

Title: PSI 17/18 - Relativity - Lecture 12

Date: Sep 20, 2017 09:00 AM

URL: <http://pirsa.org/17090031>

Abstract:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

weak field approx
 $|h_{\mu\nu}| \ll 1$

Einstein
 field
 eqns

$$R_{\mu\nu} = 0$$

$$= -\frac{1}{2} \square h_{\mu\nu} \quad \text{in harmonic gauge}$$

$$g^{\mu\nu} \Gamma_{\mu\nu}^\lambda = 0$$

$$\Leftrightarrow \square x^\lambda = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

weak field approx
 $|h_{\mu\nu}| \ll 1$

Einstein
 field
 eqns

$$R_{\mu\nu} = 0$$

Minkowski

$= -\frac{1}{2} \square h_{\mu\nu}$ in harmonic gauge
 to first order in $h_{\mu\nu}$.

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu$$

$$x^\alpha \rightarrow x'^\alpha(x) \neq \frac{\partial x'^\alpha}{\partial x^\beta} x^\beta$$

$$\left(\begin{array}{l} g^{\mu\nu} \Gamma_{\mu\nu}^\lambda = 0 \text{ (Minkowski)} \\ \Leftrightarrow \square x^\lambda = 0 \end{array} \right) \xrightarrow{\text{wfa}} \text{GENERAL}$$

$$g^{\mu\nu} \nabla_\mu \nabla_\nu x^\lambda$$

Trace as 4-indices
 "Hams trace"

$$\downarrow \quad \overline{h}^\mu_\nu \text{ trace reversed } h^\mu_\nu$$

$$\downarrow$$

$$\partial_\mu \left(h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda \right) = 0$$

$$\downarrow$$

$$ik_\mu$$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Einstein field eqs $R_{\mu\nu} = 0$

$\mathbb{D} = \eta^{\mu\nu} \partial_\mu \partial_\nu$

$x^\mu \rightarrow x'^\mu(x) \neq \frac{\partial x'^\mu}{\partial x^\alpha} x^\alpha$

weak field approx $|h_{\mu\nu}| \ll 1$

$x^\mu \rightarrow x'^\mu(x)$

$\left(g^{\mu\nu} \Gamma_{\mu\nu}^\lambda = 0 \right) \xrightarrow{\text{wfa}} \partial_\mu \left(h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda \right) = 0$

$\Rightarrow \mathbb{D} x^\lambda = 0$ GENERAL

$\bar{h}^\mu_\nu$ trace reversed h^μ_ν

$\text{Trace as 4-dim "transverse"}$

$\text{Solve (1): } h_{\mu\nu}(t, \mathbf{x}) = \sum_{\mathbf{k}} \left(\epsilon_{\mu\nu}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{c.c.} \right) \text{ satisfies (1)}$

Minkowski (1)

$= -\frac{1}{2} \mathbb{D} h_{\mu\nu}$ in harmonic gauge to first order in $h_{\mu\nu}$.

weak field approx
 $|h_{\mu\nu}| \ll 1$

Einstein field eqs
 $R_{\mu\nu} = 0$ ①
 $= -\frac{1}{2} \square h_{\mu\nu}$ in harmonic gauge
 to first order in $h_{\mu\nu}$.

$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu$

$x^\mu \rightarrow x'^\mu(x)$ transform as if coordinates
 $\left(g^{\mu\nu} \Gamma_{\mu\nu}^\lambda = 0 \right)_{\text{harmonic}}$ GENERAL
 $\Leftrightarrow \square x^\lambda = 0$

$\downarrow$ trace reversed $\bar{h}^\mu_\nu$
 $\partial_\mu \left(h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda \right) = 0$
 $\downarrow$ (k^t + k_x x)
 $i k_\mu$

$x^\nu = (t, x)$
 $k x = \eta_{\mu\nu} k^\mu x^\nu$

Solve ①: $h_{\mu\nu}(t, x) = \sum_k \left(\epsilon_{\mu\nu}(k) e^{i k x} + c.c. \right)$ ②
 amplitude and phase

satisfies ① if $k^2 = 0$
 $\vec{k} = \frac{k}{|\vec{k}|} \Rightarrow k^0 = \pm |\vec{k}|$. wlog, can choose $k^0 = |\vec{k}|$ in ②.

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ weak field approx
 $|h_{\mu\nu}| \ll 1$

$x^\mu \rightarrow x'^\mu(x)$ transform as + coordinate
// transform

$\bar{h}_{\mu\nu}$ trace reversed $h_{\mu\nu}$

$\partial_\mu (h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda) = 0$ (1)
 $\downarrow$
 $ik_\mu (h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda) = 0$ (2)

$x^\nu = (t, \mathbf{x})$
 $kx = \eta_{\mu\nu} k^\mu x^\nu$

$\square x^\lambda = 0$ GENERAL

$\square h_{\mu\nu} = 0$ in harmonic gauge
to first order in $h_{\mu\nu}$.

$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu$

$x^\lambda \rightarrow x'^\lambda(x) \neq \frac{\partial x'^\lambda}{\partial x^\mu} x^\mu$

$\text{Solve (1): } h_{\mu\nu}(t, \mathbf{x}) = \sum_{\mathbf{k}} \left(\epsilon_{\mu\nu}(\mathbf{k}) e^{ikx} + \text{c.c.} \right)$ satisfies (1) if $k^2 = 0$
amplitude and phase

$k^2 = k^\mu k_\mu = 0 \Rightarrow k^0 = \pm |\mathbf{k}|$. wlog, can choose $k^0 = |\mathbf{k}|$ in (1)

$\text{Must take gauge condition into account i.e., } k_\mu \epsilon^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} \epsilon^\lambda_\lambda = 0 \quad \forall \mathbf{k},$ (3)
 $k^\mu = (k^0, \mathbf{k}) = (\mathbf{k}, \mathbf{k})$
 $k_\mu = (-\mathbf{k}, \mathbf{k})$

$$x^\mu \rightarrow x'^\mu(x) \neq \frac{\partial x'^\mu}{\partial x^\nu} x^\nu$$

Solve ①: $h_{\mu\nu}(t, \vec{x}) = \sum_{\vec{k}} \underbrace{(\epsilon_{\mu\nu}(\vec{k}) e^{i\vec{k}\cdot\vec{x}} + c.c.)}_{\text{amplitude and phase}}$ satisfies ① if $\vec{k} = 0$
 $\vec{k} = \frac{\vec{k}}{|\vec{k}|} \Rightarrow k^0 = \pm |\vec{k}|$. wlog, can choose $\boxed{k^0 = |\vec{k}|}$ in ①.

must take gauge condition into account i.e., $k_\mu (\epsilon^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} \epsilon^\lambda{}_\lambda) = 0 \quad \forall \vec{k}$, $k^\mu = (k^0, \vec{k}) = (k, \vec{k})$
 $k_\mu = (-k, \vec{k})$

Still have residual coordinate freedom, namely if $x^\mu \rightarrow x'^\mu = x^\mu + \epsilon^\mu(x)$, then $\square x'^\mu = 0$ if $\square \epsilon^\mu = 0$.

Under a coordinate transfmⁿ $g'_{\mu\nu}(x) = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$, if $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ and $g'_{\mu\nu} = \eta_{\mu\nu} + h'_{\mu\nu}$, and $x'^\mu = x^\mu + \epsilon^\mu \Leftrightarrow x^\mu = x'^\mu - \epsilon^\mu(x)$
 $\approx x'^\mu - \epsilon^\mu(x')$
to first order in ϵ^μ .

$$x^\mu \rightarrow x'^\mu(x) \neq \frac{\partial x'^\mu}{\partial x^\nu} x^\nu$$

Solve ①: $h_{\mu\nu}(t, x) = \sum_k \underbrace{(\epsilon_{\mu\nu}(k))}_{\text{amplitude and phase}} e^{ikx} + c.c.$ satisfies ① if $k^2 = 0$
 $k^2 = k_\mu k^\mu \Rightarrow k^0 = \pm |\mathbf{k}|$. wlog, can choose $\boxed{k^0 = |\mathbf{k}|}$ in ①

must take gauge condition into account i.e., $k_\mu (\epsilon^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} \epsilon^\lambda{}_\lambda) = 0 \quad \forall k$, $k^\mu = (k^0, \mathbf{k}) = (k, \mathbf{k})$
 $k_\mu = (-k, \mathbf{k})$

Still have residual coordinate freedom, namely if $x^\mu \rightarrow x'^\mu = x^\mu + \epsilon^\mu(x)$, then $\square x'^\mu = 0$ if $\square \epsilon^\mu = 0$.

Under a coordinate transfm: $g'_{\mu\nu}(x) = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$, if $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ and $g'_{\mu\nu} = \eta_{\mu\nu} + h'_{\mu\nu}$, and $x'^\mu = x^\mu + \epsilon^\mu \Leftrightarrow x^\mu = x'^\mu - \epsilon^\mu(x)$
 $\approx x'^\mu - \epsilon^\mu(x')$
 to first order in ϵ^μ .

We treat ϵ^μ as the same order as $h_{\mu\nu}$.

must take gauge condition into account i.e., $k_\mu (\epsilon^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} \epsilon^\lambda{}_\lambda) = 0 \quad \forall \epsilon$, $k^\mu = (k^0, \vec{k}) = (\vec{k}, \vec{k})$
 $k_\mu = (-k, \vec{k})$

Still have residual coordinate freedom, namely if $x^\mu \rightarrow x'^\mu = x^\mu + \epsilon^\mu(x)$, then $\square x^\mu = 0$ if $\square \epsilon^\mu = 0$.

Under a coordinate transfm. $g'_{\mu\nu}(x) = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$, If $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $g'_{\mu\nu} = \eta_{\mu\nu} + h'_{\mu\nu}$, and $x'^\mu = x^\mu + \epsilon^\mu \Leftrightarrow x^\mu = x'^\mu - \epsilon^\mu(x)$
 $\approx x'^\mu - \epsilon^\mu(x')$
 to first order in ϵ^μ .

$$\eta_{\mu\nu} + h'_{\mu\nu} \approx \eta_{\mu\nu} - \epsilon^\alpha{}_\mu \delta^\beta_\nu \eta_{\alpha\beta} - \delta^\alpha_\mu \epsilon^\beta_\nu \eta_{\alpha\beta} = \eta_{\mu\nu} - \epsilon_{\mu,\alpha} - \epsilon_{\nu,\mu} + h_{\mu\nu}$$

(to linear order in ϵ and h)

$$\left(\delta^\alpha_\mu - \epsilon^\alpha{}_\mu \right) \left(\delta^\beta_\nu - \epsilon^\beta{}_\nu \right) \eta_{\alpha\beta}$$

$$\Rightarrow h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$$

We treat ϵ^μ as the same order as $h_{\mu\nu}$.

$$\eta_{\mu\nu} + h_{\mu\nu} \simeq \eta_{\mu\nu} - \epsilon_{\mu} \delta_{\nu}^{\lambda} \partial_{\lambda} \phi + h_{\mu\nu} \quad (\text{to linear order in } \epsilon \text{ and } h)$$

$$\left(\delta_{\mu}^{\lambda} - \epsilon_{\mu}^{\lambda} \right) \left(\delta_{\nu}^{\rho} - \epsilon_{\nu}^{\rho} \right) \eta_{\lambda\rho}$$

$$\Rightarrow h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\nu,\mu} - \epsilon_{\mu,\nu} \quad (3)$$

If ϵ^{μ} satisfies $\partial^{\mu} \epsilon_{\mu} = 0$, we remain in harmonic gauge. , can express

$$\epsilon^{\mu}(x) = \sum_{\mathbf{k}} (i c_{\mu}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} + \text{c.c.}) \quad \text{where } k^0 = |\mathbf{k}| \text{ as before}$$

$$\text{Subst into (3), use (2)} \Rightarrow \epsilon'_{\mu\nu}(\mathbf{k}) = \epsilon_{\mu\nu} + k_{\mu} c_{\nu} + k_{\nu} c_{\mu}$$

$$\eta_{\mu\nu} + h'_{\mu\nu} \simeq \eta_{\mu\nu} - \epsilon^\alpha_{,\mu} \delta^\mu_{\nu} \eta_{\alpha\beta} - \delta^\alpha_{,\mu} \epsilon^\mu_{,\nu} \eta_{\alpha\beta} = \eta_{\mu\nu} - \epsilon_{,\mu\alpha} - \epsilon_{,\nu\alpha}$$

(to linear order in ϵ and h)

We treat ϵ^μ as the same as before

$$\left(\delta^\mu_{\alpha} - \epsilon^\mu_{,\alpha} \right) \left(\delta^\beta_{\nu} - \epsilon^\beta_{,\nu} \right) \eta_{\alpha\beta}$$

$$\Rightarrow h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{,\mu\nu} - \epsilon_{,\nu\mu} \quad (3)$$

If ϵ^μ satisfies $\partial_\mu \epsilon^\mu = 0$, we remain in harmonic gauge, can express

$$\epsilon^\mu(x) = \sum_{\vec{k}} (i c_\mu(\vec{k}) e^{i\vec{k} \cdot \vec{x}} + c.c.) \quad \text{where } k^0 = |\vec{k}| \text{ as before}$$

Subst into (3), use (2)

$$\Rightarrow \epsilon'_{\mu\nu}(\vec{k}) = \epsilon_{\mu\nu} + k_\mu \epsilon_\nu + k_\nu \epsilon_\mu$$

can check that this change preserves gauge condition

$$k^\mu \epsilon_{\mu\nu} - \frac{1}{2} k_\nu \epsilon^\mu{}_\mu = 0$$

Change is $\downarrow$

$$k^\mu (k_\mu \epsilon_\nu + k_\nu \epsilon_\mu) - \frac{1}{2} k_\nu (2 k^\mu \epsilon_\mu) = 0$$

Consider g-wave travelling in the z direction

$$k^\mu = (k, 0, 0, k)$$

$$k_\mu = (-k, 0, 0, k)$$

$$k^\mu x_\mu = (-kt + kz)$$
$$= k(z-t)$$

gauge condⁿ reads

Consider g-wave travelling in the z direction

$$k^\mu = (k, 0, 0, k)$$

$$k_\mu = (-k, 0, 0, k)$$

$$k^\mu x_\mu = (-kt + kz)$$

$$= k(z-t)$$

Component

gauge condⁿ reads $\nabla \cdot \mathbf{A} = 0$: $k(\epsilon_{00} + \epsilon_{33}) + \frac{1}{2}k(-\epsilon_{00} + \epsilon_{11} + \epsilon_{22} + \epsilon_{33}) = 0$

$$k^\mu (\epsilon_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \epsilon^\lambda{}_\lambda) = 0$$

1: $k(\epsilon_{03} + \epsilon_{30}) - \frac{1}{2}k(-\epsilon_{00} + \epsilon_{11} + \epsilon_{22} + \epsilon_{33}) = 0$

2: $k(\epsilon_{01} + \epsilon_{10}) = 0$

3: $k(\epsilon_{02} + \epsilon_{20}) = 0$

Consider g-wave travelling in the z direction

$$k^\mu = (k, 0, 0, k)$$

$$k_\mu = (-k, 0, 0, k)$$

$$k^\mu x_\mu \eta_{\mu\nu} = (-kt + kz)$$

$$= k(z-t)$$

Component

gauge condⁿ reads $\nabla \cdot \mathbf{A} = 0$: $k(\epsilon_{00} + \epsilon_{30}) + \frac{1}{2}k(-\epsilon_{00} + \epsilon_{11} + \epsilon_{22} + \epsilon_{33}) = 0$

$$k^\mu (\epsilon_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \epsilon^\lambda{}_\lambda) = 0$$

1: $k(\epsilon_{03} + \epsilon_{33}) - \frac{1}{2}k(-\epsilon_{00} + \epsilon_{11} + \epsilon_{22} + \epsilon_{33}) = 0$

2: $k(\epsilon_{01} + \epsilon_{31}) = 0$

3: $k(\epsilon_{02} + \epsilon_{32}) = 0$

}

$$\epsilon_{01} = -\epsilon_{31}$$

$$\epsilon_{02} = -\epsilon_{32}$$

$$\epsilon_{03} = -\frac{1}{2}(\epsilon_{00} + \epsilon_{33})$$

$$\epsilon_{12} = -\epsilon_{11}$$

- have eliminated 4 comps in terms of other 6.

Still have freedom $\epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + k_\mu C_\nu + k_\nu C_\mu$

$$3: \quad k(\epsilon_{02} + \epsilon_{32}) = 0$$

- have eliminated 4 constraints in terms of other 6.

Still have freedom $\epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + k_\mu C_\nu + k_\nu C_\mu$

$$\Rightarrow \epsilon'_{00} = \epsilon_{00} - 2kC_0$$

$$\epsilon'_{13} = \epsilon_{13} + kC_1$$

$$\epsilon'_{23} = \epsilon_{23} + kC_2$$

$$\epsilon'_{33} = \epsilon_{33} + 2kC_3$$

} can choose C_μ to set $\epsilon'_{00} = \epsilon'_{13} = \epsilon'_{23} = \epsilon'_{33} = 0$.

$$\Rightarrow \epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{11} & \epsilon_{12} & 0 \\ 0 & \epsilon_{21} & -\epsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

this is now transverse and traceless.

can always ^{express} a linearised gravitational wave as a superposition of such waves.

instead of 10 complex numbers $\epsilon_{\mu\nu}$,

we only have 2. $\Rightarrow$ g-waves have 2 independent polarisations.

$$\begin{aligned} \epsilon_{00} &= \epsilon_{00} - 2k\epsilon_0 \\ \epsilon'_{13} &= \epsilon_{13} + k\epsilon_1 \\ \epsilon'_{23} &= \epsilon_{23} + k\epsilon_2 \\ \epsilon'_{33} &= \epsilon_{33} + 2k\epsilon_3 \end{aligned}$$

Can choose $C_{\mu\nu}$ to set $\epsilon_{00} = \epsilon_{13} = \epsilon_{23} = \epsilon_{33} = 0$.

$$\Rightarrow \epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{11} & \epsilon_{12} & 0 \\ 0 & \epsilon_{12} - \epsilon_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

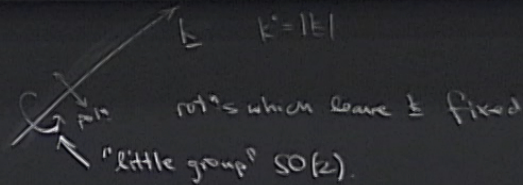
this is now transverse and traceless

instead of 10 complex numbers $C_{\mu\nu}$, we only have 2.

Can always ^{express} a linearised gravitational wave as a superposition of such waves. $\Rightarrow$ g-waves have 2 independent polarisations.

$$h \rightarrow A^\mu = (0, \epsilon_1, \epsilon_2, 0)$$

Spin of graviton.



$$\epsilon'_{ij} = O_{il} O_{jm} \epsilon_{lm} \rightarrow \epsilon' = O \epsilon O^T$$

in our case, little gp = rotations about z axis

$$\begin{pmatrix} \epsilon'_{11} & \epsilon'_{12} \\ \epsilon'_{12} - \epsilon'_{11} & \epsilon'_{22} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ -\epsilon_{12} & \epsilon_{11} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

gravitational waves

$$+h_{\mu\nu} \quad (\text{to linear order in } \epsilon \text{ and } h)$$

$$\left(\delta_{\mu}^{\alpha} - \epsilon_{\mu}^{\alpha} \right) \left(\delta_{\nu}^{\beta} - \epsilon_{\nu}^{\beta} \right) \eta_{\alpha\beta}$$

$$\Rightarrow h'_{\mu\nu} = h_{\mu\nu}$$

$SO(2)$ is abelian $\Rightarrow$ all its irreps are 1-dim^{al}.

$$\left. \begin{aligned} E_R &= E_{11} - iE_{22} \equiv E_+ - iE_x \\ E_L &= E_{11} + iE_{22} \equiv E_+ + iE_x \end{aligned} \right\} \begin{aligned} E'_R &= e^{2i\theta} E_R \\ E'_L &= e^{-2i\theta} E_L \end{aligned}$$

gravitational waves

thus.

$$+h_{\mu\nu} \quad (\text{to linear order in } \epsilon \text{ and } h)$$

$$\left(\delta_{\mu}^{\alpha} - \epsilon_{\mu}^{\alpha} \right) \left(\delta_{\nu}^{\beta} - \epsilon_{\nu}^{\beta} \right) \eta_{\alpha\beta}$$

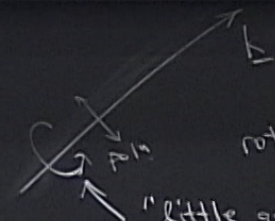
$$\Rightarrow h'_{\mu\nu} = h_{\mu\nu}$$

$SO(2)$ is abelian $\Rightarrow$ all its irreps are 1-dim^{al}.

$$\left. \begin{aligned} \epsilon_R &= \epsilon_{11} - i\epsilon_{22} \equiv \epsilon_+ - i\epsilon_x \\ \epsilon_L &= \epsilon_{11} + i\epsilon_{22} \equiv \epsilon_+ + i\epsilon_x \end{aligned} \right\} \begin{aligned} \epsilon'_R &= e^{2i\theta} \epsilon_R \\ \epsilon'_L &= e^{-2i\theta} \epsilon_L \end{aligned} \Rightarrow \text{Spin } 2$$

$$\begin{aligned} \text{EM} \\ A_\mu \rightarrow (\epsilon'_1, \epsilon'_2) &= O \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \Rightarrow \begin{aligned} \epsilon_+ &= \epsilon_1 - i\epsilon_2 \\ \epsilon_- &= \epsilon_1 + i\epsilon_2 \end{aligned} \Rightarrow \begin{aligned} \epsilon'_+ &= e^{i\theta} \epsilon_+ \\ \epsilon'_- &= e^{-i\theta} \epsilon_- \end{aligned} \end{aligned} \left. \vphantom{\begin{aligned} \epsilon'_+ &= e^{i\theta} \epsilon_+ \\ \epsilon'_- &= e^{-i\theta} \epsilon_- \end{aligned}} \right\} \text{photon is Spin } 1$$

instead of 10 complex numbers $\epsilon_{\mu\nu}$, wave as a superposition of such waves.
 we only have 2. $\Rightarrow$ g-waves have 2 independent polarisations.

uniton :  $k = |k|$
 rotations which leave k f
 "little group" $SO(2)$.

$$\epsilon'_{ij} = O_{ie} O_{jm} \epsilon_{em} \rightarrow \epsilon' = O \epsilon O^T$$

in our case, little
 gp = rotations about z
 axis

$$\begin{pmatrix} \epsilon'_{11} & \epsilon'_{12} \\ \epsilon'_{12} & \epsilon'_{22} \end{pmatrix}$$

$$\begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ -\epsilon_{12} & -\epsilon_{11} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

If e^μ satisfies $D e^\mu = 0$, we remain in harmonic gauge. can express

Gravitational Radiation

EM $\Rightarrow \partial_\mu F^{\mu\nu} = -J^\nu$
 Lorentz $\downarrow$
 $\square A^\nu$ rad
 time-varying J^ν
 leads to emission of EM waves

GR $R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G T^{\mu\nu}$
 $\Rightarrow R^{\mu\nu} = 8\pi G (T^{\mu\nu} - \frac{1}{2} g^{\mu\nu} T^\lambda{}_\lambda)$
 $= -D h_{\mu\nu}$ in TT gauge

 $G = \frac{1}{M_{Pl}^2}$
 $\rho \sim M^4$

$$\partial_\mu g^{\mu\nu} = 0 \quad \partial_\mu g^{\mu\nu} = 4$$

$\times g^{\mu\nu}$

$$R - 2R = 8\pi G T^\mu{}_\mu \Rightarrow R = -8\pi G T^\mu{}_\mu$$

$$\Rightarrow h_{\mu\nu} = -4G \int \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|} S_{\mu\nu}(\mathbf{x}', t - |\mathbf{x} - \mathbf{x}'|)$$

retarded time

$$T_{\mu\nu} = \frac{1}{2} \rho u_\mu u_\nu$$

treat $T^\mu{}_\nu$ as if it is in Minkowski.



$$P = \frac{1}{45} \frac{Q_{\mu\nu} Q^{\mu\nu}}{c^5}$$

Landau-Lifshitz formula

- confirmed for the binary pulsar in 1990s. (Hulse + Taylor).

If C^μ satisfies $D_\mu C^\mu = 0$, we remain in harmonic gauge. can express

Gravitational Radiation

$$\partial_\mu g^{\mu\nu} = \delta^\mu_\nu \quad \partial_\mu g^{\mu\mu} = 4$$

$$\times g^{\mu\nu}$$

EM $\Rightarrow \partial_\mu F^{\mu\nu} = -J^\nu$
 Lorentz $\downarrow$
 $\Delta A^\nu_{\text{rad}}$
 time-varying J^ν
 leads to emission of EM waves



GR $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G T^{\mu\nu} \Rightarrow$
 source $S^{\mu\nu}$
 $\Rightarrow R^{\mu\nu} = 8\pi G (T^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T^\lambda_\lambda)$
 radiation
 $= -Dh_{\mu\nu}$ in TT gauge

$R - 2R = 8\pi G T^\mu_\mu \Rightarrow R = -8\pi G T^\mu_\mu$
 $\Rightarrow h_{\mu\nu} = -4G \int \frac{d^3x'}{|\mathbf{x}-\mathbf{x}'|} S_{\mu\nu}(t-|\mathbf{x}-\mathbf{x}'|, \mathbf{x}')$
 retarded time
 $\frac{1}{M_{\text{Pl}}^2} \frac{M^4 L^3}{c^3} T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\lambda_\lambda$, treat T^μ_ν as if it's in Minkowski.
 $h \sim \frac{G T L^3}{c^3}$ dimensionless $\ll 1$

$G = \frac{1}{M_{\text{Pl}}^2}$
 $\rho \sim M^4 = 1$

$P = \frac{1}{45} \frac{Q_{\text{EE}} Q_{\text{EE}}}{c^5}$
 Landau-Lifshitz formula

$Q_{\text{EE}} = \frac{d^3 Q_{\text{EE}}}{dt^3}$

- confirmed for the binary pulsar in 1990s. (Hulse + Taylor)

