Title: Towards holography via quantum source-channel codes

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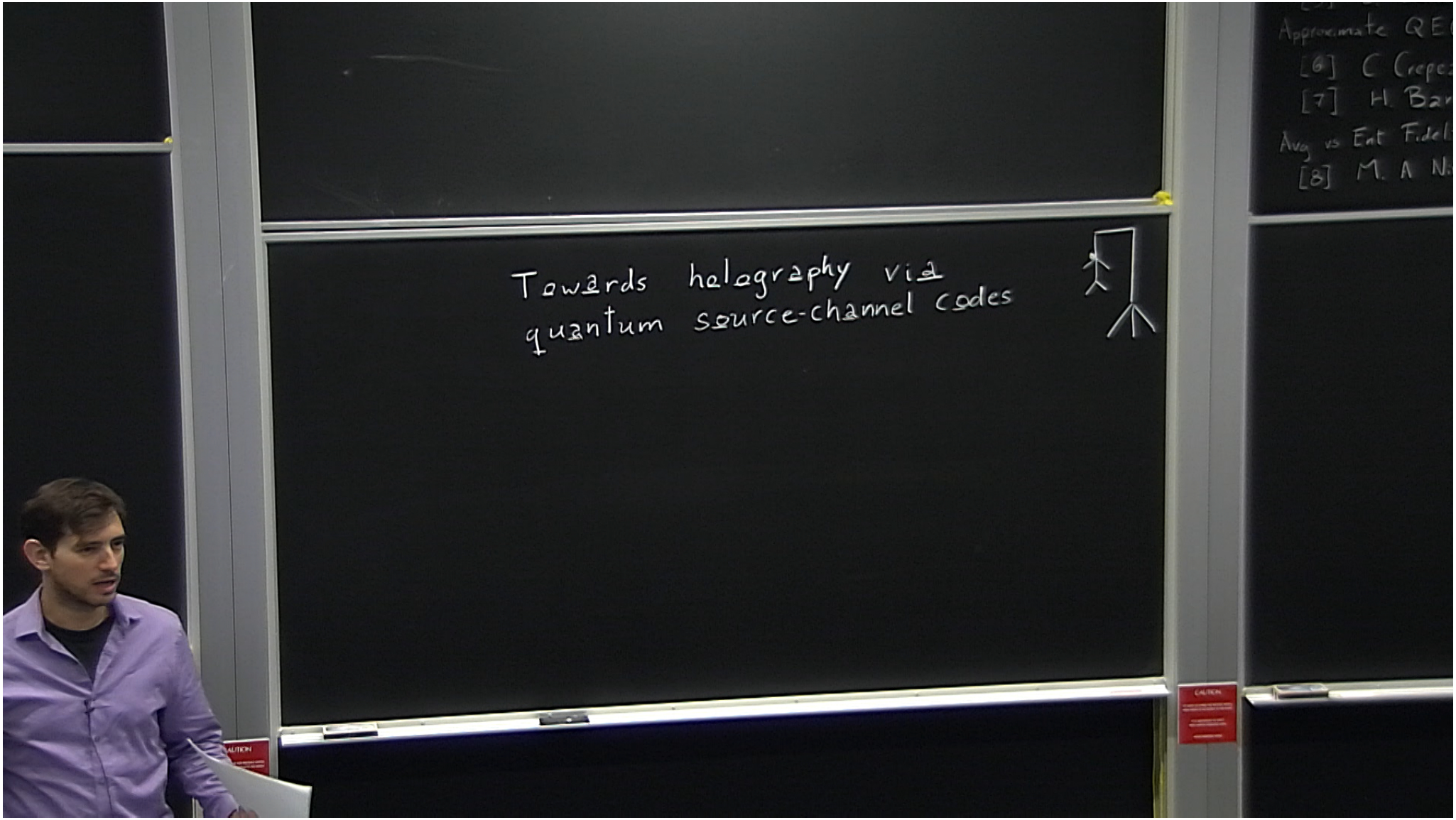
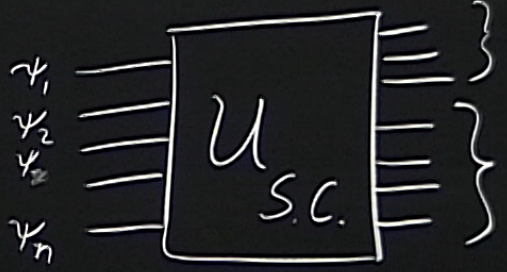
Abstract: While originally motivated by quantum computation, quantum error correction (QEC) is currently providing valuable insights into many-body quantum physics such as topological phases of matter. Furthermore, mounting evidence originating from holography research (AdS/CFT), indicates that QEC should also be pertinent for conformal field theories. With this motivation in mind, we introduce quantum source-channel codes, which combine features of lossy-compression and approximate quantum error correction, both of which are predicted in holography. Through a recent construction for approximate recovery maps, we derive guarantees on its erasure decoding performance from calculations of an entropic quantity called conditional mutual information. As an example, we consider Gibbs states of the transverse field Ising model at criticality and provide evidence that they exhibit non-trivial protection from local erasure. This gives rise to the first concrete interpretation of a bona fide conformal field theory as a quantum error correcting code. We argue that quantum source-channel codes are of independent interest beyond holography.


Towards holography via
quantum source-channel codes

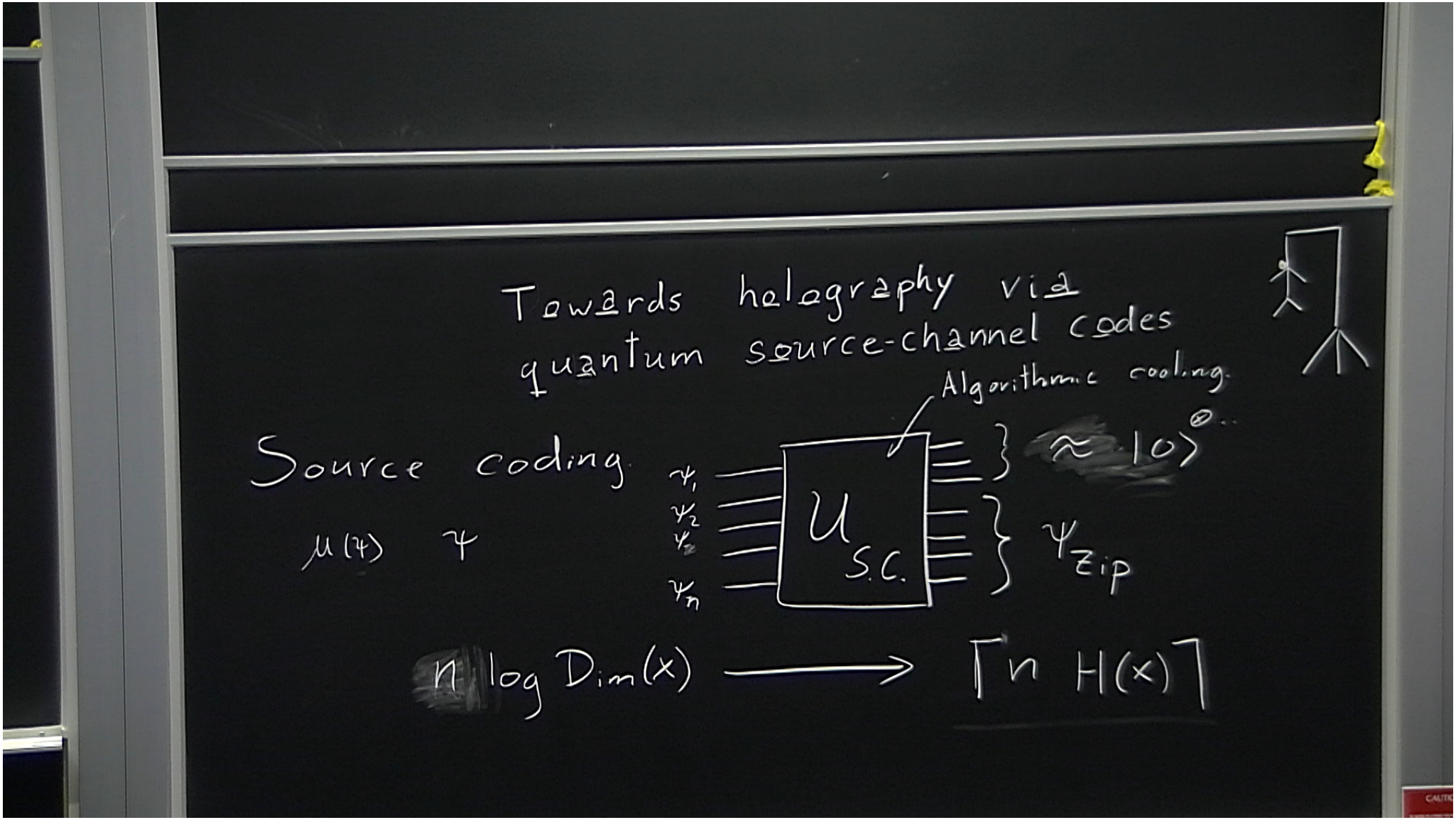
Towards holography via quantum source-channel codes

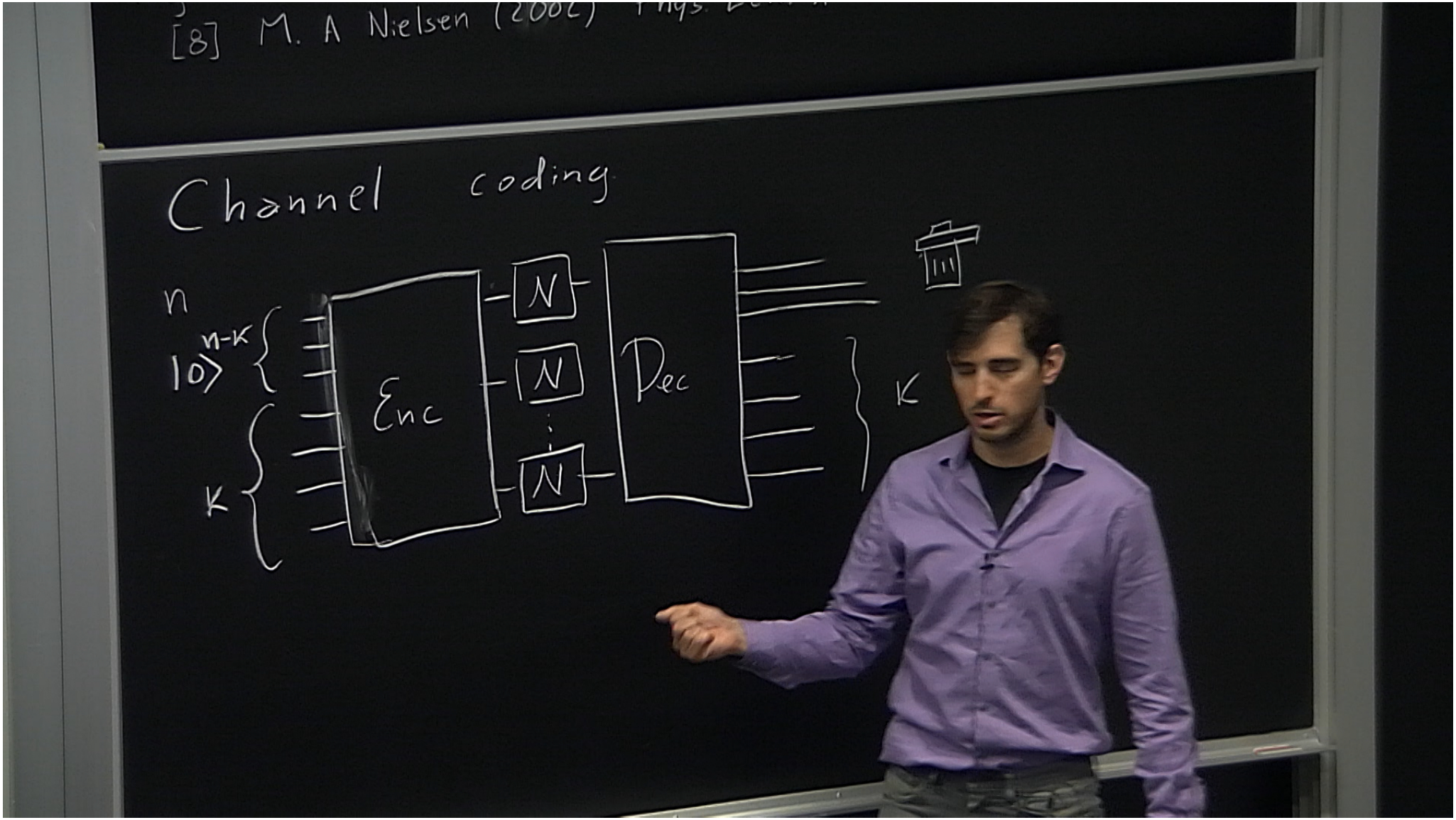
Source coding

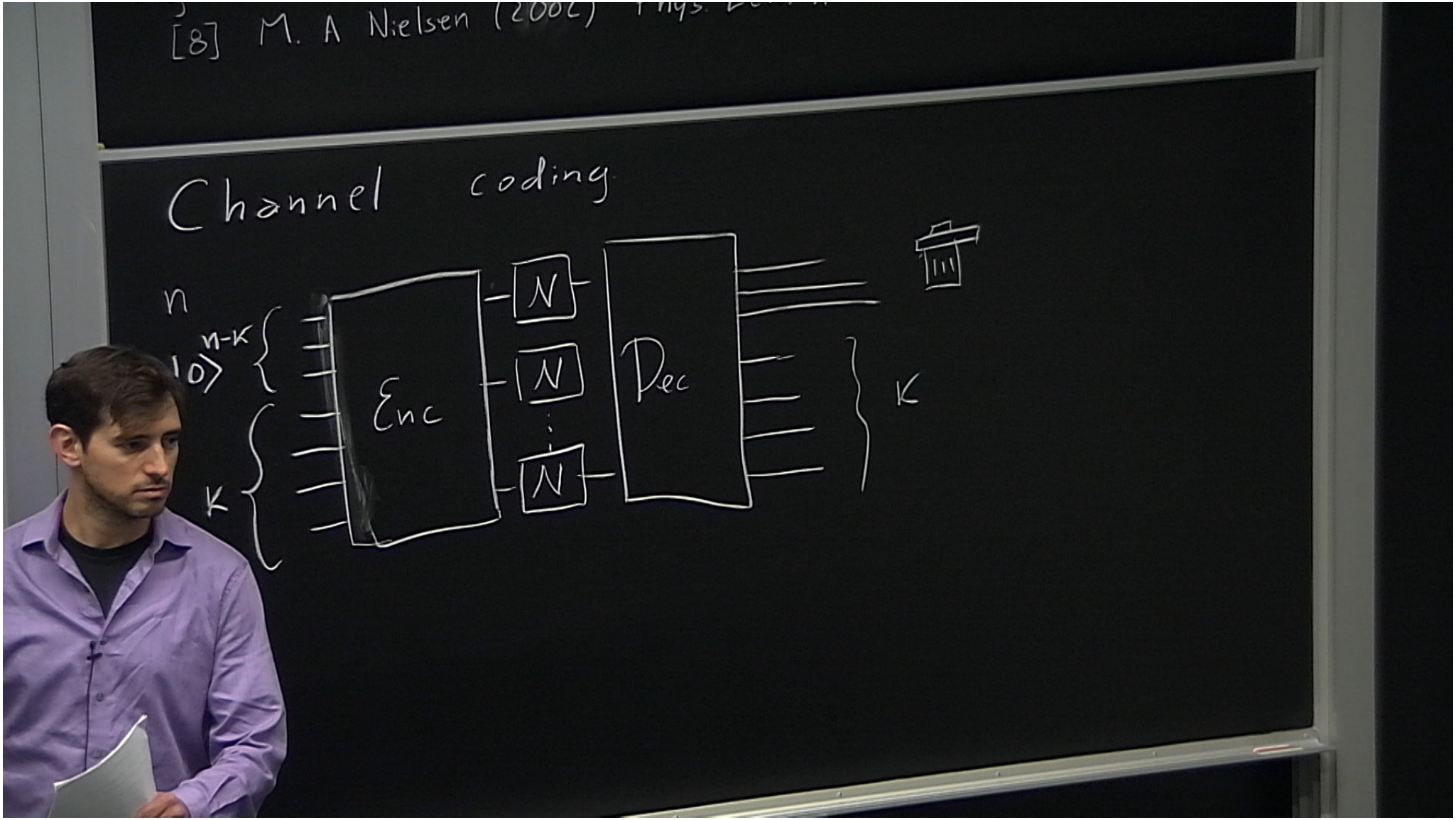
$\mu(\psi)$ ψ



$n \log \mathrm{Dim}(\mathcal{X}) \longrightarrow \lceil n\, H(\mathcal{X}) \rceil$


Towards holography via
quantum source-channel codes
Algorithmic cooling
Source coding
$\mathcal{M}(\mathcal{H})$ $\mathcal{H}$
ψ_1
ψ_2
ψ_3
ψ_n
$U_{S.C.}$
$\approx |0\rangle^{\otimes}$
ψ_{zip}
$n \log \mathrm{Dim}(\mathcal{X}) \longrightarrow \lceil n H(X) \rceil$


[8] M. A Nielsen (2002)
Channel coding
n
|0⟩ n-k
k
Enc
N
N
N
Dec
k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Channel coding
n
|0⟩ n-k
k
Enc
N
N
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Dec
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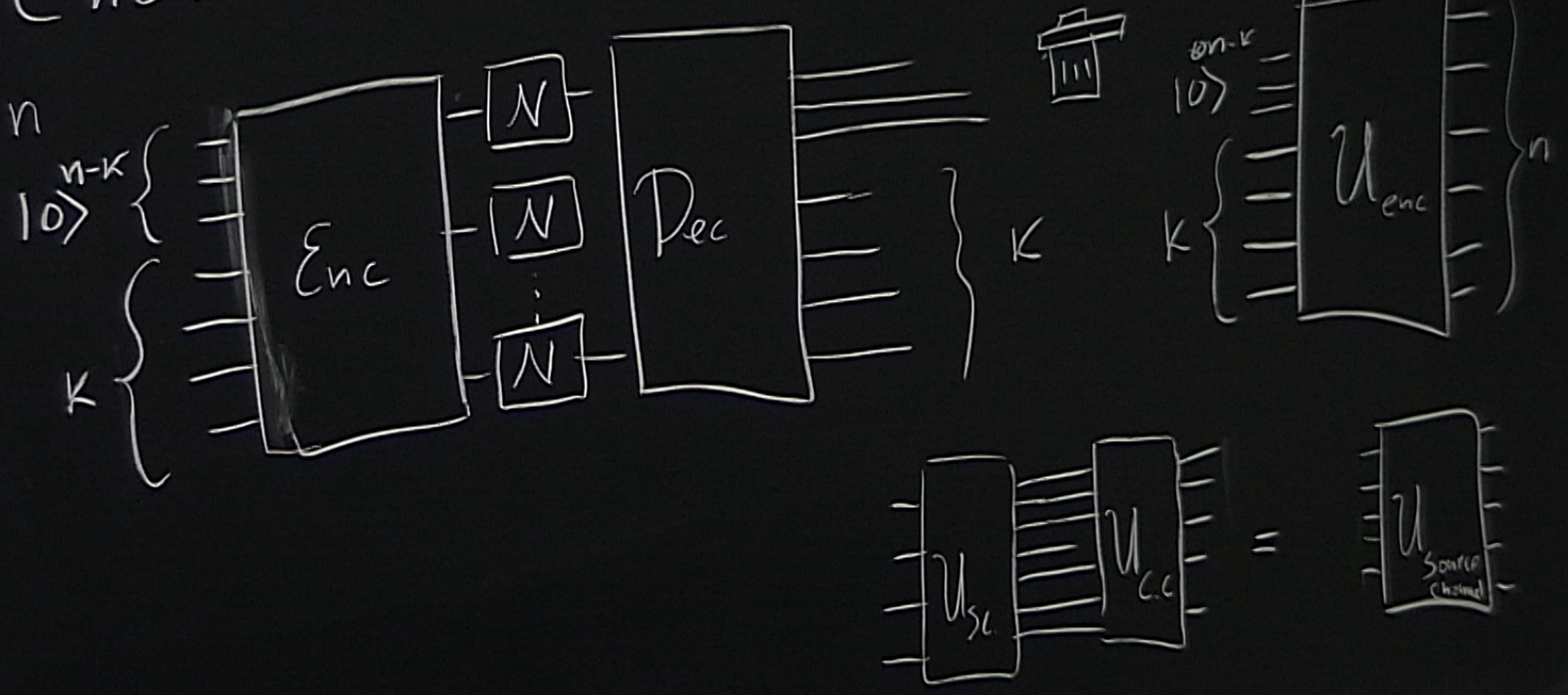
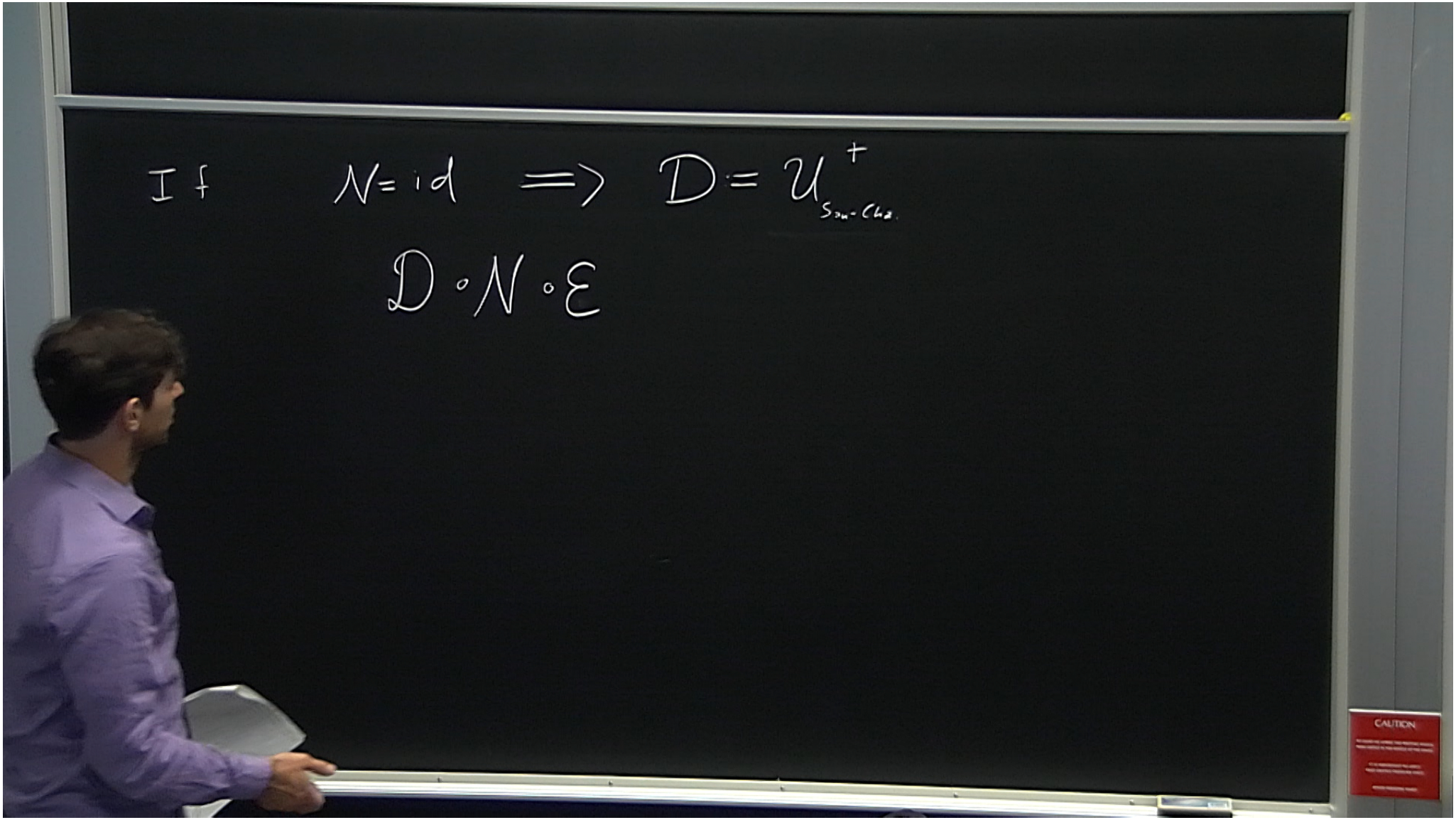
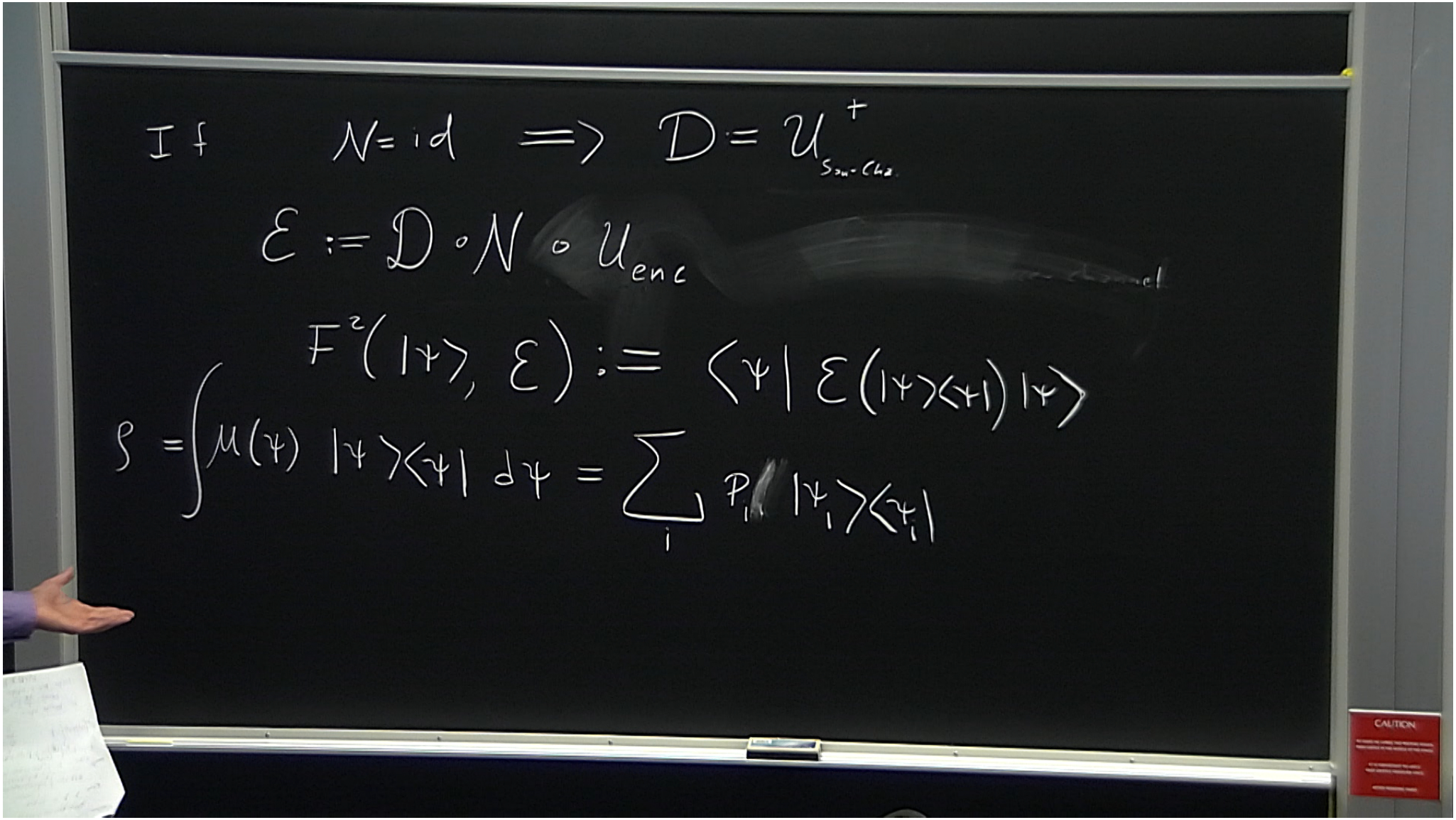
[8] M. A Nielsen (2002) Phys.

Channel coding

[n, k, d]




If $N = id \Rightarrow D = \mathcal{U}^{+}_{\text{Sym-Cha}}$
$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E}$
CAUTION


If $\mathcal{N} = id \Rightarrow \mathcal{D} = \mathcal{U}^{+}_{Sou-Cha}$
$\mathcal{E} := \mathcal{D} \circ \mathcal{N} \circ \mathcal{U}_{enc}$
$F^2(|\psi\rangle, \mathcal{E}) := \langle\psi| \mathcal{E}(|\psi\rangle\langle\psi|) |\psi\rangle$
$\rho = \int \mu(\psi) |\psi\rangle\langle\psi| \, d\psi = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
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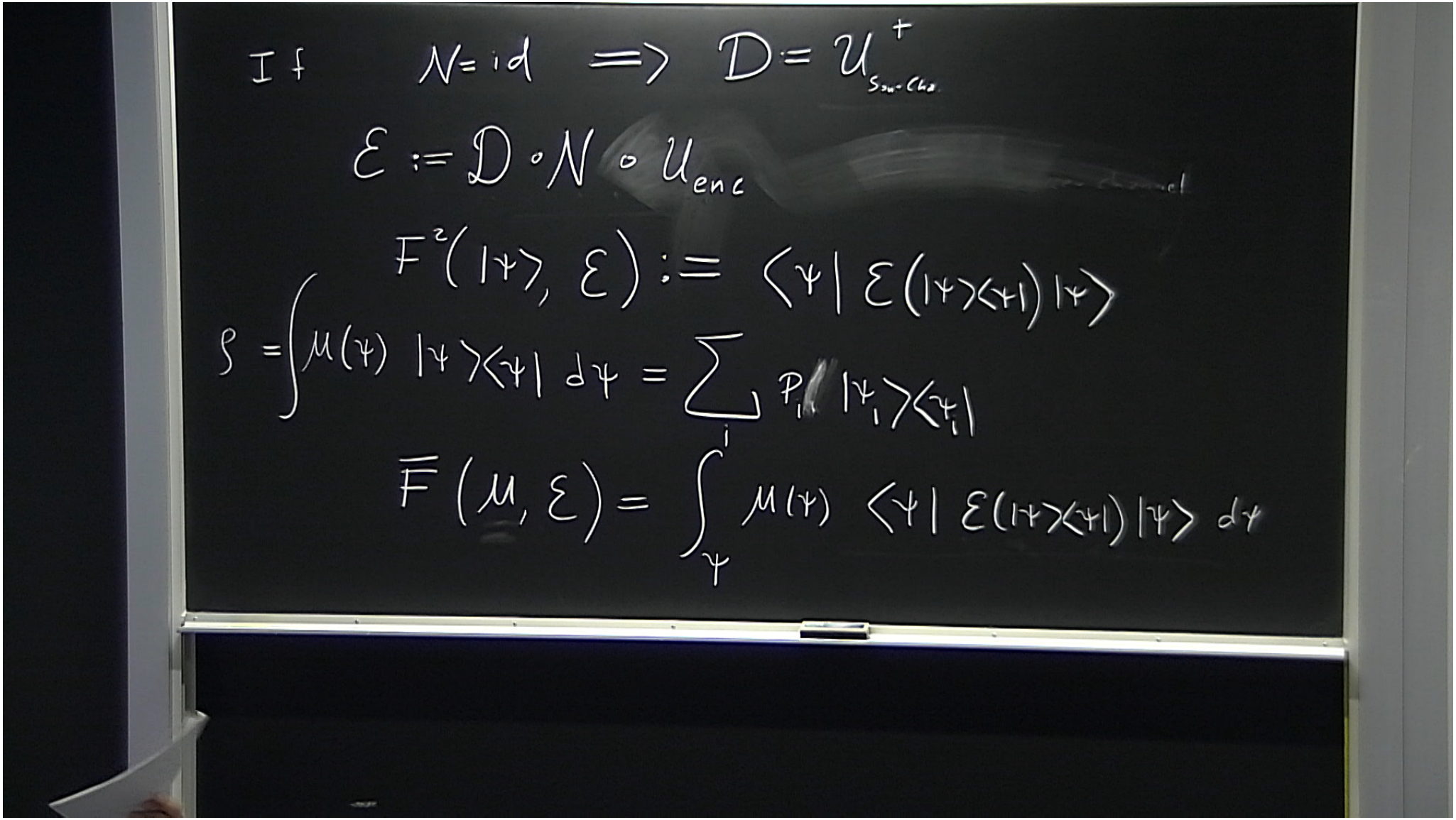
$$\bar{F}(\mu, \mathcal{E}) = \int_\psi \mu(\psi) \langle\psi| \mathcal{E}(|\psi\rangle\langle\psi|) |\psi\rangle \, d\psi$$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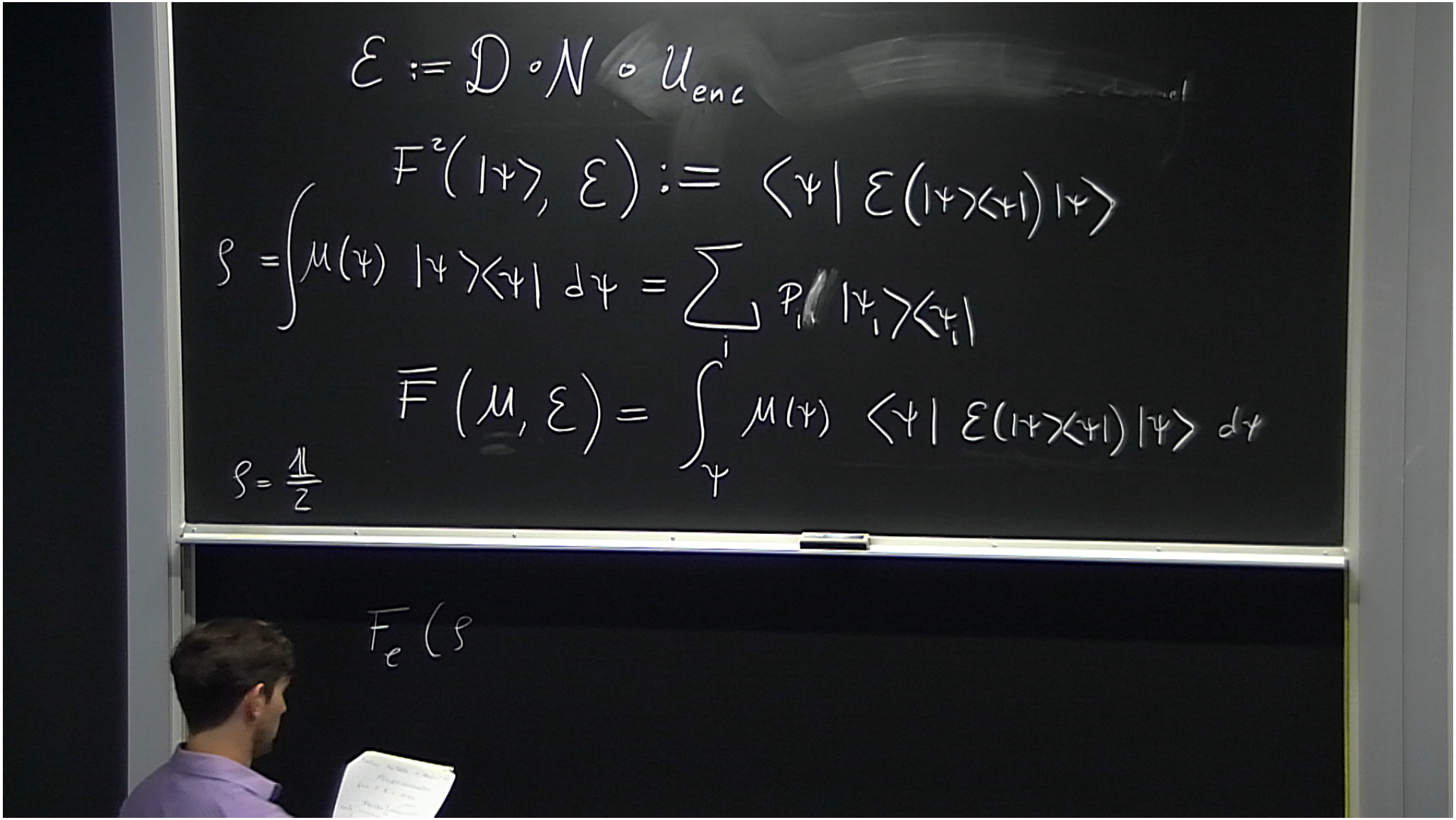
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$\rho = \frac{\mathbb{1}}{2}$
$F_e(\rho$

If $\mathcal{N} = id \Rightarrow \mathcal{D} = \mathcal{U}^{+}_{\text{Sch-Cha}}$

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$$F^2(|\psi\rangle, \mathcal{E}) := \langle\psi| \mathcal{E}(|\psi\rangle\langle\psi|) |\psi\rangle$$

$$\rho = \int \mu(\psi) |\psi\rangle\langle\psi| \, d\psi = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

don't need to be orthogonal

$$F(\mu, \mathcal{E}) = \int_\psi \mu(\psi) \langle\psi| \mathcal{E}(|\psi\rangle\langle\psi|) |\psi\rangle \, d\psi$$

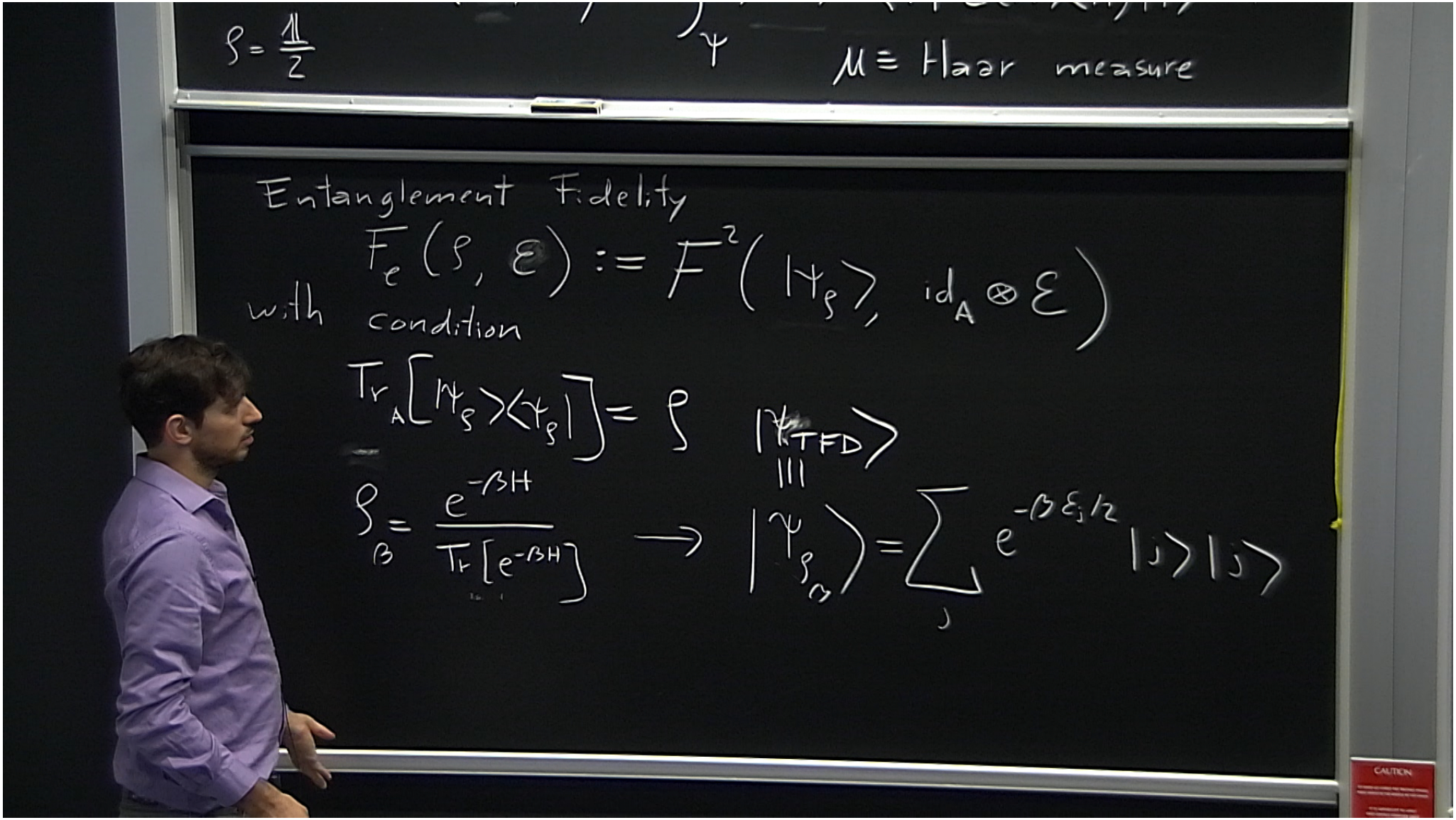
$\rho = \frac{\mathbb{1}}{2}$

$\mu \equiv$ Haar measure

Entanglement fidelity

$$F_e(\rho, \mathcal{E}) := F^2(|\psi_\rho\rangle, id_A \otimes \mathcal{E})$$

with condition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ψ
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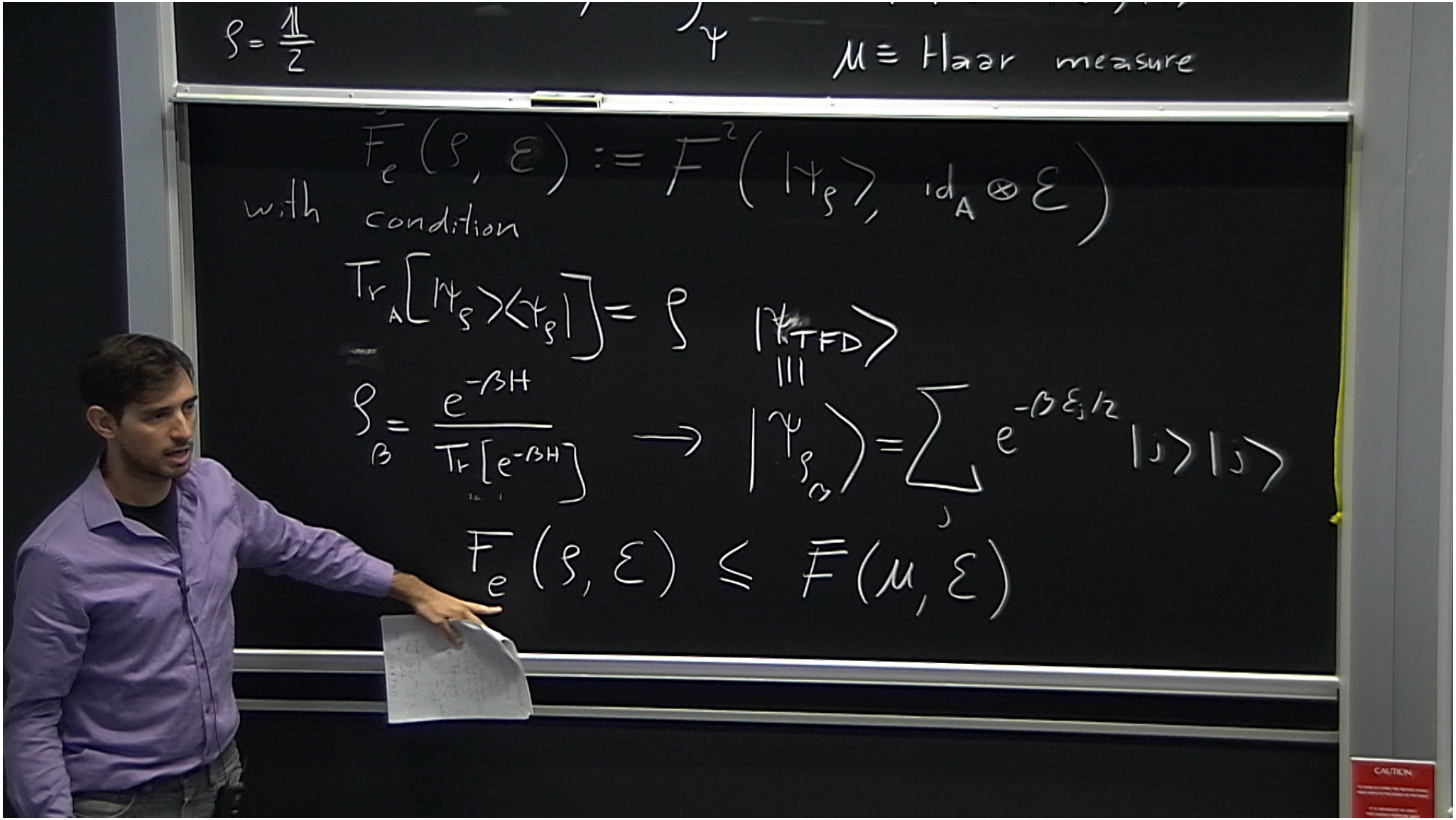
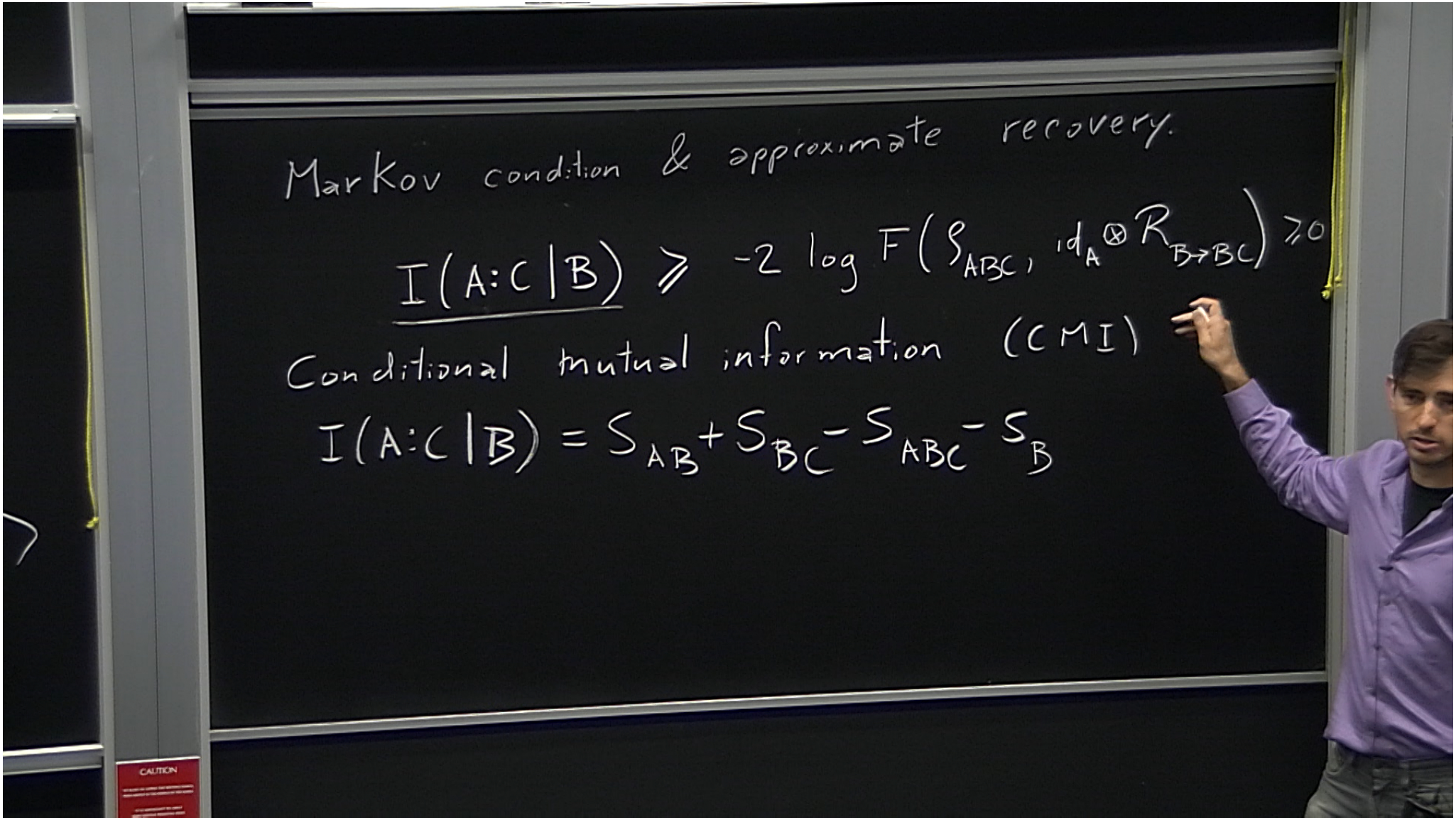
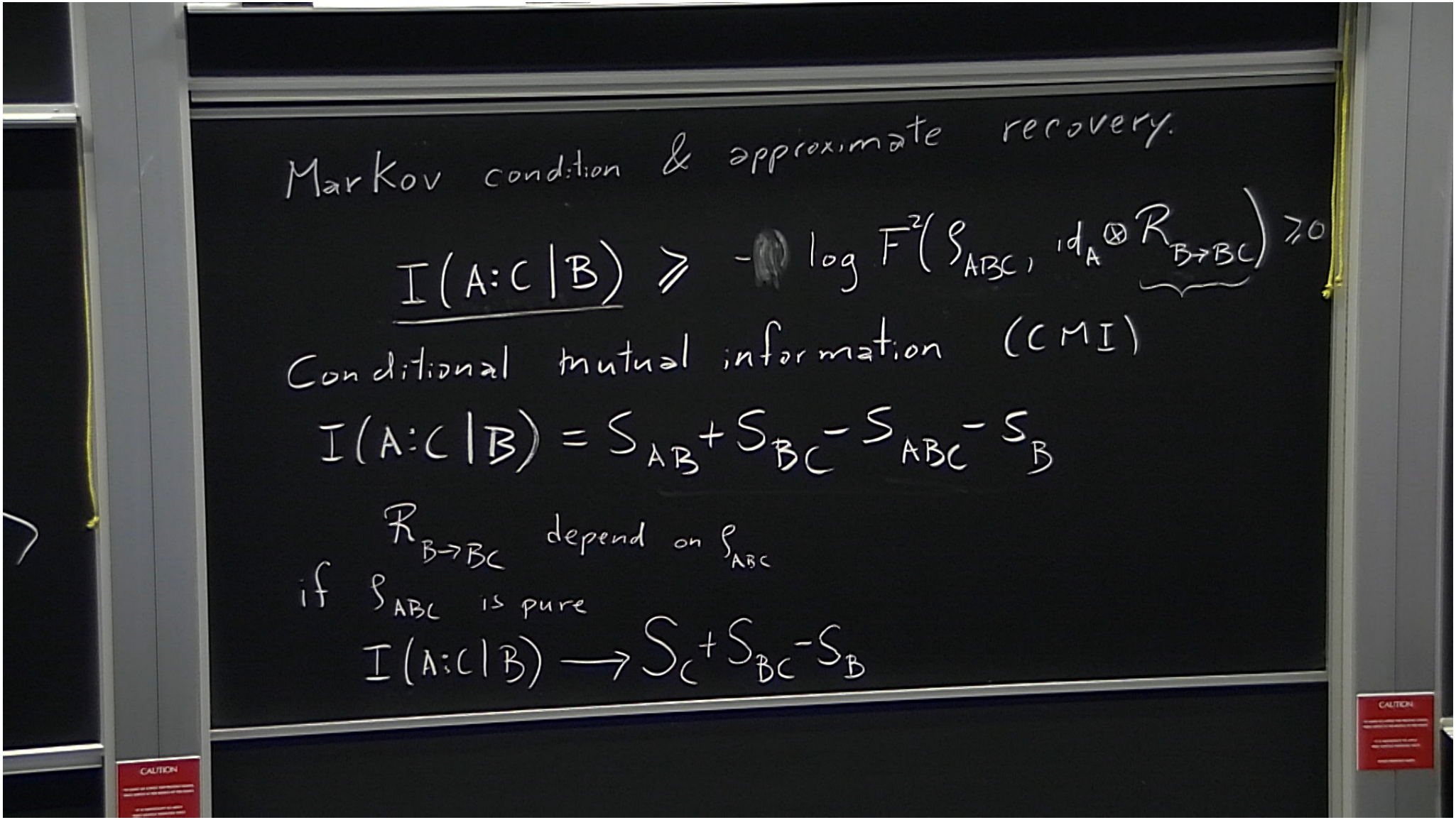
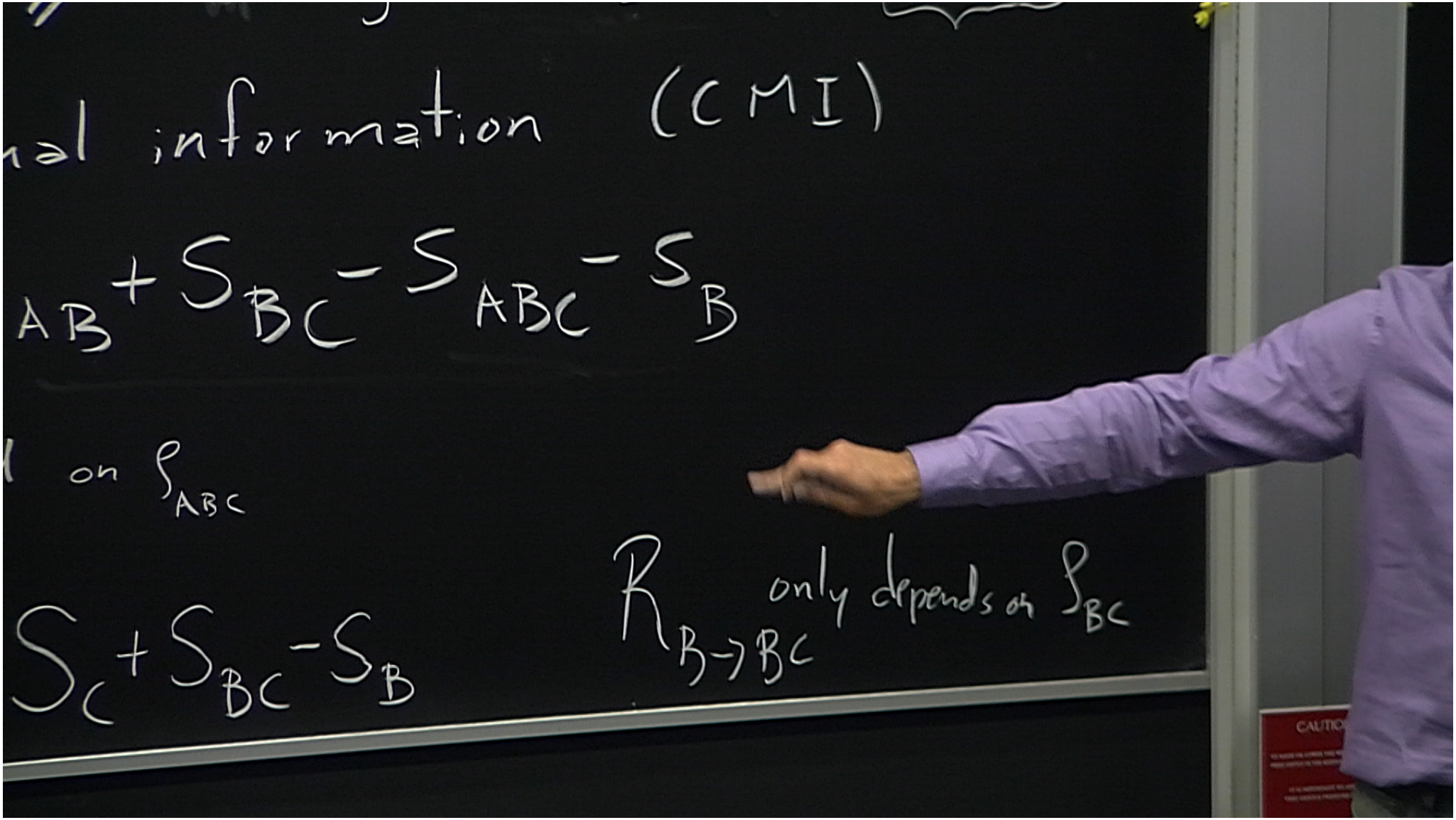
Entanglement Fidelity
$F_e(\rho, \mathcal{E}) := F^2(|\psi_\rho\rangle, id_A \otimes \mathcal{E})$
with condition
$Tr_A[|\psi_\rho\rangle\langle\psi_\rho|] = \rho$
$|\psi_{TFD}\rangle$
$\rho_\beta = \frac{e^{-\beta H}}{Tr[e^{-\beta H}]} \rightarrow |\psi_{\rho_\beta}\rangle = \sum_j e^{-\beta E_j/2} |j\rangle|j\rangle$


$\rho = \frac{\mathbb{1}}{2}$
ψ
$\mathcal{M} \equiv$ Haar measure
$F_e(\rho, \mathcal{E}) := F^2(|\psi_\rho\rangle, id_A \otimes \mathcal{E})$
with condition
$Tr_A[|\psi_\rho\rangle\langle\psi_\rho|] = \rho$
$|\psi_{TFD}\rangle$
$\rho_\beta = \frac{e^{-\beta H}}{Tr[e^{-\beta H}]} \rightarrow |\psi_{\rho_\beta}\rangle = \sum_j e^{-\beta E_j/2} |j\rangle|j\rangle$
$F_e(\rho, \mathcal{E}) \leq F(\mathcal{M}, \mathcal{E})$
CAUTION


Markov condition & approximate recovery.
$I(A:C|B) \geq -2 \log F(\rho_{ABC}, id_A \otimes R_{B \to BC}) \geq 0$
Conditional mutual information (CMI)
$I(A:C|B) = S_{AB} + S_{BC} - S_{ABC} - S_B$
CAUTION


Markov condition & approximate recovery.
$I(A:C|B) \geq -\log F^2(\rho_{ABC}, id_A \otimes R_{B\to BC}) \geq 0$
Conditional mutual information (CMI)
$I(A:C|B) = S_{AB} + S_{BC} - S_{ABC} - S_B$
$R_{B\to BC}$ depend on ρ_{ABC}
if ρ_{ABC} is pure
$I(A:C|B) \longrightarrow S_C + S_{BC} - S_B$

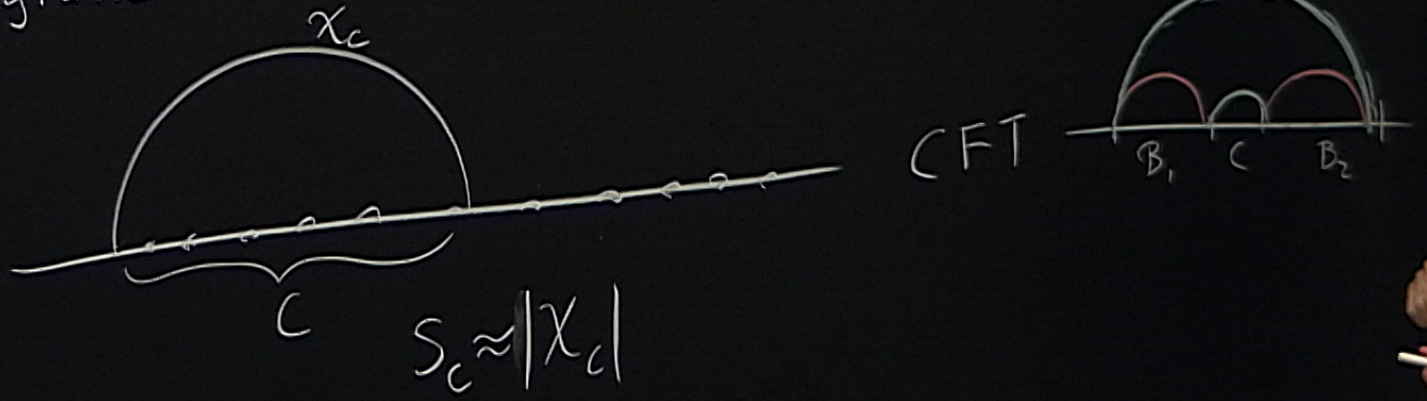

nal information (CMI)
$S_{AB} + S_{BC} - S_{ABC} - S_B$
on ρ_{ABC}
$S_C + S_{BC} - S_B$
$R_{B \to BC}$ only depends on ρ_{BC}

If ρ_{ABC} pure

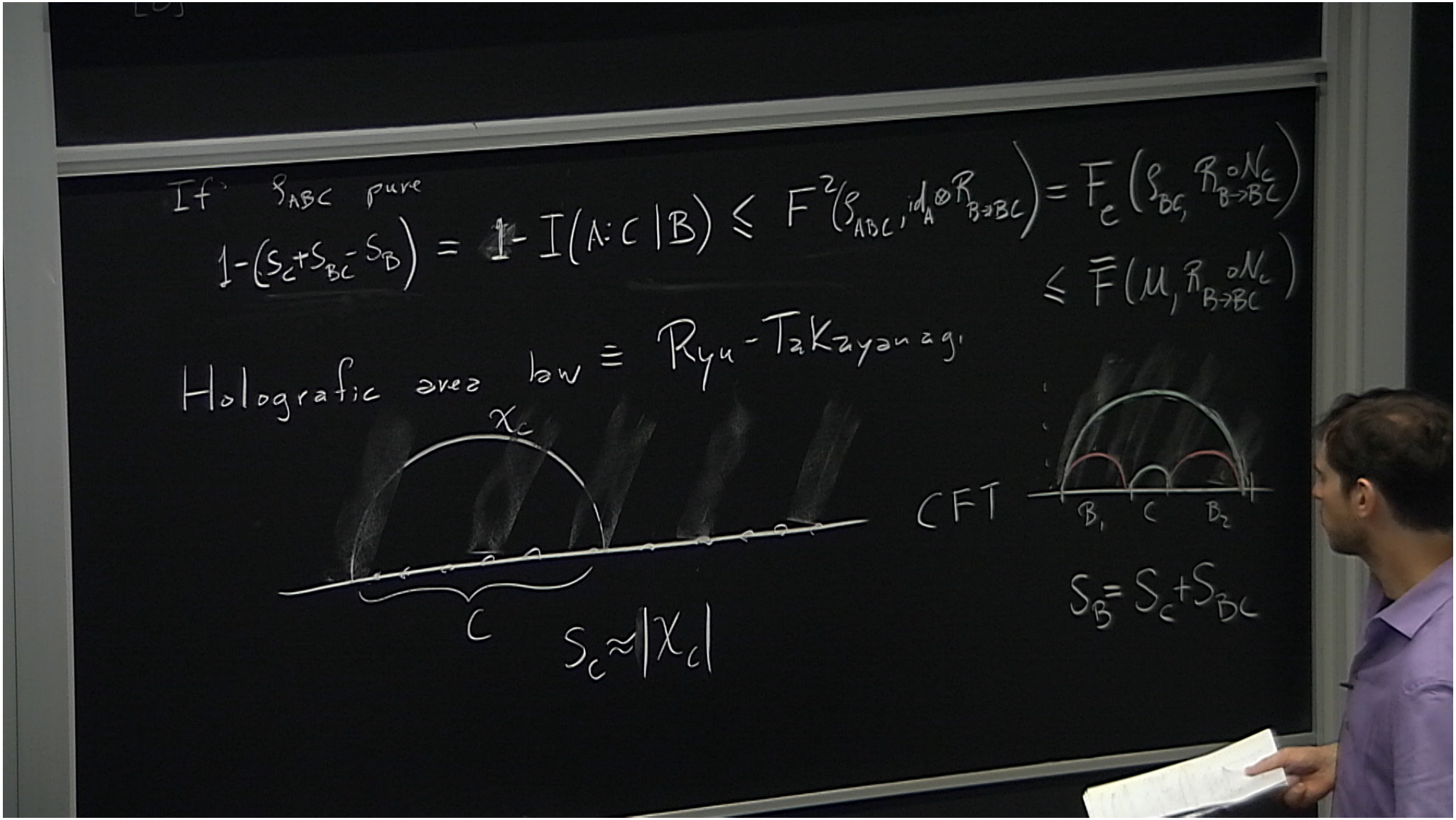
$$1-(S_C+S_{BC}-S_B) = I(A:C|B) \leq F^2(\rho_{ABC}, id_A \otimes \mathcal{R}_{B\to BC}) = F_e(\rho_{BC}, \mathcal{R}_{B\to BC}\circ \mathcal{N}_C)$$

$$\leq \bar{F}(\mu, \mathcal{R}_{B\to BC}\circ \mathcal{N}_C)$$

Holografic area law $\equiv$ Ryu-Takayanagi



$$S_C \approx |\chi_C|$$

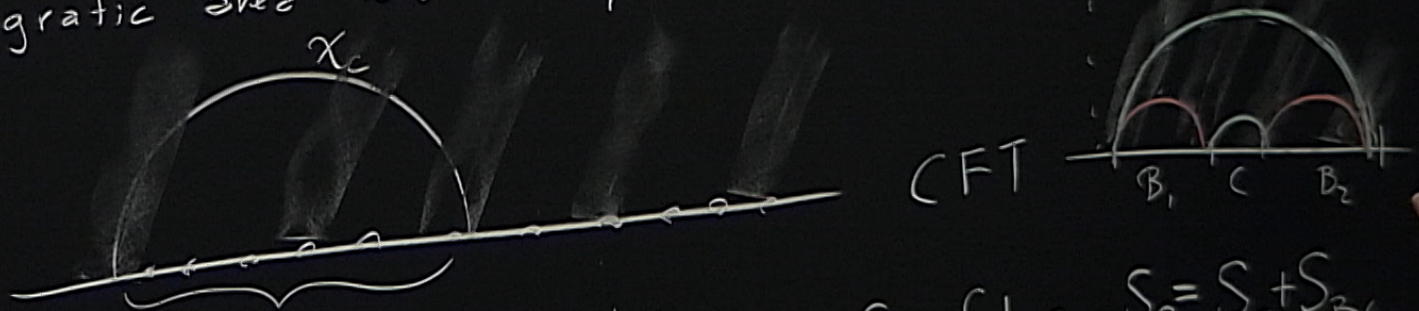

If ρ_{ABC} pure
$1-(S_C+S_{BC}-S_B) = 1-I(A:C|B) \leq F^2(\rho_{ABC}, id_A \otimes R_{B\to BC}) = F_e(\rho_{BC}, R_{B\to BC}\circ N_C)$
$\leq \bar{F}(\mathcal{M}, R_{B\to BC}\circ N_C)$
Holografic area law $\equiv$ Ryu-Takayanagi
χ_C
C
$S_C \approx |\chi_C|$
CFT
B_1 C B_2
$S_B = S_C + S_{BC}$

If ρ_{ABC} pure

$$1-(S_C+S_{BC}-S_B) = 1-I(A:C|B) \leq F^2(\rho_{ABC}, id_A \otimes \mathcal{R}_{B\to BC}) = F_e(\rho_{BC}, \mathcal{R}_{B\to BC}\circ \mathcal{N}_C)$$

$$\leq \bar{F}(\mathcal{M}, \mathcal{R}_{B\to BC}\circ \mathcal{N}_C)$$

Holografic area law $\equiv$ Ryu-Takayanagi



$$S_C \approx |\chi_C| \qquad S_{2\ell} \overset{\sim}{\approx} S_\ell + \underbrace{\frac{c}{6}\log 2} \qquad S_B = S_C + S_{BC}$$

$\rho = \frac{\mathbb{1}}{D_{\mathcal{H}_A}}$ $\quad \mu \equiv$ Haar $\quad \bar{F}(\varepsilon) = F_e(\varepsilon) + \frac{1 - F_e(\varepsilon)}{D_{\mathcal{H}_A}^{+1}}$

Transverse field Ising (CFT)

$$H_{TF} = \sum_{j=1}^{n} -\sigma_j^x \sigma_{j+1}^x - \sigma_j^z \qquad P := \bigotimes_{j=1}^{n} \sigma_i^z$$

$$P_{even} = \frac{P + \mathbb{1}}{2} \qquad P_{odd} = \frac{-P + \mathbb{1}}{2}$$

$$H_{maj}^{(odd, even)} = \sum$$

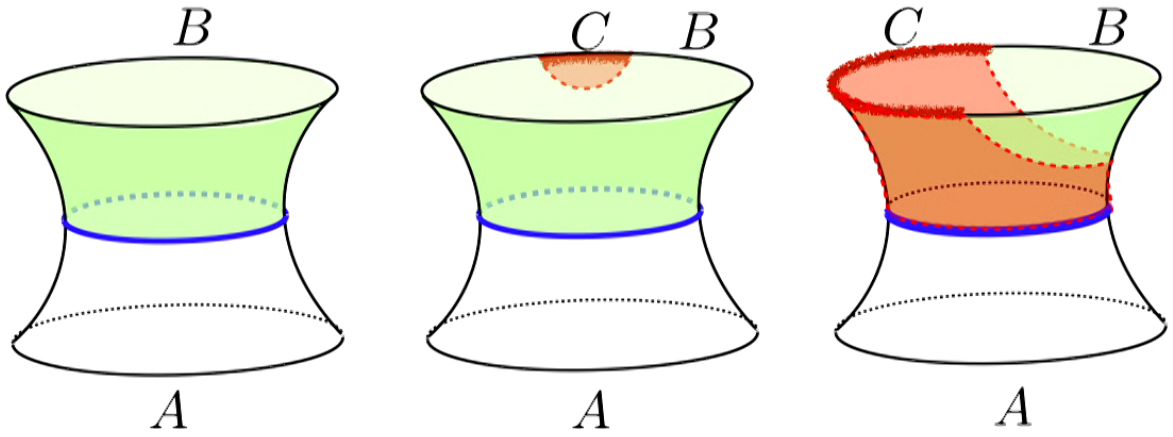
$\rho = \frac{\mathbb{1}}{D_{\mathcal{H}_A}}$ $\quad \mu \equiv$ Haar $\quad \bar{F}(\mathcal{E}) = F_e(\mathcal{E}) + \frac{1 - F_e(\mathcal{E})}{D_{\mathcal{H}_A}^2 + 1}$

Transverse field Ising (CFT)

$$H_{TF} = \sum_{j=1}^{n} -\sigma_j^x \sigma_{j+1}^x - \sigma_j^z \qquad P := \bigotimes_{j=1}^{n} \sigma_i^z$$

$$H_{maj}^{(odd,even)} = \left(\sum_{j=1}^{2n-1} i w_j w_{j+1} \right) \pm i w_{2n} w_1 \qquad P_{even} = \frac{P + \mathbb{1}}{2} \quad P_{odd} = \frac{-P + \mathbb{1}}{2}$$

Thermal CFT geometric interpretation

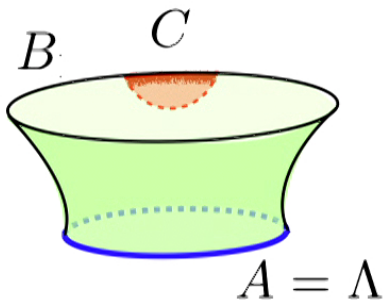


$$S_{ABC} = 0 \quad \Rightarrow \quad S(A:C|B) = S_C + S_{BC} - S_B$$

- BTZ black hole dual to CFT thermal state.
- No **proper** subspace supporting the thermal state.
- Interpret a CFT thermal state as a source-channel code.

Calculate on your favorite CFT

Study conditional mutual information as a function of system size *n*. (lattice Hamiltonian)



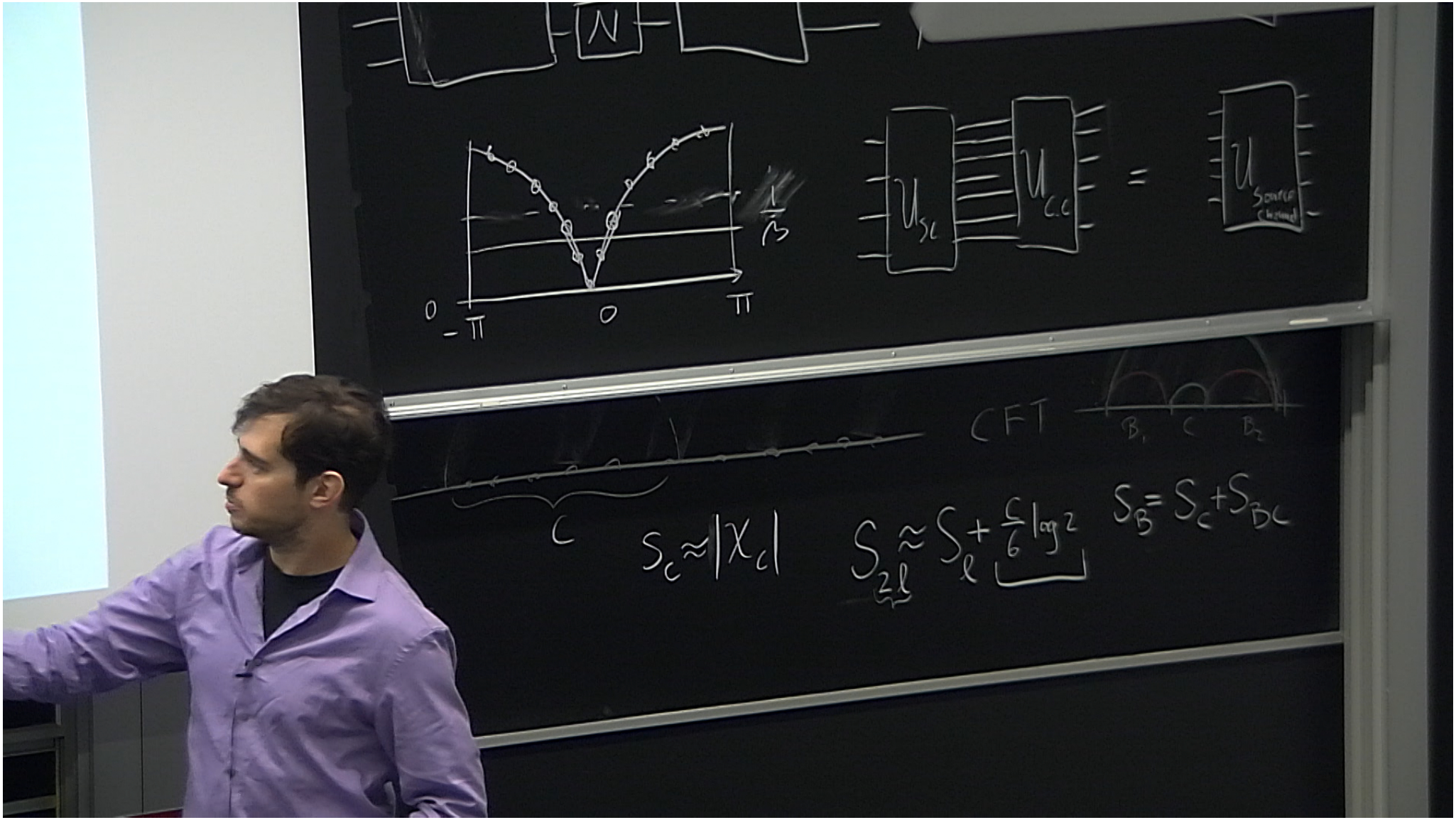
Constant temperature correspond to BH horizon a constant distance from boundary.

Scale inverse temperature with *n*. $\beta \propto n^q \qquad q \in (0, 1]$

For constant $S(\rho_\beta)$ thermal entropy $\beta \propto n$

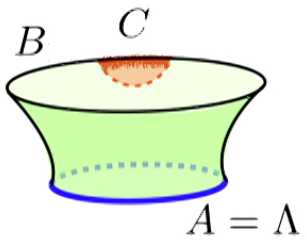
Now calculate CMI on your favorite CFT!!!

Non-standard limit with comparable quantities.


N
0
-π
0
π
1/β
U_SC
U_CC
=
U_source channel
CFT
B_1
C
B_2
C
S_C ≈ |χ_C|
S_2l ≈ S_l + c/6 log 2
S_B = S_C + S_BC

Critical transverse field Ising

$$H = \sum_{j=1}^{n} Z_j + X_j \otimes X_{j+1}$$



Equivalent to free fermions within given parity sector.

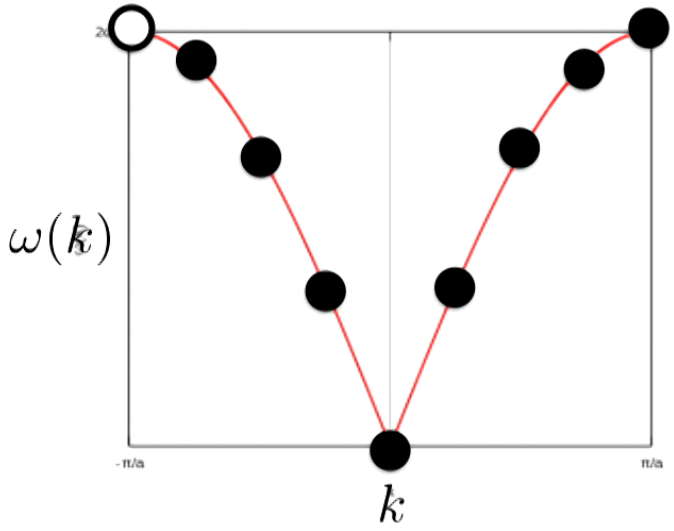
$$\rho_{BC} = \rho_{\text{even}}^{(\beta)} := \frac{P_{\text{even}} e^{-\beta H_{TF}}}{\text{tr}\left[P_{\text{even}} e^{-\beta H_{TF}}\right]}.$$

Free theory c=1/2 hence not holographic.

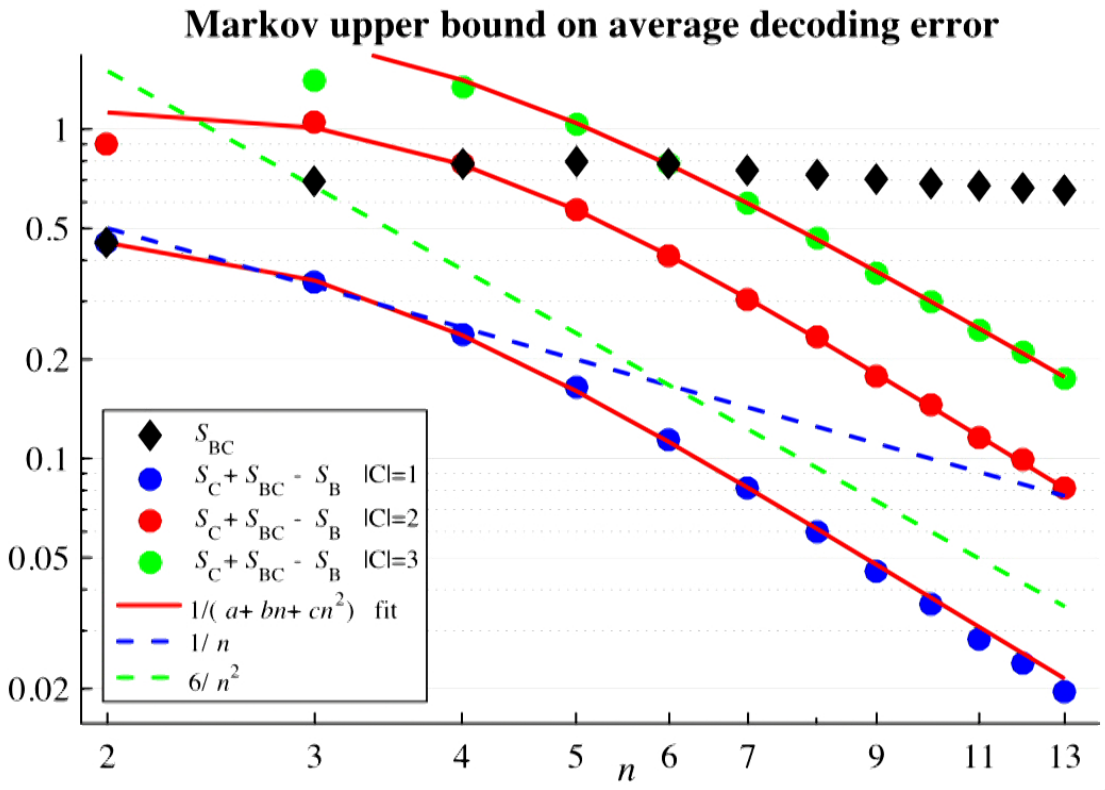
Dispersion relation:

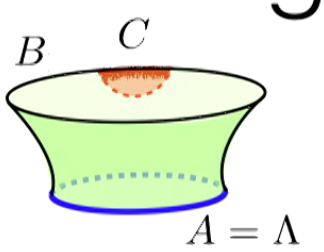
$\beta \propto n$ ~constant k

$S(\rho_\beta)$



Critical transverse field Ising





$\beta \propto n$

~constant k

$S(\rho_\beta)$

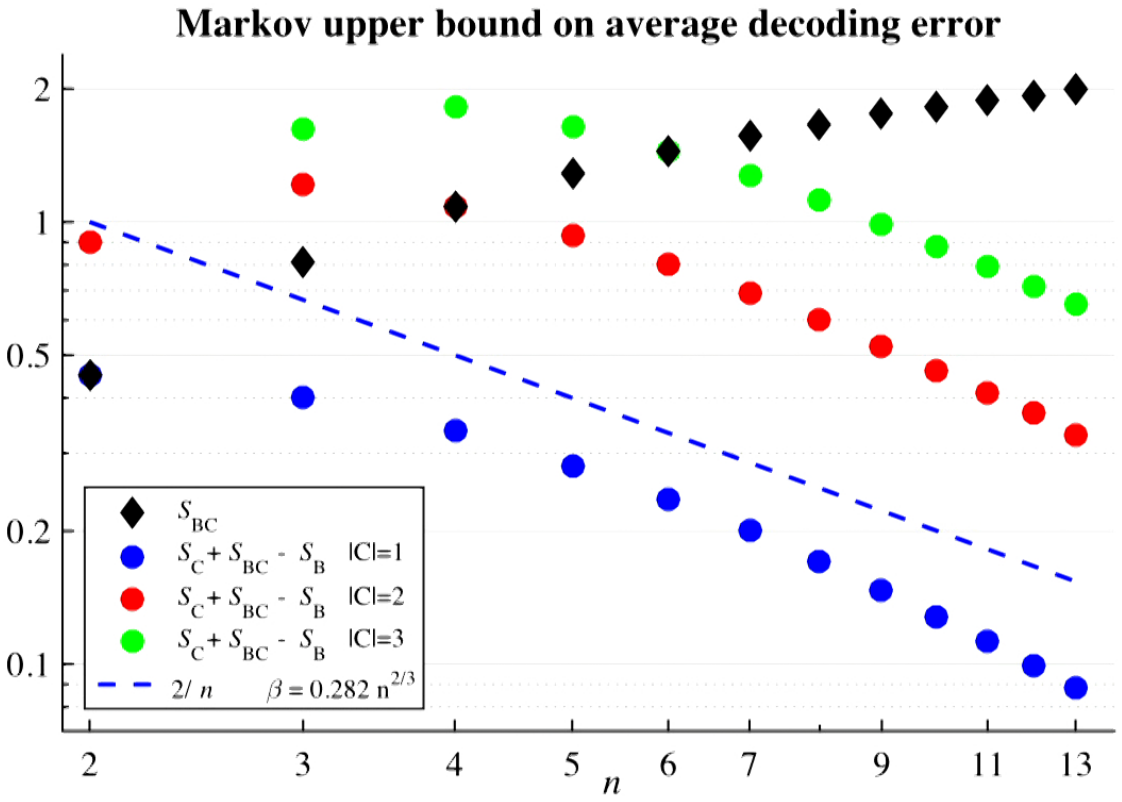
constant |C|

$\epsilon \propto 1/n^2$

Parity hack to forbid unphysical errors.

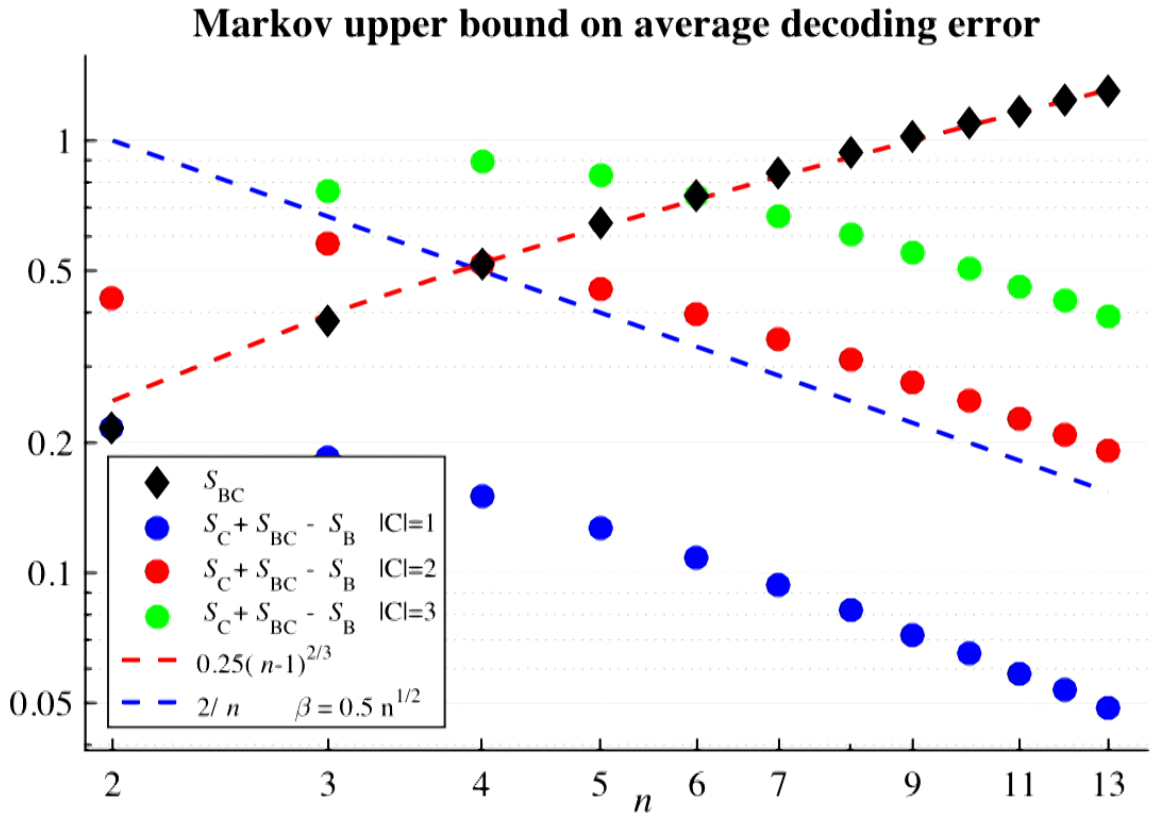
$$\rho_{BC} = \rho_{\text{even}}^{(\beta)} := \frac{P_{\text{even}} e^{-\beta H_{TF}}}{\text{tr}\left[P_{\text{even}} e^{-\beta H_{TF}}\right]}.$$

Larger BH = more logical Inf.



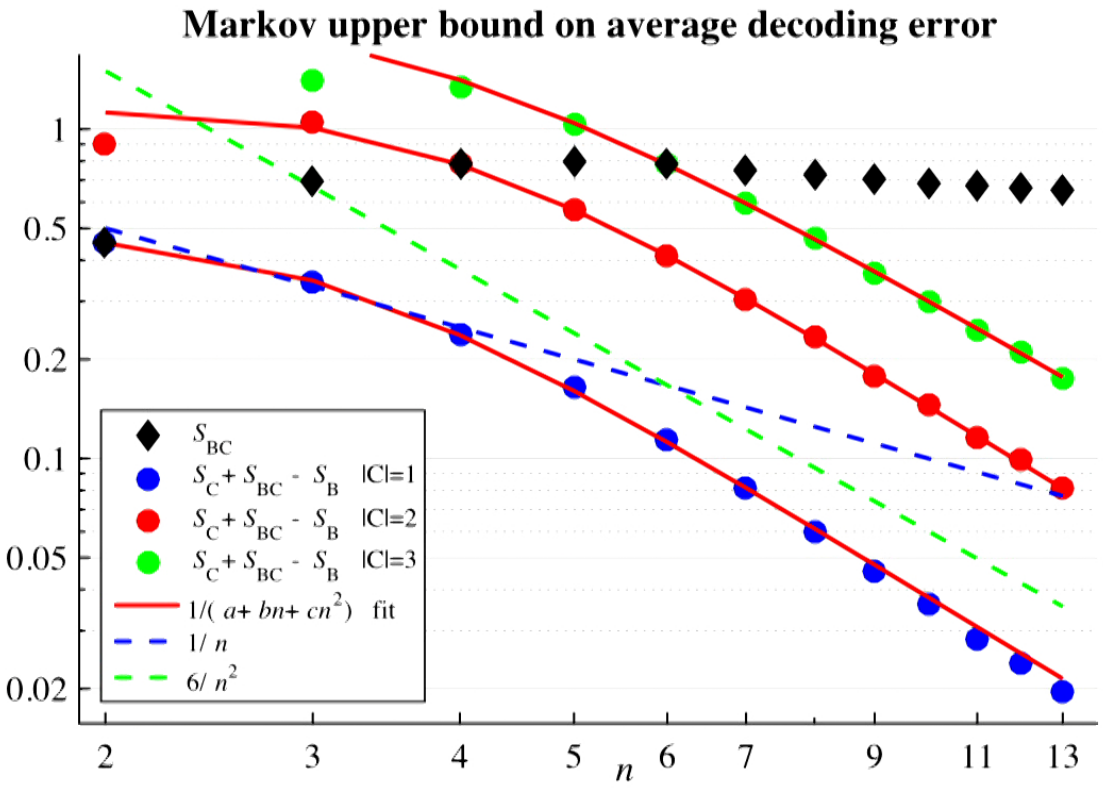
$$\beta \propto n^{2/3}$$

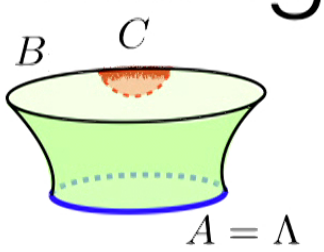
Larger BH = more logical Inf.



$$\beta \propto n^{1/2}$$

Critical transverse field Ising





$\beta \propto n$

~constant k
$S(\rho_\beta)$

constant |C|
$\epsilon \propto 1/n^2$

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