Title: PSI 17/18 - Relativity - Lecture 13

Date: Sep 21, 2017 09:00 AM

URL: http://pirsa.org/17090032

Abstract:

Black holes !

Einstein equation with $T^{\mu\nu} = 0 \implies$ g-waves $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow R_{\mu\nu} \approx -\frac{1}{2}\Box h_{\mu\nu} = 0$

Today we will solve $R_{\mu\nu} = 0$ nonlinearly and with a different symmetry

last time $h \sim e^{-ikt+ikz}$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow R_{\mu\nu} \approx -\frac{1}{2}\Box h_{\mu\nu} = 0$

different symmetry - spherical symmetry (looking for a blob).

last time $h \sim e^{-ikt+ikz}$

- waves $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow R_{\mu\nu} \approx -\frac{1}{2}\Box h_{\mu\nu} = 0$

Karl Schwarz

with a different symmetry - spherical symmetry (looking for a blob).

coords, t, $\underline{x}$: $\underline{x} \to x_i' = O_{ij} x_j$ O rotation matrix.

want line element to be invariant under rotations $SO(3)$.

Black holes!

Einstein equation with $T^{\mu\nu} = 0 \Rightarrow$ g-waves $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow$

Today we will solve $R_{\mu\nu} = 0$ nonlinearly and with a different symmetry.

line element $g^{(x)}_{\mu\nu} dx^\mu dx^\nu$: have four coords, t, $\underline{x}$: want line

$\Rightarrow$ can write $dt, t, r^2 = \underline{x}^2, r, r dr = \underline{x} \cdot d\underline{x} \Rightarrow dr, d\underline{x}^2 =$

$\Rightarrow$ most general sph. symm. line element is

(convenience)

$$ds^2 = -C(r,t)dt^2 + E(r,t)dr^2 - 2D(r,t)dtdr$$

$$= (dt, dr, d\theta, d\varphi) \begin{pmatrix} -C & -D & 0 & 0 \\ -D & +E & 0 & 0 \\ 0 & 0 & Fr^2 & 0 \\ 0 & 0 & 0 & Fr^2 \sin^2\theta \end{pmatrix} \begin{pmatrix} dt \\ dr \\ d\theta \\ d\varphi \end{pmatrix}$$

$$dr, \quad d\underline{x}^2 = dr^2 + r^2\overbrace{(d\theta^2 + \sin^2\theta\, d\varphi^2)}^{d\Omega_2^2}$$

↑ spherical polars

$$x = r\sin\theta\cos\varphi$$
$$y = r\sin\theta\sin\varphi$$
$$z = r\cos\theta.$$

$$\ldots D(r,t)\,dt\,dr + F(r,t)\,r^2 d\Omega_2^2$$

must have $\det g \neq 0$

one negative, three positive evalues.

$$\begin{pmatrix} dt \\ dr \\ d\theta \\ d\varphi \end{pmatrix}$$

$\det g \neq 0$ must have

one negative, three positive evalues

$+ F(r,t)\, r^2 d\Omega_2^2$

r^2

$r' \equiv \sqrt{F(r,t)}\, r$

$\Rightarrow t' = t$

$dr' = \# dr + \# dt$

$dt' = dt$

$dr = \# dr' + \# dt'$

etc

$$ds^2 = -C'(r';t')\,dt'^2 + E'(r';t')\,dr'^2 - 2D'(r';t')\,dt'dr' + r'^2 d\Omega_2^2$$

area of orbits of SO(3) is $4\pi r^2$

$-2D(r,t)\,dt\,dr + r^2 d\Omega_2^2$

$D(r,t)$ by a coordinate transformation?

$$C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dt\,dr + r^2 d\Omega_2^2$$

(area of orbits of)

Q: can I remove $D(r,t)$ by a coordinate transformation?

define t' by $dt' = f(r,t)\,(C(r,t)\,dt + D(r,t)\,dr)$ (arbitrary fn.)

$$ds^2 = -\frac{dt'^2}{f^2(r,t)\,C(r,t)} + \left(\frac{D^2(r,t)}{C(r,t)} + E(r,t)\right)dr^2 + r^2 d\Omega_2^2$$

$$= -C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dt\,dr + r^2 d\Omega_2^2$$

← area of a

rep primes. ; Q: can I remove $D(r,t)$ by a coordinate transform

let's define t' by $dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right)$ ← arb fn

$$ds^2 = -\frac{dt'^2}{f^2(r,t)\,C(r,t)} + \left(\frac{D^2(r,t)}{C(r,t)} + E(r,t)\right)dr^2 + r^2 d\Omega_2^2$$

$$= -A(r,t)\,dt'^2$$

arb fn of $(r,t) = B(r,t)$

$D(r,t')\,dt'dr' + r'^2 d\Omega_2^2$ ← area of orbits of SO(3) is $4\pi r^2$

(r,t) by a coordinate transformation?

← arbitrary fn. – integrating factor

$(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right)$ → possible?

i.e. is there a function $t'(r,t)$ s.t. $dt' \propto C dt + D dr$?

$\left(\frac{D(r,t)}{C(r,t)} + E(r,t)\right)dr^2 + r^2 d\Omega_2^2$

arbitrary fn. of $(r,t) = B(r,t)$

If t' exists, then $dt' = \frac{\partial t'}{\partial t} dt + \frac{\partial t'}{\partial r} dr = f C dt + f D dr$

$$\Rightarrow \frac{\partial t'}{\partial t} = f(r,t)\, C(r,t)$$

$$\frac{\partial t'}{\partial r} = f(r,t)\, D(r,t)$$

$$\frac{\partial t'}{\partial r}\,dr = f C\,dt + f D\,dr$$

$$= f(r,t)\,C(r,t)$$

$$= f(r,t)\,D(r,t)$$

for consistency, must have:

$$\frac{\partial^2 t'}{\partial r\,\partial t} = \frac{\partial^2 t'}{\partial t\,\partial r}$$

partial derivatives commute

$$\frac{\partial}{\partial r}\left(f(r,t)\,C(r,t)\right) = \frac{\partial}{\partial t}\left(f(r,t)\,D(r,t)\right)$$

$$\Rightarrow \frac{\partial t'}{\partial t} = f(r,t)\,C(r,t)$$
$$\frac{\partial t'}{\partial r} = f(r,t)\,D(r,t)$$

} for consistency, must have: $\frac{\partial^2 t'}{\partial r \partial t} = \frac{\partial^2 t'}{\partial t \partial r}$

partial derivati... commu...

$$\frac{\partial}{\partial r}\big(f(r,t)\,C(r,t)\big) = \frac{\partial}{\partial t}\big(f(r,t)\,D(r,t)\big)$$

1

$$\partial_r f D + f \partial_r D = \partial_t f C + f \partial_t C$$

Can we always s...

yes, at least locally : imagine fixing $f(r,0)$

$C(r,t)$

$D(r,t)$

for consistency, must have: $\frac{\partial^2 t'}{\partial r \partial t} = \frac{\partial^2 t'}{\partial t \partial r}$

partial derivatives commute

$$\frac{\partial}{\partial r}\left(f(r,t)\,C(r,t)\right) = \frac{\partial}{\partial t}\left(f(r,t)\,D(r,t)\right)$$

$$\partial_t f D + f \partial_t D = \partial_r f C + f \partial_r C$$

Can we always solve this?

$\Rightarrow \partial_t f(r,0)$

fast locally: imagine fixing $f(r,0)$

for consistency, must have: $\frac{\partial^2 t'}{\partial r \partial t} = \frac{\partial^2 t'}{\partial t \partial r}$

partial derivatives commute

$$\frac{\partial}{\partial r}\left(f(r,t)C(r,t)\right) = \frac{\partial}{\partial t}\left(f(r,t)D(r,t)\right)$$

$\partial_r f D + f \partial_r D = \partial_t f C + f \partial_t C$ (*) Can we always solve this?

imagine fixing $f(r,0) \Rightarrow \partial_t f(r,0)$, simply $\frac{\partial}{\partial t}(*) \Rightarrow \frac{\partial^2 f}{\partial t^2}$ in terms of f

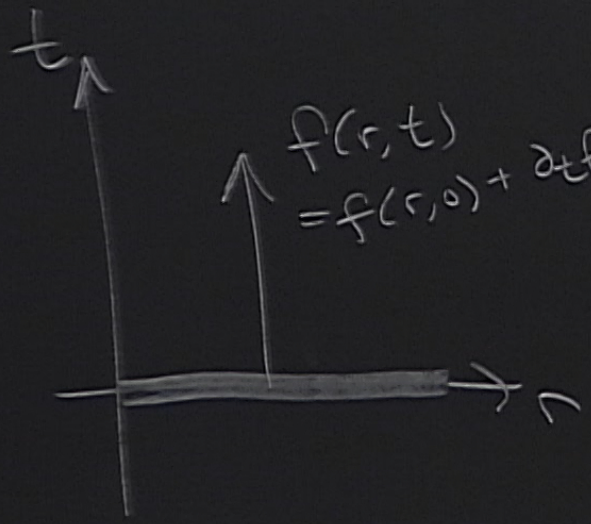
exists, then

$$\Rightarrow \frac{\partial t'}{\partial t} = f(r,t)\,C(r,t)$$
$$\frac{\partial t'}{\partial r} = f(r,t)\,D(r,t)$$

} for con

mus

$\frac{\partial}{\partial r}$



$$f(r,t) = f(r,0) + \partial_t f(r,t)\Big|_{t=0} + \frac{1}{2}\partial_t^2 f(r,t)\Big|_{t=0} \quad \ldots \text{ etc.}$$

$\partial_t f\, D + f$

yes, at least locally : imagine

drop prime on t

$\Longrightarrow$ left with $ds^2 = -A(r,t)dt^2 + B(r,t)dr^2 + r^2 d\Omega$

now, use $R_{\mu\nu} = 0$ to fix A, B. We shall prove that most genera

notation: $\partial_t A \equiv \dot{A}$, $\partial_r A = A'$ etc.

Thomas Hartle Software for book

$$\Gamma^{\mu}_{\lambda\nu} : \quad \Gamma^{t}_{tt} = \frac{\dot{A}}{2A}, \quad \Gamma^{t}_{rr} = \frac{\dot{B}}{2A}, \quad \Gamma^{t}_{tr} = \frac{A'}{2A}, \quad \Gamma^{r}_{tt} = \frac{A'}{2B}, \quad \Gamma^{r}_{tr} =$$

$\frac{1}{2} g^{\mu\alpha}\left(g_{\lambda\alpha,\nu} + g_{\nu\alpha,\lambda} - g_{\nu\lambda,\alpha}\right)$

all other components are zero.

$\ldots + B(r,t)\,dr^2 + r^2 d\Omega_2^2$

(Birkhoff's theorem)

I prove that most general sph. symm. Soln. of $R_{\mu\nu} = 0$ is static!

$\Gamma^t_{tr} = \frac{A'}{2A}$, $\Gamma^r_{tt} = \frac{A'}{2B}$, $\Gamma^r_{tr} = \frac{\dot{B}}{2B}$, $\Gamma^r_{rr} = \frac{B'}{2B}$, $\Gamma^r_{\theta\theta} = -\frac{r}{B}$, $\Gamma^r_{\phi\phi} = -\frac{r\sin^2\theta}{B}$, $\Gamma^\theta_{r\theta} = \frac{1}{r}$, $\Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta$

$\Gamma^\phi_{r\phi} = \frac{1}{r}$, $\Gamma^\phi_{\phi\theta} = \cot\theta$.

all other components are zero.

③:

$$R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB} \quad ①$$

$$R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{Br} + \frac{\ddot{B}}{2A} - \frac{\dot{A}\dot{B}}{4A^2} - \frac{\dot{B}^2}{4AB} \quad ②$$

$$R_{tr} = \frac{\dot{B}}{Br} \quad ③$$

$$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB} \quad ④$$

$$R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta} \quad ⑤$$

$$R^{\rho}_{\lambda\mu\nu} \sim \partial\Gamma + \Gamma\Gamma - (\mu \leftrightarrow \nu)$$

$$R_{\lambda\nu} = R^{\rho}_{\lambda\rho\nu} : \quad R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB} \quad (1)$$

$$R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{Br} + \frac{\ddot{B}}{2A} - \frac{\dot{A}\dot{B}}{4A^2} - \frac{\dot{B}^2}{4AB} \quad (2)$$

$$R_{tr} = \frac{\dot{B}}{Br} \quad (3)$$

$$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB} \quad (4)$$

$$R_{\varphi\varphi} = \sin^2\theta\, R_{\theta\theta} \quad (5)$$

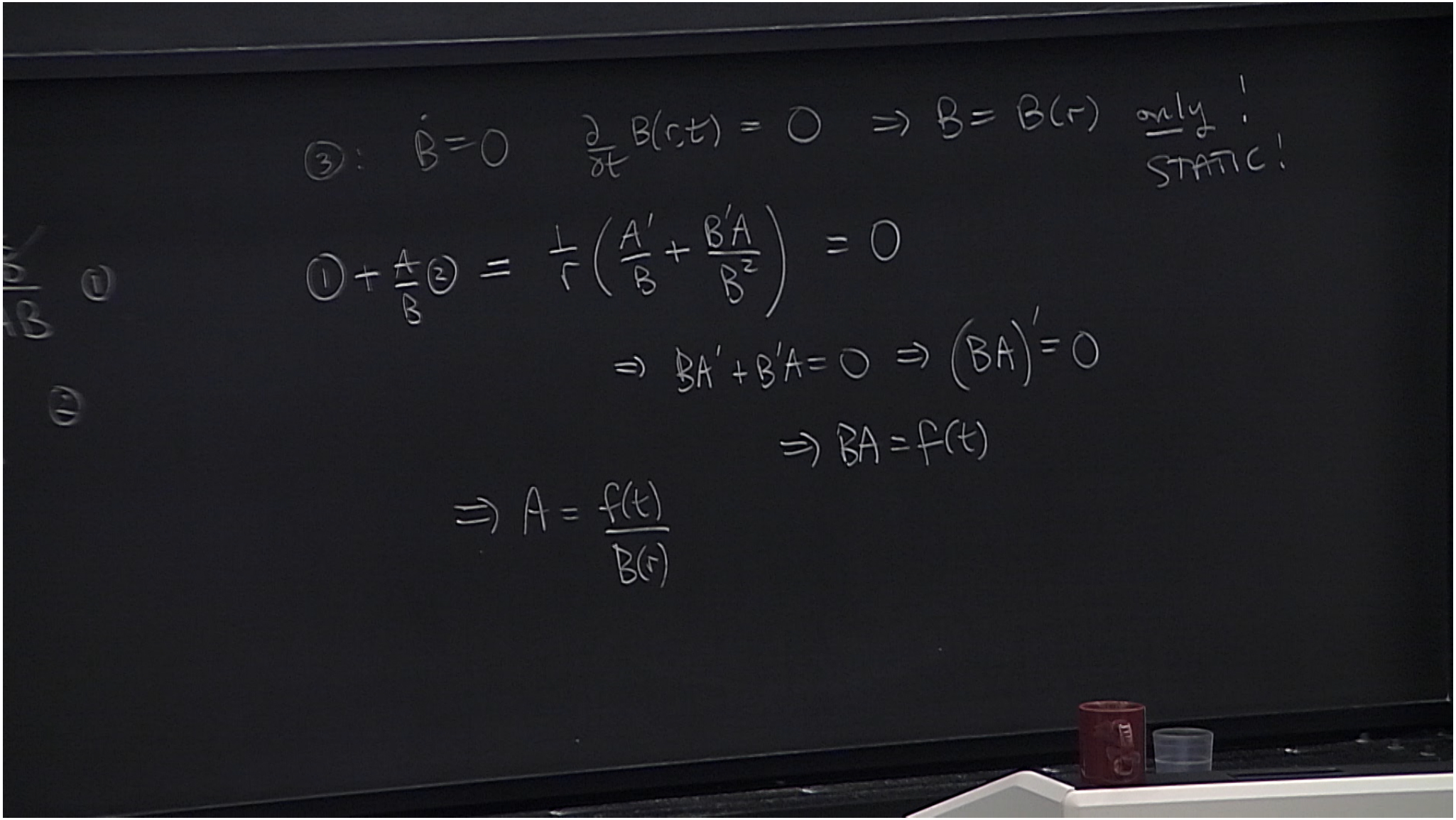
$$R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB} \quad (1)$$

$$R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{Br} + \frac{\ddot{B}}{2A} - \frac{\dot{A}\dot{B}}{4A^2} - \frac{\dot{B}^2}{4AB} \quad (2)$$

$$R_{tr} = \frac{\dot{B}}{Br} \quad (3)$$

$$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB} \quad (4)$$

$$R_{\varphi\varphi} = \sin^2\theta \, R_{\theta\theta} \quad (5)$$


③: $\dot{B}=0$ $\frac{\partial}{\partial t}B(r,t)=0 \Rightarrow B=B(r)$ only! STATIC!
$① + \frac{A}{B}② = \frac{1}{r}\left(\frac{A'}{B}+\frac{B'A}{B^2}\right)=0$
$\Rightarrow BA'+B'A=0 \Rightarrow (BA)'=0$
$\Rightarrow BA=f(t)$
$\Rightarrow A=\frac{f(t)}{B(r)}$

③: $\dot{B}=0 \quad \frac{\partial}{\partial t}B(r,t)=0 \Rightarrow B=B(r)$ only! STATIC!

① $\quad ① + \frac{A}{B}② = \frac{1}{r}\left(\frac{A'}{B}+\frac{B'A}{B^2}\right)=0$

$\Rightarrow BA'+B'A=0 \Rightarrow (BA)'=0$

$\Rightarrow BA=f(t)$

$\Rightarrow A=\frac{f(t)}{B(r)}$

$\Rightarrow ds^2=-\frac{f(t)dt^2}{B(r)}+B(r)dr^2+r^2d\Omega_2^2$

$$\frac{A'^2}{4AB} \quad (1)$$

$$\frac{B'^2}{4AB} \quad (2)$$

$$(1) + \frac{A}{B}(2) = \frac{1}{r}\left(\frac{A'}{B} + \frac{BA'}{B^2}\right) = 0$$

$$\Rightarrow BA' + B'A = 0 \Rightarrow (BA)' = 0$$

$$\Rightarrow BA = f(t)$$

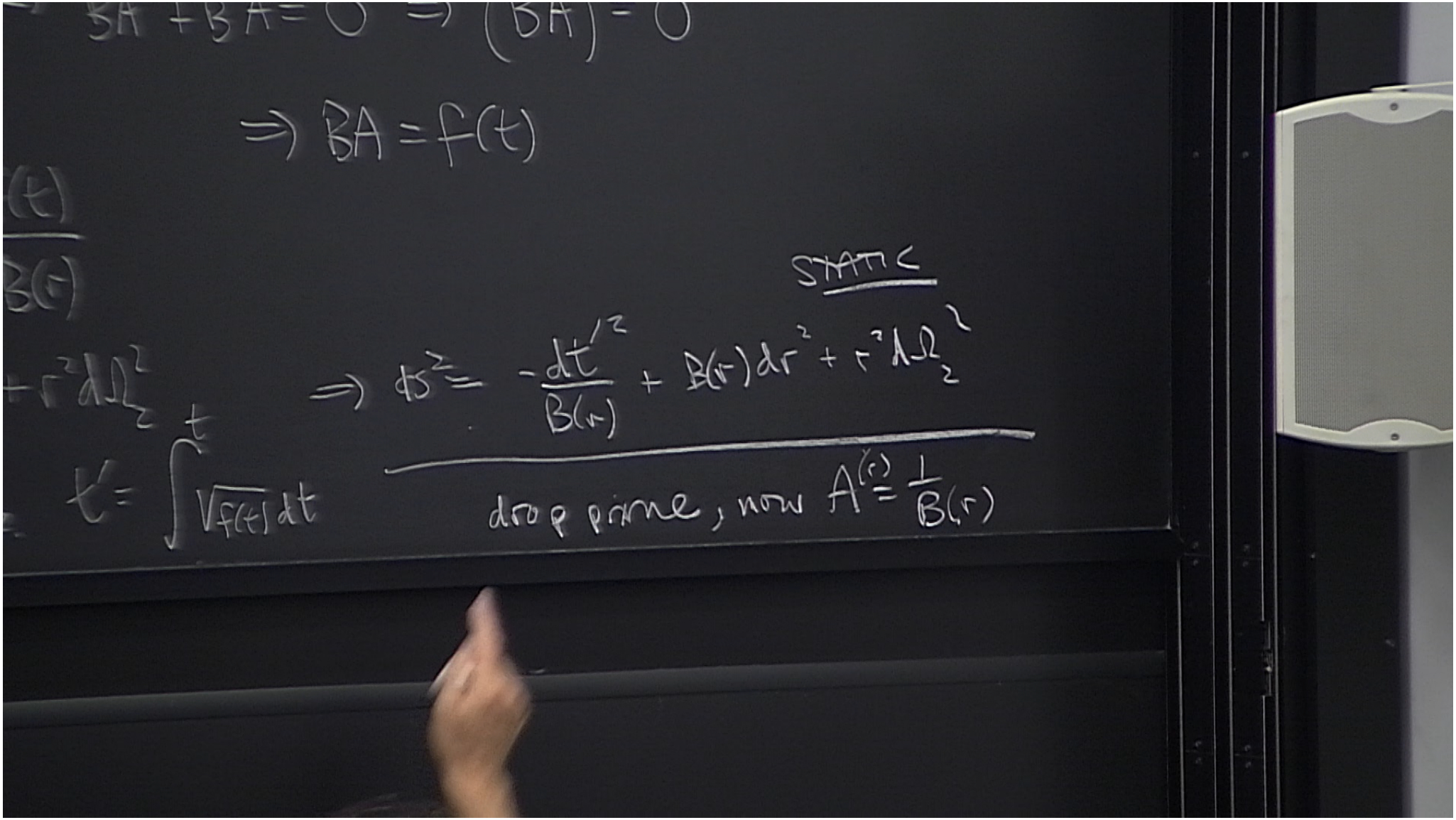
$$\Rightarrow A = \frac{f(t)}{B(r)}$$

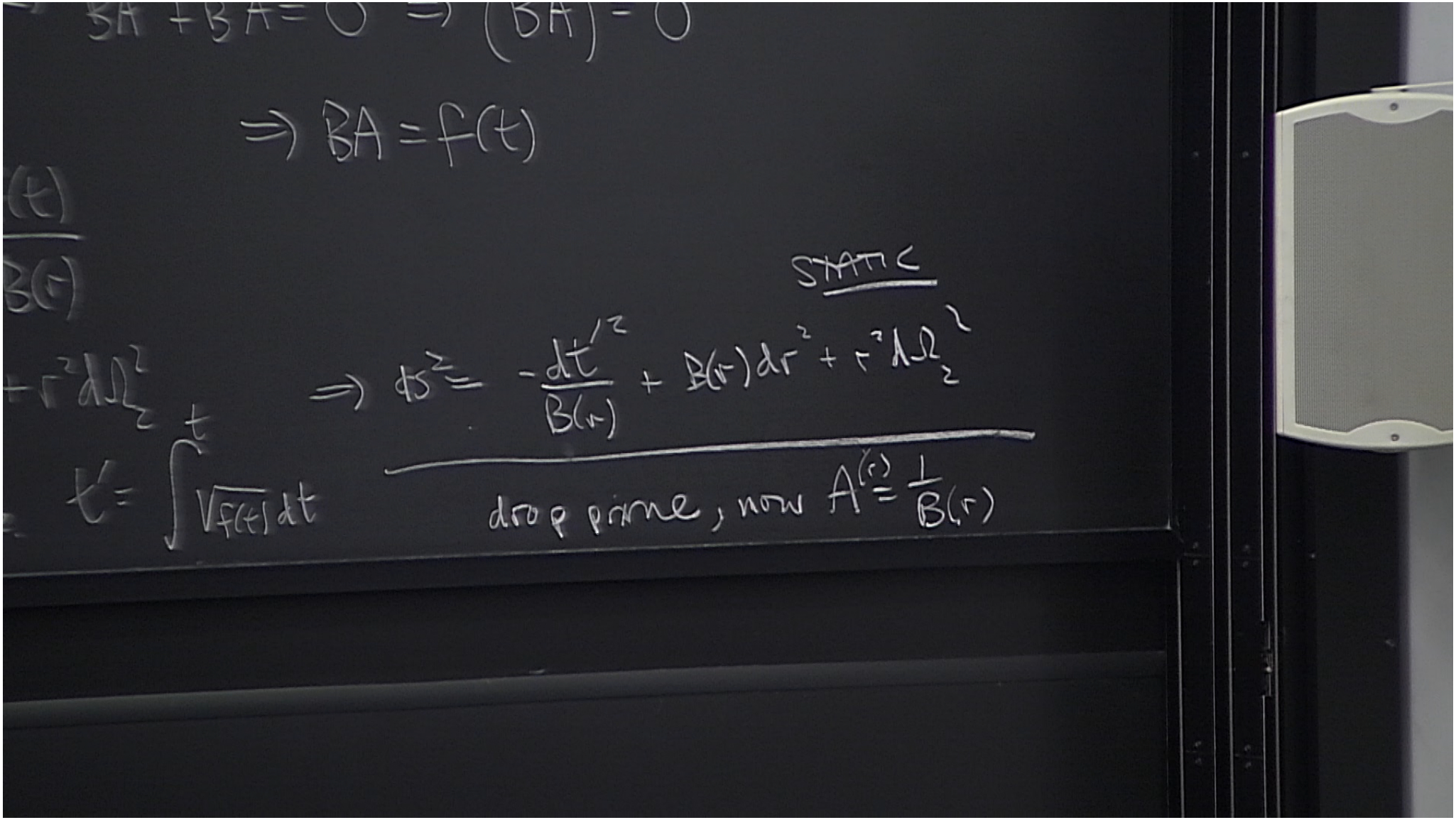
$$\Rightarrow ds^2 = -\frac{f(t)dt^2}{B(r)} + B(r)dr^2 + r^2 d\Omega_2^2$$

STATIC

$$\Rightarrow ds^2 = -\frac{dt'^2}{B(r)} + B(r)dr^2 + r^2 d\Omega_2^2$$

f must be positive so define $dt' = \sqrt{f(t)}\,dt$ i.e. $t' = \int^t \sqrt{f(t)}\,dt$


$\dot{B}A + B\dot{A} = 0 \Rightarrow (BA)^{\cdot} = 0$
$\Rightarrow BA = f(t)$
STATIC
$\Rightarrow ds^2 = -\frac{dt'^2}{B(r)} + B(r)dr^2 + r^2 d\Omega_2^2$
$t' = \int^t \sqrt{f(t)}\,dt$
drop prime, now $A(r) = \frac{1}{B(r)}$


$\dot{B}A + B\dot{A} = 0 \Rightarrow (BA)^{\cdot} = 0$
$\Rightarrow BA = f(t)$
$\frac{f(t)}{B(r)}$
$+ r^2 d\Omega_2^2$
$t' = \int^t \sqrt{f(t)}\, dt$
STATIC
$\Rightarrow ds^2 = -\frac{dt'^2}{B(r)} + B(r) dr^2 + r^2 d\Omega_2^2$
drop prime, now $A(r) = \frac{1}{B(r)}$

$A = \frac{1}{B(r)}$ ④: $1 - \frac{1}{B} + \frac{rB'}{2B^2} + \frac{rB'}{2B^2}$

$\frac{A'}{A} = -\frac{B'}{B}$ $\frac{A'}{A} = -\frac{B'}{B}$

$1 - \frac{1}{B} + \frac{rB'}{B^2} = 0$ $1 = \left(\frac{r}{B}\right)' = \frac{1}{B} - r\frac{B'}{B^2}$

$\Rightarrow \frac{r}{B} = r + k$, where k is a constant

$\Rightarrow B = \frac{1}{1 + k/r}$ $\Rightarrow A = 1 + k/r$

$\Rightarrow ds^2 = -\left(1 + \frac{k}{r}\right)dt^2 + \frac{dr^2}{\left(1 + \frac{k}{r}\right)} + r^2 d\Omega_2^2$. Schwarzschild metric : What is k?

$A = \frac{1}{B(r)}$ ④: $1 - \frac{1}{B} + \frac{rB'}{2B^2} + \frac{rB'}{2B^2}$

$\frac{A'}{A} = -\frac{B'}{B}$ $\frac{A'}{A} = -\frac{B'}{B}$

$1 - \frac{1}{B} + \frac{rB'}{B^2} = 0$ $1 = \left(\frac{r}{B}\right)' = \frac{1}{B} - r\frac{B'}{B^2}$

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large r, $g_{\mu\nu} \approx \eta_{\mu\nu}$

$A = \frac{1}{B(r)}$ ④: $1 - \frac{1}{B} + \frac{rB'}{2B^2} + \frac{rB'}{2B^2}$

$\frac{A'}{A} = -\frac{B'}{B}$ $\frac{A'}{A}$

$1 - \frac{1}{B} + \frac{rB'}{B^2} = 0$

$1 = \left(\frac{r}{B}\right)' = \frac{1}{B}$

$-dt^2 + \underbrace{dr^2 + r^2 d\Omega_2^2}_{\text{3d flat space in spherical polars}}$

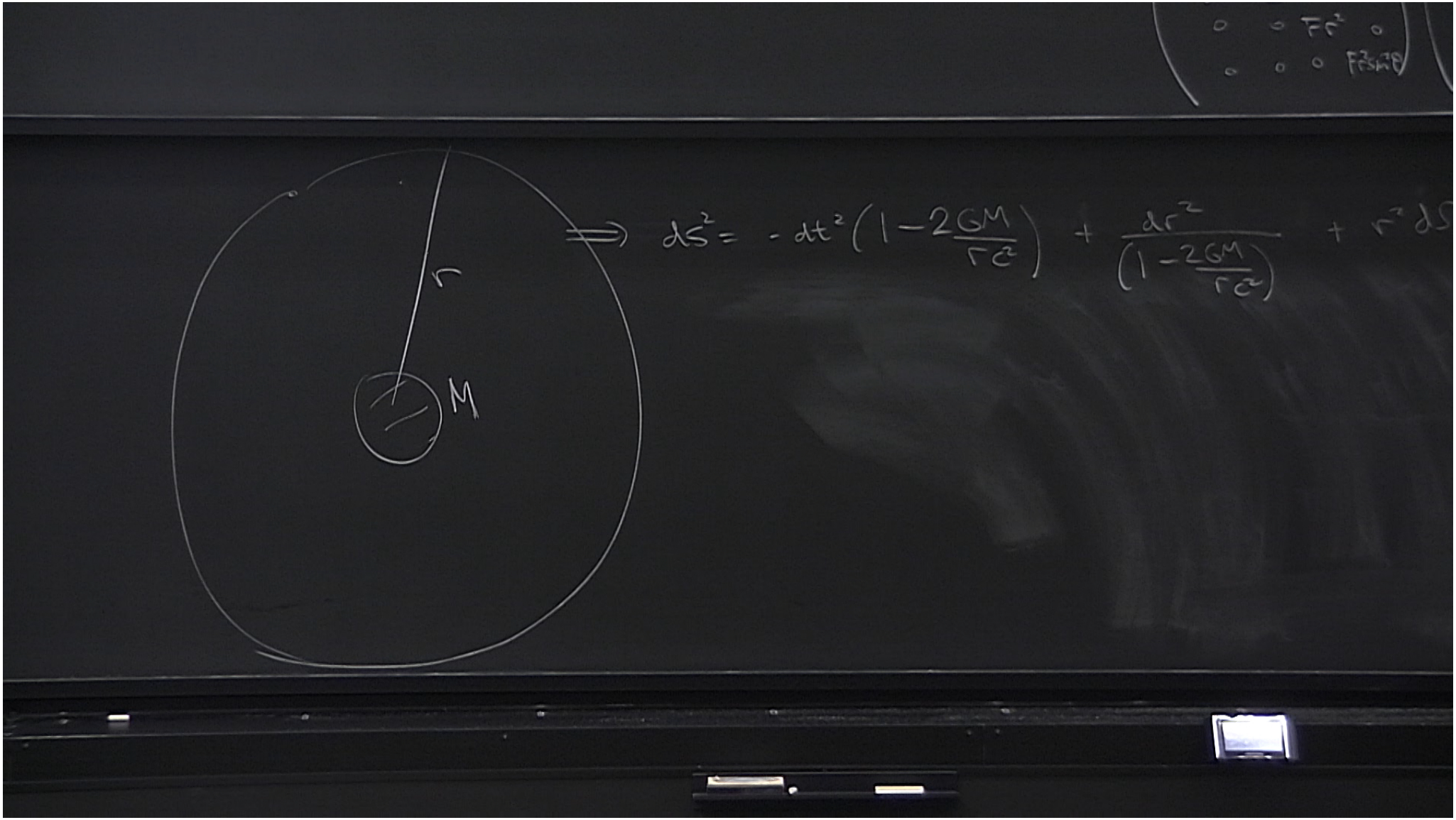
Minkowski

$\Rightarrow \frac{r}{B} = r + k$, wh

$\Rightarrow B = \frac{1}{1 + k/r}$ $\Rightarrow$ A

$\Rightarrow ds^2 = -\left(1 + \frac{k}{r}\right)dt^2 + \frac{dr^2}{\left(1 + \frac{k}{r}\right)} + r^2 d\Omega_2^2$. Schwarzschild metric : W

large r; $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow h_{00} = -\frac{k}{r} = -2\phi_g = \frac{2GM}{rc^2}$ ← Newtonian gravitational potential


r
M
$\Rightarrow ds^2 = -dt^2\left(1-\frac{2GM}{rc^2}\right) + \frac{dr^2}{\left(1-\frac{2GM}{rc^2}\right)} + r^2 d$

$$\begin{pmatrix} \cdots & \cdots & \cdots \\ 0 & Fr^2 & 0 \\ 0 & 0 & Fr^2\sin^2\theta \end{pmatrix} \begin{pmatrix} d\theta \\ d\varphi \end{pmatrix}$$

$r' \equiv \sqrt{F(t)}\, r$

$\Rightarrow t' = t$

$$\Rightarrow ds^2 = -dt^2\left(1-\frac{2GM}{rc^2}\right) + \frac{dr^2}{\left(1-\frac{2GM}{rc^2}\right)} + r^2 d\Omega_2^2$$

, where M is the "

by comparing w

$$\equiv 1 - \frac{r_s}{r}$$

, $r_s = 2\frac{GM}{c^2}$ is the Schwarzchild radius.

$= -dt^2\left(1-\frac{2GM}{rc^2}\right) + \frac{dr^2}{\left(1-\frac{2GM}{rc^2}\right)} + r^2 d\Omega_2^2$, where M is the "mass"
by comparing with Ne

$\equiv 1 - \frac{r_s}{r}$, $r_s = 2\frac{GM}{c^2}$ is the Schwarzchild radius.

Very strange things happen as r decreases to r_s and becomes smaller t

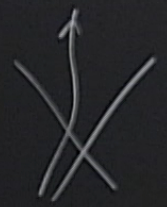
$t \rightarrow$ spatial coordinate
$r \rightarrow$ timelike coordinate!

$dr = \#\,dr + \#\,dt$

etc

...ed at large r

...nity.



$A = \frac{1}{B(r)}$ ④: $1 - \frac{1}{B} + \frac{rB'}{2B^2} + \frac{rB'}{2B^2}$

$1 - \frac{1}{B} + \frac{rB'}{B^2} = 0$

$-dt^2 + \underbrace{dr^2 + r^2 d\Omega_2^2}_{\text{3d flat space in spherical polars}}$

Minkowski

$\Rightarrow ds^2 = -\left(1 + \frac{k}{r}\right)dt^2 + \frac{dr^2}{\left(1 + \frac{k}{r}\right)} + r^2 d\Omega$

large r, $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow h_{00} = -\frac{k}{r} = -2$

$$\begin{pmatrix} -D & +E & 0 & 0 \\ 0 & 0 & Fr^2 & 0 \\ 0 & 0 & 0 & Fr^2\sin^2\theta \end{pmatrix} \begin{pmatrix} dr \\ d\theta \\ d\varphi \end{pmatrix}$$

$$\Rightarrow \begin{matrix} r' \equiv \sqrt{F(r,t)}\, r \\ t' = t \end{matrix} \Bigg\} \begin{matrix} dr' = \#\, dr + \#\, dt \\ dt' = dt \end{matrix} \Bigg\} dr = \#\, dr' + \#\, dt' \quad \text{etc}$$

$$+ \frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)} + r^2 d\Omega_2^2 \quad \text{, where } M \text{ is the "mass" as determined at large } r \text{ by comparing with Newtonian gravity.}$$

, $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius.

happen as r decreases to r_s and becomes smaller than r_s:

tial coordinate
uclike coordinate!

turns out that sing'y in metric at $r = r_s$ is an artefact of the coordinates

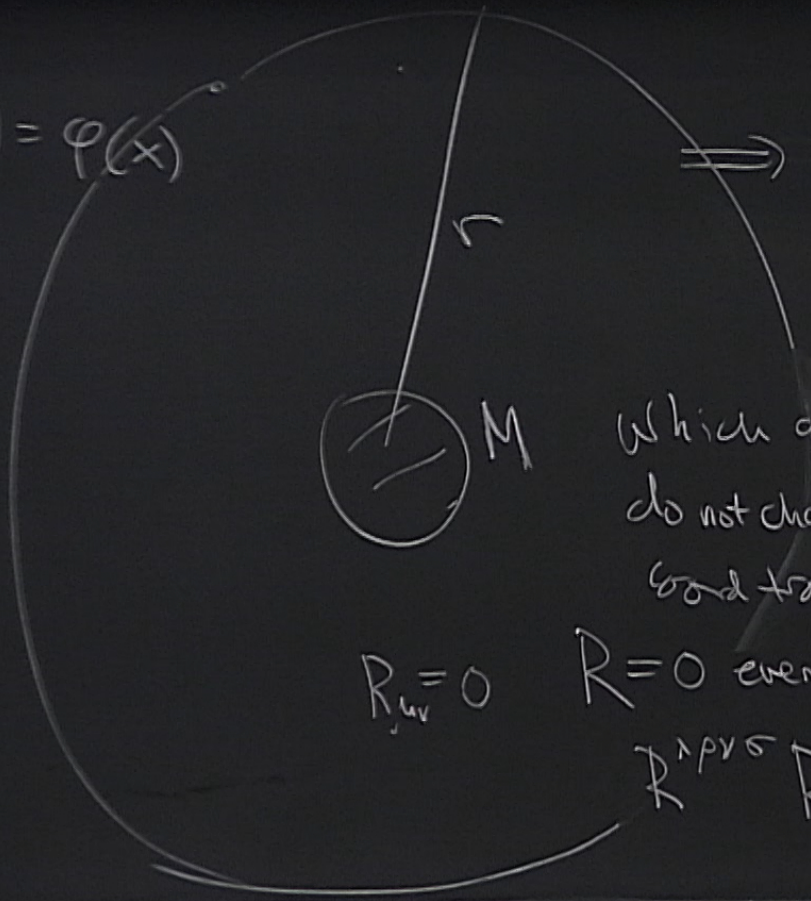
$= d\Omega_2^2$, where M is the "mass" as determined at large r
by comparing with Newtonian gravity.

Schwarzchild radius.

decreases to r_s and becomes smaller than r_s:

turns out that singy in metric at $r = r_s$ is an artefact of the coordinates
but it " " " at $r = 0$ is genuine, cannot be removed
by changing coordinates.

$\varphi'(x') = \varphi(x)$



$\Rightarrow ds^2 = -dt^2\left(1 - \frac{2GM}{rc^2}\right) + \frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)}$

$\equiv 1 - \frac{r_s}{r}$, $r_s = \frac{2GM}{c^2}$

Which quantities do not change under coord transfm^{n}s?

$R_{\mu\nu} = 0$ $R = 0$ everywhere.

$R^{\lambda\rho\nu\sigma} R_{\lambda\rho\nu\sigma} = \text{scalar} \overset{?}{=} \#\frac{(GM)^2}{r^6}$

very strange things happen a

$t \to$ spatial coordinate

$r \to$ timelike coordi

$$\Rightarrow ds^2 = -dt^2\left(1-\frac{2GM}{rc^2}\right) + \frac{dr^2}{\left(1-\frac{2GM}{rc^2}\right)} + r^2 d\Omega_2^2$$

$$\equiv 1 - \frac{r_s}{r}, \quad r_s = \frac{2GM}{c^2} \text{ is the Schwarzchil}$$

M

Which quantities do not change under good transfm^{n}s?

$R=0$ everywhere.

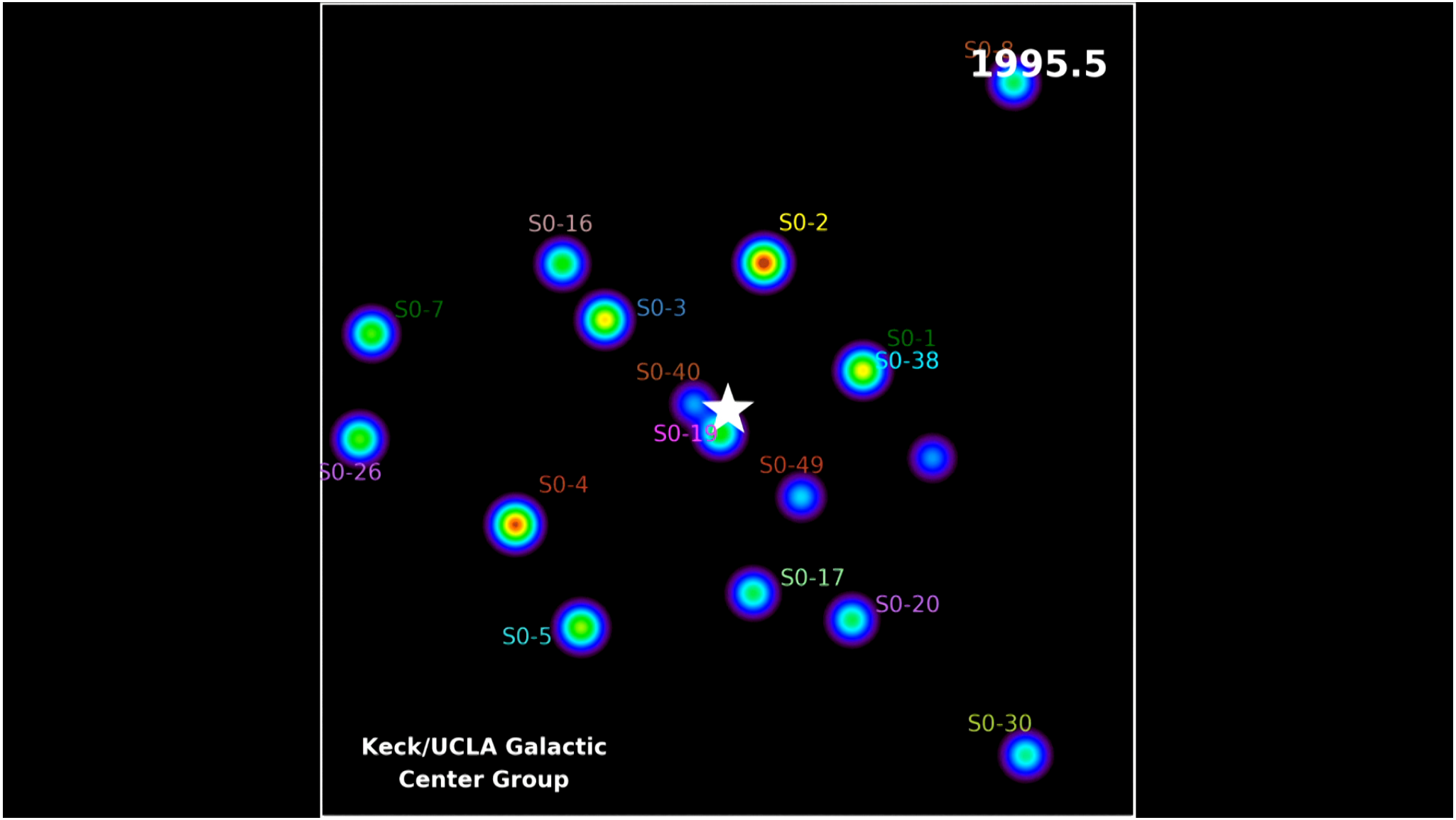
very strange things happen as r decreases to r_s

$t \rightarrow$ spatial coordinate

$r \rightarrow$ timelike coordinate!

turns out th

$R^{\lambda\rho\nu\sigma} R_{\lambda\rho\nu\sigma} = \text{scalar} \overset{?}{=} \#\frac{(GM)^2}{r^6} \Rightarrow \infty$ at $r=0$, in all coordinate systems!


1995.5
S0-16
S0-2
S0-7
S0-3
S0-1
S0-38
S0-40
S0-19
S0-26
S0-49
S0-4
S0-17
S0-20
S0-5
S0-30
Keck/UCLA Galactic
Center Group

$= (dt, dr, d\theta, d\varphi) \begin{pmatrix} -C & -D & 0 & \\ -D & +E & 0 & \\ 0 & 0 & Fr^2 & \\ 0 & 0 & 0 & \end{pmatrix}$

Andrea Ghez : supermassive black hole

$M \approx 4 \times 10^6 M_\odot$

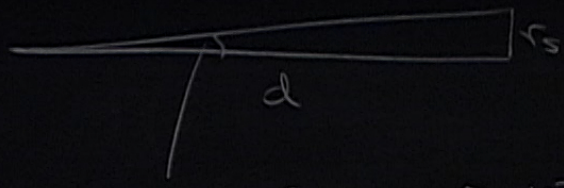
$\Rightarrow ds^2 = -dt^2 \left(1 - \frac{2GM}{rc^2}\right) + \frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)}$ +

$= -\left(1 - \frac{r_s}{\quad}\right)$ $r_s = \frac{2GM}{\quad}$

$r_s \simeq 17\, R_\odot \sim 1.2 \times 10^{10}$ m

25,000 light yrs from us $\sim 2.4 \times 10^{20}$ m

Andrea Ghez : supermassive black hole $M \approx$



$\theta \sim \frac{r_s}{d} \sim 0.5 \times 10^{-10}$ radians

$s^2 = -dt^2\left(1 - 2\frac{G}{r}\right.$

radio astronomy $\nu \sim 10^3$ GHz

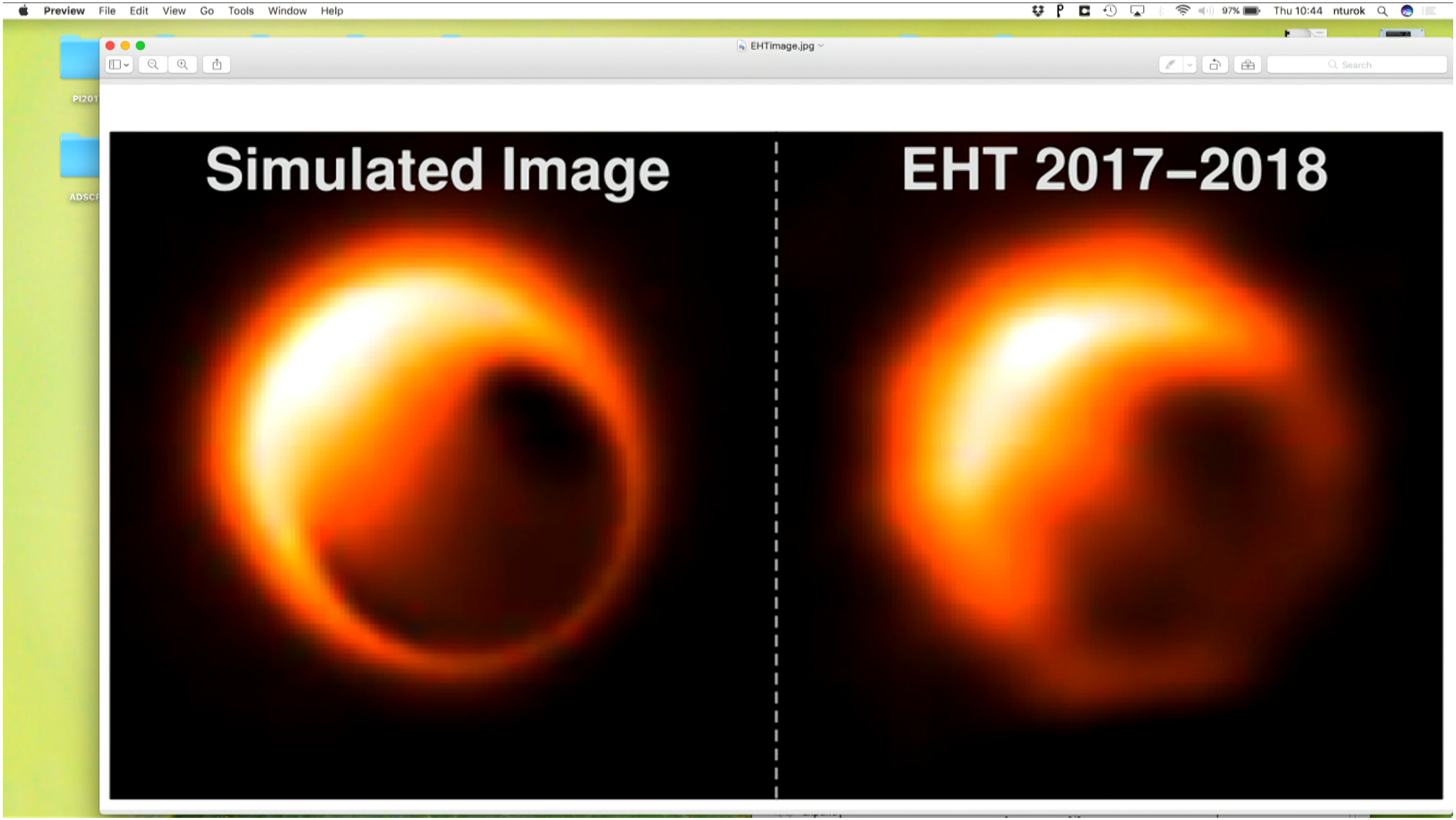
$\lambda \sim 0.3$ mm



Res^n: $\theta \sim \frac{\lambda}{D} \sim \frac{\frac{1}{3} \times 10^{-3}\ \text{m}}{10^7\ \text{m}} \sim \frac{1}{3} \times 10^{-10}$

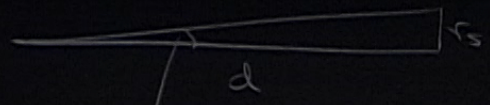
$\equiv 1 -$

very strange


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Simulated Image
EHT 2017–2018

25,000 light years

Andrea Ghez : $M \approx 4 \times 10^6 M_\odot$



$\theta \sim \frac{r_s}{d} \sim 0.5 \times 10^{-10}$ radians

$s^2 = -dt^2\left(1 - \frac{2GM}{rc^2}\right) + \frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)}$

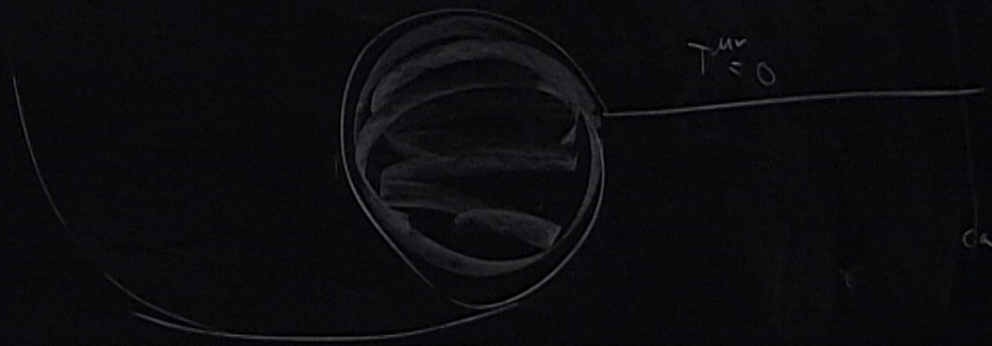
$\equiv 1 - \frac{r_s}{r}$, r_s

radio astronomy $\nu \sim 10^3$ GHz

$\lambda \sim 0.3$ mm



Resn. $\theta \sim \frac{\lambda}{D} \sim \frac{\frac{1}{3} \times 10^{-3}\ m}{10^7\ m} \sim \frac{1}{3} \times 10^{-10}$



ery strange things hap

$t \rightarrow$ spatial coo

$r \rightarrow$ timelike

calar $\stackrel{?}{=} \frac{(GM)^2}{r^6} \Rightarrow \infty$ at r=