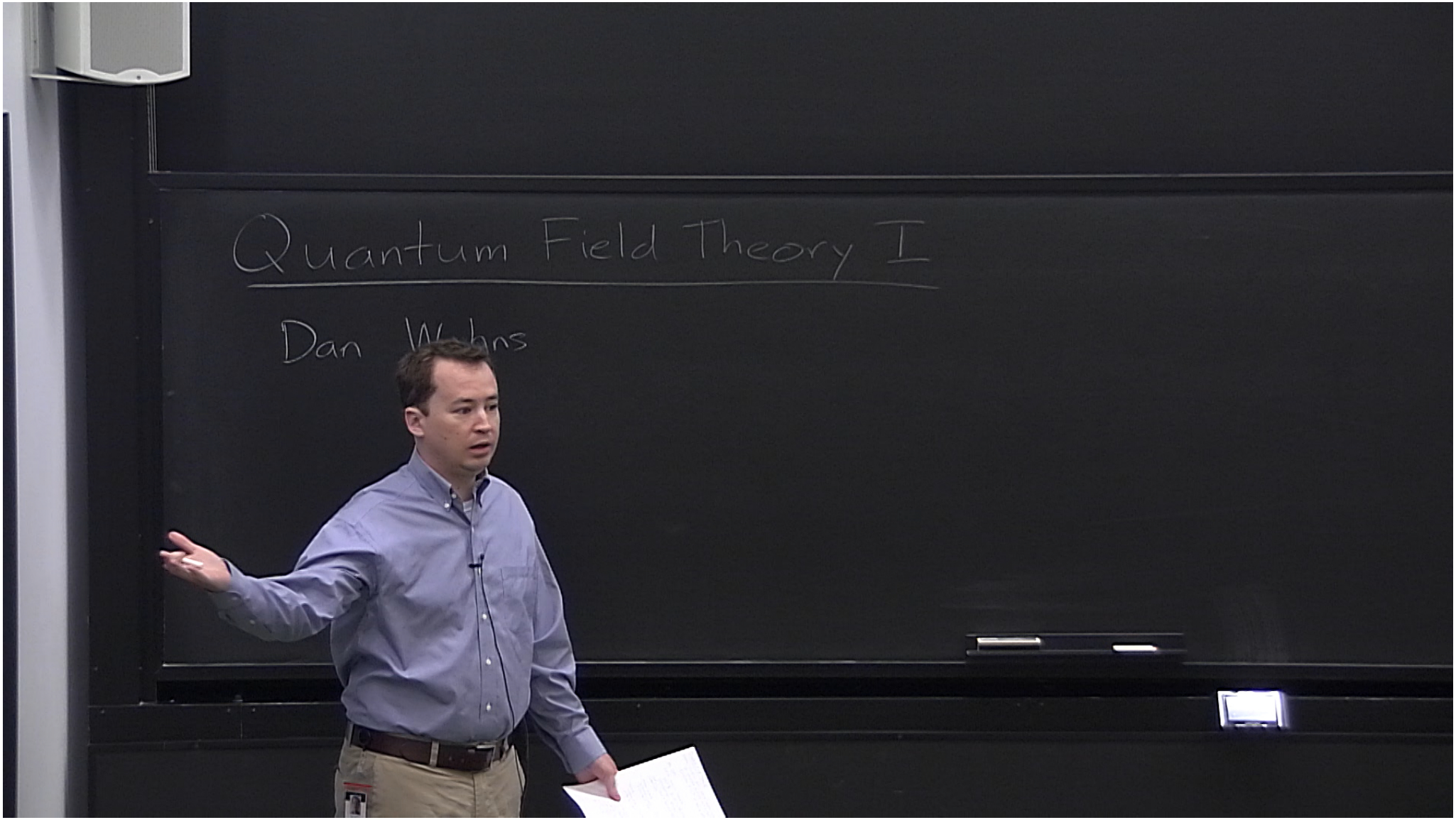
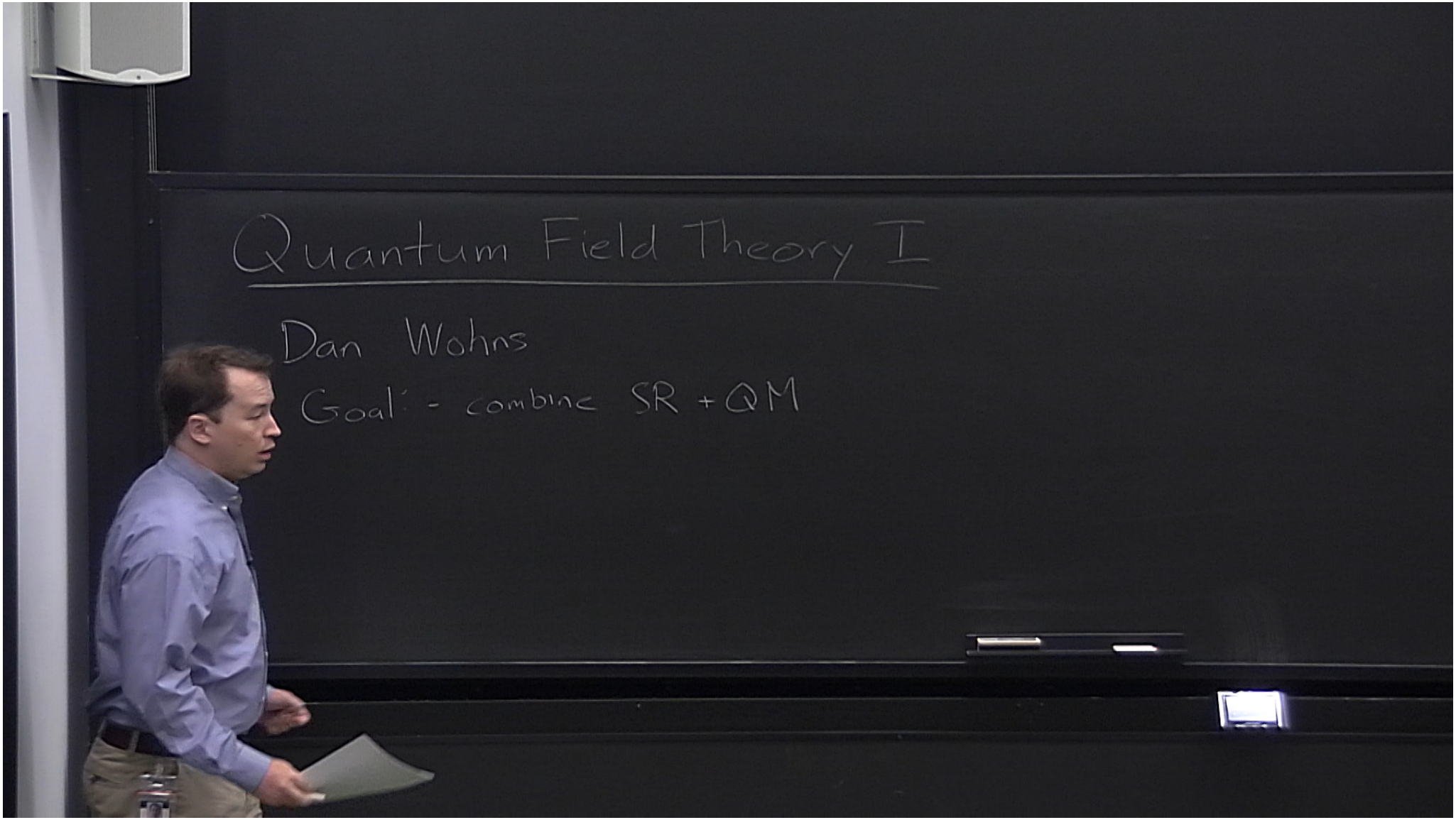
Title: PSI 17/18 - Quantum Field Theory I - Lecture 1

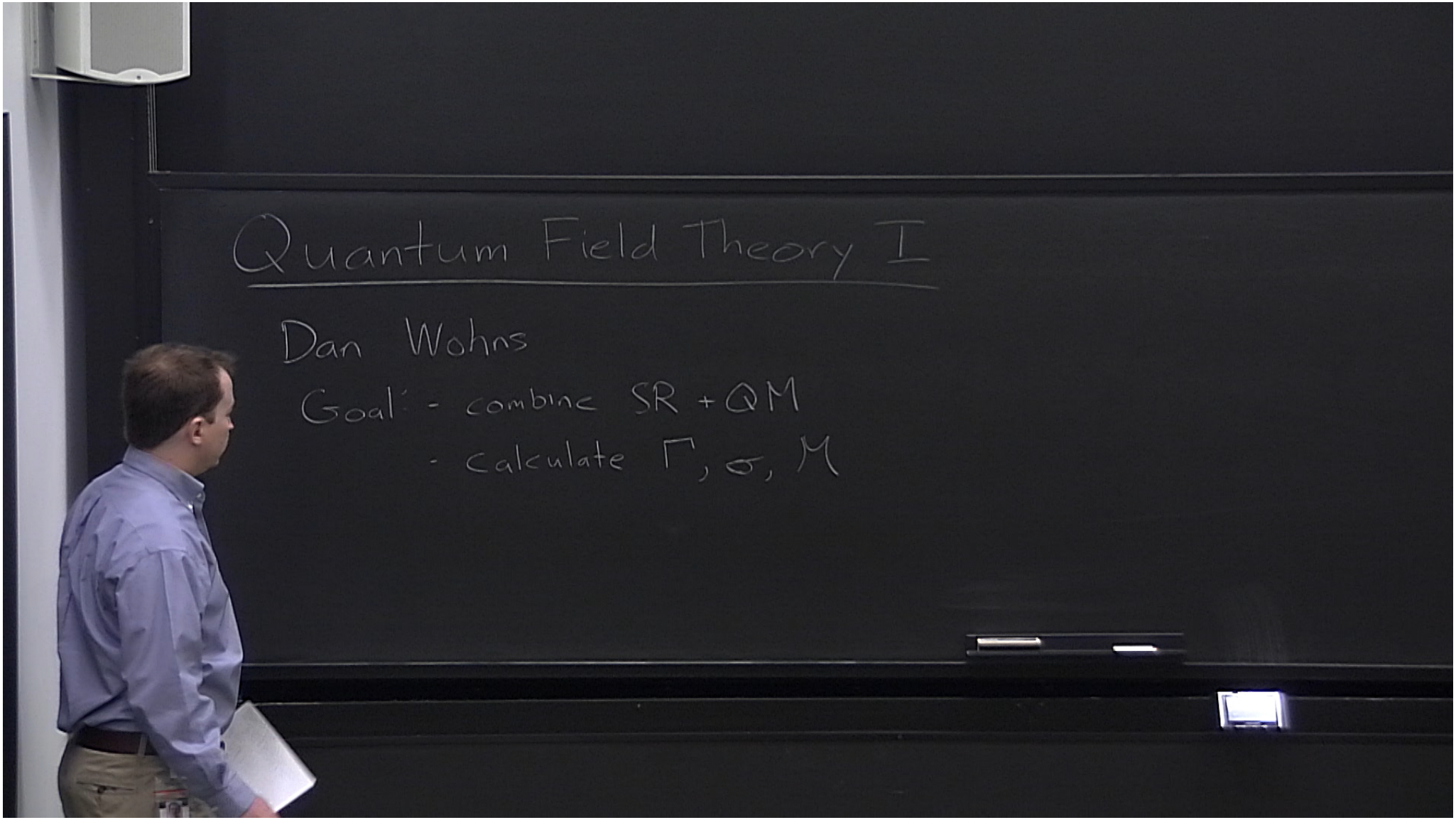
Date: Oct 10, 2017 09:00 AM

URL: http://pirsa.org/17100020

Abstract:


Quantum Field Theory I
Dan


Quantum Field Theory I
Dan Wohns
Goal: - combine SR + QM


Quantum Field Theory I
Dan Wohns
Goal: - combine SR + QM
- calculate Γ, σ, M

Fields

- particle number non-conservation

box of size L

$$\Delta E \geq \frac{\hbar c}{L}$$

$$\Delta E \geq 2mc^2$$

Fields

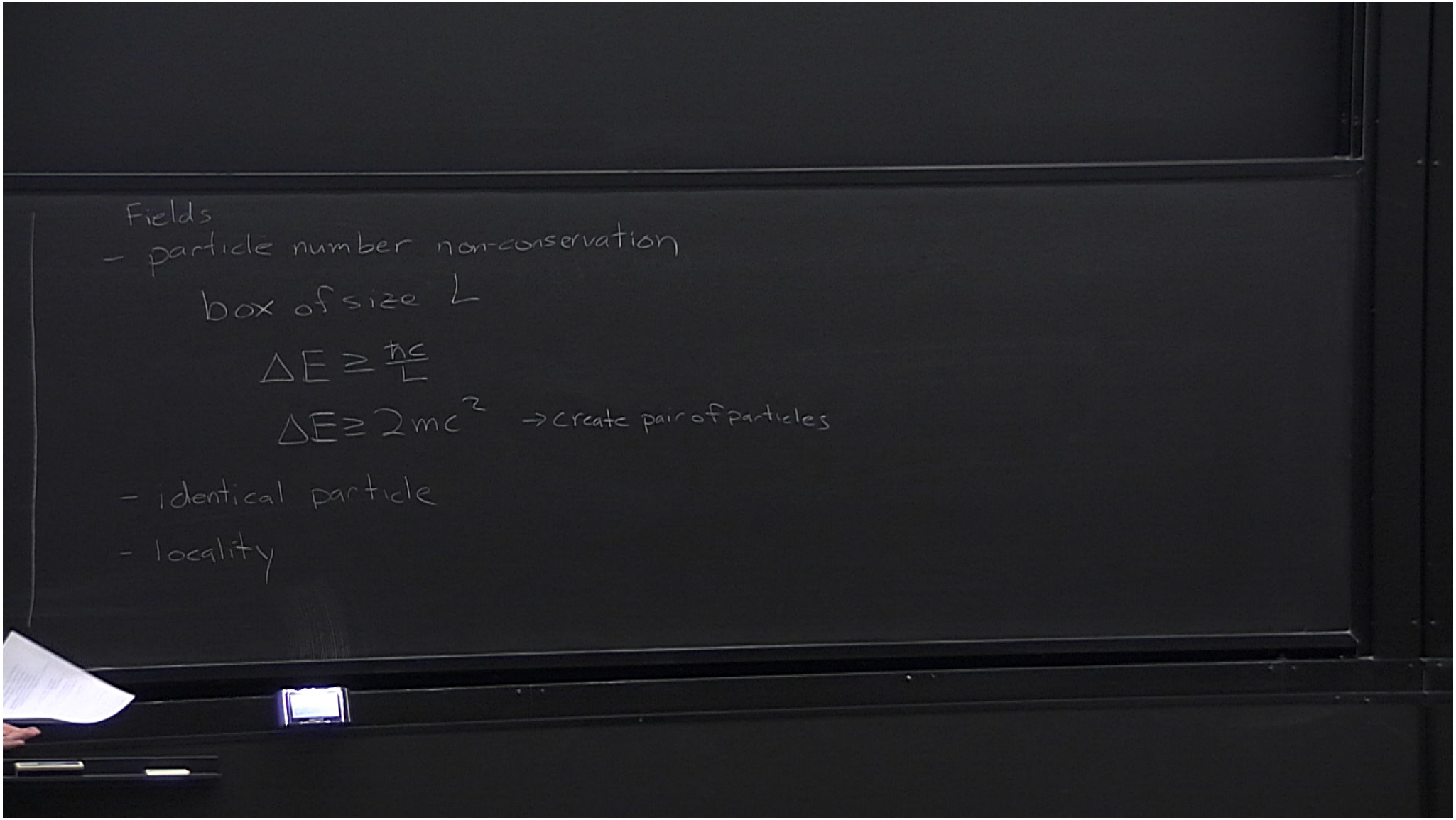
- particle number non-conservation

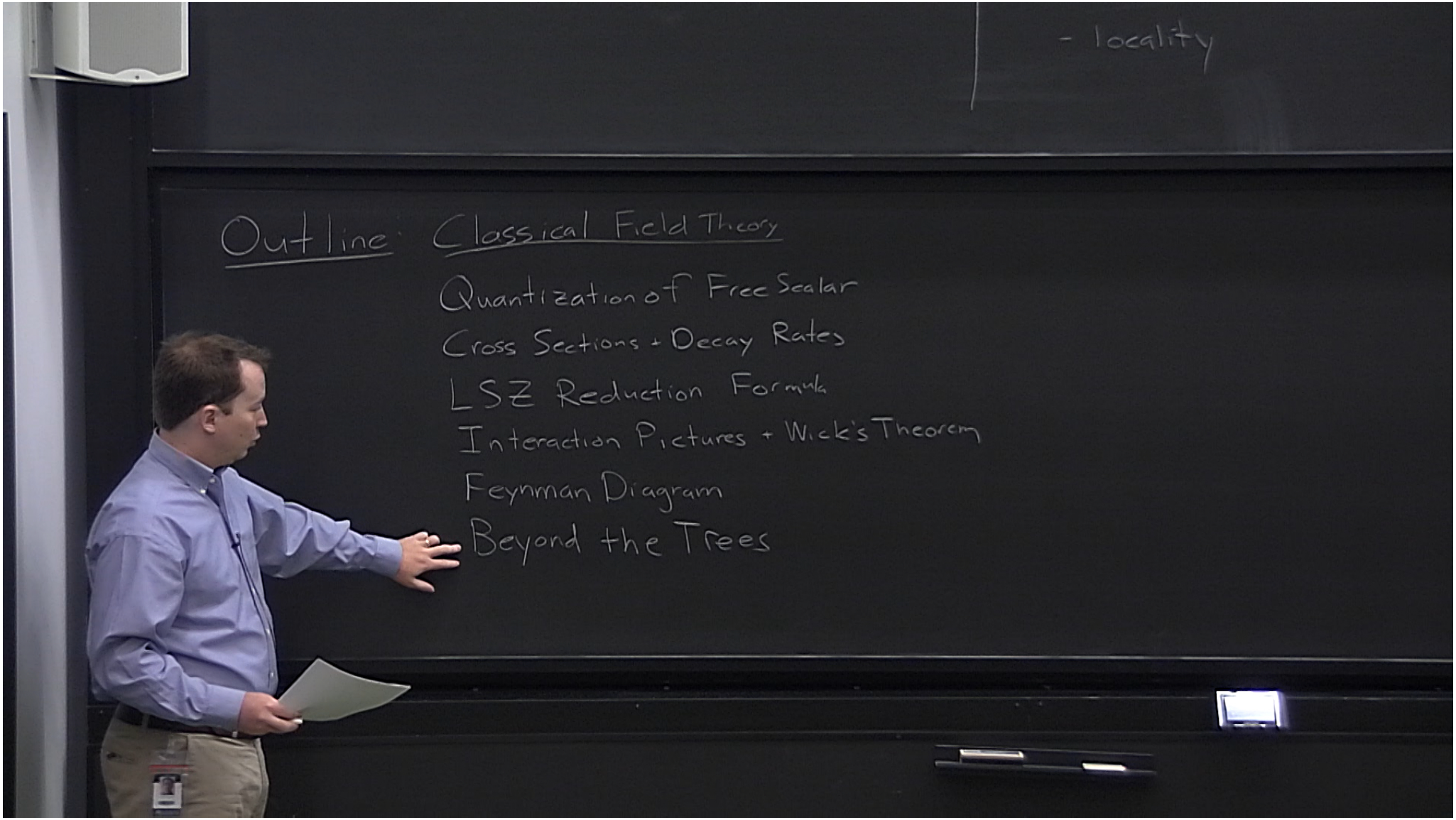
box of size L

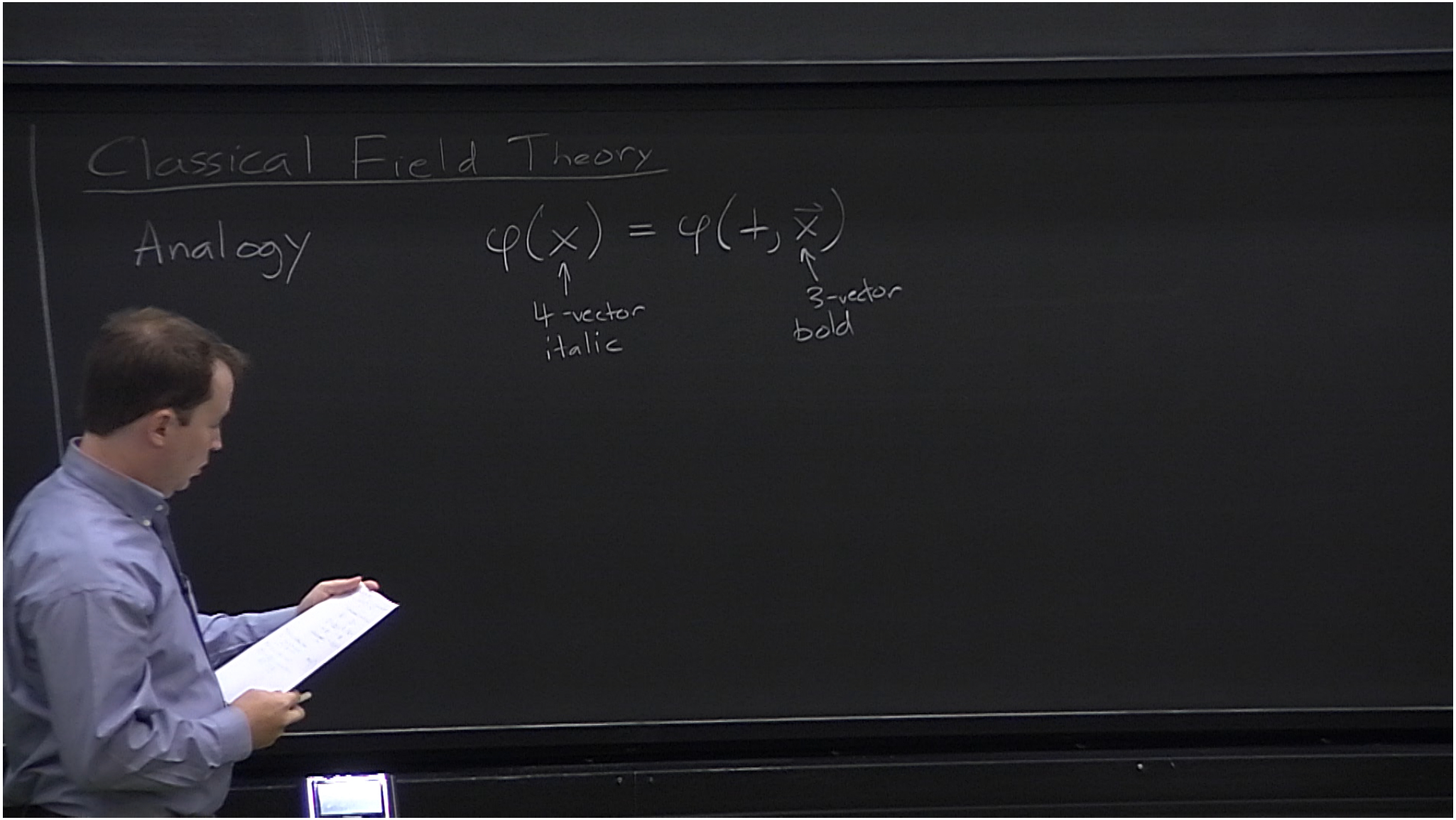
$$\Delta E \geq \frac{\hbar c}{L}$$

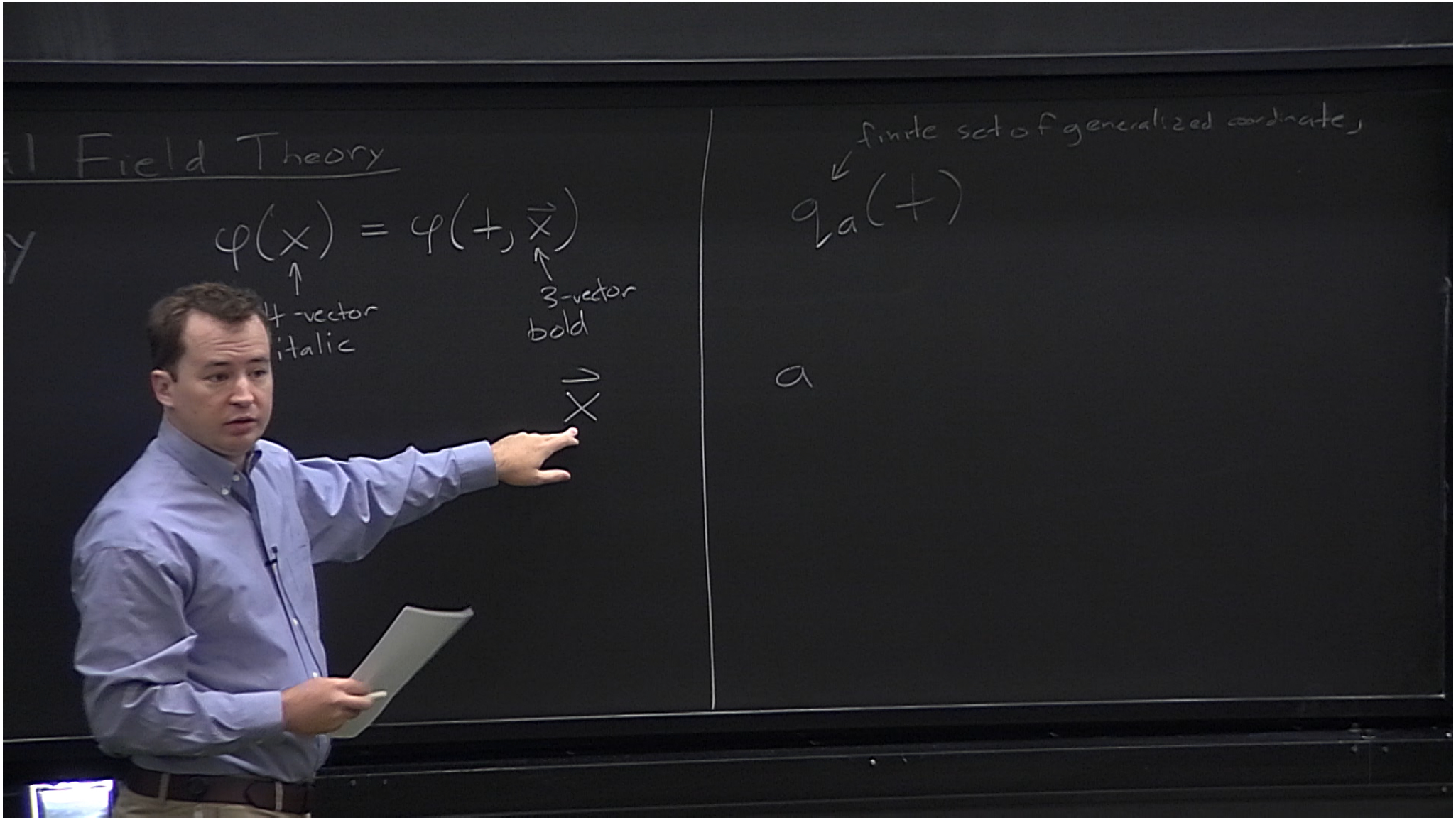
$$\Delta E \geq 2mc^2 \rightarrow \text{create pair of particles}$$

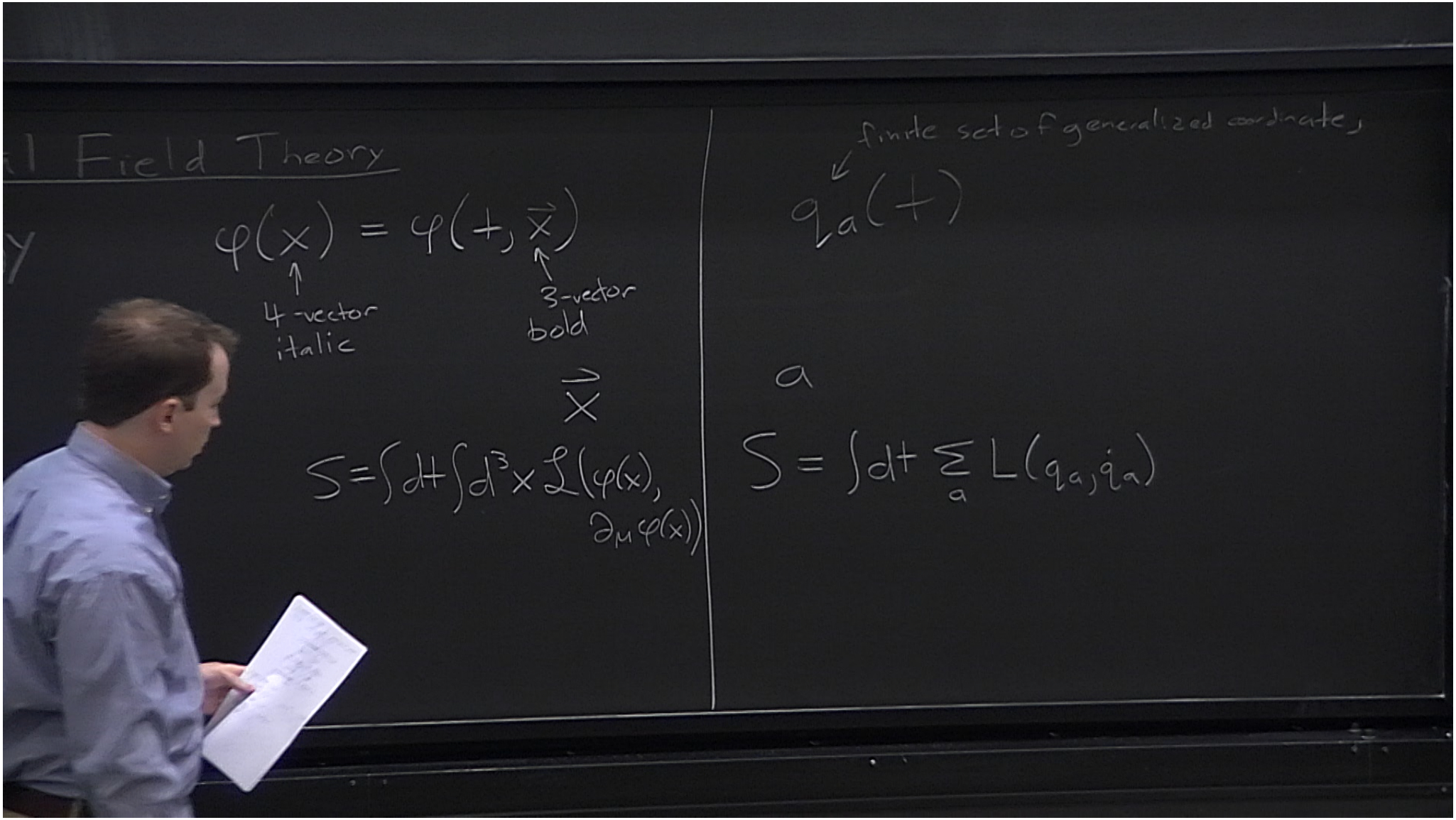
- identical particle

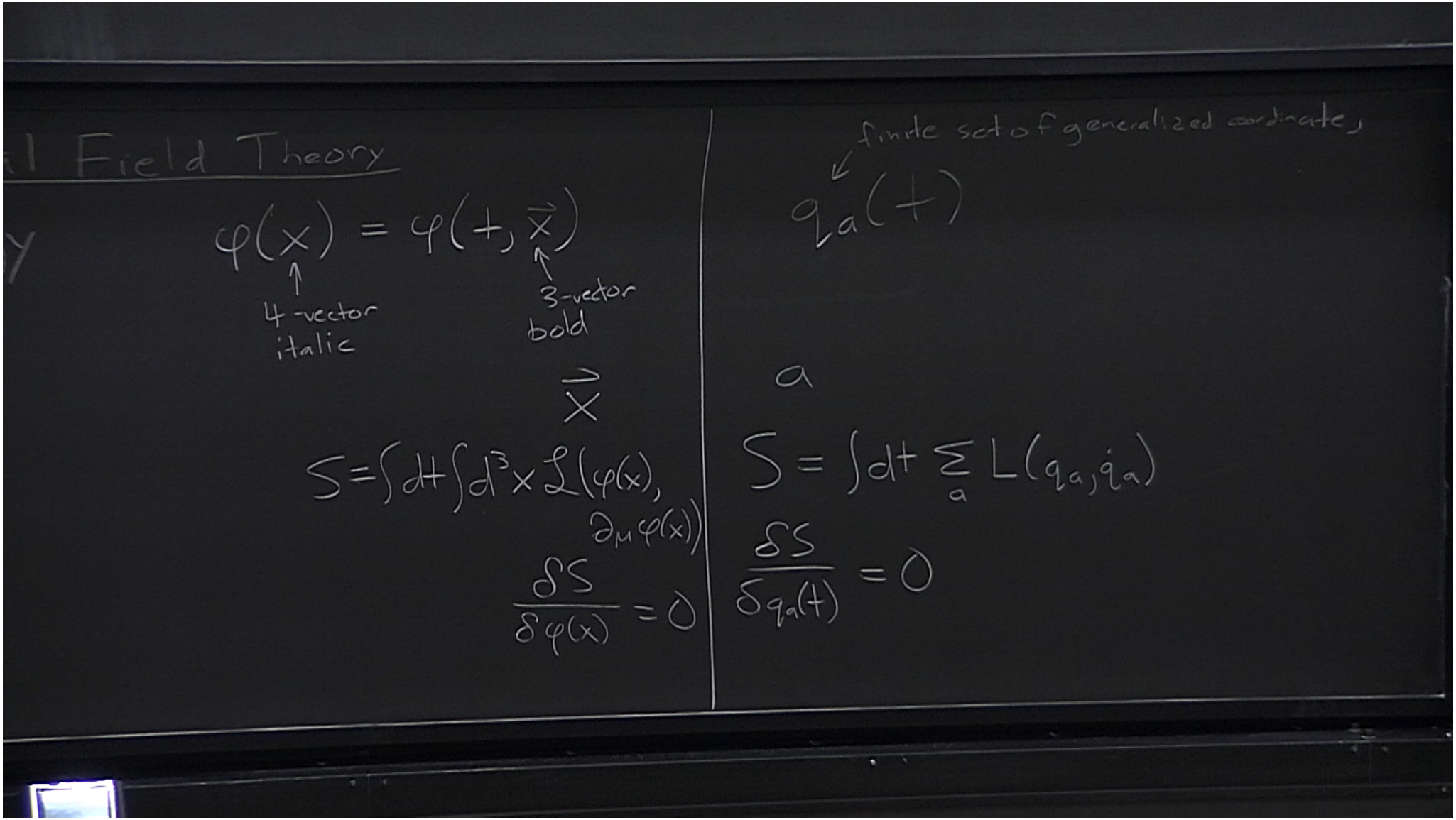

Fields
- particle number non-conservation
box of size L
$\Delta E \geq \frac{\hbar c}{L}$
$\Delta E \geq 2mc^2$ → create pair of particles
- identical particle
- locality

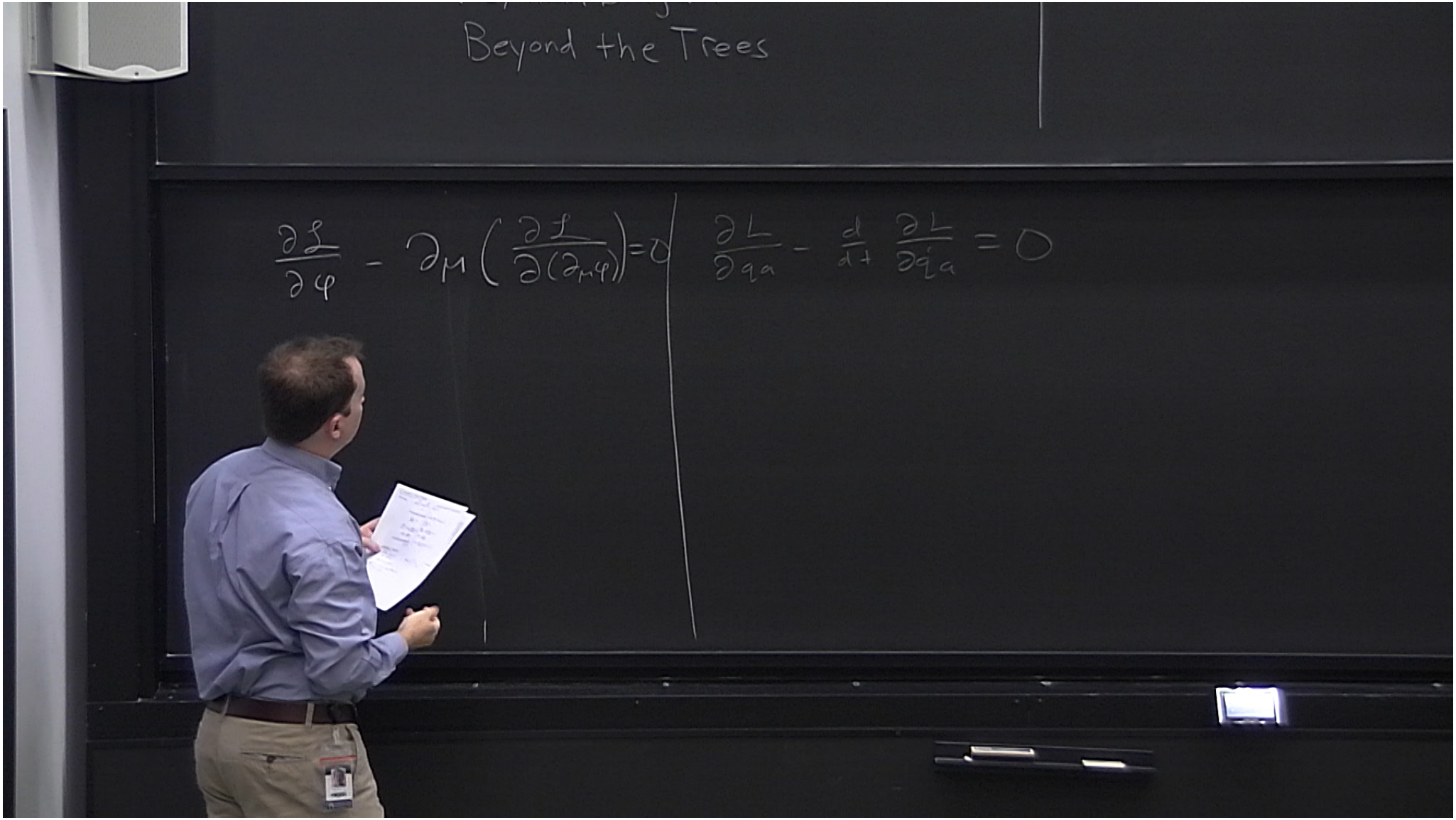

- locality
Outline: Classical Field Theory
Quantization of Free Scalar
Cross Sections + Decay Rates
LSZ Reduction Formula
Interaction Pictures + Wick's Theorem
Feynman Diagram
Beyond the Trees

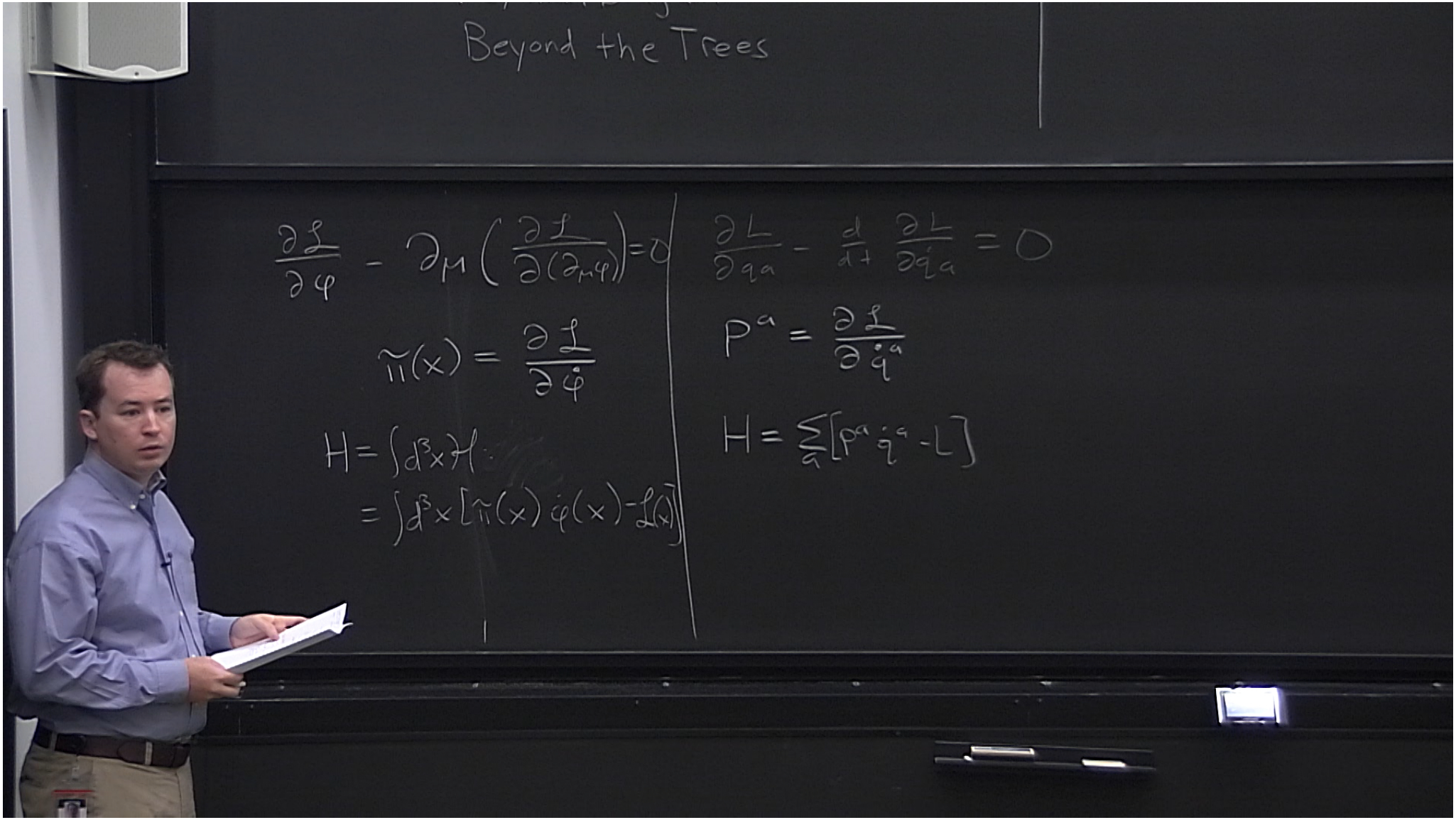

Classical Field Theory
Analogy
$\varphi(x) = \varphi(+, \vec{x})$
4-vector
italic
3-vector
bold

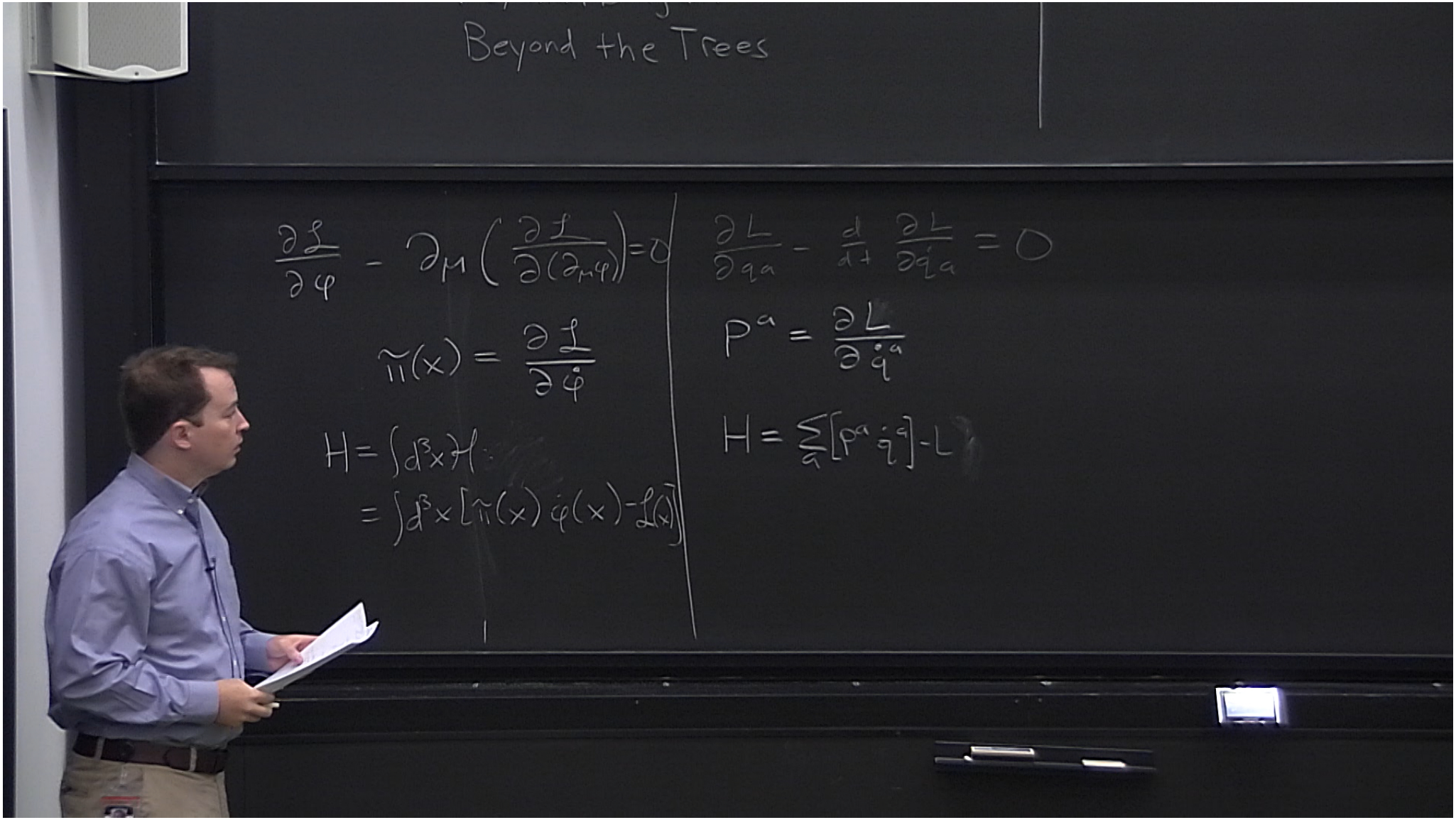

Field Theory
$\varphi(x) = \varphi(t, \vec{x})$
4-vector
italic
3-vector
bold
$\vec{x}$
finite set of generalized coordinates
$q_a(t)$
a

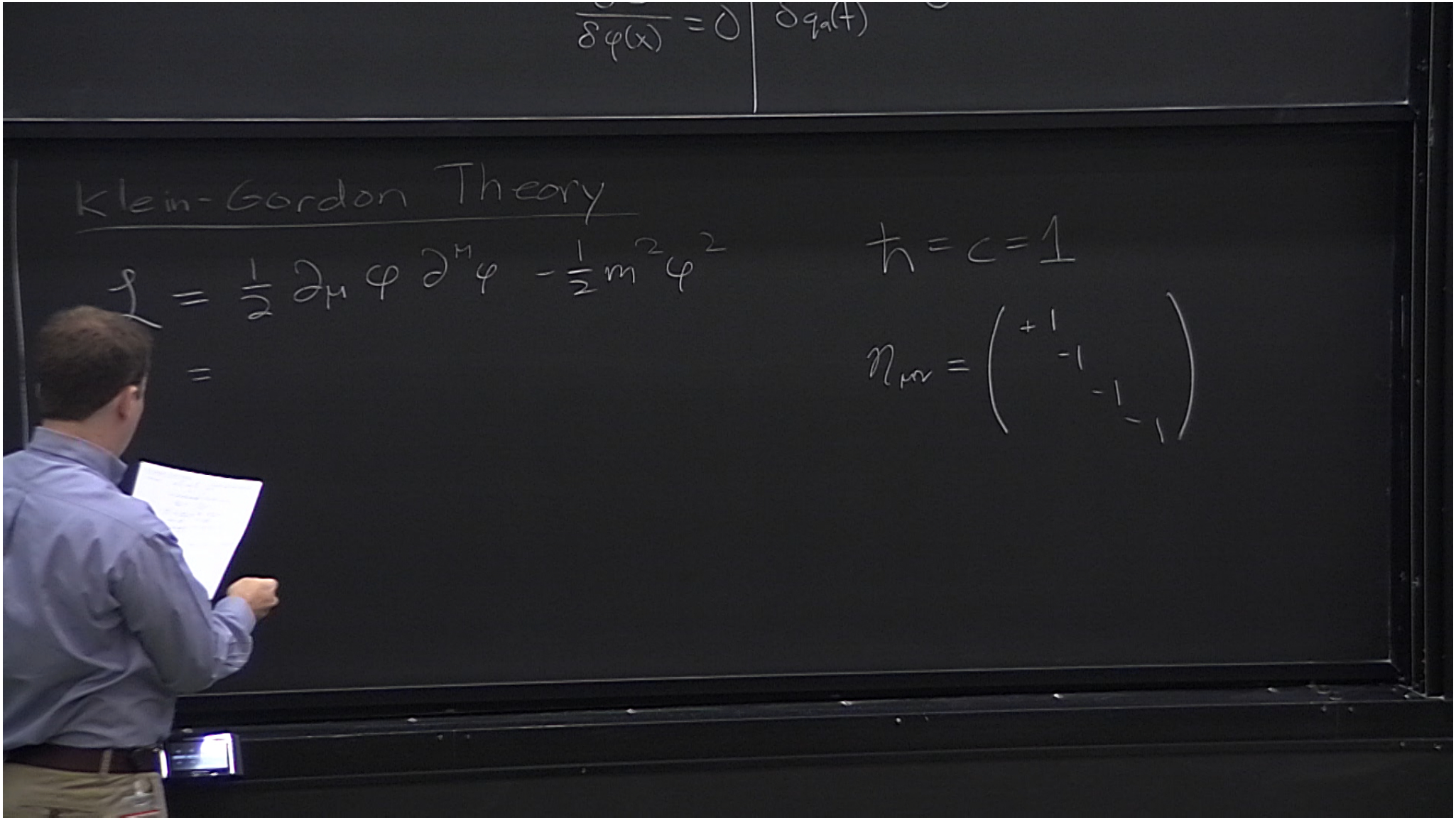

Field Theory
$\varphi(x) = \varphi(t, \vec{x})$
4-vector
italic
3-vector
bold
$\vec{x}$
$S = \int dt \int d^3x \, \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$
finite set of generalized coordinates
$q_a(t)$
a
$S = \int dt \sum_a L(q_a, \dot{q}_a)$


Field Theory
$\varphi(x) = \varphi(t, \vec{x})$
4-vector
italic
3-vector
bold
$\vec{x}$
$S = \int dt \int d^3x \, \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$
$\frac{\delta S}{\delta \varphi(x)} = 0$
finite set of generalized coordinates
$q_a(t)$
a
$S = \int dt \sum_a L(q_a, \dot{q}_a)$
$\frac{\delta S}{\delta q_a(t)} = 0$


Beyond the Trees
$\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) = 0$
$\frac{\partial L}{\partial q_a} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_a} = 0$


Beyond the Trees
$\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) = 0$
$\frac{\partial L}{\partial q_a} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_a} = 0$
$\tilde{\pi}(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$
$p^a = \frac{\partial L}{\partial \dot{q}^a}$
$H = \int d^3x \mathcal{H}$
$= \int d^3x [\tilde{\pi}(x) \dot{\varphi}(x) - \mathcal{L}(x)]$
$H = \sum_a [p^a \dot{q}^a - L]$


Beyond the Trees
$\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) = 0$
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$\frac{\delta}{\delta\varphi(x)} = 0$
$\delta q_a(t)$
Klein-Gordon Theory
$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$
$=$
$\hbar = c = 1$
$\eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$\frac{\delta}{\delta\varphi(x)} = 0$ | $\delta q_a(t)$

Klein-Gordon Theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$$

$$= \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\vec{\nabla}\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

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$$\pi = \dot{\varphi}$$

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2$$

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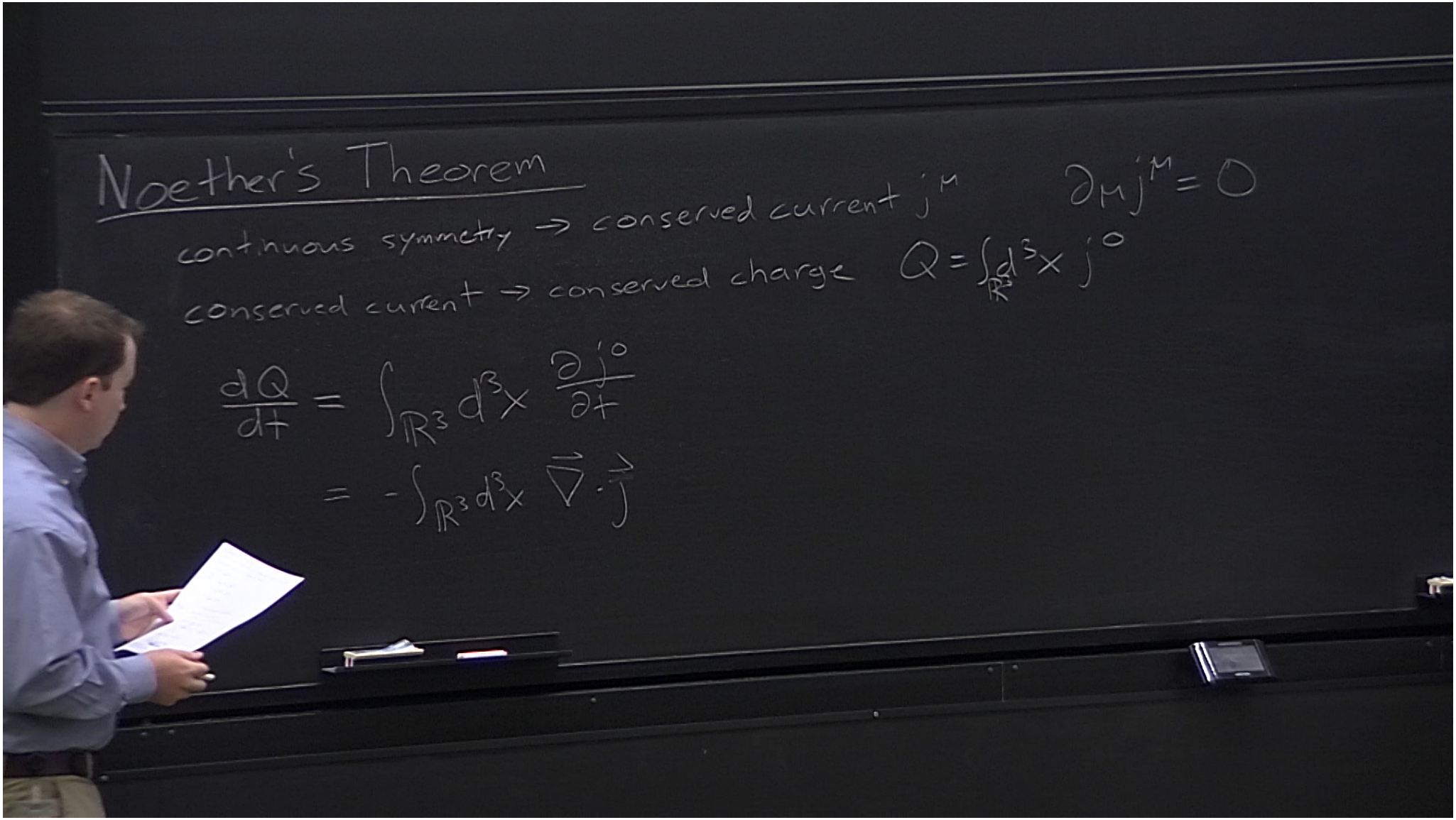
$\hbar = c = 1$

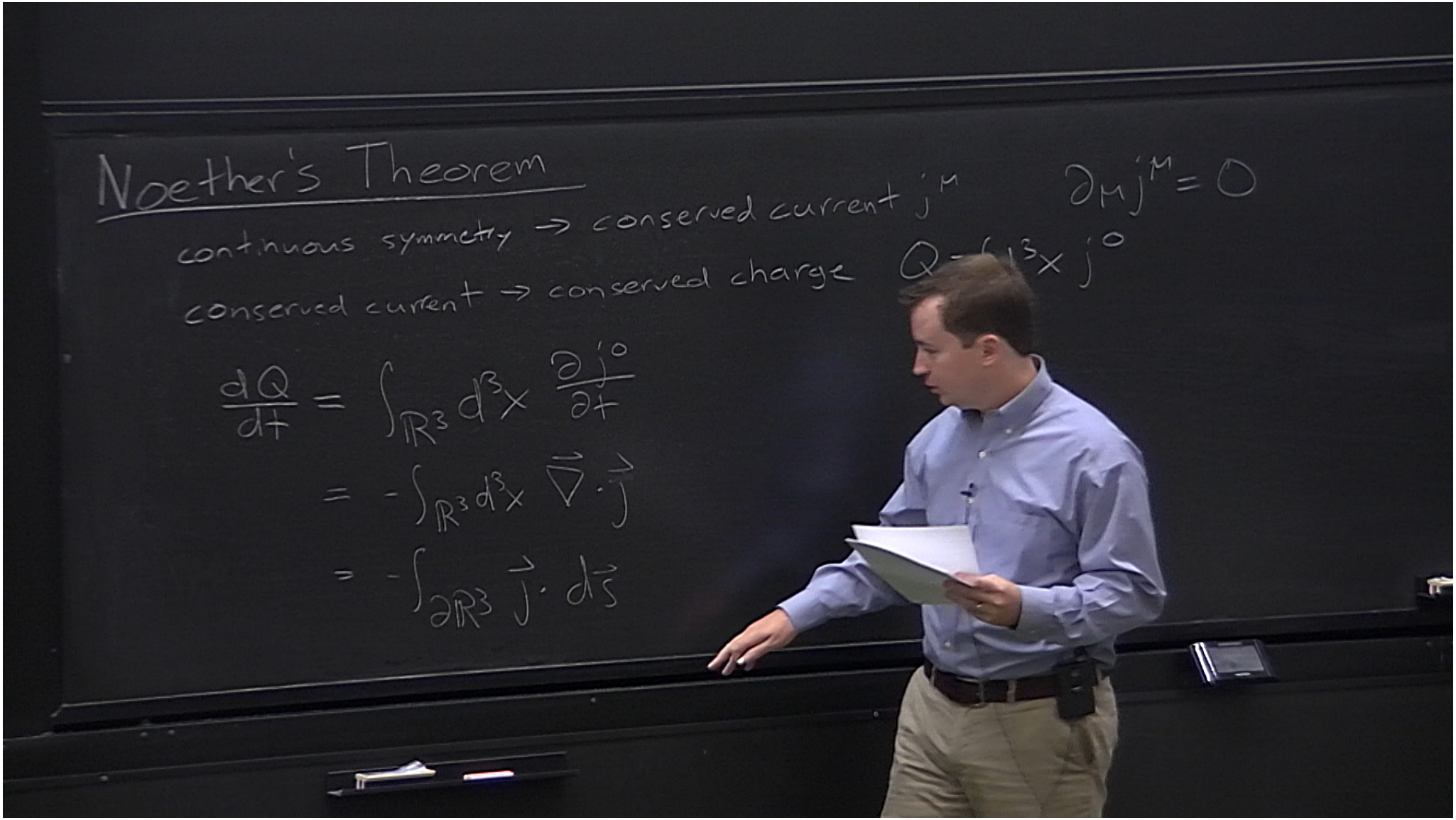
$$\eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

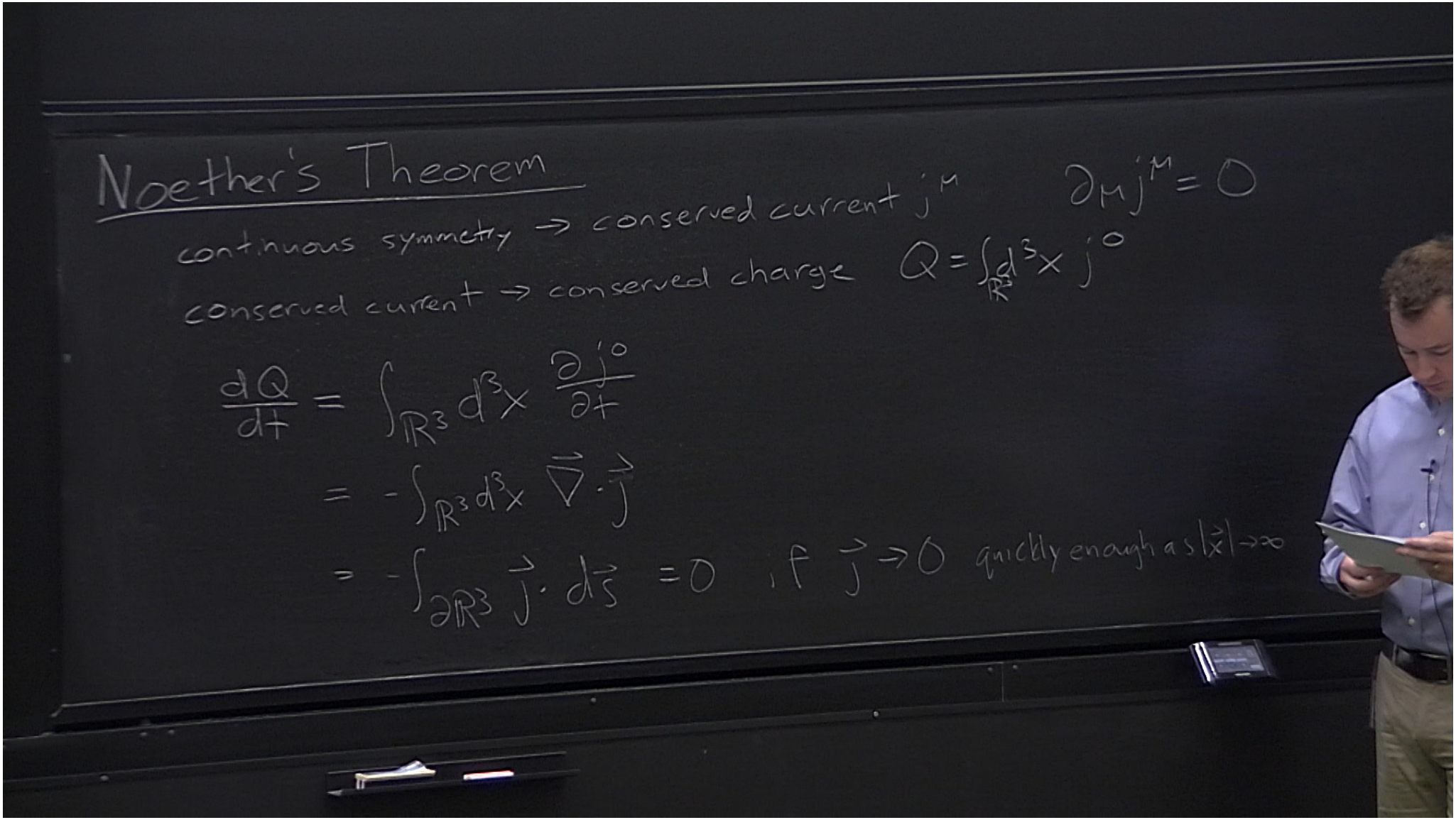
$\pi = \dot{\varphi}$

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2$$

$$\frac{\partial\mathcal{L}}{\partial\varphi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\right) = -m^2\varphi - \partial_\mu\partial^\mu\varphi = 0 \rightarrow \boxed{(\partial_\mu\partial^\mu + m^2)\varphi = 0}$$


Noether's Theorem
continuous symmetry → conserved current j^μ $\partial_\mu j^\mu = 0$
conserved current → conserved charge $Q = \int_{\mathbb{R}^3} d^3x\, j^0$
$\frac{dQ}{dt} = \int_{\mathbb{R}^3} d^3x \frac{\partial j^0}{\partial t}$
$= -\int_{\mathbb{R}^3} d^3x\, \vec{\nabla} \cdot \vec{j}$


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$= -\int_{\partial\mathbb{R}^3} \vec{j} \cdot d\vec{S}$


Noether's Theorem
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$= -\int_{\mathbb{R}^3} d^3x\, \vec{\nabla}\cdot\vec{j}$
$= -\int_{\partial\mathbb{R}^3} \vec{j}\cdot d\vec{S} = 0$ if $\vec{j} \to 0$ quickly enough as $|\vec{x}| \to \infty$

ein-Gordon Theory

$$= \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$$

$$= \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\vec{\nabla}\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

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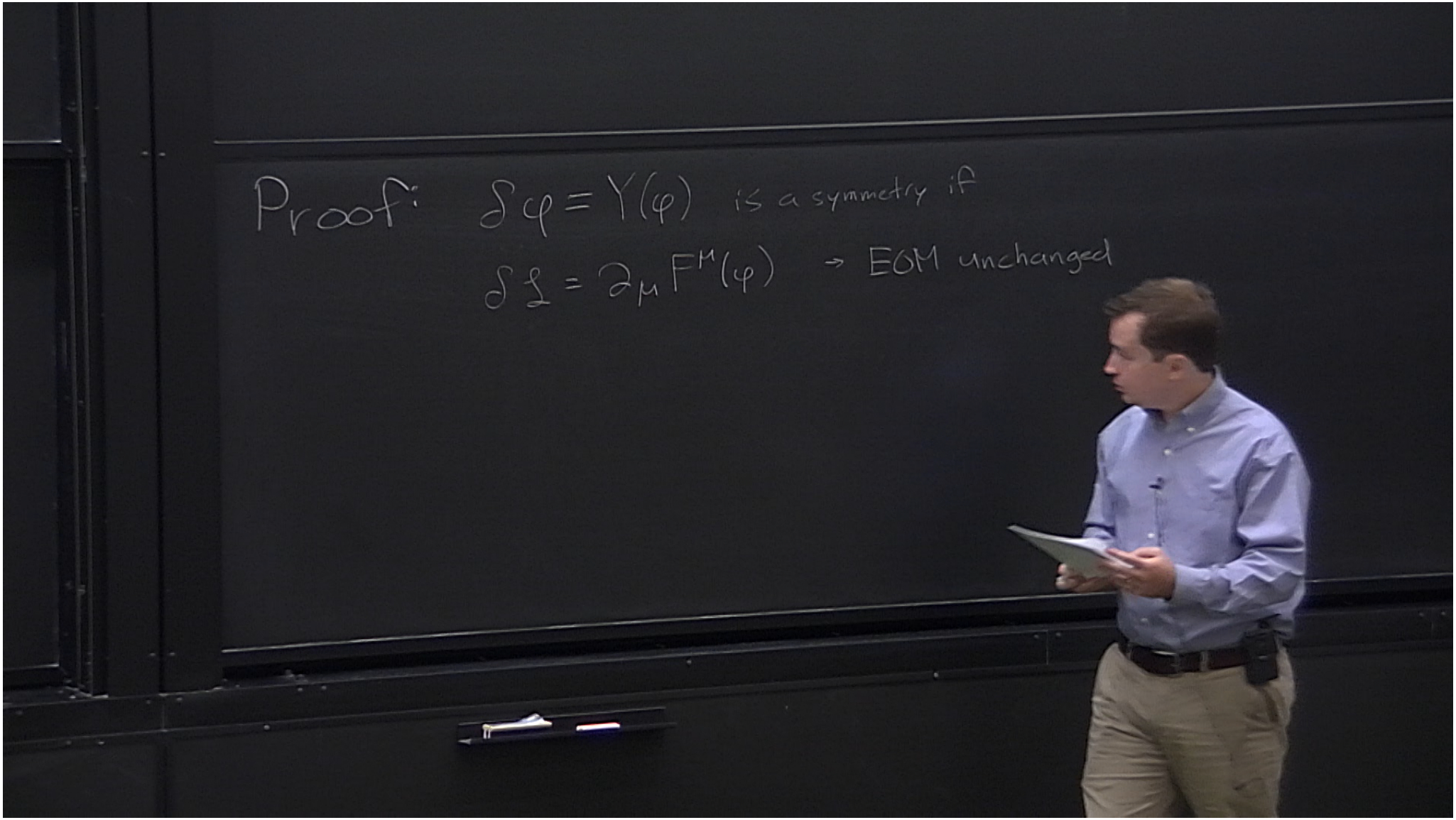
$$-\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\right) = -m^2\varphi - \partial_\mu\partial^\mu\varphi = 0$$

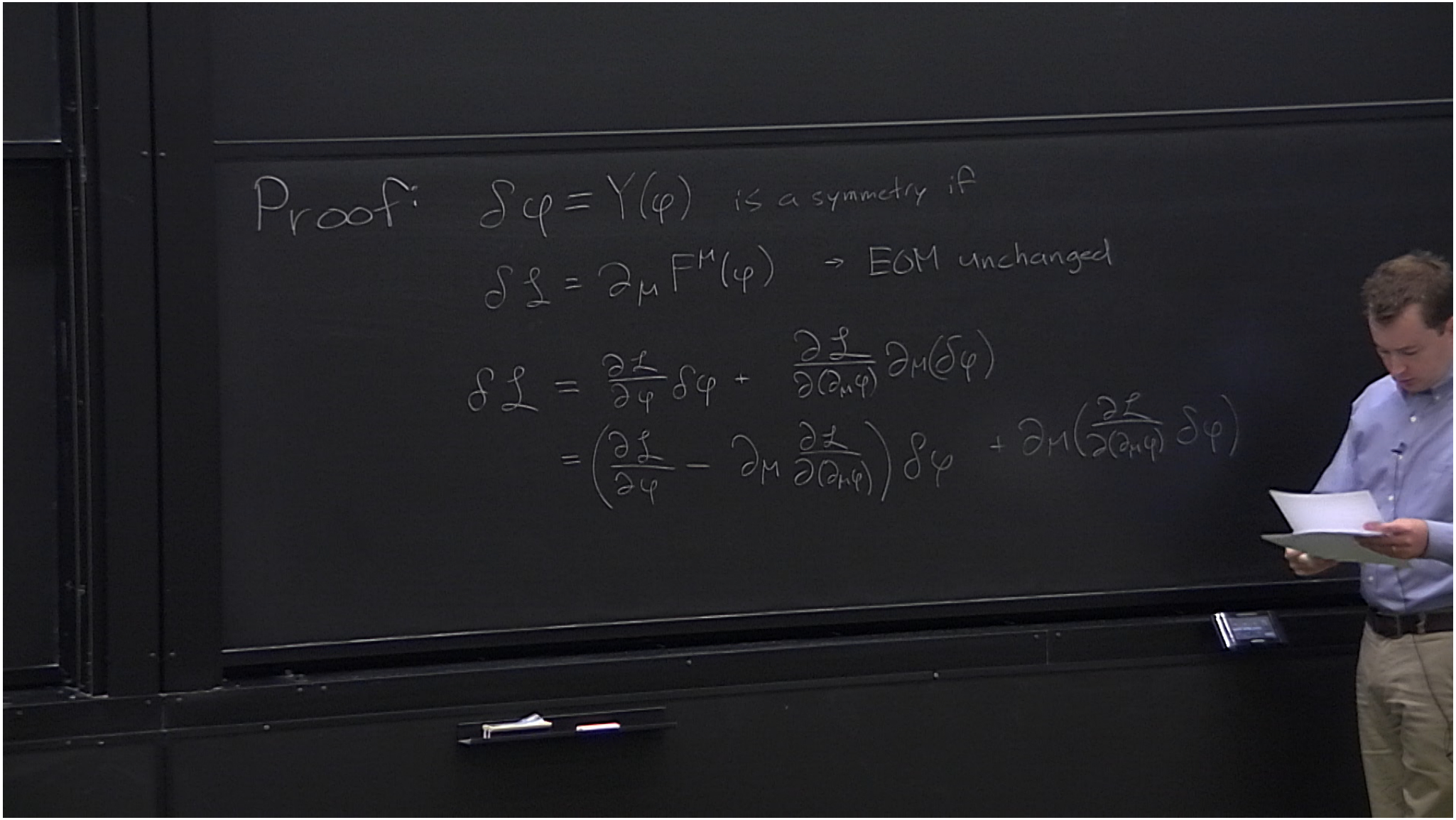
$\hbar = c = 1$

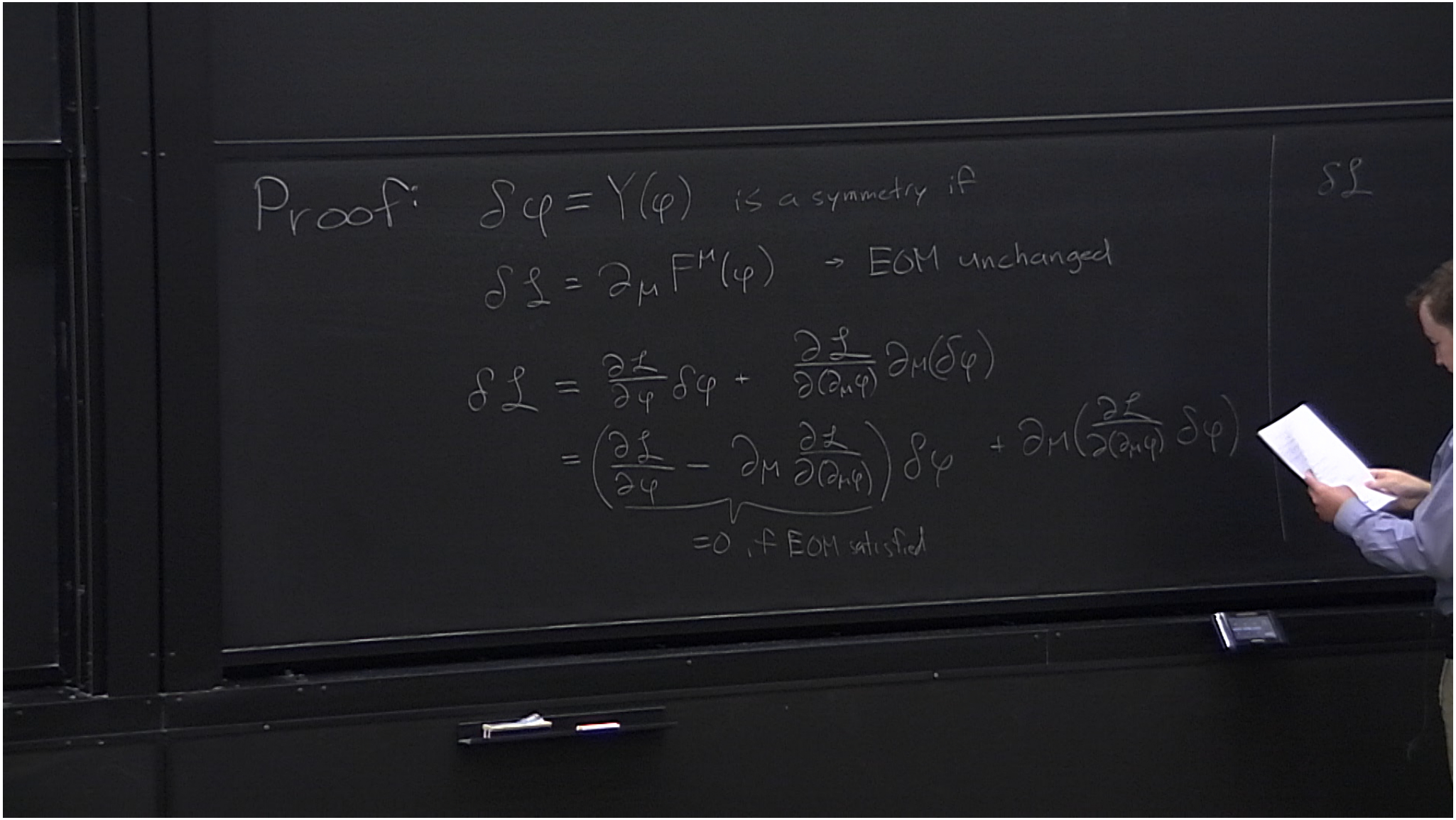
$$\eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

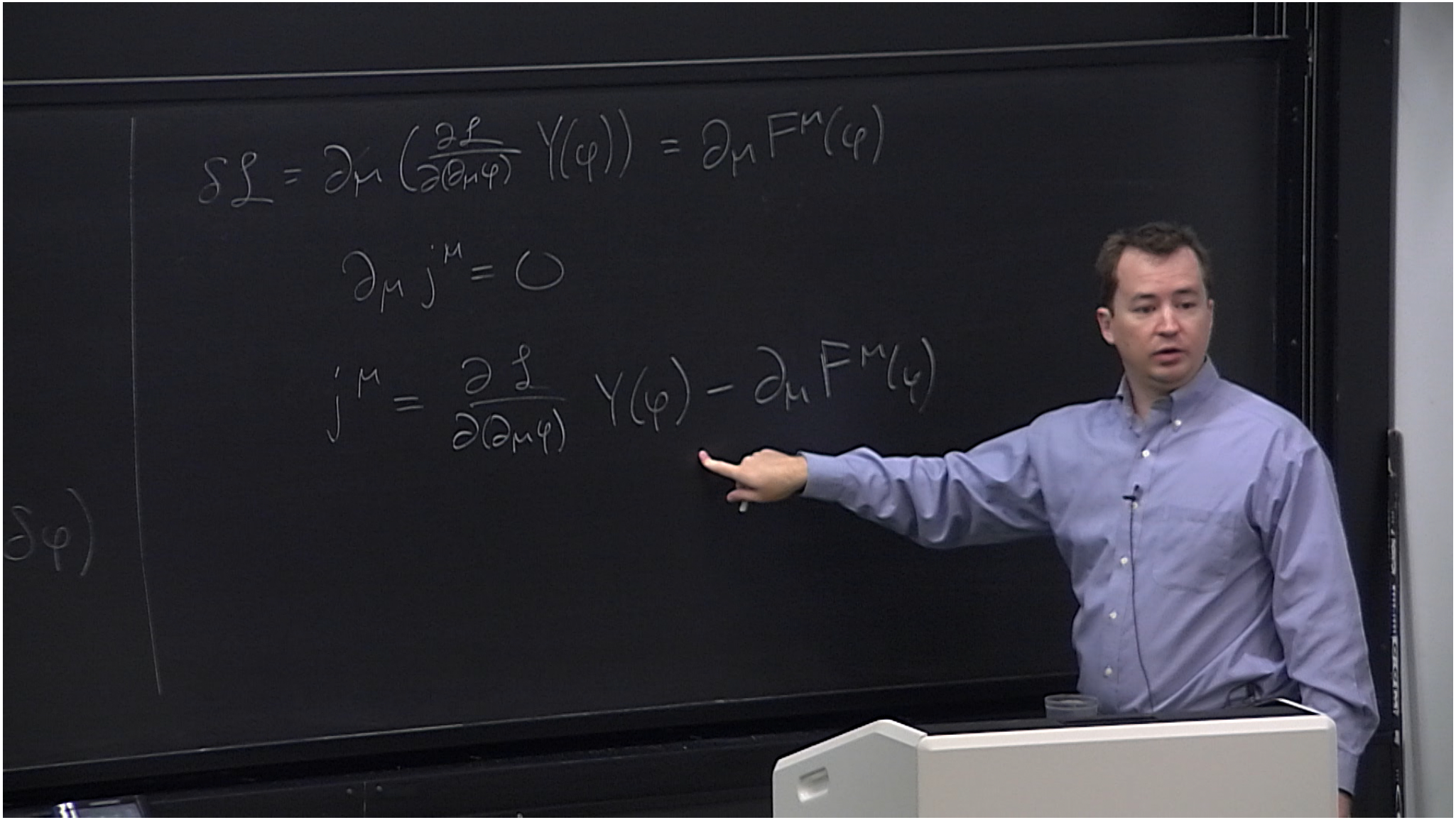
K-G equation

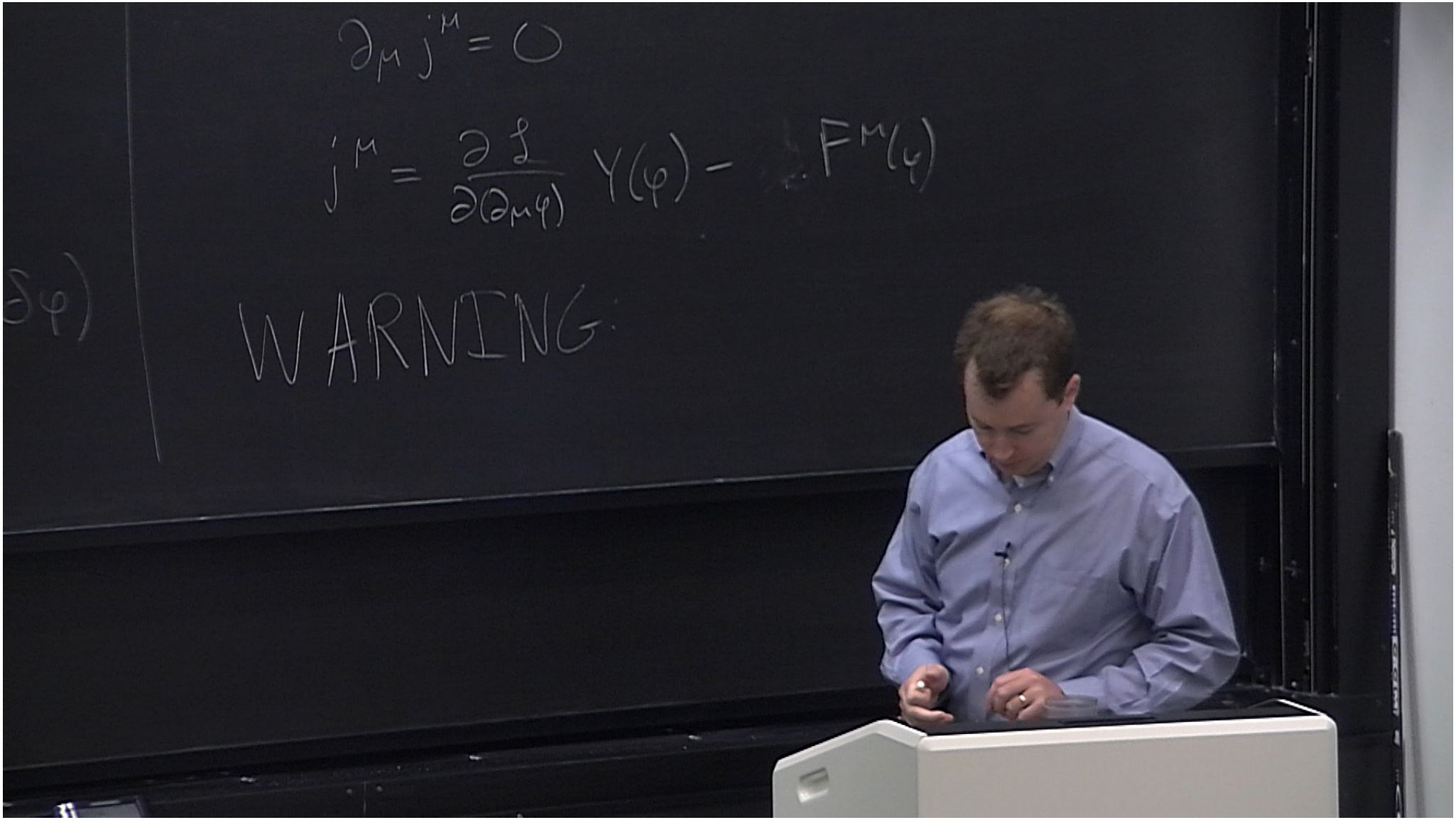
$$\boxed{(\partial_\mu\partial^\mu + m^2)\varphi = 0}$$

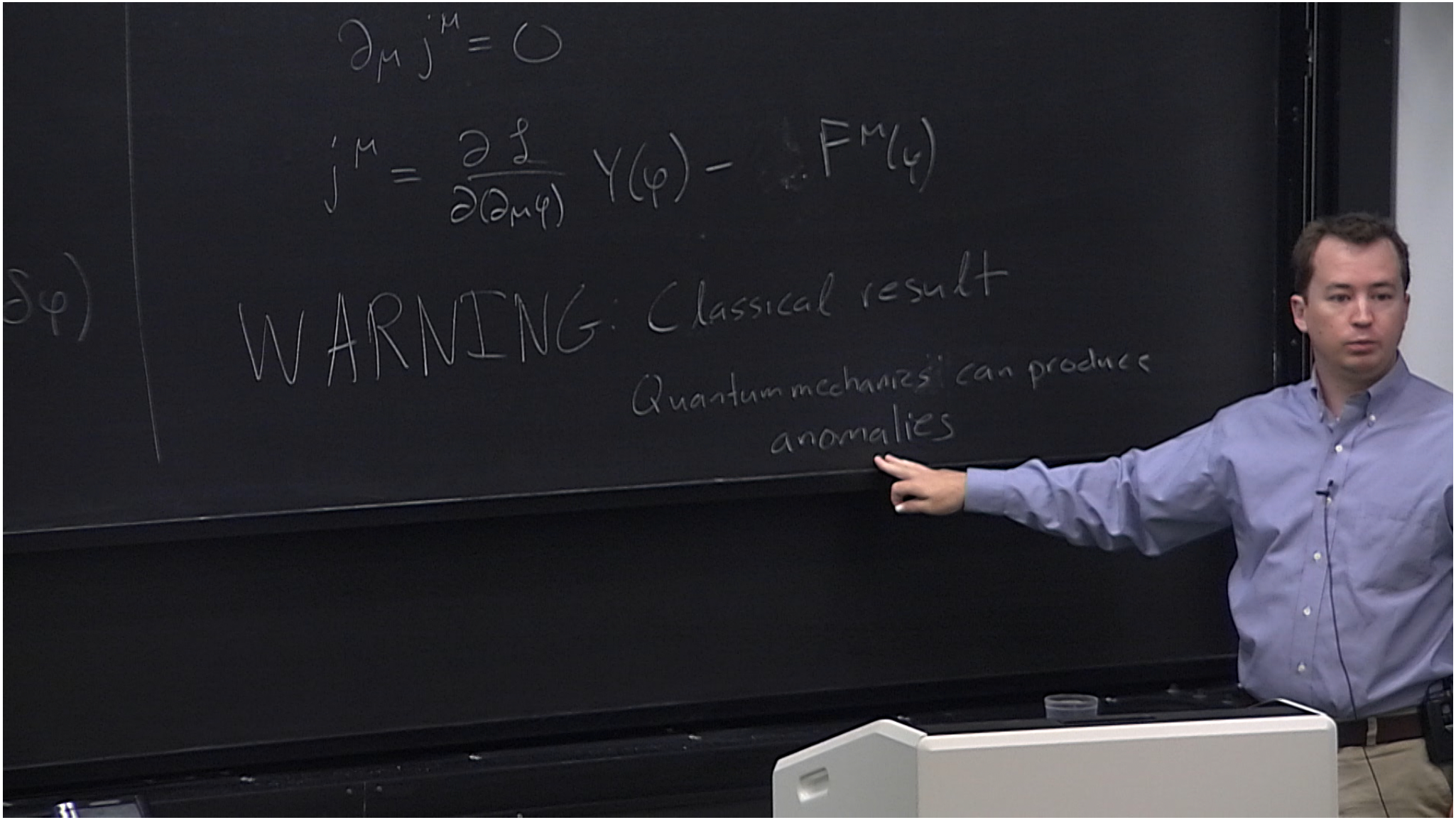

Proof: $\delta\varphi = Y(\varphi)$ is a symmetry if
$\delta\mathcal{L} = \partial_\mu F^\mu(\varphi)$ → EOM unchanged

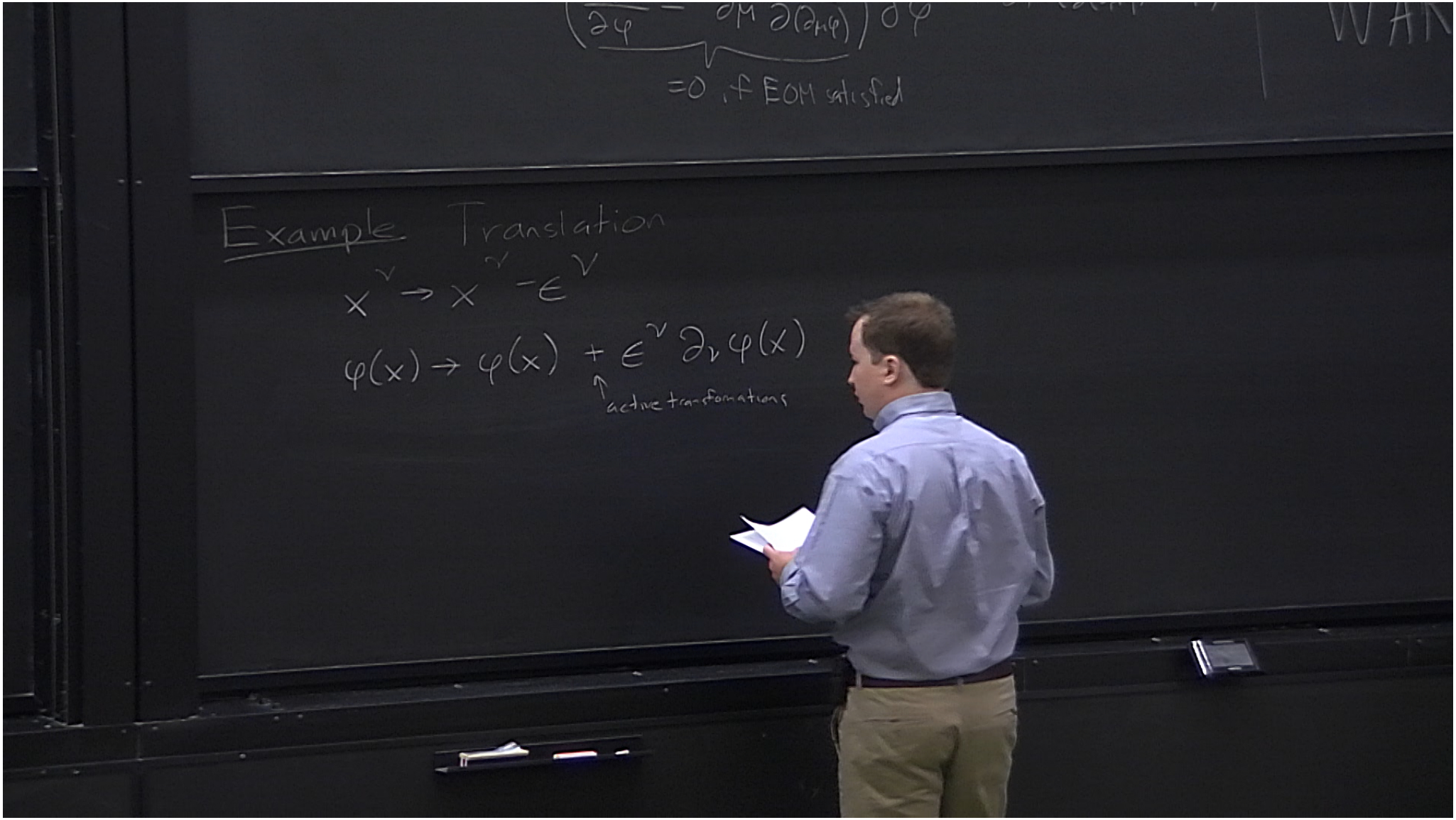

Proof: $\delta\varphi = Y(\varphi)$ is a symmetry if
$\delta\mathcal{L} = \partial_\mu F^\mu(\varphi)$ → EOM unchanged
$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi}\delta\varphi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\partial_\mu(\delta\varphi)$
$= \left(\frac{\partial\mathcal{L}}{\partial\varphi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\right)\delta\varphi + \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\delta\varphi\right)$

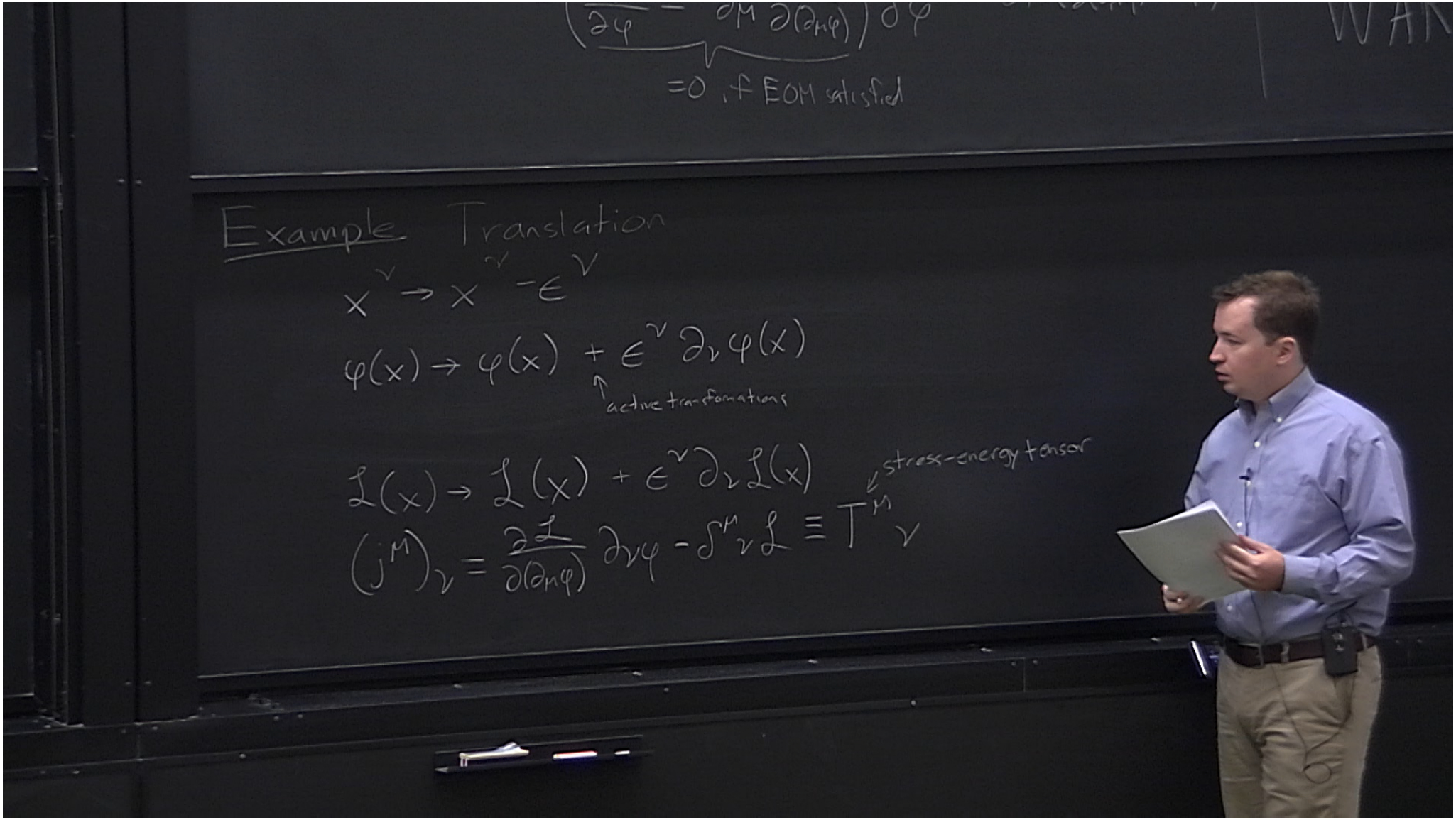

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$= \left(\frac{\partial\mathcal{L}}{\partial\varphi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\right)\delta\varphi + \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\delta\varphi\right)$
$=0$ if EOM satisfied
$\delta\mathcal{L}$


$\delta\mathcal{L} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} Y(\varphi) \right) = \partial_\mu F^\mu(\varphi)$
$\partial_\mu j^\mu = 0$
$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} Y(\varphi) - \partial_\mu F^\mu(\varphi)$
$\delta\varphi)$


$\partial_\mu j^\mu = 0$
$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} Y(\varphi) - F^\mu(\varphi)$
WARNING:


$\partial_\mu j^\mu = 0$
$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} Y(\varphi) - F^\mu(\varphi)$
WARNING: Classical result
Quantum mechanics can produce anomalies


$\left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\right) \delta\varphi$
$= 0$ if EOM satisfied
Example Translation
$x^\nu \to x'^\nu = x^\nu - \epsilon^\nu$
$\varphi(x) \to \varphi(x) + \epsilon^\nu \partial_\nu \varphi(x)$
active transformations


$\left(\frac{\partial}{\partial\varphi} - \partial_\mu \frac{\partial}{\partial(\partial_\mu\varphi)}\right)\delta\varphi$
$=0$ if EOM satisfied
Example Translation
$x^\nu \to x'^\nu = x^\nu - \epsilon^\nu$
$\varphi(x) \to \varphi(x) + \epsilon^\nu \partial_\nu \varphi(x)$
active transformations
$\mathcal{L}(x) \to \mathcal{L}(x) + \epsilon^\nu \partial_\nu \mathcal{L}(x)$
stress-energy tensor
$(j^\mu)_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi)} \partial_\nu\varphi - \delta^\mu{}_\nu \mathcal{L} \equiv T^\mu{}_\nu$

$\left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\right) \delta\varphi$

$= 0$ if EOM satisfied

Example Translation

$x^\nu \to x^\nu - \epsilon^\nu$

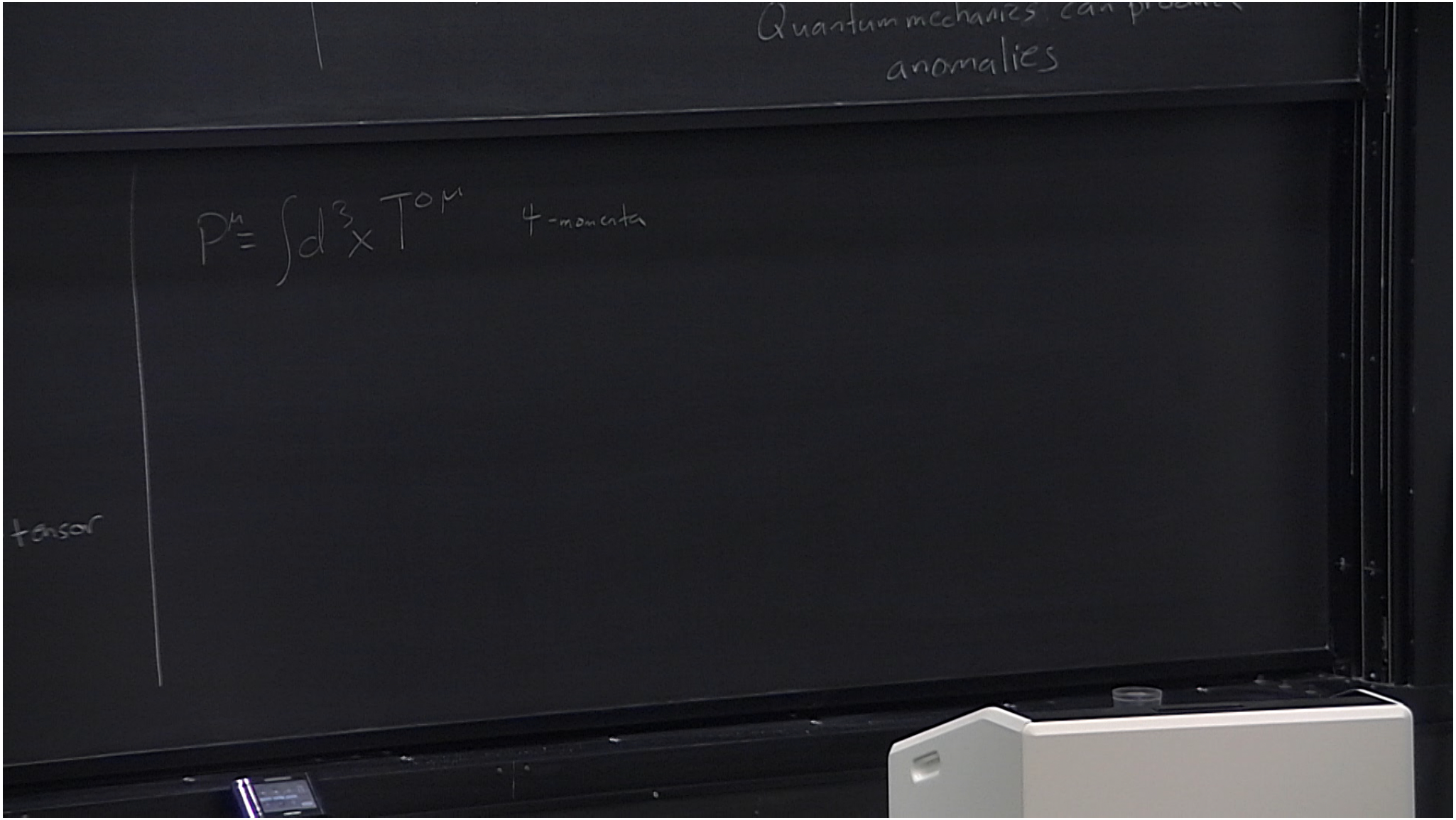
$\varphi(x) \to \varphi(x) + \boxed{\epsilon^\nu \partial_\nu \varphi(x)}$

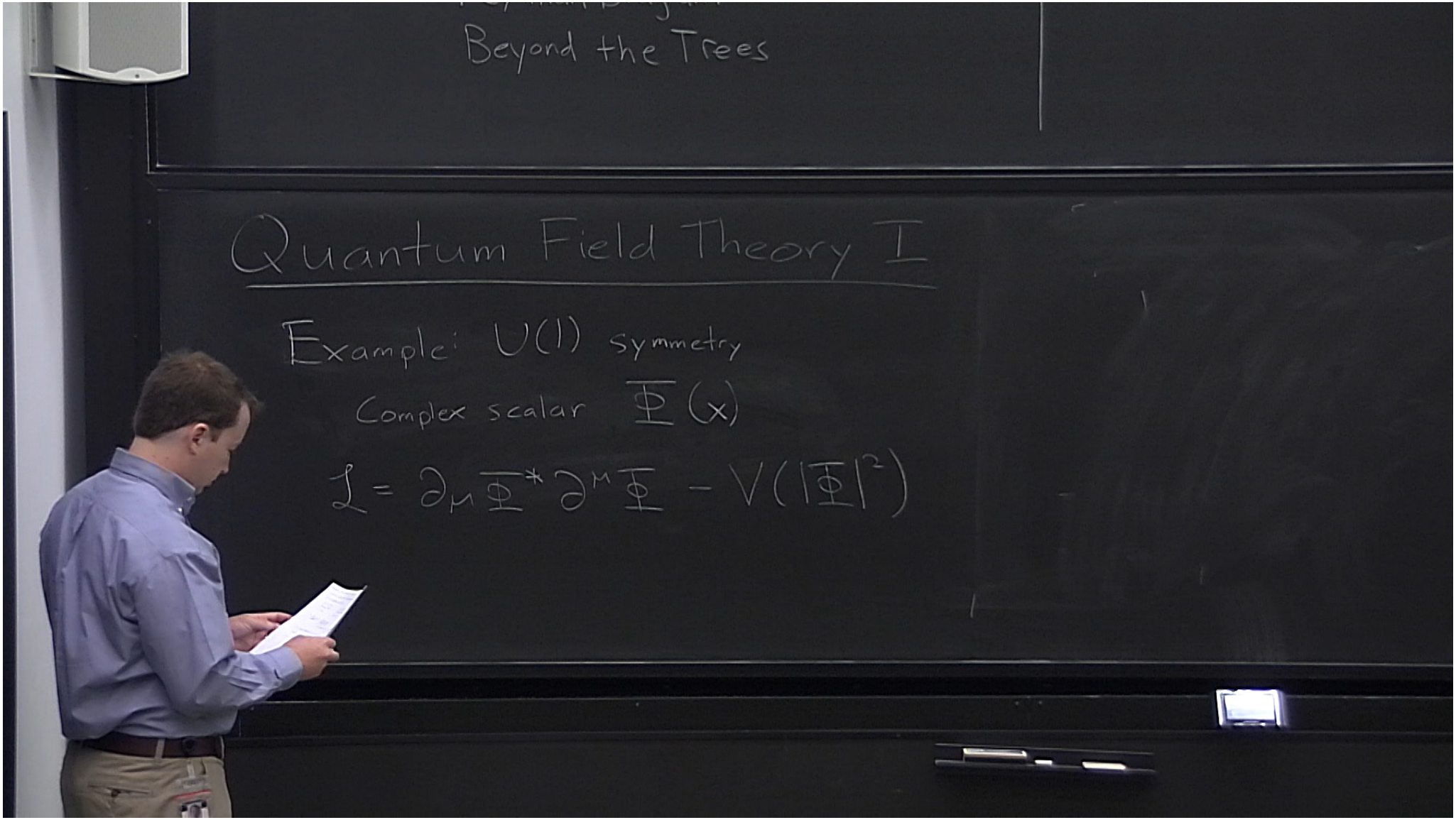
↑ active transformations

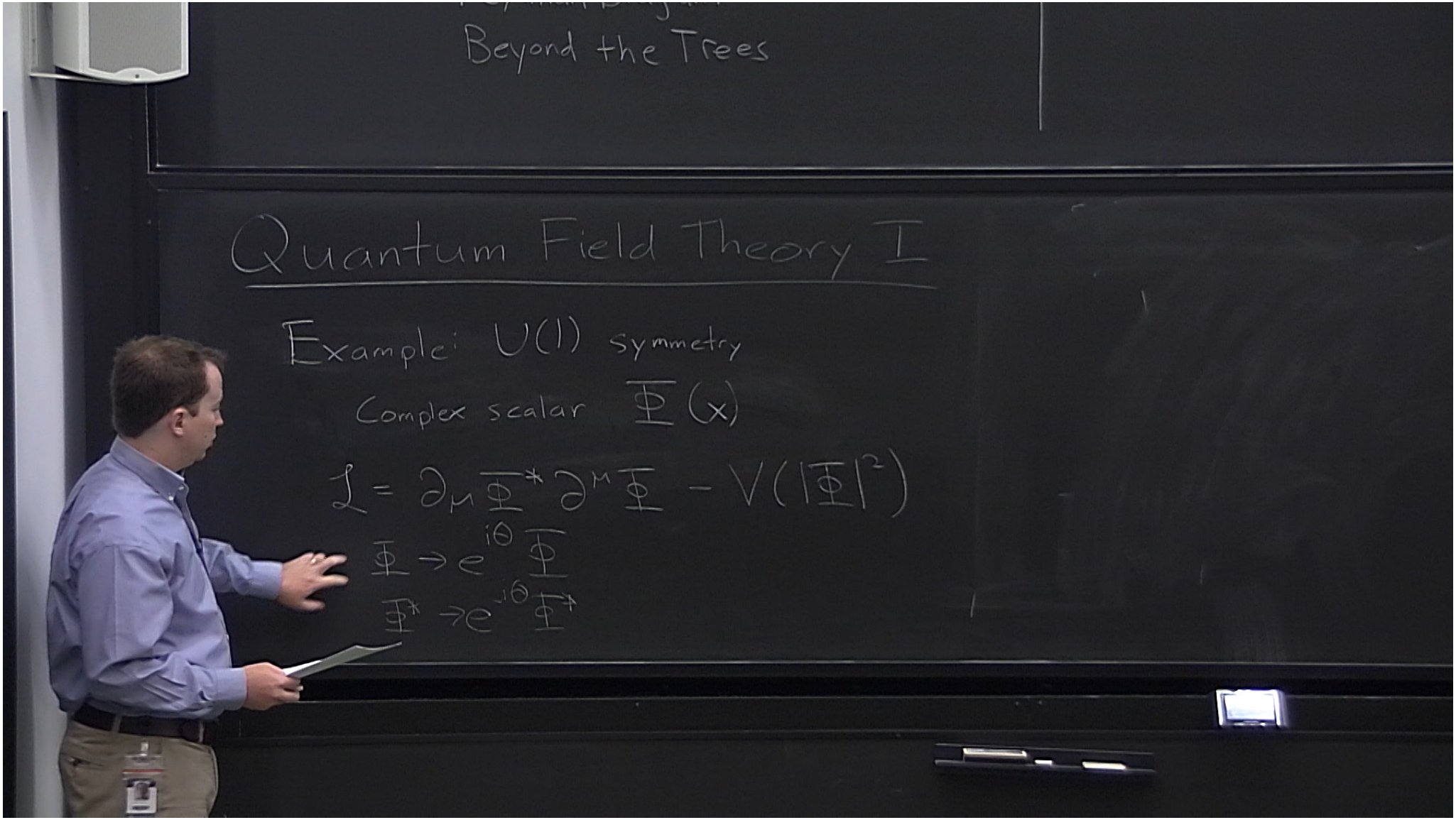
$\mathcal{L}(x) \to \mathcal{L}(x) + \boxed{\epsilon^\nu \partial_\nu \mathcal{L}(x)}$

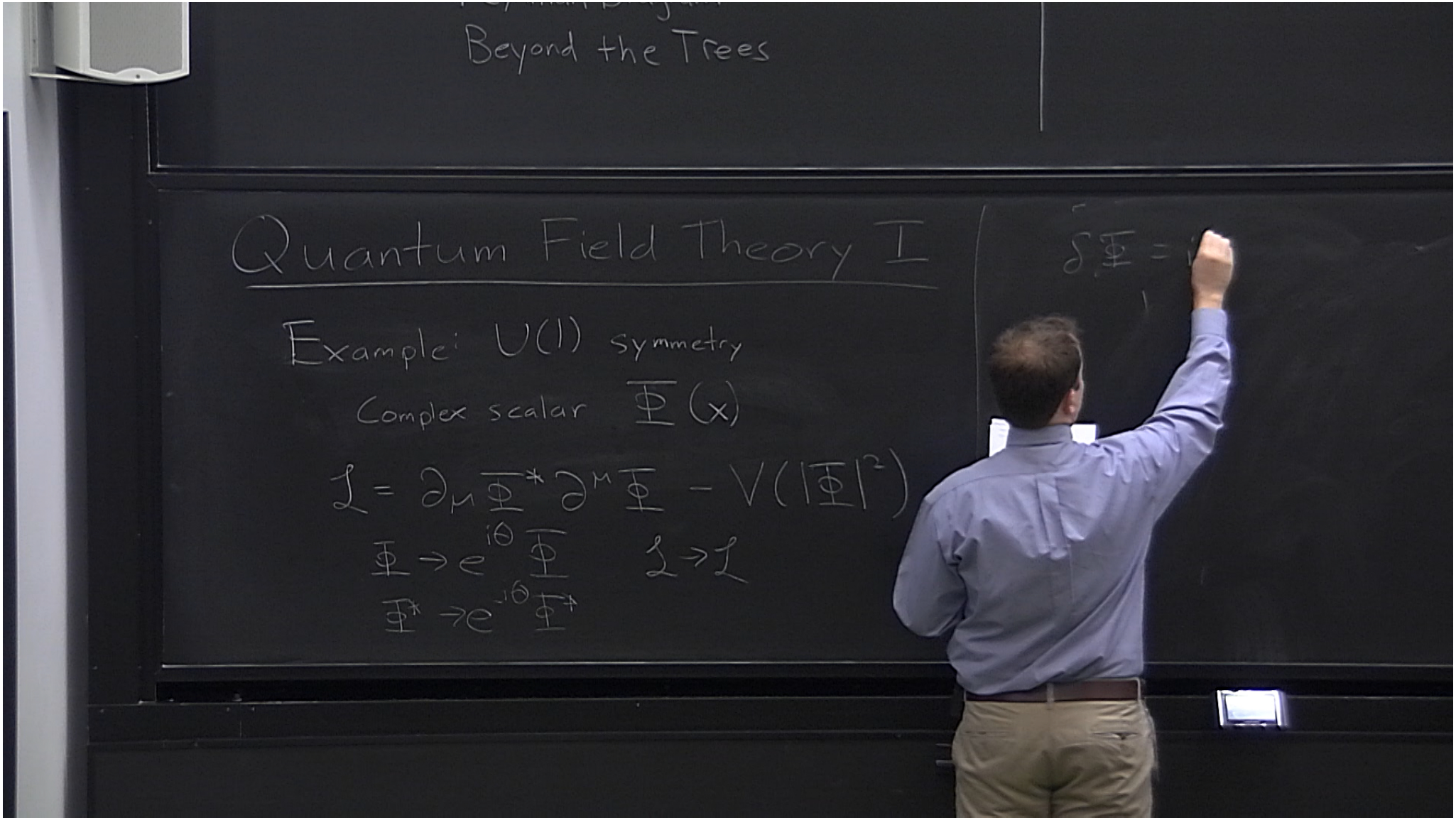
$(j^\mu)_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial_\nu \varphi - \delta^\mu{}_\nu \mathcal{L} \equiv T^\mu{}_\nu$

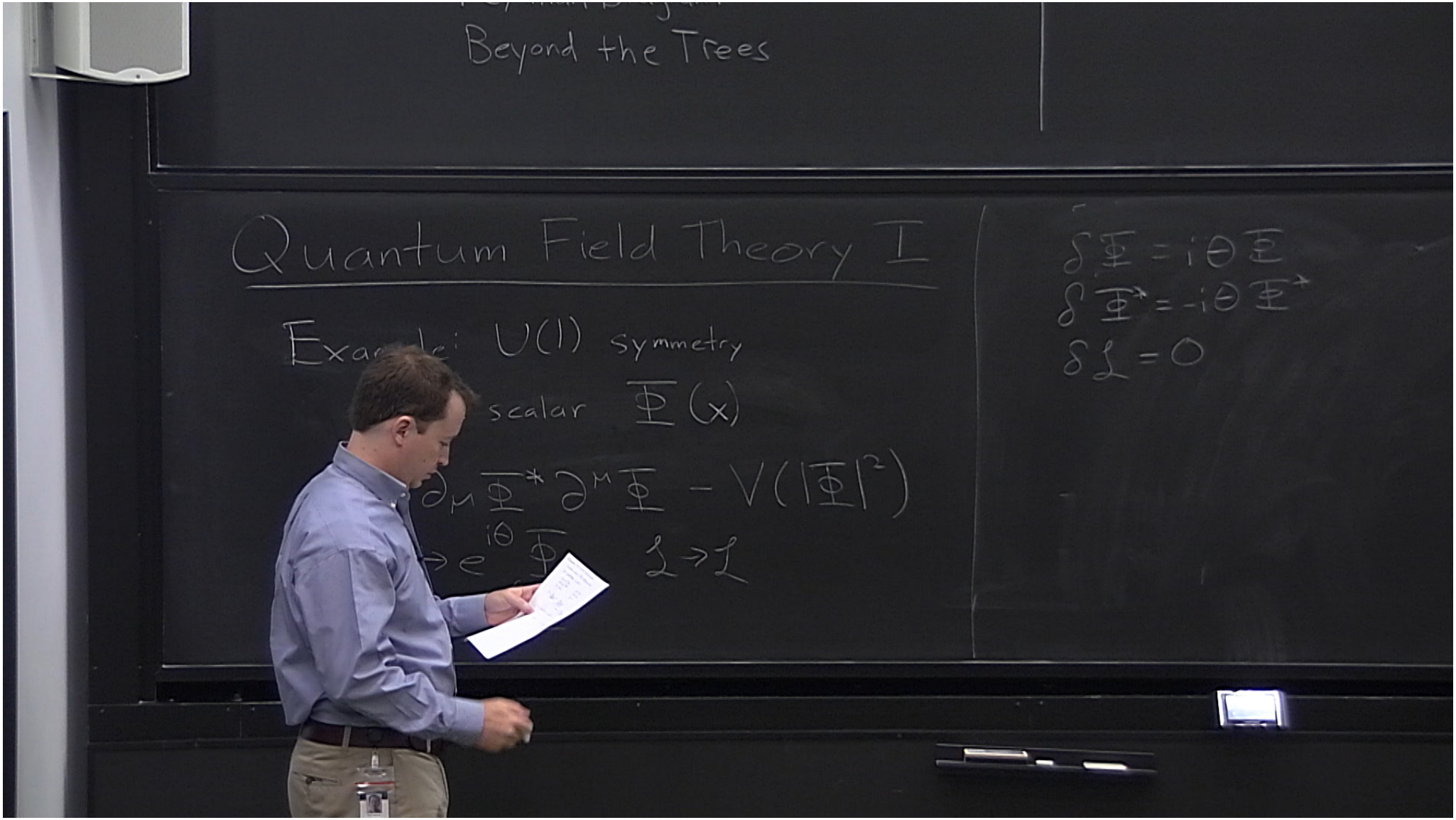
← stress-energy tensor


Quantum mechanics
anomalies
$P^\mu = \int d^3x\, T^{0\mu}$
4-momenta
tensor


Beyond the Trees
Quantum Field Theory I
Example: U(1) symmetry
Complex scalar $\Phi(x)$
$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$


Beyond the Trees
Quantum Field Theory I
Example: U(1) symmetry
Complex scalar $\Phi(x)$
$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$
$\Phi \to e^{i\theta} \Phi$
$\Phi^* \to e^{-i\theta} \Phi^*$


Beyond the Trees
Quantum Field Theory I
Example: U(1) symmetry
Complex scalar $\Phi(x)$
$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$
$\Phi \to e^{i\theta} \Phi$ $\quad \mathcal{L} \to \mathcal{L}$
$\Phi^* \to e^{-i\theta} \Phi^*$


Beyond the Trees
Quantum Field Theory I
Example: U(1) symmetry
scalar $\Phi(x)$
$\partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$
$\Phi \to e^{i\theta} \Phi$
$\mathcal{L} \to \mathcal{L}$
$\delta \Phi = i\theta \Phi$
$\delta \Phi^* = -i\theta \Phi^*$
$\delta \mathcal{L} = 0$

Beyond the Trees

Quantum Field Theory I

Example: U(1) symmetry

Complex scalar $\Phi(x)$

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$$

$$\Phi \to e^{i\theta} \Phi \qquad \mathcal{L} \to \mathcal{L}$$

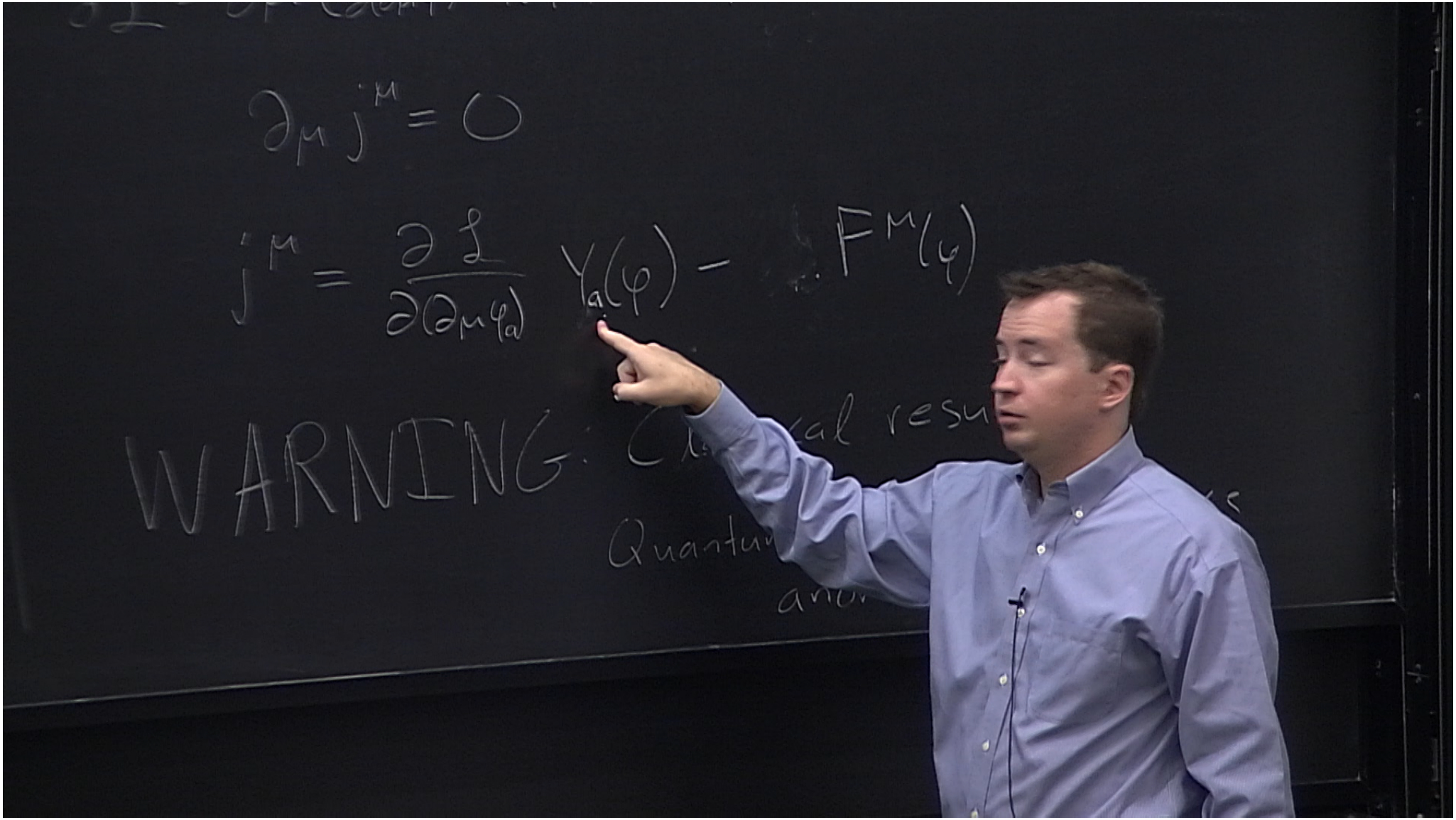
$$\Phi^* \to e^{-i\theta} \Phi^*$$

$$\delta \Phi = i\theta \Phi$$

$$\delta \Phi^* = -i\theta \Phi^*$$

$$\delta \mathcal{L} = 0$$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi)} \delta \Phi$$


$\partial_\mu j^\mu = 0$
$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} Y_a(\varphi) - F^\mu(\varphi)$
WARNING:

$$\delta\Phi = i\theta\Phi$$

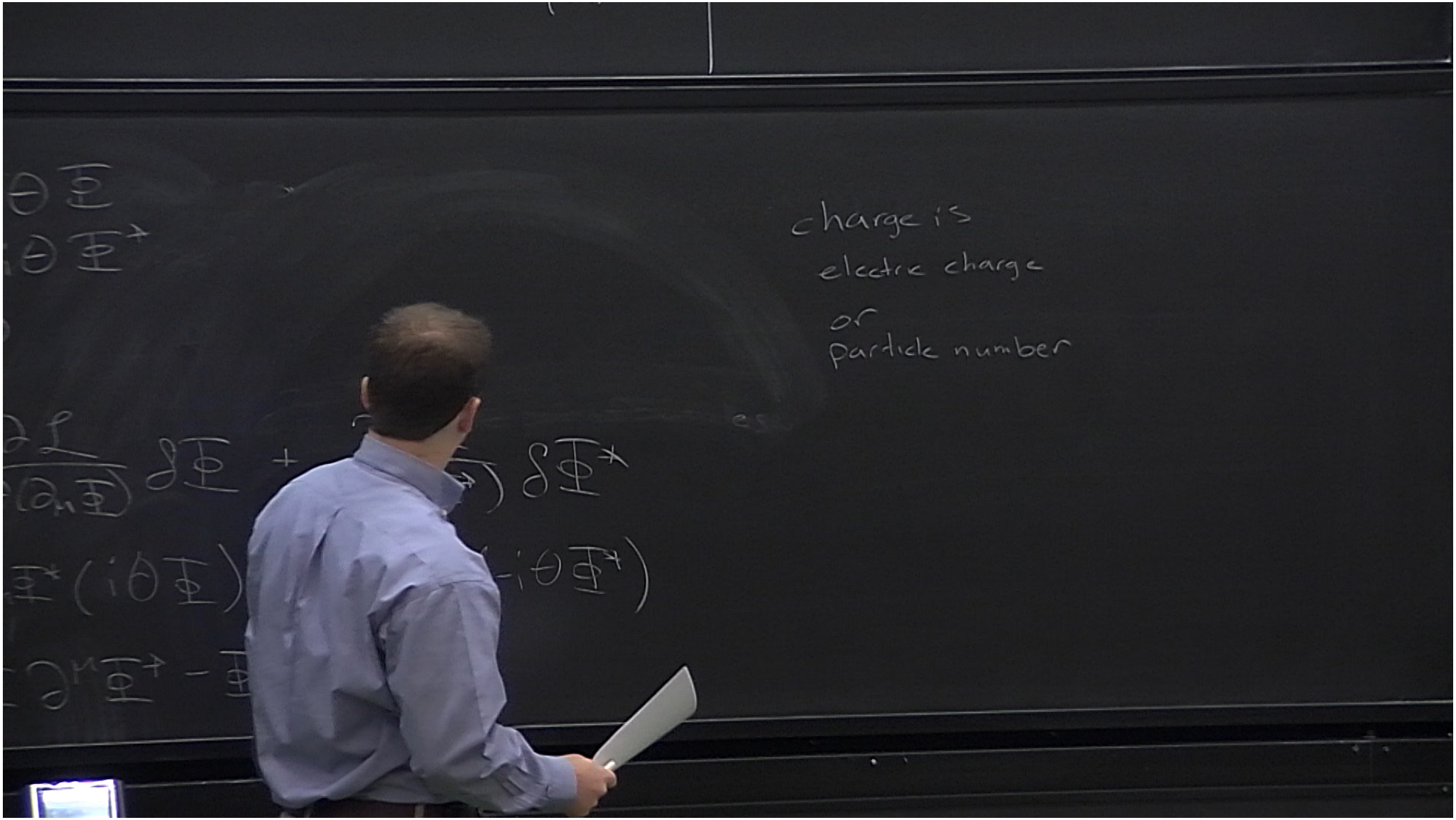
$$\delta\Phi^\dagger = -i\theta\Phi^\dagger$$

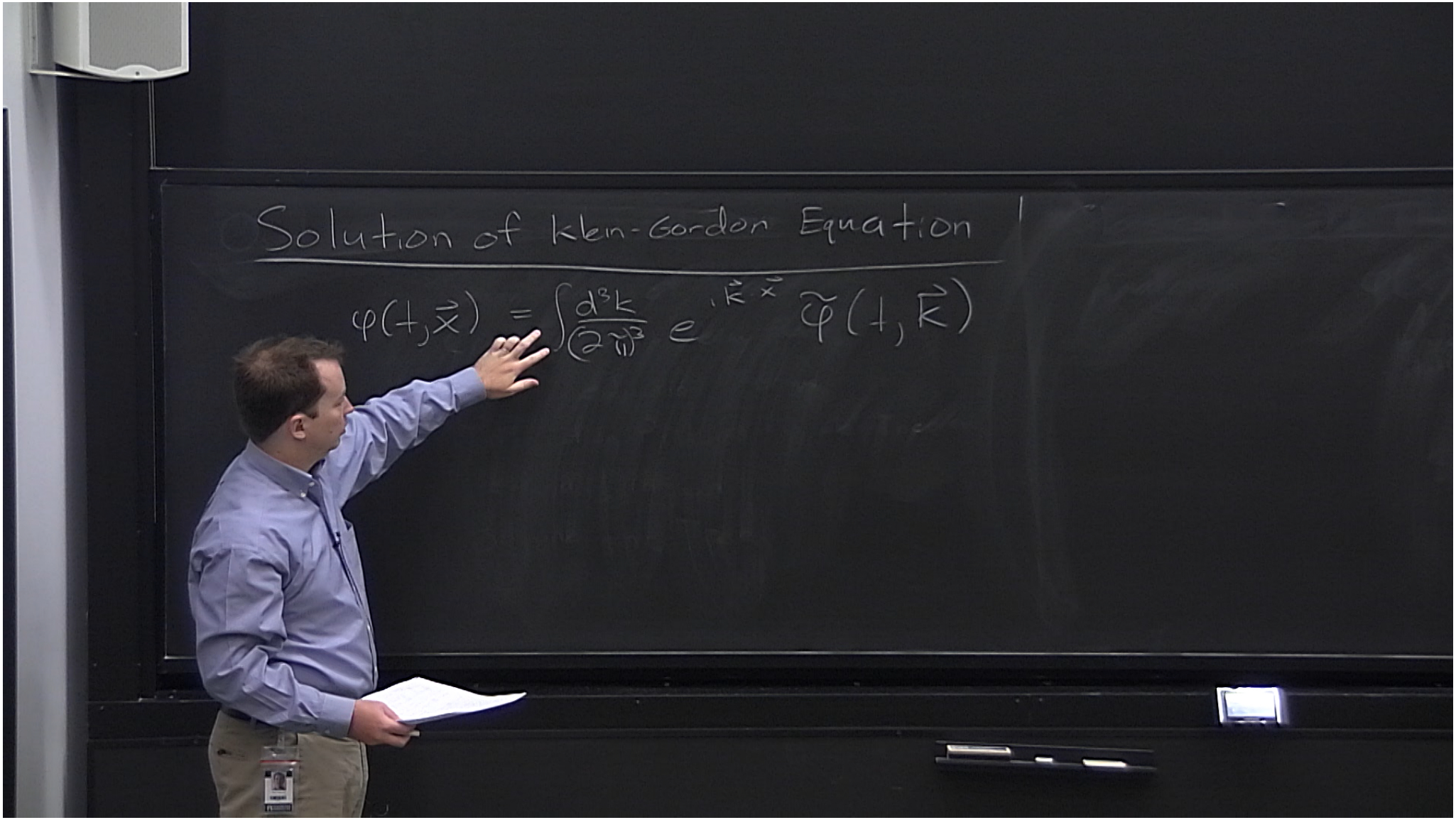
$$\delta\mathcal{L} = 0$$

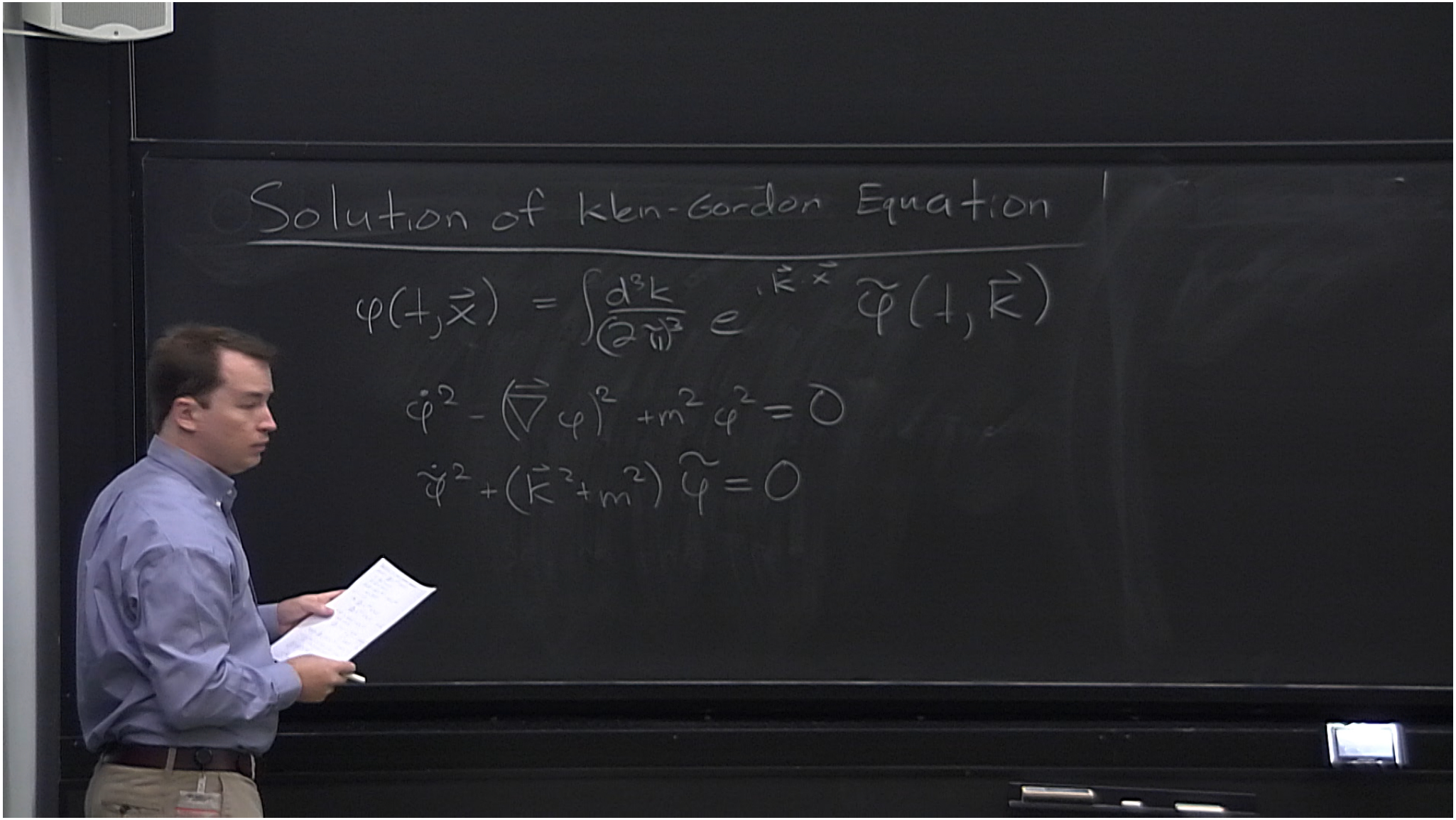
$$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi)}\delta\Phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi^\dagger)}\delta\Phi^\dagger$$

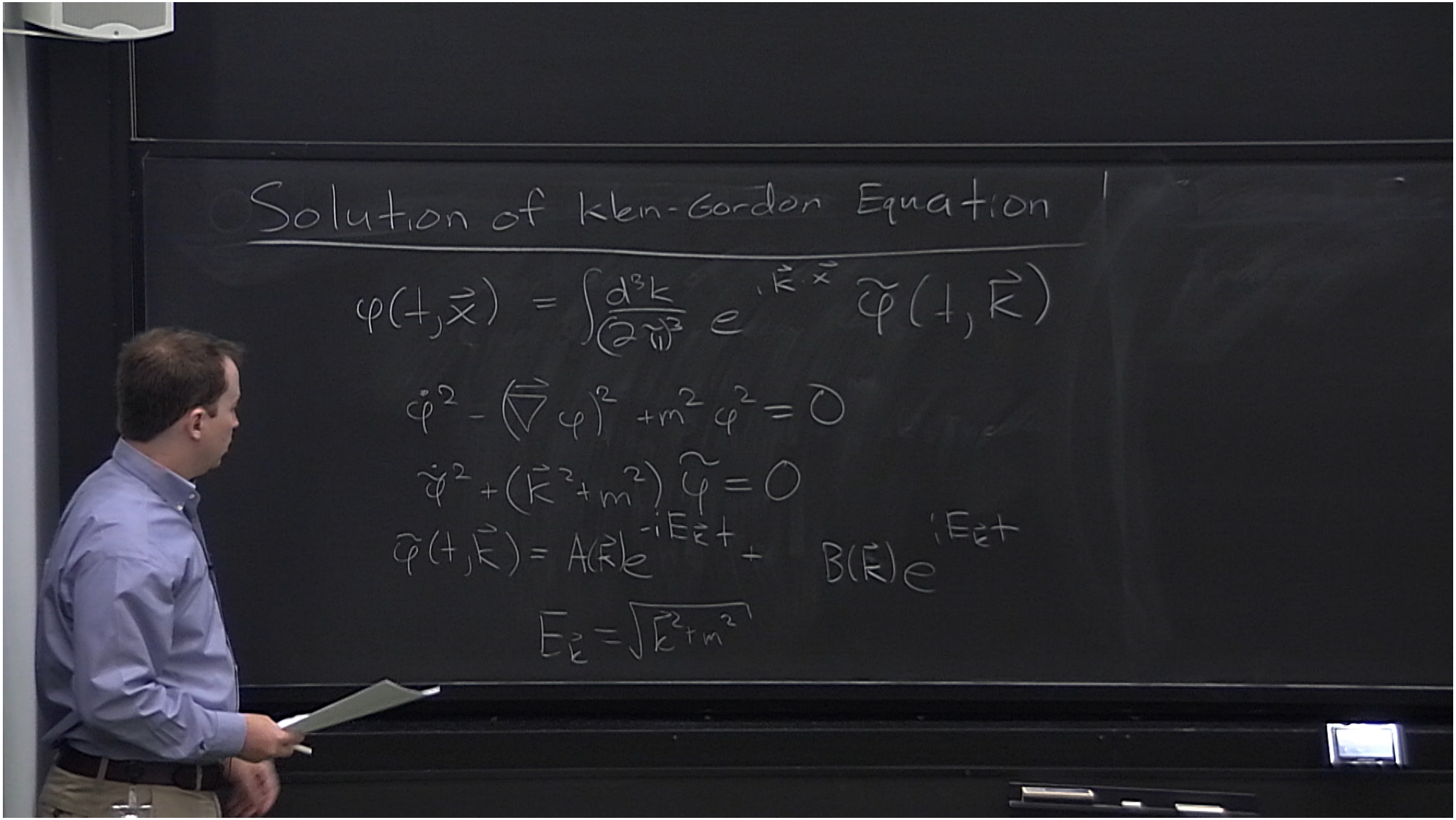
$$j^\mu = \partial_\mu\Phi^\dagger(i\theta\Phi) + (\partial^\mu\Phi)(-i\theta\Phi^\dagger)$$

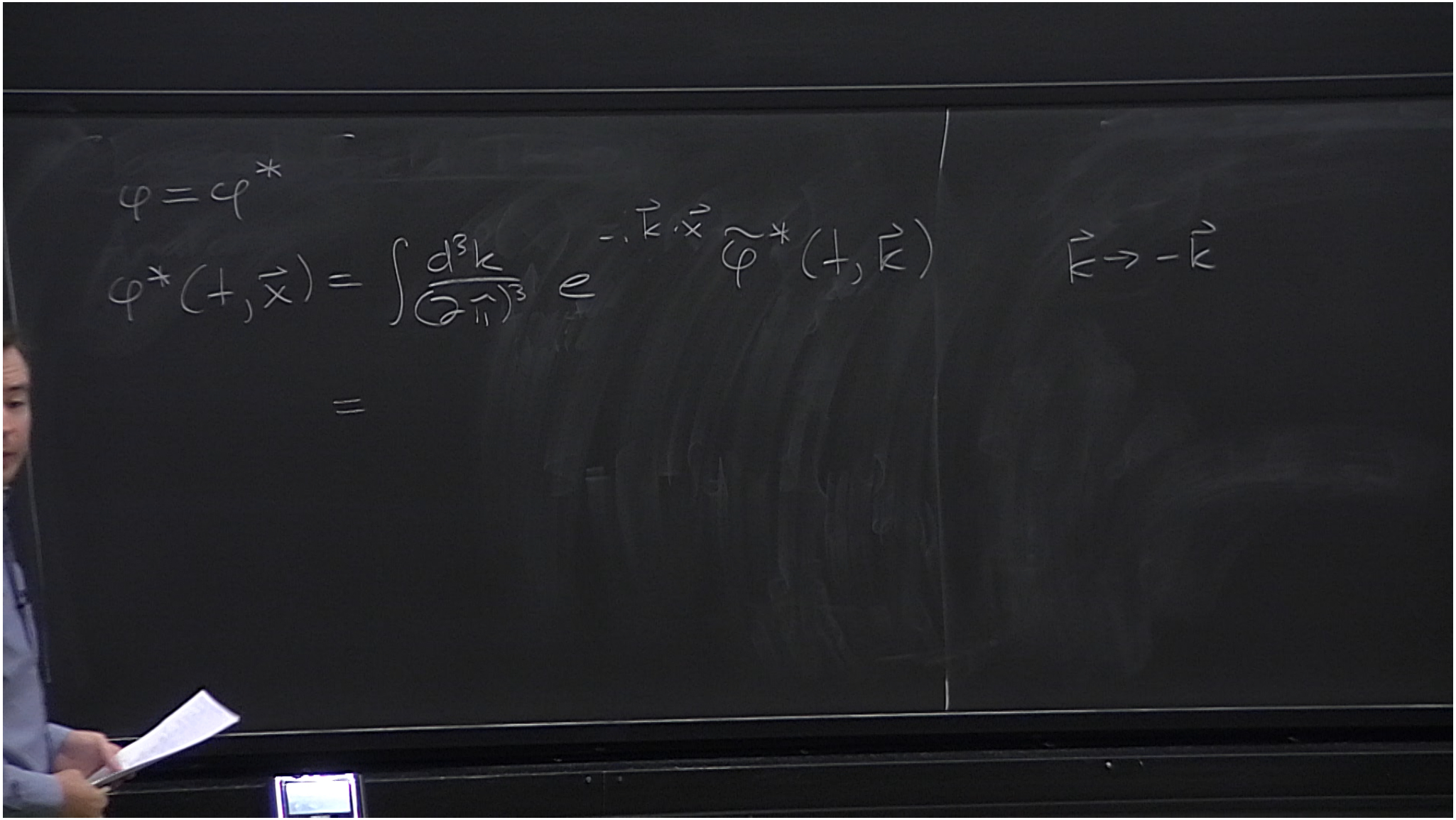
$$j^\mu = \Phi\partial^\mu\Phi^\dagger - \Phi^\dagger\partial^\mu\Phi$$


charge is
electric charge
or
particle number


Solution of Klein-Gordon Equation
$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \tilde{\varphi}(t,\vec{k})$


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$\dot{\varphi}^2 - (\vec{\nabla}\varphi)^2 + m^2\varphi^2 = 0$
$\ddot{\tilde{\varphi}}^2 + (\vec{k}^2 + m^2)\tilde{\varphi} = 0$
$\tilde{\varphi}(t,\vec{k}) = A(\vec{k})e^{-iE_k t} + B(\vec{k})e^{iE_k t}$
$E_k = \sqrt{\vec{k}^2 + m^2}$


$\varphi = \varphi^*$
$\varphi^*(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \tilde{\varphi}^*(t,\vec{k})$
$\vec{k} \to -\vec{k}$
$=$

$$\varphi = \varphi^*$$

$$\varphi^*(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \tilde{\varphi}^*(t,\vec{k}) \qquad \vec{k} \to -\vec{k}$$

$$= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot x} \tilde{\varphi}^*(t,-\vec{k})$$

$$\varphi = \varphi^*$$

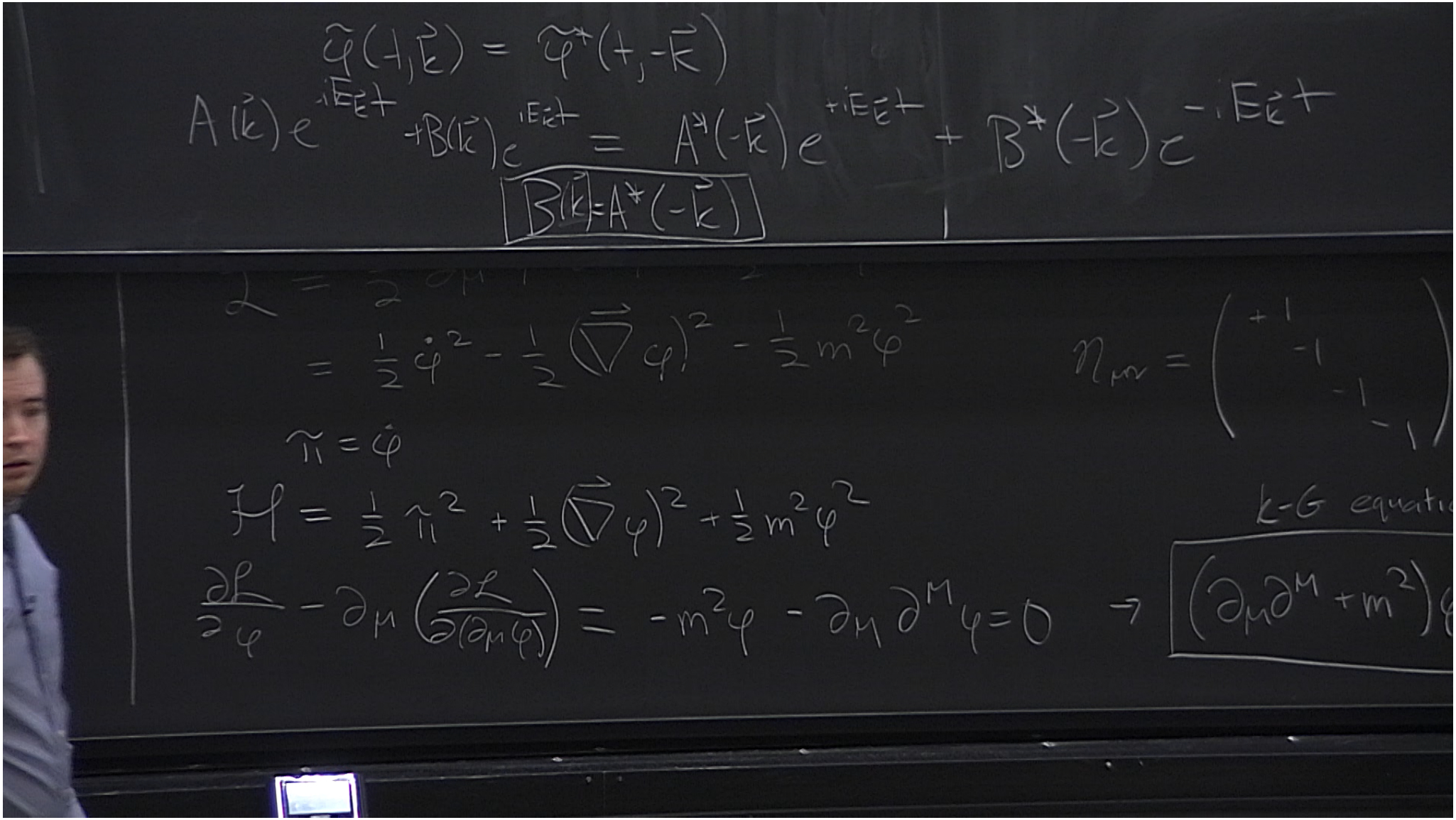
$$\varphi^*(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \tilde{\varphi}^*(t,\vec{k}) \qquad \vec{k} \to -\vec{k}$$

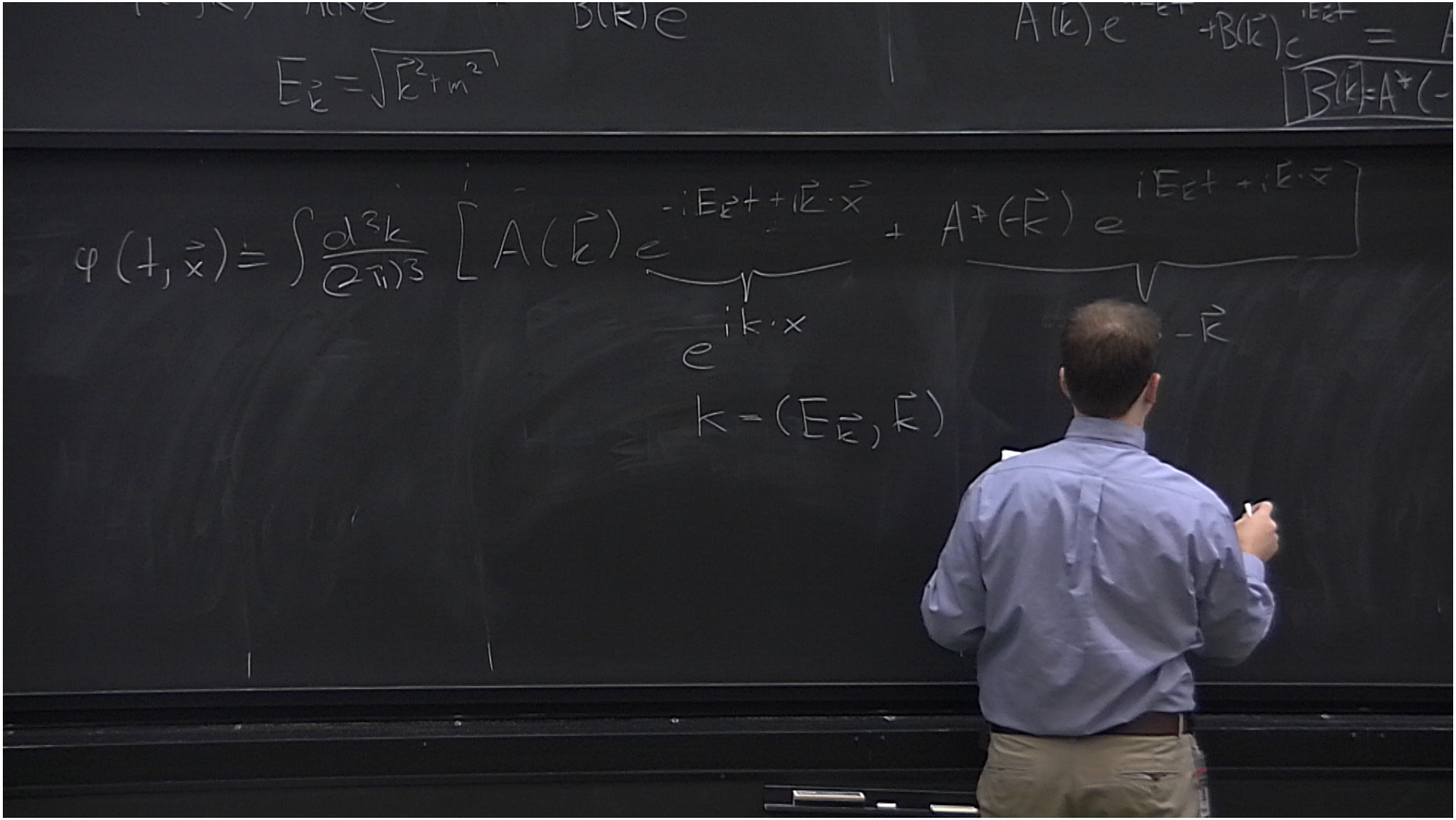
$$= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot x} \tilde{\varphi}^*(t,-\vec{k})$$

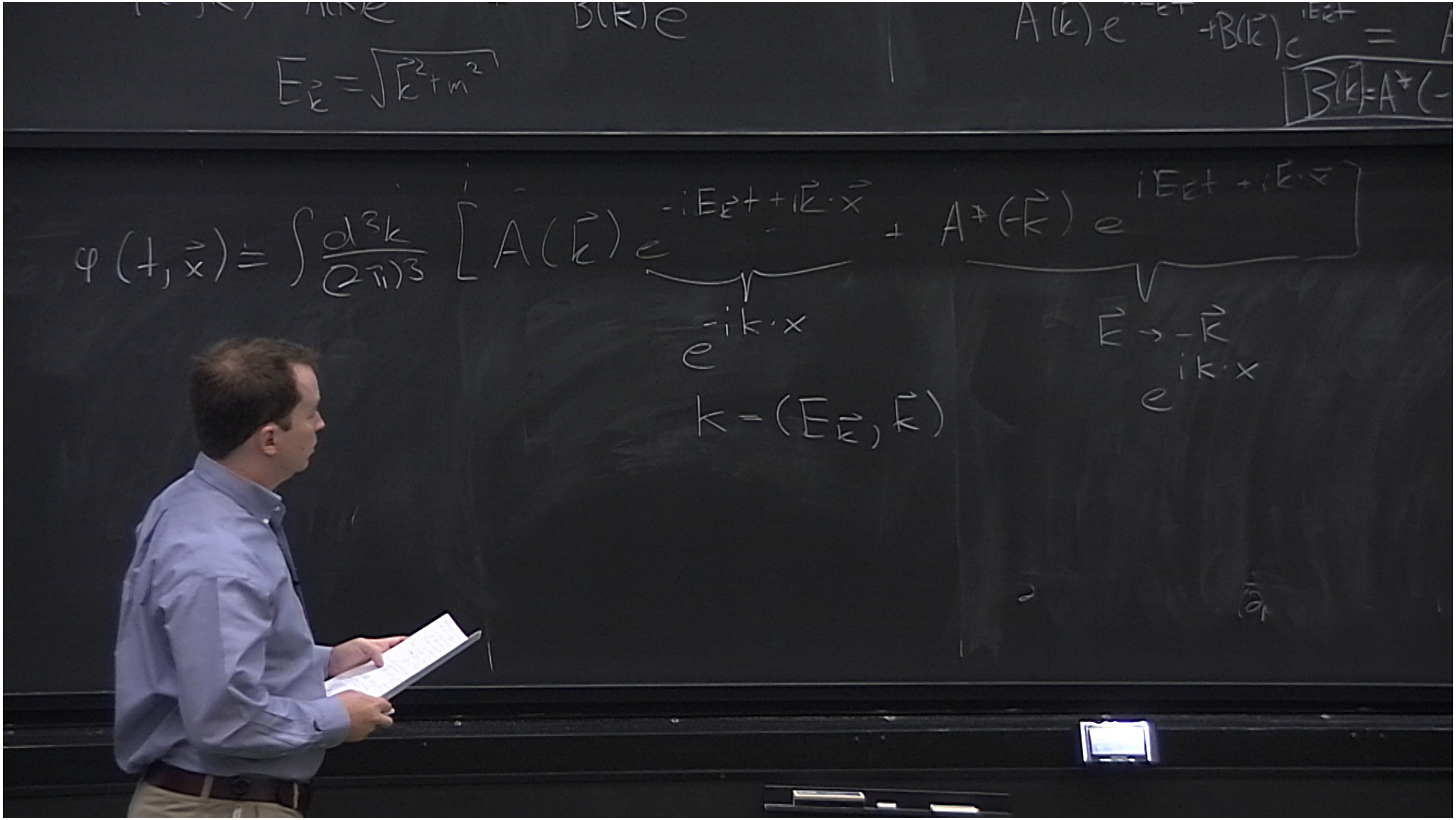
$$\tilde{\varphi}(t,\vec{k}) = \varphi^*(t,-\vec{k})$$

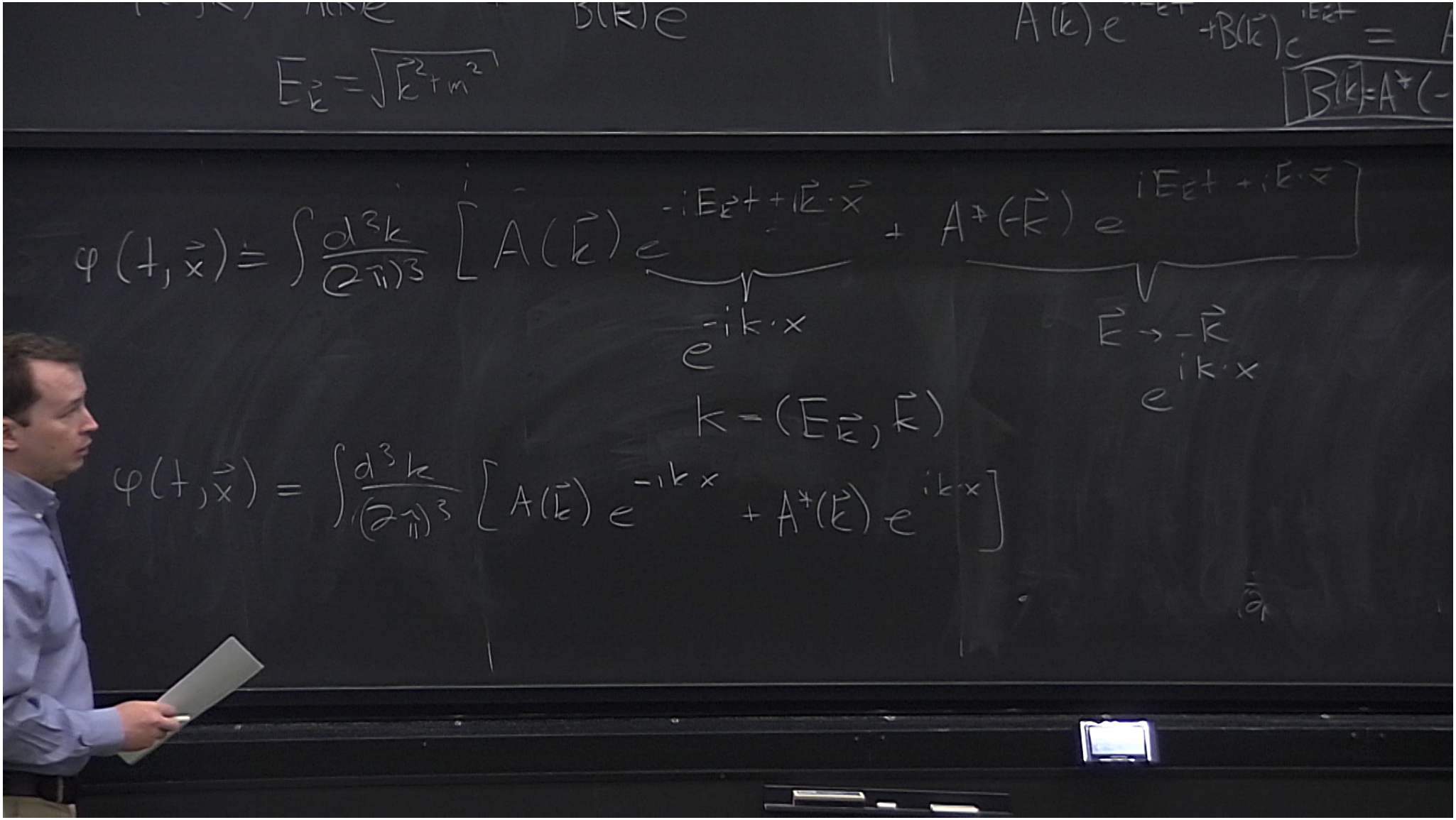
$$A(\vec{k}) e^{-iE_k t} + B(\vec{k}) e^{iE_k t} = A^*(-\vec{k}) e^{iE_k t} + B^*(-\vec{k}) e^{-iE_k t}$$

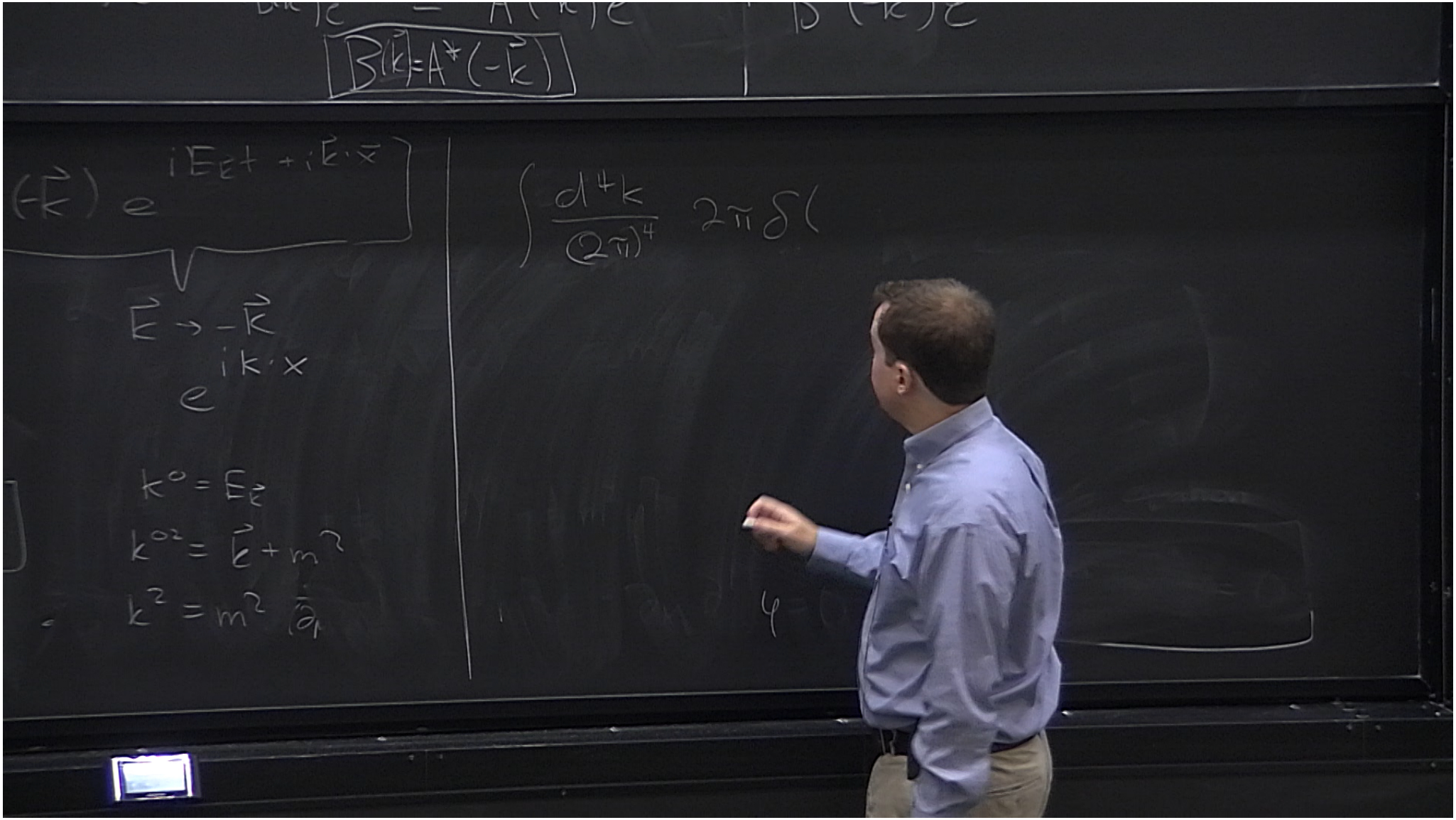
$$\boxed{B(\vec{k}) = A^*(-\vec{k})}$$

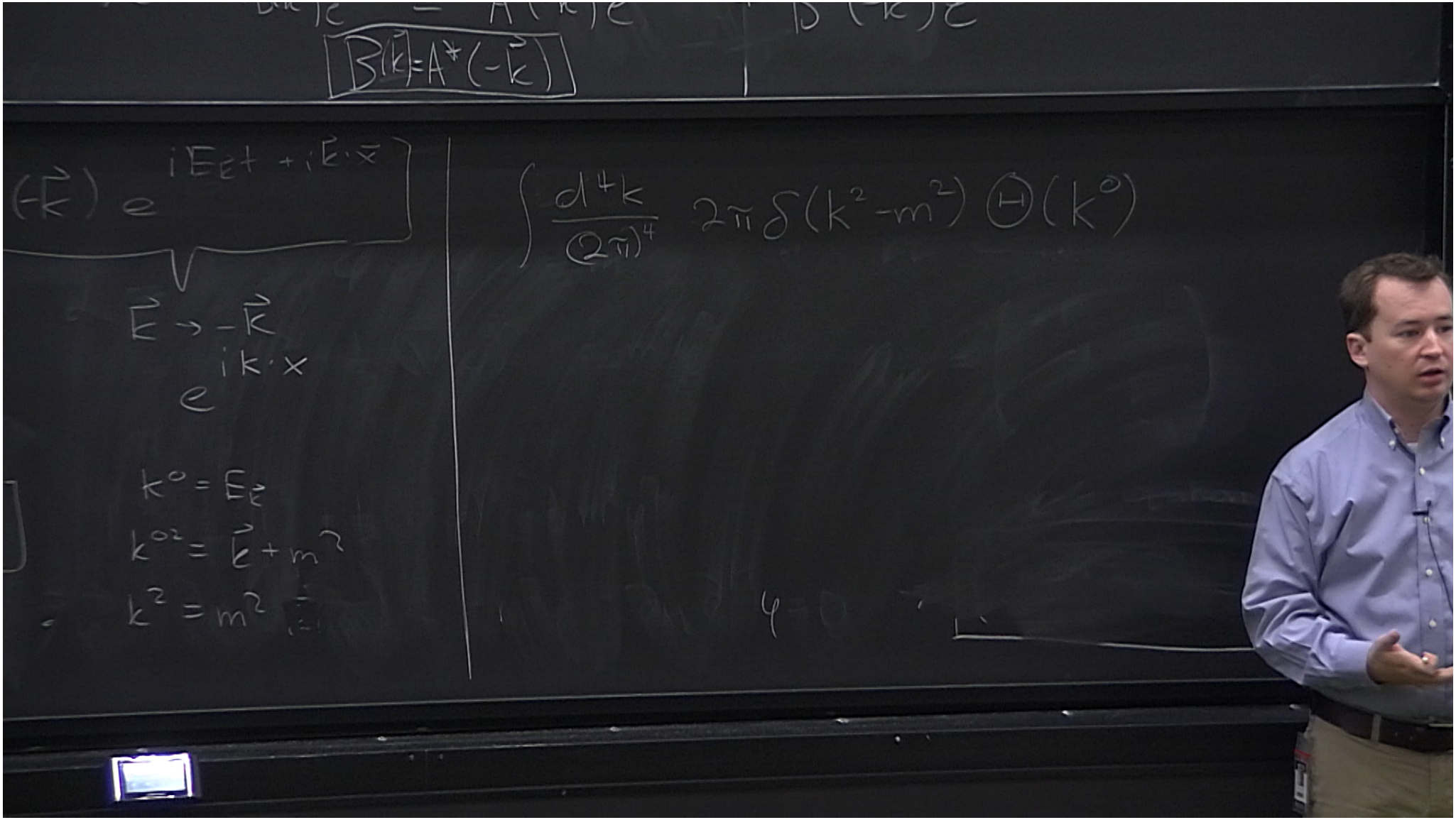

$\tilde{\varphi}(+,\vec{k}) = \tilde{\varphi}^*(+,-\vec{k})$
$A(\vec{k})e^{-iE\vec{k}t} + B(\vec{k})e^{iE\vec{k}t} = A^*(-\vec{k})e^{+iE\vec{k}t} + B^*(-\vec{k})e^{-iE\vec{k}t}$
$B(\vec{k}) = A^*(-\vec{k})$
$\mathcal{L} = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\vec{\nabla}\varphi)^2 - \frac{1}{2}m^2\varphi^2$
$\eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
$\pi = \dot{\varphi}$
$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2$
k-G equati
$\frac{\partial\mathcal{L}}{\partial\varphi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\right) = -m^2\varphi - \partial_\mu\partial^\mu\varphi = 0 \rightarrow (\partial_\mu\partial^\mu + m^2)$


$E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$
$\varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) e^{-iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}} + A^*(-\vec{k}) e^{iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}} \right]$
$e^{ik\cdot x}$
$-\vec{k}$
$k = (E_{\vec{k}}, \vec{k})$


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$e^{-ik\cdot x}$
$k = (E_{\vec{k}}, \vec{k})$
$\vec{k} \to -\vec{k}$
$e^{ik\cdot x}$


$E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$
$B(\vec{k})$
$A(\vec{k})$
$+B(\vec{k})$
$B(\vec{k}) = A^*(-$
$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) e^{-iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}} + A^*(-\vec{k}) e^{iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}} \right]$
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$B(\vec{k}) = A^*(-\vec{k})$
$(-\vec{k})\, e^{iE_k t + i\vec{k}\cdot\vec{x}}$
$\int \frac{d^4k}{(2\pi)^4}\, 2\pi\delta($
$\vec{k} \to -\vec{k}$
$e^{ik\cdot x}$
$k^0 = E_{\vec{k}}$
$k^{0^2} = \vec{k} + m^2$
$k^2 = m^2$


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$\int \frac{d^4k}{(2\pi)^4}\, 2\pi\delta(k^2 - m^2)\,\Theta(k^0)$

$B(\vec{k}) = A^*(-\vec{k})$

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$\vec{k} \to -\vec{k}$

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$k^0 = E_{\vec{k}}$

$k^{0^2} = \vec{k} + m^2$

$k^2 = m^2$

$\int \frac{d^4k}{(2\pi)^4}\, 2\pi\, \delta(k^2 - m^2)\, \Theta(k^0)$

↑ invariant under orthochronous Lorentz

$= \frac{1}{2E_{\vec{k}}} \int \frac{d^3k}{(2\pi)^3}$

$k \to -k$

$e^{ik \cdot x}$

$k^0 = E_k$

$k^{0^2} = \vec{k} + m^2$

$k^2 = m^2$

$= \frac{1}{2E_k} \int \frac{d^3k}{(2\pi)^3}$

orthochronous Lorentz

$A(k) = \frac{a(k)}{2E_k}$

$$\varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3 2E_k} \left[a(\vec{k}) e^{-ikx} + a^\dagger(\vec{k}) e^{ik \cdot x} \right]$$

$k \to -k$

$e^{ik \cdot x}$

$k^0 = E_k$

$k^{02} = \vec{k} + m^2$

$k^2 = m^2$

$= \frac{1}{2E_k} \int \frac{d^3k}{(2\pi)^3}$

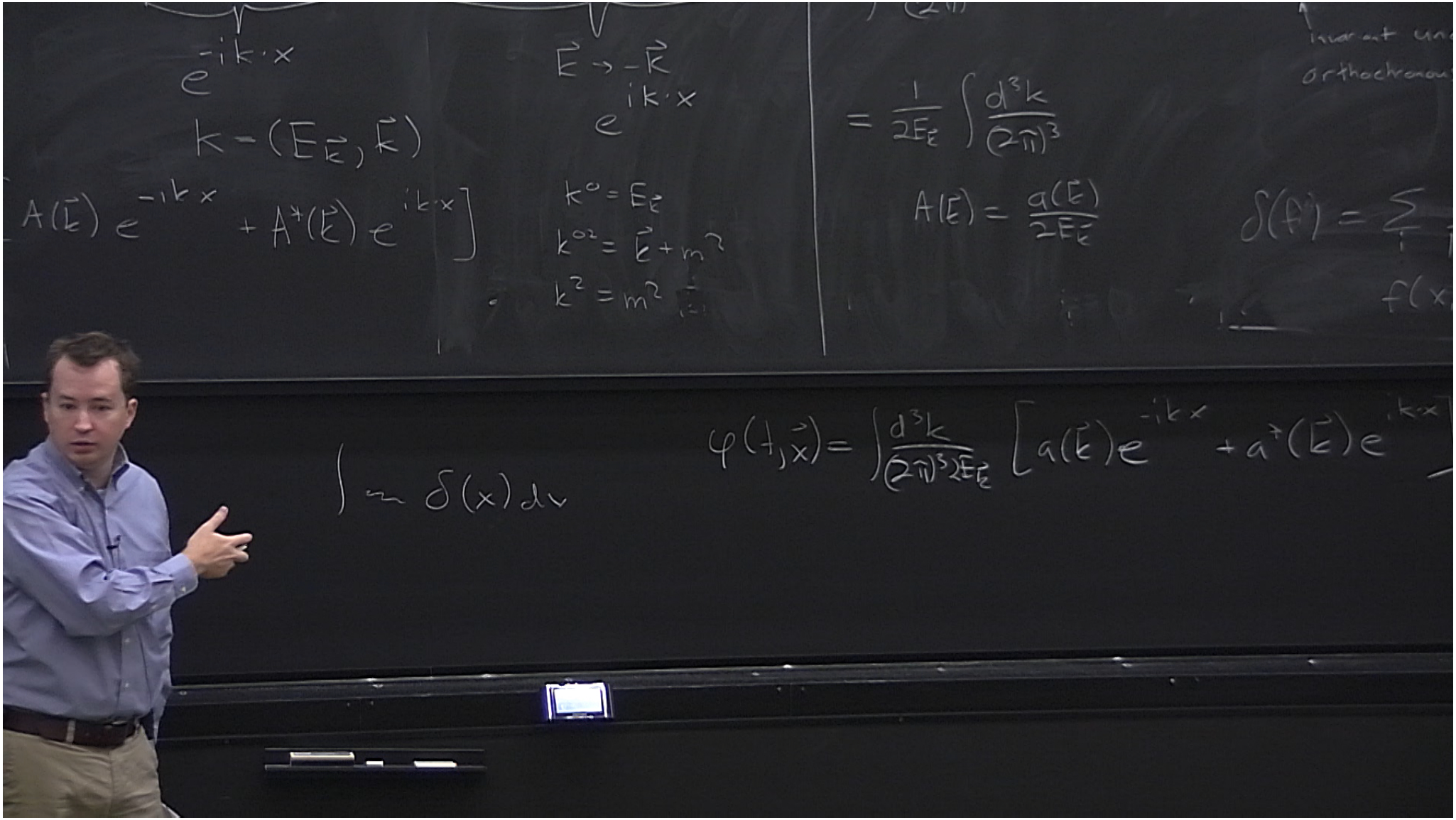
$A(k) = \frac{a(k)}{2E_k}$

orthochronous Lorentz

$\delta(f(x)) = \sum_i \frac{\delta_{x_i}}{|f'(x_i)|}$

$f(x_i) = 0$

$$\varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3 2E_k} \left[a(k) e^{-ikx} + a^\dagger(\vec{k}) e^{ik \cdot x} \right]$$


$e^{-ik\cdot x}$
$k = (E_{\vec{k}}, \vec{k})$
$A(\vec{k})\, e^{-ik\cdot x} + A^\dagger(\vec{k})\, e^{ik\cdot x}$
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$k^0 = E_{\vec{k}}$
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$= \frac{1}{2E_{\vec{k}}} \int \frac{d^3k}{(2\pi)^3}$
$A(\vec{k}) = \frac{a(\vec{k})}{2E_{\vec{k}}}$
$\int \sim \delta(x)\, dv$
$\varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left[a(\vec{k}) e^{-ik\cdot x} + a^\dagger(\vec{k}) e^{ik\cdot x} \right]$