

Title: PSI 17/18 - Quantum Theory - Lecture 11

Date: Sep 19, 2017 10:45 AM

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Abstract:

Generalized Transformations

1 Consider Mixture:

$$\Lambda(\rho) = \sum_i p_i U_i \rho U_i^\dagger$$

Goal effective
description in
system Hilbert
space
(w/o mentioning other
classical or
q. resources
leading to
"non-unitary"
dynamics)

Generalized Transformations

1 Consider Mixture:

$$\Lambda(\rho) = \sum_i p_i U_i \rho U_i^\dagger$$

Define: $A_i \equiv \sqrt{p_i} U_i$

So $\Lambda(\rho) = \sum_i A_i \rho A_i^\dagger$

Goal effective description in system Hilbert space (w/o mentioning other classical or q. resources leading to "non-unitary" dynamics)

Generalized Transformations

1 Consider Mixture:

$$\Lambda: \mathcal{D}(\mathcal{H}) \rightarrow \mathcal{D}(\mathcal{H})$$

$$\Lambda(\rho) = \sum_i p_i U_i \rho U_i^\dagger$$

Define: $A_i \equiv \sqrt{p_i} U_i$

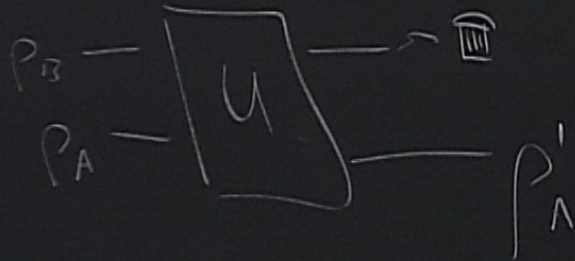
So $\boxed{\Lambda(\rho) = \sum_i A_i \rho A_i^\dagger}$

Goal effective
description in
system Hilbert
space
(w/o mentioning other
classical or
q. resource
leading
"non-"

2. Ignoring some relevant Hilbert space

$$\mathcal{H}_A \otimes \mathcal{H}_B$$

System $\mathcal{H}_A$, $\rho_A \in \mathcal{D}(\mathcal{H}_A)$



$$\rho'_A = \text{Tr}_B [U \rho_A \otimes \rho_B U^\dagger]$$

For simplicity take $\rho_B = |0\rangle\langle 0|_B$

Get $\rho'_A = \Lambda(\rho_A) = \sum_k A_k \rho A_k^\dagger$

where $A_k = \langle k | \hat{U} | 0 \rangle$

$$\text{Tr}_B [U \rho_A \otimes |0\rangle\langle 0| U^\dagger]$$

$$= \sum_k \langle k | \hat{U} \hat{\rho}_A \otimes |0\rangle\langle 0| \hat{U}^\dagger |k\rangle$$

$\{|k\rangle\}$ is an ON. basis for $\mathcal{H}_B$

$$\text{Tr}_B [U \rho_A \otimes |x\rangle\langle x| U^\dagger]$$

$$= \sum_k \langle k | \hat{U} \underbrace{(\hat{\rho}_A \otimes |x\rangle\langle x| \hat{U}^\dagger)}_{\hat{A}_k^\dagger} |k\rangle$$

$\{|k\rangle\}$ is an ON basis for $\mathcal{H}_B$

$$U \in \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B)$$

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2. Ignoring some
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$$\mathcal{H}_A \otimes \mathcal{H}_B$$

System $\mathcal{H}_A$

ρ_B

ρ_A

ρ'_A

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Generalized Transformations

1 Consider Mixtures:

$$\Lambda: \mathcal{D}(\mathcal{H}) \rightarrow \mathcal{D}(\mathcal{H})$$

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So $\Lambda(\rho) = \sum_i A_i \rho A_i^\dagger$

Goal effective description in system Hilbert space (w/o mentioning other classical or q. resources leading to "non-unitary" dynamics)

In both cases: $\sum_k A_k^\dagger A_k = \mathbb{1}$

$$1) \quad \sum_k p_k \underbrace{U_k^\dagger U_k}_{\mathbb{1}} = \sum_k p_k \underbrace{\mathbb{1}}_{=1} = \mathbb{1}_A$$

$$U \rho_A \otimes \langle \chi | U^+$$

$$\sum_k \langle k | \hat{U} \underbrace{\rho_A \otimes \langle \chi |}_{\hat{A}_k^+} \hat{U}^+ | k \rangle$$

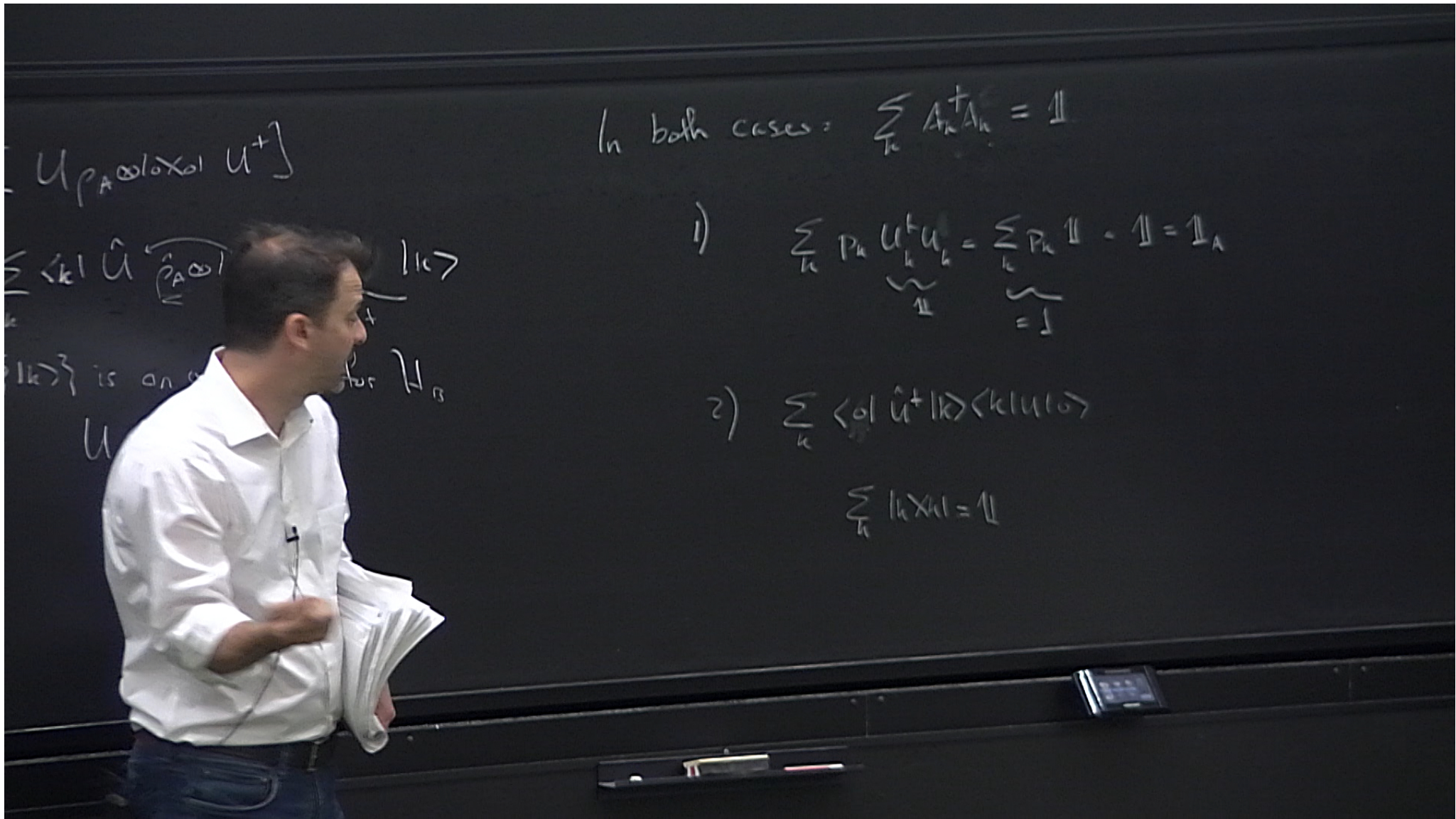
$|k\rangle\}$ is an ON. basis for $\mathcal{H}_B$

$$U \in \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B)$$

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$$1) \quad \sum_k P_k \underbrace{U_k^+ U_k}_{\mathbb{1}} = \sum_k \underbrace{P_k}_{=\mathbb{1}} = \mathbb{1} = \mathbb{1}_A$$

$$2) \quad \sum_k \langle \phi | \hat{U}^+ | k \rangle \langle k | U | \phi \rangle$$



$U \rho_A \otimes |\chi\rangle\langle\chi| U^\dagger$
 $\sum_k \langle k| \hat{U} \rho_A \otimes |\chi\rangle\langle\chi| U^\dagger |k\rangle$
 $\{|k\rangle\}$ is an orthonormal basis for $\mathcal{H}_B$
 U

In both cases: $\sum_k A_k^\dagger A_k = \mathbb{1}$

1)
$$\sum_k P_k \underbrace{U_k^\dagger U_k}_{\mathbb{1}} = \sum_k \underbrace{P_k}_{=1} \mathbb{1} = \mathbb{1}_A$$

2)
$$\sum_k \langle \phi| \hat{U}^\dagger |k\rangle \langle k| U | \phi \rangle$$

$$\sum_k |\chi_k|^2 = \mathbb{1}$$

$$U \rho_A \otimes \rho_B U^\dagger$$

$$\sum_k \langle k | \hat{U} \left(\rho_A \otimes \rho_B \right) \hat{U}^\dagger | k \rangle$$

$|k\rangle\}$ is an ON basis for $\mathcal{H}_B$

$$U \in \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B)$$

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1) $\sum_k P_k \underbrace{U_k^\dagger U_k}_{\mathbb{1}} = \sum_k P_k \underbrace{\mathbb{1}}_{=1} = \mathbb{1} = \mathbb{1}_A$

$\mathbb{1}_{A \otimes B} = \mathbb{1}_A \otimes \mathbb{1}_B$

2) $\sum_k \langle 0 | \hat{U}^\dagger | k \rangle \langle k | U | 0 \rangle = \langle 0 | \underbrace{U^\dagger U}_{\mathbb{1}_{AB}} | 0 \rangle = \mathbb{1}_A \langle 0 | \mathbb{1}_B | 0 \rangle$

$\sum_k |k\rangle \langle k| = \mathbb{1}_B \quad = \mathbb{1}_A \langle 0 | \mathbb{1}_B | 0 \rangle = \mathbb{1}_A$

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$$\sum_k |k\rangle \langle k| = \mathbb{1}_B \quad = \mathbb{1}_A \langle 0 | \mathbb{1}_B | 0 \rangle = \mathbb{1}_A$$

$$S_n \quad \Lambda(\rho) = \sum_i A_i \rho A_i^\dagger$$

"non-unitary"
dynamics)

Kraus' Theorem

Any map of the form

$$\Lambda(\rho) = \sum_n A_n \rho A_n^\dagger, \text{ st. } \sum_n A_n^\dagger A_n = \mathbb{1}$$

$\Rightarrow$ called an "operator-sum" decomposition,

$$S_n \quad \Lambda(\rho) = \sum_i A_i \rho A_i^\dagger$$

"non-unitary"
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Kraus' Theorem

A map is of the form

$$\Lambda(\rho) = \sum_n A_n \rho A_n^\dagger, \text{ st. } \sum_n A_n^\dagger A_n = \mathbb{I}$$

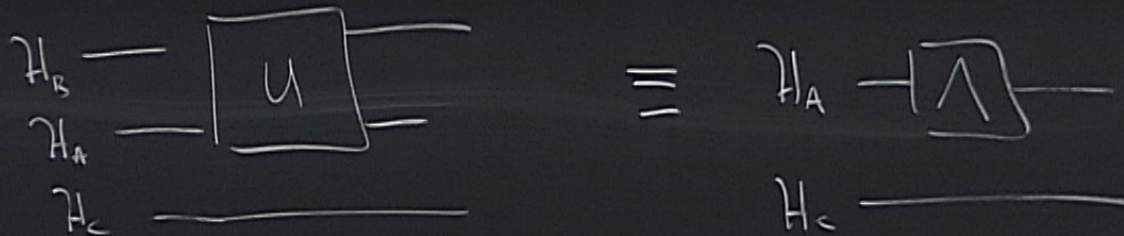
$\Rightarrow$ called an "operator-sum" decomposition,

iff it is a completely-positive trace-preserving (CPTP) map

Comp

$\mathcal{H}_B$ —
 $\mathcal{H}_A$ —
 $\mathcal{H}_C$ —

Completely-positive means

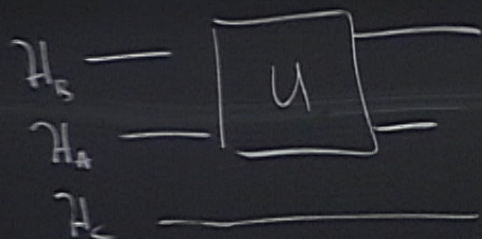


$$|\chi\rangle \in \mathcal{H}_B \otimes \mathcal{H}_A \otimes \mathcal{H}_C$$

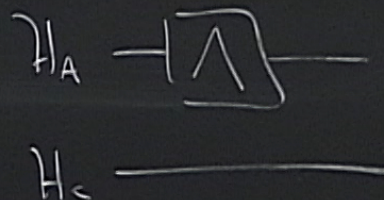
$$\text{Let } |\chi\rangle = |0\rangle_B \otimes |\Phi\rangle_{AC}$$

(CPTP) map

Completely-positive means



$\equiv$



}

ρ'_{Ac}

Λ is CP if

$$\forall |\Phi\rangle \in H_A \otimes H_C$$

$$\rho'_{Ac} = \Lambda \otimes I_C (|\Phi\rangle\langle\Phi|)$$

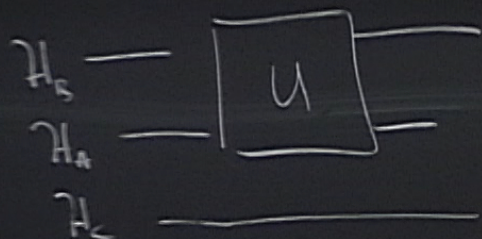
is positive

$$|\chi\rangle \in H_B \otimes H_A \otimes H_C$$

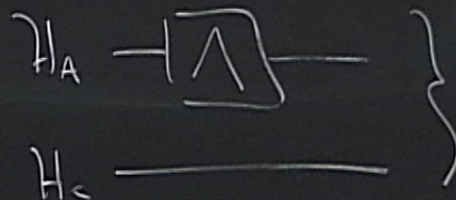
$$\text{Let } |\chi\rangle = |0\rangle_B \otimes |\Phi\rangle_{Ac}$$

$$\rho_A = \text{Tr}_C (|\Phi\rangle\langle\Phi|_{Ac})$$

Completely-positive means



$\equiv$



ρ'_{Ac}

$$\rho'_{Ac} = \Lambda \otimes I_c (|\Phi\rangle\langle\Phi|)$$

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$$|\chi\rangle \in H_B \otimes H_A \otimes H_C$$

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$$\rho_A = \text{Tr}_C (|\Phi\rangle\langle\Phi|_{Ac})$$

Λ is positive if

$$\forall \rho_A \in \mathcal{D}(H_A)$$

$$\rho'_A = \Lambda(\rho_A) \text{ is positive}$$

$$\text{So } \Lambda(\rho) = \sum_i A_i \rho A_i^\dagger$$

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A map is of the form

$$\Lambda(\rho) = \sum_n A_n \rho A_n^\dagger, \text{ st. } \sum_n A_n^\dagger A_n = \mathbb{I}$$

$\Rightarrow$ called an "operator-sum" decomposition,

iff Λ is a completely-positive trace-preserving (CPTP) map.

$$\begin{aligned} \text{Tr}(\rho') &= 1 \\ \text{if } \text{Tr}(\rho) &= 1 \end{aligned}$$

Comp

$\mathcal{H}_B$ —

$\mathcal{H}_A$ —

$\mathcal{H}_C$ —

$|\chi\rangle \in$

Let $|\chi\rangle$

Example Two qubits

$$U \in \mathcal{U}(\mathbb{C}^2 \otimes \mathbb{C}^2)$$

$$U_{\text{CNOT}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Example Two qubits

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$$U_{\text{CNOT}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|0\rangle \cong |+\rangle_z$$

$$|1\rangle \cong |-\rangle_z$$

$$|00\rangle = |0\rangle \otimes |0\rangle$$

$$|01\rangle$$

$$|10\rangle$$

$$|11\rangle$$

$$\uparrow \uparrow$$

$$A \ B$$

Initial state

$$|\psi\rangle = |0\rangle$$

$$U_{\text{CNOT}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$|\psi\rangle = \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} \otimes |0\rangle = \frac{1}{\sqrt{2}} (|0\rangle|0\rangle + |1\rangle|0\rangle) \leftarrow \text{not ent}$$

$$U_{CNOT} |\psi\rangle \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Initial state

$$|4\rangle = |0\rangle \otimes |0\rangle$$

$$U_{\text{NOT}} |4\rangle \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|4\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes |0\rangle = \frac{1}{\sqrt{2}} (|0\rangle|0\rangle + |1\rangle|0\rangle) \leftarrow \text{not entangled}$$

$$U_{\text{NOT}} |4\rangle \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

qubits

C^2

00
01
10

00 = |00>
01 = |01>
10 = |10>
11 = |11>
↑↑
A B

Initial state

$$|\psi\rangle = |0\rangle \otimes |0\rangle$$

$$U_{\text{CNOT}} |\psi\rangle \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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If $\exists |\alpha\rangle, |\beta\rangle$ s.t.
 $|\psi\rangle = |\alpha\rangle \otimes |\beta\rangle$

Initial state

$$|\psi\rangle = |0\rangle \otimes |0\rangle$$

$$U_{\text{CNOT}} |\psi\rangle \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|\psi\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes |0\rangle = \frac{1}{\sqrt{2}} (|0\rangle|0\rangle + |1\rangle|0\rangle) \leftarrow \text{not entangled}$$

$$U_{\text{CNOT}} |\psi\rangle \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}} (|0\rangle|0\rangle + |1\rangle|1\rangle) \leftarrow \text{entangled}$$

- If $\exists |\alpha\rangle, |\beta\rangle$ s.t.

$$|\psi\rangle = |\alpha\rangle \otimes |\beta\rangle$$

then $\text{Tr}_B(|\psi\rangle\langle\psi|) = \rho_A = |\alpha\rangle\langle\alpha| \Rightarrow \text{Pure state}$

$$\text{If } |\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\rho_A = \text{Tr}_B[|\psi\rangle\langle\psi|] = \sum_k \langle k| \frac{(|00\rangle + |11\rangle)(\langle 00| + \langle 11|)}{2} |k\rangle$$

$\Leftarrow$ entangled

then $\text{Tr}_B(|\psi\rangle\langle\psi|) = \rho_A = |\alpha|^2 |\alpha\rangle\langle\alpha| \Rightarrow \text{Pure state}$

$$\text{If } |\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\begin{aligned}\rho_A &= \text{Tr}_B[|\psi\rangle\langle\psi|] = \sum_{k=0}^1 \langle k| \frac{(|00\rangle + |11\rangle)(\langle 00| + \langle 11|)}{2} |k\rangle \\ &= \frac{|0\rangle\langle 0|}{2} + \frac{|1\rangle\langle 1|}{2} = \frac{11}{2} \Rightarrow \text{maximally mixed state}\end{aligned}$$

entangled

then $\text{Tr}_B(|\psi\rangle\langle\psi|) = \rho_A = |\alpha|^2 |\alpha|^2 \Rightarrow \text{Pure state}$

If $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \Rightarrow \text{is entangled}$

$$\rho_A = \text{Tr}_B[|\psi\rangle\langle\psi|] = \sum_{k=0}^1 \frac{\langle k| (|00\rangle + |11\rangle) (\langle 00| + \langle 11|)}{2} |k\rangle$$

$$= \frac{|0\rangle\langle 0|}{2} + \frac{|1\rangle\langle 1|}{2} = \frac{11}{2} \Rightarrow \text{maximally mixed state}$$

entangled

iff Λ is a completely-positive trace-preserving (CPTP) map

Let's calculate the Kraus operator-sum
for $\underline{U_{\text{CNOT}}}$

Recall $\Rightarrow$ the map Λ will also depend
on input state of system B

$$\Lambda: D(\mathcal{H}_A) \rightarrow D(\mathcal{H}_A)$$

$$A_k = \langle k | U |$$

is a completely-positive trace-preserving (CPTP) map

the Kraus operator-sum

$$\hat{A}_k = \langle k | \hat{U} | 0 \rangle$$

↑
assumed $|0\rangle$ is input state
to system B

op A will also depend
state of system B

$$A = D(\mathcal{H}_A) \rightarrow D(\mathcal{H}_A)$$

$$k=0, A_0 = \langle 0 | U | 0 \rangle$$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

is a completely-positive trace-preserving (CPTP) map

the Kraus operator-sum

$$\hat{A}_k = \langle k | \hat{U} | 0 \rangle$$

$$A_0 =$$

↑
assumed $|0\rangle$ is input state
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op A will also depend
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$$k=0, A_0 = \langle 0 | U | 0 \rangle$$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\langle 0 | \leftrightarrow (1 \ 0)$$

$$|0\rangle \leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Lambda: D(\mathcal{H}_A) \rightarrow D(\mathcal{H}_A)$$

a completely-positive trace-preserving (CPTP) map.

the Kraus operators are

$$A_k = \sqrt{p_k} U_k$$

$$A_0 =$$

↑
assumed $|0\rangle$ is input state
to system B

ρ_A will also depend
state of system B

$$\Lambda: D(\mathcal{H}_A) \rightarrow D(\mathcal{H}_A)$$

$$k=0, A_0 = \langle 0|U|0\rangle$$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$|0\rangle \leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle \leftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$$

a completely-positive trace-preserving (CPTP) map

the various operation-sum

$$A|k\rangle = \sum_i U_{ki} |i\rangle$$

$$A|0\rangle =$$

↑
assumed $|0\rangle$ is input state
to system B

sp A will
state of

$$A = D(\gamma)$$

$$k=0, A_0 = \langle 0|U|0\rangle$$

$$U_{\text{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$|0\rangle \leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle \leftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle i|U_{\text{CNOT}}|k\rangle$$

$$\langle 11|U_{\text{CNOT}}|10\rangle$$

$$U_{\text{CNOT}} = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$$

vector sum

$$\hat{A}_k = \langle k | \hat{U} | 0 \rangle$$



assumed $|0\rangle$ is input state
to system B

$$A_0 =$$

depend

B

$$k=0, A_0 = \langle 0 | U | 0 \rangle$$

$$U_{\text{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned} \langle 0 | &\leftrightarrow (\\ | 0 \rangle &\leftrightarrow \end{aligned}$$

eg. $\langle 1 | U_{\text{CNOT}} | k \rangle$
 $\langle 1 | U_{\text{CNOT}} | 0 \rangle$

$$U_{\text{CNOT}} = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 11|$$

map

$$\rho'_A = \Lambda(\rho_A)$$

$$\hat{A}_k = \langle k | \hat{U} | 0 \rangle$$

↑
assumed $|0\rangle$ is input state
to system B

$$k=0, A_0 = \langle 0 | U | 0 \rangle$$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$|0\rangle \leftrightarrow (1\ 0)$$

$$|1\rangle \leftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$$

$$A_0 = \langle 0 |_B U_{\text{CNOT}} | 0 \rangle_B$$

$$= |0\rangle_A \langle 0|_A$$

$$A_1 = \langle 1 |_B U_{\text{CNOT}} | 0 \rangle_B$$

$$= |1\rangle_A \langle 1|_A$$

$\rho'_A = \Lambda(\rho_A)$ is positive

$|10\rangle$

$|0\rangle$ is input state
to system B

$\langle 0|U|0\rangle$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$

$\langle 0| \leftrightarrow (1 \ 0)$

$|0\rangle \leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$

$$A_0 = \langle 0|_B U_{\text{CNOT}} |0\rangle_B$$

$$= |0\rangle_A \langle 0|_A \Leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_1 = \langle 1|_B \hat{U}_{\text{CNOT}} |0\rangle_B$$

$$= |1\rangle_A \langle 1|_A \Leftrightarrow \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Lambda_{\text{CNOT}}(\rho_A) = A_0 \rho_A A_0^\dagger + A_1 \rho_A A_1^\dagger \Leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \rho \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \rho \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\rho'_A = \Lambda(\rho_A) \text{ is positive}$$

$$A_0 = \langle 0|_B U_{\text{CNOT}} |0\rangle_B$$

$$= |0\rangle_A \langle 0|_A \Leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_1 = \langle 1|_B \hat{U}_{\text{CNOT}} |0\rangle_B$$

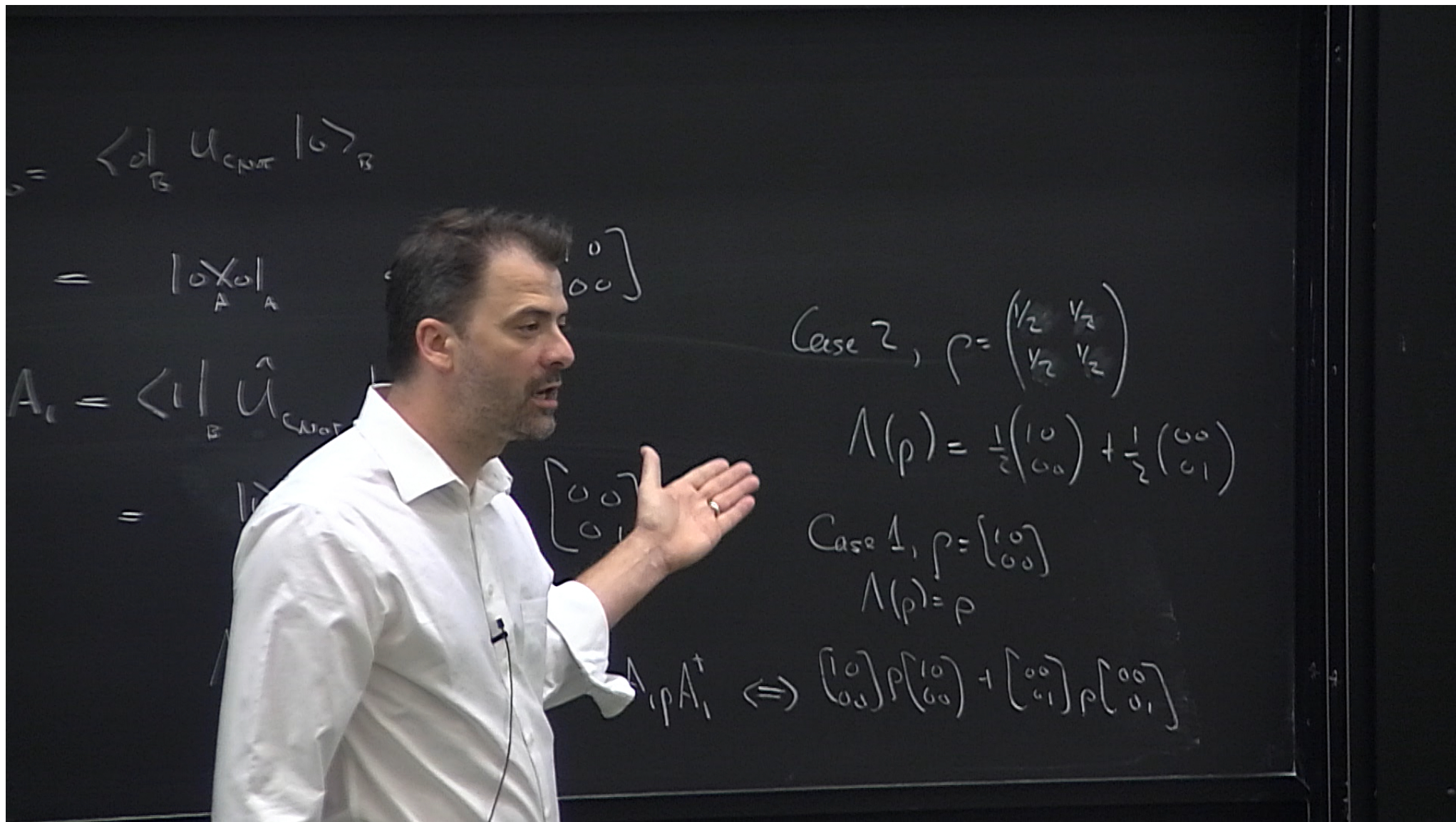
$$= |1\rangle_A \langle 1|_A \Leftrightarrow \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Case 1, } \rho = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Lambda(\rho) = \rho$$

$$\Lambda_{\text{CNOT}}(\rho_A) = A_0 \rho A_0^\dagger + A_1 \rho A_1^\dagger \Leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \rho \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \rho \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$





Stinespring Dilation Theorem

Any CPTP map can be
"realized" as a unitary
transformation on a larger
Hilbert space

Stinespring Dilation Theorem

Any CPTP map can be
"realized" as a unitary
transformation on a larger

$\mathcal{H}_A \otimes \mathcal{H}_S$ ← Hilbert space, for some
input state in the ancilla space $\mathcal{H}_S$

Postulate 3 (generalized)

Any ^q transformation
can be described by
a CPTP map.

iff $\mathcal{T}$ is a completely-positive trace-preserving (CPTP) map

Counter-example to Postulate 3

Consider Hamiltonian

$$U = e^{-iHt}$$

$$\tau = \tau_1 + \tau_2$$

Let $H\tau_1$ be chosen so that

$$U = e^{-iH\tau_1} = U_{\text{CNOT}}$$

iff $\mathcal{A}$ is a completely-positive trace-preserving (CPTP) map

example to Postulate 3

consider Hamiltonian

$$U = e^{-iHt}$$

$$\tau = \tau_1 + \tau_2$$

Let $H\tau_1$ be chosen so that

$$U = e^{-iH\tau_1} = U_{\text{CNOT}}$$

Let $H\tau_2$ also be same
 $\tau_1 = \tau_2$

From $t=0$ to τ_1

$$|7_i\rangle = U_{\text{CNOT}} |7\rangle$$

From $t=\tau_1$ to $t=2\tau_1 = \tau_1 + \tau_2$

$$|4_i\rangle = U_{\text{CNOT}} |7_i\rangle$$

fully-positive trace-preserving (CPTP) map

Postulate 3

Let H_{τ_1} also be same
 $\tau_1 = \tau_2$

From $t=0$ to τ_1

$$|\psi_1\rangle = U_{\text{evolution}} |\psi\rangle$$

From $t=\tau_1$ to $t=2\tau_1 = \tau_1 + \tau_2$

$$|\psi_2\rangle = U_{\text{evolution}} |\psi_1\rangle$$

I can describe this situation
on $H = \mathbb{C}^2 \otimes \mathbb{C}^2$ during any
time interval

using the ideal/fundamental
Postulates

$$\rho_A' = \Lambda(\rho_A) \text{ is positive}$$

using the ideal/fundamental postulates

What about generalized postulates?

Description on $\mathcal{H}_A = \mathbb{C}^n$ alone

From $t=0$ to $t=\tau_1$,

$$\underline{t=0}: \rho_A \in \mathcal{D}(\mathcal{H}_A)$$

$$\underline{t=0 \text{ to } t=\tau_1}: \Lambda \text{ a CPTP map } \mathcal{D}(\mathcal{H}_A) \rightarrow \mathcal{D}(\mathcal{H}_A)$$

$$\underline{t=\tau_1}: \rho_A(\tau_1) \in \mathcal{D}(\mathcal{H}_A)$$

$$\rho_A' = \Lambda(\rho_A) \text{ is positive}$$

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What about generalized postulates?

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$$\underline{t=\tau_1}: \rho_A(\tau_1) \in \mathcal{D}(\mathcal{H}_A)$$

$$\underline{t=\tau_1 \text{ to } t=\tau_2}: \Lambda_2, \text{ a CPTP map } \Lambda_2: \mathcal{D}(\mathcal{H}_A) \rightarrow \mathcal{D}(\mathcal{H}_A)$$

$$\rho_A' = \Lambda(\rho_A) \text{ is positive}$$

using the ideal/fundamental postulates

What about generalized postulates?

Description on $\mathcal{H}_A = \mathbb{C}^n$ alone

From $t=0$ to $t=\tau$,

$$\underline{t=0}: \rho_A \in \mathcal{D}(\mathcal{H}_A)$$

$$\underline{t=0 \text{ to } t=\tau_1}: \Lambda_1 \text{ a CPTP map } \mathcal{D}(\mathcal{H}_A) \rightarrow \mathcal{D}(\mathcal{H}_A)$$

$$\underline{t=\tau_1}: \rho_A(\tau_1) \in \mathcal{D}(\mathcal{H}_A)$$

$$\times \underline{t=\tau_1 \text{ to } t=\tau_2}: \Lambda_2 \text{ a CPTP map } \Lambda_2: \mathcal{D}(\mathcal{H}_A) \rightarrow \mathcal{D}(\mathcal{H}_A)$$

Observe that

$$U_{\text{CNOT}} \cdot U_{\text{CNOT}} = \mathbb{I}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$\Rightarrow p_A(\tau_2) = p_A(\psi=0)$$

Observe that

$$U_{CNOT} \cdot U_{CNOT} = \mathbb{1}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \rho_A(\tau_2) = \rho_A(\tau=0)$$

$$\text{also } |\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$

$$|\psi(\tau, \tau_1)\rangle = |\psi(\tau, 0)\rangle$$

Observe that

$$U_{CNOT} \cdot U_{CNOT} = \mathbb{I}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$|\psi(t=\tau_1)\rangle = |\psi(t=0)\rangle$$

At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle \langle \psi(t=\tau_1)|]$$

Observe that

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$$\Rightarrow \rho_A(\tau_2) = \rho_A(t=0)$$

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$$|\psi(t=\tau_1)\rangle = |\psi(t=0)\rangle$$

At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle \langle \psi(t=\tau_1)|]$$

Choose $|\psi(t=0)\rangle$ s.t.

$$\rho_A(t=\tau_1) = \frac{\mathbb{1}_A}{2}$$

Observe that

$$U_{CNOT} \cdot U_{CNOT} = \mathbb{I}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \rho_A(\tau_2) = \rho_A(t=0)$$

$$\text{also } |\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$

$$|\psi(t-\tau_1)\rangle = |\psi(t=0)\rangle$$

At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle\langle\psi(t=\tau_1)|]$$

I.

Choose $|\psi(t=0)\rangle$ s.t.

$$\rho_A(t=\tau_1) =$$

$$|\psi\rangle = \frac{(|0\rangle + |1\rangle)}{\sqrt{2}}$$

Observe that

$$U_{CNOT} \cdot U_{CNOT} = \mathbb{I}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \rho_A(\tau_2) = \rho_A(t=0)$$

$$\text{also } |\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$

$$|\psi(t-\tau_1)\rangle = |\psi(t=0)\rangle$$

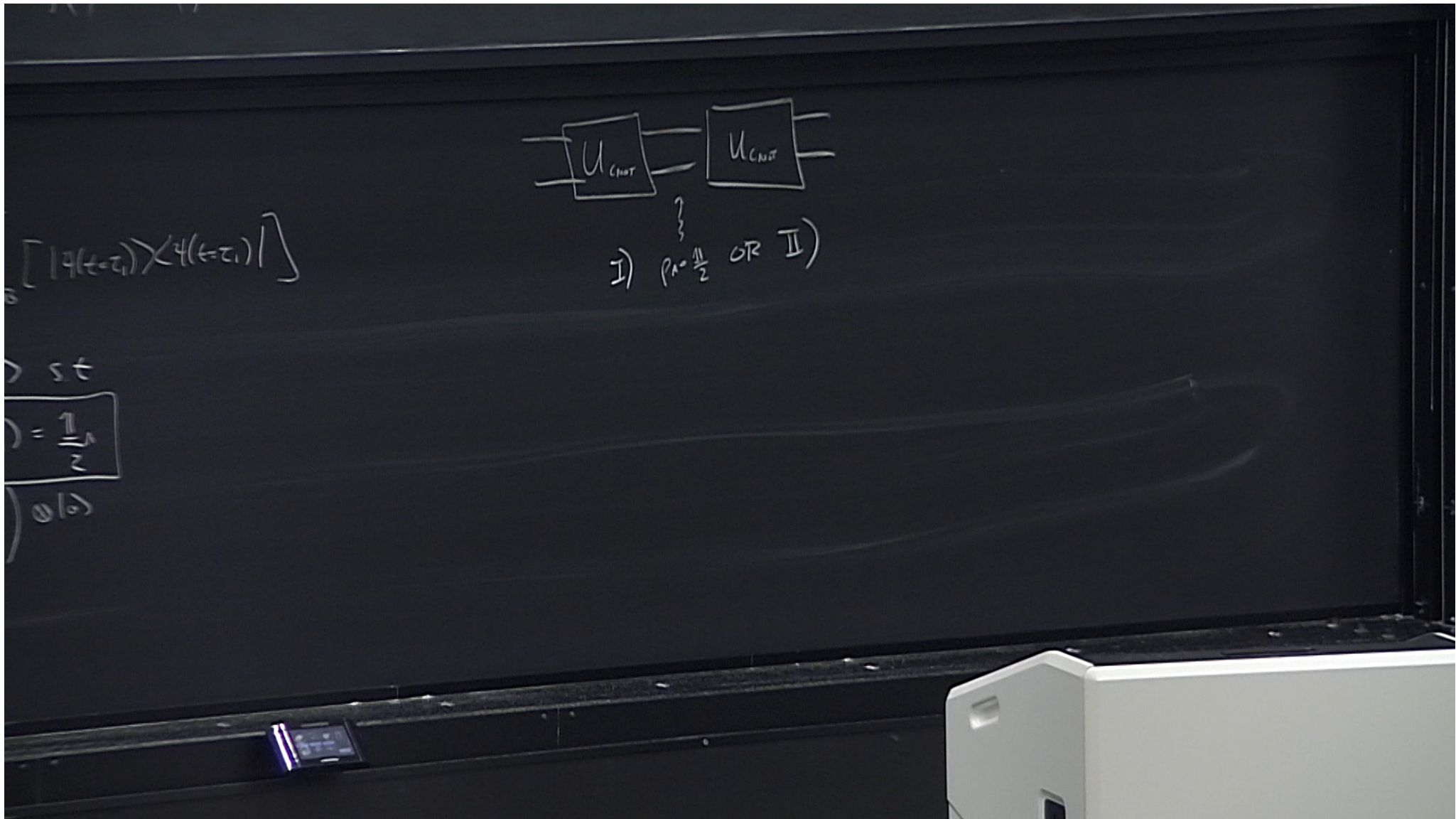
At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle \langle \psi(t=\tau_1)|]$$

I. Choose $|\psi(t=0)\rangle$ s.t.

$$\rho_A(t=\tau_1) = \frac{\mathbb{I}_A}{2}$$

$$|\psi\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes |0\rangle$$



$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

At intermediate time

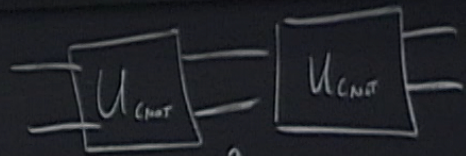
$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle \langle \psi(t=\tau_1)|]$$

I Choose $|\psi(t=0)\rangle$ s.t

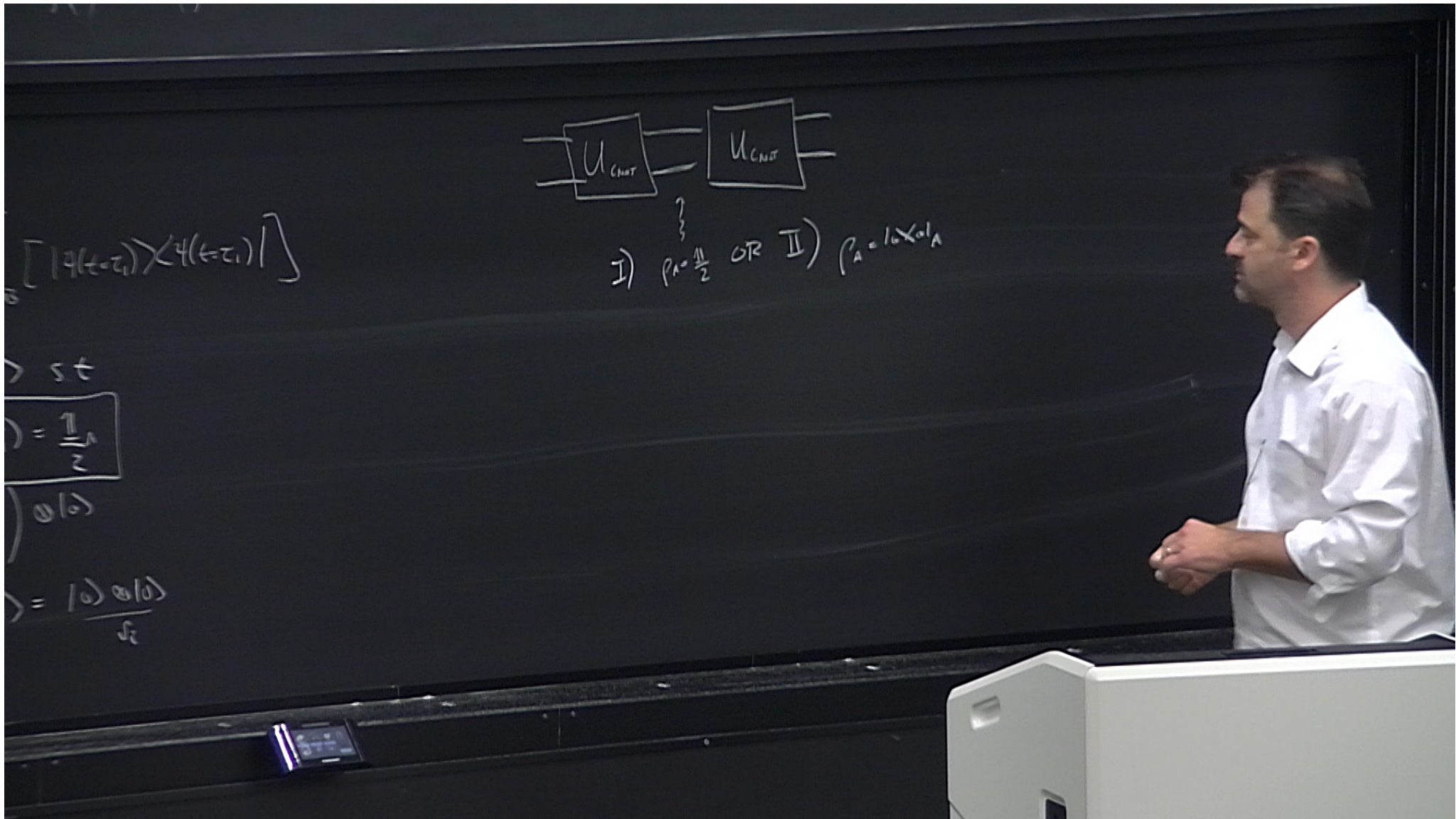
$$\rho_A(t=\tau_1) = \frac{1}{2} I_A$$

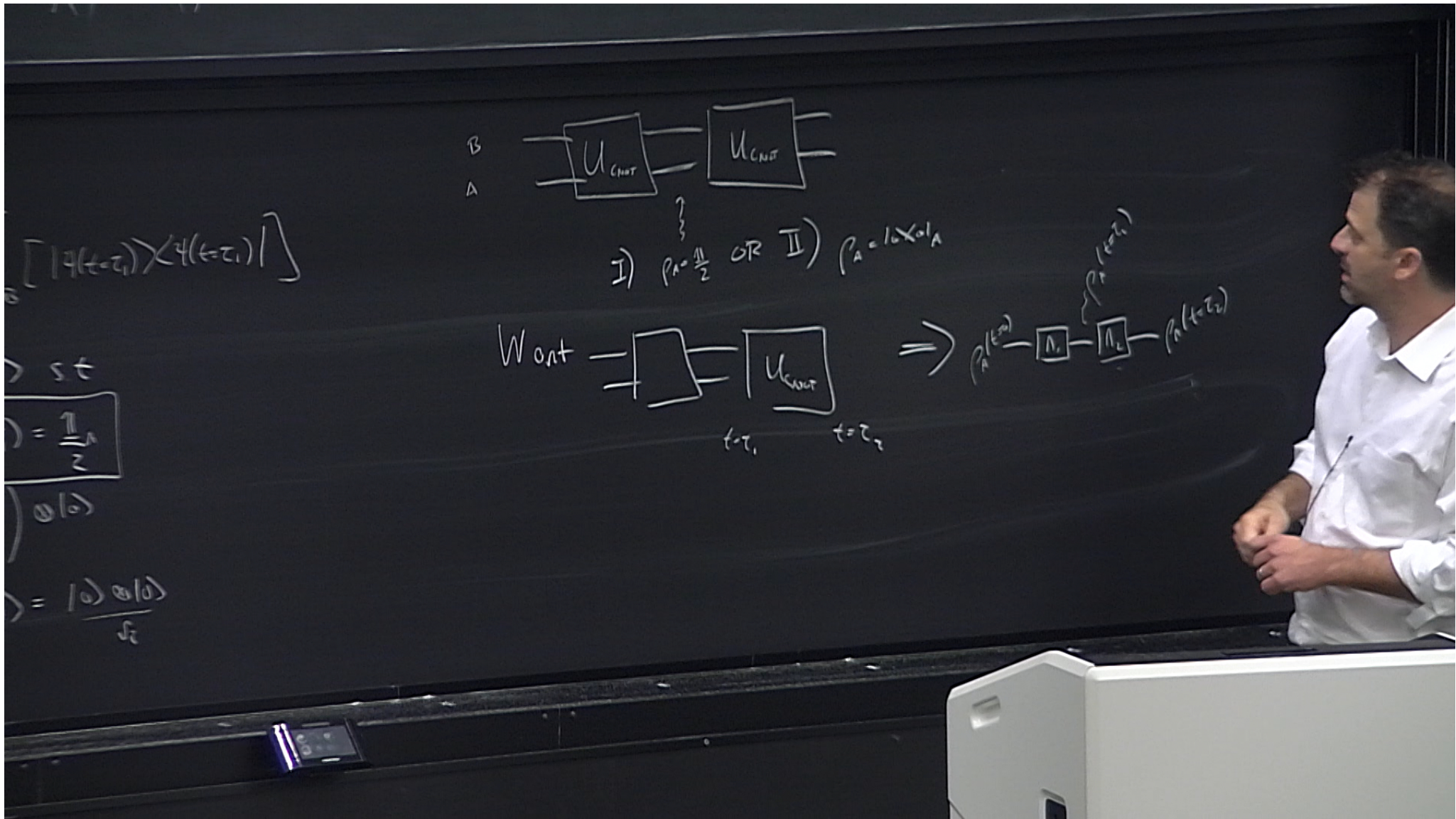
$$|\psi(t=0)\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes |0\rangle$$

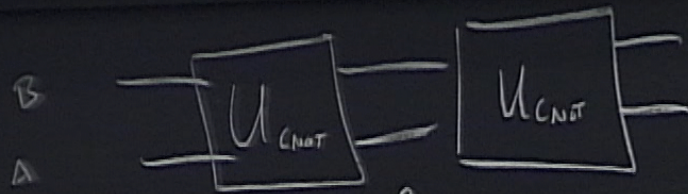
II Choose $|\psi(t=0)\rangle = \frac{|0\rangle \otimes |0\rangle}{\sqrt{2}}$



I) $\rho_A = \frac{1}{2} I$ OR II)

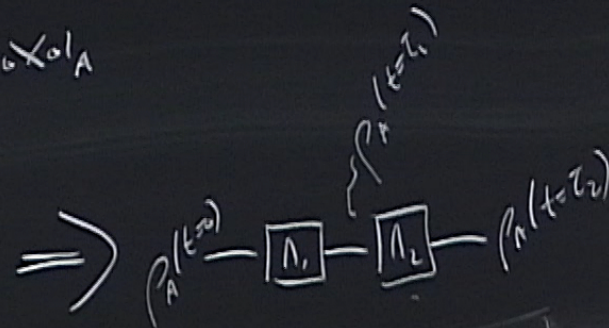
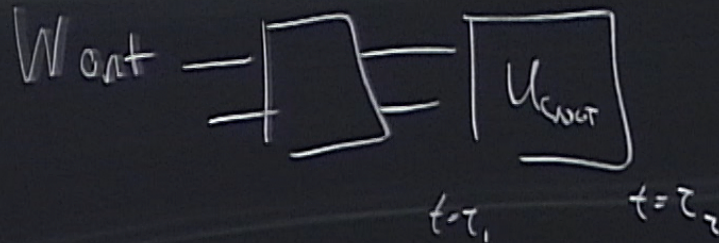


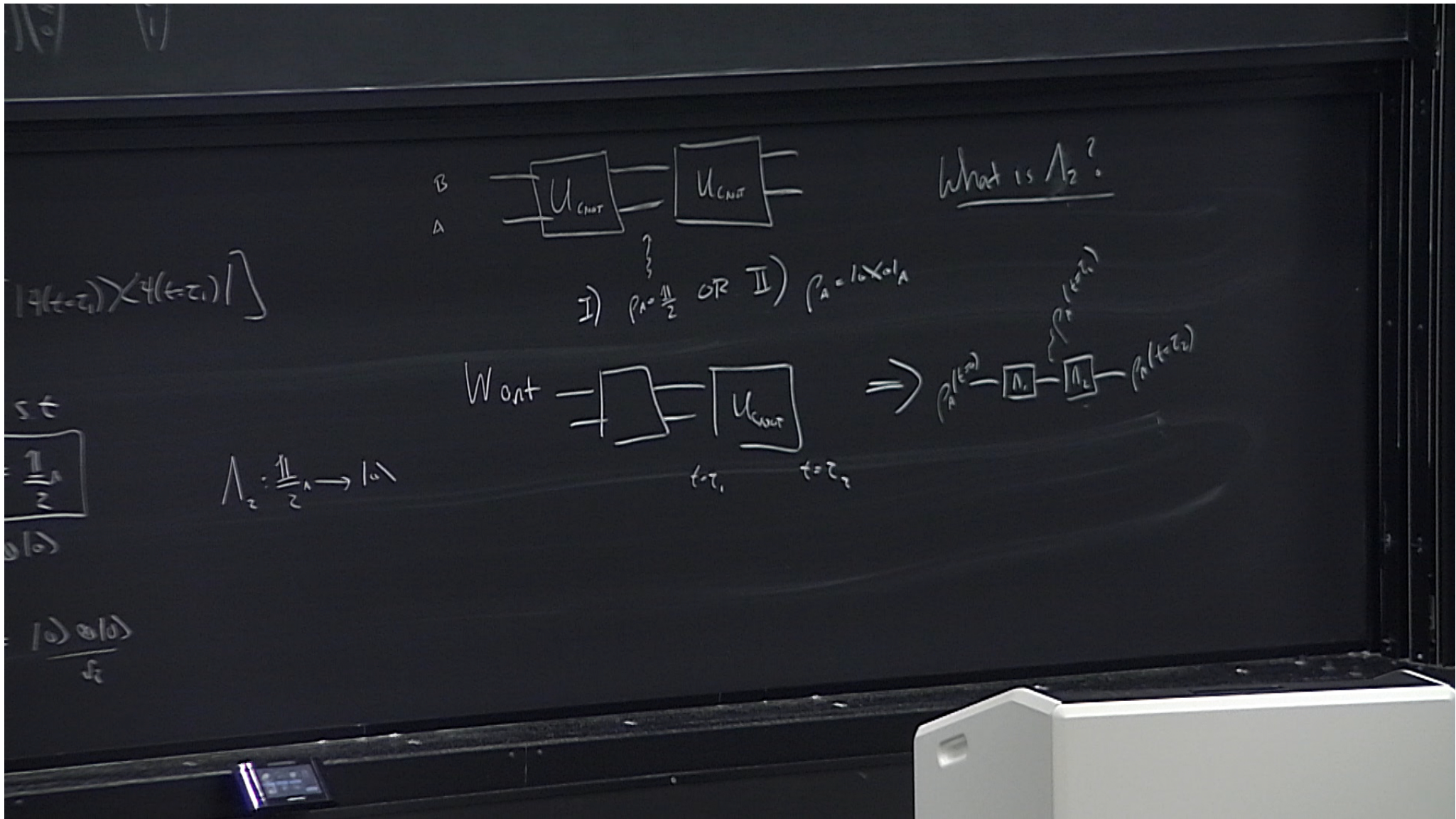




What is Λ_2 ?

I) $\rho_A = \frac{11}{2}$ OR II) $\rho_A = 16 \times 01_A$





At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle \langle \psi(t=\tau_1)|]$$

I. Choose $|\psi(t=0)\rangle =$

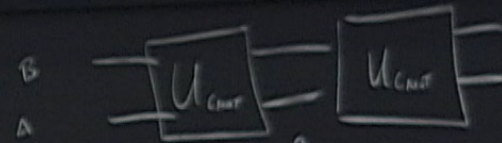
$$\rho_A(t=\tau_1) = \frac{1}{2} \Lambda$$

$$|\psi(t=0)\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes |0\rangle$$

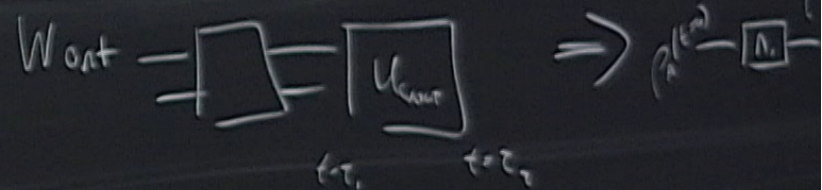
I $\Lambda_2: \frac{1}{2} \Lambda \rightarrow |0\rangle$

II. $\Lambda_2: |0\rangle \rightarrow |0\rangle$

II. Choose $|\psi(t=0)\rangle = \frac{|0\rangle \otimes |0\rangle}{\sqrt{2}}$



I) $\rho_A = \frac{1}{2}$ OR II) $\rho_A = |0\rangle\langle 0|_A$



What is

At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle\langle\psi(t=\tau_1)|]$$

I. Choose $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$

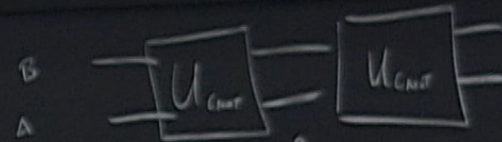
$$\rho_B(t=\tau_1) = \frac{1}{2} \mathbb{I}$$

$$|\psi(t=0)\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |0\rangle$$

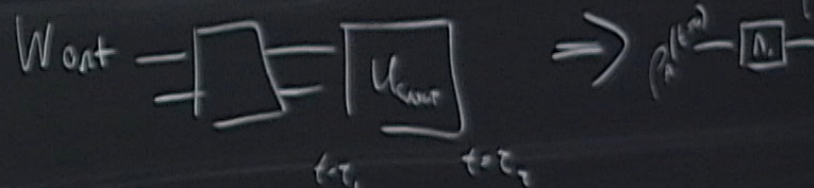
II. Choose $|\psi(t=0)\rangle = \frac{|0\rangle \otimes |0\rangle}{\sqrt{2}}$

I $\Lambda_2: \frac{1}{2} \mathbb{I} \rightarrow |0\rangle\langle 0|$

II $\Lambda_2: |0\rangle\langle 0| \rightarrow |0\rangle\langle 0|$



I) $\rho_A = \frac{1}{2} \mathbb{I}$ OR II) $\rho_A = |0\rangle\langle 0|$



At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle\langle\psi(t=\tau_1)|]$$

I. Choose $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$

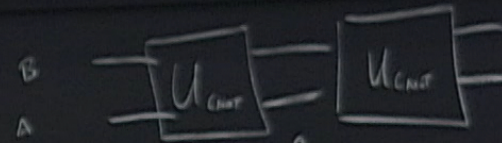
$$\rho_B(t=0) = |0\rangle\langle 0|, \quad \rho_B(t=\tau_1) = \frac{1}{2} \mathbb{I}$$

$$|\psi(t=0)\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |0\rangle$$

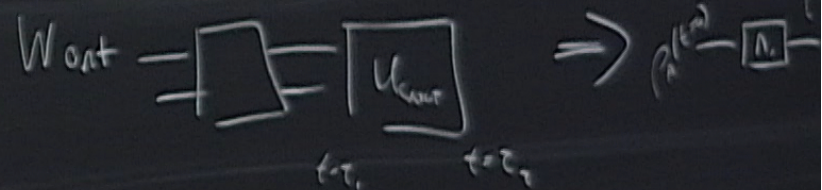
I $\Lambda_z: \frac{1}{2} \mathbb{I} \rightarrow |0\rangle\langle 0|$

II $\Lambda_z: |0\rangle\langle 0| \rightarrow |0\rangle\langle 0|$

II. Choose $|\psi(t=0)\rangle = \frac{|0\rangle \otimes |0\rangle}{\sqrt{2}}$



I) $\rho_A = \frac{1}{2} \mathbb{I}$ OR II) $\rho_A = |0\rangle\langle 0|$



At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle\langle\psi(t=\tau_1)|]$$

I. Choose $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$

$$\rho_B(t=0) = |0\rangle\langle 0|_B$$

$$\rho_B(t=\tau_1) = \frac{1}{2} \mathbb{I}_B$$

$$|\psi(t=0)\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes |0\rangle$$

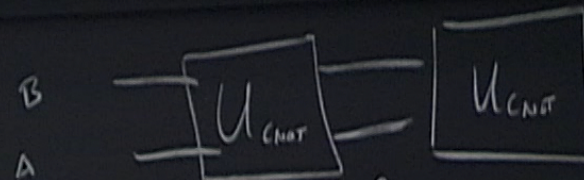
I $\Lambda_2: \frac{1}{2} \mathbb{I}_A \rightarrow |0\rangle\langle 0|$

II. $\Lambda_2: |0\rangle\langle 0| \rightarrow |0\rangle\langle 0|$

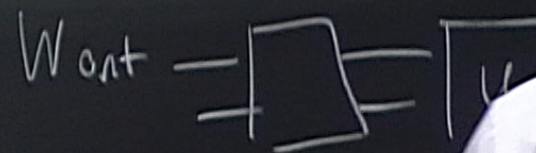
II. Choose $|\psi(t=0)\rangle = |0\rangle \otimes |0\rangle$

$$\rho_B(t=0) = |0\rangle\langle 0|_B$$

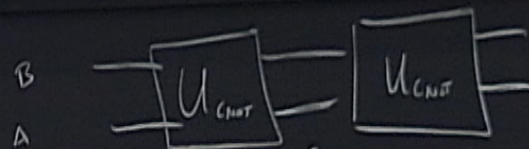
$$\rho_B(t=\tau_1) = |0\rangle\langle 0|_B$$



I) $\rho_A = \frac{1}{2} \mathbb{I}_A$ OR II

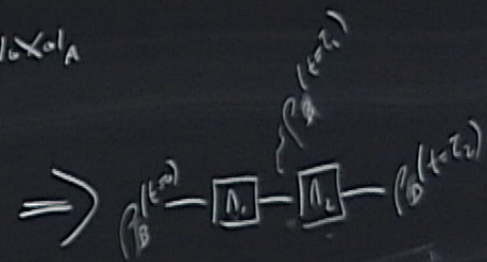
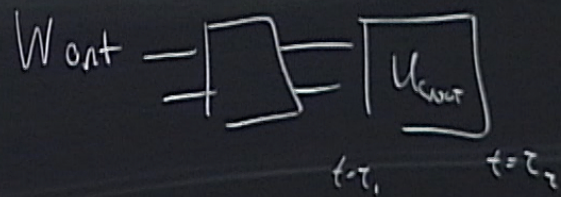


$$[|\psi(t=\tau_1)\rangle\langle\psi(t=\tau_1)|]$$



What is Λ_2 ?

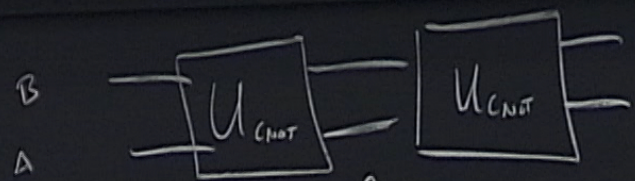
I) $P_A = \frac{11}{2}$ OR II) $P_A = 10 \times 1_A$



I $\Lambda_2: \frac{11}{2} \rightarrow 10$

II $\Lambda_2: 10 \rightarrow 10$

$$| \psi(t=\tau_1) \rangle \langle \psi(t=\tau_1) |$$



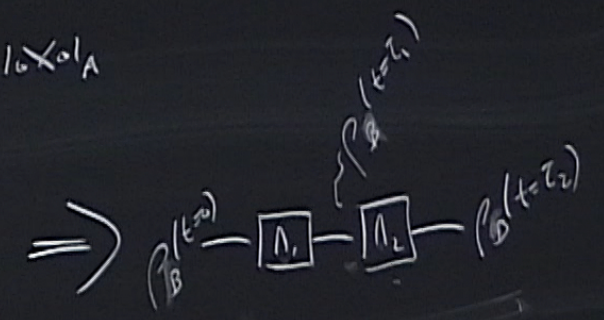
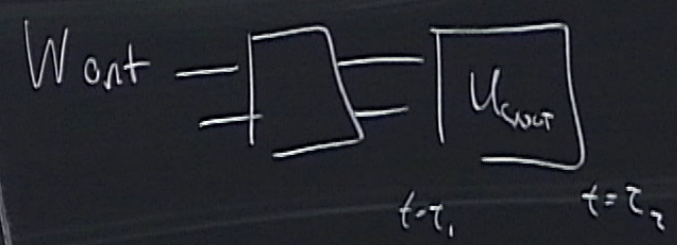
What is Λ_2 ?

I) $\rho_A = \frac{11}{2}$ OR II) $\rho_A = |0\rangle\langle 0|_A$

For $t=\tau_1$ to $t=\tau_2$

I $\Lambda_2^{(B)} = \frac{1}{2} \mathbb{I}_B \rightarrow |0\rangle\langle 0|$

II $\Lambda_2^{(B)} = |0\rangle\langle 0|_B \rightarrow |0\rangle\langle 0|$



the ideal/fundamental
states

What about generalized postulates?

Description on $\mathcal{H}_B = \mathbb{C}^2$ alone

From $t=0$ to $t=\tau_1$

$$\underline{t=0} \quad \rho_B \in \mathcal{D}(\mathcal{H}_B)$$

$$\underline{t=0 \text{ to } t=\tau_1}: \Lambda_1 \text{ a CPTP map } \mathcal{D}(\mathcal{H}_B) \rightarrow \mathcal{D}(\mathcal{H}_B)$$

$$\underline{t=\tau_1}: \rho_B(\tau_1) \in \mathcal{D}(\mathcal{H}_B)$$

$$\times \quad \underline{t=\tau_1 \text{ to } t=\tau_2}: \Lambda_2 \text{ a CPTP map} \\ \Lambda_2: \mathcal{D}(\mathcal{H}_B) \rightarrow \mathcal{D}(\mathcal{H}_B)$$

At intermediate time

$$\rho_A(t=\tau_1) = \text{Tr}_B [|\psi(t=\tau_1)\rangle\langle\psi(t=\tau_1)|]$$

I Choose $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$

$$\rho_B(t=0) = |0\rangle\langle 0|_B$$

$$|\psi(t=0)\rangle = \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} \otimes |0\rangle$$

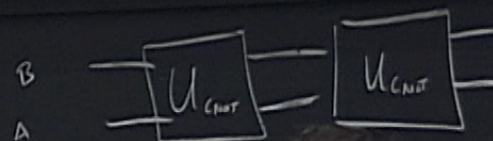
II Choose $|\psi(t=0)\rangle = |0\rangle \otimes |0\rangle$

$$\rho_B(t=0) = |0\rangle\langle 0|_B$$

For $t=\tau_1$ to $t=\tau_2$

I $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \rightarrow |0\rangle \otimes |0\rangle$

II $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \rightarrow |0\rangle \otimes |0\rangle$

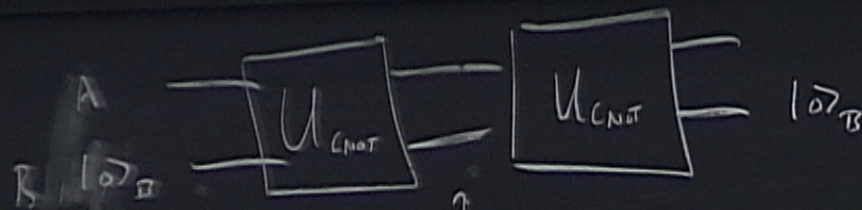


What is Λ_2 ?

I) $\rho_A = |0\rangle\langle 0|_A$



$$\Rightarrow \rho_B(t=0) \rightarrow \Lambda_1 \rightarrow \Lambda_2$$



What is Λ_2 ?

I) $\rho_B = \frac{11}{2}$ OR II) $\rho_B = |0\rangle\langle 0|_B$

