Title: A walk through quantum field theory

Date: Sep 06, 2017 02:00 PM

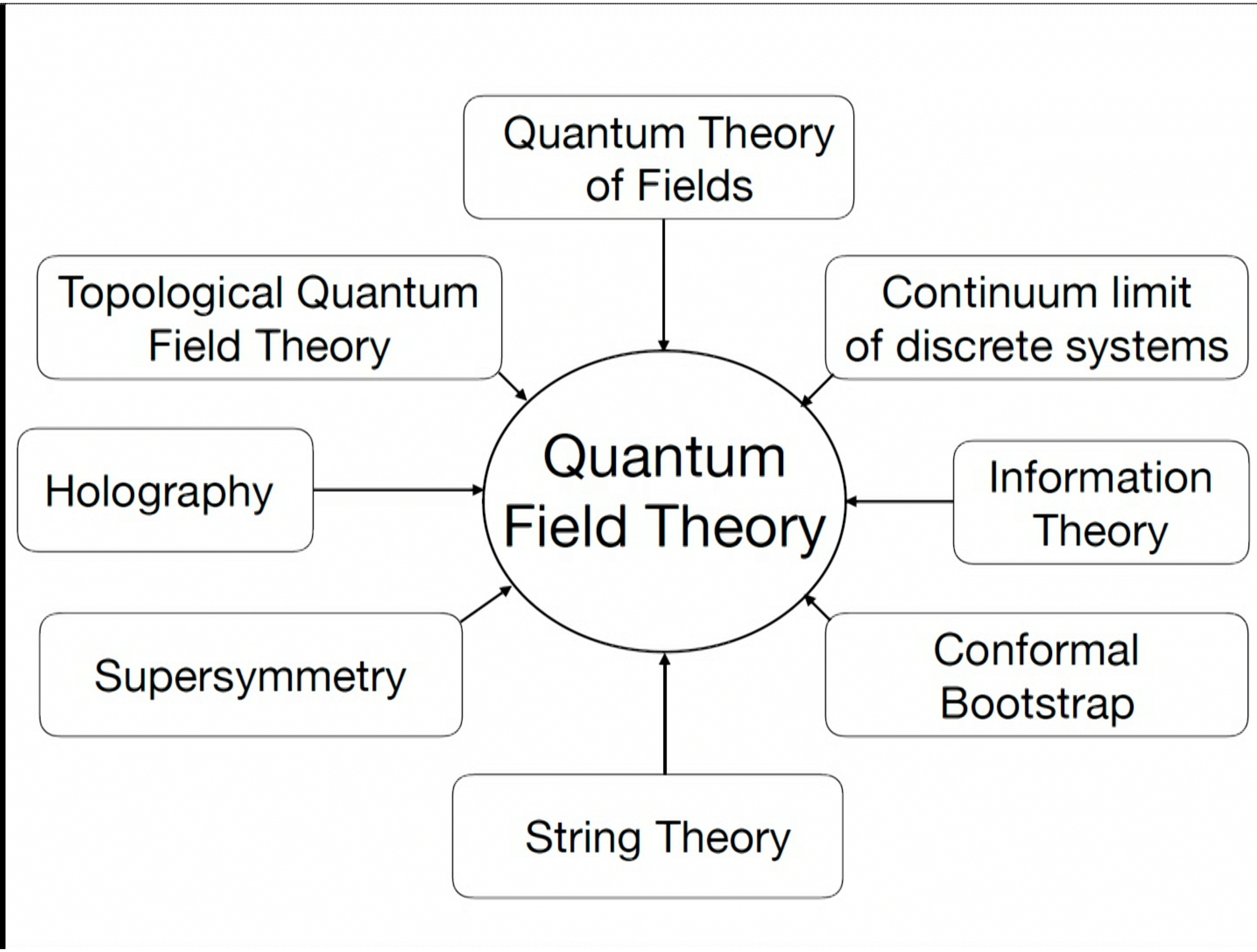
URL: http://pirsa.org/17090050

Abstract: <p>I will discuss various aspects of quantum field theory, with particular focus on the role of topological and supersymmetric examples.</p>

A walk through Quantum Field Theory

Examples

- Topological field theories can be sometimes described in terms of quantum fields, but it often obfuscates their properties.
- Dualities of supersymmetric quantum field theories and string theory constructions often involves exotic quantum field theories.
- The Conformal Bootstrap program may soon provide a direct (numerical) definition of QFTs, with no reference to a path integral.


Quantum Theory of Fields
Topological Quantum Field Theory
Continuum limit of discrete systems
Holography
Quantum Field Theory
Information Theory
Supersymmetry
Conformal Bootstrap
String Theory

Axioms for QFT?

- Quantum field theories contain answers to many different questions.
- There are subtle consistency requirements both on the questions one can ask and on the answers.
- What is a minimal self-consistent collection of questions and answers which can be used to formulate and answer all other questions?
- Only understood in special situations, active area of physics and mathematics research.

Some basic properties

- A (local) QFT has local observables:
  - Algebra $\mathcal{O}_\Sigma$ of observables attached to each region Σ in space
  - $\mathcal{O}_\Sigma \subset \mathcal{O}_{\Sigma'}$ if $\Sigma \subset \Sigma'$
  - $[\mathcal{O}_\Sigma, \mathcal{O}_{\Sigma'}] = 0$ if $\Sigma \cap \Sigma' = 0$

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Local operators

- Looking at infinitesimally small regions we get a vector space $\mathcal{O}_x$ of local operators at each point x in space-time.

- Basic output: correlation functions

$$\langle O_1(x_1)O_2(x_2)\cdots O_n(x_n)\rangle$$

  - There may be better observables: scattering matrix, transport properties. Usually computable from correlation functions.

Warning

- Local operators do not capture everything.
- Some (topological) QFTs have no local operators, but are still not trivial!
- General expectation: local operators capture local dynamics, extra data is topological.

Free Quantum Field Theory

- Basic local observables: the fields ϕ_i
- Other local operators are normal ordered polynomials in derivatives, up to equations of motion

$$\partial^n \phi_i ,\ \phi_i \phi_j ,\ \partial^n \phi_i \partial^m \phi_j , \cdots$$

- Gaussian correlation function, computed by Wick contractions

Asymptotically free Quantum Field Theory

$$\int D\phi \, e^{S[\phi]}$$

- Interacting, but interactions suppressed at short distance
- Local operators (roughly) the same as in free theory
- Correlation functions at large distance can be very different!

Back to local operators

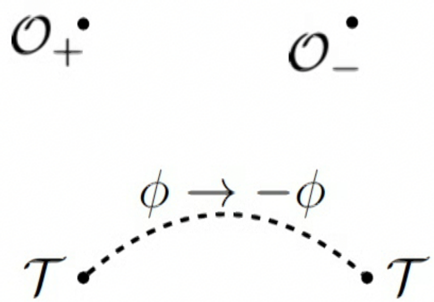
- Polynomial in the fields do NOT give all possible local operators, especially in gauge theory!

- Example 1: start from massive 2d scalar field ϕ , gauge reflection symmetry $\phi \to -\phi$ (orbifold)

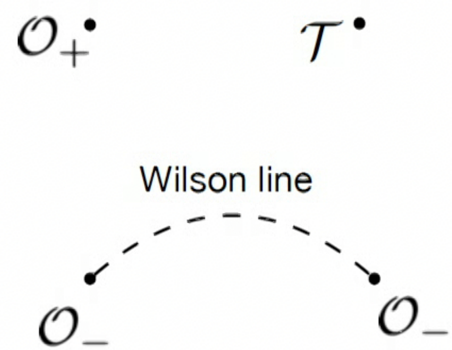
  - Remove odd local operators such as ϕ, keep even local operators such as ϕ^2

  - New "disorder operators" appear, endpoints of $\phi \to -\phi$ cut

Original Theory



Orbifold Theory



More disorder operators

- Example 2: monopole operators in 3d gauge theory
  - Remove a point from space-time, require flux through a sphere surrounding the point.
- Disorder operators are often important observables
  - They may be order parameters for phase transitions.
  - They may create solitonic excitations.
  - They may appear in effective actions, ruining our ability to do calculations.

Extended defects

- Extended observables are often important too.
  - Example: Wilson loops in gauge theory $P \exp \int_{\ell} A$
  - Example: 't Hooft loops in 4d gauge theory
  - Example: boundaries, interfaces, etc.
  - Example: coupling to lower dimensional system
- A subject for another colloquium…

Back to the orbifold

- Start with any 2d QFT with $\mathbb{Z}_2$ symmetry
- Even and odd local operators $\mathcal{O}_+, \mathcal{O}_-$
- Disorder operators can also be odd or even $\mathcal{T}_+, \mathcal{T}_-$
- Two distinct ways to "gauge $\mathbb{Z}_2$":
  - Keep $\mathcal{O}_+, \mathcal{T}_+$
  - Keep $\mathcal{O}_+, \mathcal{T}_-$

Which choices define a "good" QFT?

- Why can't we just keep $\mathcal{O}_+$?
- In 2d, "modular invariance" of the torus partition function.
- Ward identities/equations of motions for correlation functions of $\mathcal{O}_+$ operators only have multiple solutions on a space-time of general topology (say a torus).
- Going around non-trivial loops in the space of manifolds the solutions get mixed up: "good" QFTs are a choice of solution invariant under these operations (mapping class group)

Which choices define a "good" QFT?

- Consider correlation functions of $\mathcal{O}_+, \mathcal{O}_-, \mathcal{T}_+, \mathcal{T}_-$
- Multi-valued: switch sign when operators in $\mathcal{O}_-, \mathcal{T}_+$ are braided around each other, switch sign when two identical operators in $\mathcal{T}_-$ are exchanged, etc.
- Observation: the space of solutions of Ward identities in the 2d system behaves exactly as the space of states for a topological field theory in 2+1d, the topic code!
  - The braiding (and fusion) of $\mathcal{O}_+, \mathcal{O}_-, \mathcal{T}_+, \mathcal{T}_-$ reproduce these of the anionic particles in the toric code

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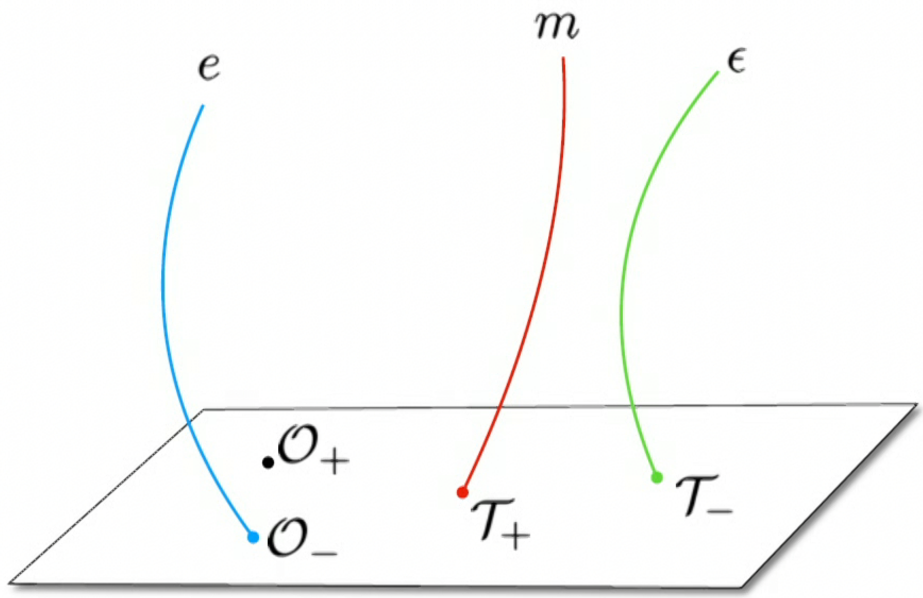
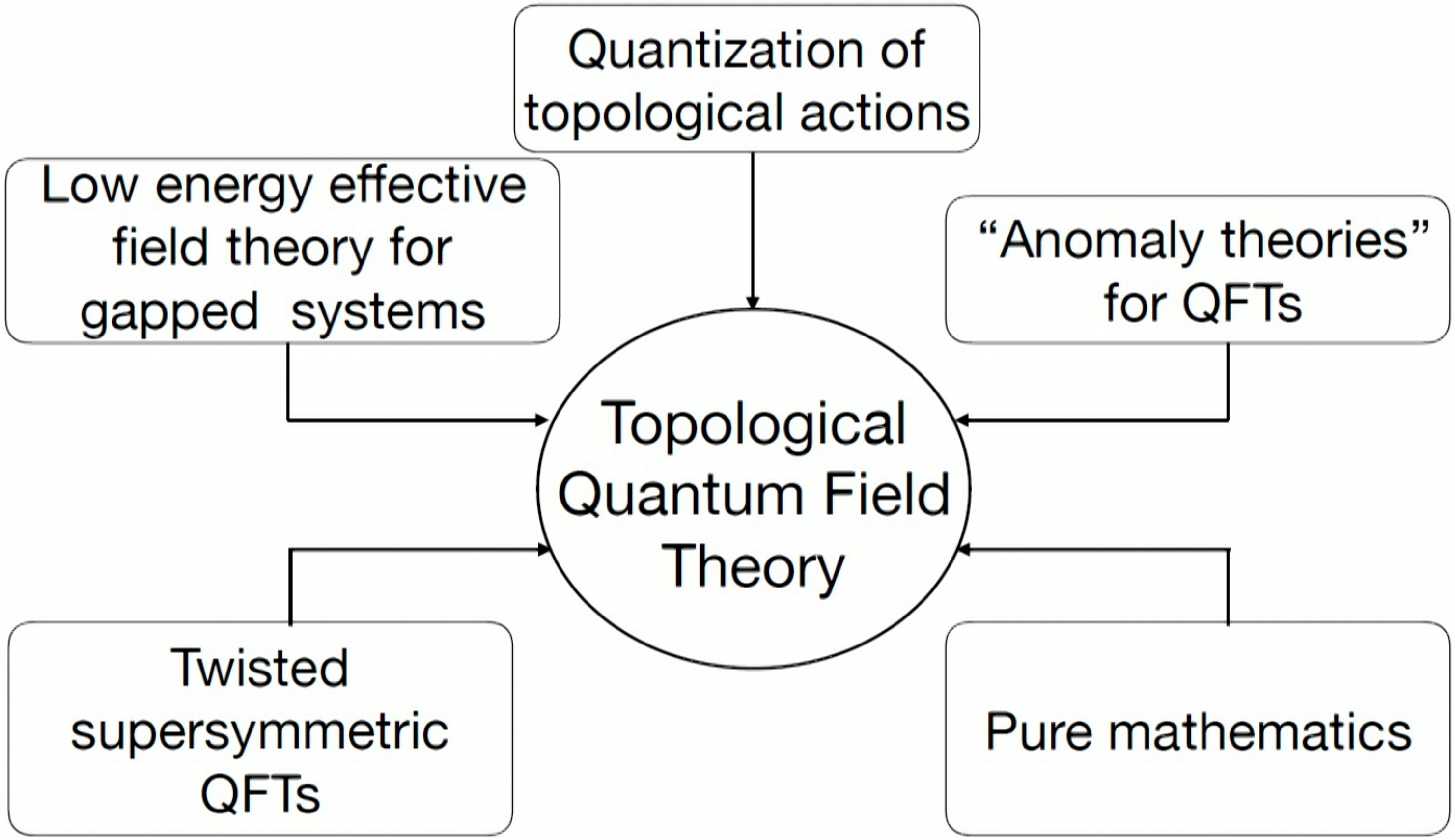


- This is a well defined 3d system, coupled to 2d degrees of freedom at one boundary.
- Choice of “good” 2d theory is a choice of a second topological boundary, where either e, m, or ϵ lines can end.

Topological field theory

- This one of the possible routes to Topological Field Theories: “incomplete” or “anomalous” QFTs live naturally at the boundary of TFTs.
  - example: the axioms for 2+1d TFTs emerged from the study of 2d RCFTs.
  - Topological phases of matter are often characterized by “edge modes”


Quantization of topological actions
Low energy effective field theory for gapped systems
"Anomaly theories" for QFTs
Topological Quantum Field Theory
Twisted supersymmetric QFTs
Pure mathematics

What is a Topological Quantum Field Theory?

- A QFT where local operators are locally constant, i.e. $\partial O \sim 0$
- No local dynamics! No excitations, no S-matrix, etc.
- Non-trivial observables:
  - spaces of ground states on manifolds of non-trivial topology
  - non-trivial statistics of heavy probes (particles, strings, etc.)
  - non-trivial edge theories

Looking for axioms

- Topological field theory is very rich, but simple enough that we should be able to fully characterize it.
- Very active research topic in
  - High Energy Physics
  - Mathematics
  - Condensed Matter Physics
  - Quantum Information
- See talk at Simons Summer Workshop….

Asymptotically Free vs Asymptotically Conformal

- Some quantum field theories are intrinsically interacting: do not become free at any energy scale.
- Conformal field theory: a QFT which does not depend on the overall scale of the metric (Weyl invariant).

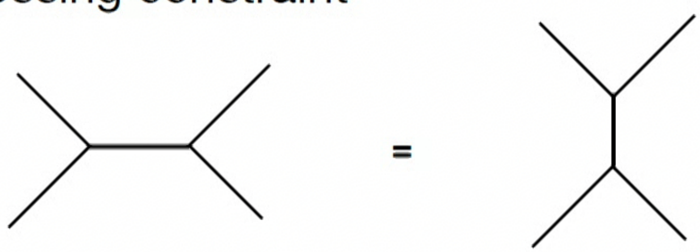
CFT data

- The algebra of local operators in a CFT is fully characterized by two pieces of data:
  - The list of scaling dimensions and spin of local operators
  - The "three point function coefficients" which appear in the OPE expansion:

$$O_i(x)O_j(0) \sim \sum_k c_{ij}^k \frac{O_k(0)}{|x|^{\Delta_i+\Delta_j-\Delta_k}} + [\partial O]$$

OPE expansion and crossing symmetry

- The OPE expansion is convergent.
- N-point correlation functions on a sphere can be recursively computed from the CFT data.
- Expansion of four point functions in different channels give a crossing constraint



Conformal bootstrap

- How constraining are crossing equations?
  - Before recently, analytic efforts only worked in 2d RCFT
  - Considerable recent progress with intensive numerics
- Are crossing equations enough?
  - Given a collection of operators closed under OPE, we should be left with topological choices only.
  - TO DO: CFT data —> TFT data in one dimension higher

Supersymmetric QFTs

- Supersymmetry: a symmetry which mixes bosons and fermions.
- Spin-statistics theorem forces supercharges to be spinors, anti commute to translations:

$$\{Q_a, Q_b\} = \gamma^{\mu}_{ab} P_{\mu}$$

- Up to sixteen supercharges are possible in QFT

Protected by SUSY

- Supersymmetry restricts how certain observables can depend on the parameters of the SQFT.
- Protected observables may be computed exactly by convenient deformations of the SQFT, overall self-consistency, etc.
- SQFT often come in families:
  - Moduli spaces of vacua
  - Parameter spaces of exactly marginal deformations
  - SQFTs defined by path integrals may be continuously connected to “exotic” SQFTs with no Lagrangian descriptions

SUSY and Mathematics

- The protected data of SQFTs may form rich mathematical structures.

- QFT facts are mapped to non-trivial mathematical results

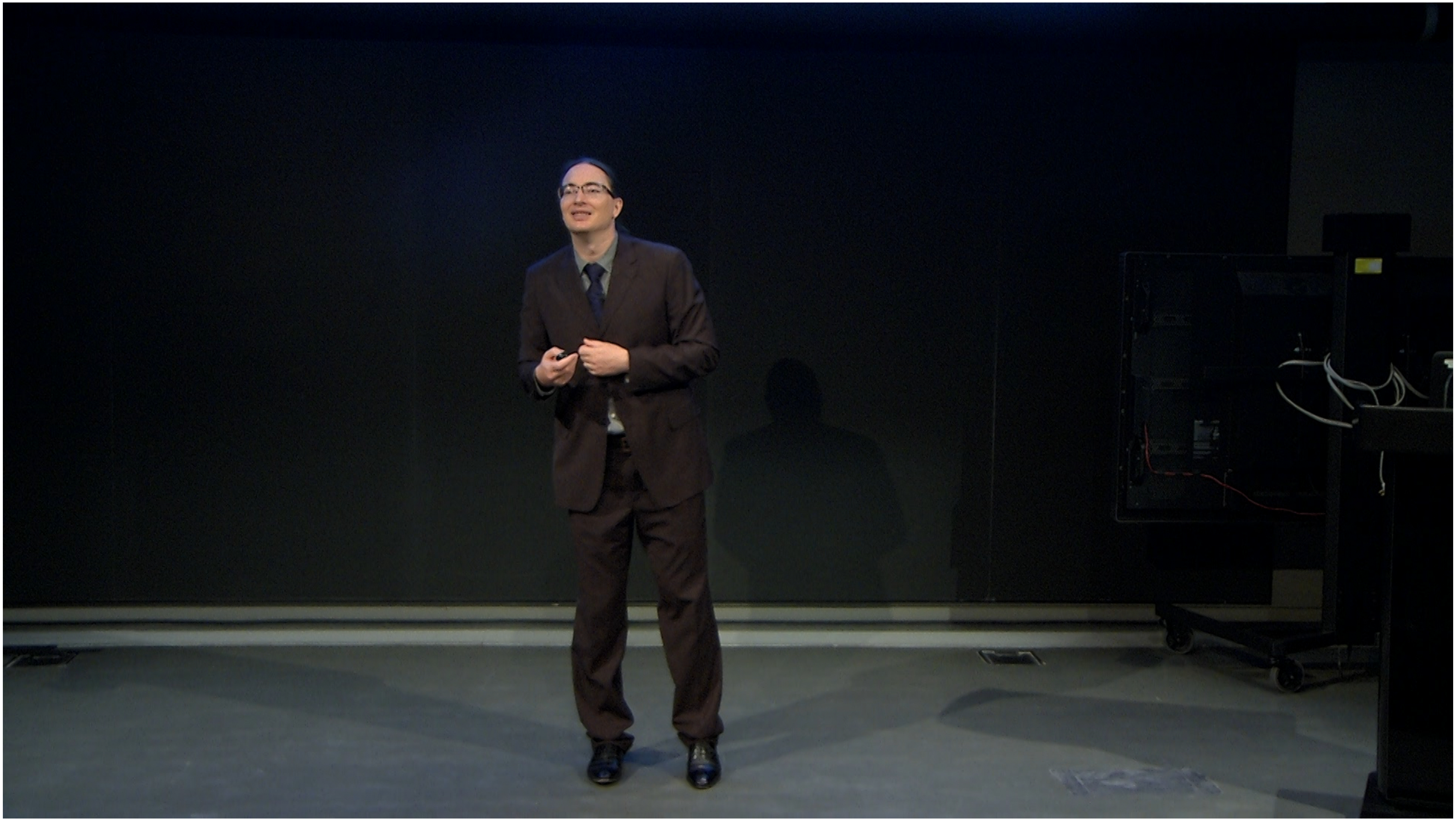
  - 2d “N=(2,2)” SQFT
  - 3d “N=4” SQFT
  - 4d “N=4” SQFT
  - 4d “N=2” SQFT

⇨

  - Homological Mirror Symmetry
  - Symplectic duality
  - Geometric Langlands, knot homology….
  - Cluster algebra, Hitchin systems, ….

Predicted by String Theory

- String theory is a source of many results in SQFT
  - Holography, AdS/CFT for SUSY gauge theories
  - Dualities
  - Exotic SQFTs
- Example: maximally supersymmetric 6d SCFTs



The power of compactification

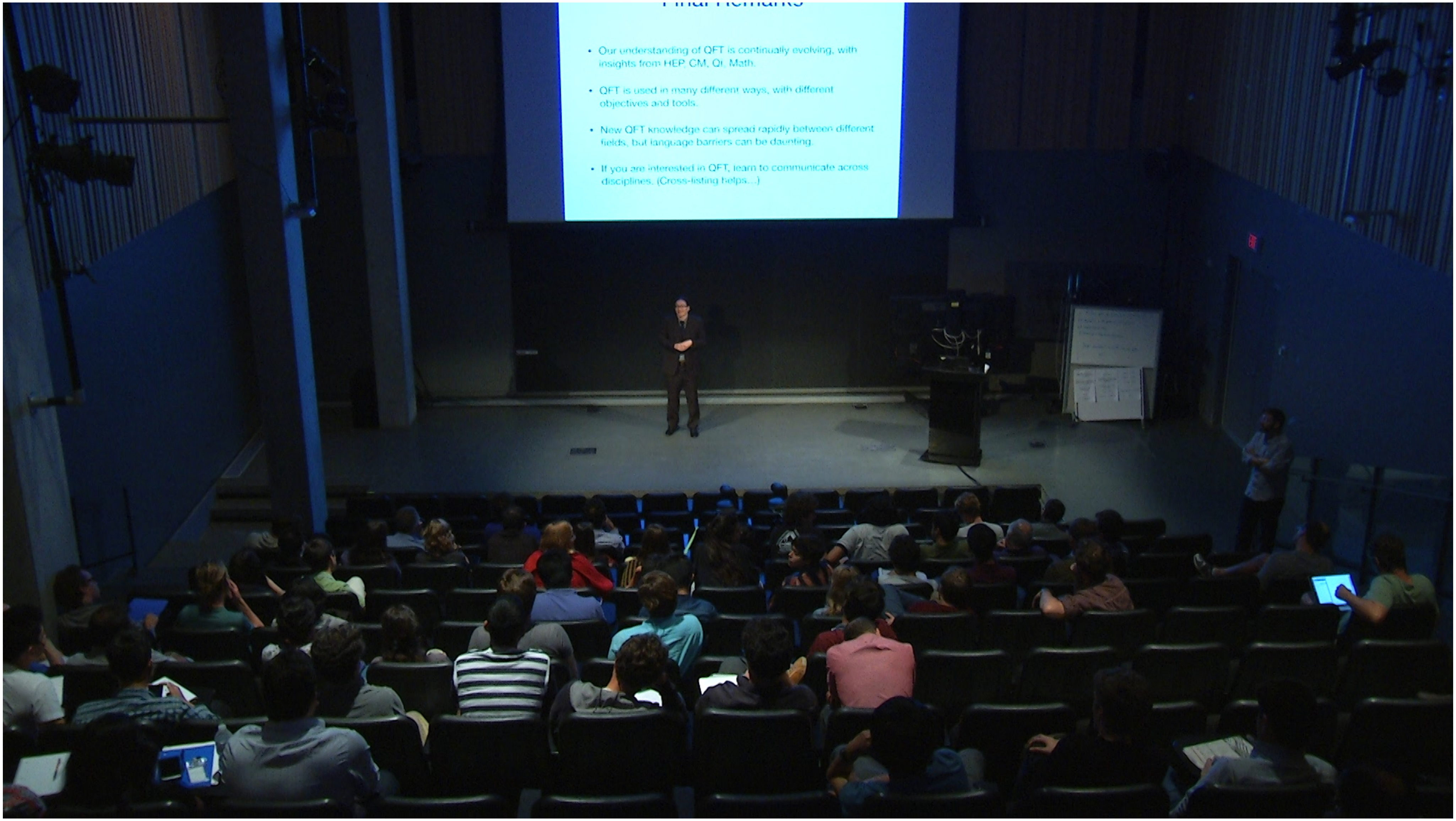
- Some SQFT can be compactified super-symmetrically.
- RG flow to lower dimensional theory, protected quantities remember geometry of UV compactification.
- Example: 6d “(2,0)” SCFT —> 4d “N=2” SQFT
  - Very large family of 4d theories labelled by geometric data.
  - Includes almost all Lagrangian SQFTs, but they are a minority!
  - In this example, quantum theories of fields are a small subset of all QFTs….
  - Work in progress on 6d “(1,0)” SCFT —> 4d “N=1” SQFT

Topological twist

- Pick a nilpotent supercharge, i.e. a Q with $Q^2 = 0$
  - An operator is Q-closed if in the kernel of Q.
  - An operator is Q-exact if in the image of Q
- Topological twist: restrict to Q-closed operators, Q-exact operators become trivial.
  - If stress tensor is Q-exact, the simplified theory is topological

Final Remarks

- Our understanding of QFT is continually evolving, with insights from HEP, CM, QI, Math.
- QFT is used in many different ways, with different objectives and tools.
- New QFT knowledge can spread rapidly between different fields, but language barriers can be daunting.
- If you are interested in QFT, learn to communicate across disciplines. (Cross-listing helps…)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