

Title: PSI 17/18 - Relativity - Lecture 15

Date: Sep 22, 2017 02:00 PM

URL: <http://pirsa.org/17090069>

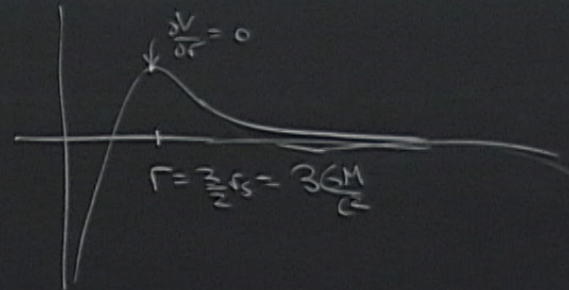
Abstract:

recall $r'^2 - \frac{r_0}{r} + \underbrace{\frac{J^2}{r^2} - \frac{r_0 J^2}{r^3}}_{V(r)} = E^2 - 1$

recall $r'^2 - \cancel{\frac{r_s}{r}} + \frac{J^2}{r^2} - \cancel{\frac{r_s J^2}{r^3}} = E^2 - V$

$$-\frac{2J^2}{r^3} + 3\frac{r_s J^2}{r^4} = 0$$

$$\Rightarrow r = \frac{3}{2} r_s$$



Fall radially into a black hole

$$J=0$$

$$\left(\frac{dr}{dt}\right)^2 + 1 - \frac{r_s}{r} = E^2$$

Start at $r = r_s$, $\frac{dr}{dt} = 0 \Rightarrow E = 0$



proper time
to reach
 $r < r_s$.

$$\tau(r) = \int_r^{r_s} \frac{dr}{\sqrt{\frac{r_s}{r} - 1}}$$

time to hit sing.

$$\int_0^E \frac{dr}{\sqrt{\frac{r_s}{r} - 1}} = \int_0^{r_s} \frac{dr \sqrt{r}}{\sqrt{r_s - r}}$$

$$\text{Survival time} = \frac{\pi r_s}{2} = \frac{\pi G M}{c^3}$$

$$\simeq 10^{-5} \left(\frac{M}{M_\odot} \right) \text{ seconds}$$

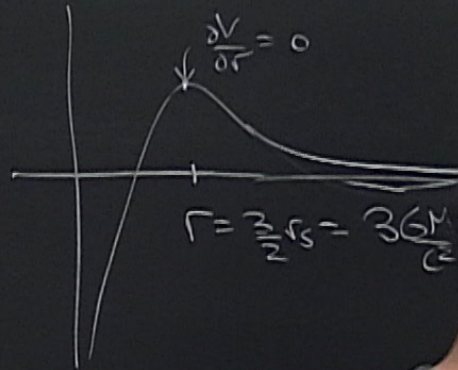
$$r = r_s \sin^2 \theta \quad 2 \int_0^{\pi/2} d\theta \underbrace{\sin^3 \theta}_{\frac{1 - \cos 2\theta}{2}} = \frac{r_s \pi}{2}$$

$$+\frac{J^2}{r^2} - \frac{r_s J^2}{r^3} = E^2 - 1$$

$V(r)$

$$-\frac{2J^2}{r^3} + 3\frac{r_s J^2}{r^4} = 0$$

$$\Rightarrow r = \frac{3}{2} r_s$$



Fall radially into
 $J=0$

$$\left(\frac{dr}{dt}\right)^2 + 1 -$$

start at $r =$

proper time
to reach
 $r < r_s$.

$$\tau(r) = \int_r^{r_s} \frac{dr}{\sqrt{\frac{r_s}{r} - 1}}$$

Survival time

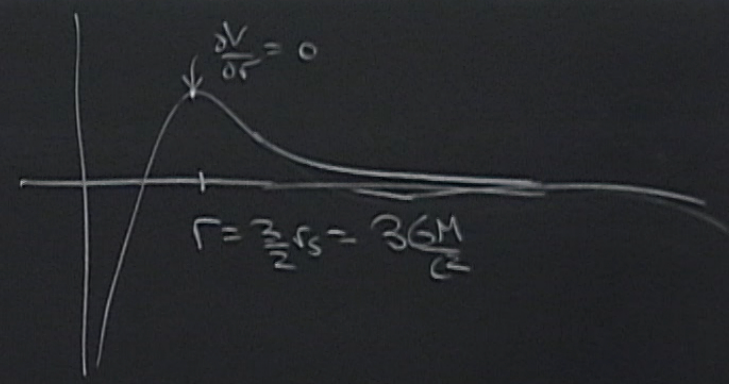
$$\simeq 10^{-5} \left(\frac{M}{M_\odot} \right)$$

$$+ \frac{J^2}{r^2} - \frac{r_s J^2}{r^3} = E^2 - 1$$

$$\underbrace{\quad}_{V(r)}$$

$$-\frac{2J^2}{r^3} + 3\frac{r_s J^2}{r^4} = 0$$

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Fall radially into
 $J=0$

$$\left(\frac{dr}{dt}\right)^2 + 1 -$$

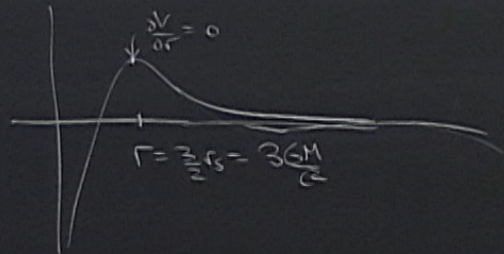
start at $r =$

for time
 to reach
 $r < r_s$.

$$\tau(r) = \int_r^{r_s} \frac{dr}{r \sqrt{\frac{r_s}{r} - 1}}$$

Survival time

$$\frac{M}{M_\odot}$$



Fall radially into a black hole

$J=0$

$$d\tau = \frac{dr}{\sqrt{\frac{r}{r_s} - 1}}$$

$$\left(\frac{dr}{dt}\right)^2 + 1 - \frac{r_s}{r} = E^2$$

start at $r = r_s$, $\frac{dr}{dt} = 0 \Rightarrow E = 0$

proper time
to reach
 $r < r_s$

$$\tau(r) = \int_r^{r_s} \frac{dr}{\sqrt{\frac{r}{r_s} - 1}}$$

time to hit sing?



$$\int_0^{r_s} \frac{dr}{\sqrt{\frac{r}{r_s} - 1}} = \int_0^{r_s} \frac{dr \sqrt{r}}{\sqrt{r_s - r}}$$

Survival time = $\frac{\tau(r_s)}{2} = \frac{\pi GM}{c^3}$

$\approx 10^{-5} \left(\frac{M}{M_\odot}\right) \text{ seconds}$

$r = \frac{r_s}{2} (1 + \cos 2\theta)$

$$2 \int_0^{\pi/2} \frac{r_s \sin^2 \theta}{1 - \cos 2\theta} d\theta$$

$$= \frac{r_s \pi}{2}$$

Geometry of the Schwarzschild spacetime

$$ds^2 = -\alpha(r) dt^2 + \frac{1}{\alpha(r)} dr^2 + r^2 d\Omega^2, \quad \alpha(r) = 1 - \frac{r_s}{r}.$$

What happens at $r = r_s$? For $r < r_s$, r is a timelike coordinate, Schwarzschild is not static!

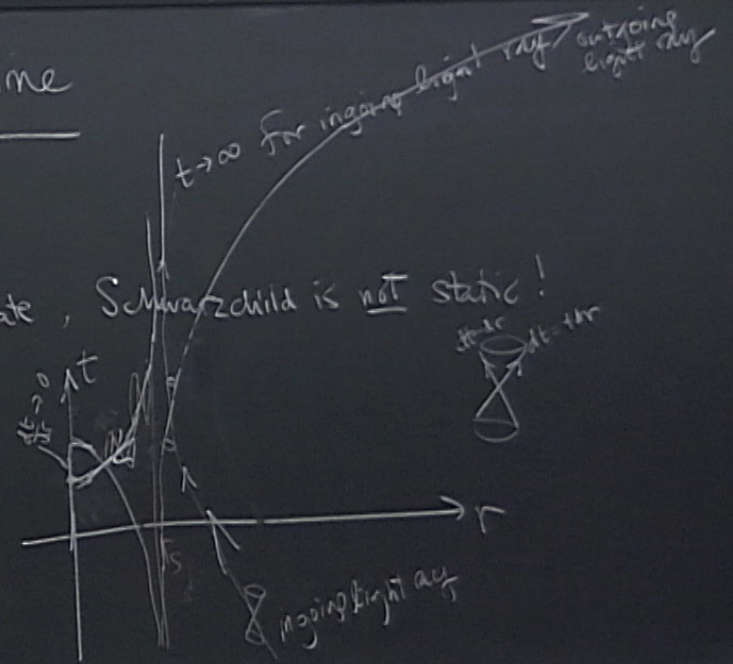
Geometry of the Schwarzschild spacetime

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Radial light signals
(null trajectories)

$$ds^2 = 0 \Rightarrow dt = \pm \frac{dr}{\alpha(r)} = \pm dr \left(1 + \frac{r_s}{r-r_s} \right)$$



Geometry of the Schwarzschild spacetime

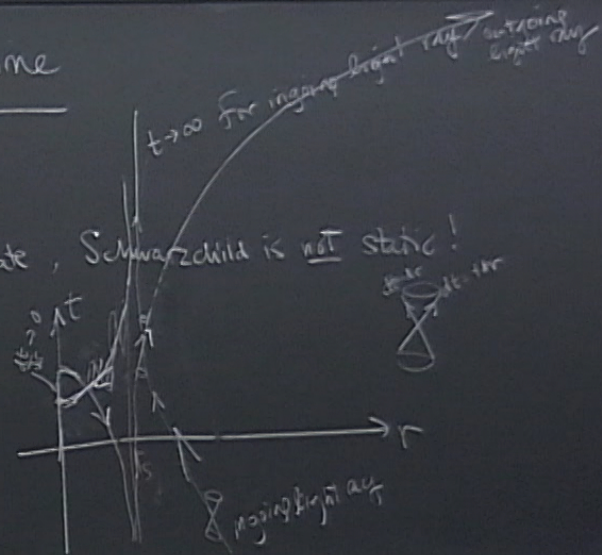
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①



Idea: change coords to coordinates in which the metric is nonsingular at $r = r_s$

Integrate ①

$$t = \pm \left(r + r_s \ln \left(\frac{r}{r_s} - 1 \right) \right) + \text{const} \quad \text{for } r > r_s$$

$$= \pm \left(r + r_s \ln \left(1 - \frac{r}{r_s} \right) \right) + \text{const} \quad \text{for } r < r_s$$

Geometry of the Schwarzschild spacetime

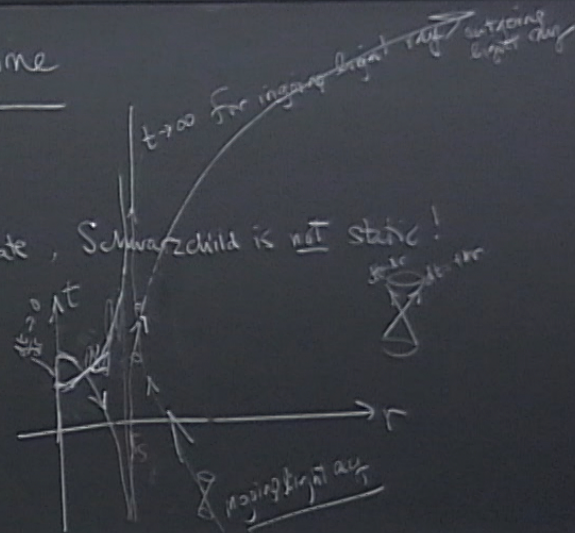
$$ds^2 = -\alpha(r) dt^2 + \frac{1}{\alpha(r)} dr^2 + r^2 d\Omega_2^2, \quad \alpha(r) = 1 - \frac{r_s}{r}.$$

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radial light signals
(null trajectories)

$$ds^2 = 0 \Rightarrow dt = \pm \frac{dr}{\alpha(r)} = \pm dr \left(1 + \frac{r_s}{r-r_s} \right)$$

①



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Integrate ① $t = \pm \left(r + r_s \ln \left| \frac{r}{r_s} - 1 \right| \right) + \text{const}$ for $r > r_s$
 $= \pm \left(r + r_s \ln \left(1 - \frac{r_s}{r} \right) \right) + \text{const}$ for $r < r_s$

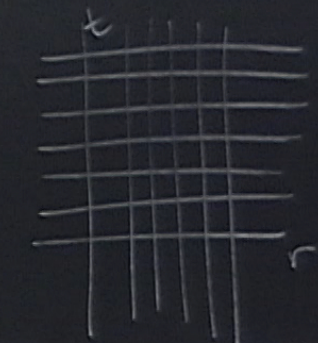
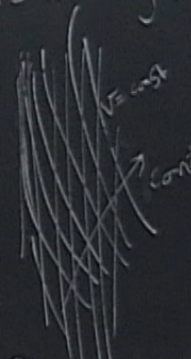
Ingoing light ray: $t = -r - r_s \ln \left| \frac{r}{r_s} - 1 \right| + \text{const}$

Idea: change coords to coordinates in which the metric is nonsingular at $r=r_s$

Integrate ① $t = \pm \left(r + r_s \ln \left(\frac{r}{r_s} - 1 \right) \right) + \text{const}$ for $r > r_s$
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Ingoing light ray: $t = -r - r_s \ln \left| \frac{r}{r_s} - 1 \right| + \text{const}$

Choose new "ingoing coordinate" $v = t + r + r_s \ln \left| \frac{r}{r_s} - 1 \right|$, $r' = r$



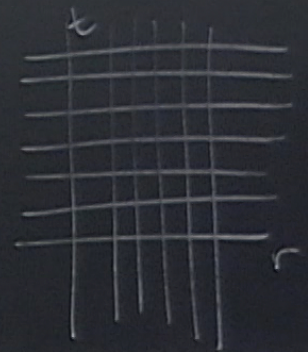
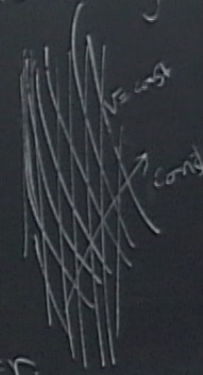
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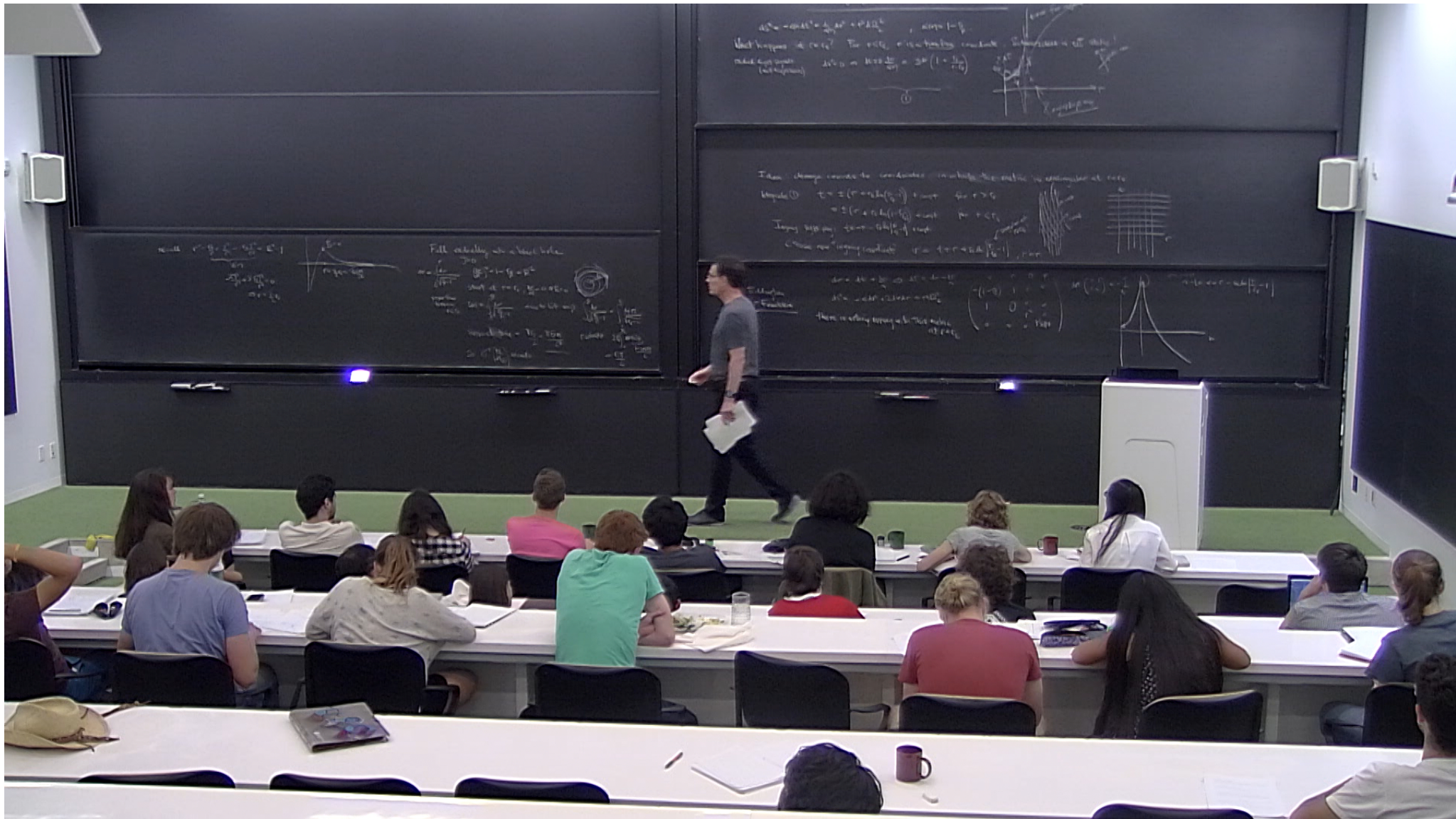
Integrate ① $t = \pm \left(r + r_s \ln \left| \frac{r}{r_s} - 1 \right| \right) + \text{const}$ for $r > r_s$
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$$dv = dt + \frac{dr}{\alpha}$$





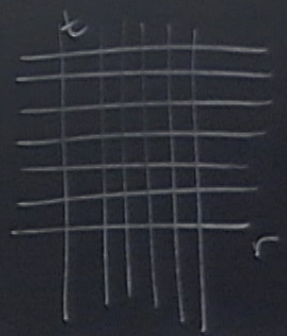
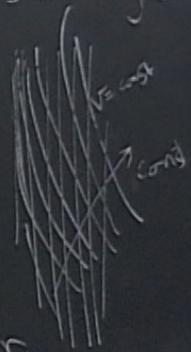
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 $= \pm (r + r_s \ln(1 - \frac{r}{r_s})) + \text{const}$ for $r < r_s$

Ingoing light ray: $t = -r - r_s \ln|\frac{r}{r_s} - 1| + \text{const}$

Choose new "ingoing coordinate" $v = t + r + r_s \ln|\frac{r}{r_s} - 1|$, $r' = r$

singular at $r=r_s$!



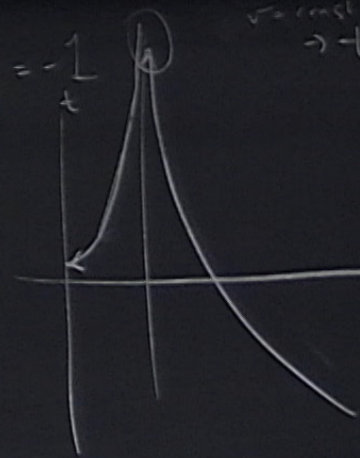
$$dv = dt + \frac{dr}{\alpha} \Rightarrow dt = dv - \frac{dr}{\alpha}$$

$$ds^2 = -\alpha^2 dv^2 + 2dvdr + r^2 d\Omega^2$$

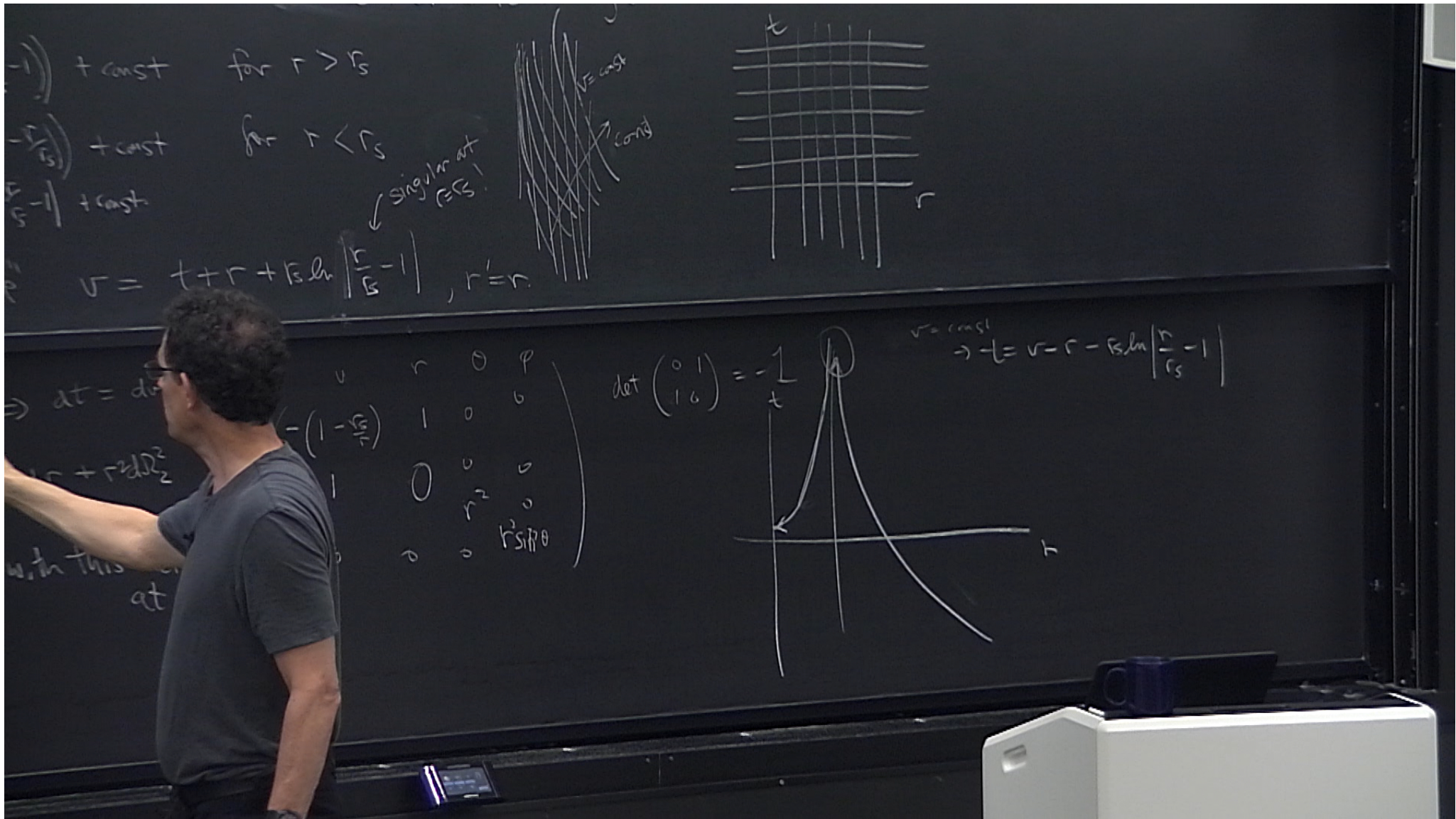
there is nothing wrong with this metric at $r=r_s$.

$$\begin{pmatrix} v & r & \theta & \phi \\ -\left(1 - \frac{r_s}{r}\right) & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$$

$$\det \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -1$$



Eddington
- Finkelstein



Outgoing light rays : $u = t - (r + r_s \ln|\frac{r}{r_s} - 1|) = \text{const.}$

$$ds^2 = -\alpha du^2 - 2du dr + r^2 d\Omega^2 \quad \text{also nonsingular at } r=r_s.$$

$$u = \text{const} \Rightarrow t = r + r_s \ln(1 - \frac{r}{r_s}) \quad r < r_s$$

$= \text{const.}$

also nonsingular at $r=r_s$.

$r < r_s$

$u = \text{const.}$

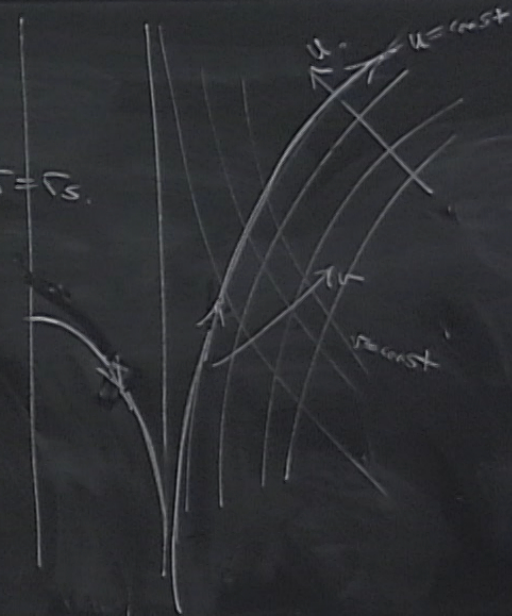
$r = \text{const.}$

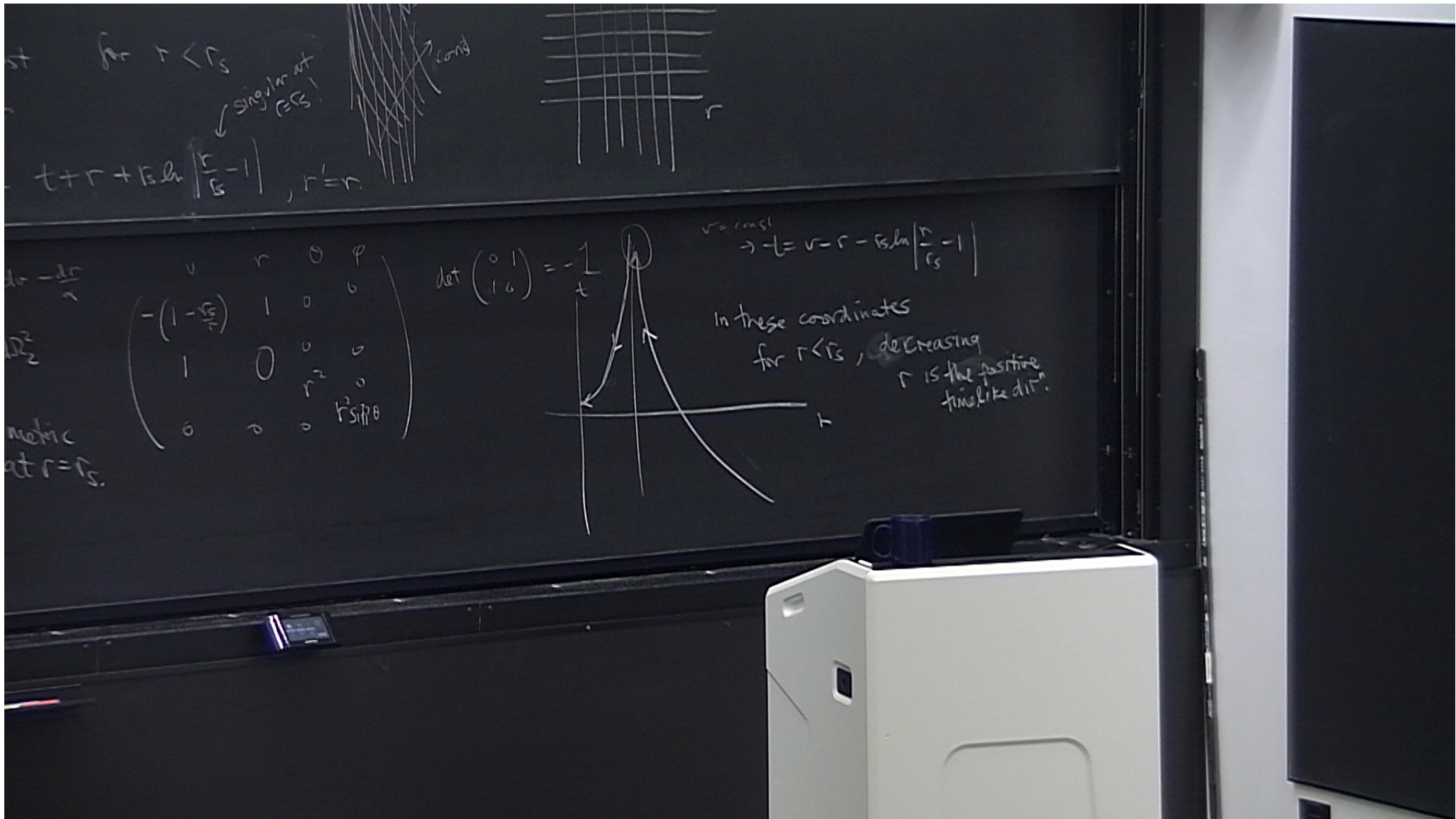
$$u = t - \left(r + r_s \ln \left| \frac{r}{r_s} - 1 \right| \right) = \text{const.}$$

$$ds^2 = -\alpha du^2 - 2du dr + r^2 d\Omega^2 \quad \text{also nonsingular at } r=r_s.$$

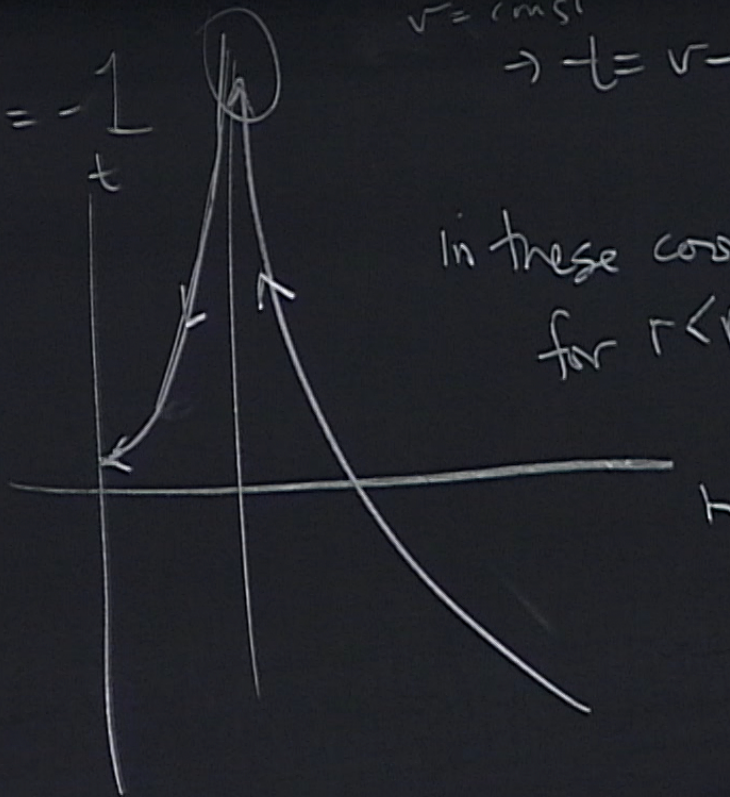
$$u = \text{const} \Rightarrow t = r + r_s \ln \left| 1 - \frac{r}{r_s} \right| \quad r < r_s$$

flow of time is in the increasing r direction
for $r < r_s$





$$\det \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -1$$



$$v = c \sin \theta$$

$$\rightarrow -t = v - r - r_s \ln \left| \frac{r}{r_s} - 1 \right|$$

In these coordinates

for $r < r_s$, decreasing
 r is the positive
 timelike dirⁿ.

flow of time is in the increasing r direction
for $r < r_s$
 $\Rightarrow$ terrible contradiction.

will turn out that both are true but neither is complete.

what does other trajectory do in ingoing coords?

$$2^{\text{nd}} \text{ null trajectory } dr = 2 \frac{dr}{\alpha}$$

$$v = 2 \left(r + r_s \ln \left| \frac{r}{r_s} - 1 \right| \right) + \text{const}$$

$$= t + r + r_s \ln \left| \frac{r}{r_s} - 1 \right|$$

$$\Rightarrow t = r + r_s \ln \left| \frac{r}{r_s} - 1 \right|$$

flow of time is in the increasing r direction
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what does other trajectory do ingoing coords?

$$2nd \text{ null trajectory } dv = 2 \frac{dr}{r}$$

$$v = 2 \left(r + r_s \ln \left| \frac{r}{r_s} - 1 \right| \right) + const$$

$$= t + r + r_s \ln \left| \frac{r}{r_s} - 1 \right|$$

$$\Rightarrow t = r + r_s \ln \left| \frac{r}{r_s} - 1 \right| + const$$

$$= t + r + r_s \ln \left| \frac{r}{r_s} - 1 \right|$$

$$\Rightarrow t = r + r_s \ln \left| \frac{r}{r_s} - 1 \right| + \text{const} \quad \rightarrow \text{"out"} \quad a$$

M. Kruskal Put this together : for $r > r_s$

$$v = t + r + r_s \ln \left(\frac{r}{r_s} - 1 \right)$$

$$u = t - r - r_s \ln \left(\frac{r}{r_s} - 1 \right)$$

Phys Rev

119, 1743 (1960)

$$dr = dt + \frac{dr}{1 - r_s/r}$$

$$du = dt - \frac{dr}{1 - r_s/r}$$

$$\Rightarrow ds^2 = -\left(1 - \frac{r_s}{r}\right) du dr + r^2 d\Omega^2$$

$$\Rightarrow t = r + r_s \ln \left| \frac{r}{r_s} - 1 \right| + \text{const}$$

$\Rightarrow$ outside horizon with
arrow of time reversed.

for $r > r_s$

$$v = t + r + r_s \ln \left(\frac{r}{r_s} - 1 \right)$$

$$u = t - r - r_s \ln \left(\frac{r}{r_s} - 1 \right)$$

$$dv = dt + \frac{dr}{1 - r_s/r}$$

$$du = dt - \frac{dr}{1 - r_s/r}$$

$$\Rightarrow ds^2 = -(1 - r_s/r) du dv + r^2 d\Omega^2$$

where r defined by

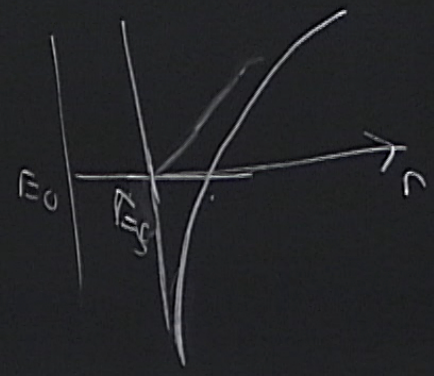
$$r + r_s \ln \left(\frac{r}{r_s} - 1 \right) = \frac{v - u}{2}$$

$$r_s \ln\left(\frac{r}{r_s} - 1\right)$$

$$- r_s \ln\left(\frac{r}{r_s} - 1\right)$$

$$\Rightarrow ds^2 = -\left(1 - \frac{r_s}{r}\right) du dr + r^2 d\Omega^2$$

where r defined by

$$r + r_s \ln\left(\frac{r}{r_s} - 1\right) = \frac{r - u}{2}$$


$$= t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$$

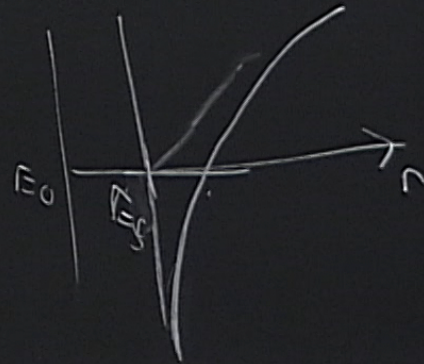
$$= t - r - r_s \ln\left(\frac{r}{r_s} - 1\right)$$

$$\Rightarrow ds^2 = -\left(1 - \frac{r_s}{r}\right) du dv + r^2 d\Omega^2$$

where r defined by

$$r + r_s \ln\left(\frac{r}{r_s} - 1\right) = \frac{v - u}{2}$$

$$1 - \frac{r_s}{r} = \frac{r_s}{r} e^{-\frac{r}{r_s}} e^{\frac{v-u}{2r_s}}$$



$$\Rightarrow ds^2 = -\frac{r_s}{r} e^{-\frac{r}{r_s}} e^{\frac{r-u}{2r_s}} du dv + r^2 d\Omega^2$$

define $U = r_s e^{-\frac{u}{2r_s}}$ $V = r_s e^{+\frac{v}{2r_s}}$

$$U = X - T$$

$$V = X + T$$

$$dU dV = dX^2 - dT^2$$

$$\Rightarrow ds^2 = + 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} dU dV + r^2 d\Omega^2$$

$$= 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega^2$$

Kruskal metric

flow of time is in the increasing direction for $r < r_s$
 $\Rightarrow$ terrible contradiction.

will turn out that both are true but neither is complete.

$$V = X + T$$

$$\underline{dU dV = dX^2 - dT^2}$$

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$$= 4\frac{r_s}{r} e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega^2$$

Kruskal metric

$$t = \frac{u+v}{2} = r_s \ln \left(\frac{v}{u} \right)$$

$$ds^2 = -\frac{r}{r_s} e^{-\frac{r}{r_s}} (du dr + r d\Omega^2)$$

define $U = r_s e^{-\frac{u}{2r_s}}$ $V = r_s e^{+\frac{v}{2r_s}}$

$$\rightarrow UV = r_s^2 e^{\frac{v-u}{2r_s}} = r_s^2 = X^2 - T^2$$

$$U = X - T$$

$$V = X + T$$

$$dU dV = dX^2 - dT^2$$

$$\Rightarrow ds^2 = + 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} dU dV + r^2 d\Omega^2$$

$$= 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega^2$$

Kruskal metric

$$t = \frac{u+v}{2} = r_s \ln \left(\frac{V}{U} \right)$$

ing light rays :

$$u = t - \left(r + r_s \ln \left| \frac{r}{r_s} - 1 \right| \right) = \text{const.}$$

$$ds^2 = du^2 - 2 du dr + r^2 d\Omega^2 \quad \text{also nonsingular at } r=r_s.$$

$$\Rightarrow t = r + r_s \ln \left| \frac{r}{r_s} - 1 \right| \quad r < r_s$$

$$= r_s e^{+\frac{r}{2r_s}}$$

$$+ 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} du dv + r^2 d\Omega^2$$

$$= 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} (dx^2 - dT^2) + r^2 d\Omega^2$$

Kruskal metric

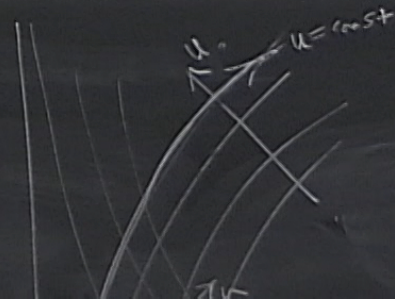
$$\rightarrow uv = r^2 e^{\frac{r-u}{2r_s}} = r_s^2 e^{\frac{r}{r_s}} \left(\frac{r}{r_s} - 1 \right) \quad \text{defines } r(x,T)$$

$$= X^2 - T^2$$

$$t = \frac{u+v}{2} = r_s \ln \left(\frac{v}{u} \right) = r_s \ln \left(\frac{x+T}{x-T} \right)$$

$$= t - \left(r + r_s \ln \left| \frac{r}{r_s} - 1 \right| \right) = \text{const.}$$

nonsingular at $r=r_s$.



$$\text{define } U = r_s e^{-\frac{1}{2} \frac{r}{r_s}} \quad V = r_s e^{+\frac{1}{2} \frac{r}{r_s}}$$

$$\rightarrow UV = r_s^2 e^{\frac{r-u}{2r_s}} = X^2 - T^2$$

$$U = X - T$$

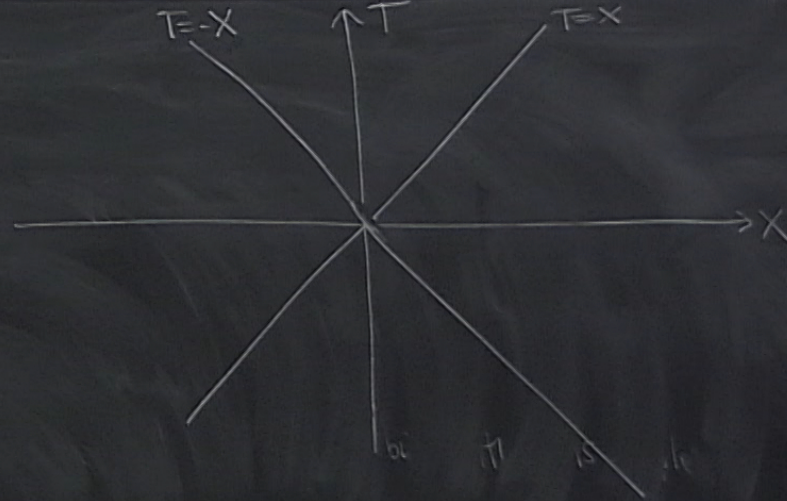
$$V = X + T$$

$$dU dV = dX^2 - dT^2$$

$$\Rightarrow ds^2 = +4r_s e^{-\frac{r}{r_s}} dU dV + r^2 d\Omega_z^2$$

$$= 4r_s e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega_z^2$$

$$t = \frac{u+v}{2} = r_s \ln \left(\frac{r}{r_s} \right)$$



define $U = r_s e^{-\frac{u}{2r_s}}$ $V = r_s e^{+\frac{v}{2r_s}}$

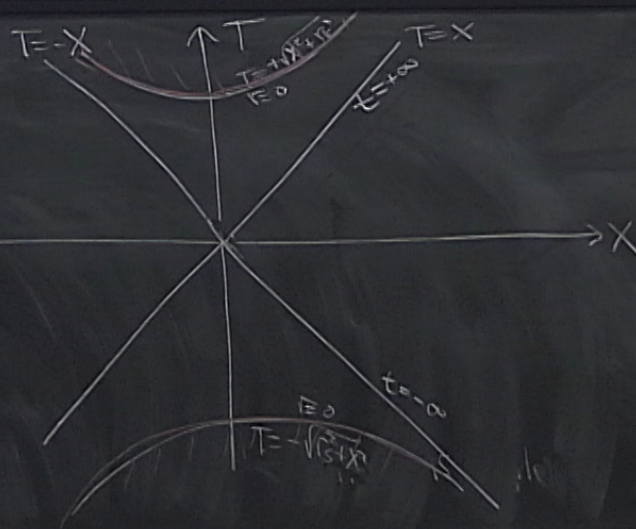
$X = T$
 $X = -T$

$dV = dX^2 - dT^2$

$\Rightarrow ds^2 = +4\frac{r_s}{r} e^{-\frac{r}{r_s}} dU dV + r^2 d\Omega^2$

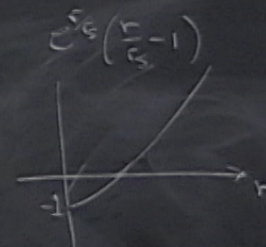
$= 4\frac{r_s}{r} e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega^2$

Kruskal diagram



$\rightarrow UV = r_s^2 e^{\frac{r-u}{2r_s}} = r_s^2 e^{\frac{r}{r_s}} \left(\frac{r}{r_s} - 1\right)$ defines $r(x, T)$
 $= X^2 - T^2$

$t = \frac{u+v}{2} = r_s \ln \left(\frac{V}{U} \right) = r_s \ln \left| \frac{X+T}{X-T} \right|$



$r=0$
 $X^2 - T^2 = -r_s^2$
 $T^2 = r_s^2 + X^2$

define $U = r_s e^{-\frac{4}{25} \phi}$ $V = r_s e^{+\frac{5}{25} \phi}$

X-T

$$X+T$$

$$dV = dx^2 - dt^2$$

$$\Rightarrow \lambda \vec{s} = + 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} \lambda u \lambda V + r^2 \lambda \Omega_z^2$$

$$= 4\frac{r_s}{r} e^{-\frac{r}{r_s}} (dx^2 - dt^2) + r^2 d\Omega_2^2$$

$$\rightarrow UV - R^2 e^{\frac{r-u}{2\sigma}} = r_s^2 e^{\frac{y}{c_s} \left(\frac{r}{r_s} - 1 \right)} \quad \text{defines } r(x, T) = X^2 - T^2$$

$$\frac{t}{\tau} = \frac{u+v}{2} = r_s \ln \left(\frac{V}{u} \right) = r_s \ln \left| \frac{x+T}{x-T} \right|$$

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$$X^2 - T^2 = -r_s^2$$

$$= r_s e^{-\frac{u}{2r_s}} \quad V = r_s e^{+\frac{v}{2r_s}}$$

$$\rightarrow UV = r_s^2 e^{\frac{v-u}{2r_s}} = r_s^2 e^{\frac{r}{r_s}} \left(\frac{r}{r_s} - 1 \right) \text{ defines } r(x, T)$$

$$= X^2 - T^2$$

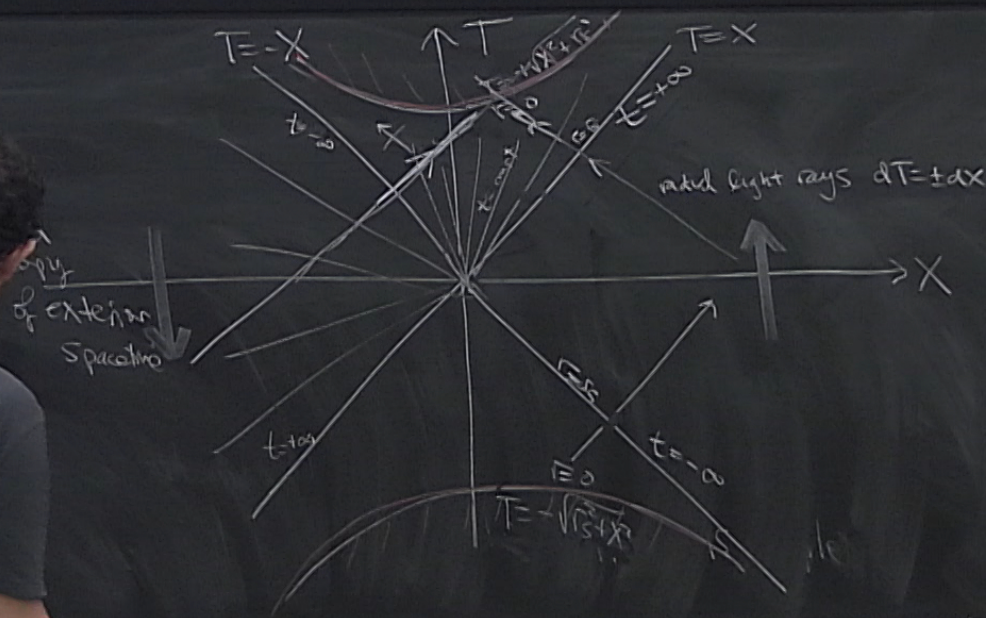
$$\Rightarrow ds^2 = + 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} du dv + r^2 d\Omega_z^2$$

$-dT^2$

$$= 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega_z^2$$

Kruskal metric

$$t = \frac{u+v}{2} = r_s \ln \left(\frac{V}{U} \right) = r_s \ln \left| \frac{X+T}{X-T} \right|$$



$$e^{\frac{r}{r_s}} \left(\frac{r}{r_s} - 1 \right)$$

$$r = 0$$

$$X^2 - T^2 = -r_s^2$$

$$T^2 = r_s^2 + X^2$$

$$= r_s e^{-u/2r_s} \quad V = r_s e^{+v/2r_s}$$

$$\rightarrow UV = r_s^2 e^{\frac{v-u}{2r_s}} = r_s^2 e^{\frac{r}{r_s}} \left(\frac{r}{r_s} - 1 \right) \text{ defines } r(x, T)$$

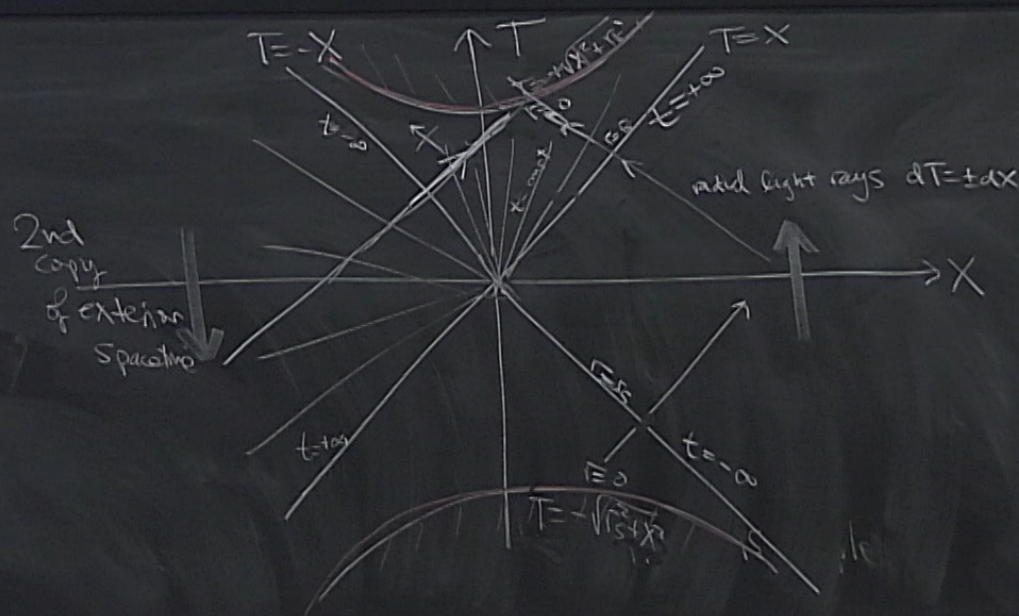
$$= X^2 - T^2$$

$$\Rightarrow ds^2 = + 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} du dv + r^2 d\Omega^2$$

$$t = \frac{u+v}{2} = r_s \ln \left(\frac{v}{u} \right) = r_s \ln \left| \frac{x+T}{x-T} \right|$$

$$= 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega^2$$

Kruskal metric



$$e^{\frac{r}{r_s}} \left(\frac{r}{r_s} - 1 \right)$$

$r = 0$

$$X^2 - T^2 = -r_s^2$$

$$T^2 = r_s^2 + X^2$$

$$= r_s e^{-\frac{u}{2r_s}} \quad V = r_s e^{+\frac{v}{2r_s}}$$

$$\rightarrow UV = r_s^2 e^{\frac{v-u}{2r_s}} = r_s^2 e^{\frac{r}{r_s}} \left(\frac{r}{r_s} - 1 \right) \text{ defines } r(x, T)$$

$$= X^2 - T^2$$

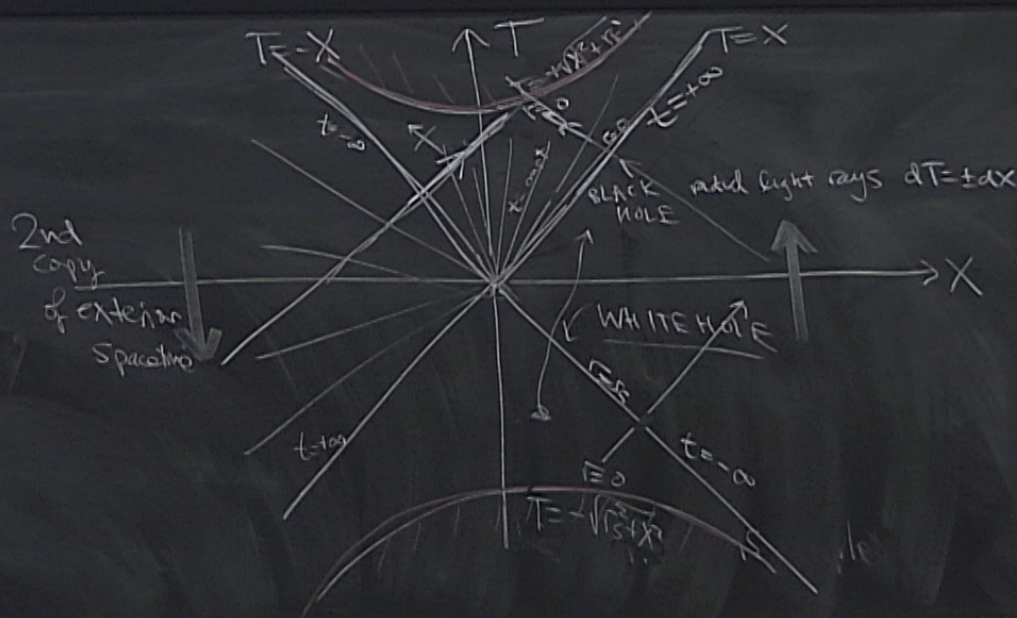
$$\Rightarrow ds^2 = + 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} du dv + r^2 d\Omega^2$$

$$t = \frac{u+v}{2} = r_s \ln \left(\frac{v}{u} \right) = r_s \ln \left| \frac{x+T}{x-T} \right|$$

$-dT^2$

$$= 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega^2$$

Kruskal diagram



$$e^{\frac{r}{r_s}} \left(\frac{r}{r_s} - 1 \right)$$

$$r=0$$

$$X^2 - T^2 = -r_s^2$$

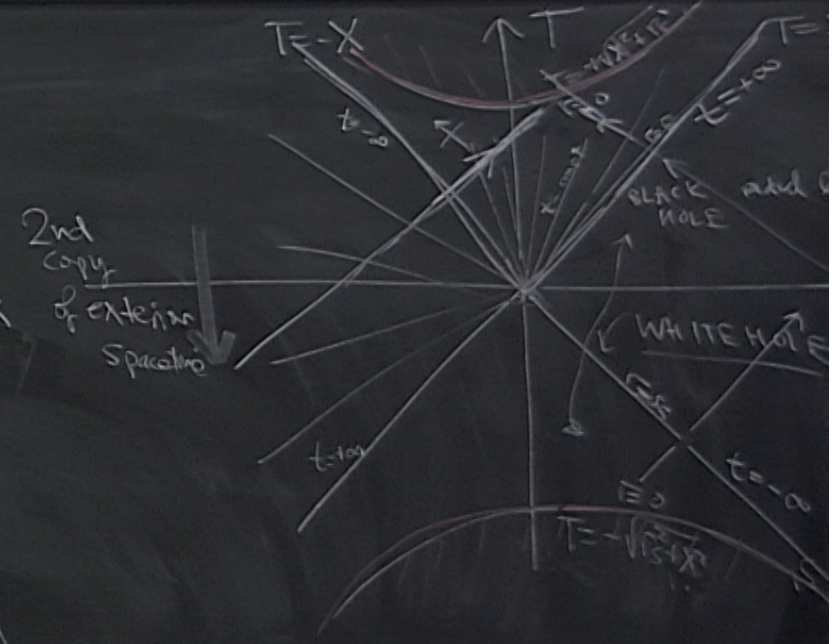
$$T^2 = r_s^2 + X^2$$

$$dU dV = dX^2 - dT^2$$

$$\Rightarrow ds^2 = + 4 \frac{r_s}{r} e^{\frac{r_s}{r}} dU dV + r^2 d\Omega^2$$

$$= 4 \frac{r_s}{r} e^{-\frac{r}{r_s}} (dx^2 - dt^2) + r^2 d\Omega^2$$

Kantel media



$$V = dX^2 - dT^2$$

$$= 4\frac{r_s}{r} e^{-\frac{r}{r_s}} (dX^2 - dT^2) + r^2 d\Omega^2$$

Kruskal metric

