0\rangle + \langle 0|T\varphi_1\varphi_2\left(-\frac{i\lambda}{4!}\right)\int d^4y\,\varphi_y^4|0\rangle + \cdots$
3 terms like $\varphi_1\varphi_2\,\varphi_y\varphi_y\,\varphi_y\varphi_y$
4·3 terms like $\varphi_1\varphi_2\,\varphi_y\varphi_y\,\varphi_y\varphi_y$


$\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi_y^4|0\rangle = 3\left(\frac{-i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}\Delta_{yy}$

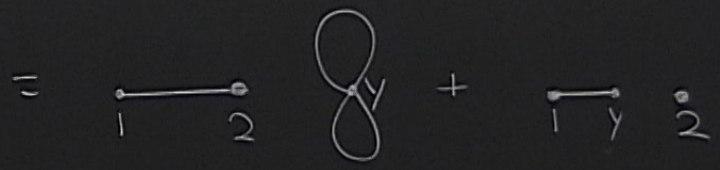

$= -\frac{\lambda}{4!}\varphi^4$
$\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi_y^4|0\rangle = 3\left(\frac{-i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}\Delta_{yy}$
$+12\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{yy}$

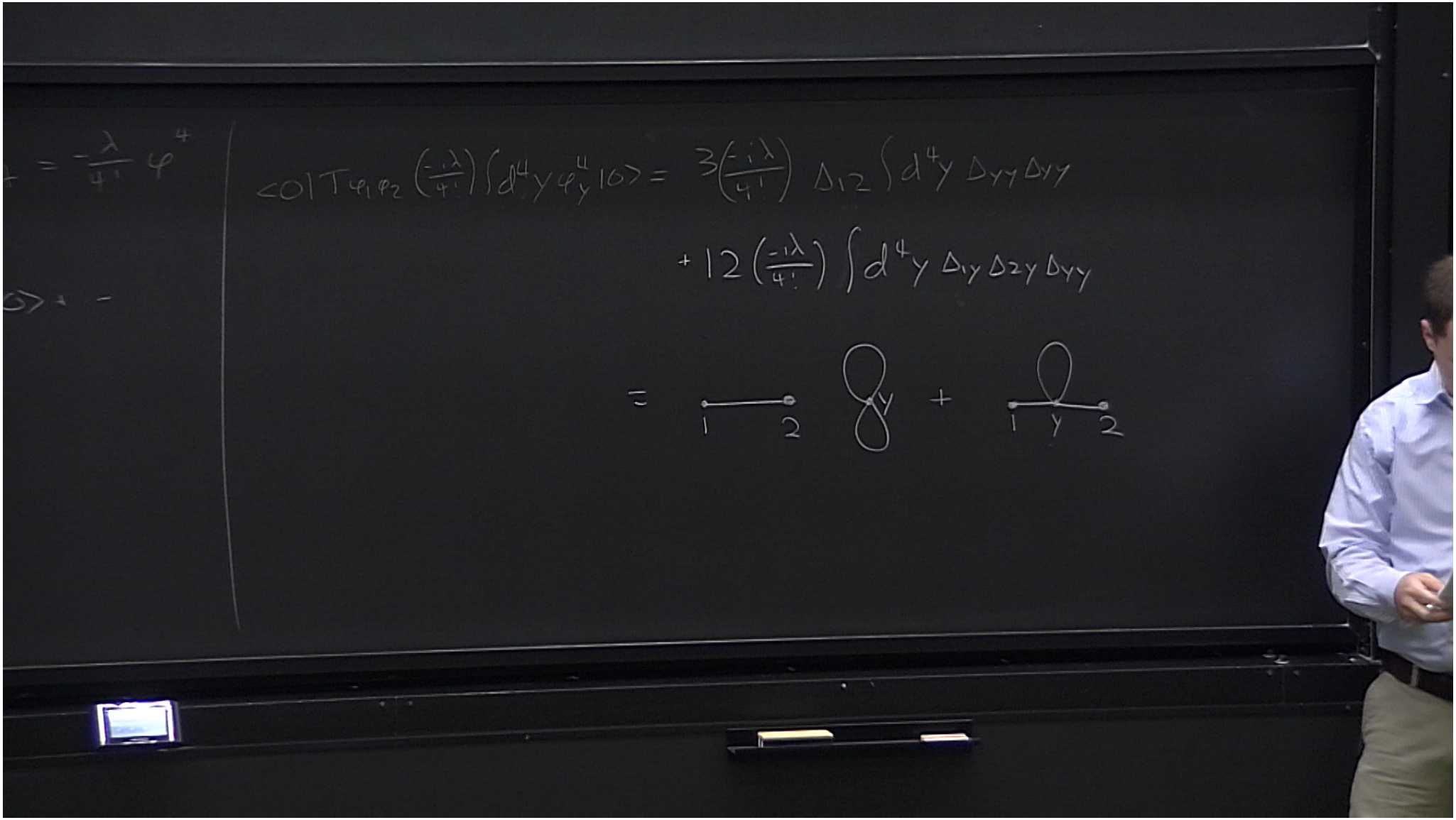

$\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi_y^4|0\rangle = 3\left(\frac{-i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}\Delta_{yy}$
$+12\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{yy}$
$=$
1
2


$= -\frac{\lambda}{4!}\varphi^4$
$\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi_y^4|0\rangle = 3\left(\frac{-i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}\Delta_{yy}$
$+12\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{yy}$
$=$
1
2
y

$$\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi_y^4|0\rangle = 3\left(\frac{-i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}\Delta_{yy}$$

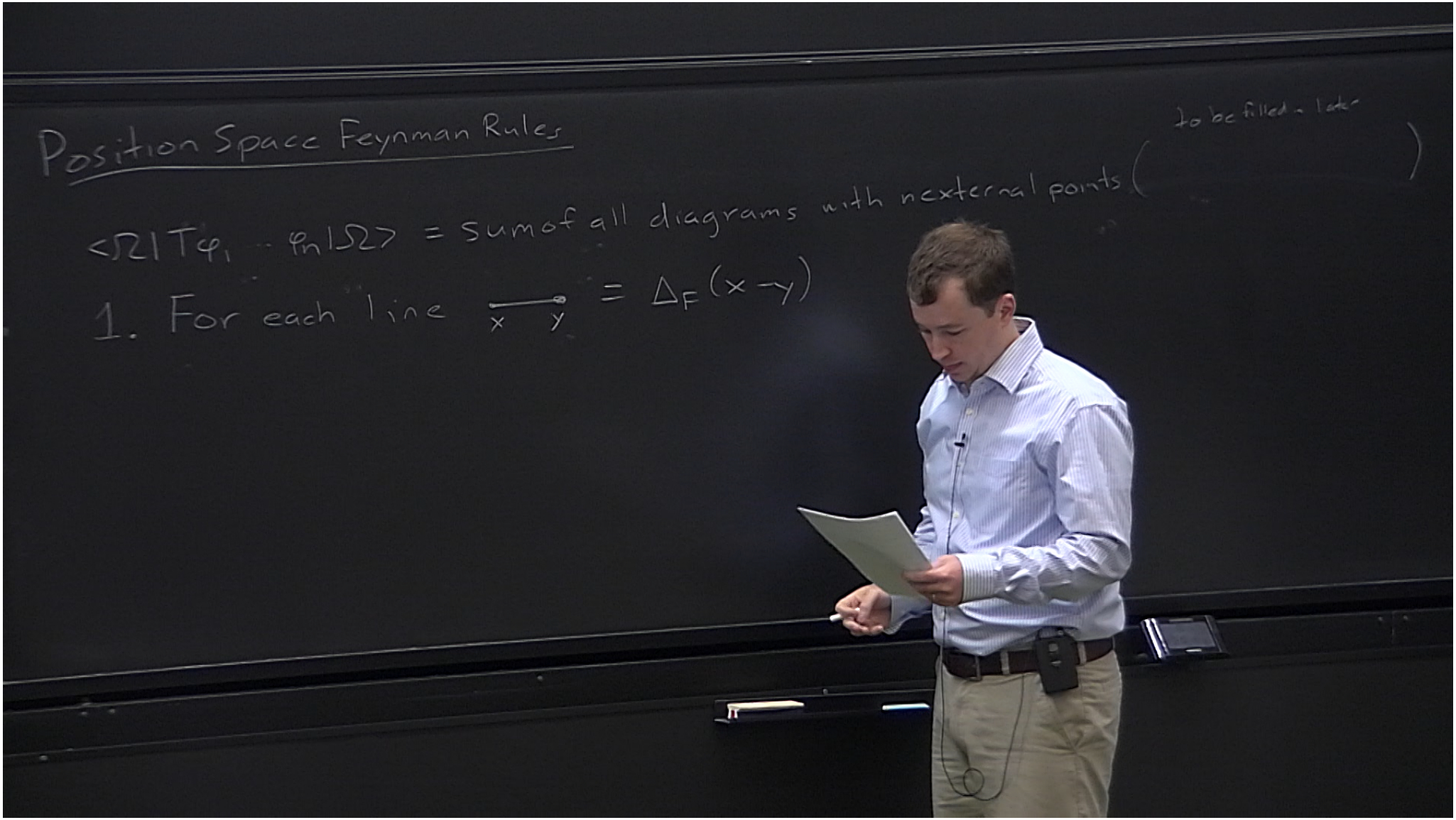
$$+12\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{yy}$$




$= -\frac{\lambda}{4!}\varphi^4$
$\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi_y^4|0\rangle = 3\left(\frac{-i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}\Delta_{yy}$
$+12\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{yy}$
$=$
1
2
y
+
1
y
2

Position Space Feynman Rules

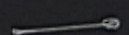
$\langle \Omega | T\varphi_1 \cdots \varphi_n | \Omega \rangle$ = sum of all diagrams with n external points (to be filled - later)

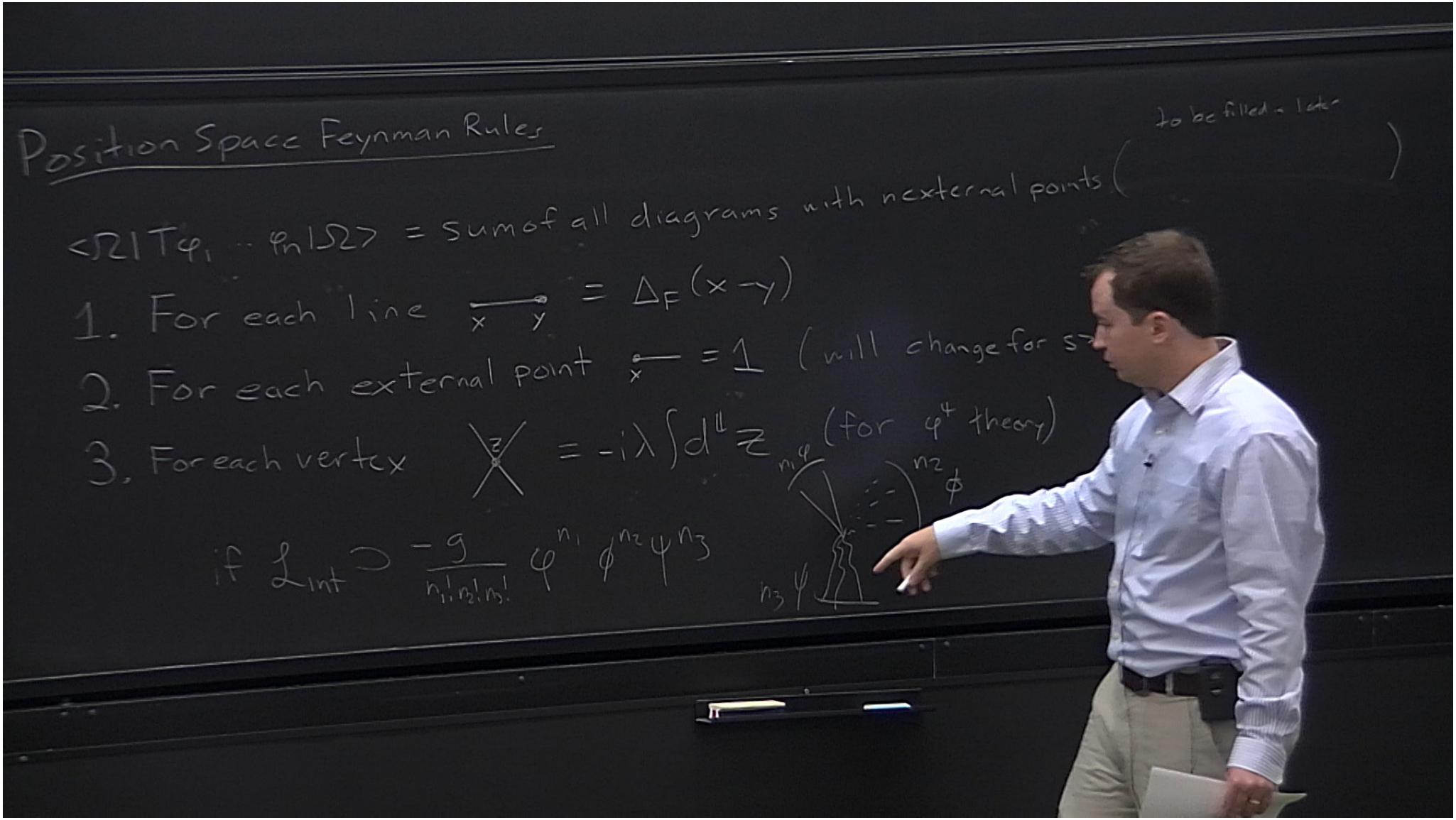

Position Space Feynman Rules
⟨Ω|T φ1 ... φn|Ω⟩ = sum of all diagrams with n external points (to be filled - later)
1. For each line x y = ΔF(x − y)

Position Space Feynman Rules

to be filled -

$\langle \Omega | T\phi_1 \cdots \phi_n | \Omega \rangle$ = sum of all diagrams with n external points (

1. For each line  $= \Delta_F(x-y)$
2. For each external point  $= 1$ (will change for $s>0$)
3. For each vertex  $= -i\lambda \int d^4 z$


Position Space Feynman Rules
to be filled - later
$\langle \Omega | T\varphi_1 \cdots \varphi_n | \Omega \rangle$ = sum of all diagrams with n external points (
1. For each line x y $= \Delta_F(x-y)$
2. For each external point x $= 1$ (will change for S
3. For each vertex $= -i\lambda \int d^4z$ (for φ^4 theory)
$n_1 \varphi$
$n_2 \phi$
$n_3 \psi$
if $\mathcal{L}_{int} \supset \frac{-g}{n_1! n_2! n_3!} \varphi^{n_1} \phi^{n_2} \psi^{n_3}$

$\Omega|S\rangle$ = sum of all diagrams with

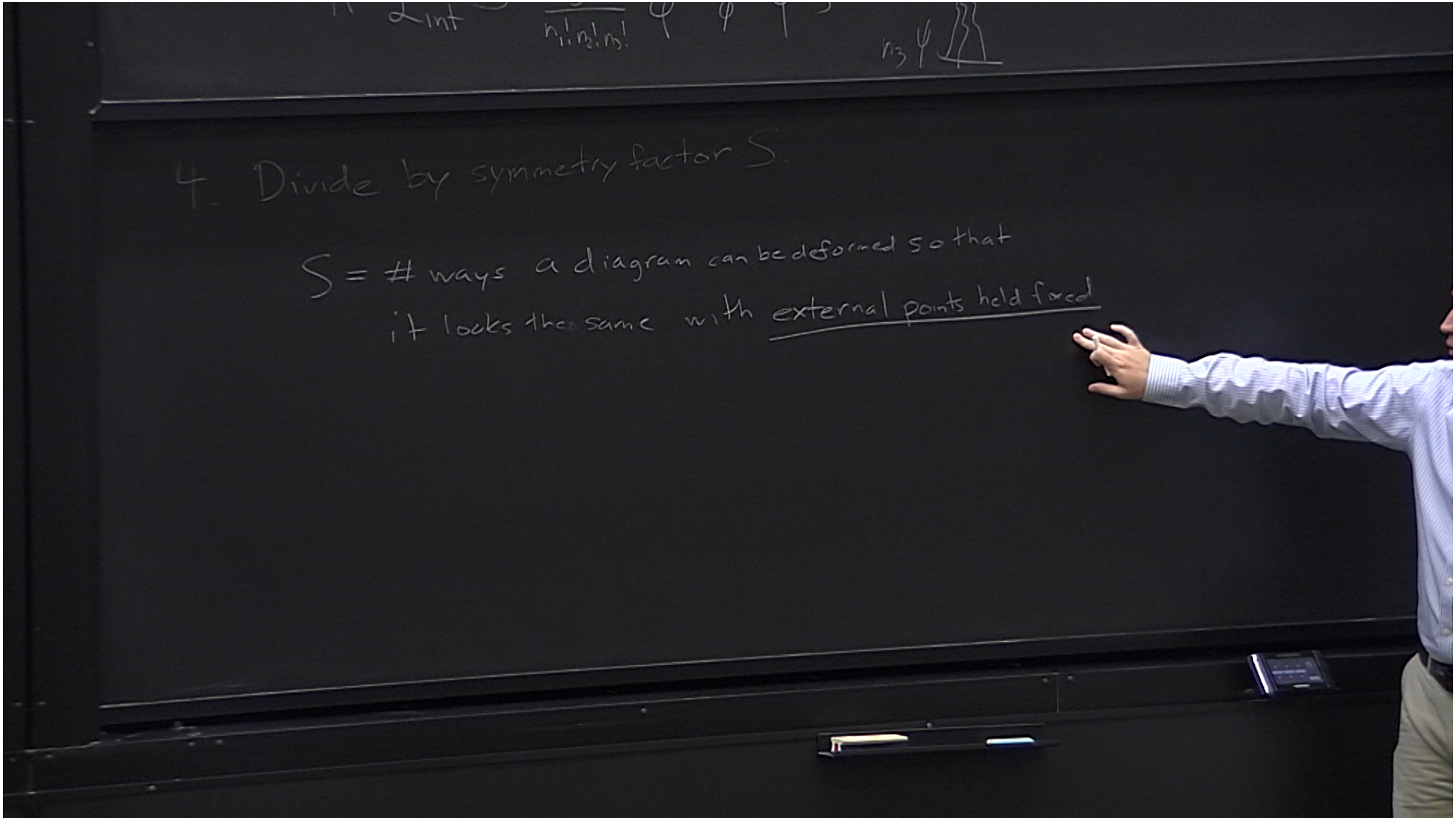
each line $x \quad y$ $= \Delta_F(x-y)$

each external point x $= 1$ (will change for $s>0$)

each vertex z $= -i\lambda \int d^4z$ (for φ^4 theory)

$n_1\varphi$ $n_2\phi$ $n_3\psi$ z

$\mathcal{L}_{int} \supset \frac{-g}{n_1! n_2! n_3!} \varphi^{n_1} \phi^{n_2} \psi^{n_3}$


4. Divide by symmetry factor S.
S = # ways a diagram can be deformed so that
it looks the same with external points held fixed

4. Divide by symmetry factor S.

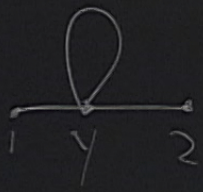
S = # ways a diagram can be deformed so that it looks the same with external points held fixed

$$\frac{1}{S} = \frac{\#\text{ Wick contractions}}{\text{factorials in Lett}}$$

4. Divide by symmetry factor S.

S = # ways a diagram can be deformed so that it looks the same with external points held fixed

$$\frac{1}{S} = \frac{\#\text{ Wick contractions}}{\text{factorials in Lint}}$$



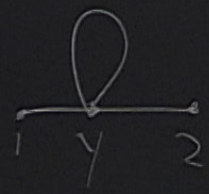
$$\frac{1}{S} = \frac{1}{2}$$

4. Divide by symmetry factor S.

S = # ways a diagram can be deformed so that it looks the same with external points held fixed

$$\frac{1}{S} = \frac{\#\text{ Wick contractions}}{\text{factorials in Lint}}$$

exchange of the ends of y-y line



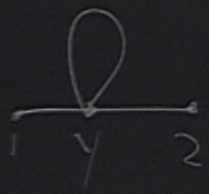
$$\frac{1}{S} = \frac{1}{2} = \frac{12}{4!}$$

4. Divide by symmetry factor S.

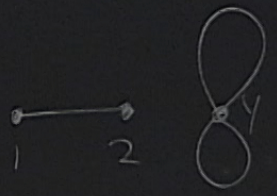
S = # ways a diagram can be deformed so that it looks the same with external points held fixed

$$\frac{1}{S} = \frac{\#\text{ Wick contractions}}{\text{factorials in Lint}}$$

exchange of two ends of y-y line



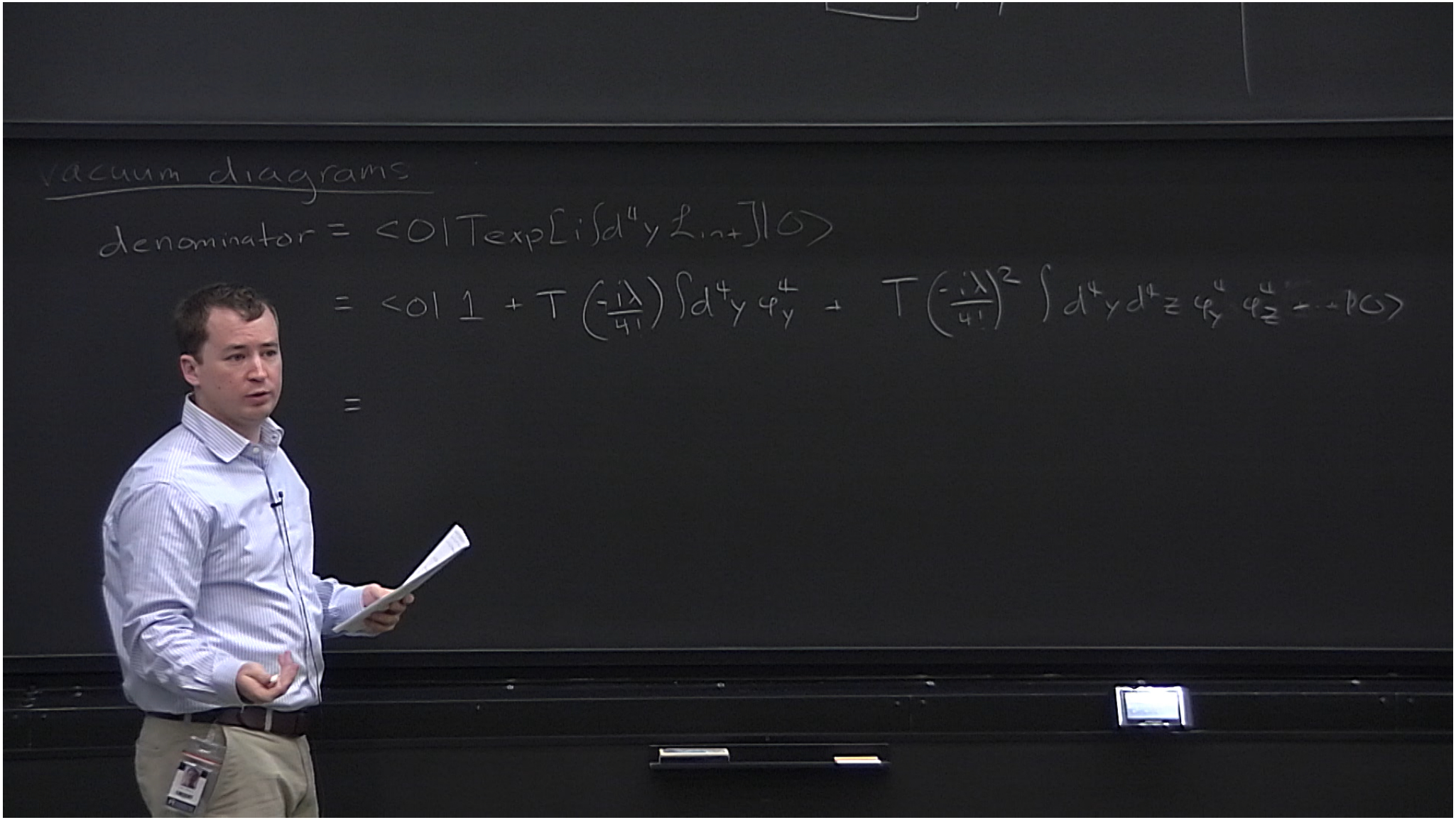
$$\frac{1}{S} = \frac{1}{2} = \frac{12}{4!}$$

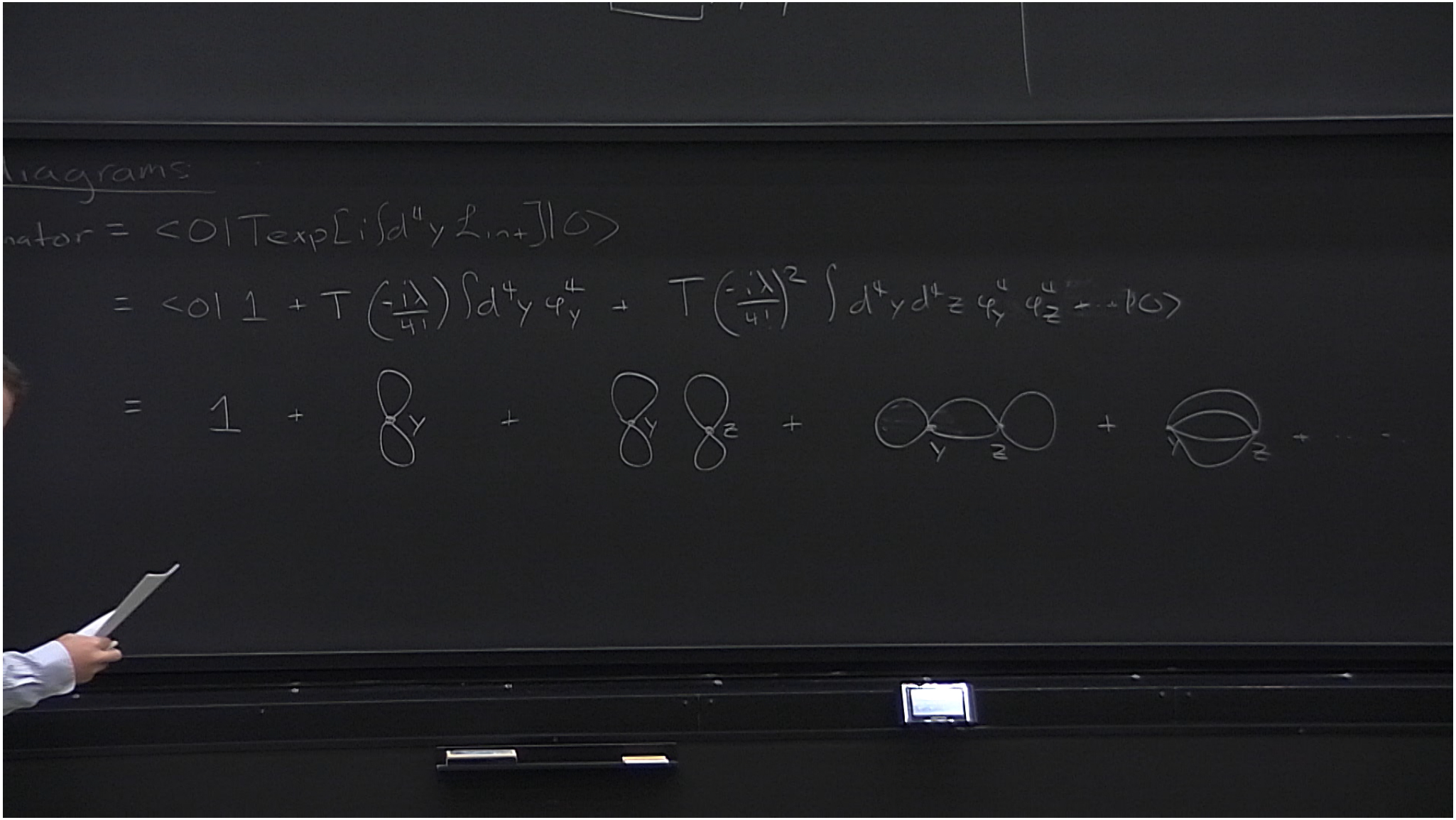


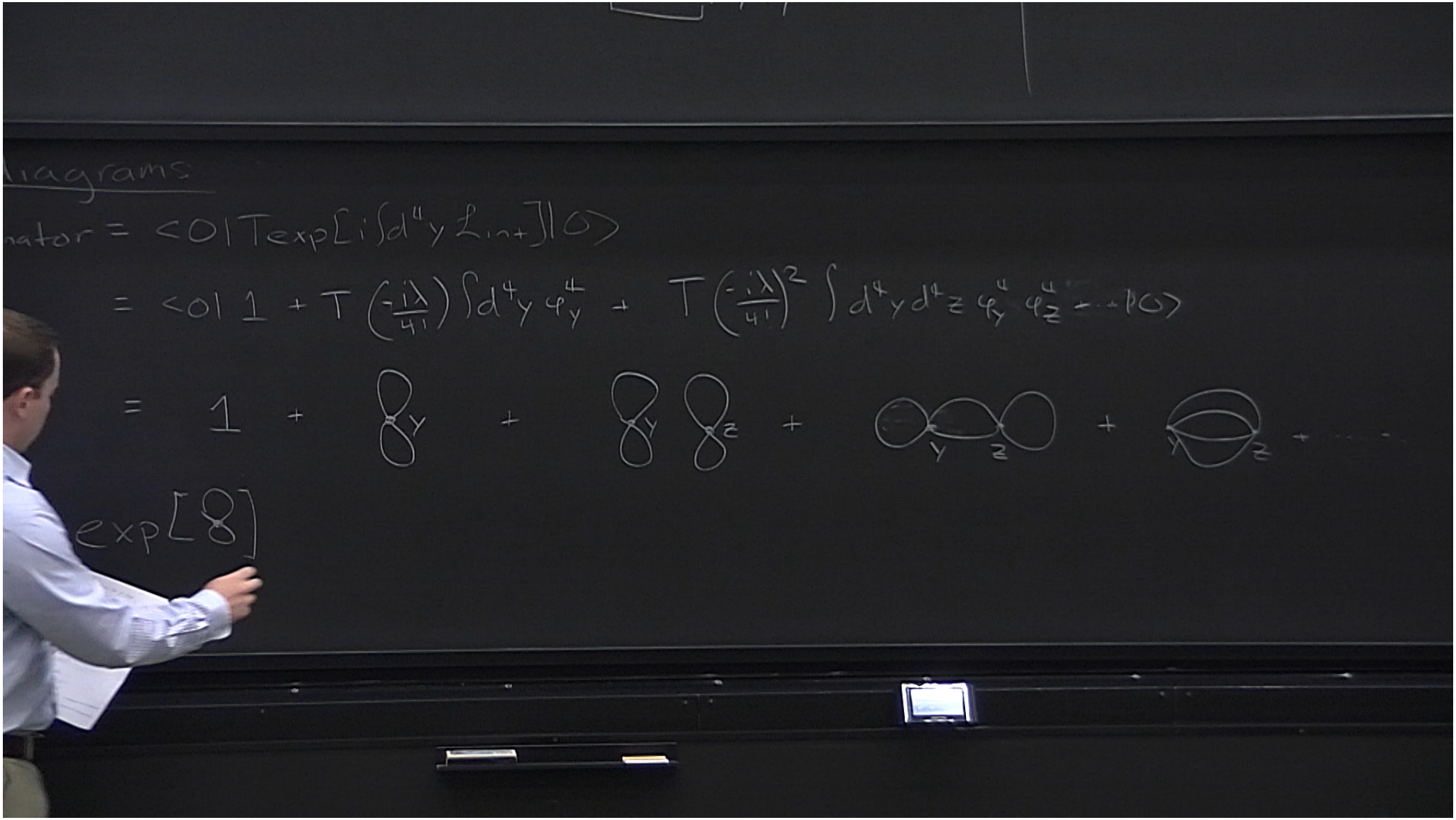
ends of upper line · ends of lower line · exchange upper + lower lines

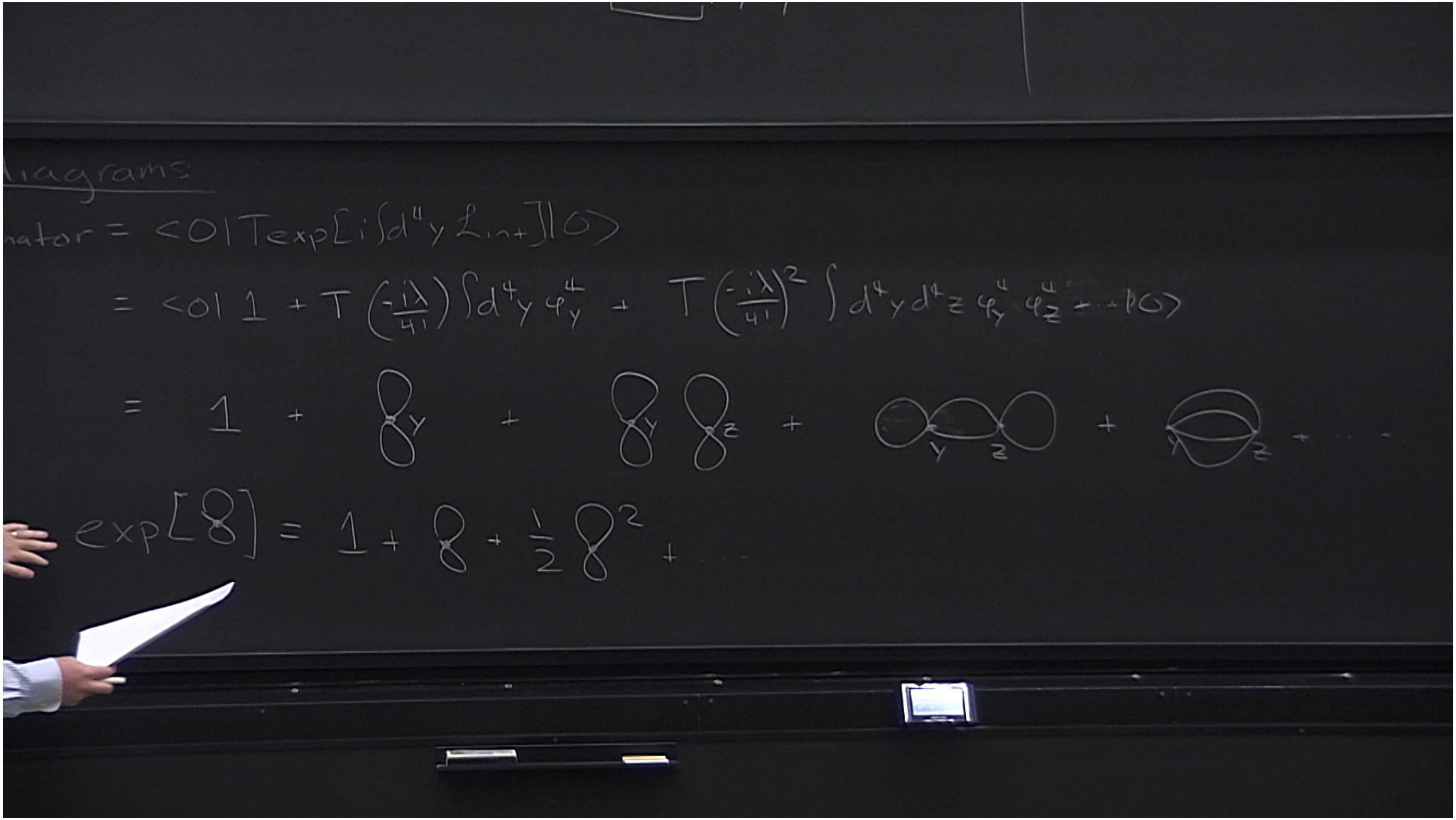
$$S = 2 \cdot 2 \cdot 2$$

$$\frac{1}{S} = \frac{1}{8} = \frac{3}{4!}$$


vacuum diagrams
denominator = <0|T exp[i ∫d⁴y ℒ_int]|0>
= <0| 1 + T(-iλ/4!) ∫d⁴y φ_y⁴ + T(-iλ/4!)² ∫d⁴y d⁴z φ_y⁴ φ_z⁴ + ...|0>
=


iagrams
nator = <0|Texp[i∫d⁴y ℒ_int]|0>
= <0| 1 + T(-iλ/4!)∫d⁴y φ_y⁴ + T(-iλ/4!)² ∫d⁴y d⁴z φ_y⁴ φ_z⁴ + ...|0>
= 1 +
y
+
y
z
+
y
z
+
y
z
+ ...

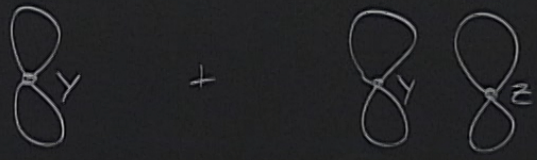
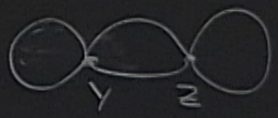

iagrams
nator = <0|Texp[i∫d⁴y ℒ_int]|0>
= <0| 1 + T(-iλ/4!)∫d⁴y φ_y⁴ + T(-iλ/4!)² ∫d⁴y d⁴z φ_y⁴ φ_z⁴ + ...|0>
= 1 +
y
+
y z
+
y z
+ ...
exp[8]


iagrams
nator = ⟨0|T exp[i∫d⁴y ℒ_int]|0⟩
= ⟨0| 1 + T(−iλ/4!)∫d⁴y φ_y⁴ + T(−iλ/4!)²∫d⁴y d⁴z φ_y⁴ φ_z⁴ + ...|0⟩
= 1 + + + + + ...
y
y z
y z
exp[8] = 1 + 8 + ½ 8² + ...

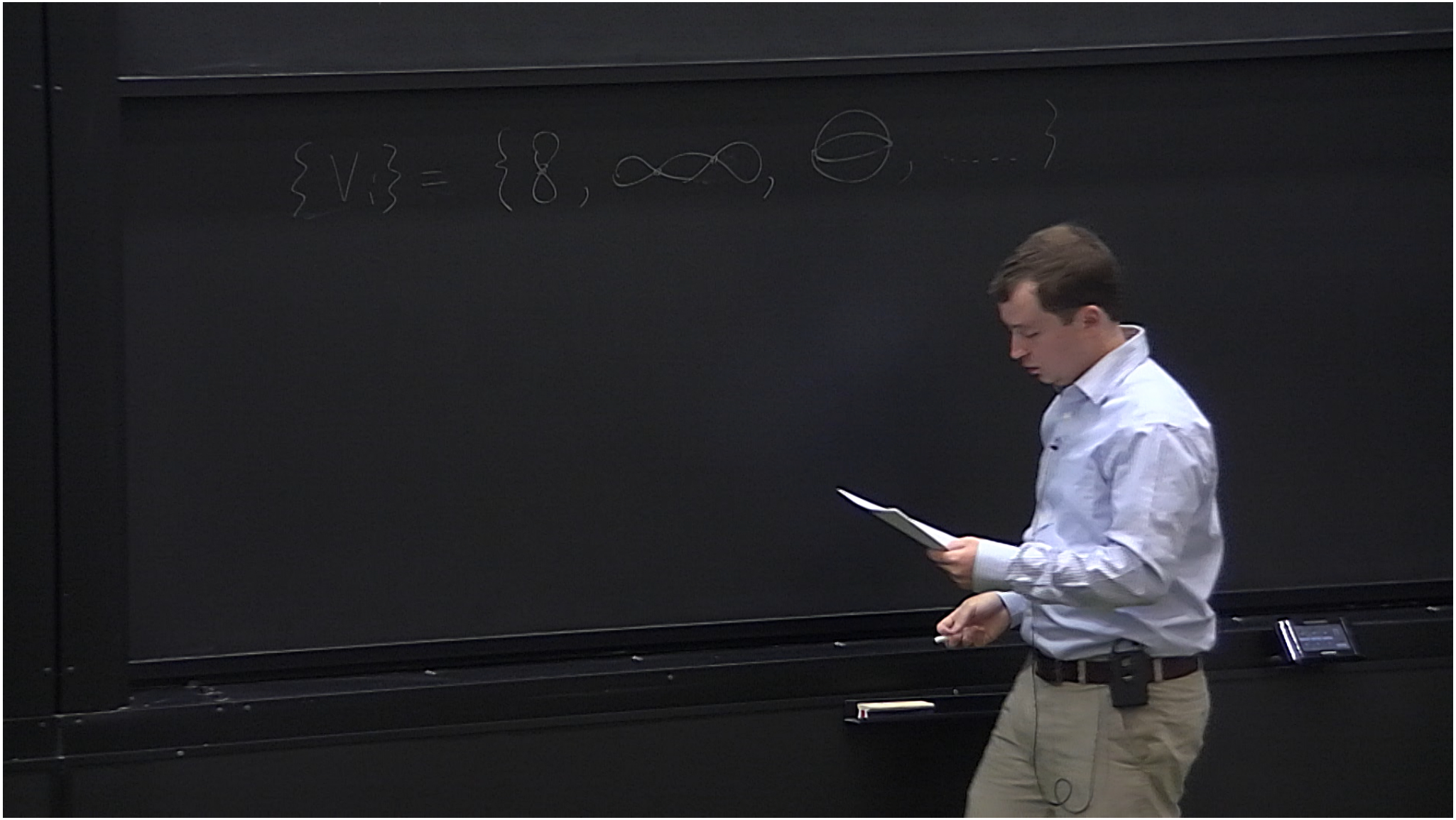
agrams

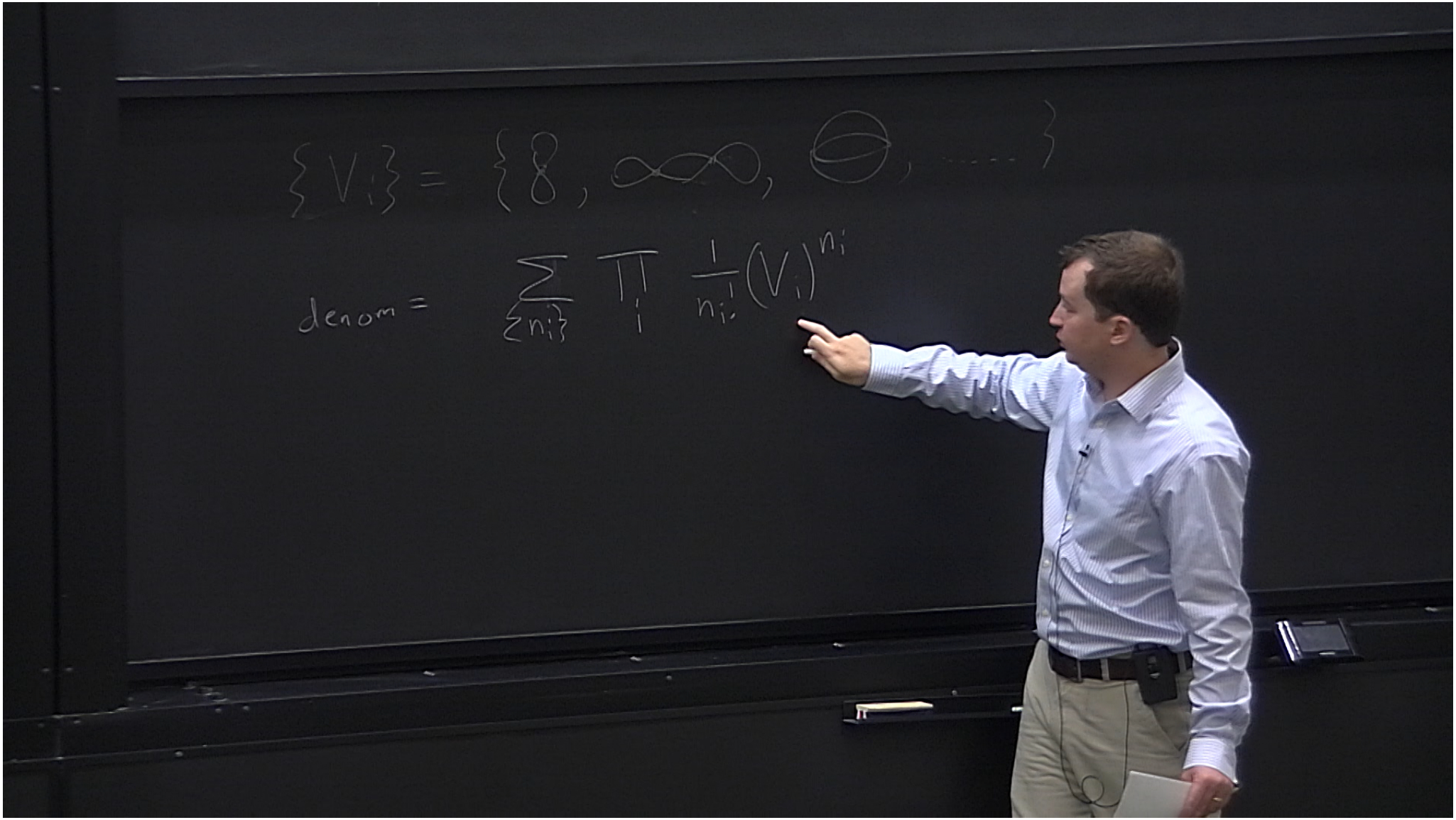
tor = $\langle 0|T\exp[i\int d^4y\,\mathcal{L}_{int}]|0\rangle$

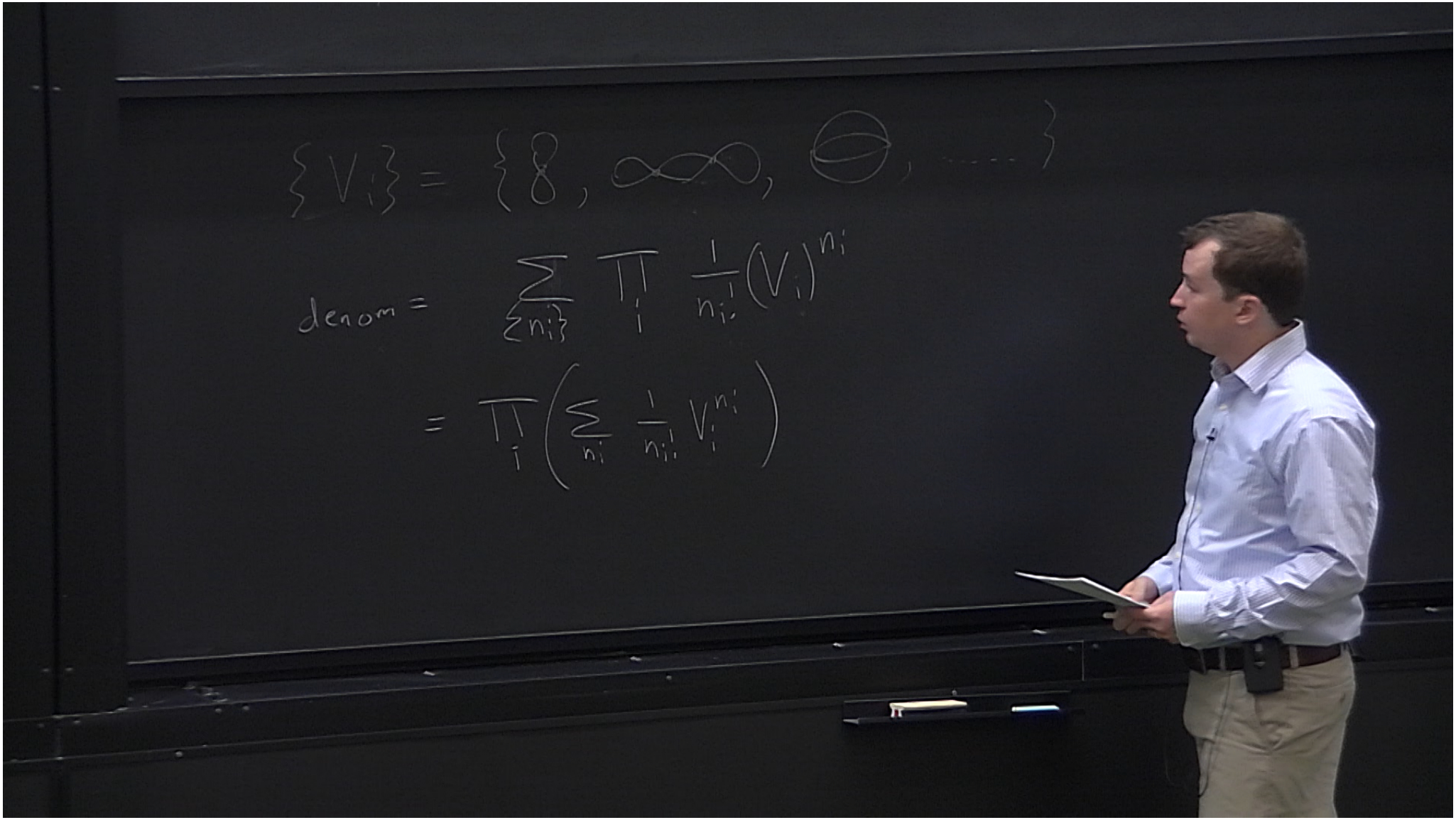
$= \langle 0|1 + T\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi_y^4 + \frac{1}{2}T\left(\frac{-i\lambda}{4!}\right)^2\int d^4y\,d^4z\,\varphi_y^4\,\varphi_z^4 + \cdots|0\rangle$

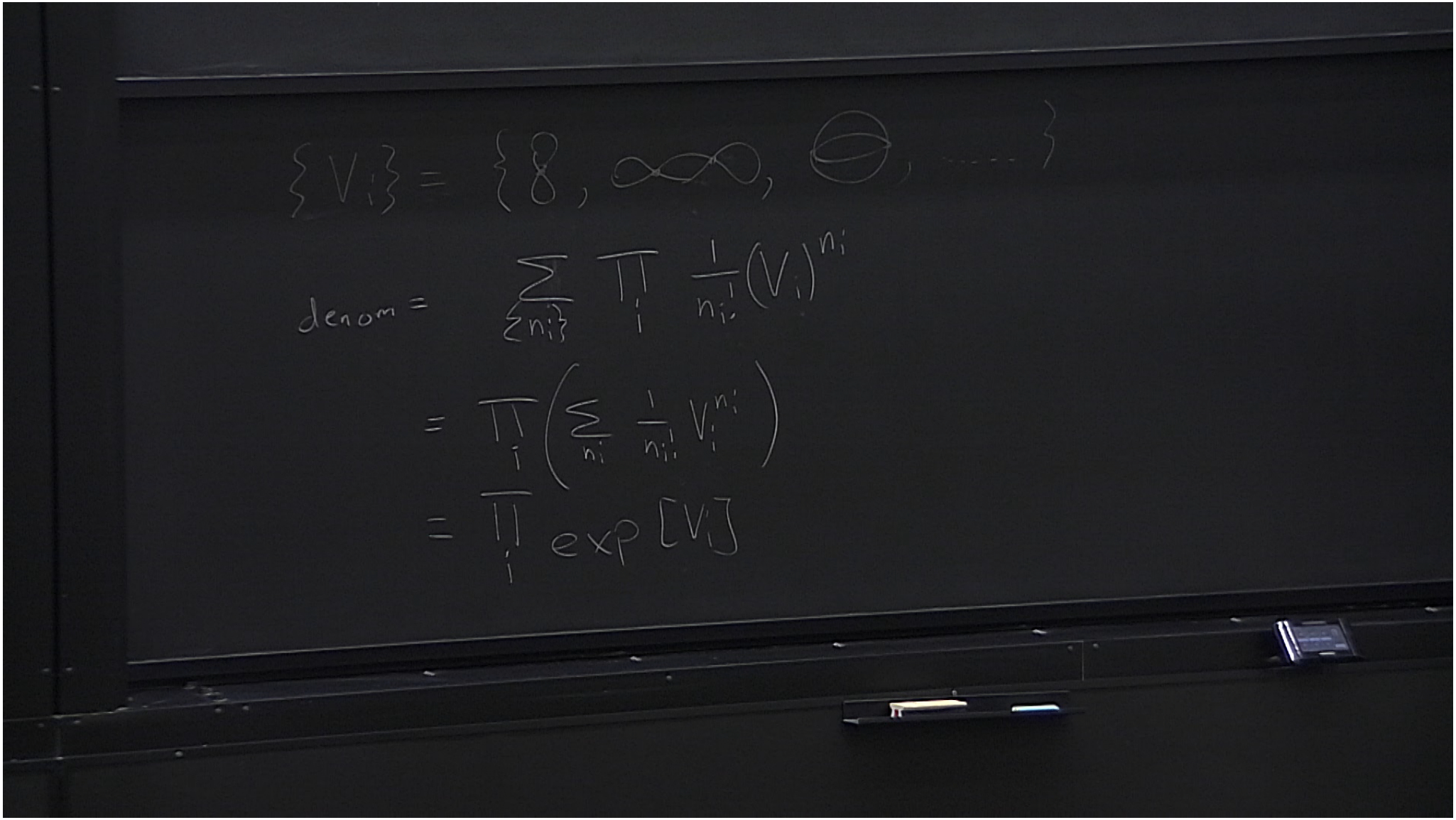
$=$ 1 +  + +  +  + ...

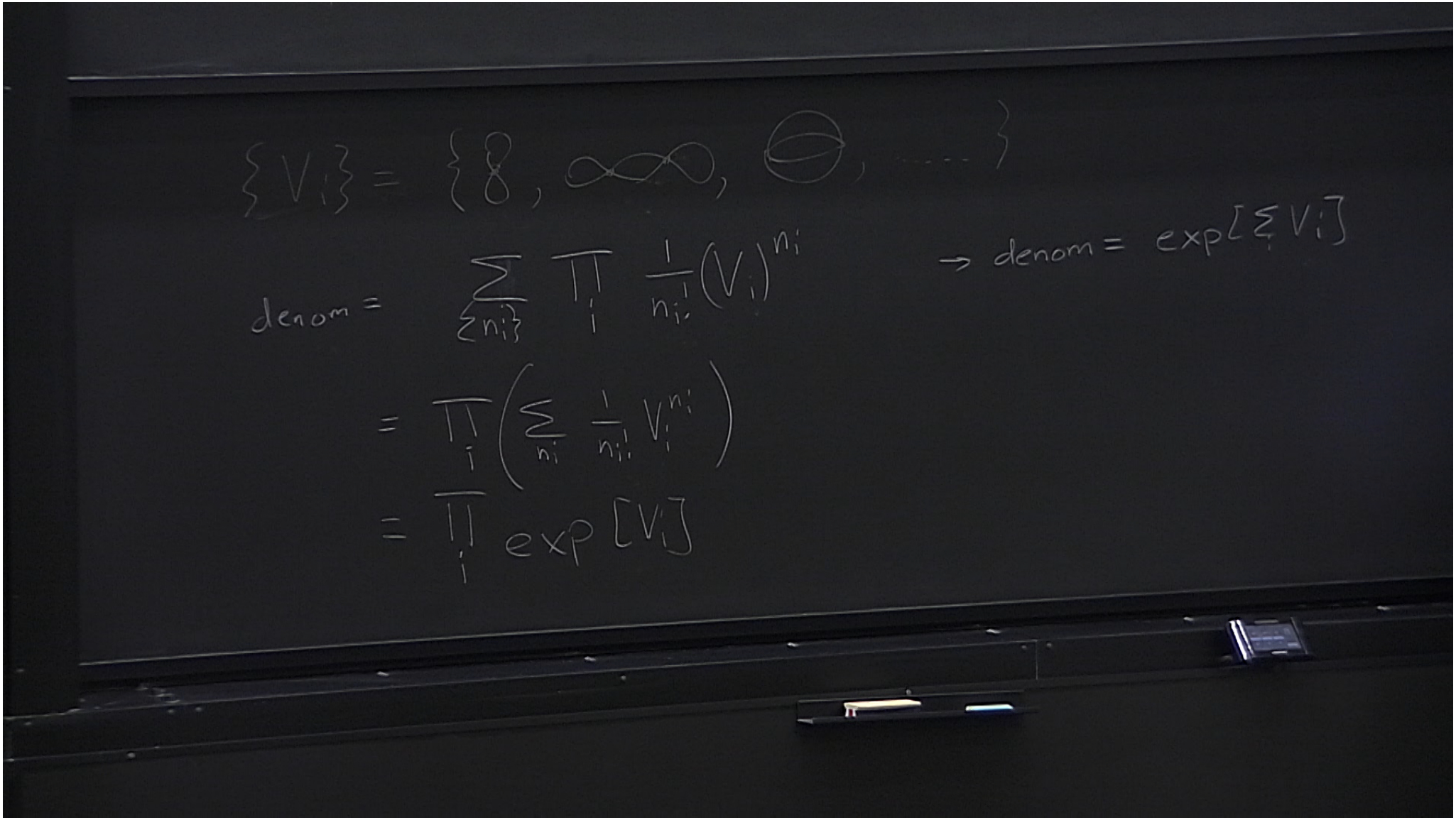
$\exp[8] = 1 + 8 + \frac{1}{2}8^2 +$ ...


{V_i} = {8, ∞∞, ⊖, ...}

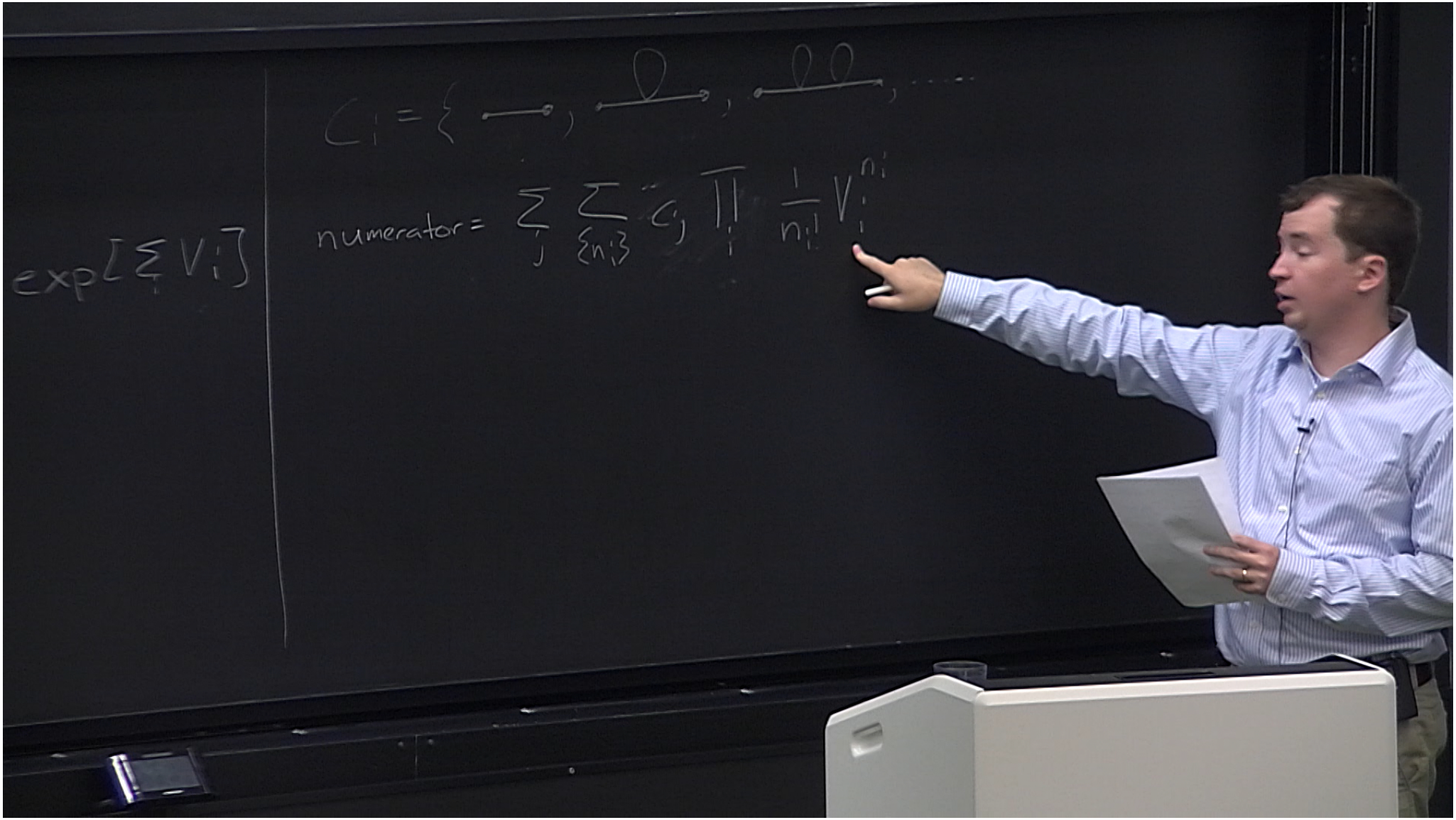

{V_i} = {8, ∞∞, ⊖, ...}
denom = Σ_{Σn_i} Π_i 1/n_i! (V_i)^{n_i}

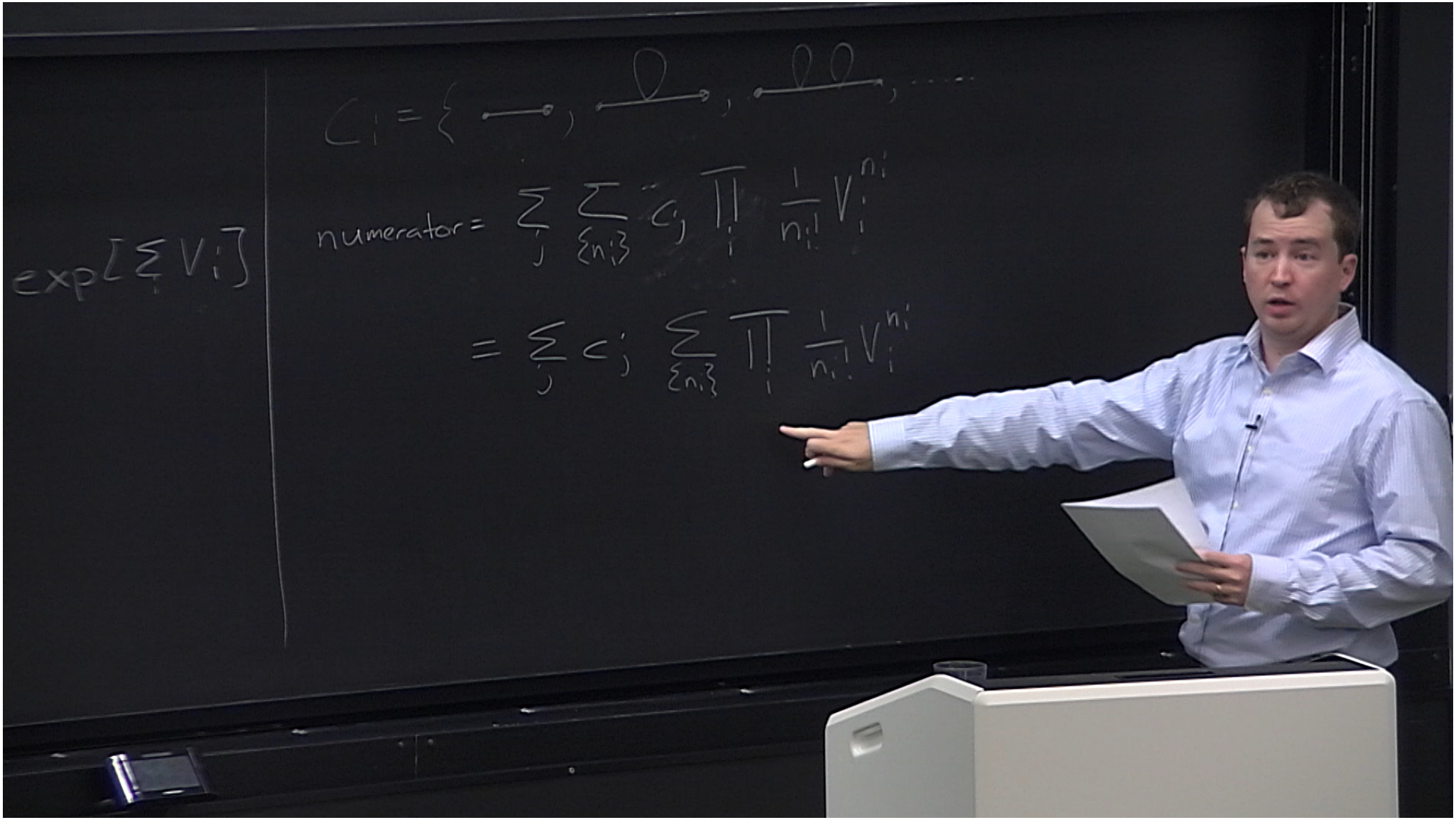

$\{V_i\} = \{8, \infty\infty, \ominus, \ldots\}$
$\text{denom} = \sum_{\{n_i\}} \prod_i \frac{1}{n_i!} (V_i)^{n_i}$
$= \prod_i \left(\sum_{n_i} \frac{1}{n_i!} V_i^{n_i} \right)$

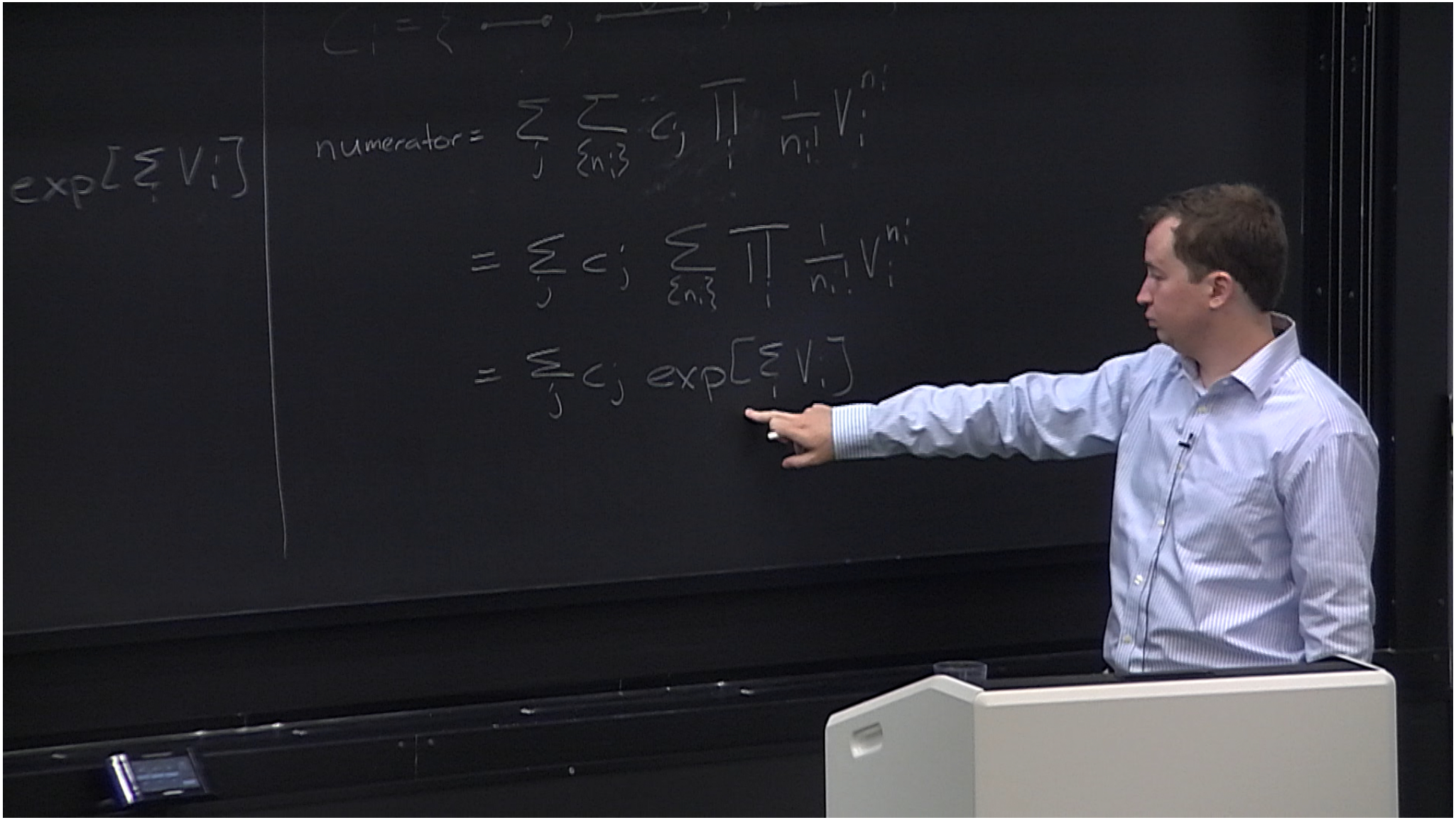

$\{V_i\} = \{$ 8, ∞, ⊖, ... $\}$
denom = $\sum_{\{n_i\}} \prod_i \frac{1}{n_i!} (V_i)^{n_i}$
$= \prod_i \left(\sum_{n_i} \frac{1}{n_i!} V_i^{n_i} \right)$
$= \prod_i \exp[V_i]$


$\{V_i\} = \{$ 8, ∞∞, ⊖, ... $\}$
denom = $\sum_{\{n_i\}} \prod_i \frac{1}{n_i!}(V_i)^{n_i}$
→ denom = $\exp[\sum_i V_i]$
$= \prod_i \left(\sum_{n_i} \frac{1}{n_i!} V_i^{n_i} \right)$
$= \prod_i \exp[V_i]$


exp[Σ Vi]


$C_i = \{$, , , ...
$\exp[\sum_i V_i]$
numerator $= \sum_j \sum_{\{n_i\}} S_j \prod_i \frac{1}{n_i!} V_i^{n_i}$


$C_i = \{$, , , ...
$\exp[\sum_i V_i]$
numerator $= \sum_j \sum_{\{n_i\}} c_j \prod_i \frac{1}{n_i!} V_i^{n_i}$
$= \sum_j c_j \sum_{\{n_i\}} \prod_i \frac{1}{n_i!} V_i^{n_i}$


exp[Σ_i V_i]
numerator = Σ_j Σ_{n_i} c_j ∏_i 1/n_i! V_i^{n_i}
= Σ_j c_j Σ_{n_i} ∏_i 1/n_i! V_i^{n_i}
= Σ_j c_j exp[Σ_i V_i]

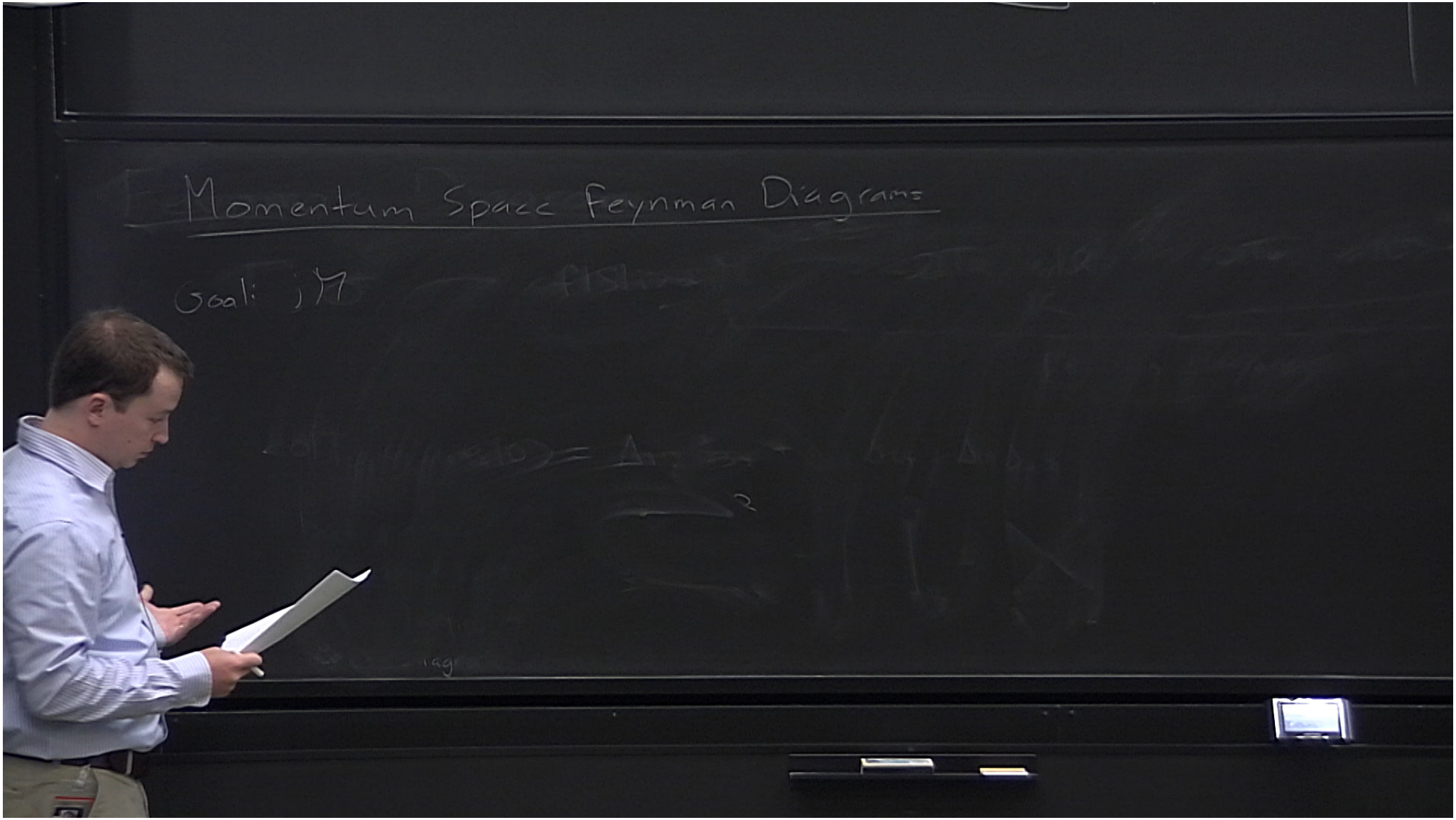
Position Space Feynman Rules

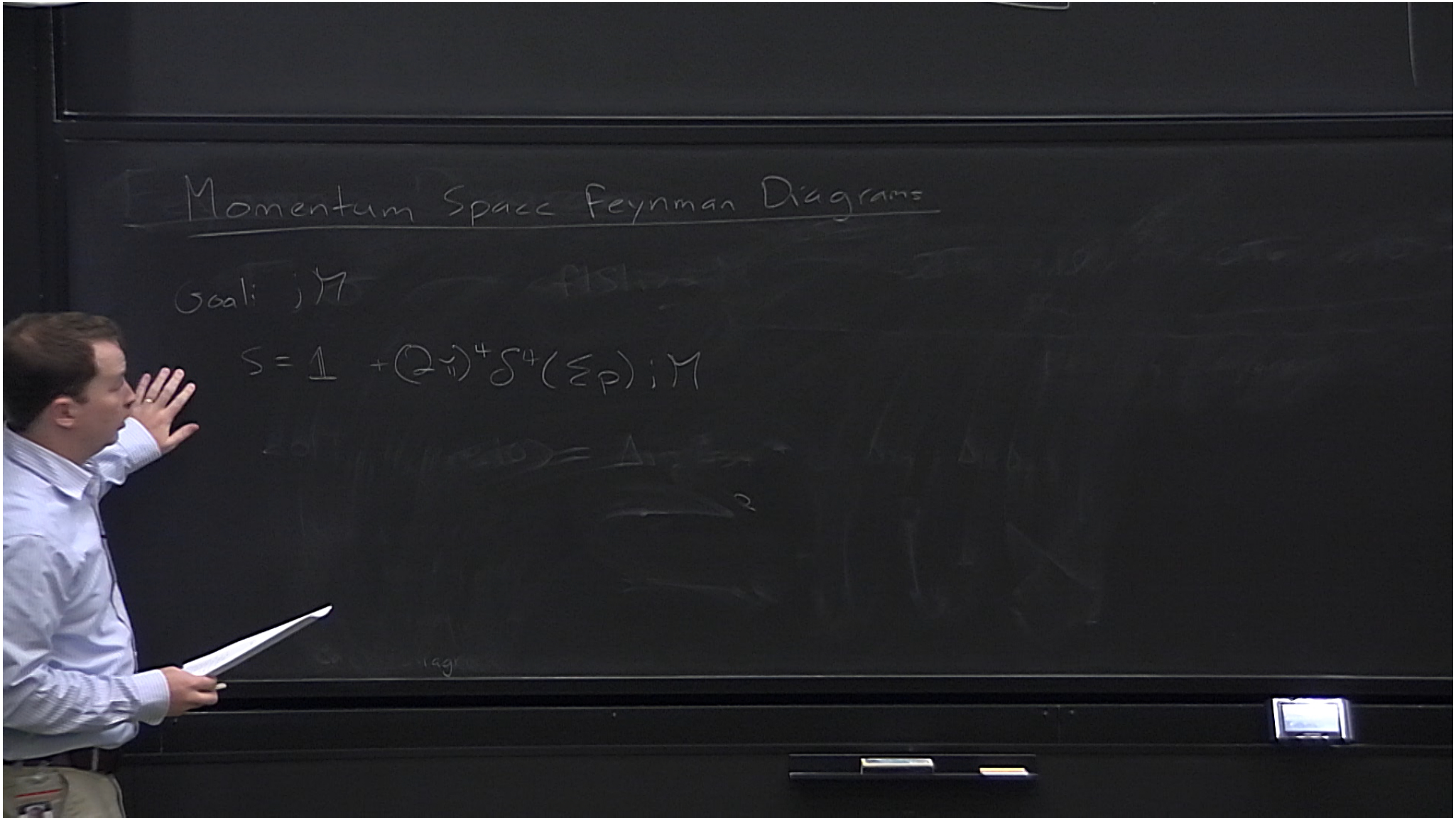
$\langle \Omega | T\varphi_1 \cdots \varphi_n | \Omega \rangle$ = sum of all diagrams with n external points + no vacuum diagrams

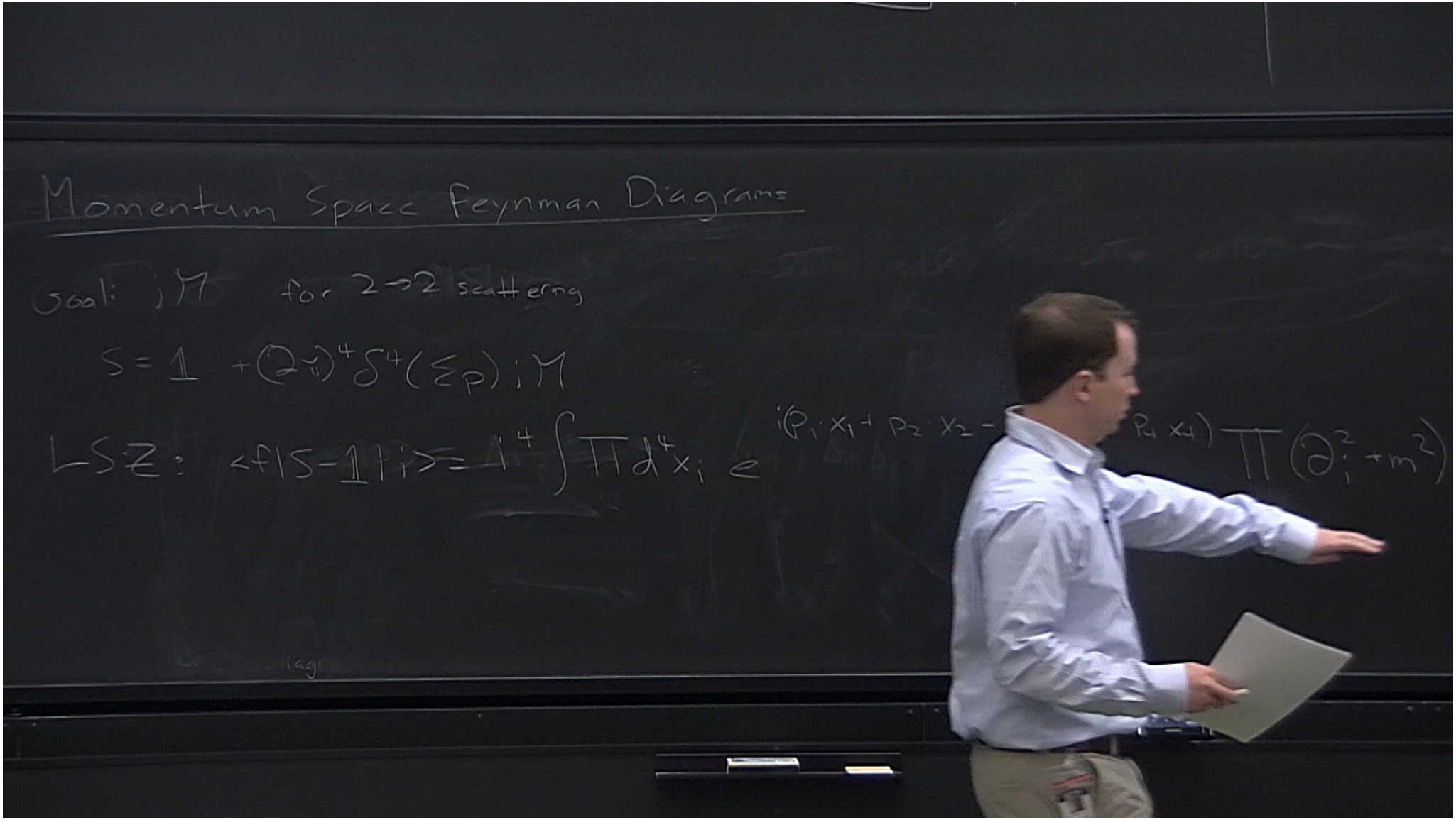
1. For each line $x \quad y$ $= \Delta_F(x-y)$
2. For each external point x $= 1$ (will change for $s>0$)
3. For each vertex z $= -i\lambda \int d^4 z$ (for φ^4 theory)

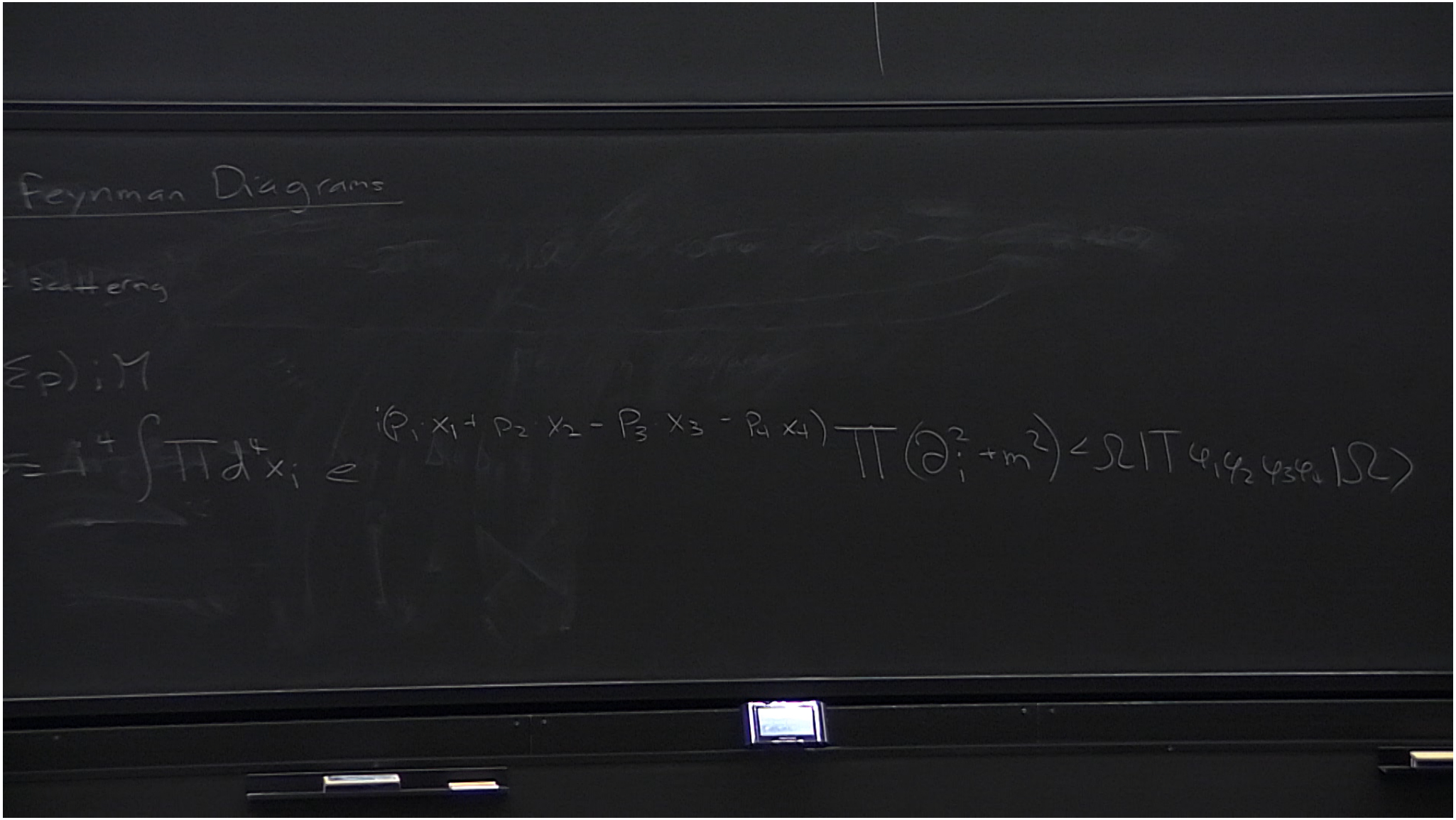


if $\mathcal{L}_{int} \supset \frac{-g}{n_1! n_2! n_3!} \varphi^{n_1} \phi^{n_2} \psi^{n_3}$


Momentum Space Feynman Diagrams
Goal:


Momentum Space Feynman Diagrams
Goal: iM
S = 1 + (2πi)^4 δ^4(Σp) iM


Momentum Space Feynman Diagrams
Goal: iM for 2→2 scattering
S = 1 + (2π)⁴ δ⁴(Σp) iM
LSZ: <f|S-1|i> = i⁴ ∫ Π d⁴x_i e^(i(p1·x1 + p2·x2 - p4·x4)) Π (∂_i² + m²)


Feynman Diagrams
scattering
$\langle p \rangle ; M$
$= i^4 \int \prod d^4 x_i \, e^{i(p_1 \cdot x_1 + p_2 \cdot x_2 - p_3 \cdot x_3 - p_4 \cdot x_4)} \prod (\partial_i^2 + m^2) \langle \Omega | T \phi_1 \phi_2 \phi_3 \phi_4 | \Omega \rangle$

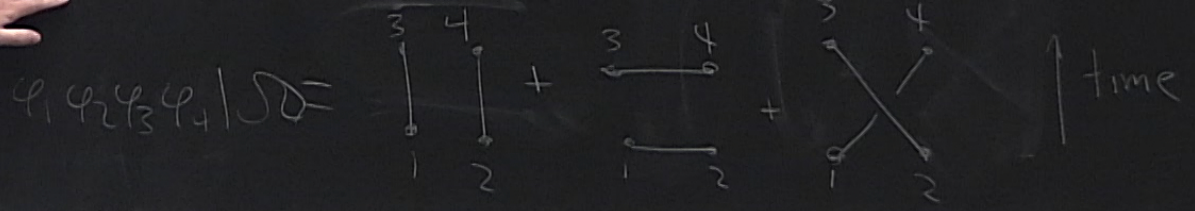
Momentum Space Feynman Diagrams

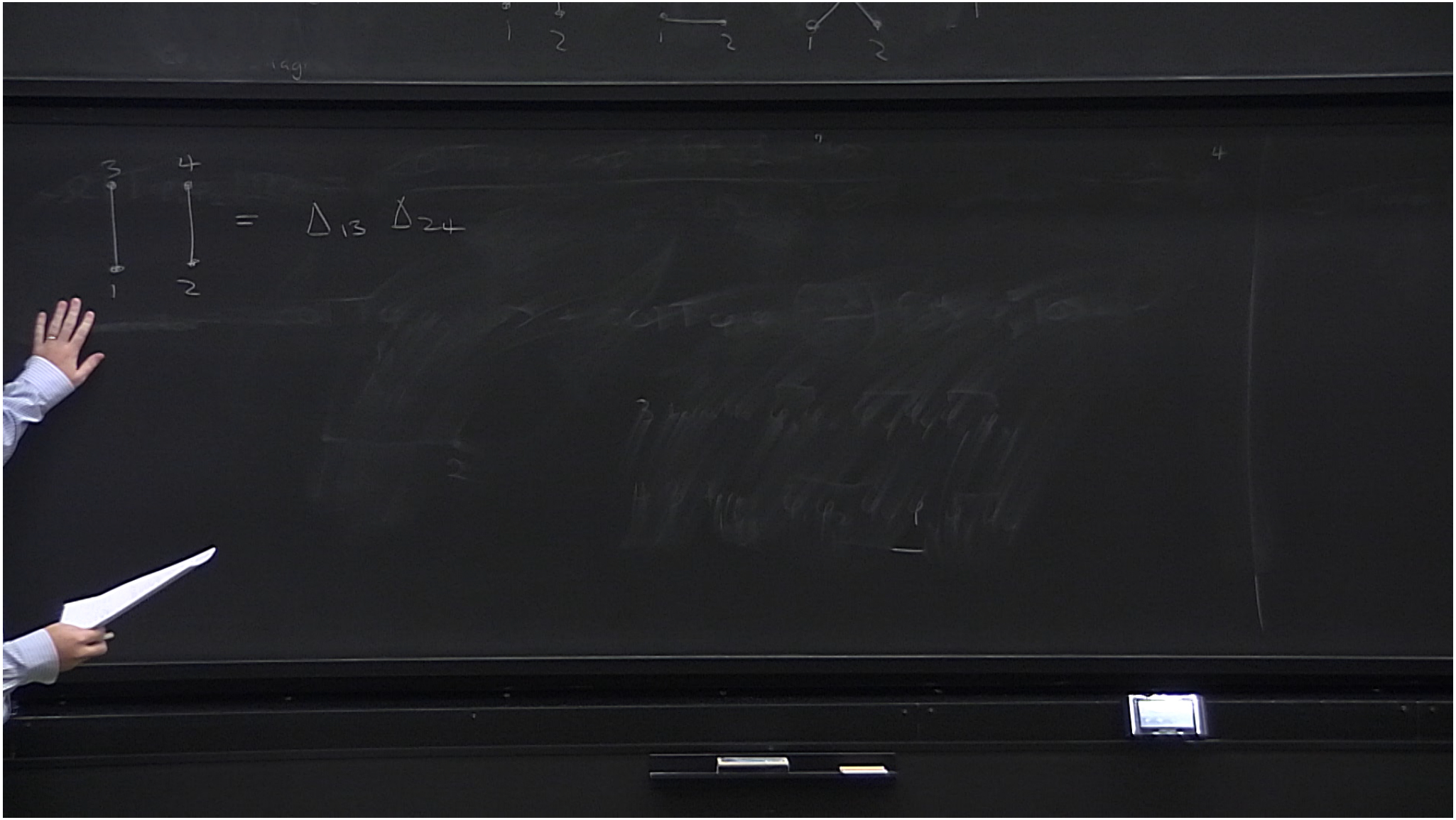
Goal: $i\mathcal{M}$ for $2 \to 2$ scattering

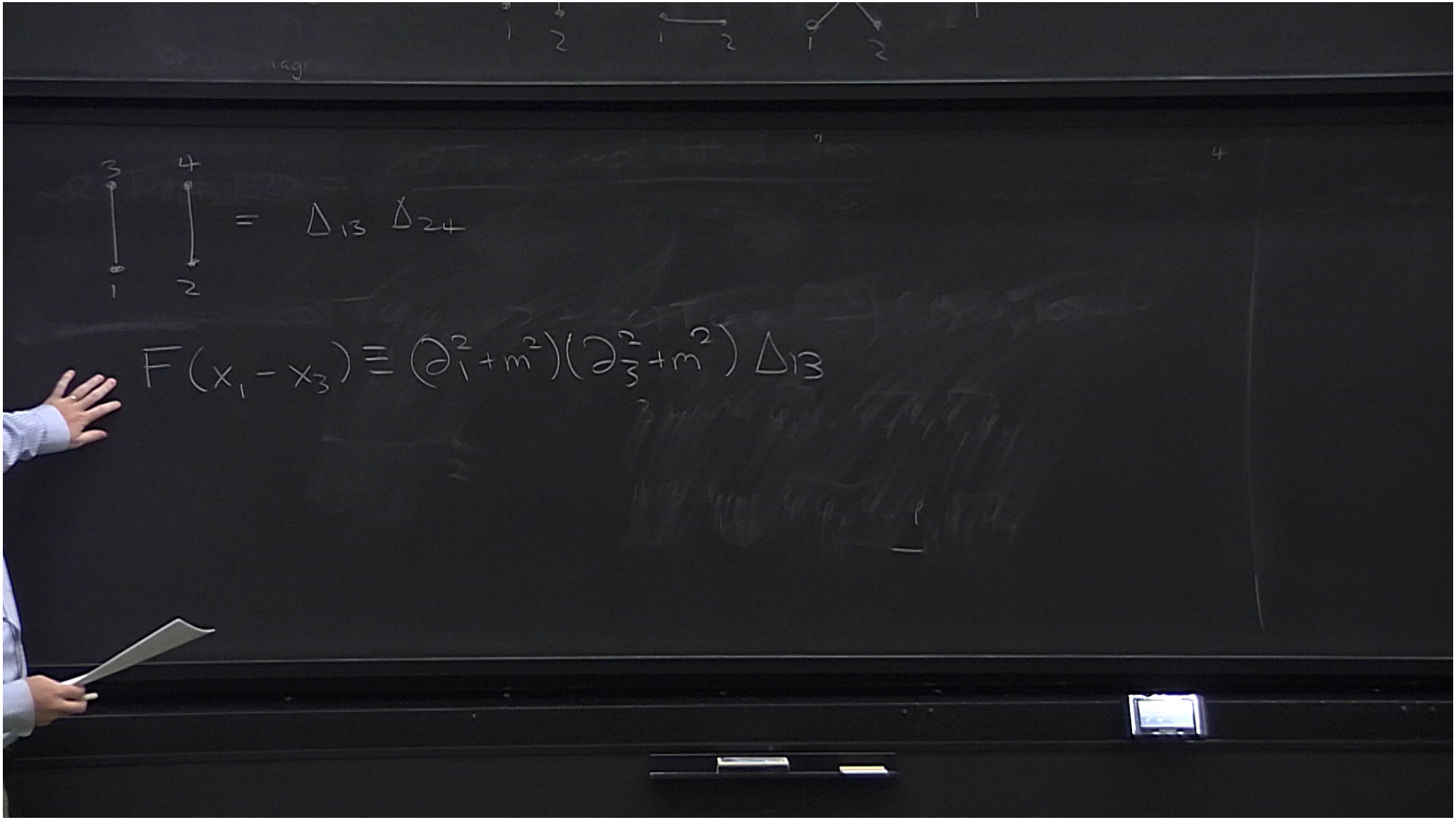
$$S = 1 + (2\pi)^4 \delta^4(\textstyle\sum p)\, i\mathcal{M}$$

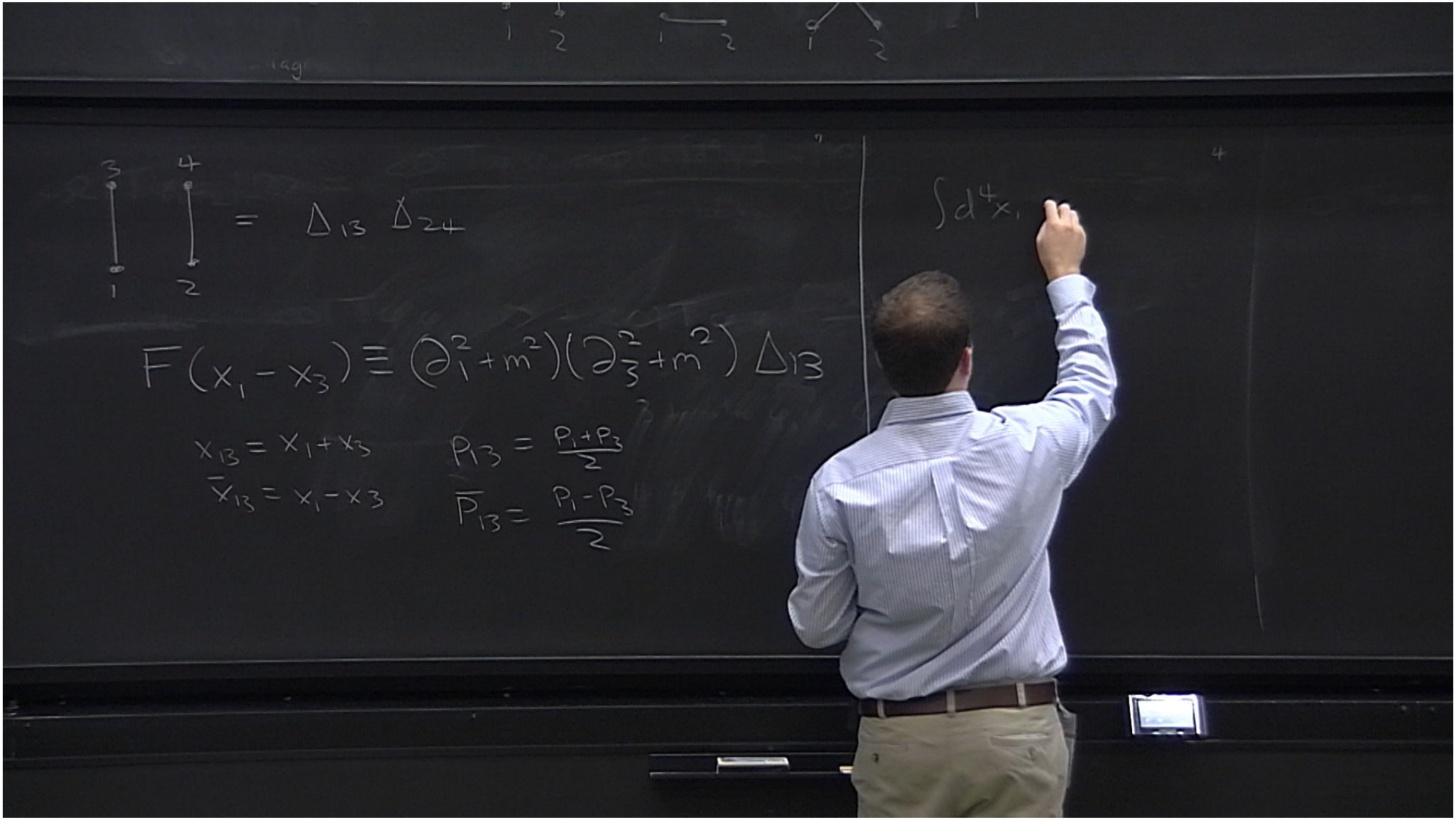
LSZ: $\langle f|S-1|i\rangle = A^4 \int \prod d^4x_i \, e^{i(p_1 x_1 + p_2 x_2 - p_3 x_3 - p_4 x_4)} \prod (\partial_i^2 + m^2)$

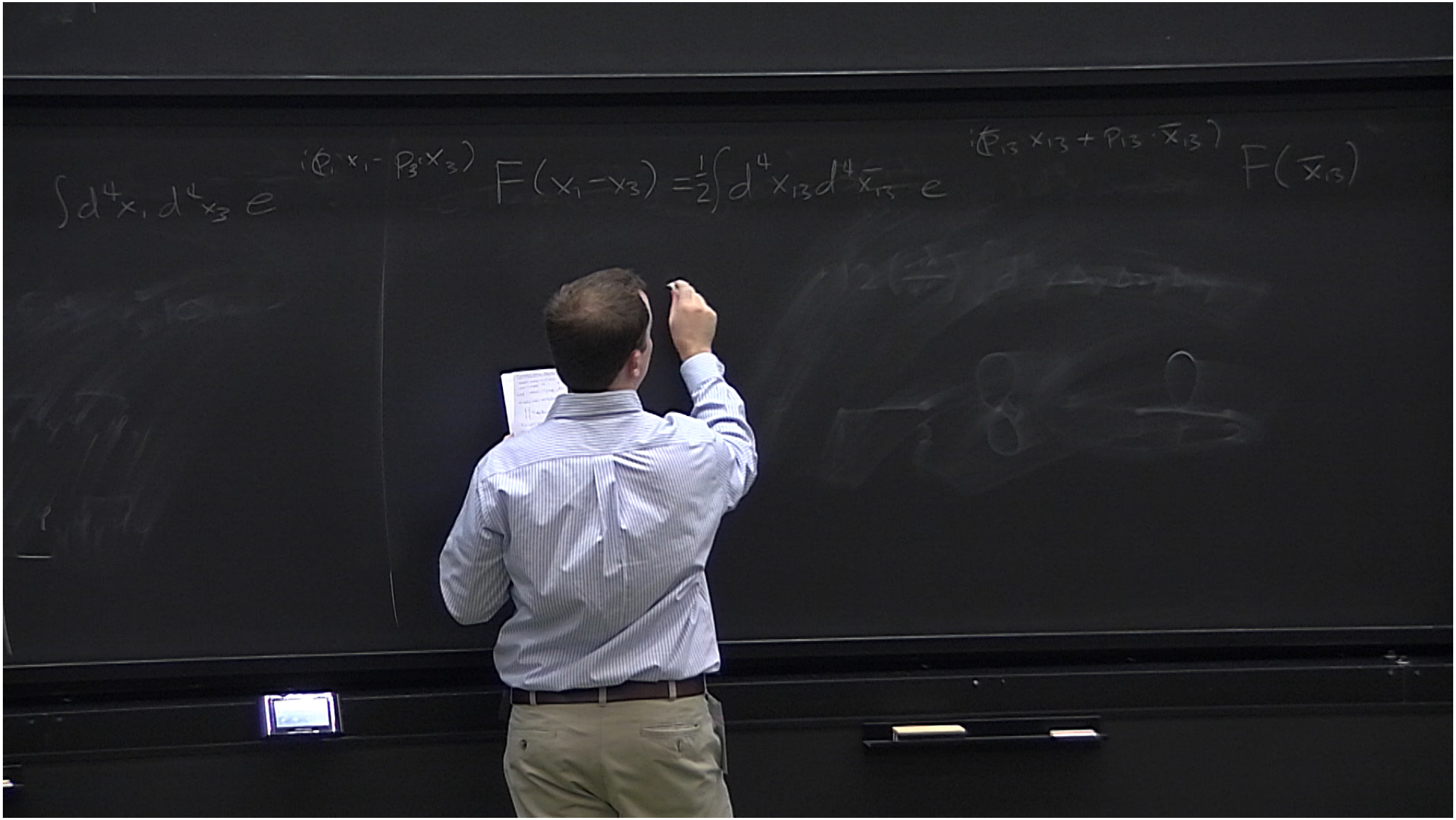
$\langle \Omega | T\{\varphi_1 \varphi_2 \varphi_3 \varphi_4\} | \Omega \rangle =$

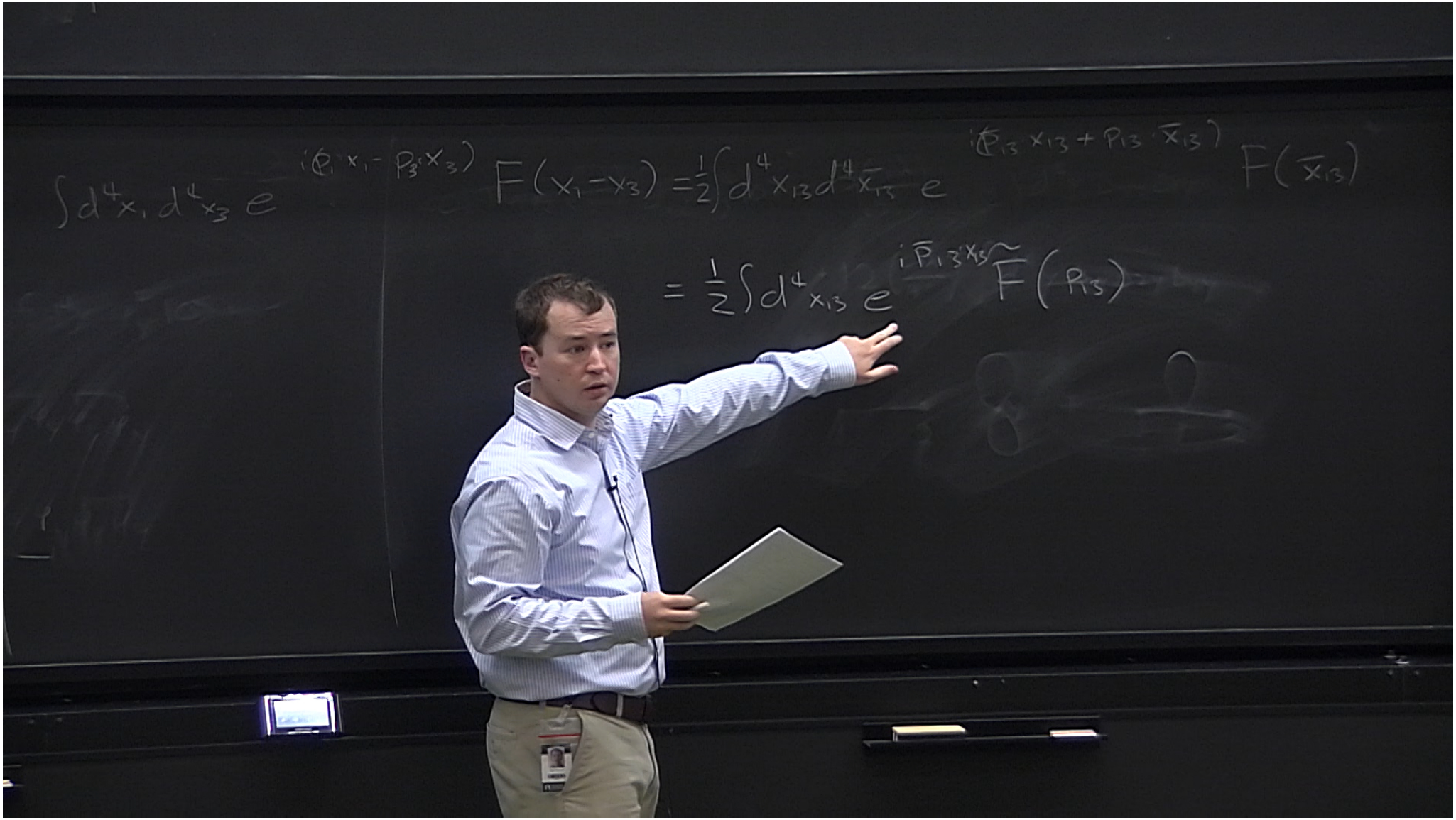


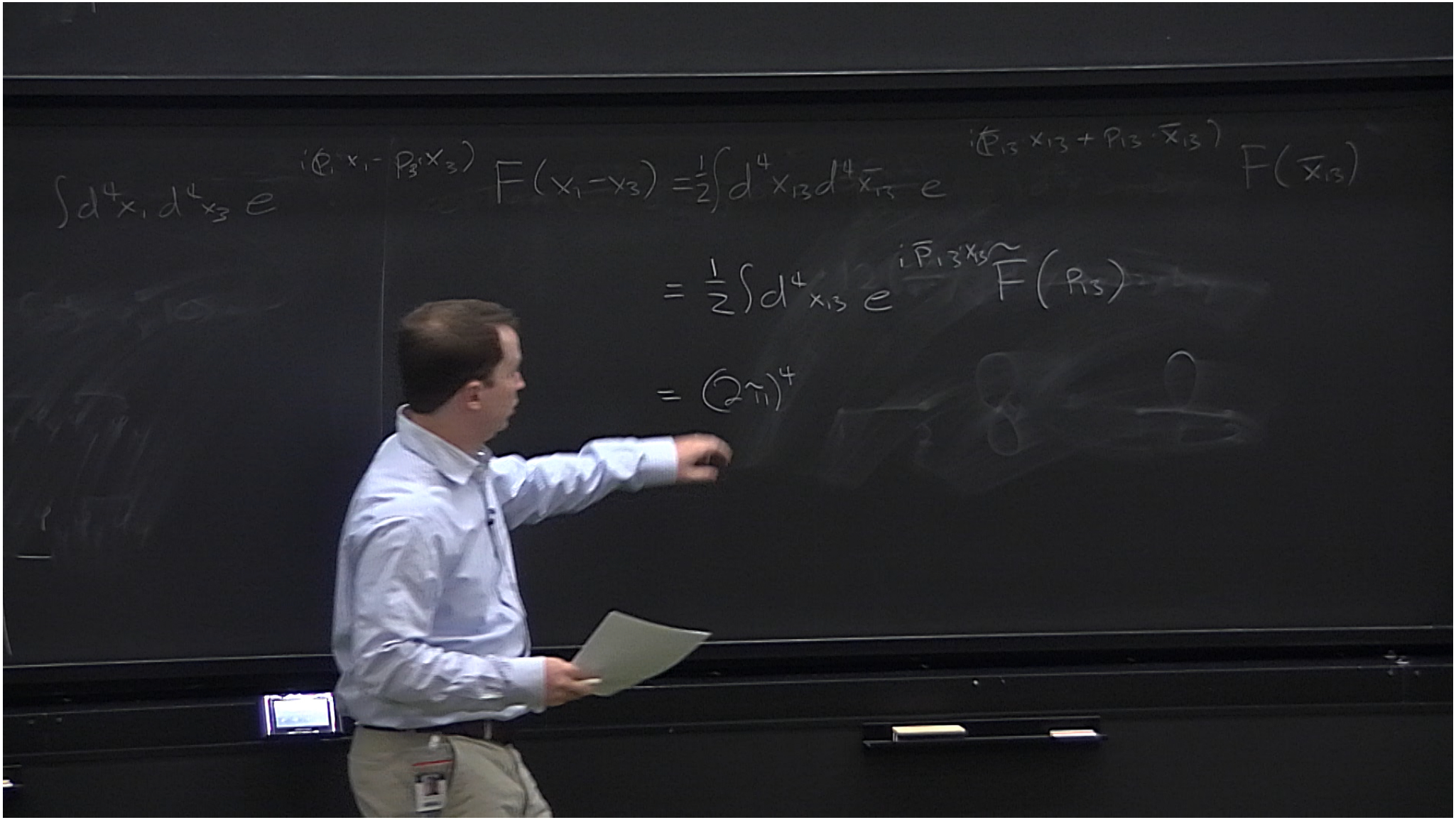

3
4
1
2
= $\Delta_{13}\,\Delta_{24}$

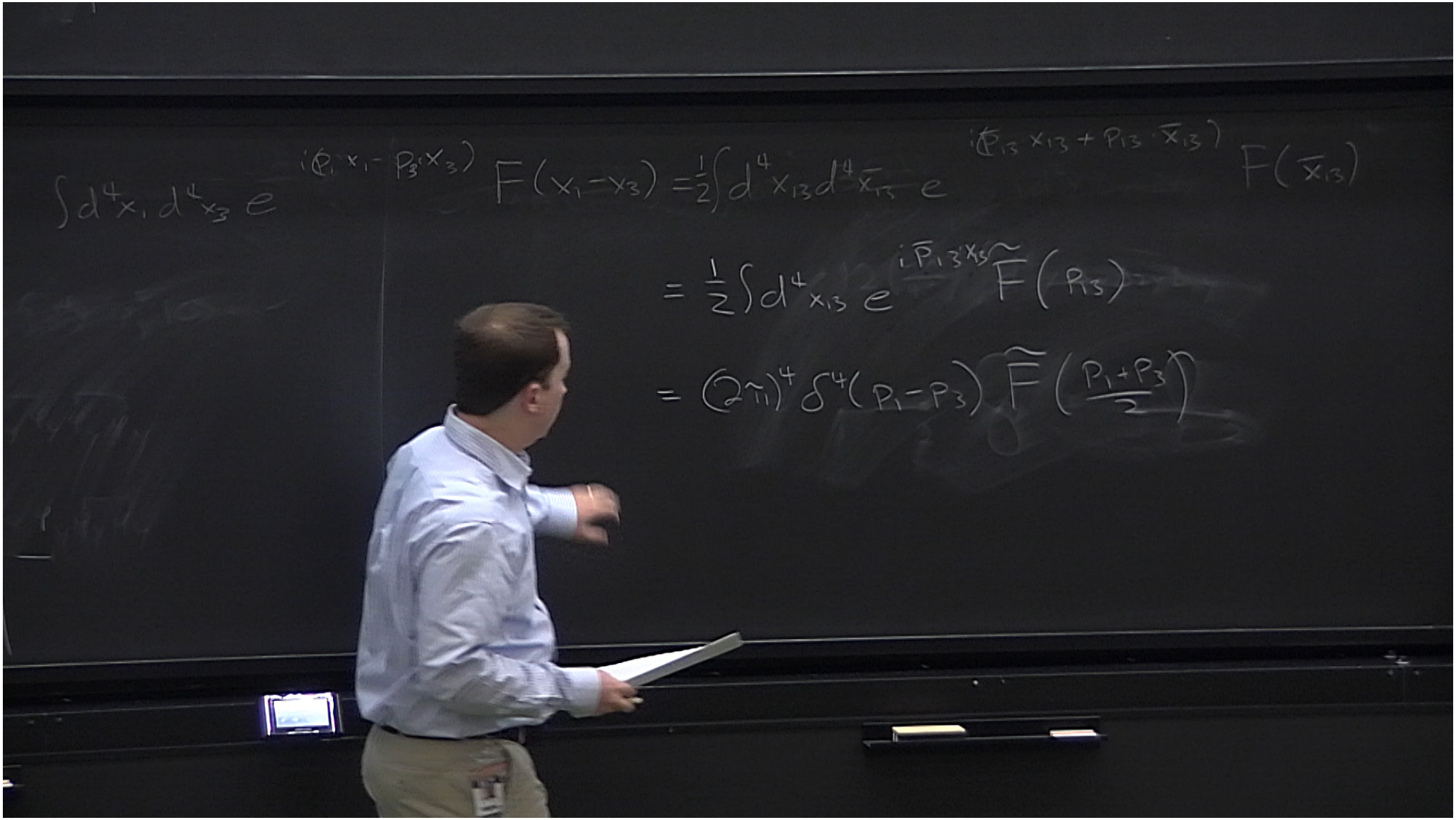

3
4
1
2
= $\Delta_{13}\,\Delta_{24}$
$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_{13}$


3 4
1 2
= $\Delta_{13}\,\Delta_{24}$
$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_{13}$
$x_{13} = x_1 + x_3$
$\bar{x}_{13} = x_1 - x_3$
$p_{13} = \frac{p_1 + p_3}{2}$
$\bar{p}_{13} = \frac{p_1 - p_3}{2}$
$\int d^4x_1$


$\int d^4x_1 d^4x_3 e^{i(p_1 x_1 - p_3 x_3)} F(x_1 - x_3) = \frac{1}{2}\int d^4x_{13} d^4\bar{x}_{13} e^{i(p_{13} x_{13} + p_{13} \cdot \bar{x}_{13})} F(\bar{x}_{13})$


$\int d^4x_1 d^4x_3 e^{i(p_1 x_1 - p_3 x_3)} F(x_1 - x_3) = \frac{1}{2}\int d^4x_{13} d^4\bar{x}_{13} e^{i(p_{13} x_{13} + p_{13} \bar{x}_{13})} F(\bar{x}_{13})$
$= \frac{1}{2}\int d^4x_{13} e^{i\bar{p}_{13} x_{13}} \tilde{F}(p_{13})$


$\int d^4x_1 d^4x_3 e^{i(p_1 x_1 - p_3 x_3)} F(x_1 - x_3) = \frac{1}{2}\int d^4x_{13} d^4\bar{x}_{13} e^{i(p_{13} x_{13} + p_{13} \cdot \bar{x}_{13})} F(\bar{x}_{13})$
$= \frac{1}{2}\int d^4x_{13} e^{i\bar{p}_{13} x_{13}} \tilde{F}(p_{13})$
$= (2\pi)^4$

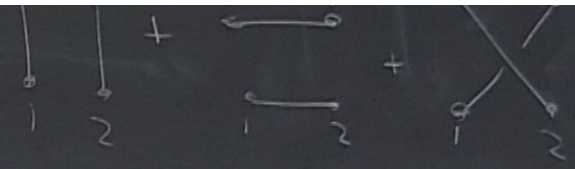

$\int d^4x_1 d^4x_3 e^{i(p_1 x_1 - p_3 x_3)} F(x_1 - x_3) = \frac{1}{2}\int d^4x_{13} d^4\bar{x}_{13} e^{i(p_{13} x_{13} + p_{13} \bar{x}_{13})} F(\bar{x}_{13})$
$= \frac{1}{2}\int d^4x_{13} e^{i\bar{p}_{13} x_{13}} \tilde{F}(p_{13})$
$= (2\pi)^4 \delta^4(p_1 - p_3) \tilde{F}\left(\frac{p_1 + p_3}{2}\right)$

$$\int d^4x_1\, d^4x_3\, e^{i(p_1 x_1 - p_3 x_3)} F(x_1 - x_3) = \frac{1}{2}\int d^4x_{13}\, d^4\bar{x}_{13}\, e^{i(p_{13} x_{13} + p_{13}\cdot \bar{x}_{13})} F(\bar{x}_{13})$$

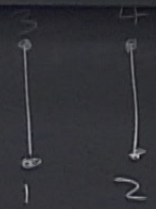
$$= \frac{1}{2}\int d^4x_{13}\, e^{i\bar{p}_{13} x_{13}} \tilde{F}(p_{13})$$

$$= (2\pi)^4 \delta^4(p_1 - p_3)\, \tilde{F}\left(\frac{p_1+p_3}{2}\right)$$

$$\langle f|S|i\rangle_1 \propto \delta^{(4)}(p_1 - p_3)\, \delta^{(4)}(p_2 - p_4)$$

$\langle \Omega | T \phi_1 \phi_2 \phi_3 \phi_4 | \Omega \rangle =$  + +

time

 $= \Delta_{13}\,\Delta_{24}$

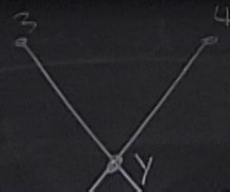
$$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_{13}$$

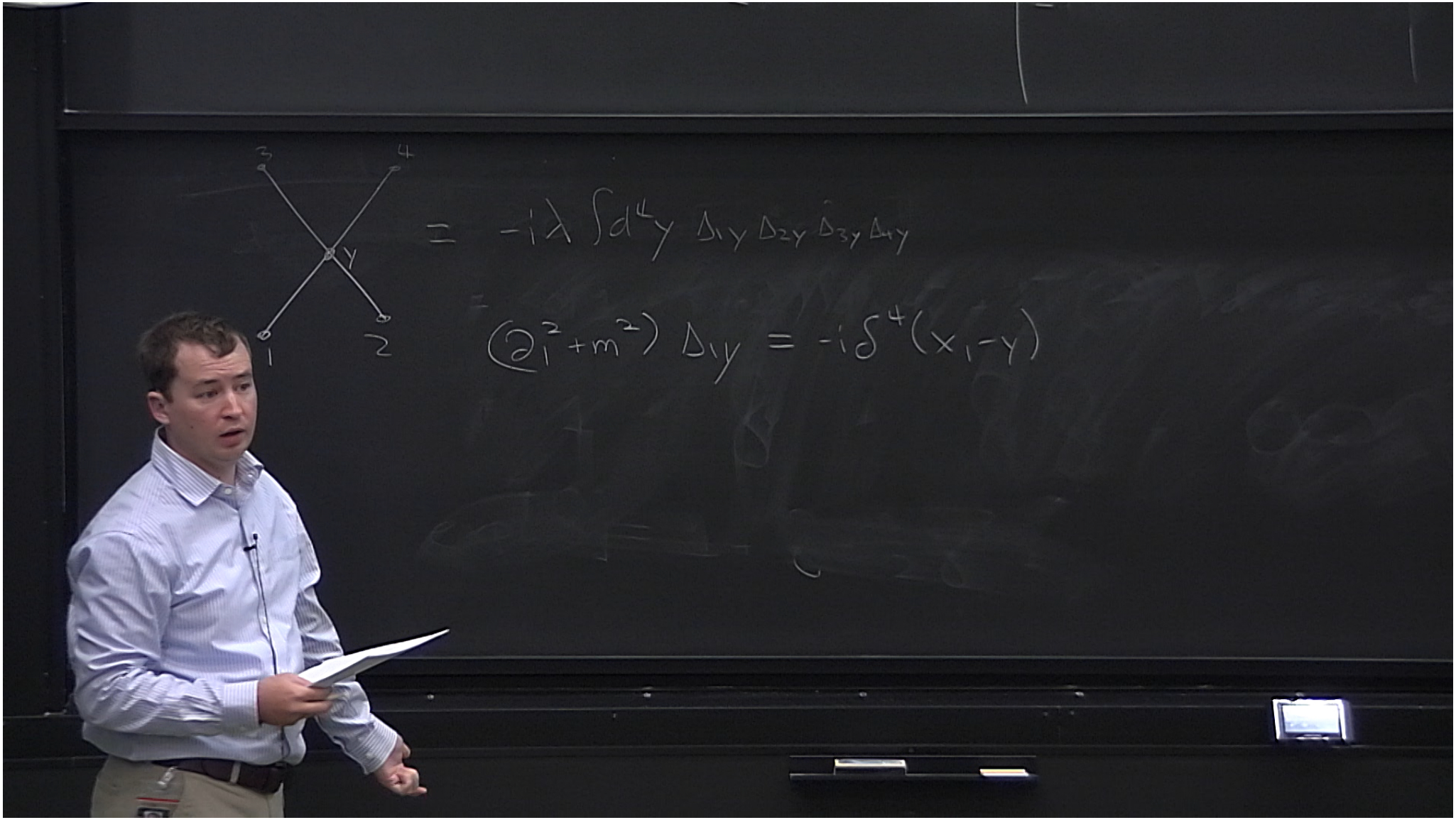
$$x_{13} = x_1 + x_3 \qquad p_{13} = \frac{p_1 + p_3}{2}$$

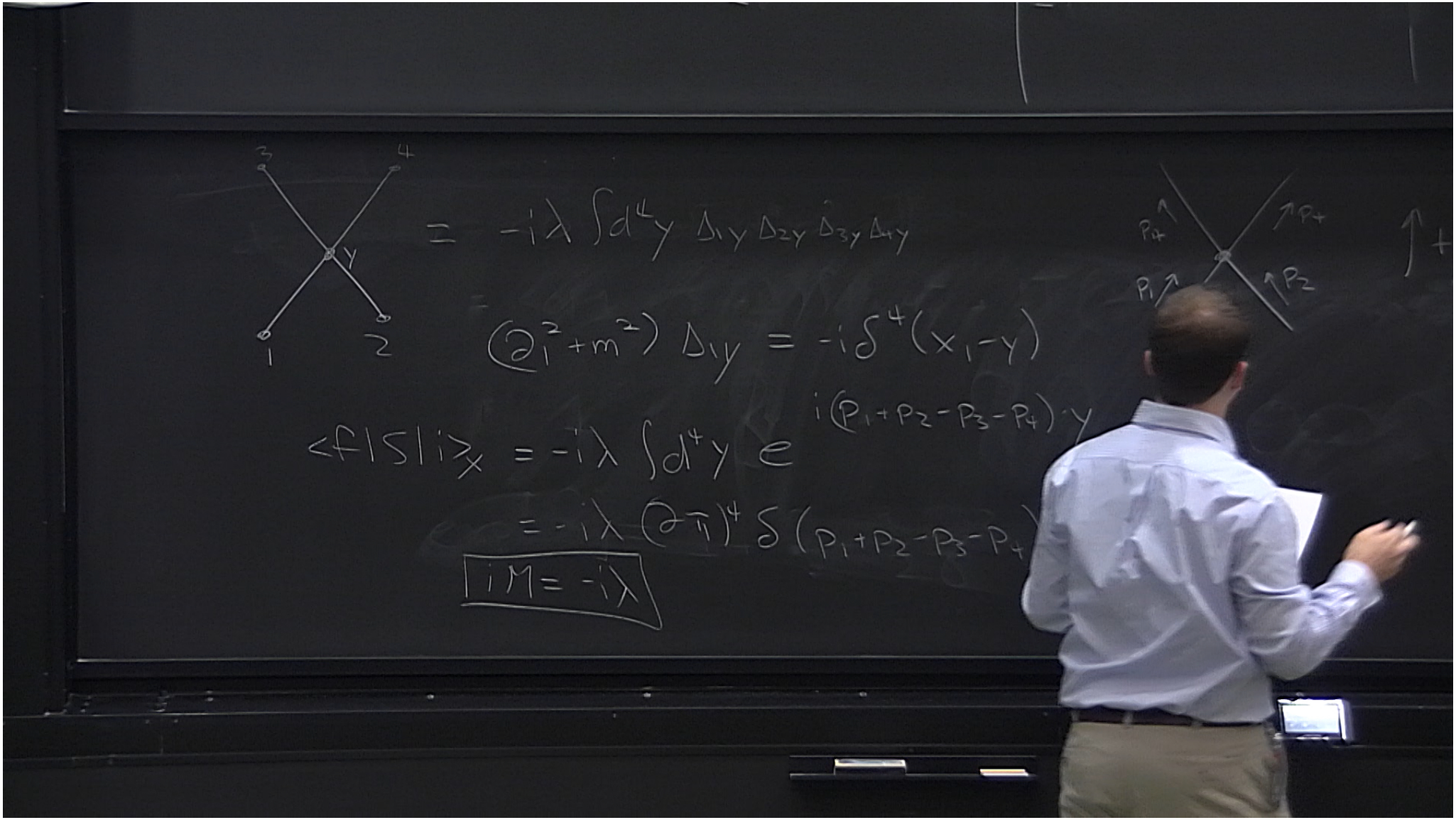
$$\bar{x}_{13} = x_1 - x_3 \qquad \bar{p}_{13} = \frac{p_1 - p_3}{2}$$

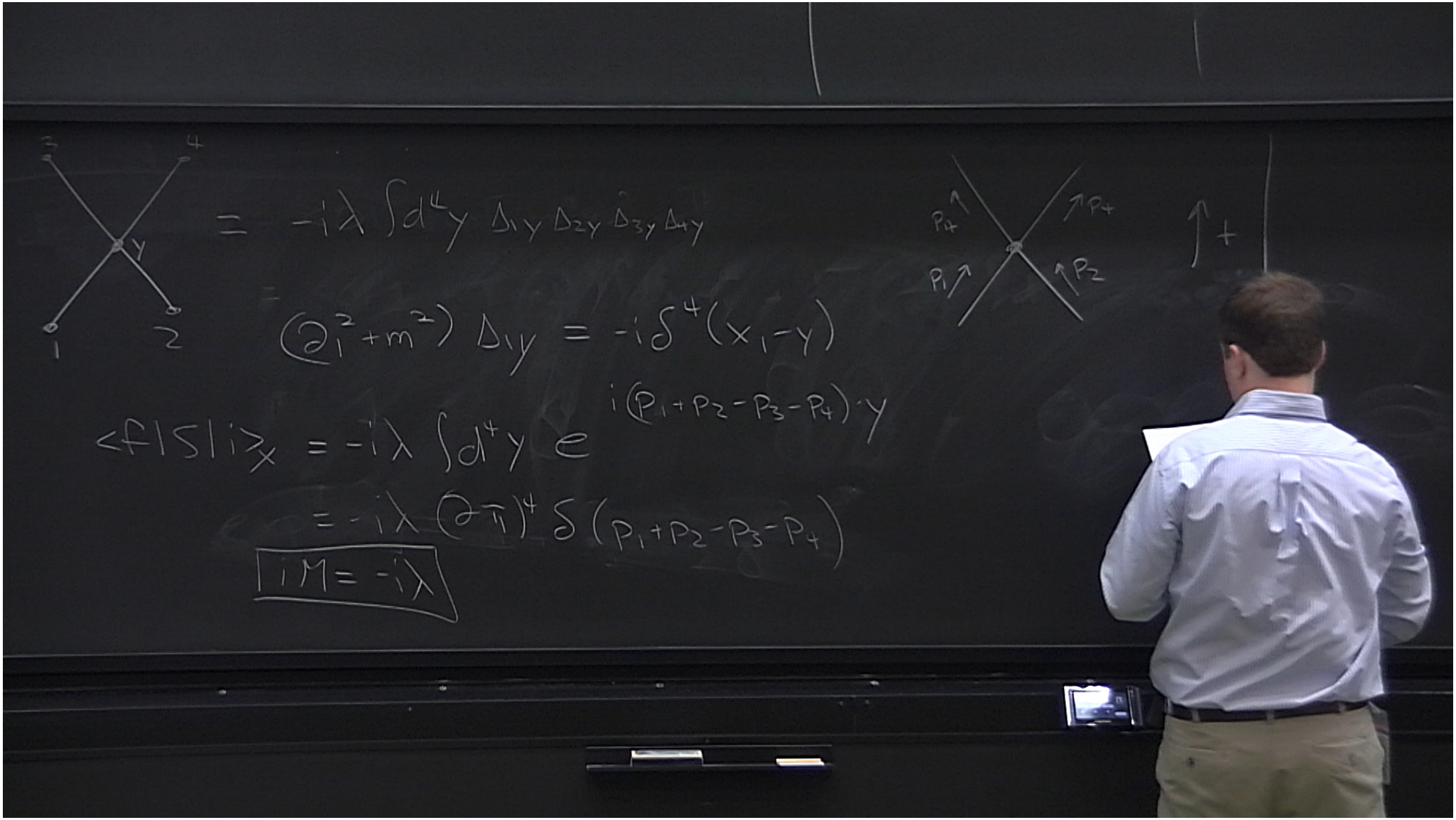
$\int d^4x_1\, d^4x_3\, e$

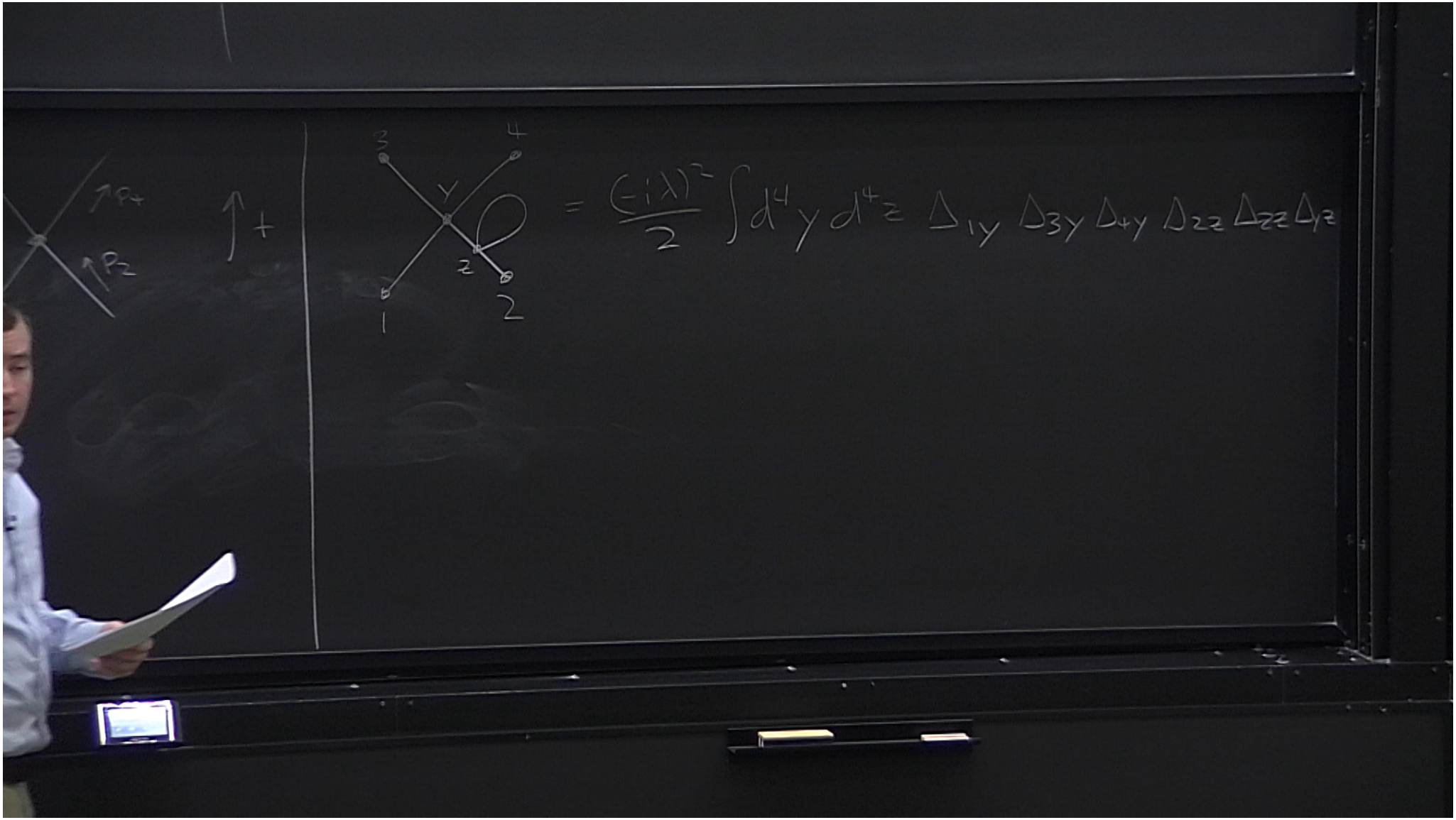
$\langle f | S | i \rangle_1 \propto \delta^{(4)}$

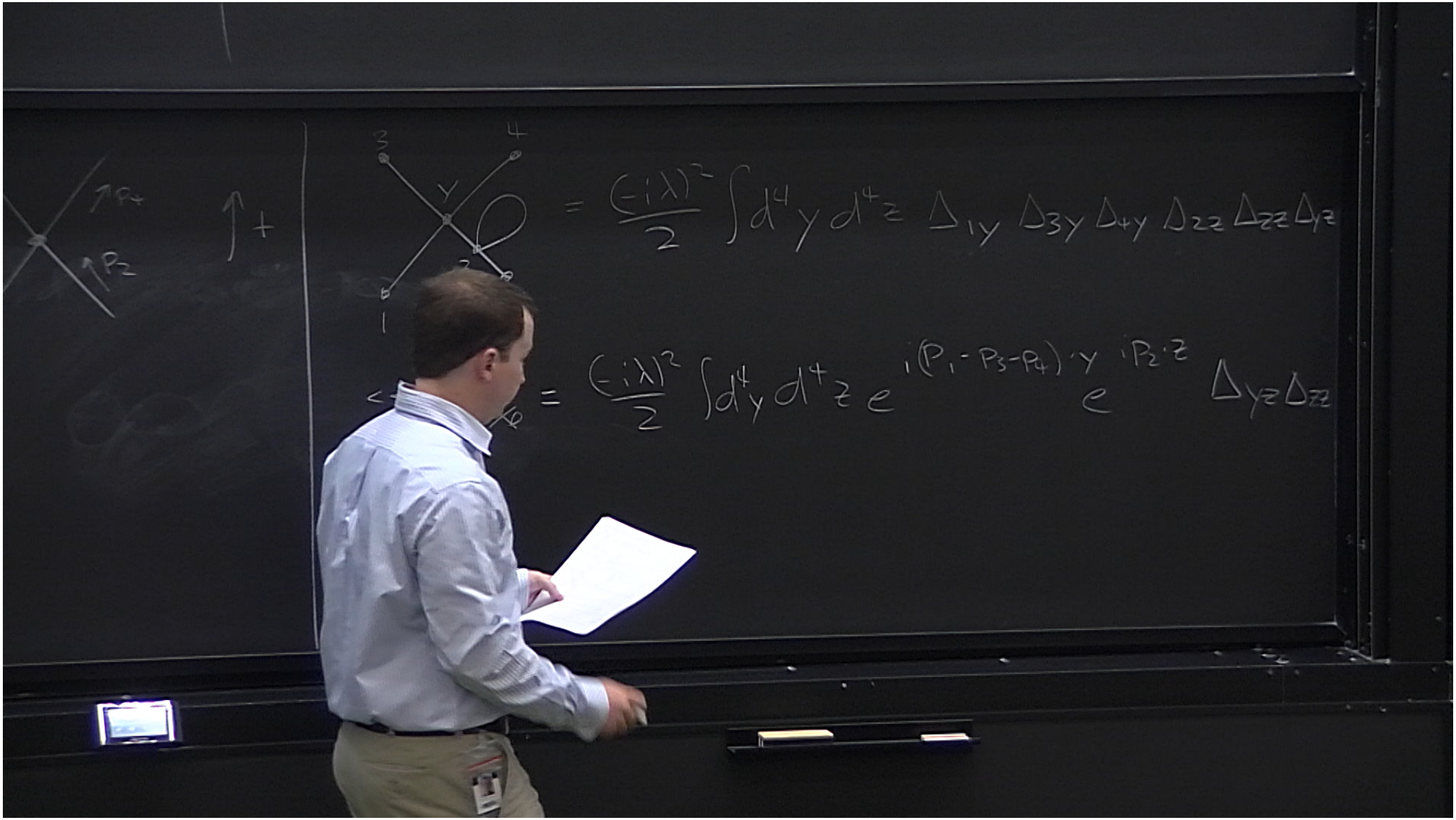
 $= -i\lambda \int d^4y\, \Delta_{1y}\,\Delta_{2y}\,\Delta_{3y}\,\Delta_{4y}$

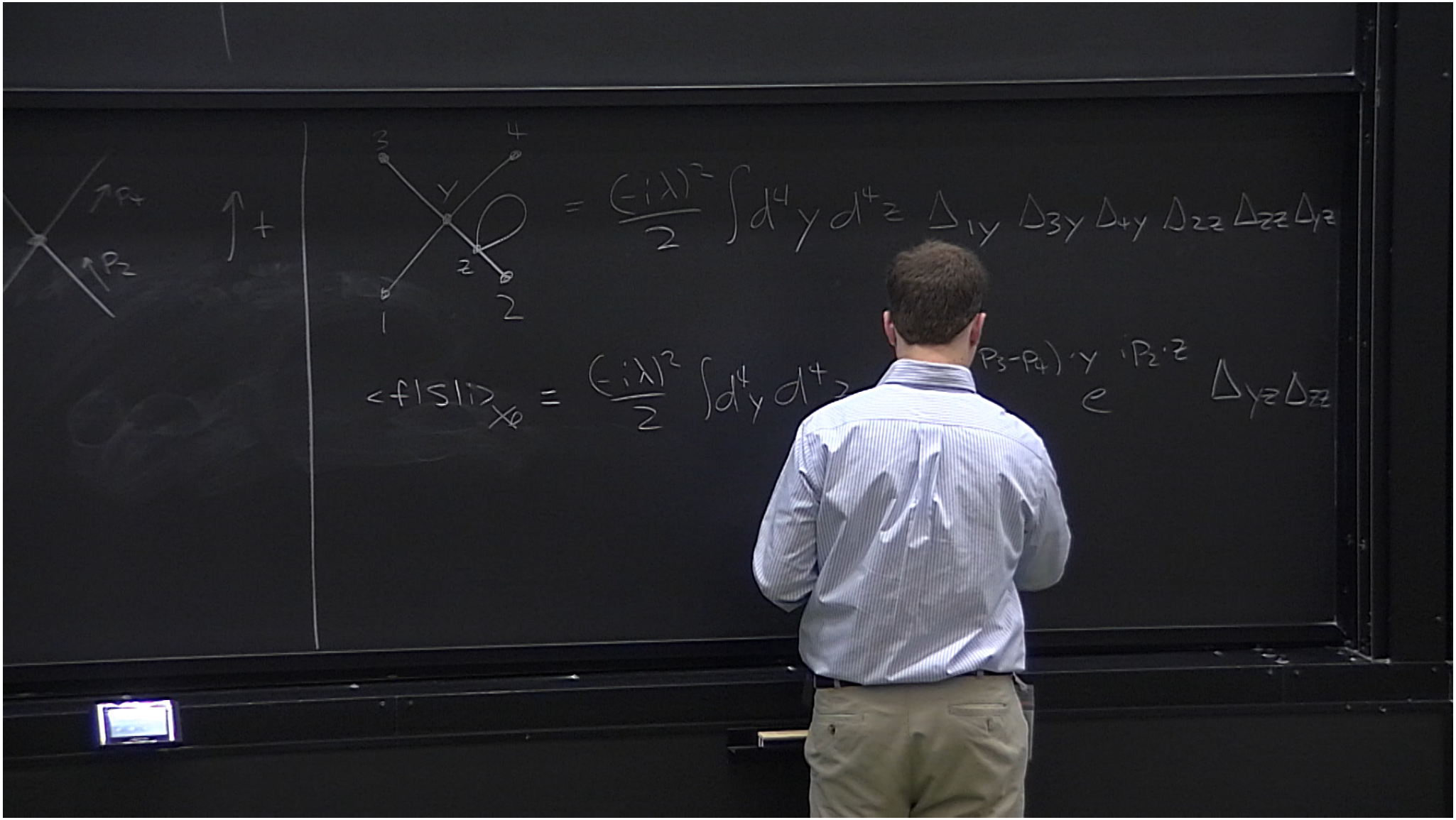

3
4
y
1
2
$= -i\lambda \int d^4y \, \Delta_{1y} \Delta_{2y} \Delta_{3y} \Delta_{4y}$
$(\partial_1^2 + m^2) \Delta_{1y} = -i\delta^4(x_1 - y)$

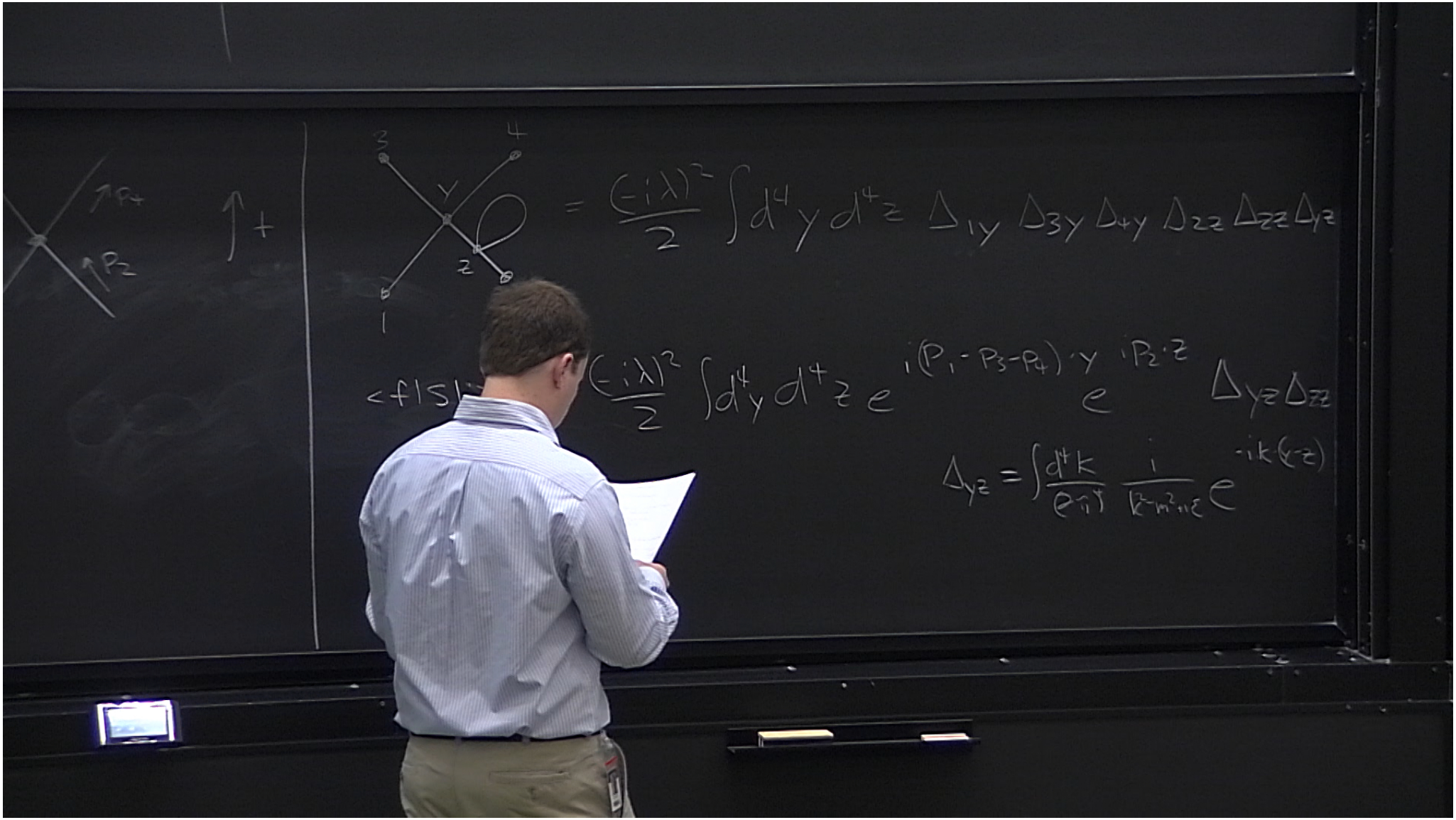

3
4
y
1
2
$= -i\lambda \int d^4y \, \Delta_{1y} \Delta_{2y} \Delta_{3y} \Delta_{4y}$
$(\partial_1^2 + m^2)\, \Delta_{1y} = -i\delta^4(x_1 - y)$
$\langle f|S|i\rangle_X = -i\lambda \int d^4y \, e^{i(p_1+p_2-p_3-p_4)\cdot y}$
$= -i\lambda (\partial_T)^4 \delta(p_1+p_2-p_3-p_4$
$iM = -i\lambda$
p_4
p_3
p_1
p_2

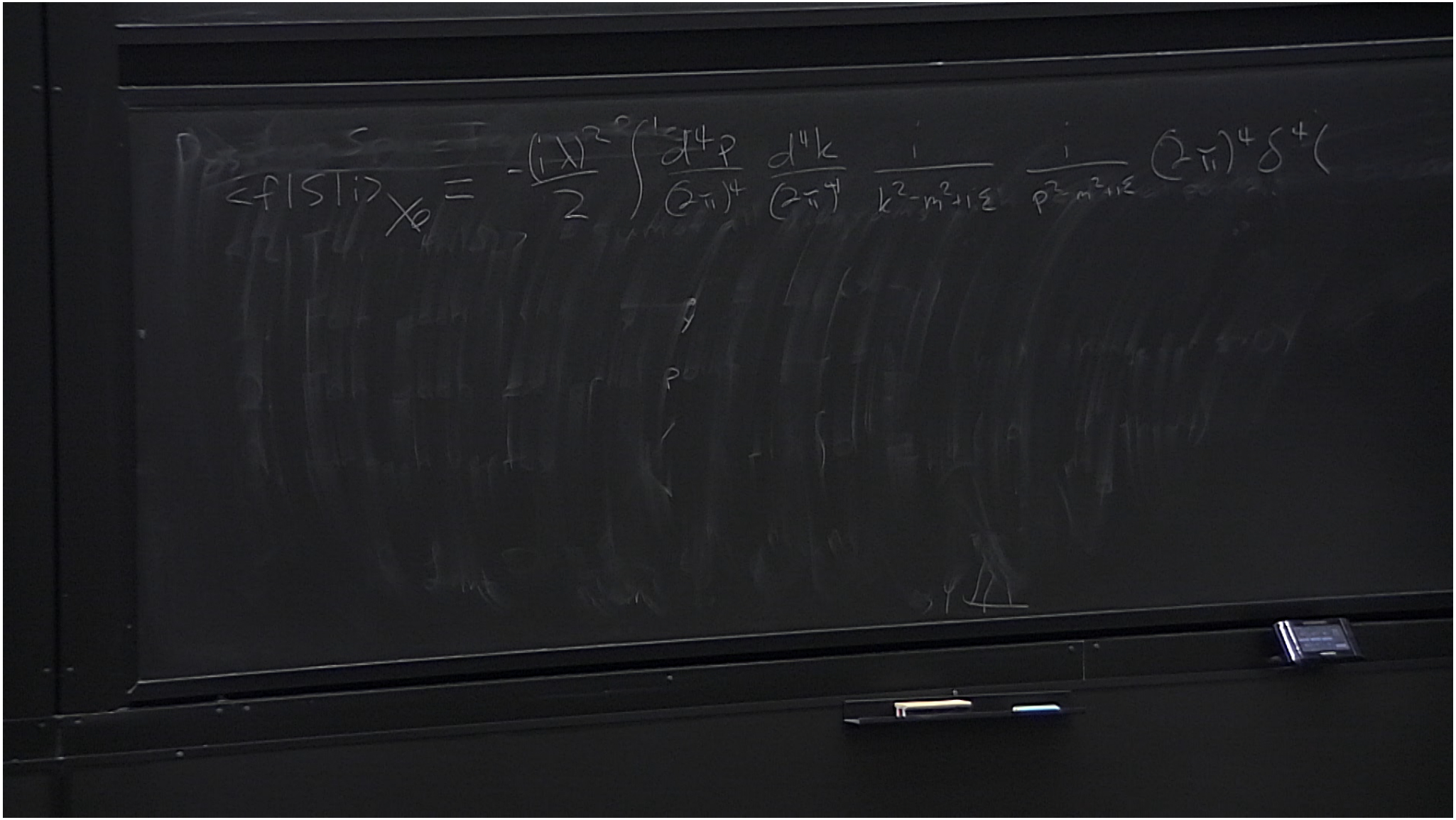

3
4
y
1
2
$= -i\lambda \int d^4y\, \Delta_{1y}\Delta_{2y}\Delta_{3y}\Delta_{4y}$
$(\partial_1^2 + m^2)\Delta_{1y} = -i\delta^4(x_1 - y)$
$\langle f|S|i\rangle = -i\lambda \int d^4y\, e^{i(p_1+p_2-p_3-p_4)\cdot y}$
$= -i\lambda (2\pi)^4 \delta(p_1+p_2-p_3-p_4)$
$iM = -i\lambda$
p_4
p_3
p_1
p_2
t

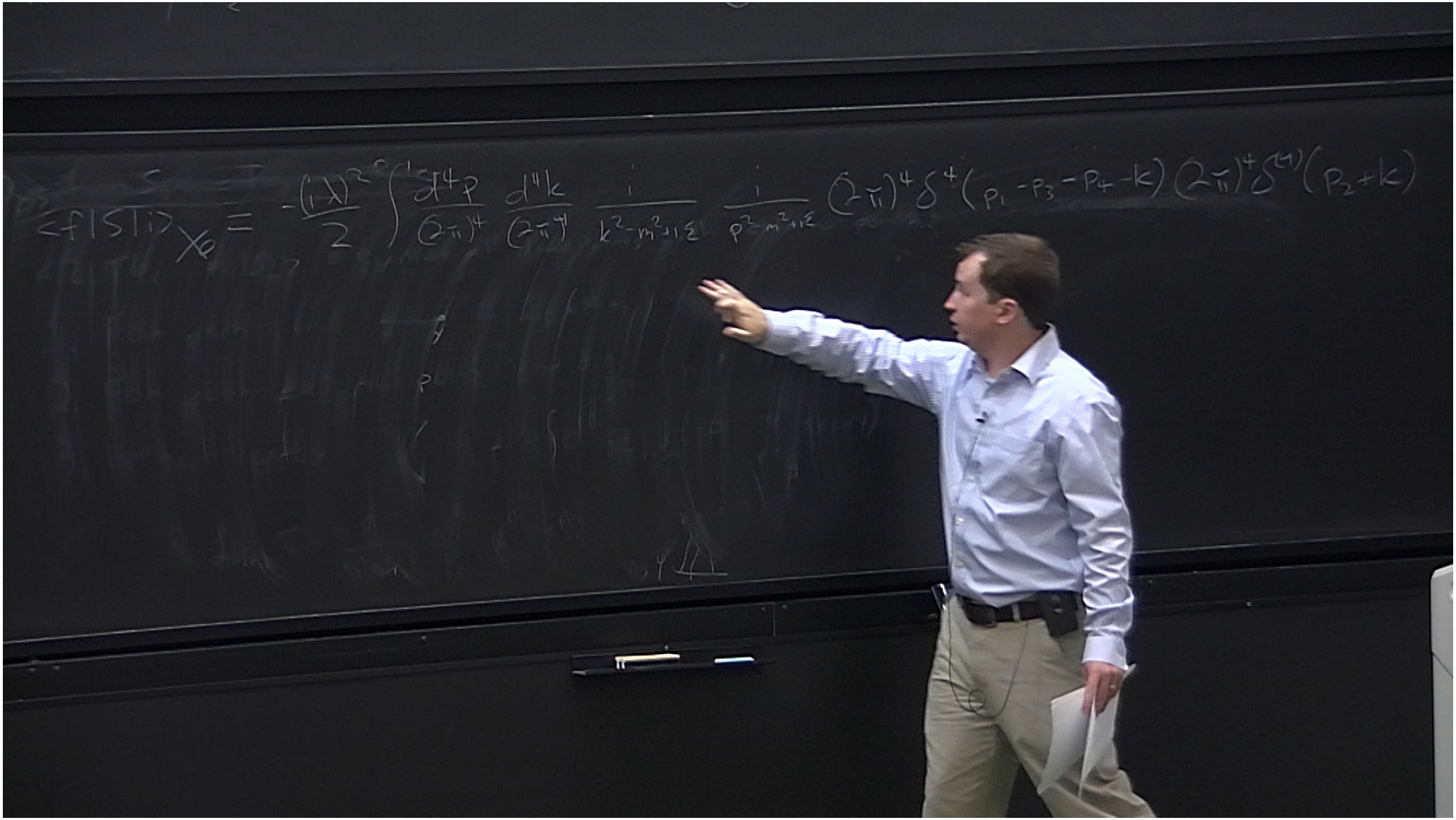

p_1
p_2
$+$
3
4
y
z
1
2
$= \frac{(-i\lambda)^2}{2} \int d^4y\, d^4z\, \Delta_{1y}\, \Delta_{3y}\, \Delta_{4y}\, \Delta_{zz}\, \Delta_{zz}\, \Delta_{y2}$

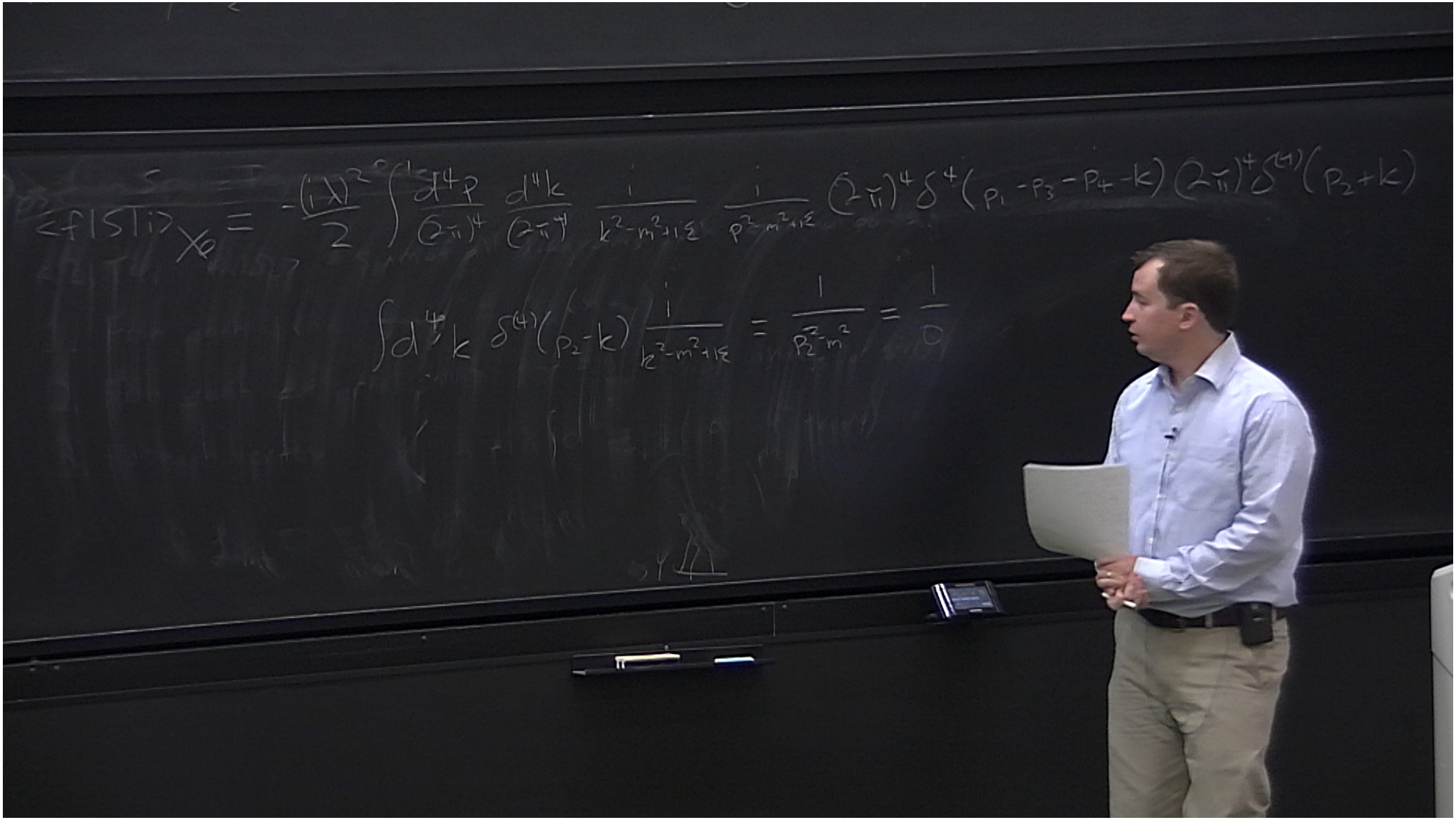

p_1
p_2
$+$
3
4
y
1
2
$= \frac{(-i\lambda)^2}{2} \int d^4y\, d^4z\, \Delta_{1y}\, \Delta_{3y}\, \Delta_{4y}\, \Delta_{zz}\, \Delta_{zz}\, \Delta_{yz}$
$= \frac{(-i\lambda)^2}{2} \int d^4y\, d^4z\, e^{i(p_1 - p_3 - p_4)\cdot y}\, e^{i p_2 \cdot z}\, \Delta_{yz}\, \Delta_{zz}$

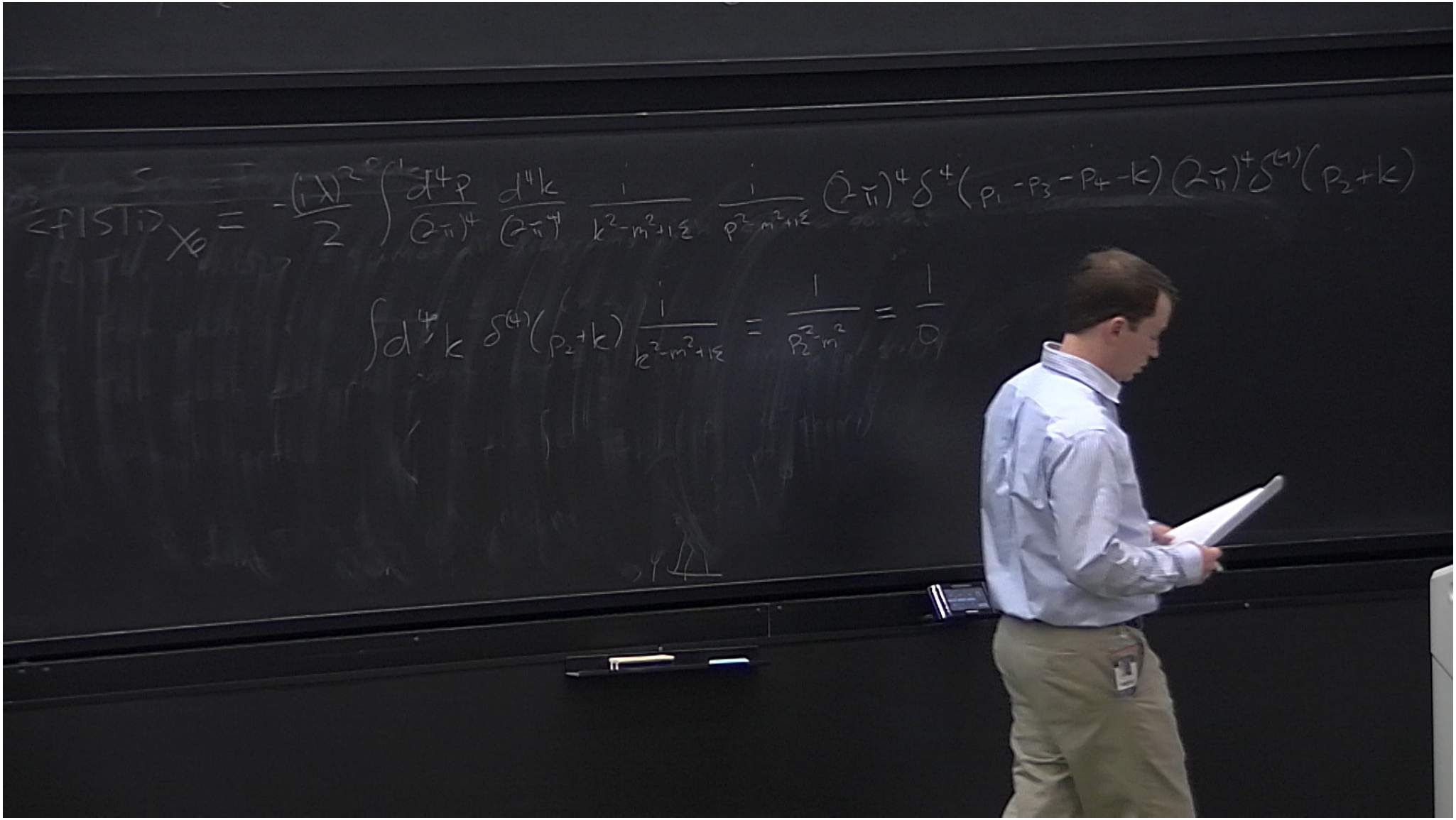

p_4
p_2
$+$
3
4
y
z
1
2
$= \frac{(-i\lambda)^2}{2} \int d^4y\, d^4z\, \Delta_{1y} \Delta_{3y} \Delta_{4y} \Delta_{zz} \Delta_{zz} \Delta_{yz}$
$\langle f|S|i\rangle_{\chi^0} = \frac{(-i\lambda)^2}{2} \int d^4y\, d^4z$
$(p_3-p_4)\cdot y$
$ip_2\cdot z$
e
$\Delta_{yz} \Delta_{zz}$

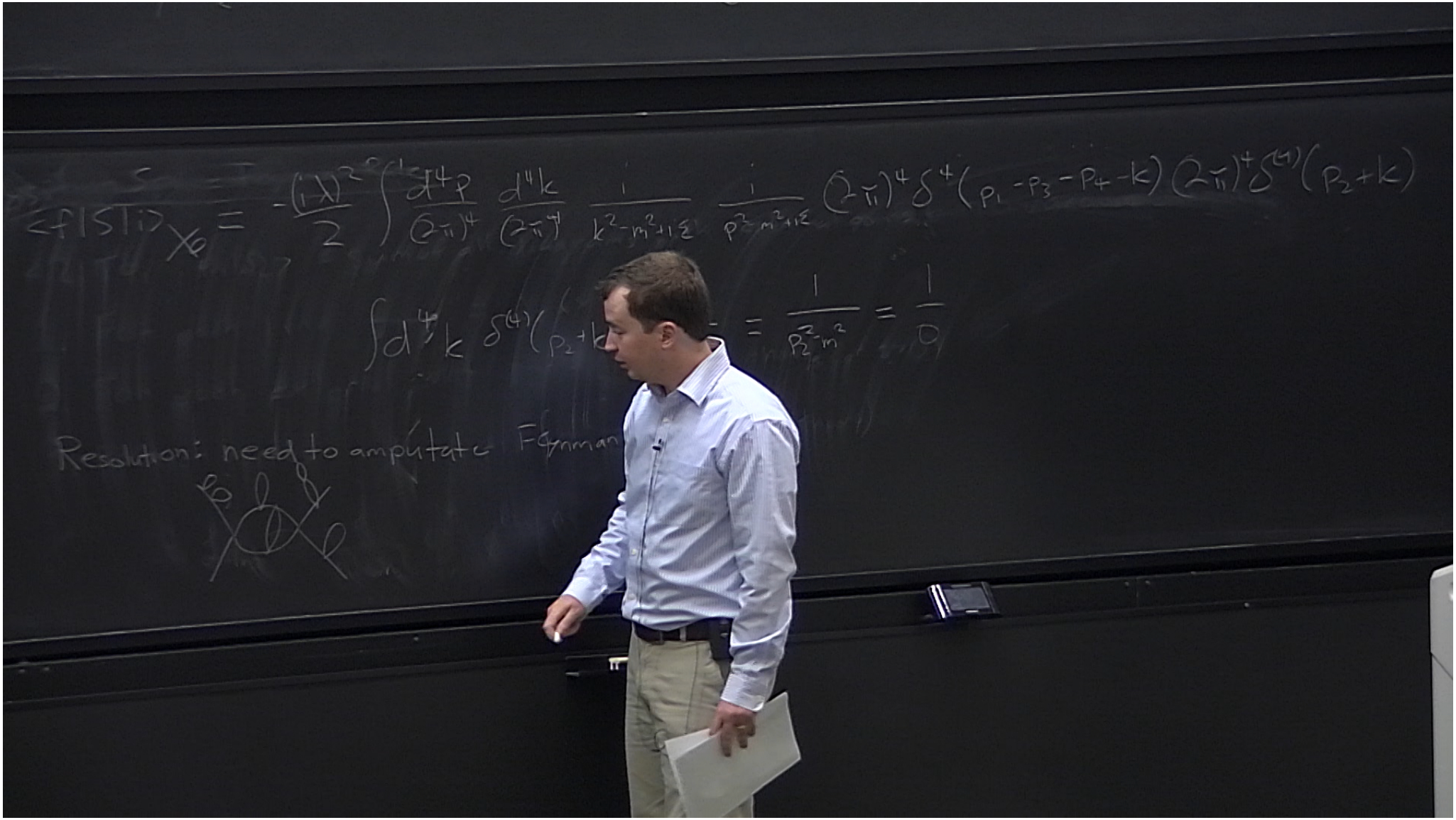

p_4
p_2
$+$
3
4
y
z
1
$= \frac{(-i\lambda)^2}{2} \int d^4y\, d^4z\, \Delta_{1y}\, \Delta_{3y}\, \Delta_{4y}\, \Delta_{zz}\, \Delta_{zz}\, \Delta_{yz}$
$\frac{(-i\lambda)^2}{2} \int d^4y\, d^4z\, e^{i(p_1 - p_3 - p_4)\cdot y}\, e^{i p_2 \cdot z}\, \Delta_{yz}\, \Delta_{zz}$
$\Delta_{yz} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\varepsilon} e^{-ik\cdot(y-z)}$


$\langle f|S|i\rangle_{X_0} = -\frac{(i\lambda)^2}{2}\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\varepsilon}\frac{i}{p^2-m^2+i\varepsilon}(2\pi)^4\delta^4($
p

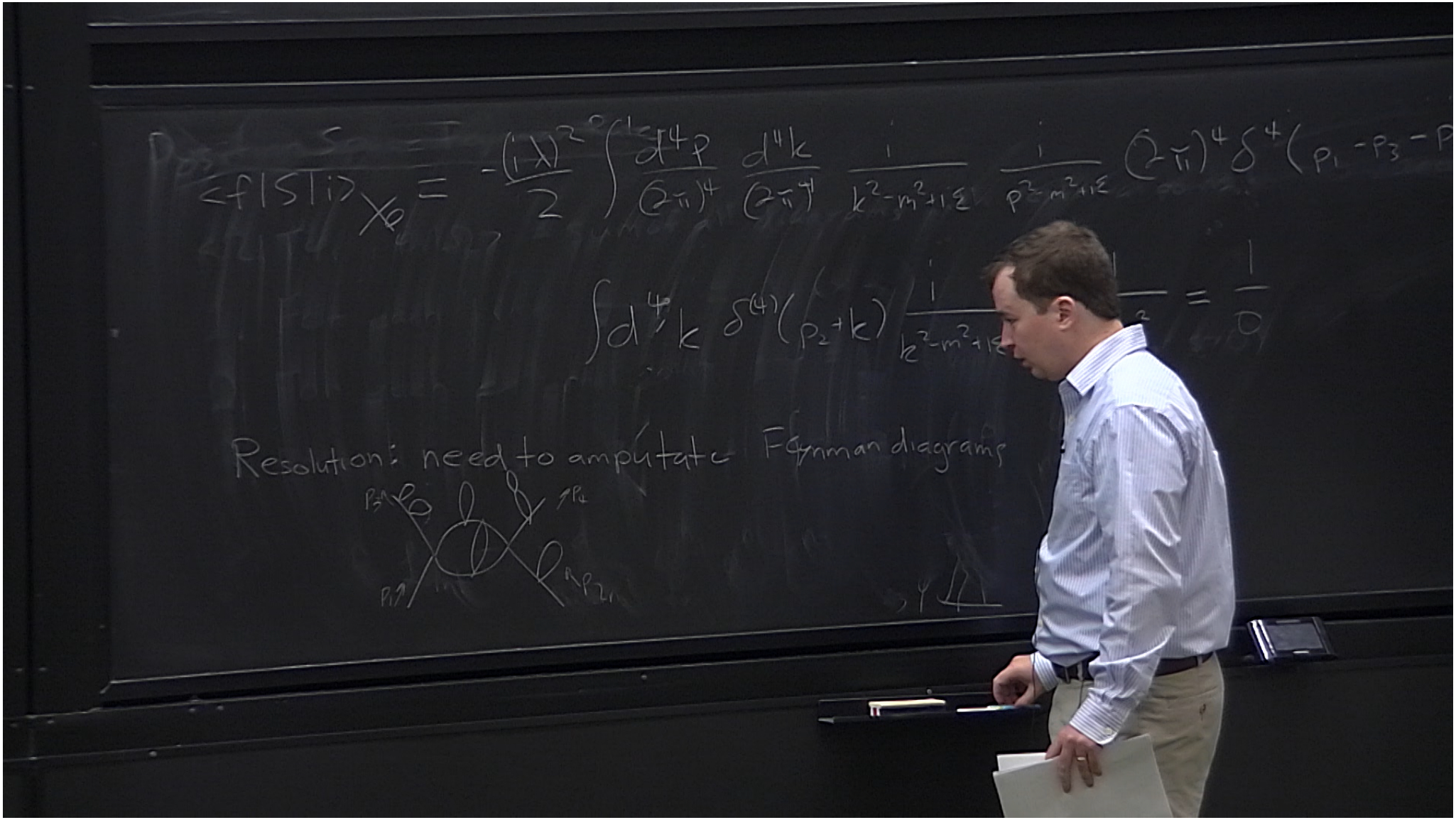

$\langle f|S|i\rangle_{X_0} = -\frac{(i\lambda)^2}{2}\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\varepsilon}\frac{i}{p^2-m^2+i\varepsilon}(2\pi)^4\delta^4(p_1-p_3-p_4-k)(2\pi)^4\delta^4(p_2+k)$
p

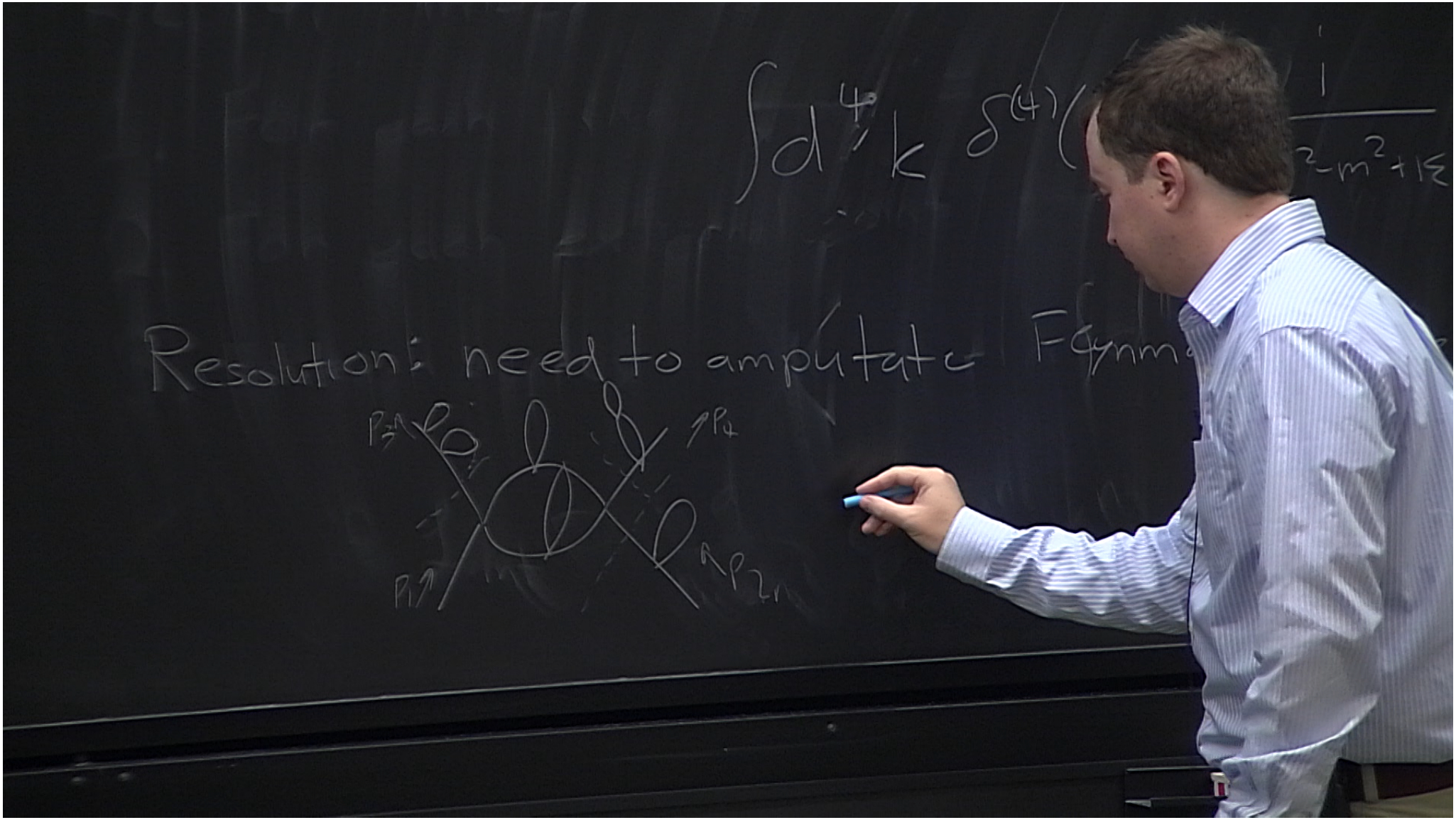

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$\int d^4k\,\delta^{(4)}(p_2-k)\frac{i}{k^2-m^2+i\varepsilon} = \frac{1}{p_2^2-m^2} = \frac{1}{0}$


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Resolution: need to amputate Feynman

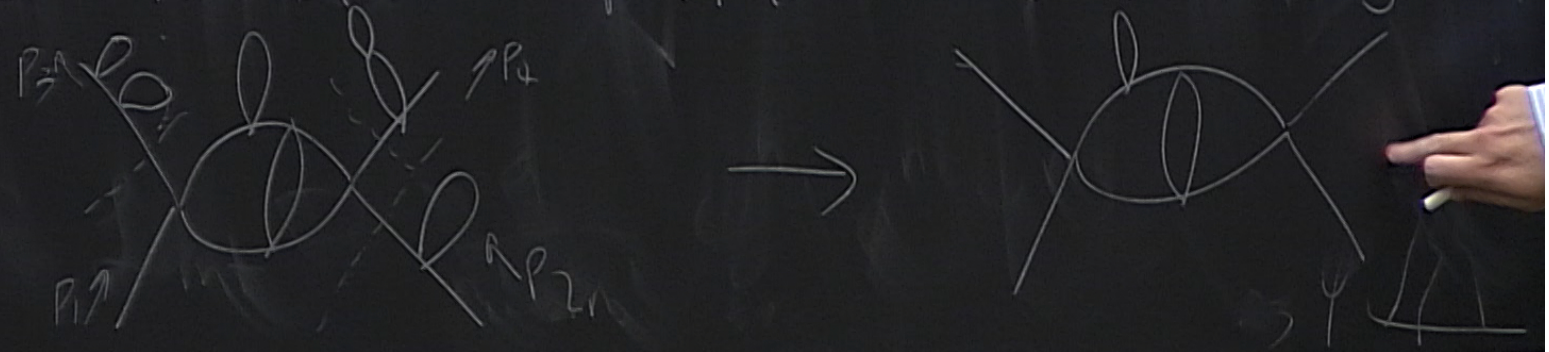

Resolution: need to

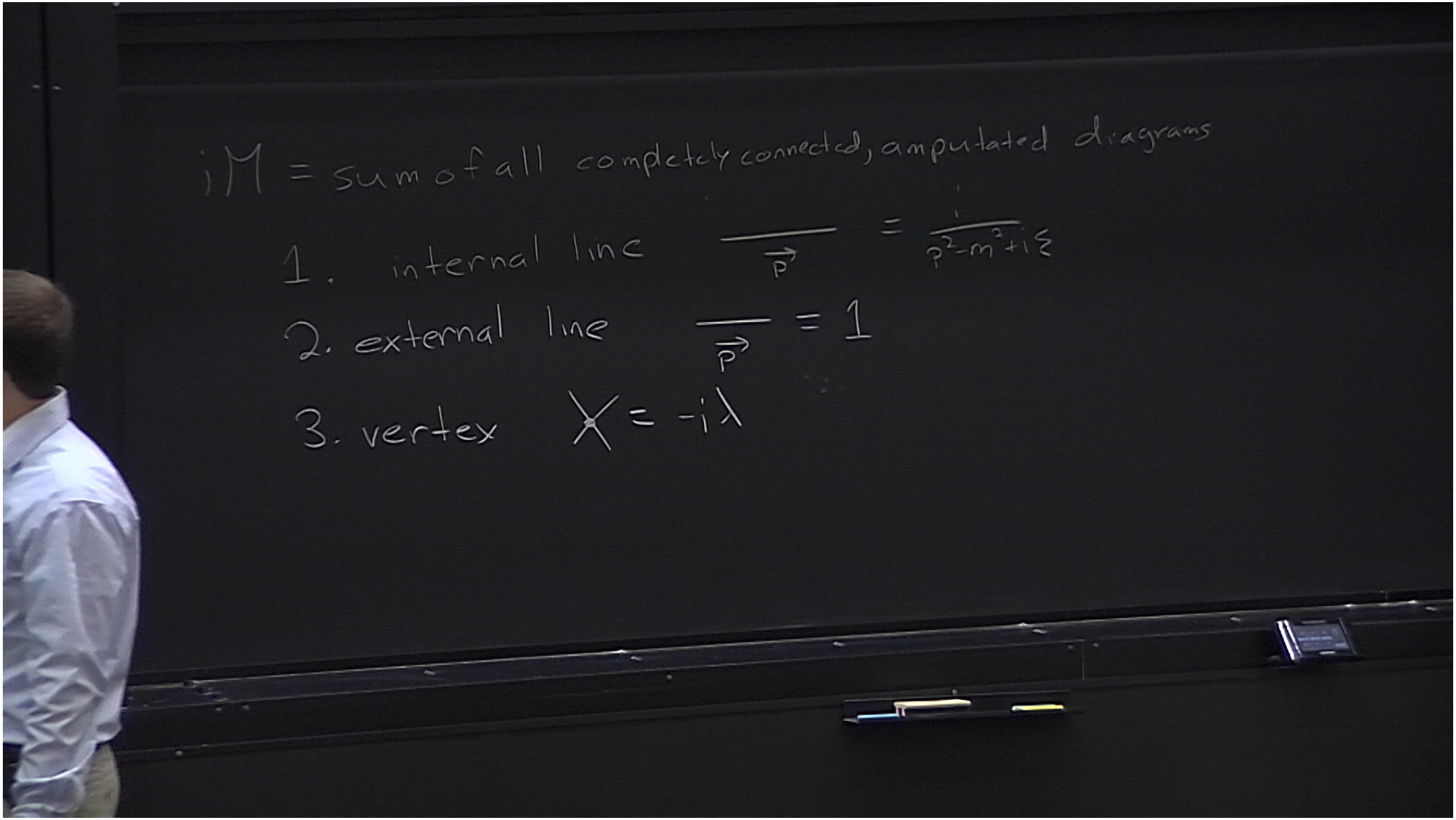

$\langle f|S|i\rangle_{X_0} = -\frac{(i\lambda)^2}{2}\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\varepsilon}\frac{i}{p^2-m^2+i\varepsilon}(2\pi)^4\delta^4(p_1-p_3-p$
$\int d^4k\,\delta^{(4)}(p_2+k)\frac{i}{k^2-m^2+i\varepsilon}$
$= \frac{1}{0}$
Resolution: need to amputate Feynman diagrams
p_3
p_4
p_1
p_2


Resolution: need to amputate Feynm

$$\int d^4k\, \delta^{(4)}(p_2+k)\, \frac{i}{k^2-m^2+i\varepsilon}$$

Resolution: need to amputate Feynman diagrams




iM = sum of all completely connected, amputated diagrams
1. internal line $\overrightarrow{p}$ $= \frac{i}{p^2 - m^2 + i\varepsilon}$
2. external line $\overrightarrow{p}$ $= 1$
3. vertex $= -i\lambda$

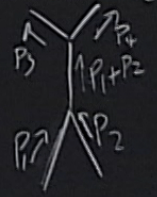
iM = sum of all completely connected, amputated diagrams

1. internal line $\overrightarrow{p}$ $= \frac{i}{p^2 - m^2 + i\epsilon}$

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3. vertex $= -i\lambda$ or similarly for other theories

4. Impose momentum conservation at each vertex



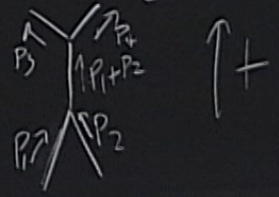
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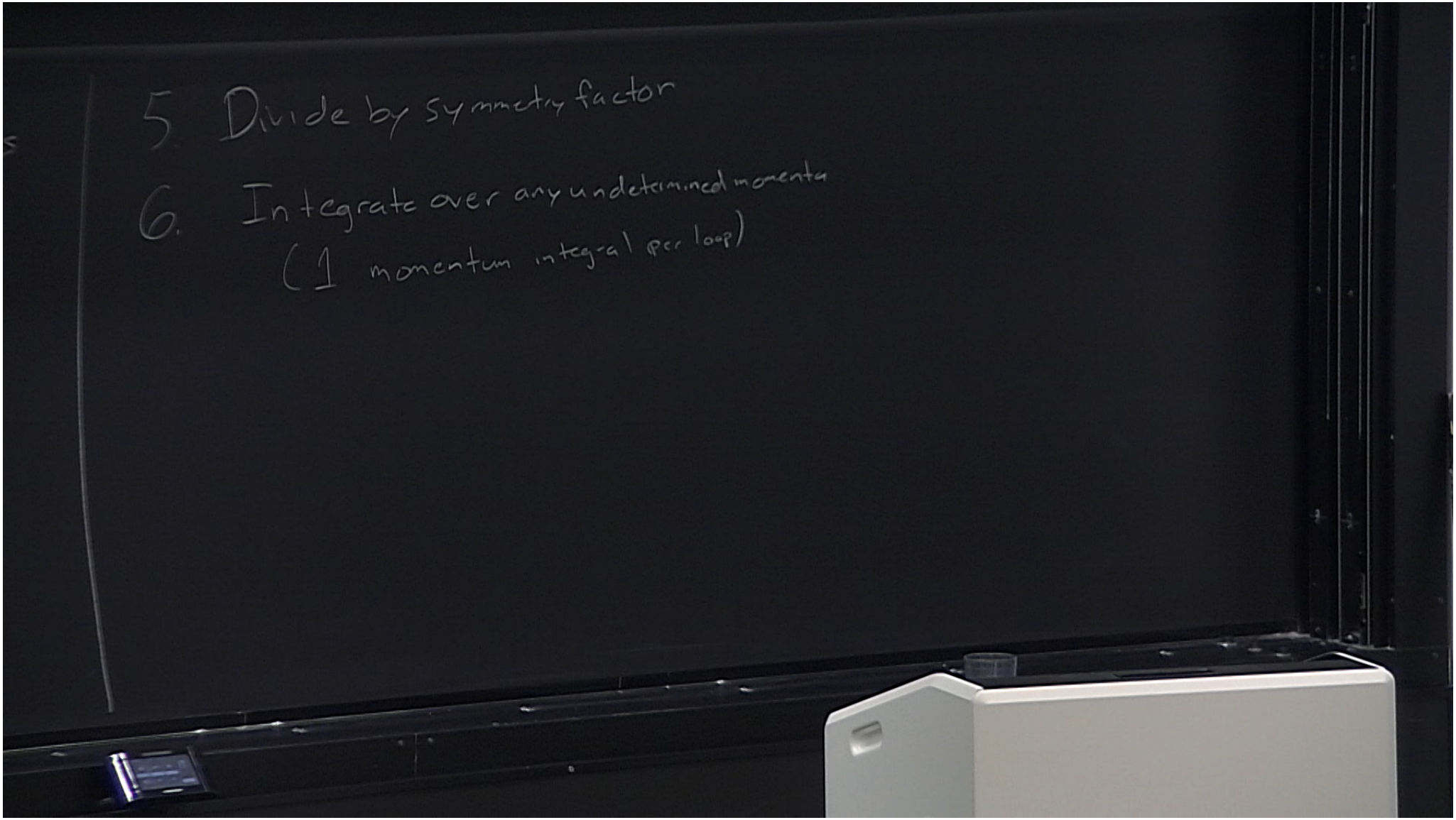
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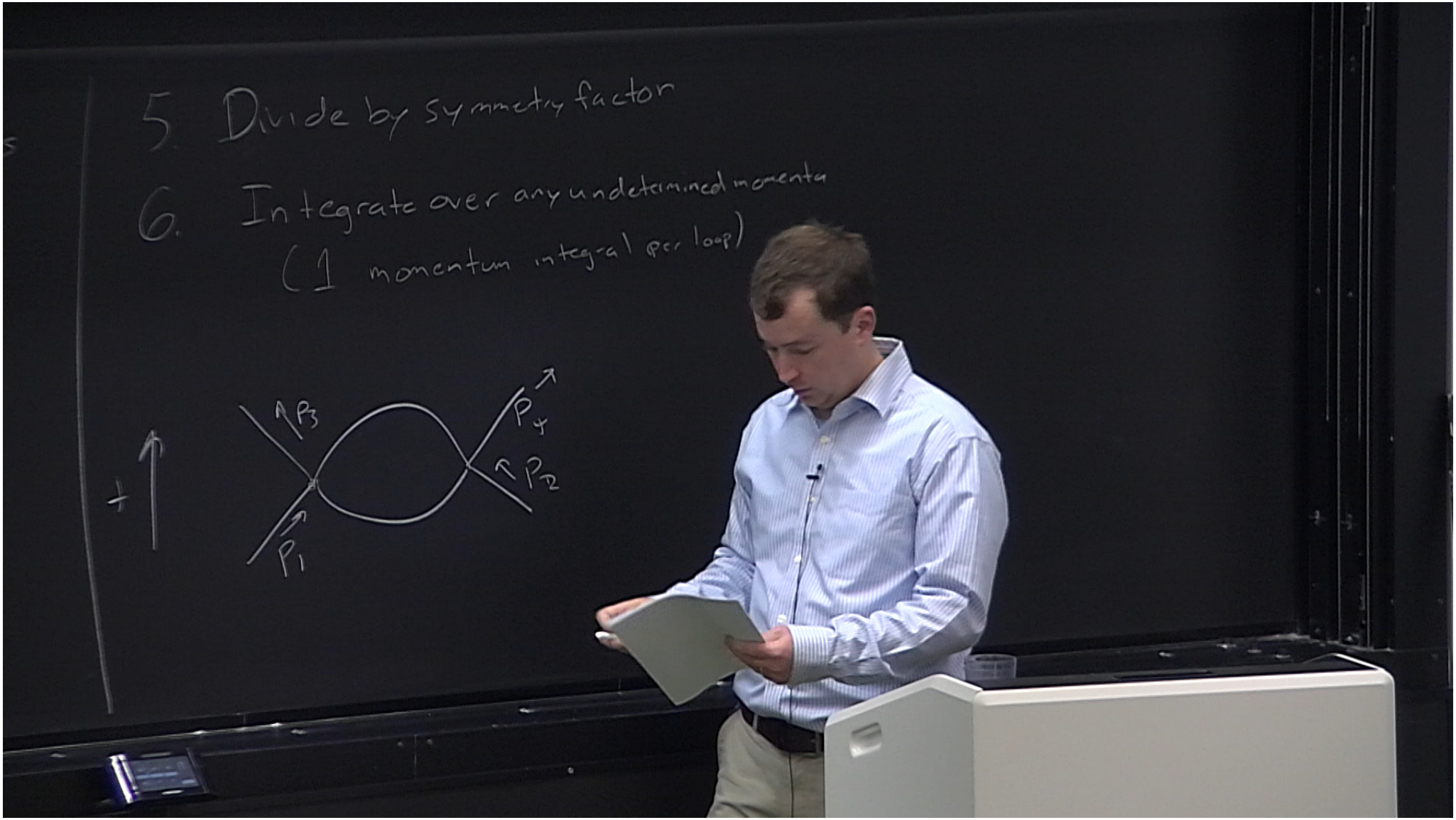
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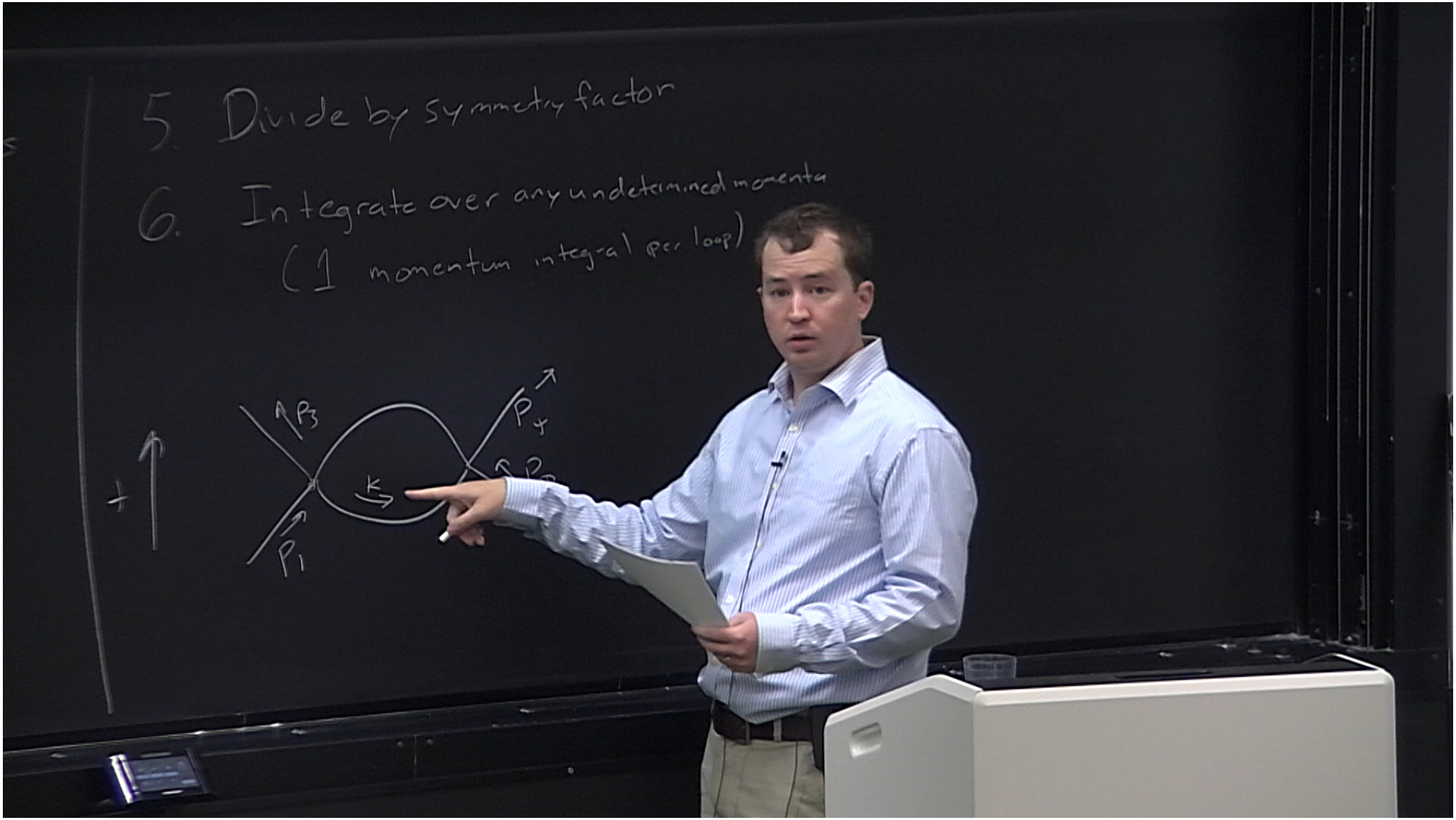
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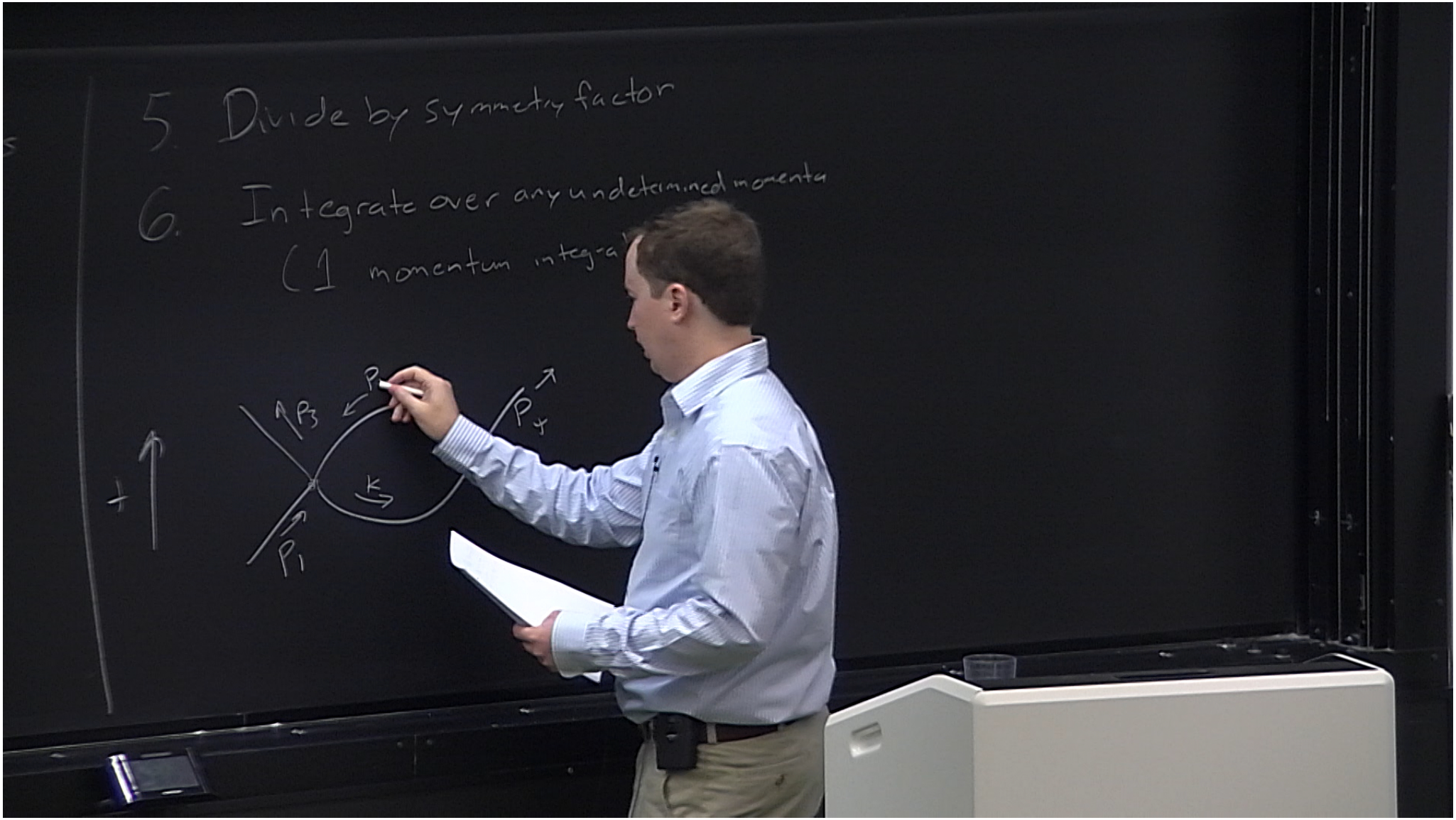
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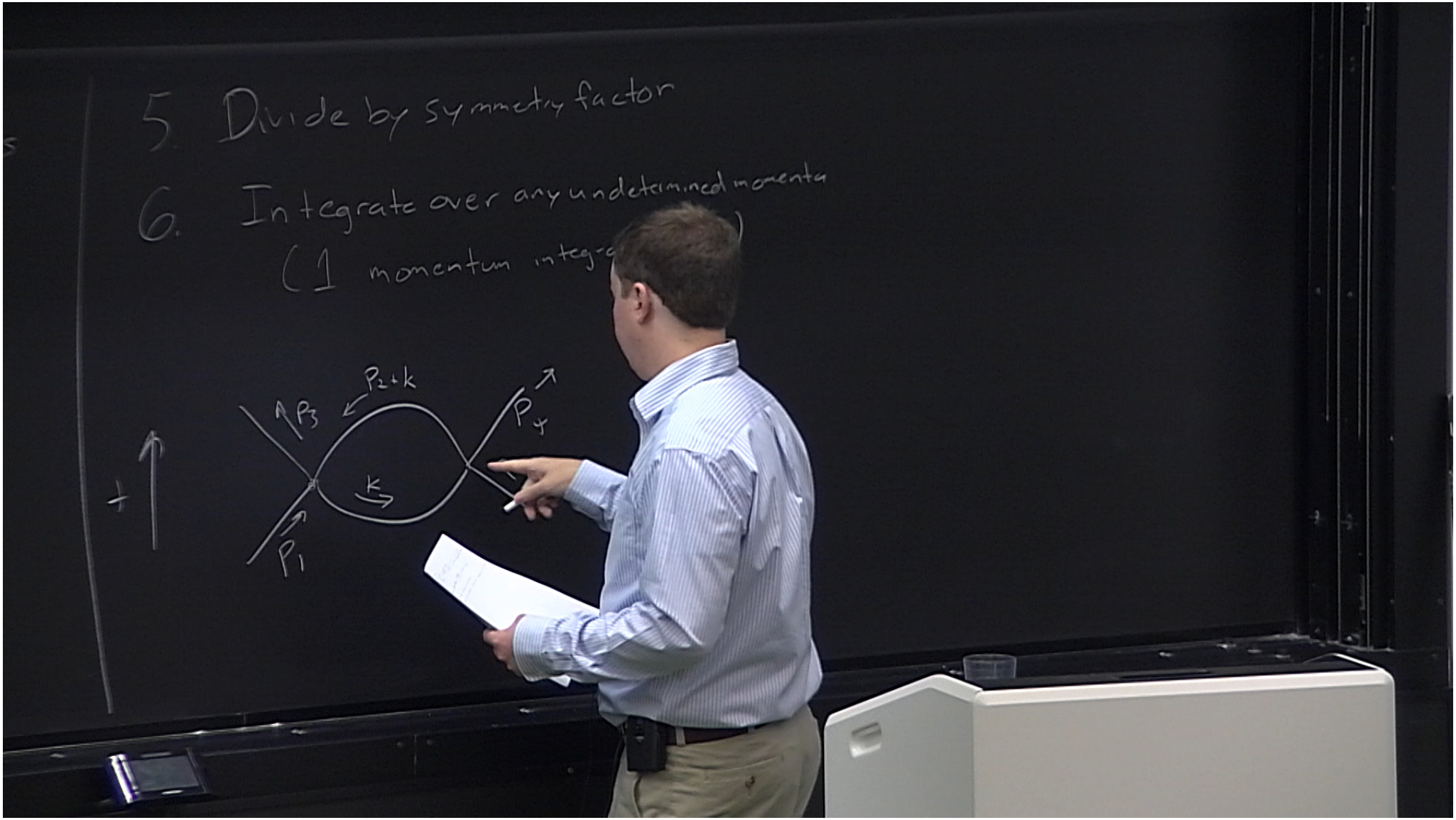


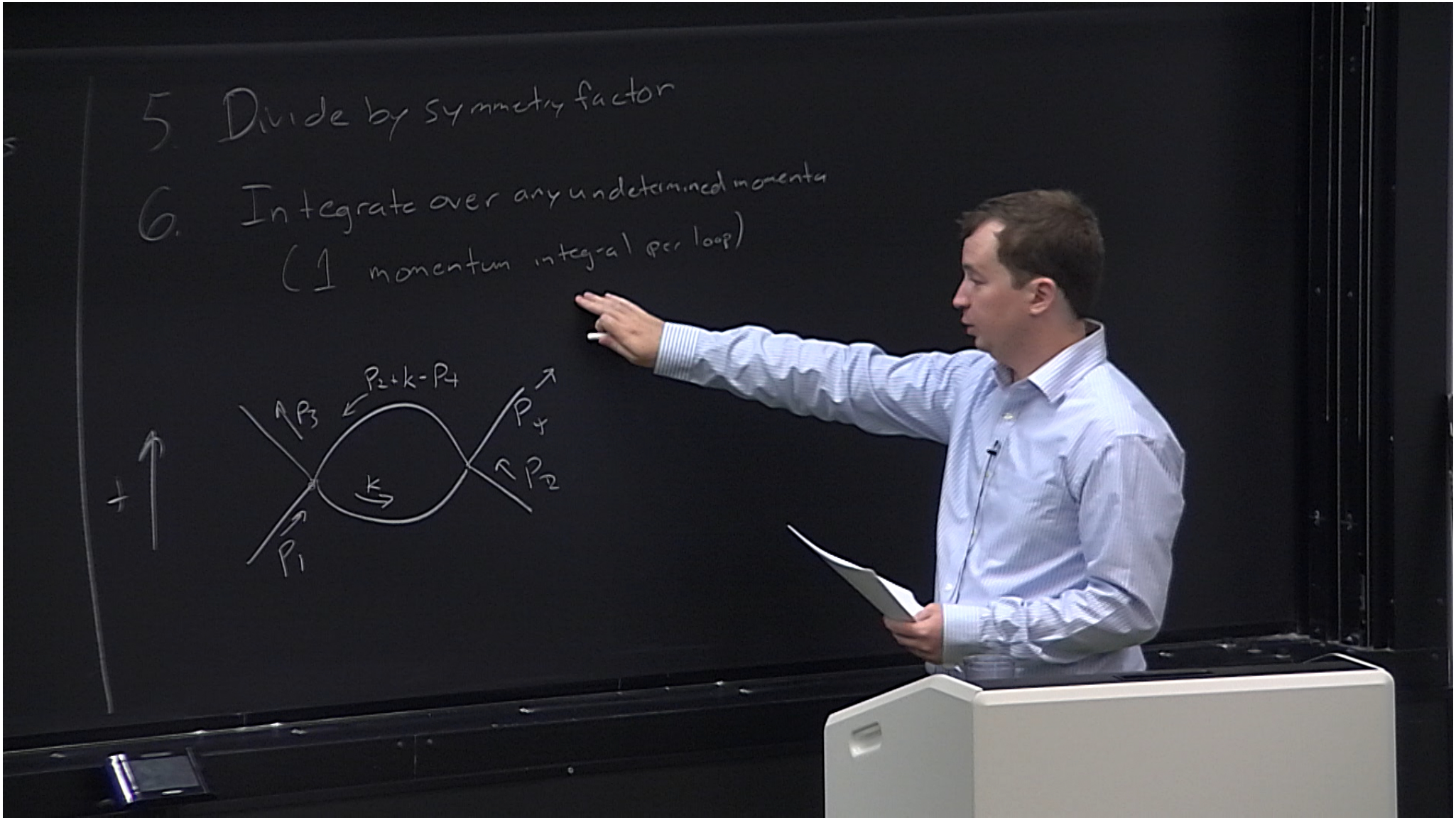

5. Divide by symmetry factor
6. Integrate over any undetermined momenta
(1 momentum integral per loop)


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6. Integrate over any undetermined momenta
(1 momentum integral per loop)
+1
p_3
p_4
p_2
p_1


5. Divide by symmetry factor
6. Integrate over any undetermined momenta
(1 momentum integral per loop)
p_3
p_4
k
p_1

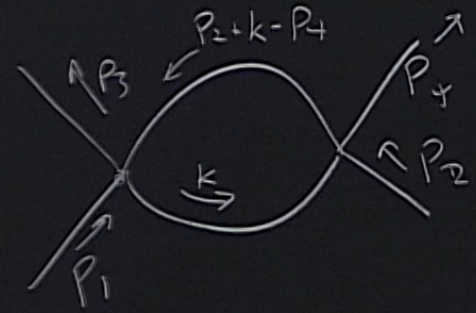

5. Divide by symmetry factor
6. Integrate over any undetermined momenta
(1 momentum integral
p_3
p
p_4
k
p_1


5. Divide by symmetry factor
6. Integrate over any undetermined momenta
(1 momentum integ
p_2+k
p_3
p_4
k
p_1

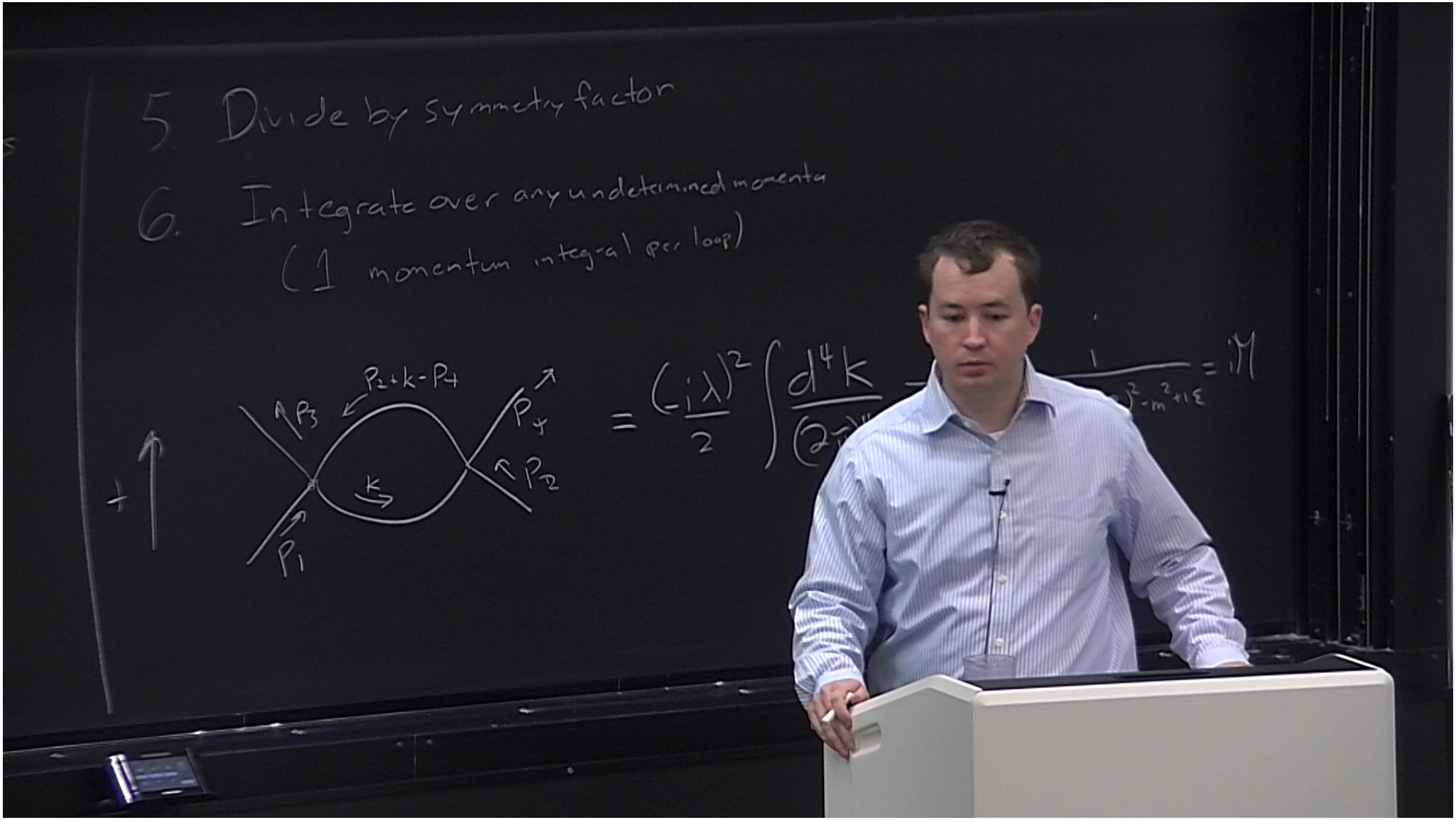

5. Divide by symmetry factor
6. Integrate over any undetermined momenta
(1 momentum integral per loop)
p_2+k-p_4
p_3
p_4
p_2
k
p_1

5. Divide by symmetry factor

6. Integrate over any undetermined momenta
(1 momentum integral per loop)



$$= \frac{(-i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\varepsilon} \frac{i}{(k+p_2-p_4)^2-m^2+i\varepsilon}$$


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6. Integrate over any undetermined momenta
(1 momentum integral per loop)
p_2+k-p_4
p_3
p_4
p_2
p_1
k
$= \frac{(-i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4}$