Title: Towards a Framework of Probabilistic Bayesian Theories

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Abstract: <p>Quantum mechanics can be seen as a set of instructions how to calculate probabilities by associating mathematical objects to physical procedures, like preparation, manipulation, and measurement of a system. Quantum theory then yields probabilities which are neutral with respect to its use, e.g., in a Bayesian or a frequentistic way. We investigate a different approach to quantum theory and physical theories in general, in which we aim for subjective predictions in the Bayesian sense. This gives a structure different from the operational framework of general probabilistic theories. We explore these differences and the according mathematical language of such probabilistic Bayesian theories. This is joint work with Adan Cabello, Giulio Chiribela, and Markus MÃ¼ller.</p>

Towards a Framework of Probabilistic Bayesian Theories

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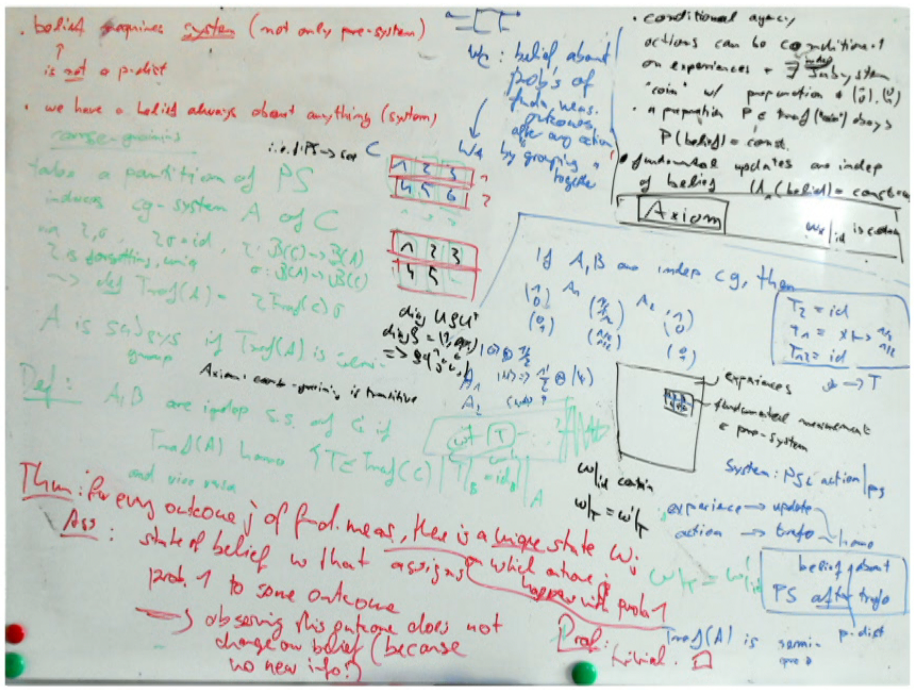

INTRODUCTION

Is quantum theory too versatile?

Quantum theory as a probabilistic theory

An experiment is described by

- **state** ω, summarizing the initial situation,
- **operation** ϕ, modeling the experimental procedures,
- **measurement outcome** f, connecting to probabilities.

All of these form convex sets. Quantum theory corresponds to a particular structure of these convex sets.

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Defining structure: The "probability"

$$P(\omega, \phi, f)$$

is linear in ω, ϕ, and f.

How to compute $P(\omega, \phi, f)$

Linearity and temporal ordering suggests either of:

Schrödinger picture: $P = f[\phi(\omega)]$.
Heisenberg picture: $P = \omega[\phi(f)]$.

☞ None is mathematically preferred, one is the dual of the other.

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Example (Quantum theory)

- Schrödinger picture:
  $\omega = \rho, \quad \phi\colon X \mapsto UXU^\dagger, \quad f\colon X \mapsto \mathrm{tr}(\Pi X)$
- Heisenberg picture:
  $f = \Pi, \quad \phi\colon X \mapsto U^\dagger XU, \quad \omega\colon X \mapsto \mathrm{tr}(X\rho).$

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☞ None is mathematically preferred, one is the dual of the other.

- The dual to the “measurement problem” is a “preparation problem?”
- The dual to the “collapse of the wave function” is a “collapse of the measurement?”

☞ Duality of concepts does not genuinely apply.

What is the probability P?

P is some real number, $0 \leq P \leq 1$, with the promise that P is the parameter for a Bernoulli trail.

☞ Allows for **frequentist** and **Bayesian** inference.

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For a parametrized model $x \mapsto P(\omega_x, \phi_x, f_x)$,

- a confidence interval $[x_{\text{low}}, x_{\text{up}}]$ can be inferred, most likely containing a plausible value for x for the past experiment.
- a distribution $x \mapsto p_x$ can be inferred, allowing to guess outcomes of future experiments.

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Maybe both are needed?

Mathematical language

- Quantum **theory** deals with states, transformations, and measurements as mathematical objects.
- Quantum **mechanics** yields the connection between the physical and mathematical objects.

☞ General probabilistic theories do not have feature a “mechanics.”

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The quantization problem

- It is only understood partially.
- There are situations where different quantization methods yield different results.

CONCEPTS

Design decisions for a Bayesian framework.

Belief based operationalism

A Bayesian agent has three key abilities:

- having **perceptions**,
- performing **actions**, and
- making **predictions**.

The ability for predictions origins in a **belief model**.

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Questions

- How many agents are there?
- Perceptions about what and actions on what?
- Are perceptions and actions different concepts?
- What is the structure of belief models?
- How much will this deviate from quantum theory?

The agent as Bayesian update machine

~~Machine learning~~

The agent has a theory according to which the belief model is kept up to date.

No quality assessment of the agent.

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No quality assessment of the agent.

Example (Naive quantum agent)

- The belief model is the quantum state ρ.
- The action is a measurement with fixed model $(\Pi_1, \Pi_2, \dots)$.

The perception of outcome i induces the belief update

$$\rho \longrightarrow \frac{\Pi_i \rho \Pi_i}{\mathrm{tr}(\Pi_i \rho)}.$$

Local tomography

Knowledge of all correlations $P(a, b|x, y)$ is sufficient to predict all global perceptions.

☞ Counterexample: Quantum theory over the real numbers.

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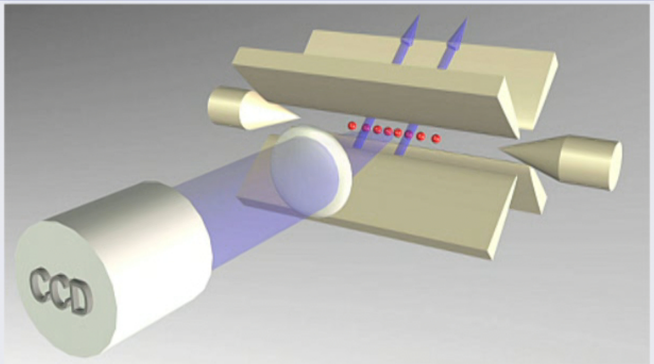
The global agent

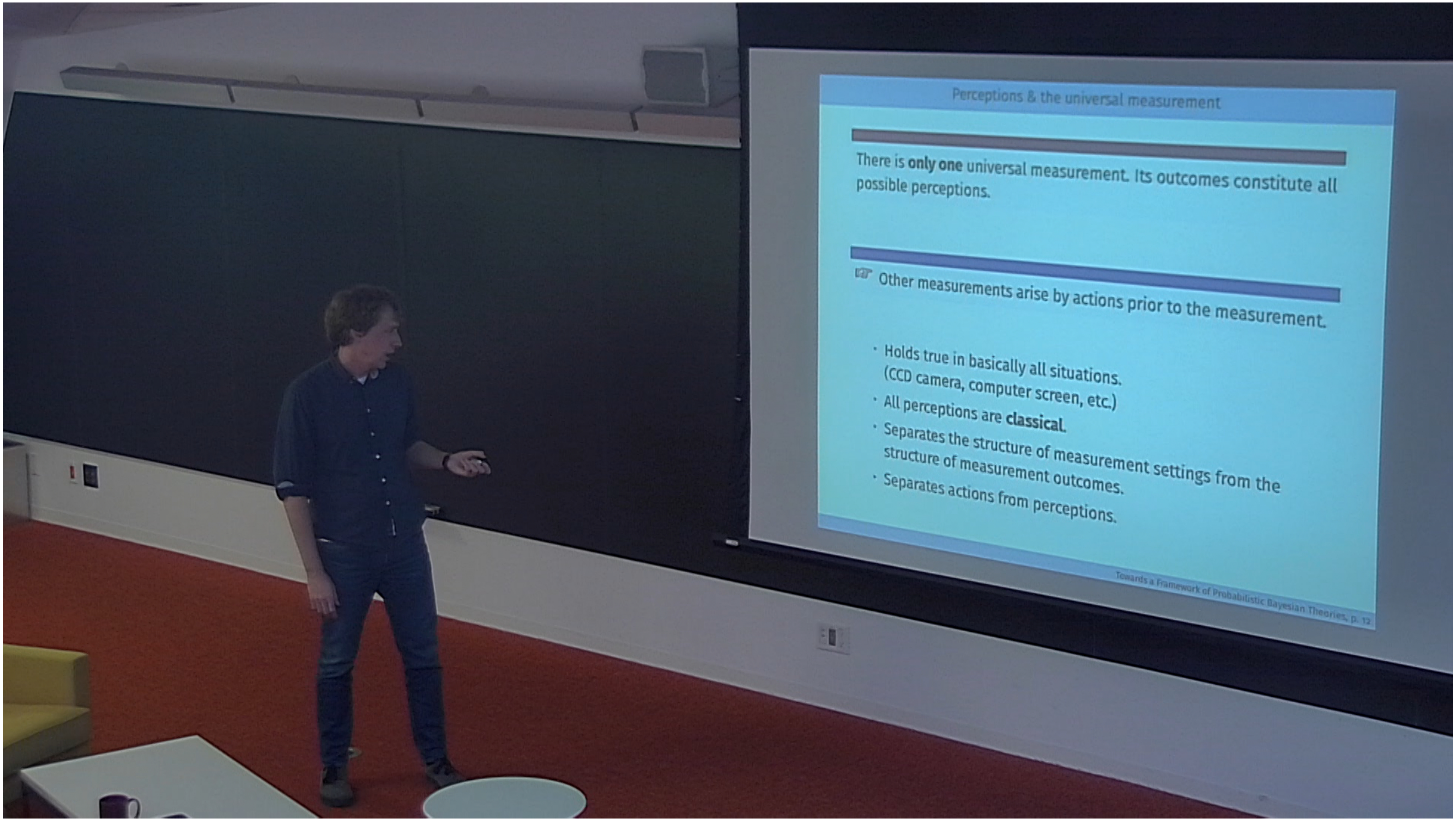
- The global agent is responsible only for a given system.
- Different global agents may be inconsistent.
- But: There is a notion of "sub-agents" which are consistent agents on certain subsets of perceptions and actions.

The ion-trap agent



- loading the trap
- initializing the CCD camera
- cooling
- changing the trap potential
- operating several lasers directed on all or particular ions
- readout data of the CCD camera

There is **only one** universal measurement. Its outcomes constitute all possible perceptions.


Perceptions & the universal measurement
There is only one universal measurement. Its outcomes constitute all possible perceptions.
Other measurements arise by actions prior to the measurement.
Holds true in basically all situations. (CCD camera, computer screen, etc.)
All perceptions are classical.
Separates the structure of measurement settings from the structure of measurement outcomes.
Separates actions from perceptions.
Towards a Framework of Probabilistic Bayesian Theories, p. 12

METHODS

Finding a mathematical language.

Summary of the agent

The agent is equipped with

- an abstract belief,
- the ability to have a perception,
- the ability to act,
- a system how to change the belief, consistent with perceptions and actions, and
- an opinion about future perceptions (prediction).

Events & predictions

- An event $e \in \mathcal{E}$ is a subset of the "things that can happen" Ω.
  ($\mathcal{E}$ is a σ-algebra on Ω)
- A prediction is a probability measure $P\colon e \to [0, 1]$ on the events $\mathcal{E}$.
  (P is σ-additive)

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  (P is σ-additive)

☞ Predictions are Kolomogorvian (as they should).

Predictions & beliefs

- Predictions are based on a belief $\beta \in \mathcal{B}$.
- The prediction $P \in \mathcal{P}$ is extracted from β via a function $X : \mathcal{B} \to \mathcal{P}$.

Event-based perceptions

A perception is an event, i.e., a **set** $e \in \mathcal{E} \subset \mathfrak{P}\Omega$.

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...why?

- Mathematical analogy to event-based predictions.
- Only partial information might be taken into consideration.
- ...it gives a lot of appropriate structure.

A perception $e \in \mathcal{E}$ changes the belief of the agent via $T_e : \mathcal{B} \to \mathcal{B}$.

Axiom 1

$T : e \mapsto T_e$ is a semigroup homomorphism with respect to $ef = e \cap f$, i.e.,

$$T_e T_f = T_{e \cap f}$$

but $T_{\{\ \}}$ is not valid.

Perception-induced belief update

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Rationale:

- Refining a perception is a consistent operation.
- Perceptions are classical and hence commute, $T_e T_f = T_f T_e$.
- Contradicting perceptions, $e \cap f = \{\ \}$, are not admissible.

Actions & time order

An action $a \in \mathcal{A}$ changes the belief of the agent via $Y_a : \mathcal{B} \to \mathcal{B}$.

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Axiom 2

$Y : a \mapsto Y_a$ is a semigroup homomorphism with respect to $ba =$ "a then b", i.e.,

$$Y_b Y_a = Y_{ba}.$$

(Also, $\mathcal{A}$ is a semigroup.)

Summary of the agent

A agent is equipped with

- a belief $\beta \in B$.
- the ability to have a perception $e \in \mathcal{E} \subset \mathfrak{P}\Omega$.
- the ability to perform an action $a \in \mathcal{A}$
- a system how to change his belief, consistent with perceptions, via T, and actions, via Y.
- an opinion about future perceptions, via X.

EXAMPLES

and comments.

Example

Assume an agent $\mathbf{W}$. Let $\mathcal{A}' = \mathcal{A} \cup \mathcal{E} \cup \{ u \} \cup$ all products ae, etc., and $Z: x \mapsto Z_x$ with $Z_{xa} = Z_x Y_a$ and $Z_{xe} = Z_x T_e$.

The belief $\beta' = \{ \beta'_f \mid f \in \mathcal{A}' \}$ has

$$\beta'_f = X Z_f \beta, \quad X' \beta' = \beta'_u,$$

$$Y'_e \beta' = \beta'_e, \quad T'_a \beta' = \beta'_a.$$

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Neat, but…

this agent is mindless: the belief is just a list of all possible future predictions.

Insufficient encapsulation

The agent is not responsible for the whole universe, which causes:

Example

Assume a state $\rho_1 \otimes \rho_2$ but the agents belief takes into account only one system,

$$\text{“}\beta = \rho_1.\text{”}$$

If “swap,” $1 \leftrightarrow 2$, is a valid action then the model is insufficient.

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Example

Assume a state $\rho_1 \otimes \rho_2$ but the agents belief takes into account only one system,

$$“\beta = \rho_1.”$$

If “swap,” $1 \leftrightarrow 2$, is a valid action then the model is insufficient.

☞ Even an agent describing all subjective actions and perception can be insufficient.

Encapsulation

The set of actions and perceptions have to be restricted (by the agent), so that insufficient encapsulation cannot occur.

Quantum agent

We choose $\Omega = \{1, \dots d\}$ and $\mathcal{E} = \mathfrak{P}\Omega$. The belief model is a **weighted quantum state** γ of a d-level system. We let

$$(X\gamma)(e) = \sum_{k \in e} \langle k|\gamma|k\rangle \,/\, \mathrm{tr}(\gamma).$$

Actions are all unitaries U on the system and

$$T_U\gamma = U\gamma U^\dagger.$$

The update rule is

$$Y_e\gamma = \sum_{k \in e} |k\rangle\langle k| \,\langle k|\gamma|k\rangle\,.$$

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$$Y_e\gamma = \sum_{k \in e} |k\rangle\langle k| \, \langle k|\gamma|k\rangle \, .$$

von Neumann state update

A perception destroys any coherence in the belief model γ.

The belief model is a **pair** $\beta = [V, q]$, where V is a **unitary** and q is a **measure** on $\mathcal{E}$. We let

$$(X[V, q])(e) = q(e)/q(\Omega).$$

and

$$T_U[V, q] = [UV, q]$$

There is an additional action "measure!," for which

$$T_{\text{measure!}}[V, q] = [\mathbb{1}, e \mapsto \sum_{k \in e,\, \ell \in \Omega} q(\{\, \ell \,\})|\langle k|V|\ell\rangle|^2].$$

The update rule is canonical, $Y_e[V, q] = [V, g \mapsto q(e \cap g)]$.

Alternative quantum agent

The belief model is a **pair** $\beta = [V, q]$, where V is a **unitary** and q is a **measure** on $\mathcal{E}$. We let

$$(X[V, q])(e) = q(e)/q(\Omega).$$

and

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The update rule is canonical, $Y_e[V, q] = [V, g \mapsto q(e \cap g)]$.

Preserves V (the basis in which the state is diagonal) when making a perception.

Summary

Aim: Framework for probabilistic Bayesian theories.

☞ Independent of quantum theory.

(i) No Heisenberg—Schrödinger duality present.

(ii) No ambiguity about inference methods.

(iii) Avoids quantization problem?

Conclusions

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Aim: Framework for probabilistic Bayesian theories.

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Open ends

- How powerful are subsystems?
- Are there circumstances where the time-ordering is emergent?
- **Will quantum mechanics emerge?**
  And if so, is this a good thing?