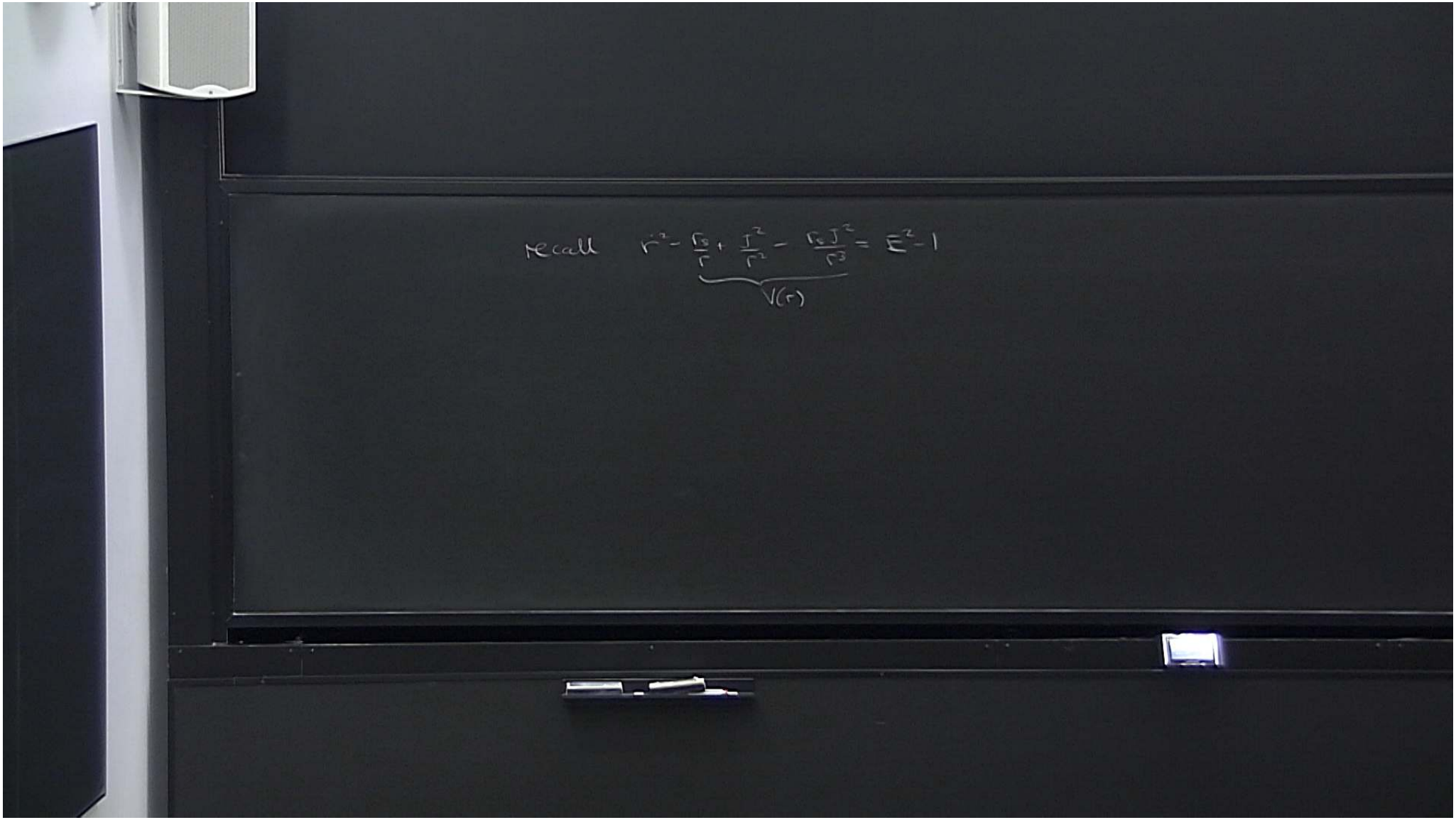
Title: PSI 17/18 - Relativity - Lecture 15

Date: Sep 22, 2017 02:00 PM

URL: http://pirsa.org/17090069

Abstract:

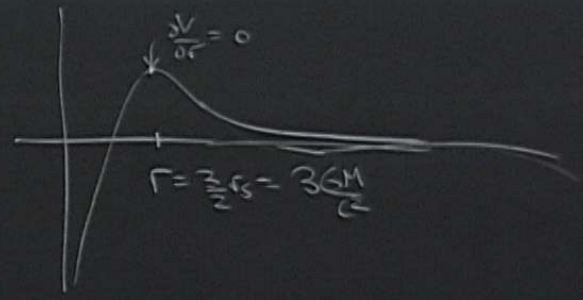

Recall $\dot{r}^2 - \underbrace{\frac{\Gamma_0}{r} + \frac{J^2}{r^2} - \frac{\Gamma_0 J^2}{r^3}}_{V(r)} = E^2 - 1$

Recall $\dot{r}^2 - \frac{r_s}{r} + \frac{J^2}{r^2} - \frac{r_s J^2}{r^3} = E^2 - 1$

$V(r)$

$$-\frac{2J^2}{r^3} + 3\frac{r_s J^2}{r^4} = 0$$

$$\Rightarrow r = \frac{3}{2} r_s$$

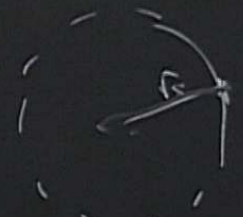


Fall radially into a black hole

$J=0$

$$\left(\frac{dr}{d\tau}\right)^2 + 1 - \frac{r_s}{r} = E^2$$

Start at $r=r_s$, $\frac{dr}{d\tau}=0 \Rightarrow E=0$



proper time to reach $r<r_s$.

$$\tau(r) = \int_r^{r_s} \frac{dr}{\sqrt{\frac{r_s}{r}-1}}$$

time to hit sing.!

$$\int_0^{r_s} \frac{dr}{\sqrt{\frac{r_s}{r}-1}} = \int_0^{r_s} \frac{dr\sqrt{r}}{\sqrt{r_s-r}}$$

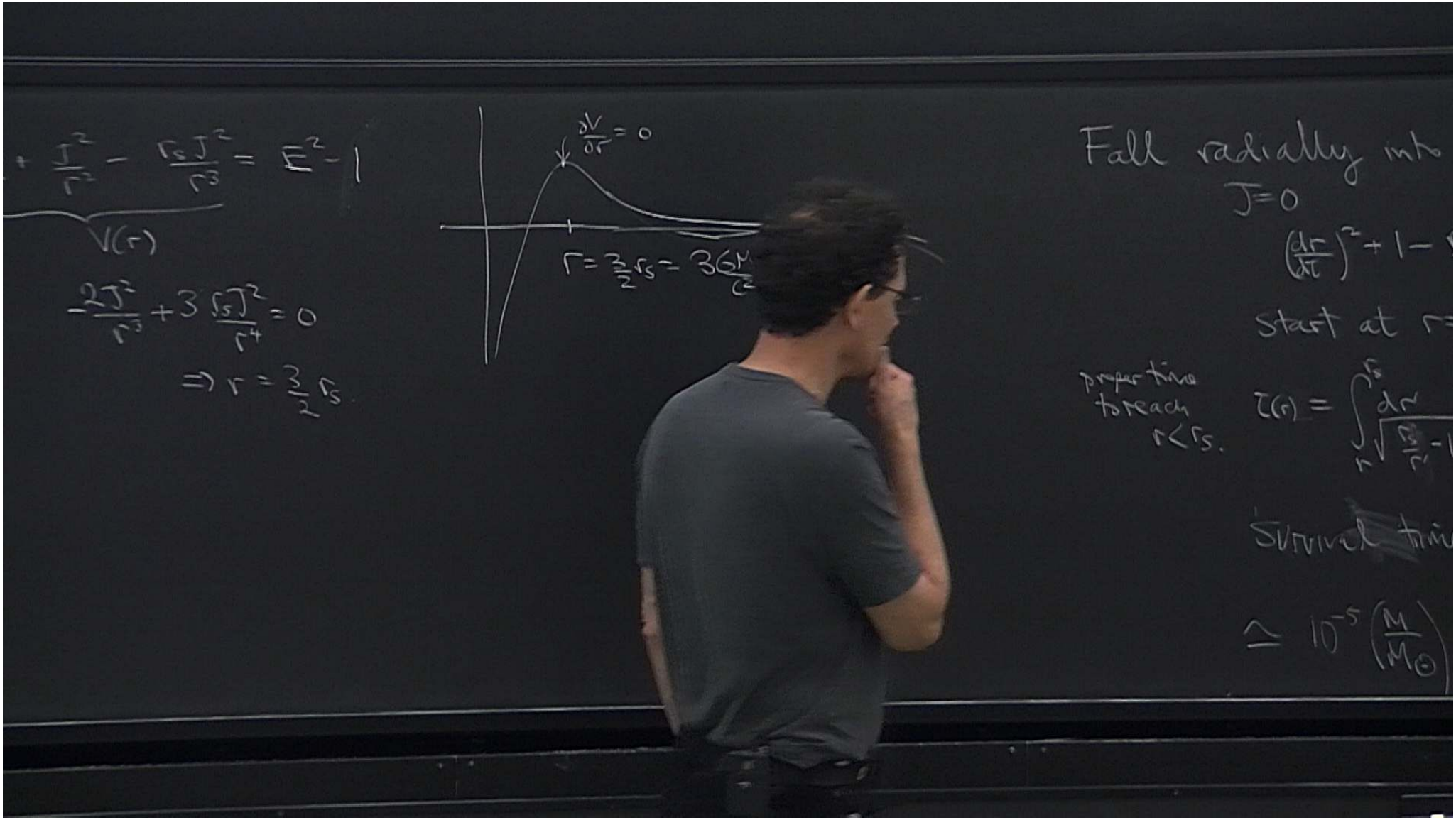
Survival time $= \frac{\pi r_s}{2} = \frac{\pi G M}{c^3}$

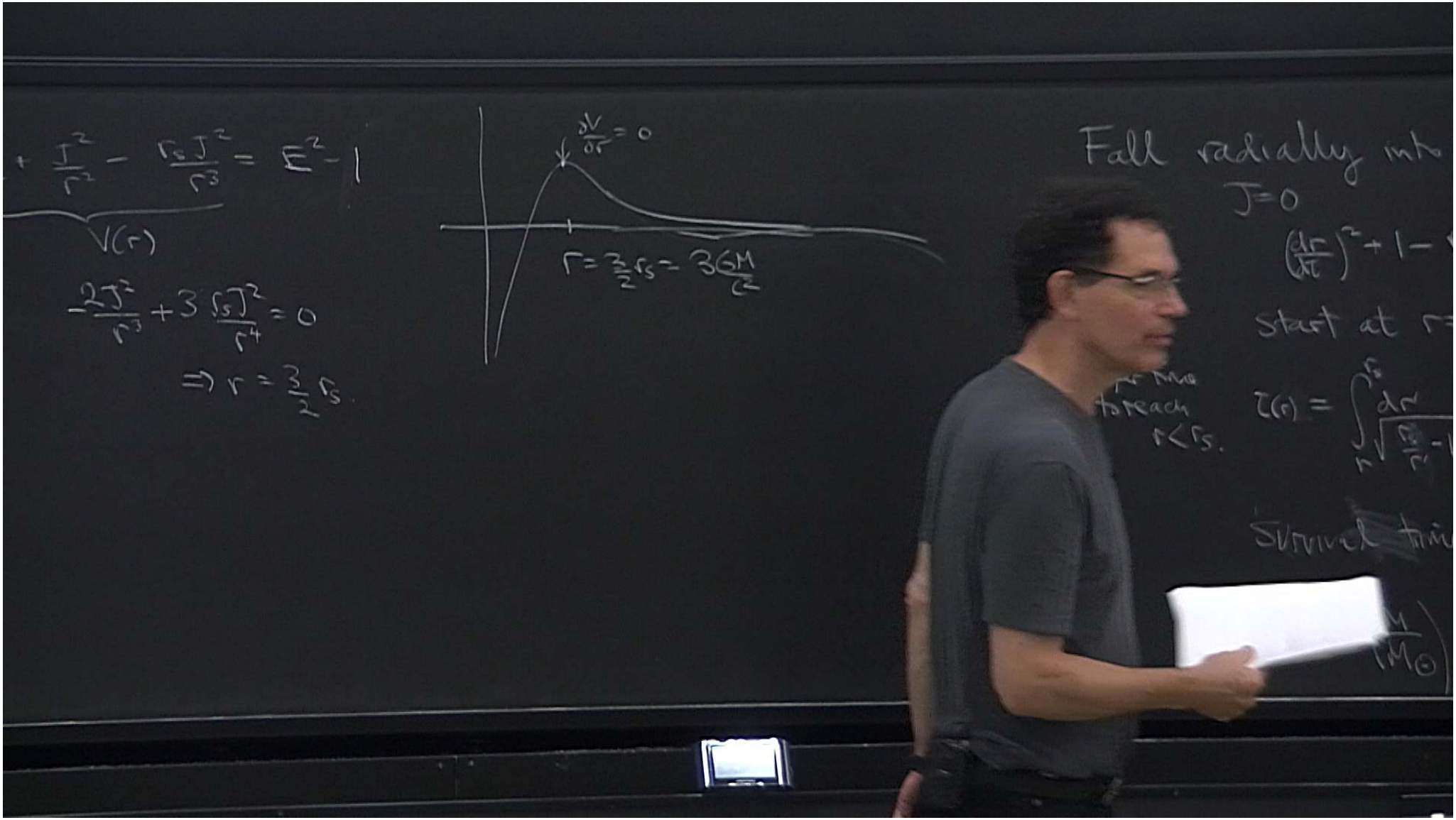
$r = r_s \sin^2\theta$

$$2r_s \int_0^{\pi/2} d\theta \sin^3\theta \qquad \sin^3\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{r_s \pi}{2}$$

$\simeq 10^{-5}\left(\frac{M}{M_\odot}\right)$ seconds


$+\frac{J^2}{r^2} - \frac{r_s J^2}{r^3} = E^2 - 1$
$V(r)$
$-\frac{2J^2}{r^3} + 3\frac{r_s J^2}{r^4} = 0$
$\Rightarrow r = \frac{3}{2} r_s$
$\frac{dV}{dr} = 0$
$r = \frac{3}{2} r_s = \frac{3GM}{c^2}$
Fall radially into
$J = 0$
$\left(\frac{dr}{d\tau}\right)^2 + 1 -$
Start at $r =$
proper time to reach $r < r_s$.
$\tau(r) = \int_r^{r_s} \frac{dr}{\sqrt{\frac{r_s}{r} - 1}}$
Survival time
$\simeq 10^{-5} \left(\frac{M}{M_\odot}\right)$


$\frac{1}{2}\frac{J^2}{r^2} - \frac{r_s J^2}{r^3} = E^2 - 1$
$V(r)$
$-\frac{2J^2}{r^3} + 3\frac{r_s J^2}{r^4} = 0$
$\Rightarrow r = \frac{3}{2} r_s$
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$r = \frac{3}{2} r_s = \frac{3GM}{c^2}$
Fall radially into
$J = 0$
$\left(\frac{dr}{dt}\right)^2 + 1 -$
Start at $r =$
to reach
$r < r_s$.
$\tau(r) = \int_r^{r_s} \frac{dr}{\sqrt{\frac{r_s}{r} - 1}}$
Survival time
$M_\odot$

$= E^2 - 1$

$\frac{\partial V}{\partial r} = 0$

$r = \frac{3}{2} r_s = 3\frac{GM}{c^2}$

$= 0$

$= \frac{3}{2} r_s$

Fall radially into a black hole

$J = 0$

$$d\tau = -\int_{r_s}^{r} \frac{dr}{\sqrt{\frac{r_s}{r} - 1}}$$

$$\left(\frac{dr}{d\tau}\right)^2 + 1 - \frac{r_s}{r} = E^2$$

Start at $r = r_s$, $\frac{dr}{d\tau} = 0 \Rightarrow E = 0$

proper time to reach $r < r_s$.

$$\tau(r) = +\int_{r}^{r_s} \frac{dr'}{\sqrt{\frac{r_s}{r'} - 1}}$$

time to hit sing'y

$$\int_0^{r_s} \frac{dr}{\sqrt{\frac{r_s}{r} - 1}} = \int_0^{r_s} \frac{dr\sqrt{r}}{\sqrt{r_s - r}}$$

Survival time $= \frac{\pi r_s}{2} = \frac{\pi G M}{c^3}$

$r = r_s \sin^2\theta$

$$2 r_s \int_0^{\pi/2} d\theta \underbrace{\sin^2\theta}_{\frac{1-\cos 2\theta}{2}} = \frac{r_s \pi}{2}$$

$\simeq 10^{-5} \left(\frac{M}{M_\odot}\right)$ seconds

Geometry of the Schwarzchild spacetime

$$ds^2 = -\alpha(r)\,dt^2 + \frac{1}{\alpha(r)}dr^2 + r^2 d\Omega_2^2 \quad , \quad \alpha(r) = 1 - \frac{r_s}{r}.$$

What happens at $r = r_s$? For $r < r_s$, r is a timelike coordinate, Schwarzchild is not static!

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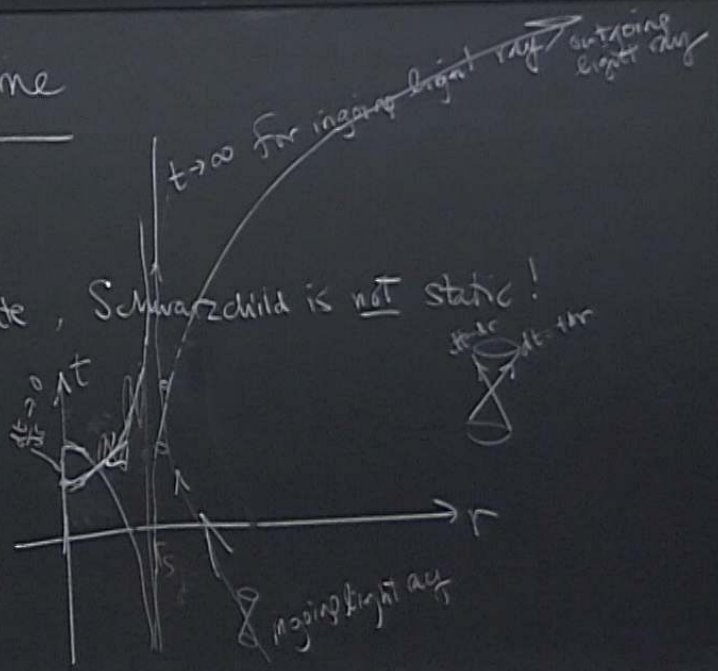
Radial light signals (null trajectories) $\quad ds^2 = 0 \Rightarrow dt = \pm \frac{dr}{\alpha(r)} = \pm dr \left(1 + \frac{r_s}{r - r_s}\right)$



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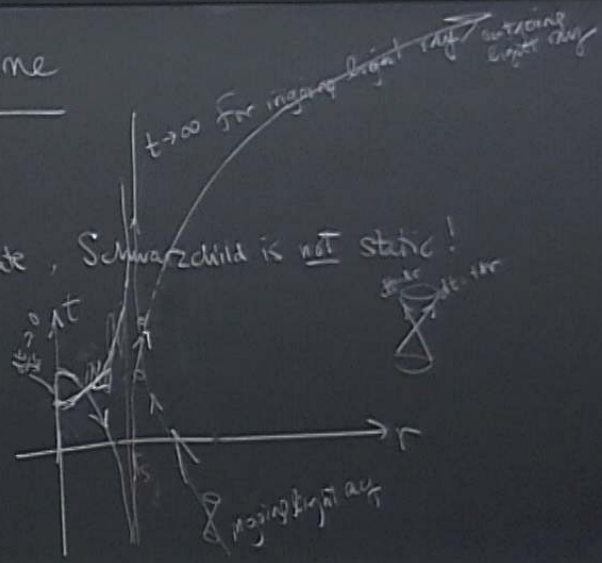

Idea: change coords to coordinates in which the metric is nonsingular at $r = r_s$

Integrate ① $\quad t = \pm\left(r + r_s \ln\left(\frac{r}{r_s} - 1\right)\right) + \text{const} \quad$ for $r > r_s$

$\qquad = \pm\left(r + r_s \ln\left(1 - \frac{r}{r_s}\right)\right) + \text{const} \quad$ for $r < r_s$

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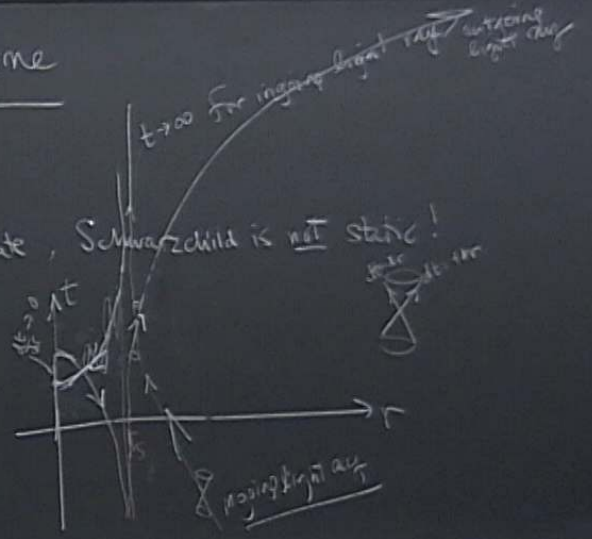
$$ds^2 = -\alpha(r)\,dt^2 + \frac{1}{\alpha(r)}dr^2 + r^2 d\Omega_2^2 \quad , \quad \alpha(r) = 1 - \frac{r_s}{r}.$$

What happens at $r = r_s$? For $r < r_s$, r is a <u>timelike</u> coordinate, Schwarzschild is <u>not</u> static!



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Ingoing light ray: $t = -r - r_s \ln\left|\frac{r}{r_s} - 1\right| + \text{const}$

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Integrate ①

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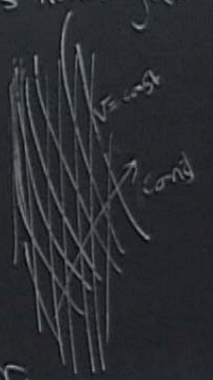
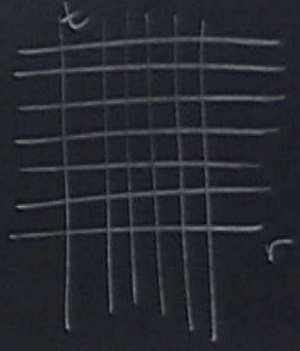
Ingoing light ray: $t = -r - r_s \ln\left|\frac{r}{r_s} - 1\right| + \text{const.}$

Choose new "ingoing coordinate" $v = t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|$, $r' = r$





Idea: change coords to coordinates in which the metric is nonsingular at $r = r_s$

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$= \pm\left(r + r_s \ln\left(1 - \frac{r}{r_s}\right)\right) + \text{const}$ for $r < r_s$

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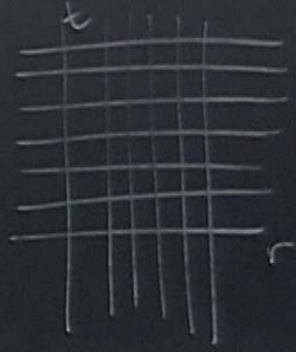
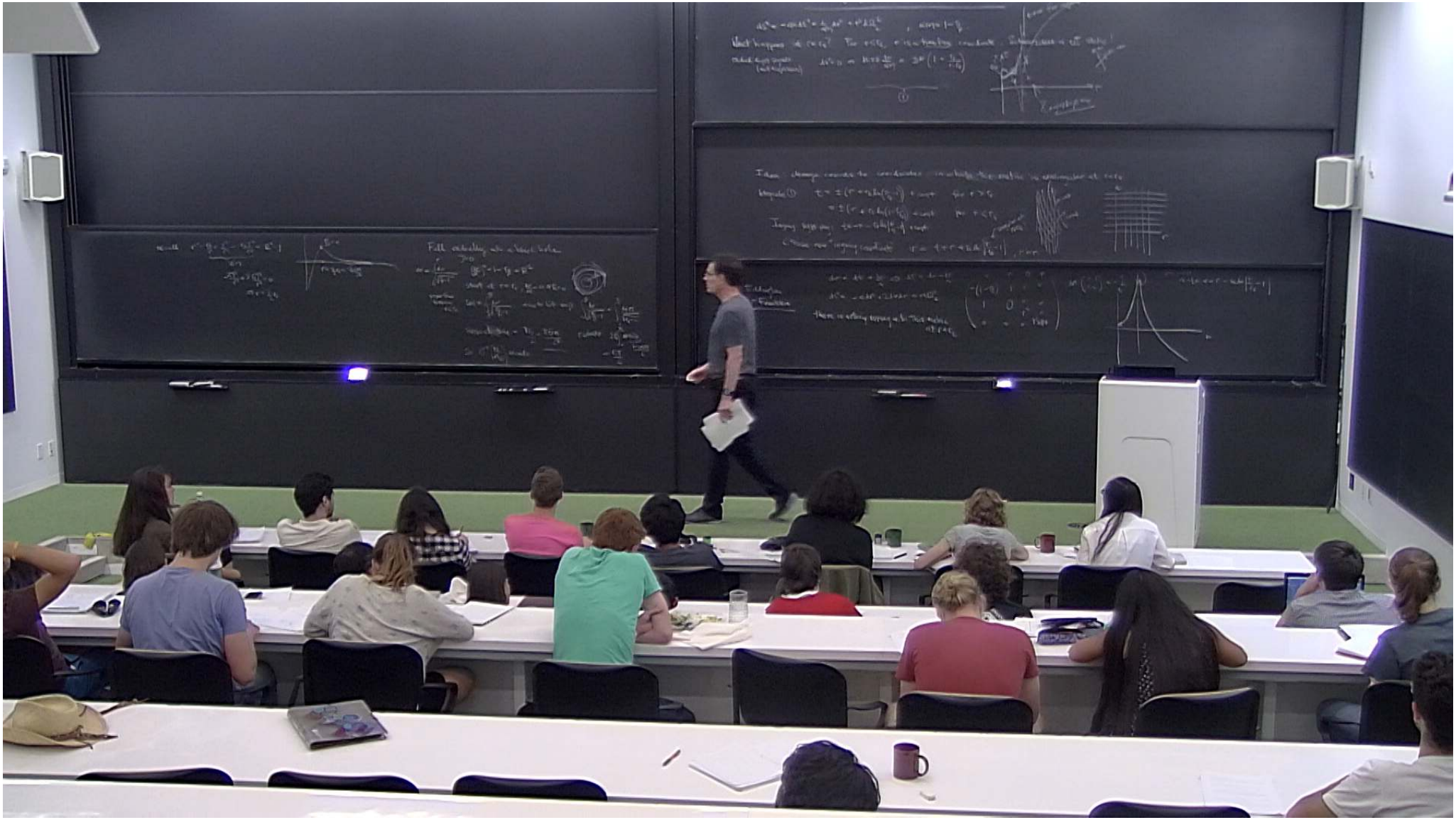
↙ singular at $r = r_s$!



$dv = dt + \frac{dr}{\alpha}$



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Integrate ①
$$t = \pm\left(r + r_s \ln\left(\frac{r}{r_s} - 1\right)\right) + \text{const} \quad \text{for } r > r_s$$
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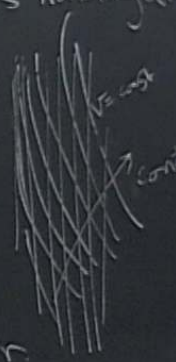
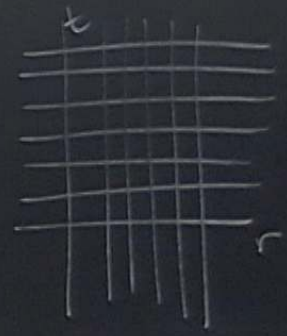
Choose new "ingoing coordinate" $v = t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|$, $r' = r$ ← singular at $r = r_s$!





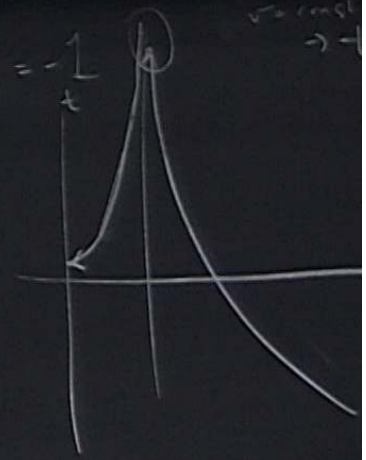
$$dv = dt + \frac{dr}{\alpha} \Rightarrow dt = dv - \frac{dr}{\alpha}$$

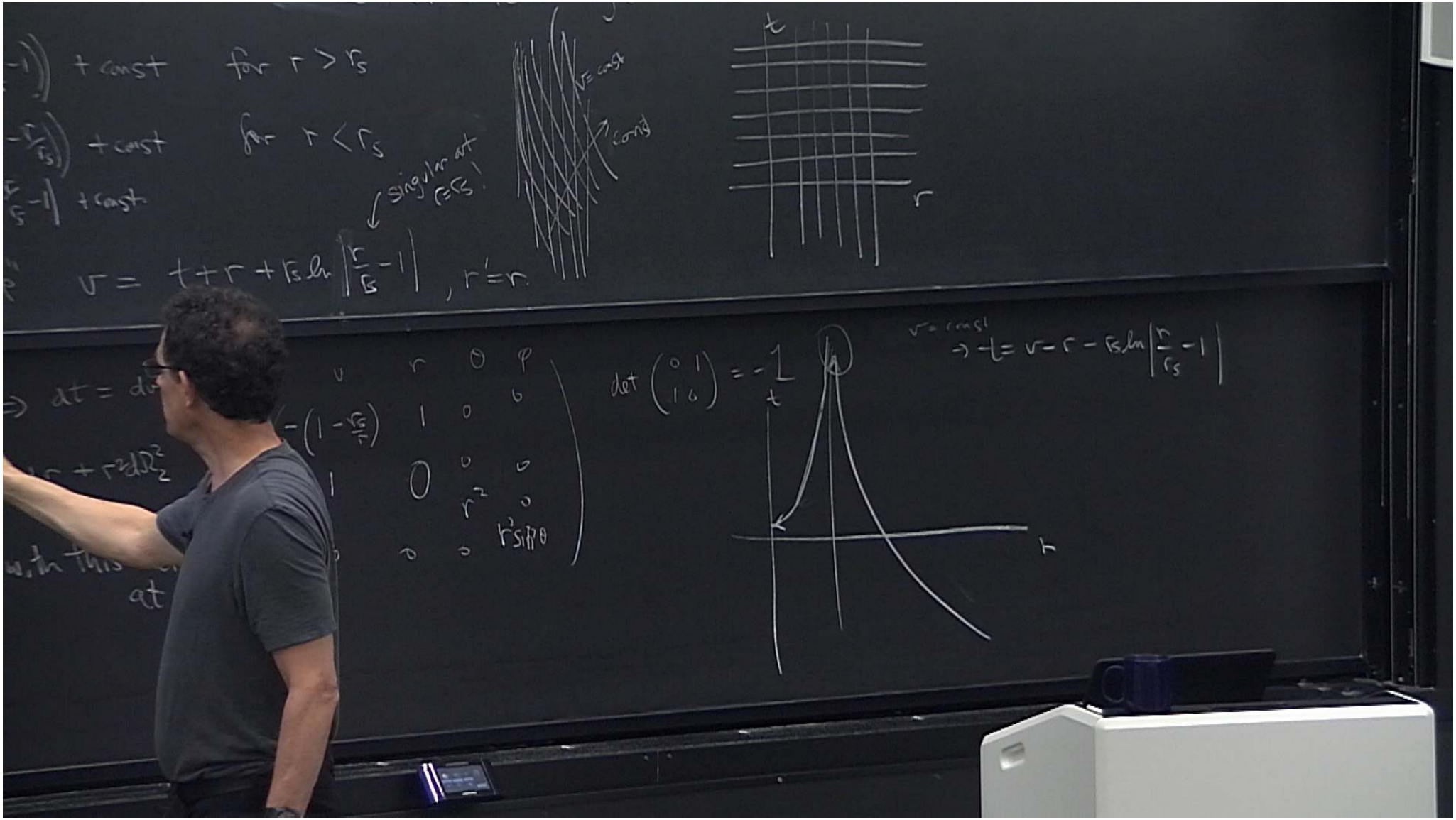
$$ds^2 = -\alpha\, dv^2 + 2\, dv\, dr + r^2 d\Omega_2^2$$

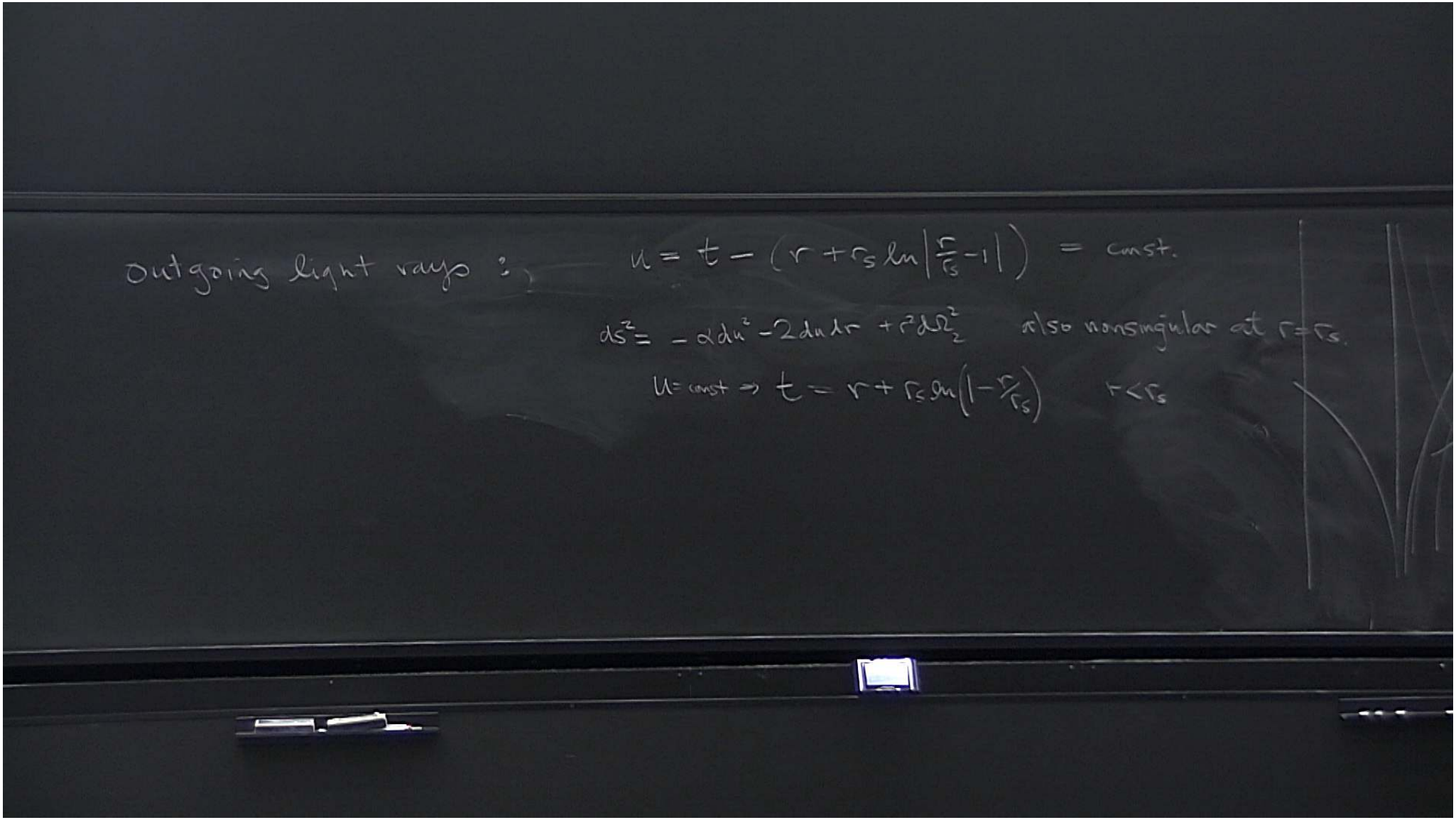
Eddington-Finkelstein

there is nothing wrong with this metric at $r = r_s$.

$$\begin{pmatrix} -\left(1 - \frac{r_s}{r}\right) & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \qquad \det\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -1$$



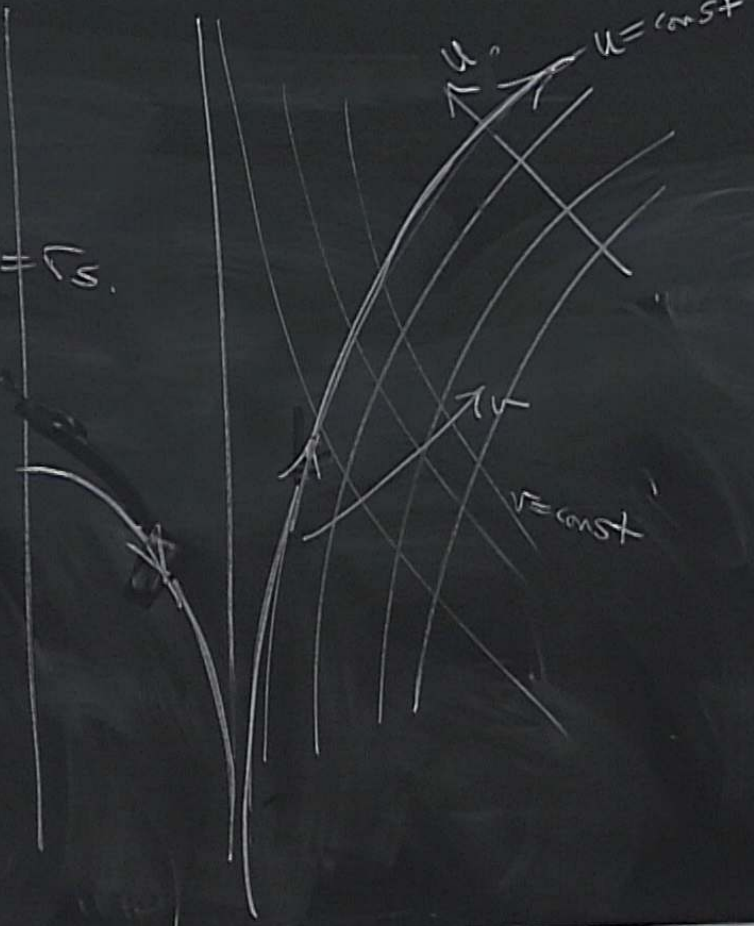

+ const for r > rs
+ const for r < rs
+ const
singular at r=rs!
v = t + r + rs ln |r/rs - 1| , r' = r
v = const
const
t
r
det (0 1 / 1 0) = -1
v = const
→ -t = v - r - rs ln |r/rs - 1|
t
r


outgoing light rays :
$u = t - \left(r + r_s \ln\left|\frac{r}{r_s} - 1\right|\right) = \text{const.}$
$ds^2 = -\alpha du^2 - 2\,du\,dr + r^2 d\Omega_2^2$ also nonsingular at $r = r_s$.
$u = \text{const} \Rightarrow t = r + r_s \ln\left(1 - \frac{r}{r_s}\right)$ $r < r_s$

$= \text{const.}$

also nonsingular at $r = r_s$.

$r < r_s$

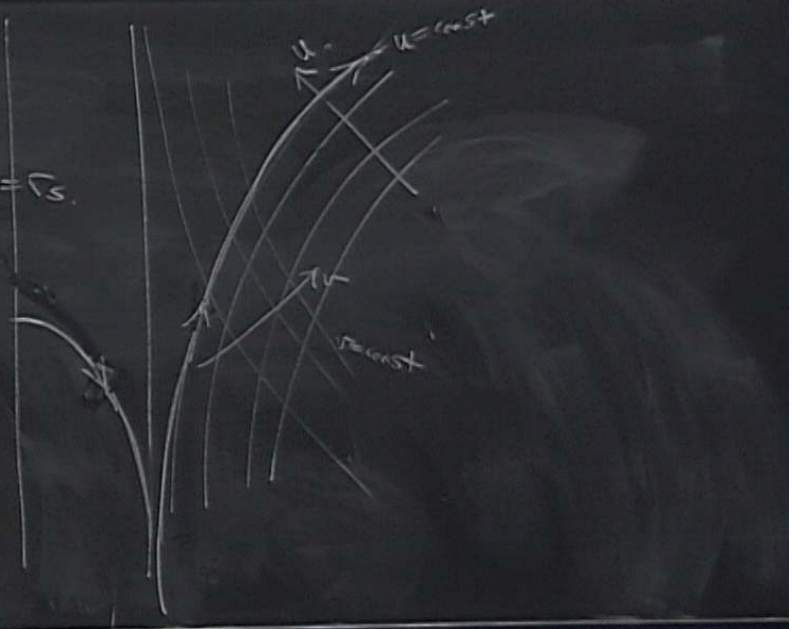


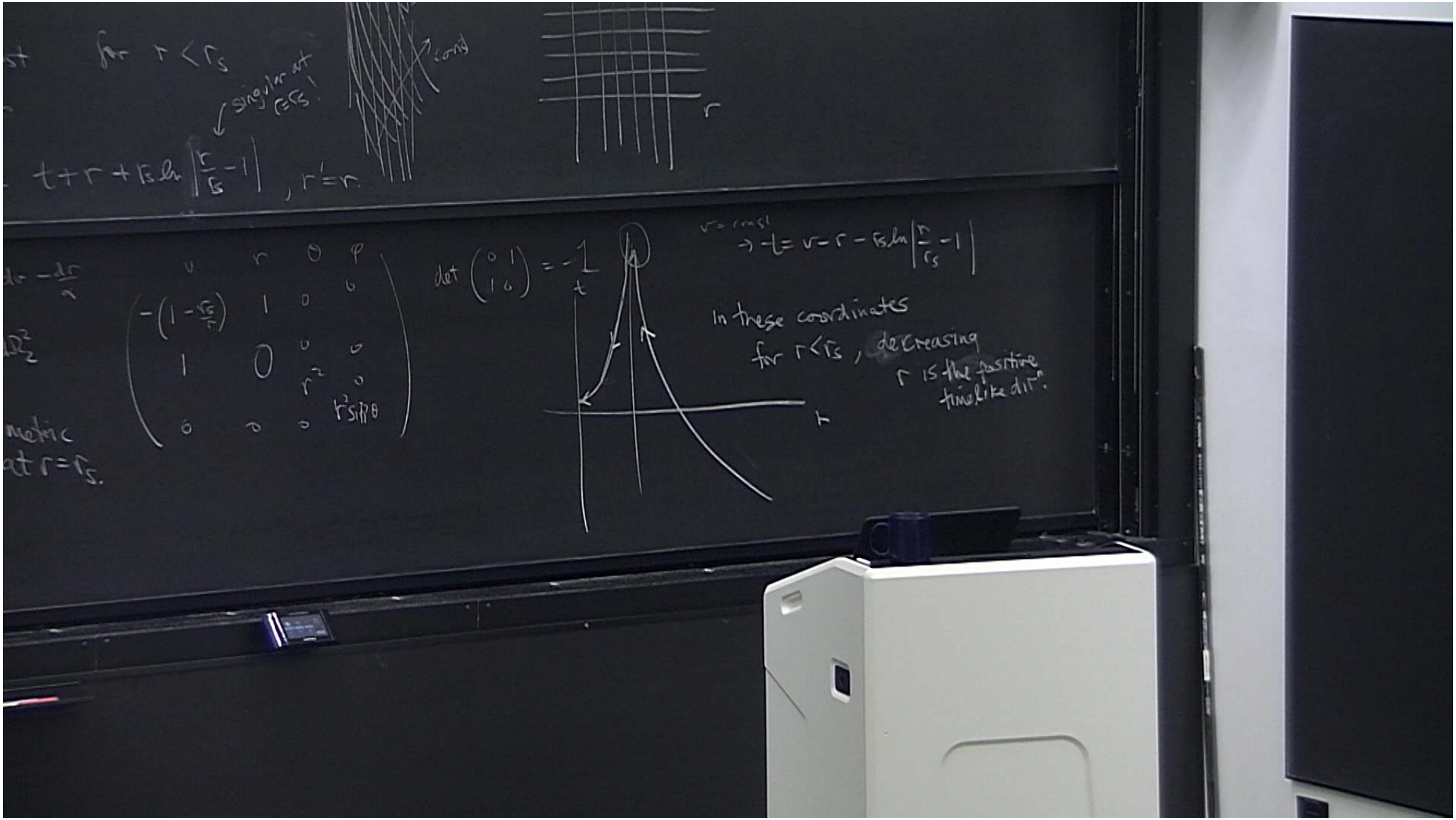
$$u = t - \left(r + r_s \ln\left|\frac{r}{r_s} - 1\right| \right) = \text{const.}$$

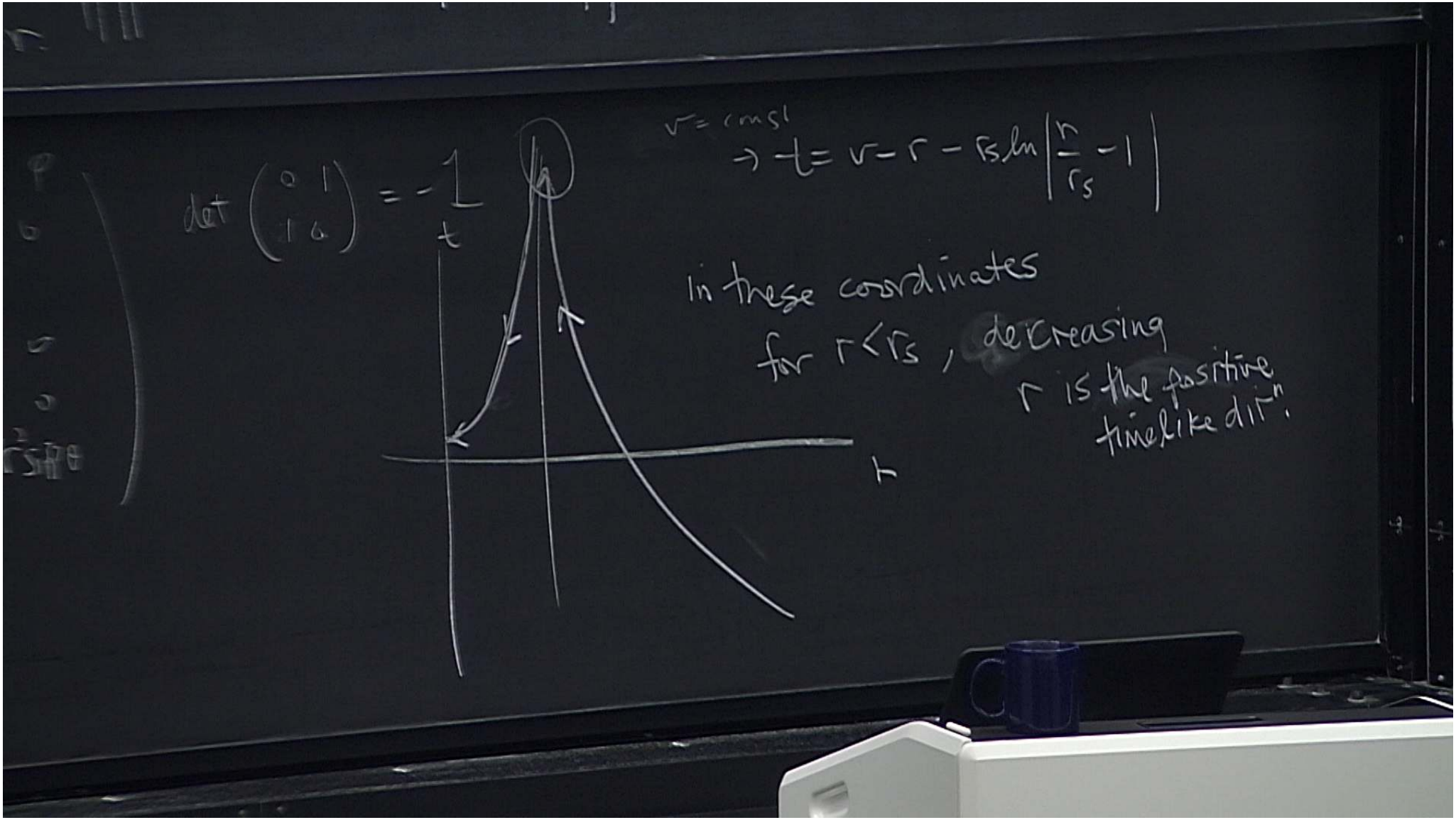
$$ds^2 = -\alpha du^2 - 2 du\, dr + r^2 d\Omega_2^2 \qquad \text{also nonsingular at } r = r_s.$$

$$u = \text{const} \Rightarrow t = r + r_s \ln\left|1 - \frac{r}{r_s}\right| \qquad r < r_s$$

flow of time is in the increasing r direction for $r < r_s$




for r < rs
singular at r=rs!
const
r
t + r + rs ln |r/rs − 1| , r' = r.
det (0 1 / 1 0) = −1
v = const
→ −t = v − r − rs ln |r/rs − 1|
In these coordinates
for r < rs , decreasing
r is the positive
timelike dir^n.
metric
at r = rs.


$\det \begin{pmatrix} 0 & 1 \\ 1 & a \end{pmatrix} = -\frac{1}{t}$
$v = ct + r$
$\Rightarrow -t = v - r - r_s \ln\left|\frac{r}{r_s} - 1\right|$
in these coordinates
for $r < r_s$, decreasing
r is the positive
timelike dirⁿ.
t

flow of time is in the increasing r direction for $r < r_s$

$\Rightarrow$ terrible contradiction.

will turn out that both are true but neither is complete.

what does other trajectory do in ingoing coords?

2nd null trajectory $dv = 2\frac{dr}{\alpha}$

$$v = 2\left(r + r_s \ln\left|\frac{r}{r_s} - 1\right|\right) + \text{const}$$

$$= t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|$$

$$\Rightarrow t = r + r_s \ln\left|\frac{r}{r_s} - 1\right|$$

flow of time is in the increasing r direction for $r < r_s$

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2nd null trajectory $dv = 2\frac{dr}{\alpha}$

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$$= t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|$$

$$\Rightarrow t = r + r_s \ln\left|\frac{r}{r_s} - 1\right| + \text{const}$$

$= t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|)$ → "outg

$\Rightarrow t = r + r_s \ln\left|\frac{r}{r_s} - 1\right| + \text{const}$

M. Kruskal
Phys Rev
119, 1743 (1960)

Put this together : for $r > r_s$ $\quad v = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$

$u = t - r - r_s \ln\left(\frac{r}{r_s} - 1\right)$

$dv = dt + \frac{dr}{1 - r_s/r}$

$du = dt - \frac{dr}{1 - r_s/r}$

$\rightarrow ds^2 = -\left(1 - \frac{r_s}{r}\right) du\, dv + r^2 d\Omega^2$

$\Rightarrow t = r + r_s \ln\left|\frac{r}{r_s} - 1\right| + \text{const}$

arrow of time reversed.

for $r > r_s$

$$v = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

$$u = t - r - r_s \ln\left(\frac{r}{r_s} - 1\right)$$

$$dv = dt + \frac{dr}{1 - r_s/r}$$

$$du = dt - \frac{dr}{1 - r_s/r}$$

$$\Rightarrow ds^2 = -\left(1 - \frac{r_s}{r}\right) du\, dv + r^2 d\Omega_2^2$$

where r defined by

$$r + r_s \ln\left(\frac{r}{r_s} - 1\right) = \frac{v - u}{2}$$

$r_s \ln\left(\frac{r}{r_s} - 1\right)$

$- r_s \ln\left(\frac{r}{r_s} - 1\right)$

$\Rightarrow ds^2 = -\left(1 - \frac{r_s}{r}\right) du\, dr + r^2 d\Omega_2^2$ where r defined by

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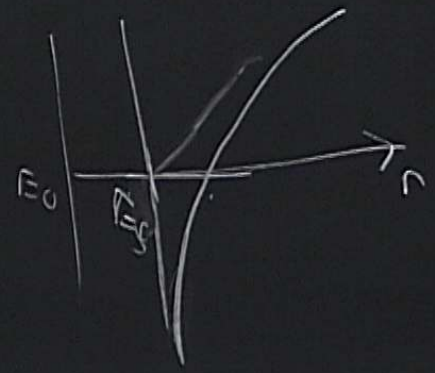


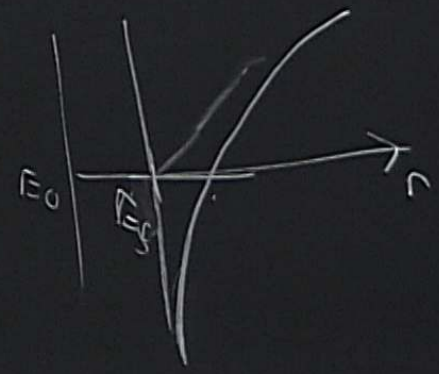
$= t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$

$= t - r - r_s \ln\left(r/r_s - 1\right)$

$\Rightarrow ds^2 = -\left(1 - r_s/r\right) du\, dv + r^2 d\Omega_2^2$ where r defined by

$r + r_s \ln\left(\frac{r}{r_s} - 1\right) = \frac{v-u}{2}$

$1 - \frac{r_s}{r} = \frac{r_s}{r} e^{-r/r_s} e^{\frac{v-u}{2r_s}}$



$$\Rightarrow ds^2 = -\frac{r_s}{r} e^{-r/r_s} e^{\frac{v-u}{2r_s}} du\,dv + r^2 d\Omega_2^2 \qquad \frac{r}{r_s} - 1 = e^{\frac{r}{r_s}} e^{\frac{r}{2r_s}}$$

define $U = r_s e^{-u/2r_s}$ $\quad V = r_s e^{+v/2r_s}$

$U = X - T$
$V = X + T$

$dU\,dV = dX^2 - dT^2$

$$\Rightarrow ds^2 = +4\frac{r_s}{r} e^{-r/r_s} dU\,dV + r^2 d\Omega_2^2$$

$$= 4\frac{r_s}{r} e^{-r/r_s} (dX^2 - dT^2) + r^2 d\Omega_2^2$$

Kruskal metric

flow of time is in the increasing r direction for $r < r_s$

$\Rightarrow$ terrible contradiction.

will turn out that both are true but neither is complete.

$V = X + T$

$dU\,dV = dX^2 - dT^2$

$\Rightarrow ds^2 = \ldots + 4\frac{r_s}{r} e^{\ldots} dU\,dV + \ldots$

$= 4\frac{r_s}{r} e^{-r/r_s}\left(dX^2 - dT^2\right) + r^2 d\Omega_2^2$

Kruskal metric

$t = \frac{U+V}{2} = r_s \ln\left(\frac{V}{U}\right)$

define $U = r_s e^{-u/2r_s}$ $\quad V = r_s e^{+v/2r_s}$ $\quad \rightarrow \quad UV - r_s^2 e^{\frac{r-u}{2r_s}} = r_s^2$

$= X^2 - T^2$

$t = \frac{u+v}{2} = r_s \ln\left(\frac{V}{U}\right)$

$U = X - T$

$V = X + T$

$dU\,dV = dX^2 - dT^2$

$\Rightarrow ds^2 = + 4\frac{r_s}{r} e^{-r/r_s} dU\,dV + r^2 d\Omega_2^2$

$= 4\frac{r_s}{r} e^{-r/r_s}\left(dX^2 - dT^2\right) + r^2 d\Omega_2^2$

Kruskal metric

ing light rays :) $\quad u = t - \left(r + r_s \ln\left|\frac{r}{r_s} - 1\right|\right) = \text{const.}$

ds^2 $\quad du^2 - 2\,du\,dr + r^2 d\Omega_2^2$ $\quad$ also nonsingular at $r = r_s$.

$\Rightarrow t = r + r_s \ln\left|\frac{r}{r_s} - 1\right|$ $\quad r < r_s$

$= r_s e^{+r/2r_s}$

$\longrightarrow \quad UV - r_s^2 e^{\frac{r-u}{2r_s}} = r_s^2 e^{r/r_s}\left(\frac{r}{r_s}-1\right)$ defines $r(x,T)$

$= X^2 - T^2$

$+ 4\frac{r_s}{r} e^{-r/r_s} dU dV + r^2 d\Omega_2^2$

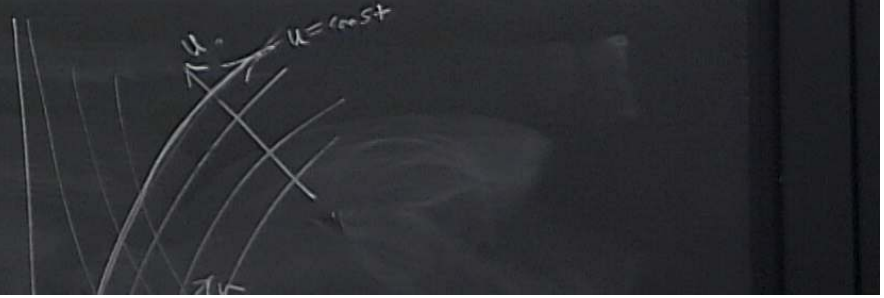
$t = \frac{u+v}{2} = r_s \ln\left(\frac{V}{U}\right) = r_s \ln\left(\frac{X+T}{X-T}\right)$

$= 4\frac{r_s}{r} e^{-r/r_s}\left(dX^2 - dT^2\right) + r^2 d\Omega_2^2$

Kruskal metric

$= t - \left(r + r_s \ln\left|\frac{r}{r_s}-1\right|\right) = \text{const.}$

nonsingular at $r = r_s$.



define $U = r_s e^{-u/2r_s}$ $\quad V = r_s e^{v/2r_s}$ $\quad \rightarrow UV = r_s^2 e^{\frac{v-u}{2r_s}} = -(X^2 - T^2)$

$t = \frac{u+v}{2} = r_s \ln$

$U = X - T$

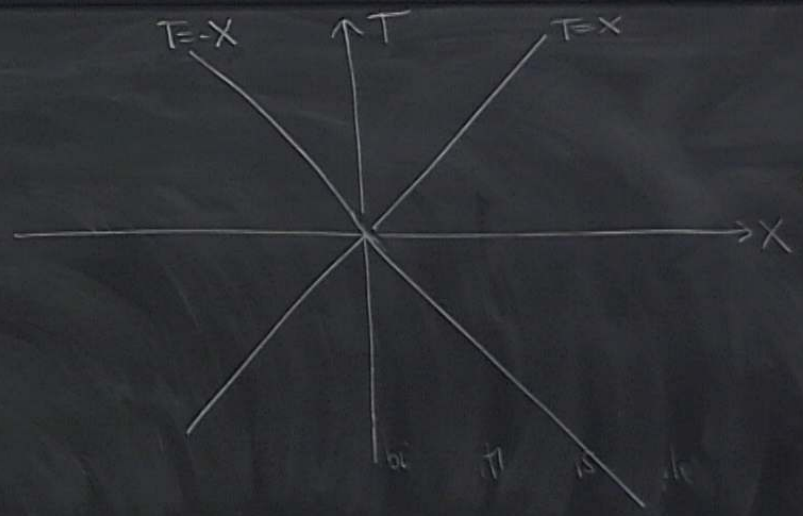
$V = X + T$

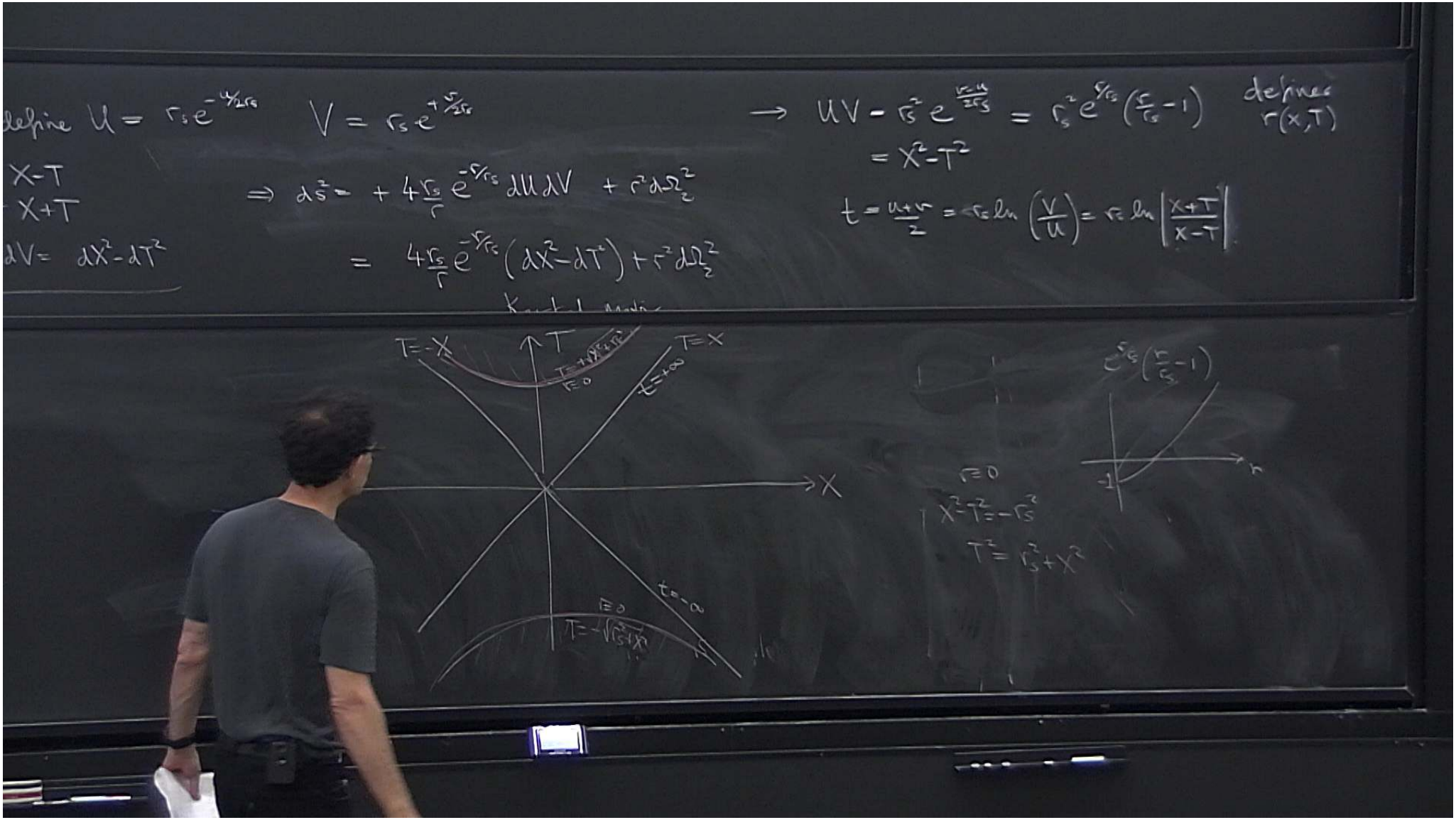
$dU\,dV = dX^2 - dT^2$

$\Rightarrow ds^2 = +\frac{4r_s}{r} e^{-r/r_s} dU\,dV + r^2 d\Omega_2^2$

$= \frac{4r_s}{r} e^{-r/r_s} (dX^2 - dT^2) + r^2 d\Omega_2^2$

Kruskal




define $U = r_s e^{-u/2r_s}$ $V = r_s e^{+v/2r_s}$
$\rightarrow UV - r_s^2 e^{\frac{v-u}{2r_s}} = r_s^2 e^{r/r_s}\left(\frac{r}{r_s}-1\right)$
$= X^2 - T^2$
defines $r(X,T)$
$X = T$
$X + T$
$dV = dX^2 - dT^2$
$\Rightarrow ds^2 = + 4\frac{r_s}{r} e^{-r/r_s} dU dV + r^2 d\Omega_2^2$
$= 4\frac{r_s}{r} e^{-r/r_s}(dX^2 - dT^2) + r^2 d\Omega_2^2$
$t = \frac{u+v}{2} = r_s \ln\left(\frac{V}{U}\right) = r_s \ln\left|\frac{X+T}{X-T}\right|$
T=-X
T
T=X
r=0
t=+∞
X
t=-∞
r=0
$T = -\sqrt{r_s^2 + X^2}$
$e^{r/r_s}\left(\frac{r}{r_s}-1\right)$
r
-1
r=0
$X^2 - T^2 = -r_s^2$
$T^2 = r_s^2 + X^2$

define $U = r_s e^{-u/2r_s}$ $\quad V = r_s e^{+v/2r_s}$ $\quad \rightarrow \quad UV - r_s^2 e^{\frac{r-u}{2r_s}} = r_s^2 e^{r/r_s}\left(\frac{r}{r_s}-1\right)$ defines $r(x,T)$

$X-T$

$X+T$

$= X^2 - T^2$

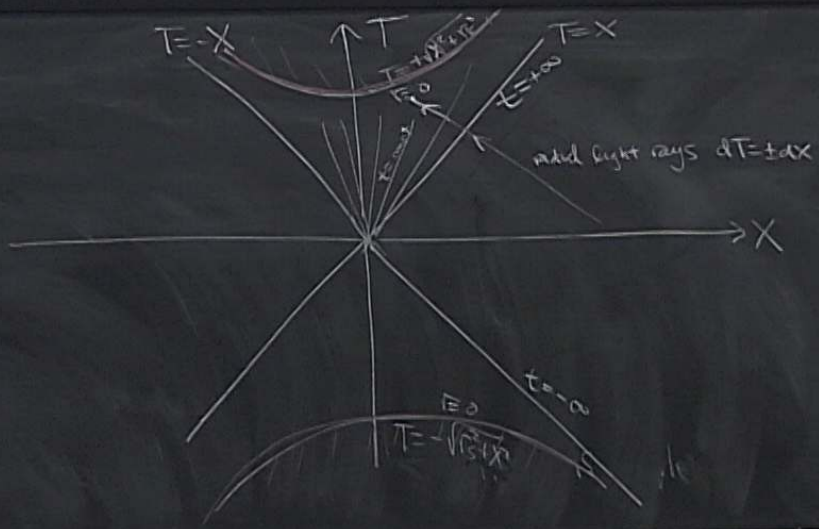
$t = \frac{u+v}{2} = r_s \ln\left(\frac{V}{U}\right) = r_s \ln\left|\frac{X+T}{X-T}\right|$

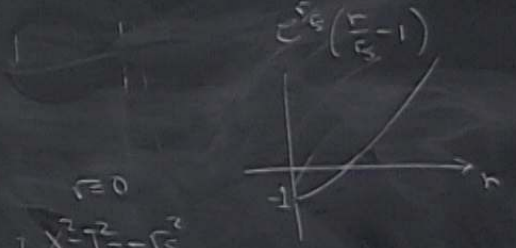
$dV = dX^2 - dT^2$

$\Rightarrow ds^2 = + 4\frac{r_s}{r} e^{-r/r_s} dU\,dV + r^2 d\Omega_2^2$

$= 4\frac{r_s}{r} e^{-r/r_s}\left(dX^2 - dT^2\right) + r^2 d\Omega_2^2$

Kruskal spacetime





$r=0$

$X^2 - T^2 = -r_s^2$

$T^2 = r_s^2 + X^2$

$= r_s e^{-u/2r_s}$ $\quad V = r_s e^{+v/2r_s}$ $\quad \longrightarrow \quad UV - r_s^2 e^{\frac{v-u}{2r_s}} = r_s^2 e^{r/r_s}\left(\frac{r}{r_s}-1\right)$ defines $r(x,T)$

$= X^2 - T^2$

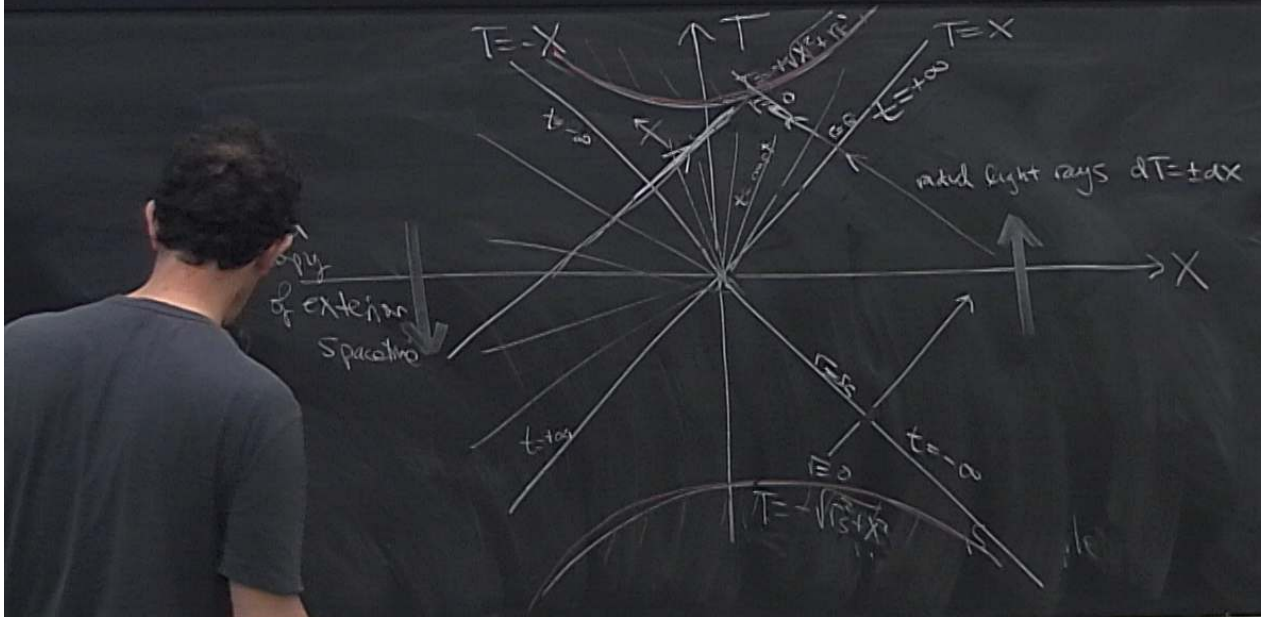
$\Rightarrow ds^2 = +\frac{4r_s}{r} e^{-r/r_s} du\,dV + r^2 d\Omega_2^2$

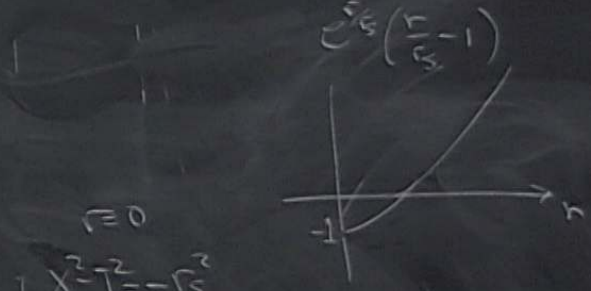
$t = \frac{u+v}{2} = r_s \ln\left(\frac{V}{U}\right) = r_s \ln\left|\frac{X+T}{X-T}\right|$

$-dT^2$

$= \frac{4r_s}{r} e^{-r/r_s}\left(dX^2 - dT^2\right) + r^2 d\Omega_2^2$

Kruskal-Szekeres





$= r_s e^{-u/2r_s}$ $\quad V = r_s e^{+v/2r_s}$ $\quad \longrightarrow \quad UV - r_s^2 e^{\frac{r}{r_s}} = r_s^2 e^{r/r_s}\left(\frac{r}{r_s} - 1\right)$ defines $r(x,T)$

$= X^2 - T^2$

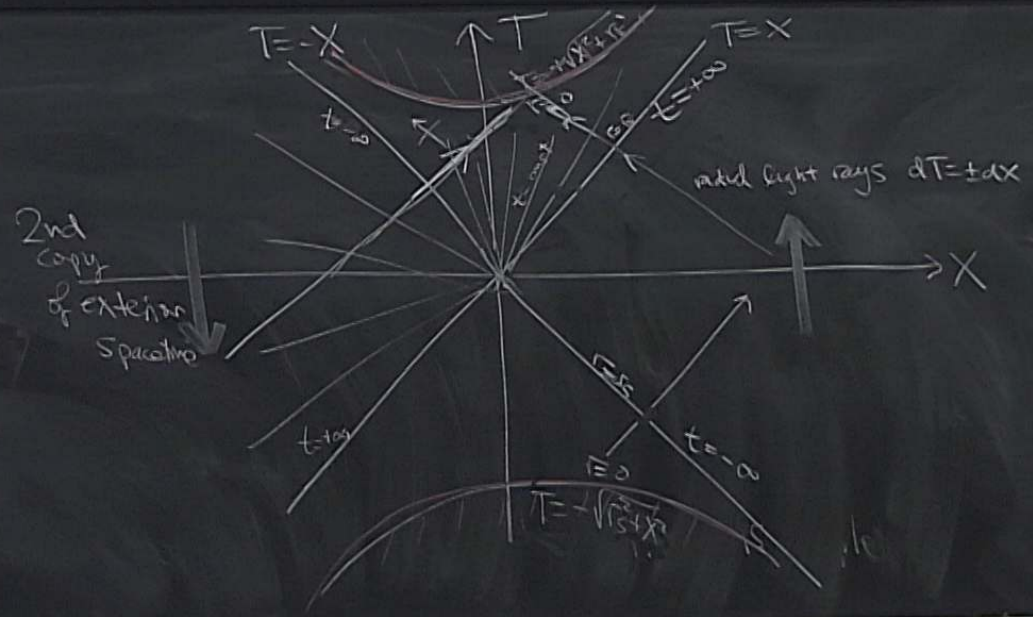
$\Rightarrow ds^2 = + 4\frac{r_s}{r} e^{-r/r_s} du\, dV + r^2 d\Omega_2^2$

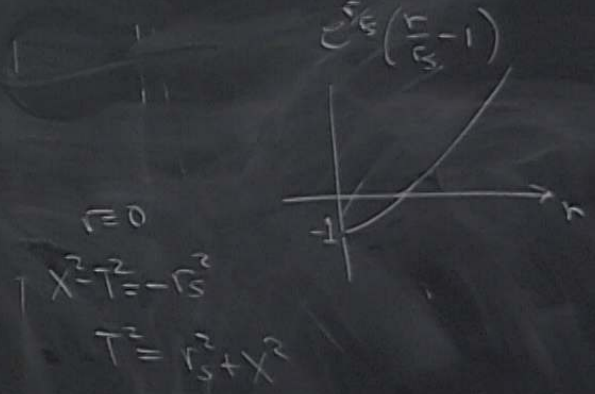
$t = \frac{u+v}{2} = r_s \ln\left(\frac{V}{U}\right) = r_s \ln\left|\frac{X+T}{X-T}\right|$

$-dT^2$

$= 4\frac{r_s}{r} e^{-r/r_s}\left(dX^2 - dT^2\right) + r^2 d\Omega_2^2$

Kruskal Metric





$= r_s e^{-u/2r_s}$ $\quad V = r_s e^{+v/2r_s}$ $\quad \longrightarrow \quad UV - r_s^2 e^{\frac{v-u}{2r_s}} = r_s^2 e^{r/r_s}\left(\frac{r}{r_s}-1\right)$ defines $r(x,T)$

$= X^2 - T^2$

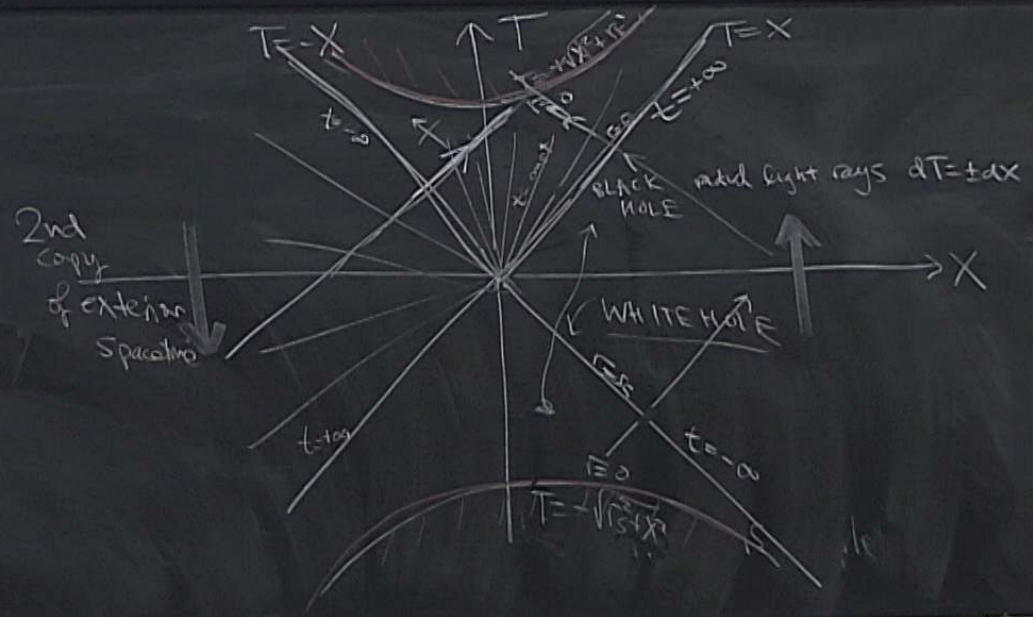
$\Rightarrow ds^2 = + \frac{4r_s}{r} e^{-r/r_s} du\, dv + r^2 d\Omega_2^2$

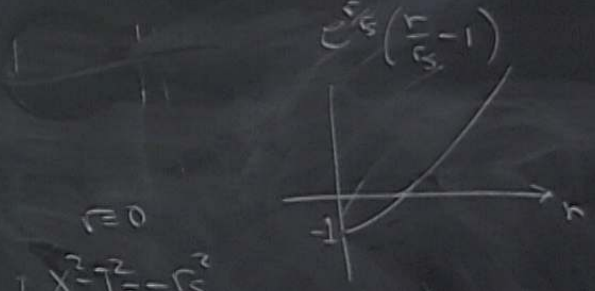
$t = \frac{u+v}{2} = r_s \ln\left(\frac{V}{U}\right) = r_s \ln\left|\frac{X+T}{X-T}\right|$

$-dT^2$

$= \frac{4r_s}{r} e^{-r/r_s}\left(dX^2 - dT^2\right) + r^2 d\Omega_2^2$

Kruskal Metric





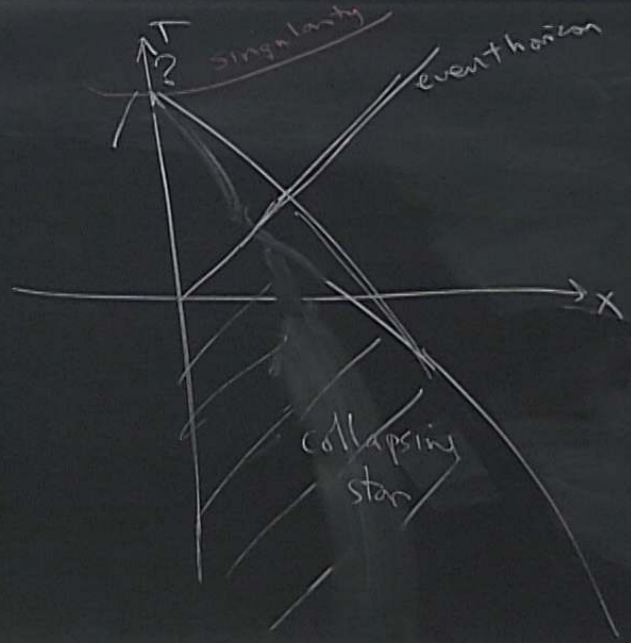
$U = X - T$

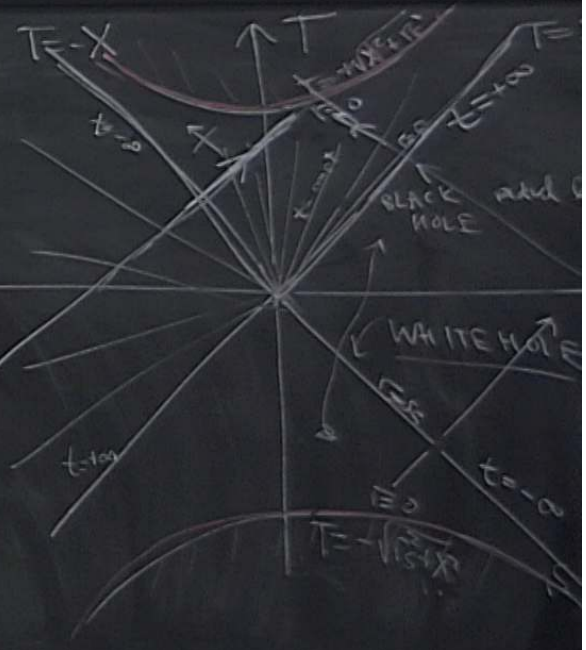
$V = X + T$

$dU\,dV = dX^2 - dT^2$

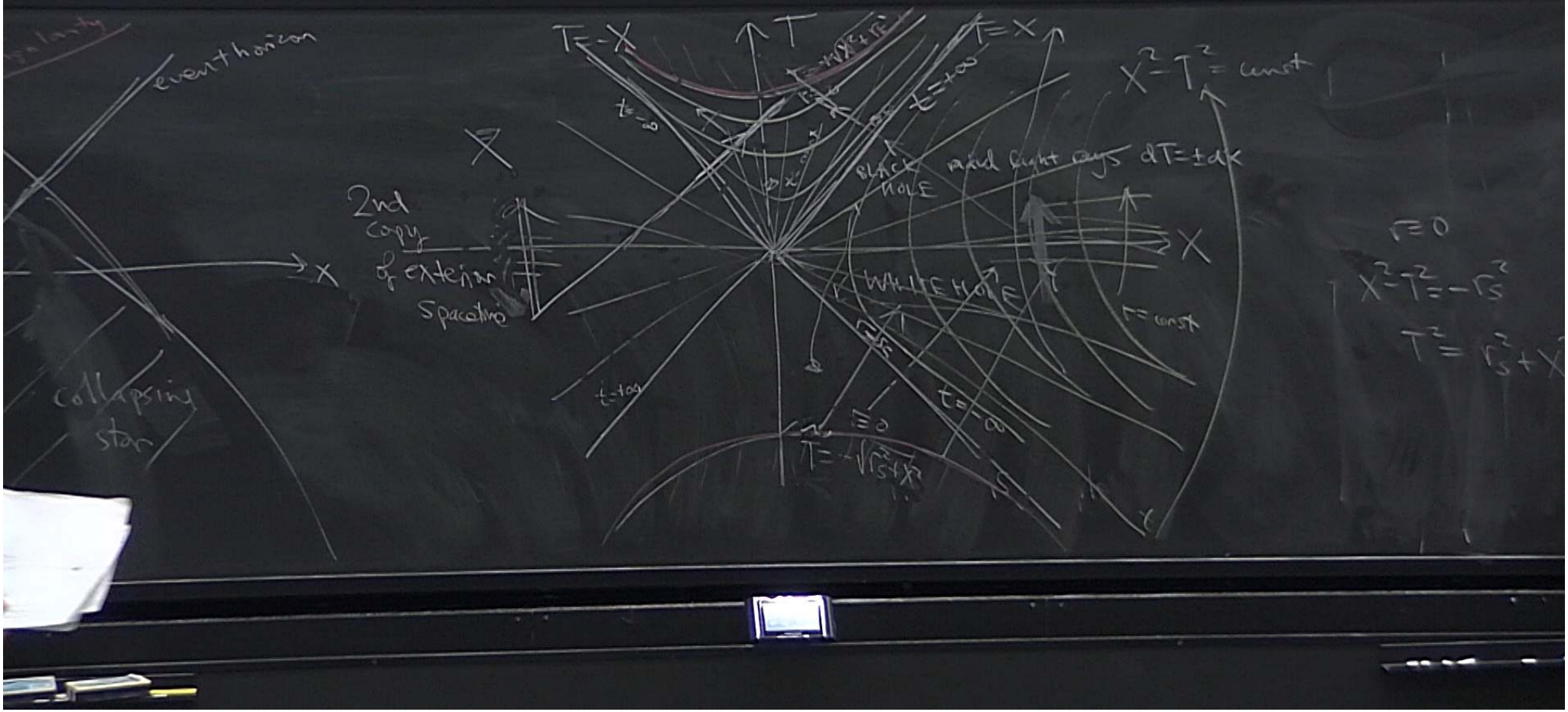
$\Rightarrow ds^2 = +\frac{4r_s}{r} e^{-r/r_s}\, dU\,dV + r^2 d\Omega^2$

$= \frac{4r_s}{r} e^{-r/r_s}(dX^2 - dT^2) + r^2 d\Omega^2$





$V = dX^2 - dT^2$

$= 4\frac{r_s}{r} e^{-r/r_s}(dX^2 - dT^2) + r^2 d\Omega_2^2$

Kruskal Manifold