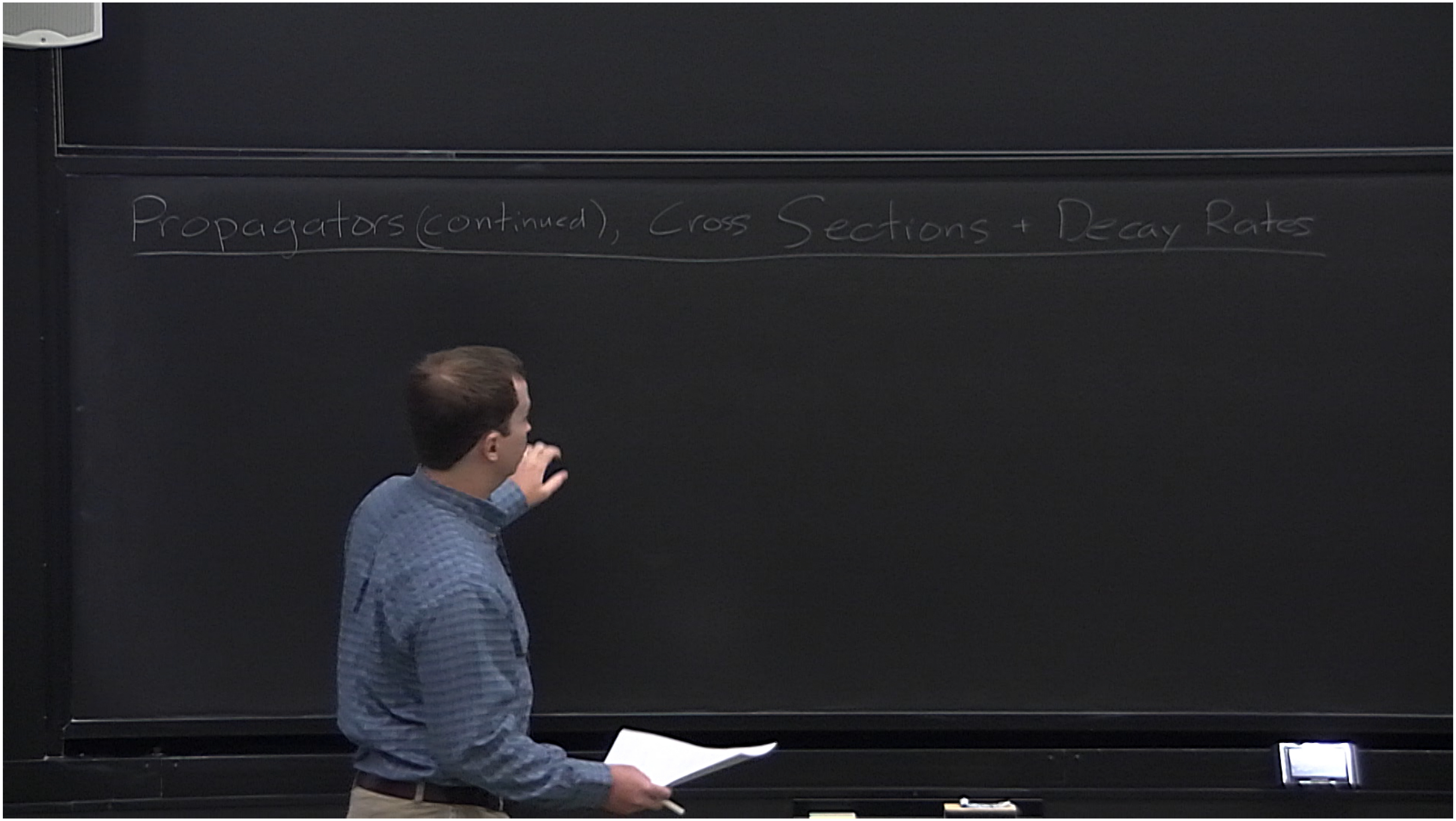
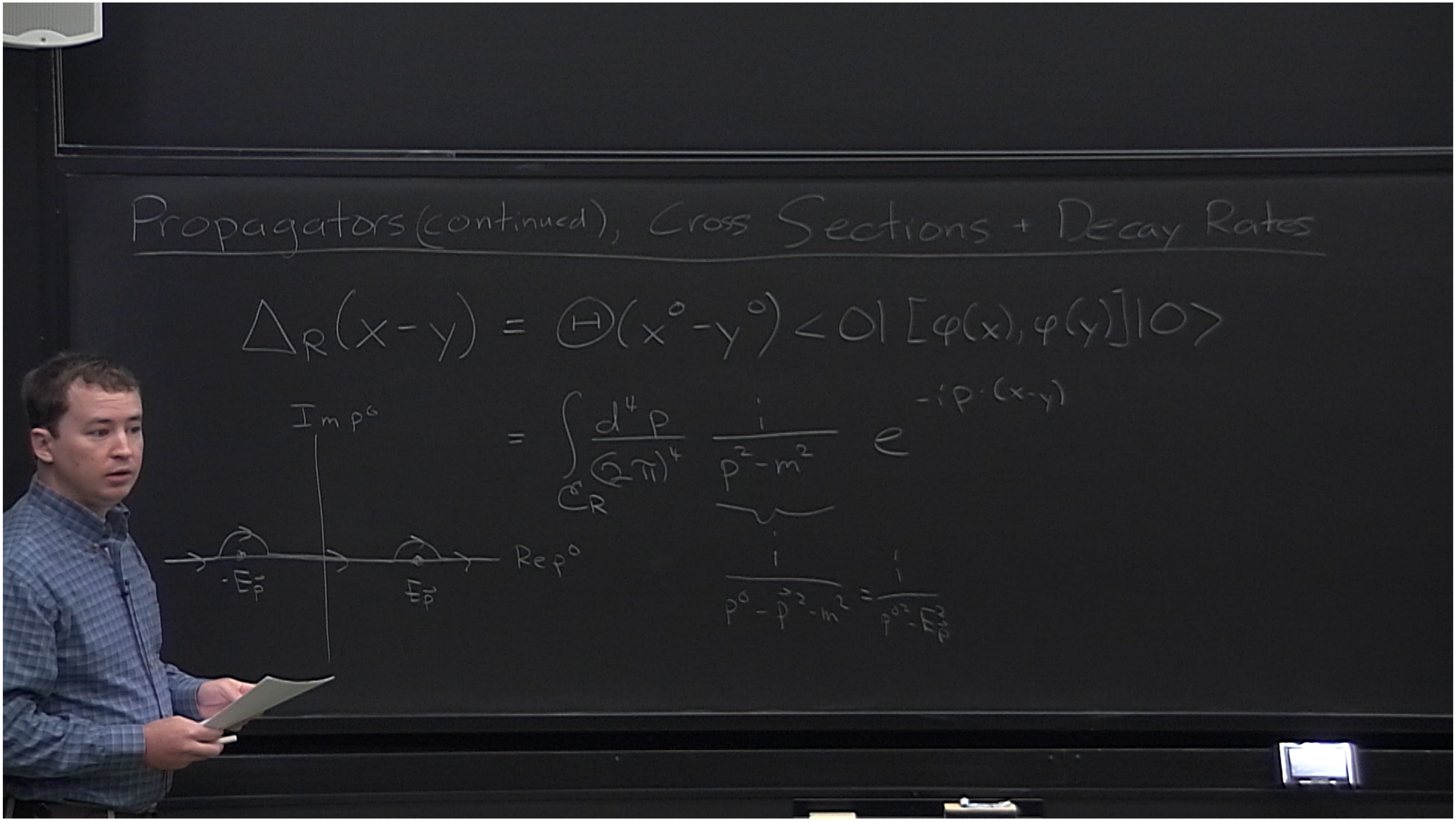
Title: PSI 17/18 - Quantum Field Theory I - Lecture 3

Date: Oct 12, 2017 09:00 AM

URL: http://pirsa.org/17100022

Abstract:


Propagators (continued), Cross Sections + Decay Rates

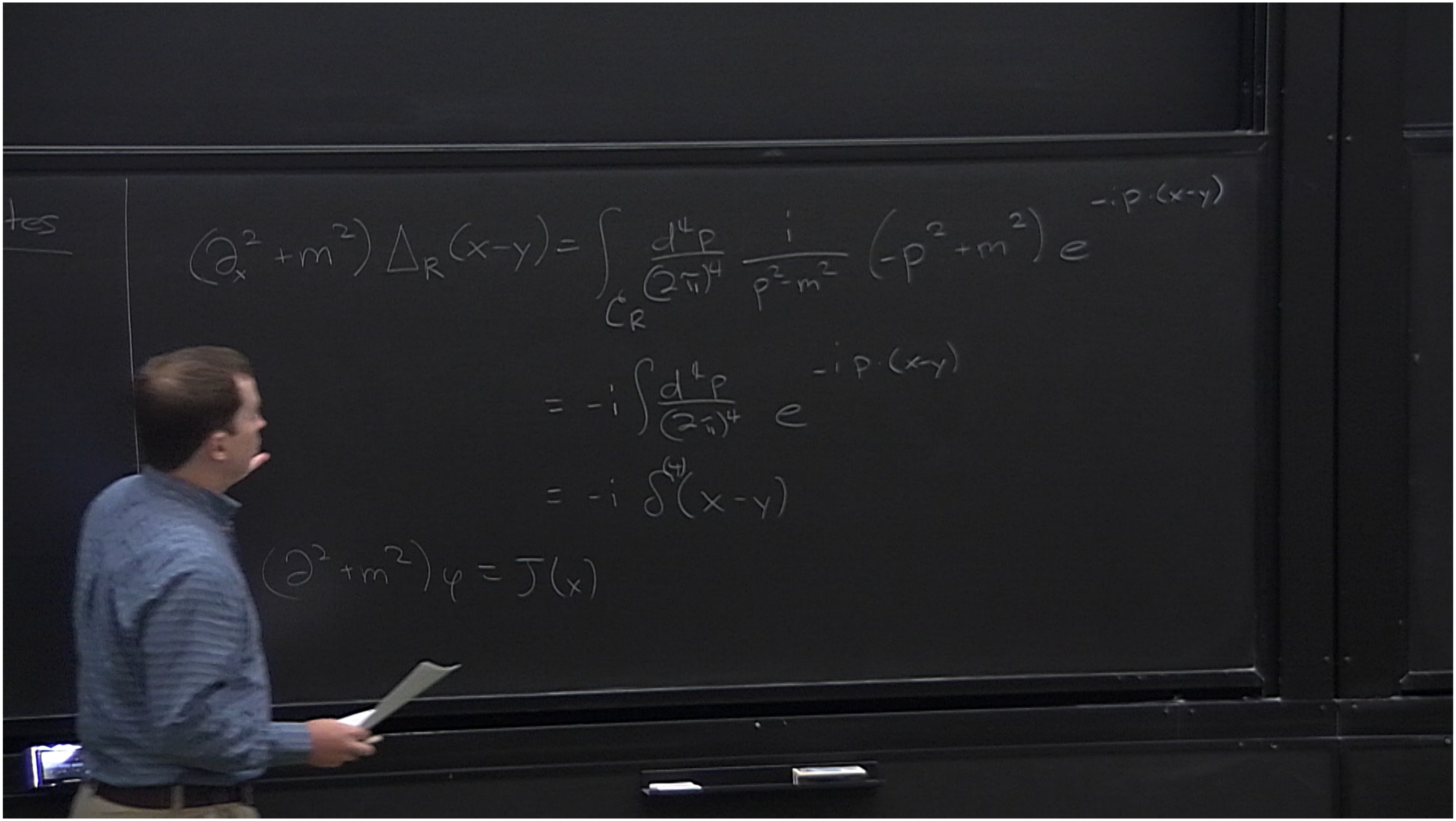

Propagators (continued), Cross Sections + Decay Rates
$\Delta_R(x-y) = \Theta(x^0-y^0)\langle 0|[\varphi(x),\varphi(y)]|0\rangle$
$= \int_{C_R} \frac{d^4p}{(2\pi)^4} \underbrace{\frac{i}{p^2-m^2}}_{\frac{i}{p^{0\,2}-\vec{p}^{\,2}-m^2} = \frac{i}{p^{0\,2}-E_{\vec{p}}^2}} e^{-ip\cdot(x-y)}$
$\mathrm{Im}\, p^0$
$\mathrm{Re}\, p^0$
$-E_{\vec{p}}$
$E_{\vec{p}}$

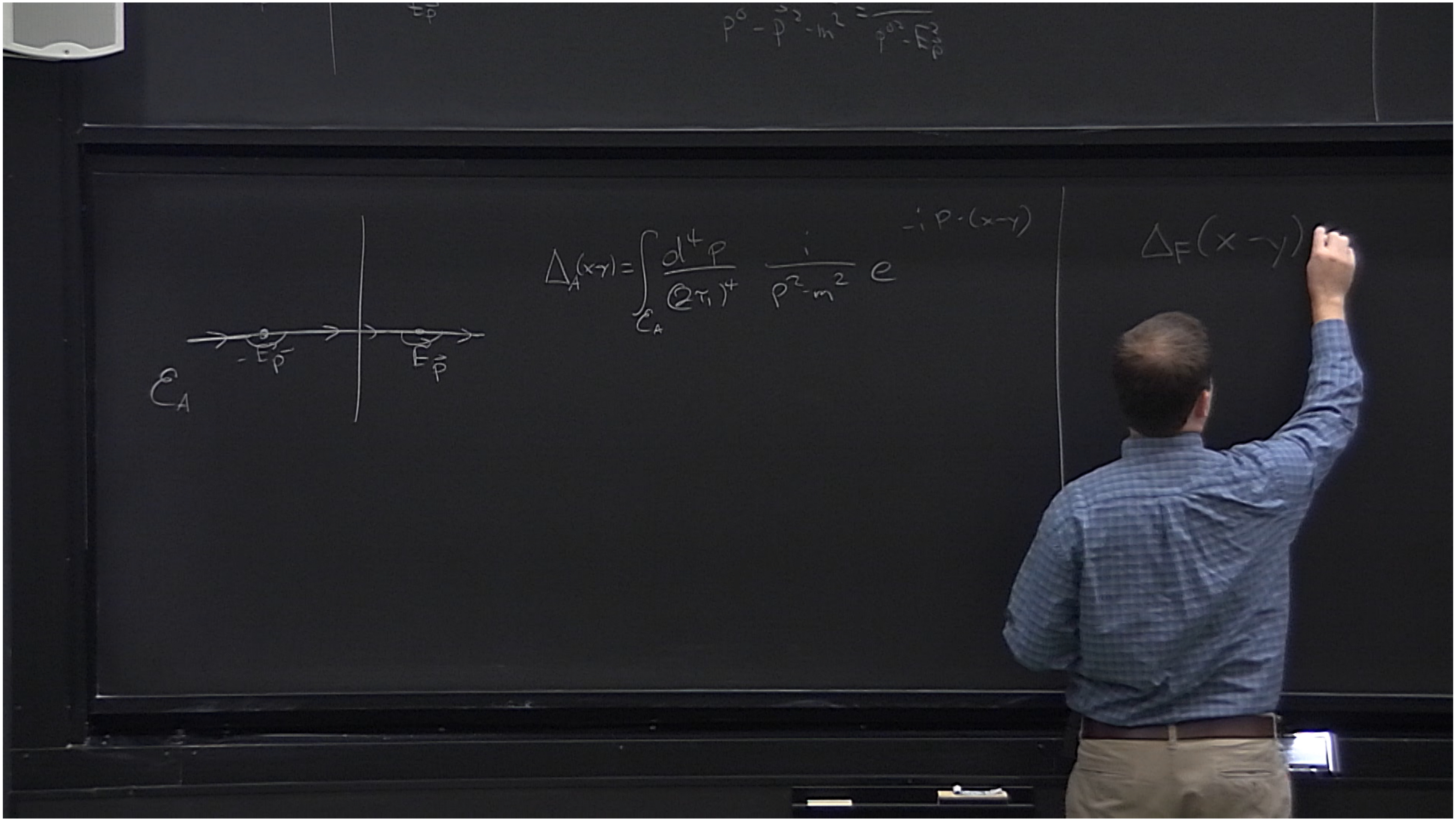
Rates

$$(\partial_x^2 + m^2)\,\Delta_R(x-y) = \int_{C_R} \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-m^2}\,(-p^2+m^2)\,e^{-ip\cdot(x-y)}$$

$$= -i\int \frac{d^4p}{(2\pi)^4}\,e^{-ip\cdot(x-y)}$$

$$= -i\,\delta^{(4)}(x-y)$$


tes
$(\partial_x^2 + m^2)\,\Delta_R(x-y) = \int_{C_R} \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2}\,(-p^2+m^2)\, e^{-ip\cdot(x-y)}$
$= -i\int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}$
$= -i\,\delta^{(4)}(x-y)$
$(\partial^2+m^2)\varphi = J(x)$



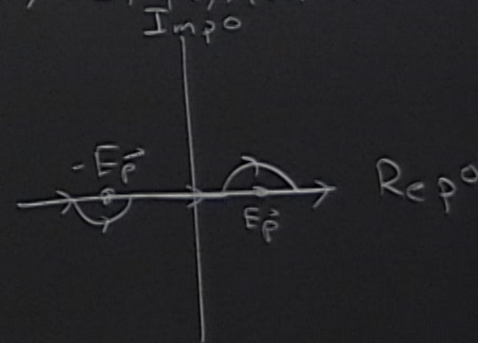
$\Delta_A(x-y) = \int_{\mathcal{C}_A} \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} e^{-ip\cdot(x-y)}$
$-E_{\vec{p}}$
$E_{\vec{p}}$
$\mathcal{C}_A$
$\Delta_F(x-y)$

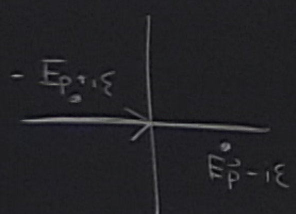
$$\Delta_F(x-y) = \langle 0 | T \varphi(x)\varphi(y) | 0 \rangle$$

time order   latest to left

$$= \Theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \Theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle$$

$$= \int_{C_F} \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} e^{-ip\cdot(x-y)}$$



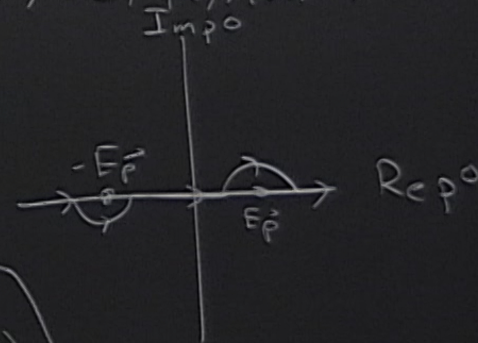


$$= \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon} e^{-ip\cdot(x-y)}$$

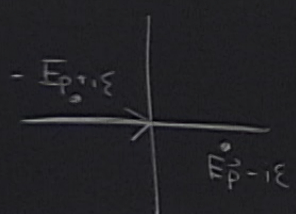
$$\Delta_F(x-y) = \langle 0 | T\,\varphi(x)\varphi(y) | 0\rangle$$

time order   latest to left

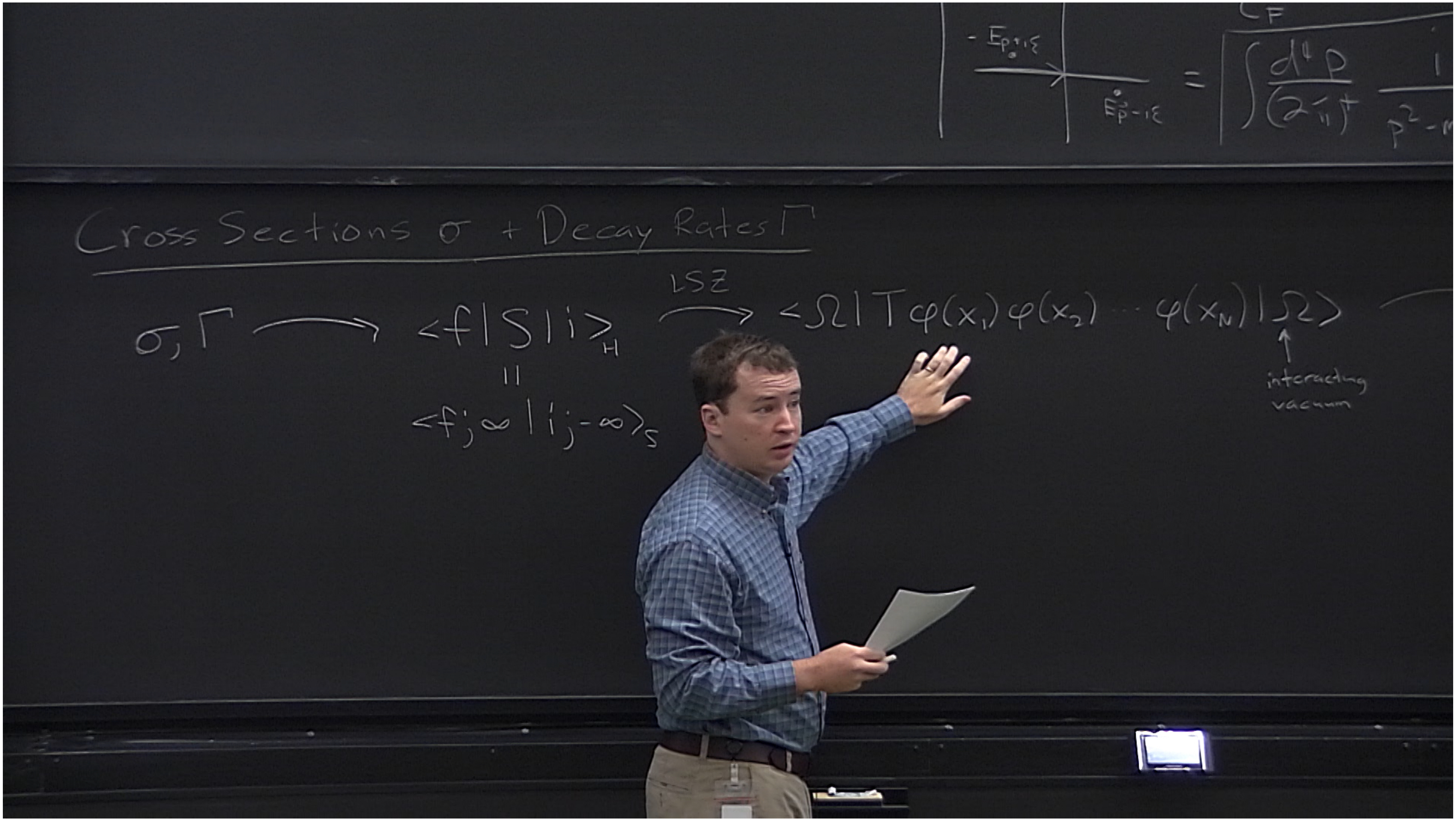
$$= \Theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \Theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle$$



$$= \int_{C_F} \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-m^2}\,e^{-ip\cdot(x-y)}$$



$$= \int \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-m^2+i\epsilon}\,e^{-ip\cdot(x-y)} = \Delta_F(x-y)$$

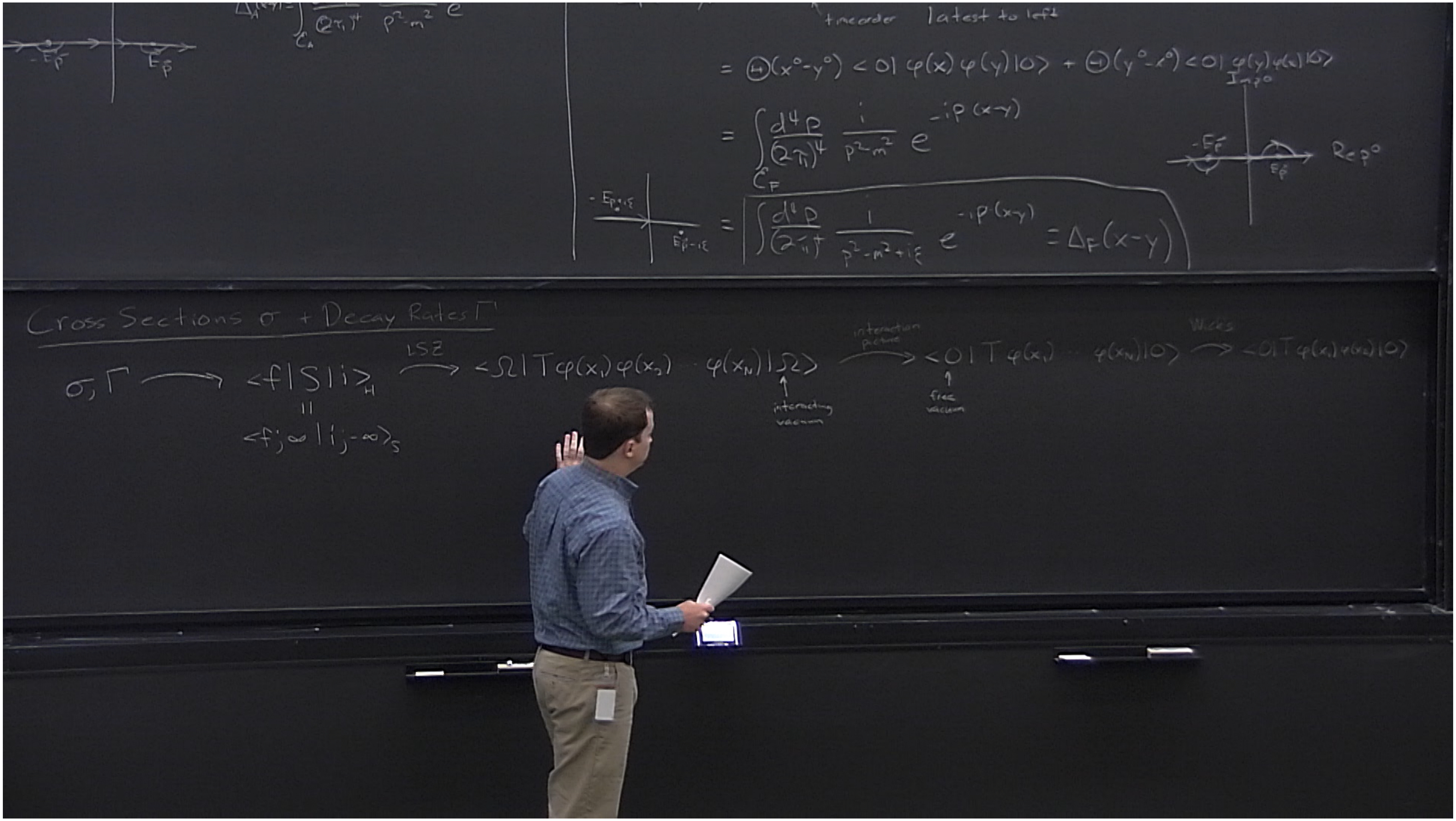

Cross Sections σ + Decay Rates Γ
σ, Γ ⟶ <f|S|i>_H ⟶ LSZ <Ω|T φ(x_1) φ(x_2) ··· φ(x_N)|Ω>
= <f; ∞|i; -∞>_S
interacting vacuum

$$\frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\varepsilon} e^{-ip\cdot(x-y)} = \Delta_F(x-y)$$

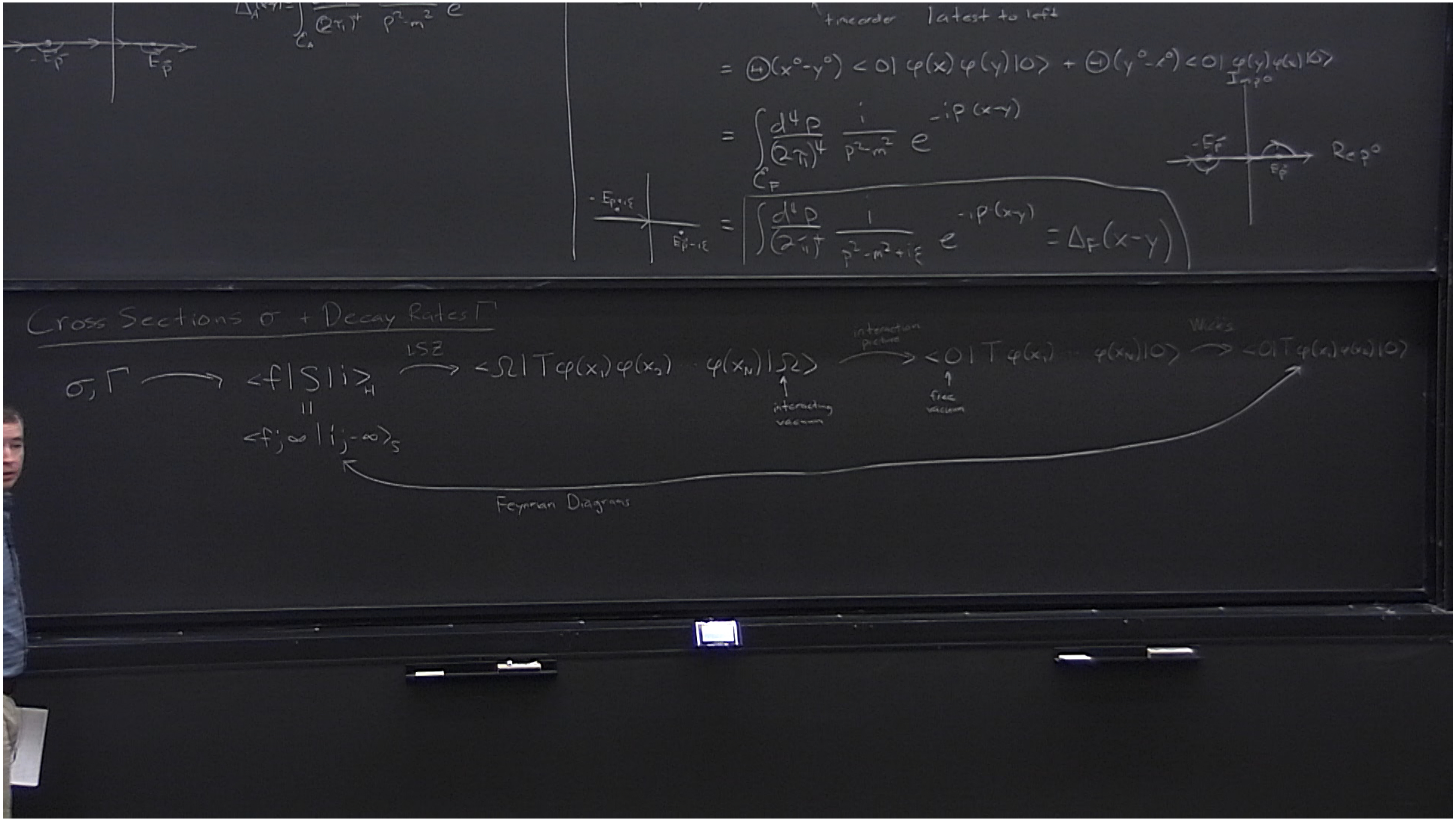
$$\Omega\rangle \xrightarrow{\text{interaction picture}} \langle 0 | T \varphi(x_1) \cdots \varphi(x_N) | 0 \rangle \xrightarrow{\text{Wick's}} \langle 0 | T \varphi(x_1) \varphi(x_2) | 0 \rangle$$

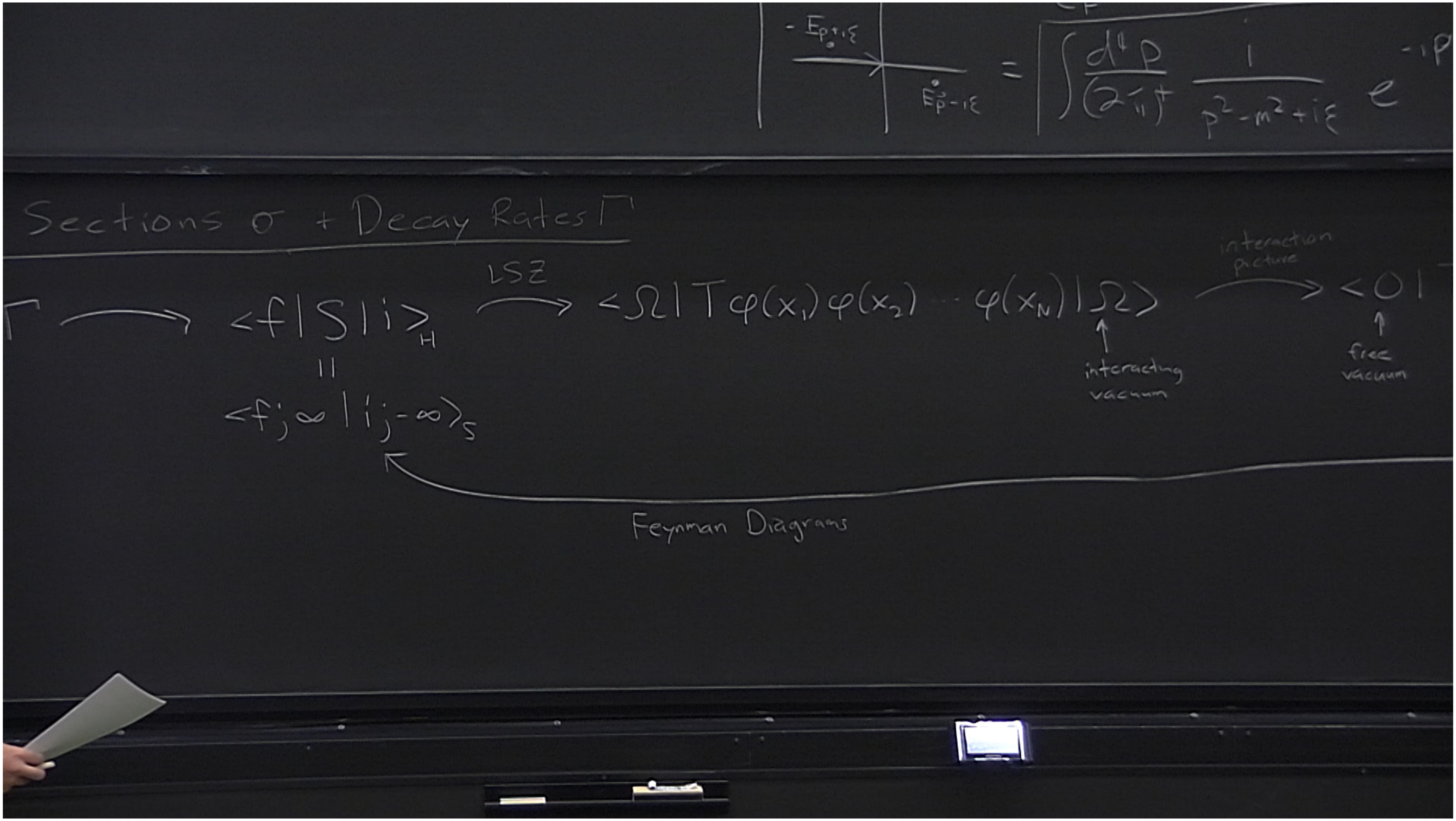
interacting vacuum

free vacuum

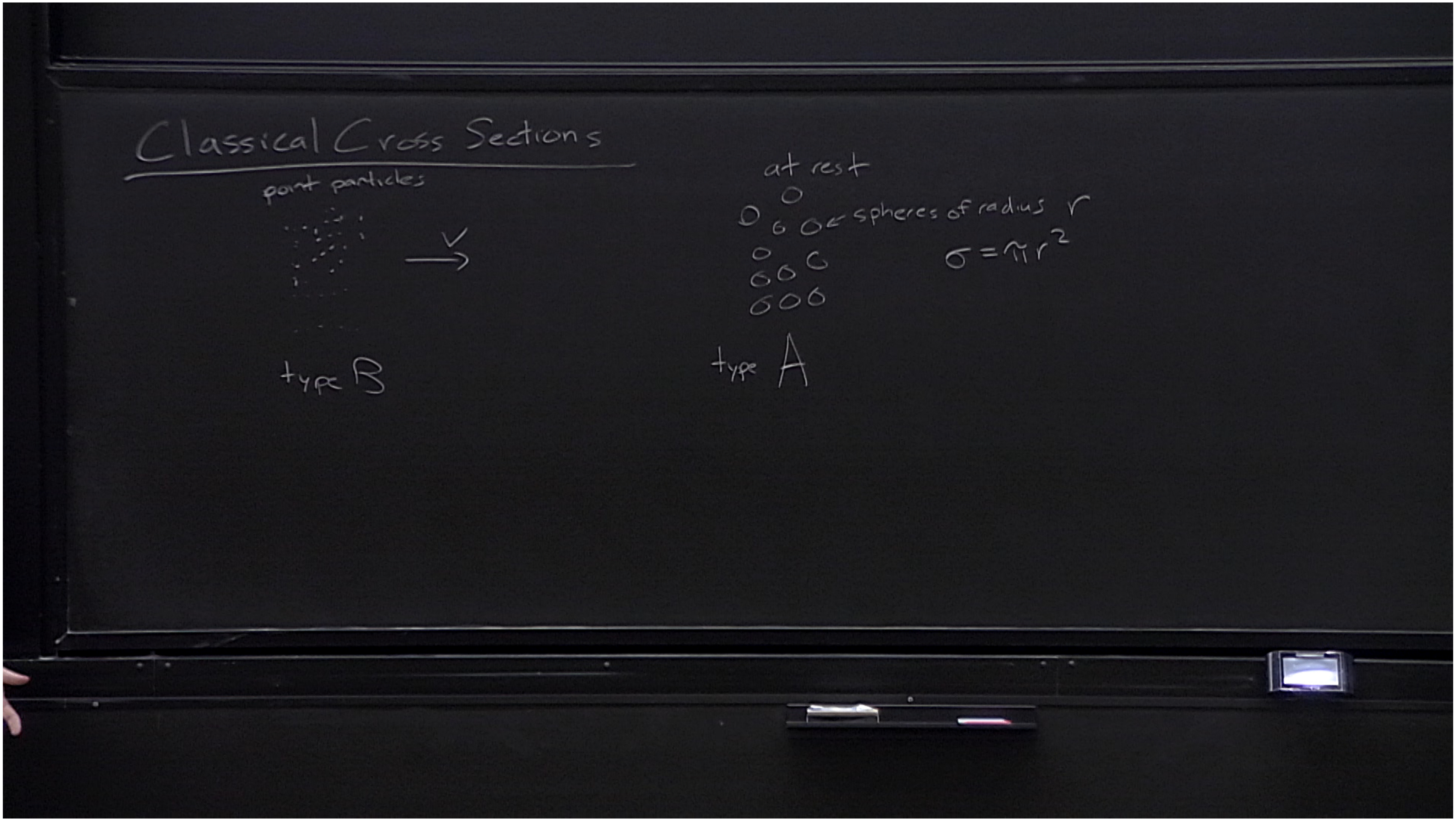


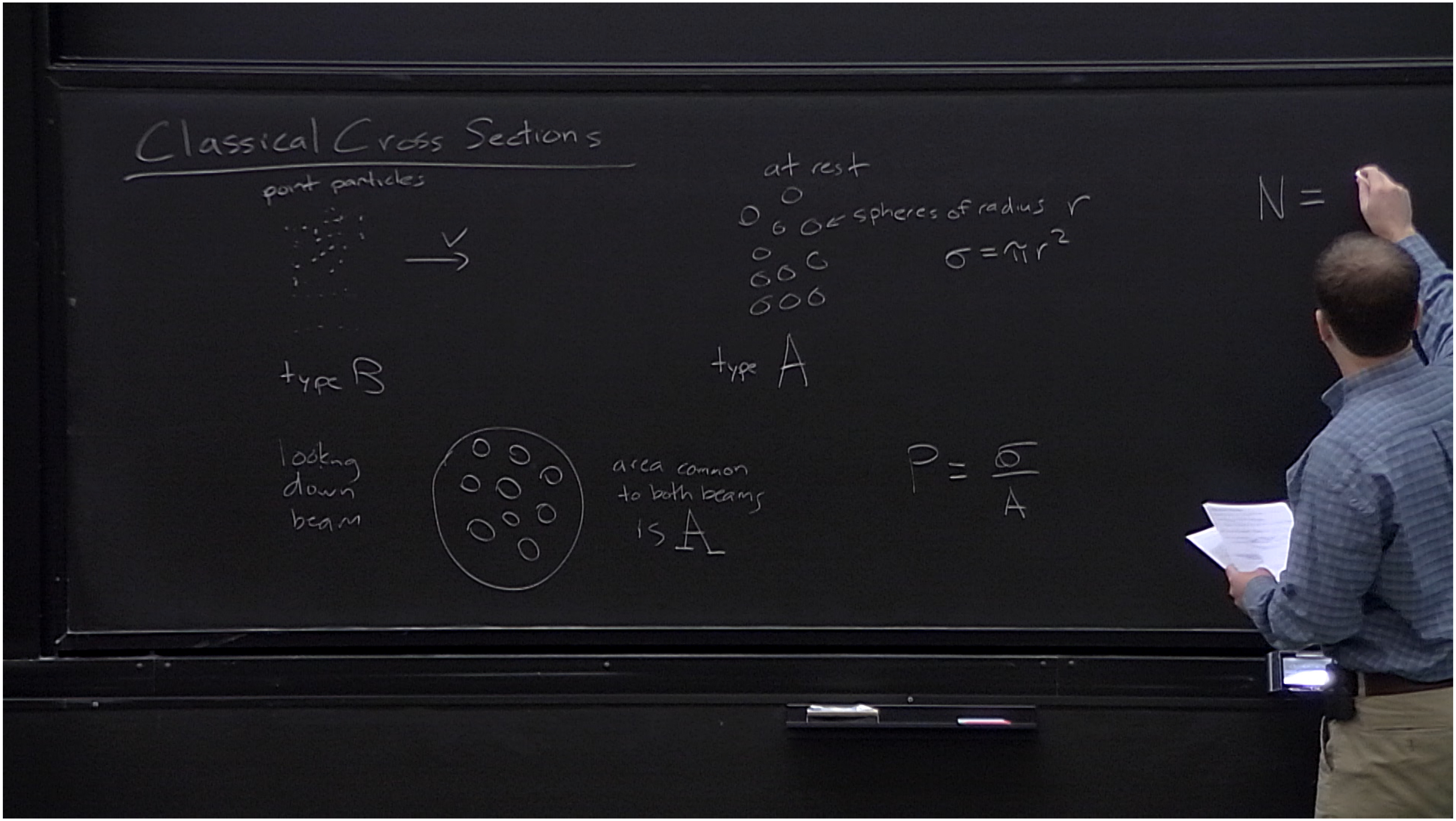
time order latest to left
Cross Sections σ + Decay Rates Γ
LSZ
interaction picture
Wicks
interacting vacuum
free vacuum
Im p°
Re p°

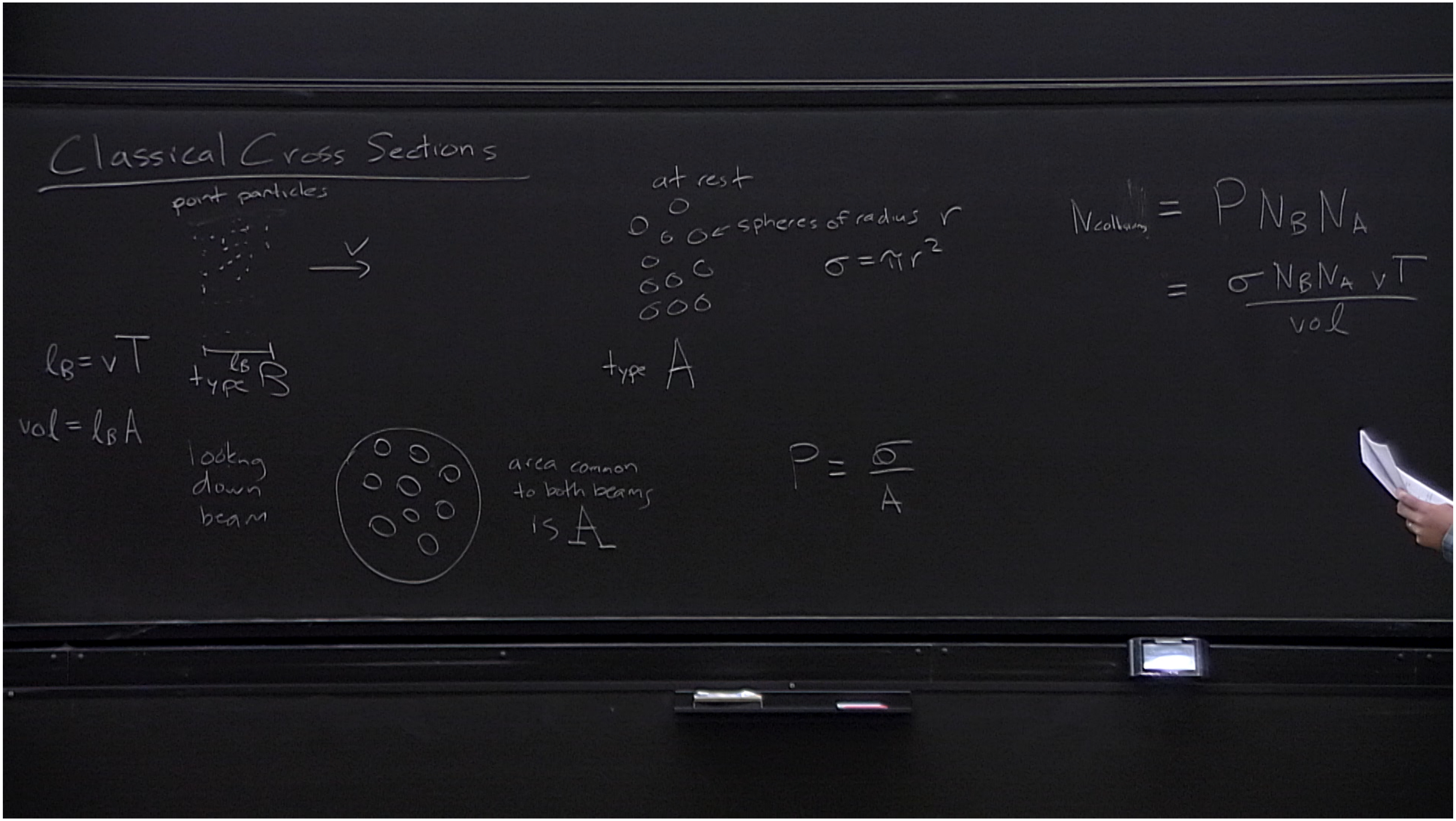

time order latest to left
$= \Theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \Theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle$
$= \int_{C_F} \frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2}e^{-ip(x-y)}$
$= \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2+i\epsilon}e^{-ip\cdot(x-y)} = \Delta_F(x-y)$
Cross Sections σ + Decay Rates Γ
$\sigma,\Gamma \longrightarrow \langle f|S|i\rangle_H \xrightarrow{LSZ} \langle\Omega|T\varphi(x_1)\varphi(x_2)\cdots\varphi(x_N)|\Omega\rangle \xrightarrow{\text{interaction picture}} \langle 0|T\varphi(x_1)\cdots\varphi(x_N)|0\rangle \xrightarrow{\text{Wick's}} \langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle$
$= \langle f;\infty|i;-\infty\rangle_S$
interacting vacuum
free vacuum
Feynman Diagrams

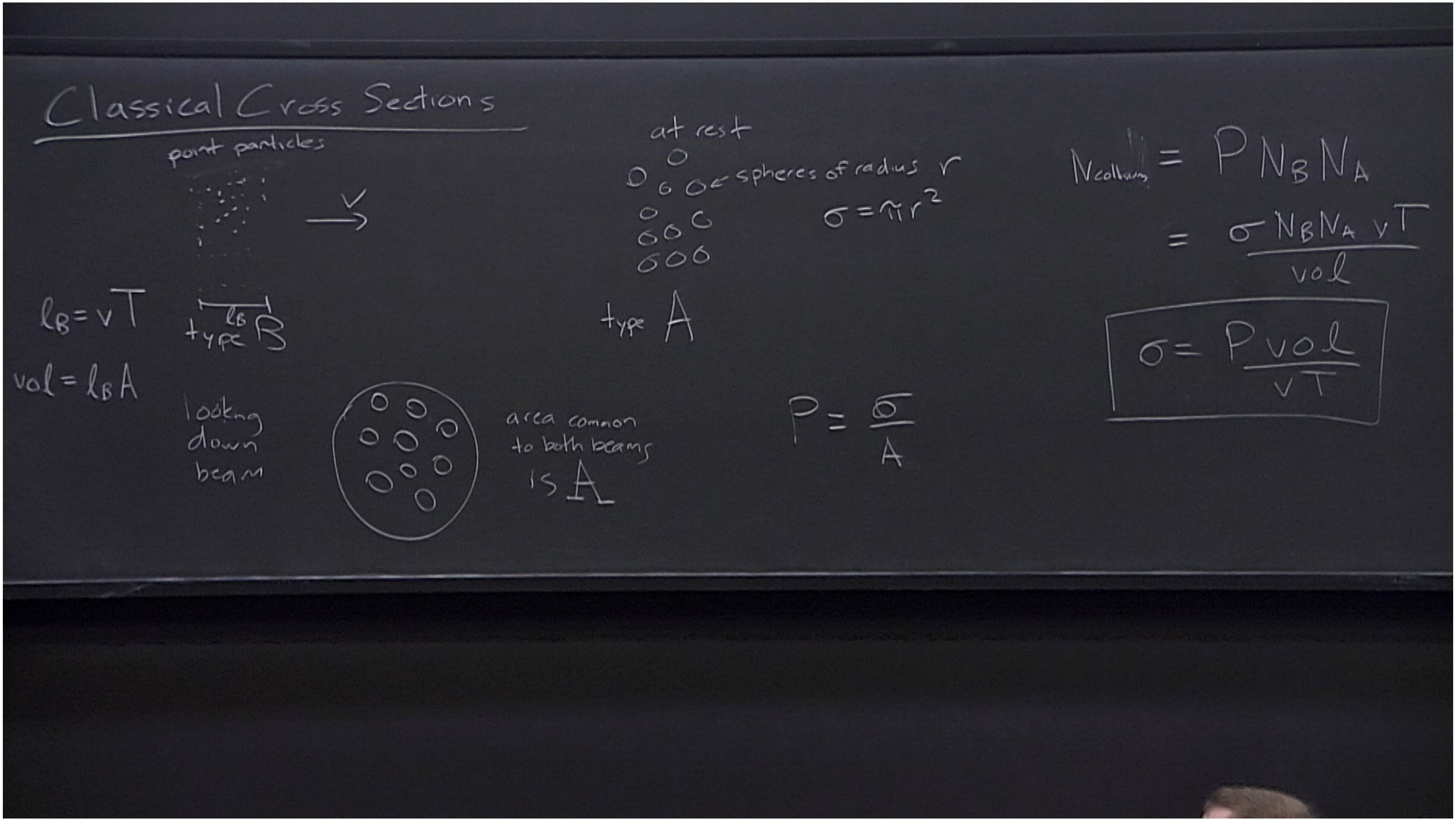


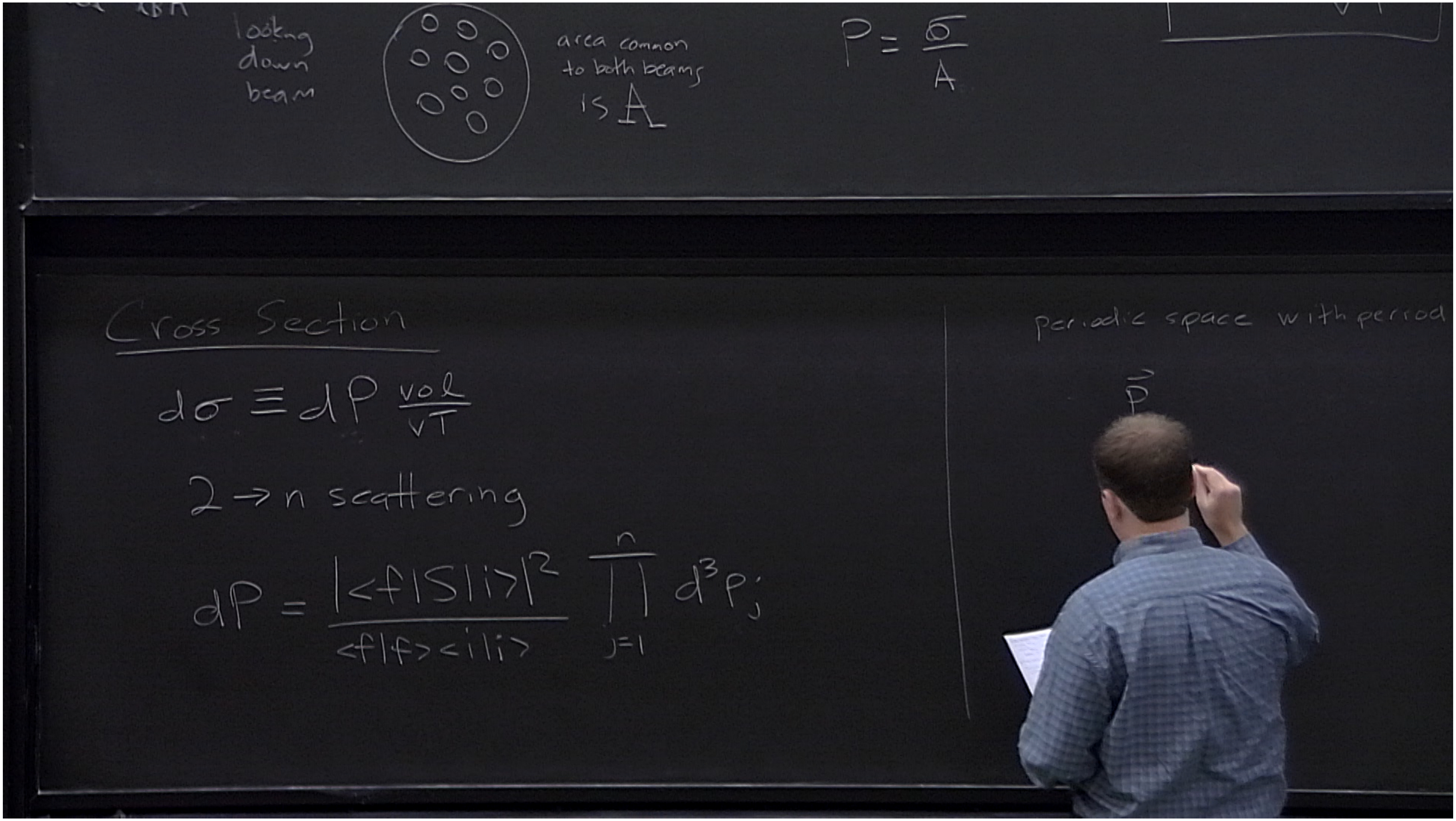
$-E_{\vec p}+i\varepsilon$
$E_{\vec p}-i\varepsilon$
$= \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\varepsilon} e^{-ip}$
Sections σ + Decay Rates Γ
$\Gamma \longrightarrow \langle f | S | i \rangle_H \xrightarrow{LSZ} \langle \Omega | T \varphi(x_1) \varphi(x_2) \cdots \varphi(x_N) | \Omega \rangle \xrightarrow{\text{interaction picture}} \langle 0 |$
$= \langle f; \infty | i; -\infty \rangle_S$
interacting vacuum
free vacuum
Feynman Diagrams


Classical Cross Sections
point particles
v
type B
at rest
spheres of radius r
σ = πr²
type A


Classical Cross Sections
point particles
type B
at rest
spheres of radius r
σ = πr²
type A
N =
looking down beam
area common to both beams is A
P = σ / A


Classical Cross Sections
point particles
at rest
spheres of radius r
σ = πr²
N_coll = P N_B N_A
= σ N_B N_A vT / vol
l_B = vT
vol = l_B A
l_B
type B
type A
looking
down
beam
area common
to both beams
is A
P = σ / A


Classical Cross Sections
point particles
v
$\ell_B = vT$
ℓ_B
type B
$vol = \ell_B A$
looking down beam
area common to both beams is A
at rest
spheres of radius r
$\sigma = \pi r^2$
type A
$P = \frac{\sigma}{A}$
$N_{colliding} = P N_B N_A$
$= \frac{\sigma N_B N_A vT}{vol}$
$\boxed{\sigma = \frac{P\,vol}{vT}}$


looking down beam
area common to both beams is A
$P = \frac{\sigma}{A}$
Cross Section
$d\sigma \equiv dP \frac{vol}{vT}$
2 → n scattering
$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle} \prod_{j=1}^{n} d^3p_j$
periodic space with period
$\vec{p}$

A

periodic space with period L $\quad$ $\mathrm{vol} = L^3$

$$\vec{p}_i = \frac{2\pi}{L}\,\vec{n}_i$$

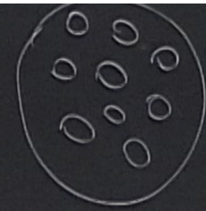
vector with integer components

$$\sum_{\vec{n}_i} \rightarrow \int \frac{d^3 p_i}{(2\pi)^3}\,\mathrm{vol}$$

Rep°

$\varphi(x_1)\varphi(x_2)|0\rangle$

looking down beam



area common to both beams is A

$P = \frac{\sigma}{A}$

Cross Section

$$d\sigma \equiv dP \, \frac{vol}{vT}$$

$2 \to n$ scattering

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle} \prod_{j=1}^{n} \left(\frac{d^3p_j}{(2\pi)^3} vol \right)$$

periodic spac

$\vec{p}_i = 2$

$\sum_{n_j} \to$

$$dP = \frac{\ldots}{\langle f|f\rangle\langle i|i\rangle} \prod_{j=1} \left(\frac{d^3 p_j}{(2\pi)^3} V \ldots \right)$$

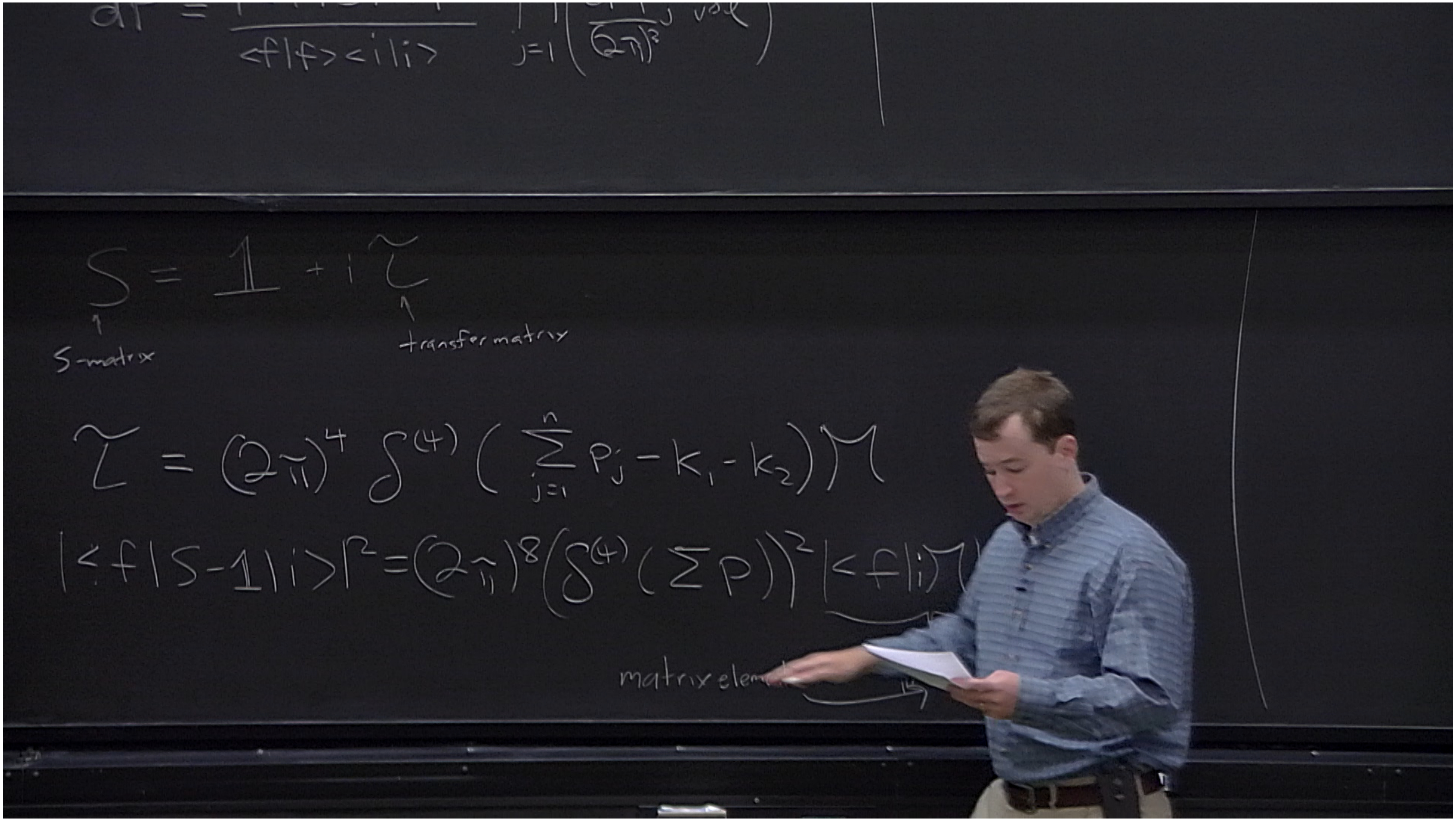
$$S = \mathbb{1} + i\mathcal{T}$$

S-matrix

transfer matrix

$$\mathcal{T} = (2\pi)^4 \, \delta^{(4)}\left(\sum_{j=1}^{n} p_j - k_1 - k_2 \right) \mathcal{M}$$

$$|\langle f|S-\mathbb{1}|i\rangle|^2 = (2\pi)^8 \left(\delta^{(4)}\left(\sum p\right)\right)^2 \underbrace{|\langle f|\mathcal{M}|i\rangle|^2}_{\equiv |\mathcal{M}|^2 \equiv |\mathcal{M}_{i\to f}|^2}$$



<f|f><i|i>
S = 1 + i T
S-matrix
transfer matrix
T = (2π)^4 δ^(4)(Σ_{j=1}^{n} p_j − k_1 − k_2) M
|<f|S−1|i>|^2 = (2π)^8 (δ^(4)(Σp))^2
matrix

$$dP = \frac{\cdots}{\langle f|f\rangle\langle i|i\rangle} \prod_{j=1} \left(\frac{d^3p_j\, Vol}{(2\pi)^3}\right)$$

$$\underbrace{S}_{\text{S-matrix}} = \mathbb{1} + i\underbrace{\mathcal{T}}_{\text{transfer matrix}}$$

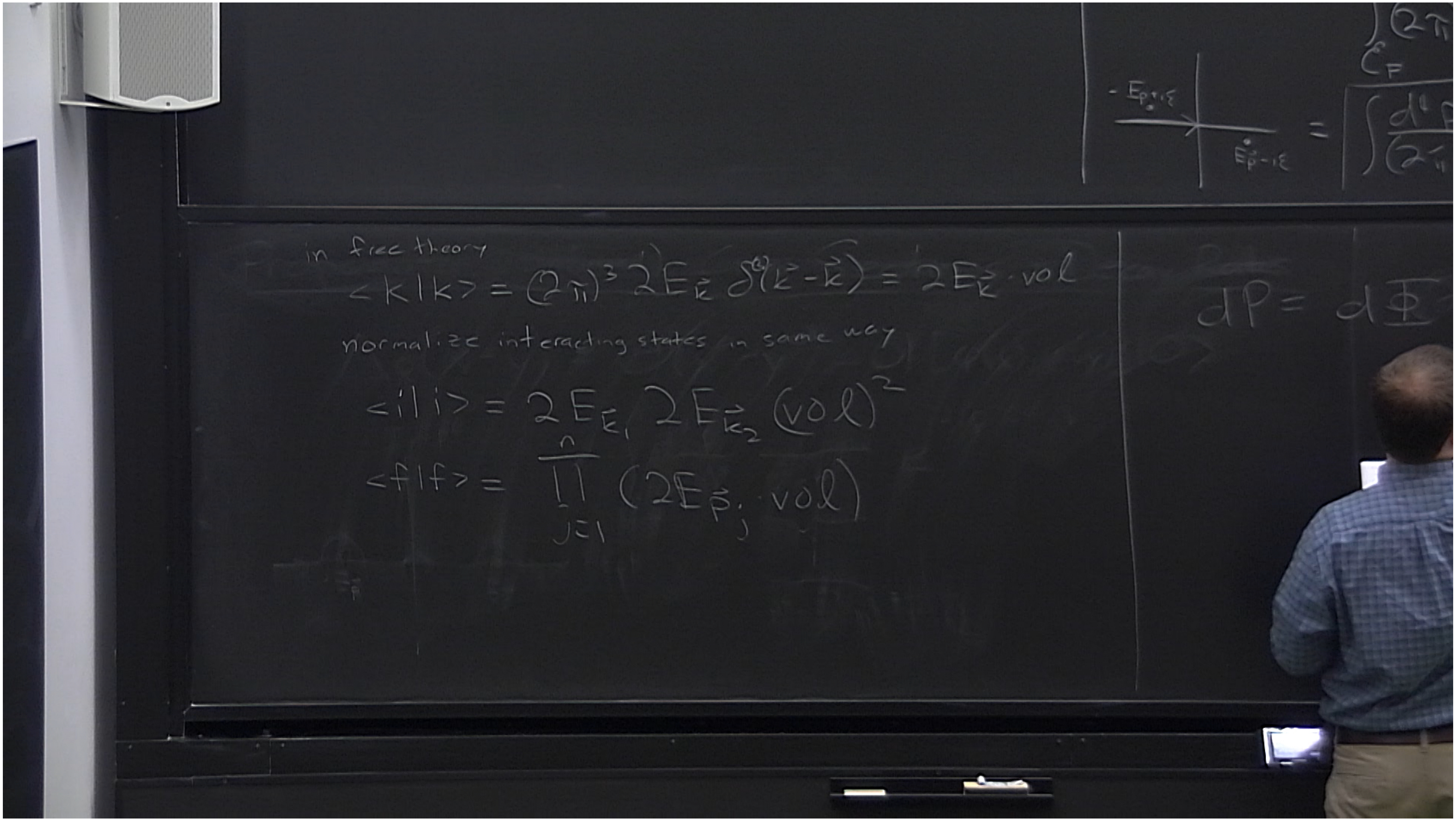
$$\mathcal{T} = (2\pi)^4\, \delta^{(4)}\left(\sum_{j=1}^{n} p_j - k_1 - k_2\right)\mathcal{M}$$

$$|\langle f|S-\mathbb{1}|i\rangle|^2 = (2\pi)^8 \left(\delta^{(4)}\left(\sum p\right)\right)^2 \underbrace{|\langle f|i\mathcal{M}|i\rangle|^2}_{\equiv |i\mathcal{M}|^2 \equiv |i\mathcal{M}_{i\to f}|^2}$$

matrix element

$(2\pi)^4$

$$(2\pi)^4 \delta^{(4)}(0) = \int d^4x \, e^{ip\cdot x}\Big|_{p=0} = (\mathrm{vol})\,T$$


in free theory
$\langle k|k\rangle = (2\pi)^3\, 2E_{\vec{k}}\, \delta^3(\vec{k}-\vec{k}) = 2E_{\vec{k}} \cdot vol$
normalize interacting states in same way
$\langle i|i\rangle = 2E_{\vec{k}_1}\, 2E_{\vec{k}_2}\, (vol)^2$
$\langle f|f\rangle = \prod_{j=1}^{n} (2E_{\vec{p}_j} \cdot vol)$
$dP = d\Phi$

$$\frac{\quad}{E_{\vec{p}} - i\epsilon} = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\epsilon} e^{-ip\cdot(x-y)} = \Delta_F(x-y)$$

$$dP = d\Phi \, \frac{|iM|^2 \, \mathrm{vol}\, T \, \cancel{(\mathrm{vol})^n}}{2E_{\vec{k}_1} 2E_{\vec{k}_1} \cdot (\mathrm{vol})^2 \, \cancel{(\mathrm{vol})^n}}$$

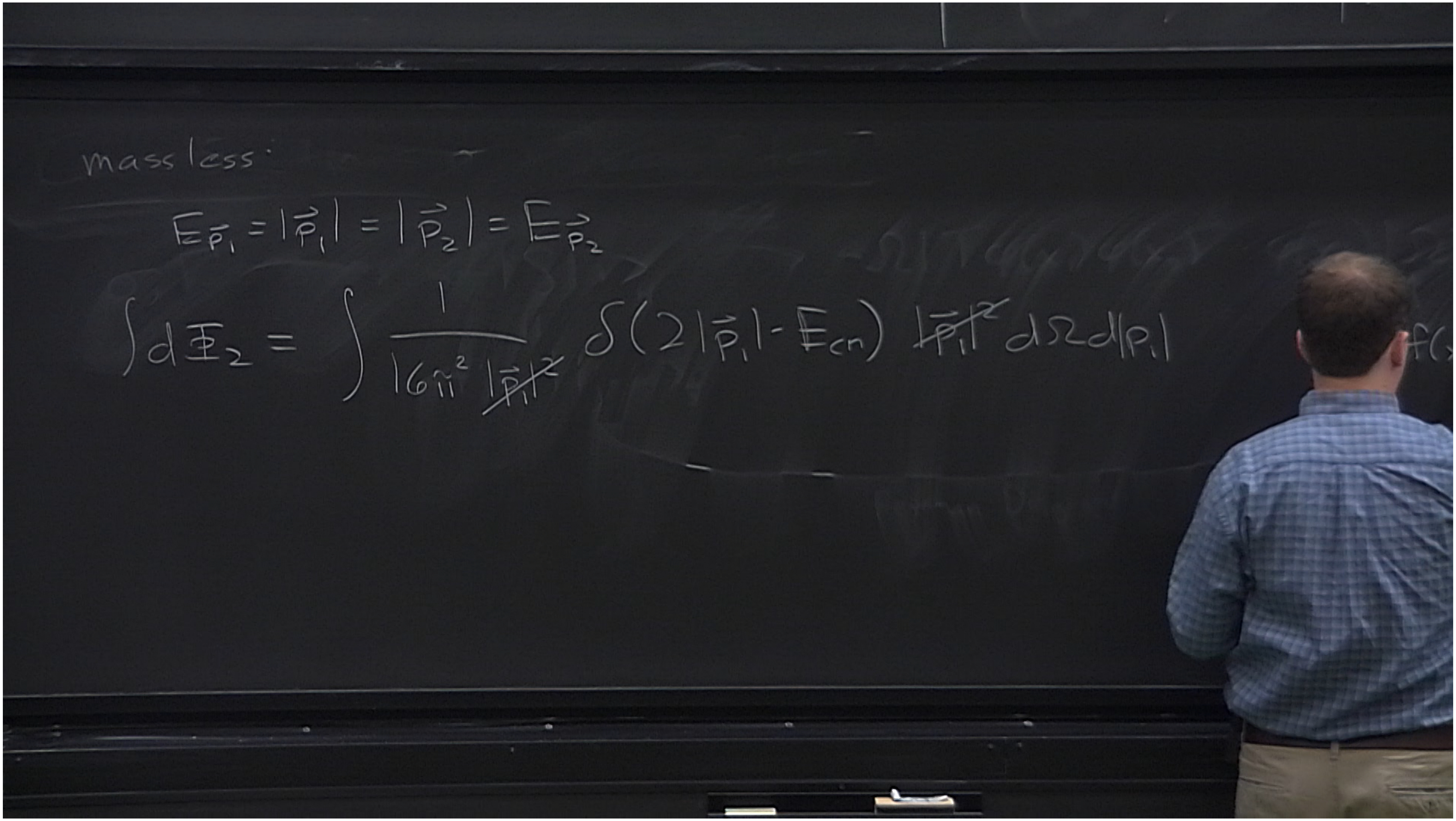
d^3p

$\langle f | f \rangle$

$$d\Phi = (2\pi)^4 \delta^{(4)}\Big(\sum_{i,f} p\Big) \prod_{j=1}^{n} \frac{d^3 p_j}{(2\pi)^3 \, 2E_{\vec{p}_j}}$$

phase space

$$\boxed{d\sigma = d\Phi \, \frac{|iM|^2}{2E_{\vec{k}_1} 2E_{\vec{k}_2} \cdot V}}$$


massless:
$E_{\vec{p}_1} = |\vec{p}_1| = |\vec{p}_2| = E_{\vec{p}_2}$
$\int d\Phi_2 = \int \frac{1}{16\pi^2 \cancel{|\vec{p}_1|^2}} \delta(2|\vec{p}_1| - E_{cm}) \cancel{|\vec{p}_1|^2} d\Omega \, d|p_1|$

massless:

$$E_{\vec{p}_1} = |\vec{p}_1| = |\vec{p}_2| = E_{\vec{p}_2}$$

$$\int d\Phi_2 = \int \frac{1}{16\pi^2 \cancel{|\vec{p}_1|^2}} \delta(2|\vec{p}_1| - E_{cm}) \cancel{|\vec{p}_1|^2} \, d\Omega \, d|\vec{p}_1|$$

$$= \int \frac{d\Omega}{32\pi^2}$$

$$2E_{\vec{p}_1}\big|_{CM}$$

$$\int d\Phi_2 \; \frac{1}{(2\pi)^2\, 2E_{\vec{p}_1} 2E_{\vec{p}_2}} \, \delta\left(E_{\vec{p}_1}+E_{\vec{p}_2}-E_{CM}\right) d^3p_1$$

massless:

$$E_{\vec{p}_1} = |\vec{p}_1| = |\vec{p}_2| = E_{\vec{p}_2}$$

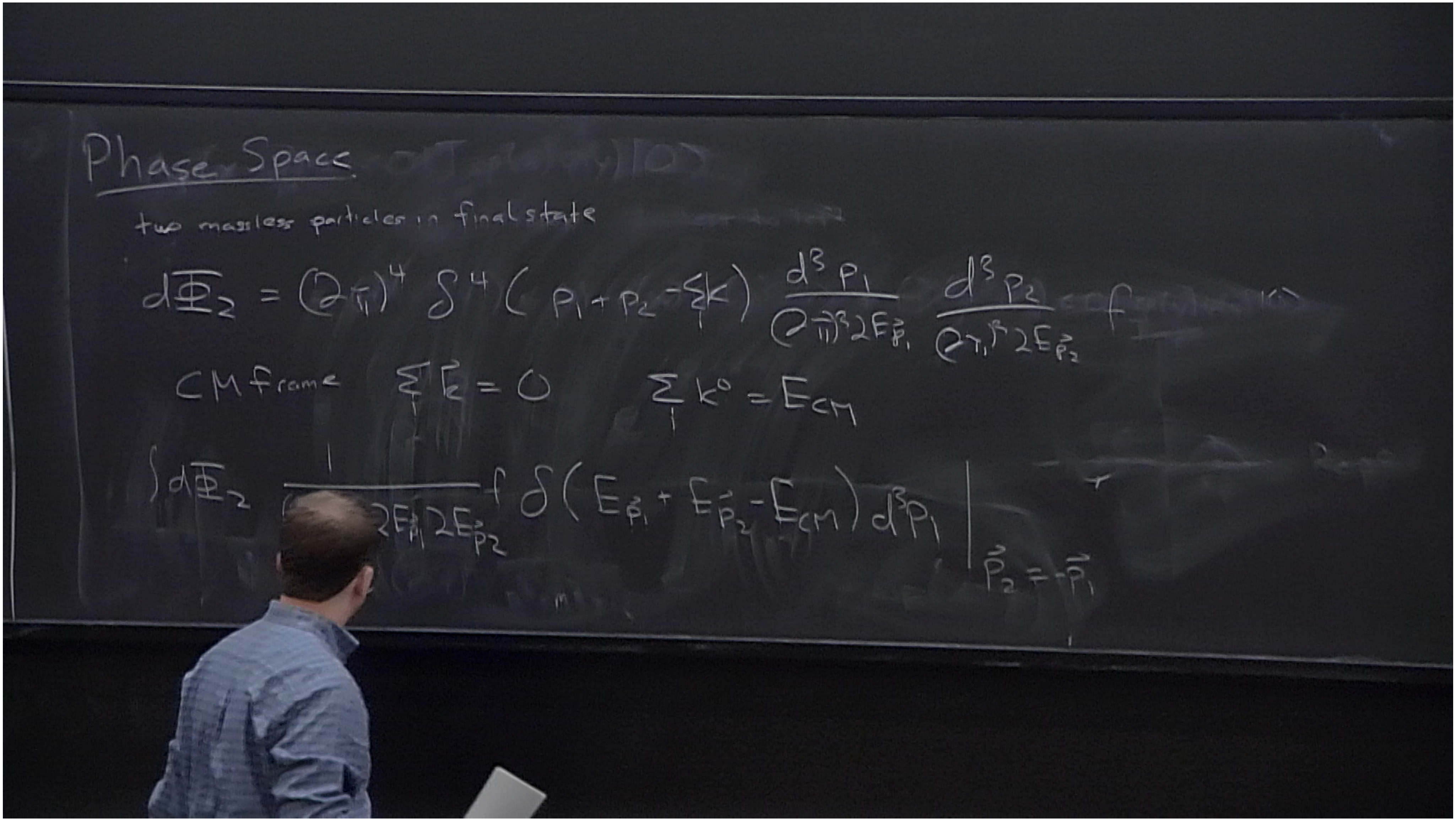
$$\int d\Phi_2 = \int \frac{1}{16\pi^2 \cancel{|\vec{p}_1|^2}} \, \delta\left(2|\vec{p}_1| - E_{cm}\right) \cancel{|\vec{p}_1|^2} \, d\Omega \, d|p_1|$$

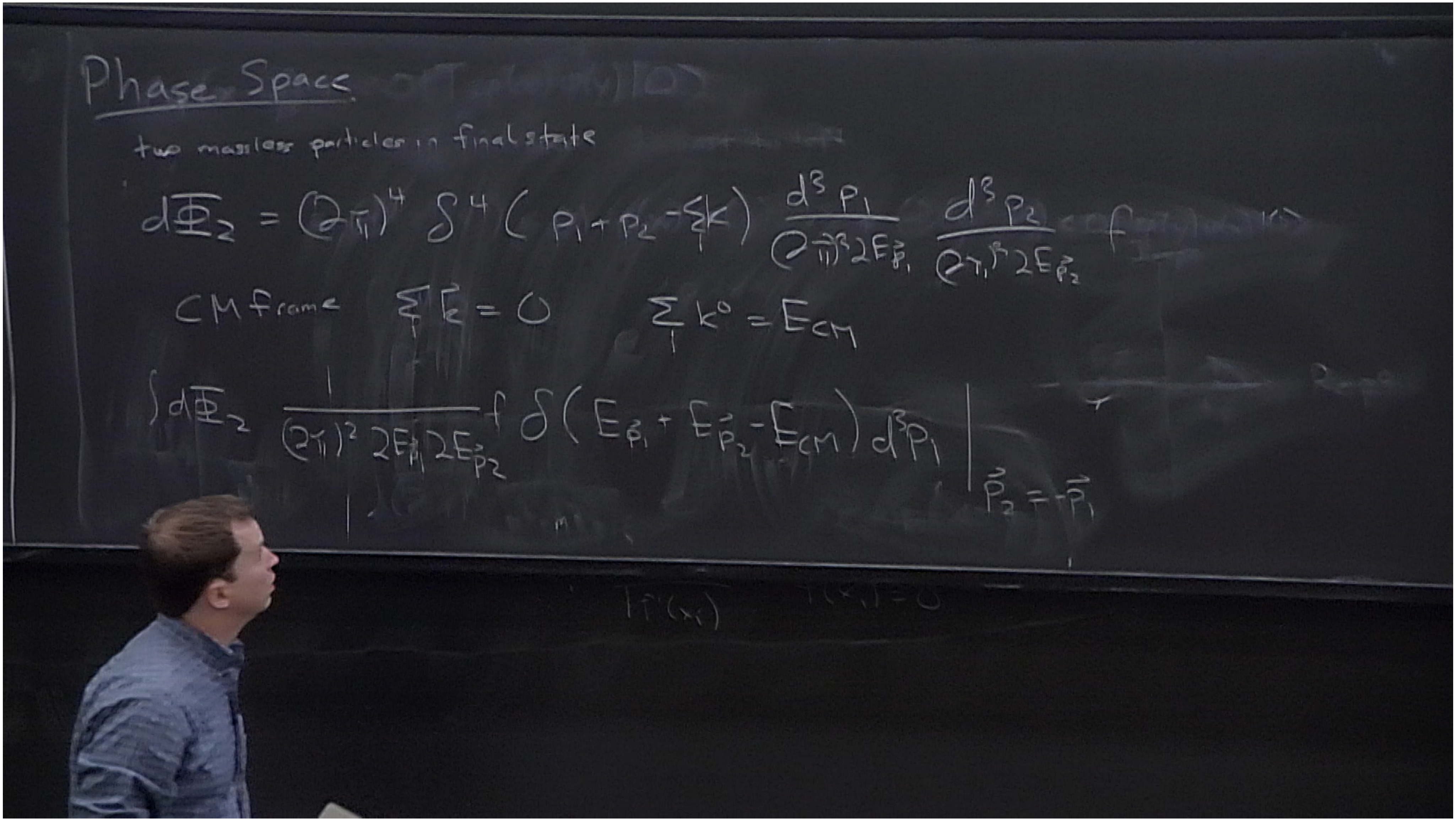
$$\delta(f(x)) = \sum_i \frac{\delta(x-x_i)}{|f'(x_i)|} \qquad f(x_i)=0$$

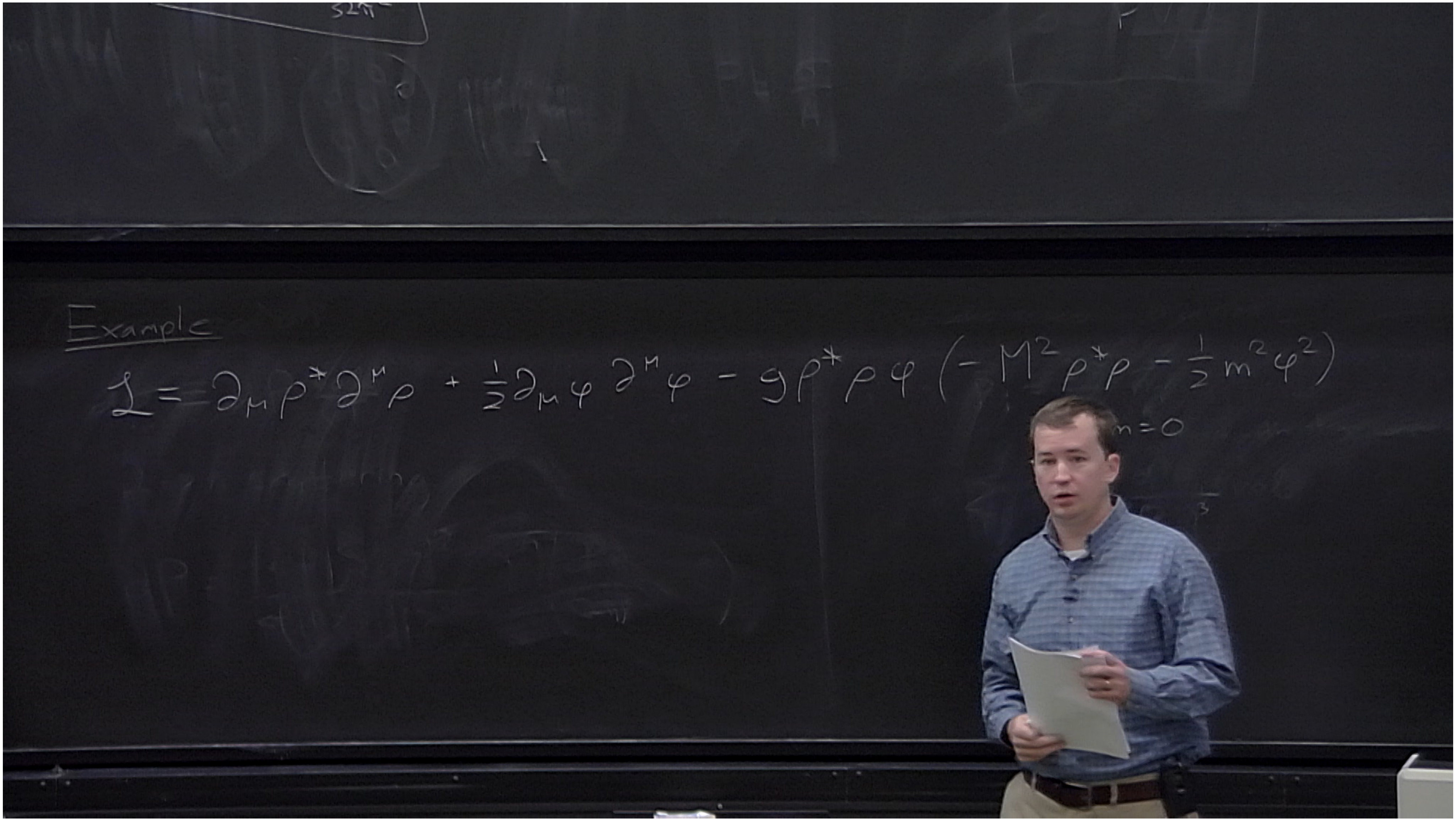
$$= \int \frac{d\Omega}{32\pi^2}$$

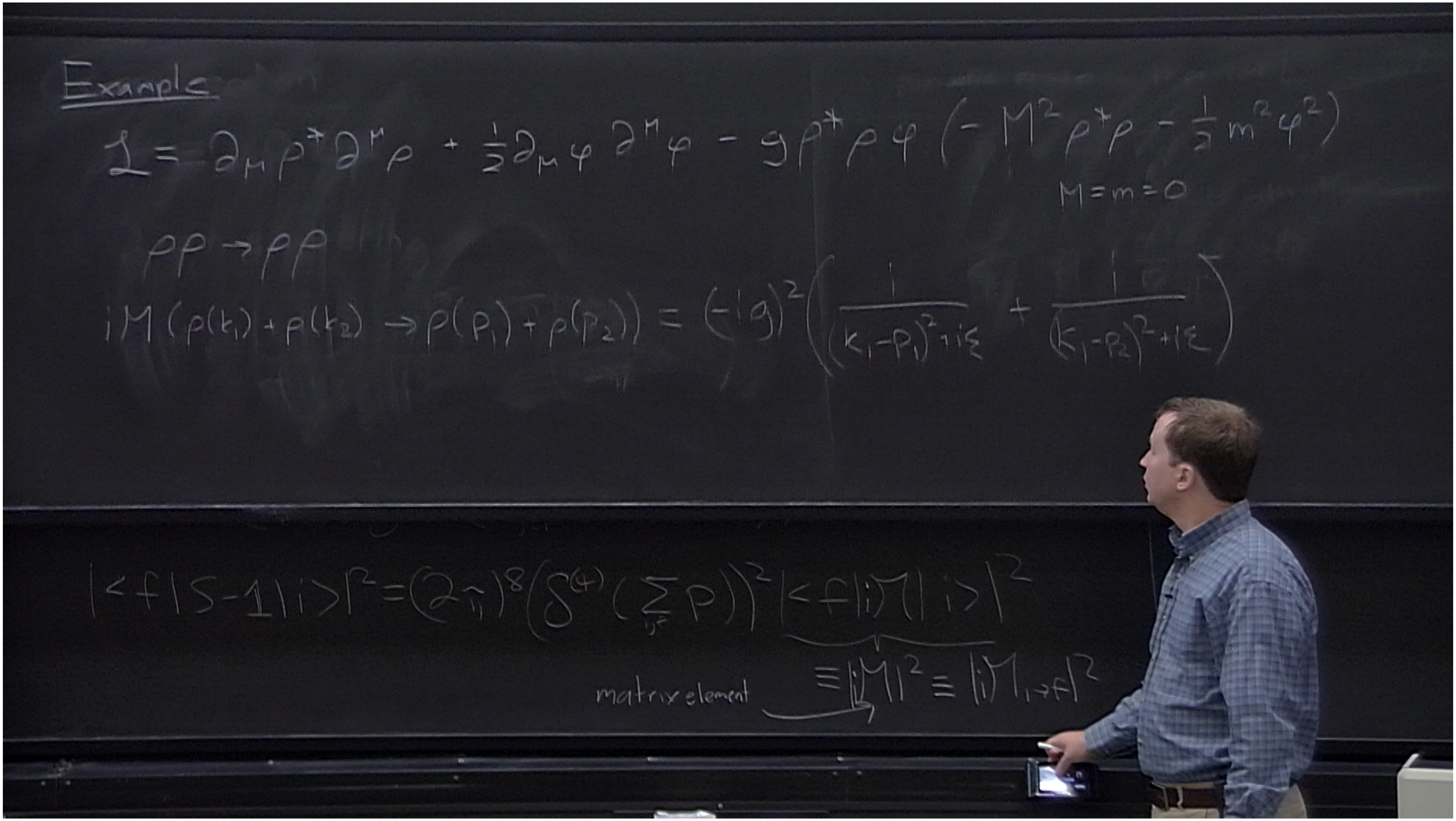
$$\int d\Phi_2\, f(p_1, p_2) = \int \frac{d\Omega}{32\pi^2}\, f(p_1, p_2)\Big|_{\substack{\vec{p}_2 = -\vec{p}_1 \\ |\vec{p}_1| = \frac{1}{2}E_{cm}}}$$

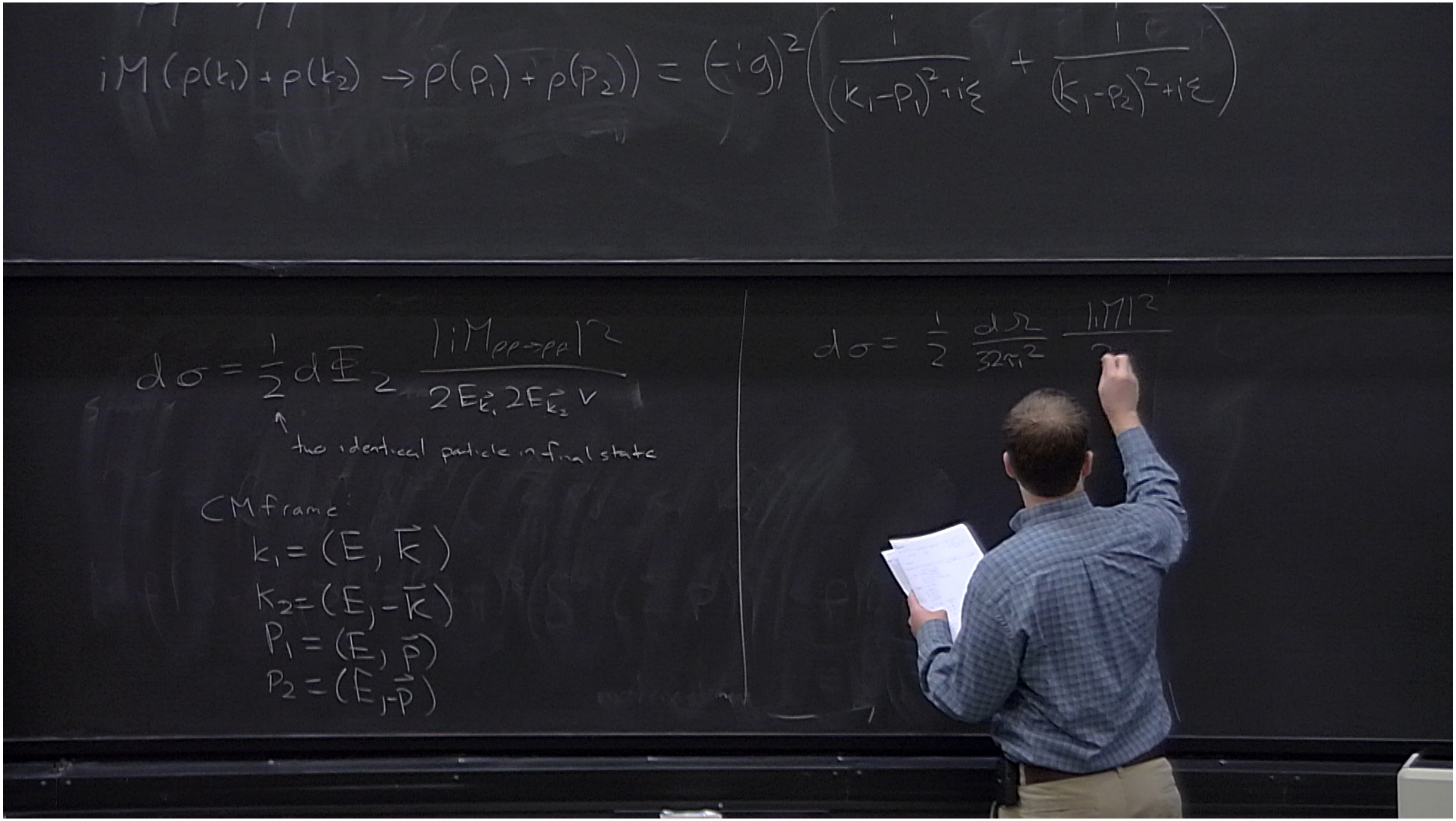
$$\boxed{d\Phi_2 = \frac{d\Omega}{32\pi^2}}$$


Phase Space
two massless particles in final state
$d\Phi_2 = (2\pi)^4 \delta^4(p_1 + p_2 - \sum_i k) \frac{d^3 p_1}{(2\pi)^3 2E_{\vec{p}_1}} \frac{d^3 p_2}{(2\pi)^3 2E_{\vec{p}_2}}$
CM frame $\sum_i \vec{k} = 0$ $\sum_i k^0 = E_{CM}$
$\int d\Phi_2 \frac{1}{\ldots 2E_{\vec{p}_1} 2E_{\vec{p}_2}} f\, \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - E_{CM})\, d^3 p_1 \Big|_{\vec{p}_2 = -\vec{p}_1}$

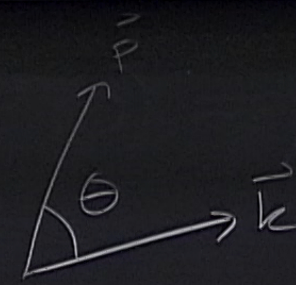

Phase Space
two massless particles in final state
$d\Phi_2 = (2\pi)^4 \delta^4(p_1 + p_2 - \sum_i k) \frac{d^3 p_1}{(2\pi)^3 2E_{\vec{p}_1}} \frac{d^3 p_2}{(2\pi)^3 2E_{\vec{p}_2}}$
CM frame $\sum_i \vec{k} = 0$ $\sum_i k^0 = E_{CM}$
$\int d\Phi_2 \frac{1}{(2\pi)^2 2E_{\vec{p}_1} 2E_{\vec{p}_2}} f \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - E_{CM}) d^3 p_1 \Big|_{\vec{p}_2 = -\vec{p}_1}$


Example
$\mathcal{L} = \partial_\mu \rho^* \partial^\mu \rho + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - g\rho^* \rho \varphi \left(-M^2 \rho^* \rho - \frac{1}{2} m^2 \varphi^2\right)$
$m=0$


Example
$\mathcal{L} = \partial_\mu \rho^* \partial^\mu \rho + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - g\rho^*\rho\varphi \left(- M^2 \rho^*\rho - \frac{1}{2} m^2 \varphi^2\right)$
$M = m = 0$
$\rho\rho \to \rho\rho$
$i\mathcal{M}(\rho(k_1) + \rho(k_2) \to \rho(p_1) + \rho(p_2)) = (-ig)^2 \left(\frac{1}{(k_1-p_1)^2 + i\varepsilon} + \frac{1}{(k_1-p_2)^2 + i\varepsilon} \right)$
$|\langle f | S - 1 | i \rangle|^2 = (2\pi)^8 \left(\delta^{(4)}\left(\sum_{i,f} p\right)\right)^2 \underbrace{|\langle f | i\mathcal{M} | i \rangle|^2}_{\equiv |i\mathcal{M}|^2 \equiv |i\mathcal{M}_{i\to f}|^2}$
matrix element


$iM(\rho(k_1)+\rho(k_2)\to\rho(p_1)+\rho(p_2)) = (-ig)^2\left(\frac{i}{(k_1-p_1)^2+i\varepsilon}+\frac{i}{(k_1-p_2)^2+i\varepsilon}\right)$
$d\sigma = \frac{1}{2} d\Phi_2 \frac{|iM_{\rho\rho\to\rho\rho}|^2}{2E_{\vec{k}_1} 2E_{\vec{k}_2} v}$
two identical particle in final state
CM frame
$k_1 = (E, \vec{k})$
$k_2 = (E, -\vec{k})$
$p_1 = (E, \vec{p})$
$p_2 = (E, -\vec{p})$
$d\sigma = \frac{1}{2} \frac{d\Omega}{32\pi^2} |iM|^2$

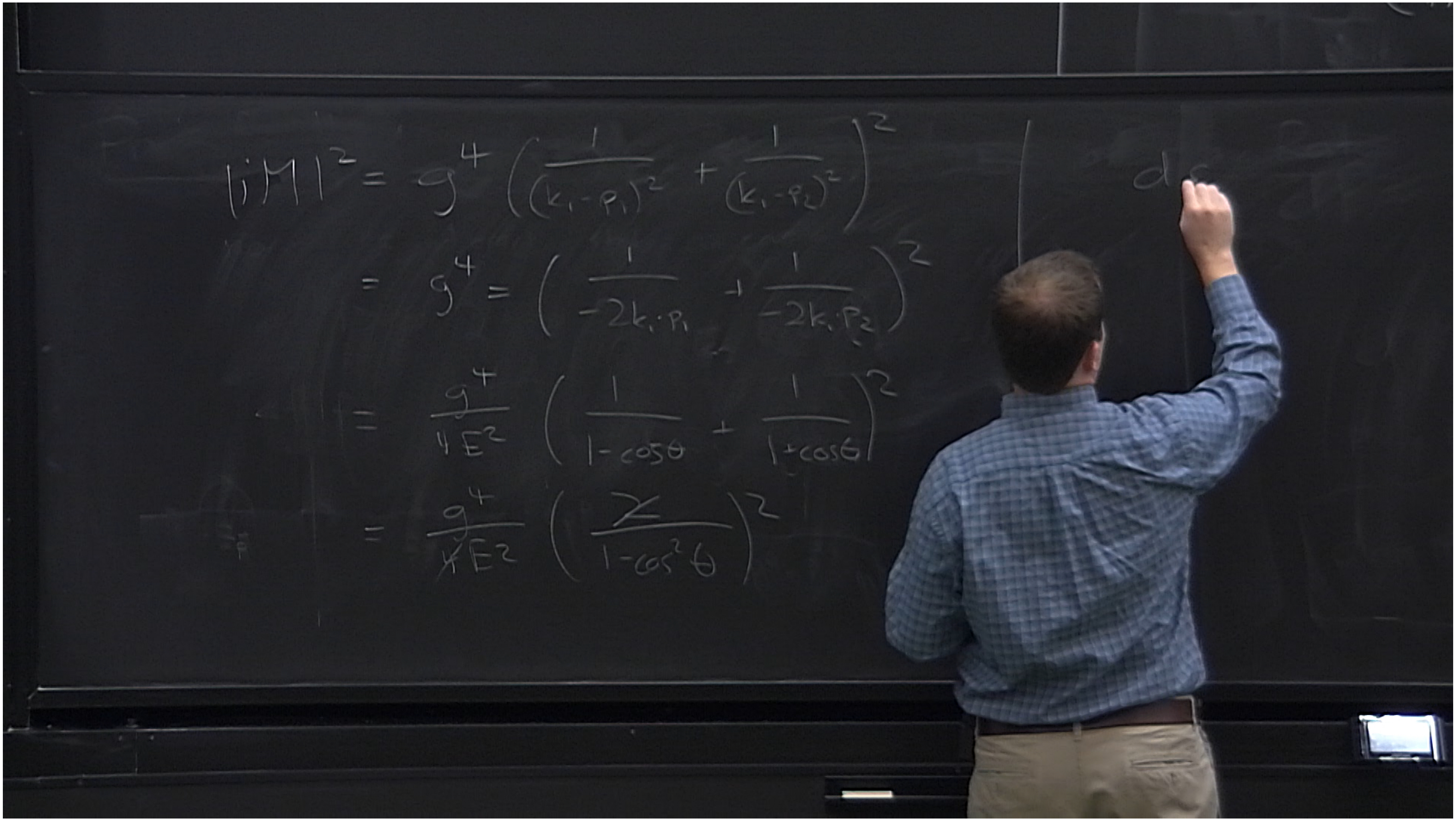
$$d\sigma = \frac{1}{2}\,\frac{d\Omega}{32\pi^2}\,\frac{|M|^2}{8E^2}$$

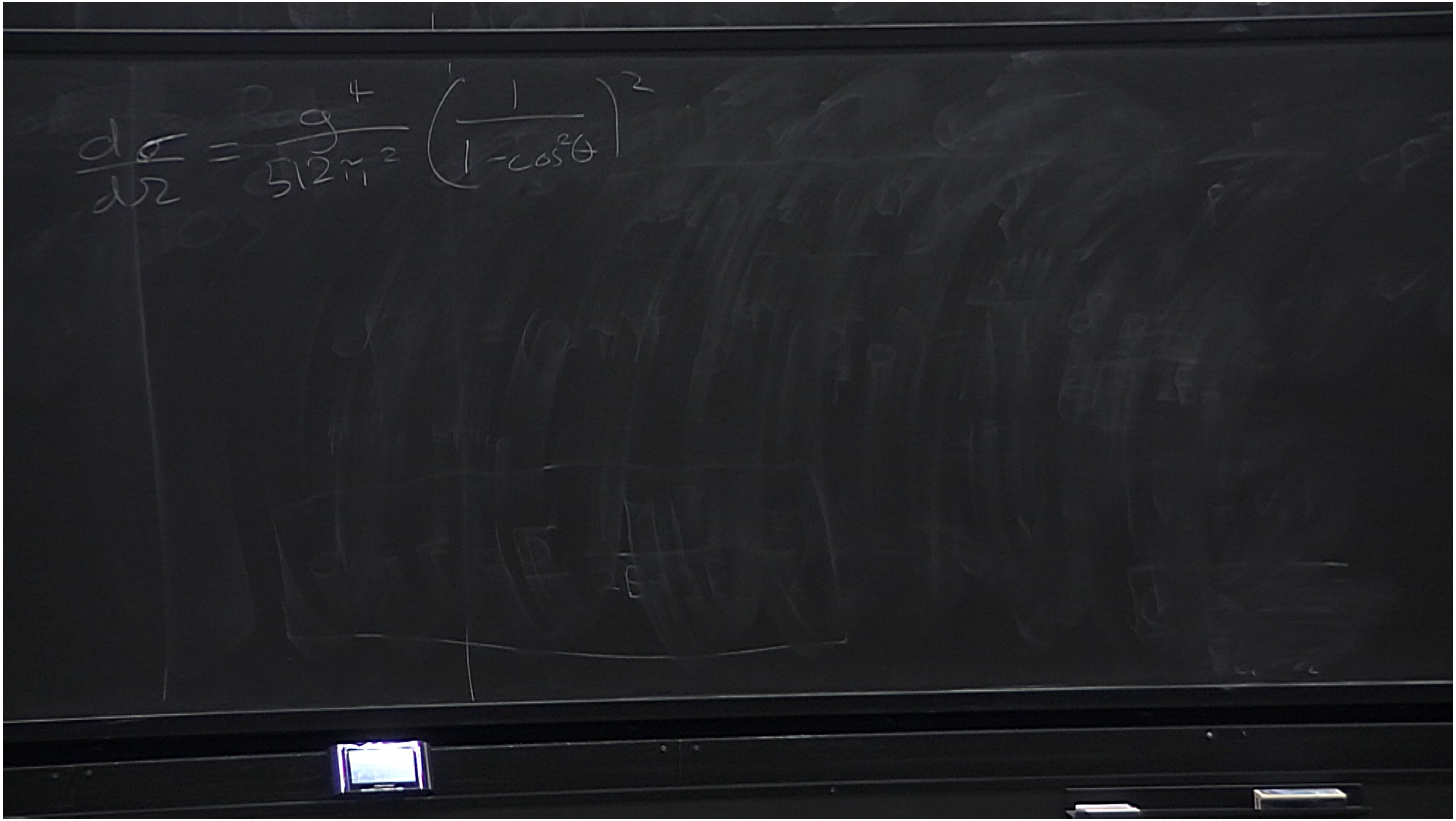


$$k_1^2 = k_2^2 = p_1^2 = p_2^2 = 0$$

$$k_1 \cdot p_1 = E^2 - \vec{k}\cdot\vec{p} = E^2(1-\cos\theta)$$

$$k_1 \cdot p_2 = E^2 + \vec{k}\cdot\vec{p} = E^2(1+\cos\theta)$$


$|iM|^2 = g^4 \left(\frac{1}{(k_1-p_1)^2} + \frac{1}{(k_1-p_2)^2} \right)^2$
$= g^4 = \left(\frac{1}{-2k_1 \cdot p_1} + \frac{1}{-2k_1 \cdot p_2} \right)^2$
$= \frac{g^4}{4E^2} \left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} \right)^2$
$= \frac{g^4}{\cancel{4}E^2} \left(\frac{\cancel{2}}{1-\cos^2\theta} \right)^2$


$\frac{d\sigma}{d\Omega} = \frac{g^4}{512 m^2} \left(\frac{1}{1-\cos^2\theta} \right)^2$