Title: Causal Sets in 2 dimensions

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Abstract: <p>Calculating the path integral over all causal sets will take a lot of computing power, and requires a way to suppress non-manifold like causal sets. To work towards these goals we can start by taking the path integral over a restricted class of causal sets, the 2d orders. Using these have found a phase transition and been able to study the Hartle-Hawking wave function in causal sets. In this talk I will present these recent results (arXiv:1410.8775 and arXiv:1706.06432), and some preliminary data exploring how the behavior of the 2d orders is influenced by coupling them to an Ising model.</p>

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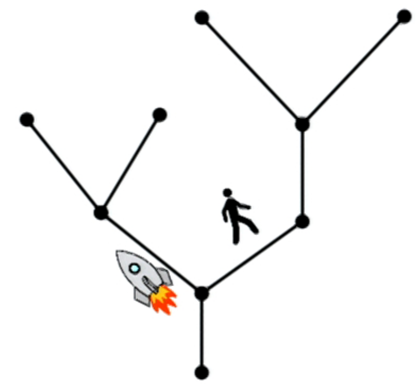
Causal Sets in 2 dimensions

Radboud University

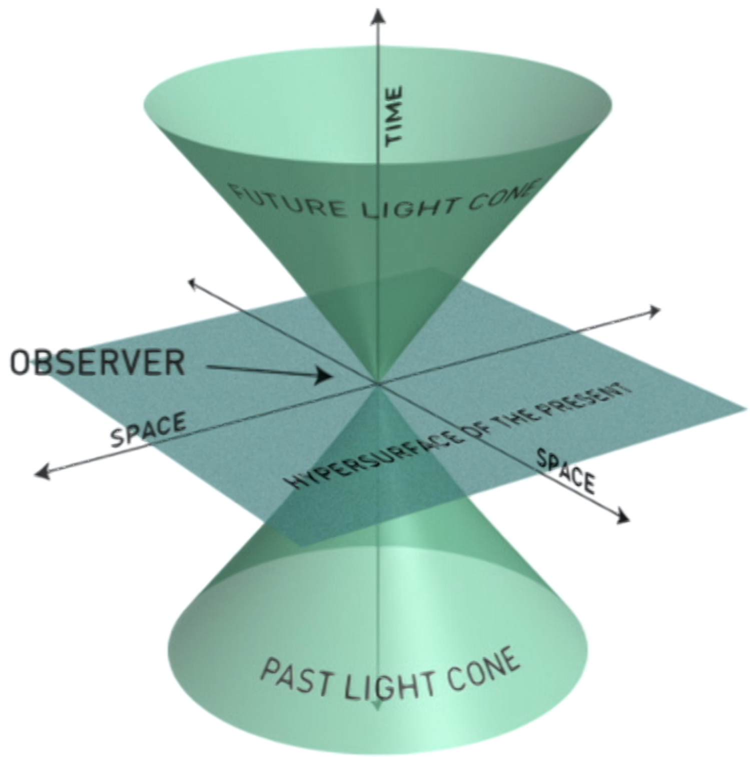
Lisa Glaser
September 14, 2017



Content:

What is a causal set?
2d orders
Hartle-Hawking wave function
Ising model on Causal sets

Causal Structure

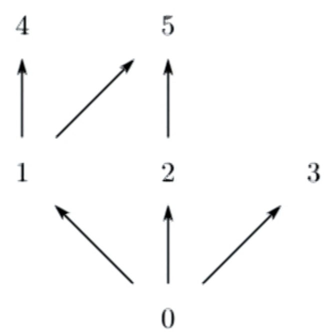


Malaments Theorem

Causal structure + conformal factor = Geometry

Number + Order = Geometry

Causal set = partially ordered set
Partial order relations = causal relations



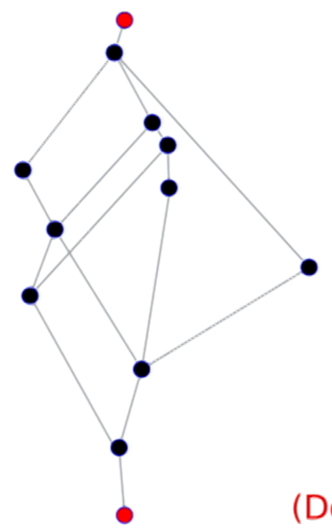
- **reflexive** for all $x \in \mathcal{C}$ $x \preceq x$
- **transitive** for all $x, y, z \in \mathcal{C}$ and $x \preceq y$ and $y \preceq z$ then $x \preceq z$
- **antisymmetric** if $x, y \in \mathcal{C}$ and $x \preceq y \preceq x$ then $x = y$
- **locally finite** for all $x, y \in \mathcal{C}$ $|I(x, y)| < \infty$

Wave operators on causal set

Wave equation on causal set

Use covariant observables only:

- Volume of interval between two causet elements



(Dowker & LG arXiv:1305.2588)

(Aslanbeigi et.al. arXiv:1403.1622)

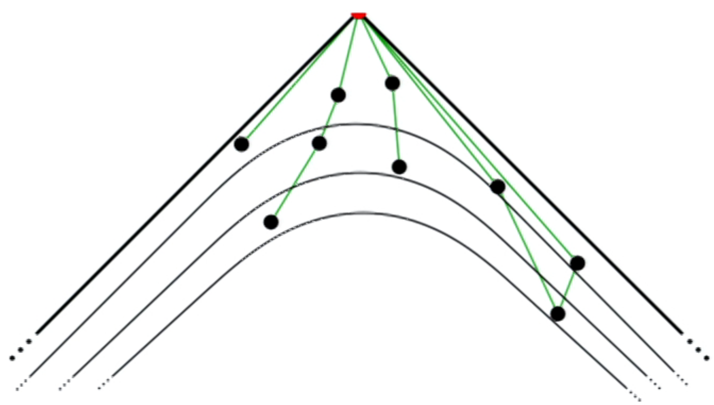
Wave operators on causal set

Wave equation on causal set

Use covariant observables only:

- Volume of interval between two causet elements

$$\rho^{-\frac{2}{D}}(B_{\rho}^{(D)}\phi)(x) = a\phi(x) + \sum_{n=0}^{L_{max}} b_n \sum_{y\in I_n(x)} \phi(y)$$



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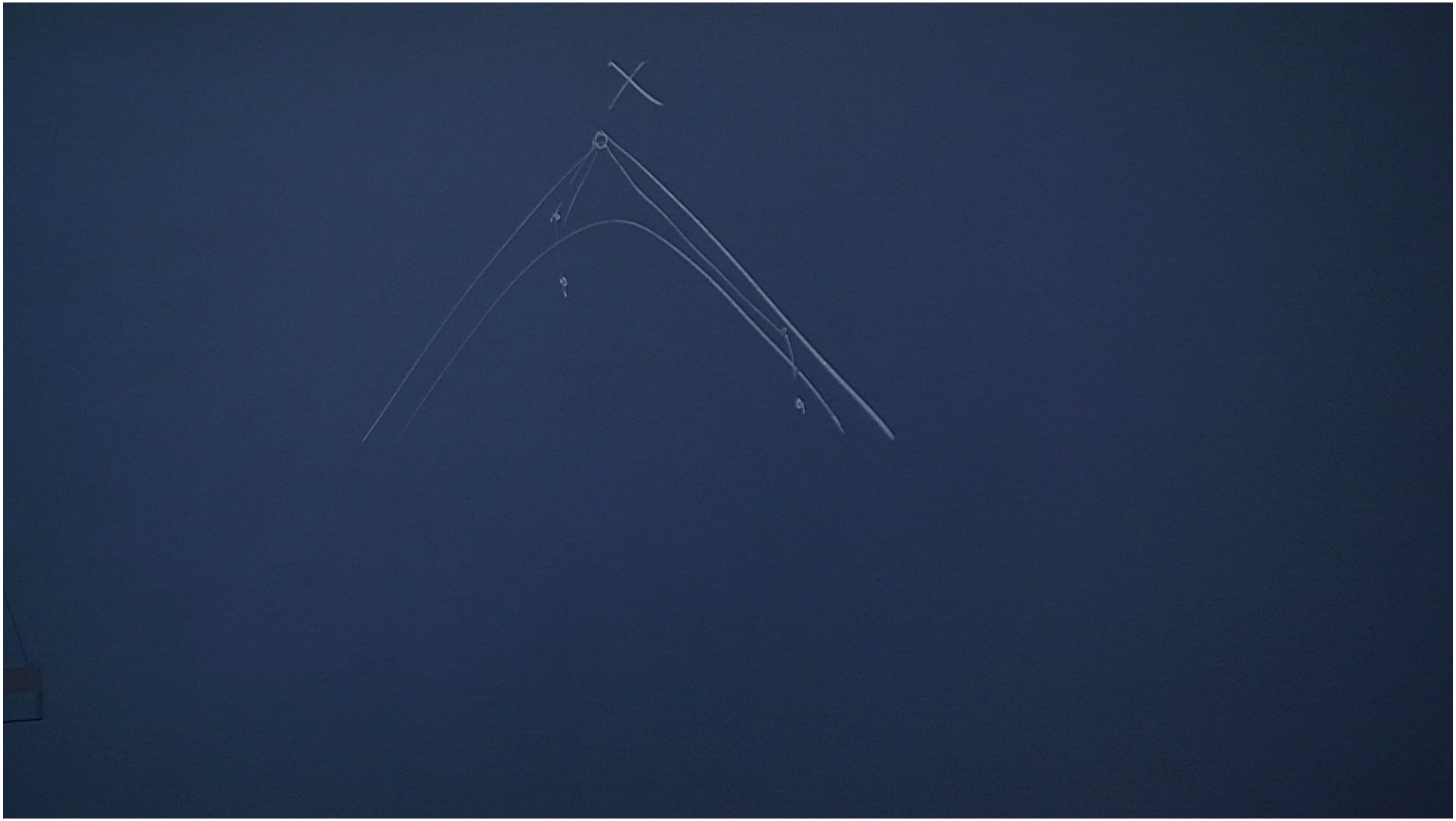

The action

Assuming the discreteness is at the Planck scale $\rho = \frac{1}{l_p^D}$ we can define an action by acting $B_\rho^{(D)}$ on a constant

$$l_p^2 S_\rho^{(D)} = l_p^2 \sum_{\mathcal{C}} B_\rho^{(D)}(-2) = -2 \sum_{\mathcal{C}} a + \sum_{n=0}^{L_{max}} b_n \sum_{y \in I_n(x)}$$

$$\to \int \mathrm{d}V \sqrt{-g} R$$

For $D = 2$ the action is

$$\frac{1}{\hbar} S_{2D} = N - 2N_0 + 4N_1 - 2N_2$$

(Benincasa & Dowker arXiv:1001.2725)

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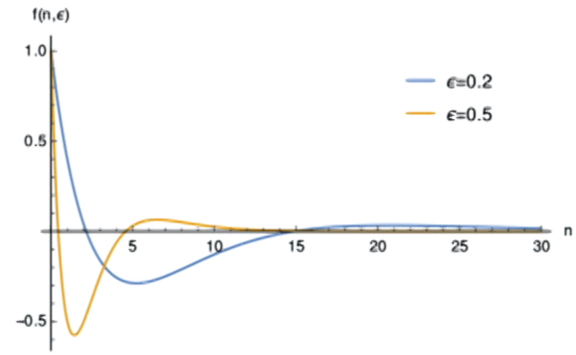
The action

To suppress fluctuations introduce intermediate scale ϵ

$$f(n,\epsilon) = (1-\epsilon)^n \sum_{j=0} b_j \binom{n}{j} \left(\frac{\epsilon}{1-\epsilon}\right)^j$$



$$l_p^2 S_\rho^{(D)} = -2\sum_{\mathcal{C}} a + \sum_{n=0} N_n f(n,\epsilon)$$

(Benincasa & Dowker arXiv:1001.2725)

Path integral in causal sets

$$\mathcal{Z}_{cont} = \int \mathcal{D}[\mathcal{M}] e^{\frac{i}{\hbar}\mathcal{S}(\mathcal{M},g)} \quad \rightarrow \quad \mathcal{Z}_{\Omega_{2d}} = \sum_{\mathcal{C}\in\Omega_{2d}} e^{-\beta S_{2d}(\mathcal{C},\epsilon)}$$

- $\mathcal{D}[\mathcal{M}] \rightarrow \sum_{\mathcal{C}\in\Omega_{2d}}$ continuum to discrete
- $\frac{i}{\hbar} \rightarrow -\beta$ analytic continuation
- $\mathcal{S}(\mathcal{M},g) \rightarrow S_{2d}(\mathcal{C},\epsilon)$ EH action to BD action

A statistical ensemble of Lorentizan geometries!

A simpler class of causal sets

0 to 10 define a total order
Two total orders define a coordinate grid

(Brightwell et.al. arXiv:0706.0375)

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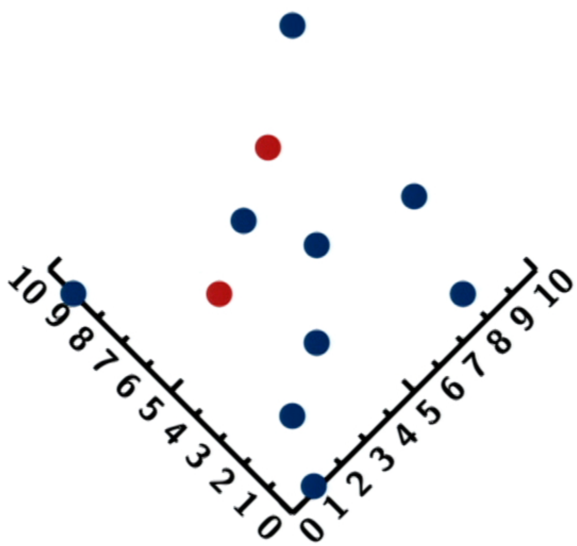
A statistical ensemble of Lorentizan geometries!

A simpler class of causal sets

Total order on numbers defines a partial order on elements



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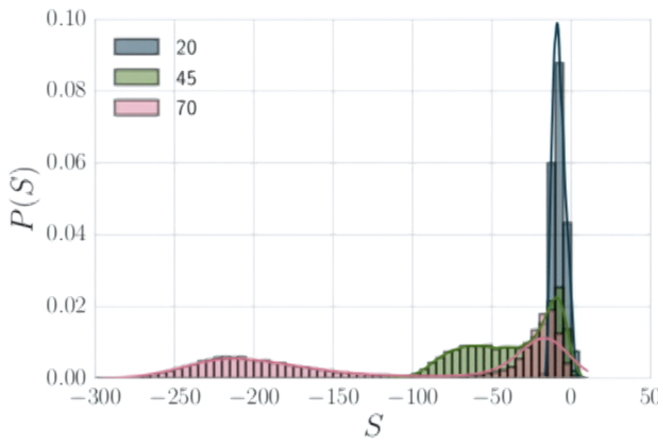
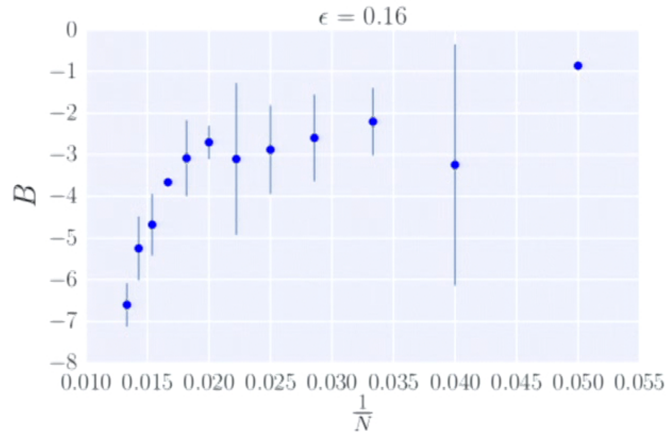
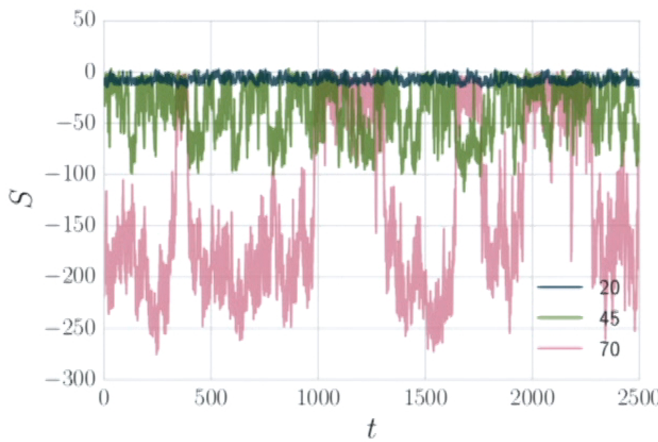
Order of the phase transition





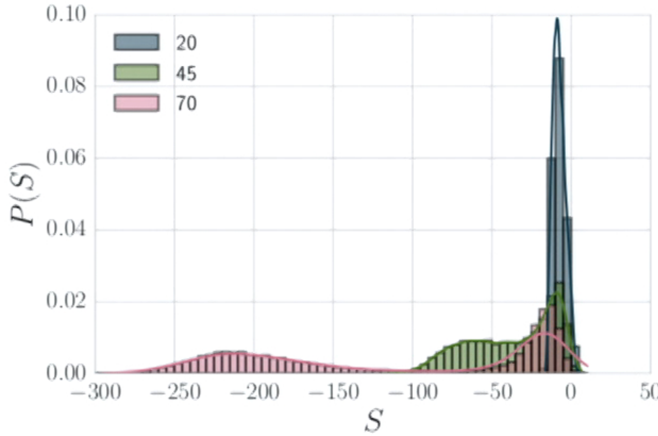


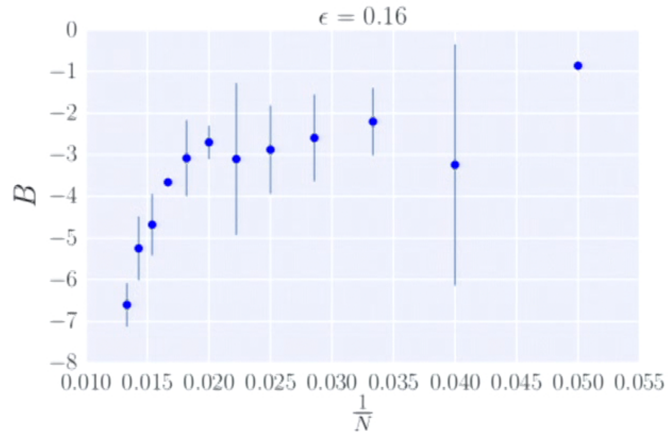
$$B = \frac{1}{3}\left(1 - \frac{\langle \mathcal{S}^4 \rangle}{\langle \mathcal{S}^2 \rangle^2}\right)$$

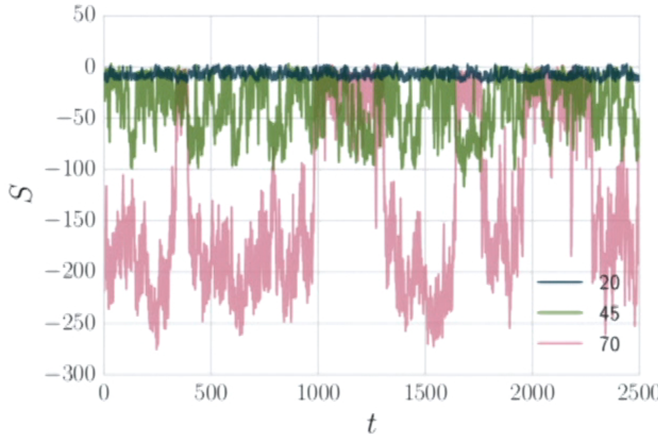
for 2nd order P.T. $B \to 0$ and coexistence disappears as $N \to \infty$

(LG,O'Connor,Surya, arXiv:1706.06432)

Order of the phase transition







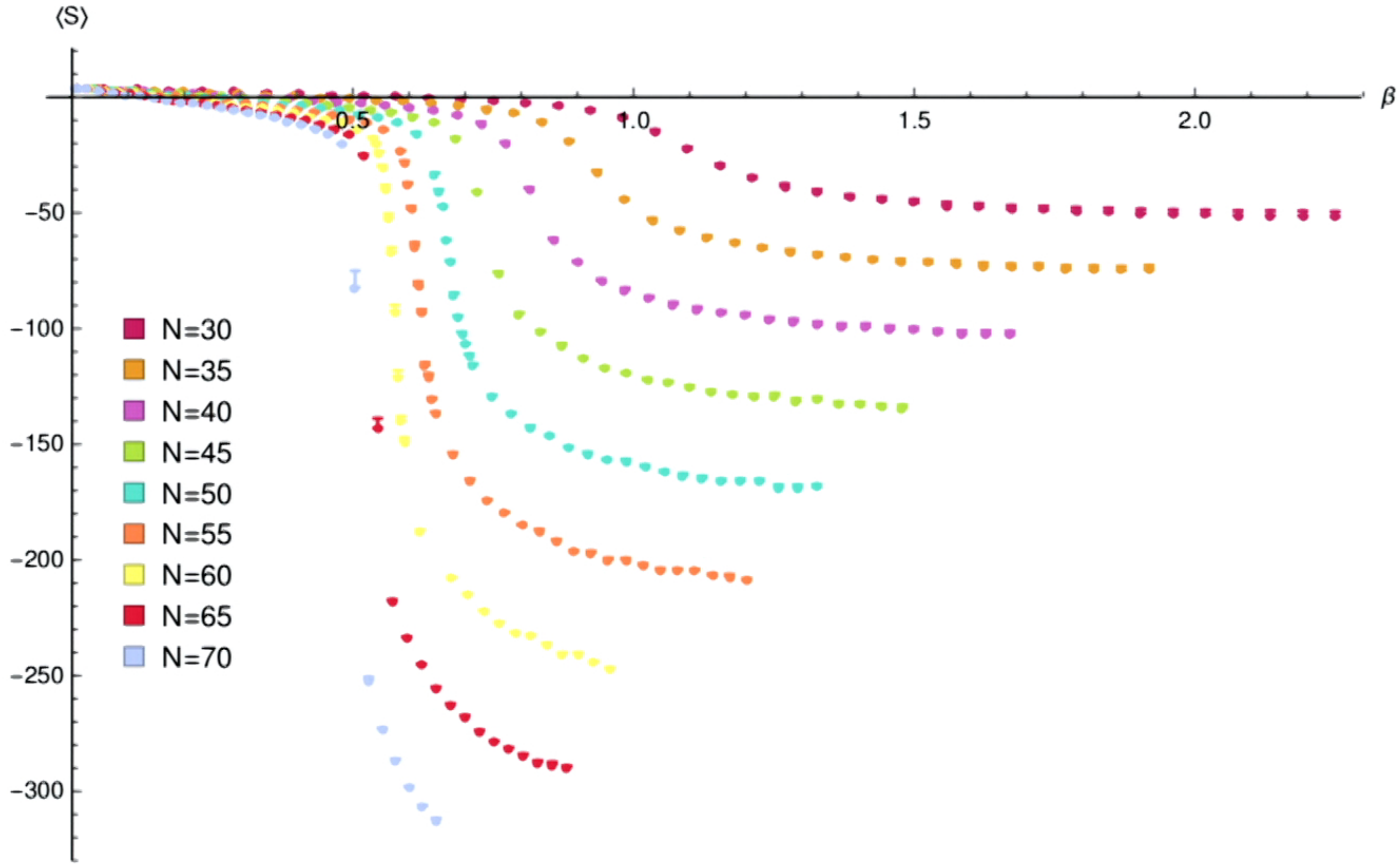
Transition is 1st order!

Remark:

Since we only want a continuum approximation not a continuum limit we do not require a higher order PT.

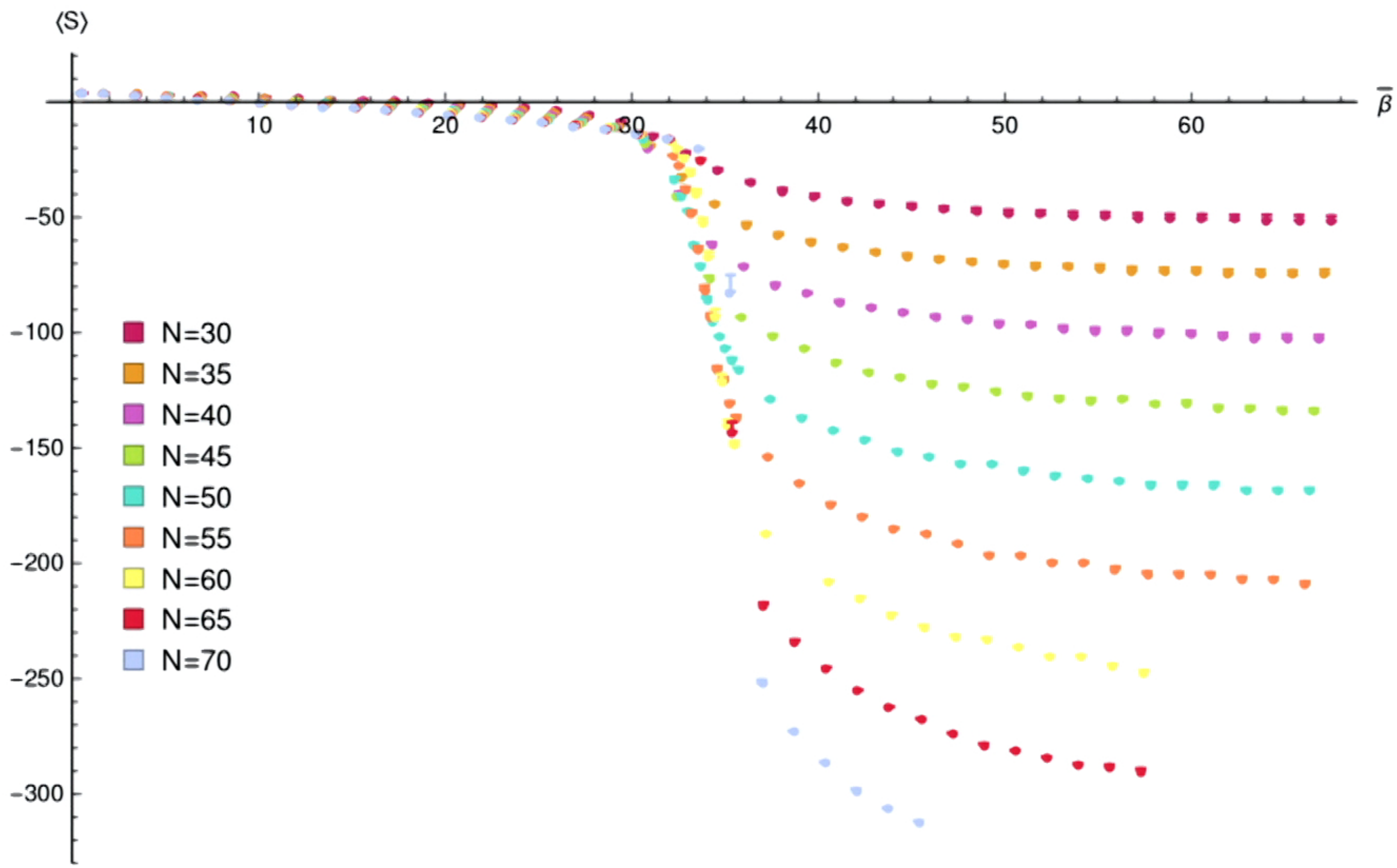
(LG,O'Connor,Surya, arXiv:1706.06432)

Scaling of the phase transition



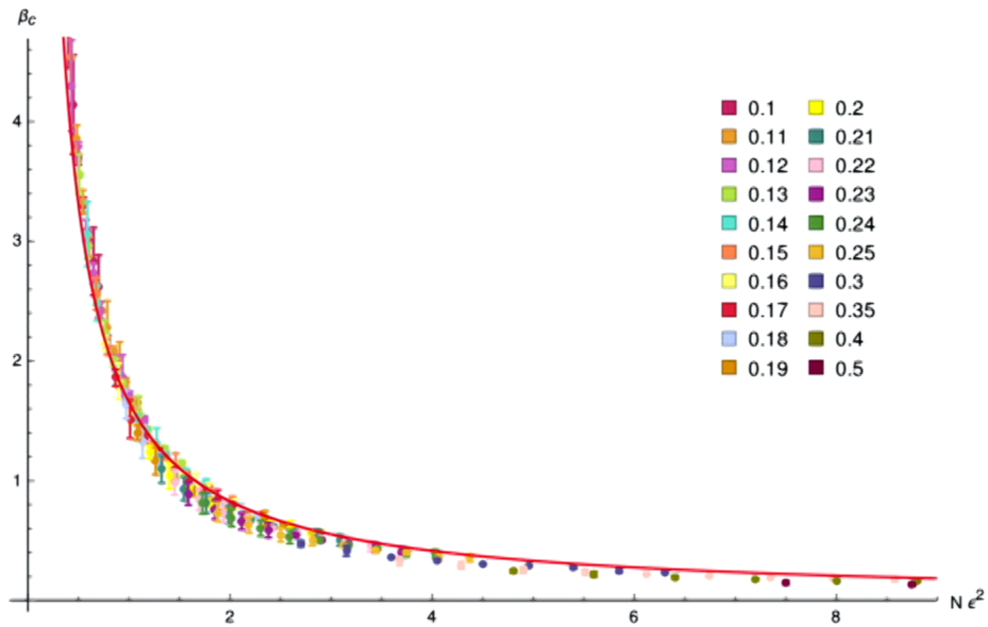
Location of the phase transition depends on N

Scaling of the phase transition



Location of the phase transition depends on N, $\bar{\beta} = \beta N$

Scaling of the phase transition

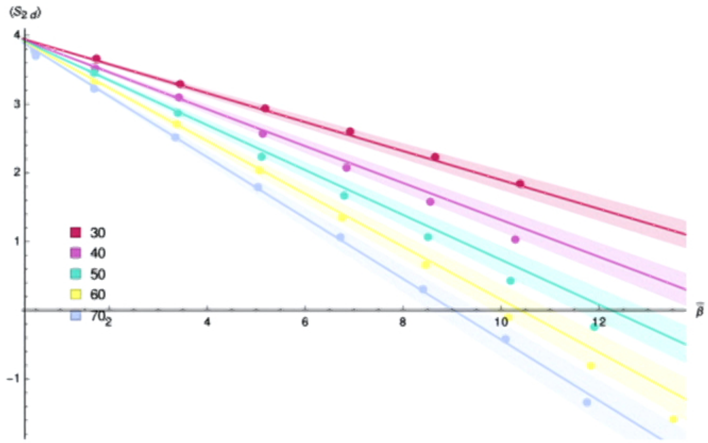


$$\beta_c(N, \epsilon) \approx \frac{1.66}{N\epsilon^2}$$

→ Scaling w. N, so β_c exists for all finite N

(LG,O'Connor,Surya, arXiv:1706.06432)

Scaling in the continuum phase

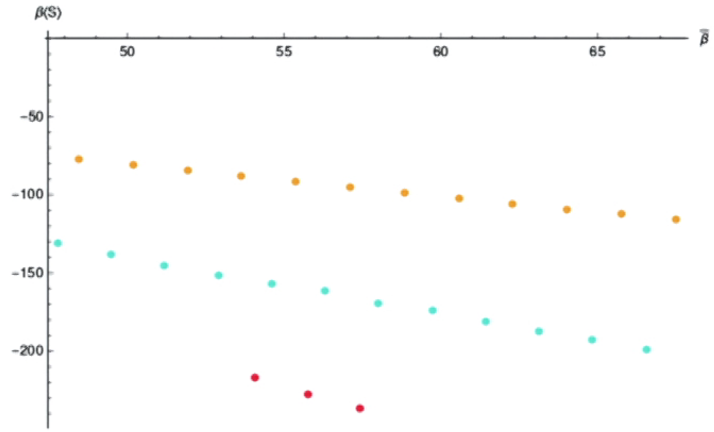


$\epsilon = 0.21$

- linear behavior
- $\mathcal{S}^- - 4 = (b_0^-(\epsilon) + b_1^-(\epsilon)N)\bar{\beta}$
- A negative Λ?

(LG,O'Connor,Surya, arXiv:1706.06432)

Scaling in the discrete phase

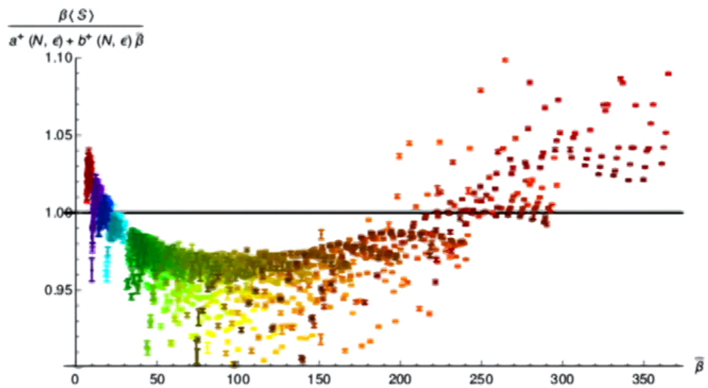


$\epsilon = 0.21$

- $\beta\langle S\rangle$ is linear in $\bar{\beta}$
- $\beta\mathcal{S}^{+} = a^{+}(N,\epsilon) + b^{+}(N,\epsilon)\bar{\beta}$
- $a^{+}(N,\epsilon), b^{+}(N,\epsilon)$ are linear in N
- Consistent w. analytic expectation from bilayer orders

(LG,O'Connor,Surya, arXiv:1706.06432)

Scaling in the discrete phase



all ϵ, N

- $\beta\langle S\rangle$ is linear in $\bar{\beta}$
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Hartle Hawking wave function

Continuum

$$\Psi_0(h_{ab}, \Sigma) = A \sum_M \int dg^E e^{-I_E(g)}$$

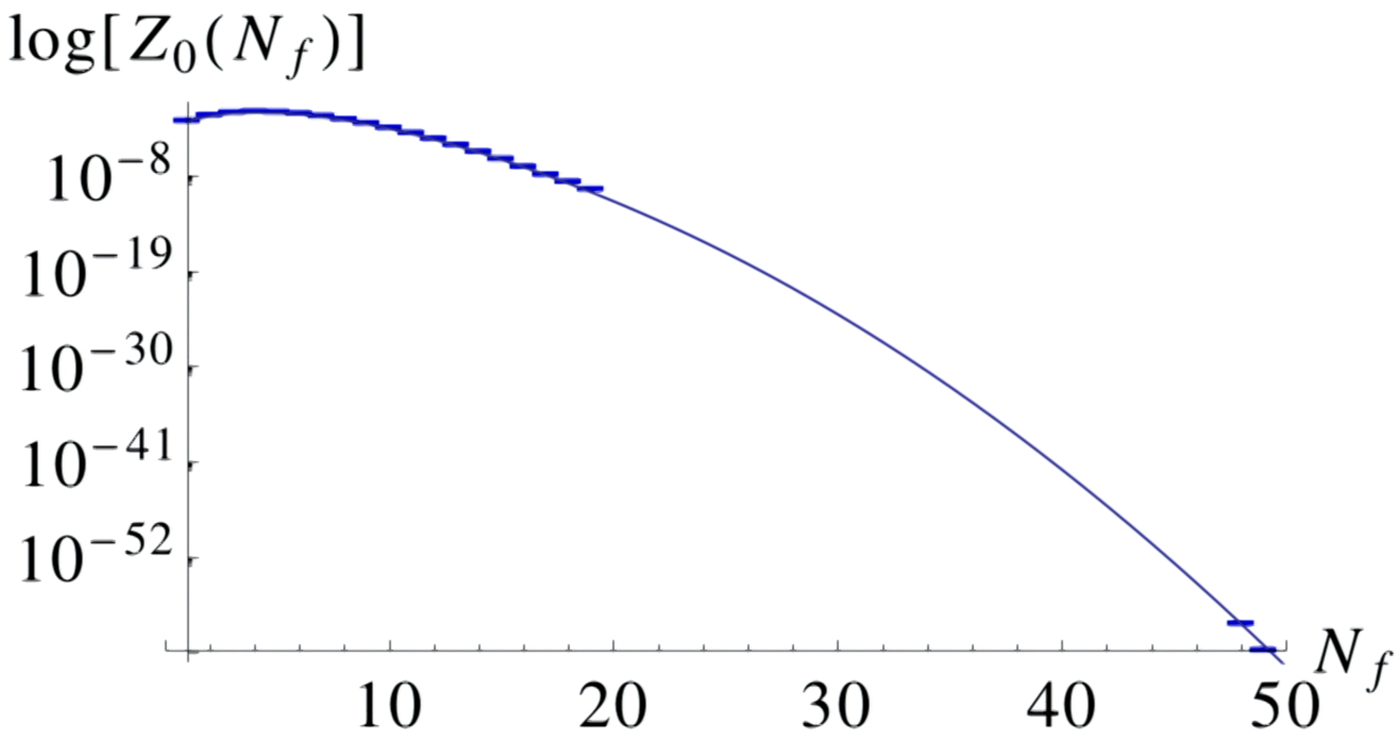
- Integrate over euclidean geometries
- **zero boundary condition** to final geometry (h_{ab}, Σ)

Normalisation factor $\mathcal{Z}_0$

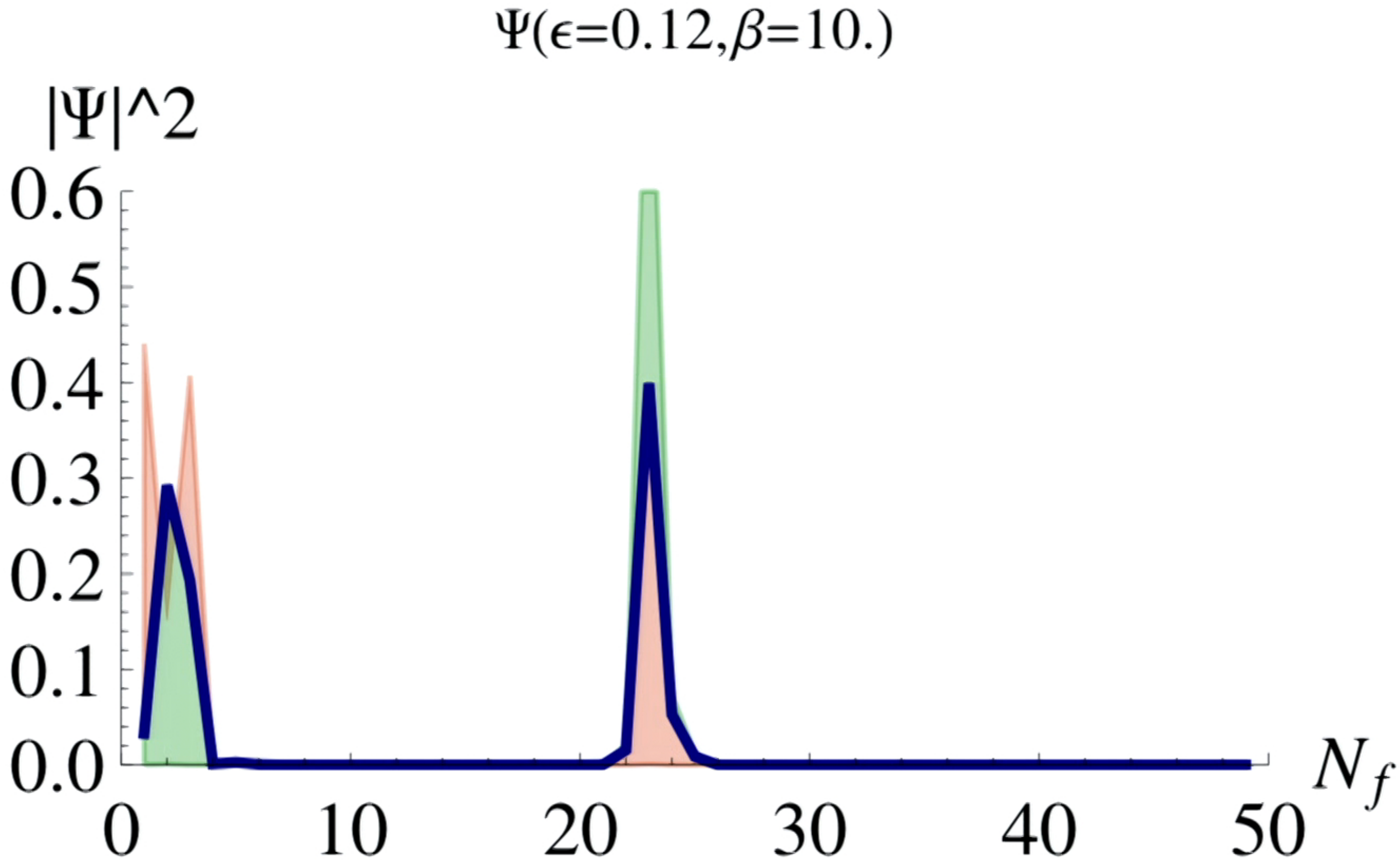
$$\mathcal{Z}_0(\text{free}) = \sum_{\mathcal{N}_f} \mathcal{Z}_0(\mathcal{N}_f)$$

- $\mathcal{Z}_0(\text{free})$ is the ensemble of all 2d orders
- The relative frequency for orders with $\mathcal{N}_f$ final elements in $\mathcal{Z}_0(\text{free})$ does then give us $\mathcal{Z}_0(\mathcal{N}_f)$ up to an overall constant

Normalisation factor $\mathcal{Z}_0$

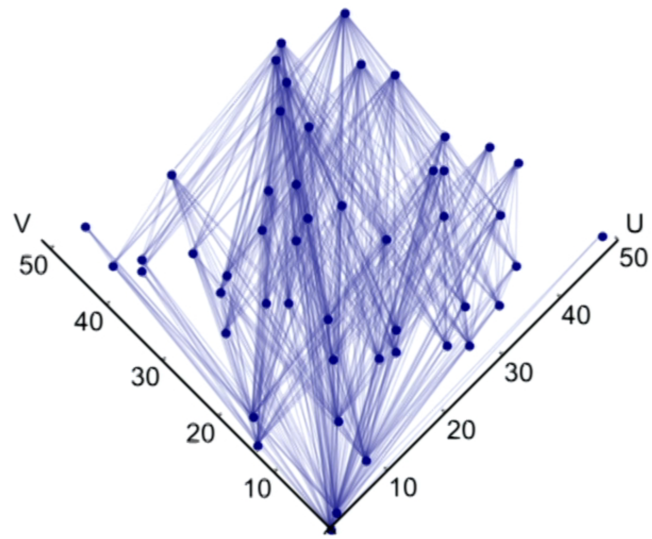


The wave function



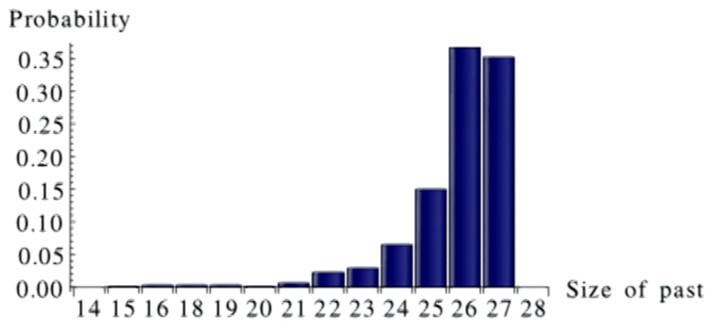
Geometry in the 1st peak (low β)

- Continuum type 2-d order
- Dominated by $\mathcal{Z}_0$
- 'as high as wide'

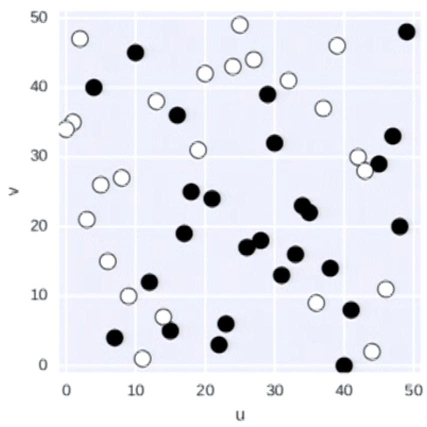


Geometry in the 2nd peak (high β)

- Crystalline structure
- Fast expansion
- Homogeneous pasts



The setup



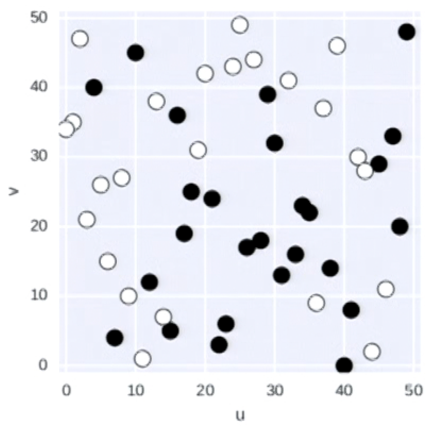
- spins s_i on elements
- nearest neighbour = linked
- spin coupling j

$$\beta \mathcal{S}(\mathcal{C}) = \beta \left(\mathcal{S}_{BD}(\mathcal{C}) + \mathcal{S}_{Ising}(\mathcal{C}) \right)$$

$$\mathcal{S}_{Ising}(\mathcal{C}) = j \sum_{i,k \; i \prec k} s_i s_k$$

(using code by Will Cunningham 1709.03013)

The setup



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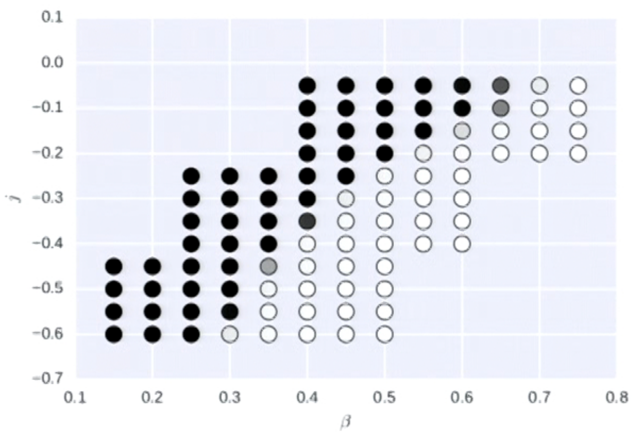
Phase diagram



- transition in β and j
- transition at lower β, j for larger N

Explore phase transition along fixed lines.

Summary & Conclusion

- Phase transition in the 2d orders
  - Entropy vs action, 1st order phase transition
  - consistent scaling w. N.
- Hartle Hawking wave function
  - The HH-wavefunction in the simple model shows signs of inflation
- Ising model and causal set
  - Phase transition moves slightly
  - order of PT?