Title: PSI 17/18 - Relativity - Lecture 11

Date: Sep 19, 2017 09:00 AM

URL: http://pirsa.org/17090030

Abstract:

recap

$$R^{\sigma}{}_{\rho\mu\nu} = -\Gamma^{\sigma}_{\rho\mu,\nu} + \Gamma^{\sigma}_{\rho\mu}\Gamma^{\beta}_{\rho\nu} - (\mu \leftrightarrow \nu)$$

$$\simeq \frac{1}{2}\eta^{\sigma\lambda}\left(g_{\nu\lambda,\rho\mu} - g_{\rho\nu,\lambda\mu} - g_{\mu\lambda,\rho\nu} + g_{\rho\mu,\lambda\nu}\right)$$

recap

$$R^{\sigma}{}_{\rho\mu\nu} = -\Gamma^{\sigma}_{\rho\mu,\nu} + \Gamma^{\sigma}_{\rho\mu}\Gamma^{\beta}_{\rho\nu} - (\mu \leftrightarrow \nu)$$

$$g_{\mu\nu} \approx \eta_{\mu\nu} + \tfrac{1}{2} g^{(0)}_{\mu\nu,\rho\lambda} x^{\rho} x^{\lambda}$$

$$\simeq \tfrac{1}{2}\eta^{\sigma\lambda}\left( g_{\nu\lambda,\rho\mu} - g_{\rho\nu,\lambda\mu} - g_{\mu\lambda,\rho\nu} + g_{\rho\mu,\lambda\nu}\right)$$

"locally Minkowski" coordinates

recap

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$$\Downarrow$$

$$g^{(0)}_{\mu\nu,\lambda} = 0$$

$$\Downarrow$$

$$\Gamma^{\lambda}_{\mu\nu}(0) = 0$$

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recap

in locally Minkowski coordinates

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$$g^{(0)}_{\mu\nu,\lambda} = 0$$

$$\Gamma^{\sigma}_{\rho\mu}(0) = 0$$

$R_{\sigma\rho\mu\nu}$ : 1) symm under exchange of 1st pair and 2nd pair of indices $(= R_{\mu\nu\sigma\rho})$

2) antisymm on 1st pair and 2nd pair of indices $(= -R_{\rho\sigma\mu\nu}, = -R_{\sigma\rho\nu\mu})$

recap

in locally Minkowski coordinates

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$$g^{(0)}_{\mu\nu,\lambda} = 0$$

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1) symm under exchange of 1st pair and 2nd pair of indices $(= R_{\mu\nu\sigma\rho})$

2) antisymm on 1st pair and 2nd pair of indices $(= -R_{\rho\sigma\mu\nu}, = -R_{\sigma\rho\nu\mu})$

3) cyclic in last 3 indices $R_{\sigma\rho\mu\nu} + R_{\sigma\mu\nu\rho} + R_{\sigma\nu\rho\mu} = 0.$

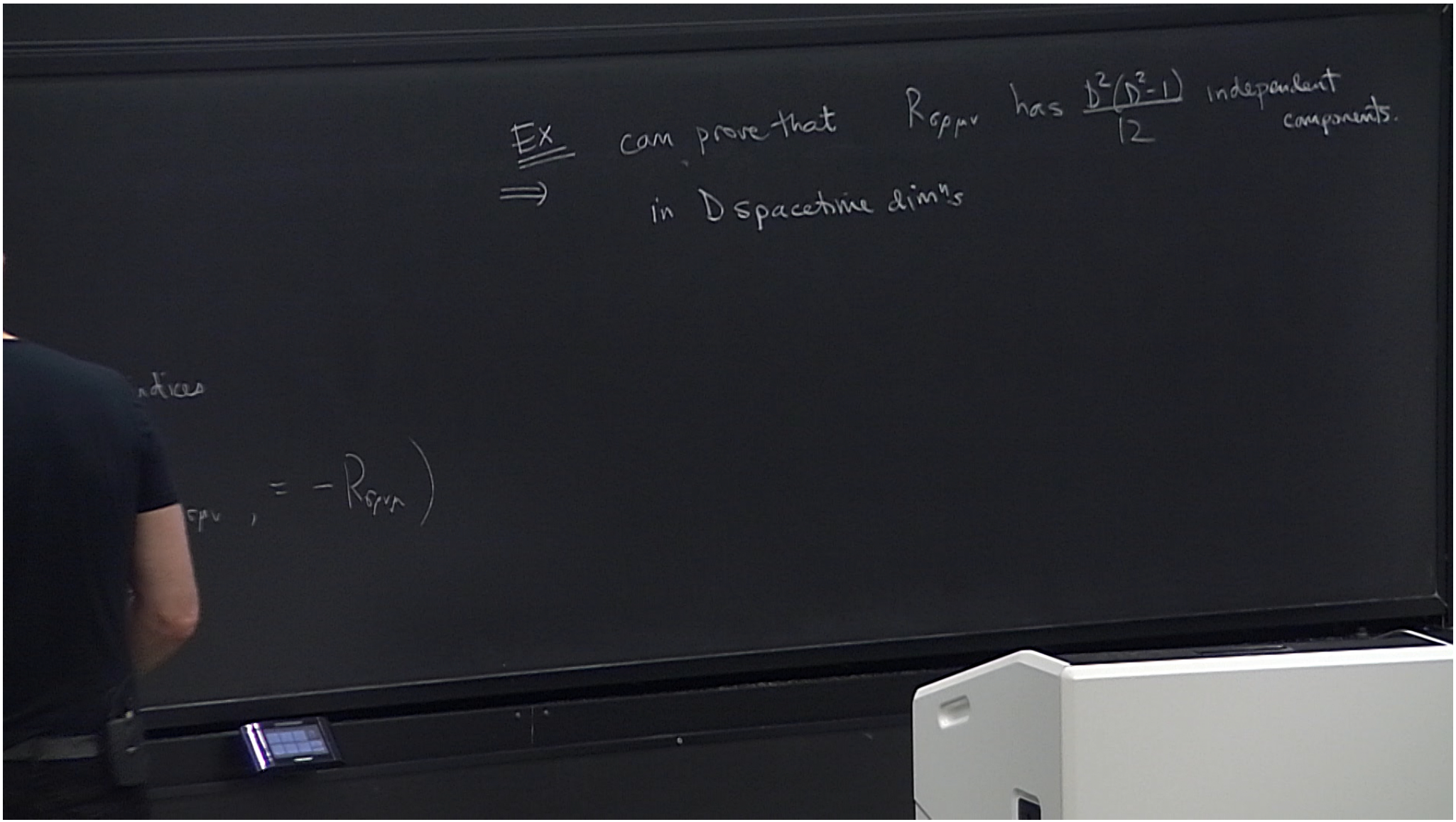
$\underline{\underline{\text{Ex}}}$ can prove that $R_{\sigma\rho\mu\nu}$ has $\frac{D^2(D^2-1)}{12}$ independent components.

$\Longrightarrow$

d pair of indices

$\left(= -R_{\rho\sigma\mu\nu}\ ,\ = -R_{\sigma\rho\nu\mu}\right)$

$R_{\sigma\nu\rho\mu} = 0.$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ndices
$\sigma\rho\mu\nu$ , $= -R_{\sigma\rho\nu\mu}$ )

recap:

$$A X^{\mu\nu} = 8\pi G T^{\mu\nu}$$

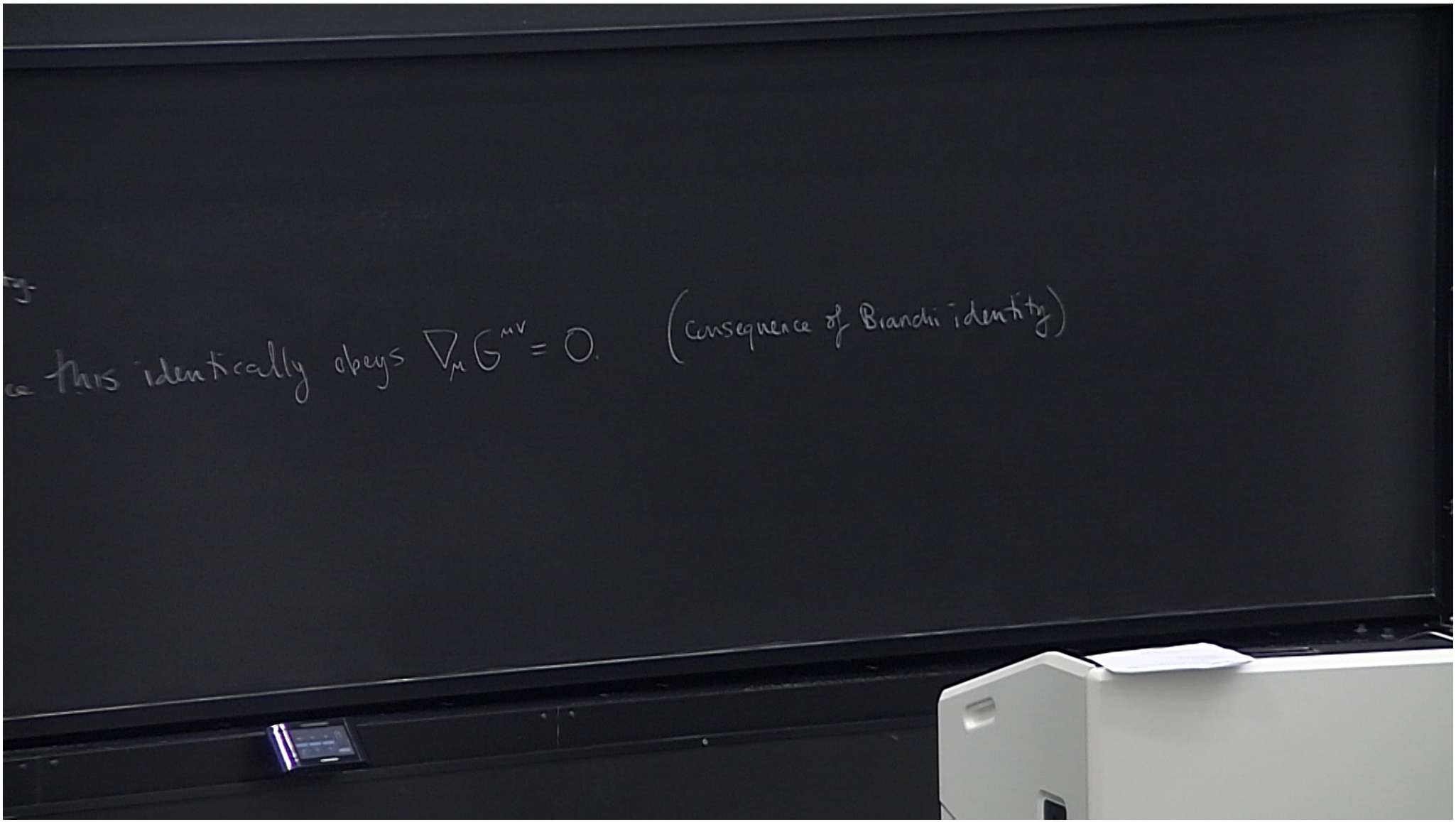
where $\nabla_\mu X^{\mu\nu} = 0 \quad \forall\, g_{\mu\nu}$.

Recap:

$$A X^{\mu\nu} = 8\pi G T^{\mu\nu}$$

where $\nabla_\mu X^{\mu\nu} = 0 \quad \forall\, g_{\mu\nu}$. as an Identity.

Candidate for $X^{\mu\nu} = G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$ , since this identically obeys $\nabla_\mu G^{\mu\nu} = 0$.


this identically obeys $\nabla_\mu G^{\mu\nu} = 0$.
(consequence of Bianchi identity)

recap:

Poisson $\longleftrightarrow$

$\nabla^2 \phi_g = 4\pi G \rho_m$

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recap:

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would like to fix const A by comparing with the Newtonian limit.

First, consider spatial components of (1): $|T^{ij}| \ll T^{00}$

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st A by comparing with the Newtonian limit.

sider spatial components of (1) : $|T^{ij}| \ll T^{00} \Rightarrow G^{ij}$ is small i.e. $R^{ij} \approx \frac{1}{2}\delta^{ij} R$

but $R = g^{\mu\nu} \underbrace{R_{\mu\nu}}_{\sim \partial^2 h_{\mu\nu}} \approx \eta^{\mu\nu} R_{\mu\nu} = -R_{00} + R_{ii}$

$g \sim \eta + h$

st A by comparing with the Newtonian limit.

nsider spatial components of ①: $|T^{ij}| \ll T^{00} \Rightarrow G^{ij}$ is small i.e. $R^{ij} \approx \frac{1}{2}\delta^{ij} R$ (*)

$$R = g^{\mu\nu} \underbrace{R_{\mu\nu}}_{\sim \partial^2 h_{\mu\nu}} \approx \eta^{\mu\nu} R_{\mu\nu} = -R_{00} + R_{ii} = -R_{00} + \frac{3}{2} R \quad \text{from } (*).$$

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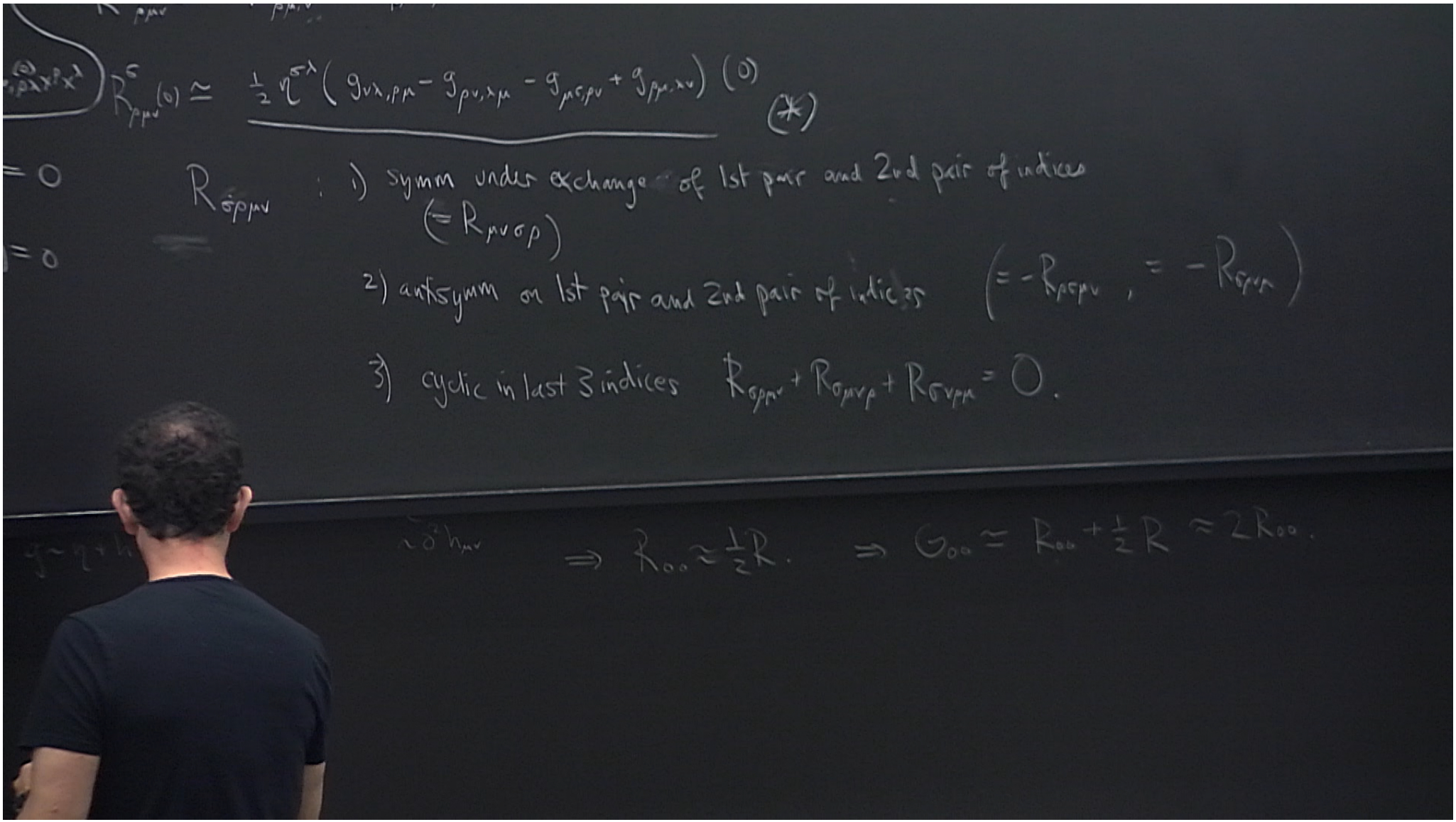
$g \sim \eta + h$ $\quad \Rightarrow R_{00} \approx \frac{1}{2} R. \quad \Rightarrow G_{00} \simeq R_{00} + \frac{1}{2} R \approx 2 R_{00}.$


$R^{\sigma}_{\rho\mu\nu}(0) \simeq \frac{1}{2}\eta^{\sigma\lambda}\left(g_{\nu\lambda,\rho\mu} - g_{\rho\nu,\lambda\mu} - g_{\mu\sigma,\rho\nu} + g_{\rho\mu,\lambda\nu}\right)(0)$ (*)
$R_{\sigma\rho\mu\nu}$ : 1) symm under exchange of 1st pair and 2nd pair of indices
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2) antisymm on 1st pair and 2nd pair of indices $(= -R_{\rho\sigma\mu\nu}, = -R_{\sigma\rho\nu\mu})$
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$\sim \partial^2 h_{\mu\nu}$ $\Rightarrow R_{00} \approx \frac{1}{2} R. \quad \Rightarrow G_{00} \simeq R_{00} + \frac{1}{2} R \approx 2 R_{00}.$

But from (*), $R_{\sigma\rho\mu\nu} = \frac{1}{2}\left(h_{\nu\sigma,\rho\mu} - h_{\rho\nu,\sigma\mu} - h_{\mu\sigma,\rho\nu} + h_{\rho\mu,\sigma\nu}\right) + o(h^2)$

$\rightarrow R_{00} = \eta^{\sigma\mu} R_{\sigma 0\mu 0}$

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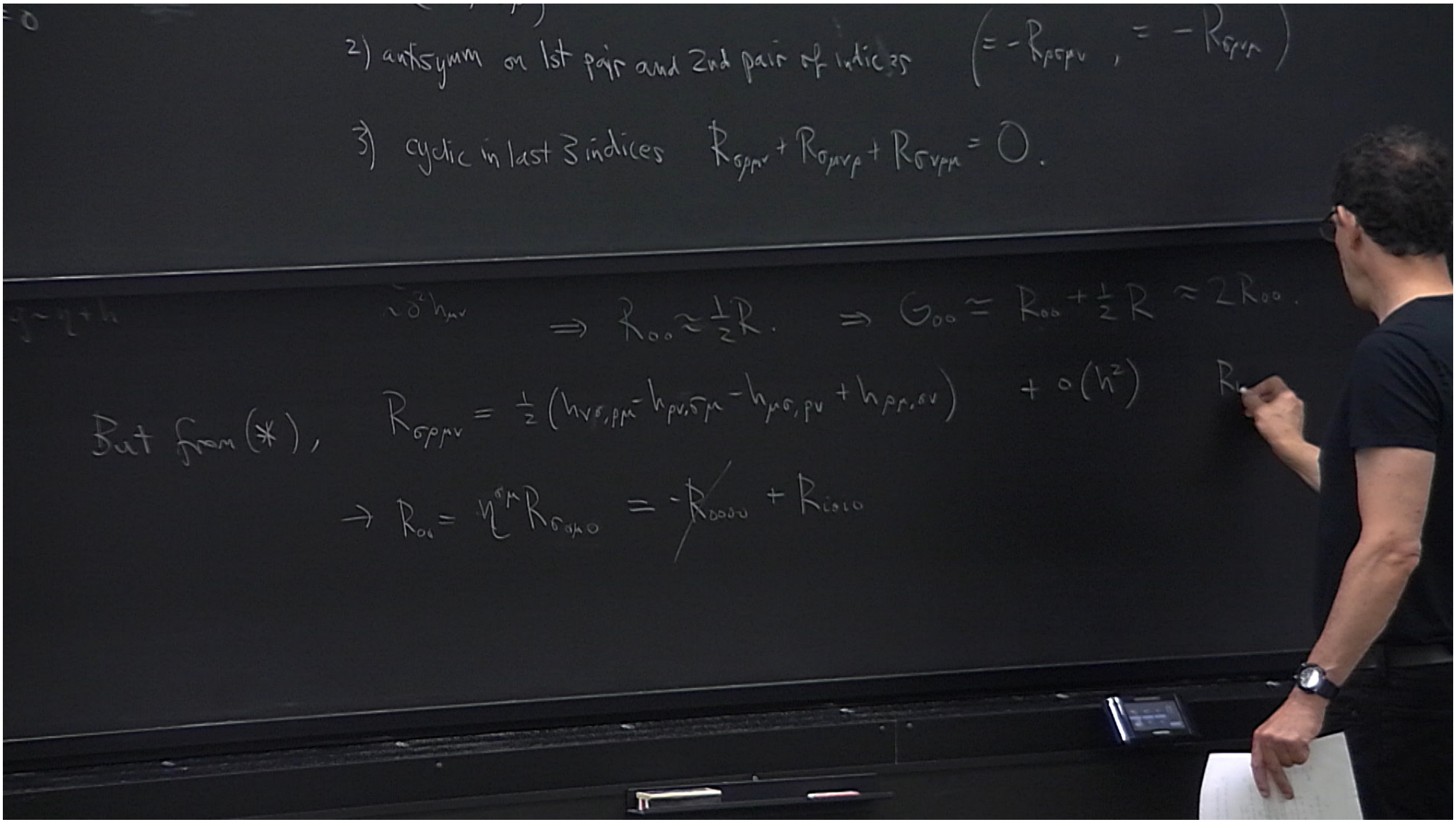
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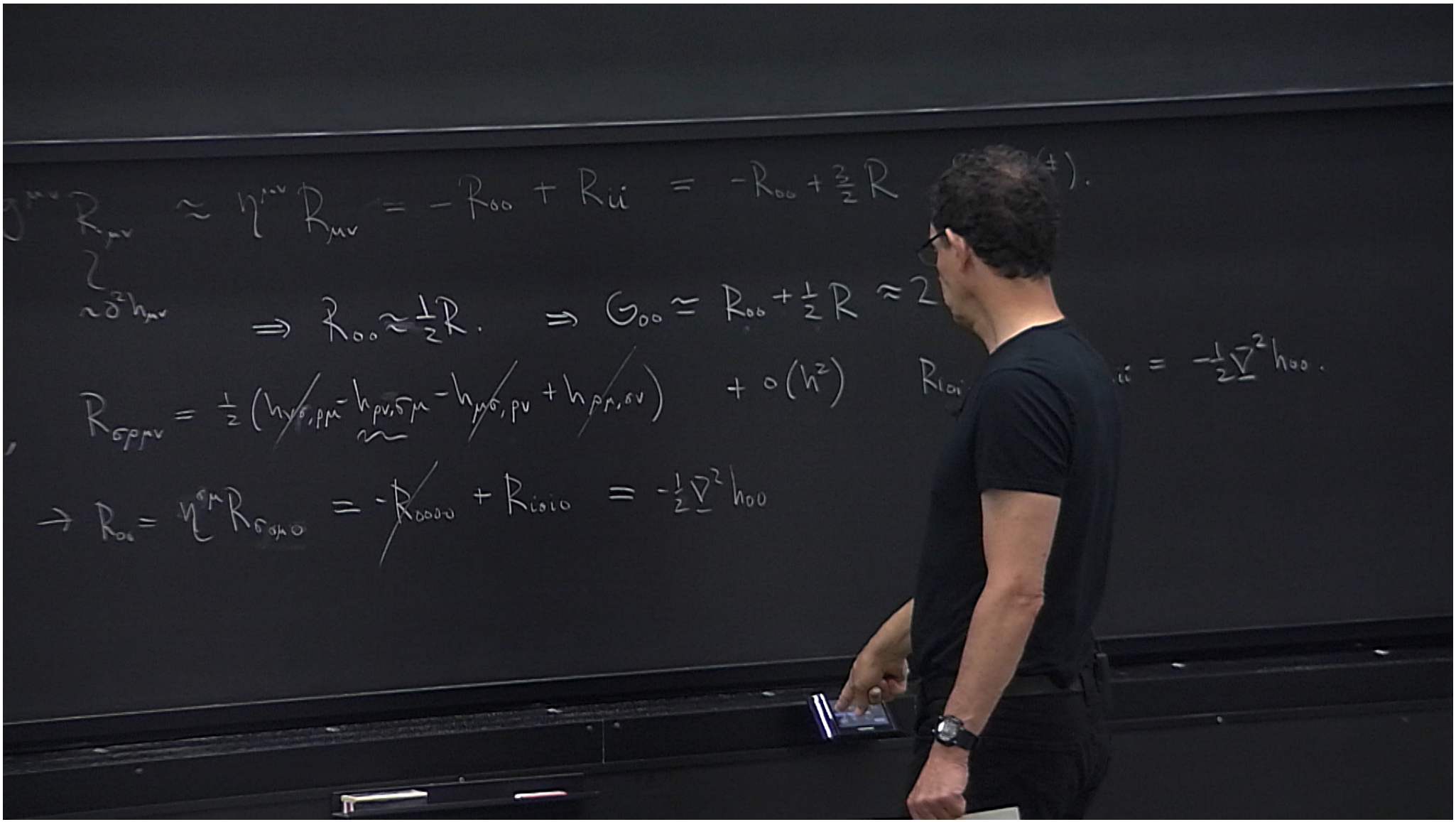
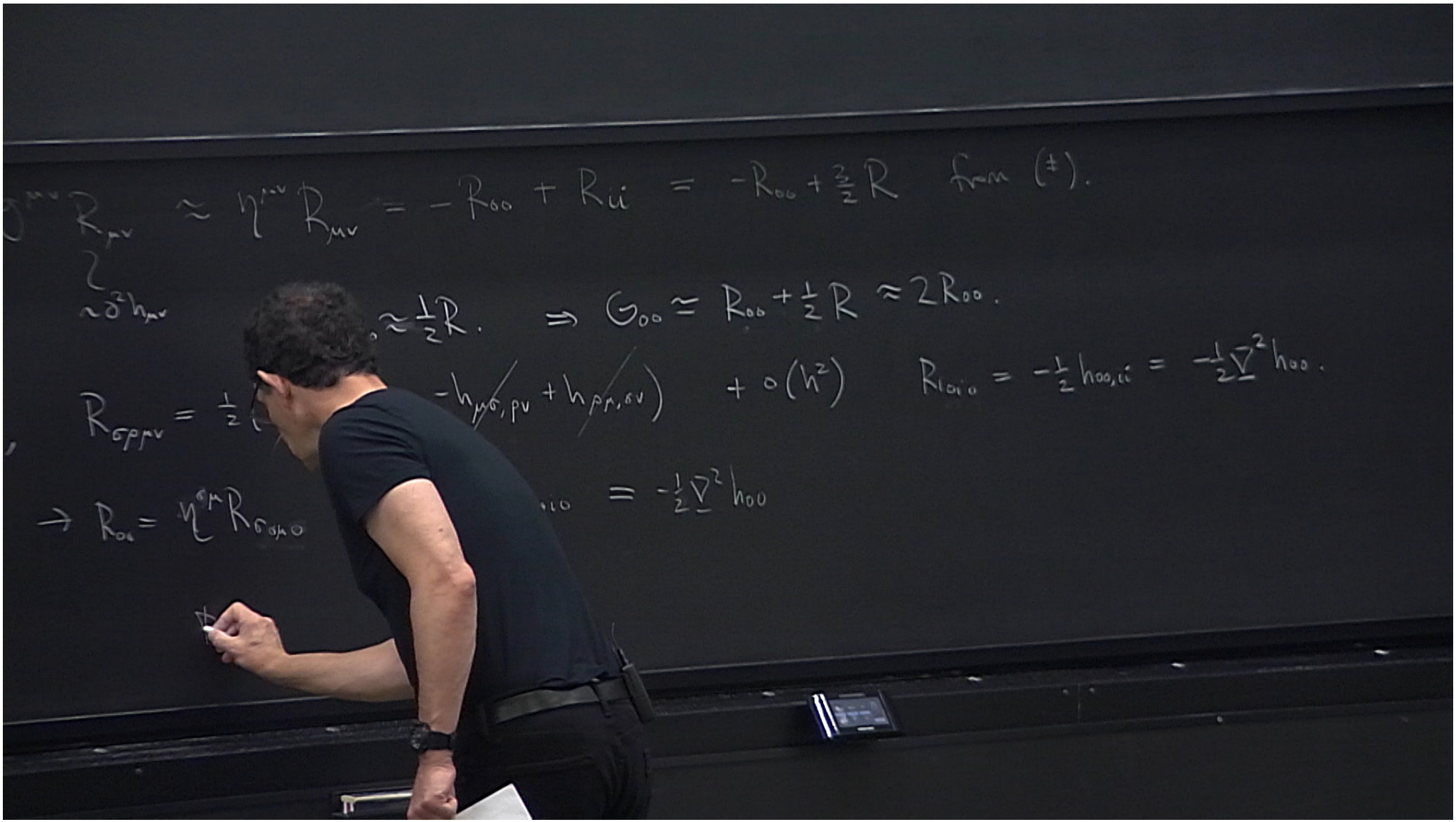
$\rightarrow R_{00} = \eta^{\sigma\mu} R_{\sigma 0\mu 0} = -\cancel{R_{0000}} + R_{i0i0}$


$g^{\mu\nu} R_{\mu\nu} \approx \eta^{\mu\nu} R_{\mu\nu} = -R_{00} + R_{ii} = -R_{00} + \frac{3}{2} R$
$\sim \partial^2 h_{\mu\nu}$
$\Rightarrow R_{00} \approx \frac{1}{2} R. \quad \Rightarrow G_{00} \simeq R_{00} + \frac{1}{2} R \approx 2$
$R_{\sigma\rho\mu\nu} = \frac{1}{2}(h_{\mu\nu,\rho\mu} - h_{\rho\nu,\sigma\mu} - h_{\mu\sigma,\rho\nu} + h_{\rho\mu,\sigma\nu}) + o(h^2)$
$\rightarrow R_{00} = \eta^{\sigma\mu} R_{\sigma 0\mu 0} = -R_{0000} + R_{i0i0} = -\frac{1}{2}\nabla^2 h_{00}$
$= -\frac{1}{2}\nabla^2 h_{00}.$


$g^{\mu\nu} R_{\mu\nu} \approx \eta^{\mu\nu} R_{\mu\nu} = -R_{00} + R_{ii} = -R_{00} + \frac{3}{2} R$ from (‡).
$\sim \partial^2 h_{\mu\nu}$
$\approx \frac{1}{2} R$. $\Rightarrow G_{00} \simeq R_{00} + \frac{1}{2} R \approx 2 R_{00}$.
$R_{\sigma\rho\mu\nu} = \frac{1}{2} ( \ldots - h_{\mu\sigma,\rho\nu} + h_{\rho\mu,\sigma\nu} ) + o(h^2)$
$R_{i0i0} = -\frac{1}{2} h_{00,ii} = -\frac{1}{2} \nabla^2 h_{00}$.
$\rightarrow R_{00} = \eta^{\sigma\mu} R_{\sigma 0 \mu 0} \ldots = -\frac{1}{2} \nabla^2 h_{00}$

$$g^{\mu\nu} R_{\mu\nu} \approx \eta^{\mu\nu} R_{\mu\nu} = -R_{00} + R_{ii} = -R_{00} + \tfrac{3}{2} R \quad \text{from } (\ddagger).$$

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$$\rightarrow R_{00} = \eta^{\sigma\mu} R_{\sigma 0\mu 0} = -\cancel{R_{0000}} + R_{i0i0} = -\tfrac{1}{2} \underline{\nabla}^2 h_{00}$$

Back to ①: $$A G^{00} \overset{(\ddagger)}{\approx} 2 A R^{00} \approx -A \underline{\nabla}^2 h_{00} \overset{?}{=} 8\pi G T^{00} = 8\pi G \rho_m$$

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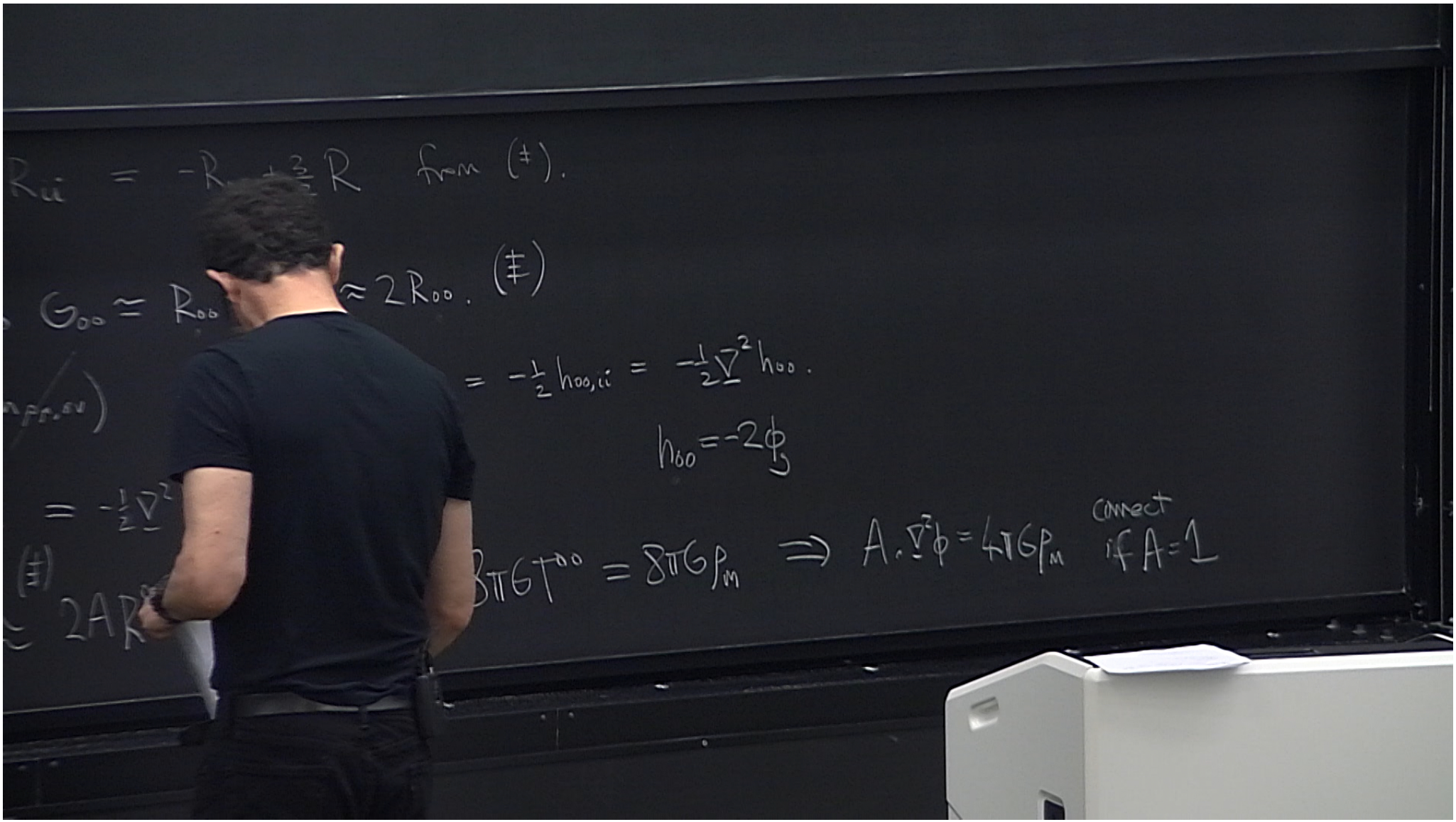
$$h_{00} = -2\phi$$

$$\rightarrow R_{00} = \eta^{\sigma\mu} R_{\sigma 0\mu 0} = -\cancel{R_{0000}} + R_{i0i0} = -\tfrac{1}{2} \underline{\nabla}^2 h_{00}$$

Back to ①: $$A G^{00} \overset{(\ddagger)}{\approx} 2 A R^{00} \approx -A \underline{\nabla}^2 h_{00} \overset{?}{=} 8\pi G T^{00} = 8\pi G \rho_m$$


Rii = -R + 3/2 R from (‡).
G00 ≃ R00 ≈ 2R00. (‡)
= -1/2 h00,ii = -1/2 ∇² h00.
h00 = -2φ
= -1/2 ∇²
(‡)
≃ 2AR
8πGT00 = 8πGρM ⇒ A·∇²φ = 4πGρM
connect if A=1

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \Longrightarrow \text{ Poisson eq}^{n} \text{ in Newtonian limit.}$$

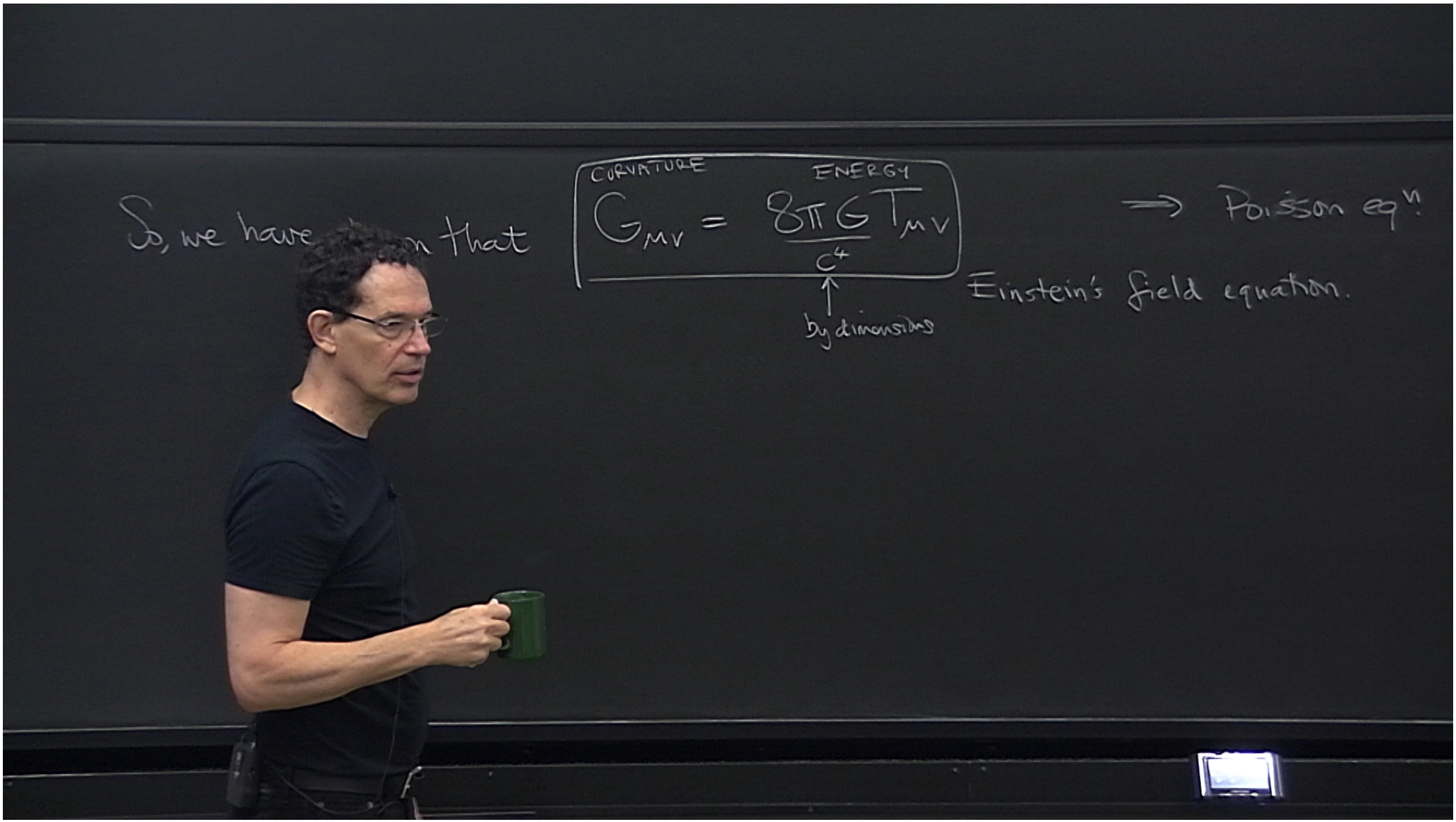
So, we have shown that $\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$ $\Longrightarrow$ Poisson eq$^{n}$!

Einstein's field equation.

So, we have shown that $\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$ $\Longrightarrow$ Poisson eq$^n$!

Einstein's field equation.

by dimensions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CURVATURE
ENERGY
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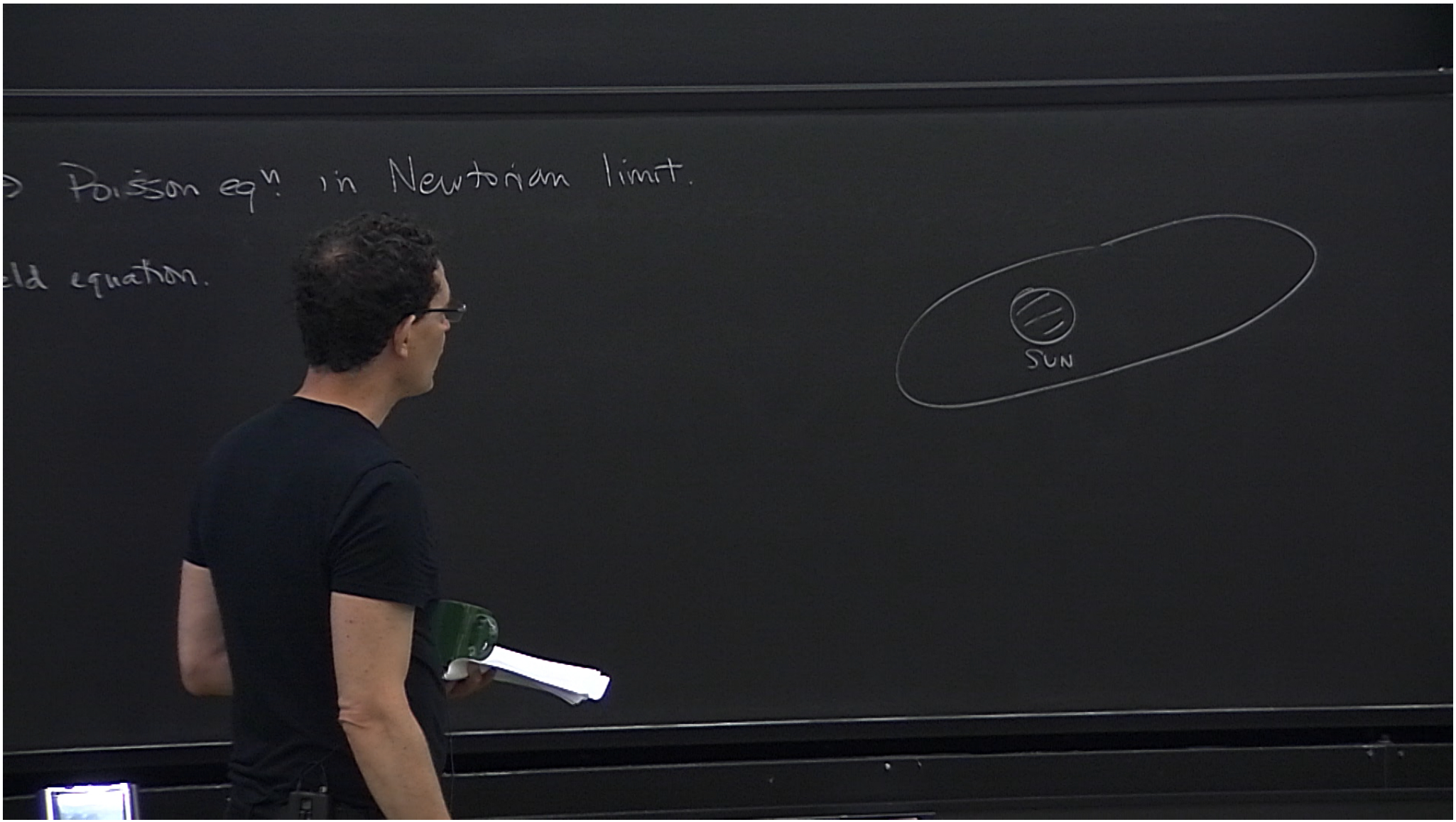
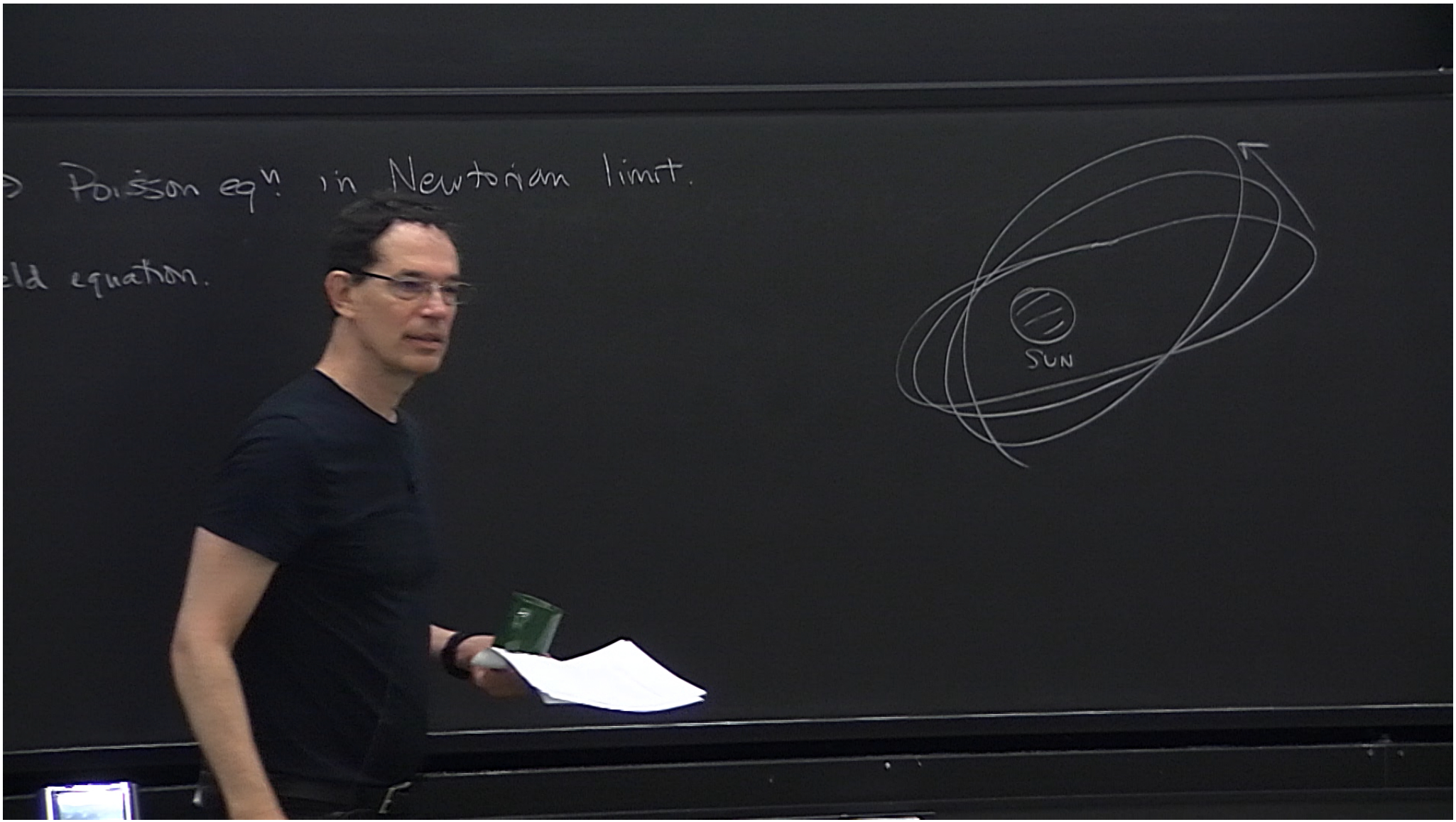
by dimensions
→ Poisson eq$^n$!
Einstein's field equation.


Poisson eqn in Newtonian limit.
eld equation.
SUN


Poisson eqn in Newtonian limit.
eld equation.
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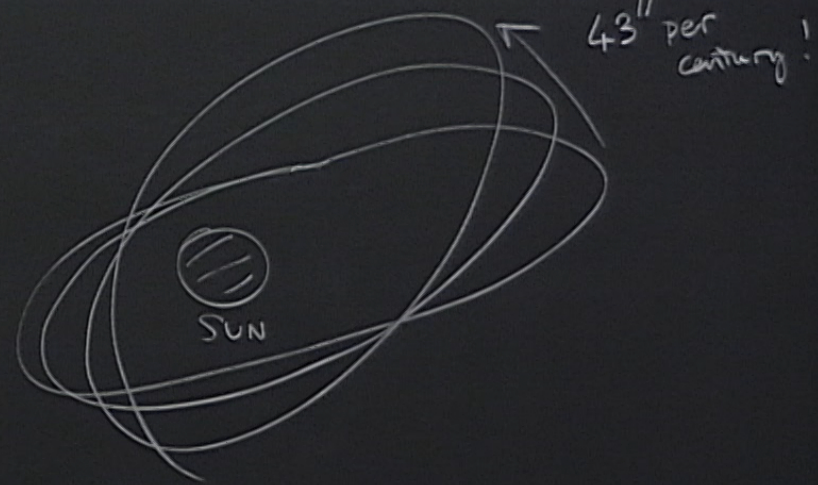
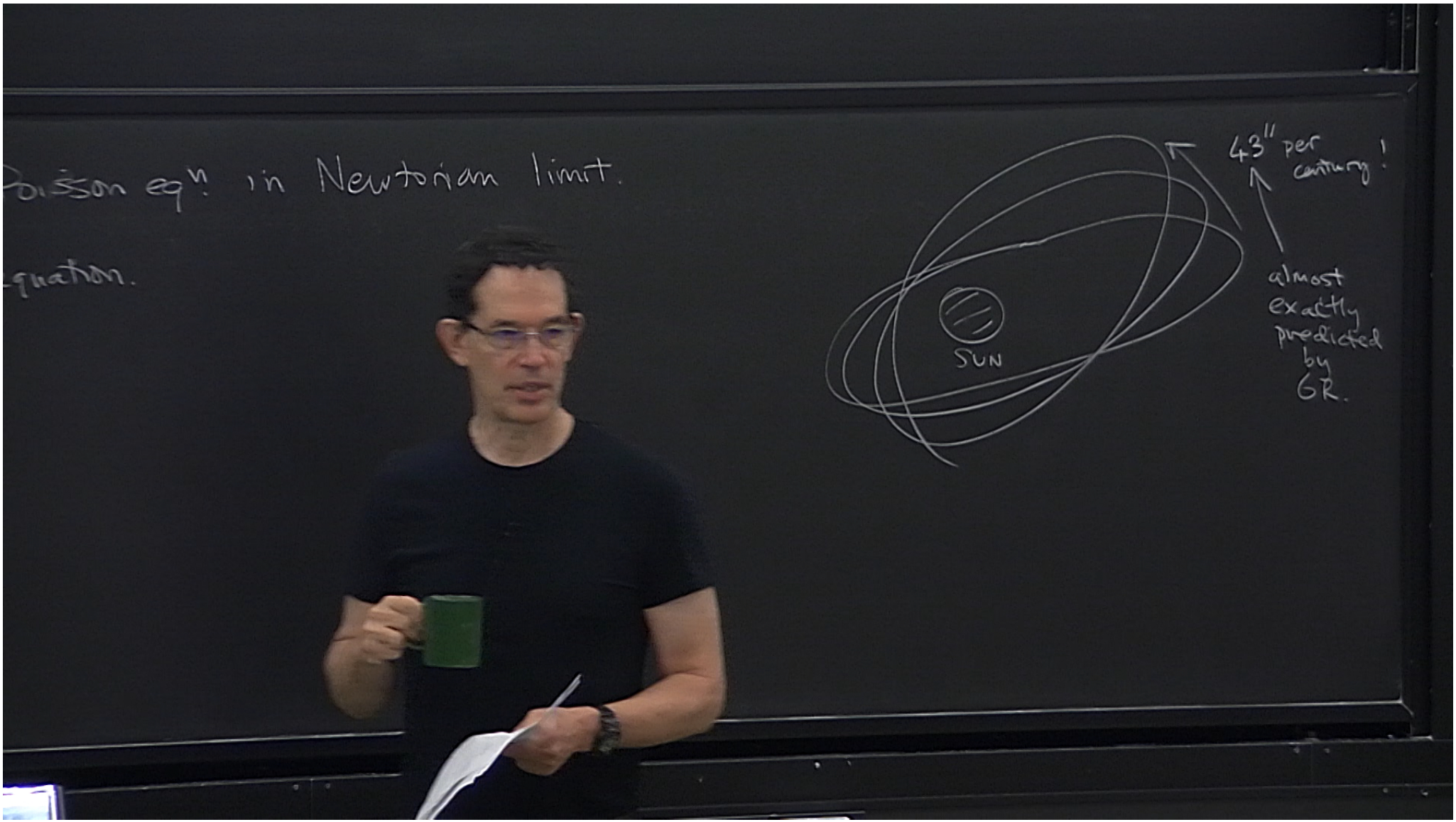
Poisson eq$^{n}$ in Newtonian limit.

equation.




Poisson eqn in Newtonian limit.
equation.
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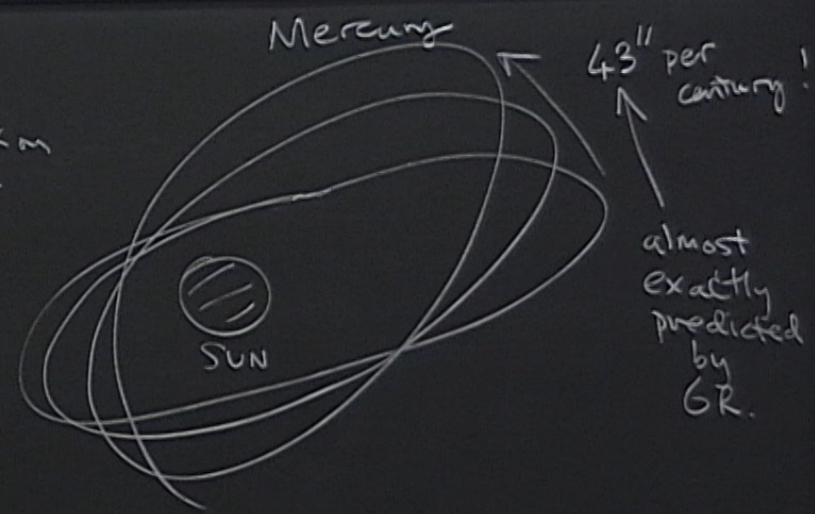
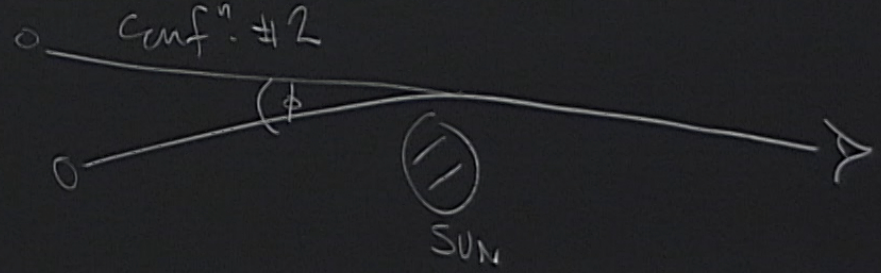
43" per century!
almost exactly predicted by GR.

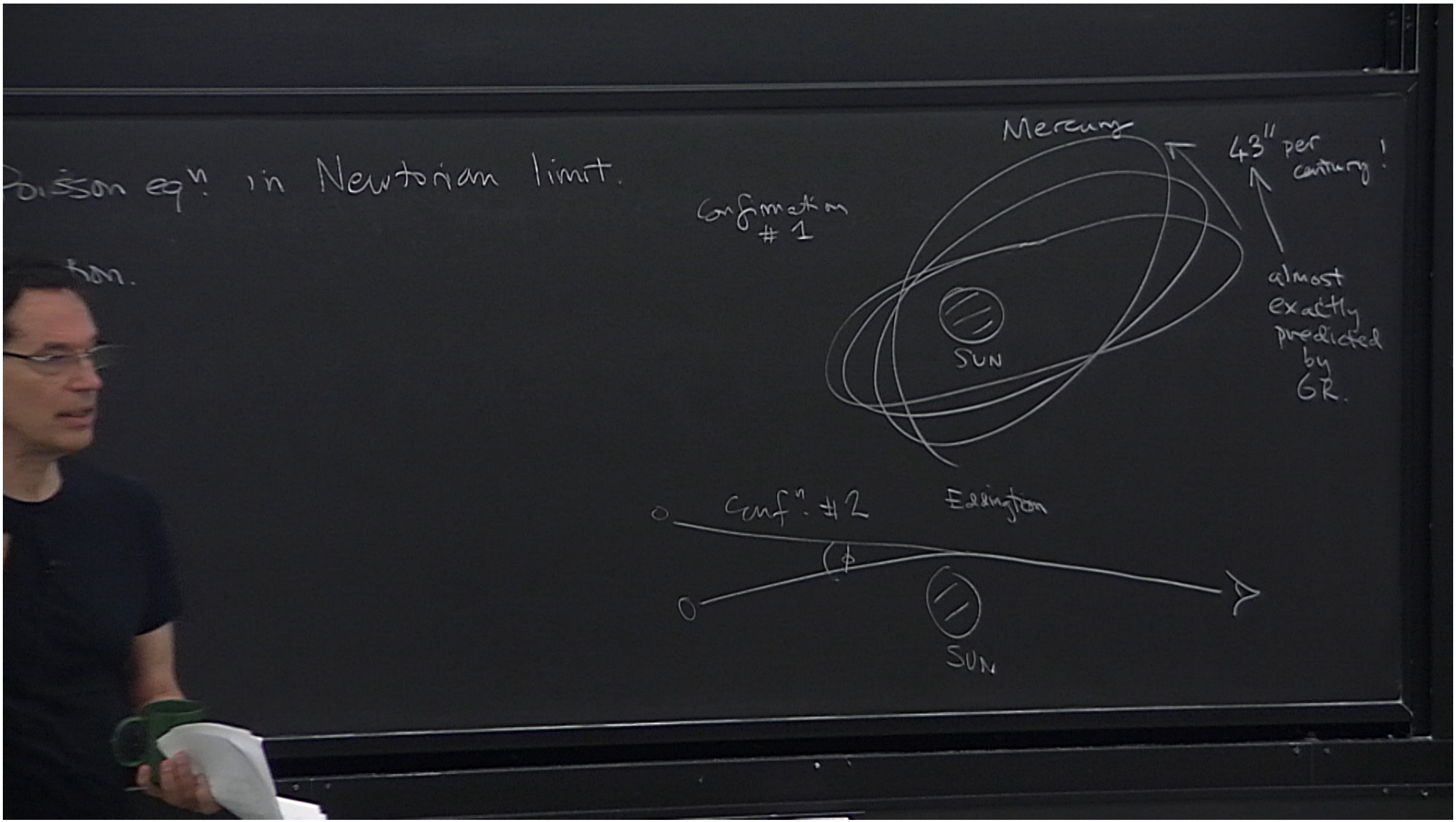
Poisson eq$^n$ in Newtonian limit.

equation.

Configuration # 1




Poisson eqn in Newtonian limit.
tion.
Mercury
43'' per century!
Confirmation # 1
SUN
almost exactly predicted by GR.
Conf^n. #2
Eddington
φ
SUN

So, we have shown that

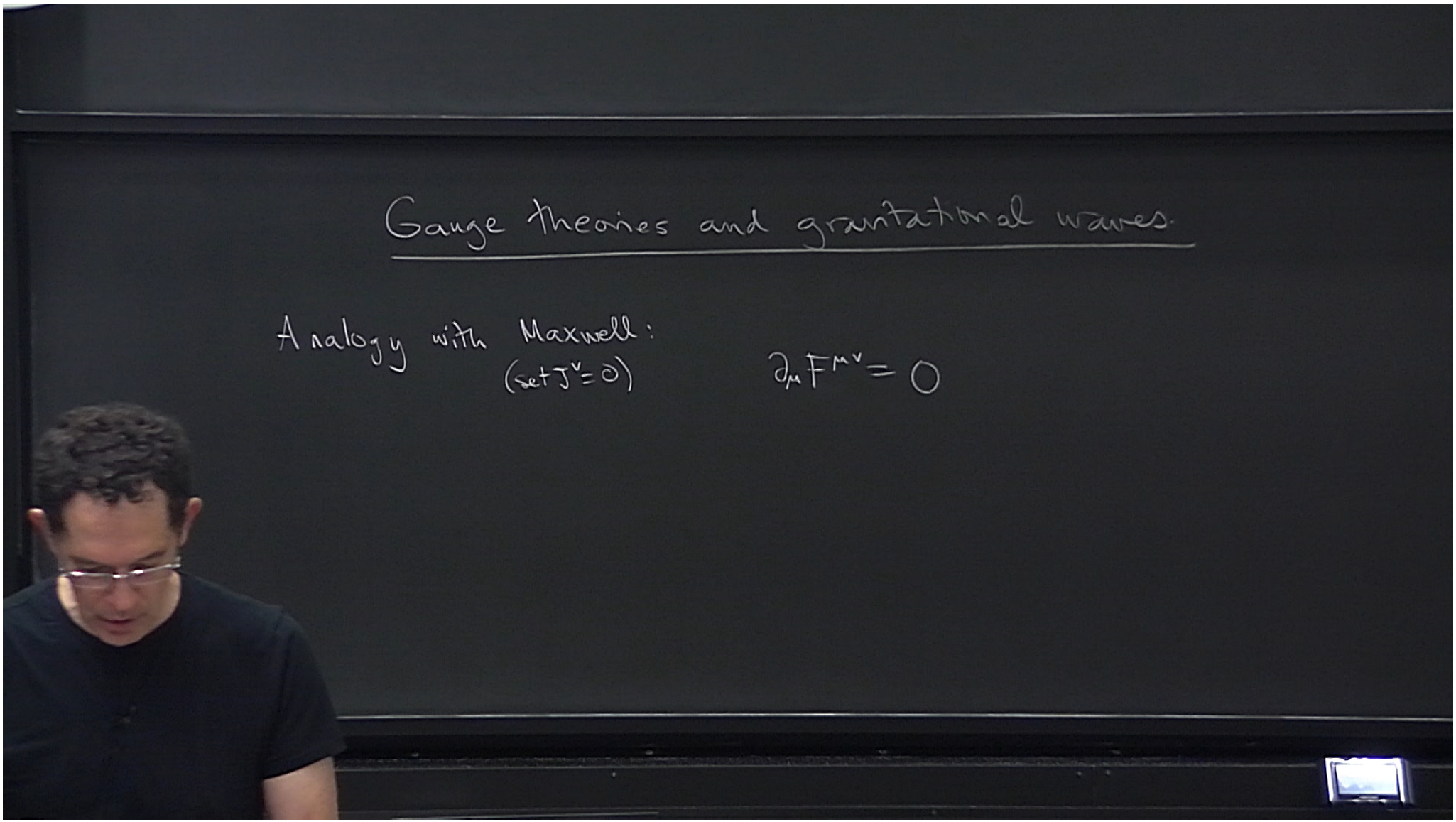
$$\boxed{\underbrace{G_{\mu\nu}}_{\text{CURVATURE}} = \frac{8\pi G}{c^4}\underbrace{T_{\mu\nu}}_{\text{ENERGY}}}$$

$\Longrightarrow$ Poisson eq$^{n}$.

Einstein's field equation.

by dimensions

— now the foundation for cosmology.


Gauge theories and gravitational waves.
Analogy with Maxwell:
(set $J^\nu = 0$)
$\partial_\mu F^{\mu\nu} = 0$

al waves.

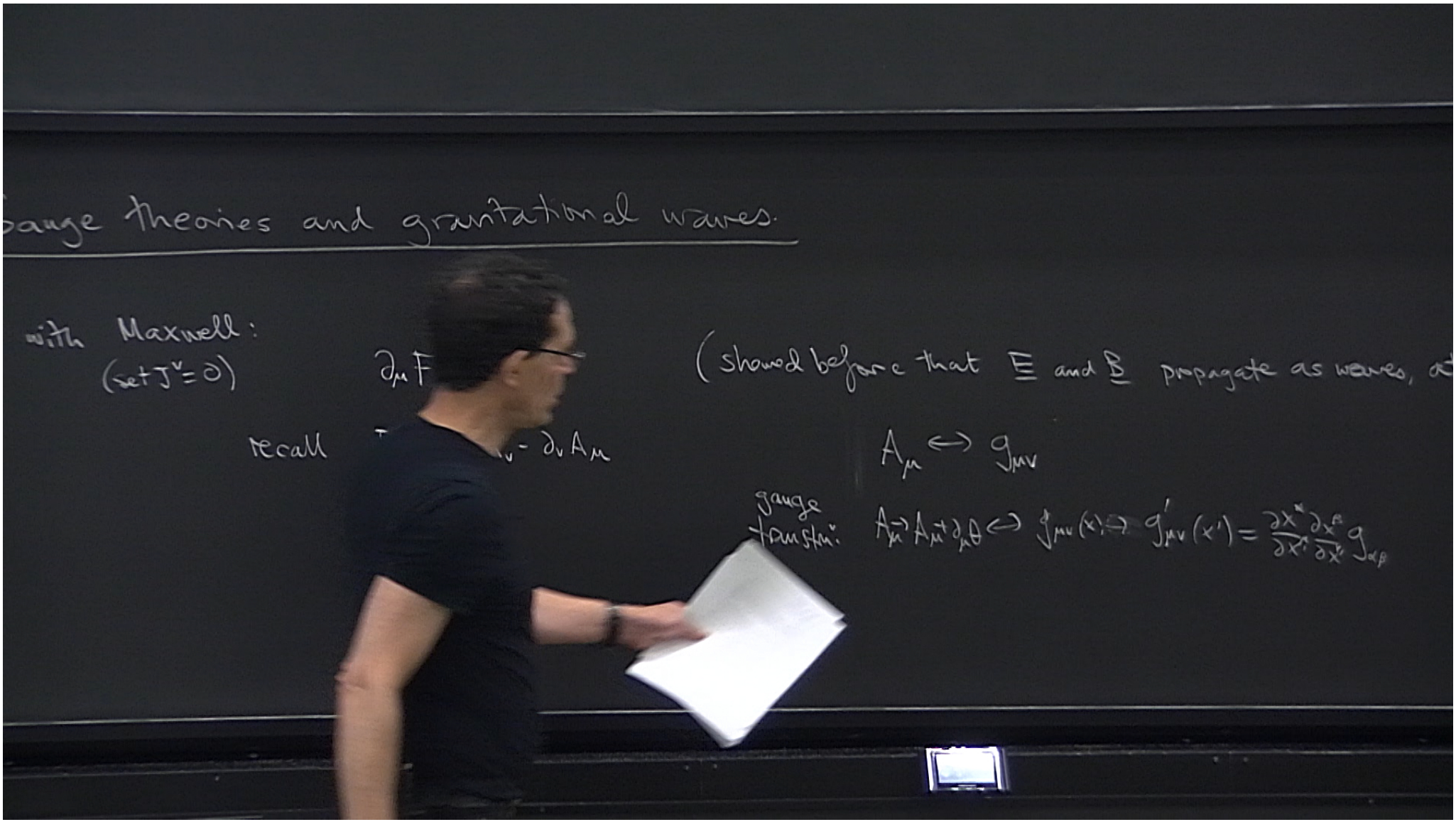
(showed before that $\underline{E}$ and $\underline{B}$ propagate as waves, at speed of light).

# Gauge theories and gravitational waves.

Analogy with Maxwell:

(set $J^\nu = 0$) $\quad \partial_\mu F^{\mu\nu} = 0$ $\quad$ (showed before that $\underline{E}$ and $\underline{B}$ propagate

recall $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$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with Maxwell:
(set $J^\nu = 0$)
$\partial_\mu F$
(showed before that $\underline{E}$ and $\underline{B}$ propagate as waves, a
recall
$\partial_\nu A_\mu$
$A_\mu \leftrightarrow g_{\mu\nu}$
gauge transfm:
$A_\mu \to A_\mu + \partial_\mu \theta \leftrightarrow g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$

## Gauge theories and gravitational waves.

Maxwell:

(set $J^\nu = 0$)

$$\partial_\mu F^{\mu\nu} = 0$$

(showed before that $\underline{E}$ and $\underline{B}$ propagate as waves, a

recall $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$A_\mu \leftrightarrow g_{\mu\nu}$$

gauge transfm: $A_\mu \to A_\mu + \partial_\mu \theta \leftrightarrow g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$

# Gauge theories and gravitational waves.

Analogy with Maxwell:

(set $J^\nu = 0$) $\quad \partial_\mu F^{\mu\nu} = 0 \quad (*)$ (showed before that $\underline{E}$ and $\underline{B}$ propa

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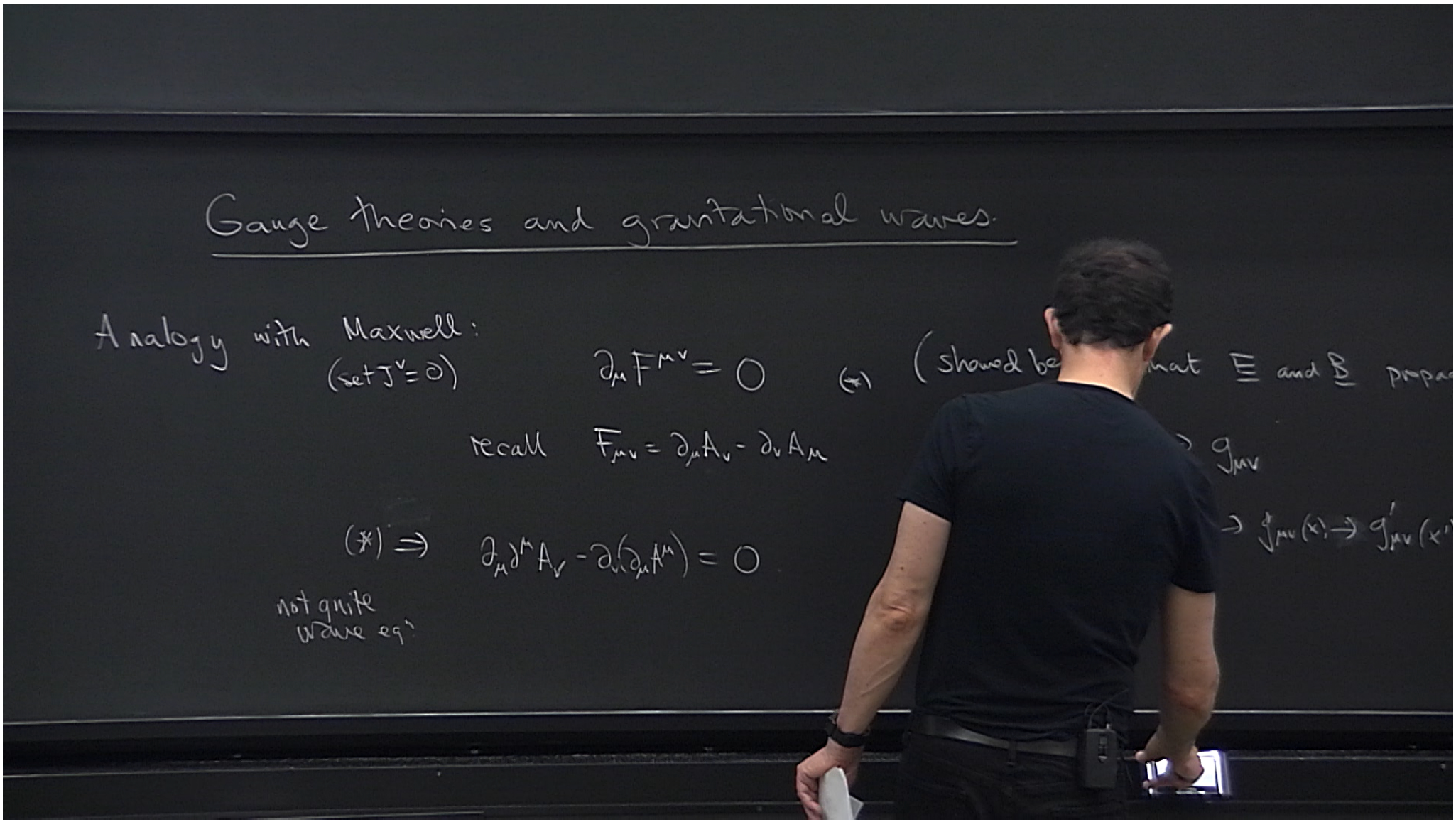
$(*) \Rightarrow \quad \partial_\mu \partial^\mu A_\nu - \partial_\nu(\partial_\mu A^\mu) = 0$


Gauge theories and gravitational waves.
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Fourier Transform $\quad A_\mu = \sum \epsilon_\mu e^{\pm k\cdot x}$

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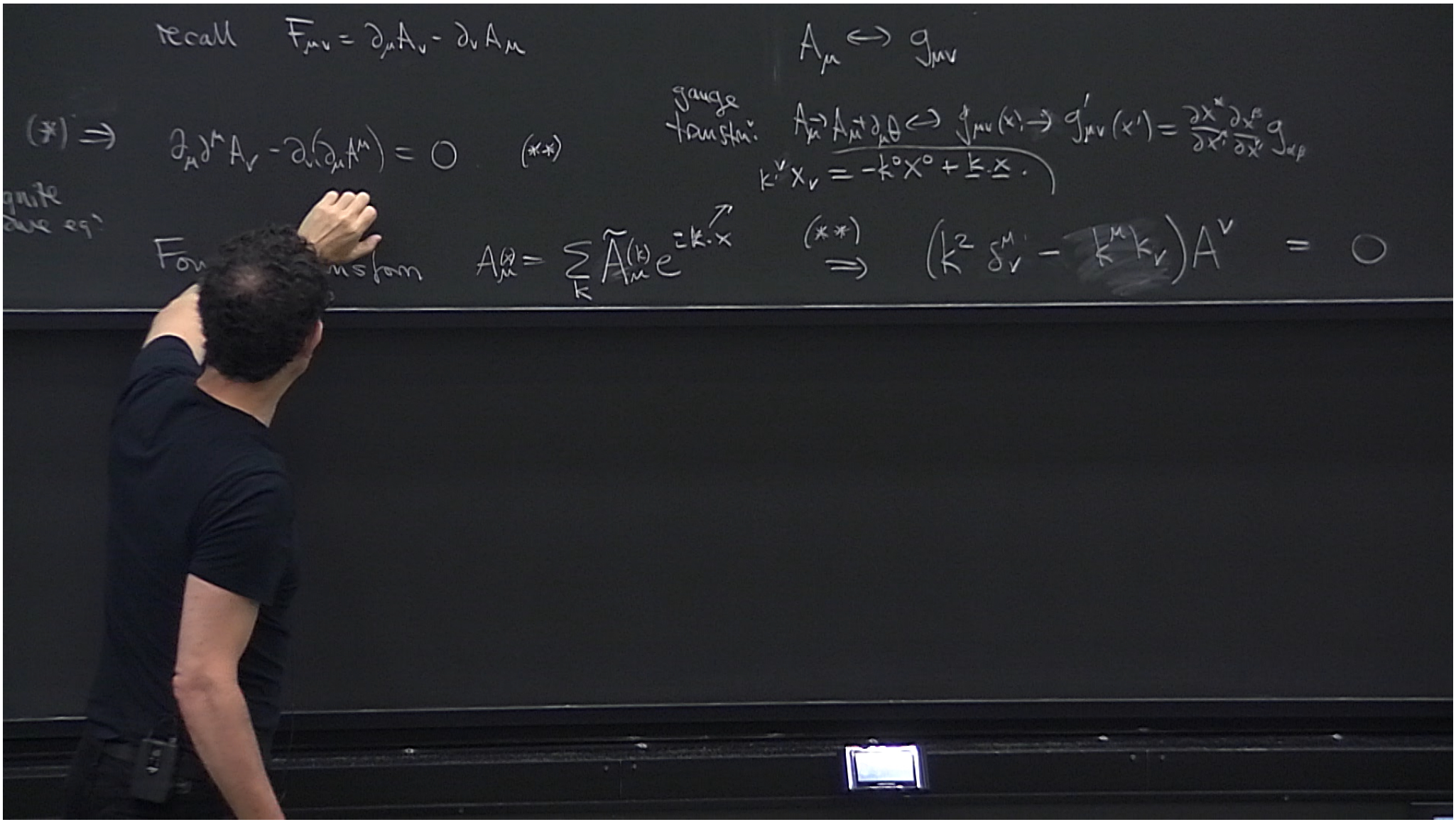
not quite wave eq:

Fourier Transform $\quad A_\mu(x) = \sum_k \tilde{A}_\mu(k) e^{ik\cdot x}$

$(**) \Rightarrow \quad (k^2 \delta^\mu_\nu - k^\mu k_\nu) A^\nu = 0$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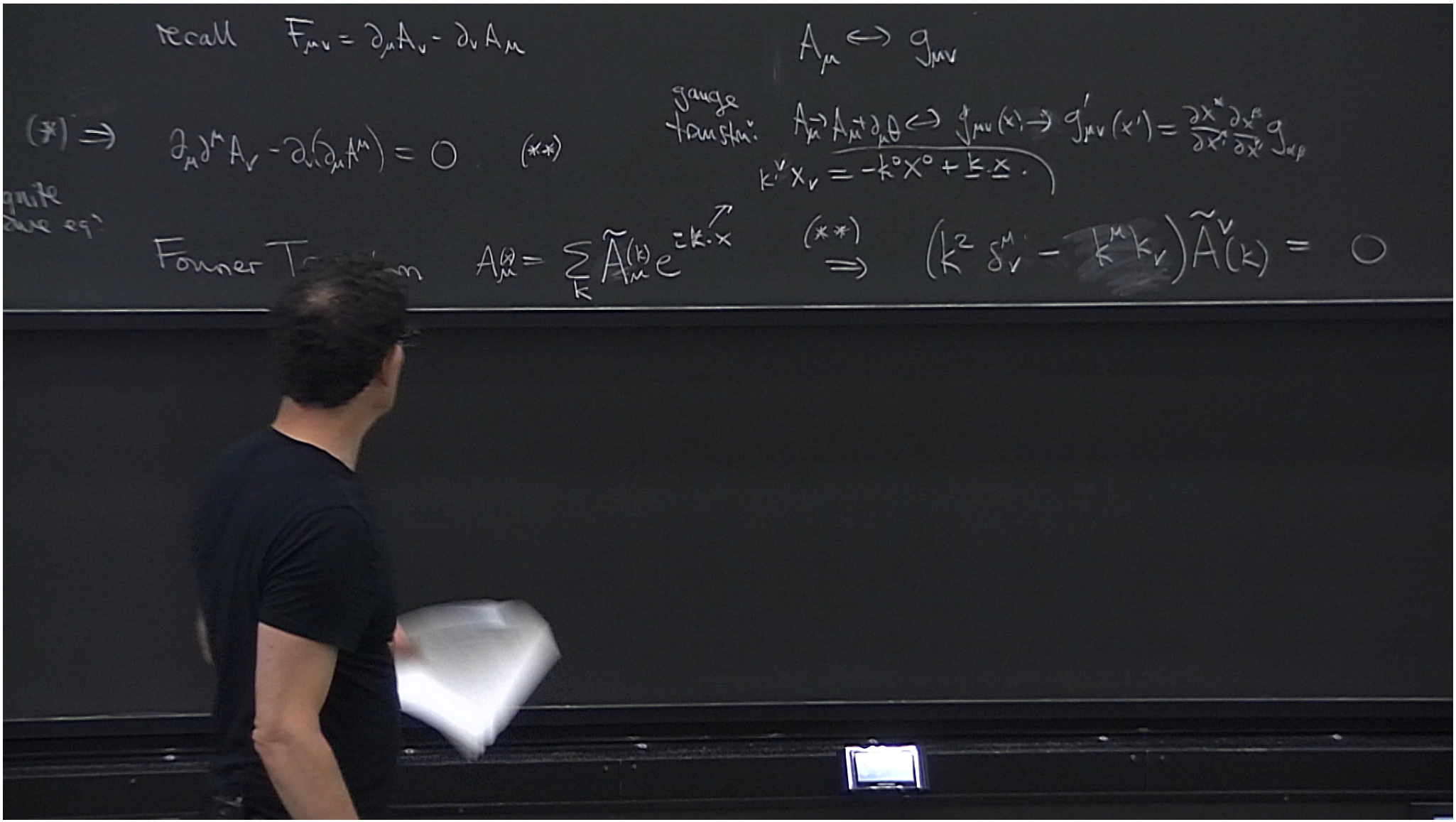
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$g_{\mu\nu}$

$A_\mu + \partial_\mu \theta \leftrightarrow g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$

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algebraic eqn

$(*) \Rightarrow \quad (k^2 \delta^\mu_\nu - k^\mu k_\nu)\tilde{A}^\nu(k) = 0$

matrix operation

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$A_\mu x^\mu \partial_\lambda \theta \leftrightarrow g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$

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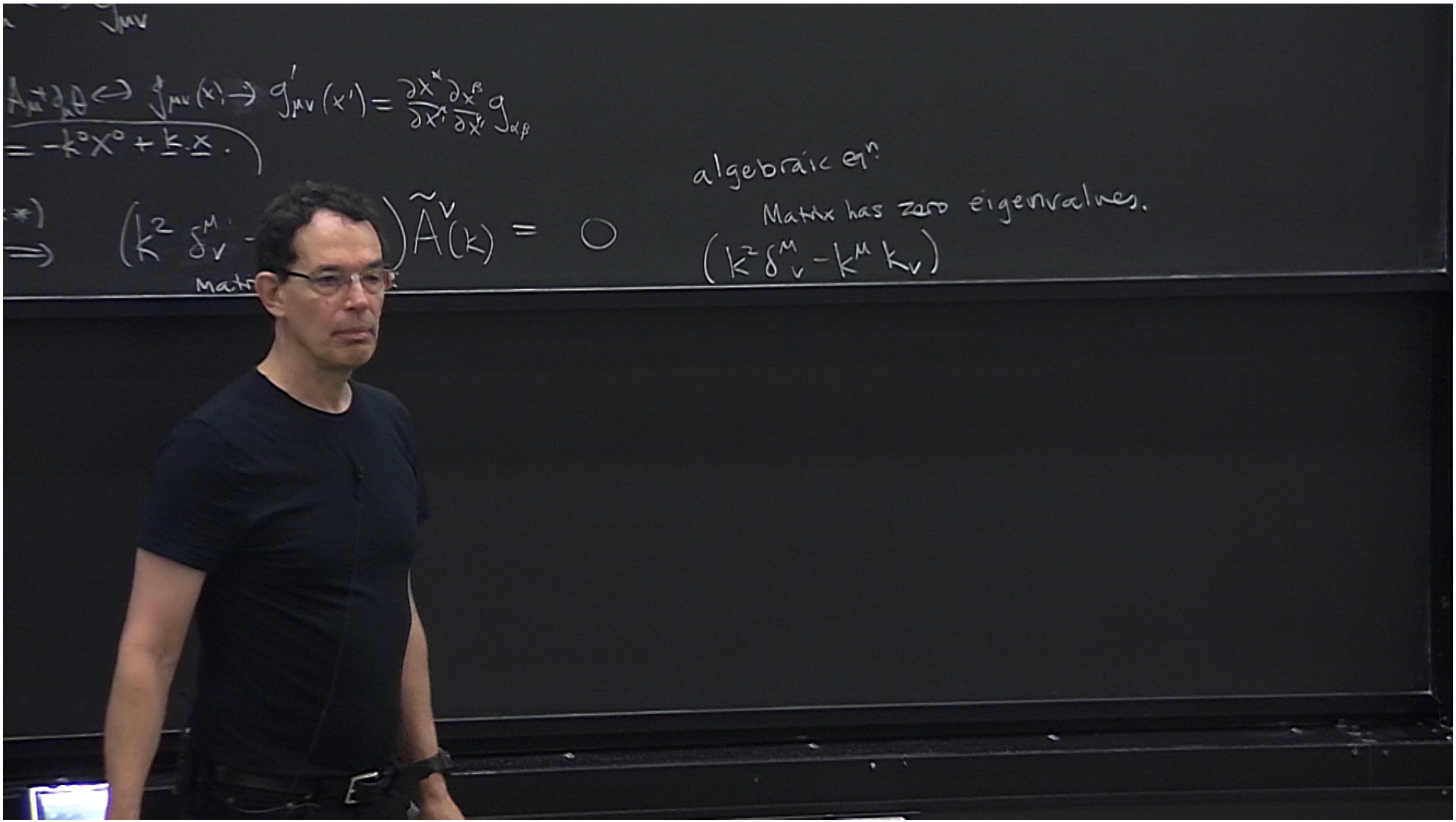
Matrix operation

algebraic eqn

Matrix has zero eigenvalues.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algebraic eqn

Matrix has

$(k^2 \delta^\mu_\nu - k^\mu k$

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$(**) \Rightarrow (k^2 \delta^\mu_\nu - k^\mu k_\nu)\tilde{A}^\nu(k) = 0$

Matrix operator

$\tilde{A}^\nu(k) \to \tilde{A}^\nu(k) + \epsilon(k) k^\nu$ is still a solution, for all $\epsilon$.

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To solve Maxwell, choose $\Theta$ (or $\epsilon$) to set $\partial_\mu A^\mu = 0 \leftrightarrow k_\mu \tilde{A}^\mu = 0$

$g_{\mu\nu}$

$A_\mu + \partial_\mu \theta \leftrightarrow g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$

$= -k^0 x^0 + \underline{k}\cdot\underline{x}.$

$(**)$

$$\Rightarrow \quad \underbrace{\left(k^2 \delta^\mu_\nu - k^\mu k_\nu\right)}_{\text{Matrix operator}} \tilde{A}^\nu(k) = 0 \qquad (***)$$

algebraic eqn

Matrix has zero eigenvalues.

$$\left(k^2 \delta^\mu_\nu - k^\mu k_\nu\right) k^\nu = 0.$$

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$\epsilon$) to set $\partial_\mu A^\mu = 0 \leftrightarrow k_\mu \tilde{A}^\mu = 0$, then $(**)$ or $(***)$ solved if $k^2 = 0$

$$\| $$

$$k^0{}^2 - \underline{k}^2 = 0$$

$$\underline{k^0 = \pm |\underline{k}|}$$

not quite wave eq$^n$

Fourier Transform $A_\mu(x) = \sum_k \tilde{A}_\mu(k) e^{ik\cdot x}$ $\quad (**) \Rightarrow \quad (k^2 \delta^\mu_\nu -$

matrix

Gauge fixing:

1) Set $\partial_\mu A^\mu = 0$ Lorentz gauge

2) residual gauge freedom

$A_\mu \to A_\mu + \partial_\mu \theta$, with $\partial^2 \theta = 0$

$\tilde{A}_\mu \to \tilde{A}_\mu + i\epsilon(k) k_\mu$ with $k^2 = 0$.

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Back to ①: $AG^{00} \approx 2AR^{00} \approx -A\nabla^2 h_{00} = 8\pi G\rho = 8\pi G\rho_m$

So for EM, $A_\mu$ (4 components): $4 = 2 + 1 + 1$

physical modes

residual gauge freedom

fixed by Lorentz gauge

Back to ①: $AG^{00} \approx 2AR^{00} \approx -A\nabla h_{00}$

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GR: first problem is to fix a gauge.

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Back to ①: $A\Theta \approx 2\pi k \approx$

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GR: first problem is to fix a gauge.

$g_{\mu\nu}$

$\Gamma^{\alpha}_{\mu\nu}$

$R_{\cdot\cdot}$

Back to ①: $A_0 \approx \partial_\mu A^\mu$

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Back to ①: $A_\mu \simeq \partial_\mu \Lambda$

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$$h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$$

Back to ①: $A_\mu \sim \partial_\mu \Lambda \sim \ldots$

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Ex

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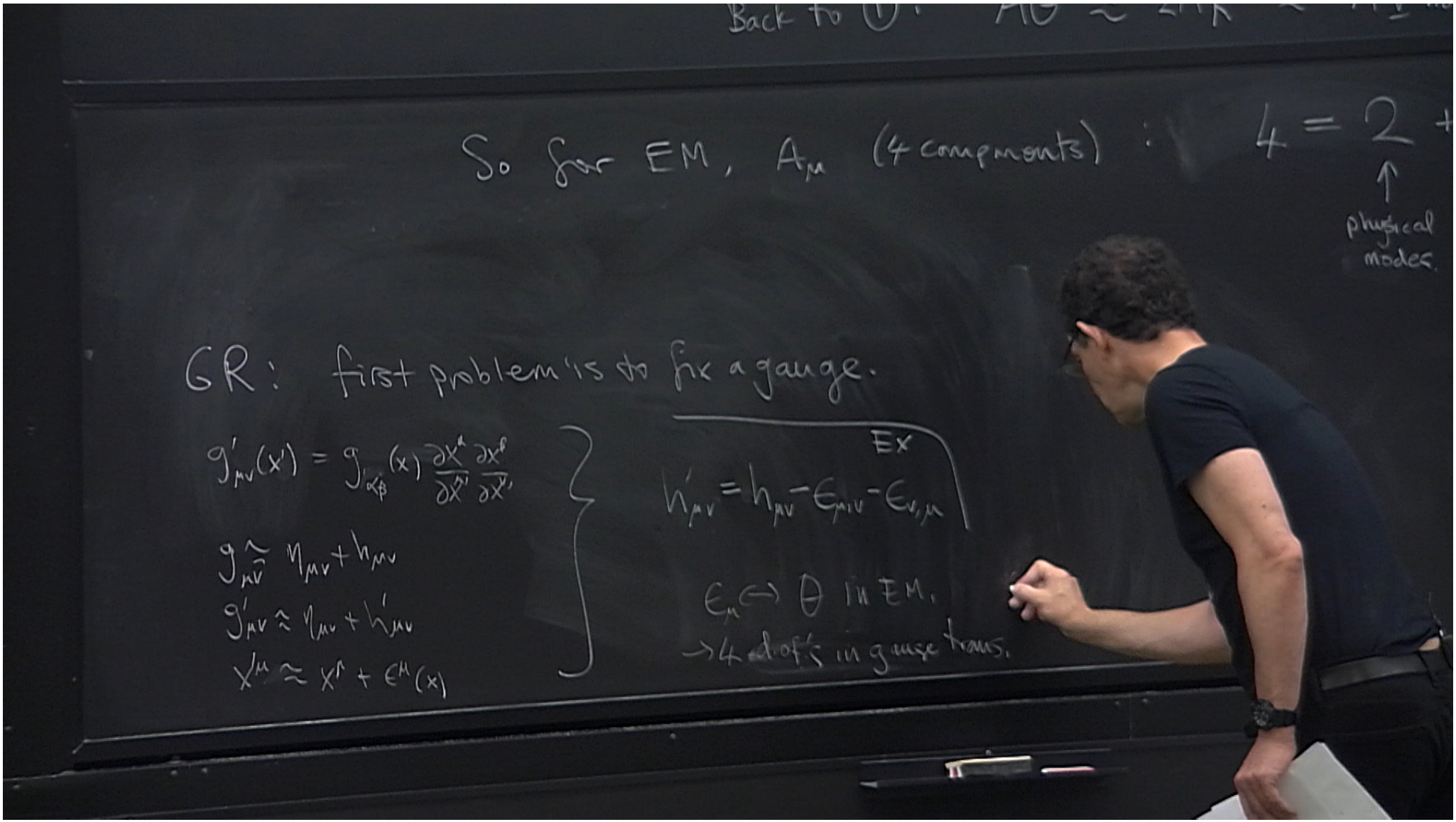
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Ex
$h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$
$\epsilon_\mu \leftrightarrow \theta$ in EM.
$\rightarrow$ 4 d.o.f.'s in gauge trans.

Back to ①: $A\theta \sim 2\pi k$

So for EM, $A_\mu$ (4 components) : $4 = 2 +$

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So for EM, $A_\mu$ (4 components) : $4 = 2 +$

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$g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$

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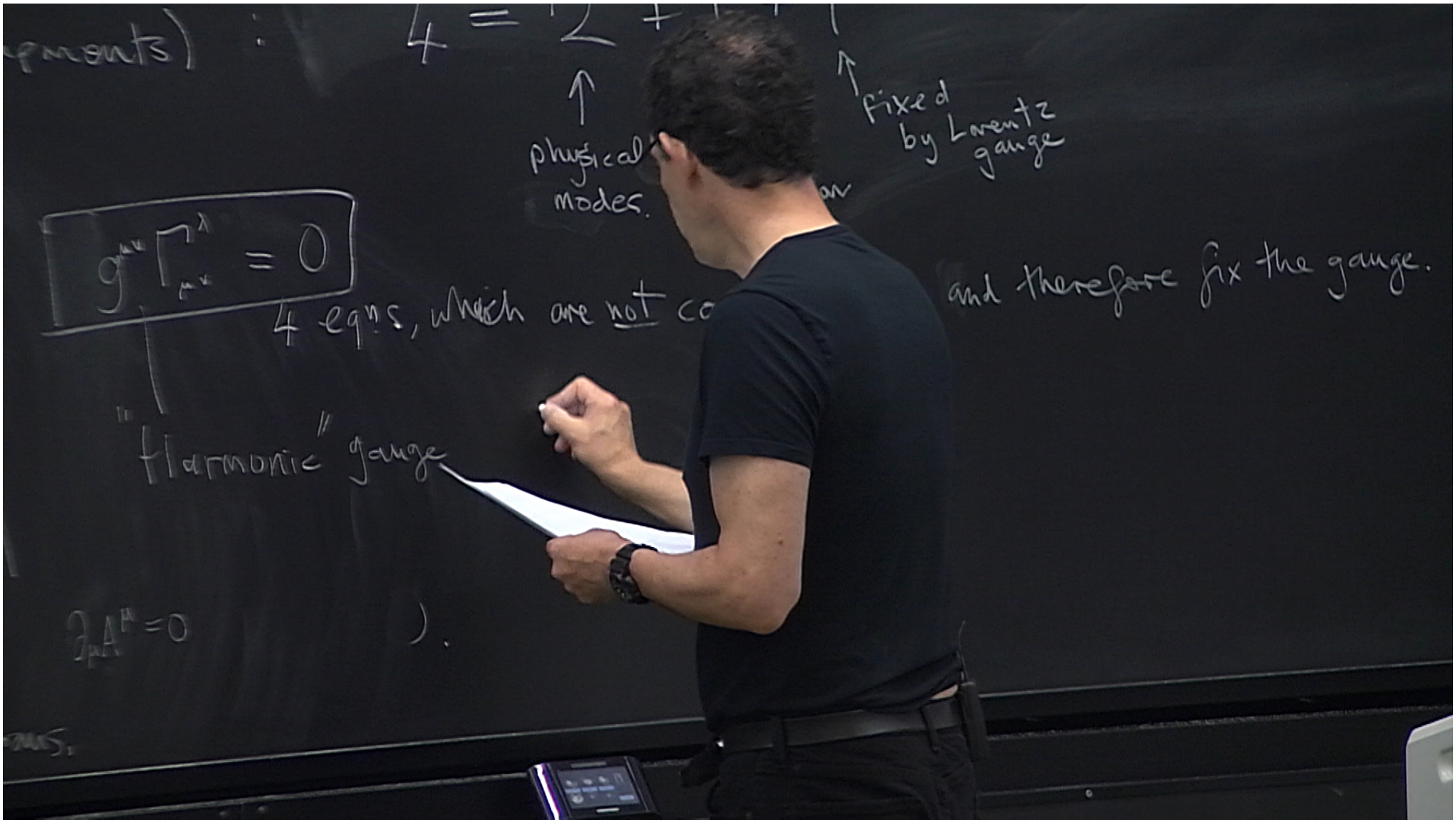
$$4 = 2 + 1 + 1$$

physical modes. residual gauge freedom fixed by Lorentz gauge

$$g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0$$

4 eqns, which are <u>not</u> coord invariant, and therefore fix the gauge.

$$\partial_\mu A^\mu = 0$$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physical modes.
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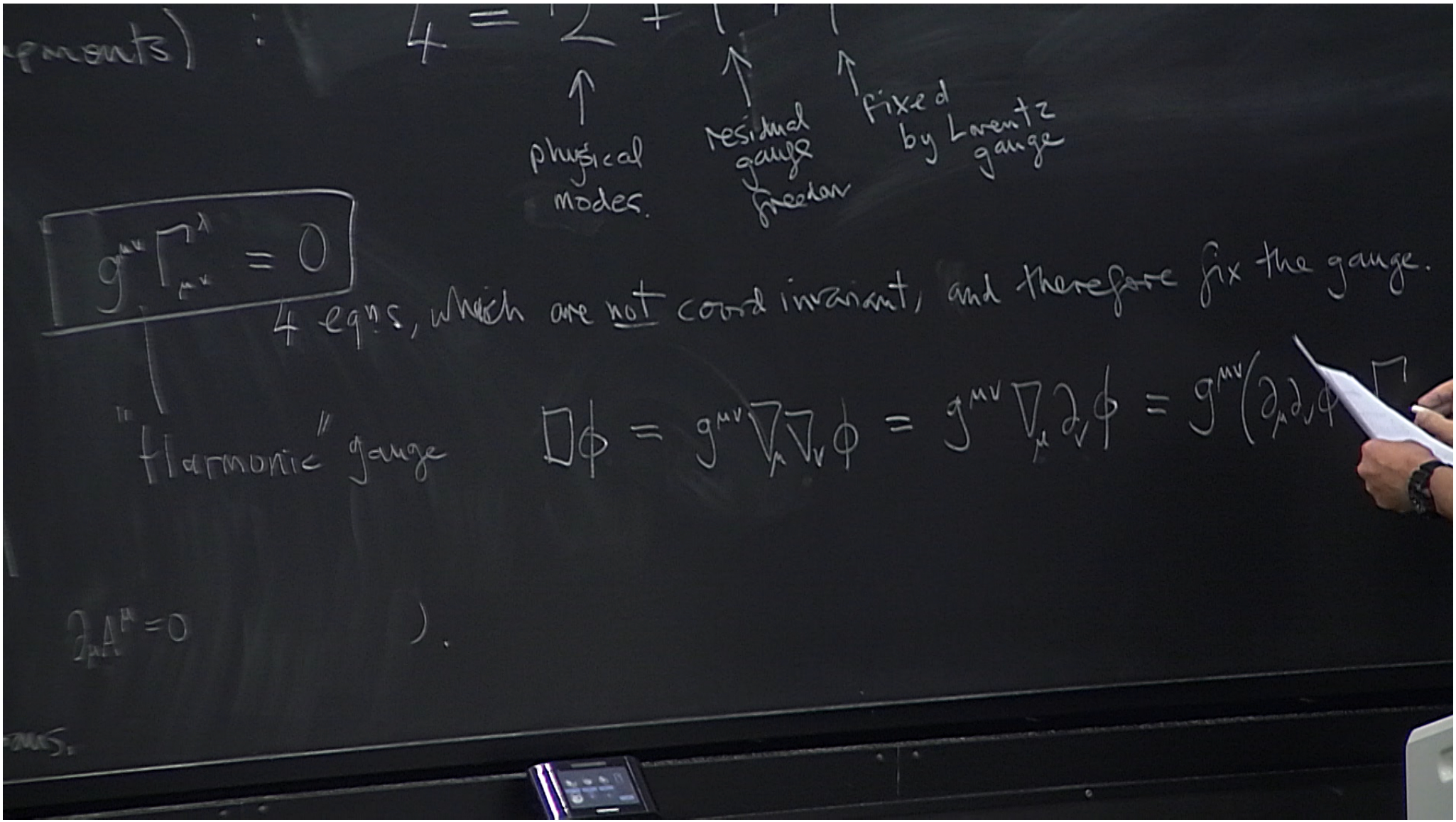
"Harmonic" gauge
$\partial_\mu A^\mu = 0$


4 = 2 + 1 + 1
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"Harmonic" gauge
$\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = g^{\mu\nu}\nabla_\mu\partial_\nu\phi = g^{\mu\nu}(\partial_\mu\partial_\nu\phi$
$\partial_\mu A^\mu = 0$

$= 2 + 1 + 1$

physical modes

residual gauge freedom

fixed by Lorentz gauge

are not coord invariant, and therefore fix the gauge.

in harmonic gauge

$$\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = g^{\mu\nu}\nabla_\mu\partial_\nu\phi = g^{\mu\nu}\left(\partial_\mu\partial_\nu\phi - \Gamma^\lambda_{\mu\nu}\partial_\lambda\phi\right) = g^{\mu\nu}\partial_\mu\partial_\nu\phi .$$

"Harmonic" gauge $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = g^{\mu\nu}\nabla_\mu\partial_\nu\phi = g^{\mu\nu}(\partial_\mu\partial_\nu\phi - \Gamma^\lambda_{\mu\nu}\partial_\lambda\phi) = g^{\mu\nu}\partial_\mu\partial_\nu\phi$.

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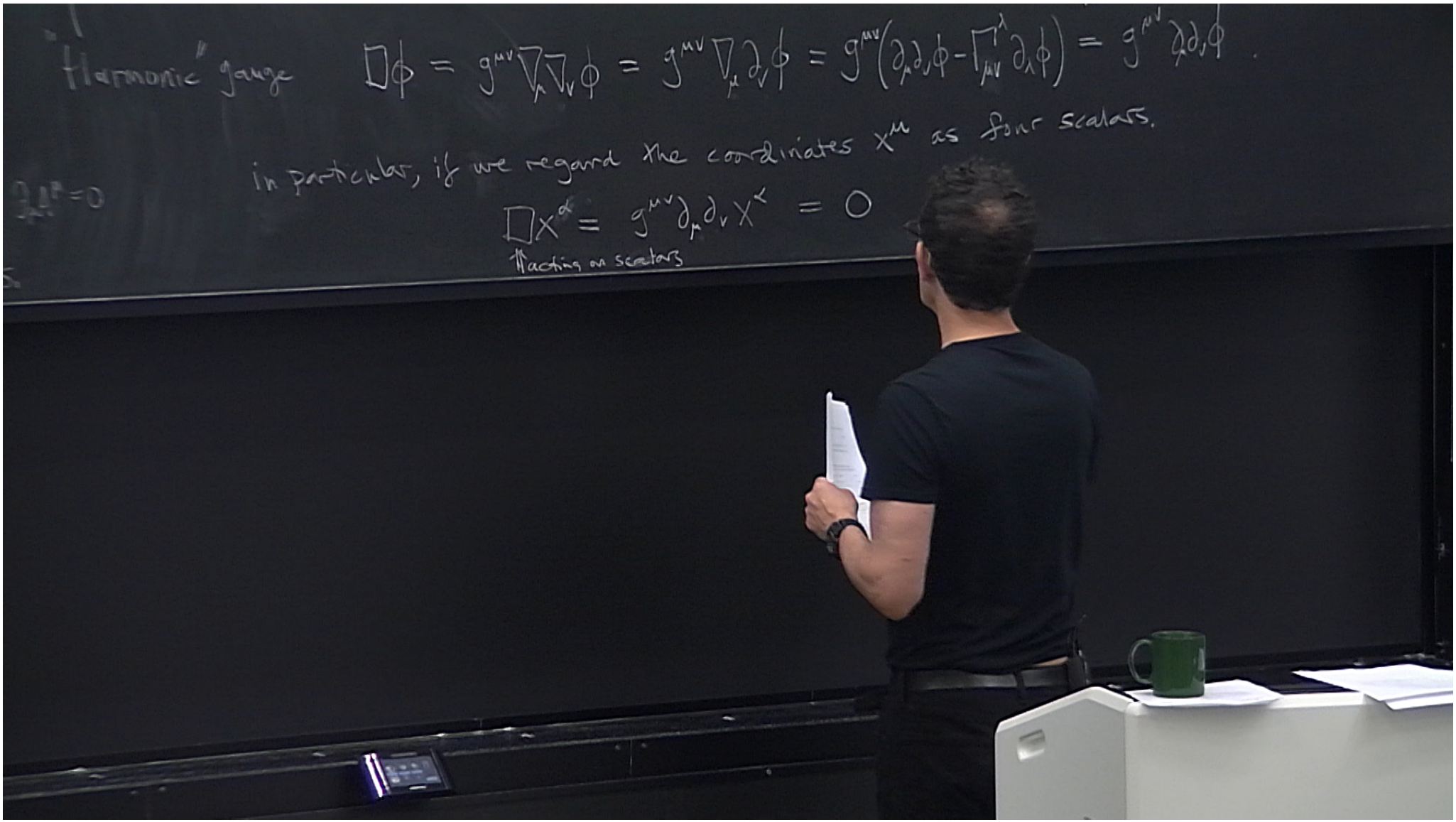
in particular, if we regard the coordinates $x^\mu$ as four scalars.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in particular, if we regard the coordinates $x^\mu$ as four scalars.

$$\Box x^\alpha = g^{\mu\nu}\partial_\mu\partial_\nu x^\alpha = 0 \quad \text{in } \underline{\text{harmonic}} \text{ gauge}$$

acting on scalars

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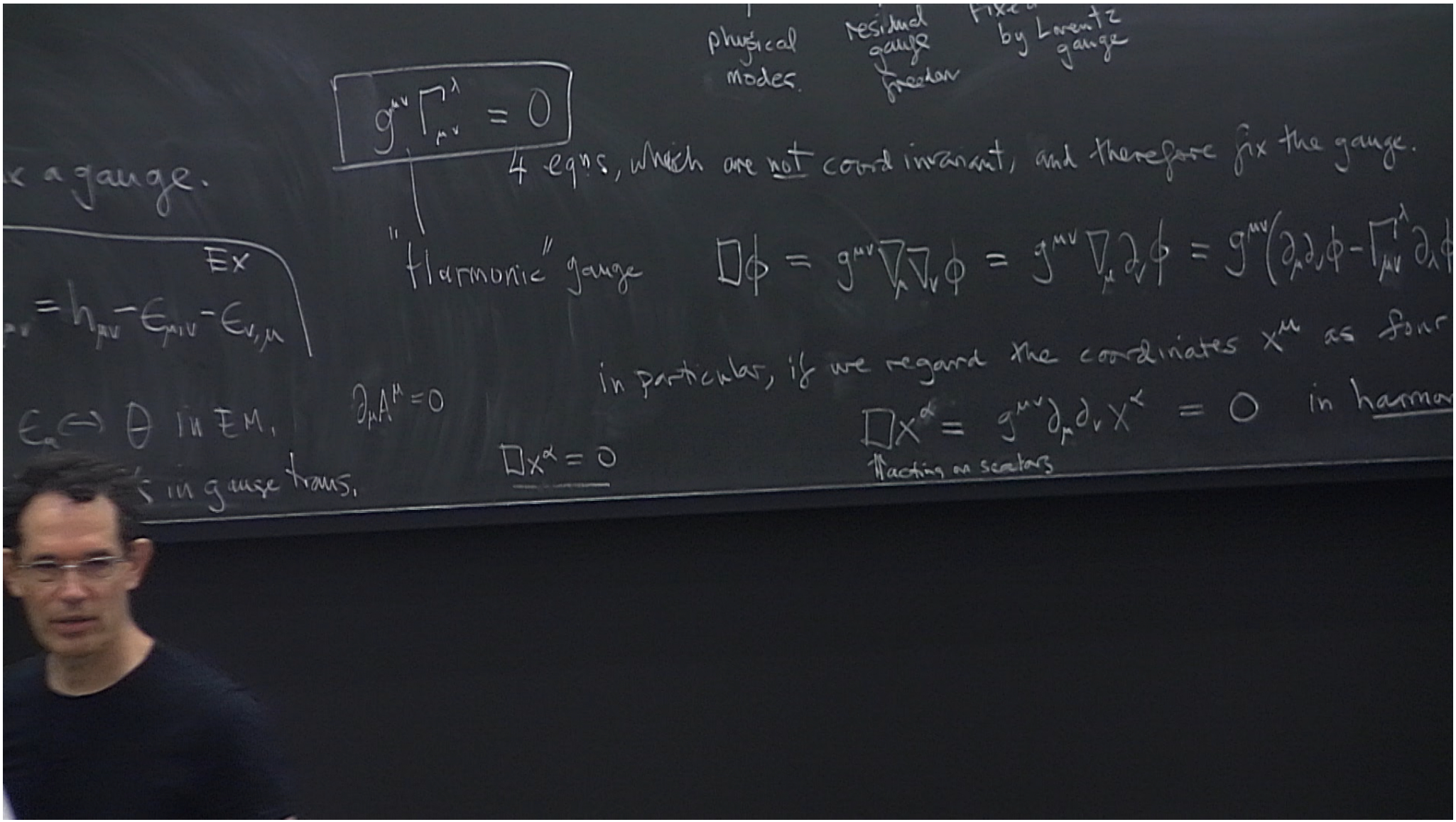
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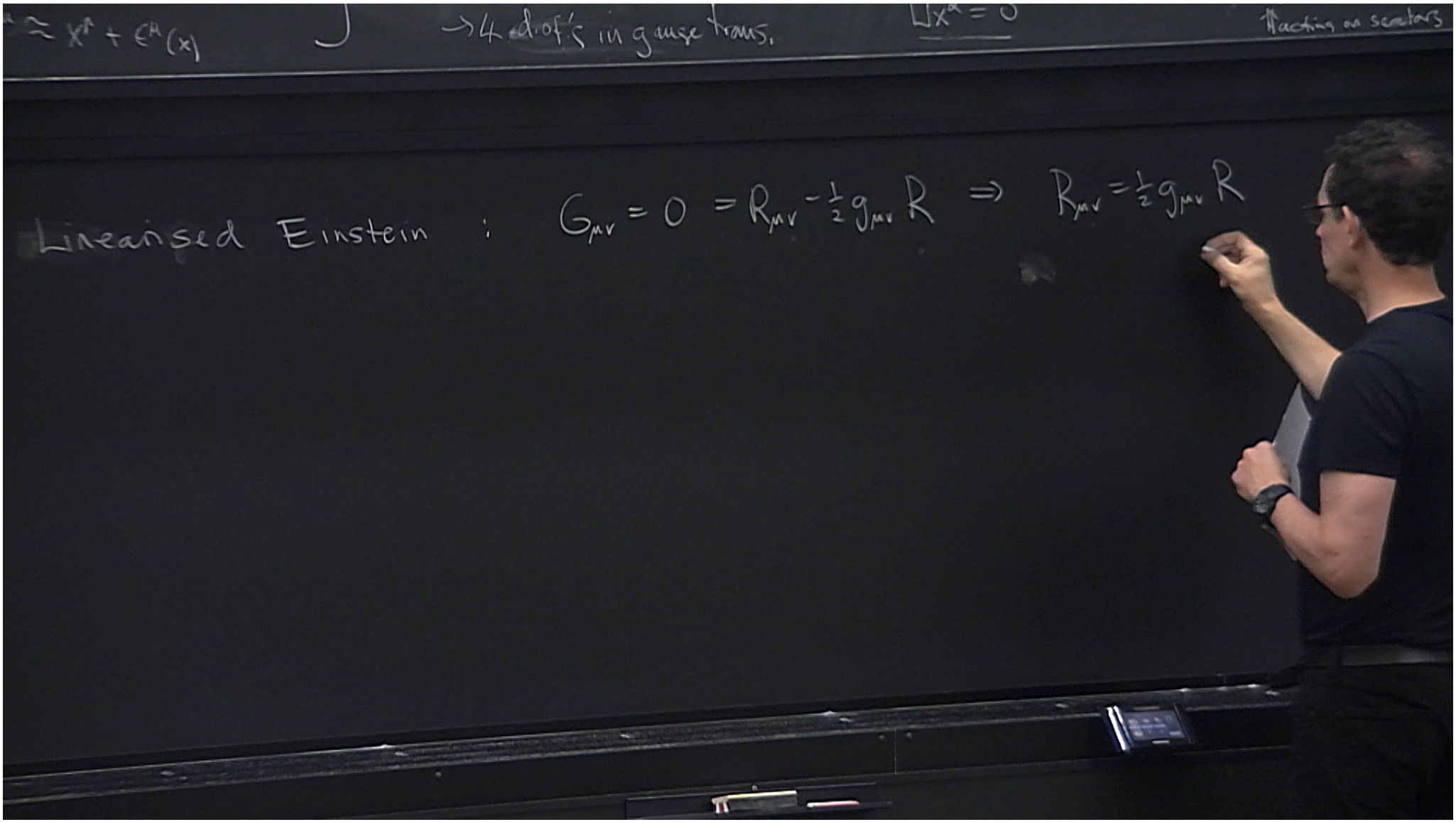
x a gauge.
Ex
$= h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$
in particular, if we regard the coordinates $x^\mu$ as four
$\epsilon_\mu \leftrightarrow \theta$ in EM.
$\partial_\mu A^\mu = 0$
$\Box x^\alpha = g^{\mu\nu}\partial_\mu\partial_\nu x^\alpha = 0$ in harmon
acting on scalars
$\Box x^\alpha = 0$
's in gauge trans.


$\simeq x^\mu + \epsilon^\mu(x)$
$\rightarrow$ 4 d.o.f.'s in gauge trans.
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Linearised Einstein : $G_{\mu\nu} = 0 = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \Rightarrow R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R$

d.o.f. in gauge trans. $\Box x^\mu = 0$ $\Box$ acting on scalars

$$G_{\mu\nu} = 0 = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \Rightarrow R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R, \quad \times g^{\mu\nu} \Rightarrow R = 2R \Rightarrow R = 0$$

$$g^{\mu\nu} g_{\mu\nu} = \delta^\mu_\mu = 4$$

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$x^\mu \simeq x^\mu + \epsilon^\mu(x)$ $\rightarrow$ 4 d.of's in gauge trans. $\Box x^\mu = 0$ Acting on scalar

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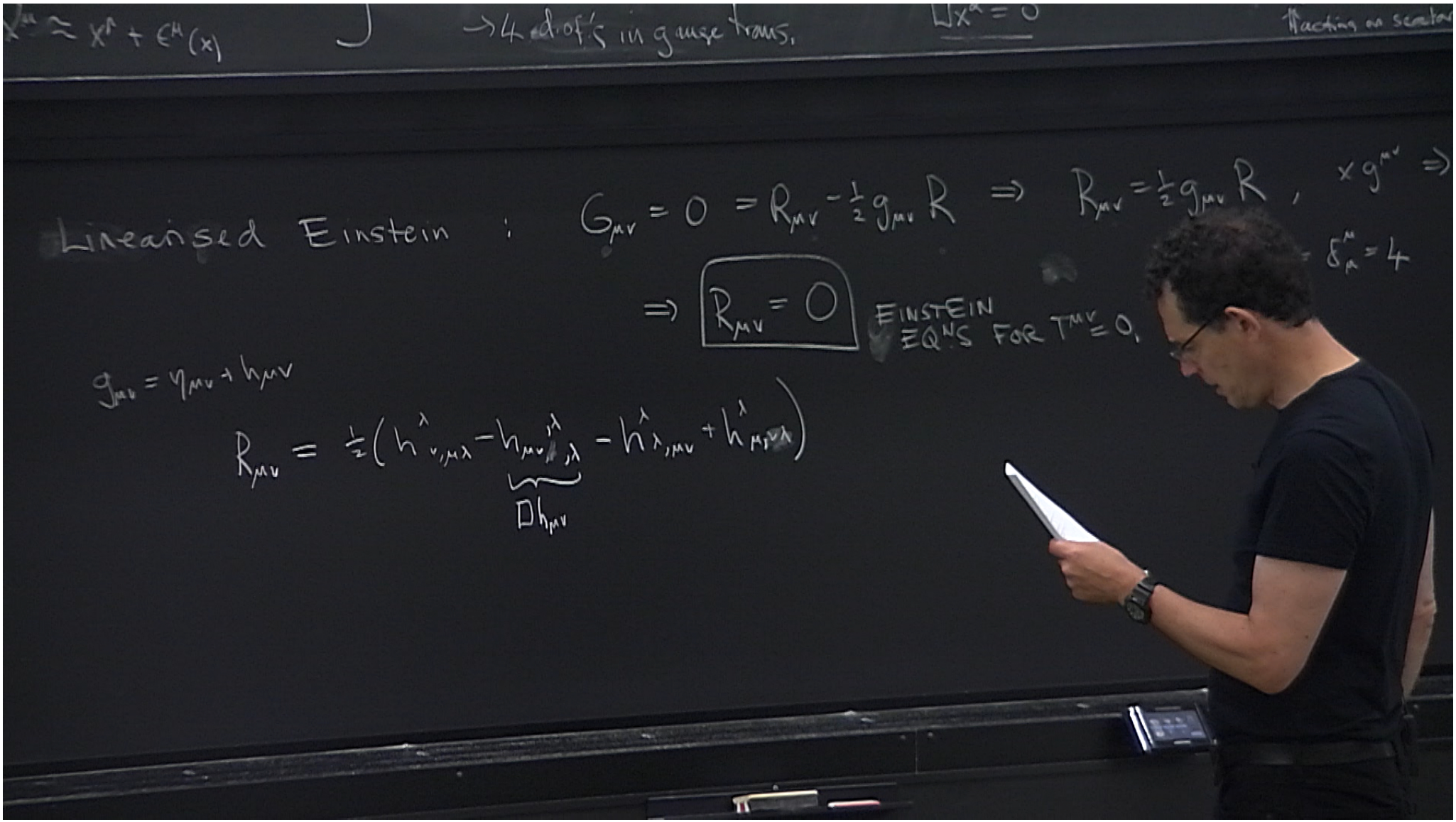
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$R_{\mu\nu} = \frac{1}{2}\left( h^\lambda{}_{\nu,\mu\lambda} - h_{\mu\nu}{}^{,\lambda}{}_{,\lambda} - h^\lambda{}_{\lambda,\mu\nu} + h^\lambda{}_{\mu,\nu\lambda} \right)$$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$x^\mu \simeq x^\mu + \epsilon^\mu(x)$ $\rightarrow$ 4 d.o.f.'s in gauge trans. $\Box x^\mu = 0$ Acting on scalar

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...of $\xi$ in gauge trans. $\Box x^\lambda = 0$ $\uparrow$ acting on scalars

$$G_{\mu\nu} = 0 = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R \Rightarrow R_{\mu\nu} = \tfrac{1}{2} g_{\mu\nu} R, \quad \times g^{\mu\nu} \Rightarrow R = 2R \Rightarrow R = 0$$

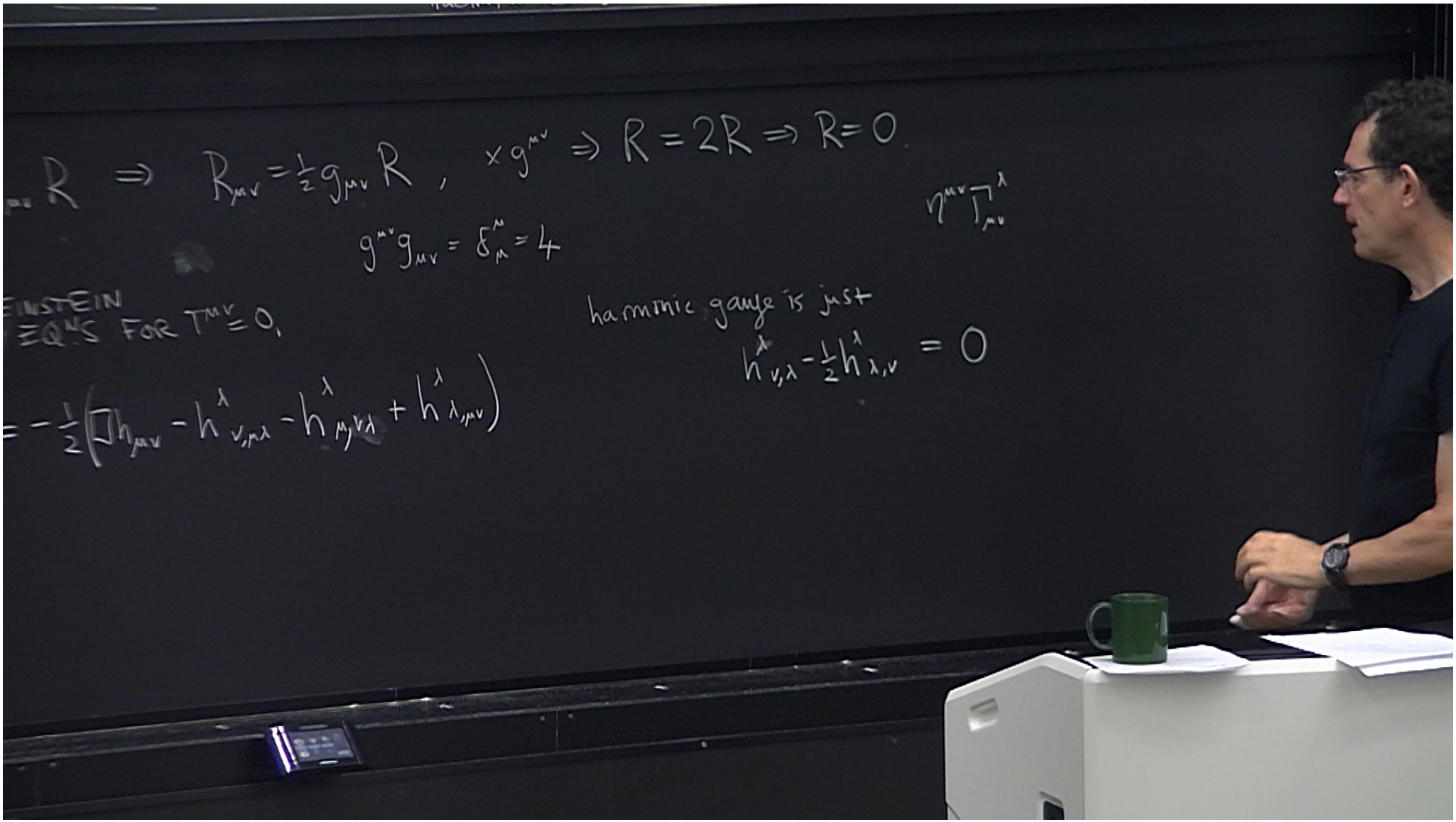
$$g^{\mu\nu} g_{\mu\nu} = \delta^\mu_\mu = 4$$

$$\Rightarrow \boxed{R_{\mu\nu} = 0}$$ EINSTEIN EQ^NS FOR $T^{\mu\nu} = 0$.

harmonic gauge is just

$$h^\lambda{}_{\nu,\lambda} - \tfrac{1}{2} h^\lambda{}_{\lambda,\nu} = 0$$

$$\underbrace{h_{\mu\nu}{}^{,\lambda}{}_{,\lambda}}_{\Box h_{\mu\nu}} - h^\lambda{}_{\lambda,\mu\nu} + h^\lambda{}_{\mu,\nu\lambda}) = -\tfrac{1}{2}\left(\Box h_{\mu\nu} - h^\lambda{}_{\nu,\mu\lambda} - h^\lambda{}_{\mu,\nu\lambda} + h^\lambda{}_{\lambda,\mu\nu}\right)$$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EINSTEIN
EQ^NS FOR $T^{\mu\nu} = 0$,
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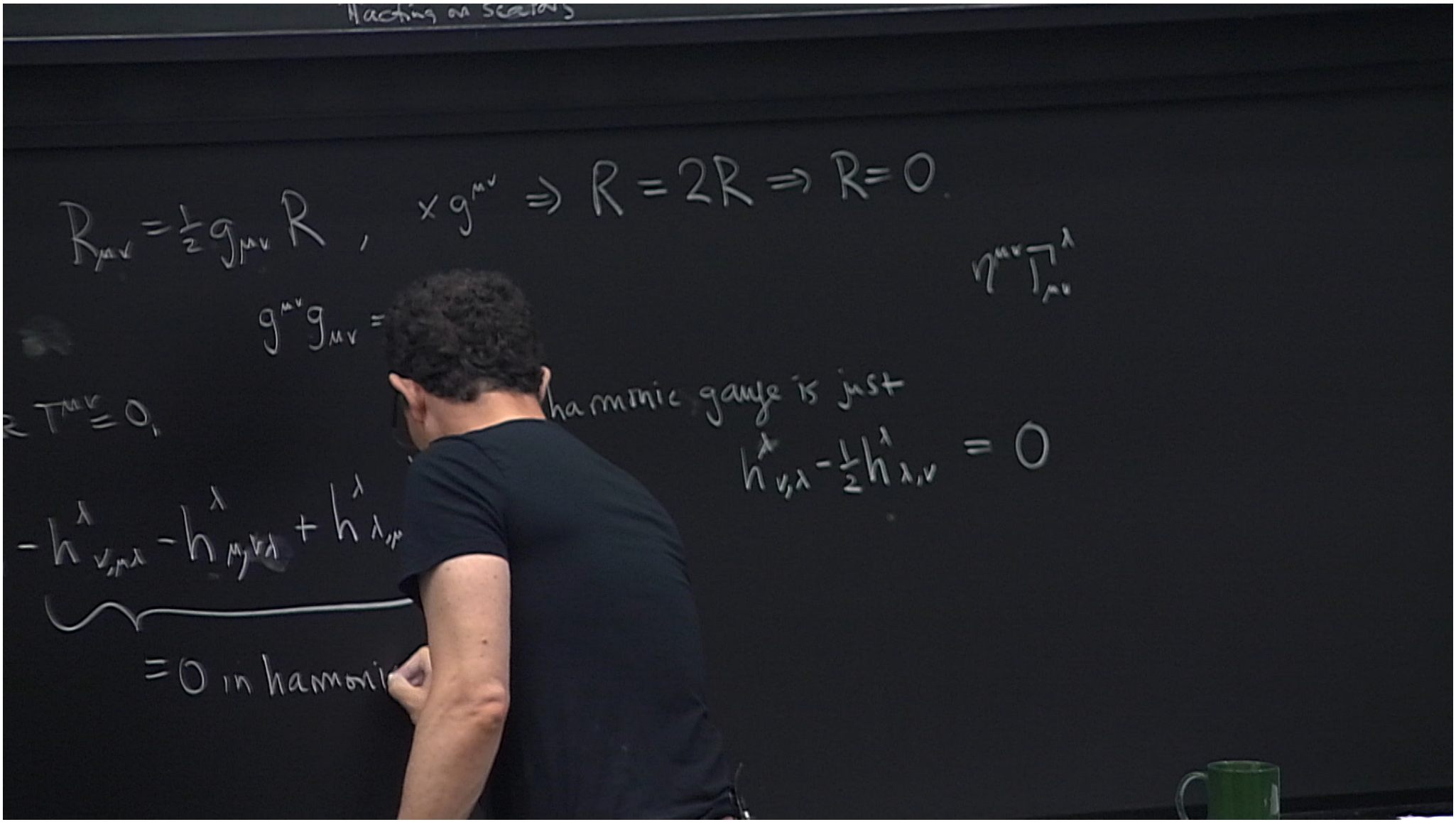
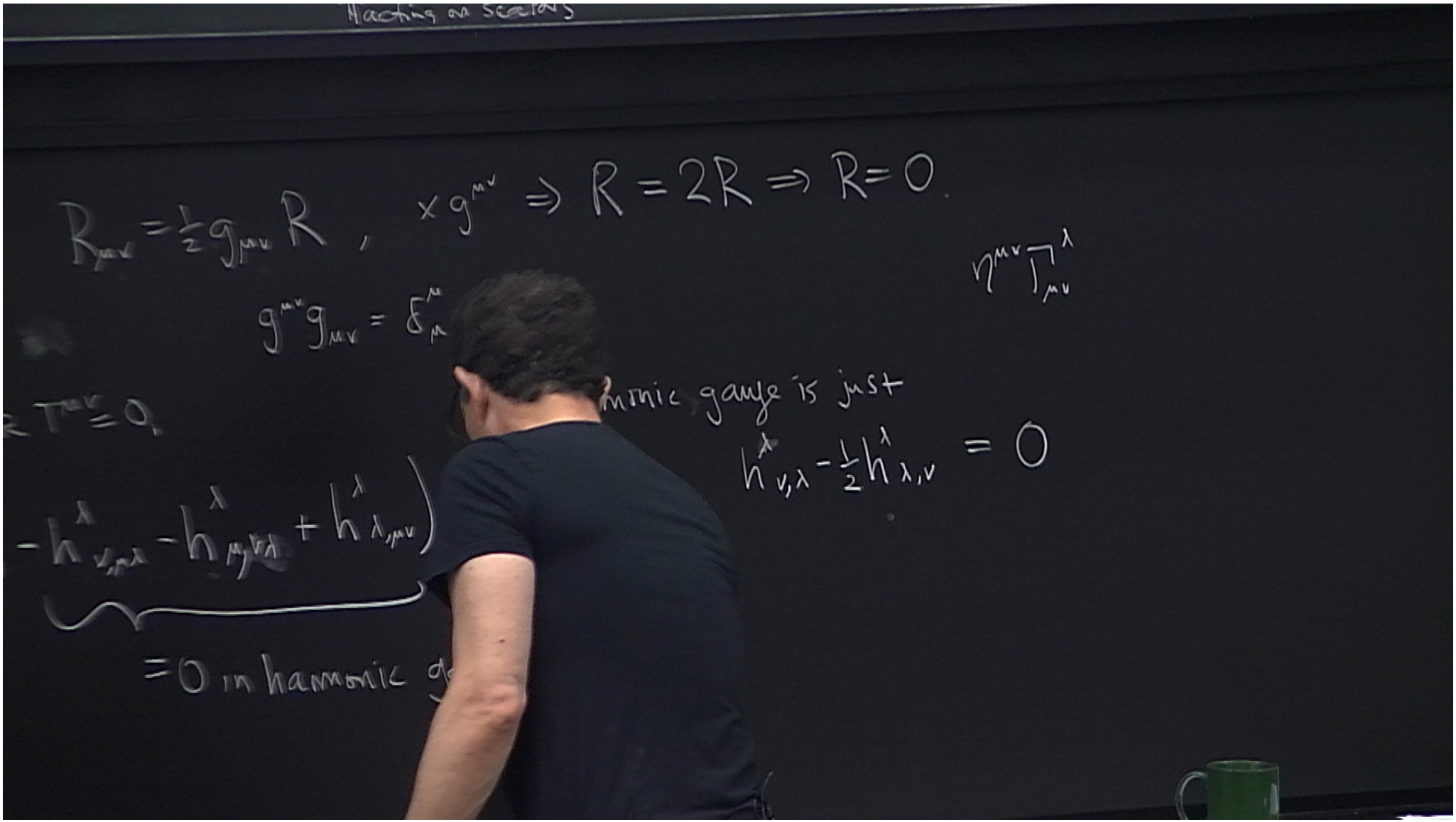
$h^{\lambda}_{\nu,\lambda} - \frac{1}{2} h^{\lambda}_{\lambda,\nu} = 0$
$= -\frac{1}{2}\left(\Box h_{\mu\nu} - h^{\lambda}_{\nu,\mu\lambda} - h^{\lambda}_{\mu,\nu\lambda} + h^{\lambda}_{\lambda,\mu\nu}\right)$


Acting on scalars
$R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R$ , $\times g^{\mu\nu} \Rightarrow R = 2R \Rightarrow R = 0$
$\eta^{\mu\nu} \Gamma^{\lambda}_{\mu\nu}$
$g^{\mu\nu} g_{\mu\nu} =$
harmonic gauge is just
$T^{\mu\nu} = 0$
$h^{\lambda}_{\nu,\lambda} - \frac{1}{2} h^{\lambda}_{\lambda,\nu} = 0$
$- h^{\lambda}_{\nu,\mu\lambda} - h^{\lambda}_{\mu,\nu\lambda} + h^{\lambda}_{\lambda,\mu}$
$= 0$ in harmoni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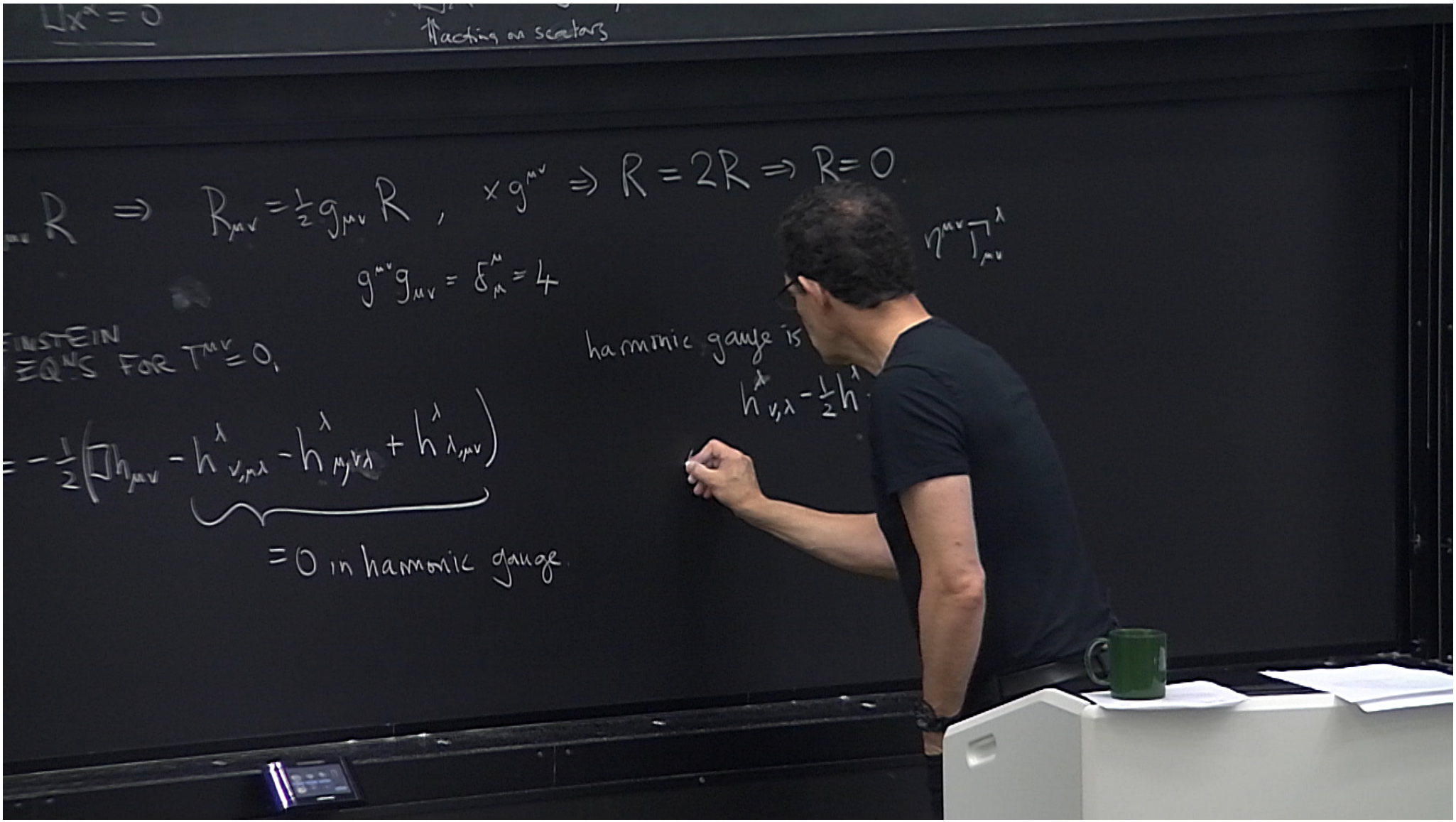
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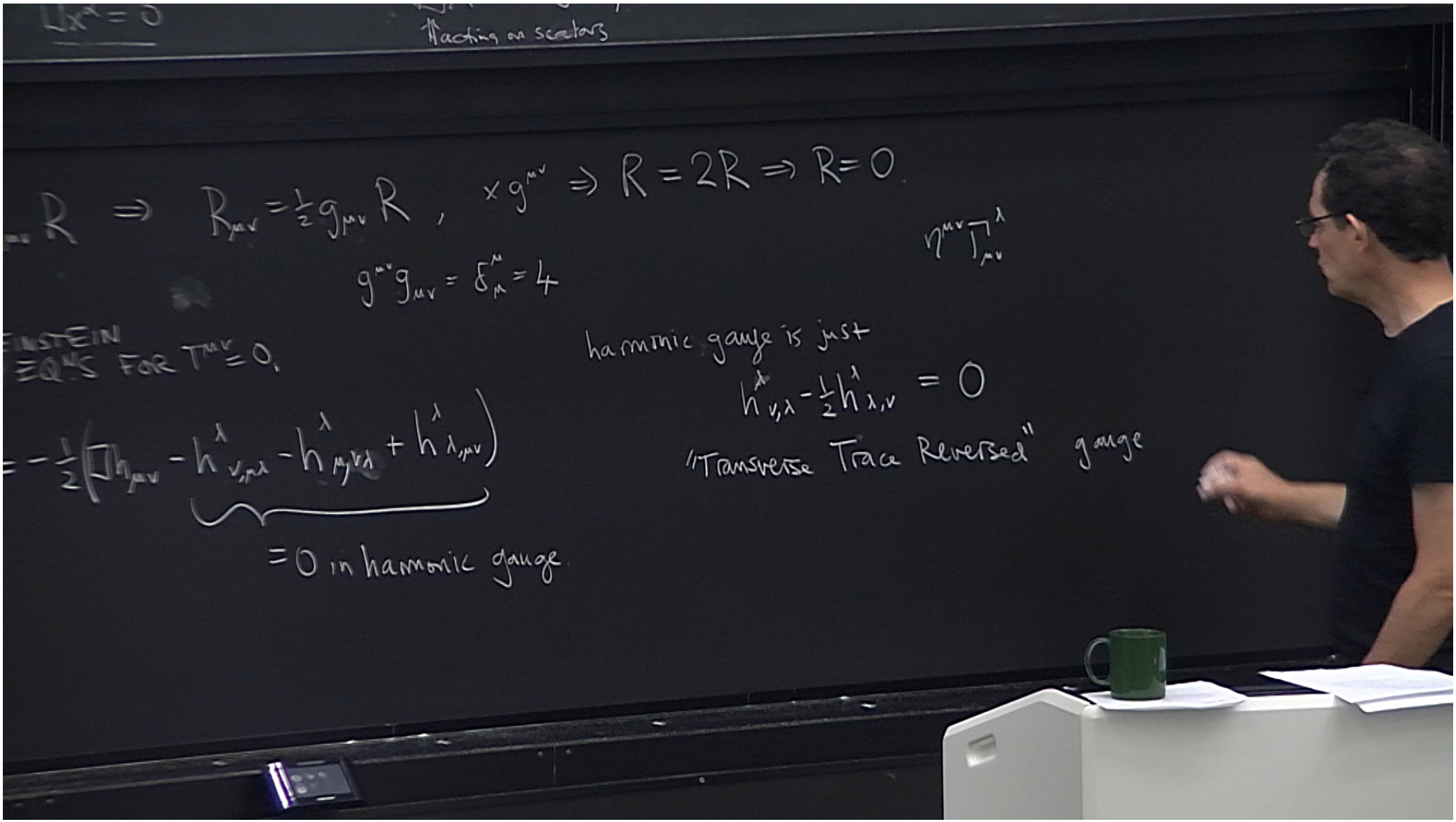
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$h^{\lambda}_{\nu,\lambda} - \frac{1}{2} h^{\lambda}_{\lambda,\nu} = 0$
"Transverse Trace Reversed" gauge
$= -\frac{1}{2}\left(\Box h_{\mu\nu} - h^{\lambda}_{\nu,\mu\lambda} - h^{\lambda}_{\mu,\nu\lambda} + h^{\lambda}_{\lambda,\mu\nu}\right)$
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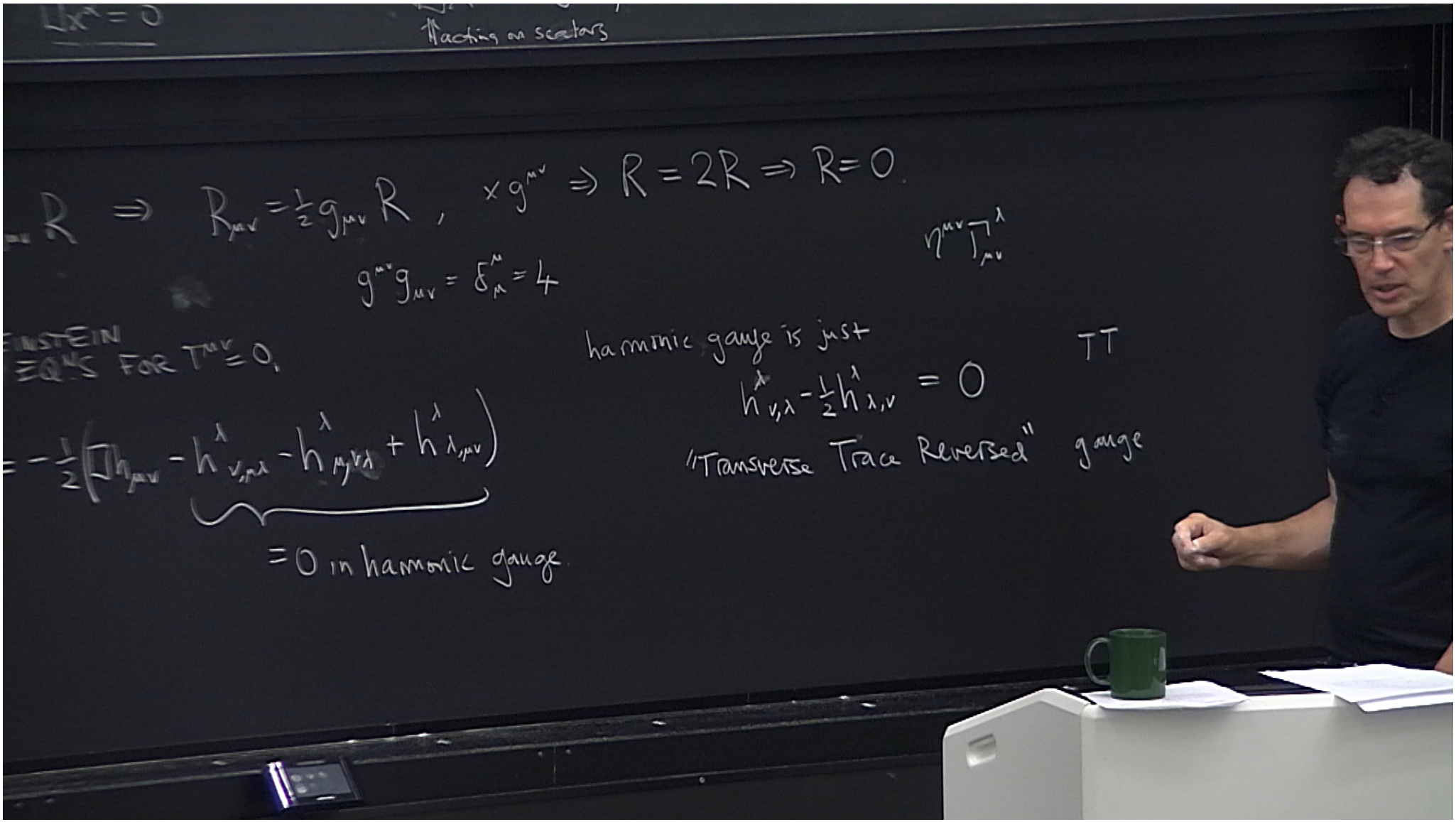
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$\Box x^\lambda = 0$

acting on scalars

$R_{\mu\nu} R \Rightarrow R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R \,, \quad \times g^{\mu\nu} \Rightarrow R = 2R \Rightarrow R = 0$

$g^{\mu\nu} g_{\mu\nu} = \delta^\mu_\mu = 4$

EINSTEIN EQ'NS FOR $T^{\mu\nu} = 0$.

$$= -\frac{1}{2}\Big(\Box h_{\mu\nu} - \underbrace{h^\lambda{}_{\nu,\mu\lambda} - h^\lambda{}_{\mu,\nu\lambda} + h^\lambda{}_{\lambda,\mu\nu}}_{=0 \text{ in harmonic gauge}}\Big)$$

harmonic gauge is just

$$h^\lambda{}_{\nu,\lambda} - \frac{1}{2} h^\lambda{}_{\lambda,\nu} = 0$$

"Transverse Trace Reversed" gauge

Consider

$$h^\lambda{}_\nu - \frac{1}{2} h^\alpha{}_\alpha \delta^\lambda_\nu$$

"Trace reversed"

$\eta^{\mu\nu} \Gamma^\lambda_{\mu\nu}$

TT

$\Box x^\alpha = 0$

acting on scalars

$R_{\mu\nu} R \Rightarrow R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R \,, \quad \times g^{\mu\nu} \Rightarrow R = 2R \Rightarrow R = 0$

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EINSTEIN EQ'NS FOR $T^{\mu\nu} = 0$.

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"Transverse Trace Reversed" gauge

$\eta^{\mu\nu} \Gamma^\lambda_{\mu\nu}$

TT

Consider

$h^\lambda{}_\nu - \frac{1}{2} h^\alpha{}_\alpha \delta^\lambda_\nu$

"Trace reversed"

$h^\beta_\beta - 2 h^\alpha_\alpha = -h^\alpha_\alpha$

$\Box x^\alpha = 0$

$\Uparrow$ acting on scalars

$R_{\mu\nu} R \Rightarrow R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R \,, \quad \times g^{\mu\nu} \Rightarrow R = 2R \Rightarrow R = 0$

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harmonic gauge is just

$$h^\lambda{}_{\nu,\lambda} - \frac{1}{2} h^\lambda{}_{\lambda,\nu} = 0$$

"Transverse Trace Reversed" gauge

$$\partial_\lambda \Big( \underbrace{h^\lambda{}_\nu - \frac{1}{2} h^\alpha{}_\alpha \delta^\lambda_\nu}_{\text{trace reversed.}} \Big) = 0$$

$\eta^{\mu\nu} \Gamma^\lambda_{\mu\nu}$

TT

Consider

$$h^\lambda{}_\nu - \frac{1}{2} h^\alpha{}_\alpha \delta^\lambda_\nu$$

"Trace reversed"

$$h^\beta{}_\beta - 2 h^\alpha{}_\alpha = -h^\alpha{}_\alpha$$

$x^\mu \approx x^\mu + \epsilon^\mu(x)$ $\quad \rightarrow$ 4 d.o.f's in gauge trans. $\quad \Box x^\mu = 0$

Linearised Einstein : $G_{\mu\nu} = 0 = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \Rightarrow R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R$

$g^{\mu\nu} g$

$$\Rightarrow \boxed{R_{\mu\nu} = 0}$$ EINSTEIN EQ^NS FOR $T^{\mu\nu} = 0$.

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$\Box x^\alpha = 0$

$\downarrow$

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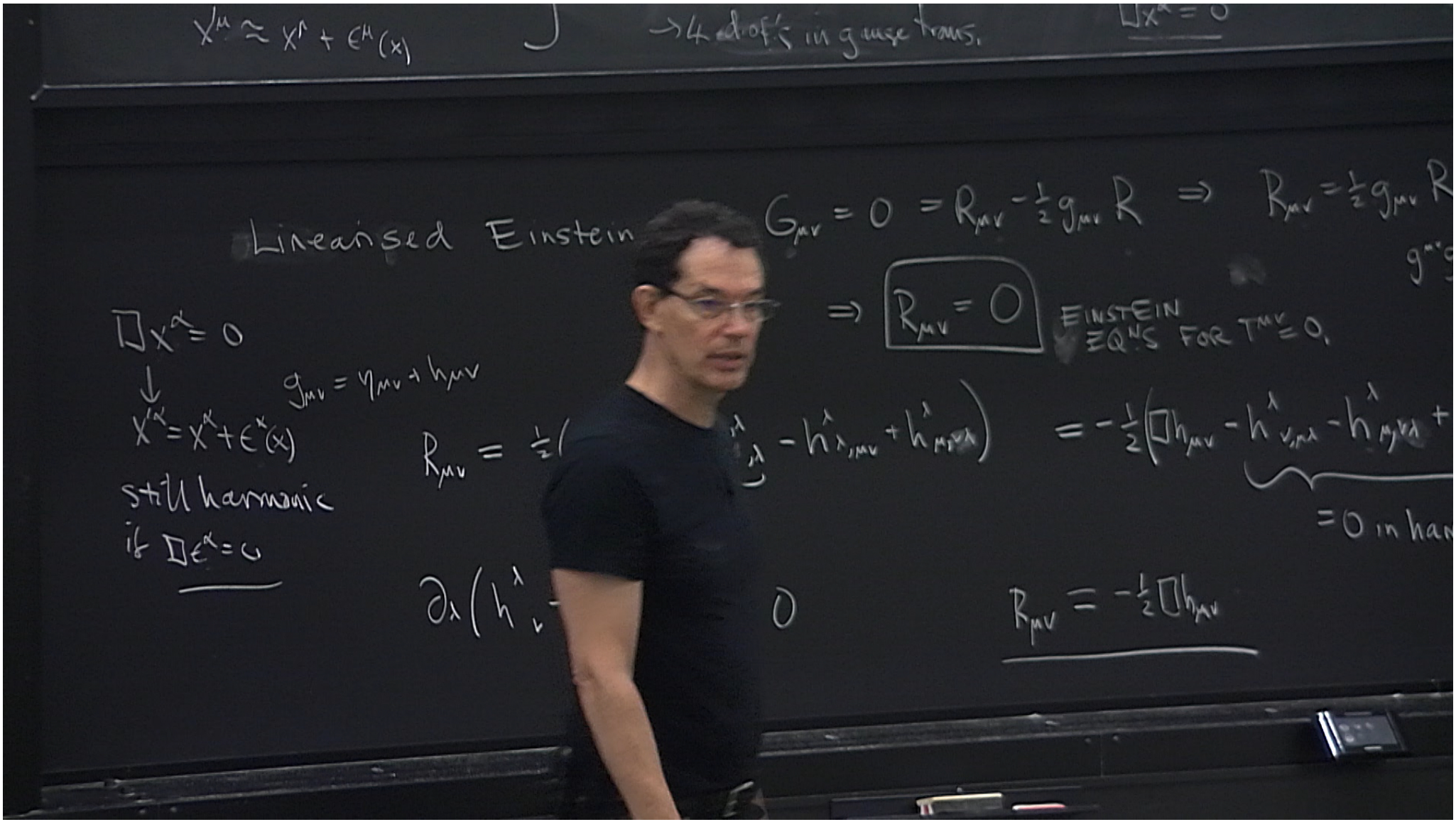
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EINSTEIN EQ^NS FOR $T^{\mu\nu} = 0$.
$\Box x^\alpha = 0$
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
$x'^\alpha = x^\alpha + \epsilon^\alpha(x)$
still harmonic if $\Box \epsilon^\alpha = 0$
$R_{\mu\nu} = \frac{1}{2}($
$= -\frac{1}{2}(\Box h_{\mu\nu} - h^\lambda{}_{\nu,\mu\lambda} - h^\lambda{}_{\mu,\nu\lambda} +$
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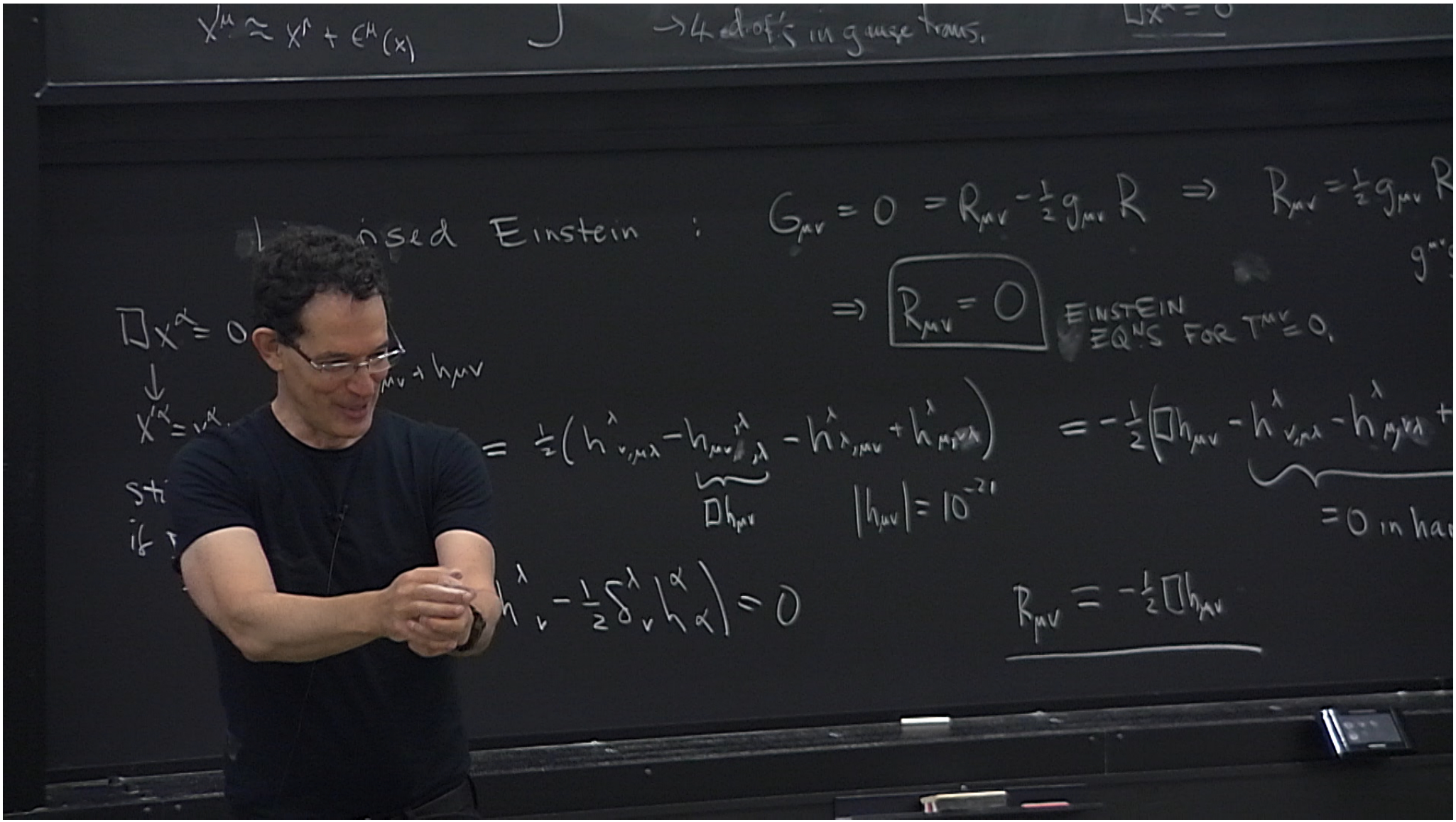
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$= \frac{1}{2}(h^\lambda{}_{\nu,\mu\lambda} - h_{\mu\nu}{}^{,\lambda}{}_{,\lambda} - h^\lambda{}_{\lambda,\mu\nu} + h^\lambda{}_{\mu,\nu\lambda})$
$\Box h_{\mu\nu}$
$|h_{\mu\nu}| = 10^{-21}$
$= -\frac{1}{2}(\Box h_{\mu\nu} - h^\lambda{}_{\nu,\mu\lambda} - h^\lambda{}_{\mu,\nu\lambda} +$
$= 0$ in ha
$h^\lambda{}_\nu - \frac{1}{2}\delta^\lambda_\nu h^\alpha{}_\alpha) = 0$
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