

Title: Transport bounds: from resistor networks to quantum chaos

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Abstract: <p>The Kovtun-Son-Starinets conjecture that the ratio of the viscosity to the entropy density was bounded from below by fundamental constants has inspired over a decade of conjectures about fundamental bounds on the hydrodynamic and transport coefficients of strongly interacting quantum systems.&nbsp; I will present two complementary and (relatively) rigorous approaches to proving bounds on the transport coefficients of strongly interacting systems.&nbsp;&nbsp; Firstly, I will discuss lower bounds on the conductivities (and thus, diffusion constants) of inhomogeneous fluids, based around the principle that transport minimizes the production of entropy.&nbsp;&nbsp; I will show explicitly how to use this principle in classical theories, and in theories with a holographic dual. Secondly, I will derive lower bounds on sound velocities and diffusion constants arising from the consistency of hydrodynamics with quantum decoherence and chaos, in large N theories.&nbsp;&nbsp; I will discuss the possible tension of such bounds with (some) holographic theories, and discuss resolutions to some existing puzzles.</p>

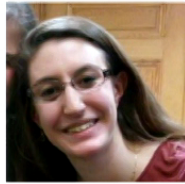
# Transport bounds: from resistor networks to quantum chaos

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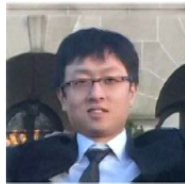
September 25, 2017



Julia Steinberg  
Harvard Physics



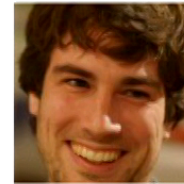
Subir Sachdev  
Harvard Physics & Perimeter Institute



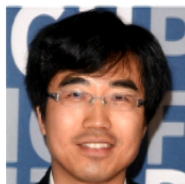
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- ▶ **hydrodynamics:** effective field theory of relaxing to thermal equilibrium:



## The Diffusion Equation

- ▶ **hydrodynamics:** effective field theory of relaxing to thermal equilibrium:
  - ▶ small parameter:  $\ell_{\text{mfp}} \nabla!$
- ▶ degrees of freedom: conserved quantities:

$$\partial_t \rho + \nabla \cdot \mathbf{J} = 0.$$

- ▶ many-body limit, gradient expansion:

$$\langle \mathbf{J} \rangle = -D(\rho) \nabla \langle \rho \rangle - \dots$$

$$\partial_t \rho = \nabla \cdot (D \nabla \rho)$$

- ▶ local second law of thermodynamics:

$$D \geq 0, \quad (\text{dissipation only})$$

## Resistivity and Viscosity

- ▶ hydrodynamics in quantum systems:
  - ▶ cold atomic gases
  - ▶ quark-gluon plasma
  - ▶ electrons in metals
- ▶ diffusion constants measurable via static response:

**viscosity:**

$$T_{xy} = -\eta \partial_x v_y$$

$$\eta = (\epsilon + P) D_{\text{momentum}}$$

**(incoherent) conductivity:**

$$J_i = \sigma E_i$$

$$\sigma = \chi D_{\text{charge}}$$

- ▶ these hydrodynamic coefficients are very hard to compute in interesting, interacting quantum systems:

$$D \sim \ell_{\text{mfp}} v$$

## A Viscosity Bound?

- ▶ from AdS/CFT, a conjecture: [Kovtun, Son, Starinets; hep-th/0405231]

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}.$$

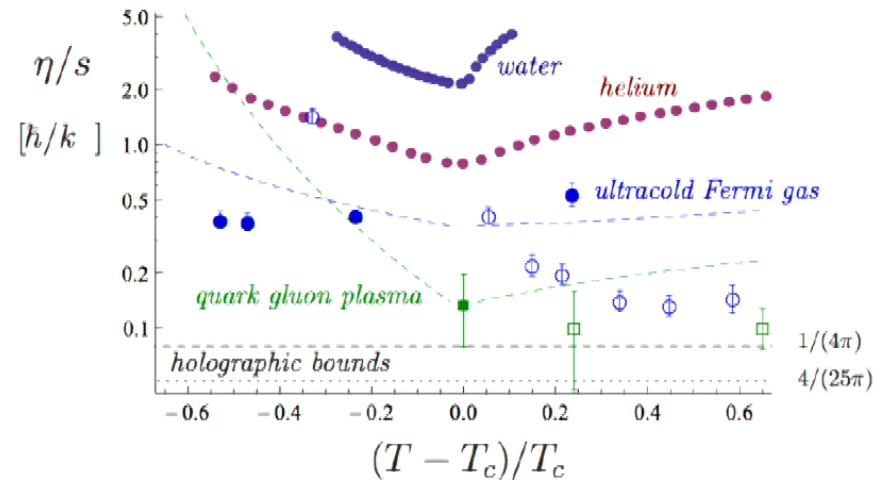
- ▶ a heuristic motivation (speed  $c = 1$ ):

$$\eta = TsD$$

$$D \sim \ell_{\text{mfp}} \sim \Delta t_{\text{thermal}} \gtrsim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{k_B T}$$

$$\frac{\eta}{s} \sim T \ell_{\text{mfp}} \gtrsim 1.$$

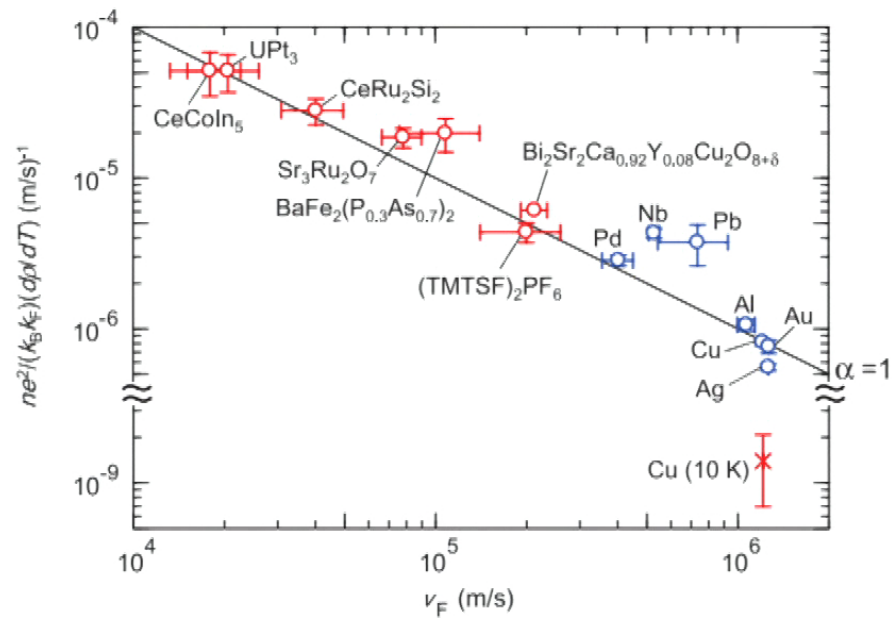
- ▶ bound consistent with experiment: [Adams *et al*; 1205.5180]



- ▶ theoretically, bound has been violated
  - ▶ by O(1) constant: higher derivative gravity [Brigante *et al*; 0712.0805]
  - ▶ parametrically: broken translation invariance (but how to define  $\eta$ ?) [Hartnoll *et al*; 1601.02757]; [Alberte *et al*; 1601.003384]; [Burikham *et al*; 1601.04624]

## Resistivity of Strange Metals

$$\rho \lesssim \frac{m}{ne^2} \frac{k_B T}{\hbar}$$



[Bruin, Sakai, Perry, Mackenzie (2013)]

## A Diffusion Bound?

- ▶ can *all* prior bounds be recast as [Hartnoll, 1405.3651]

$$D \gtrsim v^2 \tau_T = v^2 \frac{\hbar}{k_B T}?$$

- ▶ resistivity upper bound:

$$\rho \sim \frac{1}{\chi D} \sim \frac{T}{v^2} \times \frac{\partial \mu}{\partial n}$$

(in a metal,  $\partial \mu / \partial n$ ,  $v$  are  $T$ -independent?)

- ▶ viscosity bound ( $v = c = 1$ ):

$$\frac{\eta}{s} = \frac{\epsilon + P}{s} D \gtrsim \frac{\epsilon + P}{T s} \gtrsim 1$$

- ▶ but, holographic observation: extremal charged RN black hole

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad \epsilon + P \sim s \sim T^0$$



## Summary of the Talk

**challenges for the diffusion bound  $D \gtrsim v^2/T$ :**

1. what is  $v$ ?
2. what physical principle underlies this bound?
3. when is such a bound valid? (insulators where  $D = 0$  exist...)

**the punchlines of this talk:**

1. lower bounds on  $D$  can come from **minimizing entropy production** (but not written in the form  $D \gtrsim v^2/T$ )
2. (at least in large  $N$  theories) quantum consistency requires *upper bounds* on diffusion constants:

$$D \lesssim v_B^2 \tau$$

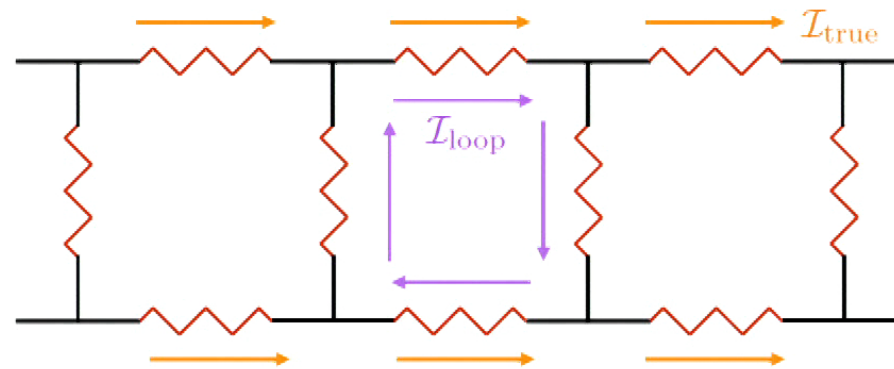
3.  $v_B$  is the butterfly velocity (“speed of quantum chaos”)
4.  $\tau$  is the time beyond which Fick’s law of diffusion holds (sometimes  $\tau \gg 1/T$ )

## Resistor Network Bounds

- ▶ Thomson's principle: resistance of resistor network obeys

$$I_0^2 R_{\text{eff}} \leq \sum_{\text{edges } e} \mathcal{I}_e^2 R_e$$

for arbitrary **conserved** currents  $\mathcal{I}_e$ :



- ▶ Thomson's principle in the continuum limit:

$$\frac{1}{\sigma} = \rho \leq \frac{1}{J_{x,\text{avg}}^2} \times \int \frac{d^d \mathbf{x}}{V} \frac{\mathbf{J}^2}{\sigma_{\text{loc}}(\mathbf{x})}, \quad \nabla \cdot \mathbf{J} = 0.$$



- ▶ in (homogeneous) kinetic theory: if

$$\frac{df(p)}{dt} = - \sum_{p'} W(p, p') f(p'),$$

$$J_i = \sum_p q v_p f_p = \sum_p j_p f_p$$

then the resistivity obeys [Ziman (1960)]

$$\rho \leq \frac{f_p W_{pq} f_q}{(f_p j_p)^2}, \quad (\text{index sum implied})$$

- ▶  $\rho$  arises from minimizing dissipation:

$$f_p W_{pq} f_q = T \dot{s}$$

## Transport from Entropy Production

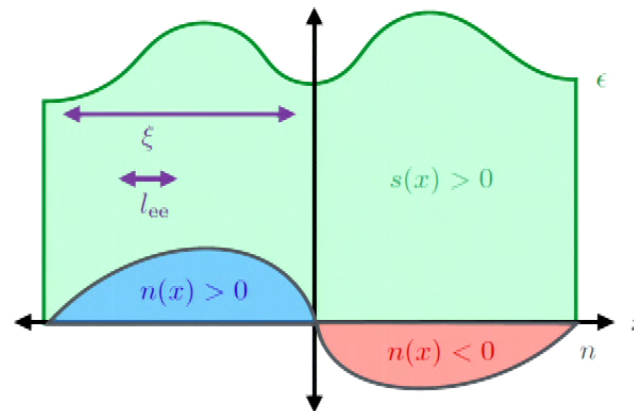
- ▶ a ‘general’ semiclassical principle: [Lucas, Hartnoll, 1706.04621]

$$\rho_{xx} \leq \frac{(T\dot{s})_{\text{avg}}}{J_x^2} \Big|_{\nabla \cdot \mathbf{J}=0, \text{ for all conserved currents}}$$

with variations taken over inversion-odd quantities

- ▶ **hydrodynamic** limit: [Lucas, 1506.02662]

$$(T\dot{s})_{\text{avg}} = \int \frac{d^d \mathbf{x}}{V_d} \left[ (J_i^a - n^a v_i) (\Sigma^{-1})^{ab} (J_i^b - n^b v_i) + \eta_{ijkl} \partial_i v_j \partial_k v_l \right]$$



## Application 1: Holographic Models

- ▶ mathematical tricks [Donos, Gauntlett, 1506.01360] allow us to use this mathematics in holographic models
- ▶ for a certain 2 + 1D QFT ( $\sim$  ABJM/M2 brane theory)  
[Grozdhanov, Lucas, Sachdev, Schalm, 1507.00003]  
[Grozdhanov, Lucas, Schalm, 1511.05970]

electrical conductivity:  $\sigma \geq 1$

thermal conductivity:  $\kappa \geq \frac{4\pi^2 T}{3}$ .

**disorder cannot cause metal-insulator transition**

- ▶ other models: weaker bounds, but often  $\sigma > 0$

## Application 2: Do Interactions Increase or Decrease Transport?

qualitative results are easy to find:

$$(T\dot{s})_{\text{avg}} = \int \frac{d^d \mathbf{x}}{V_d} \left[ (J_i^a - n^a v_i) (\Sigma^{-1})^{ab} (J_i^b - n^b v_i) + \eta_{ijkl} \partial_i v_j \partial_k v_l \right]$$

- ▶ if charge and momentum conserved: ansatz  $J_i = n v_i$ :

$$\rho \sim \int \frac{d^d \mathbf{x}}{V_d} \eta \left( \nabla \frac{1}{n} \right)^2 \sim \frac{D_{\text{mom}}}{\xi^2} \sim \frac{\ell_{\text{ee}}}{\xi^2}$$

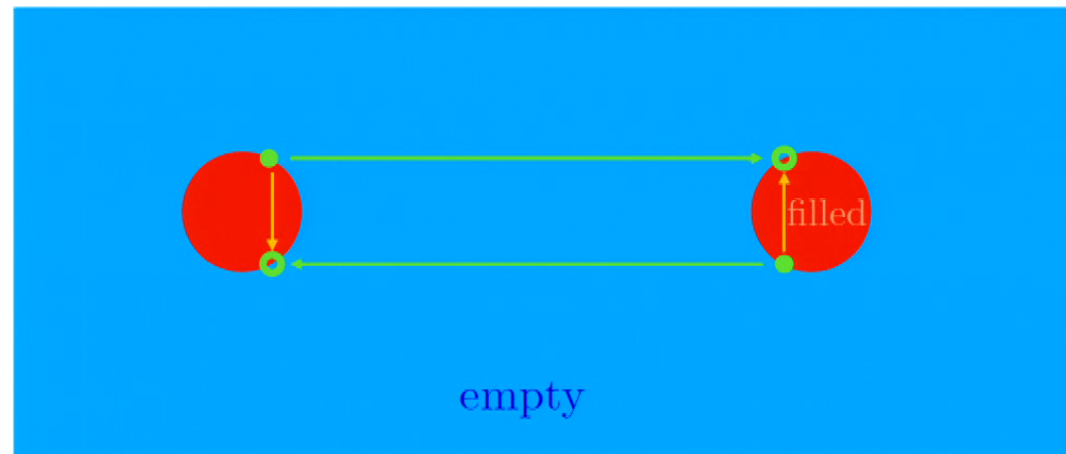
if  $\xi$  is length scale of inhomogeneity

- ▶ but if we have an extra conservation law (energy):

$$\rho \sim \int \frac{d^d \mathbf{x}}{V_d} \left[ \frac{1}{\Sigma^{22}} \left( J^2 - \frac{n^2}{n^1} J^1 \right)^2 + \eta \left( \nabla \frac{1}{n^1} \right)^2 \right]$$

and in the hydrodynamic limit,  $\ell_{\text{ee}} \ll \xi$ :

$$\rho \sim \frac{1}{\chi^{22} D^{22}} \sim \frac{1}{\ell_{\text{ee}}}$$

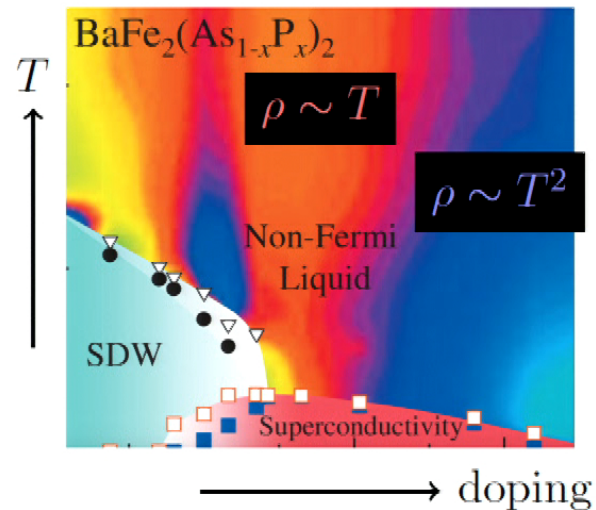


- ▶ quasiparticle 2-body collisions conserve momentum  $\implies$   
 QPs on each pocket separately conserved
  - ▶  $\Sigma_{\text{imb}} \sim \chi_{\text{imb}} D_{\text{imb}}, \quad \chi_{\text{imb}} \sim T^0$
  - ▶  $T$ -dependence of  $\rho$  from  $T$ -dep. of  $\ell_{ee}$
  - ▶ (weak?) higher order effects spoil this new conservation law

[Lucas, Hartnoll, 1704.07384]

## Phenomenology: Enhanced Resistivity near Criticality

- ▶ sample phase diagram:



- ▶ our theory: imbalance diffusion causes

$$\rho \sim \frac{1}{\tau_{ee}} \sim \begin{cases} T & \text{strange metal} \\ T^2 & \text{Fermi liquid} \end{cases} .$$



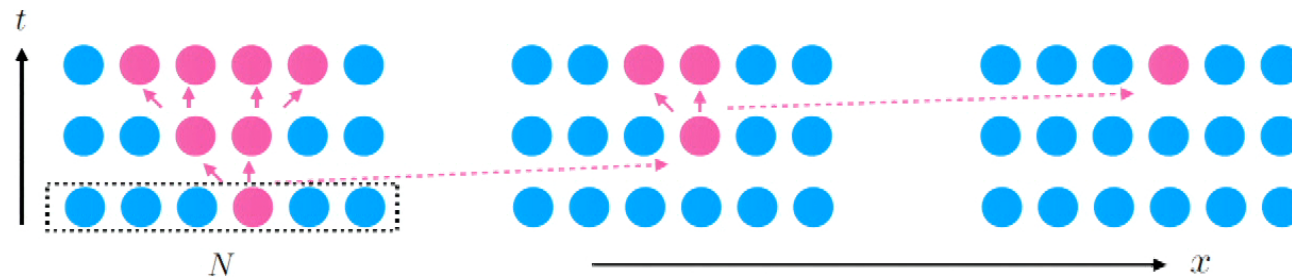
## Out-of-Time-Ordered Correlators

- ▶ **conjecture:** in a large  $N$  QFT, out-of-time-ordered correlators (OTOCs) obey

$$\langle A(x, t)[B(x, t), C(0)]D(0) \rangle_{\beta} \leq \frac{\mathcal{P}}{N} \exp \left[ \frac{v_B t - |x|}{v_B \tau_L} \right] + O \left( \frac{1}{N^2} \right)$$

$\tau_L$  is the Lyapunov time;  $v_B$  is the butterfly velocity

- ▶ quantum information is scrambled among DOF:



## Hydrodynamic Operators

- ▶ in a large  $N$  QFT, hydrodynamic operators often take the schematic form

$$\rho = \sum_I \Phi_I^\dagger \Phi_I$$

- ▶ if we define

$$G_{\rho\rho}^{\text{R}}(x, t) \equiv i\Theta(t) \langle [\rho(x, t), \rho(0, 0)] \rangle_\beta$$

then if there is hydrodynamic diffusion at late times:

[Kadanoff, Martin (1963)]

$$G_{\rho\rho}^{\text{R}}(k, \omega) = \frac{\chi D k^2}{D k^2 - i\omega} + \dots$$

- ▶ if  $\rho$  instead couples to sound waves:

$$G_{\rho\rho}^{\text{R}}(k, \omega) = \frac{w k^2}{v_s^2 k^2 - \omega^2 - i\omega k^2 \Gamma_s} + \dots$$



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## Consistency of Chaos and Hydrodynamics

- ▶ from the quantum butterfly effect,  $\rho = \sum \Phi_I^\dagger \Phi_I$ :

$$\begin{aligned}
 |\langle [\rho(x, t), \rho(0, 0)] \rangle_\beta| &= \sum_{IJ} \langle \Phi_I^\dagger(x, t) [\Phi_I(x, t), \Phi_J^\dagger(0, 0)] \Phi_J(0, 0) \rangle_\beta \\
 &\quad + 3 \text{ similar terms} \\
 &\leq 4\mathcal{P}N \exp \left[ \frac{v_B t - |x|}{v_B \tau_L} \right] + \mathcal{O}(N^0)
 \end{aligned}$$

- ▶ if there is a sound wave,

$$G_{\rho\rho}^R(x, t) \sim \partial_x^2 \exp \left[ -\frac{(|x| - v_s t)^2}{4\Gamma_s t} \right], \quad (d = 1)$$

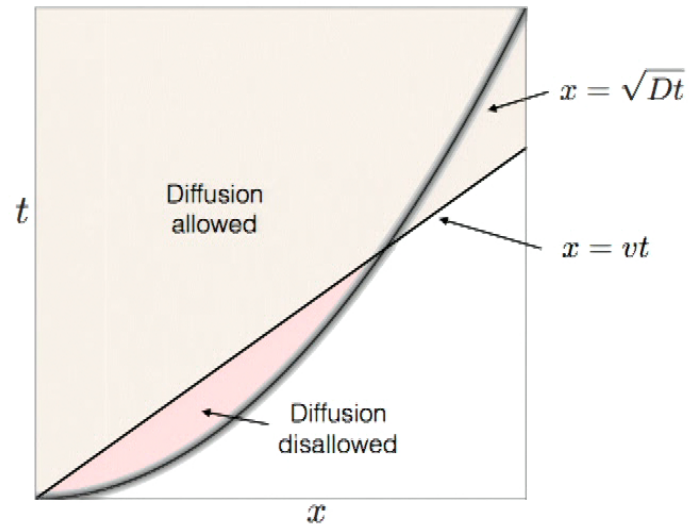
- ▶ these two equations are only consistent if

$$\boxed{v_s \leq v_B}$$

[Lucas, to appear]

## Consistency with Diffusion

- ▶ diffusion exponentially suppressed when  $x^2 \gtrsim Dt$ :



- ▶ if hydrodynamic diffusion is valid for  $t \geq \tau$ :

[Hartman, Hartnoll, Mahajan, 1706.00019]

$$D \leq v^2 \tau$$

with  $v = v_B$  (at leading order in  $1/N$ ) [Lucas, to appear]

## A Puzzle: Holographic Charge Diffusion

- ▶ in “scaling” holographic theories: [Blake, 1603.08510]

$$D \sim \frac{v_B^2}{T} \left( \frac{\Lambda}{T} \right)^{\max(0, -\Delta/z)} .$$

- ▶  $v_B$  computed at horizon, because charge decouples from gravity in linear response
- ▶ if  $\Delta > 0$ , then

$$\tau \sim \tau_L \sim \frac{1}{T}$$

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- ▶ but if  $\Delta < 0$ , hydrodynamics fails unexpectedly early?

$$\tau \sim \frac{1}{T} \left( \frac{\Lambda}{T} \right)^{-\Delta/z}$$

rigorous bounds on transport and hydrodynamics:

- ▶ semiclassical lower bounds on  $\sigma$  (indirectly,  $D$ ):

$$\sigma \geq \frac{J_x^2}{(T\dot{s})_{\text{avg}}} \Big|_{\nabla \cdot \mathbf{J}=0, \text{ for all conserved currents}}$$

- ▶ what is the quantum generalization?
- ▶ upper bounds on hydrodynamics from the quantum butterfly effect:

$$v_s \leq v_B, \quad D \leq v_B^2 \tau$$

- ▶ we find that many strange metals have unexpectedly rapid failure of hydrodynamics...**why?**