Title: Transport bounds: from resistor networks to quantum chaos

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Abstract: <p>The Kovtun-Son-Starinets conjecture that the ratio of the viscosity to the entropy density was bounded from below by fundamental constants has inspired over a decade of conjectures about fundamental bounds on the hydrodynamic and transport coefficients of strongly interacting quantum systems. I will present two complementary and (relatively) rigorous approaches to proving bounds on the transport coefficients of strongly interacting systems.   Firstly, I will discuss lower bounds on the conductivities (and thus, diffusion constants) of inhomogeneous fluids, based around the principle that transport minimizes the production of entropy.   I will show explicitly how to use this principle in classical theories, and in theories with a holographic dual. Secondly, I will derive lower bounds on sound velocities and diffusion constants arising from the consistency of hydrodynamics with quantum decoherence and chaos, in large N theories.   I will discuss the possible tension of such bounds with (some) holographic theories, and discuss resolutions to some existing puzzles.</p>

Transport bounds: from resistor networks to quantum chaos

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The Diffusion Equation

- **hydrodynamics:** effective field theory of relaxing to thermal equilibrium:

The Diffusion Equation

- **hydrodynamics:** effective field theory of relaxing to thermal equilibrium:
  - small parameter: $\ell_{\mathrm{mfp}}\nabla$!
- degrees of freedom: conserved quantities:

$$\partial_t \rho + \nabla \cdot \mathbf{J} = 0.$$

- many-body limit, gradient expansion:

$$\langle \mathbf{J} \rangle = -D(\rho)\nabla\langle\rho\rangle - \cdots$$

$$\partial_t \rho = \nabla \cdot (D\nabla\rho)$$

- local second law of thermodynamics:

$$D \geq 0, \qquad \text{(dissipation only)}$$

Resistivity and Viscosity

- hydrodynamics in quantum systems:
  - cold atomic gases
  - quark-gluon plasma
  - electrons in metals
- diffusion constants measurable via static response:

viscosity:

$$T_{xy} = -\eta \partial_x v_y$$

$$\eta = (\epsilon + P) D_{\text{momentum}}$$

(incoherent) conductivity:

$$J_i = \sigma E_i$$

$$\sigma = \chi D_{\text{charge}}$$

- these hydrodynamic coefficients are very hard to compute in interesting, interacting quantum systems:

$$D \sim \ell_{\text{mfp}} v$$

A Viscosity Bound?

- from AdS/CFT, a conjecture: [Kovtun, Son, Starinets; hep-th/0405231]

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_{\mathrm{B}}}.$$

- a heuristic motivation (speed $c = 1$):

$$\eta = TsD$$

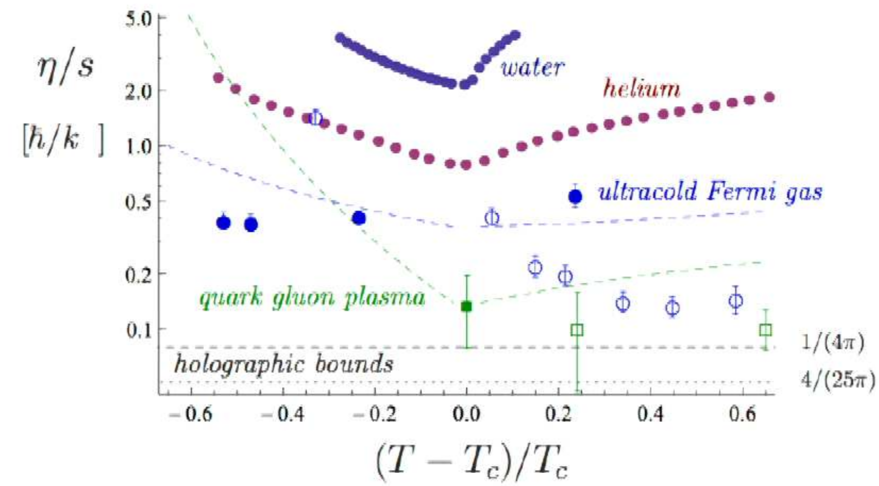
$$D \sim \ell_{\mathrm{mfp}} \sim \Delta t_{\mathrm{thermal}} \gtrsim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{k_{\mathrm{B}} T}$$

$$\frac{\eta}{s} \sim T\ell_{\mathrm{mfp}} \gtrsim 1.$$

Conjectured Transport Bounds

Viscosity Bound: A Status Report

- bound consistent with experiment: [Adams *et al*; 1205.5180]

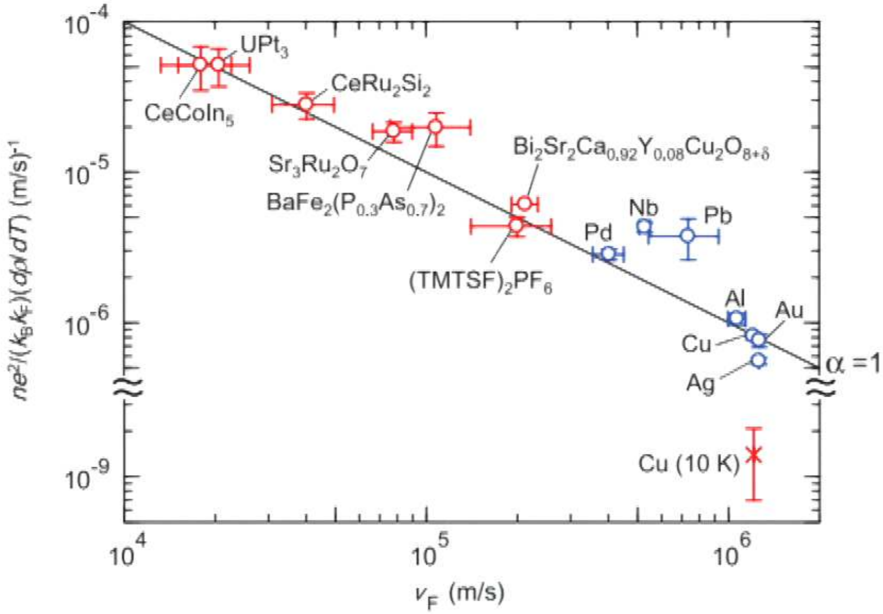


- theoretically, bound has been violated
  - by O(1) constant: higher derivative gravity [Brigante *et al*; 0712.0805]
  - parametrically: broken translation invariance (but how to define η?) [Hartnoll *et al*; 1601.02757]; [Alberte *et al*; 1601.003384]; [Burikham *et al*; 1601.04624]

Conjectured Transport Bounds

Resistivity of Strange Metals

$$\rho \lesssim \frac{m}{ne^2}\frac{k_B T}{\hbar}$$



[Bruin, Sakai, Perry, Mackenzie (2013)]

Conjectured Transport Bounds

A Diffusion Bound?

- can *all* prior bounds be recast as [Hartnoll, 1405.3651]

$$D \gtrsim v^2 \tau_T = v^2 \frac{\hbar}{k_B T}?$$

- resistivity upper bound:

$$\rho \sim \frac{1}{\chi D} \sim \frac{T}{v^2} \times \frac{\partial \mu}{\partial n}$$

(in a metal, $\partial \mu / \partial n$, v are T-independent?)

- viscosity bound ($v = c = 1$?):

$$\frac{\eta}{s} = \frac{\epsilon + P}{s} D \gtrsim \frac{\epsilon + P}{Ts} \gtrsim 1$$

  - but, holographic observation: extremal charged RN black hole

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad \epsilon + P \sim s \sim T^0$$

challenges for the diffusion bound $D \gtrsim v^2/T$:

1. what is v?
2. what physical principle underlies this bound?
3. when is such a bound valid? (insulators where $D = 0$ exist...)

the punchlines of this talk:

1. lower bounds on D can come from **minimizing entropy production** (but not written in the form $D \gtrsim v^2/T$)
2. (at least in large N theories) quantum consistency requires *upper bounds* on diffusion constants:

$$D \lesssim v_{\mathrm{B}}^2 \tau$$

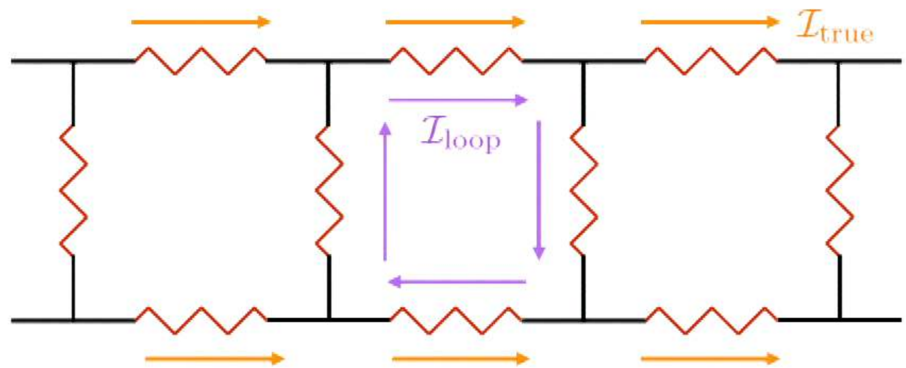
3. v_{B} is the butterfly velocity ("speed of quantum chaos")
4. τ is the time beyond which Fick's law of diffusion holds (sometimes $\tau \gg 1/T$)

Resistor Network Bounds

- Thomson's principle: resistance of resistor network obeys

$$I_0^2 R_{\text{eff}} \leq \sum_{\text{edges } e} \mathcal{I}_e^2 R_e$$

for arbitrary **conserved** currents $\mathcal{I}_e$:



- Thomson's principle in the continuum limit:

$$\frac{1}{\sigma} = \rho \leq \frac{1}{J_{x,\text{avg}}^2} \times \int \frac{\mathrm{d}^d\mathbf{x}}{V} \frac{\mathbf{J}^2}{\sigma_{\text{loc}}(\mathbf{x})}, \quad \nabla \cdot \mathbf{J} = 0.$$

Kinetic Theory

- in (homogeneous) kinetic theory: if

$$\frac{\mathrm{d}f(p)}{\mathrm{d}t} = -\sum_{p'} W(p,p')f(p'),$$

$$J_i = \sum_p q v_p f_p = \sum_p j_p f_p$$

then the resistivity obeys [Ziman (1960)]

$$\rho \le \frac{f_p W_{pq} f_q}{(f_p j_p)^2}, \quad \text{(index sum implied)}$$

- ρ arises from minimizing dissipation:

$$f_p W_{pq} f_q = T\dot{s}$$

Upper Bounds on Resistivity

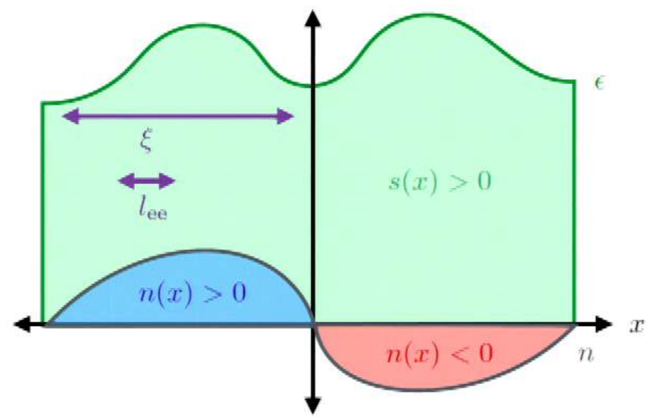
Transport from Entropy Production

- a 'general' semiclassical principle: [Lucas, Hartnoll, 1706.04621]

$$\rho_{xx} \le \left.\frac{(T\dot{s})_{\mathrm{avg}}}{J_x^2}\right|_{\nabla\cdot\mathbf{J}=0,\ \text{for all conserved currents}}$$

  with variations taken over inversion-odd quantities
- **hydrodynamic** limit: [Lucas, 1506.02662]

$$(T\dot{s})_{\mathrm{avg}} = \int \frac{\mathrm{d}^d\mathbf{x}}{V_d}\left[(J_i^a - n^a v_i)(\Sigma^{-1})^{ab}(J_i^b - n^b v_i) + \eta_{ijkl}\partial_i v_j \partial_k v_l\right]$$



Application 1: Holographic Models

- mathematical tricks [Donos, Gauntlett, 1506.01360] allow us to use this mathematics in holographic models
- for a certain $2+1$D QFT ($\sim$ ABJM/M2 brane theory)
  [Grozdanov, Lucas, Sachdev, Schalm, 1507.00003]
  [Grozdanov, Lucas, Schalm, 1511.05970]

$$\text{electrical conductivity: } \sigma \geq 1$$

$$\text{thermal conductivity: } \kappa \geq \frac{4\pi^2 T}{3}.$$

disorder cannot cause metal-insulator transition

- other models: weaker bounds, but often $\sigma > 0$

Application 2: Do Interactions Increase or Decrease Transport?

qualitative results are easy to find:

$$(T\dot{s})_{\text{avg}} = \int \frac{\mathrm{d}^d\mathbf{x}}{V_d}\left[(J_i^a - n^a v_i)(\Sigma^{-1})^{ab}(J_i^b - n^b v_i) + \eta_{ijkl}\partial_i v_j \partial_k v_l\right]$$

- if charge and momentum conserved: ansatz $J_i = nv_i$:

$$\rho \sim \int \frac{\mathrm{d}^d\mathbf{x}}{V_d}\eta\left(\nabla\frac{1}{n}\right)^2 \sim \frac{D_{\text{mom}}}{\xi^2} \sim \frac{\ell_{\text{ee}}}{\xi^2}$$

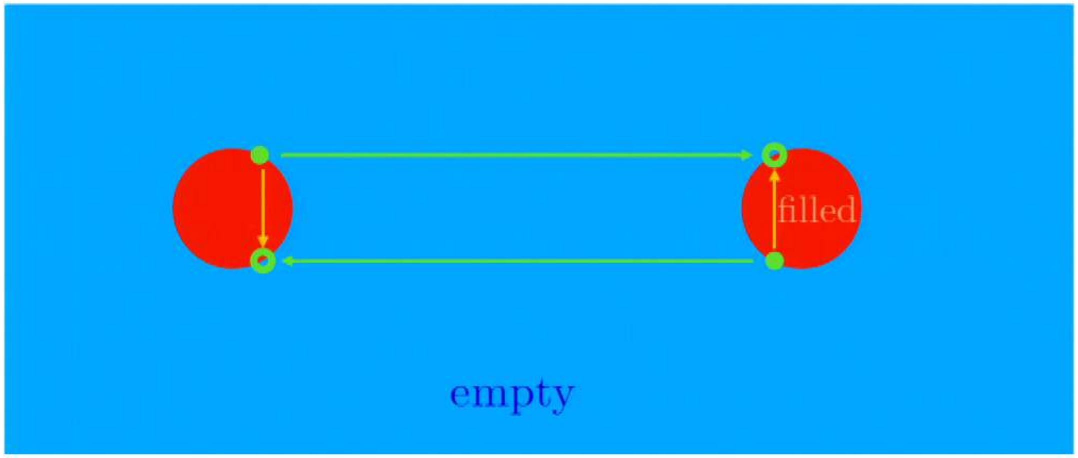
  if ξ is length scale of inhomogeneity
- but if we have an extra conservation law (energy):

$$\rho \sim \int \frac{\mathrm{d}^d\mathbf{x}}{V_d}\left[\frac{1}{\Sigma^{22}}\left(J^2 - \frac{n^2}{n^1}J^1\right)^2 + \eta\left(\nabla\frac{1}{n^1}\right)^2\right]$$

  and in the hydrodynamic limit, $\ell_{\text{ee}} \ll \xi$:

$$\rho \sim \frac{1}{\chi^{22}D^{22}} \sim \frac{1}{\ell_{\text{ee}}}$$

Imbalance Diffusion

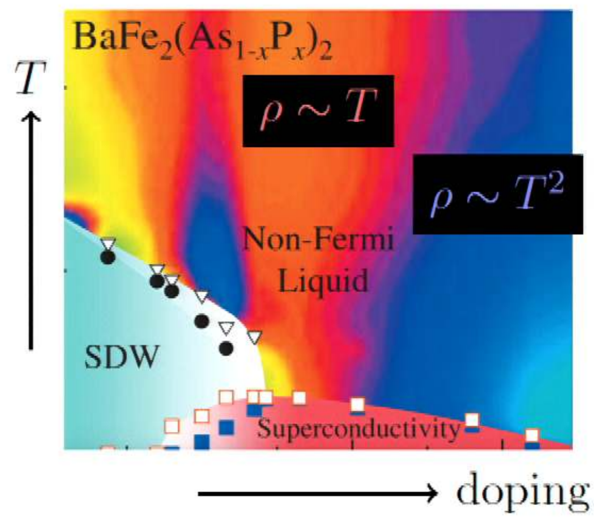


- quasiparticle 2-body collisions conserve momentum $\Longrightarrow$ QPs on each pocket separately conserved
  - $\Sigma_{\mathrm{imb}} \sim \chi_{\mathrm{imb}} D_{\mathrm{imb}}, \quad \chi_{\mathrm{imb}} \sim T^0$
  - T-dependence of ρ from T-dep. of ℓ_{ee}
  - (weak?) higher order effects spoil this new conservation law

[Lucas, Hartnoll, 1704.07384]

Phenomenology: Enhanced Resistivity near Criticality

- sample phase diagram:



- our theory: imbalance diffusion causes

$$\rho \sim \frac{1}{\tau_{\mathrm{ee}}} \sim \begin{cases} T & \text{strange metal} \\ T^2 & \text{Fermi liquid} \end{cases} .$$

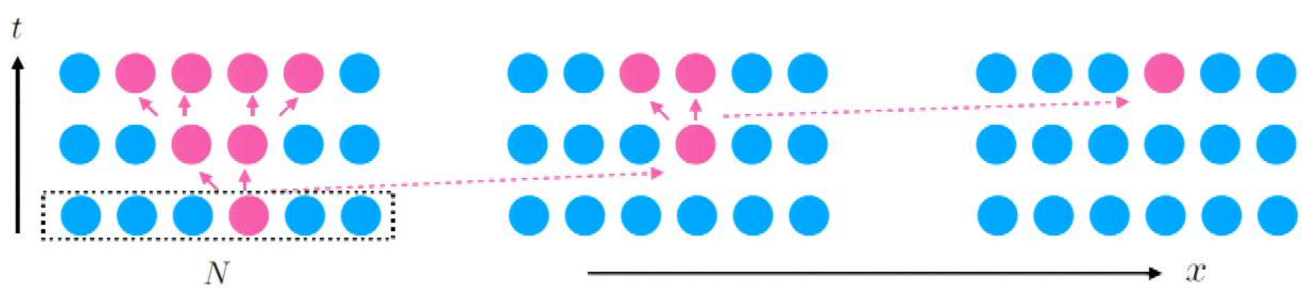
Out-of-Time-Ordered Correlators

- **conjecture**: in a large N QFT, out-of-time-ordered correlators (OTOCs) obey

$$\langle A(x,t)[B(x,t),C(0)]D(0)\rangle_\beta \le \frac{\mathcal{P}}{N}\exp\left[\frac{v_{\mathrm{B}}t-|x|}{v_{\mathrm{B}}\tau_{\mathrm{L}}}\right]+\mathrm{O}\left(\frac{1}{N^2}\right)$$

  τ_{L} is the Lyapunov time; v_{B} is the butterfly velocity

- quantum information is scrambled among DOF:



Hydrodynamic Operators

- in a large N QFT, hydrodynamic operators often take the schematic form

$$\rho = \sum_I \Phi_I^\dagger \Phi_I$$

- if we define

$$G^{\mathrm{R}}_{\rho\rho}(x,t) \equiv \mathrm{i}\Theta(t)\langle[\rho(x,t),\rho(0,0)]\rangle_\beta$$

then if there is hydrodynamic diffusion at late times: [Kadanoff, Martin (1963)]

$$G^{\mathrm{R}}_{\rho\rho}(k,\omega) = \frac{\chi D k^2}{Dk^2 - \mathrm{i}\omega} + \cdots$$

- if ρ instead couples to sound waves:

$$G^{\mathrm{R}}_{\rho\rho}(k,\omega) = \frac{wk^2}{v_{\mathrm{s}}^2 k^2 - \omega^2 - \mathrm{i}\omega k^2 \Gamma_{\mathrm{s}}} + \cdots$$

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Consistency of Chaos and Hydrodynamics

- from the quantum butterfly effect, $\rho = \sum \Phi_I^\dagger \Phi_I$:

$$
\begin{aligned}
|\langle[\rho(x,t),\rho(0,0)]\rangle_\beta| &= \sum_{IJ} \langle \Phi_I^\dagger(x,t)[\Phi_I(x,t),\Phi_J^\dagger(0,0)]\Phi_J(0,0)\rangle_\beta \\
&\quad + 3 \text{ similar terms} \\
&\leq 4\mathcal{P}N \exp\left[\frac{v_\mathrm{B}t-|x|}{v_\mathrm{B}\tau_\mathrm{L}}\right] + \mathrm{O}\left(N^0\right)
\end{aligned}
$$

- if there is a sound wave,

$$
G^\mathrm{R}_{\rho\rho}(x,t) \sim \partial_x^2 \exp\left[-\frac{(|x|-v_\mathrm{s}t)^2}{4\Gamma_\mathrm{s}t}\right], \quad (d=1)
$$

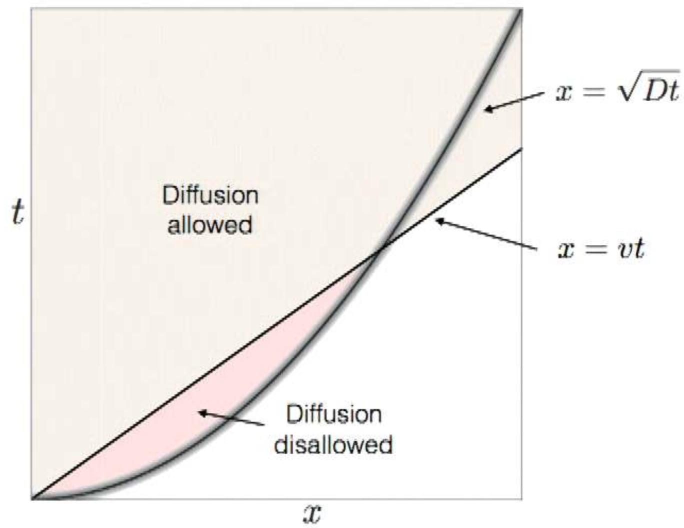
- these two equations are only consistent if

$$
\boxed{v_\mathrm{s} \leq v_\mathrm{B}}
$$

[Lucas, to appear]

Consistency with Diffusion

- diffusion exponentially suppressed when $x^2 \gtrsim Dt$:



- if hydrodynamic diffusion is valid for $t \geq \tau$:
[Hartman, Hartnoll, Mahajan, 1706.00019]

$$\boxed{D \leq v^2 \tau}$$

with $v = v_\mathrm{B}$ (at leading order in $1/N$) [Lucas, to appear]

A Puzzle: Holographic Charge Diffusion

- in "scaling" holographic theories: [Blake, 1603.08510]

$$D \sim \frac{v_{\mathrm{B}}^2}{T}\left(\frac{\Lambda}{T}\right)^{\max(0,-\Delta/z)}.$$

  - v_{B} computed at horizon, because charge decouples from gravity in linear response

- if $\Delta > 0$, then

$$\tau \sim \tau_{\mathrm{L}} \sim \frac{1}{T}$$

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  - v_{B} computed at horizon, because charge decouples from gravity in linear response
- if $\Delta > 0$, then

$$\tau \sim \tau_{\mathrm{L}} \sim \frac{1}{T}$$

- but if $\Delta < 0$, hydrodynamics fails unexpectedly early?

$$\tau \sim \frac{1}{T}\left(\frac{\Lambda}{T}\right)^{-\Delta/z}$$

Conclusions

rigorous bounds on transport and hydrodynamics:

- semiclassical lower bounds on σ (indirectly, D):

$$\sigma \geq \left. \frac{J_x^2}{(T\dot{s})_{\text{avg}}} \right|_{\nabla \cdot \mathbf{J}=0,\ \text{for all conserved currents}}$$

  - what is the quantum generalization?

- upper bounds on hydrodynamics from the quantum butterfly effect:

$$v_{\text{s}} \leq v_{\text{B}}, \qquad D \leq v_{\text{B}}^2 \tau$$

  - we find that many strange metals have unexpectedly rapid failure of hydrodynamics...**why?**