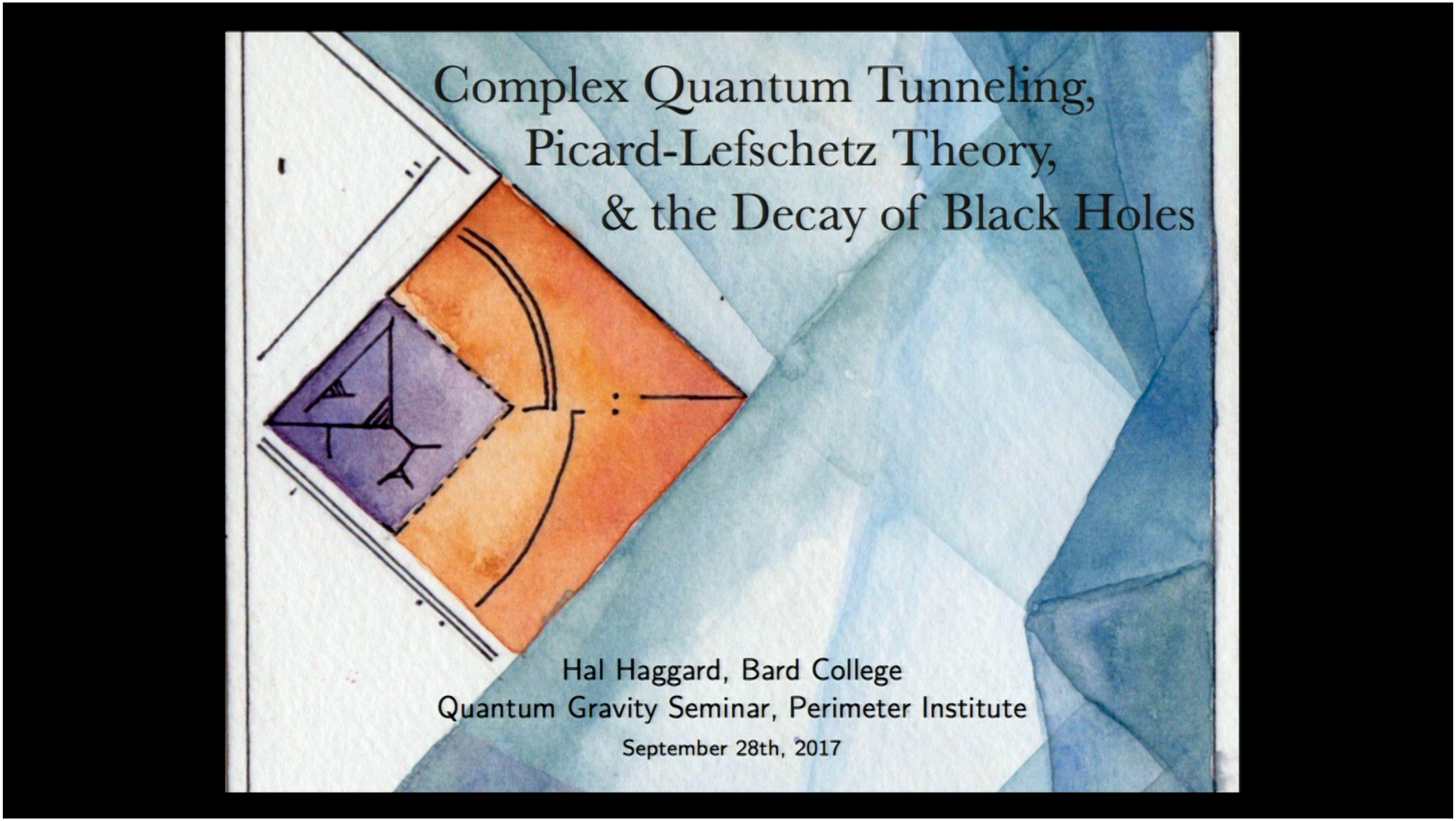
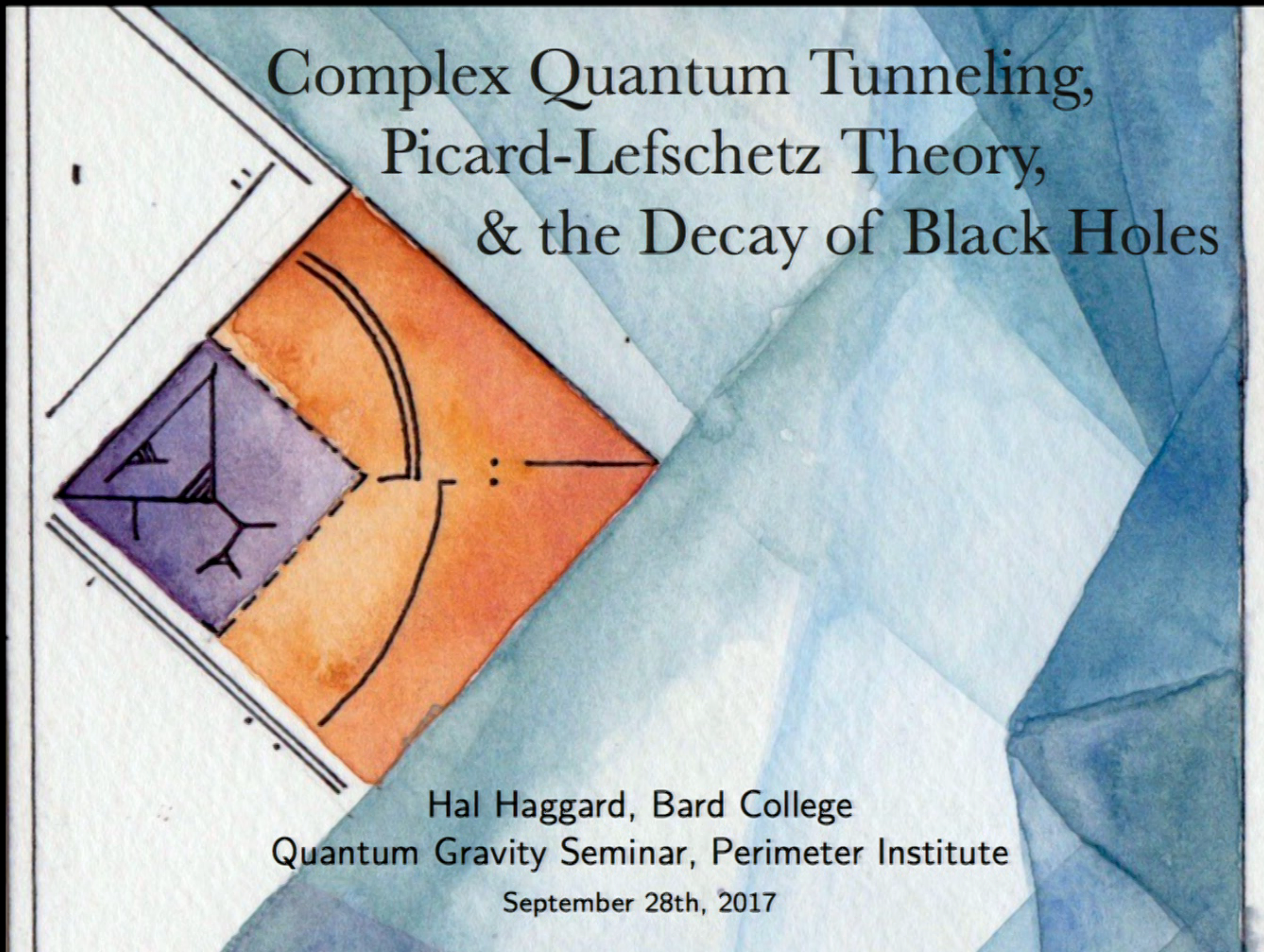
Title: Complex Quantum Tunneling, Picard-Lefschetz Theory, and the Decay of Black Holes

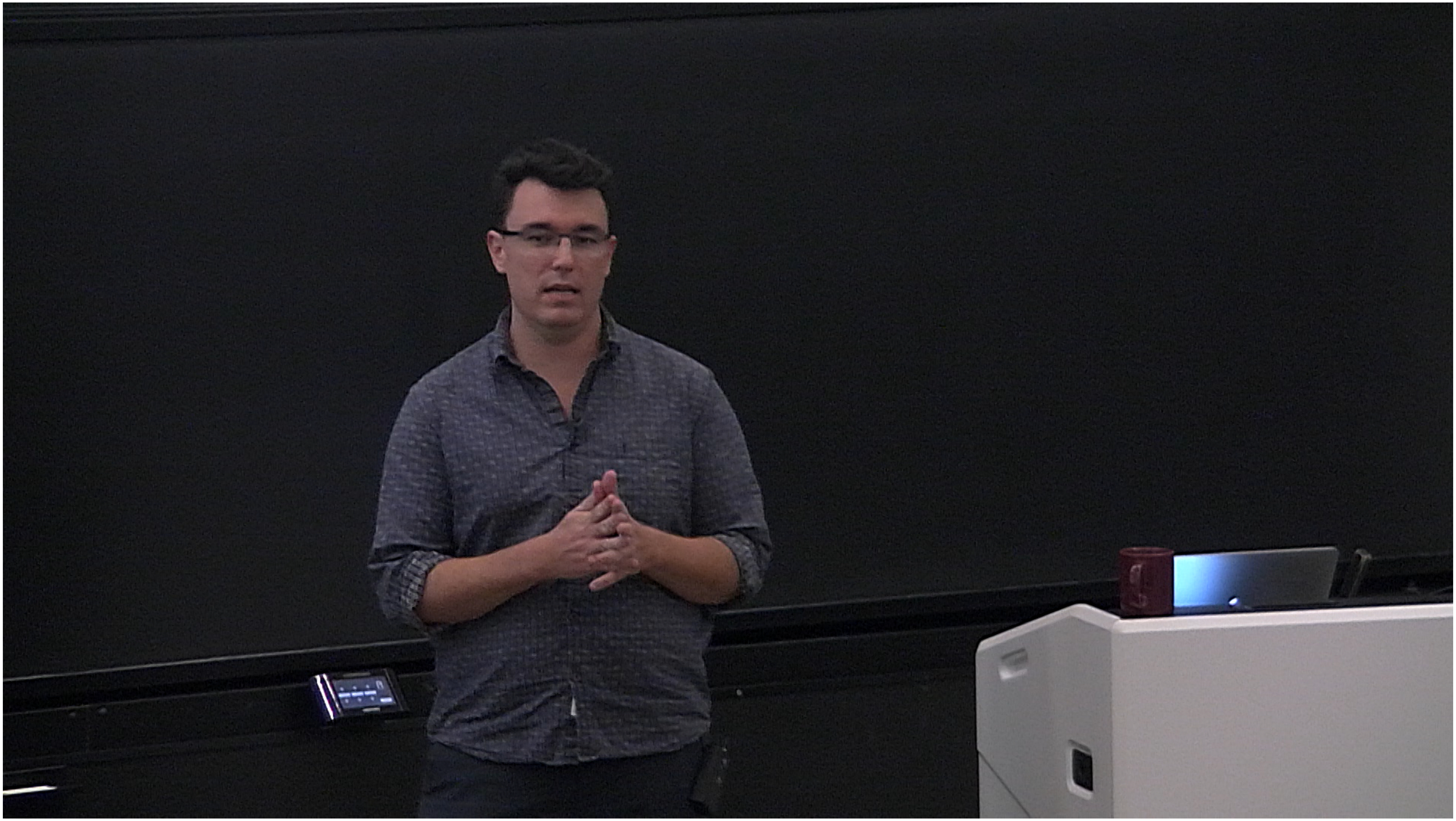
Date: Sep 28, 2017 02:30 PM

URL: http://pirsa.org/17090071

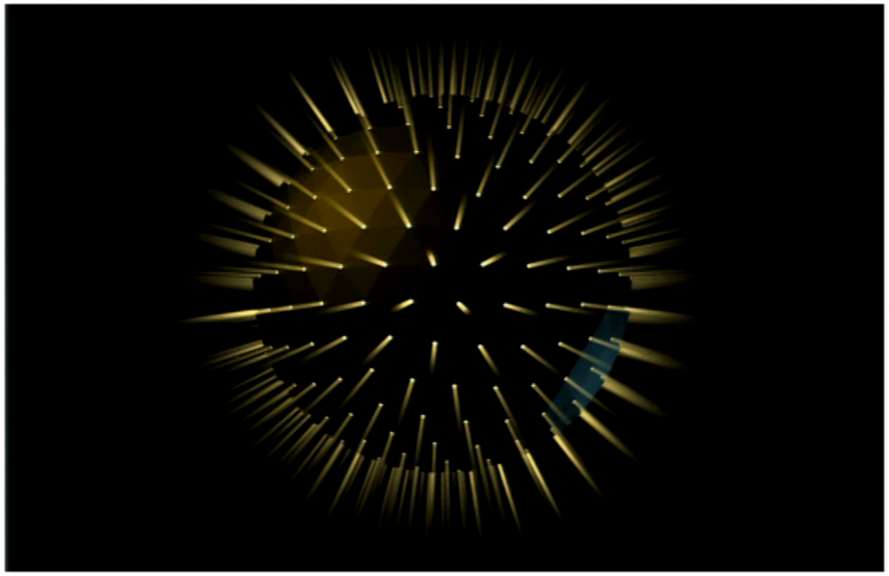
Abstract: <p>Quantum effects render black holes unstable. In addition to Hawking radiation, which leads to the prediction of a long lifetime, there is the possibility of quantum tunneling of the black hole geometry itself. A robust possibility for treating the quantum tunneling of a spacetime geometry is through a complex path integral and Picard-Lefschetz theory. I will illustrate the semiclassical approximation of complex path integrals using these techniques with an analytically solvable 1D quantum potentialâ€"the inverse square barrierâ€"and describe the setup of the calculation for spherically symmetric black holes. While the black hole calculation is incomplete, I will be able to describe a surprising extension of this setup to the more astrophysical rotating Kerr black hole.</p>


Complex Quantum Tunneling,
Picard-Lefschetz Theory,
& the Decay of Black Holes
Hal Haggard, Bard College
Quantum Gravity Seminar, Perimeter Institute
September 28th, 2017


Complex Quantum Tunneling,
Picard-Lefschetz Theory,
& the Decay of Black Holes
Hal Haggard, Bard College
Quantum Gravity Seminar, Perimeter Institute
September 28th, 2017



Quantum mechanics allows black holes to evaporate via Hawking radiation



Is this the only mode of evolution? Is it the dominant one?

The standard argument for the lifetime of a black hole goes roughly as follows

If

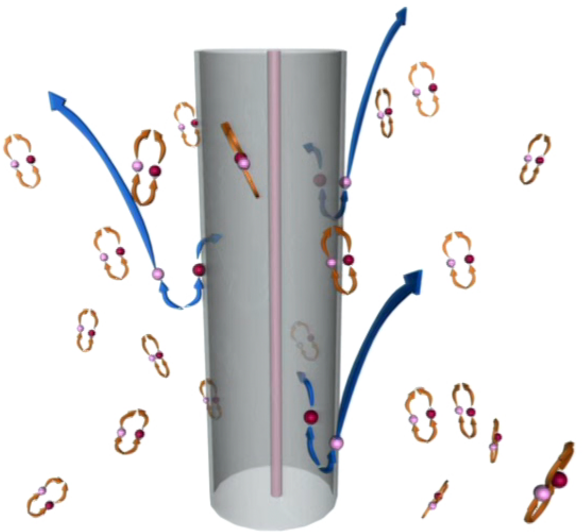
$$T_H \propto \frac{1}{M} \qquad \text{and} \qquad P_H \propto A_{BH} T_H^4 \propto \frac{1}{M^2},$$

then

$$\frac{dM}{dt} \propto -P_H \propto -\frac{1}{M^2} \implies \tau_{BH} \propto M^3.$$

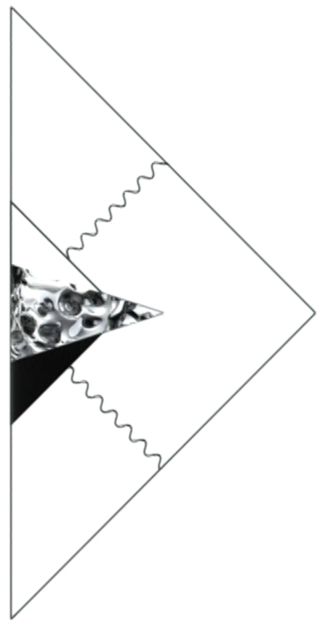
The approximation used in this argument, which has a hybrid character, is not semiclassical.

This is a very, very slow process. For a solar mass black hole it takes $\tau_{BH} = 10^{75}$ secs. The age of the universe is $\tau_U = 10^{17}$ secs.



Understanding Hawking radiation as a tunneling process explains this unhurried pace. Is there no other physics before this time?

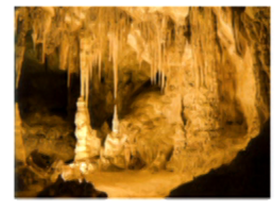
Is it possible for the geometry of a black hole to tunnel? Can a black hole in the past be continued to a white hole in the future?



4

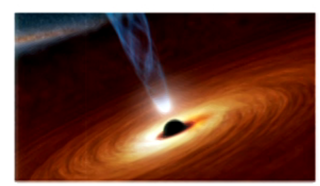
Outline

I. Complex Quantum Tunneling



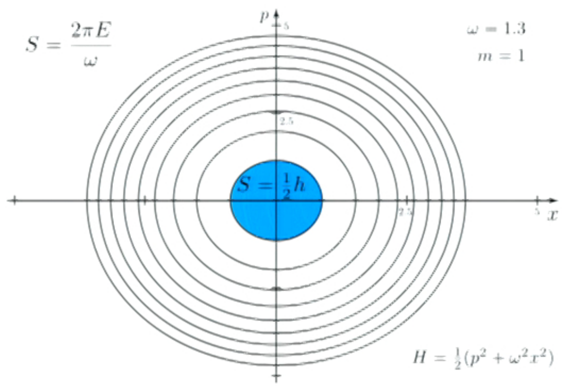
II. Black to White Hole Transitions

III. The Rotating Case



People say there's no classical correspondent to quantum tunneling —*not strictly true.*

Bohr-Sommerfeld rule:

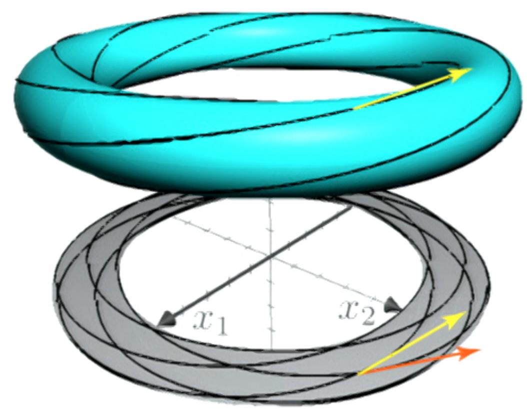


Require

$$S = \oint_\gamma p dx = (n + \tfrac{1}{2})h$$

$$\rightsquigarrow \quad E_n = (n + \tfrac{1}{2})\hbar\omega$$

EBK Quantization:



Einstein moves to $\vec{p} \cdot d\vec{x}$ and requires

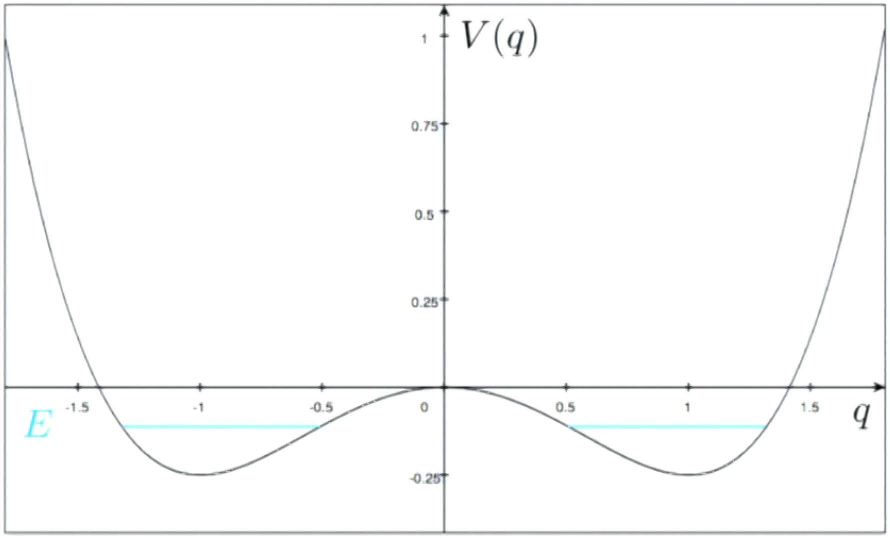
$$S_i = \oint_{C_i} \vec{p} \cdot d\vec{x} = n_i h.$$

6

Capture tunneling classically by complexifying classical variables

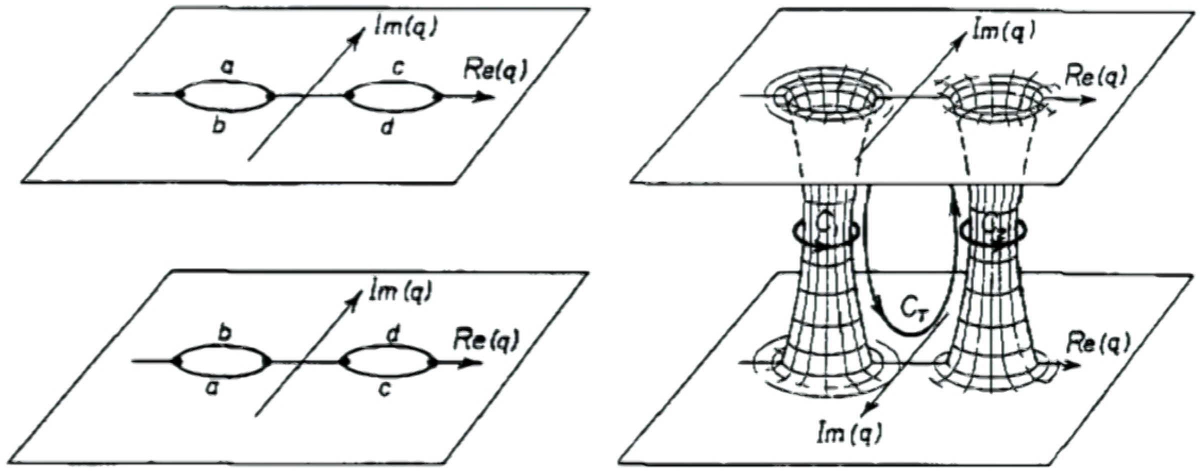
Following S. Creagh [J. Phys. A '94], return to 1D and consider

$$\mathcal{H} = \frac{p^2}{2} - \frac{q^2}{2} + \frac{q^4}{4} = E$$



For real E dynamics is stuck in each well and E's are degenerate.

As a complex condition $p = \sqrt{2(E-V)}$ is a torus!



Upon compactification, just 2 top. independent cycles, C_T & C_1.

Requiring a single-valued WKB wavefunction connects the action integrals, S_1 & S_T, along these cycles

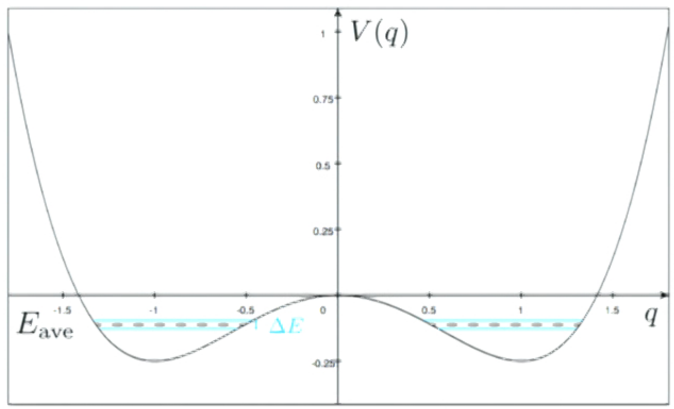
$$S_1(E) = 2\pi\hbar(n + \tfrac{1}{2}) \pm \hbar\, e^{-S_T/2\hbar}.$$

Expanding

$$S_1(E_{\mathsf{ave}} + \Delta E) = 2\pi\hbar(N + \tfrac{1}{2}) \pm \hbar\, e^{-S_T/2\hbar},$$

to first order

$$\frac{dS_1}{dE}\Delta E = 2\hbar\, e^{-S_T/2\hbar} \qquad \rightsquigarrow \qquad \Delta E = \frac{2\hbar}{\tau} e^{-S_T/2\hbar}.$$



Nonperturbative effects are encoded in the analytic continuation of classical mechanics!

What changes when we allow our classical variables to be complex numbers? Which variables should we allow to take such values? Can or should we complexify time?

To try and address these questions we will turn to the path integral

$$\Psi(x_f, t_f) = \int \mathcal{D}x \int dx_i \, e^{\frac{i}{\hbar} S[x_f, t_f; x_i, t_i]} \Psi(x_i, t_i)$$

This is an infinite dimensional integral and it is useful to warm up on a finite analog. We'll use Picard-Lefschetz theory.

[See e.g. Witten, Tanizaki & Koike, Cherman & Ünsal, Feldbrugge, Lehners, & Turok]

Oscillatory integrals like

$$I = \int_{\mathcal{R}} dx \; e^{\frac{i}{\hbar} S(x)}$$

are dominated by saddles in the complex plane

Continue to $\mathbb{C}$ and consider

$$\mathcal{E}(z) = \tfrac{i}{\hbar} S(z) \equiv s + i\tilde{s}.$$

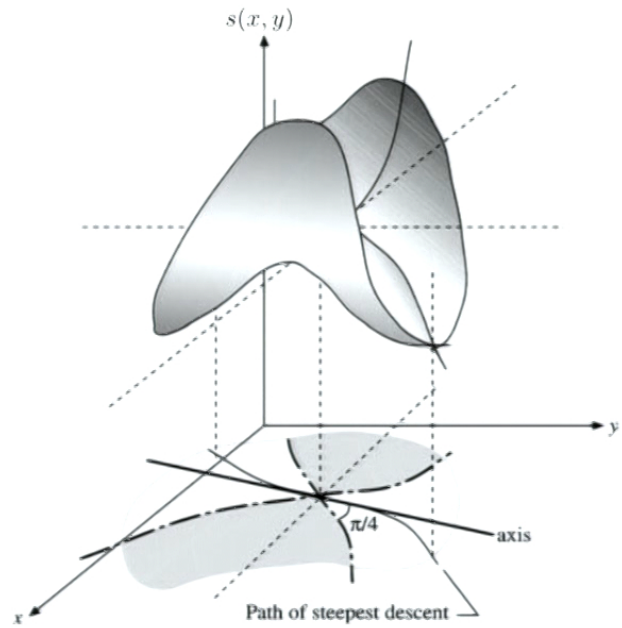
For differentiability we need

$$\frac{\partial s}{\partial x} = \frac{\partial \tilde{s}}{\partial y} \quad \& \quad \frac{\partial s}{\partial y} = -\frac{\partial \tilde{s}}{\partial x}.$$

Critical pts $\partial_z \mathcal{E}\big|_{z_*} = 0$ have

$$\frac{\partial^2 s}{\partial x^2} = \frac{\partial^2 \tilde{s}}{\partial x \partial y} = -\frac{\partial^2 s}{\partial y^2}$$

hence are saddles of s.

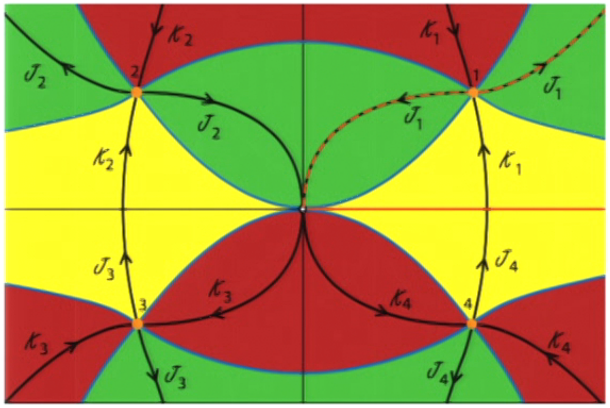


Developing this theory leads to a useful result

$$I = \int_{\mathcal{R}} dx\, e^{\frac{i}{\hbar}S(x)} = \sum_{\text{saddles } \sigma} n_\sigma e^{i\tilde{s}(z_\sigma)} \int_{\mathcal{J}_\sigma} d\lambda \frac{dz}{d\lambda}\, e^{s}$$

$$\approx \sum_{\sigma} n_\sigma e^{\frac{i}{\hbar}S(z_\sigma)} [R_\sigma + \mathcal{O}(\hbar)],$$

with R_σ the leading gaussian integral about the critical point z_σ.



[picture from Feldbrugge, Lehners, & Turok '17]

Nothing in the path integral forbids extemization by complex $x(t)$

$$\Psi(x_f, t_f) = \int \mathcal{D}x \int dx_i \, e^{\frac{i}{\hbar} S(x_f, t_f; x_i, t_i)} \Psi(x_i, t_i)$$

Semiclassically, the initial condition of the classical trajectory of stationary phase is found by variation with respect to x_i [Turok '14]

$\rightsquigarrow$ boundary conditions could be complex

Indeed, semiclassical theory of the propagator

$$K(x_f, x_i, t = t_f - t_i) = \int \mathcal{D}x \, e^{\frac{i}{\hbar} S[x]}$$

is well developed:

$$K(x_f, x_i, t) \sim \sum_{\text{traj}} \sqrt{\frac{D}{2\pi i \hbar}} e^{\frac{i}{\hbar} S(x_f, x_i, t) - i \frac{\pi \nu}{2}},$$

the Van Vleck-Gutzwiller (VVG) propagator

Can VVG be used to treat tunneling? Diverse explorations...

Grossman & Heller: real $E \rightsquigarrow$ one trajectory and inaccurate approx

Many authors: In coherent state rep, need $\mathbb{C}$ trajectories even for classically allowed processes

Bender, Hook et al: real t and complex E, clear evidence of tunneling behavior

Ankerhold & Seltzer: real t along trajectory $\rightsquigarrow$ difficulty meeting complex b.c.s

What variables should be complexified? K. Kay: Consistent perspective, which I draw from here.

14

To treat K semiclassically start from E-dependent Green's function

$$K(x_f, x_i, t) = \frac{i}{2\pi} \int dE \; G(x_f, x_i, E) e^{-iEt/\hbar}$$

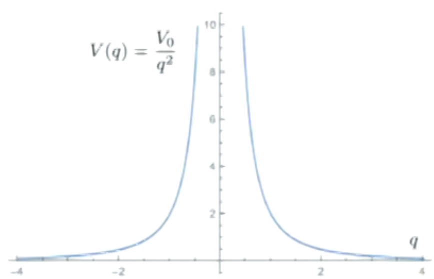
and Gutzwiller's formula

$$G(x_f, x_i, E) = \tfrac{1}{i\hbar} \textstyle\sum_{\mathsf{traj}} \frac{\exp\{iW(x_f, x_i, E)/\hbar - i\pi\mu/2\}}{|\dot{q}(T)\dot{q}(0)|}.$$

Here $W = \int p\, dq = \int_{x_i}^{x_f} \sqrt{2m(E-V)}\, dq$ and

$$T(x_f, x_i, E) = \frac{\partial W}{\partial E} = \int_{x_i}^{x_f} \frac{m}{\sqrt{2m(E-V)}}\, dq = \int \frac{dq}{\dot{q}} = \int dt.$$

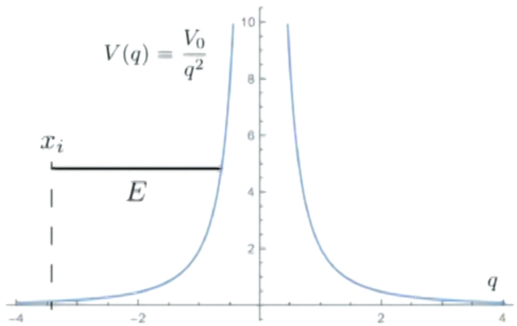
Model potential

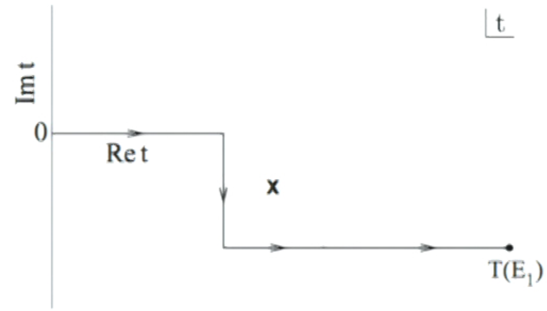


Below the barrier no real time trajectory exists with good b.c.

$$W = \int p\, dq = \int_0^T p(q(t), E)\dot{q}(q(t), E)dt$$

Instead, between turning points a and b turn into complex t-plane





Along the vertical segment

$$\tau(E) = \sqrt{\frac{m}{2}} \int_a^b \frac{1}{\sqrt{V(q) - E}} dq$$

and

$$T(x_f, x_i, E) = T_1 - i\tau + T_2.$$

Putting the Gutzwiller formula into the propagator

$$K(x_f, x_i, t) = \sum_{\text{traj}} \frac{1}{2\pi\hbar} \int \frac{\exp\{i\phi/\hbar - i\pi\mu/2\}}{|\dot{q}(T)\dot{q}(0)|} dE,$$

with $\phi = -Et + W(x_f, x_i, E)$, and requiring stationary phase:

$$\frac{\partial \phi}{\partial E} = -t + T(x_f, x_i, E) = 0.$$

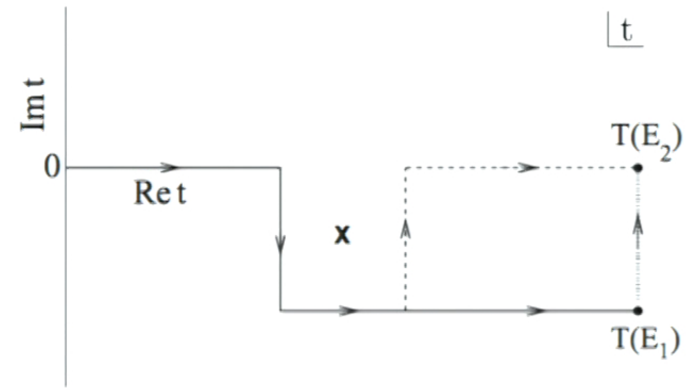
For t real, stationary phase points exist only if T is real

$\rightsquigarrow$ complex E

$\rightsquigarrow$ complex p

$\rightsquigarrow$ complex q

Choosing a complex E can bring the contour back to a real b.c.,



but cannot cross over the branch point of p and q, the pt **X**

Can identify these branch points using

$$t_* = \int_{x_i}^{q_*} \frac{dq}{\dot{q}} \qquad \text{where} \qquad q_* \quad \text{is s.t.} \quad V(q_*) = \infty.$$

18

All so intricate—can we actually do it?

For the inverse square barrier the exact propagator is known

$$K(x_f, x_i, t) = \frac{\sqrt{x_i x_f}}{i\hbar t} \exp\left(\frac{x_i^2 + x_f^2}{2\hbar t}\right) I_\nu\left(-i\frac{x_i x_f}{\hbar t}\right)$$

where I_ν is the Bessel function with order $\nu = \frac{1}{2}\sqrt{1 + 8V_0/\hbar^2}\ldots$

…and we can carry out the semiclassical calculations analytically

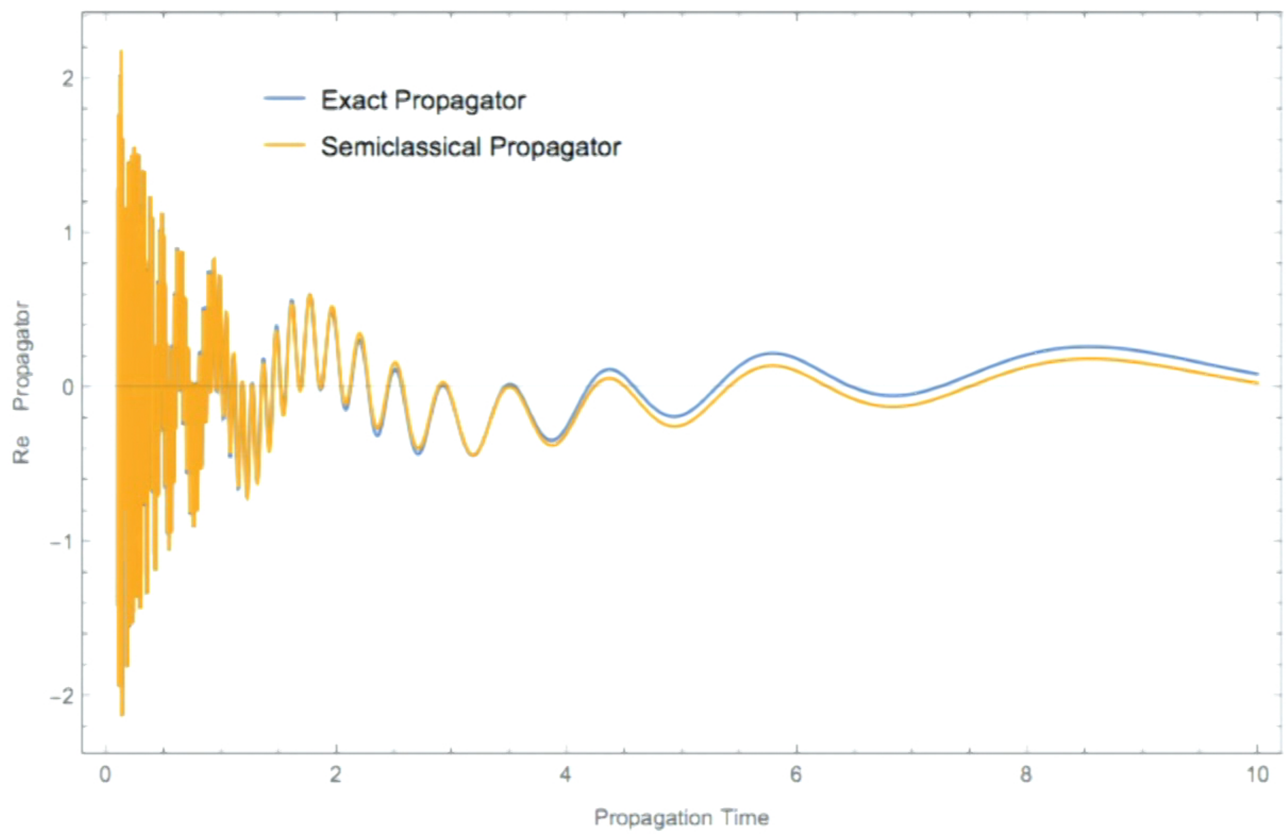
$$S(x_f, x_i, t) = Et - \sqrt{2mV_0}\sin^{-1}\left(\frac{\sqrt{\frac{2V_0}{m}}t}{x_f x_i}\right),$$

with $E(x_f, x_i, t) = m/2t^2\left(x_f^2 + x_i^2 \pm 2\sqrt{x_f^2 x_i^2 - 2V_0 t^2/m}\right)$

to show that…

19

... the complex trajectory VVG formula works!

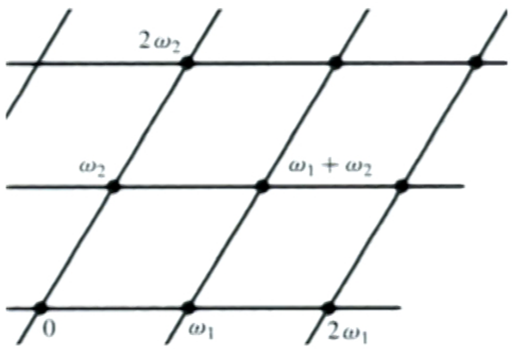


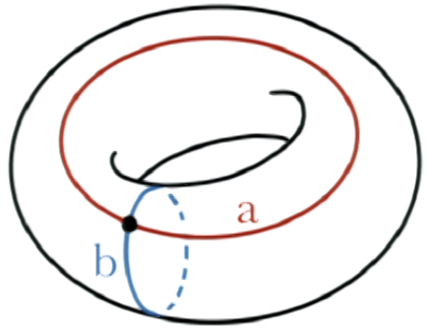
20

Basar, Unsal, & Dunne [JHEP '17]: Complex analytic structure can determine behavior to all orders in $\hbar$.

For $V(x)$ quartic the complex Lagrangian manifold is an elliptic curve, i.e. a torus

$$p^2 = 2m[E - V(x)]$$



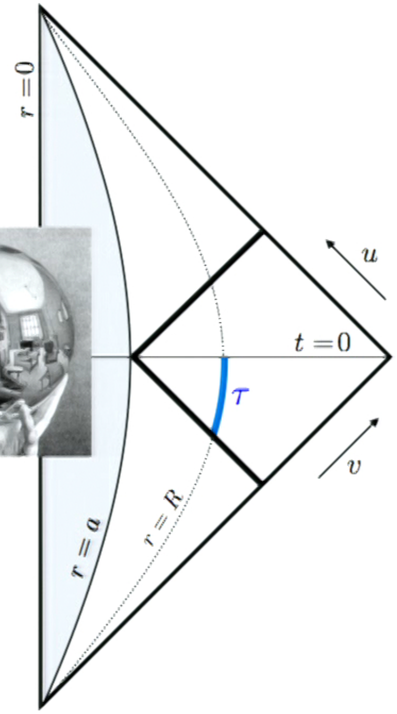


Analytic dependence on E connects the two actions S_a & S_b, both solve the Picard-Fuchs equation. You can find tunneling effects at each order that you can treat perturbative relations.

Gravitational time dilation: "Slouching clocks run slow"

How long for a shell of light to bounce off a mirrored ball?

$$\tau_R = \sqrt{1 - \frac{2M}{R}}\Big(R - a - 2M \ln \frac{a - 2M}{R - 2M}\Big)$$



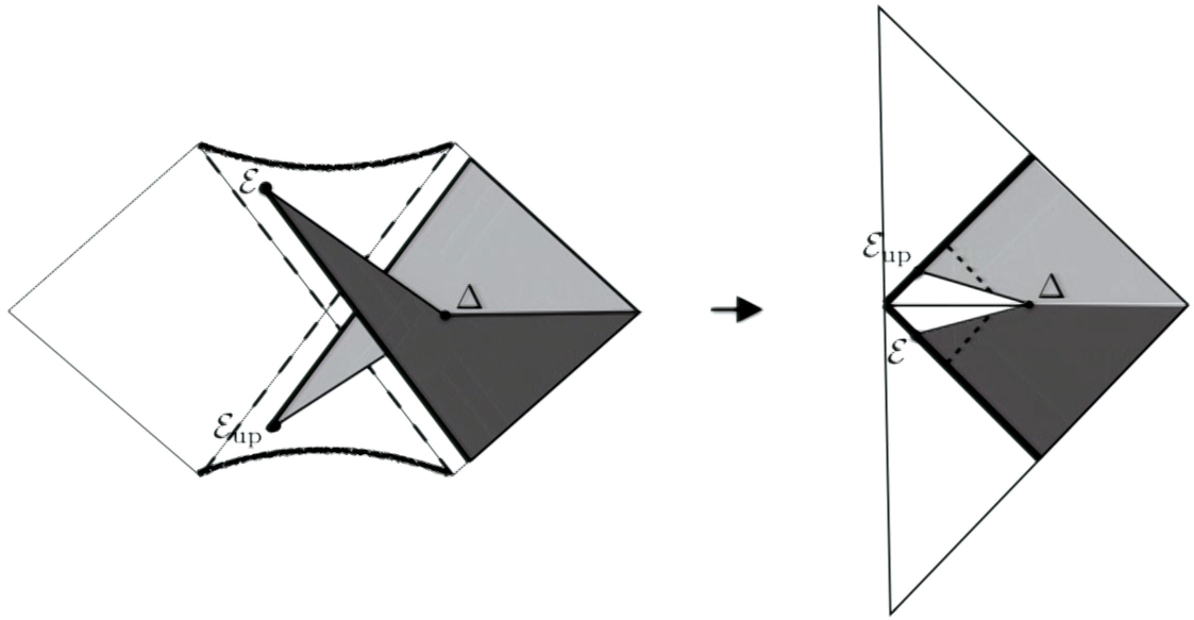
Naïve classicality parameter

$$q = \ell_{\mathsf{Pl}}\, \mathcal{R}_K\, \tau_R,$$

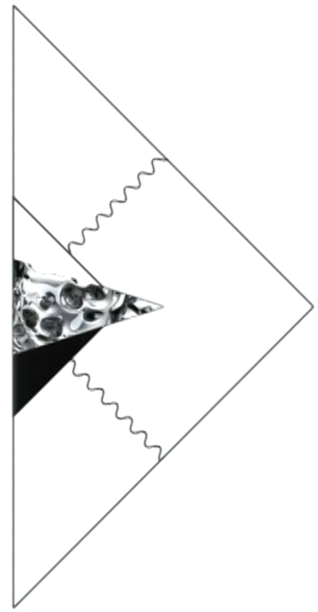
with $\mathcal{R}_K \sim \frac{M}{R^3}$ the curvature scale and $q << 1$ means classical.

q can be near 1. It has a maximum at $R_q = \frac{7}{6}(2M)$ (out horizon). Requiring $q \sim 1$ gives $\tau_q \sim M^2$.

The spacetime



This metric sets up boundary conditions to do a complex tunneling calculation for black to white hole transitions



Strategy: study $\mathbb{C}$ solutions to the symmetry reduced Einstein eqns with these b.c. This midisuperspace approach looks promising for full path integral calculation...but work is still in progress.

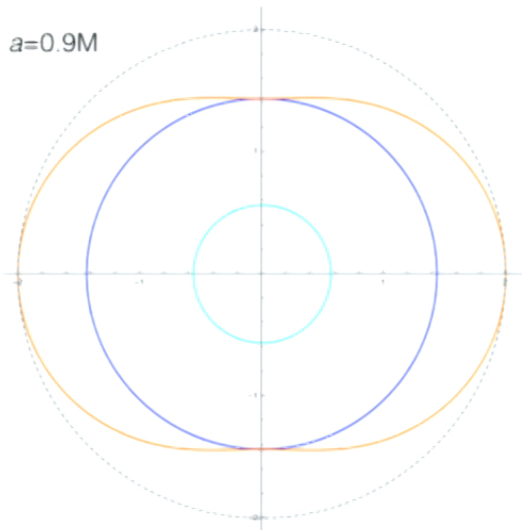
27

$$ds^2_{\mathsf{Kerr}} = -\frac{\rho^2\Delta}{\Sigma}dt^2 + \frac{\Sigma}{\rho^2}\sin^2\theta(d\phi - \omega dt)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2,$$

where $\rho^2 = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2Mr + a^2$, and

$$\Sigma = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta$$

$$\omega \equiv -\frac{g_{t\phi}}{g_{\phi\phi}} = -\frac{2Mar}{\Sigma} \underset{r\to\infty}{\sim} \frac{2J}{r^3}, \quad M = \text{ADM mass}, \quad \& \quad a \equiv \frac{J}{M}.$$



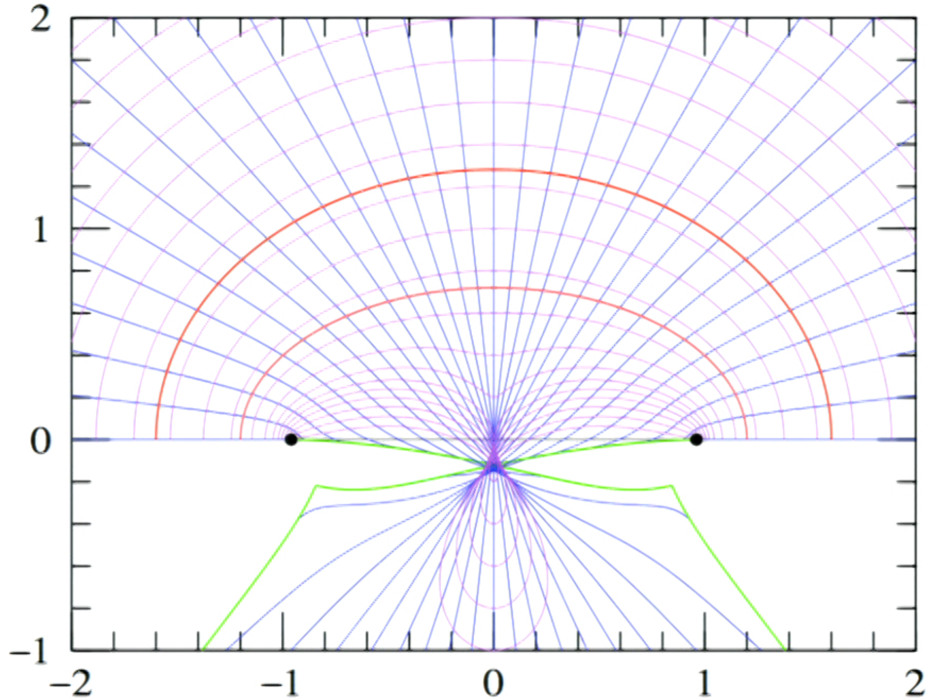
$$r_\pm = M \pm \sqrt{M^2 - a^2}$$

$$R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} \equiv \mathcal{R}_K$$

$$= \frac{48M^2(r^2 - a^2\cos^2\theta)(\rho^4 - 16a^2r^2\cos^2\theta)}{\rho^{12}}$$

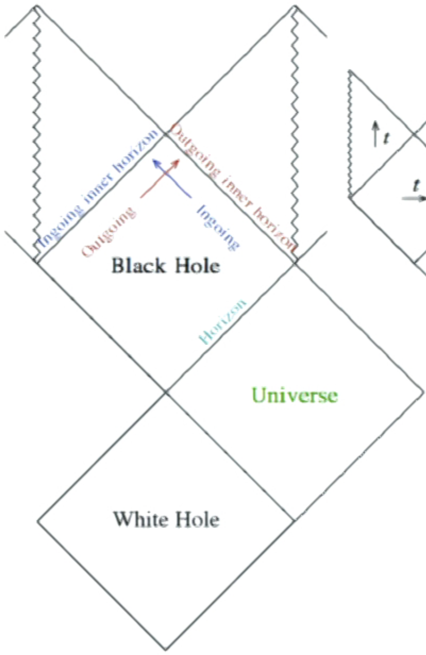
30

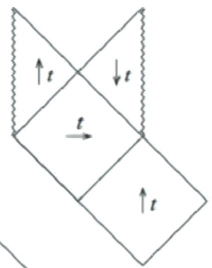
Pretorius & Israel [CQG '98] remedy this and construct a hypersurface orthogonal congruence, the quasi-spherical lightcones

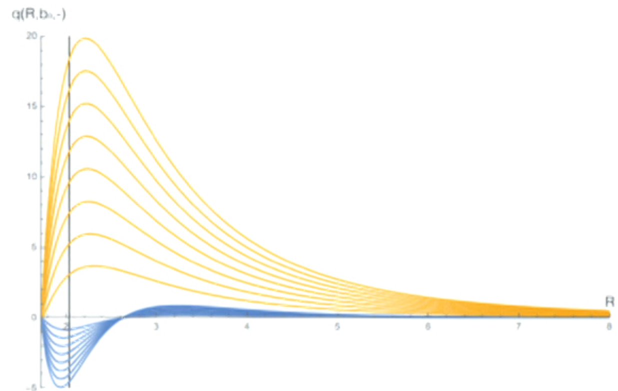


Key idea: allow Q to vary over the members of the congruence

Pretorius & Israel light cones open the way to rotating Kerr fireworks:







$R_{q,\mathrm{lim}} \approx 2.03$

Calculation of classicality estimate $q \equiv \ell_{\mathsf{Pl}}^2 \mathcal{R}^2 \tau_{R,\theta}^2$ suggests again that there could be quantum effects *outside* Kerr horizon.

- Collapsing matter bounces in a short time locally but a long time from far away, $\sim M^2$.
  Solar mass: $\tau_q \sim 10^{32}$ sec, $\tau_H \sim 10^{75}$ sec, $\tau_U \sim 10^{17}$ sec.

- Possible to describe using a metric with no singularity, two trapped regions, and all matter exiting $\rightsquigarrow$ all info escapes

- Could a black hole be a bouncing star seen in super slow motion? With the black to white hole metric we can attack this question rigorously using complex tunneling techniques.

- It would be particularly exciting if quantum black holes leave observable signatures in FRBs or an imprint on the near horizon structure measured with the Event Horizon Telescope