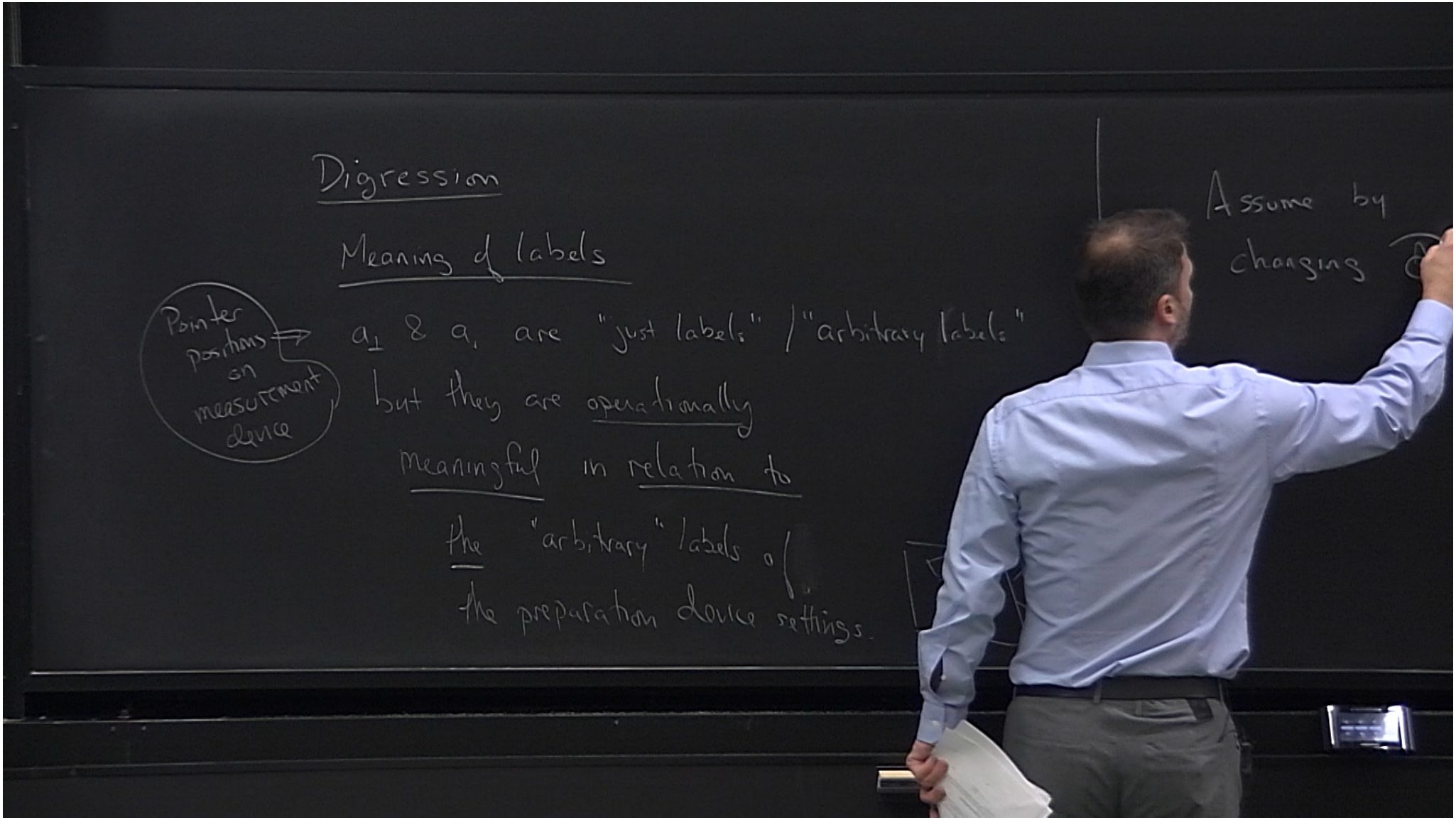
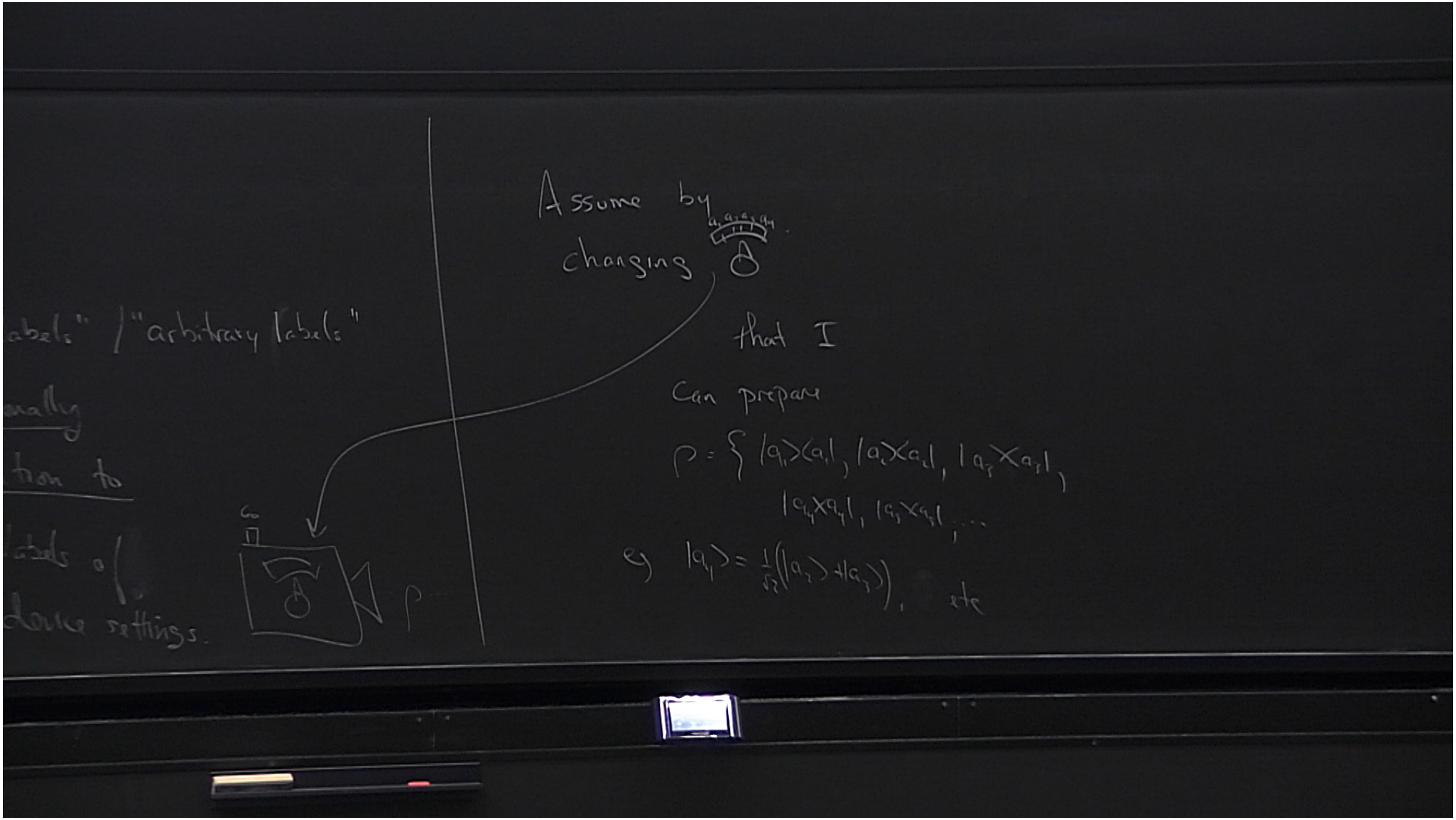
Title: PSI 17/18 - Quantum Theory - Lecture 5

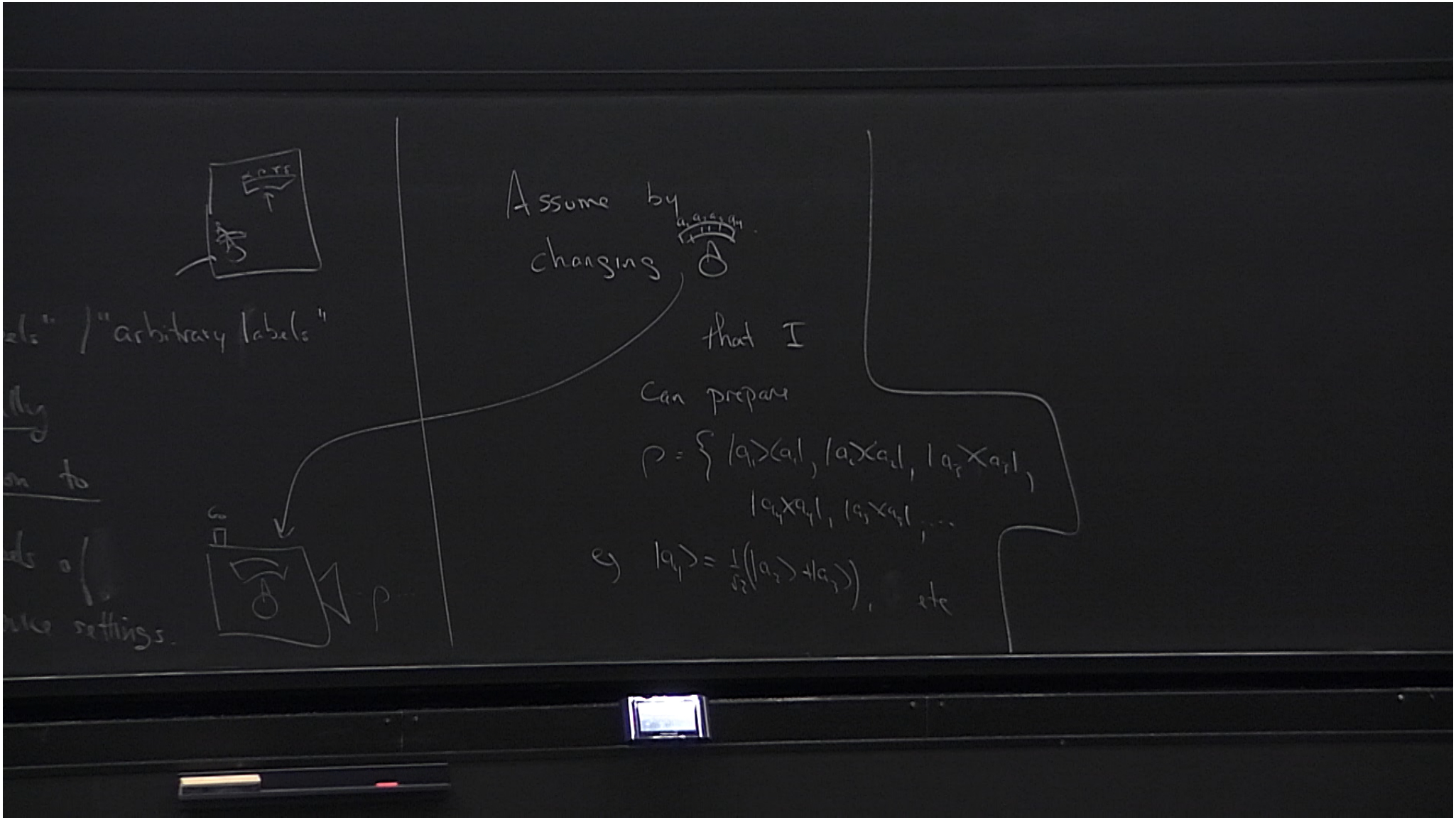
Date: Sep 11, 2017 10:45 AM

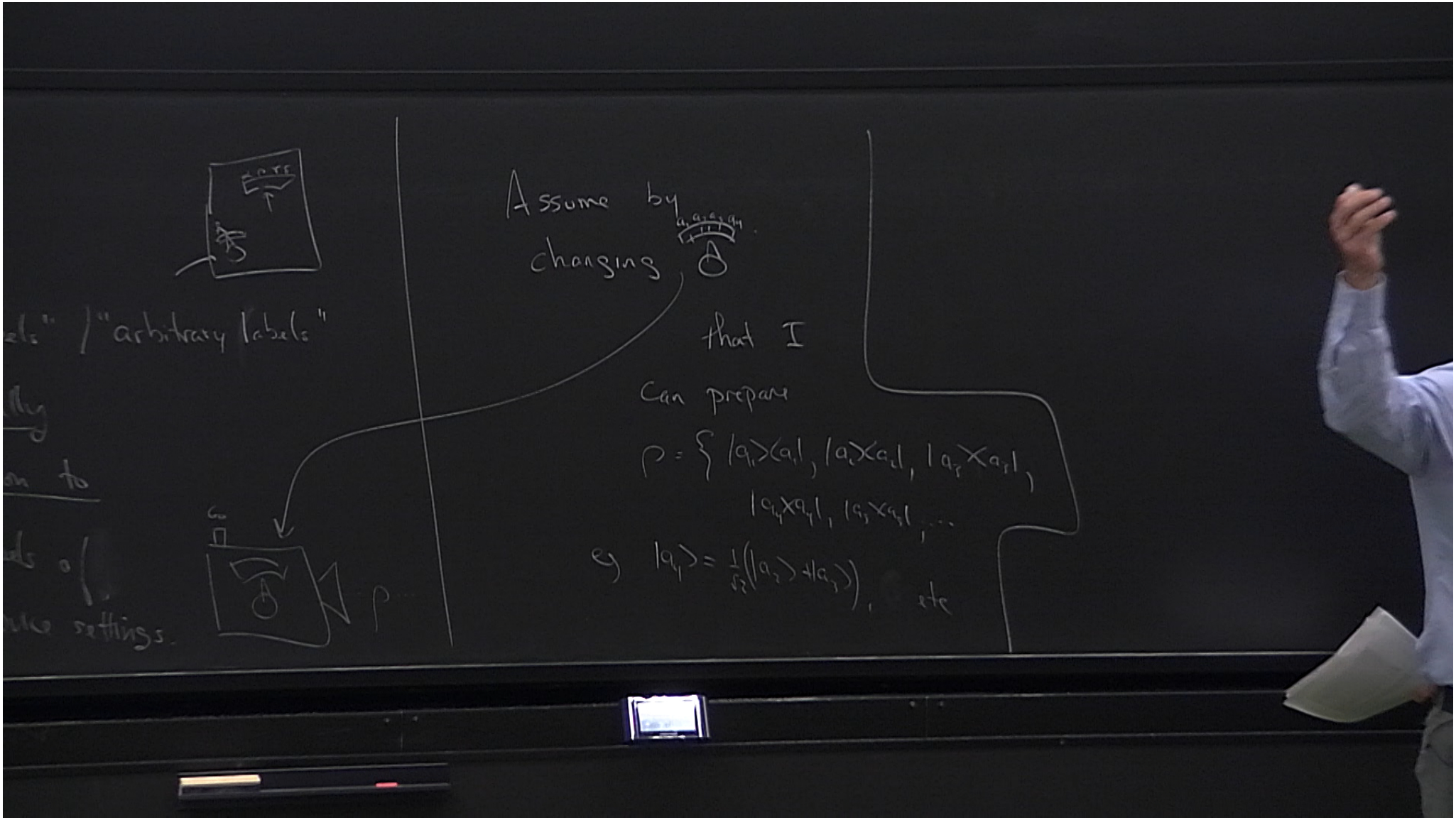
URL: http://pirsa.org/17090038

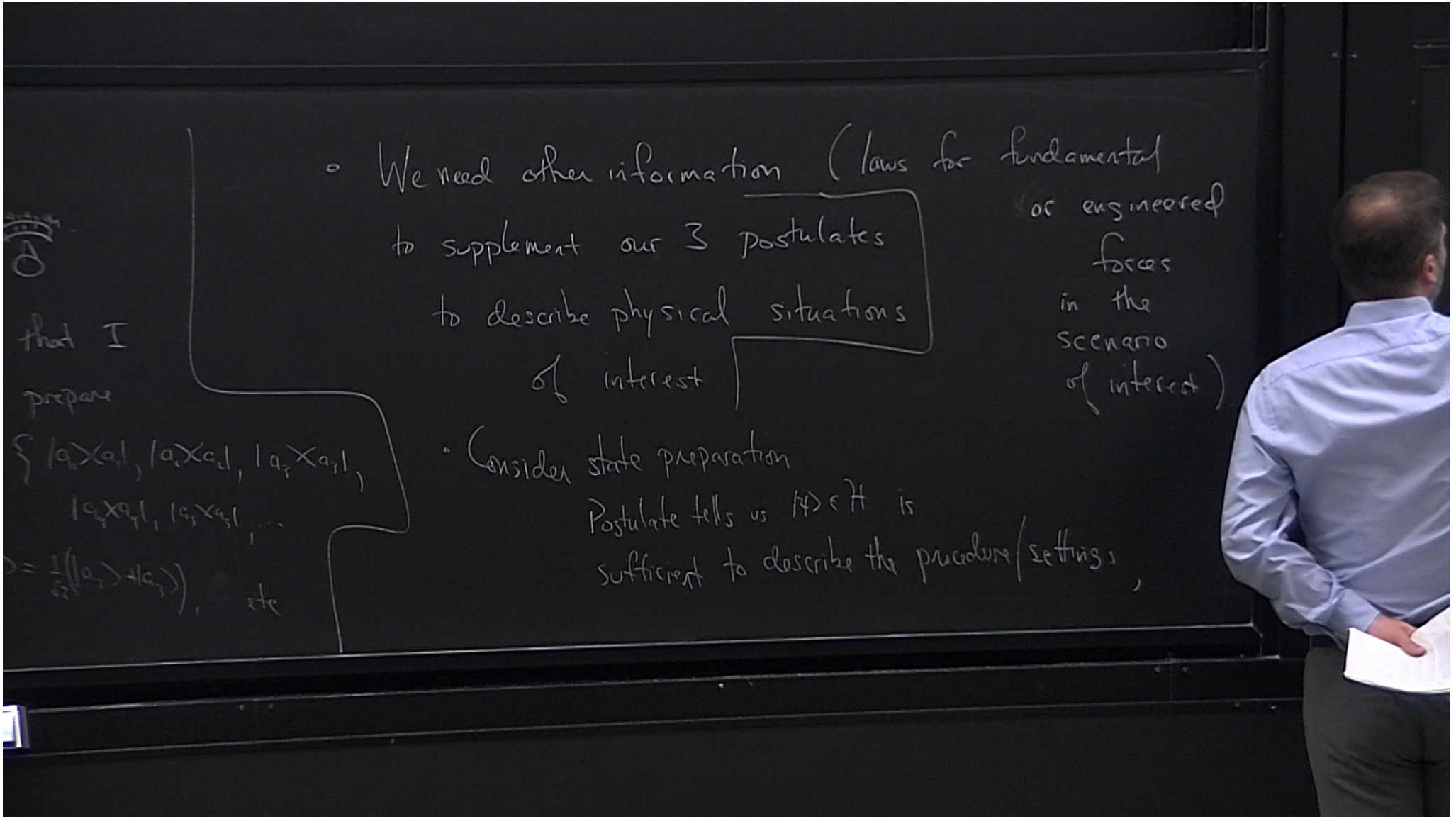
Abstract:

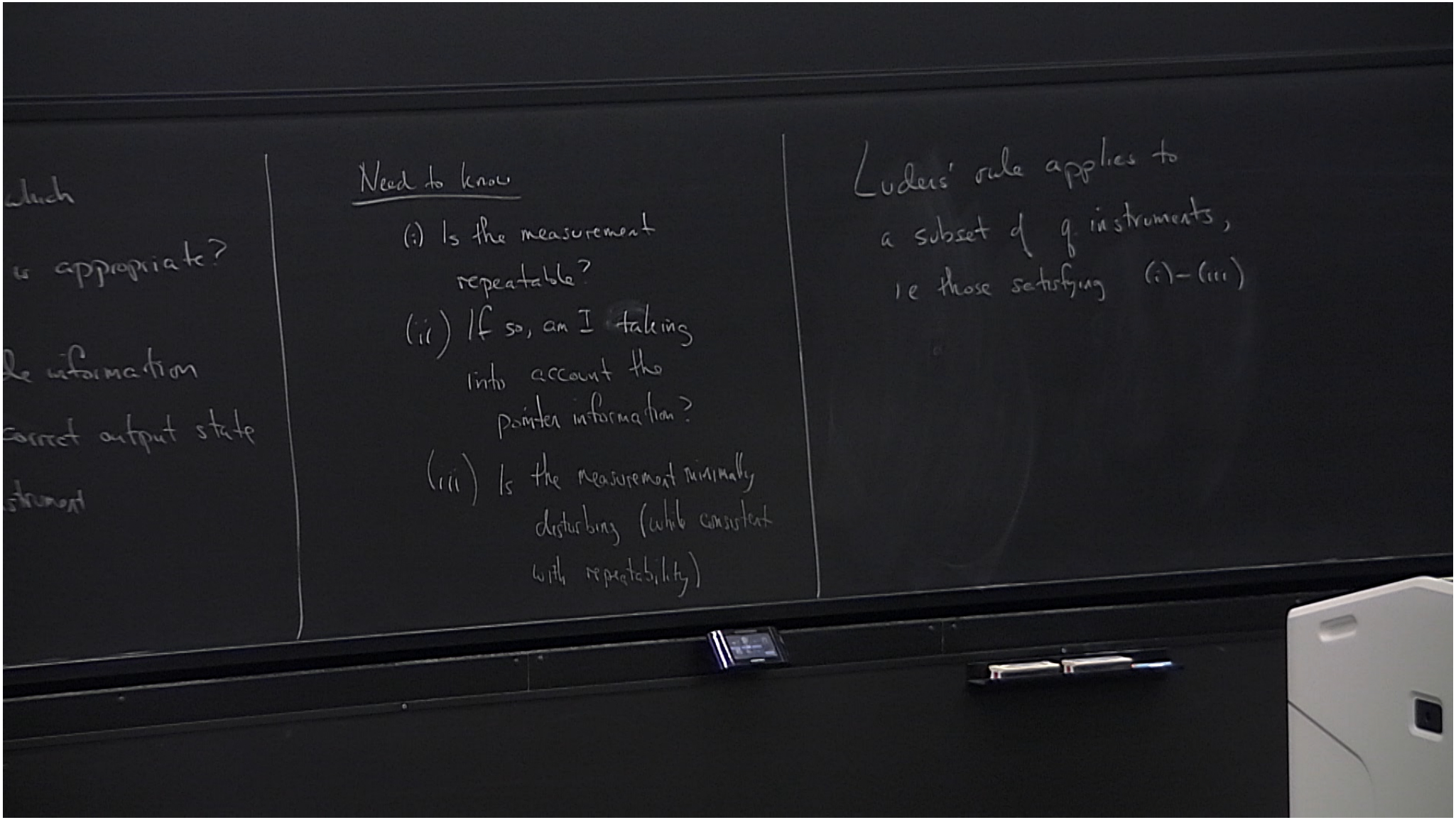

Digression
Meaning of labels
Pointer positions on measurement device
a_1 & a_i are "just labels" / "arbitrary labels"
but they are operationally meaningful in relation to the "arbitrary" labels of the preparation device settings.
Assume by changing

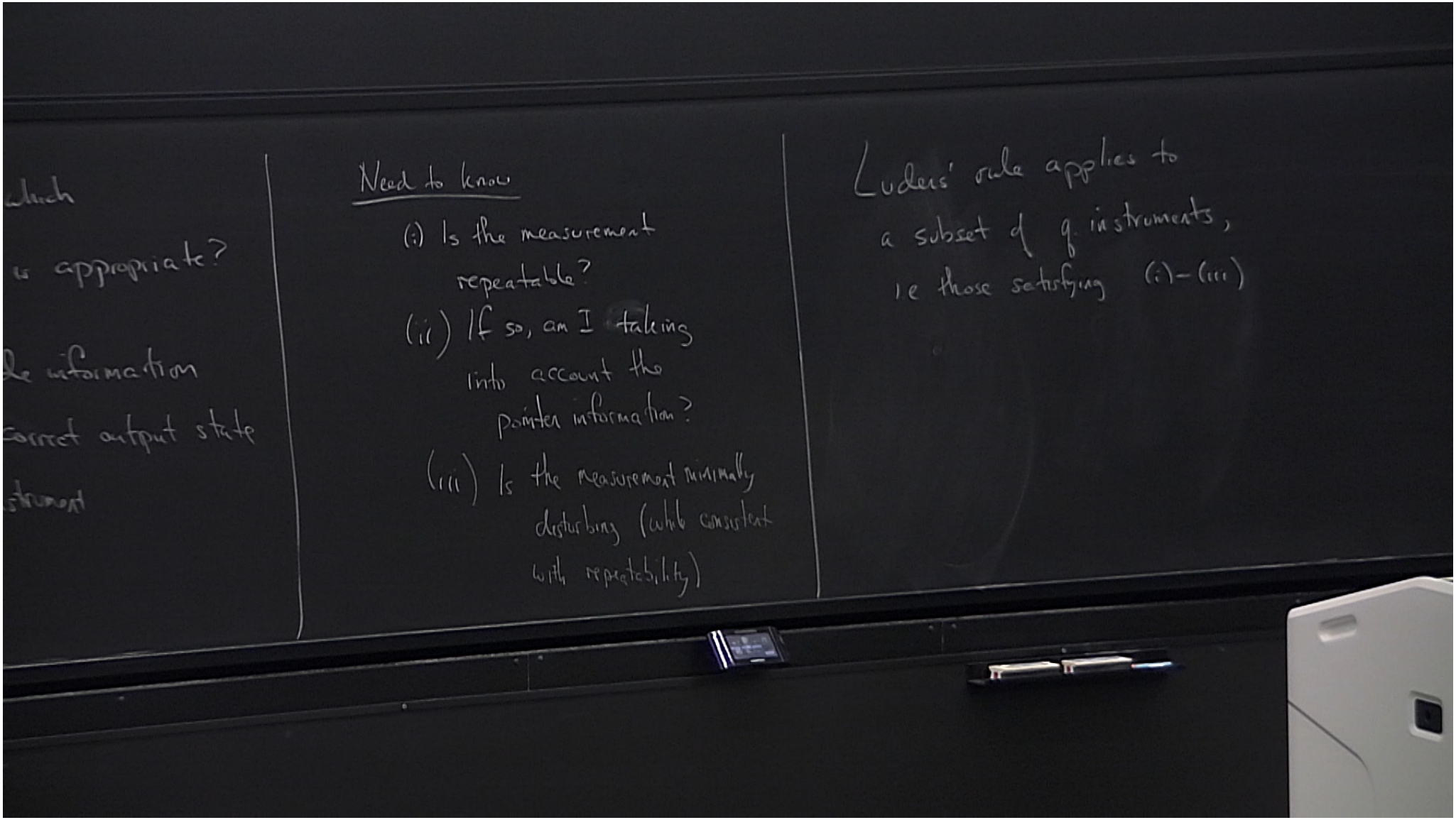

Assume by
changing
that I
Can prepare
$\rho = \{|a_1\rangle\langle a_1|, |a_2\rangle\langle a_2|, |a_3\rangle\langle a_3|,$
$|a_4\rangle\langle a_4|, |a_5\rangle\langle a_5|, \ldots$
eg $|a_4\rangle = \frac{1}{\sqrt{2}}(|a_2\rangle + |a_3\rangle)$, etc
ρ


Assume by
chasing
that I
can prepare
$\rho = \{|a_1\rangle\langle a_1|, |a_2\rangle\langle a_2|, |a_3\rangle\langle a_3|, |a_4\rangle\langle a_4|, |a_5\rangle\langle a_5|, \ldots\}$
eg $|a_1\rangle = \frac{1}{\sqrt{2}}(|a_2\rangle + |a_3\rangle)$, etc
ρ

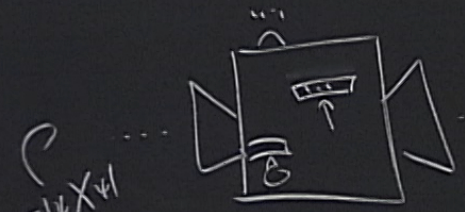

"arbitrary labels"
Assume by
chasing
that I
Can prepare
$\rho = \{|a_1\rangle\langle a_1|, |a_2\rangle\langle a_2|, |a_3\rangle\langle a_3|,$
$|a_4\rangle\langle a_4|, |a_5\rangle\langle a_5|, \ldots$
eg $|a_1\rangle = \frac{1}{\sqrt{2}}(|a_2\rangle + |a_3\rangle)$, etc
ρ


• We need other information (laws for fundamental or engineered forces in the scenario of interest)
to supplement our 3 postulates
to describe physical situations of interest
• Consider state preparation
Postulate tells us $|\psi\rangle \in \mathcal{H}$ is sufficient to describe the procedure/settings,
that I prepare
$\{|a_1\rangle\langle a_1|, |a_2\rangle\langle a_2|, |a_3\rangle\langle a_3|, |a_4\rangle\langle a_4|, |a_5\rangle\langle a_5|, \ldots$
$= \frac{1}{\sqrt{2}}(|a_2\rangle + |a_3\rangle)$, etc


Need to know
(i) Is the measurement repeatable?
(ii) If so, am I taking into account the pointer information?
(iii) Is the measurement minimally disturbing (while consistent with repeatability)
Luders' rule applies to a subset of q. instruments, ie those satisfying (i)–(iii)


which
is appropriate?
the information
correct output state
strument
Need to know
(i) Is the measurement repeatable?
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Luders' rule applies to a subset of q. instruments, ie those satisfying (i)-(iii)

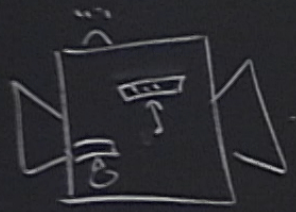
Conditional/post-selected scenario



$\rho = |+\rangle\langle+|$ $\quad\quad$ $\rho' = |x'\rangle\langle x'|$

If measurement repeatable, (i) & (ii),

Conditional/post-selected scenario



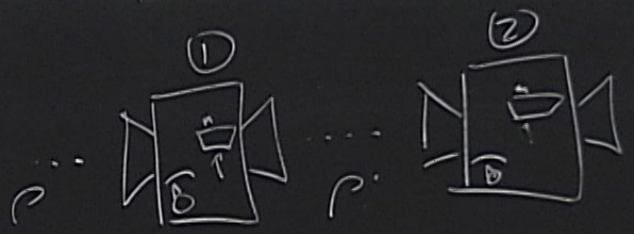
$\rho' = |x\rangle\langle x|$

If measurement repeatable & post-selected } (i) & (ii)

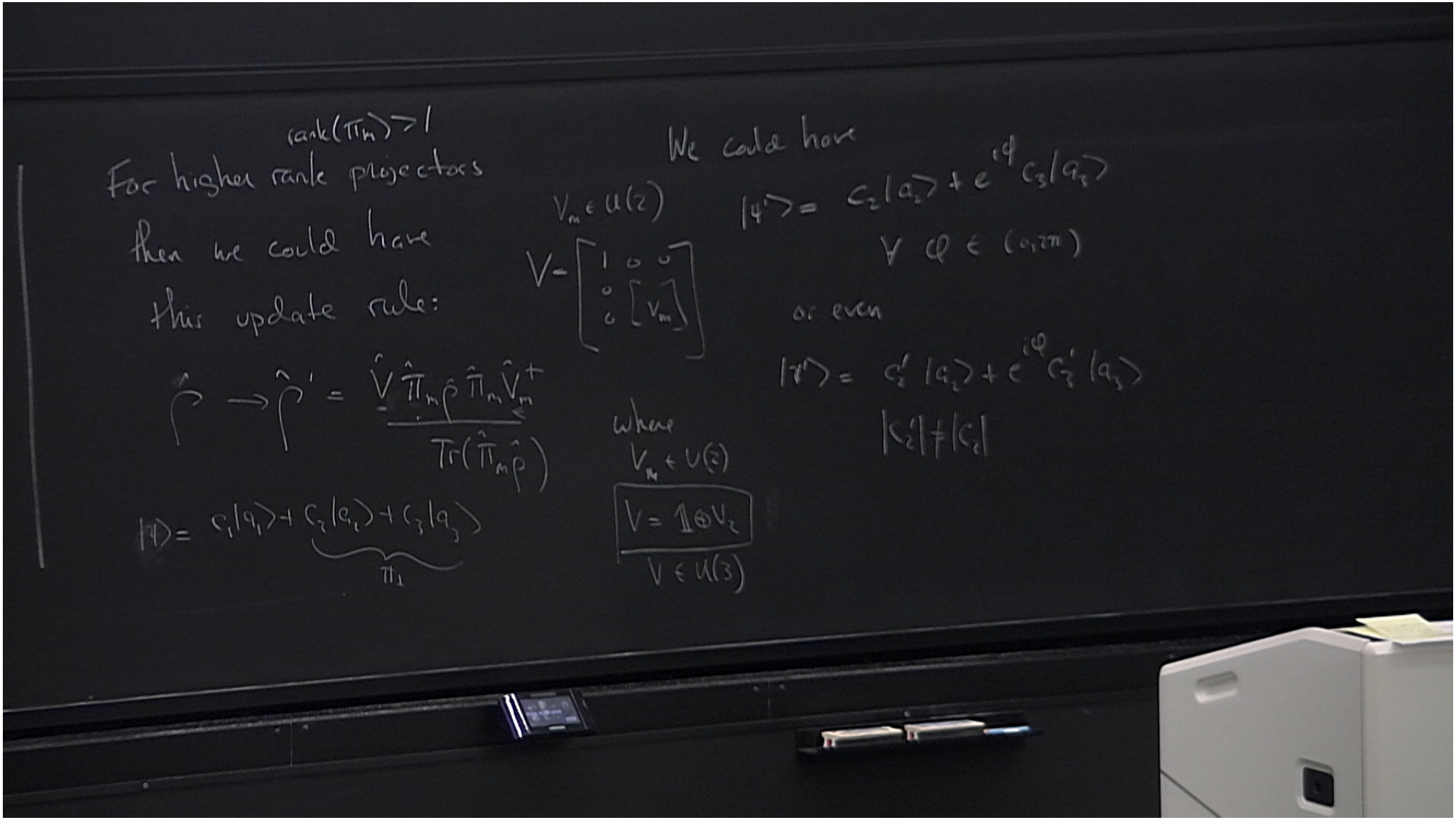
then given outcome "m" $\rho' = \rho_m = \frac{\Pi_m \rho \Pi_m}{Tr(\rho \Pi_m)}$

this must be true because repeatability encodes the assument that if ① and get "m" as classical output & then ② I get outcome "

be true because
ability encodes the
ument that if ①
get "m" as classical
output & then ②
I get outcome "m" with Prob = 1



$\Rightarrow \rho' \in V_m$ = subspace of $\mathcal{H}$
with eigenvalue 1
under π_m
$= \text{Image}(\pi_m)$


rank(Π_m) > 1
For higher rank projectors
then we could have
this update rule:
ρ̂ → ρ̂' = V̂ Π̂_m ρ̂ Π̂_m V̂_m† / Tr(Π̂_m ρ̂)
|ψ⟩ = c_1|q_1⟩ + c_2|q_2⟩ + c_3|q_3⟩
Π_1
We could have
V_m ∈ U(2)
V = [1 0 0; 0 0; 0 [V_m]]
where
V_m ∈ U(2)
V = 𝟙 ⊕ V_2
V ∈ U(3)
|ψ'⟩ = c_2|q_2⟩ + e^{iφ} c_3|q_3⟩
∀ φ ∈ (0, 2π)
or even
|ψ'⟩ = c_2'|q_2⟩ + e^{iφ} c_3'|q_3⟩
|c_2'| ≠ |c_2|

$$|\psi'\rangle\langle\psi'| = \frac{V_m \Pi_m |\psi\rangle\langle\psi| \Pi_m V_m^\dagger}{\mathrm{Tr}(\Pi_m \rho)}$$

$$|\tilde{\psi}\rangle = \Pi_m |\psi\rangle \quad \forall\, |\psi\rangle$$

Want to demand that the q instrument satisfies

$\forall\, |\psi\rangle$, $\max_{\{V_m\}} |\langle \psi' | \tilde{\psi} \rangle| \Rightarrow V_m = \frac{\mathbb{1}}{\sqrt{k}}$ $\forall\, m$.

$k = \mathrm{rank}(\Pi_m)$

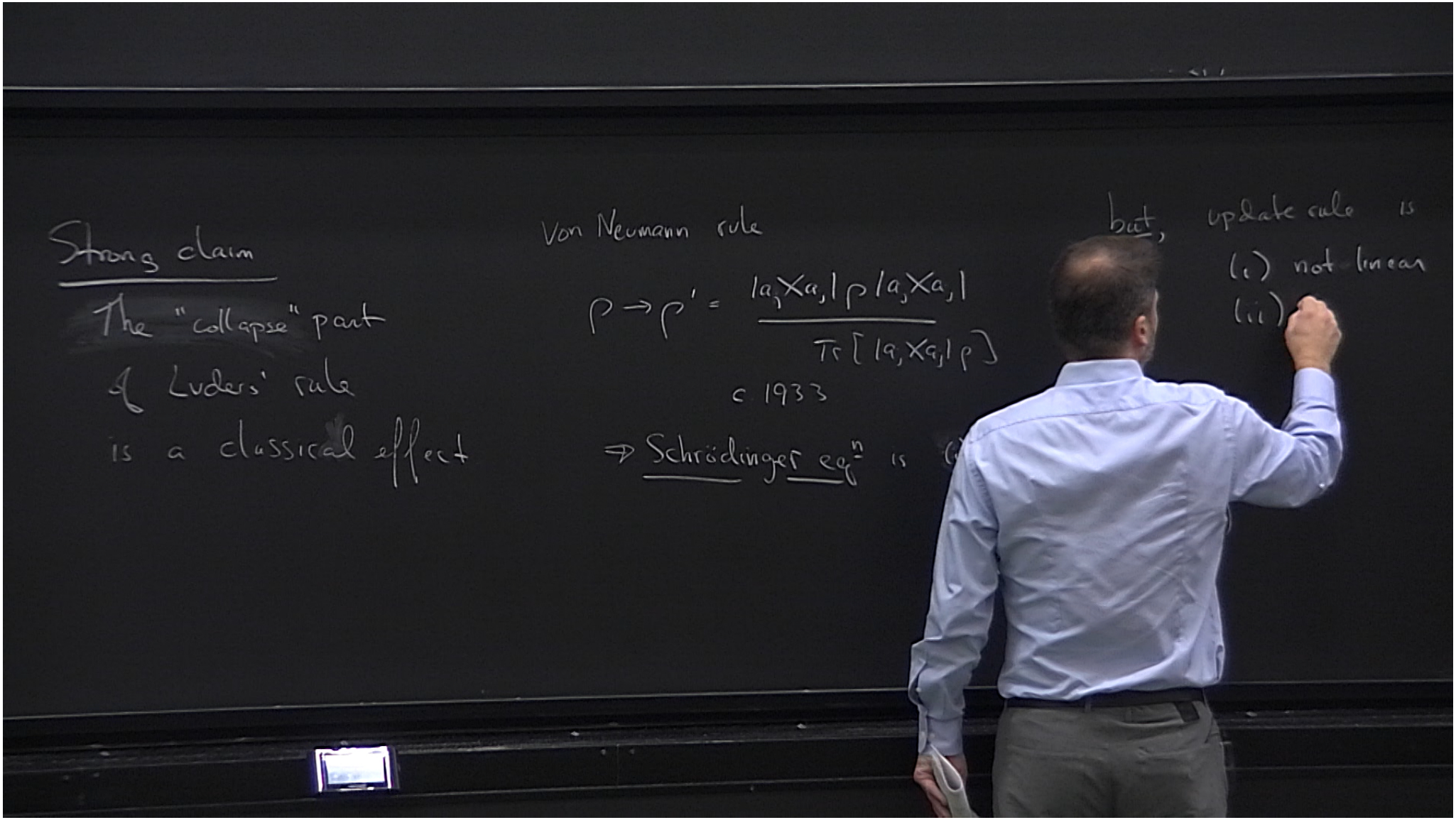
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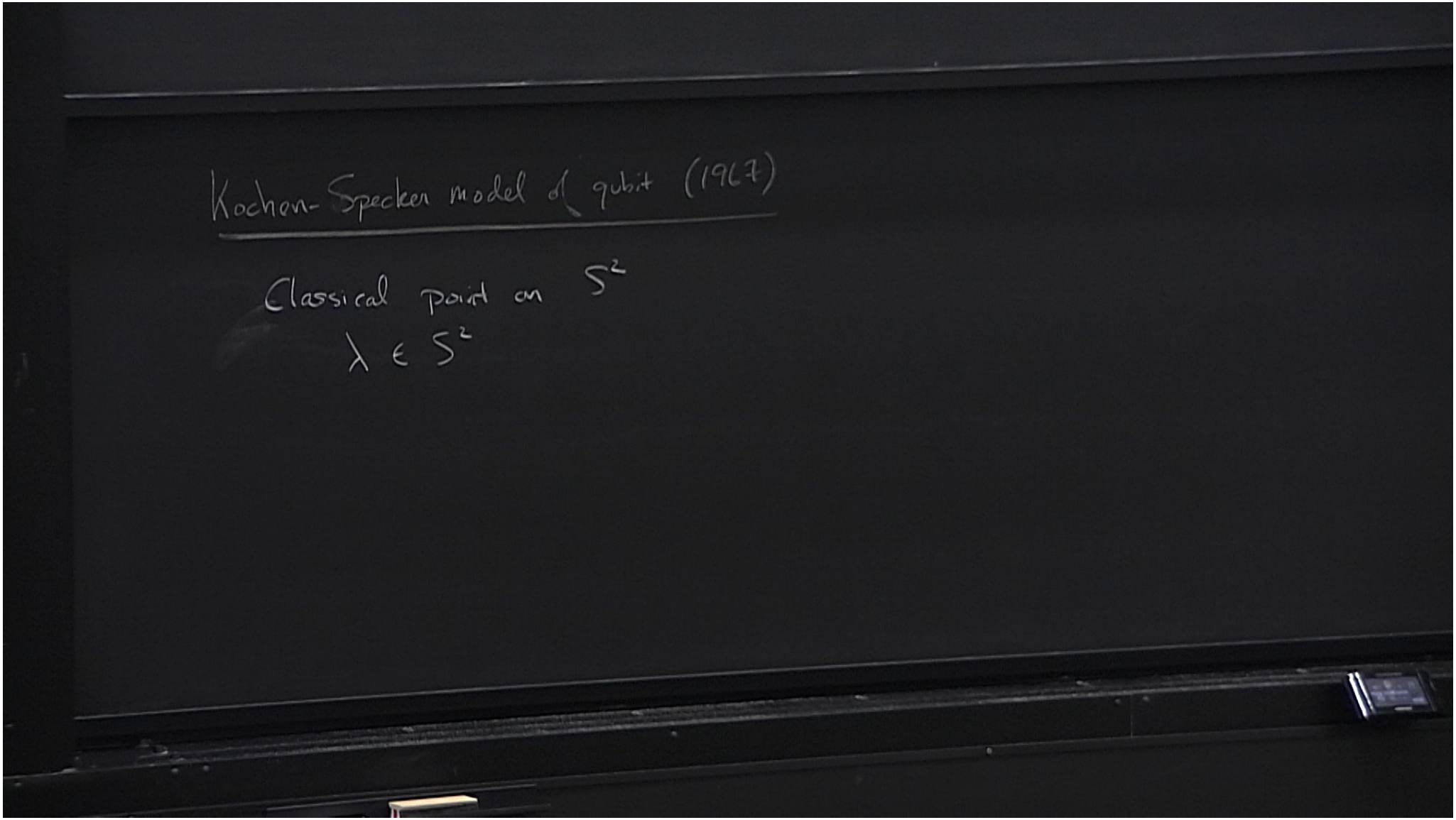
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$k = \mathrm{rank}(\Pi_m)$


Strong claim
The "collapse" part
of Lüders' rule
is a classical effect
Von Neumann rule
$\rho \to \rho' = \frac{|a_i\rangle\langle a_i| \rho |a_i\rangle\langle a_i|}{\mathrm{Tr}[|a_i\rangle\langle a_i| \rho]}$
c 1933
⇒ Schrödinger eqⁿ is
but, update rule is
(i) not linear
(ii)


Kochen-Specker model of qubit (1967)
Classical point on S^2
$\lambda \in S^2$

Kochen-Specker model of qubit (1967)

Classical point on S^2

$\vec{\lambda} \in S^2$, $\lambda = \lambda(\theta, \varphi)$

↑ physical state

Give me a preparation procedure $+\hat{z}$, $\mu_{+\hat{z}}(\lambda)\,d\lambda$

$= \Theta(\vec{\lambda} \cdot \hat{e}_z)$



$\mu_{+\hat{z}}(\lambda)\,d\lambda = \cos(\theta)\,d\lambda$ for $\theta \in [0, \frac{\pi}{2})$

$= 0$ for $\theta \in [\frac{\pi}{2}, \pi)$

Along direction $\vec{v}$, prepare $\mu_{+\hat{z}}$, $v = v(\theta', \varphi')$

& rotate sphere $R_z(\alpha) R_y(\beta) R_z(\gamma)$

$\rightarrow \mu_{\vec{v}}(\lambda)\,d\lambda$

Using Euler angles,

$\beta = \theta'$, $\alpha = \varphi'$

Kochen-Specker model of qubit (1967)

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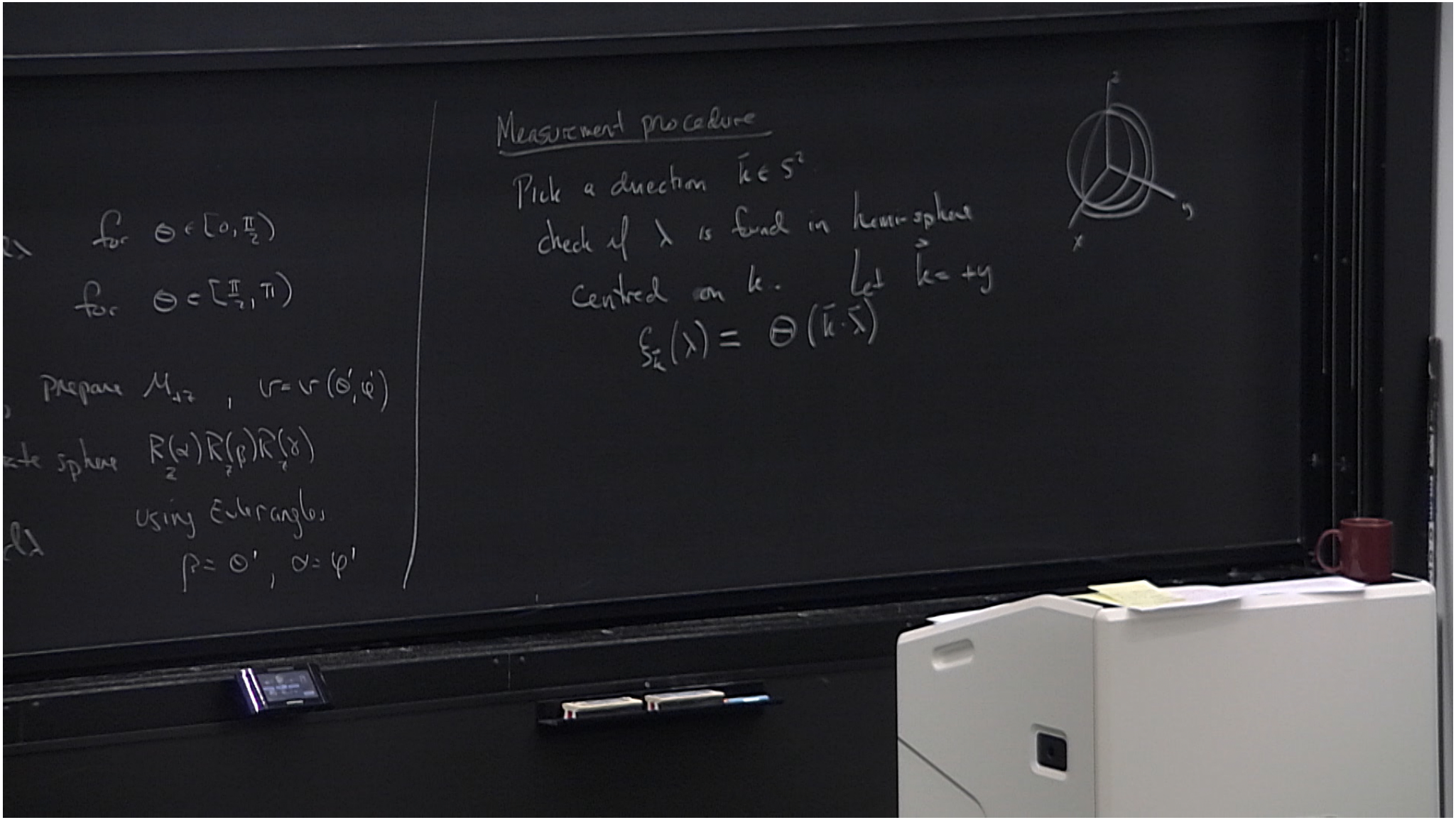
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$\Rightarrow \mu_{\vec{v}}(\lambda)\,d\lambda$

using Euler angles

$\beta = \theta'$, $\alpha = \varphi'$


for $\theta \in [0, \frac{\pi}{2})$
for $\theta \in [\frac{\pi}{2}, \pi)$
Prepare $M_{\pm z}$, $\vec{v} = \vec{v}(\theta', \varphi')$
$R_z(\alpha) R_y(\beta) R_z(\gamma)$
using Euler angles
$\beta = \theta'$, $\alpha = \varphi'$
Measurement procedure
Pick a direction $\vec{k} \in S^2$
check if λ is found in hemi-sphere
centred on k. Let $\vec{k} = +y$
$\xi_{\vec{k}}(\lambda) = \Theta(\vec{k} \cdot \vec{\lambda})$
z
y
x

chon-Specker model of qubit (1967)

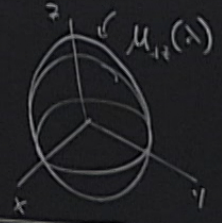
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Give me a preparation procedure

$+\hat{z}$: $\boxed{\mu_{+\hat{z}}(\lambda)\,d\lambda = \Theta(\vec{\lambda}\cdot\vec{e}_{+\hat{z}})\,\lambda\cdot\vec{e}_{+\hat{z}}}$



$\mu_{+\hat{z}}(\lambda)\,d\lambda = \cos(\theta)\,d\lambda$ for $\theta \in [0, \frac{\pi}{2})$

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Along direction $\vec{v}$, prepare $\mu_{+\hat{z}}$, $\vec{v} = \vec{v}(\theta', \varphi')$

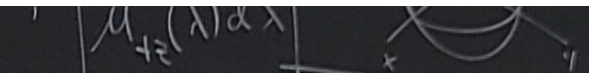
& rotate sphere $R_z(\alpha) R_y(\beta) R_z(\gamma)$

$\Rightarrow \mu_{\vec{v}}(\lambda)\,d\lambda$

using Euler angles

$\beta = \theta'$, $\alpha = \varphi'$

Measu

$\mu_{+\hat{z}}(\lambda)d\lambda$  $\Rightarrow \mu_{\vec{c}}(\lambda)d\lambda$

$= \Theta(\vec{\lambda}\cdot\vec{c}_{+\hat{z}})\,\vec{\lambda}\cdot\vec{e}_{+\hat{z}}$

$\theta = \theta'$, $\alpha = \varphi'$

Prepare a q state $H = \mathbb{C}^2$

along $\vec{c}$

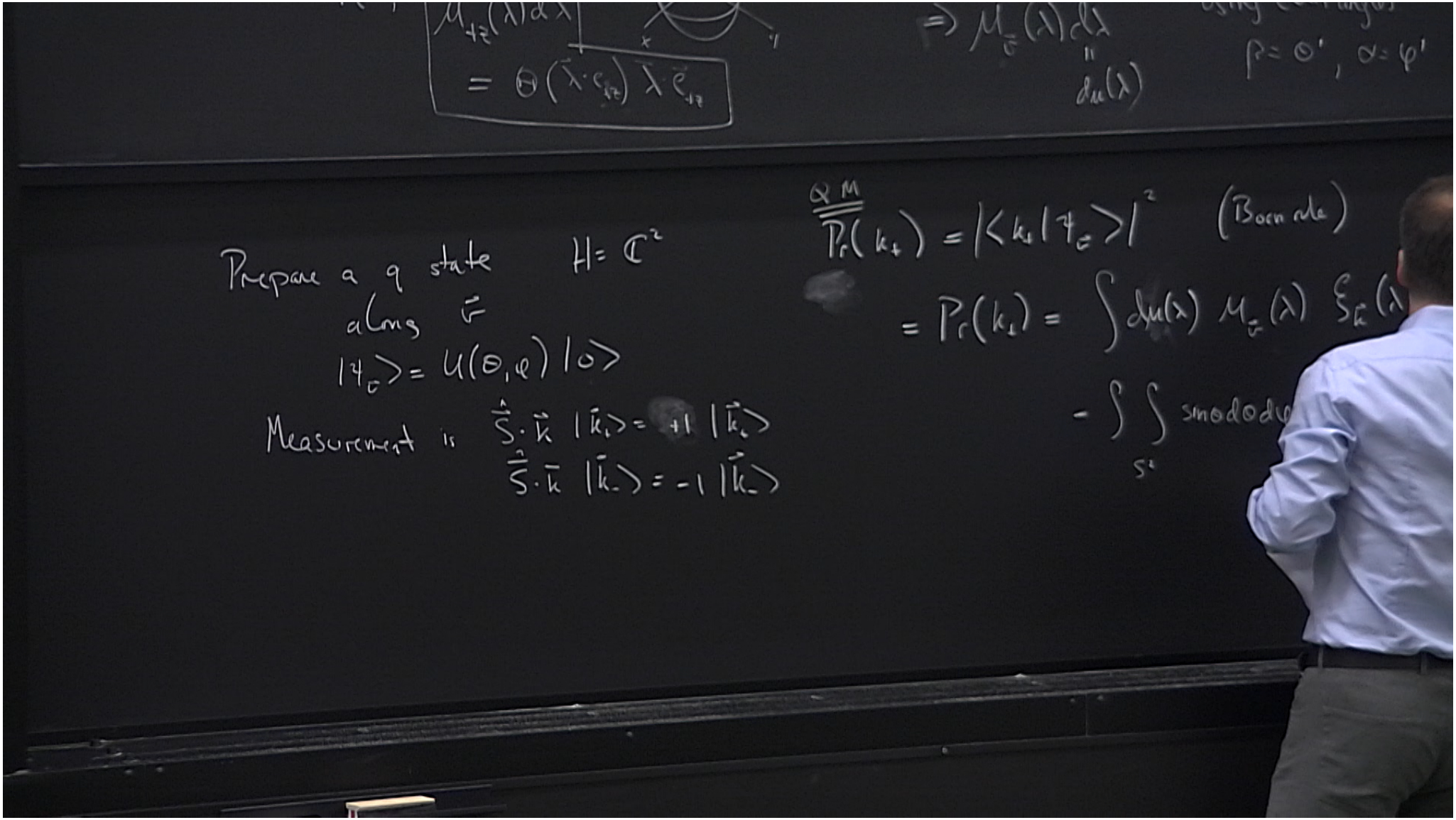
$|\psi_{\vec{c}}\rangle = U(\theta,\varphi)|0\rangle$

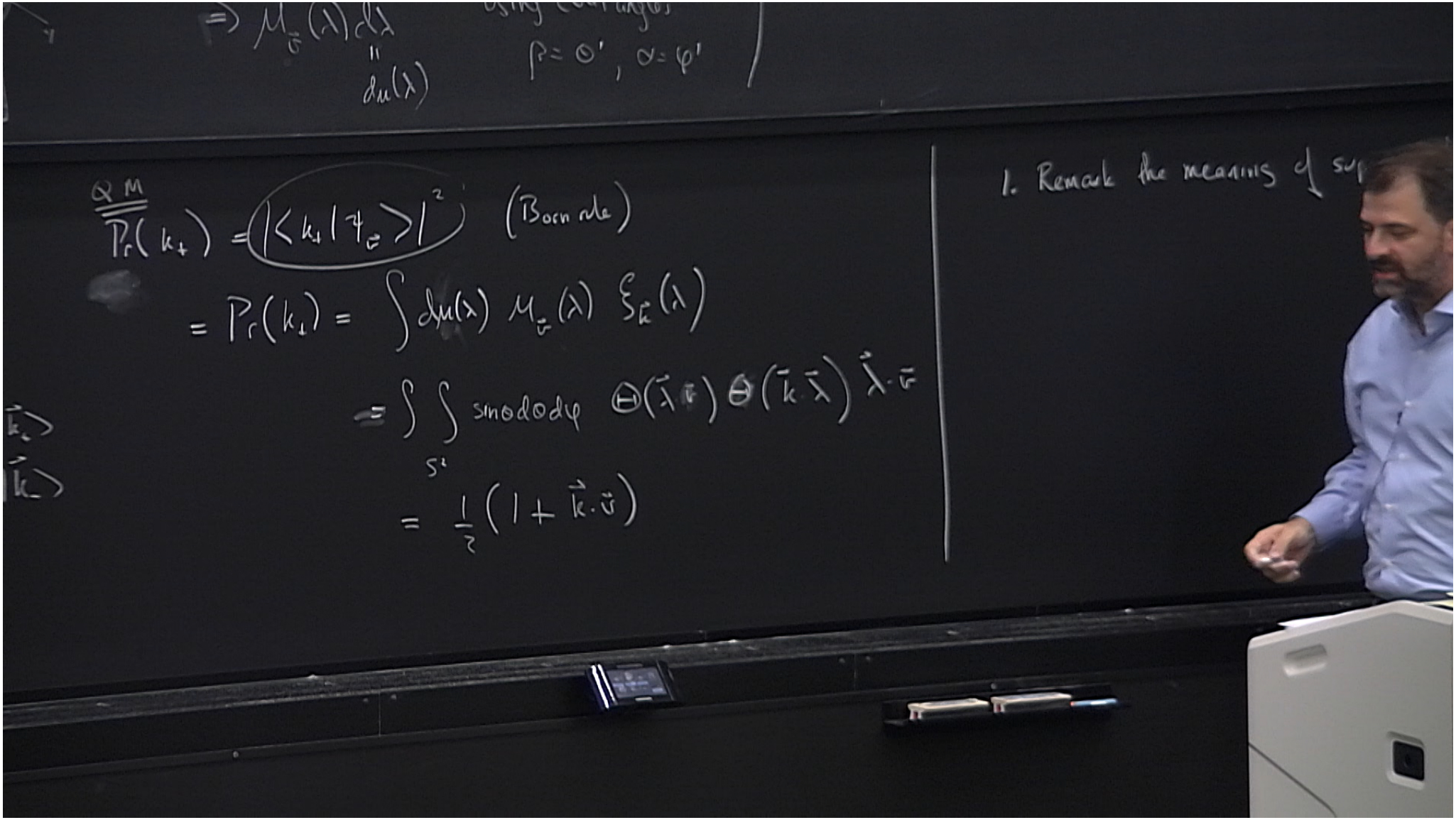
Measurement is $\hat{\vec{S}}\cdot\vec{k}\,|\vec{k}_+\rangle = +1\,|\vec{k}_+\rangle$

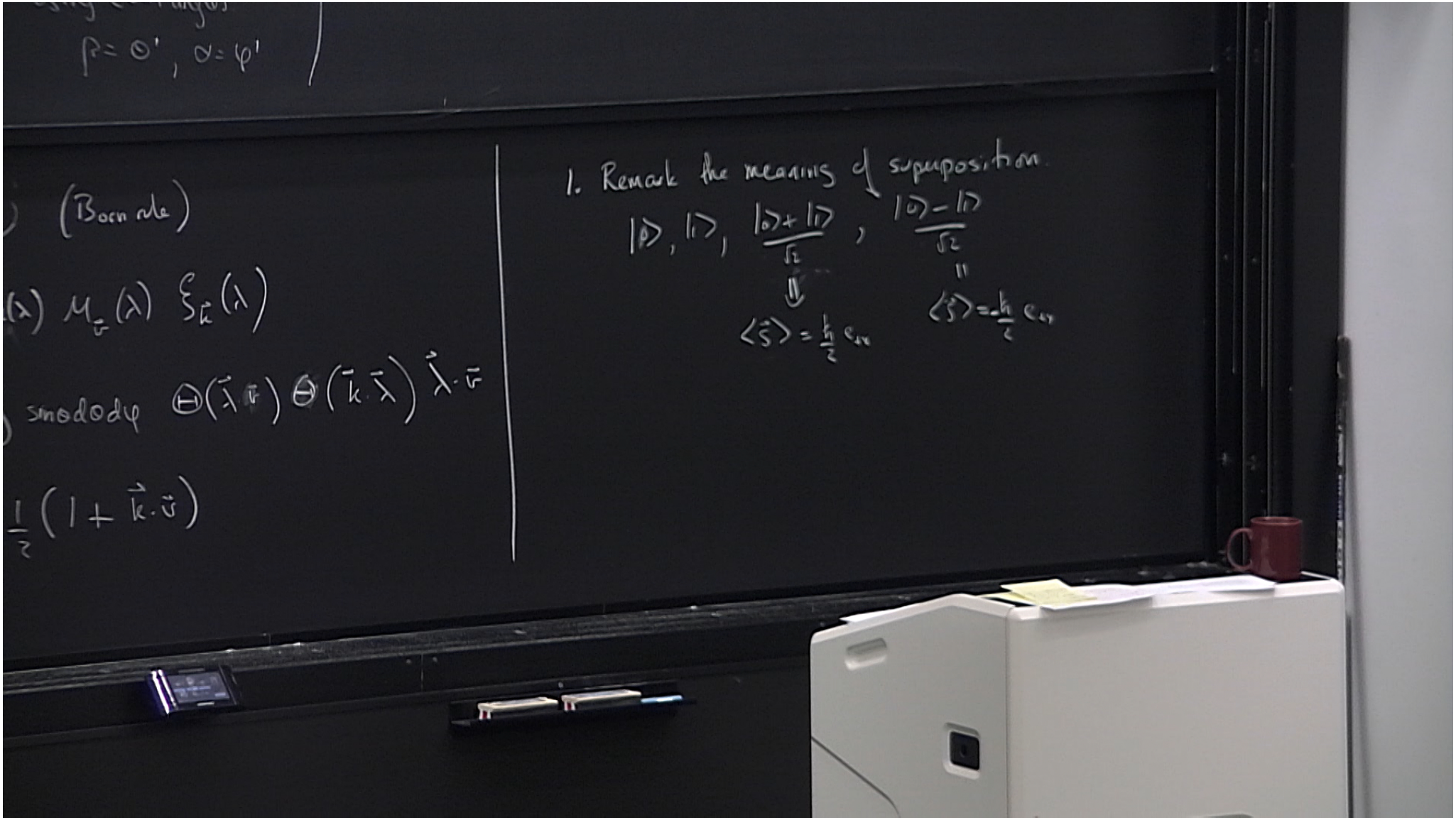
$\hat{\vec{S}}\cdot\vec{k}\,|\vec{k}_-\rangle = -1\,|\vec{k}_-\rangle$

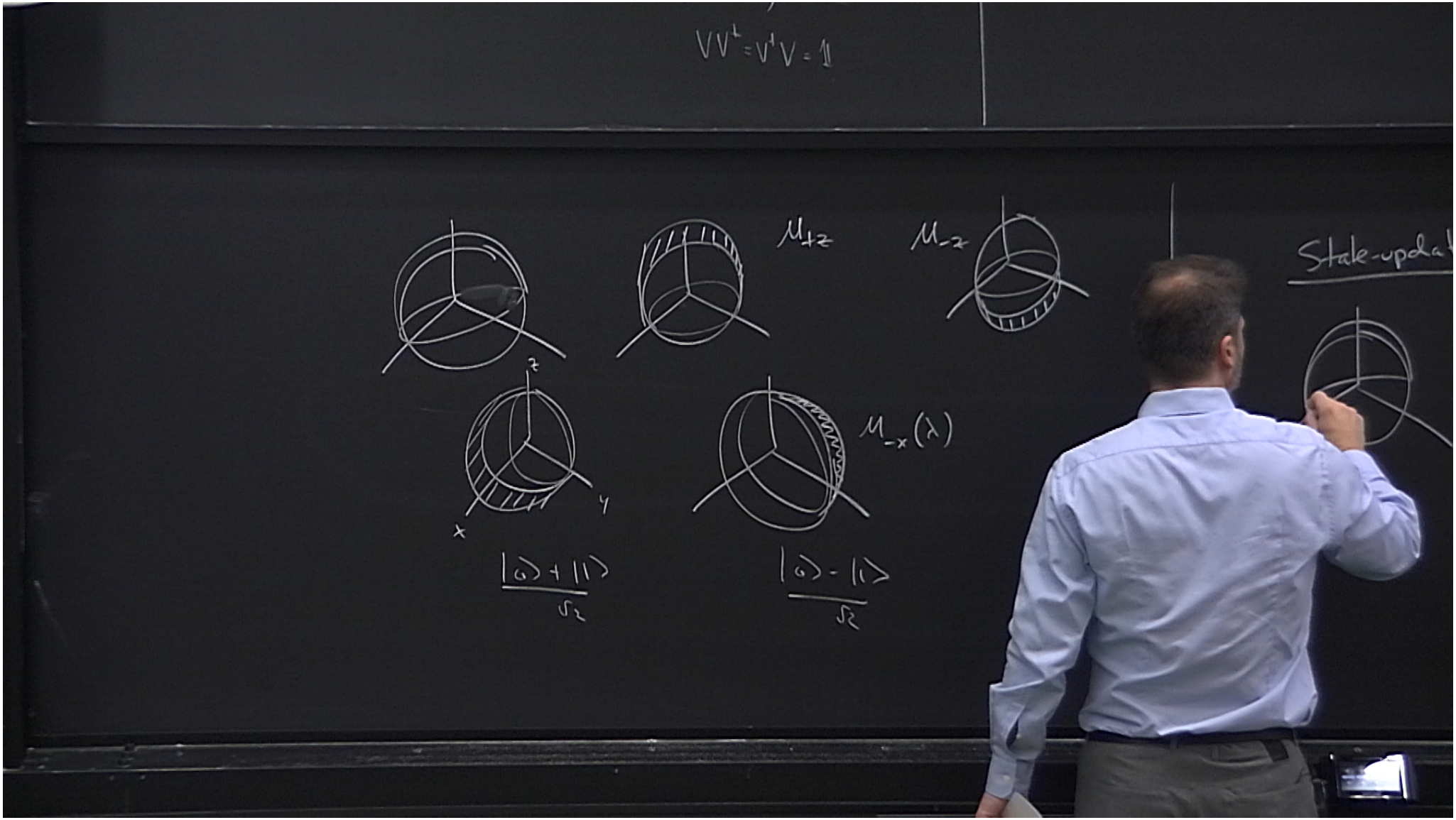
$\overset{QM}{=\!=\!=} P_r(k_+) = |\langle k_+|\psi_{\vec{c}}\rangle|^2$ (Born rule)

$= P_r($


$\mu_{+\hat{z}}(\lambda) d\lambda$
$= \Theta(\vec{\lambda}\cdot\vec{e}_{+\hat{z}})\,\vec{\lambda}\cdot\vec{e}_{+\hat{z}}$
$\Rightarrow \mu_{\vec{c}}(\lambda) d\lambda$
$= d\mu(\lambda)$
$\beta = \theta'$, $\alpha = \varphi'$
Prepare a q state $H = \mathbb{C}^2$
along $\vec{c}$
$|\psi_{\vec{c}}\rangle = U(\theta,\varphi)|0\rangle$
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QM
$P_r(k_+) = |\langle k_+|\psi_{\vec{c}}\rangle|^2$ (Born rule)
$= P_r(k_+) = \int d\mu(\lambda)\,\mu_{\vec{c}}(\lambda)\,\xi_{\vec{k}}(\lambda$
$- \int_{S^2}\int$ sinθ d

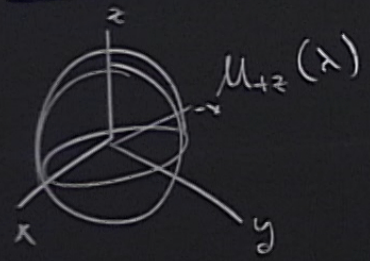

$\Rightarrow \mu_{\vec{v}}(\lambda) d\lambda$
$= d\mu(\lambda)$
$\theta = \theta'$, $\alpha = \varphi'$
QM
$P_r(k_+) = |\langle k_+ | \eta_{\vec{v}} \rangle|^2$ (Born rule)
$= P_r(k_+) = \int d\mu(\lambda)\, \mu_{\vec{v}}(\lambda)\, \xi_{\vec{k}}(\lambda)$
$= \int_{S^2} \sin\theta\, d\theta\, d\varphi\, \Theta(\vec{\lambda}\cdot\vec{v})\, \Theta(\vec{k}\cdot\vec{\lambda})\, \vec{\lambda}\cdot\vec{v}$
$= \frac{1}{2}(1 + \vec{k}\cdot\vec{v})$
1. Remark the meaning of


β = Θ' , α = φ'
(Born rule)
smodody
1. Remark the meaning of superposition
|0⟩, |1⟩, (|0⟩+|1⟩)/√2 , (|0⟩−|1⟩)/√2
⟨S⟩ = (ħ/2) e_x
⟨S⟩ = −(ħ/2) e_x


$VV^\dagger = V^\dagger V = \mathbb{1}$
$\mathcal{U}_{+z}$
$\mathcal{U}_{-z}$
State-upda
z
y
x
$\mathcal{U}_{-x}(\lambda)$
$\frac{|\Omega+\rangle}{\sqrt{2}}$
$\frac{|\Omega-\rangle}{\sqrt{2}}$

State-update

Measure along x-axis



"Dynamics under rotation are (i) linear (ii) deterministic (iii) continuous."

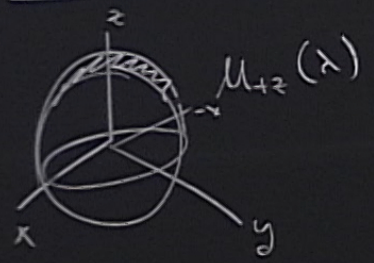
Liouville eqn

$$\frac{d\rho}{dt} = \{H, \rho\} = \frac{\partial H}{\partial q}\frac{\partial \rho}{\partial p} - \frac{\partial \rho}{\partial q}\frac{\partial H}{\partial p}$$

ρ = prob density

(i) linear
(ii) deterministic

State-update



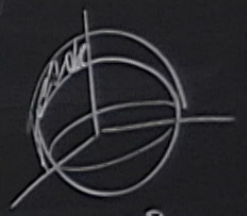
Measure along x-axis

Output/outcome "+x"

How do I update my state of knowledge classically?

$$\mu'(\lambda) \propto \mu_{+z} \cap \xi_{+x}$$

(i) not linear (renormalization)
(ii) non-deterministic
(iii) discontinuous



OR

If -x outcome



"Dynamics under rotation" are (i) linear (ii) deterministic (iii) continuous "

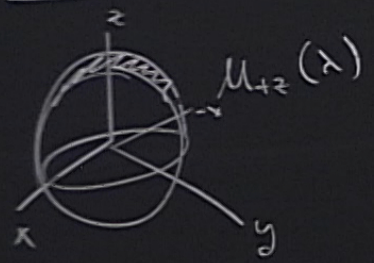
Liouville eq^n

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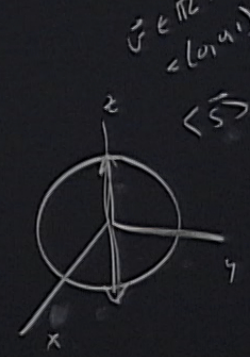
ρ = prob density

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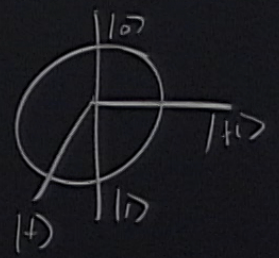
$$\langle \vec{S} \rangle \equiv \langle \psi | \vec{S} | \psi \rangle = \left(\langle \psi | \hat{S}_x | \psi \rangle, \langle \psi | \hat{S}_y | \psi \rangle, \langle \psi | \hat{S}_z | \psi \rangle \right)$$

Rosetta Stone

$\|\psi\rangle \in \mathbb{C}^2$	$\langle \vec{S} \rangle \in S^2 \subset \mathbb{R}^3$
$\|\psi\rangle_z = \|0\rangle$	$\langle \vec{S} \rangle = (0,0,1)\frac{\hbar}{2}$
$\|\psi\rangle_z = \|1\rangle$	$\langle \vec{S} \rangle = (0,0,-1)\frac{\hbar}{2}$
$\|\psi\rangle_x = \frac{\|0\rangle + \|1\rangle}{\sqrt{2}} \equiv \|+\rangle$	$\langle \vec{S} \rangle = (1,0,0)\frac{\hbar}{2}$
$\|\psi\rangle_x = \frac{\|0\rangle - \|1\rangle}{\sqrt{2}} \equiv \|-\rangle$	$\langle \vec{S} \rangle = (-1,0,0)\frac{\hbar}{2}$
$\|\psi\rangle_y = \frac{\|0\rangle + i\|1\rangle}{\sqrt{2}}$	$\langle \vec{S} \rangle = (0,1,0)\frac{\hbar}{2}$



Remark Bloch sphere



$\langle S \rangle \equiv \langle \psi | \hat{S} | \psi \rangle = ($

R

$|\psi\rangle \in$

$|+\hat{z}\rangle = |0\rangle$

$|-\hat{z}\rangle = |1\rangle$

$|+\hat{x}\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$|-\hat{x}\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$|+\hat{y}\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}}$

$\vec{\lambda} \propto \langle \vec{S} \rangle_{classical}$

$= \int d\lambda \; \vec{\lambda} \; \mu_{|z}(\lambda)$