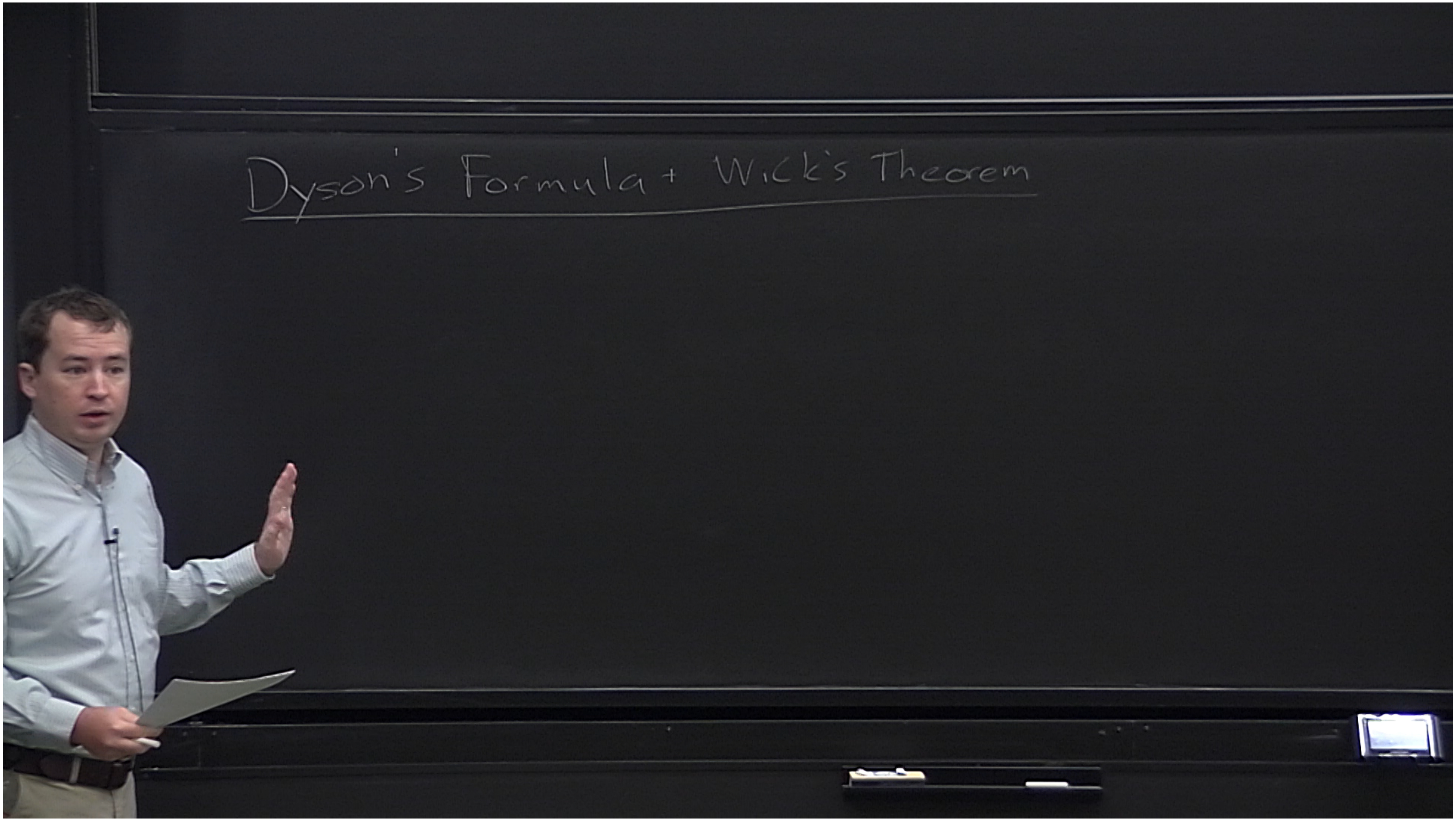
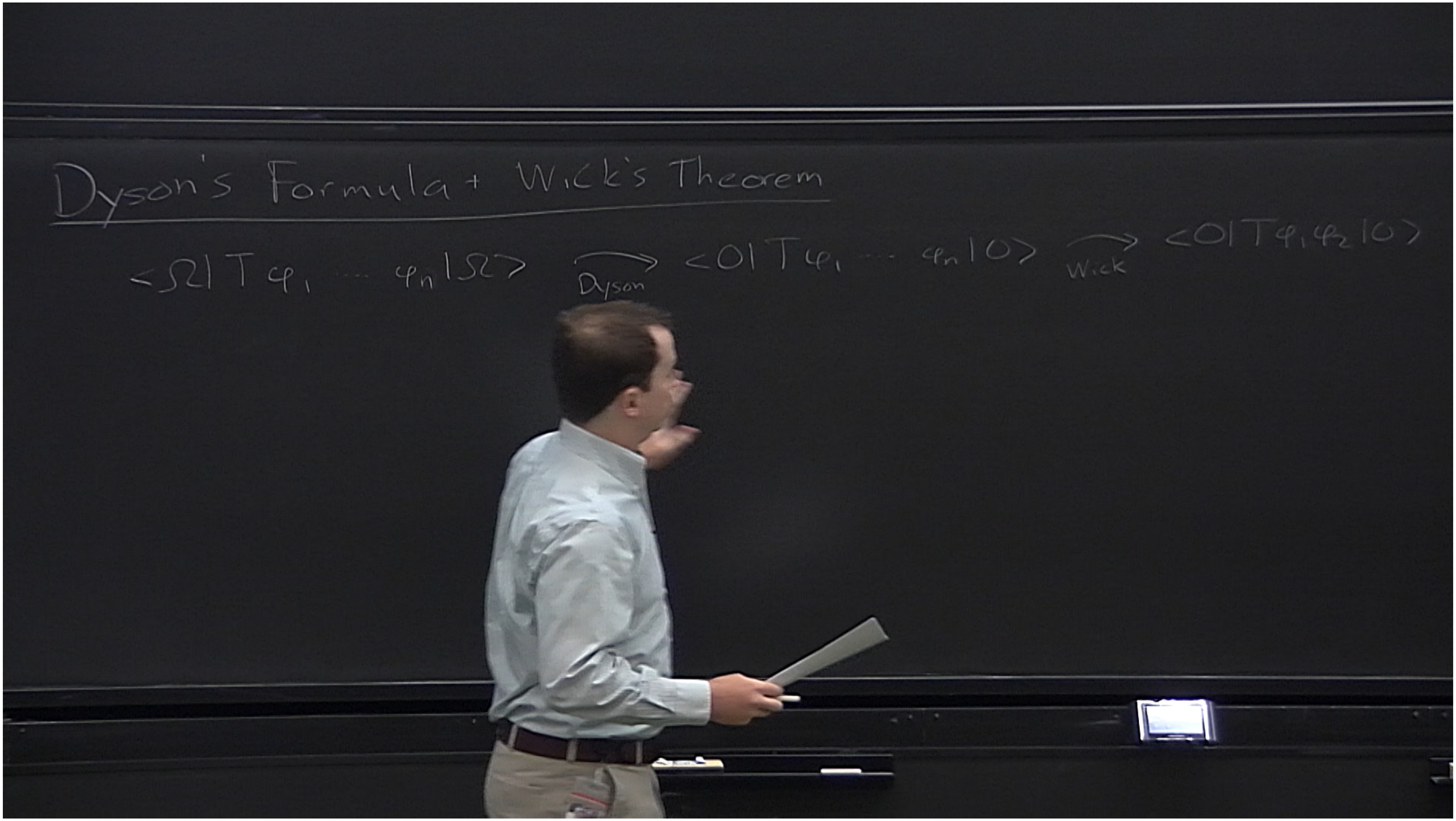
Title: PSI 17/18 - Quantum Field Theory I - Lecture 5

Date: Oct 16, 2017 09:00 AM

URL: http://pirsa.org/17100024

Abstract:


Dyson's Formula + Wick's Theorem


Dyson's Formula + Wick's Theorem
$\langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle \xrightarrow[\text{Dyson}]{} \langle 0 | T \varphi_1 \cdots \varphi_n | 0 \rangle \xrightarrow[\text{Wick}]{} \langle 0 | T \varphi_1 \varphi_2 | 0 \rangle$

Dyson's Formula + Wick's Theorem

$$\langle\Omega| T\varphi_1 \cdots \varphi_n |\Omega\rangle \xrightarrow[\text{Dyson}]{} \langle 0| T\varphi_1 \cdots \varphi_n |0\rangle \xrightarrow[\text{Wick}]{} \langle 0| T\varphi_1\varphi_2 |0\rangle$$

Dyson Series + Interaction Picture

$$H = H_0 + H_1 \quad \leftarrow \text{perturbation}$$

$$\text{e.g. } H_1 = \int d^3x \, \frac{\lambda}{4!}\varphi^4$$

$$\varphi_0(\vec{x},t) = e^{iH_0(t-t_0)}\,\varphi(\vec{x})\,e^{-iH_0(t-t_0)}$$

$$\ldots \varphi_n |0\rangle \xrightarrow[\text{Wick}]{} \langle 0 | T \varphi_1 \varphi_2 | 0 \rangle$$

$$i \partial_t \varphi(x) = [\varphi(x), H(t)]$$

$$\ldots \varphi_n|0\rangle \xrightarrow[\text{Wick}]{} \langle 0|T\varphi_1\varphi_2|0\rangle$$

$$i\partial_t \varphi(x) = [\varphi(x), H(t)]$$

$$\varphi(\vec{x},t) = S^+(t,t_0)^+ \underbrace{\varphi(\vec{x},t_0)}_{\text{Schroedinger}} \overbrace{S(t,t_0)}^{\text{time evolution operator}}$$

$$\ldots\varphi_n|0\rangle \xrightarrow{\text{Wick}} \langle 0|T\varphi_1\varphi_2|0\rangle$$

$$i\partial_t\varphi(x) = [\varphi(x), H(t)]$$

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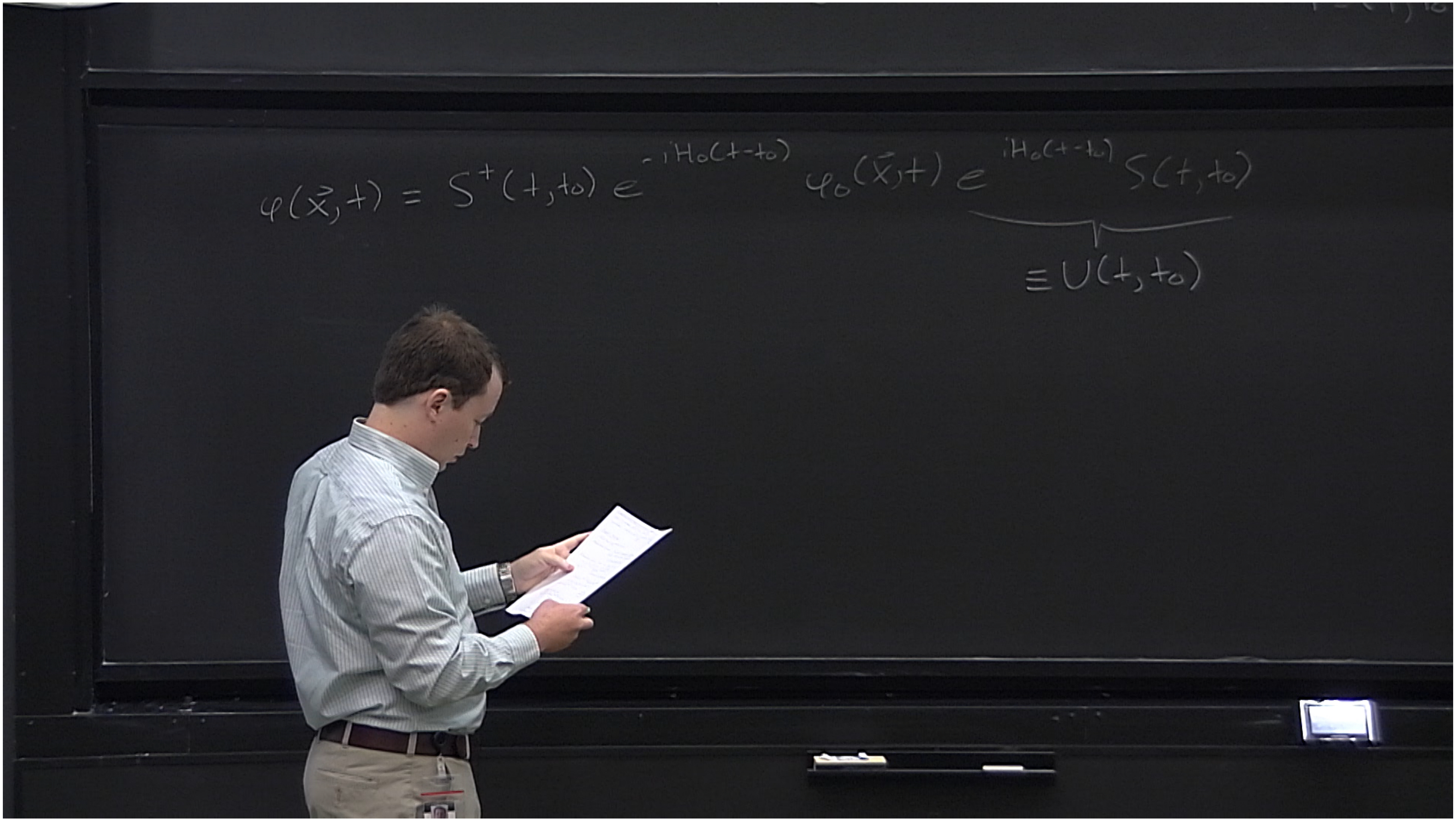
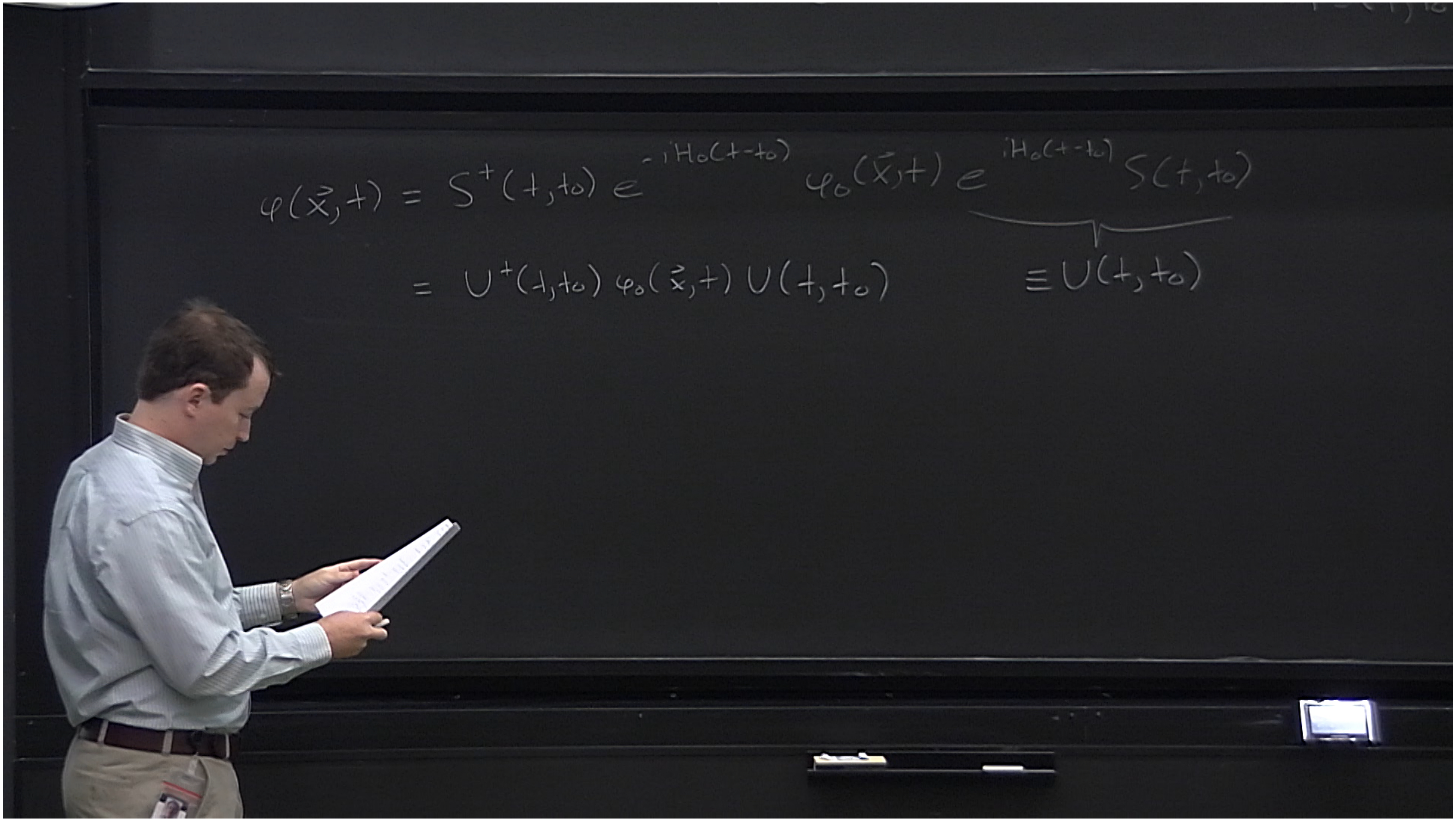
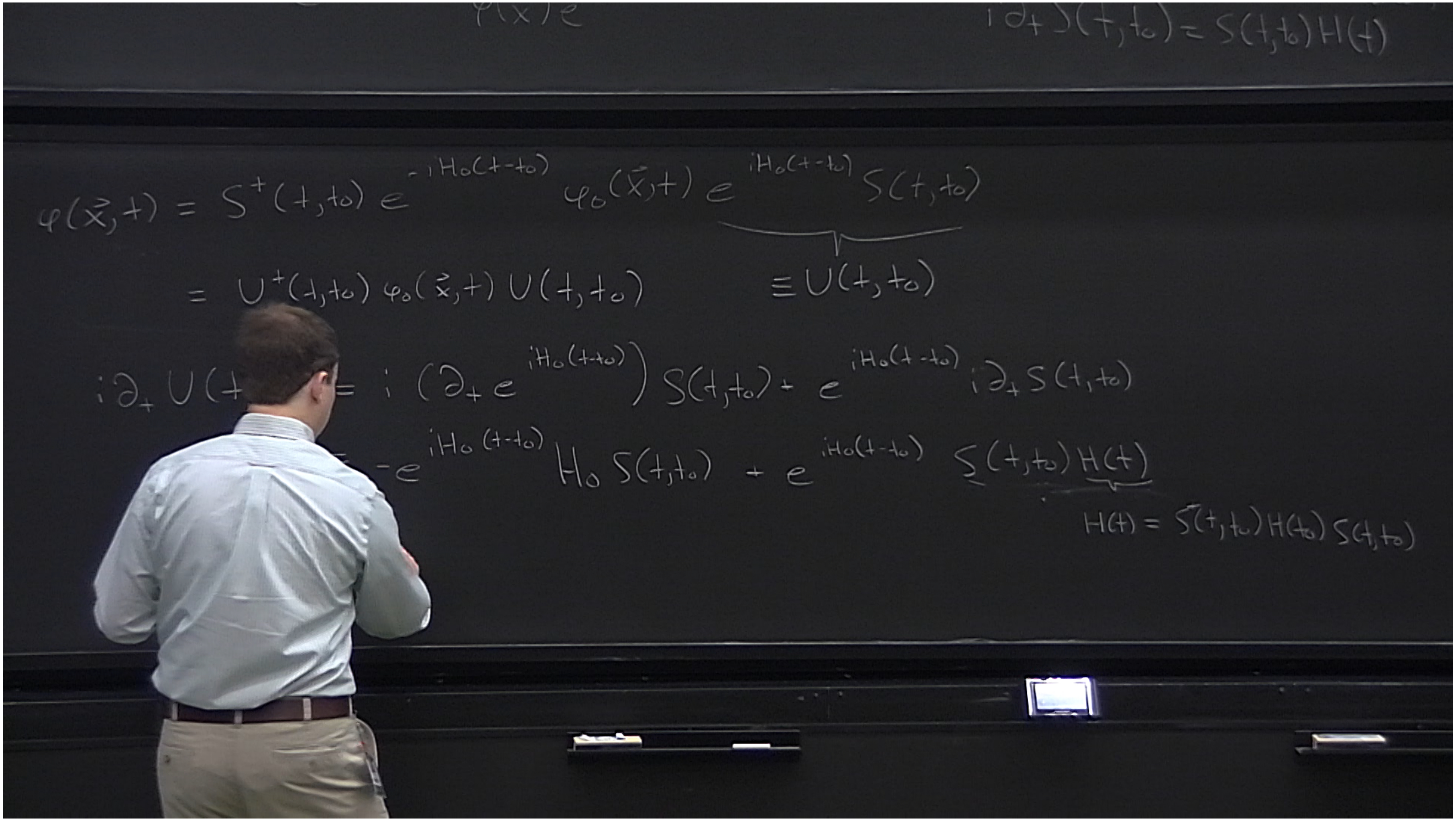
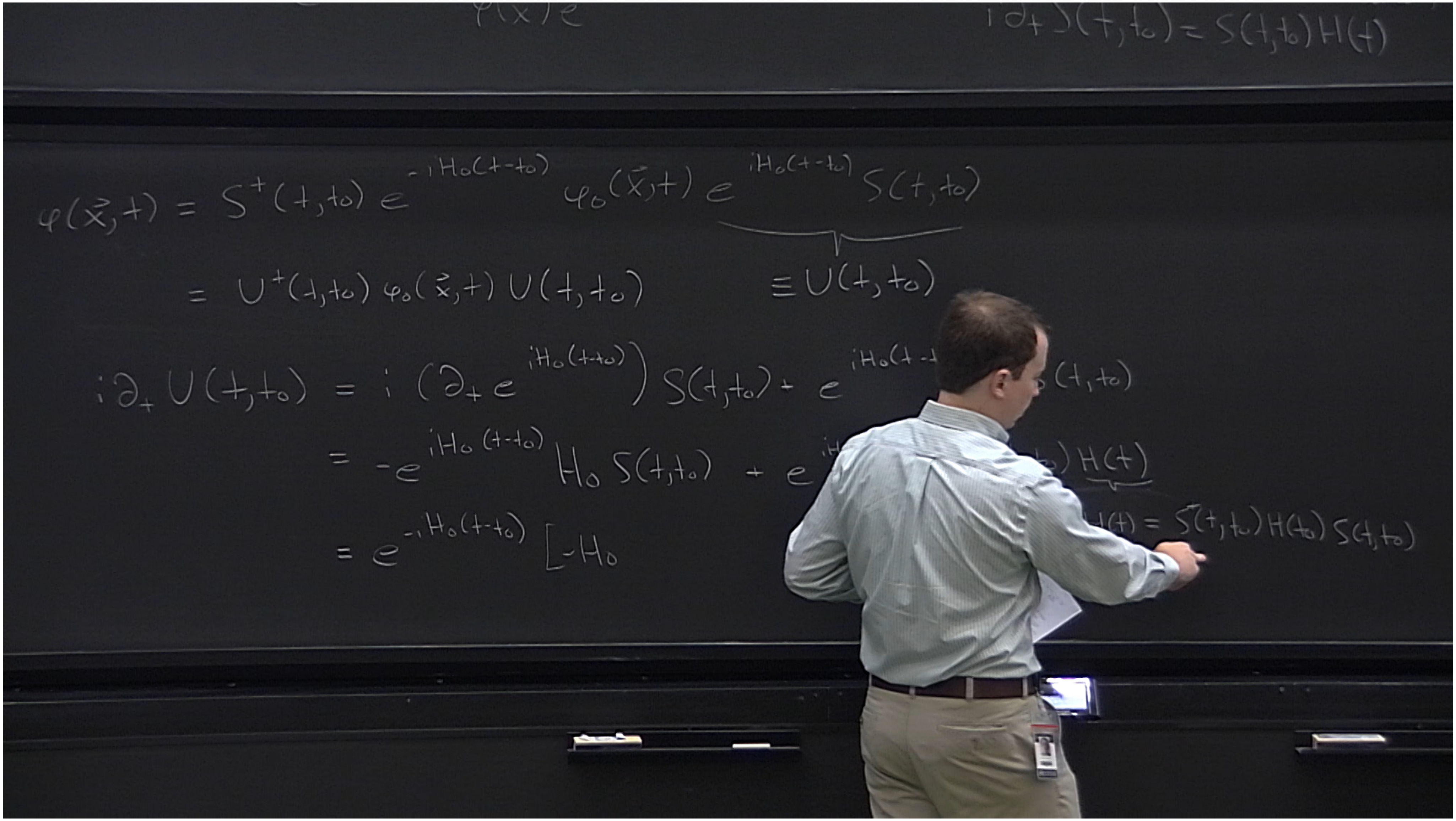
$$i\partial_t S(t,t_0) = S(t,t_0)H(t)$$

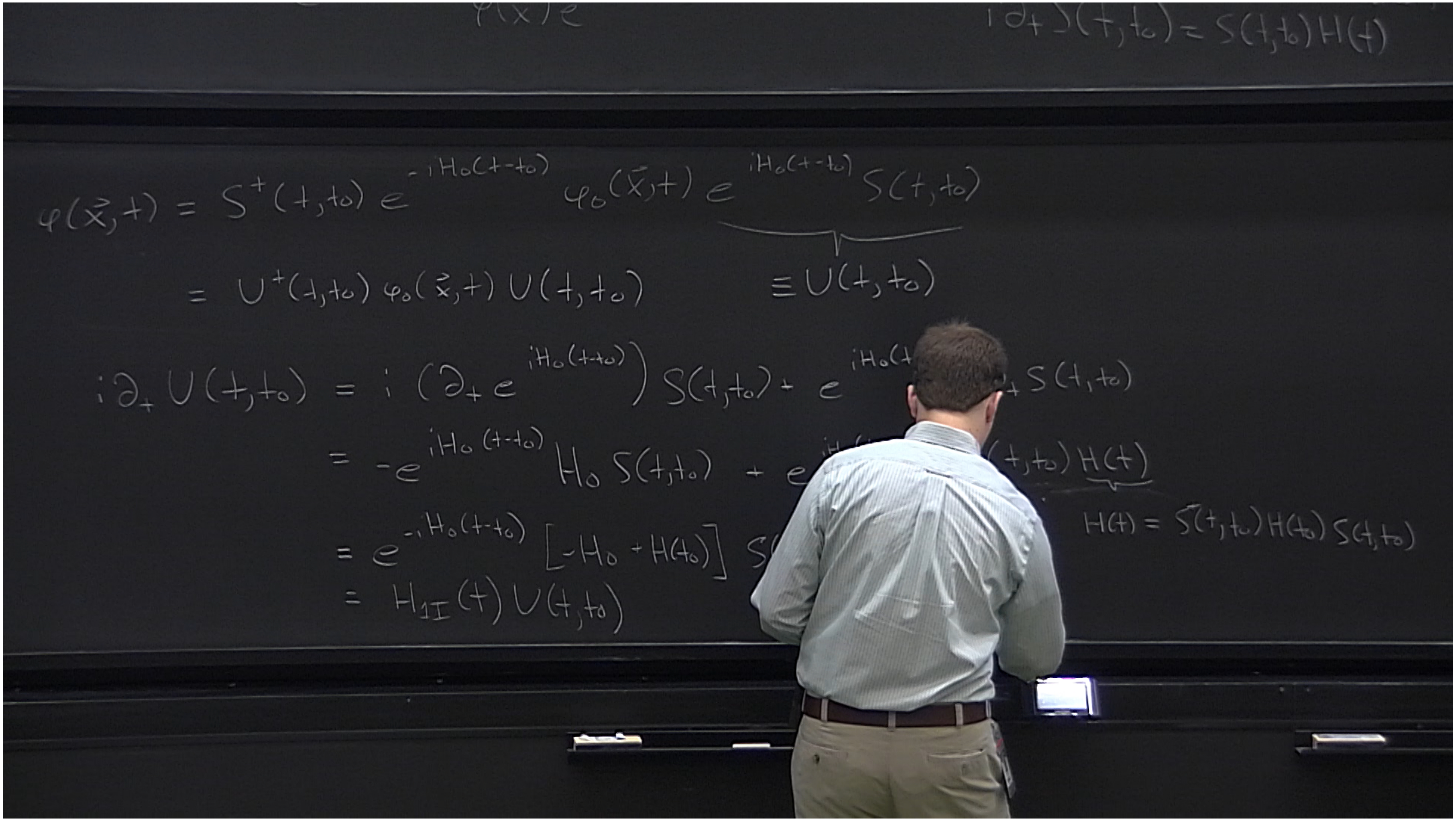

$\varphi(\vec{x},t) = S^{+}(t,t_0)\, e^{-iH_0(t-t_0)}\, \varphi_0(\vec{x},t)\, \underbrace{e^{iH_0(t-t_0)}\, S(t,t_0)}_{\equiv U(t,t_0)}$


$\varphi(\vec{x},t) = S^{+}(t,t_0)\, e^{-iH_0(t-t_0)}\, \varphi_0(\vec{x},t)\, \underbrace{e^{iH_0(t-t_0)}\, S(t,t_0)}_{\equiv U(t,t_0)}$
$= U^{+}(t,t_0)\, \varphi_0(\vec{x},t)\, U(t,t_0)$


$i\partial_t S(t,t_0) = S(t,t_0) H(t)$
$\varphi(\vec{x},t) = S^+(t,t_0)\, e^{-iH_0(t-t_0)}\, \varphi_0(\vec{x},t)\, \underbrace{e^{iH_0(t-t_0)} S(t,t_0)}_{\equiv U(t,t_0)}$
$= U^+(t,t_0)\, \varphi_0(\vec{x},t)\, U(t,t_0)$
$i\partial_t U(t$
$= i\left(\partial_t e^{iH_0(t-t_0)}\right) S(t,t_0) + e^{iH_0(t-t_0)}\, i\partial_t S(t,t_0)$
$= -e^{iH_0(t-t_0)} H_0 S(t,t_0) + e^{iH_0(t-t_0)} S(t,t_0) H(t)$
$H(t) = S^{-1}(t,t_0) H(t_0) S(t,t_0)$



$i\partial_t S(t,t_0) = S(t,t_0) H(t)$
$\varphi(\vec{x},t) = S^{+}(t,t_0)\, e^{-iH_0(t-t_0)}\, \varphi_0(\vec{x},t)\, \underbrace{e^{iH_0(t-t_0)}\, S(t,t_0)}_{\equiv U(t,t_0)}$
$= U^{+}(t,t_0)\, \varphi_0(\vec{x},t)\, U(t,t_0)$
$i\partial_t U(t,t_0) = i\left(\partial_t e^{iH_0(t-t_0)}\right) S(t,t_0) + e^{iH_0(t-t}$
$= -e^{iH_0(t-t_0)} H_0 S(t,t_0) + e$
$H(t)$
$H(t) = S^{+}(t,t_0) H(t_0) S(t,t_0)$
$= e^{-iH_0(t-t_0)} [-H_0$


$i\partial_t S(t,t_0) = S(t,t_0) H(t)$
$\varphi(\vec{x},t) = S^+(t,t_0)\, e^{-iH_0(t-t_0)}\, \varphi_0(\vec{x},t)\, e^{iH_0(t-t_0)} S(t,t_0)$
$\equiv U(t,t_0)$
$= U^+(t,t_0)\, \varphi_0(\vec{x},t)\, U(t,t_0)$
$i\partial_t U(t,t_0) = i\left(\partial_t e^{iH_0(t-t_0)}\right) S(t,t_0) + e^{iH_0(t}$
$\partial_t S(t,t_0)$
$= -e^{iH_0(t-t_0)} H_0 S(t,t_0) + e$
$(t,t_0) H(t)$
$H(t) = S^{-1}(t,t_0) H(t_0) S(t,t_0)$
$= e^{-iH_0(t-t_0)} \left[-H_0 + H(t_0)\right] S($
$= H_{1I}(t)\, U(t,t_0)$

$$\varphi(x)\, e \qquad\qquad i\partial_t S(t,t_0) = S(t,t_0)\, H(t)$$

$$t) = S^{+}(t,t_0)\, e^{-iH_0(t-t_0)}\, \varphi_0(\vec{x},t)\, \underbrace{e^{iH_0(t-t_0)}\, S(t,t_0)}_{\equiv\, U(t,t_0)}$$

$$= U^{+}(t,t_0)\, \varphi_0(\vec{x},t)\, U(t,t_0)$$

$$\partial_t U(t,t_0) = i\left(\partial_t e^{iH_0(t-t_0)}\right) S(t,t_0) + e^{iH_0(t-t_0)}\, i\partial_t S(t,t_0)$$

$$= -e^{iH_0(t-t_0)} H_0\, S(t,t_0) + e^{iH_0(t-t_0)}\, S(t,t_0)\, H(t)$$

$$H(t) = S^{+}(t,t_0)\, H(t_0)\, S(t,t_0)$$

$$= e^{-iH_0(t-t_0)} \left[-H_0 + H(t_0)\right] S(t,t_0)$$

$$= H_{1I}(t)\, U(t,t_0)$$

$$H_{1I}(t) = e^{iH(t-t_0)}\, H_1(t_0)\, e^{-iH_0(t-t_0)}$$

Schrödinger

$$i\partial_t S(t,t_0) = S(t,t_0)\,H(t)$$

$$\varphi(\vec{x},t) = S^+(t,t_0)\, e^{-iH_0(t-t_0)}\, \varphi_0(\vec{x},t)\, \underbrace{e^{iH_0(t-t_0)}\, S(t,t_0)}_{\equiv U(t,t_0)}$$

$$= U^+(t,t_0)\, \varphi_0(\vec{x},t)\, U(t,t_0)$$

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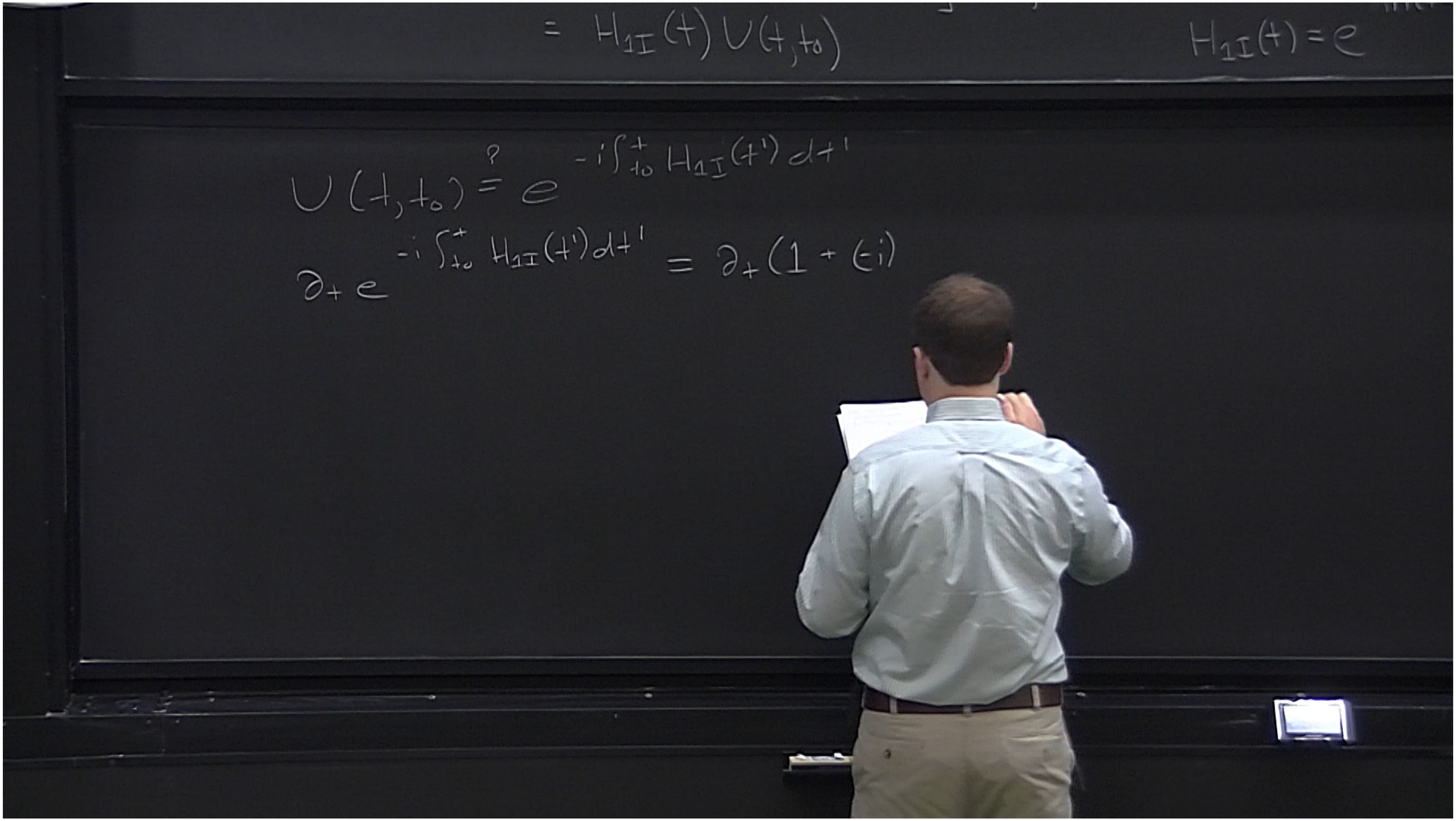
$$= -e^{iH_0(t-t_0)} H_0 S(t,t_0) + e^{iH_0(t-t_0)}\, \underline{S(t,t_0) H(t)}$$

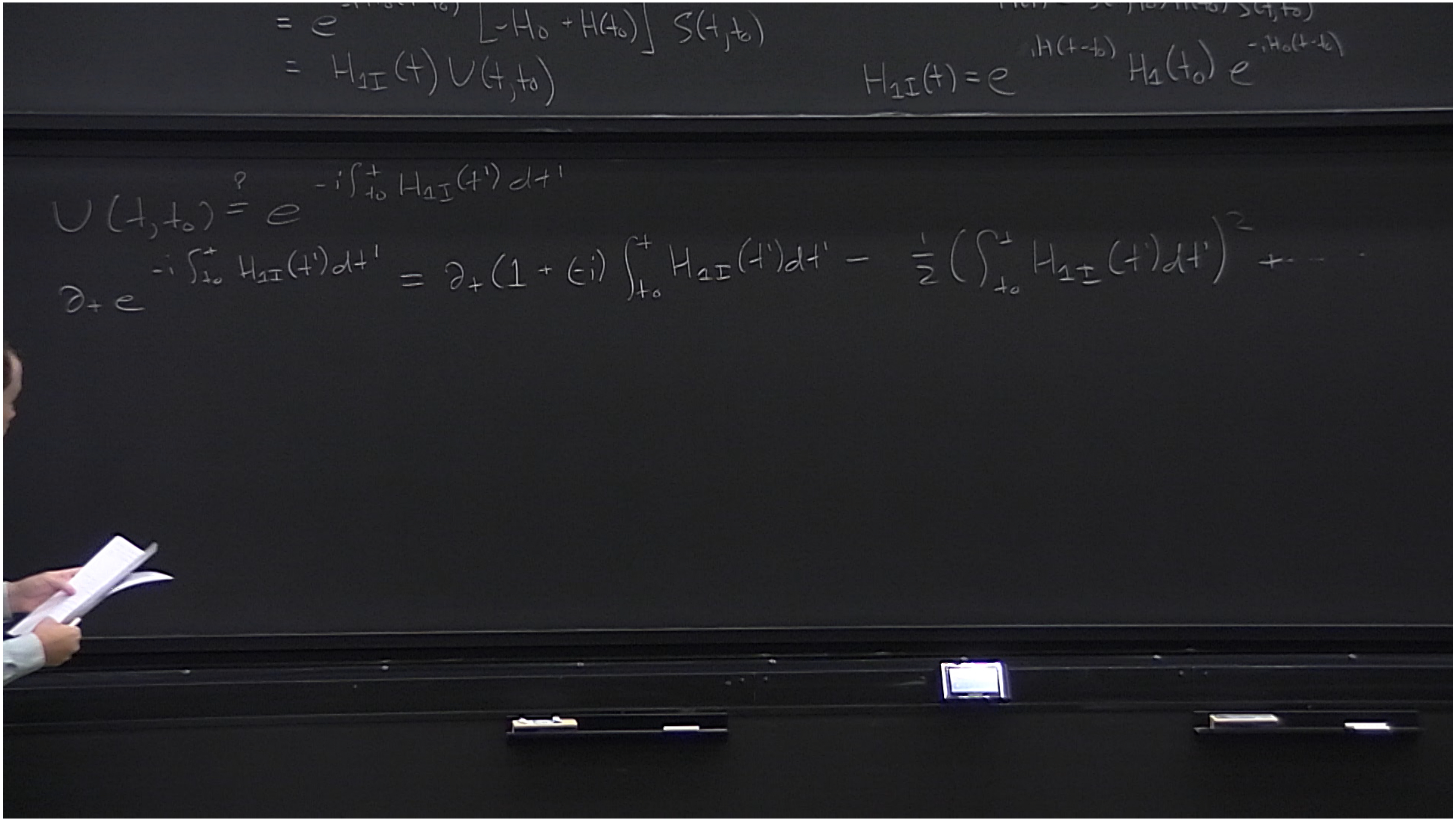
$$= e^{-iH_0(t-t_0)} \left[-H_0 + H(t_0)\right] S(t,t_0)$$

$$= H_{1I}(t)\, U(t,t_0)$$

$$H(t) = \bar{S}(t,t_0)\, H(t_0)\, S(t,t_0)$$

$$H_{1I}(t) = e^{iH(t-t_0)}\, H_1(t_0)\, e^{-iH_0(t-t_0)}$$


$= H_{1I}(t)\,U(t,t_0)$
$H_{1I}(t) = e$
$U(t,t_0) \overset{?}{=} e^{-i\int_{t_0}^{t} H_{1I}(t')\,dt'}$
$\partial_t e^{-i\int_{t_0}^{t} H_{1I}(t')\,dt'} = \partial_t (1 + (-i)$



$= e^{\cdots}\left[-H_0 + H(t_0)\right] S(t,t_0)$
$= H_{1I}(t)\, U(t,t_0)$
$H_{1I}(t) = e^{iH(t-t_0)} H_1(t_0)\, e^{-iH_0(t-t_0)}$
$U(t,t_0) \overset{?}{=} e^{-i\int_{t_0}^{t} H_{1I}(t')\,dt'}$
$\partial_t e^{-i\int_{t_0}^{t} H_{1I}(t')\,dt'} = \partial_t \Big(1 + (-i)\int_{t_0}^{t} H_{1I}(t')\,dt' - \frac{1}{2}\Big(\int_{t_0}^{t} H_{1I}(t')\,dt'\Big)^2 + \cdots$

$$= e^{\cdots}\left[-H_0 + H(t_0)\right] S(t,t_0)$$

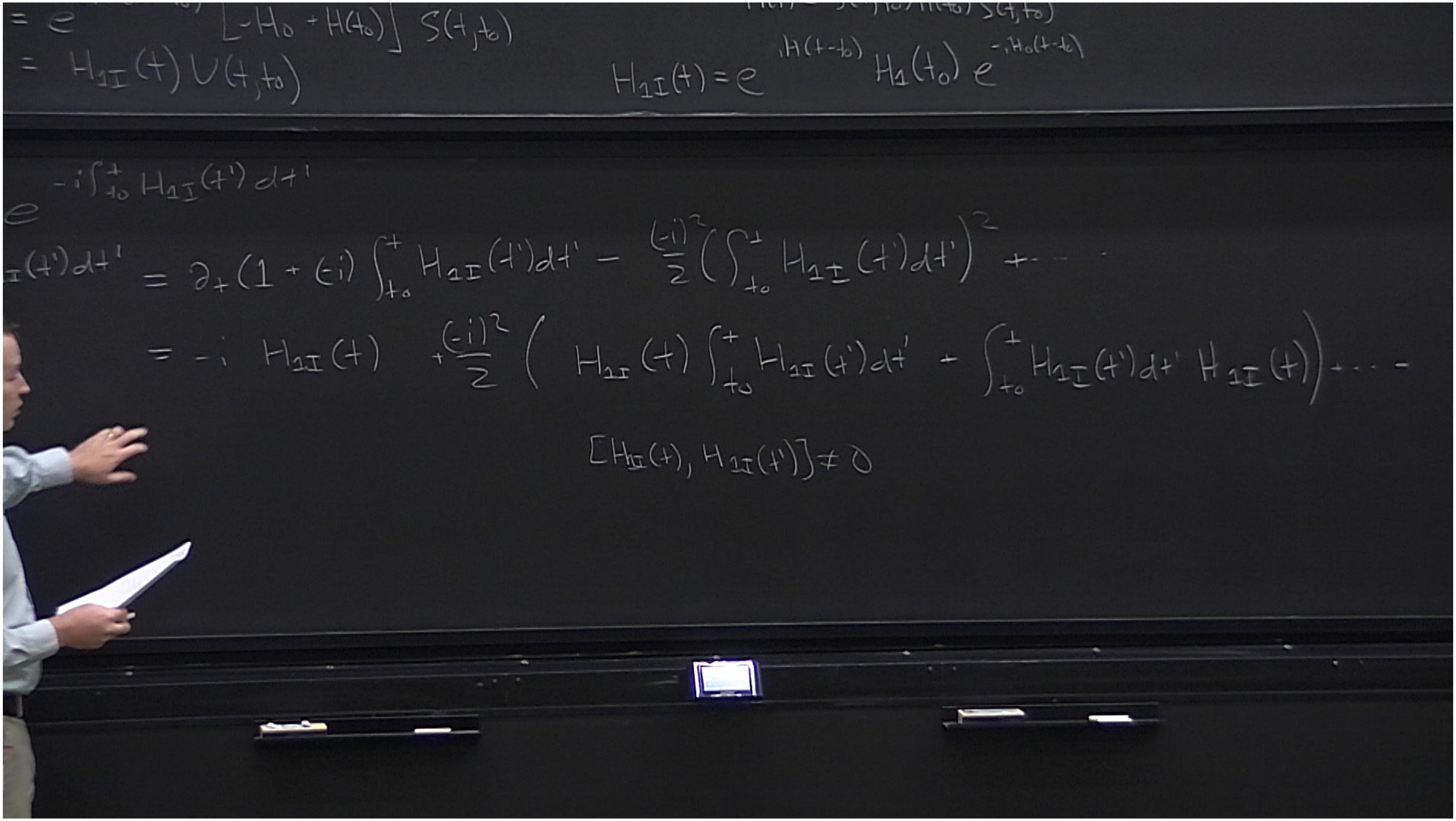
$$= H_{1I}(t)\, U(t,t_0)$$

$$H_{1I}(t) = e^{iH(t-t_0)} H_1(t_0)\, e^{-iH_0(t-t_0)}$$

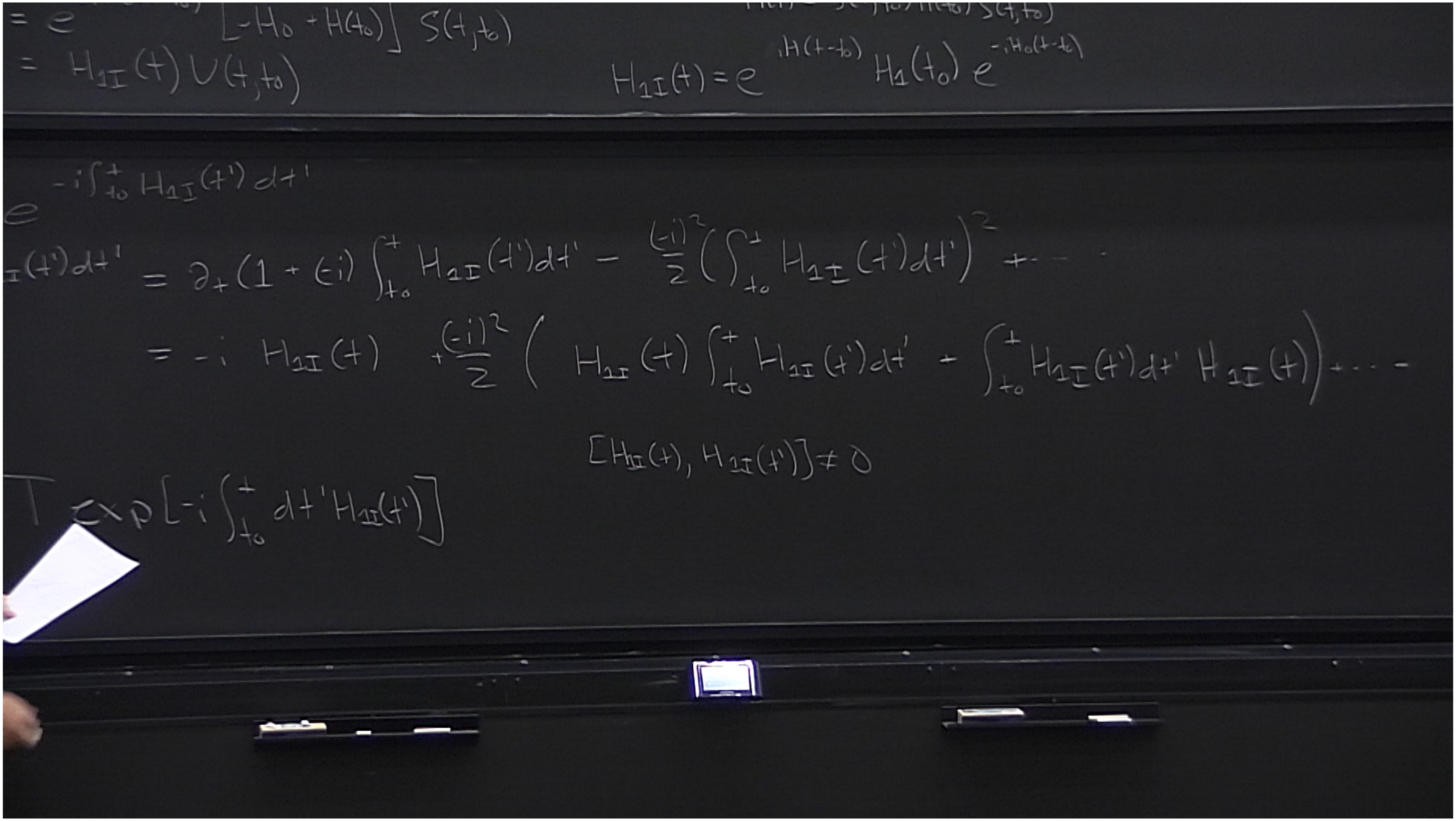
$$U(t,t_0) \overset{?}{=} e^{-i\int_{t_0}^{t} H_{1I}(t')dt'}$$

$$\partial_t e^{-i\int_{t_0}^{t} H_{1I}(t')dt'} = \partial_t\left(1 + (-i)\int_{t_0}^{t} H_{1I}(t')dt' - \frac{(-i)^2}{2}\left(\int_{t_0}^{t} H_{1I}(t')dt'\right)^2 + \cdots\right.$$

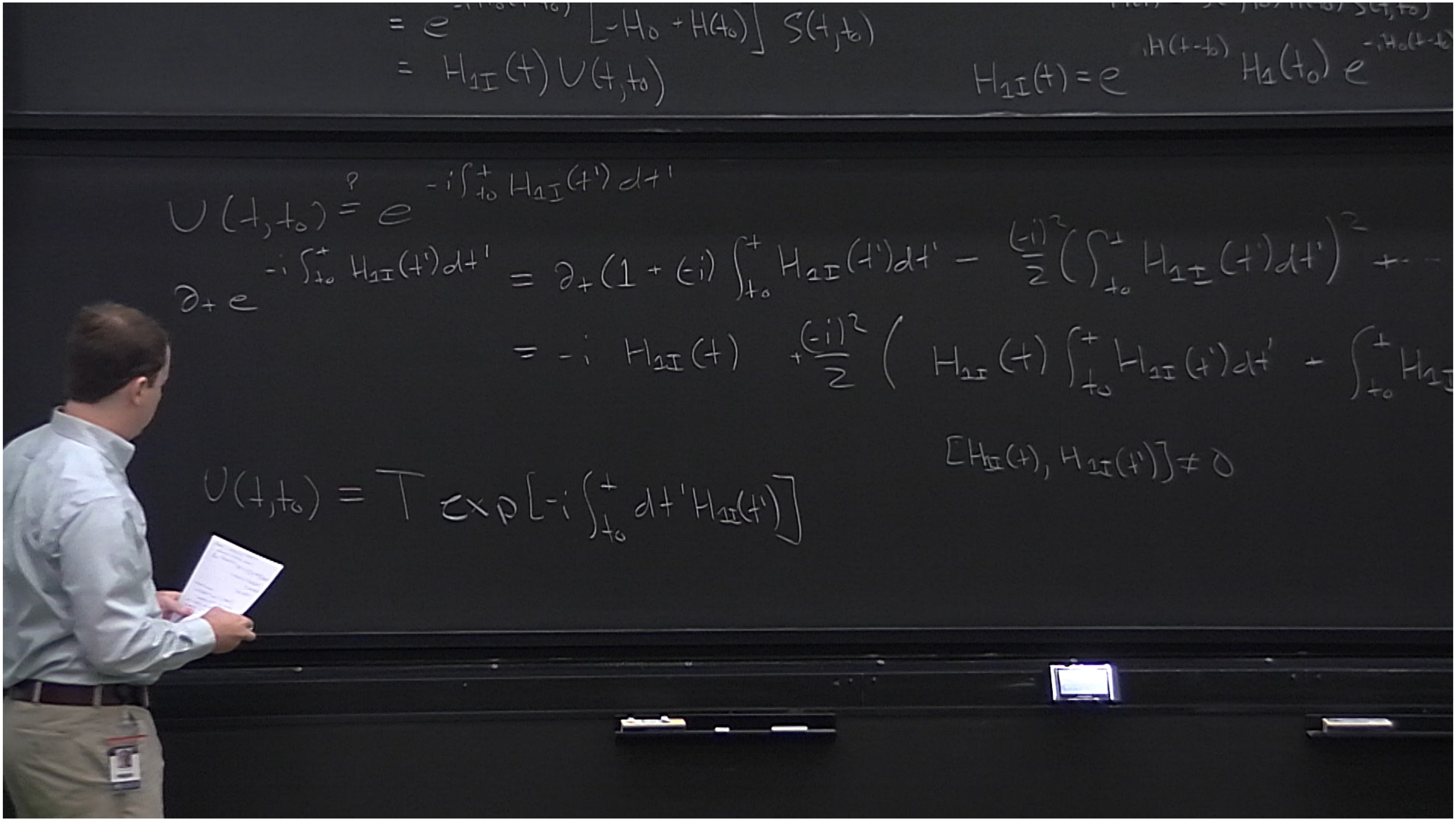
$$= -i\, H_{1I}(t) + \frac{(-i)^2}{2}\left(H_{1I}(t)\int_{t_0}^{t} H_{1I}(t')dt' + \int_{t_0}^{t} H \cdots dt'\right.$$

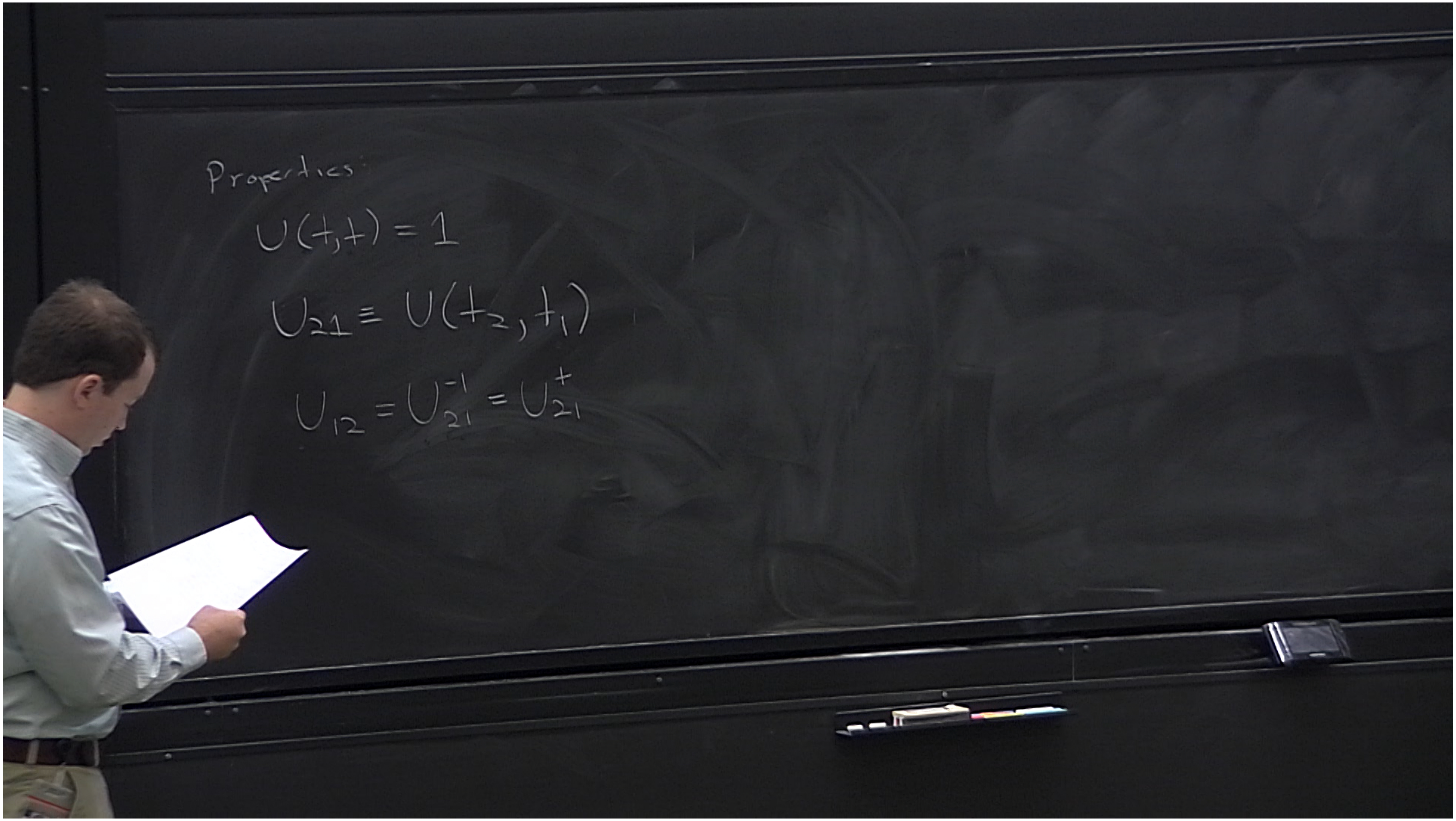


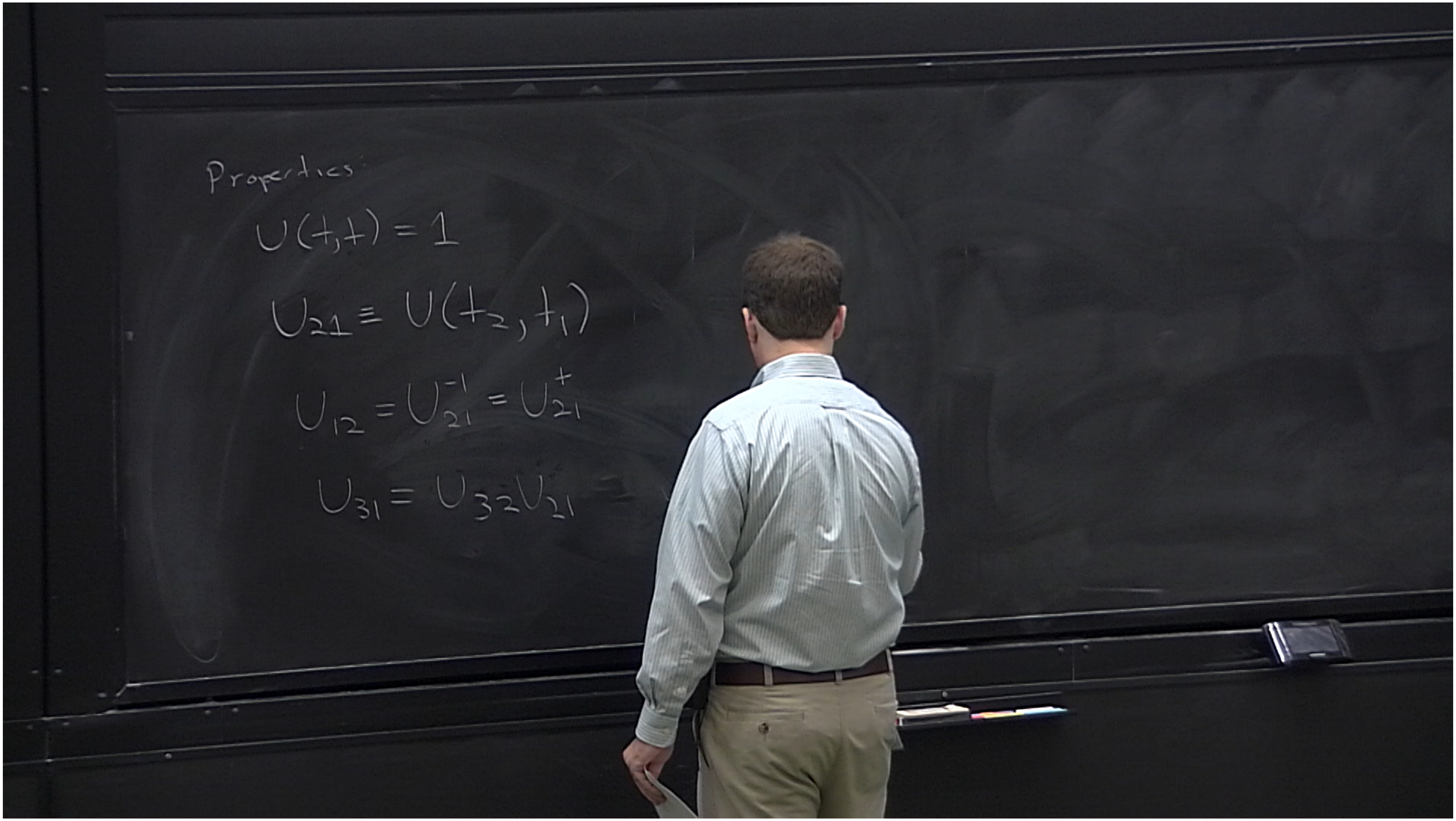
$= e \; [-H_0 + H(t_0)] \; S(t,t_0)$
$= H_{1I}(t)\, U(t,t_0)$
$H_{1I}(t) = e^{iH(t-t_0)} H_1(t_0)\, e^{-iH_0(t-t_0)}$
$e^{-i\int_{t_0}^{t} H_{1I}(t')dt'}$
$(t')dt' = \partial_t \left(1 + (-i)\int_{t_0}^{t} H_{1I}(t')dt' - \frac{(-i)^2}{2}\left(\int_{t_0}^{t} H_{1I}(t')dt'\right)^2 + \cdots\right.$
$= -i\, H_{1I}(t) + \frac{(-i)^2}{2}\left(H_{1I}(t)\int_{t_0}^{t} H_{1I}(t')dt' + \int_{t_0}^{t} H_{1I}(t')dt'\, H_{1I}(t)\right) + \cdots$
$[H_{1I}(t), H_{1I}(t')] \neq 0$



$= e \quad [-H_0 + H(t_0)] \; S(t,t_0)$
$= H_{1I}(t)\, U(t,t_0)$
$H_{1I}(t) = e^{iH(t-t_0)} H_1(t_0)\, e^{-iH_0(t-t_0)}$
$e^{-i\int_{t_0}^{t} H_{1I}(t')\,dt'}$
$(t')dt' = \partial_t \left(1 + (-i)\int_{t_0}^{t} H_{1I}(t')dt' - \frac{(-i)^2}{2}\left(\int_{t_0}^{t} H_{1I}(t')dt'\right)^2 + \cdots\right.$
$= -i\, H_{1I}(t) + \frac{(-i)^2}{2}\left(H_{1I}(t)\int_{t_0}^{t} H_{1I}(t')dt' + \int_{t_0}^{t} H_{1I}(t')dt'\, H_{1I}(t)\right) + \cdots$
$[H_{1I}(t), H_{1I}(t')] \neq 0$
$T\exp\left[-i\int_{t_0}^{t} dt'\, H_{1I}(t')\right]$


$= e^{\cdots} [-H_0 + H(t_0)] S(t, t_0)$
$= H_{1I}(t) U(t, t_0)$
$H_{1I}(t) = e^{iH(t-t_0)} H_1(t_0) e^{-iH_0(t-t_0)}$
$U(t, t_0) \stackrel{?}{=} e^{-i\int_{t_0}^{t} H_{1I}(t') dt'}$
$\partial_t e^{-i\int_{t_0}^{t} H_{1I}(t') dt'} = \partial_t \left(1 + (-i)\int_{t_0}^{t} H_{1I}(t') dt' - \frac{(-i)^2}{2}\left(\int_{t_0}^{t} H_{1I}(t') dt'\right)^2 + \cdots\right.$
$= -i H_{1I}(t) + \frac{(-i)^2}{2}\left(H_{1I}(t) \int_{t_0}^{t} H_{1I}(t') dt' + \int_{t_0}^{t} H_{1I}\right.$
$[H_{1I}(t), H_{1I}(t')] \neq 0$
$U(t, t_0) = T \exp\left[-i\int_{t_0}^{t} dt' H_{1I}(t')\right]$


Properties:
$U(t,t) = 1$
$U_{21} \equiv U(t_2, t_1)$
$U_{12} = U_{21}^{-1} = U_{21}^{\dagger}$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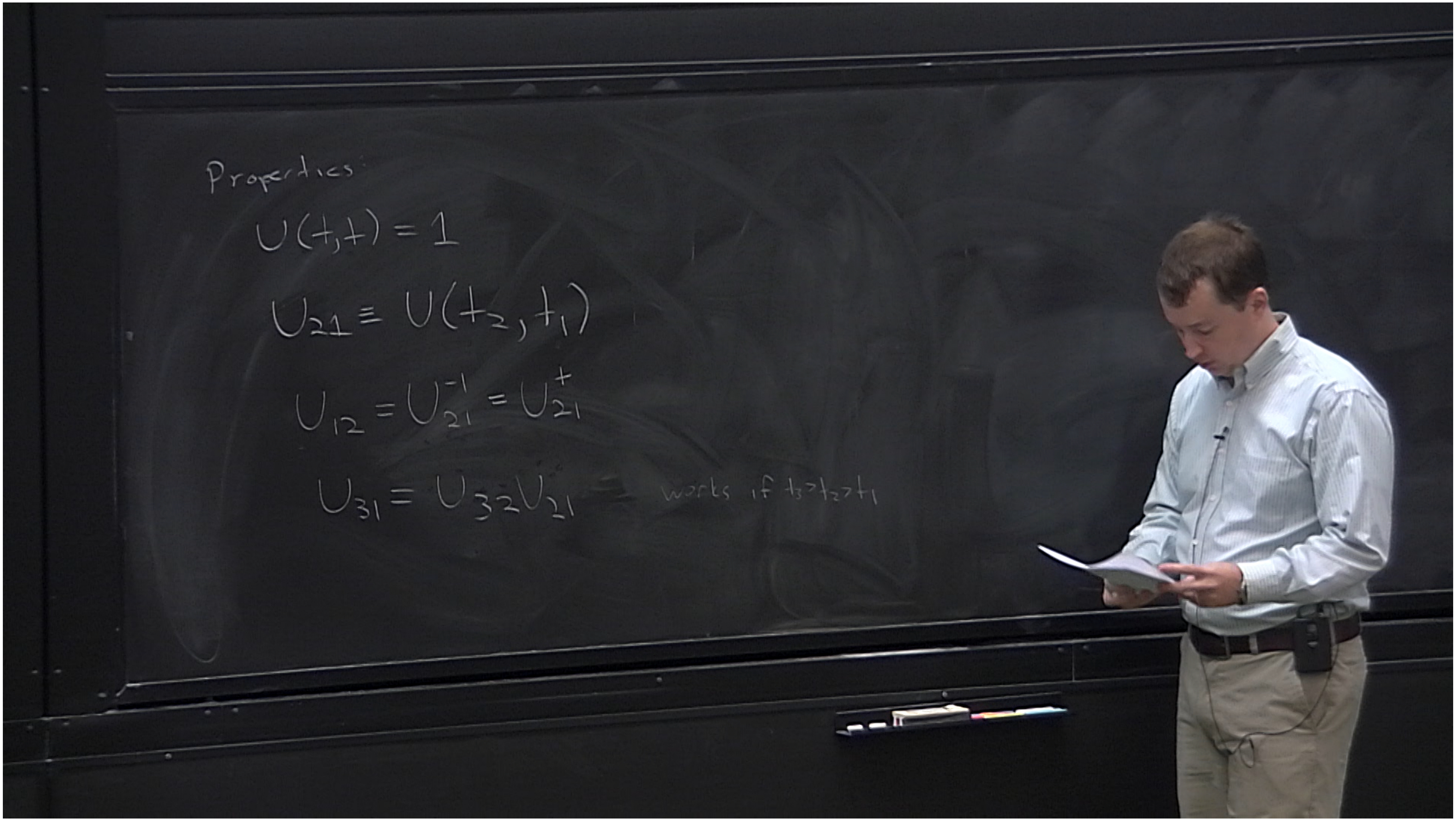
$U_{31} = U_{32} U_{21}$



Properties:
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$U_{31} = U_{32} U_{21}$
works if $t_3 > t_2 > t_1$

Properties:

$$U(t,t) = 1$$

$$U_{21} \equiv U(t_2, t_1)$$

$$U_{12} = U_{21}^{-1} = U_{21}^{\dagger}$$

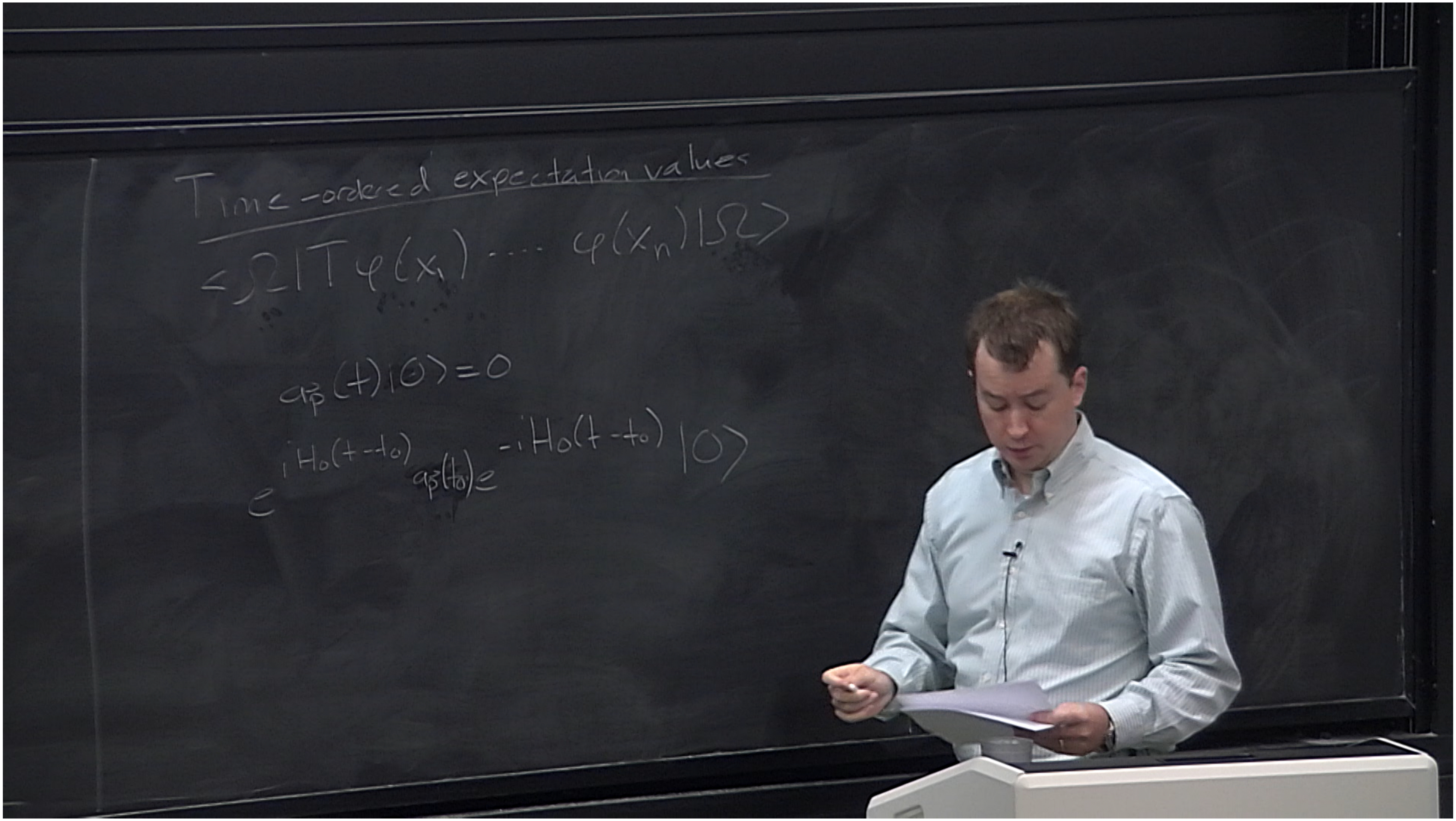
$$U_{31} = U_{32} U_{21}$$

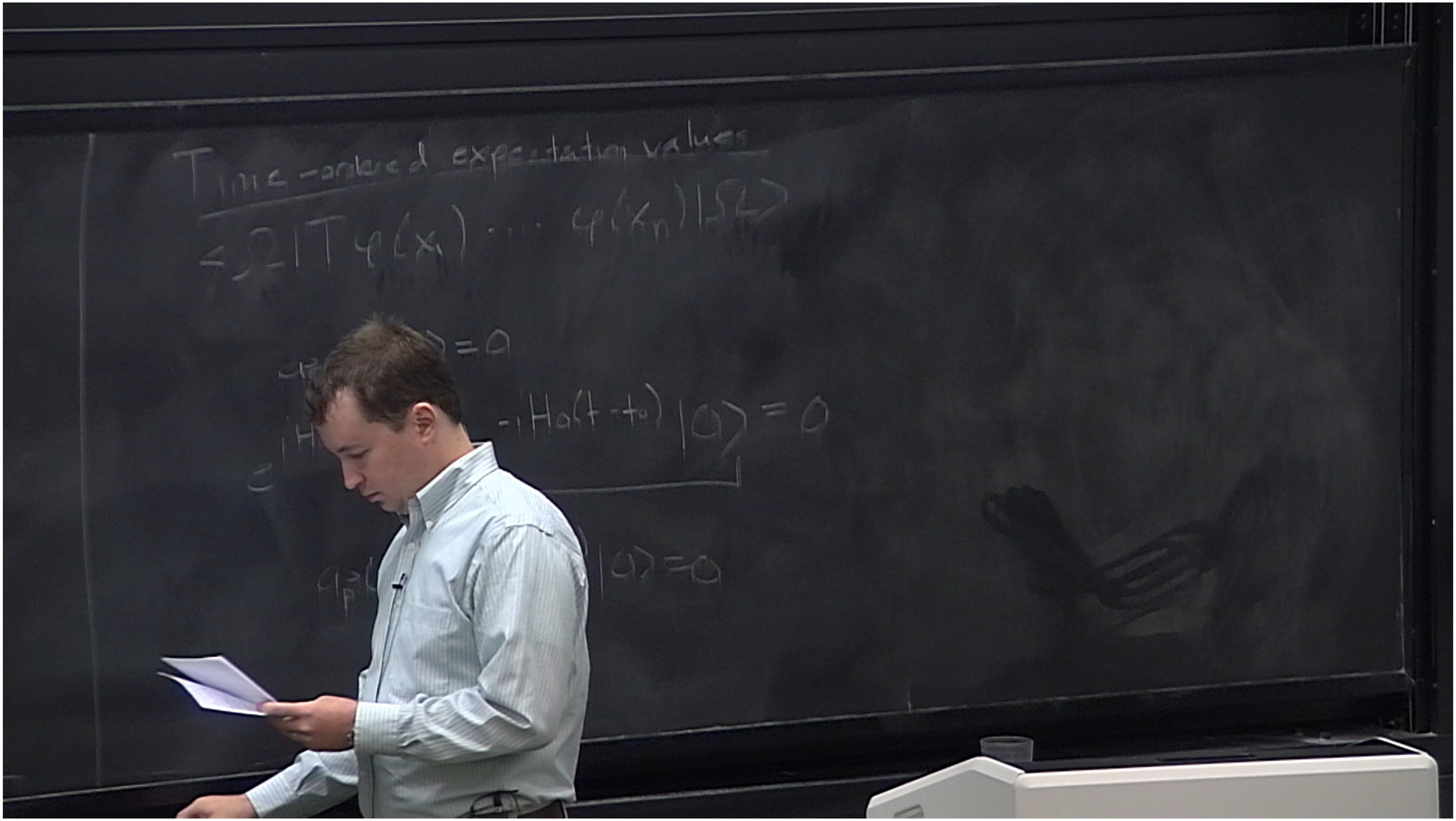
works if $t_3 > t_2 > t_1$ and for any ordering by multiplication by $\underset{\text{right}}{U_{12}}$ or $\underset{\text{left}}{U_{23}}$

$$U_{23} U_{31} = U_{23} U_{32} U_{21} = U_{21}$$

Time-ordered expectation values

$$\langle \Omega | T \varphi(x_1) \cdots \varphi(x_n) | \Omega \rangle$$


Time-ordered expectation values
$\langle \Omega | T \varphi(x_1) \dots \varphi(x_n) | \Omega \rangle$
$a_{\vec{p}}(t)|0\rangle = 0$
$e^{iH_0(t-t_0)} a_{\vec{p}}(t_0) e^{-iH_0(t-t_0)} |0\rangle$


Time-ordered expectation values
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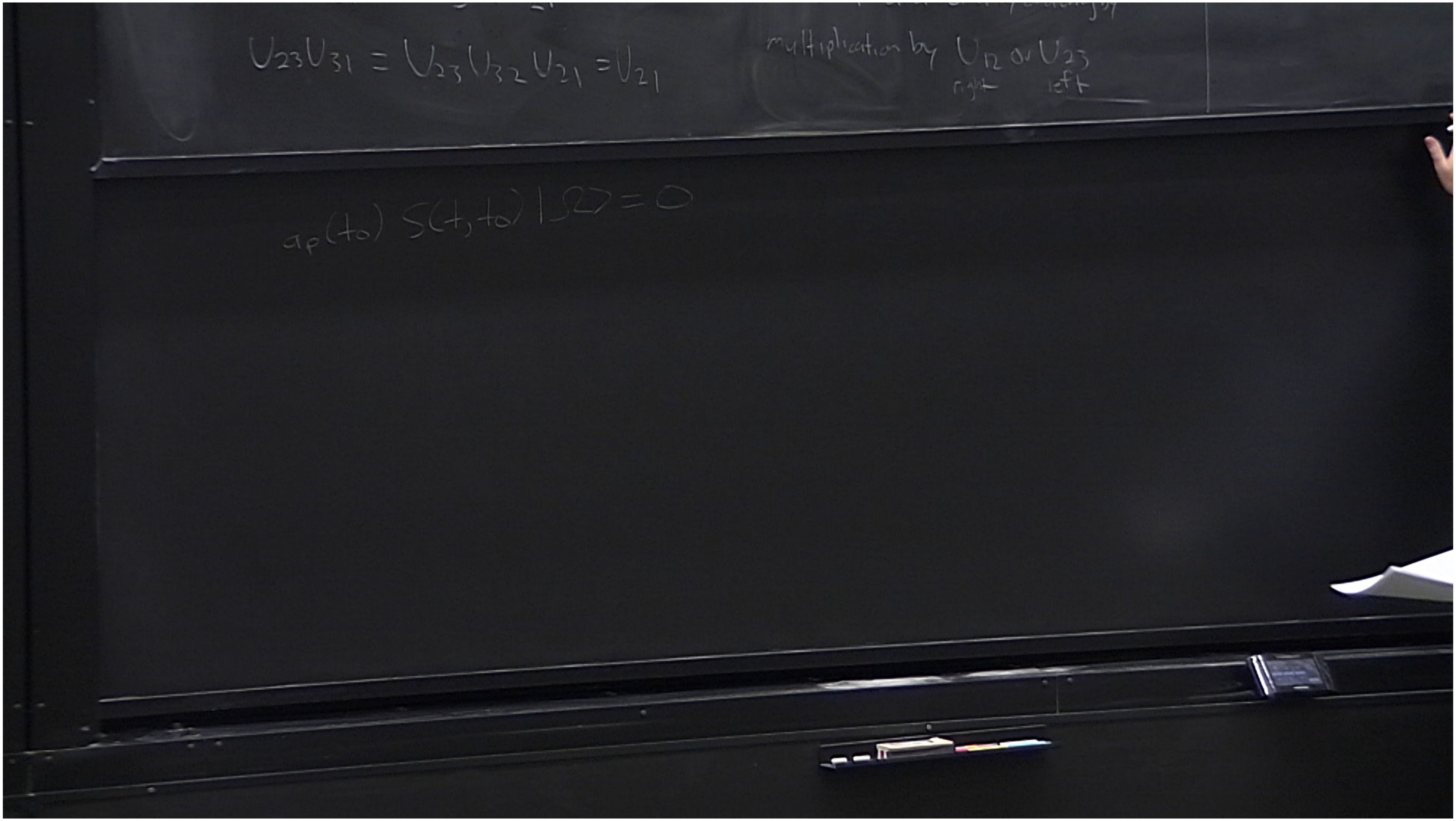
Time-ordered expectation values

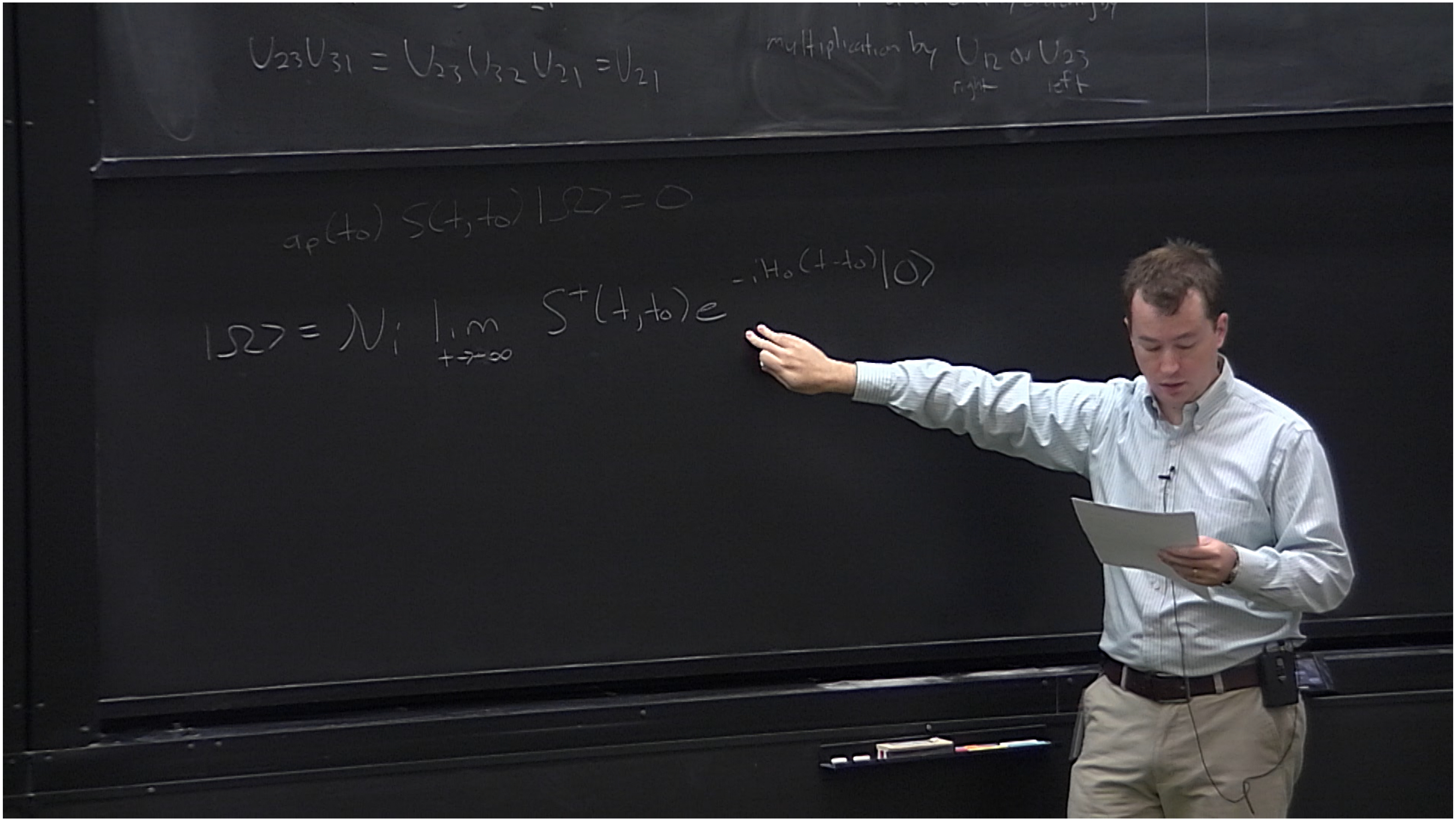
$$\langle \Omega | T \varphi(x_1) \cdots \varphi(x_n) | \Omega \rangle$$

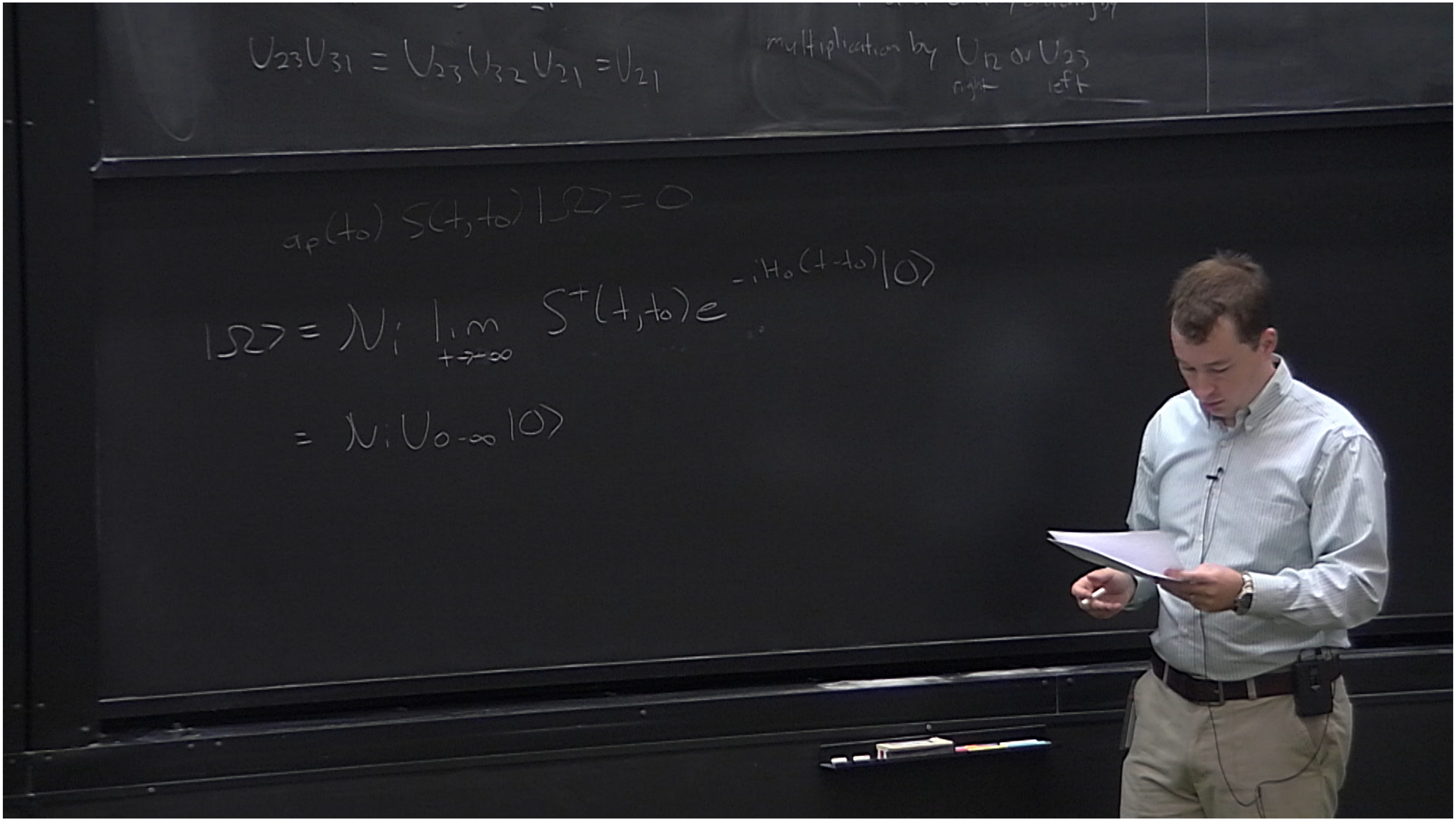
$$a_{\vec{p}}(t)|0\rangle = 0$$

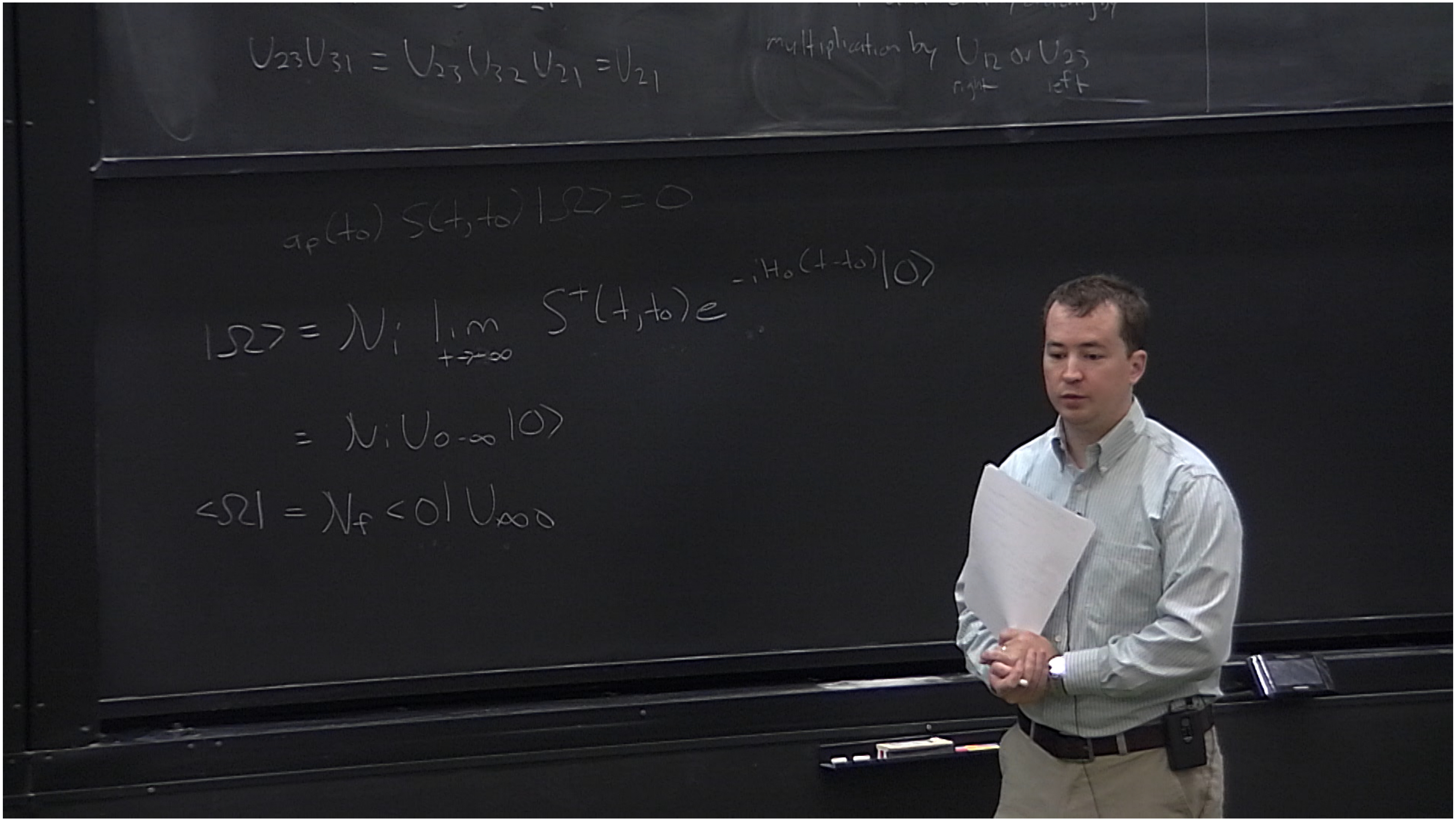
$$e^{iH_0(t-t_0)} a_{\vec{p}}(t_0) e^{-iH_0(t-t_0)} |0\rangle = 0$$

$$a_{\vec{p}}(t_0) e^{-iH_0(t-t_0)} |0\rangle = 0$$


$U_{23}U_{31} = U_{23}U_{32}U_{21} = U_{21}$
multiplication by U_{12} or U_{23}
right left
$a_p(t_0)\, S(t,t_0)|\Omega\rangle = 0$

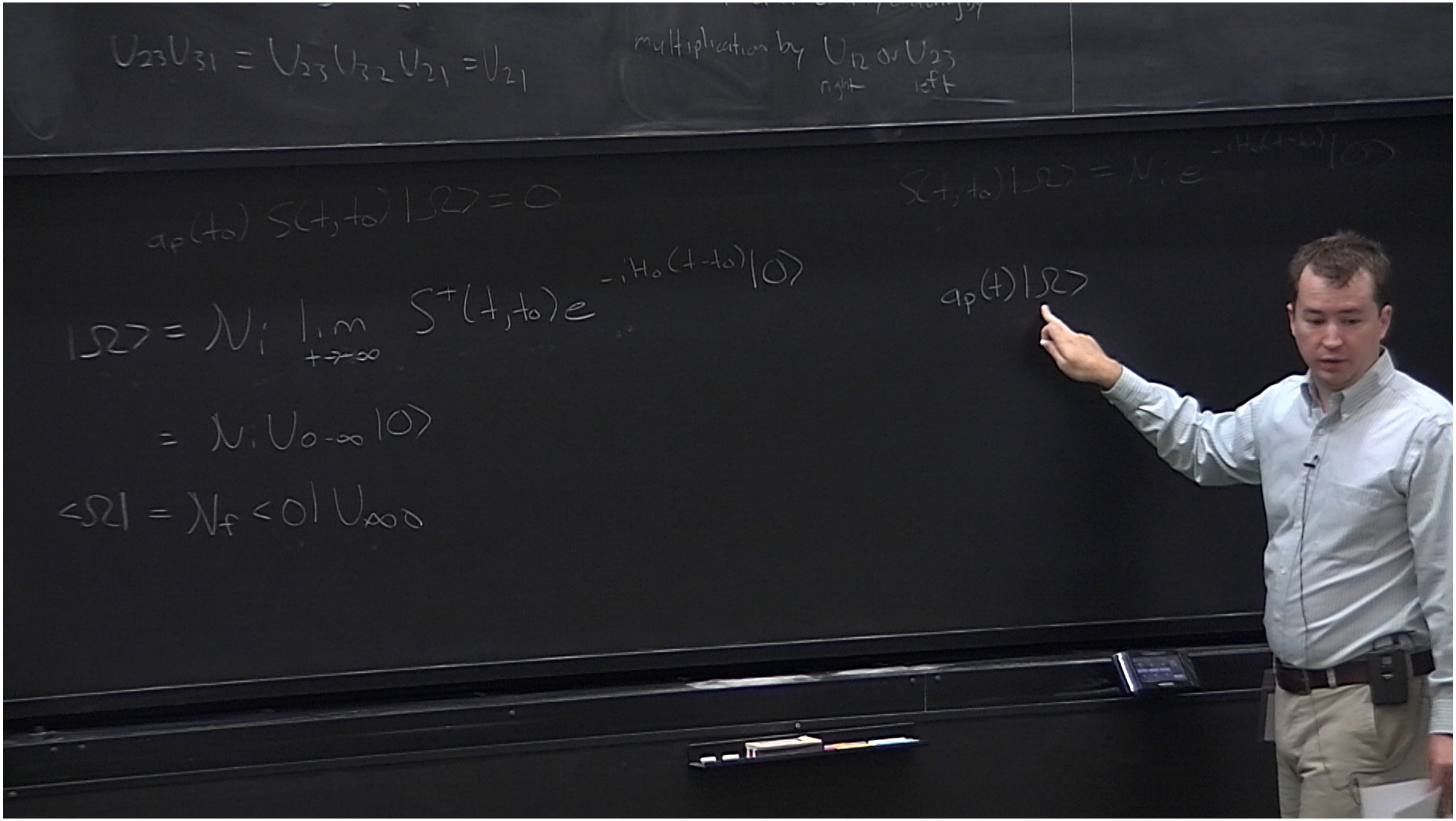

$U_{23}U_{31} = U_{23}U_{32}U_{21} = U_{21}$
multiplication by U_{12} or U_{23}
right left
$a_p(t_0)\, S(t,t_0)|\Omega\rangle = 0$
$|\Omega\rangle = N_i \lim_{t\to-\infty} S^+(t,t_0)\, e^{-iH_0(t-t_0)}|0\rangle$


U23U31 = U23U32U21 = U21
multiplication by U12 or U23
right left
ap(to) S(t,to)|Ω> = 0
|Ω> = Ni lim S+(t,to)e-iHo(t-to)|0>
t→-∞
= Ni U0-∞|0>



$U_{23}U_{31} = U_{23}U_{32}U_{21} = U_{21}$
multiplication by U_{12} or U_{23}
right left
$a_p(t_0)\, S(t,t_0)|\Omega\rangle = 0$
$|\Omega\rangle = N_i \lim_{t\to-\infty} S^+(t,t_0)\, e^{-iH_0(t-t_0)}|0\rangle$
$= N_i U_{0,-\infty}|0\rangle$
$\langle\Omega| = N_f \langle 0| U_{\infty 0}$


and for any ordering by
by U12 or U23
right left
S(t, t0)|Ω> = N e^(-iH0(t-t0))|0>
(t-t0))|0>


$U_{23}U_{31} = U_{23}U_{32}U_{21} = U_{21}$
multiplication by U_{12} or U_{23}
right left
$S(t,t_0)|\Omega\rangle = N_i e^{-iH_0(t-t_0)}|0\rangle$
$a_p(t_0)\, S(t,t_0)|\Omega\rangle = 0$
$|\Omega\rangle = N_i \lim_{t\to-\infty} S^+(t,t_0)e^{-iH_0(t-t_0)}|0\rangle$
$= N_i U_{0-\infty}|0\rangle$
$\langle\Omega| = N_f \langle 0| U_{\infty 0}$
$a_p(t)|\Omega\rangle$

$$U_{23}U_{31} = U_{23}U_{32}U_{21} = U_{21}$$

multiplication by U_{12} or U_{23}
right left

$$S(t,t_0)|\Omega\rangle = N_i e^{-iH_0(t-t_0)}|0\rangle$$

$$a_p(t_0)\, S(t,t_0)|\Omega\rangle = 0$$

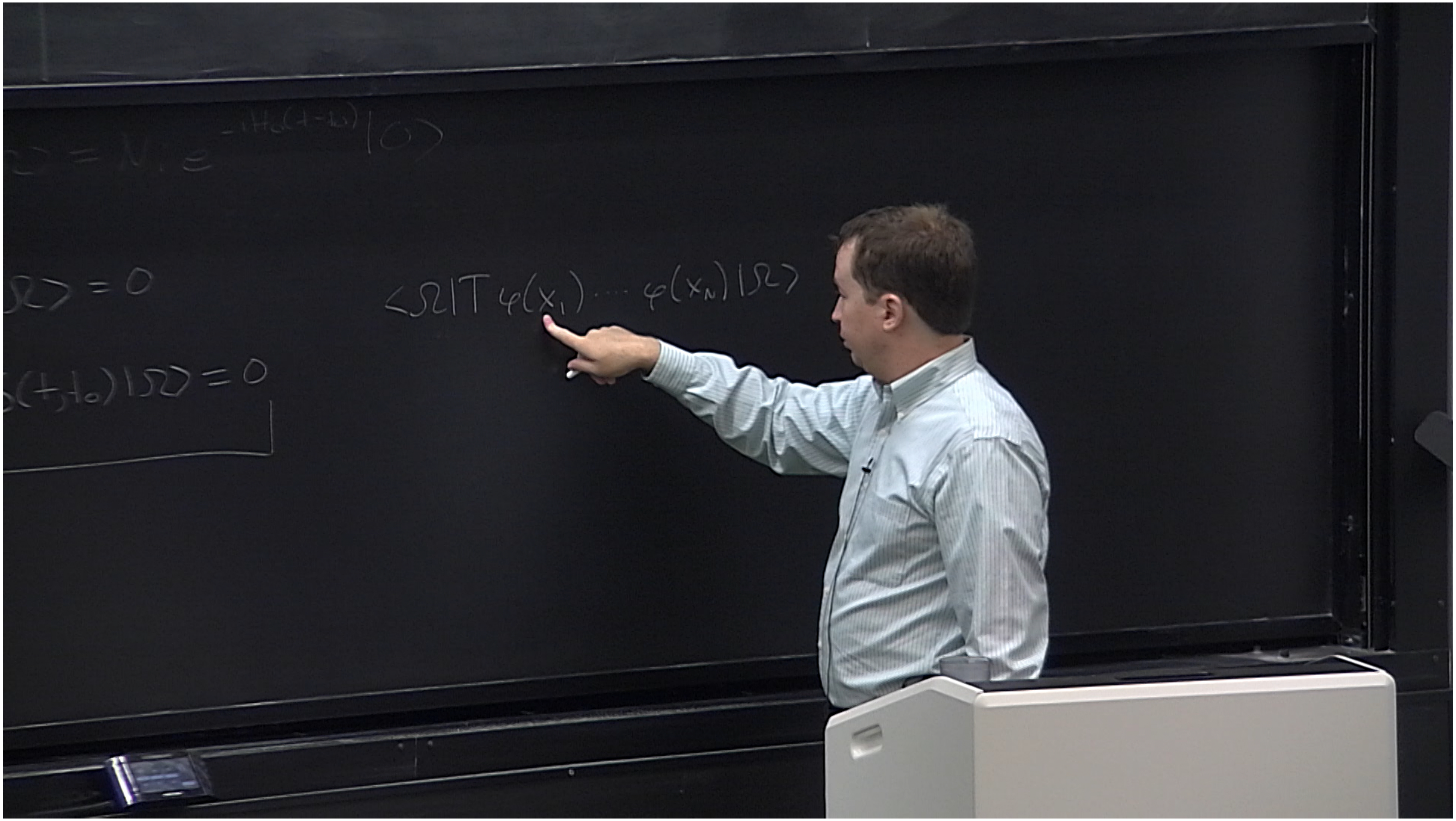
$$|\Omega\rangle = N_i \lim_{t\to-\infty} S^+(t,t_0)\, e^{-iH_0(t-t_0)}|0\rangle$$

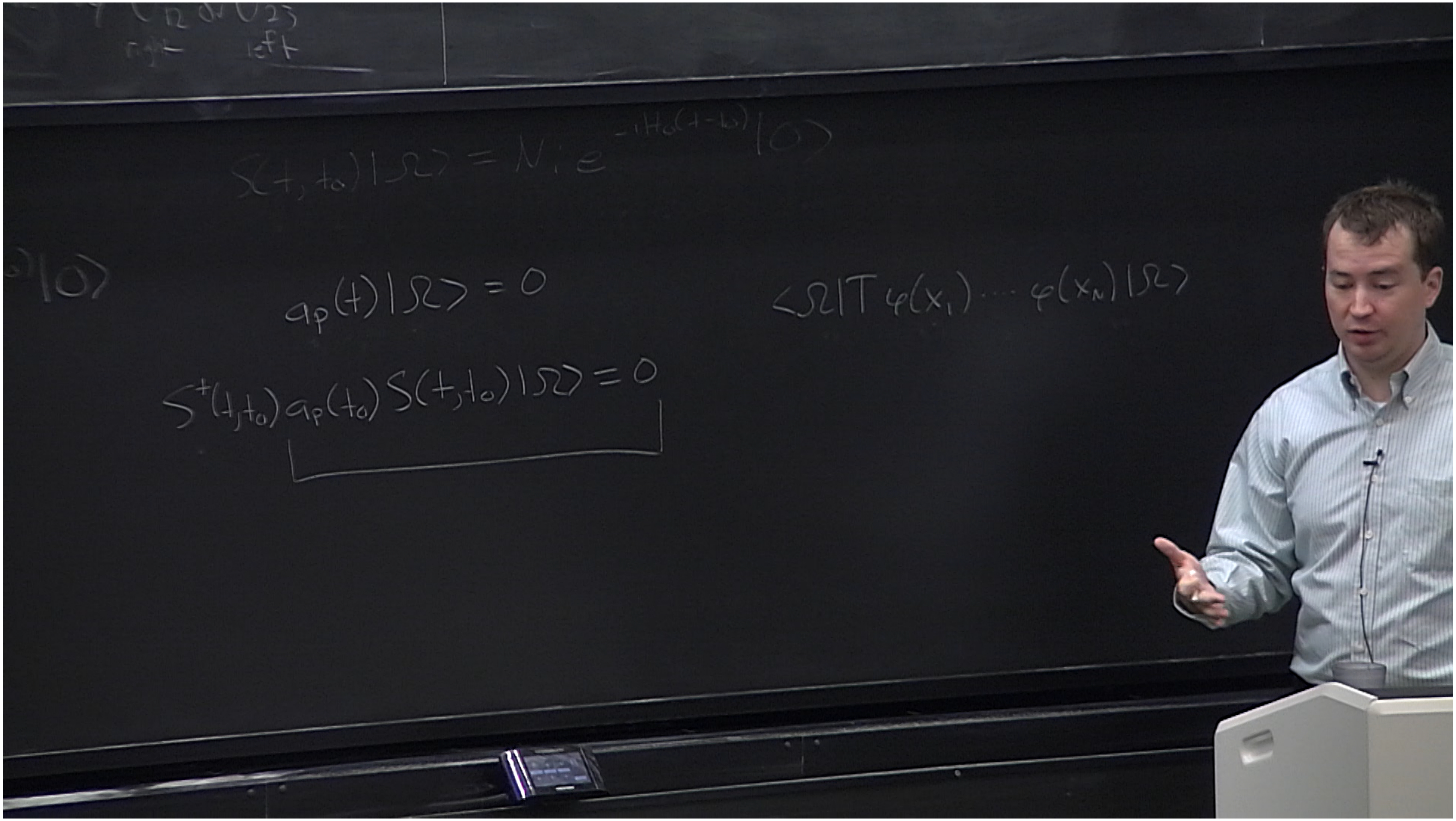
$$= N_i U_{0-\infty}|0\rangle$$

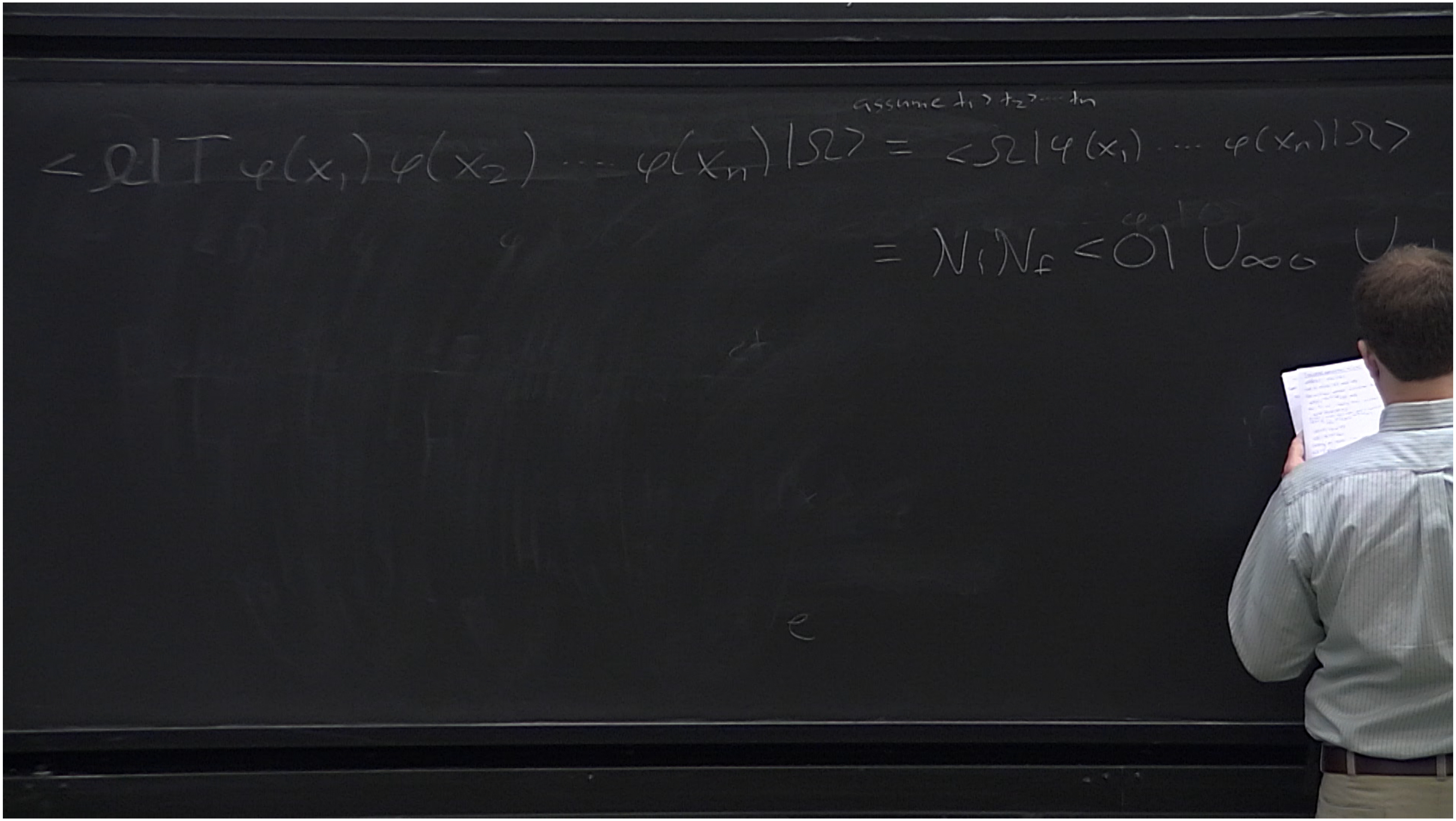
$$\langle\Omega| = N_f \langle 0| U_{\infty 0}$$

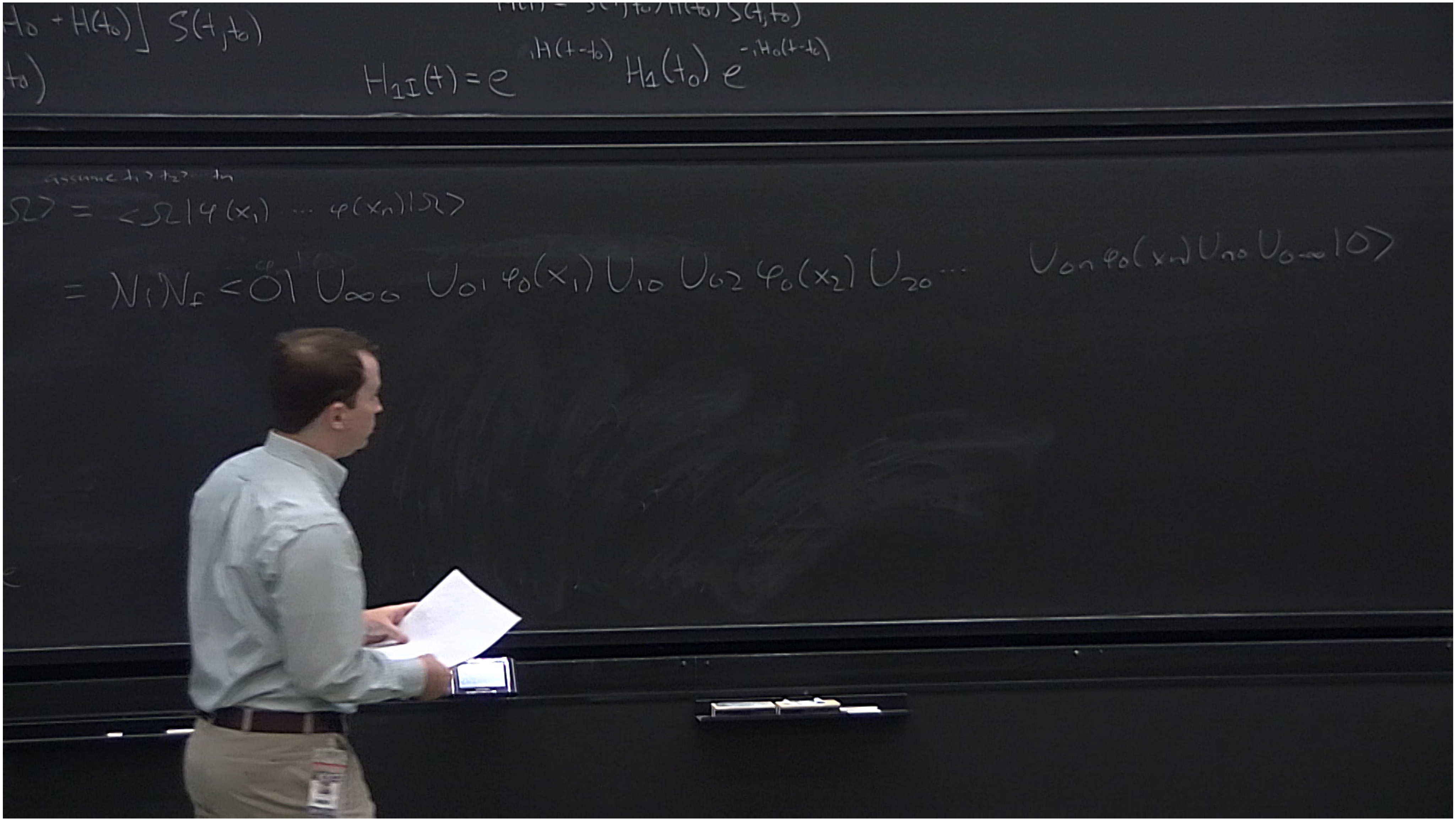
$$a_p(t)|\Omega\rangle = 0$$

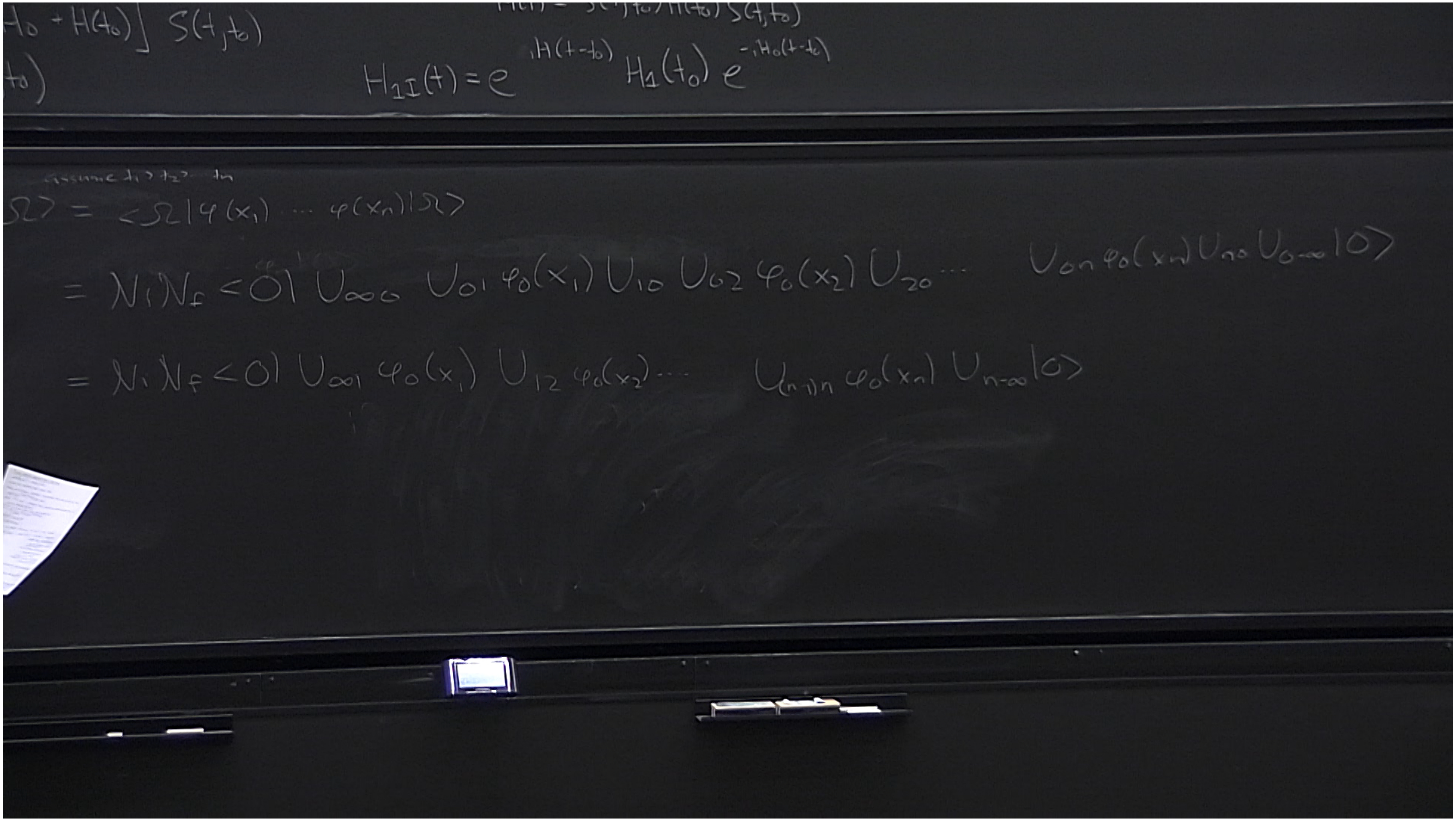
$$S^+(t,t_0)\, a_p(t_0)\, S(t,t_0)|\Omega\rangle = 0$$


<Ω|T φ(x₁) ···· φ(x_N)|Ω>

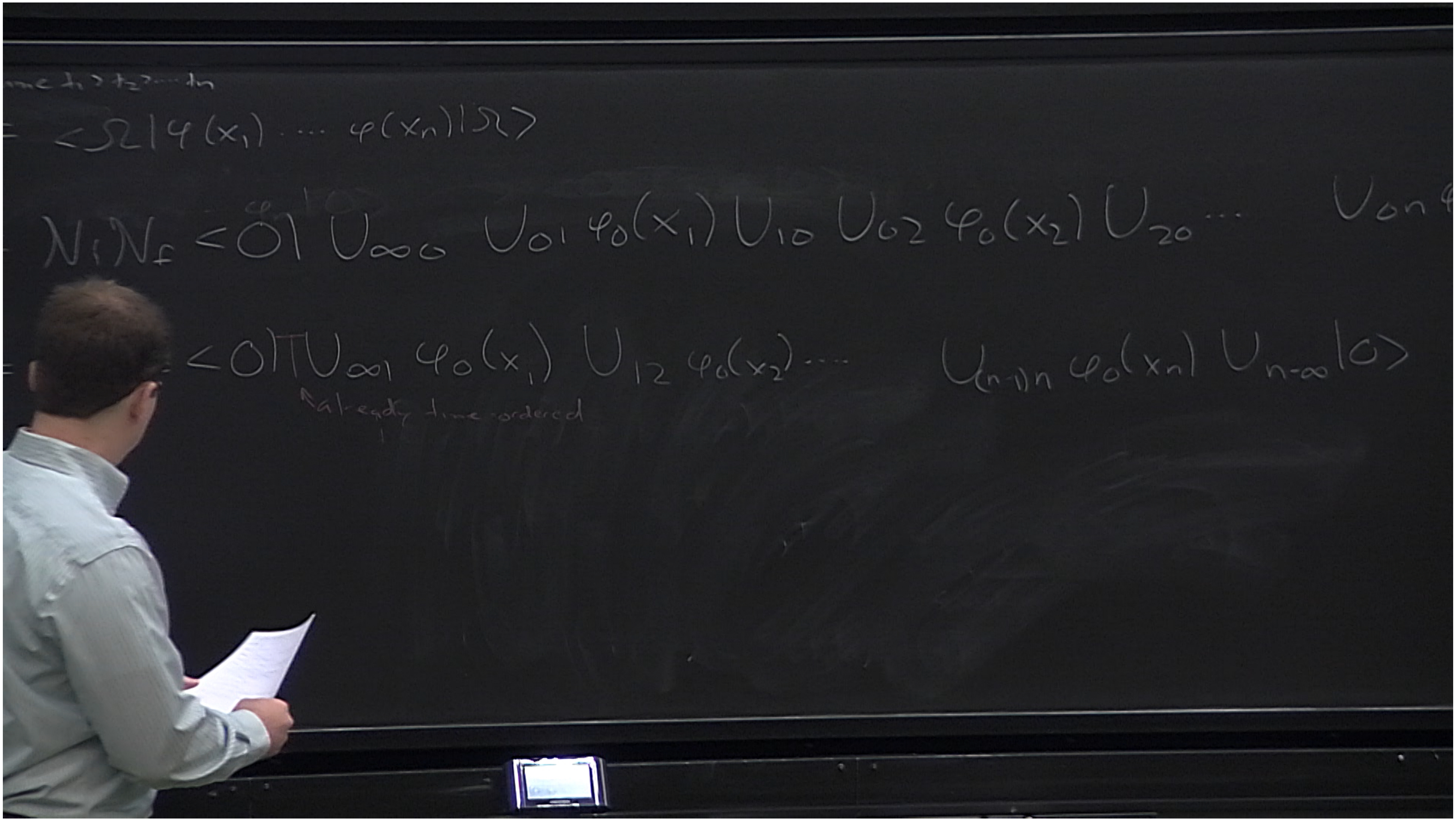

U_{12} or U_{23}
right left
$S(t,t_0)|\Omega\rangle = N_i e^{-iH_0(t-t_0)}|0\rangle$
$|0\rangle$
$a_p(t)|\Omega\rangle = 0$
$\langle\Omega|T\,\varphi(x_1)\cdots\varphi(x_N)|\Omega\rangle$
$S^{+}(t,t_0)\,a_p(t_0)\,S(t,t_0)|\Omega\rangle = 0$


assume $t_1 > t_2 > \cdots t_n$
$\langle \Omega | T \varphi(x_1) \varphi(x_2) \cdots \varphi(x_n) | \Omega \rangle = \langle \Omega | \varphi(x_1) \cdots \varphi(x_n) | \Omega \rangle$
$= N_i N_f \langle 0 | U_{\infty 0} \; U$


$H_{1I}(t) = e^{iH(t-t_0)} H_1(t_0) e^{-iH_0(t-t_0)}$
assume $t_1 > t_2 > \cdots t_n$
$\rangle = \langle \Omega | \varphi(x_1) \cdots \varphi(x_n) | \Omega \rangle$
$= N_i N_f \langle 0 | U_{\infty 0} U_{01} \varphi_0(x_1) U_{10} U_{02} \varphi_0(x_2) U_{20} \cdots U_{0n} \varphi_0(x_n) U_{n0} U_{0-\infty} | 0 \rangle$



$H_0 + H(t_0)] \, S(t,t_0)$
$H_{1I}(t) = e^{iH(t-t_0)} H_1(t_0) e^{-iH_0(t-t_0)}$
assume $t_1 > t_2 > \cdots t_n$
$\Omega\rangle = \langle\Omega|\varphi(x_1) \cdots \varphi(x_n)|\Omega\rangle$
$= N_i N_f \langle 0| U_{\infty 0} U_{01} \varphi_0(x_1) U_{10} U_{02} \varphi_0(x_2) U_{20} \cdots U_{0n} \varphi_0(x_n) U_{n0} U_{0-\infty}|0\rangle$
$= N_i N_f \langle 0| U_{\infty 1} \varphi_0(x_1) U_{12} \varphi_0(x_2) \cdots U_{(n-1)n} \varphi_0(x_n) U_{n-\infty}|0\rangle$


= <Ω|φ(x1) .... φ(xn)|Ω>
= Ni Nf <0| U∞0 U01 φ0(x1) U10 U02 φ0(x2) U20 ...
= <0|T U∞1 φ0(x1) U12 φ0(x2) .... U(n-1)n φ0(xn) Un-∞|0>
already time-ordered

$$-H(t_0)\,]\; S(t,t_0)$$

$$H_{1I}(t) = e^{iH(t-t_0)}\, H_1(t_0)\, e^{-iH_0(t-t_0)}$$

assume $t_1 > t_2 > \cdots t_n$

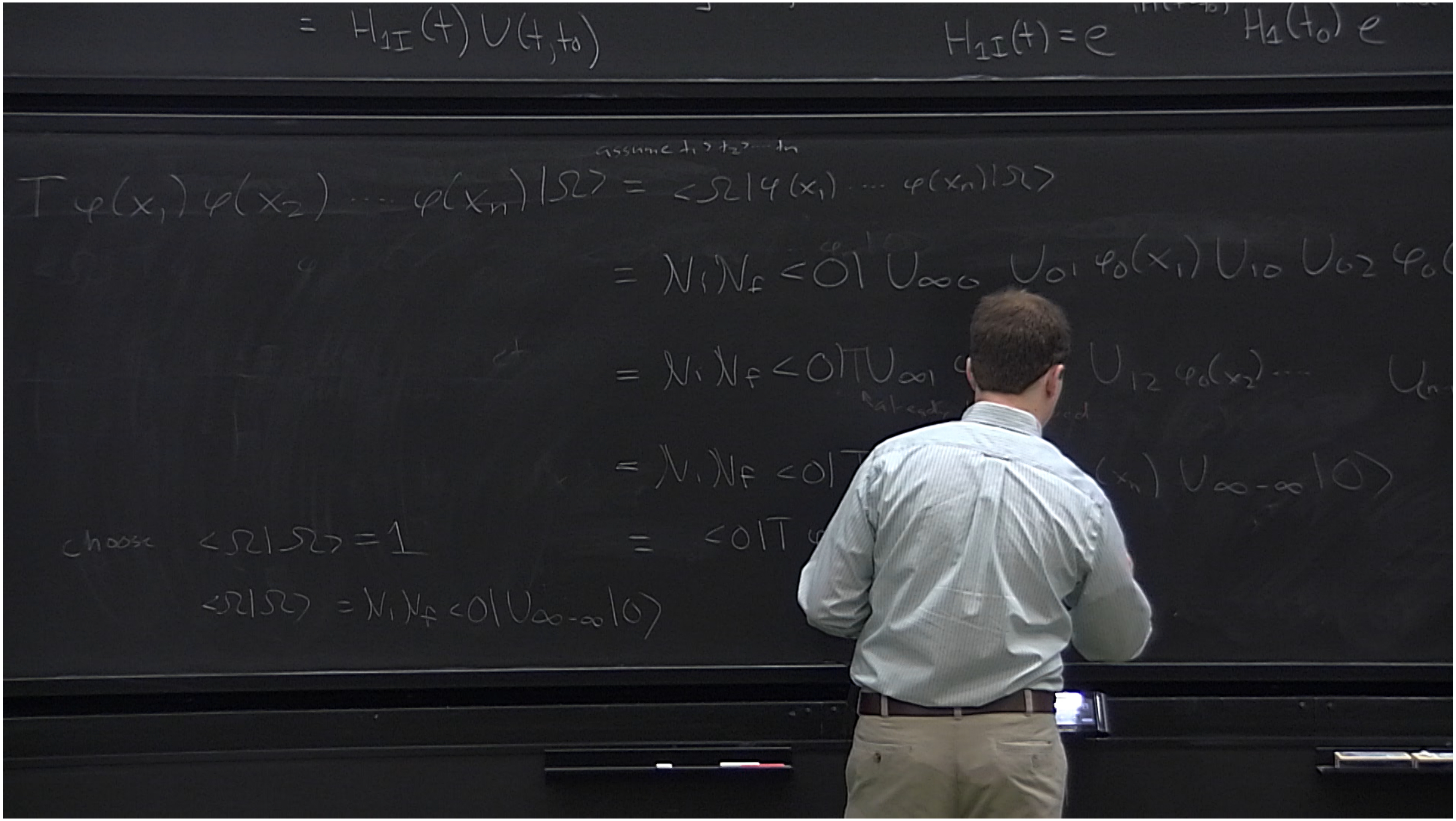
$$\Omega\rangle = \langle \Omega | \varphi(x_1) \cdots \varphi(x_n) | \Omega \rangle$$

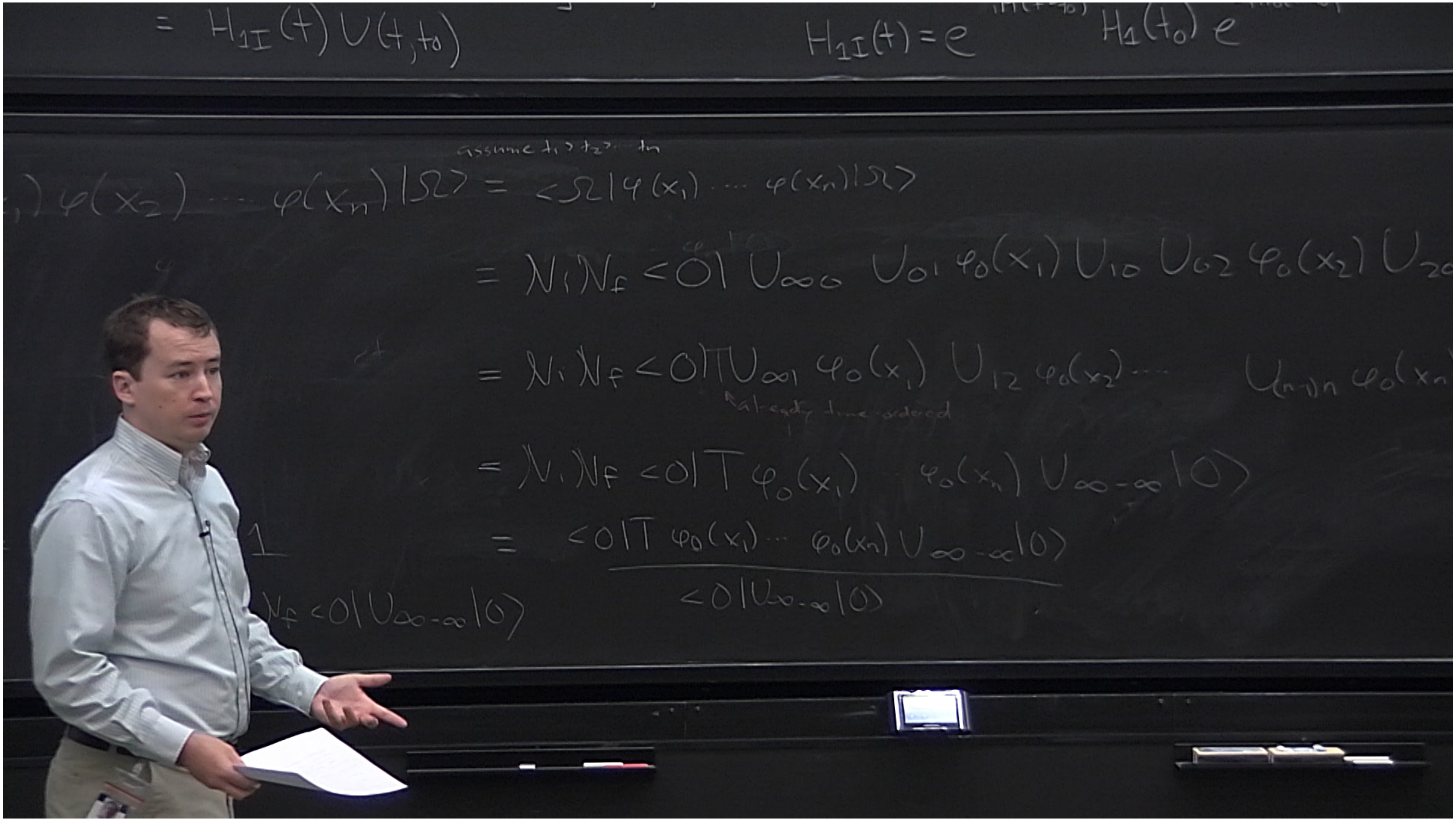
$$= N_i N_f \langle 0 | U_{\infty 0}\, U_{01}\, \varphi_0(x_1)\, U_{10}\, U_{02}\, \varphi_0(x_2)\, U_{20} \cdots U_{0n}\, \varphi_0(x_n)\, U_{n0}\, U_{0-\infty} | 0 \rangle$$

$$= N_i N_f \langle 0 | T\, U_{\infty 1}\, \varphi_0(x_1)\, U_{12}\, \varphi_0(x_2) \cdots U_{(n-1)n}\, \varphi_0(x_n)\, U_{n-\infty} | 0 \rangle$$

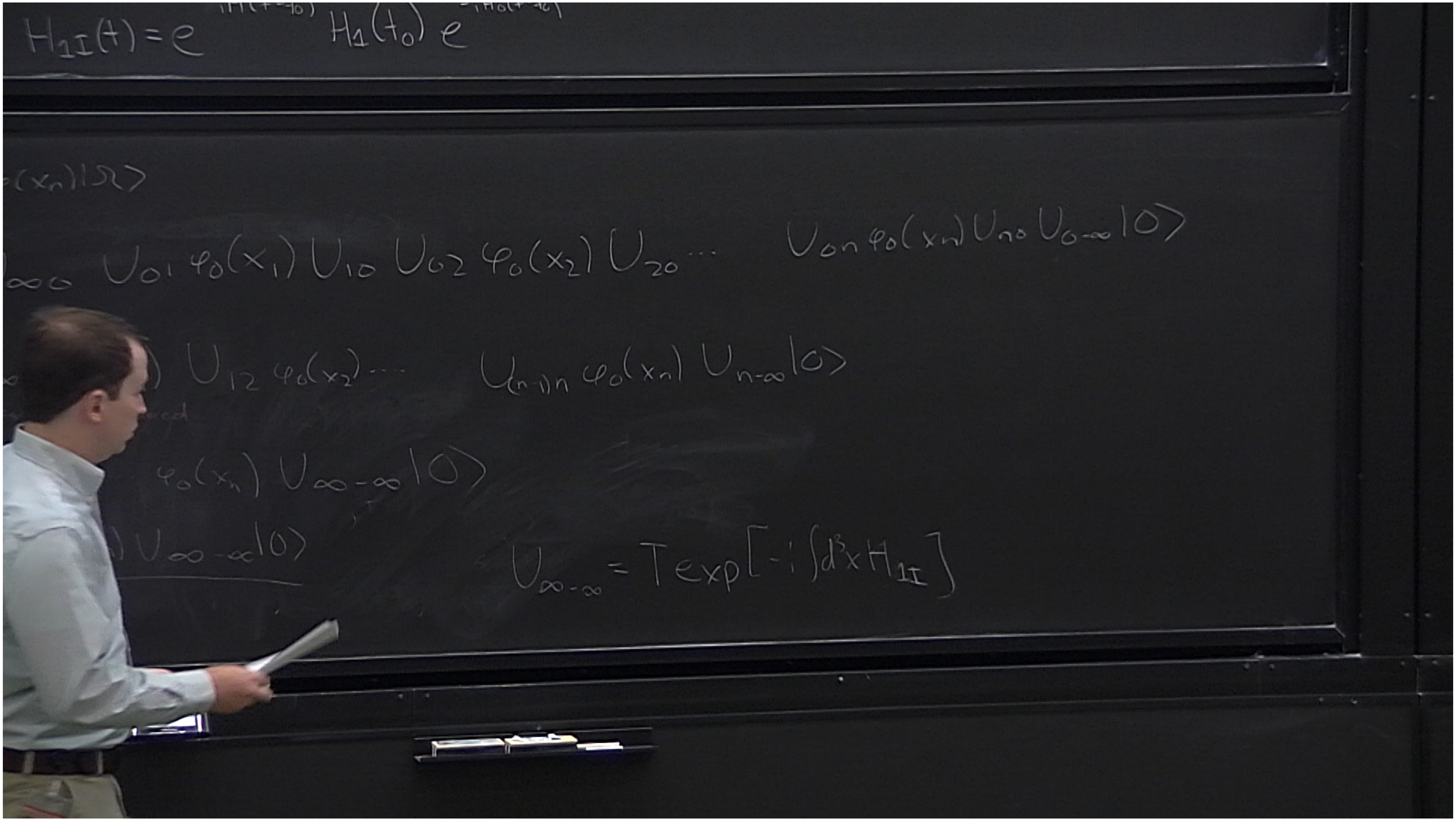
already time-ordered

$$= N_i N_f \langle 0 | T\, \varphi_0(x_1) \cdots \varphi_0(x_n)\, U_{\infty-\infty} | 0 \rangle$$

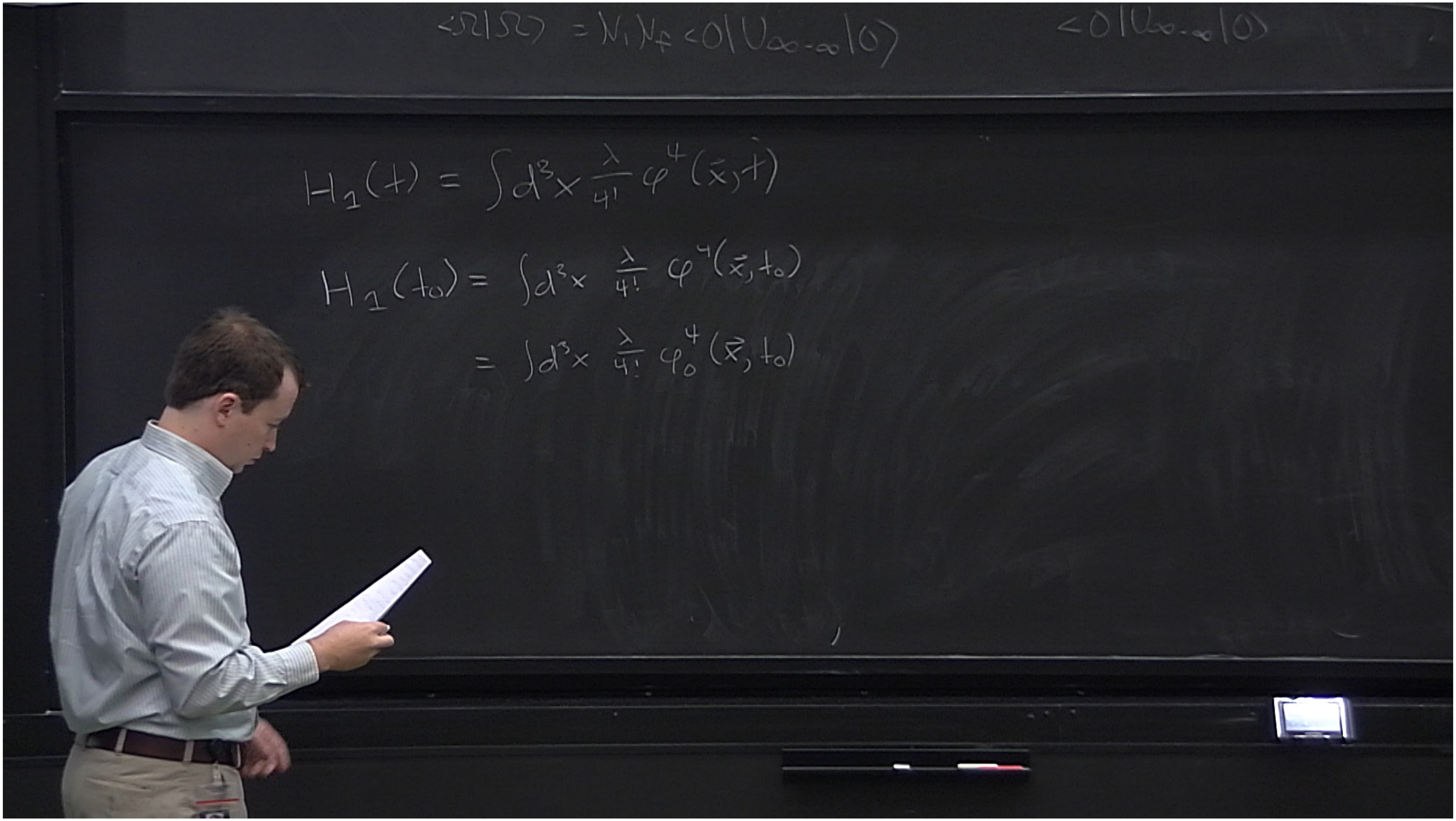

$= H_{1I}(t)\,U(t,t_0)$
$H_{1I}(t) = e \quad H_1(t_0)\, e$
assume $t_1 > t_2 > \cdots t_n$
$T\,\varphi(x_1)\,\varphi(x_2)\cdots\varphi(x_n)|\Omega\rangle = \langle\Omega|\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle$
$= N_i N_f \langle 0|\,U_{\infty 0}\;U_{01}\,\varphi_0(x_1)\,U_{10}\,U_{02}\,\varphi_0($
$= N_i N_f \langle 0|T U_{\infty 1}$
$U_{12}\,\varphi_0(x_2)\cdots$
$= N_i N_f \langle 0|T$
$(x_n)\,U_{\infty -\infty}|0\rangle$
$= \langle 0|T$
choose $\langle\Omega|\Omega\rangle = 1$
$\langle\Omega|\Omega\rangle = N_i N_f \langle 0|\,U_{\infty-\infty}|0\rangle$



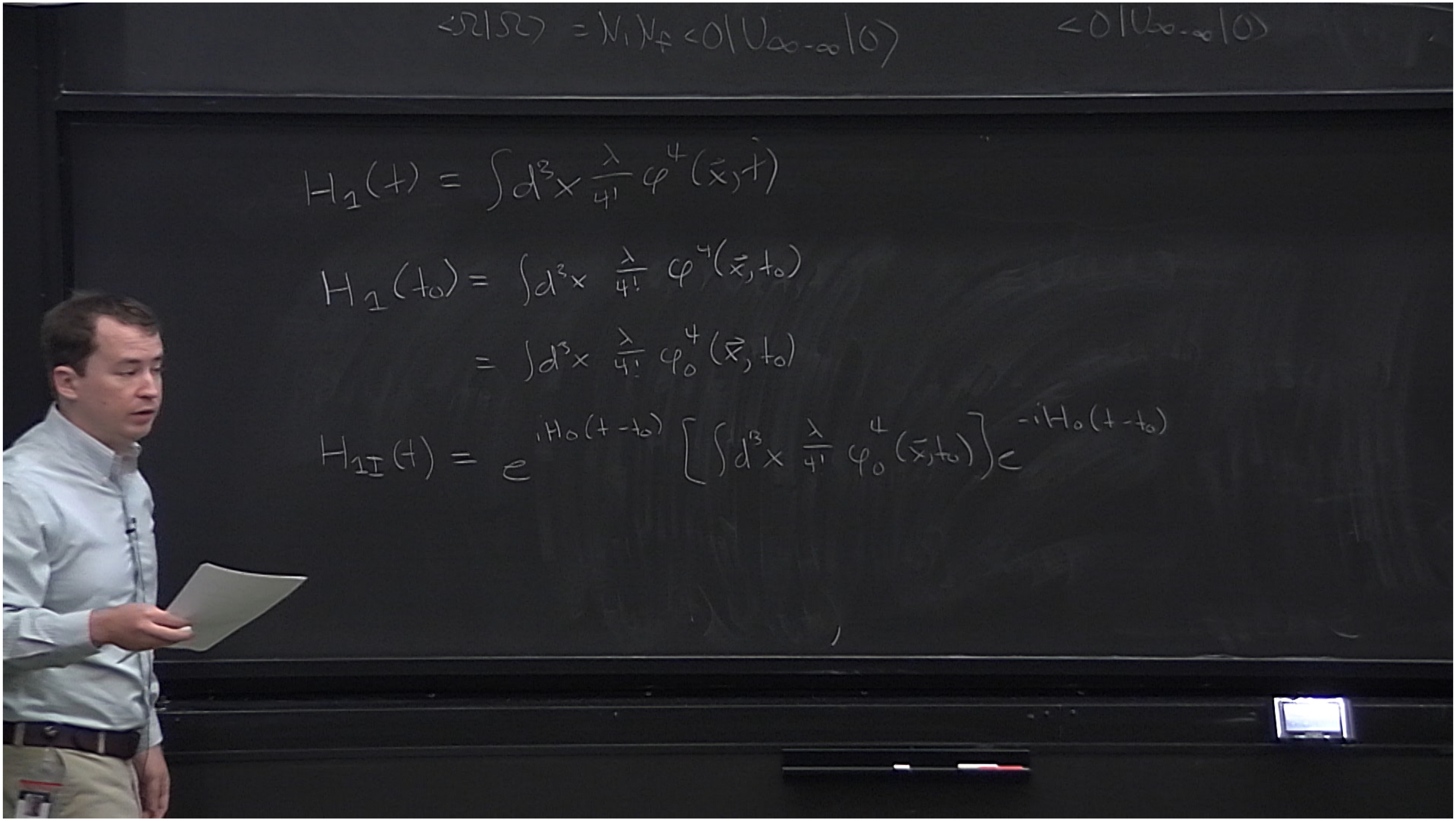
= H1I(t) U(t, t0)
H1I(t) = e H1(t0) e
assume t1 > t2 > ... tn
= ⟨Ω| φ(x1) ... φ(xn) |Ω⟩
= Ni Nf ⟨0| U∞0 U01 φ0(x1) U10 U02 φ0(x2)
= Ni Nf ⟨0| T U∞1 φ0(x1) U12 φ0(x2) ... U(n-1)n φ0(xn)
already time-ordered
= Ni Nf ⟨0| T φ0(x1) φ0(xn) U∞-∞ |0⟩
= ⟨0| T φ0(x1) ... φ0(xn) U∞-∞ |0⟩ / ⟨0| U∞-∞ |0⟩
Nf ⟨0| U∞-∞ |0⟩



$H_{1I}(t) = e^{iH_0(t-t_0)} H_1(t_0)\, e^{-iH_0(t-t_0)}$
$U_{01}\,\varphi_0(x_1)\,U_{10}\,U_{02}\,\varphi_0(x_2)\,U_{20} \cdots \; U_{0n}\,\varphi_0(x_n)\,U_{n0}\,U_{0-\infty}|0\rangle$
$U_{12}\,\varphi_0(x_2) \cdots \; U_{(n-1)n}\,\varphi_0(x_n)\,U_{n-\infty}|0\rangle$
$\varphi_0(x_n)\,U_{\infty-\infty}|0\rangle$
$U_{\infty-\infty}|0\rangle$
$U_{\infty-\infty} = T\exp\left[-i\int d^3x\, H_{1I}\right]$



$\langle \Omega | \Omega \rangle = N_i N_f \langle 0 | U_{\infty, -\infty} | 0 \rangle$
$\langle 0 | U_{\infty, -\infty} | 0 \rangle$
$H_1(t) = \int d^3x \frac{\lambda}{4!} \varphi^4(\vec{x}, t)$
$H_1(t_0) = \int d^3x \frac{\lambda}{4!} \varphi^4(\vec{x}, t_0)$
$= \int d^3x \frac{\lambda}{4!} \varphi_0^4(\vec{x}, t_0)$


$\langle \Omega | \Omega \rangle = N_i N_f \langle 0 | U_{\infty -\infty} | 0 \rangle$
$\langle 0 | U_{\infty -\infty} | 0 \rangle$
$H_1(t) = \int d^3x \frac{\lambda}{4!} \varphi^4(\vec{x}, t)$
$H_1(t_0) = \int d^3x \frac{\lambda}{4!} \varphi^4(\vec{x}, t_0)$
$= \int d^3x \frac{\lambda}{4!} \varphi_0^4(\vec{x}, t_0)$
$H_{1I}(t) = e^{iH_0(t-t_0)} \left[\int d^3x \frac{\lambda}{4!} \varphi_0^4(\vec{x}, t_0) \right] e^{-iH_0(t-t_0)}$

$$H_1(t) = \int d^3x \, \frac{\lambda}{4!} \varphi^4(\vec{x}, t)$$

$$H_1(t_0) = \int d^3x \, \frac{\lambda}{4!} \varphi^4(\vec{x}, t_0)$$

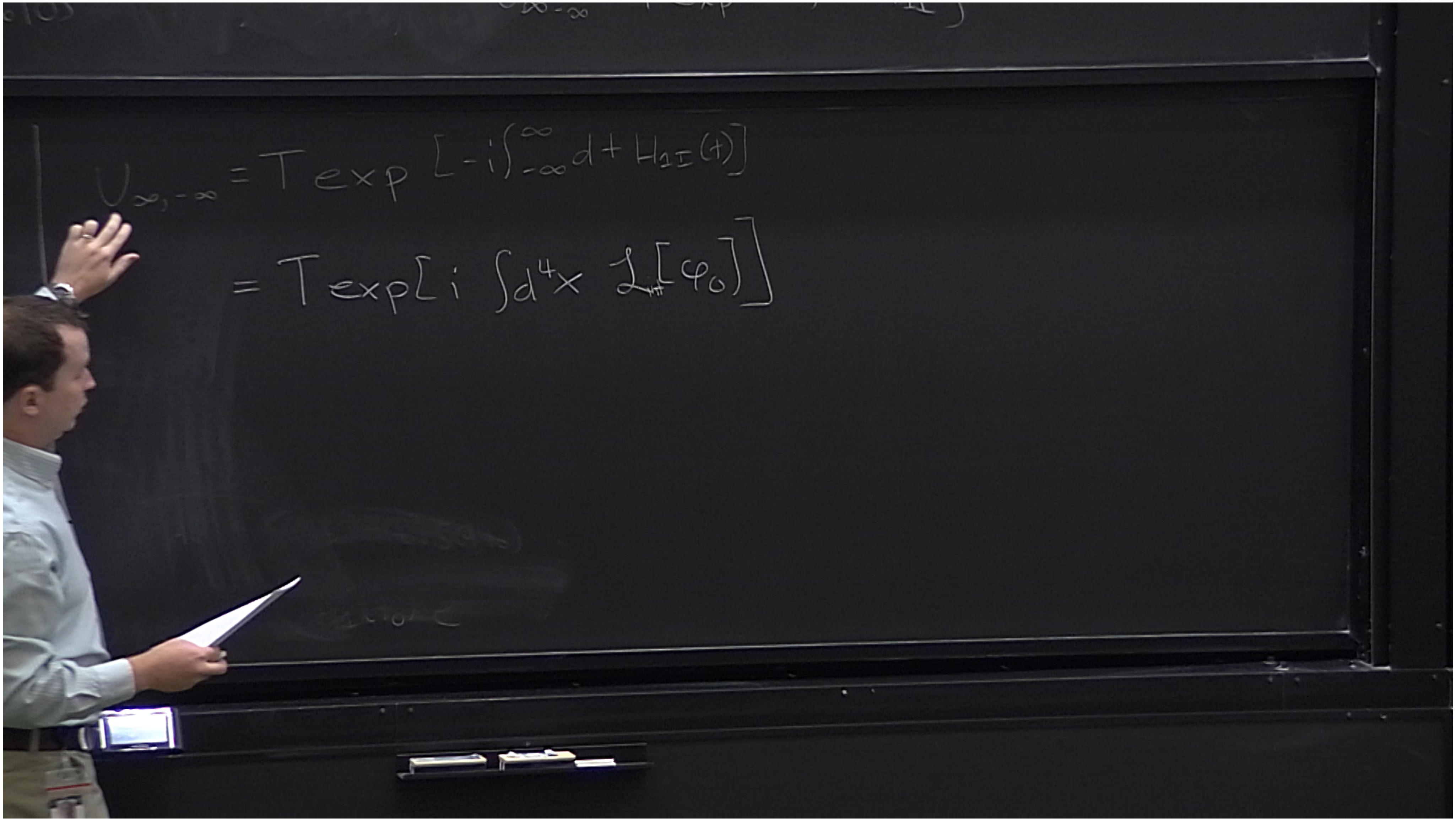
$$= \int d^3x \, \frac{\lambda}{4!} \varphi_0^4(\vec{x}, t_0)$$

$$H_{1I}(t) = e^{iH_0(t-t_0)} \left[\int d^3x \, \frac{\lambda}{4!} \varphi_0^4(\vec{x}, t_0) \right] e^{-iH_0(t-t_0)}$$

$$= \int d^3x \, \frac{\lambda}{4!} \varphi_0^4(\vec{x}, t)$$

$$U_{\infty,-\infty} = T\exp\left[-i\int_{-\infty}^{\infty} dt\, H_{1I}(t)\right]$$

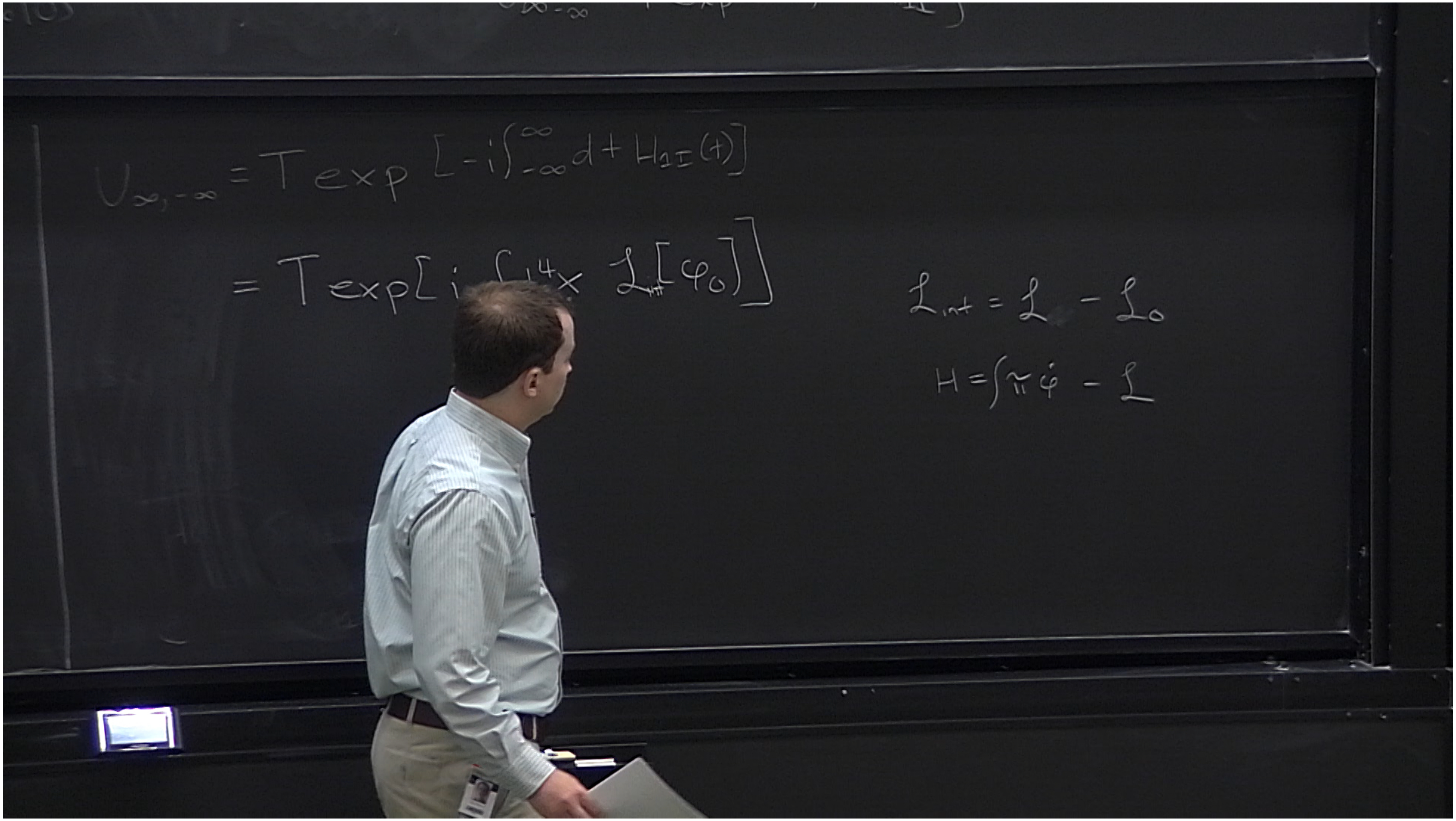
$$=$$


$U_{\infty,-\infty} = T\exp\left[-i\int_{-\infty}^{\infty} dt\, H_{1I}(t)\right]$
$= T\exp\left[i\int d^4x\, \mathcal{L}_{int}[\varphi_0]\right]$

$$U_{\infty,-\infty} = T\exp\left[-i\int_{-\infty}^{\infty} dt\, H_{1I}(t)\right]$$

$$= T\exp\left[i\int d^4x\, \mathcal{L}_{int}[\varphi_0]\right]$$

$$\mathcal{L}_{int} = \mathcal{L} - \mathcal{L}_0$$



$U_{\infty,-\infty} = T\exp[-i\int_{-\infty}^{\infty} dt\, H_{1I}(t)]$
$= T\exp[i\int d^4x\, \mathcal{L}_{int}[\varphi_0]]$
$\mathcal{L}_{int} = \mathcal{L} - \mathcal{L}_0$
$H = \int \pi\dot{\varphi} - \mathcal{L}$

$$U_{\infty,-\infty} = T\exp\left[-i\int_{-\infty}^{\infty} dt\, H_{1I}(t)\right]$$

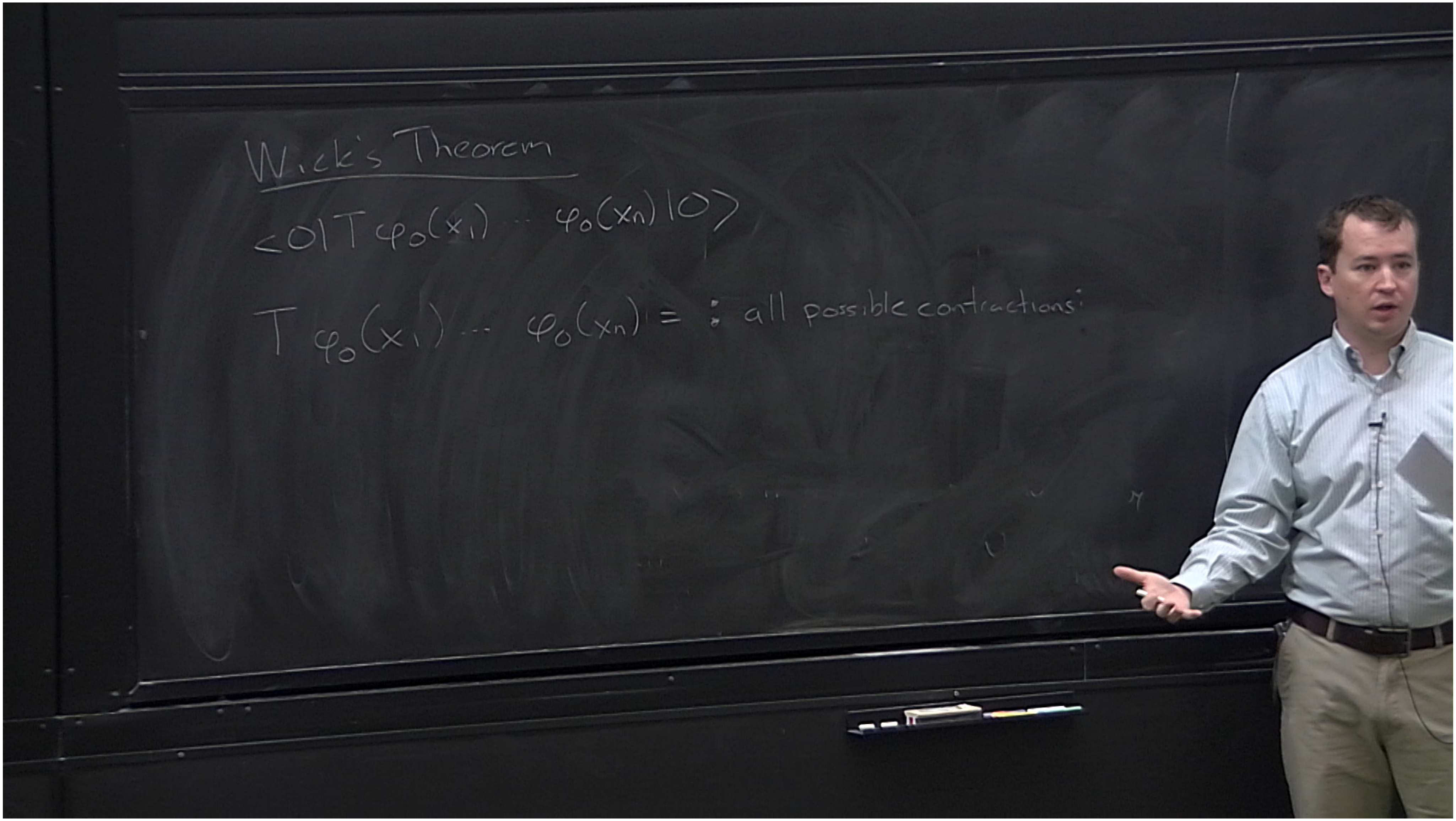
$$= T\exp\left[i\int d^4x\, \mathcal{L}_{int}[\varphi_0]\right]$$

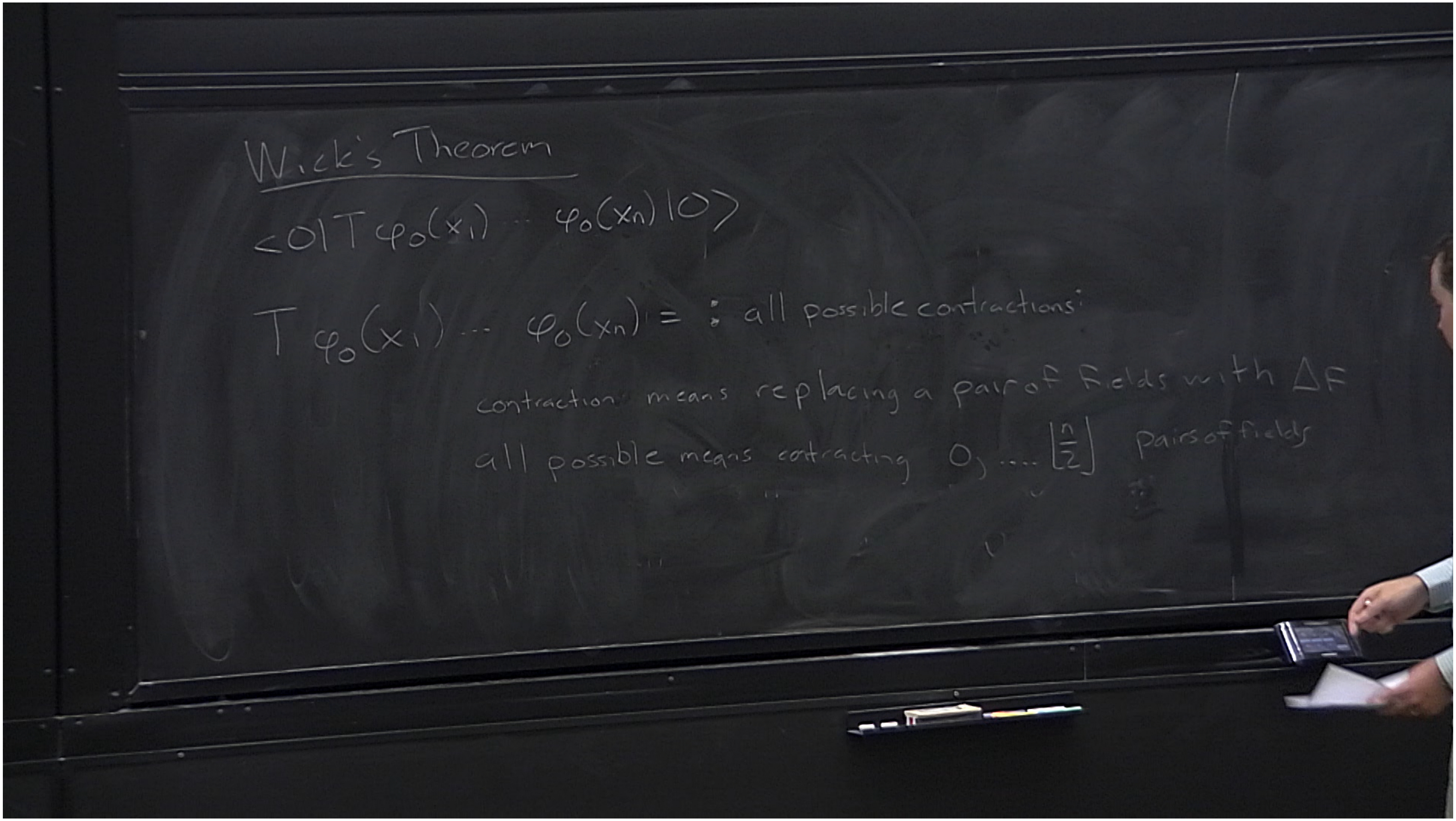
$$\mathcal{L}_{int} = \mathcal{L} - \mathcal{L}_0$$

$$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \frac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp\left[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]\right]|0\rangle}{\langle 0|T\exp\left[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]\right]|0\rangle}$$

Dyson's formula


Wick's Theorem
<0|T φo(x1) ... φo(xn)|0>


Wick's Theorem
$\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)|0\rangle$
$T\varphi_0(x_1)\cdots\varphi_0(x_n) =$: all possible contractions :


Wick's Theorem
$\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)|0\rangle$
$T\varphi_0(x_1)\cdots\varphi_0(x_n) = :\text{all possible contractions}:$
contraction means replacing a pair of fields with Δ_F
all possible means contracting $0, \ldots, \lfloor \frac{n}{2} \rfloor$ pairs of fields

Wick's Theorem

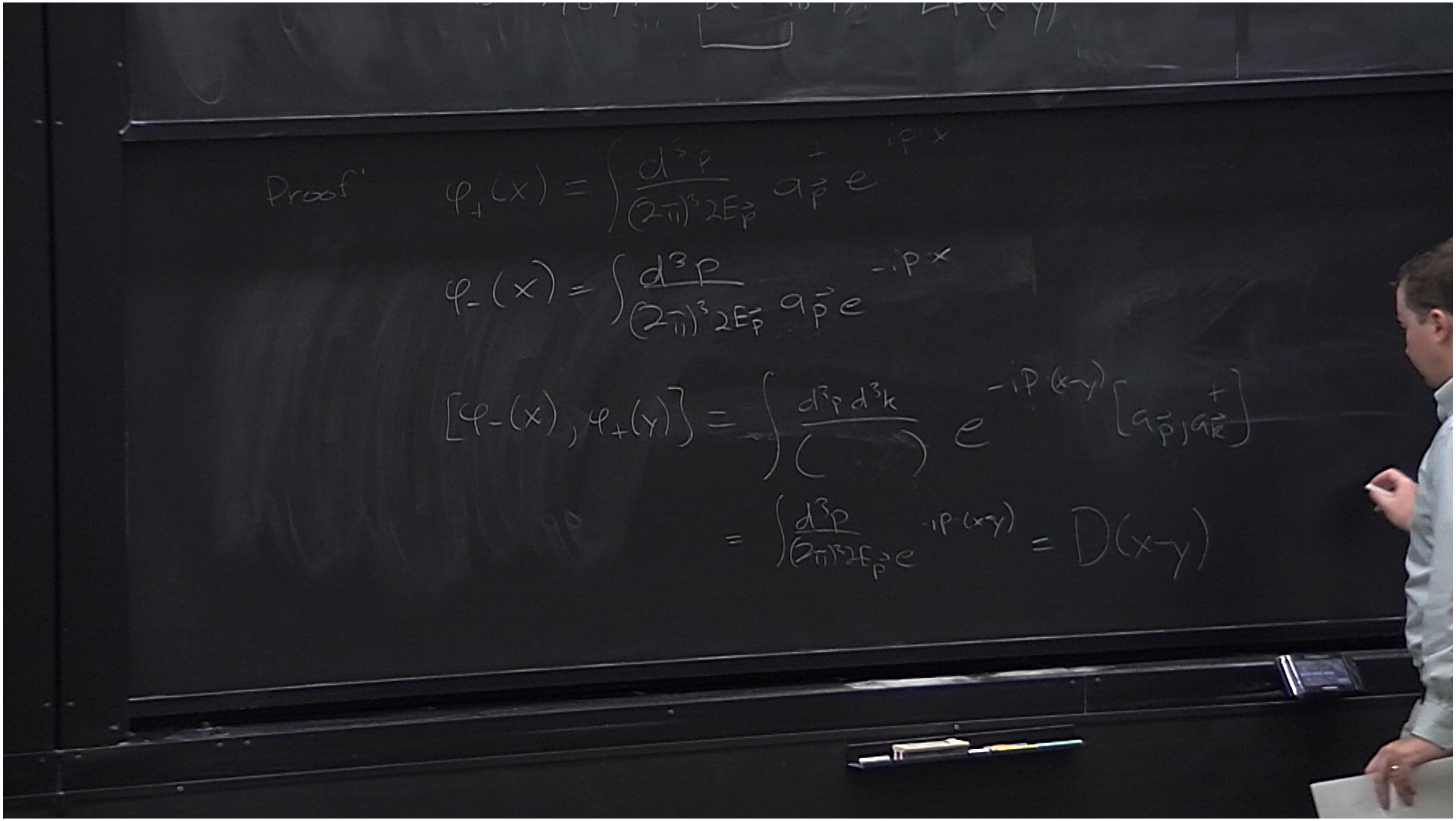
$\langle 0 | T \varphi_0(x_1) \cdots \varphi_0(x_n) | 0 \rangle$

$T \varphi_0(x_1) \cdots \varphi_0(x_n) = :$ all possible contractions $:$

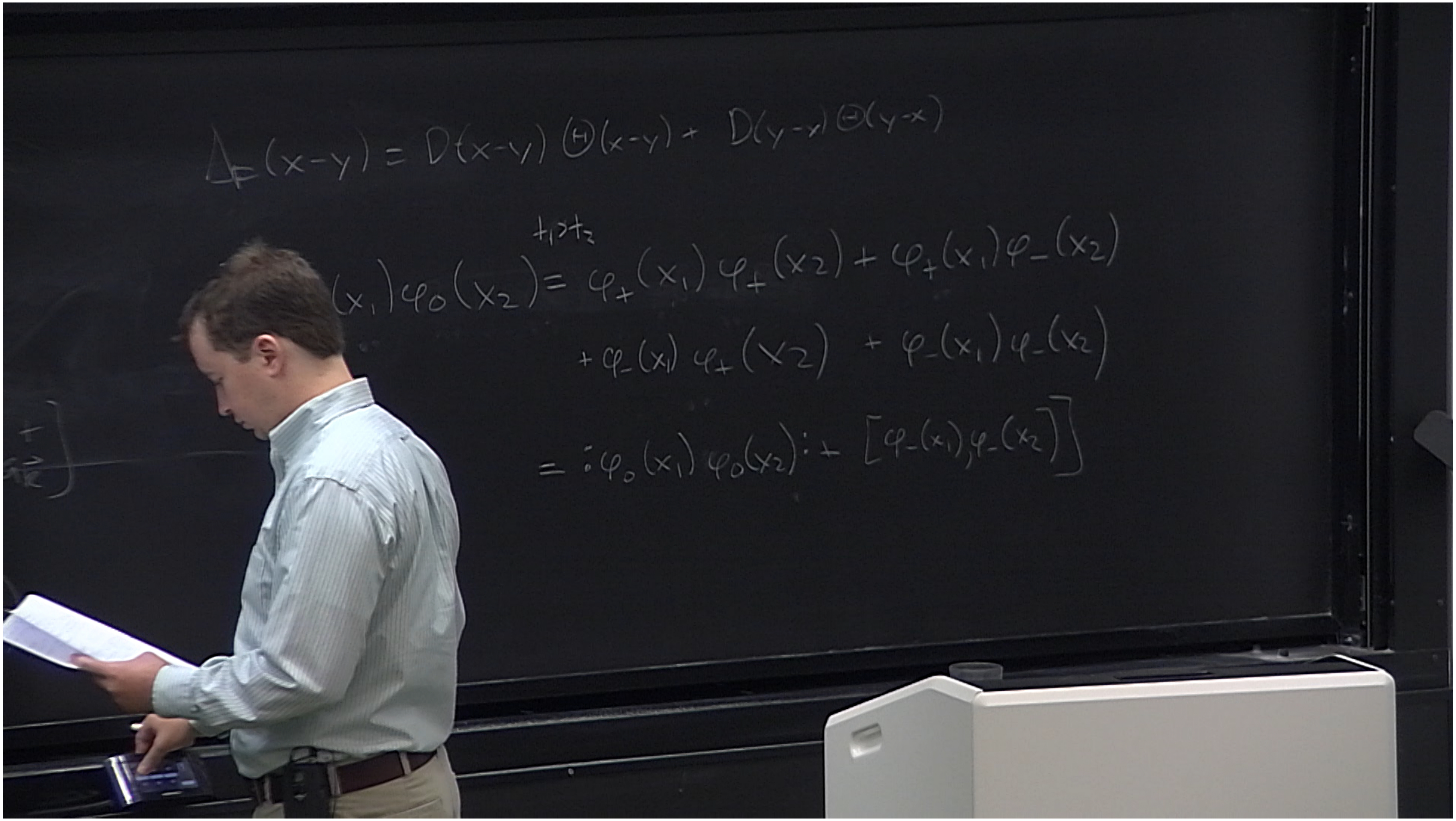
contraction means replacing a pair of fields with Δ_F

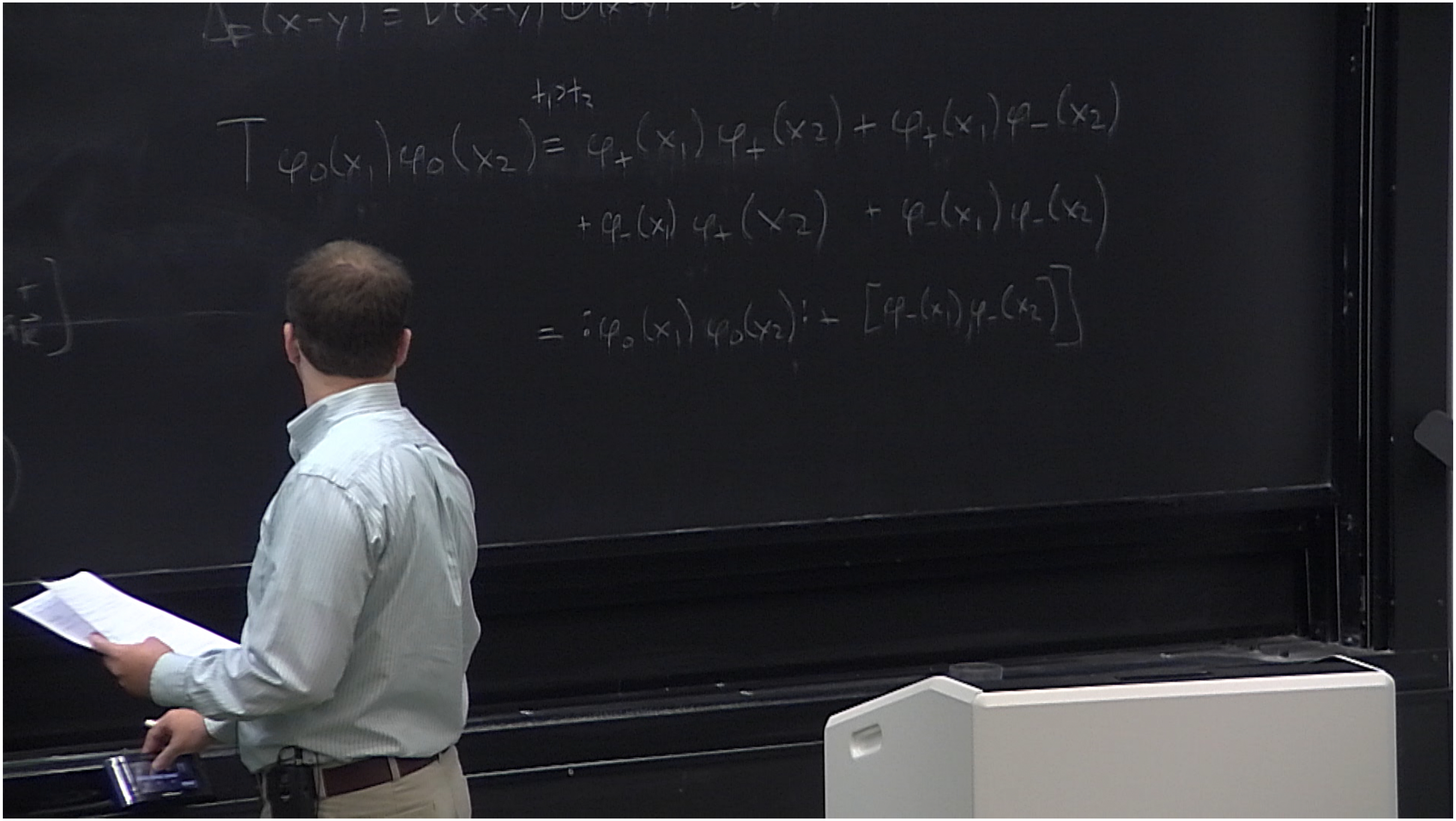
all possible means contracting $0, \ldots, \left\lfloor \frac{n}{2} \right\rfloor$ pairs of fields

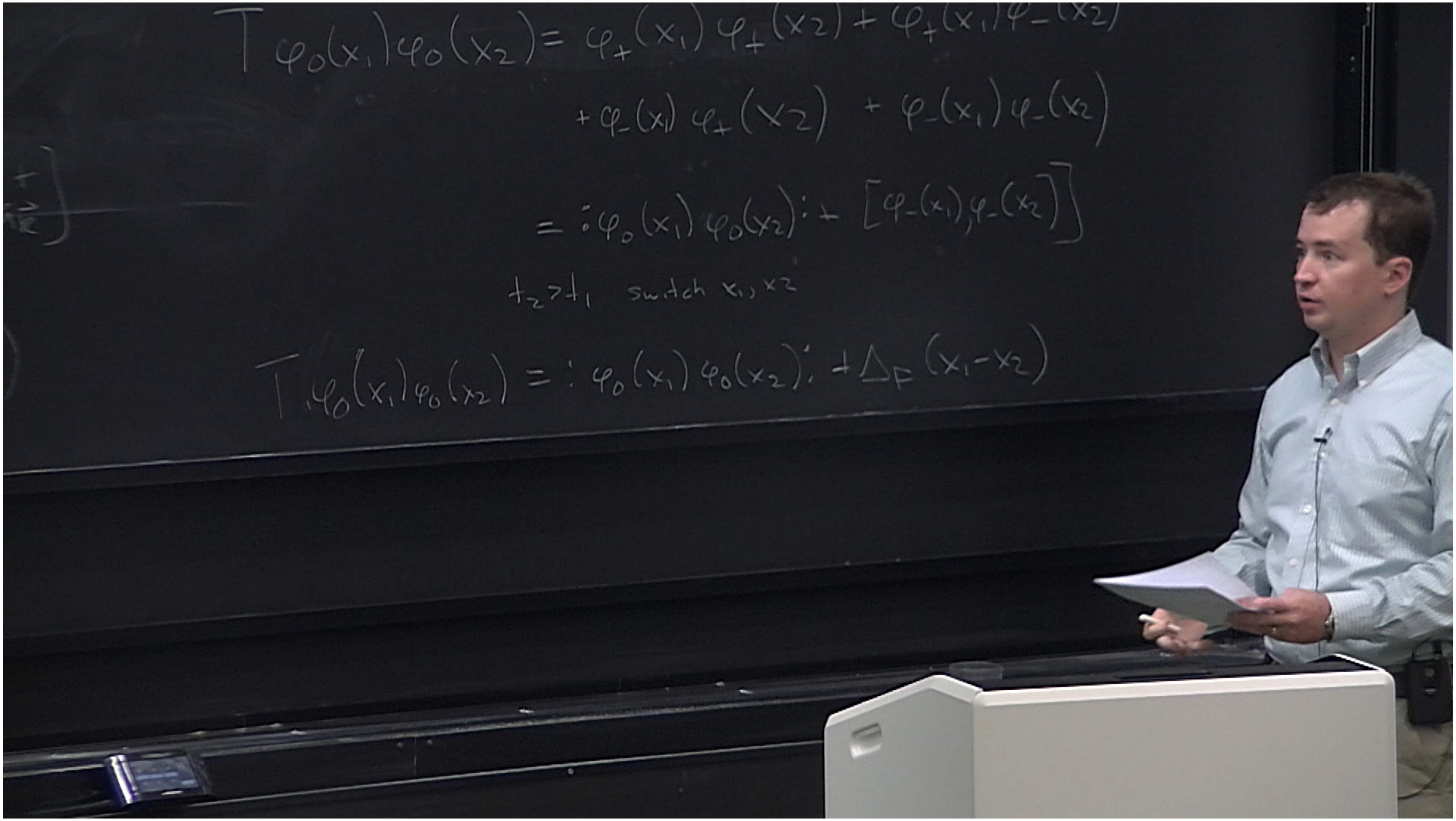
$\varphi_0(x)\varphi_0(y) = \varphi_0(x)\varphi_0(y) = \Delta_F(x-y)$


Proof:
$\varphi_+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} a^\dagger_{\vec p} e^{ip\cdot x}$
$\varphi_-(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} a_{\vec p} e^{-ip\cdot x}$
$[\varphi_-(x), \varphi_+(y)] = \int \frac{d^3p\, d^3k}{(\quad)} e^{-ip\cdot(x-y)} [a_{\vec p}, a^\dagger_{\vec k}]$
$= \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} e^{-ip\cdot(x-y)} = D(x-y)$

$$\Delta_F(x-y) = D(x-y)\,\Theta(x-y) + D(y-x)\,\Theta(y-x)$$


$\Delta_F(x-y) = D(x-y)\,\Theta(x-y) + D(y-x)\,\Theta(y-x)$
$t_1 > t_2$
$(x_1)\varphi_0(x_2) = \varphi_+(x_1)\varphi_+(x_2) + \varphi_+(x_1)\varphi_-(x_2)$
$+\varphi_-(x_1)\varphi_+(x_2) + \varphi_-(x_1)\varphi_-(x_2)$
$= :\varphi_0(x_1)\varphi_0(x_2): + [\varphi_-(x_1), \varphi_-(x_2)]$


$t_1 > t_2$
$T\,\varphi_0(x_1)\varphi_0(x_2) = \varphi_+(x_1)\varphi_+(x_2) + \varphi_+(x_1)\varphi_-(x_2)$
$+ \varphi_-(x_1)\varphi_+(x_2) + \varphi_-(x_1)\varphi_-(x_2)$
$= :\varphi_0(x_1)\varphi_0(x_2): + [\varphi_-(x_1), \varphi_-(x_2)]$



$T\varphi_0(x_1)\varphi_0(x_2) = \varphi_+(x_1)\varphi_+(x_2) + \varphi_+(x_1)\varphi_-(x_2)$
$+ \varphi_-(x_1)\varphi_+(x_2) + \varphi_-(x_1)\varphi_-(x_2)$
$= :\varphi_0(x_1)\varphi_0(x_2): + [\varphi_-(x_1),\varphi_-(x_2)]$
$t_2 > t_1$ switch x_1, x_2
$T\varphi_0(x_1)\varphi_0(x_2) = :\varphi_0(x_1)\varphi_0(x_2): + \Delta_F(x_1-x_2)$

$$\Delta_F(x-y) = D(x-y)\,\Theta(x-y) + D(y-x)\,\Theta(y-x)$$

$t_1 > t_2$

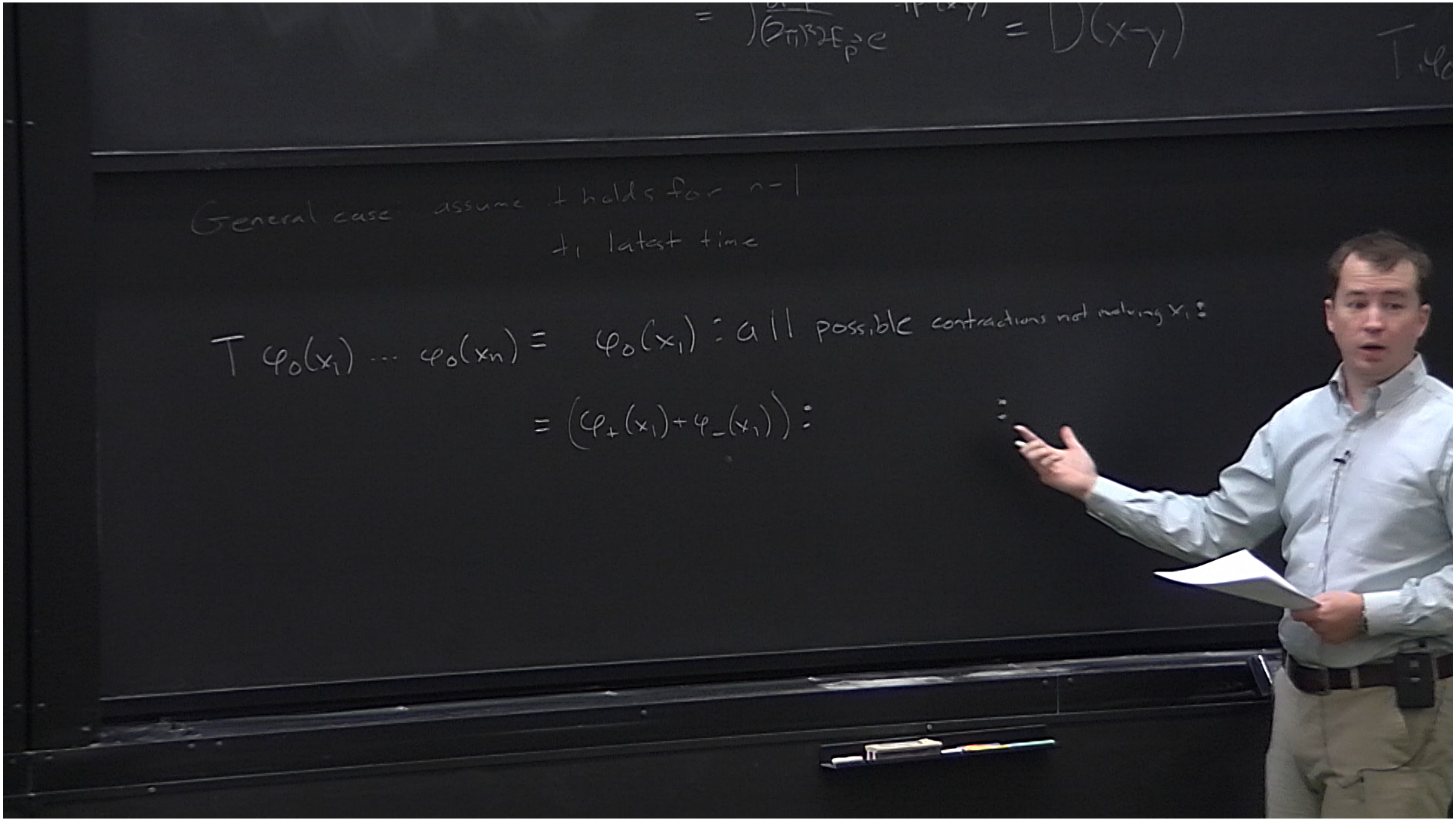
$$T\,\varphi_0(x_1)\varphi_0(x_2) = \varphi_+(x_1)\varphi_+(x_2) + \varphi_+(x_1)\varphi_-(x_2)$$

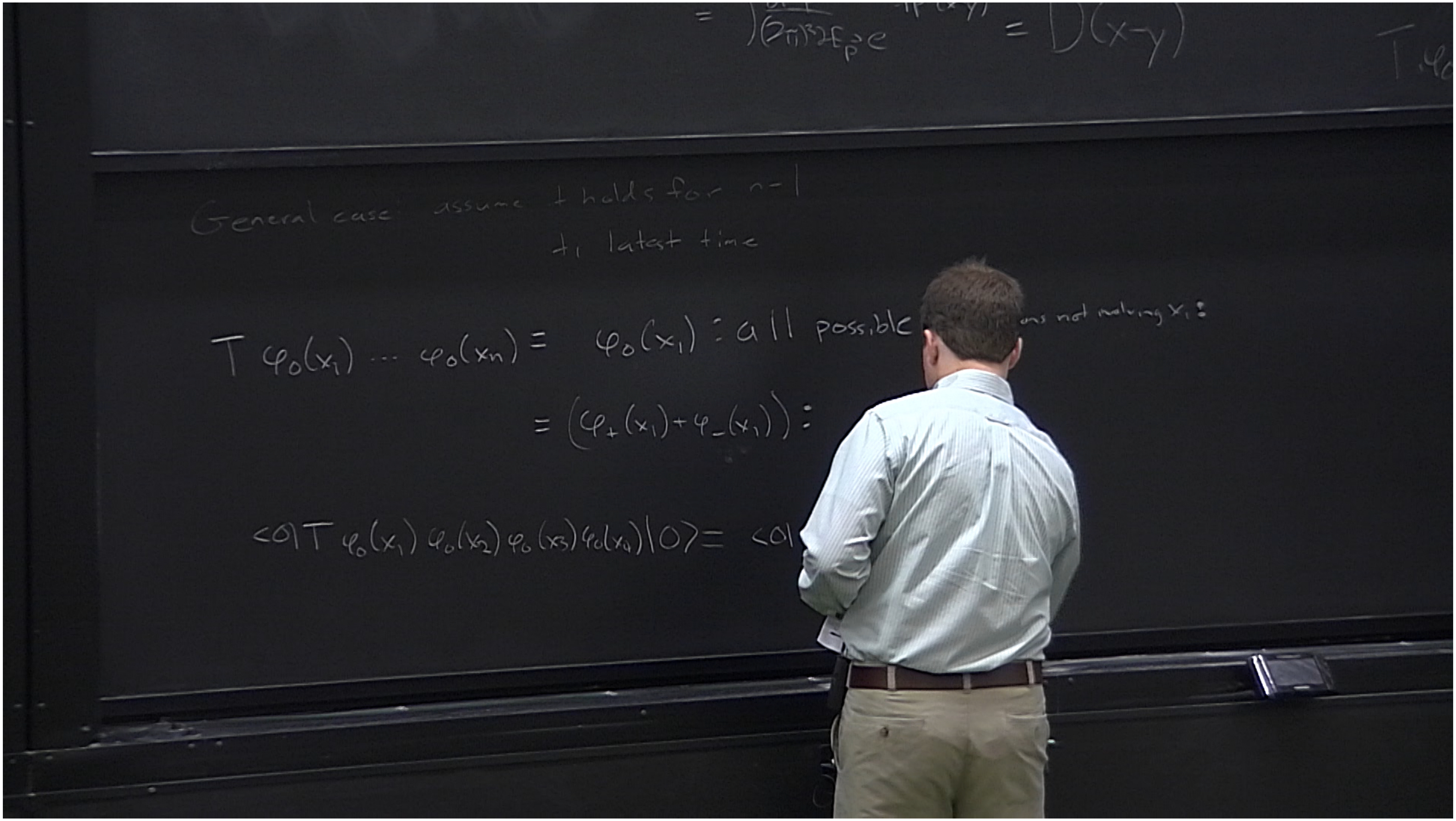
$$+\,\varphi_-(x_1)\varphi_+(x_2) + \varphi_-(x_1)\varphi_-(x_2)$$

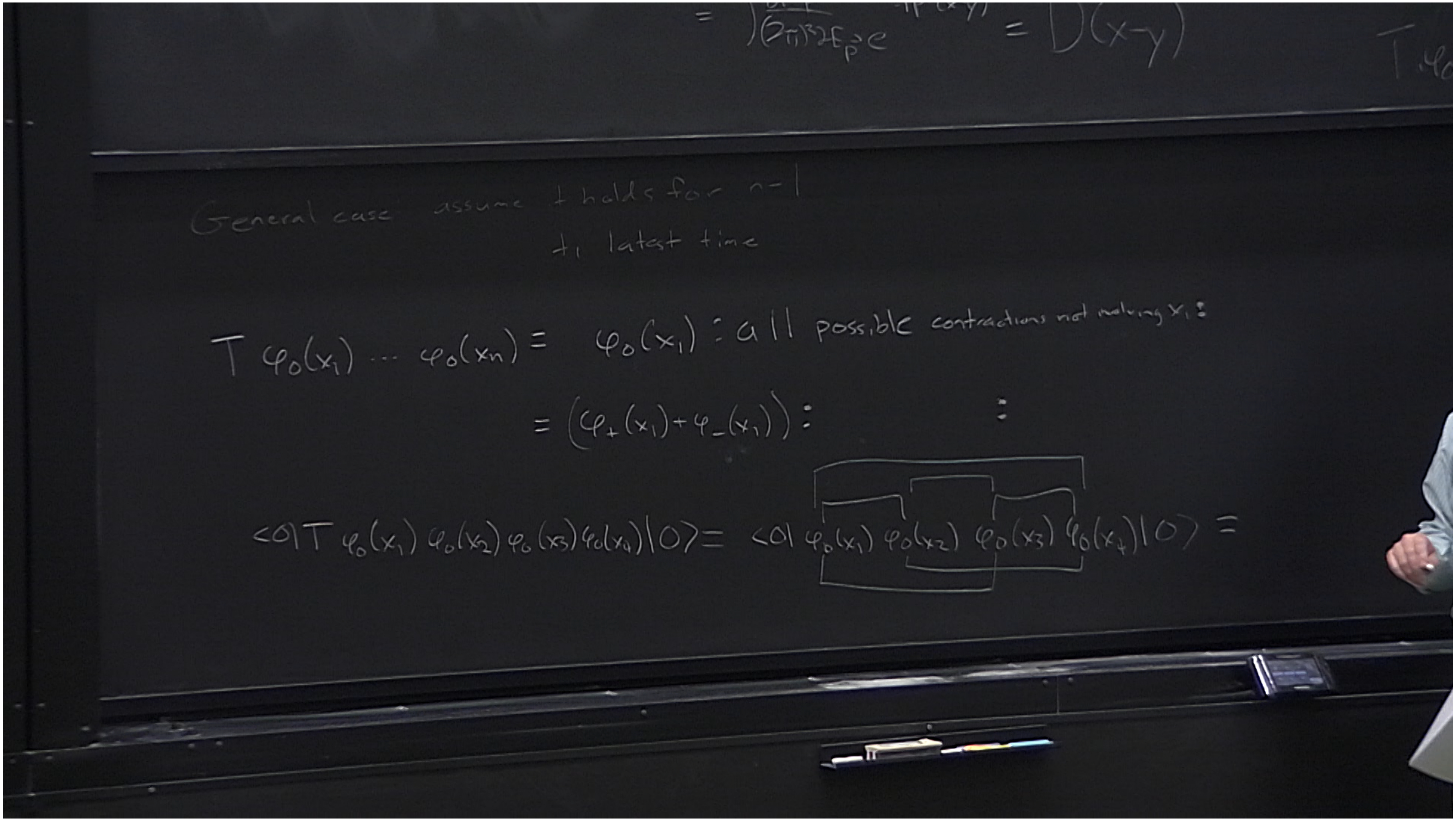
$$= :\varphi_0(x_1)\varphi_0(x_2): + \left[\varphi_-(x_1), \varphi_+(x_2)\right]$$

$t_2 > t_1$ switch x_1, x_2

$$T\,\varphi_0(x_1)\varphi_0(x_2) = :\varphi_0(x_1)\varphi_0(x_2): + \Delta_F(x_1 - x_2)$$


General case: assume it holds for n-1
t1 latest time
T φ0(x1) ... φ0(xn) = φ0(x1) :all possible contractions not involving x1:
= (φ+(x1) + φ-(x1)) :


= D(x-y)
General case: assume it holds for n-1
t1 latest time
T φ0(x1) ... φ0(xn) = φ0(x1) : all possible
= (φ+(x1) + φ-(x1)) :
<0|T φ0(x1) φ0(x2) φ0(x3) φ0(x4)|0> = <0|



$= D(x-y)$
General case assume it holds for n-1
t_1 latest time
$T\varphi_0(x_1)\cdots\varphi_0(x_n) = \varphi_0(x_1)$: all possible contractions not involving x_1 :
$= (\varphi_+(x_1)+\varphi_-(x_1))$: :
$\langle 0|T\varphi_0(x_1)\varphi_0(x_2)\varphi_0(x_3)\varphi_0(x_4)|0\rangle = \langle 0|\varphi_0(x_1)\varphi_0(x_2)\varphi_0(x_3)\varphi_0(x_4)|0\rangle =$

$\varphi_0(x_1)$: all possible contractions not involving x_1 :

$_+(x_1) + \varphi_-(x_1))$:

$(x_4)|0\rangle = \langle 0|\varphi_0(x_1)\,\varphi_0(x_2)\,\varphi_0(x_3)\,\varphi_0(x_4)|0\rangle = \Delta$

General case: assume it holds for $n-1$

t_1 latest time

$$T\,\varphi_0(x_1)\cdots\varphi_0(x_n) = \varphi_0(x_1) : \text{all possible contractions not involving } x_1 :$$

$$= \left(\varphi_+(x_1)+\varphi_-(x_1)\right) : \quad :$$

$$\langle 0|T\,\varphi_0(x_1)\,\varphi_0(x_2)\,\varphi_0(x_3)\,\varphi_0(x_4)|0\rangle = \langle 0|\,\varphi_0(x_1)\,\varphi_0(x_2)\,\varphi_0(x_3)\,\varphi_0(x_4)|0\rangle = \Delta_{12}\Delta_{34}+\Delta_{13}\Delta_{24}+\Delta_{14}\Delta_{23}$$