Title: Tunneling in Quantum Field Theory and the Ultimate Fate of our Universe

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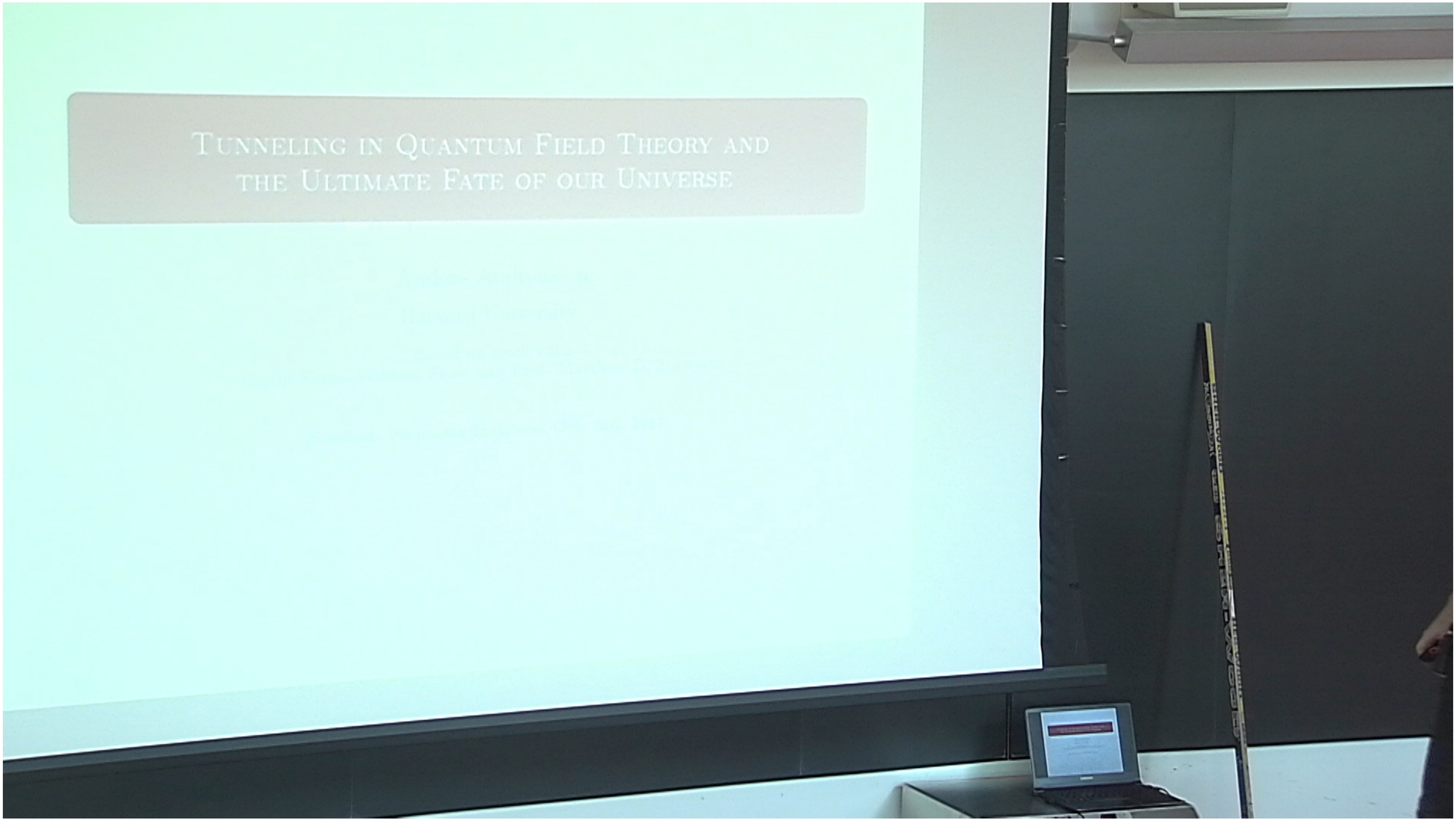
Abstract: <p>One of the most concrete implications of the discovery of the Higgs boson is that, in the absence of physics beyond the standard model, the long term fate of our universe can now be established through precision calculations. Are we in a metastable minimum of the Higgs potential or the true minimum? If we are in a metastable vacuum, what is its lifetime? To answer these questions, we need to understand tunneling in quantum field theory.</p>

<p>This talk will give an overview of the interesting history of tunneling rate calculations and all of its complications in calculating functional determinants of fluctuations around the bounce solutions. Several problems has persisted for the last four decades, and we present new solutions to these problems that enabled us to calculate exact closed-form expressions of the functional determinant. Applied to the Standard model, we then get the first-ever complete calculation of the lifetime of our universe.</p>


TUNNELING IN QUANTUM FIELD THEORY AND
THE ULTIMATE FATE OF OUR UNIVERSE

Tunneling in Quantum Field Theory and the Ultimate Fate of our Universe

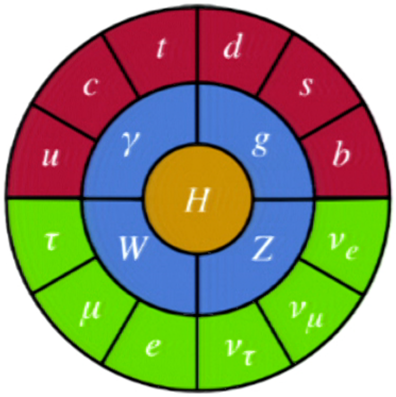
Anders Andreassen

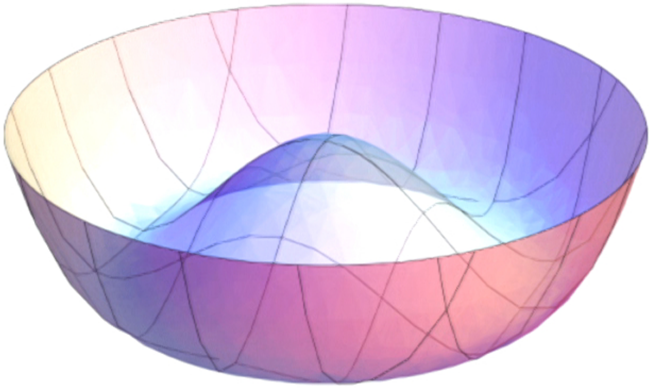
Harvard University

Based on work with
David Farhi, William Frost and Prof. Matthew D. Schwartz

Seminar, Perimeter Institute, Oct. 3rd, 2017

The Higgs Potential





- The Standard Model: $-\mathcal{L}_{\text{int}} \supset -m^2|H|^2 + \lambda|H|^4$
- 2012: ATLAS and CMS measured the Higgs boson mass
- Current best measured value is $m_H = (125.09 \pm 0.24)\text{GeV}$
- We live in a universe with spontaneous symmetry breaking with $\mathcal{L} \supset -m^2|H|^2 + \lambda|H|^4$

The Higgs Potential

Is this the whole story of the Higgs potential?

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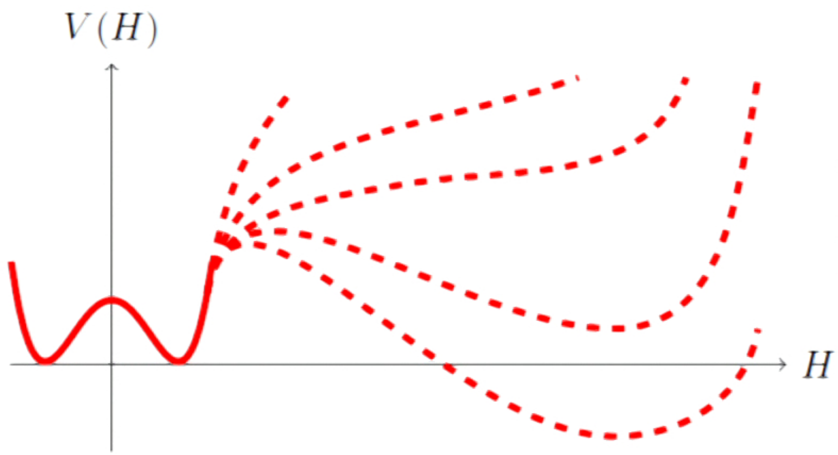
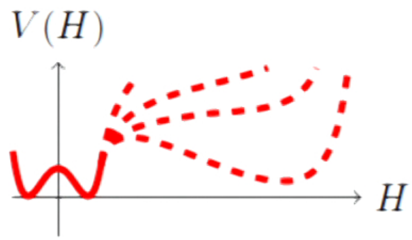


What does the potential look like for large field values?

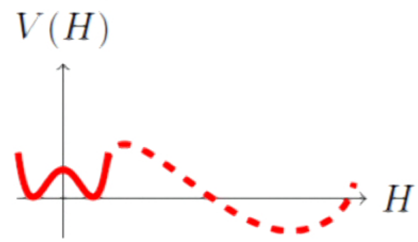
The Higgs Potential

Three different cases:

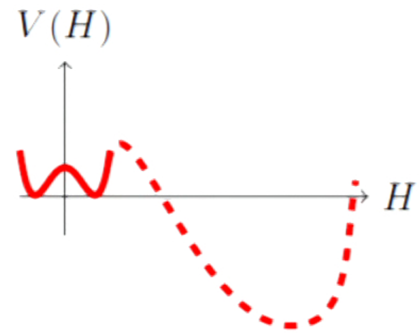
Absolute Stability:
$\tau = \infty$



Meta-Stability:
$\tau > T_U$



Instability:
$\tau < T_U$



τ = Lifetime of the system
$T_U = 10^{10}$y $\approx$ Age of the universe

Stability determined by m_{top} and m_H

Two competing effects:

- $m_H \sim v\sqrt{\lambda}$
    - Higher $m_H \to$ Increase λ
    - $V(H) \supset +\lambda H^4 \to$ more stable

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$$m_t = 173.10\text{GeV}: \ m_H > (129.1 \pm 1.5)\text{GeV}$$
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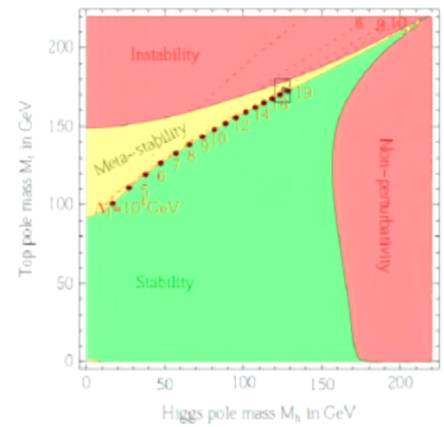

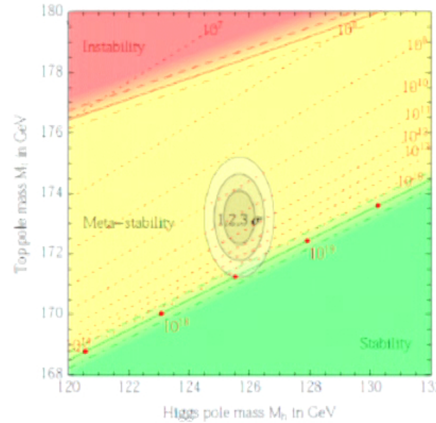

Figure taken from Buttazzo et al. arXiv:1307.3536v2

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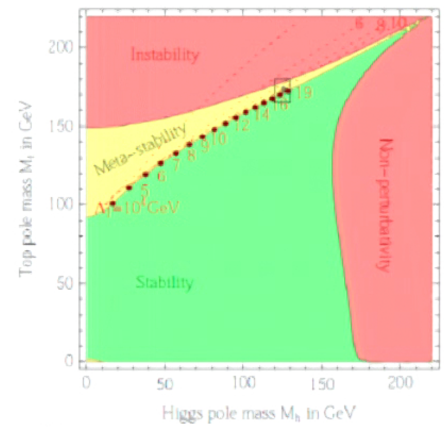
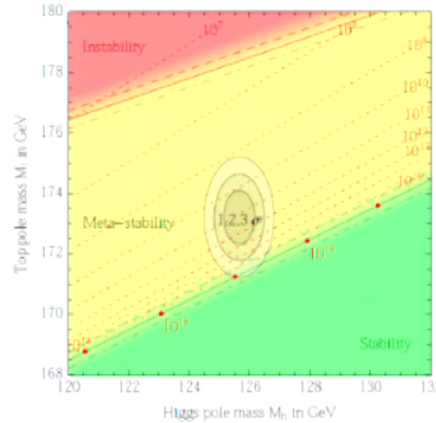
We live in a very special place!

Figure taken from Buttazzo et al. arXiv:1307.3536v2





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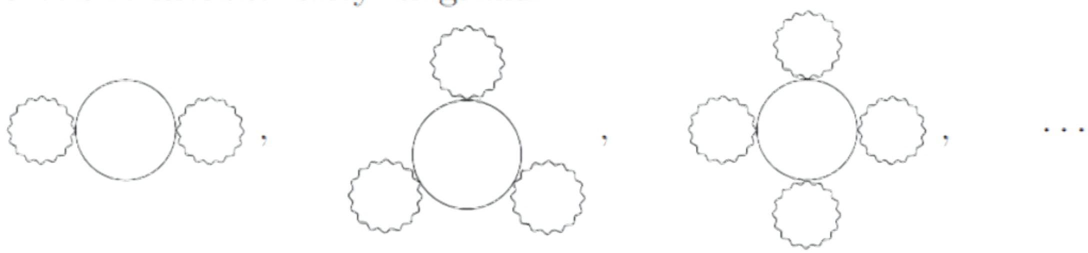
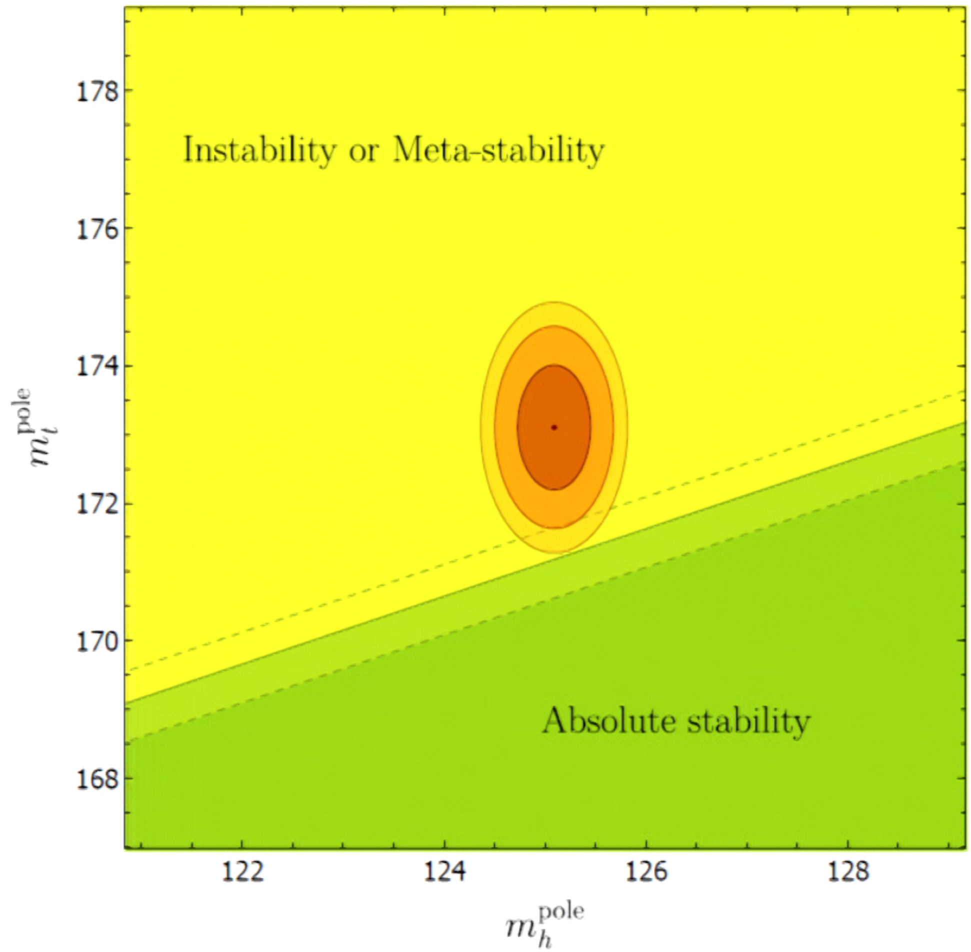
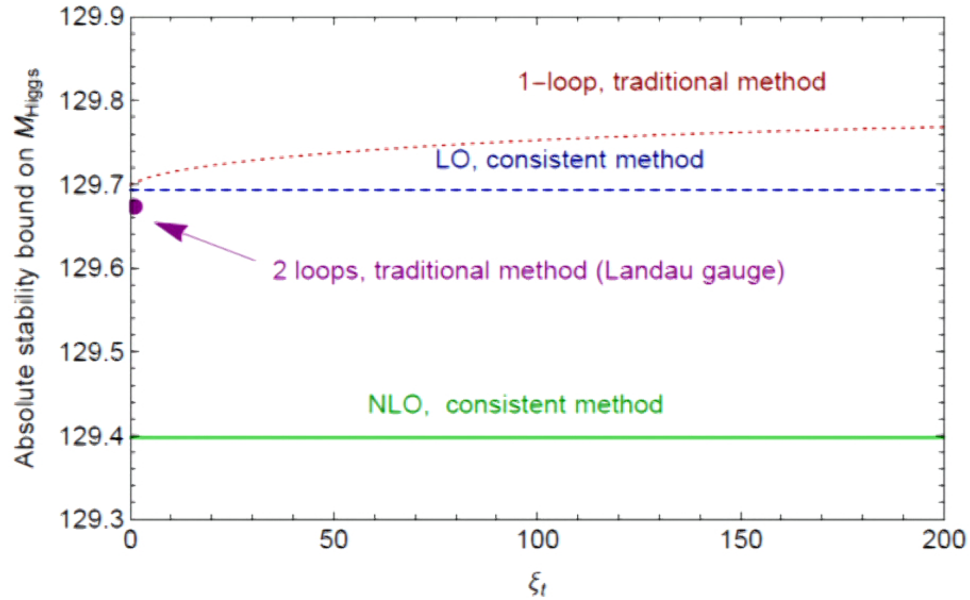
- Need to include daisy diagrams



Absolute Stability Phase Diagram



V_{min} is Gauge Invariant



Consistent Use of the Standard Model Effective Potential (PRL 113 2014), AA, W. Frost, M. Schwartz
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- Calculate Z using method of steepest descent $\Rightarrow$ **Instantons**

Analytic Continuation

$$Z = \int_{x(0)=x_0}^{x(\mathcal{T})=x_0} \mathcal{D}x e^{-S_E[x]} = \sum_E e^{-E\mathcal{T}} \phi_E(x_0)\phi_E^*(x_0)$$

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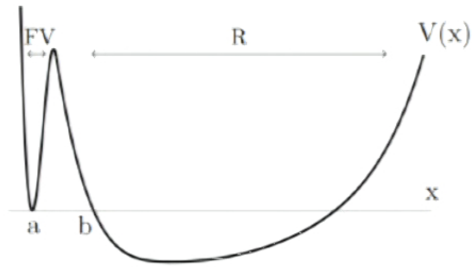
- Really: Specify complex path for path integral

AA, D. Farhi, W. Frost, M. Schwartz (2016)

Alternative Derivation



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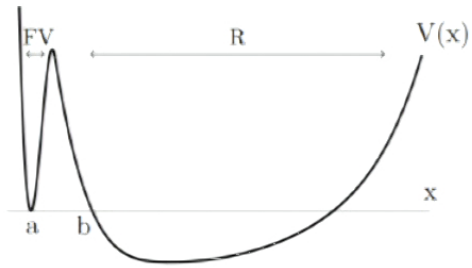

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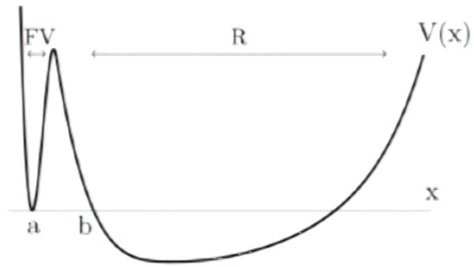

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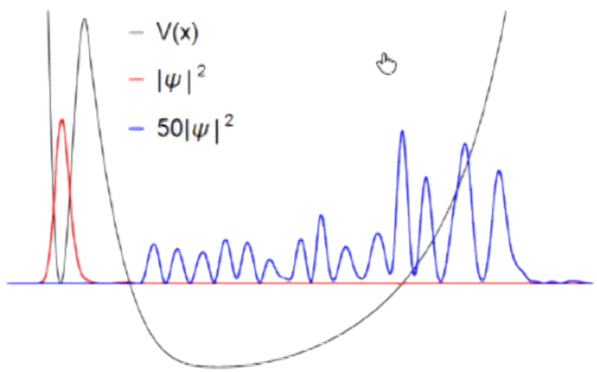

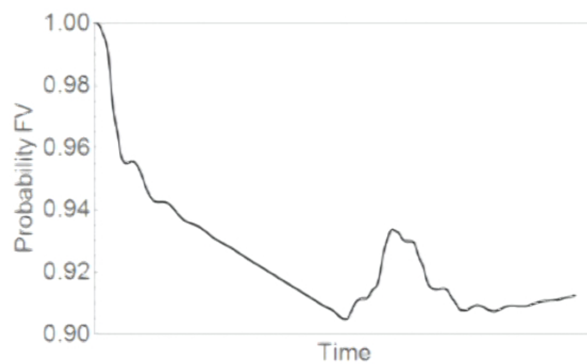

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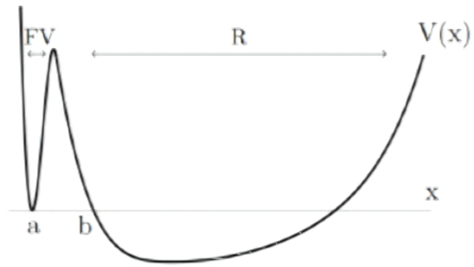

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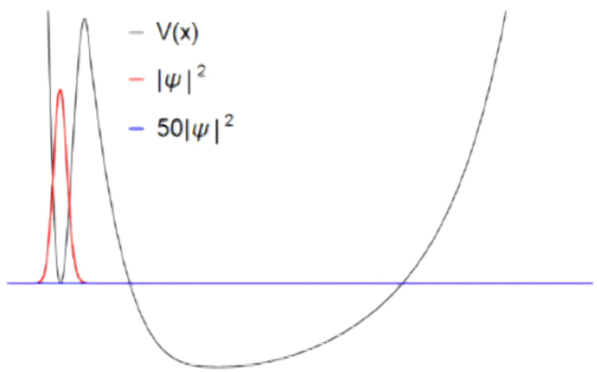

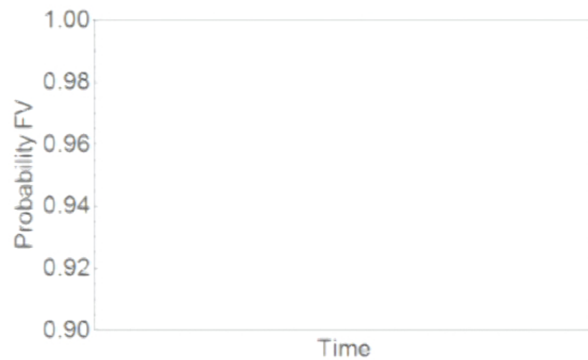

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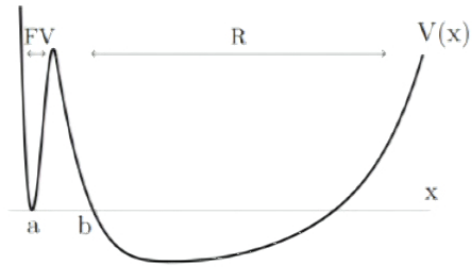

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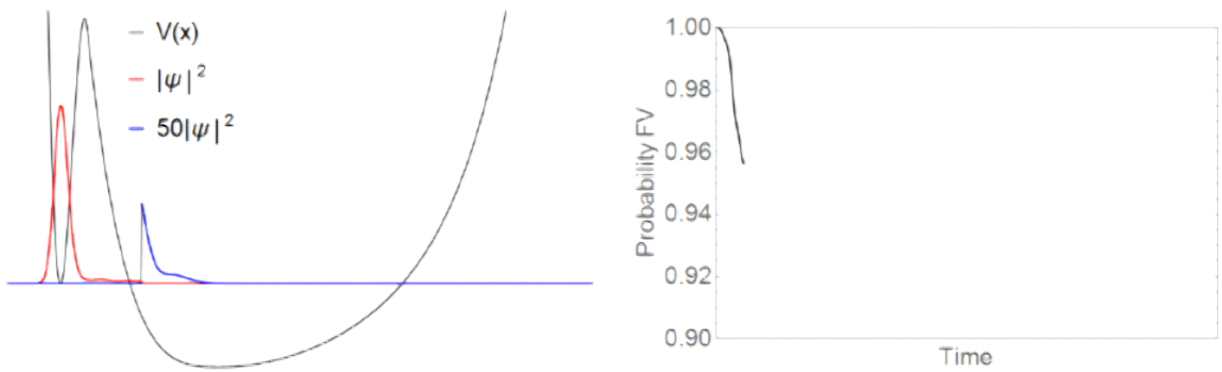

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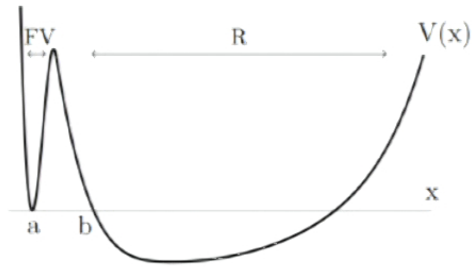

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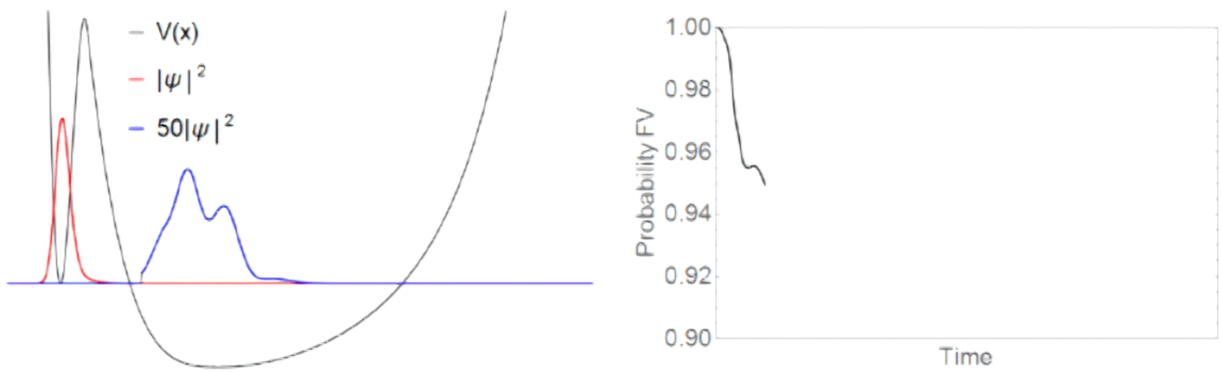
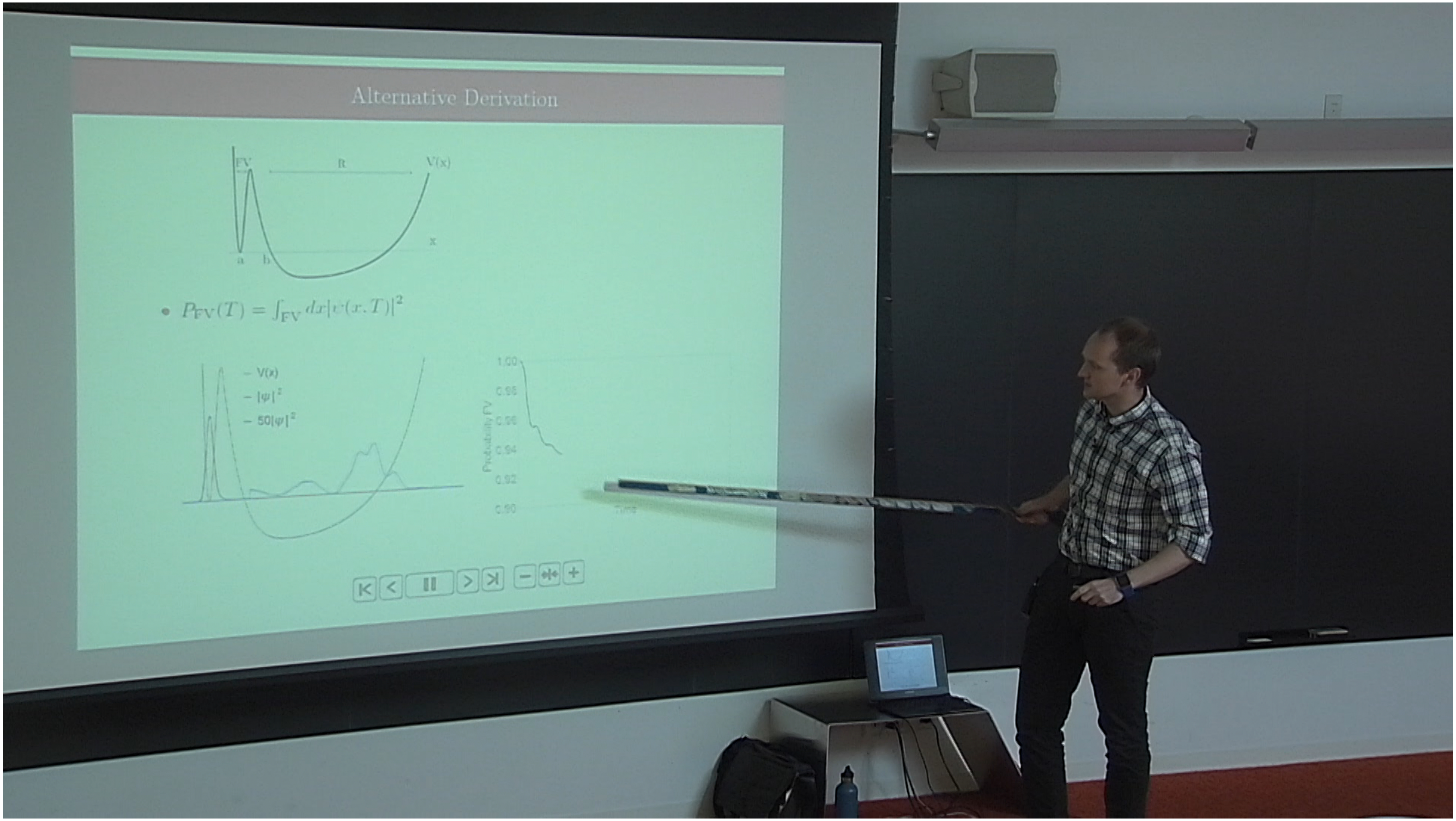
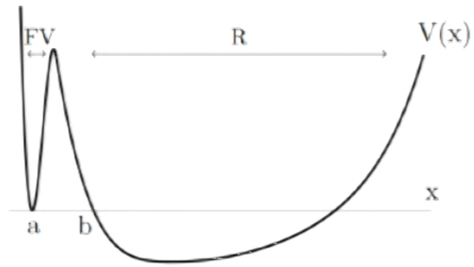



Alternative Derivation
FV
R
V(x)
x
a
b
• $P_{FV}(T) = \int_{FV} dx |\psi(x,T)|^2$
– V(x)
– $|\psi|^2$
– $50|\psi|^2$
Probability FV
1.00
0.98
0.96
0.94
0.92
0.90

Alternative Derivation



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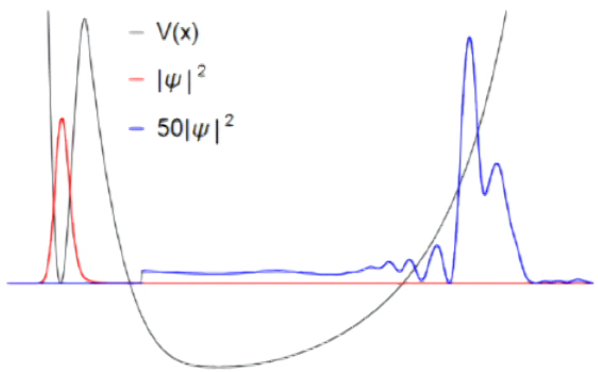

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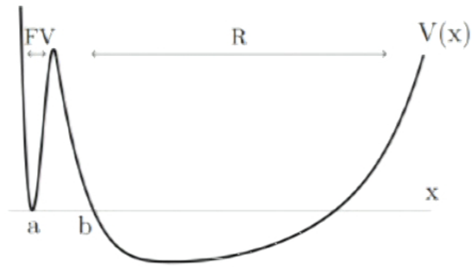

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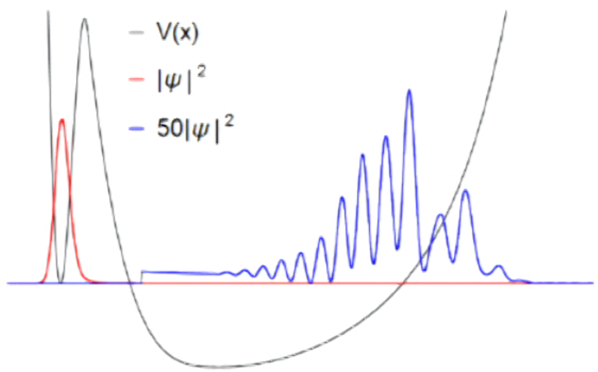

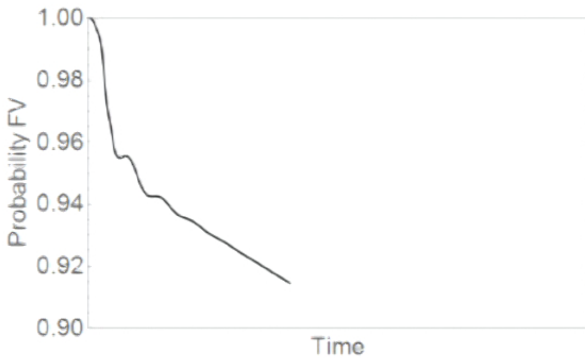

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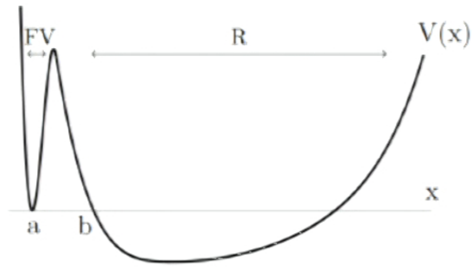

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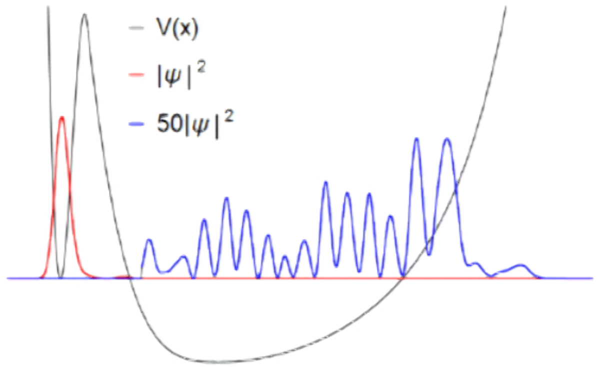

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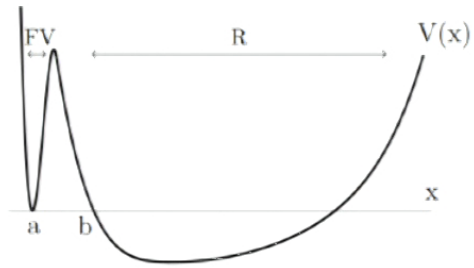

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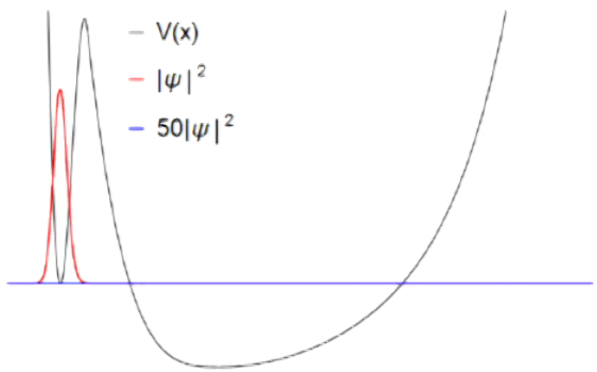

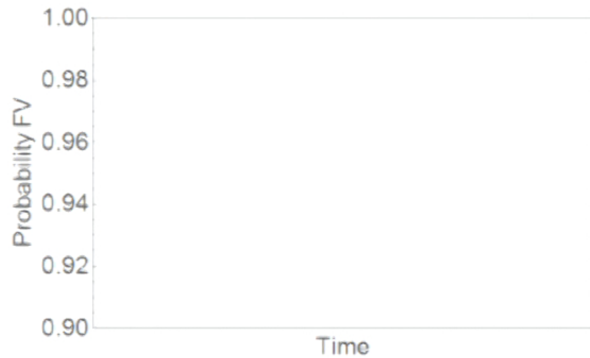

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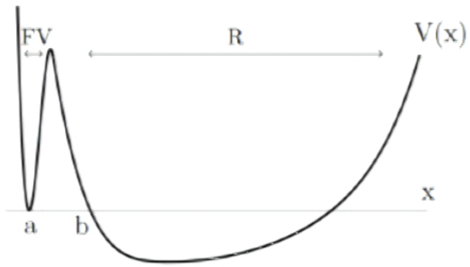
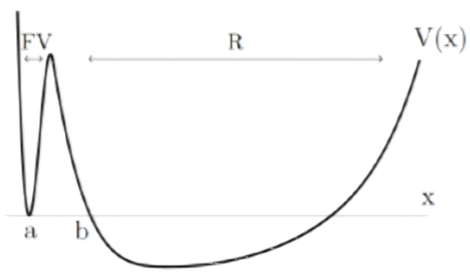
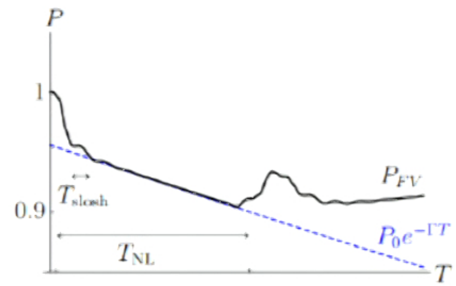


- $P_{\mathrm{FV}}(T) = \int_{\mathrm{FV}} dx |\psi(x,T)|^2$
- $P_{\mathrm{FV}}(T) \sim e^{-\Gamma T}$, for $T_{\mathrm{slosh}} \ll T \ll T_{\mathrm{NL}}$

$$\Gamma = -\frac{1}{P_{\mathrm{FV}}} \frac{d}{dT} P_{\mathrm{FV}}$$

Alternative Derivation





- $P_{\rm FV}(T) = \int_{\rm FV} dx |\psi(x,T)|^2$
- $P_{\rm FV}(T) \sim e^{-\Gamma T}$, for $T_{\rm slosh} \ll T \ll T_{\rm NL}$

$$\Gamma = -\frac{1}{P_{\rm FV}} \frac{d}{dT} P_{\rm FV}$$

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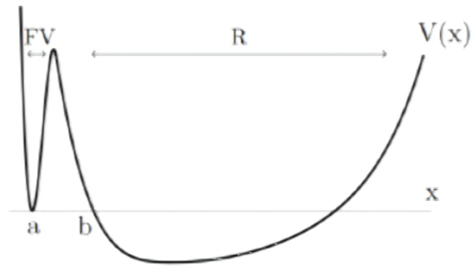

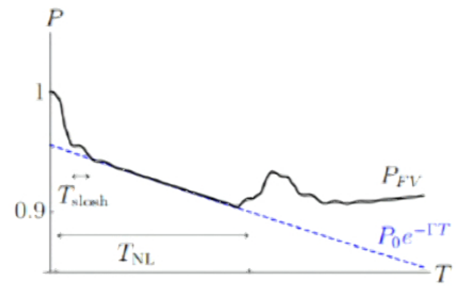

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- Alternative derivation

$$\Gamma = \lim_{T \to \infty} \left| \frac{2\mathrm{Im} \int \mathcal{D}\phi e^{-S[\phi]} \delta(\tau_{\Sigma}[\phi])}{\int \mathcal{D}\phi e^{-S[\phi]}} \right|$$

AA, D. Farhi, W. Frost, M. Schwartz (2016)

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- $\phi = \phi_b + \sum \xi_i\phi_i$ and $\langle\phi_i|\phi_j\rangle = 2\pi\delta_{ij}$
- $\int \mathcal{D}\xi e^{-\frac{1}{2}(2\pi\lambda_i)\xi_i^2} = \prod_i \sqrt{\frac{1}{\lambda_i}} = \sqrt{\frac{1}{\det S''[\phi_b]}}$

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Tunneling rate formula in QFT

$$\frac{\Gamma}{V} = e^{-S[\phi_b]+S[\phi_{\rm FV}]} \left(\frac{S[\phi_b]}{2\pi}\right)^2 \mathrm{Im}\sqrt{\frac{\det S''[\phi_{\rm FV}]}{\det' S''[\phi_b]}}$$

Zero Mode from Scale Invariance

Standard Model

$$S_E = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{4}\phi^4\right], \qquad \phi_b = \sqrt{\frac{8}{-\lambda}}\frac{R}{R^2 + r^2}$$

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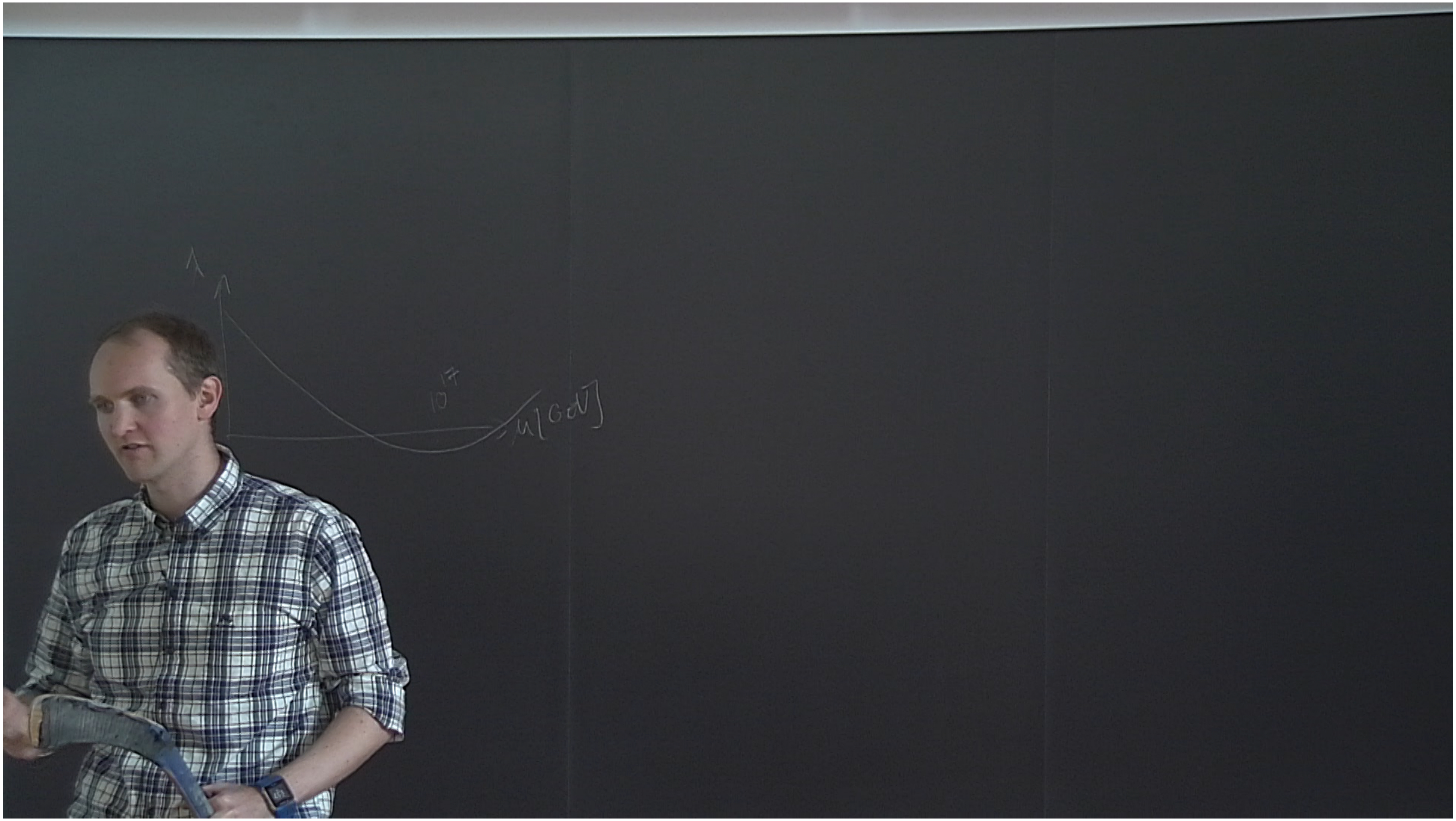
Three Problems

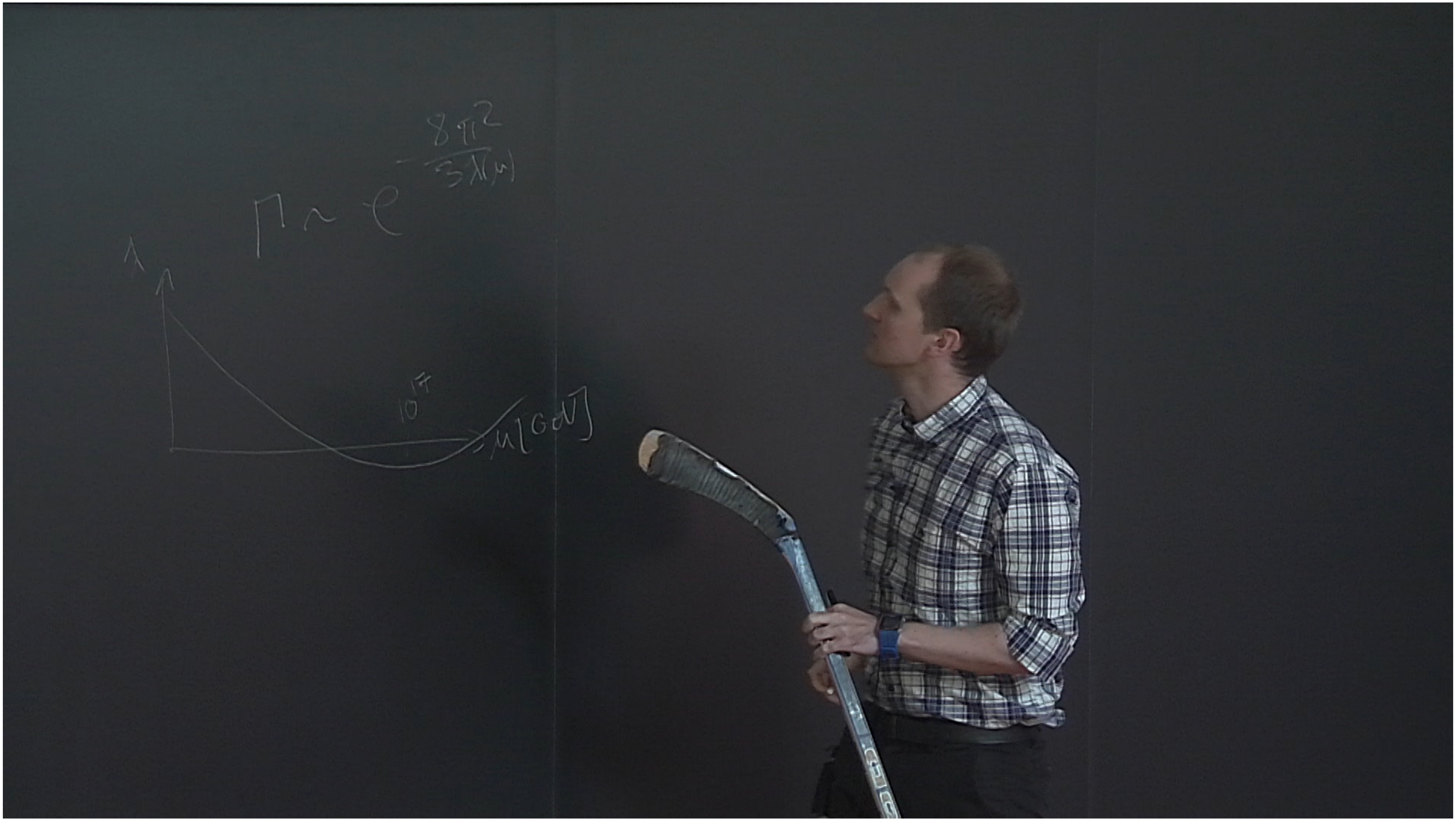
- Jacobian is infinite
- Integral over R is also infinite
- How to calculate correction from mass term in Lagrangian

Outline for the rest of the talk

- Solve Jacobian Problem
  - New basis for functional determinant
  - Exact solutions for eigenvalues
- Evaluating the Functional Determinant Three Ways
  - Sum exact eigenvalues
  - Angular momentum decomposition
  - Gelfand-Yaglom Method
- Standard Model
  - Calculate functional determinants for scalars, vectors and fermions
  - Lifetime of Standard Model Universe



λ
10^{17}
μ [GeV]


Γ ~ e^{-8π^2/3λ(μ)}
λ
10^17
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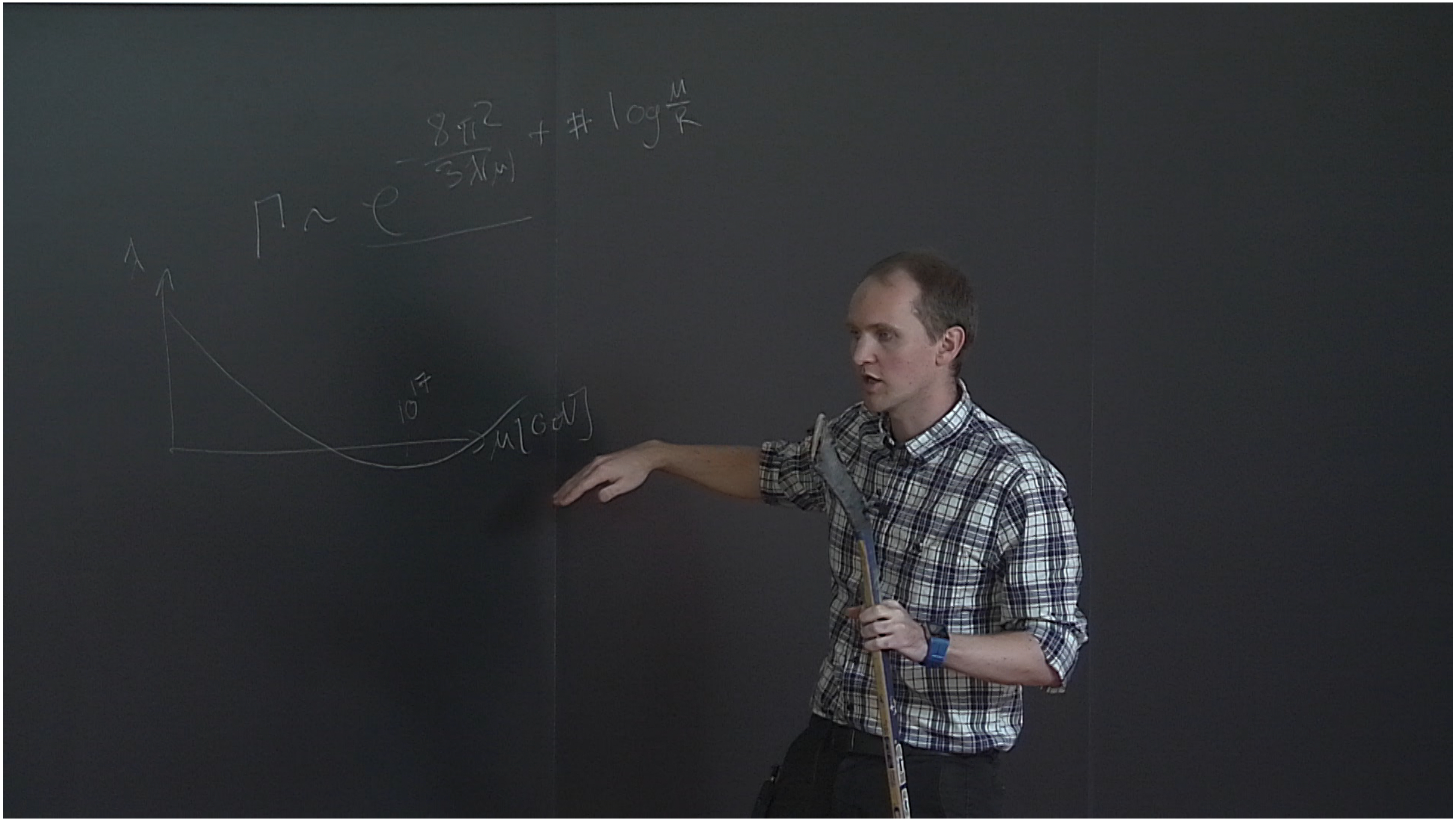

Γ ~ e^(−8π²/3λ(μ) + # log μ/R)
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$$\mathcal{O}_\phi = \frac{1}{3\lambda\phi_b^2}\Box - 1, \quad \hat{\mathcal{O}}_\phi = \frac{1}{3\lambda\phi_b^2}\Box$$

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$$\mathcal{O}_\phi \phi_{nslm} = \lambda_n^\phi \phi_{nslm}, \quad \lambda_n^\phi = \frac{(n-1)(n+4)}{6}, \quad d_n = \frac{1}{6}(n+1)(n+2)(2n+3)$$

$$\phi_{nslm}(r,\alpha,\theta,\phi) = \frac{1}{r} P_{n+1}^{-s-1}\left(\frac{R^2-r^2}{R^2+r^2}\right) Y^{slm}(\alpha,\theta,\phi)$$

$$Y^{slm}(\alpha,\theta,\phi) = \frac{1}{\sqrt{\sin\alpha}} P_{s+\frac{1}{2}}^{-l-\frac{1}{2}}(\cos\alpha) P_l^m(\cos\theta) e^{-im\phi}$$

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$$\begin{aligned}\ln D(x) &= \text{infinite sum over } n \\ &= \underbrace{(\text{infinite sum over } n) - \infty}_{\text{Finite}} + \underbrace{\infty}_{\text{Calculable using Feynman diagrams}}\end{aligned}$$

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- Isolate infinity from sum
  $S^n_{\text{sub}}(x) = \left[d_n \ln \frac{\lambda_n(x)}{\lambda_n(0)}\right]_{x,x^2} = (2n+3)x - \frac{9+6n}{n^2+3n+2}x^2$

Sum Eigenvalues

- $S''[\phi_b] = -\Box + 3\lambda\phi_b^2 \to \mathcal{M}(x) = -\Box - 3x\lambda\phi_b^2$
- $D(x) = \frac{\det \mathcal{M}(x)}{\det \mathcal{M}(0)}$, $\mathcal{O}(x) = \frac{1}{3\lambda\phi_b^2}\Box + x$
- $\mathcal{O}(x)\phi_{nslm} = \lambda_n(x)\phi_{nslm}$, $\lambda_n(x) = \widehat{\lambda}_n^\phi + x = \frac{(n+1)(n+2)}{6} + x$
- $\ln D(x) = \ln \frac{\prod_{n\geq 0}[\lambda_n(x)]^{d_n}}{\prod_{n\geq 0}[\lambda_n(0)]^{d_n}} = \sum_{n\geq 0} d_n \ln \frac{\lambda_n(x)}{\lambda_n(0)}$
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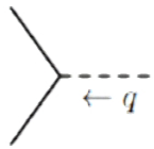
- Feynman Rule  $= 3x\lambda\widetilde{\phi_b^2}(q)$

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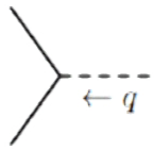
- Feynman Rule 

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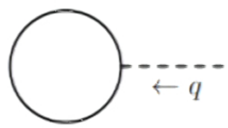
- $-S_x =$ 

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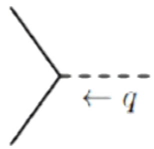
- Feynman Rule 

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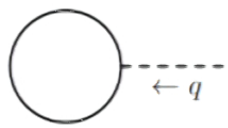
- $-S_x =$ 

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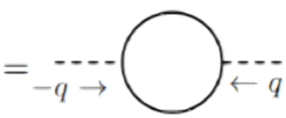
- $-S_{x^2} =$ 

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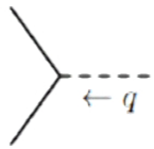
- Feynman Rule 

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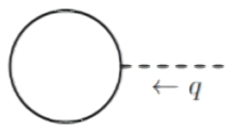
- $-S_x =$ 

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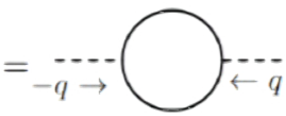
- $-S_{x^2} =$ 

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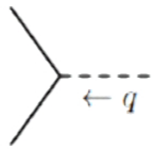
- Feynman Rule 

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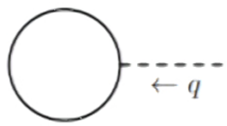
- $-S_x =$ 

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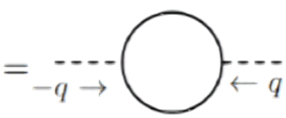
- $-S_{x^2} =$ 

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Angular Momentum Decomposition

Let us do the sum in a different order. Remember

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Gelfand-Yaglom Method

Transforms eigenvalue problem to solving a differential equation

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Powerful method for calculating functional determinants!

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We are able to:

- Calculate exact eigenvalues
- Sum eigenvalues with UV divergences for $\mathcal{M} = -\Box - 3x\lambda\phi_b^2$
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Now, let's study

- Real scalars
- Complex scalars
- Vectors
- Fermions

Real Scalars

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{4}\phi^4$$
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- Combine with Feynman loops and counterterms

$$e^{-S[\phi_b]}\text{Im}\sqrt{\frac{\det \widehat{\mathcal{O}}_\phi}{\det \mathcal{O}_\phi}} = e^{\frac{8\pi^2}{3\lambda_R}}\frac{25}{36}\sqrt{\frac{5}{6}}\exp\left[-\frac{5}{4} + 6\zeta'(-1) + 3\ln\frac{R\mu}{2}\right]$$

Complex Scalars and Global Symmetries

$$\mathcal{L} = |\partial_\mu \Phi| + V(\Phi), \quad \Phi = \frac{1}{\sqrt{2}}(\phi_b + \phi + iG)$$

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- Same calculation as before

$$\frac{\Gamma}{V} = e^{\frac{8\pi^2}{3\lambda_R}} \frac{3S[\phi_b]}{\pi^2} \int \frac{dR}{R^5} \exp\left[-\frac{4}{3} + 8\zeta'(-1) + \frac{10}{3}\ln\frac{R\mu}{2}\right]$$

Vectors and Local Symmetries

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}^2 + (\partial_\mu \Phi^* + igA_\mu \phi^*)(\partial_\mu \Phi - igA_\mu \phi) + \lambda|\Phi|^4 + \frac{1}{2\xi}(\partial_\mu A_\mu)^2 + \bar{c}\Box c$$

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$$\mathcal{M}_s^\xi = \left(1 - \frac{1}{\xi}\right)\begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -\frac{\sqrt{s(s+2)}}{r}\left(\partial_r - \frac{1}{r}\right) & 0 \\ \frac{\sqrt{s(s+2)}}{r}\left(\partial_r + \frac{3}{r}\right) & -\frac{s(s+2)}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{M}_s^T = -\Delta_s + g^2\phi_b^2 \ , \qquad x = -\frac{g^2}{3\lambda}$$

Gelfand-Yaglom For Vectors

Reminder: Gelfand-Yaglom

$$R_s = \frac{\det\left[\frac{1}{3\lambda\phi_b^2}\Delta_s + x\right]}{\det\left[\frac{1}{3\lambda\phi_b^2}\Delta_s\right]} = \left[\lim_{r\to 0}\frac{\phi_0^s(r)}{\phi_x^s(r)}\right]\left[\lim_{r\to\infty}\frac{\phi_x^s(r)}{\phi_0^s(r)}\right]$$

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$$\det \widehat{\Psi}(r) = 2s(s + s\xi + 2\xi)r^{3s}$$

Auxiliary Function Trick

Endo, Moroi, Nojiri, Shoji (2017):

$$\Psi = \begin{pmatrix} \partial_r \chi + \frac{1}{r g^2 \phi_b^2} \eta - 2 \frac{\phi_b'}{g^2 \phi_b^3} \zeta \\ \frac{\sqrt{s(s+2)}}{r} \chi + \frac{1}{\sqrt{s(s+2)} r^2 g^2 \phi_b^2} \partial_r (r^2 \eta) \\ g \phi_b \chi + \frac{1}{g \phi_b} \zeta \end{pmatrix}$$

$$\Delta_s \chi - \frac{2\phi_b'}{r g^2 \phi_b^3} \eta - \frac{2}{r^3} \partial_r \left(\frac{r^3 \phi_b'}{g^2 \phi_b^3} \zeta \right) + \xi \zeta = 0$$

$$(\Delta_s - g^2 \phi_b^2) \eta - \frac{2\phi_b'}{r^2 \phi_b} \partial_r (r^2 \eta) + \frac{2s(s+2)\phi_b'}{r \phi_b} \zeta = 0$$

$$\Delta_s \zeta = 0$$

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Sum over s and calculate divergences from Feynman diagrams as before

Fermions

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{4}\phi^4 + \bar{\psi}\not{\partial}\psi + \frac{y}{\sqrt{2}}\phi\bar{\psi}\psi$$

$$\mathcal{M}^j_{\bar{\psi}\psi} = \begin{pmatrix} \partial_r^2 - \frac{j(j-1)}{r^2} - \frac{y^2}{2}\phi_b^2 & -\frac{y}{\sqrt{2}}\phi_b' \\ -\frac{y}{\sqrt{2}}\phi_b' & \partial_r^2 - \frac{j(j+1)}{r^2} - \frac{y^2}{2}\phi_b^2 \end{pmatrix}$$

$$R_j^{\bar{\psi}\psi} = \det \frac{\mathcal{M}^j_{\bar{\psi}\psi}}{\widehat{\mathcal{M}}^j_{\bar{\psi}\psi}} = \left[\frac{\Gamma(|j| + \frac{1}{2})^2}{\Gamma\left(|j| + \frac{1}{2} + \sqrt{\frac{y^2}{\lambda}}\right)\Gamma\left(|j| + \frac{1}{2} - \sqrt{\frac{y^2}{\lambda}}\right)}\right]^2$$

Summary of Results

- Changed basis for calculating functional determinant
- Exact solutions possible
  - Directly sum λ_n
  - Gelfand-Yaglom method
- Calculated **exact** functional determinant for all fields
- Divergences calculated and subtracted using Feynman diagrams

Vacuum Stability in the Standard Model

$$\mathcal{L}_{\rm SM} = (D_\mu H)^\dagger (D_\mu H) + \lambda (H^\dagger H)^2 - \frac{1}{4}(W^a_{\mu\nu})^2 - \frac{1}{4}B^2_{\mu\nu}$$
$$+ i\bar{Q}\not{D}Q + i\bar{t}_R\not{D}t_R + i\bar{b}_R\not{D}b_R - y_t\bar{Q}Ht_R - y_t^\star\bar{t}_RH^\dagger Q - y_b\bar{Q}\widetilde{H}b_R - y_b^\star\bar{b}_R\widetilde{H}^\dagger Q + \cdots$$

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$$\frac{\Gamma}{V} = e^{-S[\phi_b]}\frac{1}{2}\mathrm{Im}\, V_{SU(2)} J_G^3 (RJ_T)^4 (RJ_d) \sqrt{\frac{\det \widehat{\mathcal{O}}_h}{\det' \mathcal{O}_h}} \sqrt{\frac{\det \widehat{\mathcal{O}}_{ZG}}{\det' \mathcal{O}_{ZG}}} \frac{\det \widehat{\mathcal{O}}_{WG}}{\det' \mathcal{O}_{WG}} \sqrt{\frac{\det \mathcal{O}_{\bar{t}t}}{\det \widehat{\mathcal{O}}_{\bar{t}t}}} \sqrt{\frac{\det \mathcal{O}_{\bar{b}b}}{\det \widehat{\mathcal{O}}_{\bar{b}b}}}$$
$$\times\, \mu_\star^4 \sqrt{-\frac{\pi S[\phi_b^\star]\lambda_\star}{\beta'_{0\star}}}\, e^{-\frac{4\lambda_\star}{S[\phi_b^\star]\beta'_{0\star}}} \left[\frac{\lambda_\star}{\lambda_{\text{1-loop}}(\hat{\mu})} - 1 - \frac{4\lambda_\star}{S[\phi_b^\star]^2\beta'_{0\star}}\right]$$

Vacuum Stability in the Standard Model

$$\frac{1}{2}\underbrace{e^{-S[\phi_b]}}_{10^{-826}}\underbrace{V_{SU(2)}}_{10^{2}}\underbrace{J_G^3}_{10^{5}}\underbrace{(RJ_T)^4(RJ_d)}_{10^{7}}\underbrace{\sqrt{\frac{\det\widehat{\mathcal{O}}_h}{\det'\mathcal{O}_h}}}_{10^{-2}}\underbrace{\sqrt{\frac{\det\widehat{\mathcal{O}}_{ZG}}{\det'\mathcal{O}_{ZG}}}}_{10^{17}}\underbrace{\frac{\det\widehat{\mathcal{O}}_{WG}}{\det'\mathcal{O}_{WG}}}_{10^{19}}\underbrace{\sqrt{\frac{\det\mathcal{O}_{\bar{t}t}}{\det\widehat{\mathcal{O}}_{\bar{t}t}}}}_{10^{25}}\underbrace{\sqrt{\frac{\det\mathcal{O}_{\bar{b}b}}{\det\widehat{\mathcal{O}}_{\bar{b}b}}}}_{0.995}$$

$$\underbrace{\mu_\star^4}_{10^{70}\ \mathrm{GeV}^4}\underbrace{\sqrt{-\frac{\pi\lambda_\star}{S[\phi_b^\star]\beta'_{0\star}}}e^{-\frac{4\lambda_\star}{S[\phi_b^\star]\beta'_{0\star}}}}_{1.09}\underbrace{S[\phi_b^\star]\left[\frac{\lambda_\star}{\lambda_{\text{1-loop}}(\hat{\mu})}-1-\frac{4\lambda_\star}{S[\phi_b^\star]^2\beta'_{0\star}}\right]}_{0.653}$$

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$$\frac{\Gamma}{V} = 10^{-683}\,\mathrm{GeV}^4 \times \begin{pmatrix}10^{-279}\\ 10^{162}\end{pmatrix}_{m_t} \times \begin{pmatrix}10^{-39}\\ 10^{35}\end{pmatrix}_{m_h} \times \begin{pmatrix}10^{-186}\\ 10^{127}\end{pmatrix}_{\alpha_s} \times \begin{pmatrix}10^{-61}\\ 10^{102}\end{pmatrix}_{\text{thr.}} \times \begin{pmatrix}10^{-2}\\ 10^{2}\end{pmatrix}_{\text{NNLO}}$$

$$= 10^{-683^{-409}_{+202}}\,\mathrm{GeV}^4$$

Vacuum Stability in the Standard Model

$$\underbrace{\frac{1}{2} e^{-S[\phi_b]}}_{10^{-826}} \underbrace{V_{SU(2)}}_{10^{2}} \underbrace{J_G^3}_{10^{5}} \underbrace{(RJ_T)^4(RJ_d)}_{10^{7}} \underbrace{\sqrt{\frac{\det \widehat{\mathcal{O}}_h}{\det' \mathcal{O}_h}}}_{10^{-2}} \underbrace{\sqrt{\frac{\det \widehat{\mathcal{O}}_{ZG}}{\det' \mathcal{O}_{ZG}}}}_{10^{17}} \underbrace{\frac{\det \widehat{\mathcal{O}}_{WG}}{\det' \mathcal{O}_{WG}}}_{10^{19}} \underbrace{\sqrt{\frac{\det \mathcal{O}_{\bar{t}t}}{\det \widehat{\mathcal{O}}_{\bar{t}t}}}}_{10^{25}} \underbrace{\sqrt{\frac{\det \mathcal{O}_{\bar{b}b}}{\det \widehat{\mathcal{O}}_{\bar{b}b}}}}_{0.995}$$

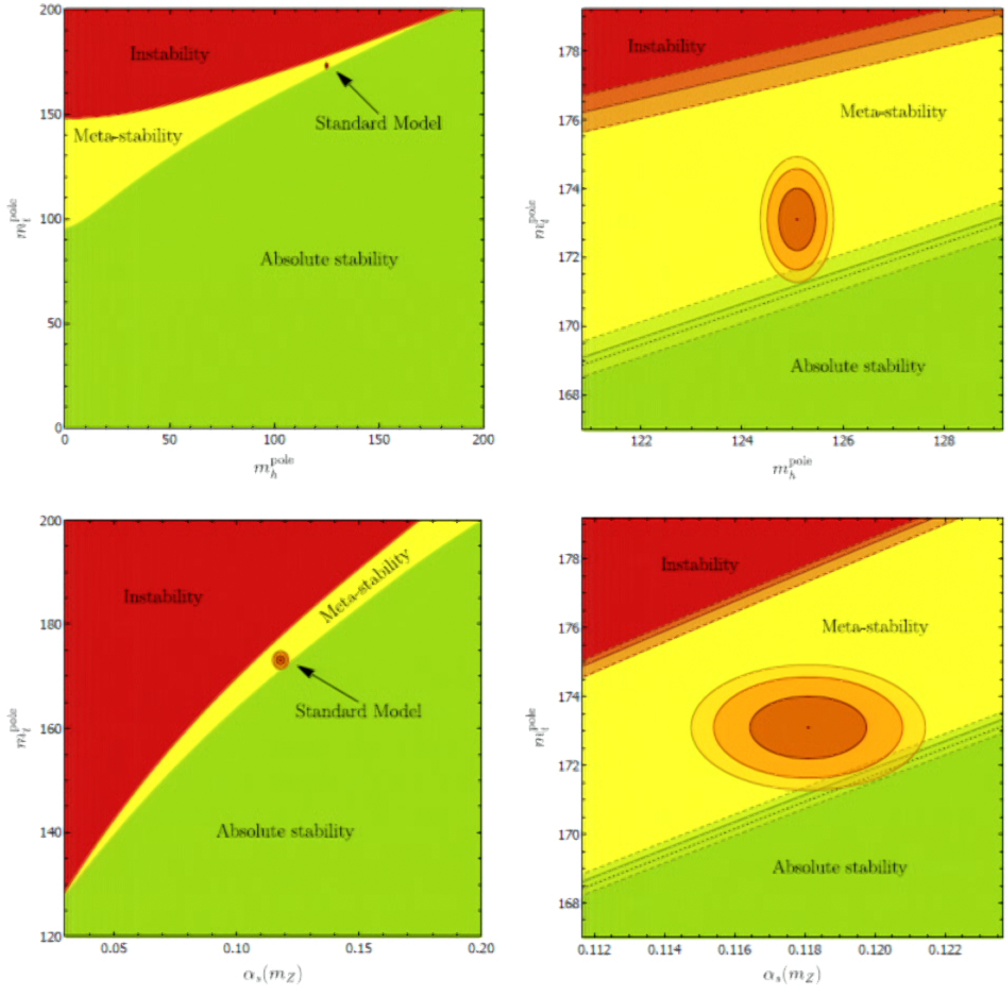
$$\underbrace{\mu_\star^4}_{10^{70}\ \mathrm{GeV}^4} \underbrace{\sqrt{-\frac{\pi\lambda_\star}{S[\phi_b^\star]\beta'_{0\star}}} e^{-\frac{4\lambda_\star}{S[\phi_b^\star]\beta'_{0\star}}}}_{1.09} \underbrace{S[\phi_b^\star]\left[\frac{\lambda_\star}{\lambda_{\text{1-loop}}(\hat{\mu})} - 1 - \frac{4\lambda_\star}{S[\phi_b^\star]^2\beta'_{0\star}}\right]}_{0.653}$$

$$\frac{\Gamma}{V} = 10^{-683}\,\mathrm{GeV}^4 \times \begin{pmatrix}10^{-279}\\10^{162}\end{pmatrix}_{m_t} \times \begin{pmatrix}10^{-39}\\10^{35}\end{pmatrix}_{m_h} \times \begin{pmatrix}10^{-186}\\10^{127}\end{pmatrix}_{\alpha_s} \times \begin{pmatrix}10^{-61}\\10^{102}\end{pmatrix}_{\text{thr.}} \times \begin{pmatrix}10^{-2}\\10^{2}\end{pmatrix}_{\text{NNLO}}$$

$$= 10^{-683^{-409}_{+202}}\,\mathrm{GeV}^4$$

$$\tau_{\text{SM}} = \left(\frac{\Gamma}{V}\right)^{-1/4} = 10^{139^{+102}_{-51}}\ \text{years}$$

Standard Model Phase Diagrams



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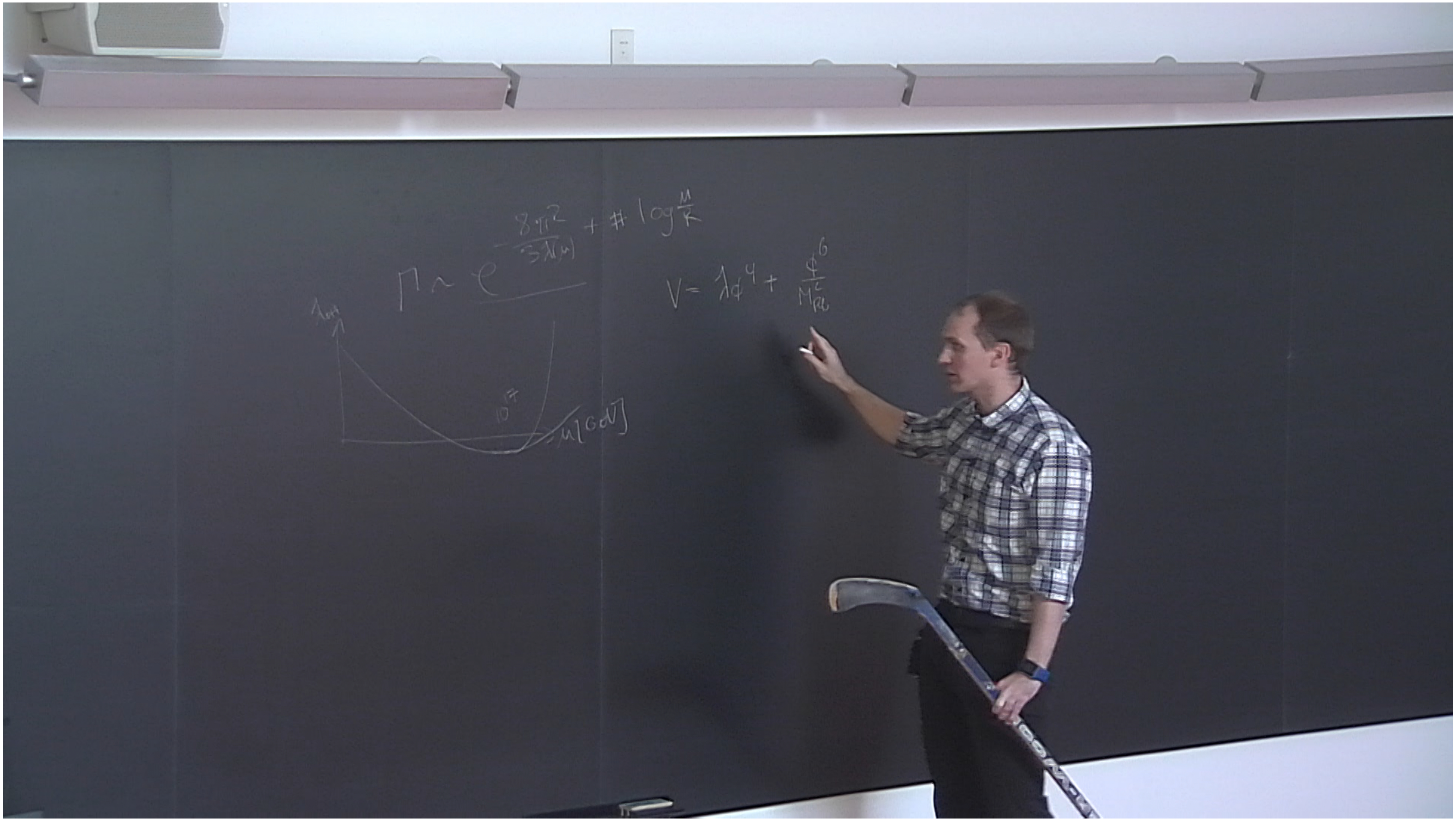
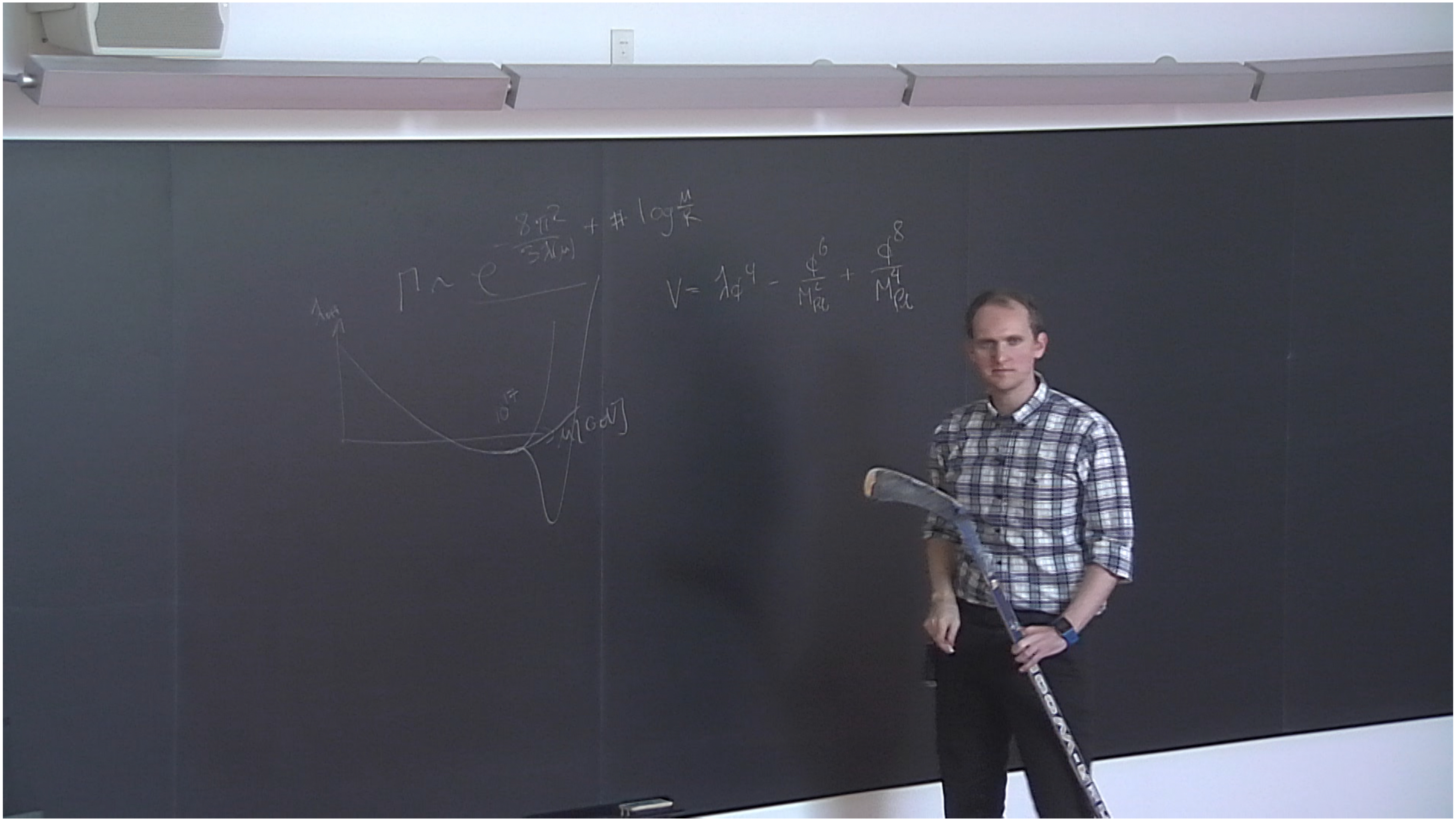
Thank you!



Γ ~ e^(−8π²/3λ(μ)) + # log μ/k
V = λφ⁴ + φ⁶/M²Pl
λeff
10¹⁷
μ[GeV]


V = λφ^4 − φ^6/M_Pl^2 + φ^8/M_Pl^4
μ [GeV]