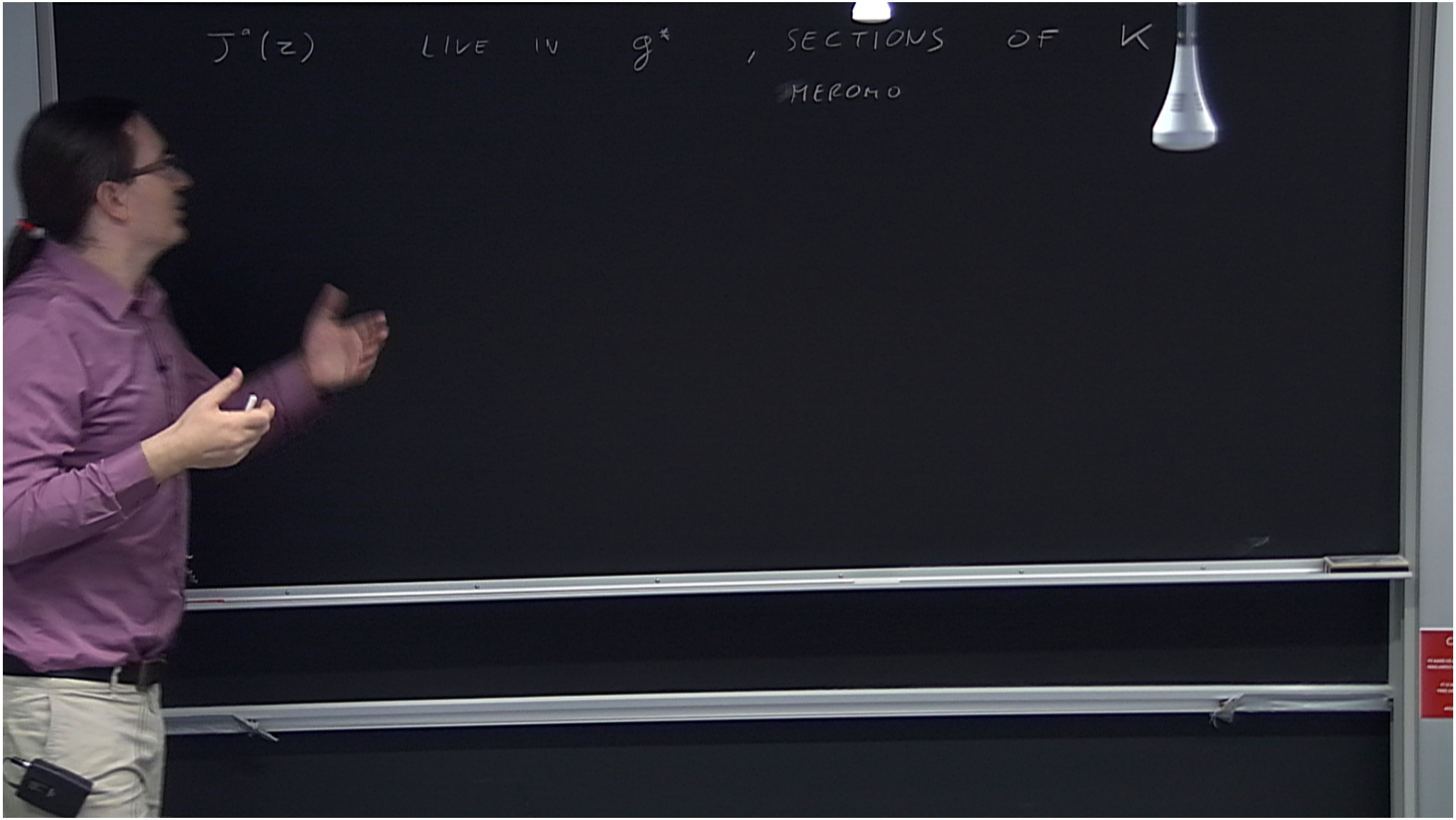
Title: Mathematical Seminar - Davide Gaiotto

Date: Sep 25, 2017 10:00 AM

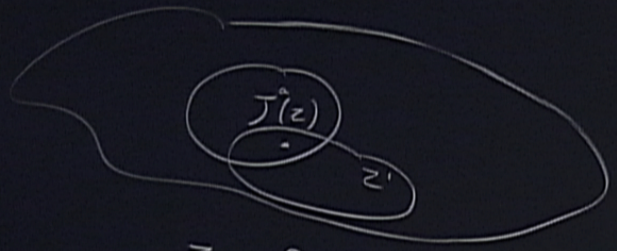
URL: http://pirsa.org/17090072

Abstract: He will discuss relations between Virasoro and Kac-Moody conformal blocks, character varieties and quantum groups, and AGT.


$J^a(z)$ LIVE IN g^*, SECTIONS OF K
MEROMO

$J^a(z)$ LIVE IN $\mathfrak{g}^*$, SECTIONS OF K

BEHAVE AS MEROMORPHIC $(1,0)$ FORMS



$z = f(z')$

$J^a(z)$ LIVE IN g^*, SECTIONS OF K

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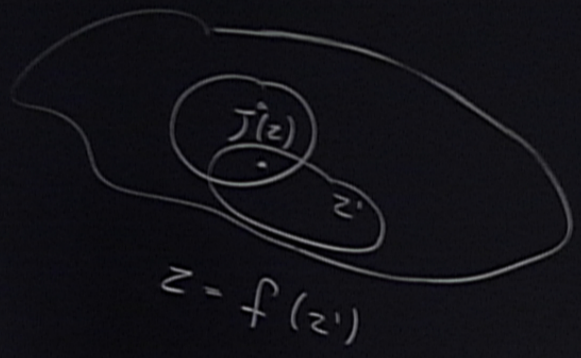
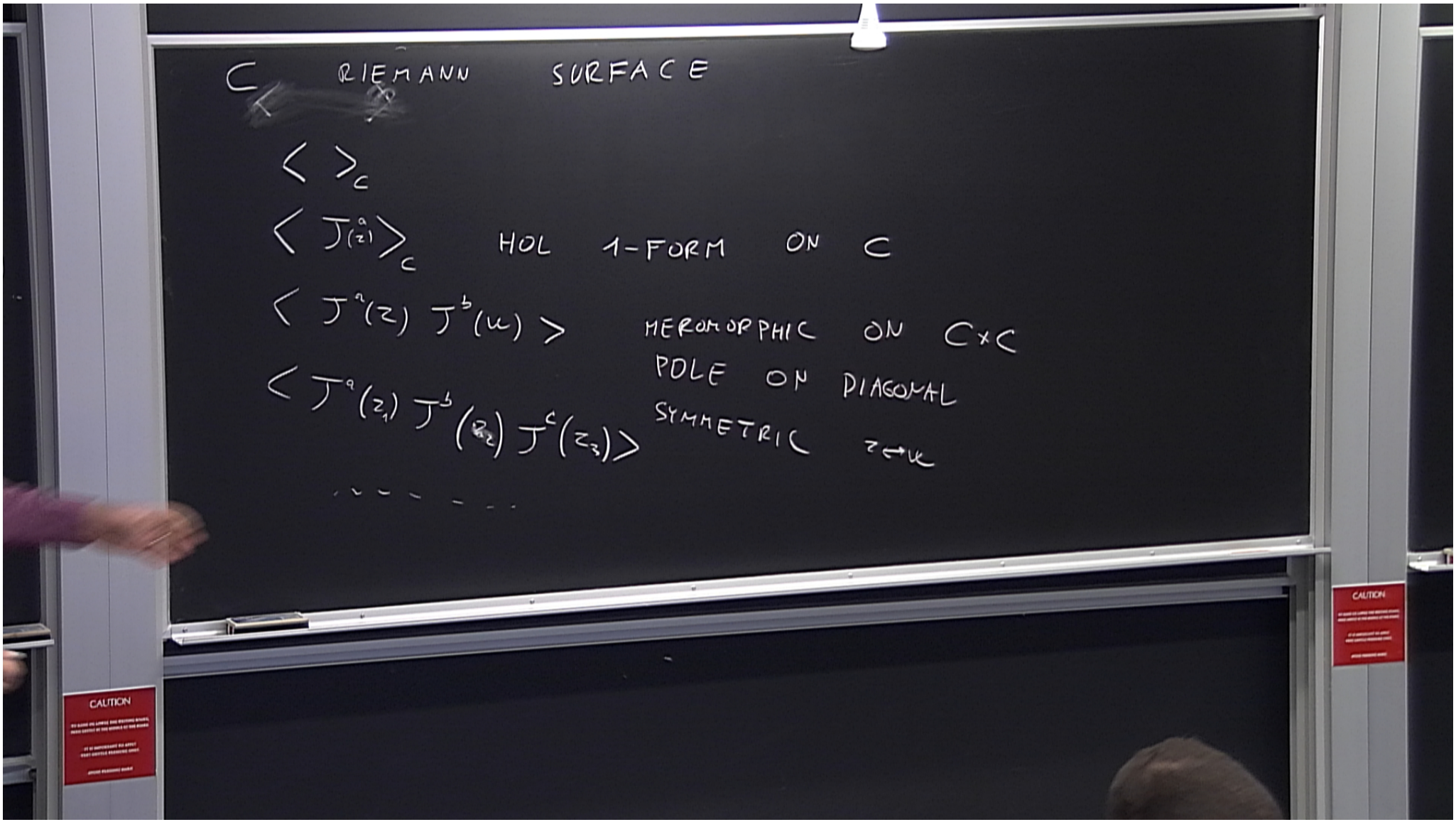
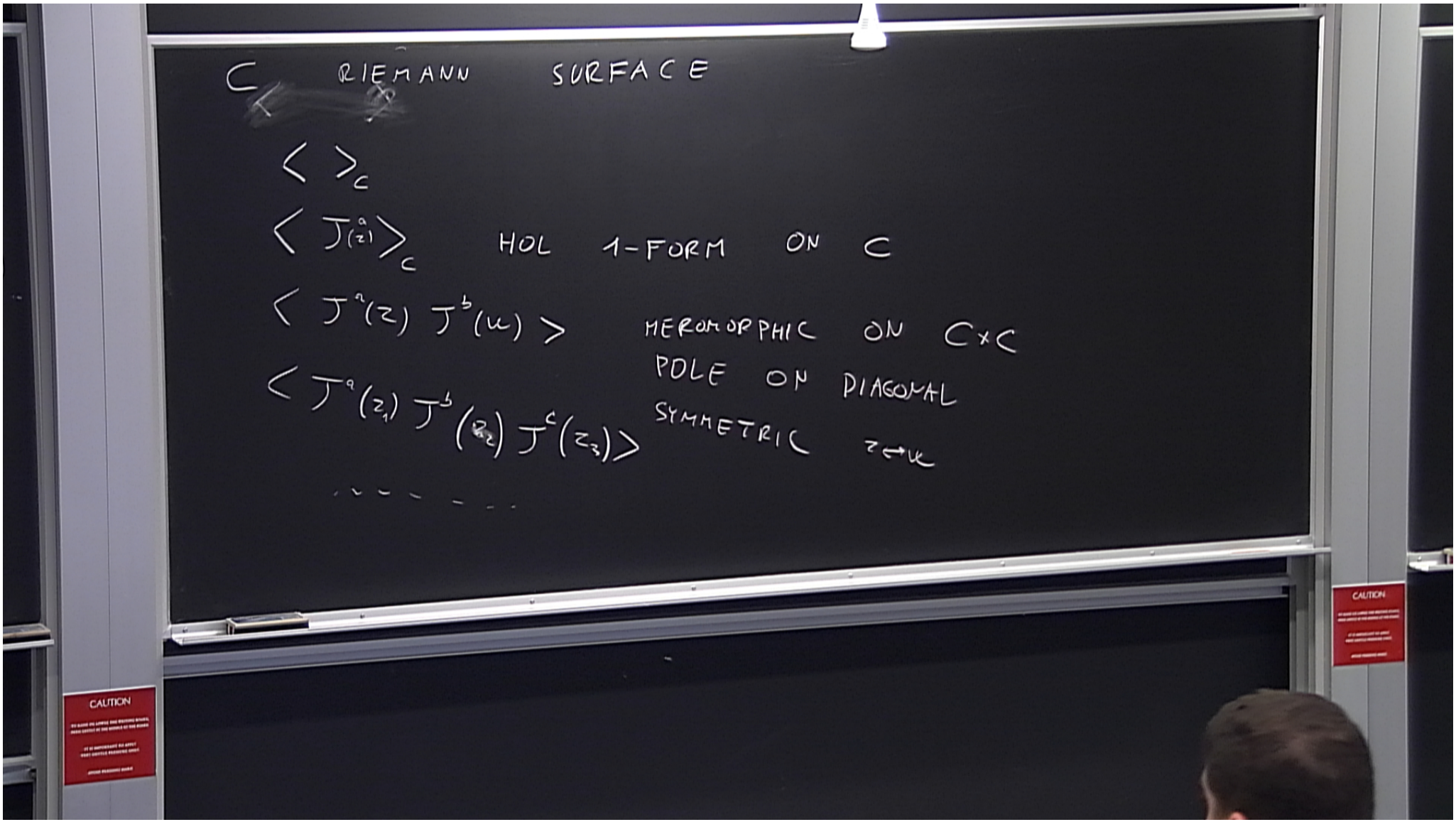


$$J^a(z)\,J^b(w) \sim \frac{\kappa\,\delta^{ab}}{(z-w)^2} + \frac{f^{ab}_{\ c}\,J^c(w)}{z-w} + \dots$$

$$J^a(z) = J^a(z')\frac{dz'}{dz}$$


C RIEMANN SURFACE
$\langle\ \rangle_C$
$\langle J^a(z) \rangle_C$ HOL 1-FORM ON C
$\langle J^a(z)\, J^b(w) \rangle$ MEROMORPHIC ON C×C
POLE ON DIAGONAL
$\langle J^a(z_1)\, J^b(z_2)\, J^c(z_3) \rangle$ SYMMETRIC $z \leftrightarrow w$
CAUTION
CAUTION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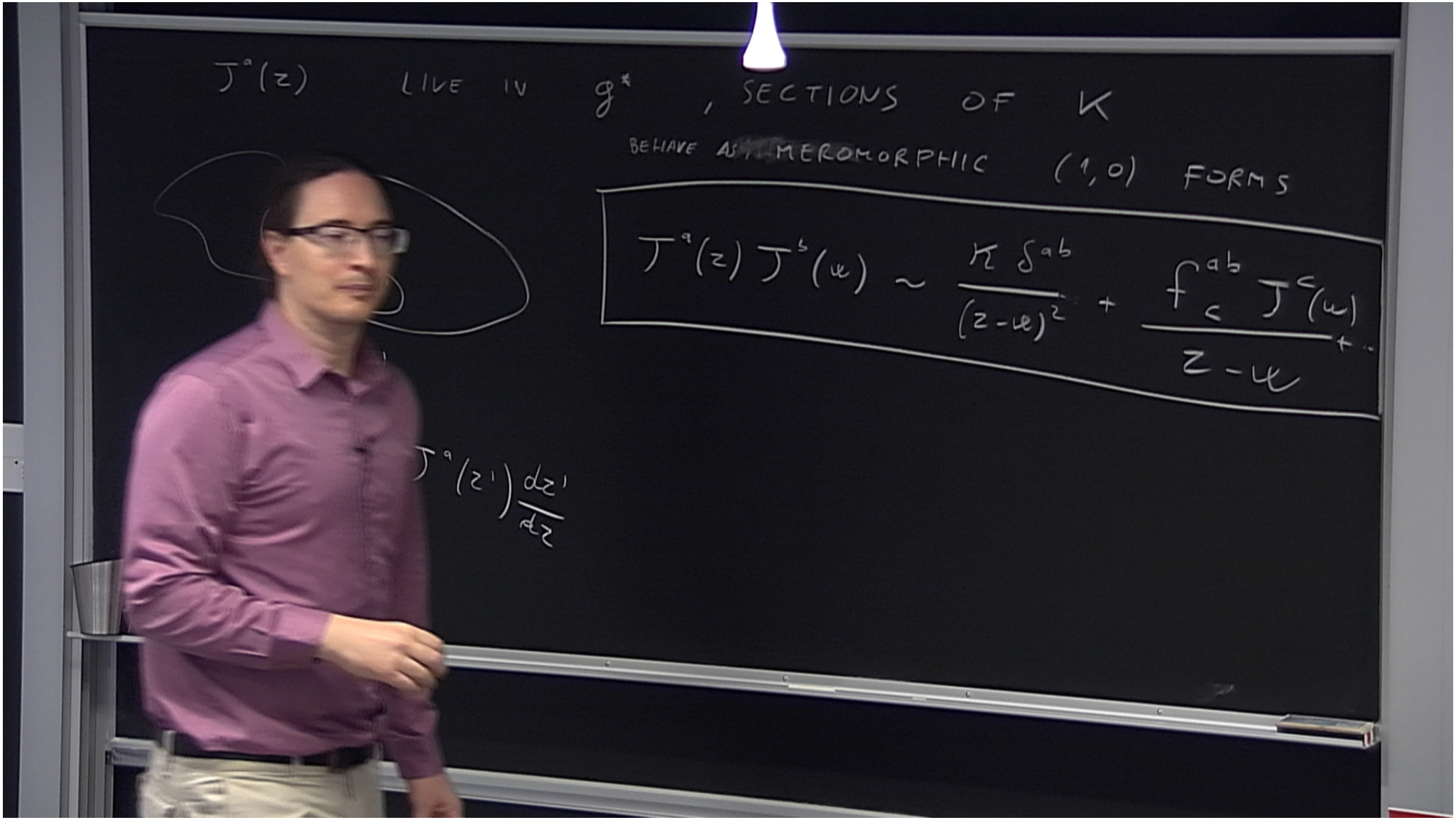
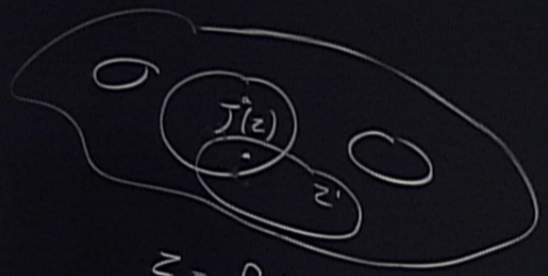

$J^a(z)$ LIVE IN g^*, SECTIONS OF K
BEHAVE AS MEROMORPHIC (1,0) FORMS
$J^a(z)\,J^b(w) \sim \frac{K\,\delta^{ab}}{(z-w)^2} + \frac{f_c^{ab}\,J^c(w)}{z-w} + \ldots$
$J^a(z')\frac{dz'}{dz}$

$J^a(z)$ LIVE IN g^*, SECTIONS OF K

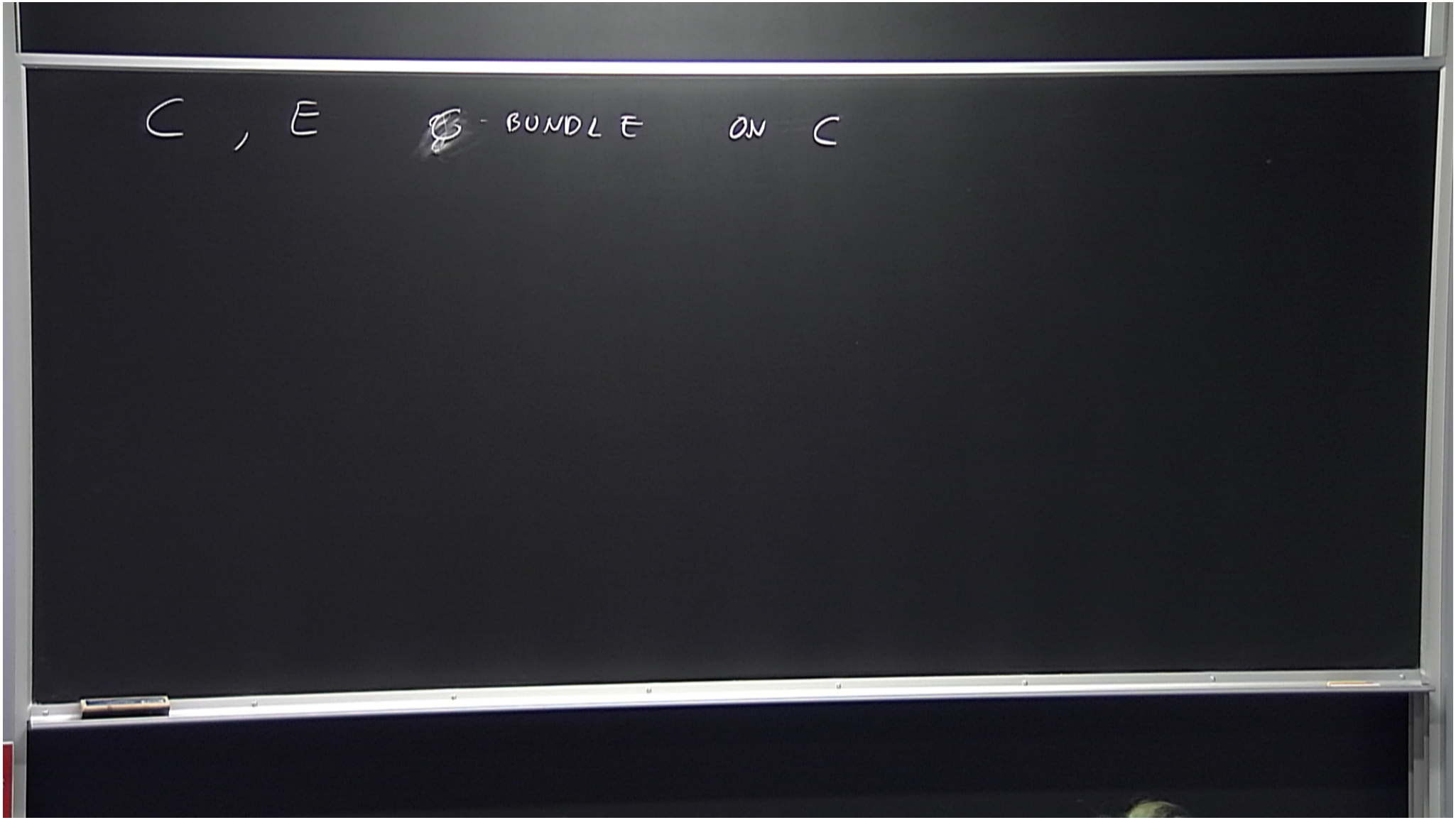
BEHAVE AS MEROMORPHIC $(1,0)$ FORMS

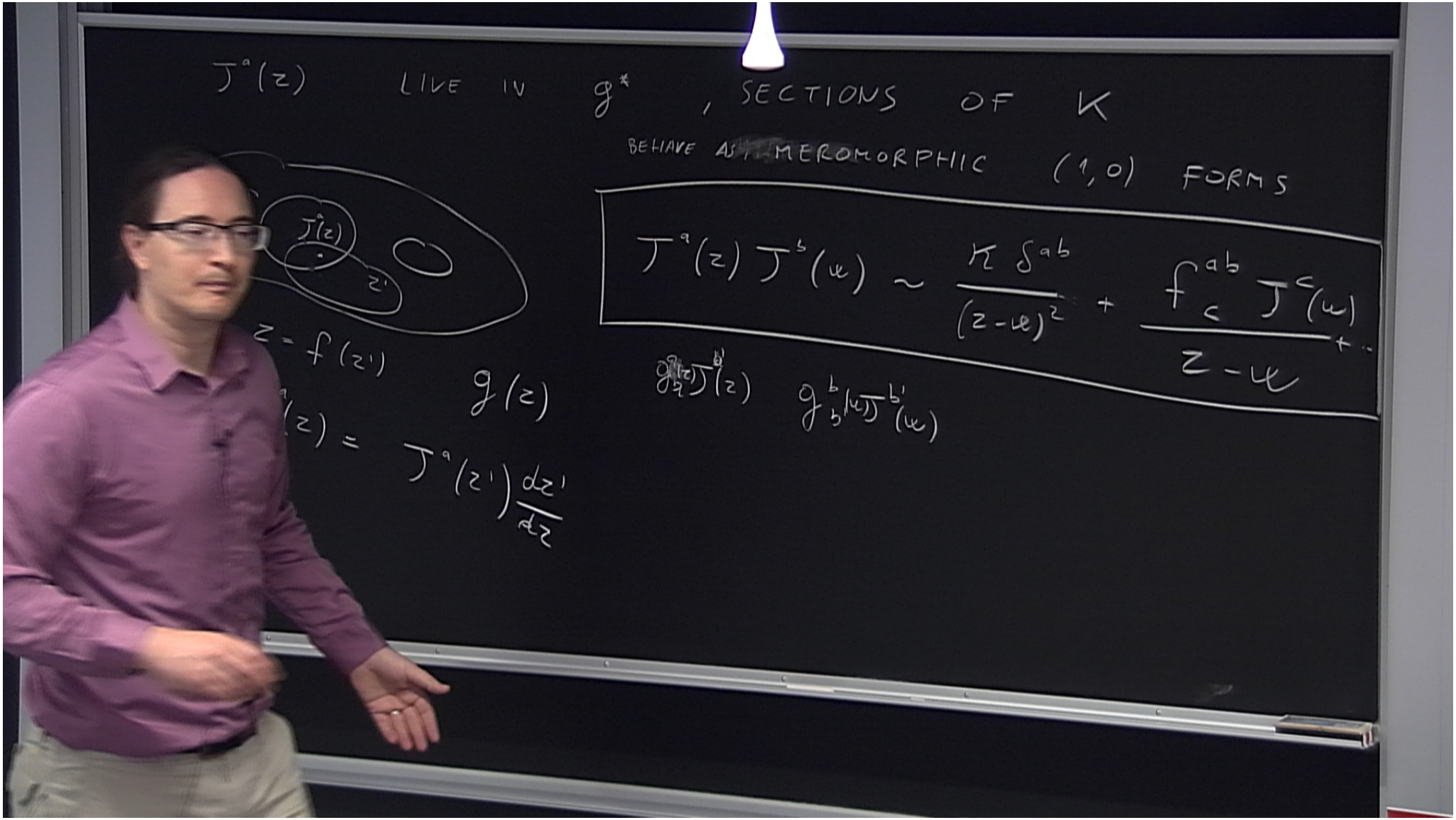


$$J^a(z)\,J^b(w) \sim \frac{\kappa\,\delta^{ab}}{(z-w)^2} + \frac{f_c^{ab}\,J^c(w)}{z-w} + \dots$$

$z = f(z')$ $\quad g(z)$

$$J^a(z) = J^a(z')\frac{dz'}{dz}$$


C , E
BUNDLE
ON
C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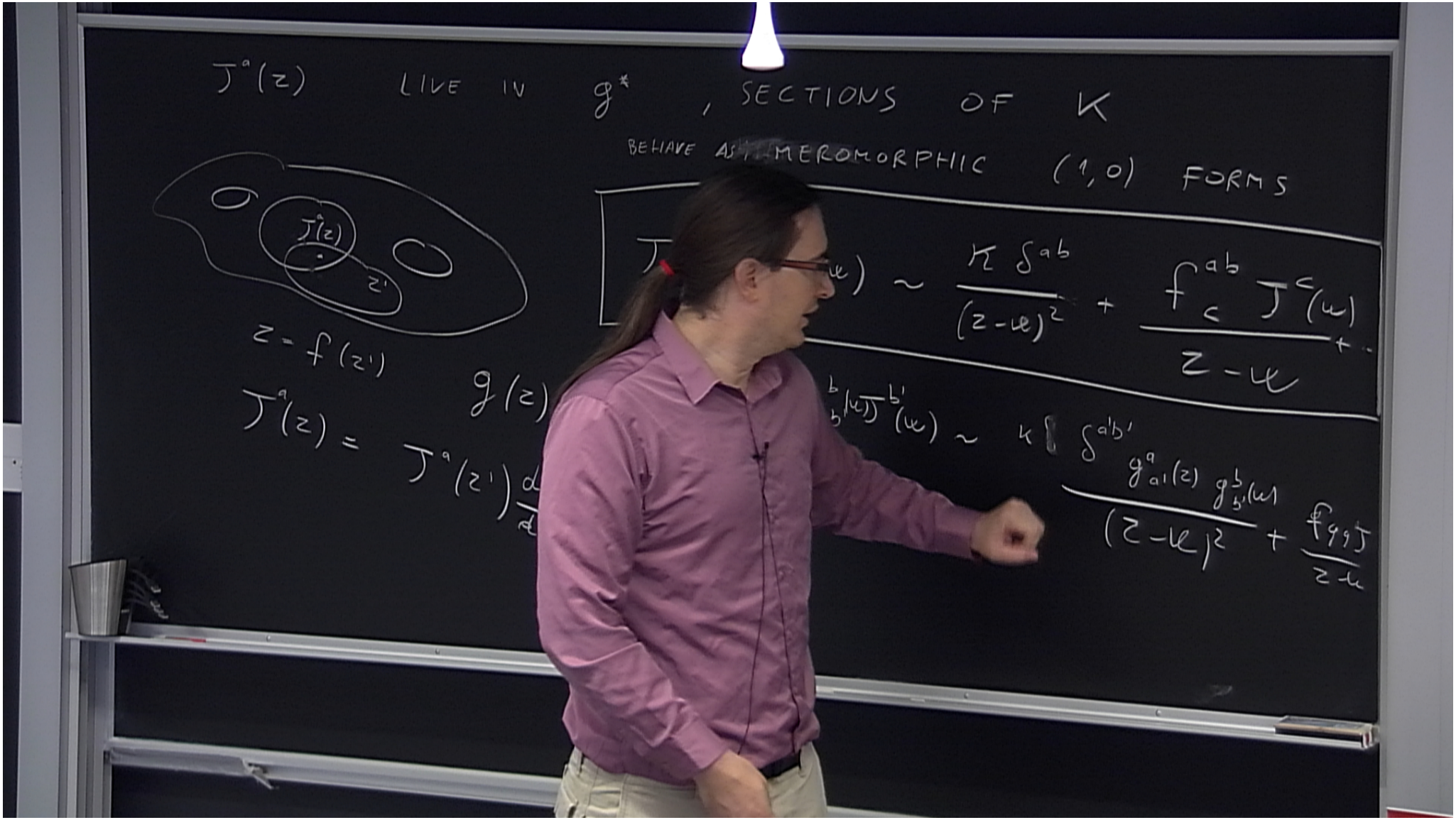
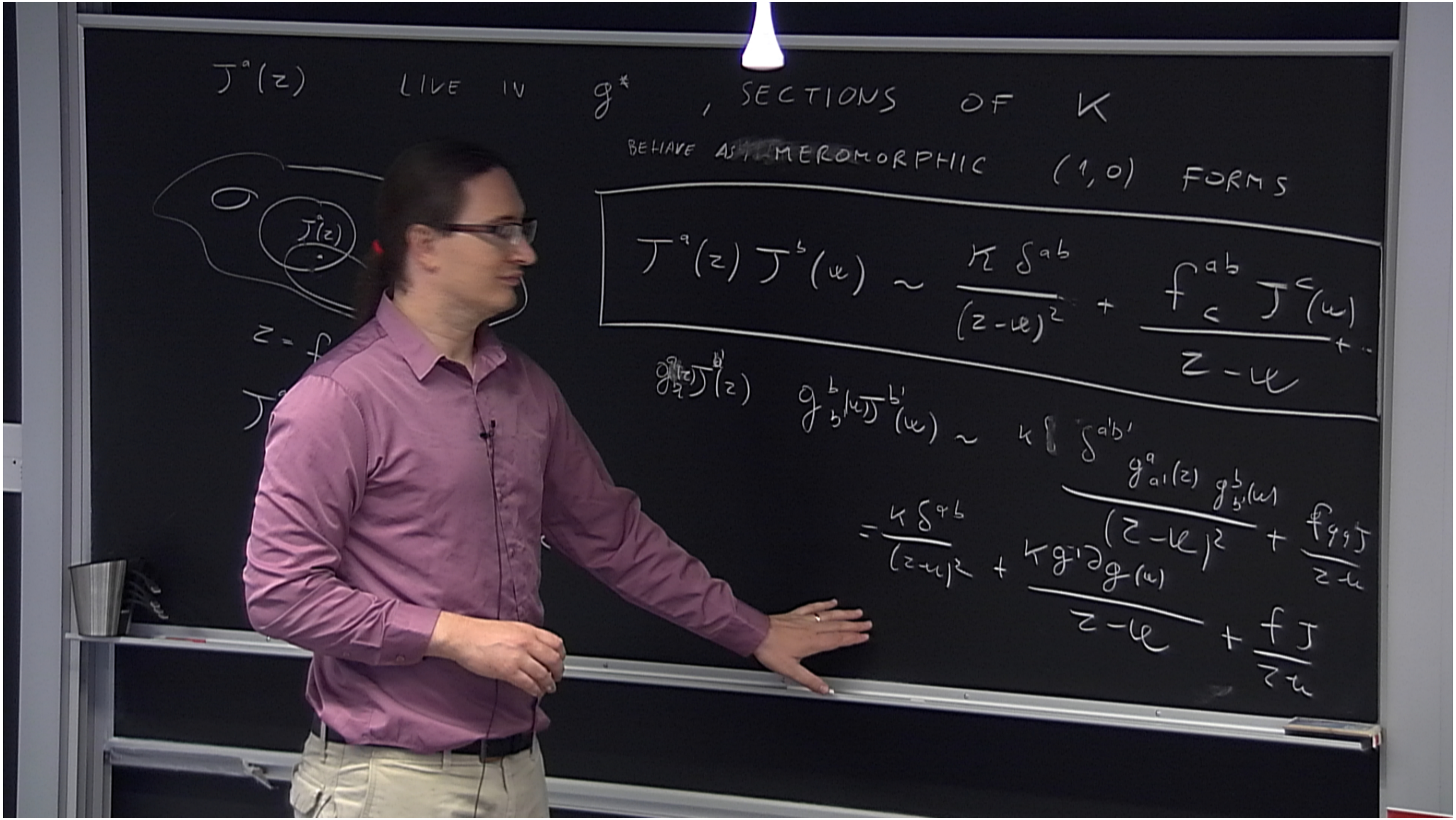
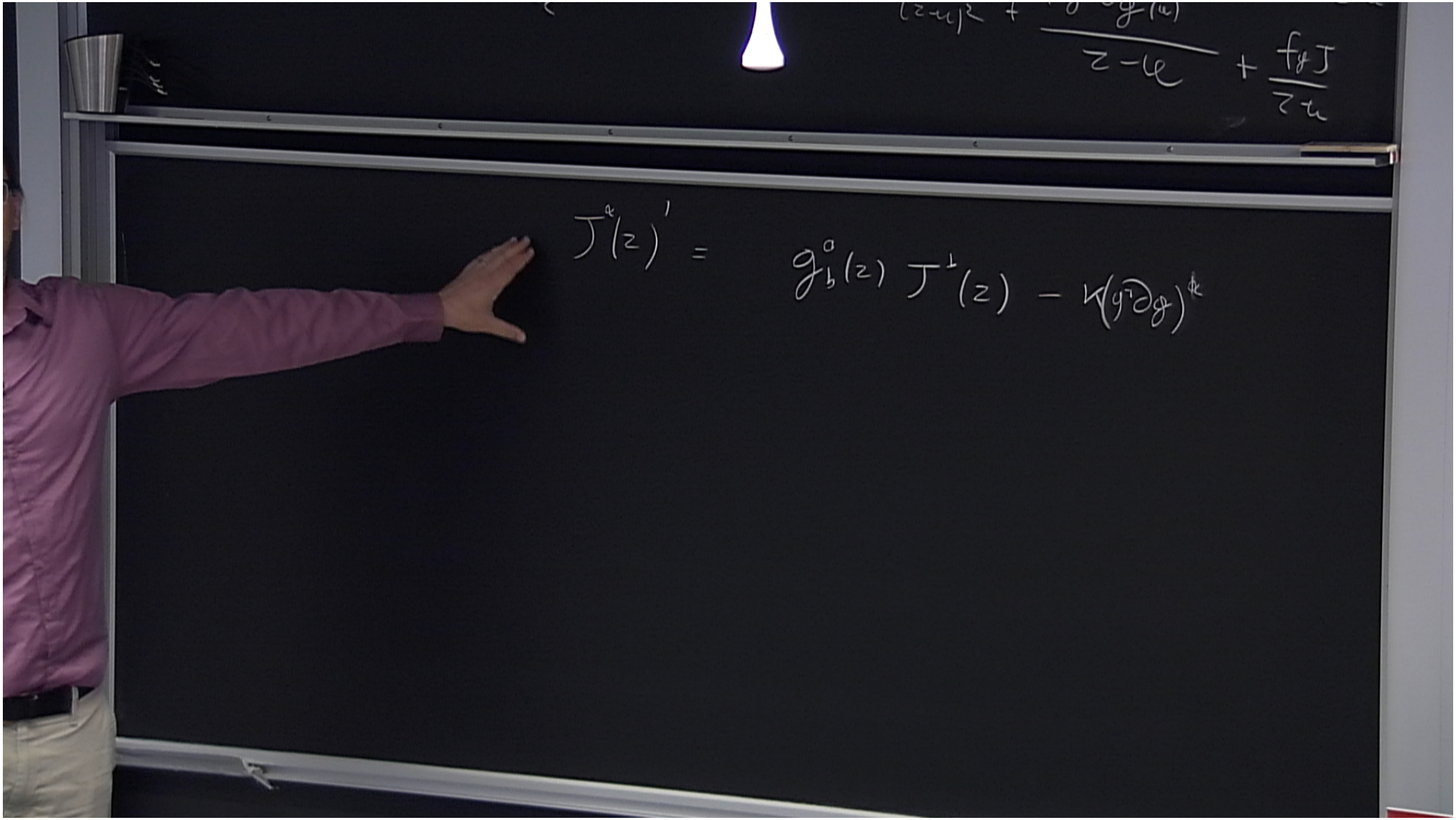
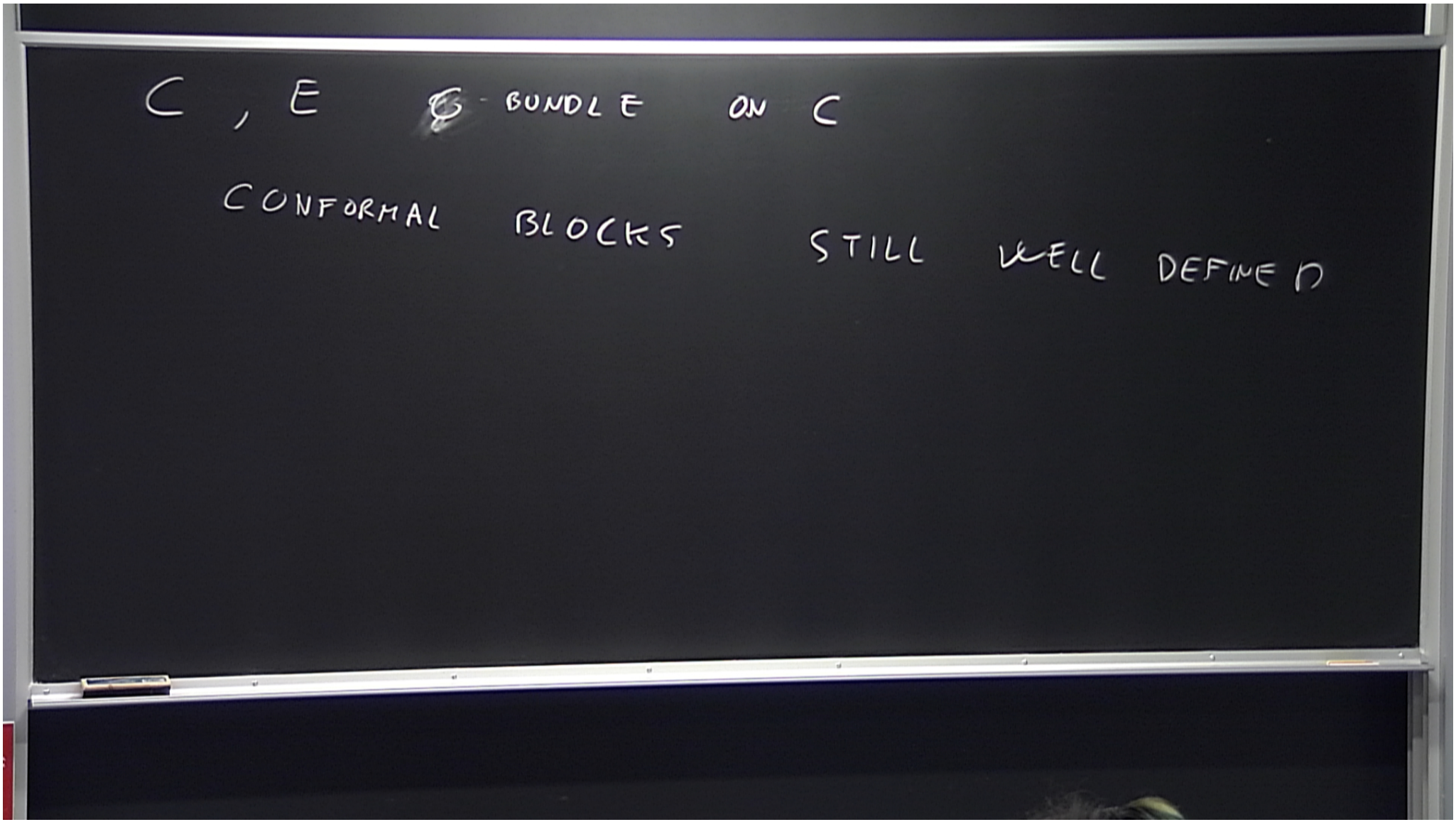
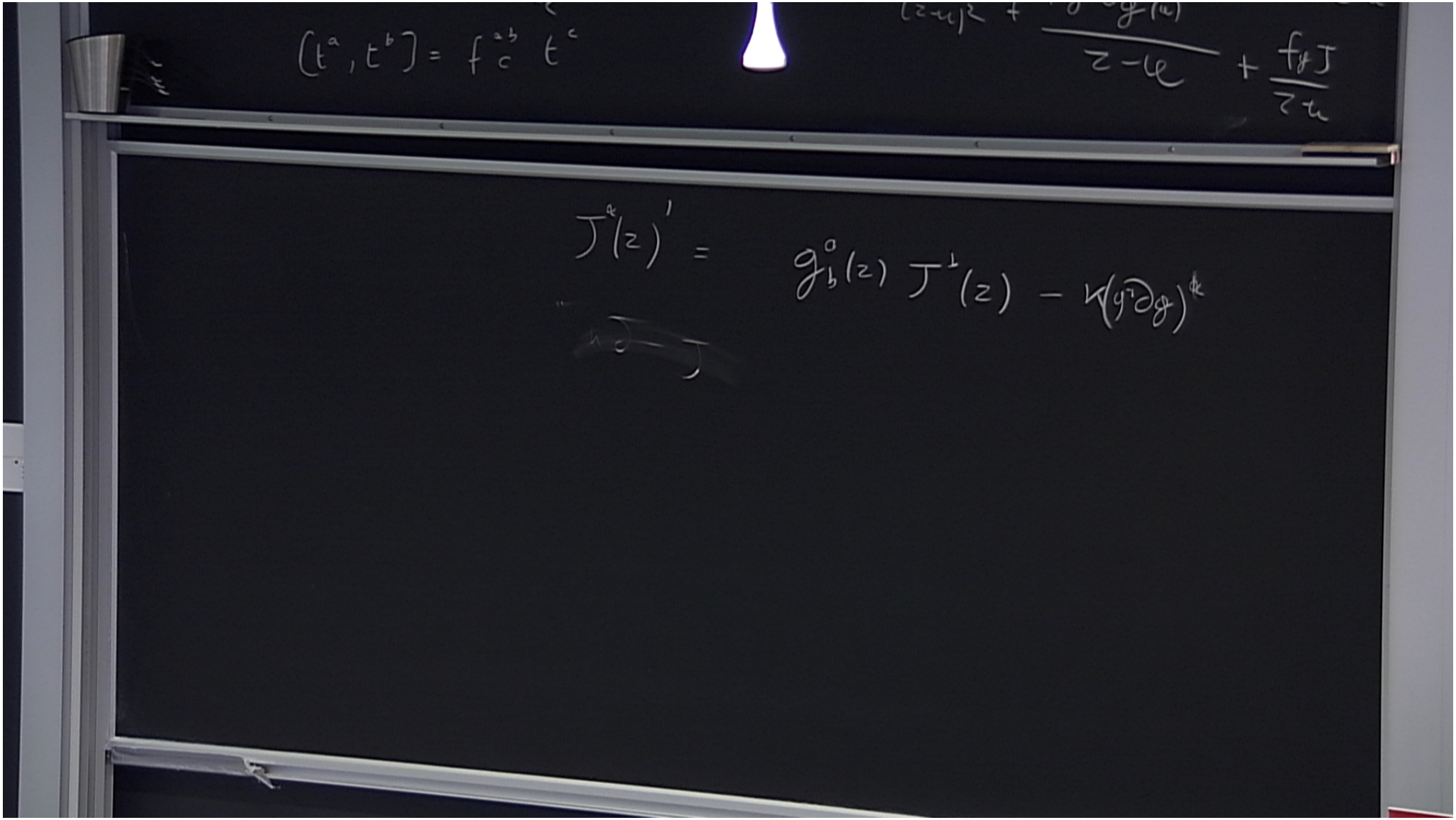
BEHAVE AS MEROMORPHIC (1,0) FORMS
$J^a(z)\,J^b(w) \sim \frac{\kappa\,\delta^{ab}}{(z-w)^2} + \frac{f_c^{ab}\,J^c(w)}{z-w} + \ldots$
$J(z)$
z'
$z = f(z')$
$g(z)$
$J^a(z) = J^{a'}(z')\frac{dz'}{dz}$
$g^{b'}_{b}(z)J^{b'}(z)$
$g^{b}_{b'}(w)J^{b'}(w)$


$J^a(z)$ LIVE IN g^*, SECTIONS OF K
BEHAVE AS MEROMORPHIC (1,0) FORMS
$J(z)$
z'
$z - f(z')$
$J^a(z) = J^a(z')$
$g(z)$
$\sim \frac{\kappa \delta^{ab}}{(z-w)^2} + \frac{f_c^{ab} J^c(w)}{z-w} + \ldots$
$\sim \kappa \left\{ \frac{\delta^{ab'} g^a_{a'}(z) g^b_{b'}(w)}{(z-w)^2} + \frac{f_{ab'}}{z-w} \right.$


$J^a(z)$ LIVE IN g^*, SECTIONS OF K
BEHAVE AS MEROMORPHIC (1,0) FORMS
$J^a(z)\,J^b(w) \sim \frac{\kappa\,\delta^{ab}}{(z-w)^2} + \frac{f_c^{ab}\,J^c(w)}{z-w} + \dots$
$g^a_{a'}(z)J^{a'}(z)$ $g^b_{b'}(w)J^{b'}(w) \sim \kappa\,\delta^{ab'}\frac{g^a_{a'}(z)\,g^b_{b'}(w)}{(z-w)^2} + \frac{f_{ab}J}{z-w}$
$= \frac{\kappa\,\delta^{ab}}{(z-w)^2} + \frac{\kappa g^{-1}\partial g(w)}{z-w} + \frac{fJ}{z-w}$

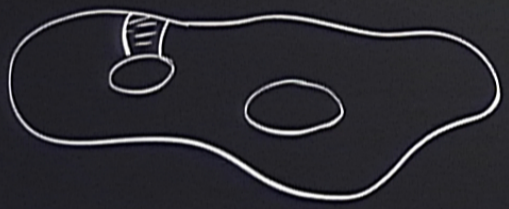

$\frac{f_{ab}J}{z-w}$
$J^a(z)' = g^0_{ab}(z)\, J^b(z) - K(g^{-1}\partial g)^a$


C , E G - BUNDLE ON C
CONFORMAL BLOCKS STILL WELL DEFINED


$[t^a, t^b] = f^{ab}_c\, t^c$
$J^a(z)' = g^a_b(z)\, J^b(z) - k(g^{-1}\partial g)^a$

C , E - BUNDLE ON C

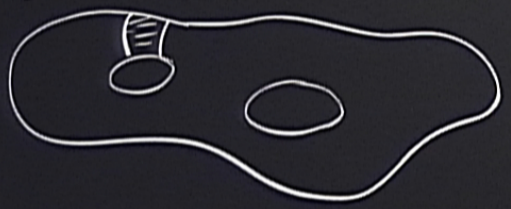
CONFORMAL BLOCKS STILL WELL DEFINED



C , E $\mathcal{E}$ - BUNDLE ON C

CONFORMAL BLOCKS STILL WELL DEFINED

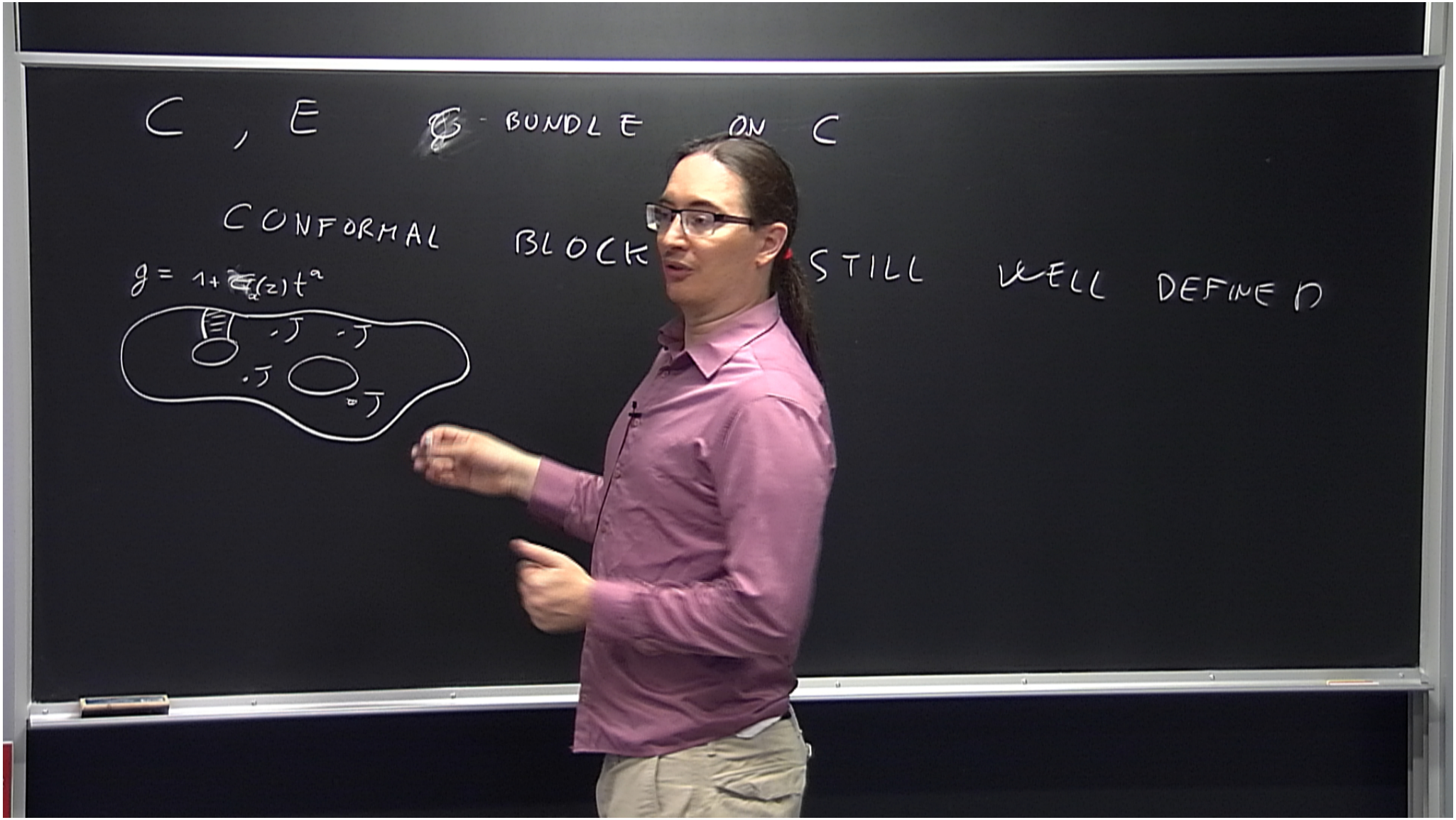
$g = 1 + \mathcal{E}(z)$



C , E $\mathfrak{g}$ - BUNDLE ON C

CONFORMAL BLOCKS STILL WELL DEFINED

$g = 1 + \epsilon_a(z) t^a$


C , E
BUNDLE ON C
CONFORMAL BLOCK
STILL WELL DEFINED
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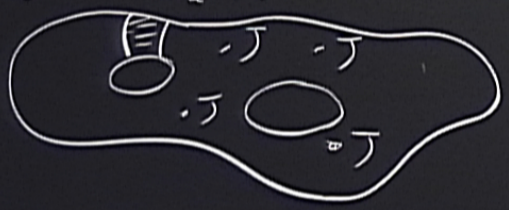
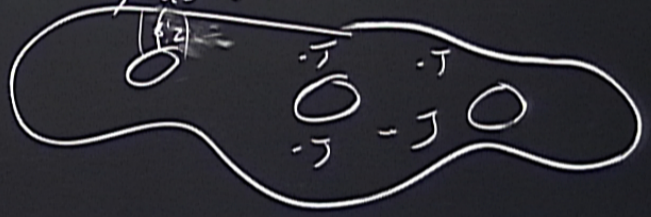
C , E $\mathfrak{g}$ - BUNDLE ON C

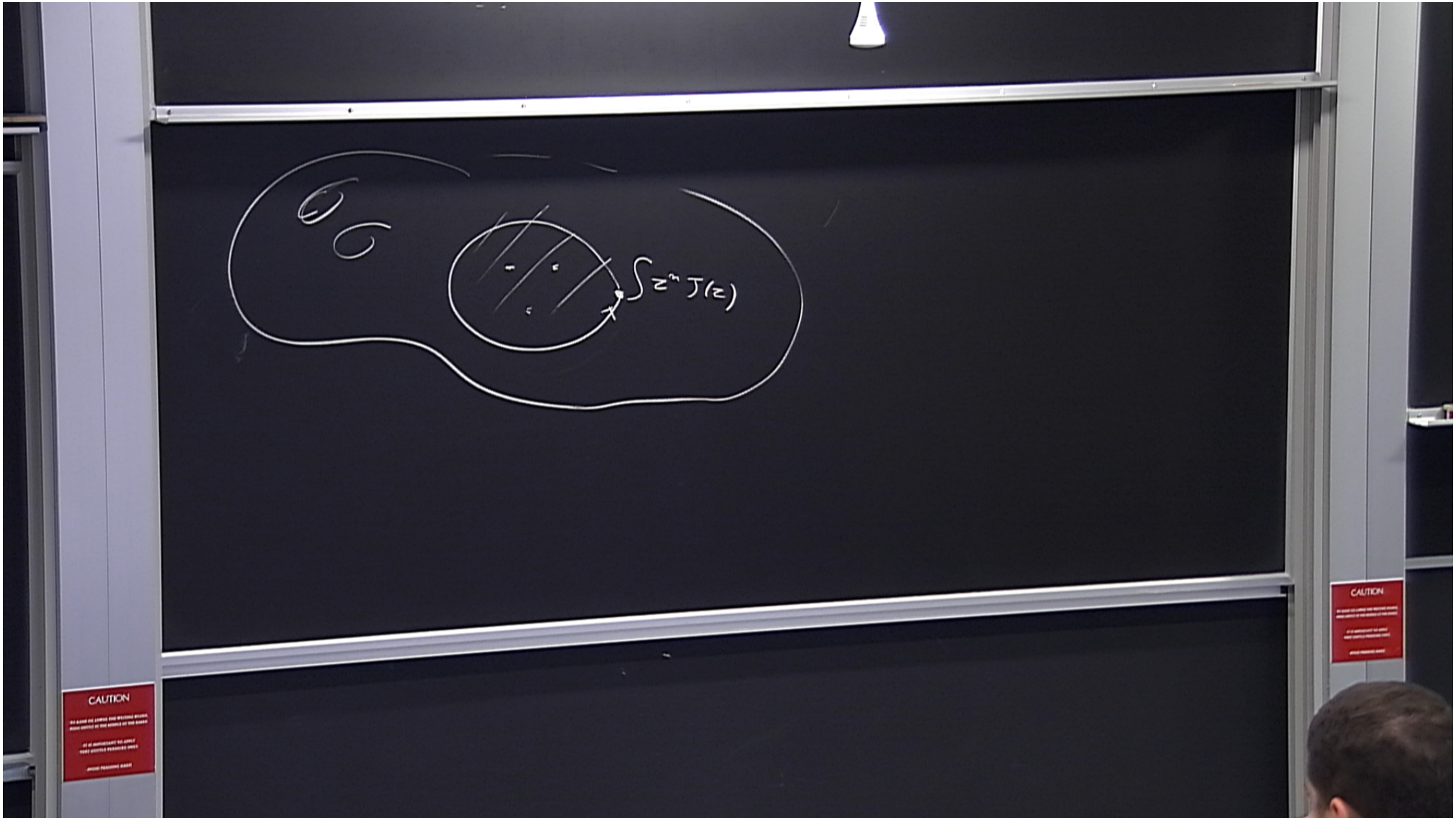
CONFORMAL BLOCKS STILL WELL DEFINED

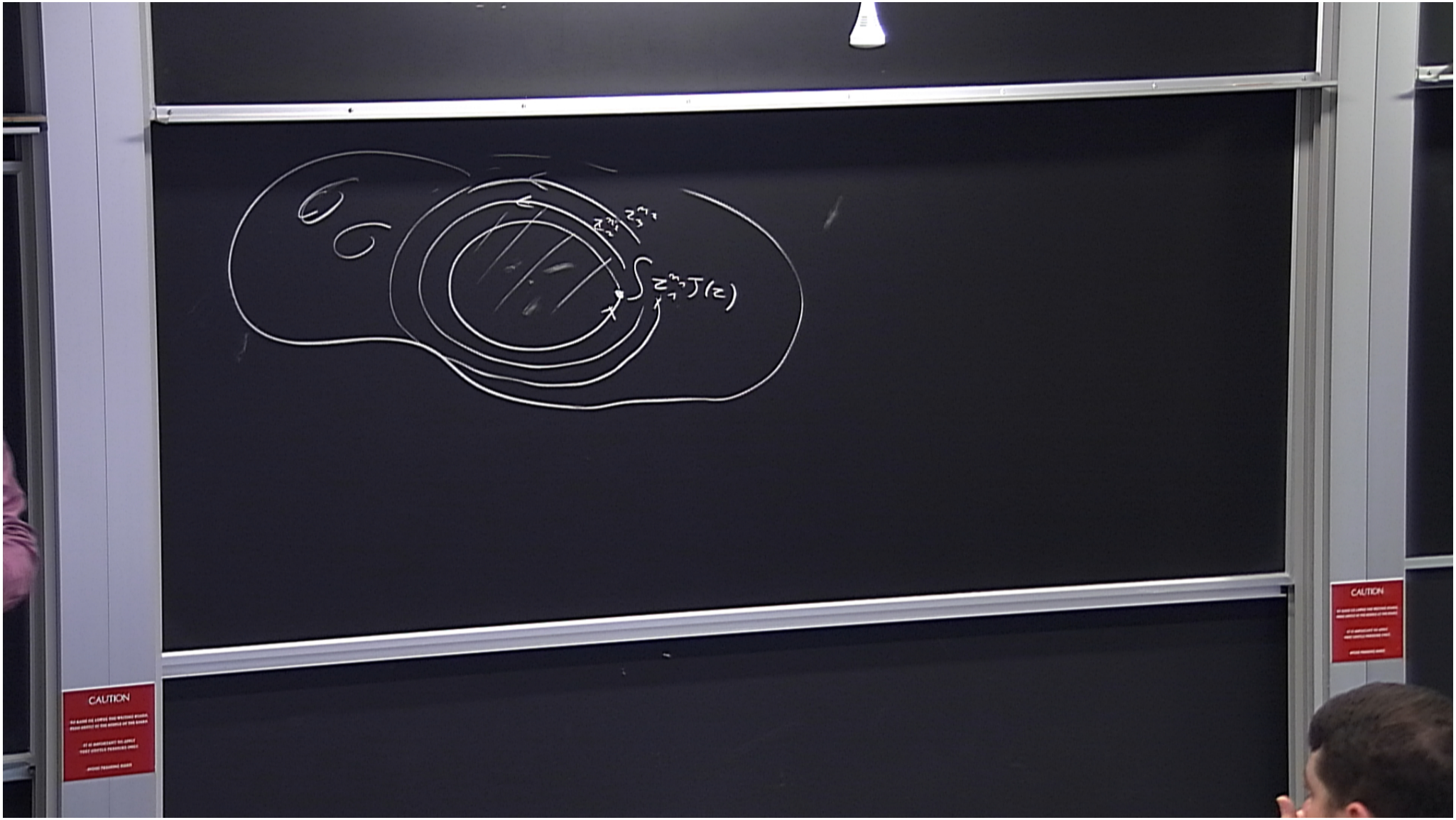
$g = 1 + \epsilon_a(z) t^a$

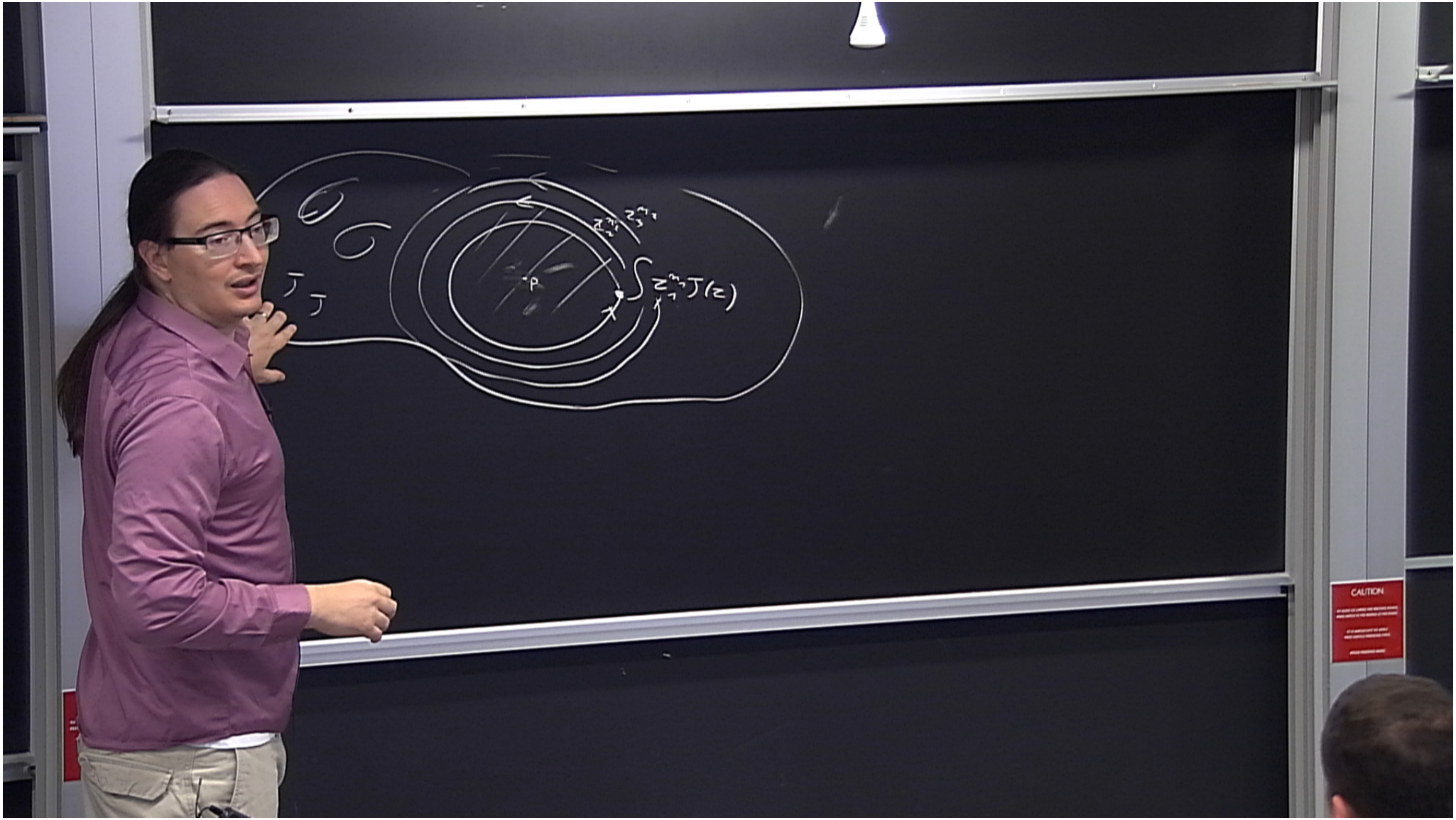


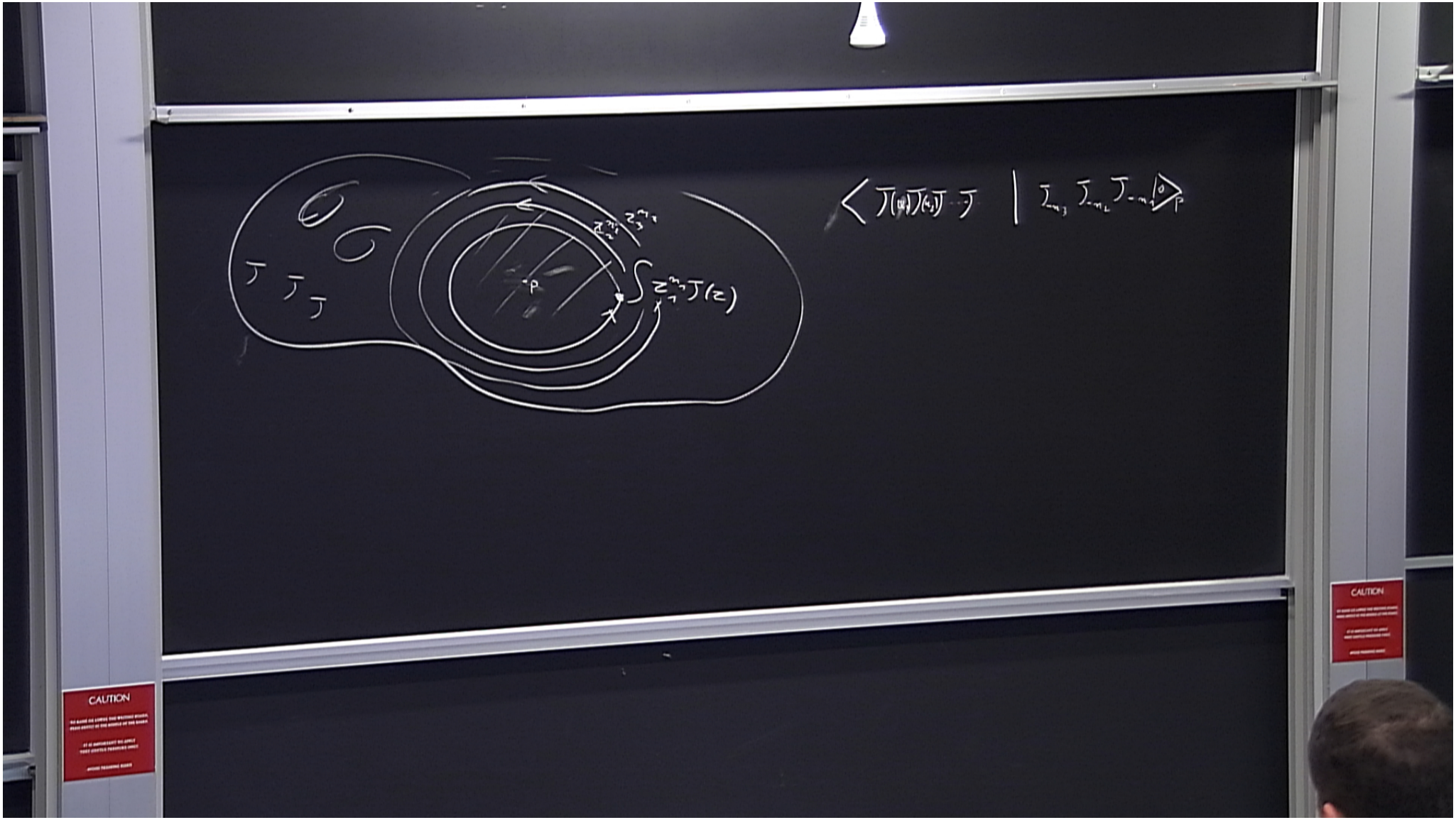
$\int \epsilon_a J^a(z)\, dz$

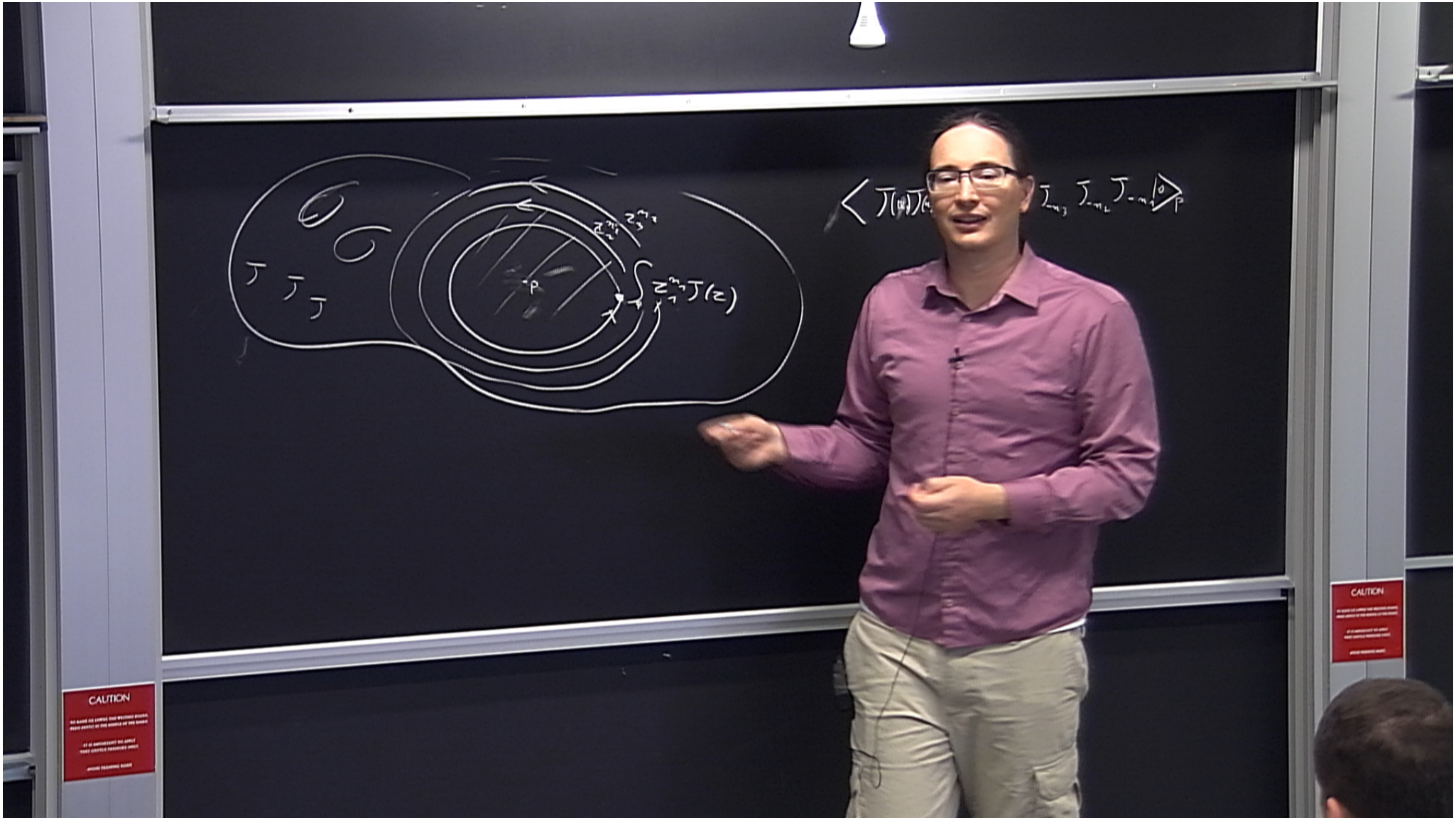


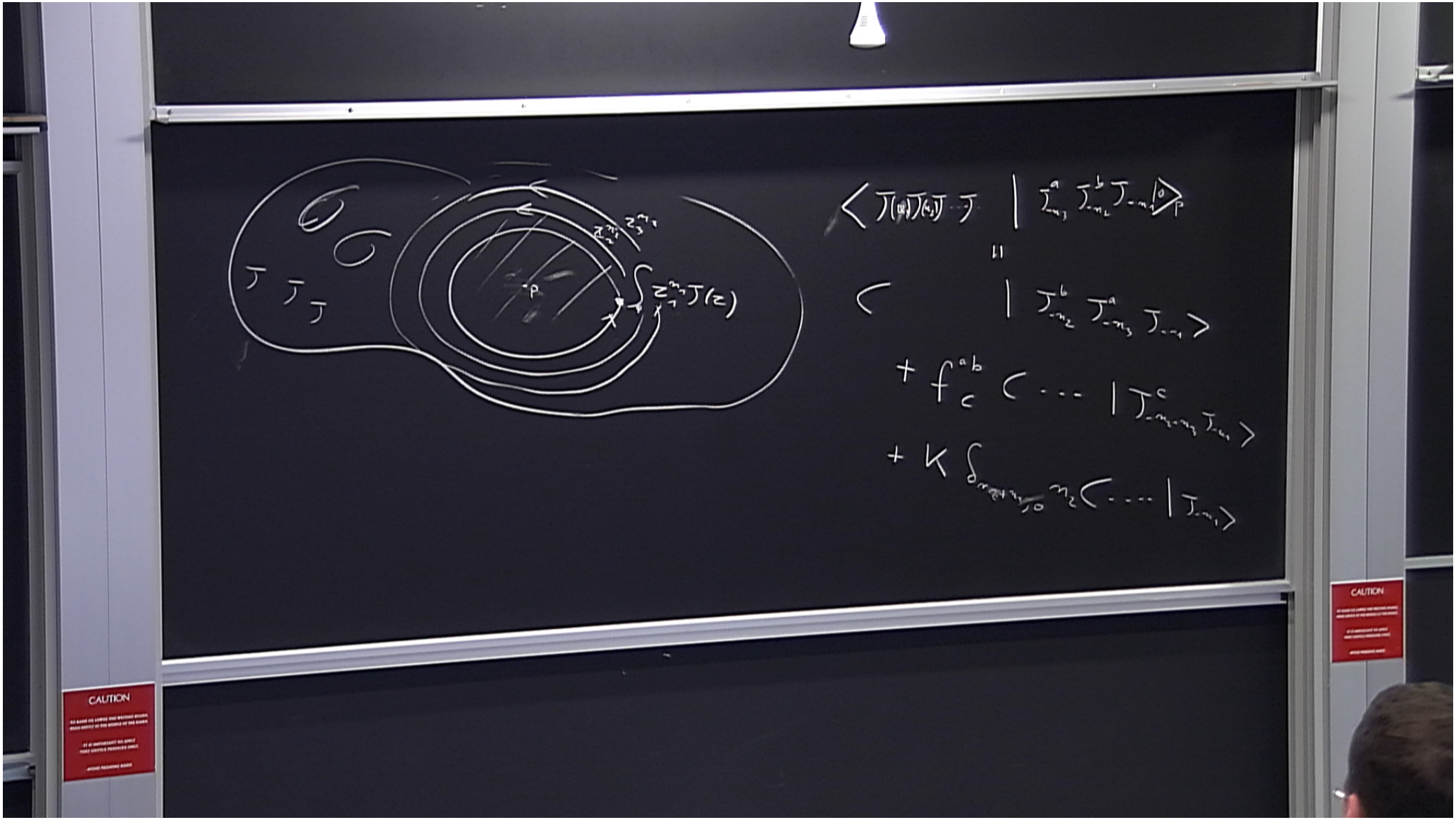

$\int z^n J(z)$
CAUTION
CAUTION


$z^3 J(z)$
CAUTION
CAUTION


P
CAUTION


P
CAUTION
CAUTION

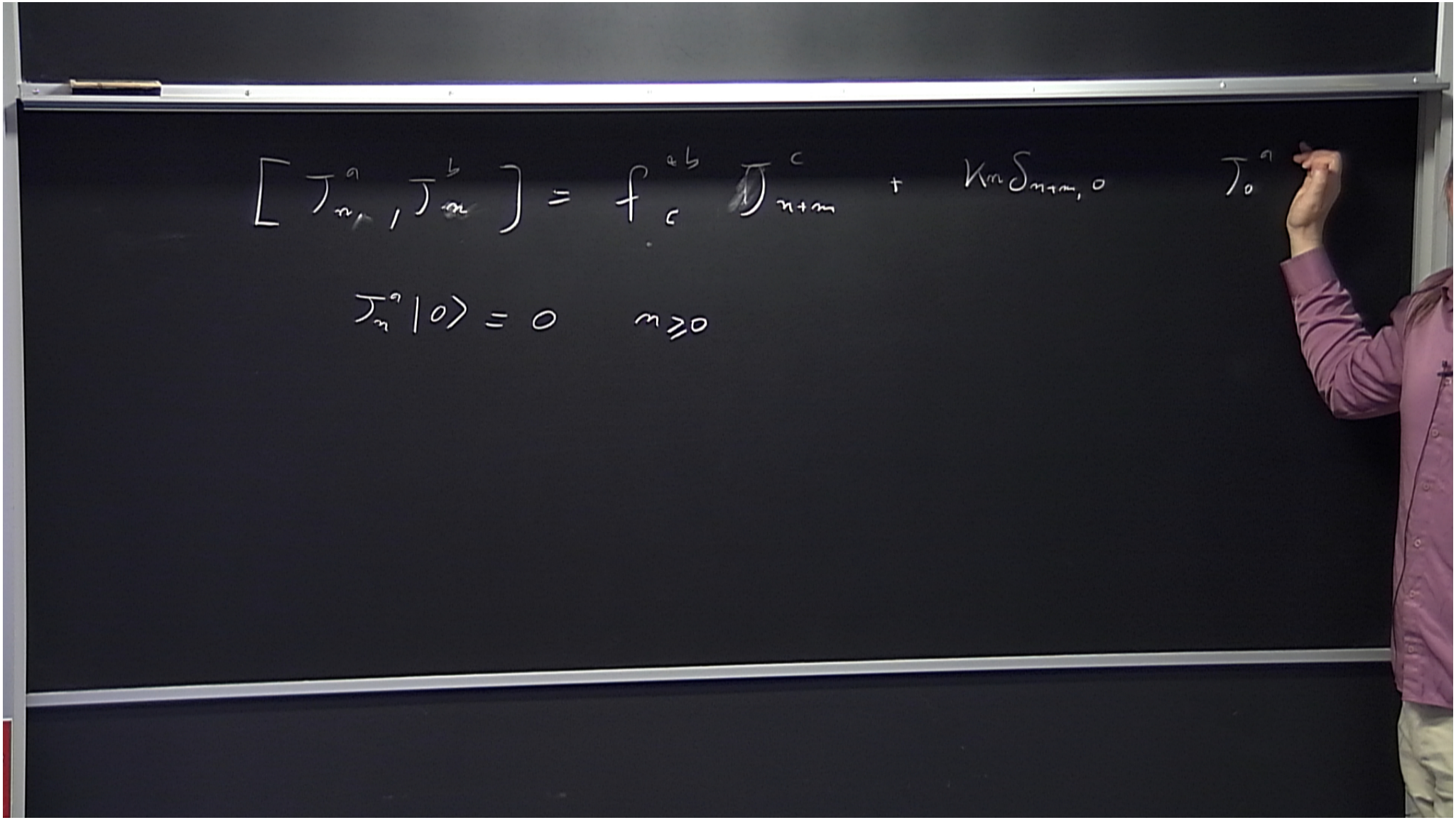

CAUTION
CAUTION


CAUTION
CAUTION

$$[J^a_n, J^b_m] = f^{ab}_c J^c_{n+m} + k n \delta_{n+m,0}$$

$$[J^a_m, J^b_n] = f^{ab}_c J^c_{m+n} + k m \delta_{m+n,0}$$

$$J^a_n |0\rangle = 0 \qquad n \geq 0$$


$[J^a_m, J^b_n] = f^{ab}_c J^c_{m+n} + k m \delta_{m+n,0}$
$J^a_n |0\rangle = 0 \quad n \geq 0$
J^a_0

$$[J^a_m, J^b_n] = f^{ab}_c J^c_{m+n} + k m \delta_{m+n,0} \quad J^a_0$$

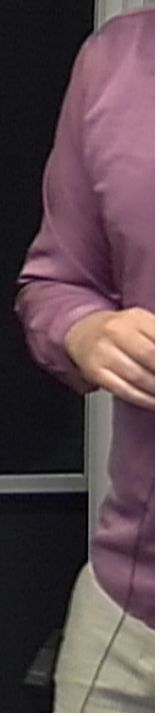
$J^a_n |0\rangle = 0 \quad n \geq 0$ VACUUM MODULE V_0

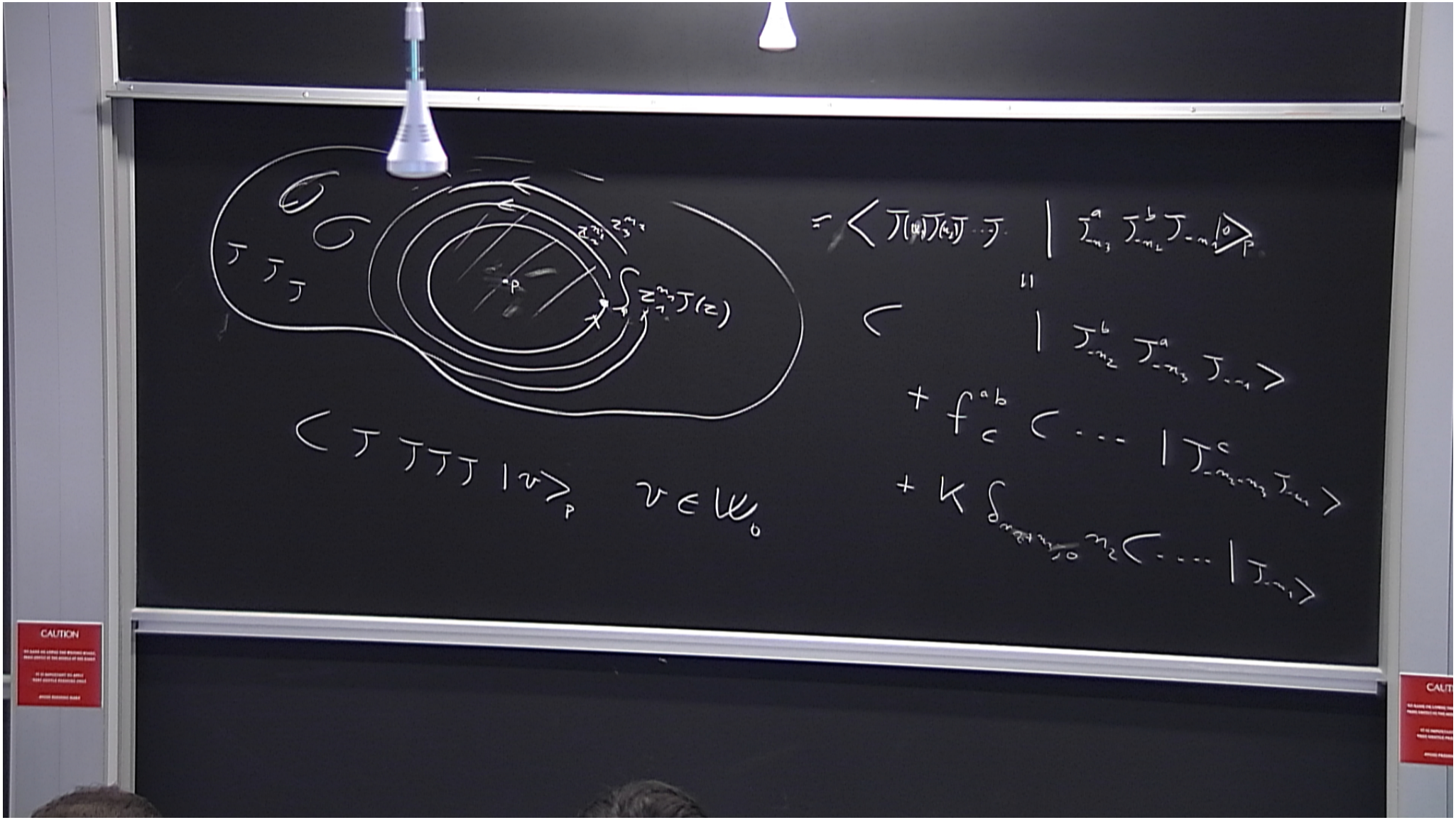
$|0\rangle$

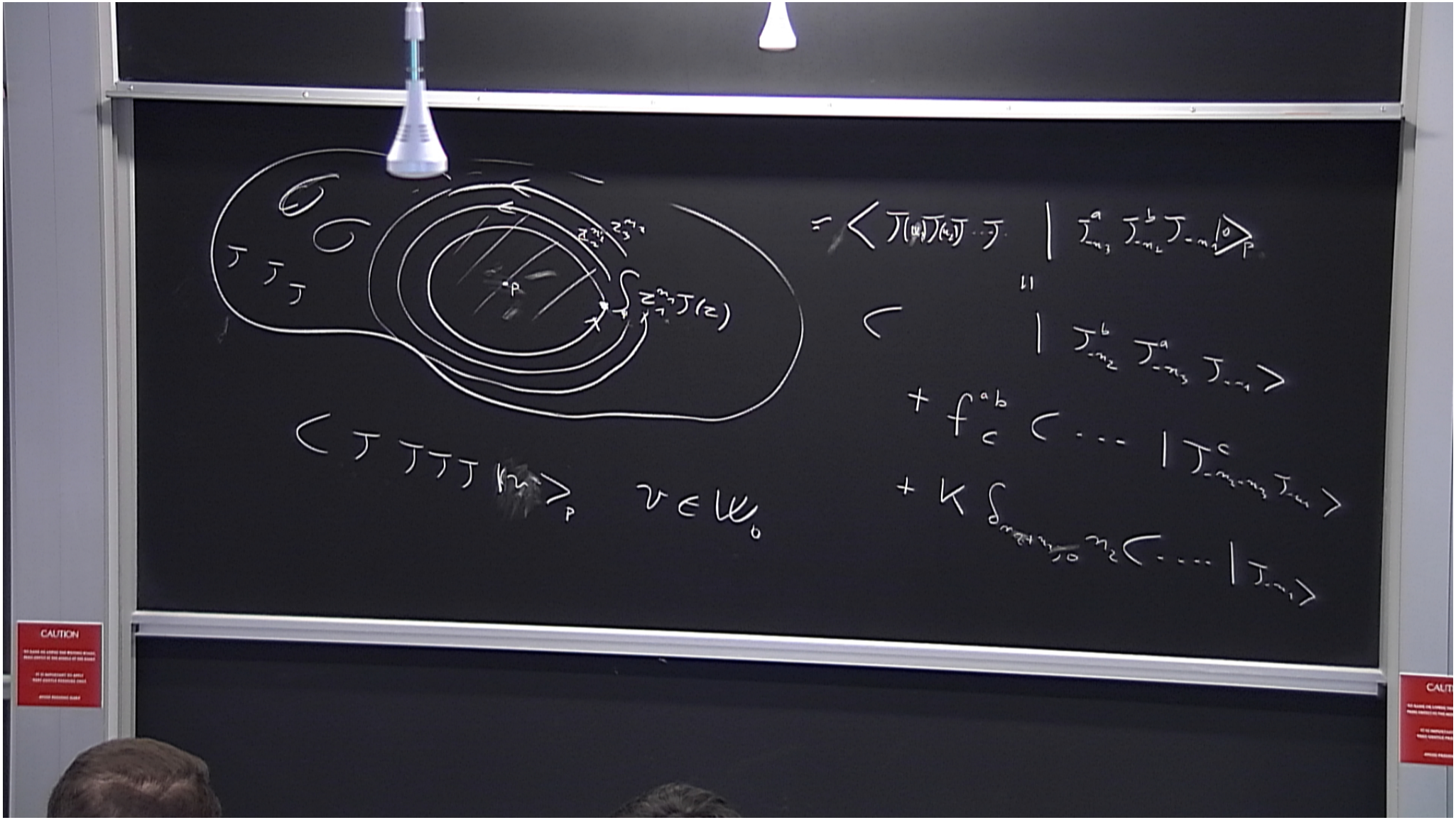
$J^a_{-1}|0\rangle$

$J^a_{-1} J^b_{-1}|0\rangle$

$J^a_{-2}|0\rangle$




P
CAUTION


P
$\int z^{m_1} J(z)$
$\langle J J J \dots |_P$
$v \in W_0$
$= \langle J(z_1) J(z_2) J \dots | J^a_{-m_3} J^b_{-m_2} J_{-m_1} |0\rangle_P$
$= \langle \dots | J^b_{-m_2} J^a_{-m_3} J_{-n} \rangle$
$+ f^{ab}_c \langle \dots | J^c_{-m_2-m_3} J_{-n} \rangle$
$+ K \delta_{m_2+m_3,0} m_2 \langle \dots | J_{-n} \rangle$

$$[J^a_m, J^b_n] = f^{ab}_c J^c_{m+n} + k m \delta_{m+n,0} \quad J^a_0$$

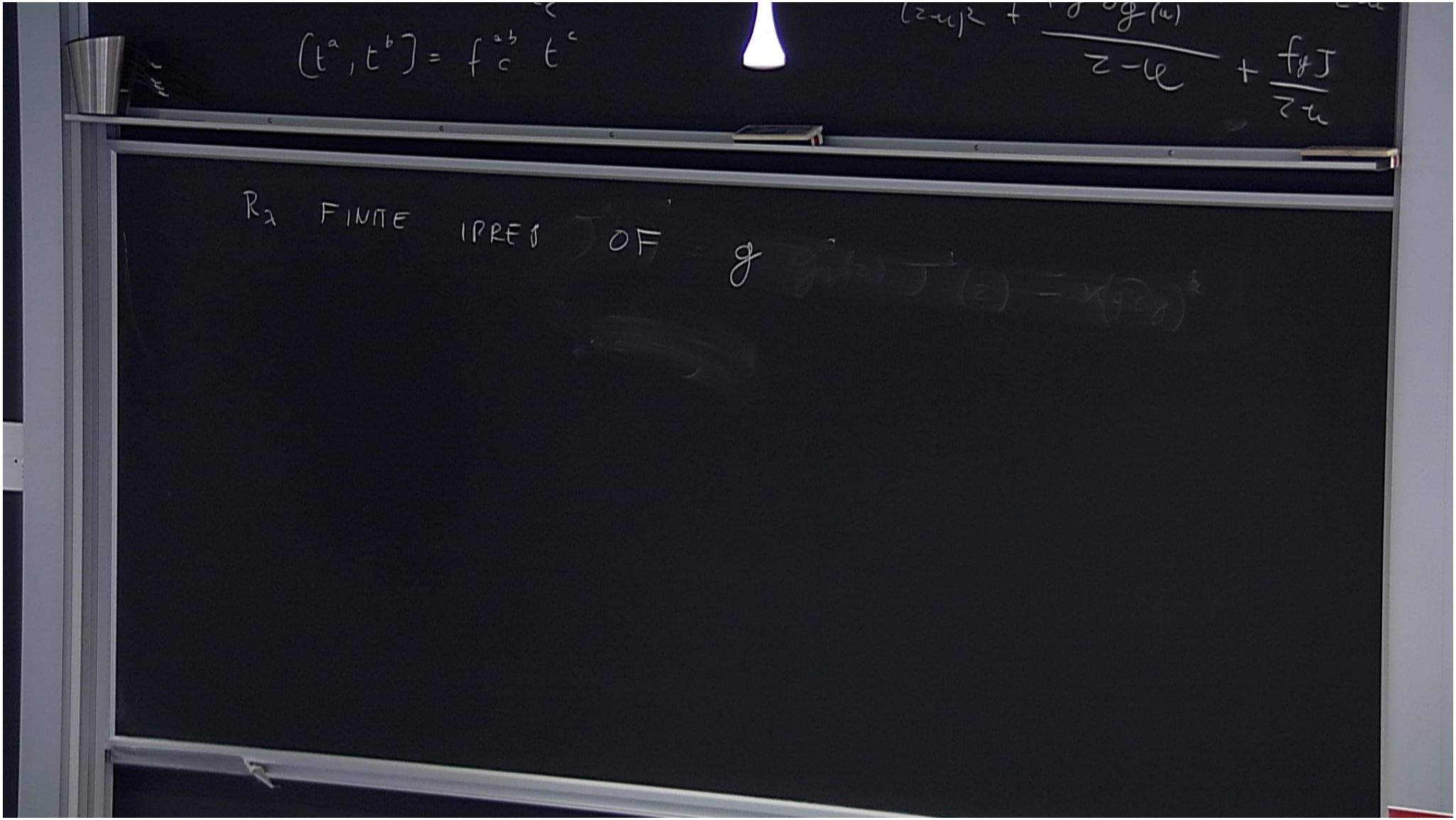
$J^a_n |0\rangle = 0 \quad n \geq 0$ VACUUM MODULE V_0

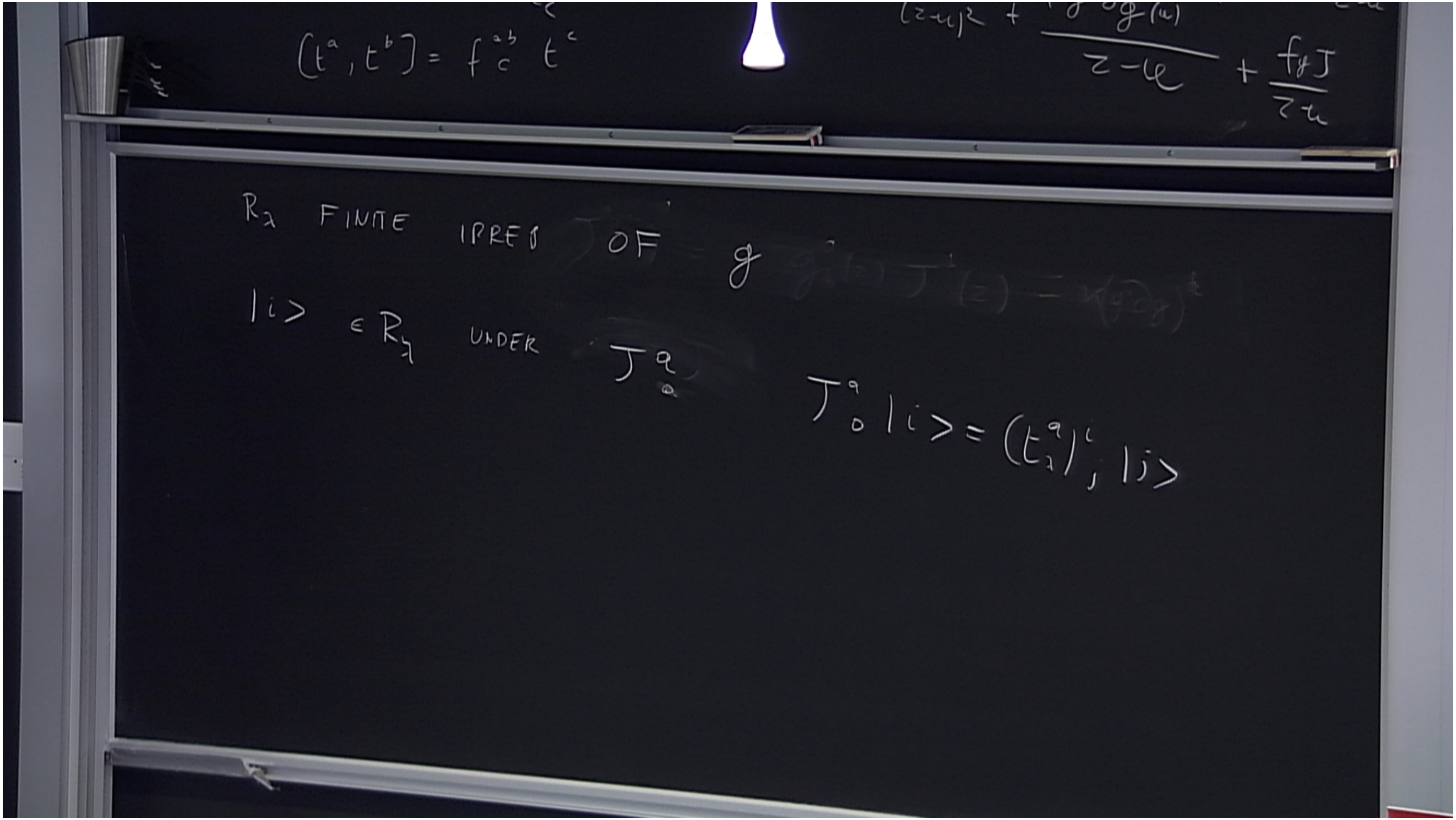
$|0\rangle$

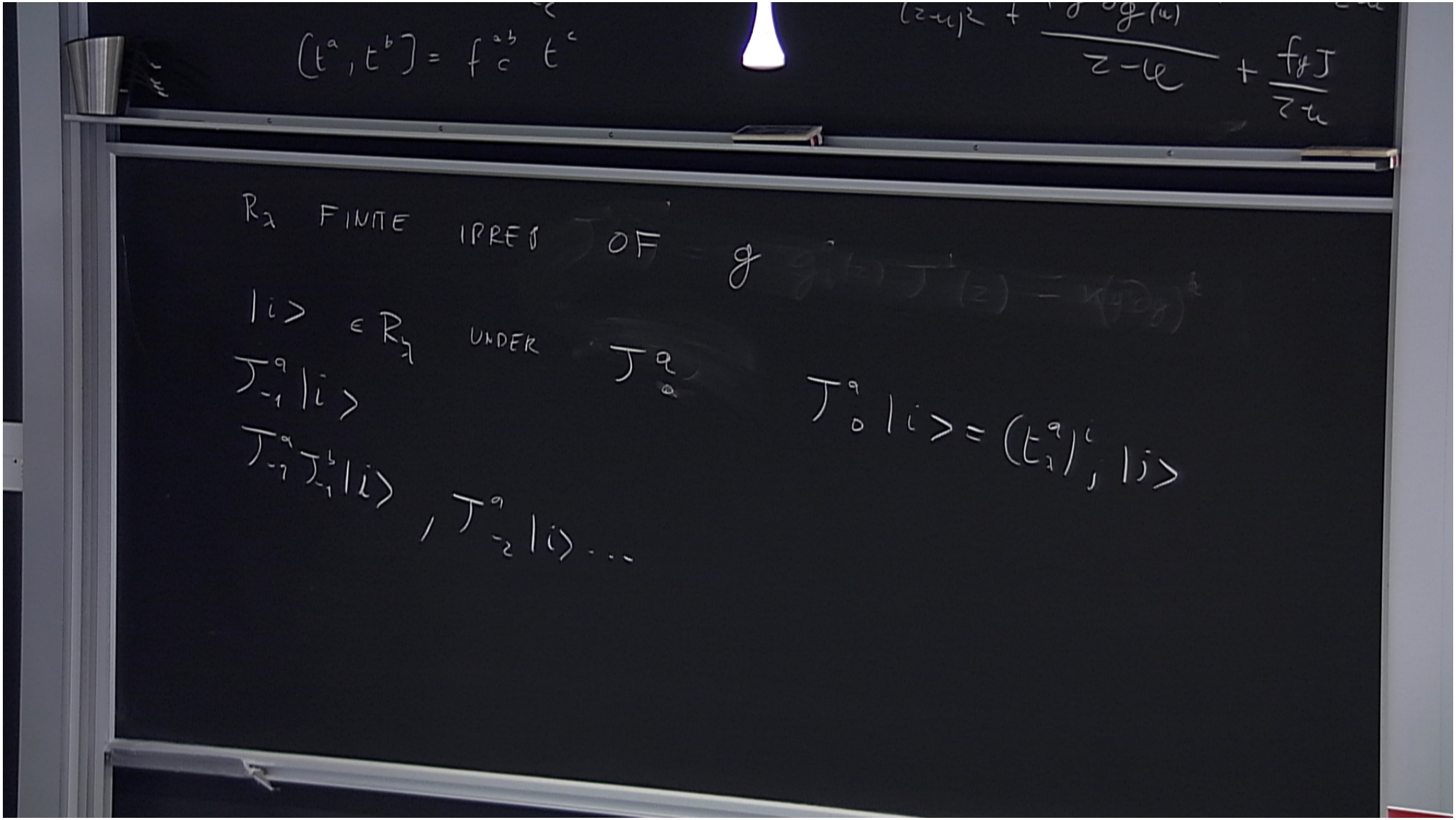
$J^a_{-1}|0\rangle$

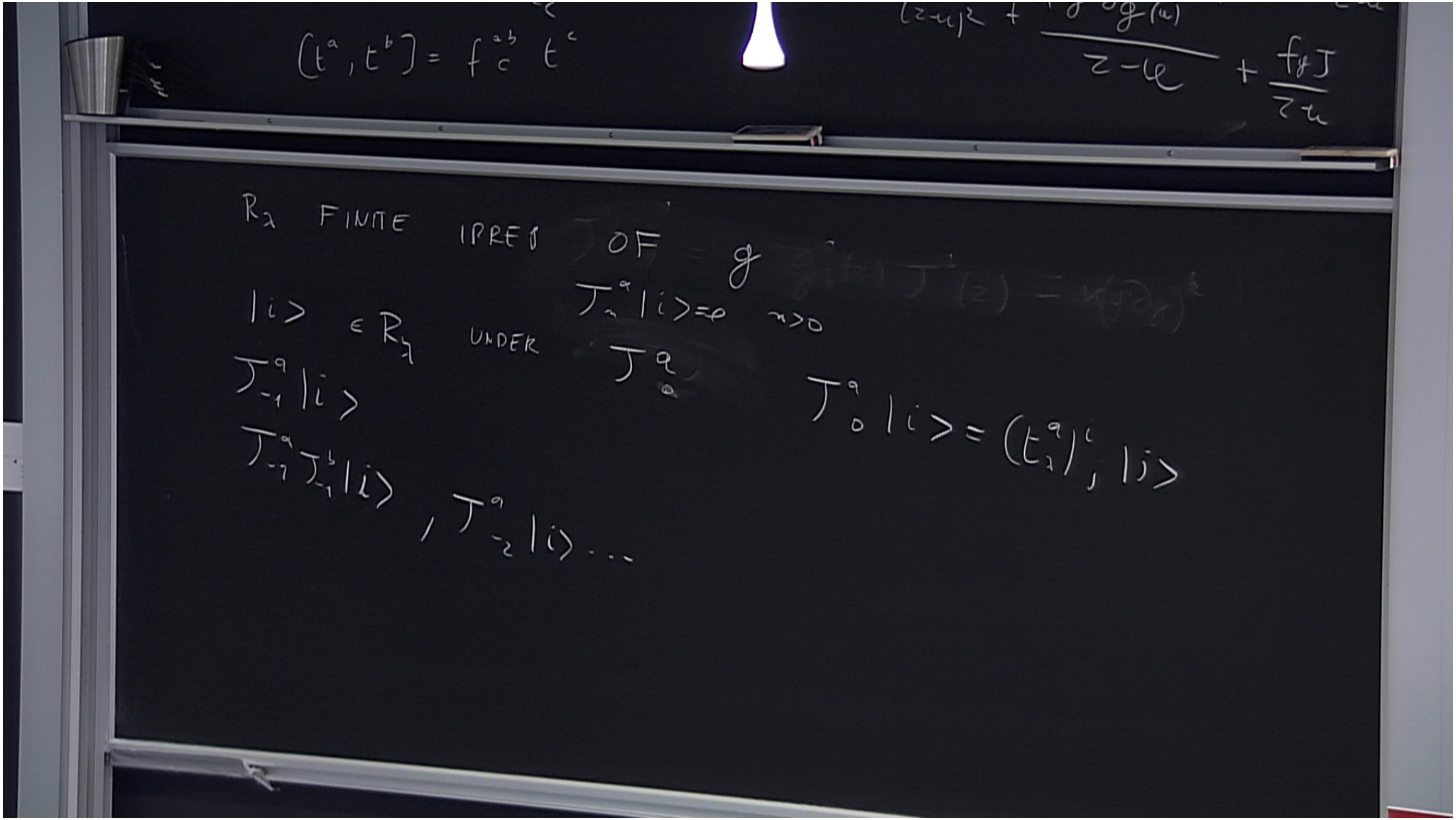
$J^a_{-1} J^b_{-1}|0\rangle$

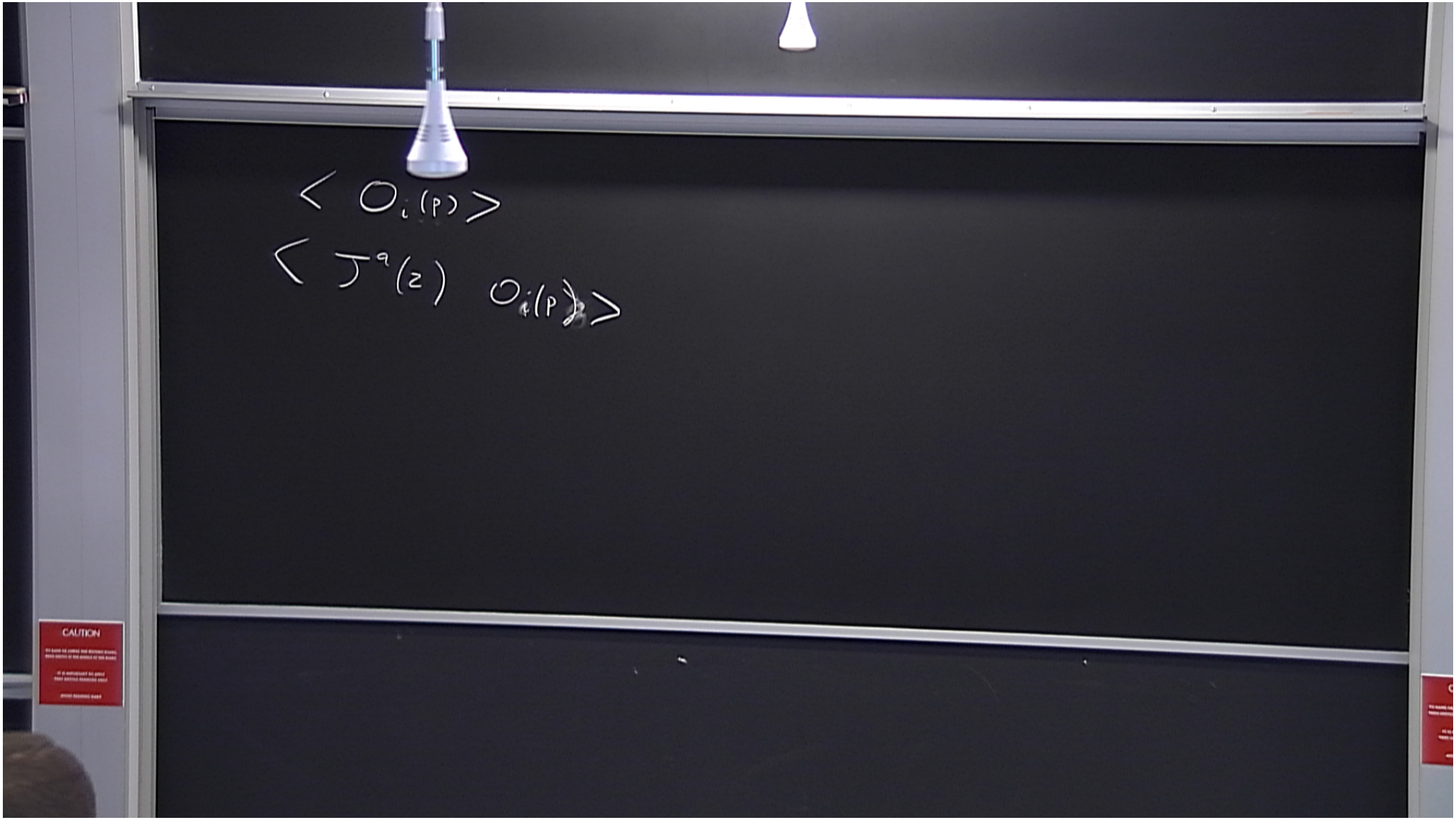
$J^a_{-2}|0\rangle$

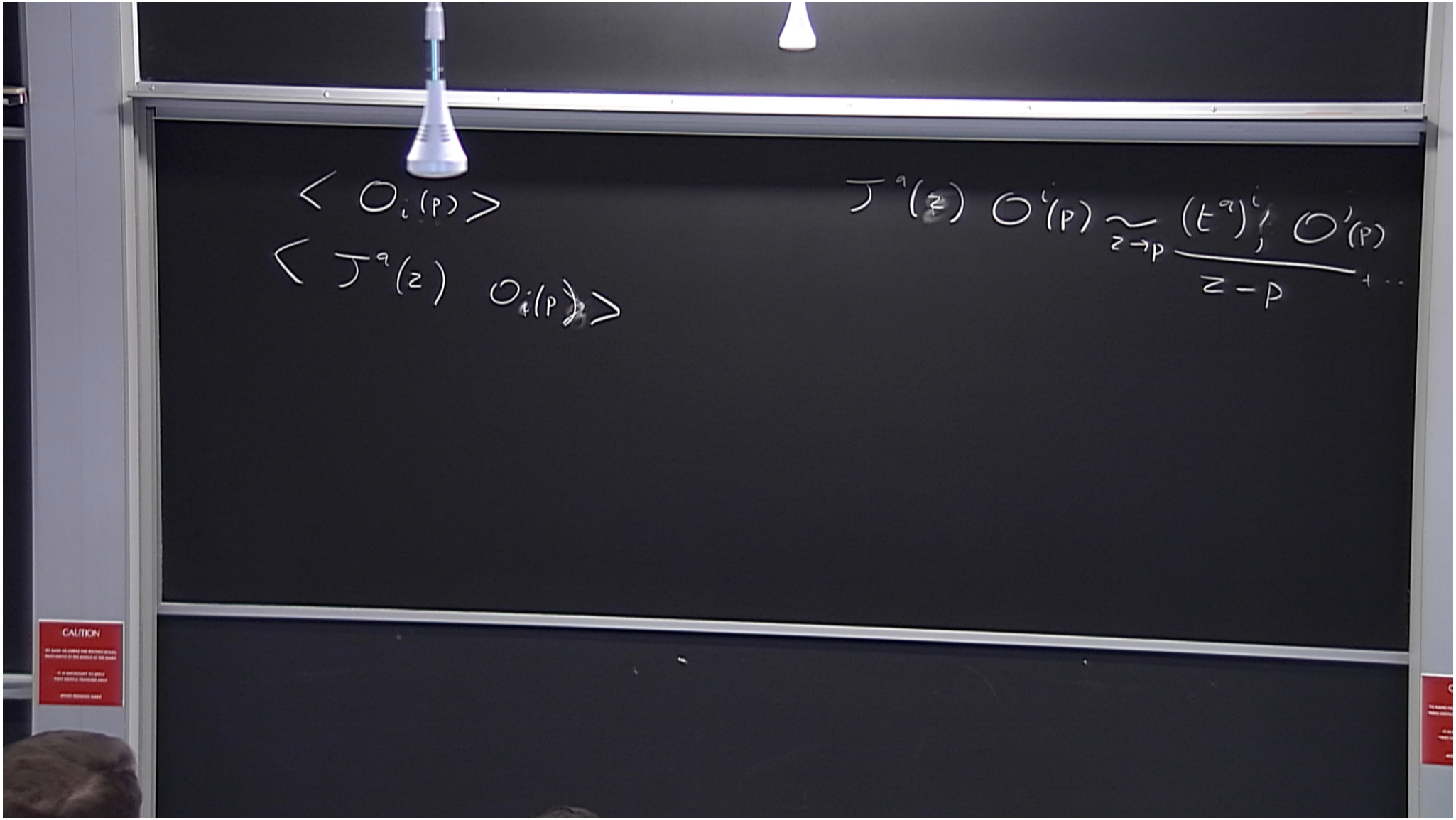

$[t^a, t^b] = f_c^{ab} t^c$
$\frac{f_{ab} J}{z-w}$
R_2 FINITE IPRES OF g

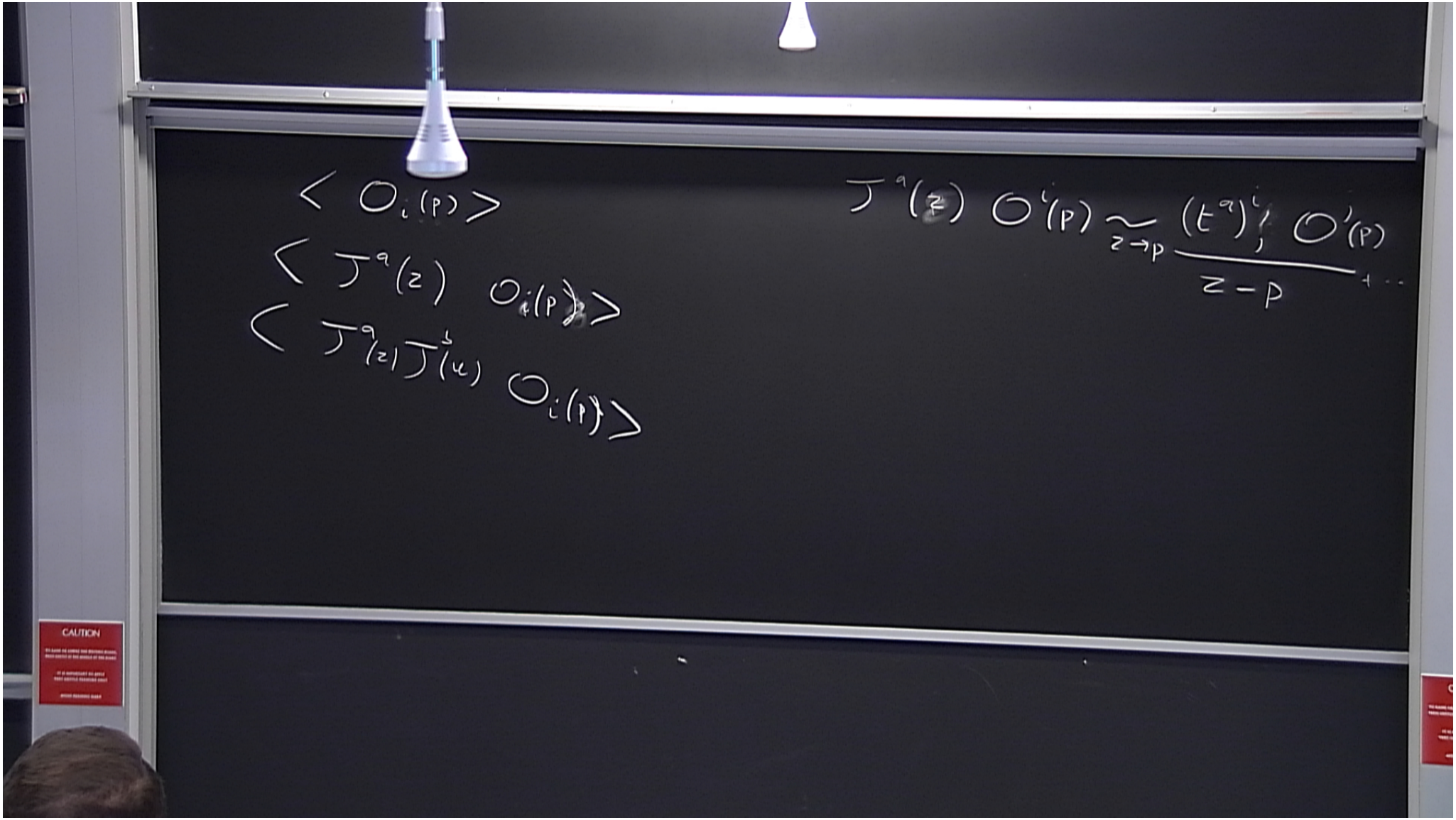

$[t^a, t^b] = f^{ab}_c t^c$
$\frac{f_{ab} J}{z-w}$
R_λ FINITE IRREP OF g
$|i\rangle \in R_\lambda$ UNDER J^a_0
$J^a_0 |i\rangle = (t^a_\lambda)^i_j |j\rangle$

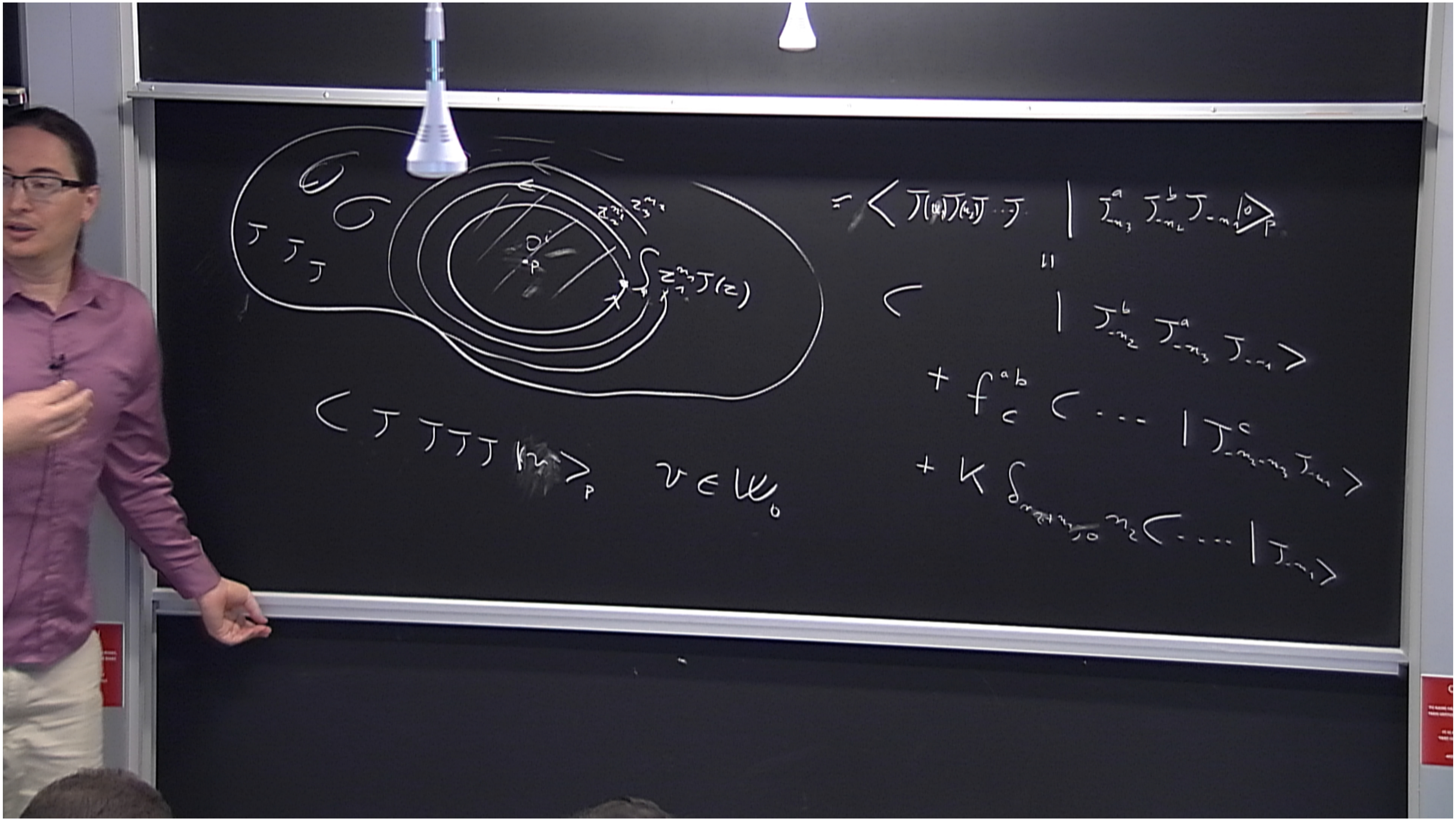

$[t^a, t^b] = f^{ab}_c t^c$
$\frac{f_{ab} J}{z-w}$
R_λ FINITE IRREP OF $\mathcal{g}$
$|i\rangle \in R_\lambda$ UNDER J^a_0
$J^a_{-1}|i\rangle$
$J^a_0 |i\rangle = (t^a_\lambda)^i_j |j\rangle$
$J^a_{-1} J^b_{-1} |i\rangle$, $J^a_{-2}|i\rangle \ldots$

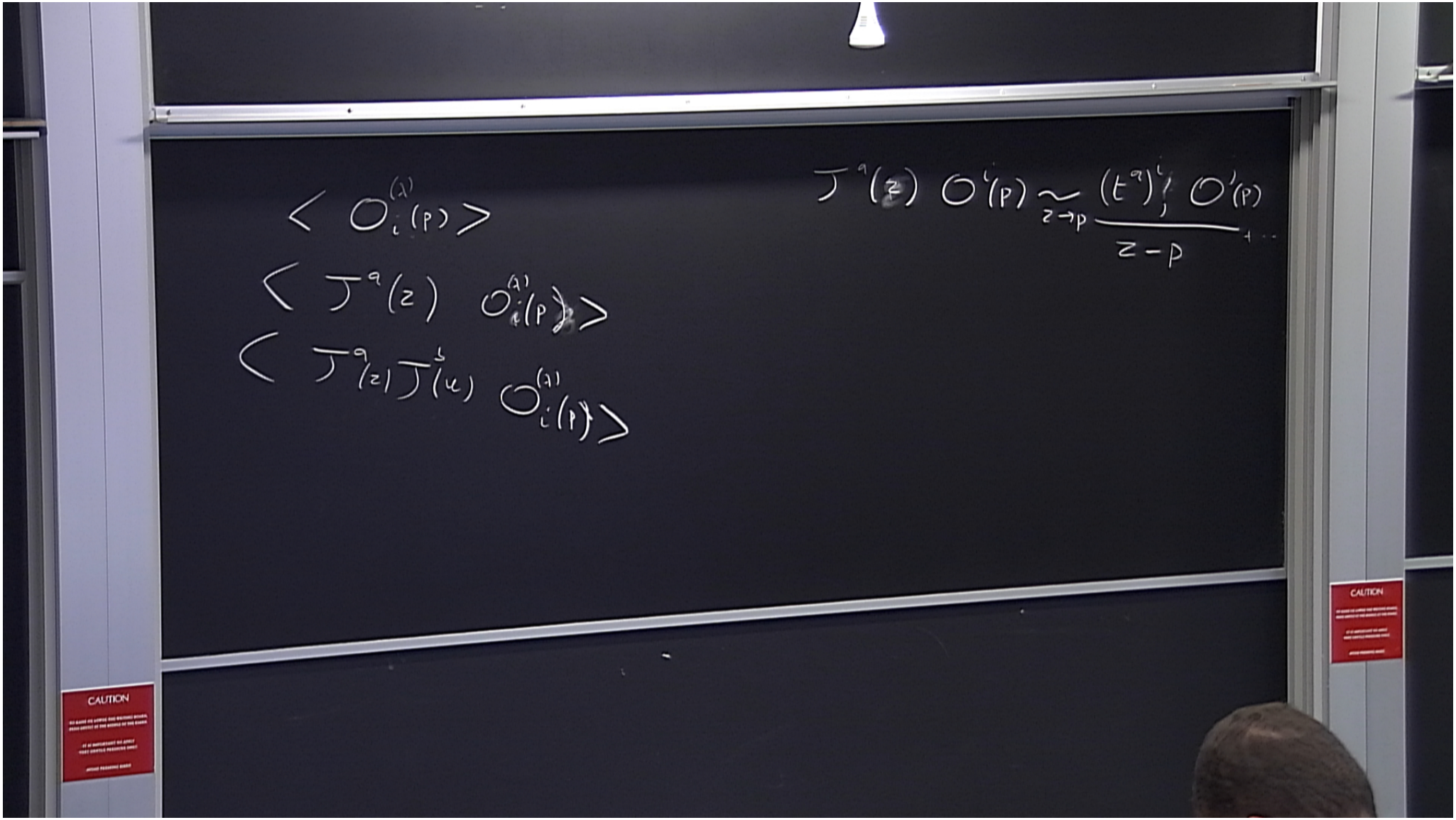

$[t^a, t^b] = f^{ab}_c t^c$
$\frac{f_{ab} J}{z-w}$
R_λ FINITE IRREP OF $\mathcal{g}$
$J_n^a |i\rangle = 0 \quad n>0$
$|i\rangle \in R_\lambda$ UNDER J_0^a
$J_{-1}^a |i\rangle$
$J_0^a |i\rangle = (t_\lambda^a)^i_j |j\rangle$
$J_{-1}^a J_{-1}^b |i\rangle$, $J_{-2}^a |i\rangle \ldots$

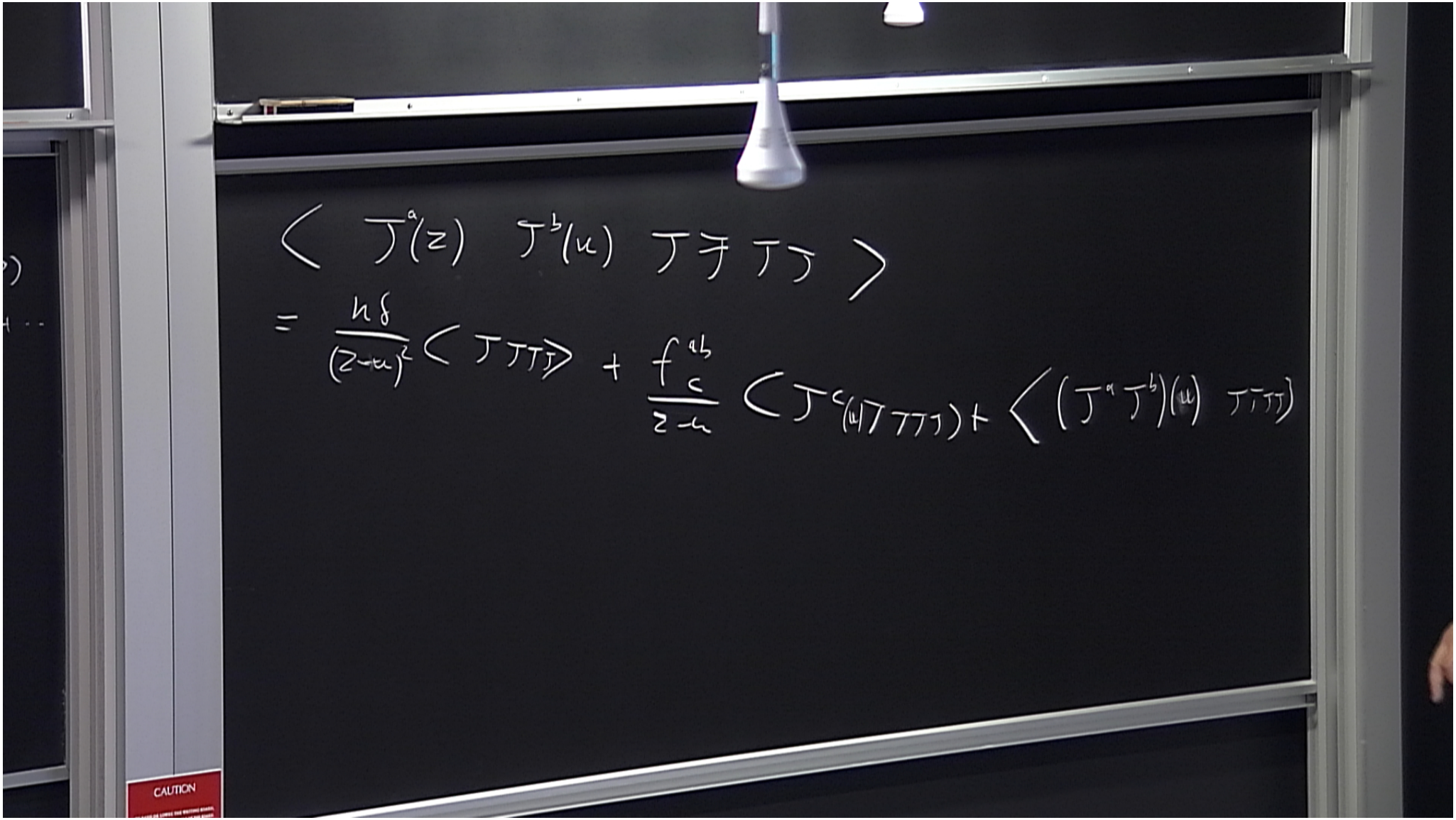

$\langle O_i(p) \rangle$
$\langle J^a(z)\ O_i(p) \rangle$
CAUTION

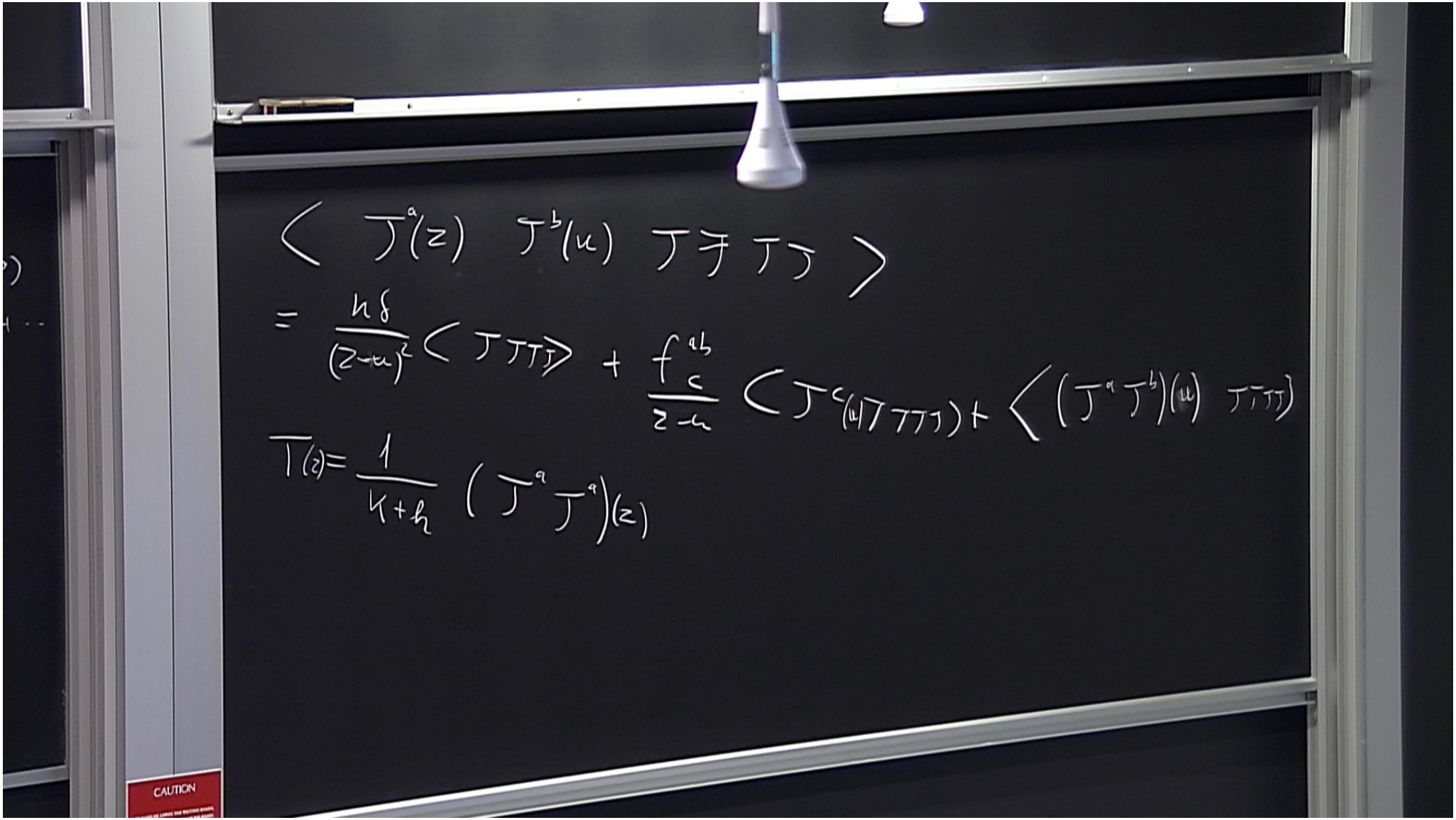

$\langle O_i(p) \rangle$
$\langle J^a(z)\ O_i(p) \rangle$
$J^a(z)\ O^i(p) \underset{z \to p}{\sim} \frac{(t^a)^i_j\ O^j(p)}{z-p} + \cdots$
CAUTION

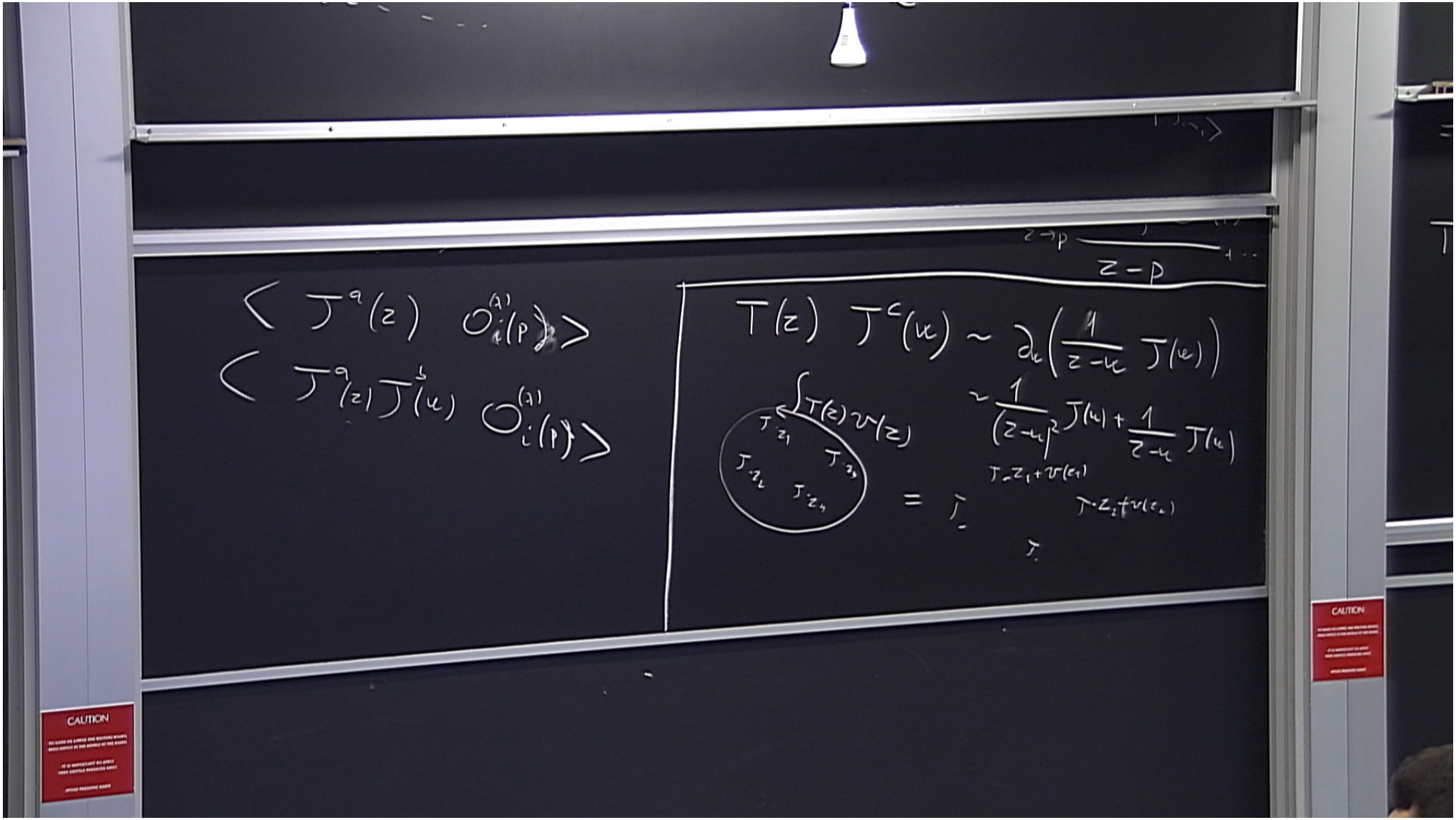

$\langle O_i(p) \rangle$
$\langle J^a(z)\ O_i(p) \rangle$
$\langle J^a(z) J^b(w)\ O_i(p) \rangle$
$J^a(z)\ O^i(p) \underset{z \to p}{\sim} \frac{(t^a)^i_j\ O^j(p)}{z-p} + \cdots$
CAUTION

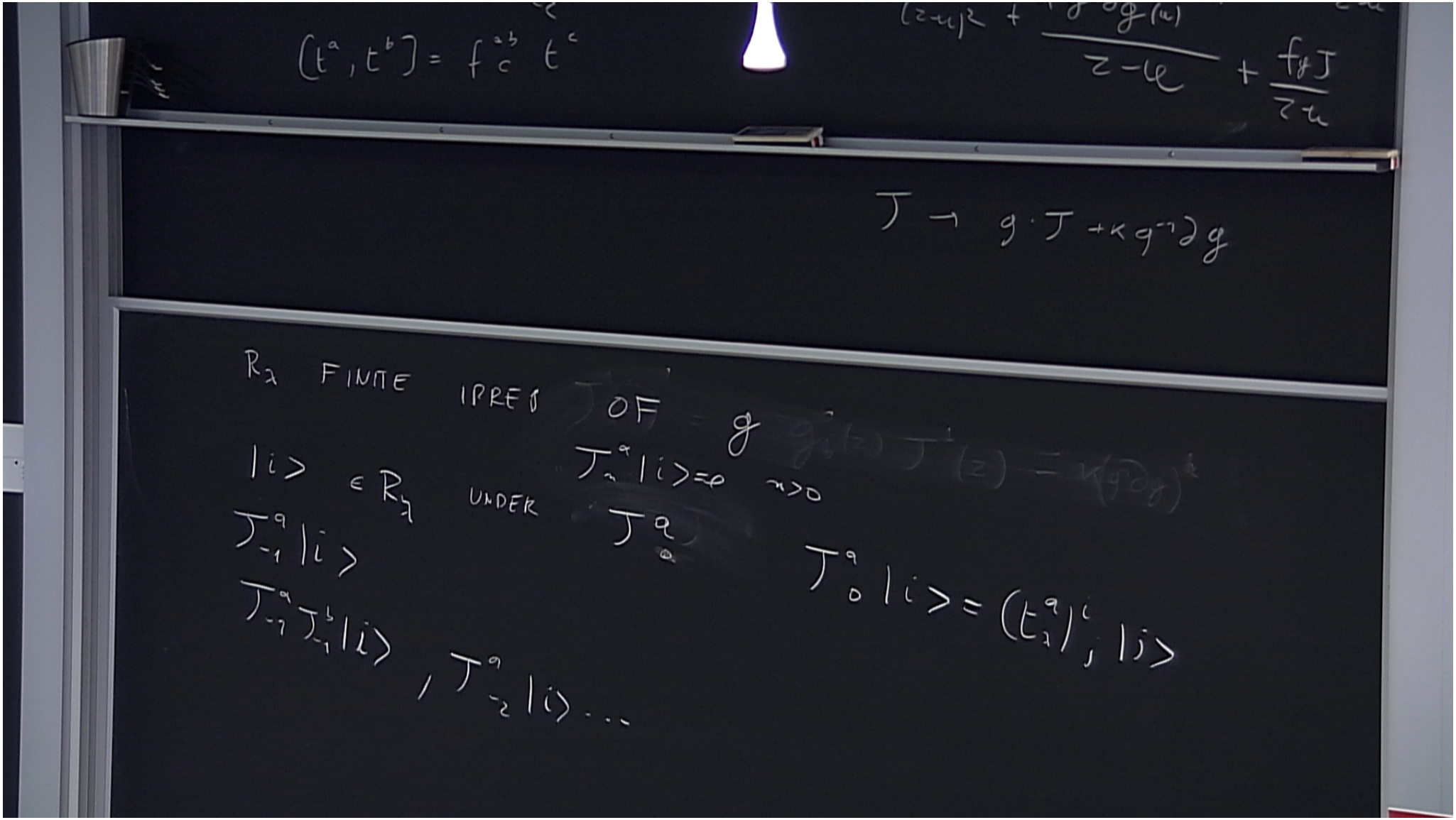

⟨ J J J J J |v⟩_p
v ∈ W_0
+ f^{ab}_c ⟨ . . . | J^c_{-n_2-n_3} J_{-n_1} ⟩
+ K δ n_2 ⟨ . . . | J_{-n_1} ⟩

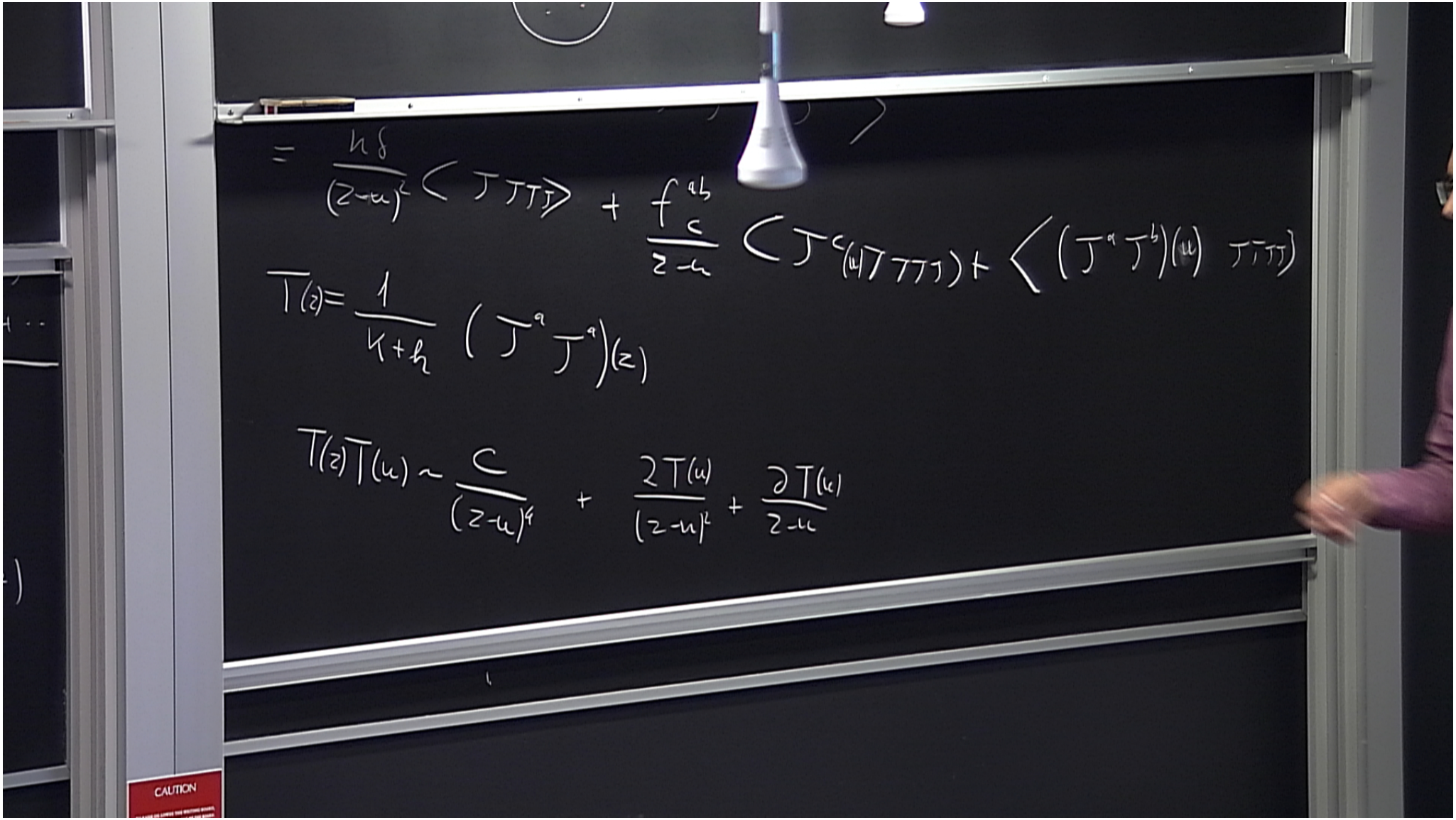

$\langle O_i^{(\lambda)}(p) \rangle$
$\langle J^a(z)\, O_i^{(\lambda)}(p) \rangle$
$\langle J^a(z) J^b(w)\, O_i^{(\lambda)}(p) \rangle$
$J^a(z)\, O_i(p) \underset{z \to p}{\sim} \frac{(t^a)_i^j\, O_j(p)}{z-p} + \cdots$
CAUTION
CAUTION

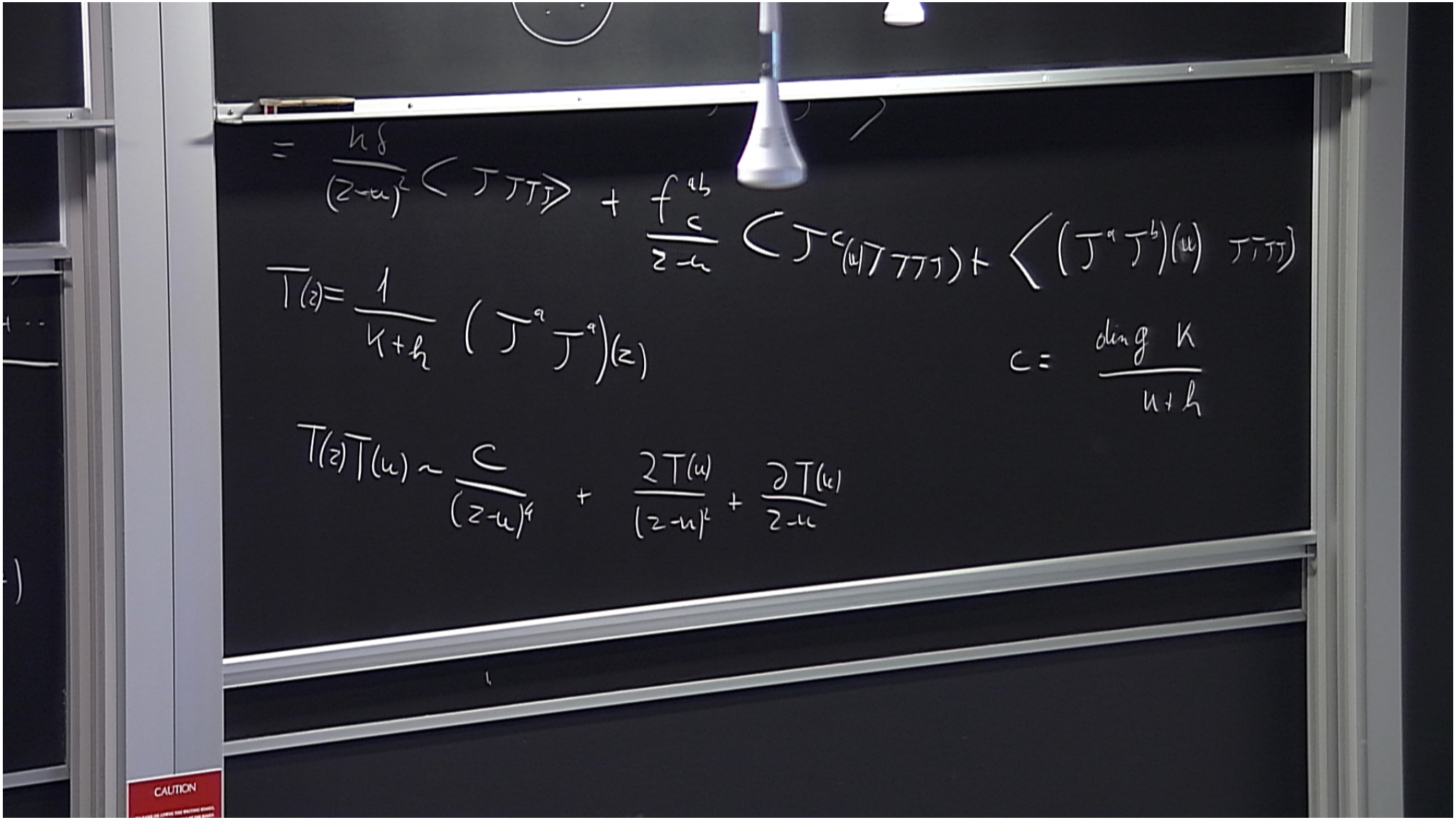

$\langle J^a(z)\ J^b(w)\ TT\ TT \rangle$
$= \frac{nk}{(z-w)^2}\langle TTTT\rangle + \frac{f^{ab}_{\ c}}{z-w}\langle J^c(w)TTTT\rangle + \langle (J^aJ^b)(w)\ TTTT\rangle$
CAUTION

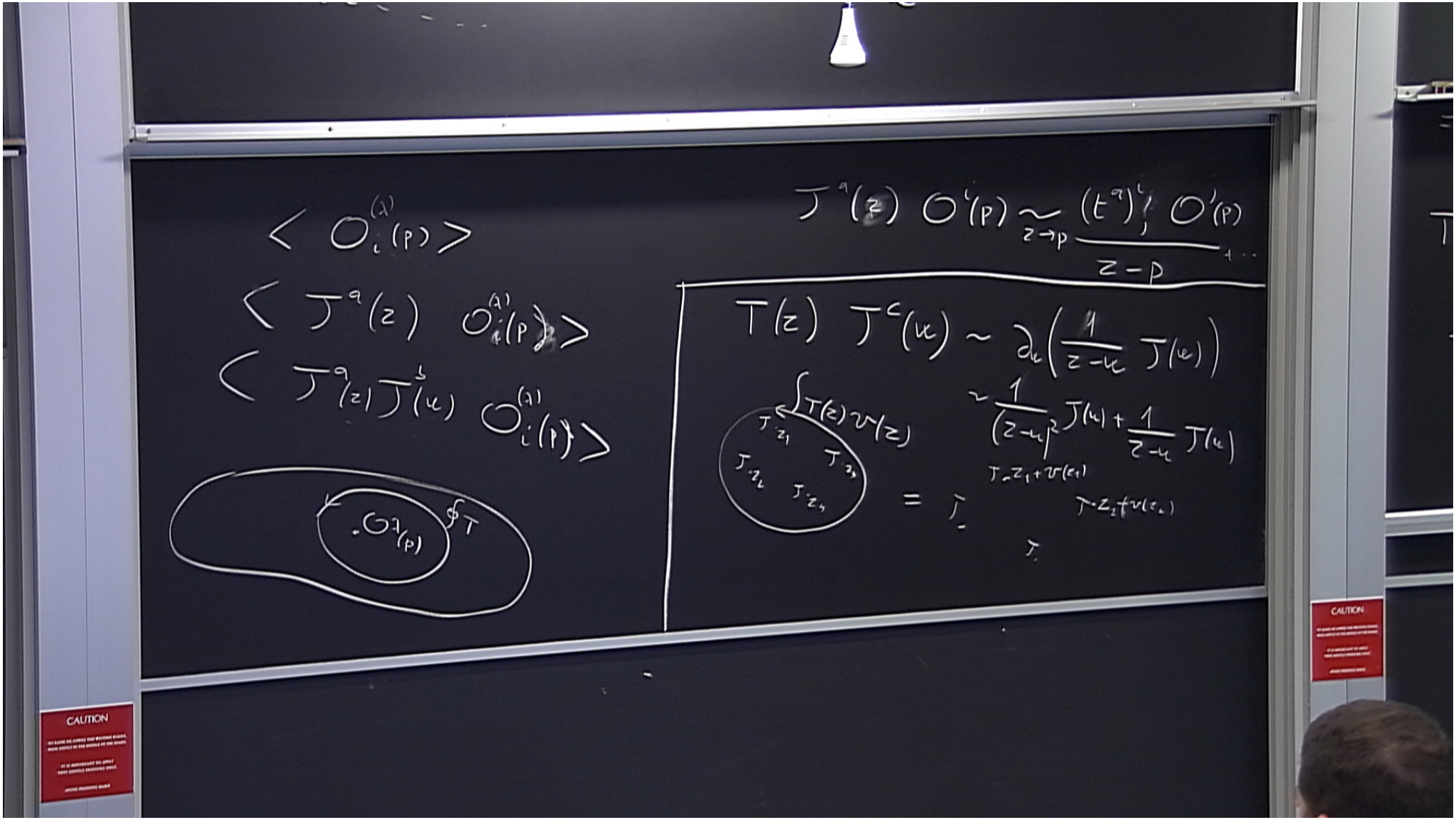

$\langle T^a(z)\ T^b(w)\ T T T \rangle$
$= \frac{k\delta}{(z-w)^2}\langle T T T \rangle + \frac{f^{ab}_{\ c}}{z-w}\langle T^c(w) T T T \rangle + \langle (T^a T^b)(w)\ T T T \rangle$
$T(z) = \frac{1}{k+h}(J^a J^a)(z)$
CAUTION

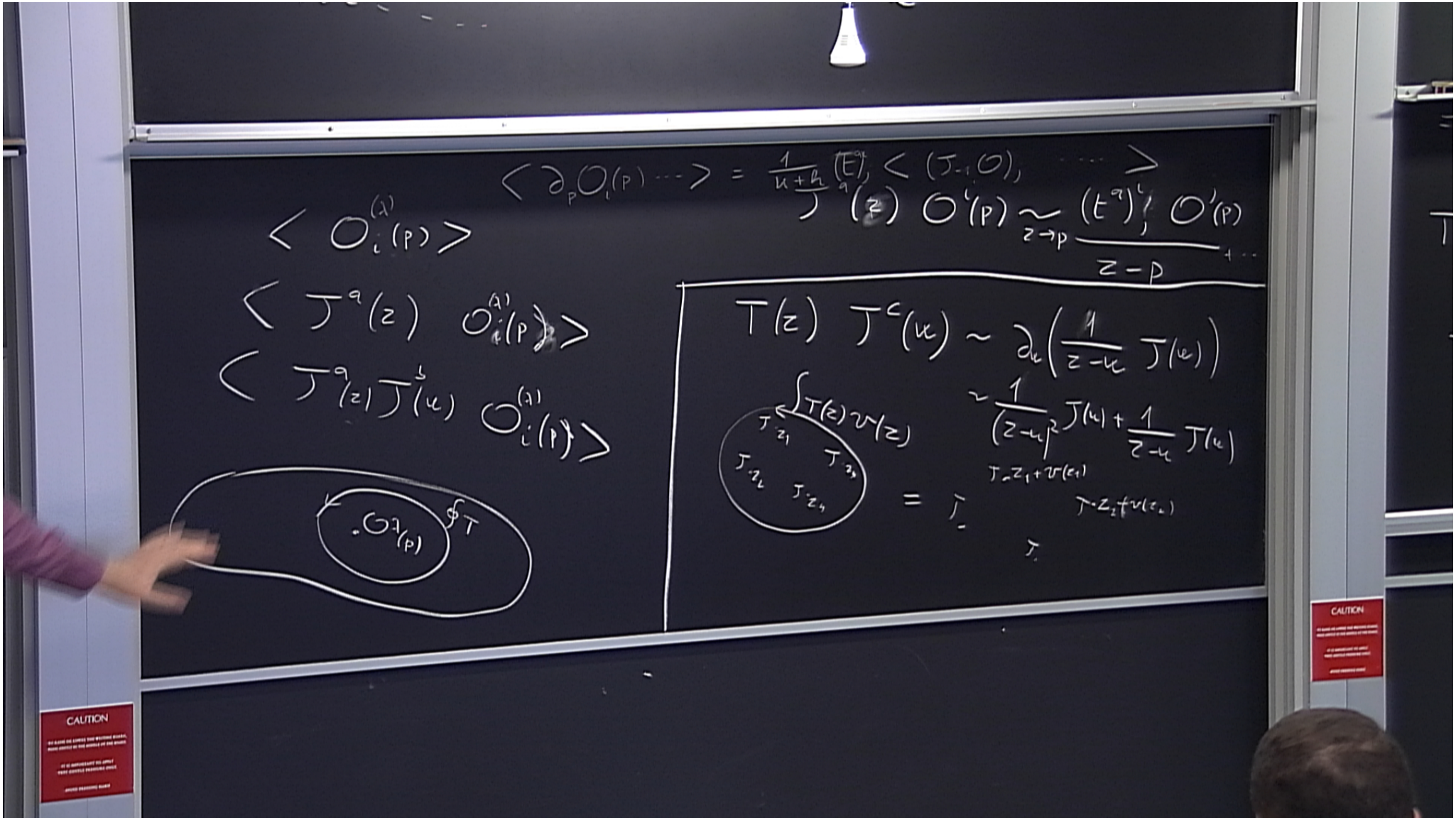

$\langle J^a(z)\ O_i^{(\lambda)}(p) \rangle$
$\langle J^a(z) J^b(w)\ O_i^{(\lambda)}(p) \rangle$
$T(z)\ J^c(w) \sim \partial_w \left(\frac{1}{z-w} J(w) \right)$
$\sim \frac{1}{(z-w)^2} J(w) + \frac{1}{z-w} J(w)$
$\oint T(z) v(z)$
$= \sum$
$z \to p$
$z - p$
CAUTION
CAUTION

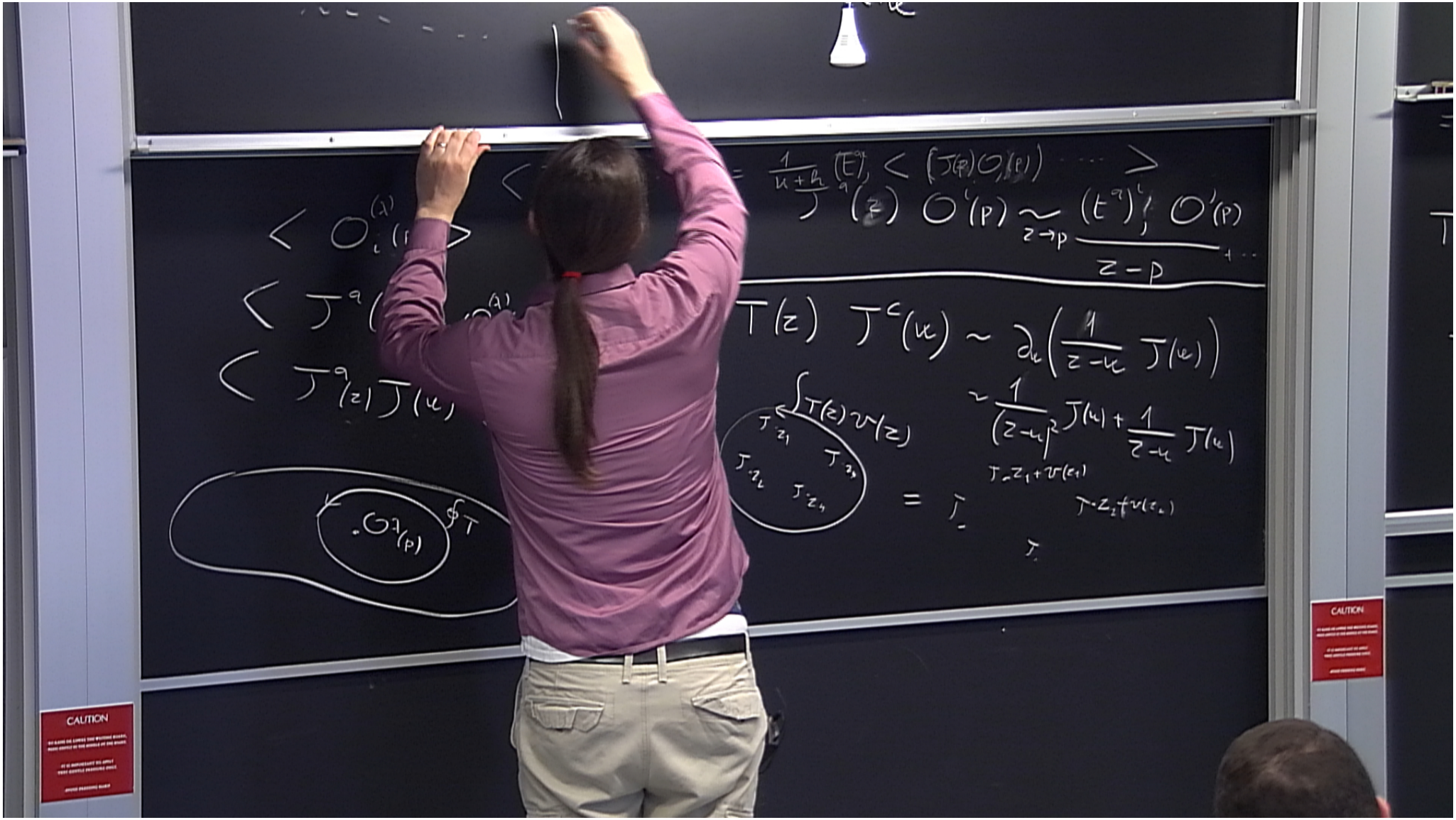

$[t^a, t^b] = f^{ab}_c t^c$
$\frac{f_{ab} J}{z-w}$
$J \to g \cdot J + \kappa g^{-1} \partial g$
R_λ FINITE IRREP OF g
$J^a_n |i\rangle = 0 \quad n > 0$
$|i\rangle \in R_\lambda$ UNDER J^a_0
$J^a_{-1} |i\rangle$
$J^a_0 |i\rangle = (t^a_\lambda)^i_j |j\rangle$
$J^a_{-1} J^b_{-1} |i\rangle$, $J^a_{-2} |i\rangle \ldots$

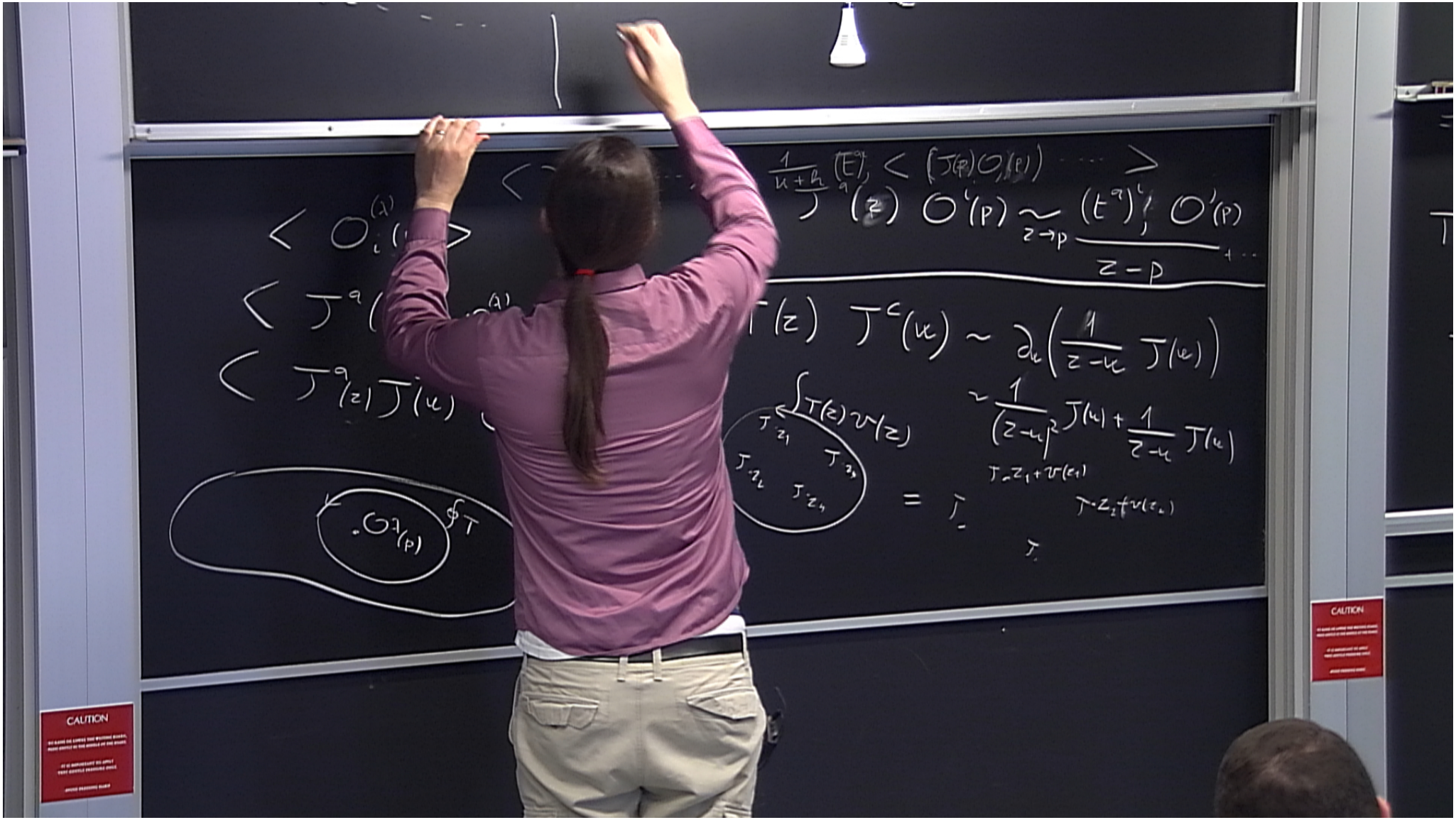

$= \frac{k\delta}{(z-u)^2} \langle JJJ \rangle + \frac{f^{ab}_{\ c}}{z-u} \langle J^c(u) JJ \rangle + \langle (J^a J^b)(u)\, JJ \rangle$
$T(z) = \frac{1}{k+h} (J^a J^a)(z)$
$T(z)T(u) \sim \frac{c}{(z-u)^4} + \frac{2T(u)}{(z-u)^2} + \frac{\partial T(u)}{z-u}$
CAUTION

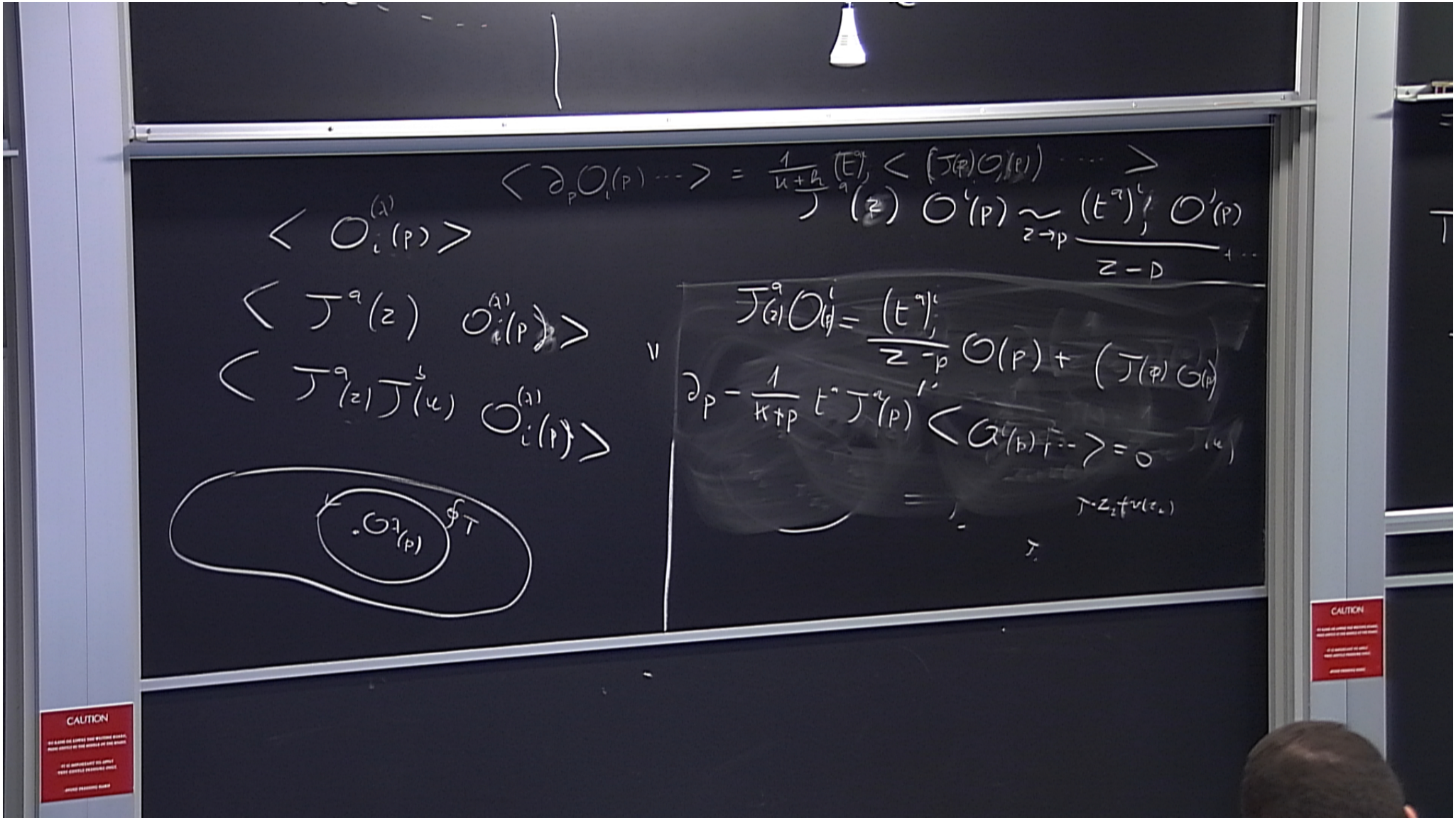

$= \frac{k\delta}{(z-u)^2} \langle JJTJ \rangle + \frac{f^{ab}_{\ c}}{z-u} \langle J^c(u) JTJJ \rangle + \langle (J^a J^b)(u)\ JTJJ \rangle$
$T(z) = \frac{1}{k+h} (J^a J^a)(z)$
$c = \frac{\dim g \cdot k}{k+h}$
$T(z)T(u) \sim \frac{c}{(z-u)^4} + \frac{2T(u)}{(z-u)^2} + \frac{\partial T(u)}{z-u}$
CAUTION

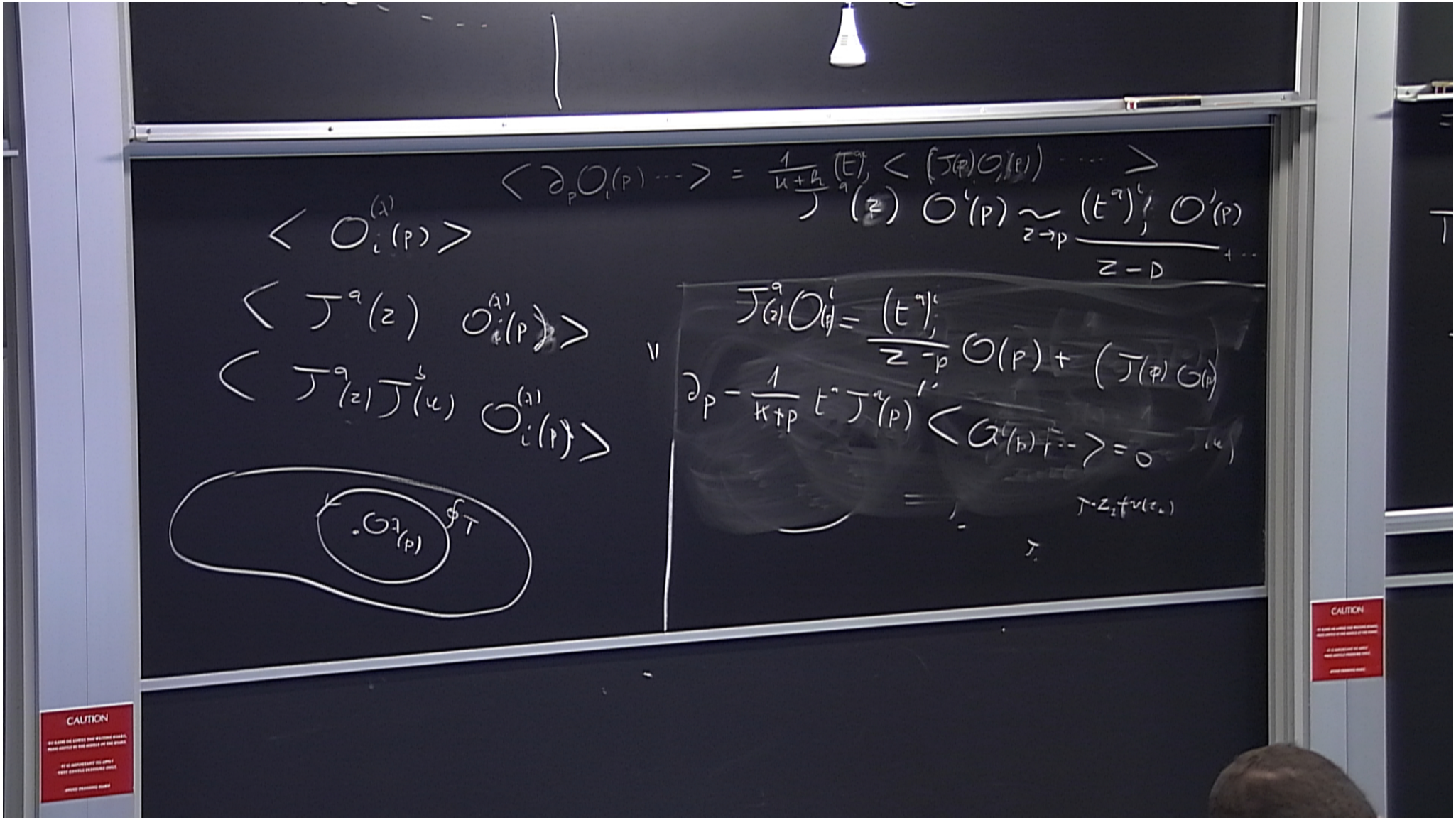

$\langle O_i^{(1)}(p) \rangle$
$\langle J^a(z)\, O_i^{(1)}(p) \rangle$
$\langle J^a(z) J^b(w)\, O_i^{(1)}(p) \rangle$
$O_i^{(1)}(p)$
$\oint T$
$J^a(z)\, O'(p) \underset{z \to p}{\sim} \frac{(t^a)^b_{\ i}\, O'(p)}{z-p} + \dots$
$T(z)\, J^c(w) \sim \partial_w \left(\frac{1}{z-w} J(w) \right)$
$\oint T(z) v(z)$
$J \cdot z_1$
$J \cdot z_2$
$J \cdot z_3$
$J \cdot z_4$
$= J \cdot z_1 + v(z_1)$
$J \cdot z_2 f v(z_2)$
$\sim \frac{1}{(z-w)^2} J(w) + \frac{1}{z-w} J(w)$
CAUTION
CAUTION

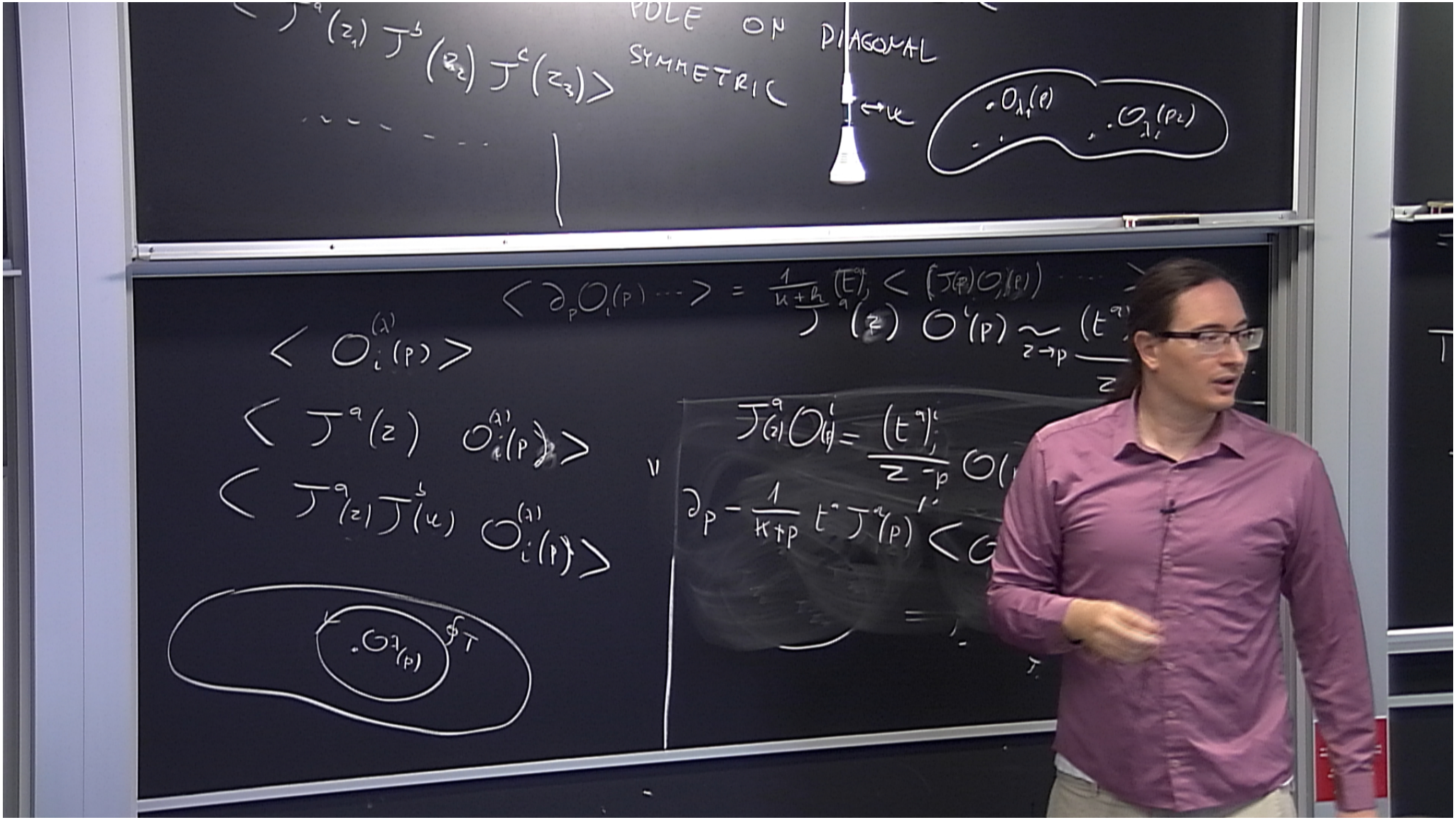

$\langle O_i^{(1)}(p) \rangle$
$\langle J^a(z)\, O_i^{(1)}(p) \rangle$
$\langle J^a(z) J^b(w)\, O_i^{(1)}(p) \rangle$
$O_i^{(1)}(p)$
$\oint T$
$\langle \partial_p O_i(p) \cdots \rangle = \frac{1}{w+h} (t^a)_j^i \langle (J_{-1} O)_j \cdots \rangle$
$J^a(z)\, O^j(p) \underset{z \to p}{\sim} \frac{(t^a)_i^j\, O^i(p)}{z-p} + \cdots$
$T(z)\, J^c(w) \sim \partial_w \left(\frac{1}{z-w} J(w) \right)$
$\sim \frac{1}{(z-w)^2} J(w) + \frac{1}{z-w} J(w)$
$\oint T(z) v(z)$
J_{z_1} J_{z_2} J_{z_3} J_{z_4}
$= J_{z_1 + v(z_1)}$
$J_{z_1 + v(z_1)}$
J_z
CAUTION
CAUTION

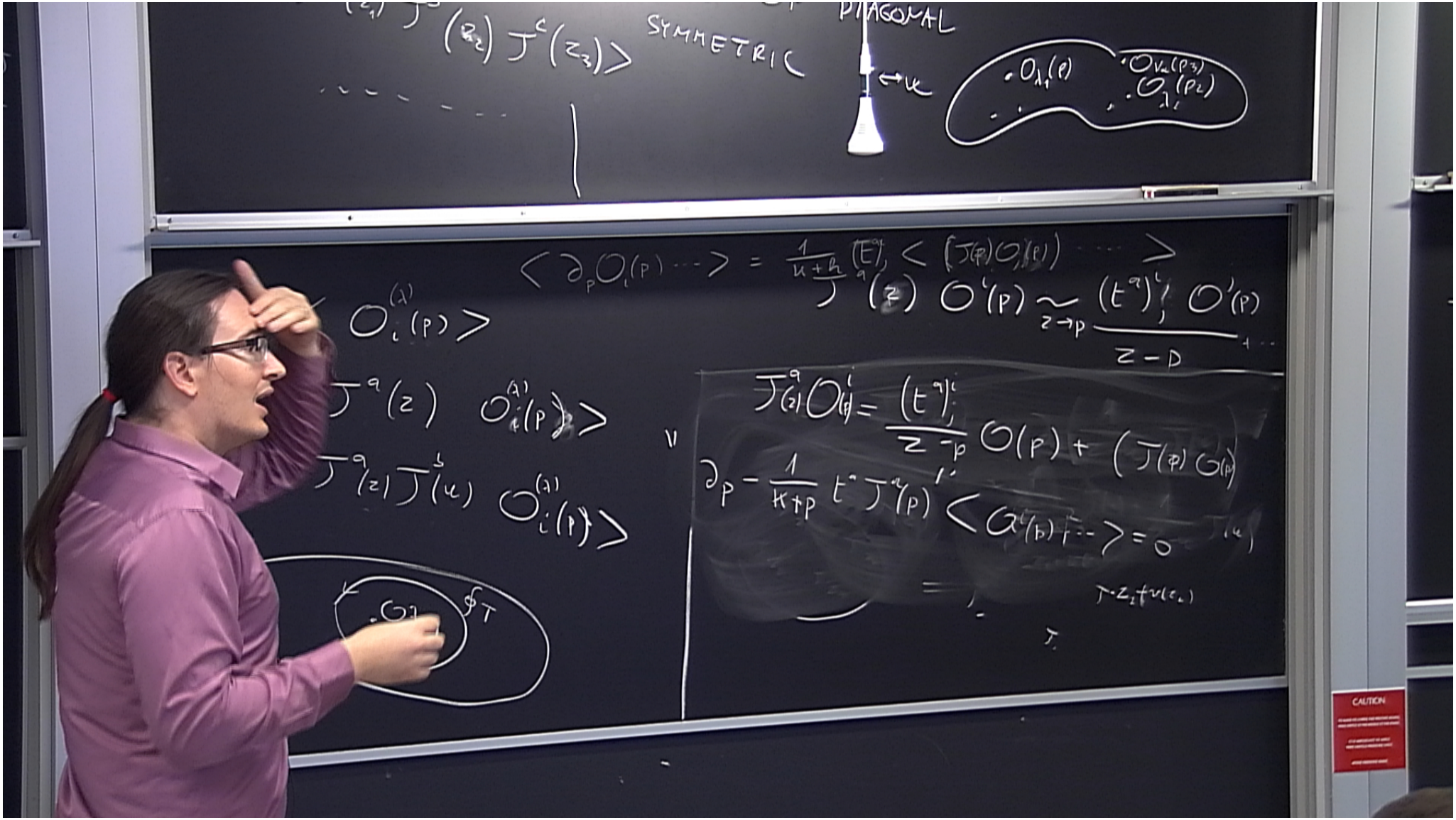

T(z) T^c(w) ~ ∂_w (1/(z-w) J(w))
CAUTION
CAUTION


CAUTION
CAUTION


$\langle \partial_p O_i(p) \cdots \rangle = \frac{1}{u+p} (t^a)^j_i \langle (J^a(p) O_j(p)) \cdots \rangle$
$\langle O_i^{(1)}(p) \rangle$
$J^a(z)\, O'(p) \underset{z \to p}{\sim} \frac{(t^a)^i_j\, O'(p)}{z-p} + \cdots$
$\langle J^a(z)\, O_i^{(1)}(p) \rangle$
$J^a(z) O_i^{(1)}(p) = \frac{(t^a)^j_i}{z-p} O(p) + (J^a(p) O_i^{(1)}(p))$
$\langle J^a(z) J^b(u)\, O_i^{(1)}(p) \rangle$
$\partial_p - \frac{1}{u+p} t^a J^a(p)$
$\langle O_i^{(1)}(p) \cdots \rangle = 0$
$O_i^{(1)}(p)$
$\oint_T$
$T = \sum_i z_i$
CAUTION
CAUTION


$\langle \partial_p O_i(p) \cdots \rangle = \frac{1}{u+p} (t^a)^j_i \langle (J^a(p) O_j(p)) \cdots \rangle$
$\langle O_i^{(1)}(p) \rangle$
$J^a(z)\, O'(p) \underset{z \to p}{\sim} \frac{(t^a)^j_i\, O'(p)}{z-p} + \cdots$
$\langle J^a(z)\, O_i^{(1)}(p) \rangle$
$J^a(z) O_i^{(1)}(p) = \frac{(t^a)^j_i}{z-p} O(p) + (J^a(p) O_i^{(1)}(p))$
$\langle J^a(z) J^b(u)\, O_i^{(1)}(p) \rangle$
$\partial_p - \frac{1}{u+p} t^a J^a(p)$
$\mathcal{O}^{(1)}(p)$
$\oint_T$
CAUTION
CAUTION


POLE ON DIAGONAL
SYMMETRIC


SYMMETRIC
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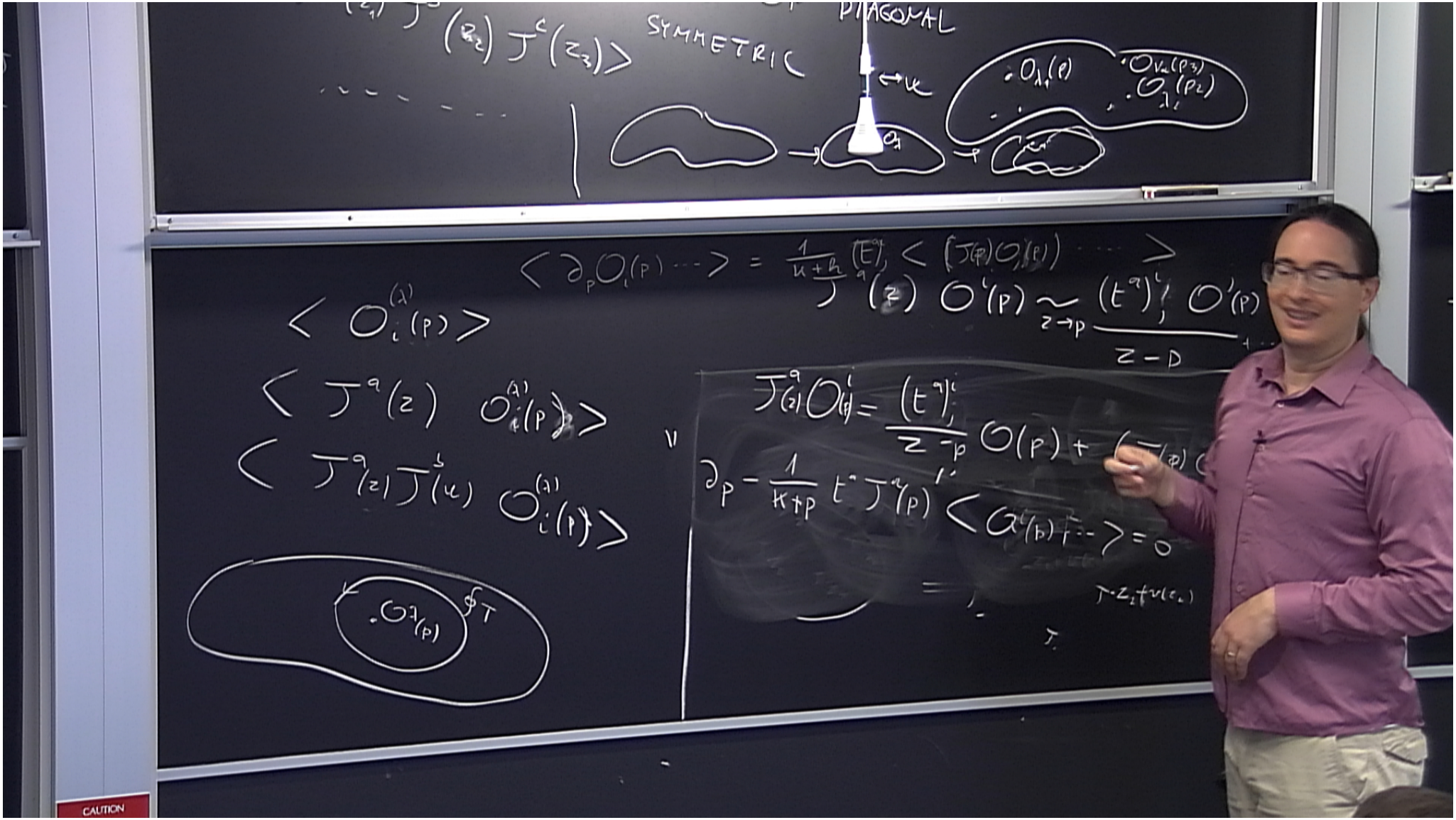
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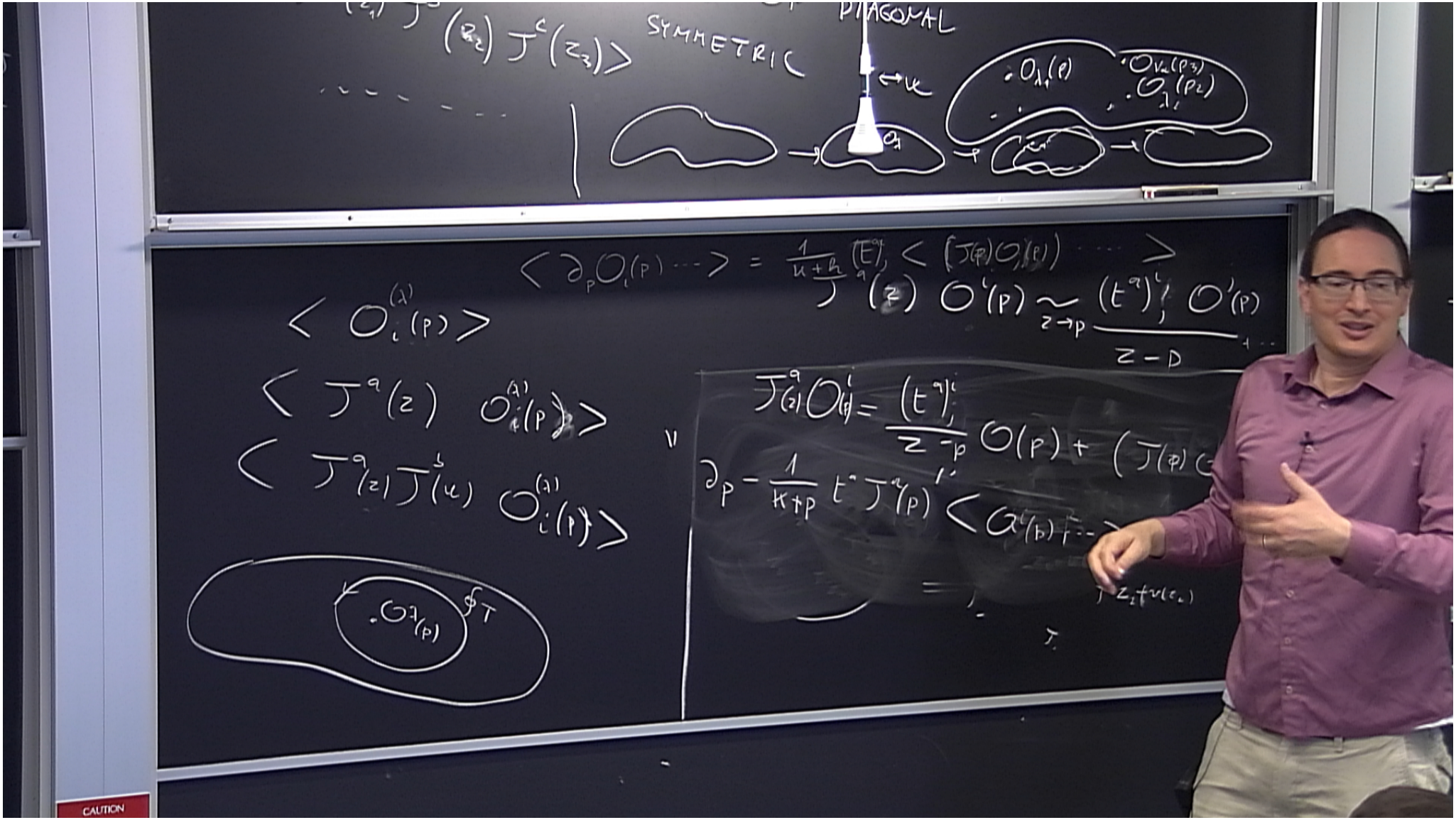
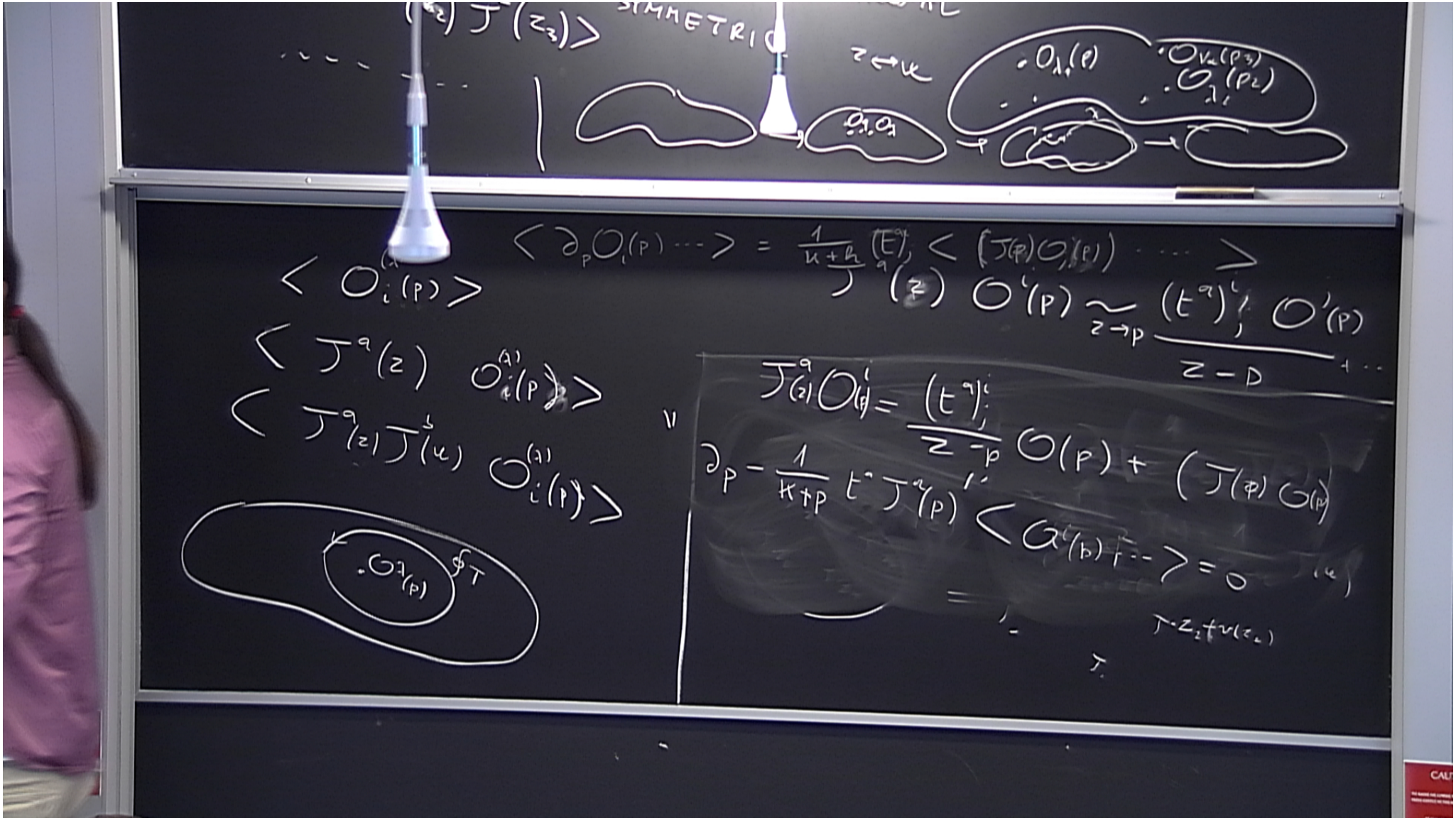
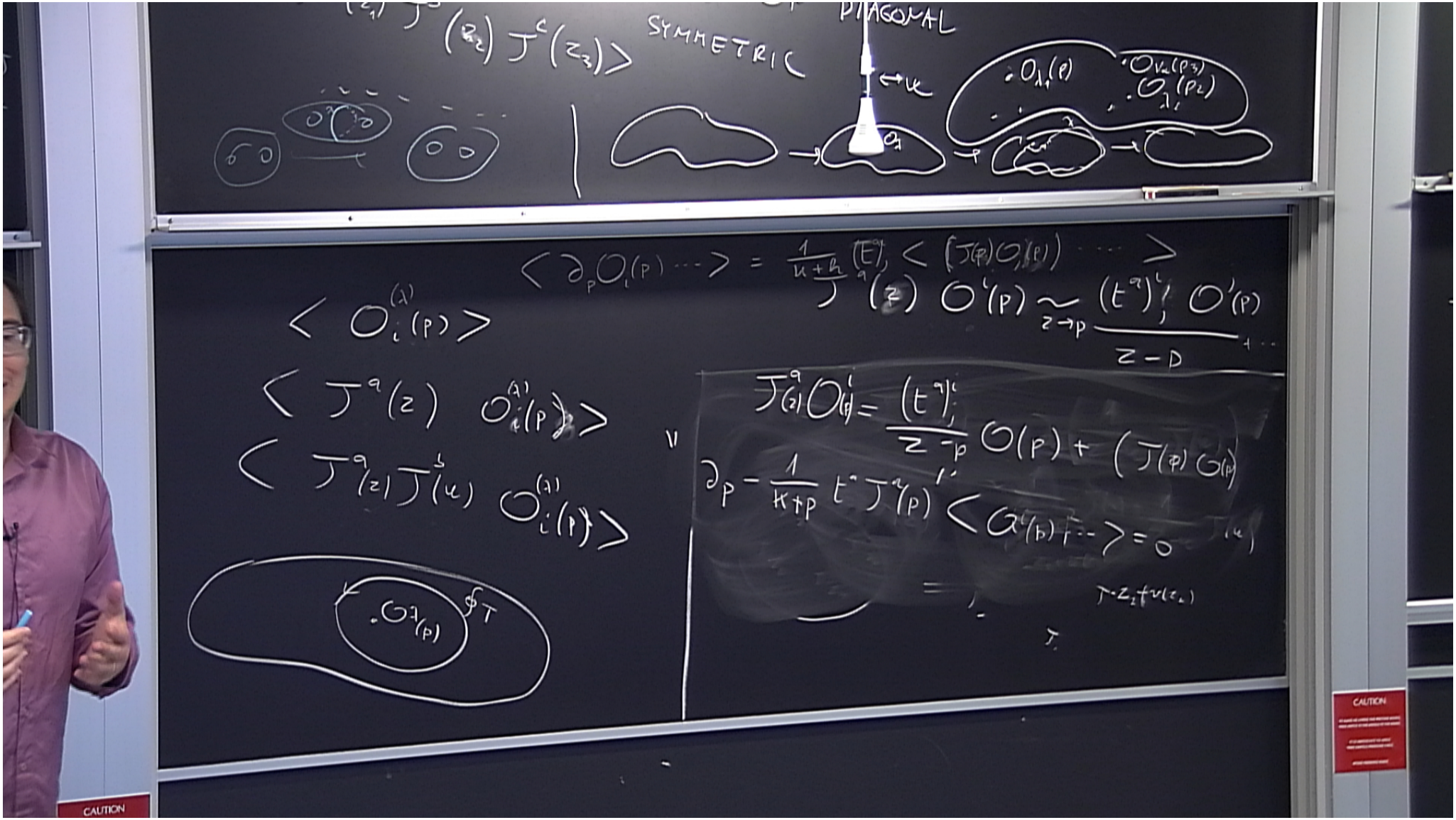
DIAGONAL
CAUTION


SYMMETRIC
DIAGONAL
CAUTION


SYMMETRIC
z ↔ w
$\langle O_i(p) \rangle$
$\langle J^a(z)\, O_i^{(a)}(p) \rangle$
$\langle J^a(z) J^b(w)\, O_i^{(a)}(p) \rangle$
$\langle \partial_p O_i(p) \cdots \rangle = \frac{1}{k+h} (t^a)^j_i \langle (J^a(p) O_j(p)) \cdots \rangle$
$J^a(z)\, O^i(p) \underset{z \to p}{\sim} \frac{(t^a)^j_i\, O^j(p)}{z-p} + \cdots$
$J^a(z) O^i(p) = \frac{(t^a)^j_i}{z-p} O^j(p) + (J^a(p) O^i(p))$
$\partial_p - \frac{1}{k+h} t^a J^a(p)$
$\langle O^i(p) \cdots \rangle = 0$
$J \cdot z_i + w(z_i)$
J_z


SYMMETRIC
DIAGONAL
CAUTION
CAUTION