

Title: PSI 17/18 - Relativity - Lecture 6

Date: Sep 12, 2017 09:00 AM

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Abstract:

$$\eta_{\mu\nu} \quad \eta^{\mu\nu}$$

$$\delta_{ij}$$

$$\vec{x}^2 = x_i x_j \delta_{ij}$$

$$\rightarrow u \cdot v = \eta_{\mu\nu} u^\mu v^\nu$$

$$\eta_{\mu\nu} \quad \eta^{\mu\nu}$$

$$\delta_{ij}$$

$$\vec{x}^2 = x_i x_i \delta_{ij}$$

$$\rightarrow u \cdot v = \eta_{\mu\nu} u^\mu v^\nu$$

Poincaré group: group of (twice differentiable) coordinate transformations $x^\mu \rightarrow x'^\mu(x^\nu)$

$$\eta_{\mu\nu} \quad \eta^{\mu\nu}$$

$$\delta_{ij}$$

$$\vec{x}^2 = x_i x_i \delta_{ij}$$

$$\rightarrow u \cdot v = \eta_{\mu\nu} u^\mu v^\nu$$

Poincaré group: group of (twice differentiable) coordinate transformations $x^\mu \rightarrow x'^\mu(x^\mu)$

transformations $x^\mu \rightarrow x'^\mu(x^\alpha)$ which leave $\eta_{\mu\nu}$ unchanged.

$$\eta_{\mu\nu} \quad \eta^{\mu\nu}$$

$$\delta_{ij}$$

$$\vec{x}^2 = x_i x_i \delta_{ij}$$

$$\rightarrow u \cdot v = \eta_{\mu\nu} u^\mu v^\nu$$

Poincaré group: group of (twice differentiable) coordinate transformations $x^\mu \rightarrow$

$$F^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta F^{\alpha\beta} \text{ etc.}$$

$$\eta_{\mu\nu} \quad \eta^{\mu\nu}$$

$$\delta_{ij}$$

$$\vec{x}^2 = x_i x_i \delta_{ij}$$

$$\rightarrow u \cdot v = \eta_{\mu\nu} u^\mu v^\nu$$

Poincaré group: group of (twice differentiable) coordinate transformations $x^\mu \rightarrow$

$$F^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta F^{\alpha\beta} \text{ etc.}$$

$$\text{require } \eta'^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta \eta^{\alpha\beta} = \eta^{\mu\nu} \Rightarrow \text{L.Ts.}$$

$$\eta_{\mu\nu} \quad \eta^{\mu\nu}$$

$$\delta_{ij}$$

$$\vec{x}^2 = x_i x_j \delta_{ij}$$

$$\rightarrow u \cdot v = \eta_{\mu\nu} u^\mu v^\nu$$

Poincaré group: group of (twice differentiable) coordinate transformations $x^\mu \rightarrow$

$$F^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta F^{\alpha\beta} \text{ etc.}$$

$$\text{require } \eta'^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta \eta^{\alpha\beta} = \eta^{\mu\nu} \Rightarrow \text{L.Ts.}$$

$$(x'^\mu = \Lambda^\mu_\nu x^\nu)$$

Consider $x^\mu \rightarrow x'^\mu(x^\alpha)$

satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$

Consider $x^\mu \rightarrow x'^\mu(x^\alpha)$

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} dx^\alpha$$

satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$

Consider $x^\mu \rightarrow x'^\mu(x^\alpha)$

satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} dx^\alpha$$

$$\frac{\partial}{\partial x'^\alpha} = \frac{\partial x'^\mu}{\partial x'^\alpha} \frac{\partial}{\partial x^\mu}$$

$$dx'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} dx^{\alpha} \quad \frac{\partial}{\partial x'^{\alpha}} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial}{\partial x^{\mu}}$$

Claim: most general such coord trans. is an element of Poincare group $x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + a^{\mu}$
 with $\Lambda^{\mu}_{\nu} \Lambda^{\rho}_{\sigma} \eta_{\mu\rho} = \eta_{\nu\sigma}$.

$$\frac{\partial}{\partial x^\alpha} = \frac{\partial x^\mu}{\partial x^\alpha} \frac{\partial}{\partial x^\mu}$$

most general such coord trans. is an element of Poincare group $x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$
 with $\Lambda^\mu_\alpha \Lambda^\nu_\beta \eta^{\alpha\beta} = \eta^{\mu\nu}$

Consider $x^\mu \rightarrow x'^\mu(x^\alpha)$

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} dx^\alpha \quad \frac{\partial}{\partial x'^\alpha} = \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial}{\partial x^\mu}$$

Satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$. Claim: most general such coord trans. is an isometry

Proof: $\frac{\partial}{\partial x^\epsilon} \nearrow \Rightarrow 0 = \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\delta} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\delta} \frac{\partial^2 x'^\beta}{\partial x^\delta \partial x^\epsilon}$

Consider $x^\mu \rightarrow x'^\mu(x^\alpha)$

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} dx^\alpha$$

$$\frac{\partial}{\partial x'^\alpha} = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial}{\partial x'^\mu}$$

Satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$

Claim: most general such coord trans. is an isometry

Proof: $\frac{\partial}{\partial x^\epsilon} \nearrow \Rightarrow 0 = \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\delta} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\delta} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\delta}$

add w $\gamma \leftrightarrow \epsilon$
subtract w $\delta \leftrightarrow \epsilon$

Consider $x^\mu \rightarrow x'^\mu(x^\alpha)$

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} dx^\alpha \quad \frac{\partial}{\partial x'^\alpha} = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial}{\partial x'^\mu}$$

Satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$. Claim: most general such coord trans. is an

Proof: $\frac{\partial}{\partial x^\epsilon} \Rightarrow 0 = \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\delta} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\delta} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\delta}$

add w $\gamma \leftrightarrow \epsilon \rightarrow + \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon} \frac{\partial x'^\beta}{\partial x^\delta \partial x^\delta}$

subtract w $\gamma \leftrightarrow \epsilon \rightarrow - \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\delta \partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\epsilon} - \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\epsilon \partial x^\delta}$

consider $x^M \rightarrow x'^M(x^\alpha)$

$$dx'^M = \frac{\partial x'^M}{\partial x^\alpha} dx^\alpha \quad \frac{\partial}{\partial x'^\alpha} = \frac{\partial x'^M}{\partial x'^\alpha} \frac{\partial}{\partial x'^M}$$

satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$. Claim: most general such coord trans. is an

Proof: $\frac{\partial}{\partial x^\epsilon} \Rightarrow 0 =$

$$\begin{aligned} & \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\delta} \\ & \text{add w } \gamma \leftrightarrow \epsilon \rightarrow + \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon} \frac{\partial x'^\beta}{\partial x^\gamma \partial x^\delta} \\ & \text{subtract w } \delta \leftrightarrow \epsilon \rightarrow - \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\gamma \partial x^\delta} \frac{\partial x'^\beta}{\partial x^\epsilon} - \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\gamma} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\delta} \end{aligned}$$

consider $x^\mu \rightarrow x'^\mu(x^\alpha)$

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} dx^\alpha$$

$$\frac{\partial}{\partial x'^\alpha} = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial}{\partial x'^\mu}$$

satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$

Claim: most general such coord trans. is an isometry

Proof: $\frac{\partial}{\partial x^\epsilon} \Rightarrow 0 =$

add w $\gamma \leftrightarrow \epsilon$

subtract w $\delta \leftrightarrow \epsilon$

$$\begin{aligned} & \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\delta} \\ & + \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\delta} \frac{\partial x'^\beta}{\partial x^\gamma} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\delta} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\gamma} \\ & - \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\delta} \frac{\partial x'^\beta}{\partial x^\gamma} - \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\delta} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\gamma} \end{aligned}$$

Consider $x^M \rightarrow x'^M(x^A)$

$$dx'^M = \frac{\partial x'^M}{\partial x^A} dx^A \quad \frac{\partial}{\partial x'^A} = \frac{\partial x'^M}{\partial x'^A} \frac{\partial}{\partial x^M}$$

Satisfying $\eta_{rs} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^r} \frac{\partial x'^\beta}{\partial x^s}$. Claim: most general such coord trans. is an isometry

Proof: $\frac{\partial}{\partial x^\epsilon} \Rightarrow 0 =$

add w $r \leftrightarrow \epsilon$
 subtract w $s \leftrightarrow \epsilon$

$$0 = \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\epsilon} \frac{\partial x'^\beta}{\partial x^\epsilon} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\epsilon} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\epsilon} + \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^r} \frac{\partial x'^\beta}{\partial x^s} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\epsilon} \frac{\partial^2 x'^\beta}{\partial x^r \partial x^s} - \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^r} \frac{\partial x'^\beta}{\partial x^s} - \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^r} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^s}$$

Consider $x^\mu \rightarrow x'^\mu(x^\alpha)$

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} dx^\alpha \quad \frac{\partial}{\partial x'^\alpha} = \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial}{\partial x^\mu}$$

Satisfying $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta}$. Claim: most general such coord trans. is an

Proof: $\frac{\partial}{\partial x^\epsilon} \Rightarrow 0 =$

$$\begin{aligned} & \left(\eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\delta} \right) \\ & \xrightarrow{\text{add w } \gamma \leftrightarrow \epsilon} + \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta} + \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\epsilon} \frac{\partial^2 x'^\beta}{\partial x^\gamma \partial x^\delta} \\ & \xrightarrow{\text{subtract w } \gamma \leftrightarrow \epsilon} - \eta_{\alpha\beta} \frac{\partial^2 x'^\alpha}{\partial x^\epsilon \partial x^\gamma} \frac{\partial x'^\beta}{\partial x^\delta} - \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial^2 x'^\beta}{\partial x^\epsilon \partial x^\delta} \end{aligned}$$

$$dx'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} dx^{\alpha} \quad \frac{\partial}{\partial x'^{\alpha}} = \frac{\partial x'^{\mu}}{\partial x'^{\alpha}} \frac{\partial}{\partial x^{\mu}}$$

$$= \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\gamma}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} \quad \text{Claim: most general such coord trans. is an element of Poincare}$$

$$\Rightarrow 0 = \left(\eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\gamma} \partial x^{\epsilon}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\delta}} \frac{\partial^2 x'^{\beta}}{\partial x^{\gamma} \partial x^{\epsilon}} \right) \Rightarrow \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\gamma} \partial x^{\epsilon}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} = 0$$

multiply by $\frac{\partial x^{\delta}}{\partial x'^{\mu}}$, use $\frac{\partial x'^{\beta}}{\partial x^{\epsilon}} \frac{\partial x^{\epsilon}}{\partial x'^{\mu}} = \delta^{\beta}_{\mu}$

$$\begin{aligned} \Rightarrow 0 &= \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\gamma} \partial x^{\epsilon}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\delta}} \frac{\partial^2 x'^{\beta}}{\partial x^{\gamma} \partial x^{\epsilon}} \\ &+ \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\epsilon} \partial x^{\gamma}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\epsilon}} \frac{\partial^2 x'^{\beta}}{\partial x^{\delta} \partial x^{\gamma}} \\ &- \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\delta} \partial x^{\epsilon}} \frac{\partial x'^{\beta}}{\partial x^{\gamma}} - \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\gamma} \partial x^{\epsilon}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} \end{aligned}$$

$$dx'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} dx^{\alpha} \quad \frac{\partial}{\partial x'^{\alpha}} = \frac{\partial x'^{\mu}}{\partial x'^{\alpha}} \frac{\partial}{\partial x^{\mu}}$$

$$= \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\gamma}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} \quad \text{Claim: most general such coord trans. is an element of Poincare}$$

$$\Rightarrow 0 = \left(\eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\gamma} \partial x^{\epsilon}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\delta}} \frac{\partial^2 x'^{\beta}}{\partial x^{\gamma} \partial x^{\epsilon}} \right) + \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\epsilon} \partial x^{\gamma}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\epsilon}} \frac{\partial^2 x'^{\beta}}{\partial x^{\gamma} \partial x^{\delta}} - \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\gamma} \partial x^{\delta}} \frac{\partial x'^{\beta}}{\partial x^{\epsilon}} - \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\gamma}} \frac{\partial^2 x'^{\beta}}{\partial x^{\epsilon} \partial x^{\delta}} \Rightarrow \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\gamma} \partial x^{\epsilon}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} = 0$$

multiply by $\frac{\partial x^{\delta}}{\partial x'^{\mu}}$, use $\frac{\partial x'^{\beta}}{\partial x^{\epsilon}} \frac{\partial x^{\epsilon}}{\partial x'^{\mu}} = \delta^{\beta}_{\mu}$

$$dx'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} dx^{\alpha} \quad \frac{\partial}{\partial x'^{\alpha}} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial}{\partial x^{\mu}}$$

$$= \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\mu}} \frac{\partial x'^{\beta}}{\partial x^{\nu}} \quad \text{Claim: most general such coord trans. is an element of Poincare}$$

$$\Rightarrow 0 = \left(\eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\mu} \partial x^{\nu}} \frac{\partial x'^{\beta}}{\partial x^{\sigma}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\mu}} \frac{\partial^2 x'^{\beta}}{\partial x^{\nu} \partial x^{\sigma}} \right) \Rightarrow \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\mu} \partial x^{\nu}} \frac{\partial x'^{\beta}}{\partial x^{\sigma}} = 0$$

multiply by $\frac{\partial x^{\sigma}}{\partial x'^{\mu}}$, use $\frac{\partial x'^{\beta}}{\partial x^{\sigma}} \frac{\partial x^{\sigma}}{\partial x'^{\mu}} = \frac{\partial x'^{\beta}}{\partial x'^{\mu}}$

$$\Rightarrow 0 = \left(\eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\mu} \partial x^{\nu}} \frac{\partial x'^{\beta}}{\partial x^{\sigma}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\mu}} \frac{\partial^2 x'^{\beta}}{\partial x^{\nu} \partial x^{\sigma}} \right) - \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\mu} \partial x^{\sigma}} \frac{\partial x'^{\beta}}{\partial x^{\nu}} - \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\nu} \partial x^{\sigma}} \frac{\partial x'^{\beta}}{\partial x^{\mu}}$$

$$\frac{\partial}{\partial x^\alpha} = \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial}{\partial x^\mu}$$

general such coord trans. is an element of Poincare group $x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$
 with $\Lambda^\mu_\alpha \Lambda^\nu_\beta \eta^{\alpha\beta} = \eta^{\mu\nu}$

$$\Rightarrow \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\delta} \frac{\partial x'^\beta}{\partial x^\epsilon} = 0$$

multiply by $\frac{\partial x^\delta}{\partial x'^\mu}$

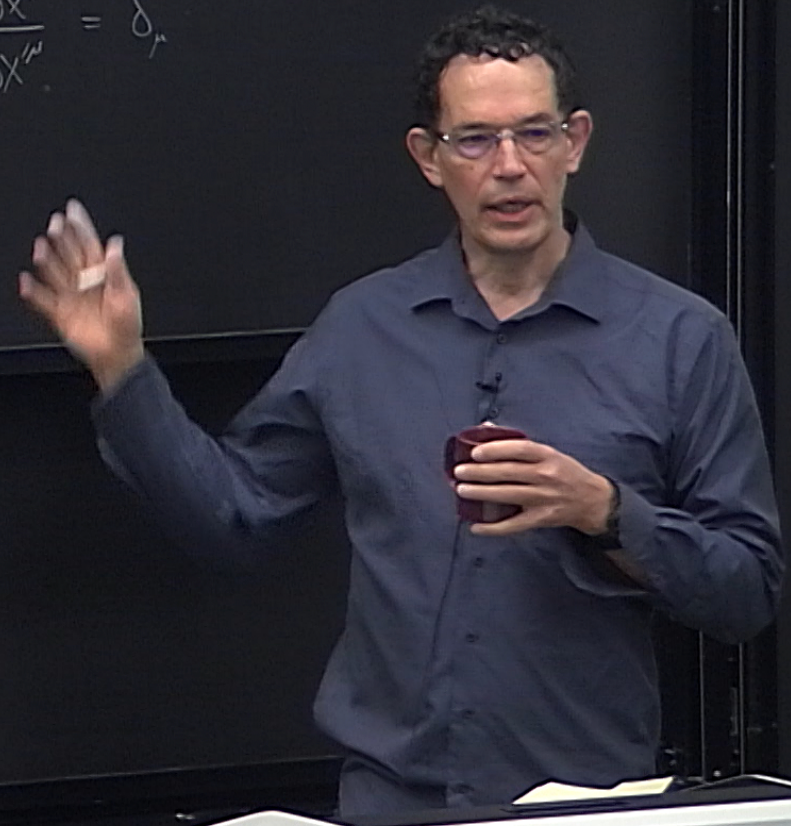
$$\text{use } \frac{\partial x'^\beta}{\partial x^\delta} \frac{\partial x^\delta}{\partial x'^\mu} = \frac{\partial x'^\beta}{\partial x'^\mu} = \delta^\beta_\mu$$

most general such coord trans. is an element of Poincare group $x^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$
 with $\Lambda^\mu_\alpha \tilde{\Lambda}^\nu_\beta \eta^{\alpha\beta} = \eta^{\mu\nu}$

$$\Rightarrow \eta_{\alpha\beta} \frac{\partial^2 x^{\alpha\beta}}{\partial x^\mu \partial x^\nu} = 0$$

multiply by $\frac{\partial x^\delta}{\partial x'^\mu}$, use $\frac{\partial x'^\beta}{\partial x^\delta} \frac{\partial x^\delta}{\partial x'^\mu} = \frac{\partial x'^\beta}{\partial x'^\mu} = \delta^\beta_\mu$

$$\Rightarrow \eta_{\alpha\mu} \frac{\partial^2 x^{\alpha\beta}}{\partial x^\mu \partial x^\nu} \Rightarrow \boxed{\frac{\partial^2 x'^\nu}{\partial x^\mu \partial x^\nu} = 0}$$

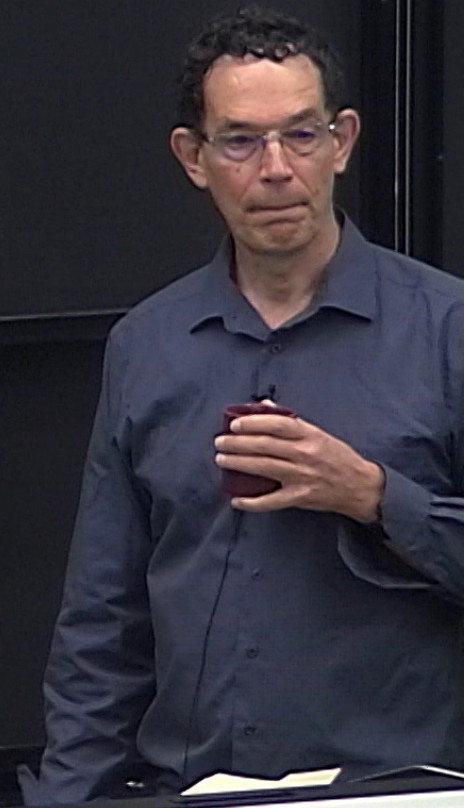


most general such coord trans. is an element of Poincare group $x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$
 with $\Lambda^\mu_\alpha \Lambda^\nu_\beta \eta^{\alpha\beta} = \eta^{\mu\nu}$

$$\Rightarrow \eta_{\alpha\beta} \frac{\partial x'^\alpha}{\partial x^\mu \partial x^\nu} \frac{\partial x'^\beta}{\partial x^\epsilon} = 0$$

multiply by $\frac{\partial x^\delta}{\partial x'^\mu}$, use $\frac{\partial x'^\beta}{\partial x^\delta} \frac{\partial x^\delta}{\partial x'^\mu} = \frac{\partial x'^\beta}{\partial x'^\mu} = \delta^\beta_\mu$

$$\Rightarrow \eta_{\alpha\mu} \frac{\partial x'^\alpha}{\partial x^\nu \partial x^\epsilon} \Rightarrow \boxed{\frac{\partial x'^\nu}{\partial x^\nu \partial x^\epsilon} = 0} \Rightarrow \frac{\partial x'^\nu}{\partial x^\epsilon} = \Lambda^\nu_\epsilon$$



most general such coord trans. is an element of Poincare group $x^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$
 with $\Lambda^\mu_\alpha \tilde{\Lambda}^\nu_\beta \eta^{\alpha\beta} = \eta^{\mu\nu}$

$$\Rightarrow \eta_{\alpha\beta} \frac{\partial x^{\alpha'}}{\partial x^\alpha} \frac{\partial x^{\beta'}}{\partial x^\beta} = 0$$

multiply by $\frac{\partial x^\delta}{\partial x^{\mu'}}$

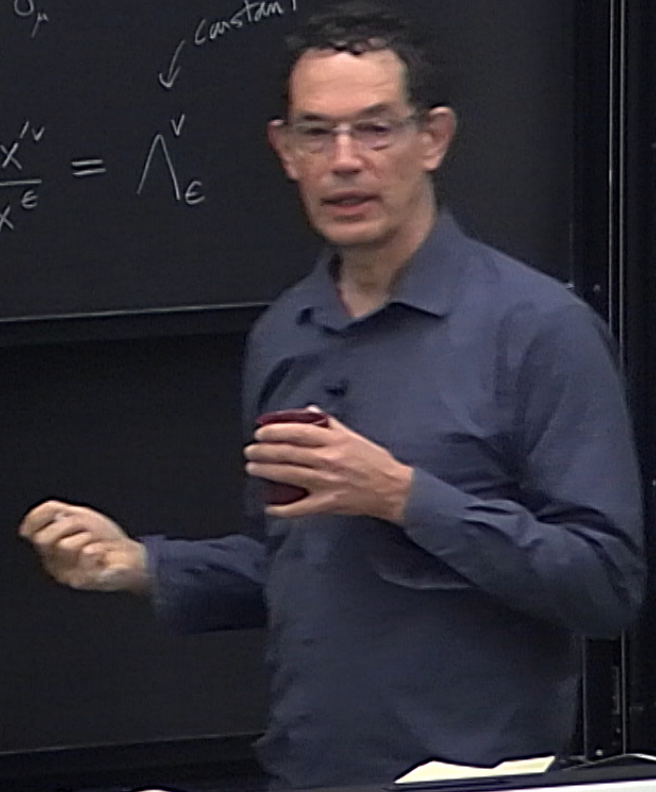
use $\frac{\partial x^{\beta'}}{\partial x^\delta} \frac{\partial x^\delta}{\partial x^{\mu'}} = \frac{\partial x^{\beta'}}{\partial x^{\mu'}} = \delta^\beta_\mu$

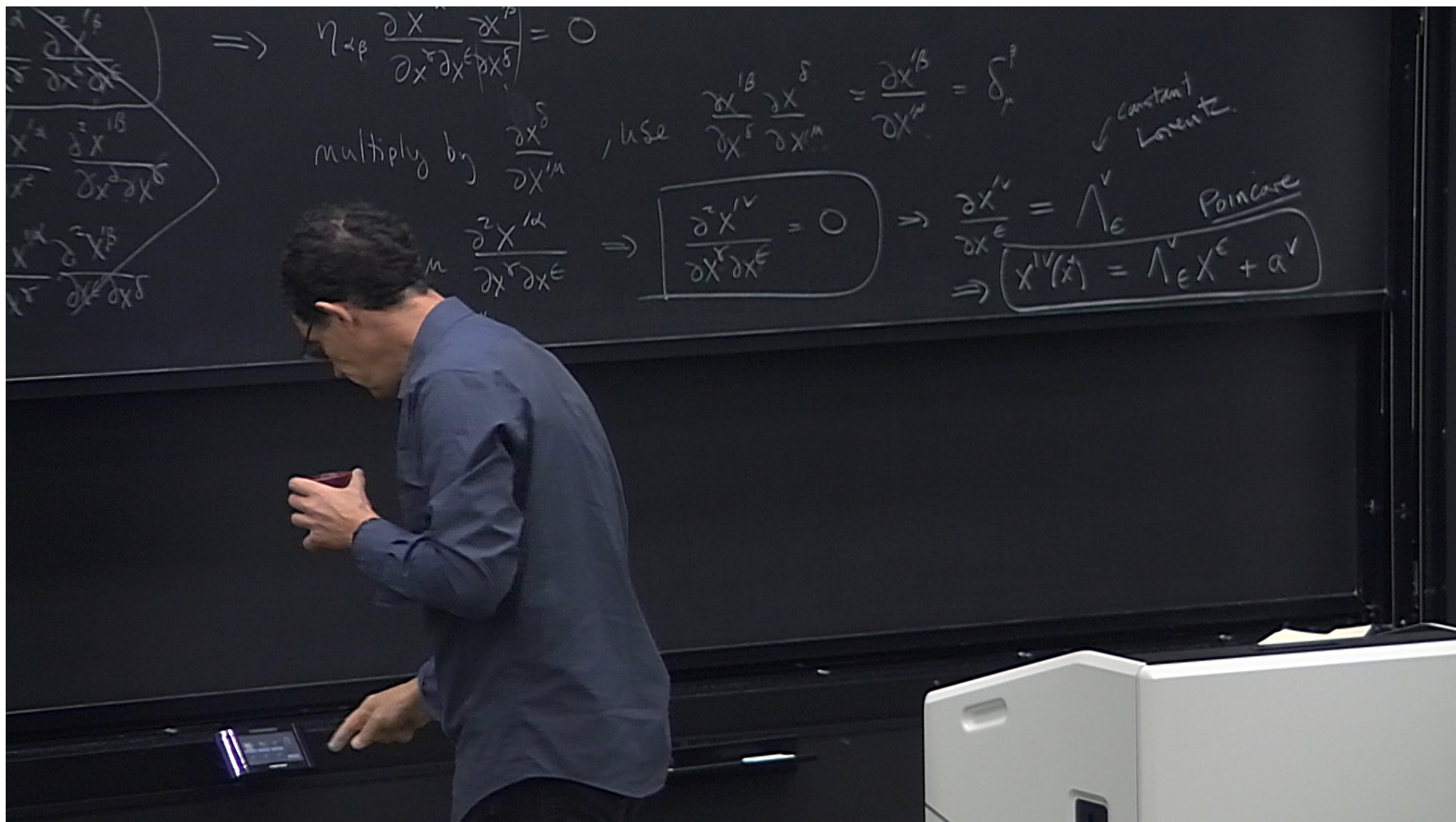
$$\Rightarrow \eta_{\alpha\mu} \frac{\partial x^{\alpha'}}{\partial x^\alpha} \frac{\partial x^{\mu'}}{\partial x^\epsilon} \times \eta^{\mu\nu}$$

$$\boxed{\frac{\partial^2 x^{\mu'}}{\partial x^\alpha \partial x^\epsilon} = 0}$$

$$\Rightarrow \frac{\partial x^{\mu'}}{\partial x^\epsilon} = \Lambda^\mu_\epsilon$$

constant





$$\frac{\partial^2 X'^\alpha}{\partial x^\mu \partial x^\nu} \frac{\partial X'^\beta}{\partial x^\sigma} + \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial x^\mu \partial x^\nu} \frac{\partial X'^\beta}{\partial x^\sigma} = 0$$

$$\frac{\partial^2 X'^\alpha}{\partial x^\mu \partial x^\nu} \frac{\partial X'^\beta}{\partial x^\sigma} + \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial x^\mu \partial x^\nu} \frac{\partial X'^\beta}{\partial x^\sigma} = 0$$

$$\frac{\partial^2 X'^\alpha}{\partial x^\mu \partial x^\nu} \frac{\partial X'^\beta}{\partial x^\sigma} - \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial x^\mu \partial x^\nu} \frac{\partial X'^\beta}{\partial x^\sigma} = 0$$

$$\Rightarrow \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial x^\mu \partial x^\nu} \frac{\partial X'^\beta}{\partial x^\sigma} = 0$$

multiply by $\frac{\partial x^\sigma}{\partial x'^\mu}$, use $\frac{\partial x'^\beta}{\partial x^\sigma} \frac{\partial x^\sigma}{\partial x'^\mu} = \frac{\partial x'^\beta}{\partial x'^\mu} = \delta_\mu^\beta$

$$\Rightarrow \eta_{\alpha\mu} \frac{\partial^2 X'^\alpha}{\partial x^\nu \partial x^\epsilon} \Rightarrow \boxed{\frac{\partial^2 X'^\nu}{\partial x^\nu \partial x^\epsilon} = 0} \Rightarrow \frac{\partial x'^\nu}{\partial x^\epsilon} = \Lambda_\epsilon^\nu$$

$$\Rightarrow x'^\nu(x) = \Lambda_\epsilon^\nu x^\epsilon + \text{Poinc}$$

$$\eta^{\mu\nu} \eta_{\alpha\beta} = \delta_\alpha^\mu$$

Constant Lorentz

SR $\eta \leftrightarrow$ Poincaré
GR $\downarrow$

SR $\eta_{\mu\nu} \leftrightarrow$ Poincaré
↓
GR $g_{\mu\nu}(x)$

SR $\eta_{\mu\nu} \leftrightarrow$ Poincaré
↓
GR $g_{\mu\nu}(x)$

$$\int \eta_{\mu\nu} dx^\mu dx^\nu$$

Poincaré

$$\sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}$$

invariant



$$\sqrt{-g_{\mu\nu}(x) dx^\mu dx^\nu}$$

SR

$\eta_{\mu\nu} \leftrightarrow$ Poincaré

GR



$g_{\mu\nu}(x)$

→ coordinate
invariant
lengths and angles
and ...

$\int \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}$
invariant



$\sqrt{-g_{\mu\nu}(x) dx^\mu dx^\nu}$

$$\eta^{\mu\nu} \eta_{\alpha\mu} = \delta^\nu_\alpha$$

$$x^\mu \longrightarrow x'^\mu(x)$$

idea: $g_{\mu\nu}(x) \longrightarrow g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$

SR

$\eta_{\mu\nu} \leftrightarrow$ Poincaré
(flat geometry)

GR

$g_{\mu\nu}(x) \rightarrow$ coordinate
invariant
lengths and angles
and ...
(curved geometry)

$\int \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}$
invariant

$\downarrow$
 $\sqrt{-g_{\mu\nu}(x) dx^\mu dx^\nu}$

idea: $g_{\mu\nu}(x)$

$\eta^{\mu\nu} \eta_{\alpha\mu} = \delta^\nu_\alpha$
 $x^\mu \rightarrow x'^\mu(x)$ arbitrary 1-1 mapping ($\frac{\partial x'^\alpha}{\partial x^\mu}$ invertible $\frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x^\mu}{\partial x'^\beta} = \delta^\alpha_\beta$ etc)
 idea: $g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$

Poincaré
(flat geometry)

→ coordinate
invariant
lengths and angles
and ...
(curved geometry)

$$\int \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}$$



$$\sqrt{-g_{\mu\nu}(x) dx^\mu dx^\nu}$$

Action

$$-mc \int \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} d\lambda$$

idea:

$$x^\mu \rightarrow x'^\mu(x^\mu) \text{ arbitrary 1-1}$$

$$g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\eta^{\mu\nu} \eta_{\mu\nu} = \delta^\mu_\mu$$

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(flat geometry)

→ coordinate
invariant
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$\int \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}$
invariant



$$\sqrt{-g_{\mu\nu}(x) dx^\mu dx^\nu}$$

Action

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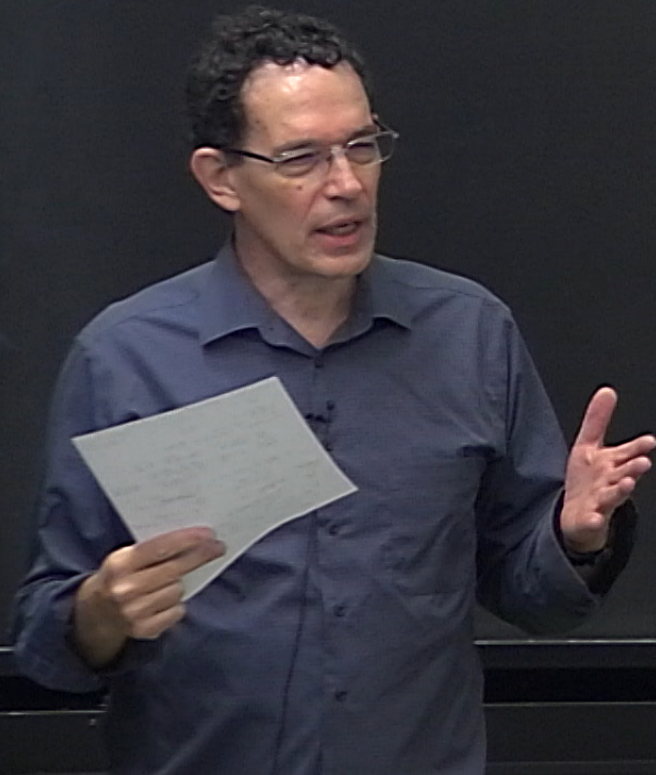
$$-mc \int \sqrt{-g_{\mu\nu}(x) \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} d\lambda$$

idea:

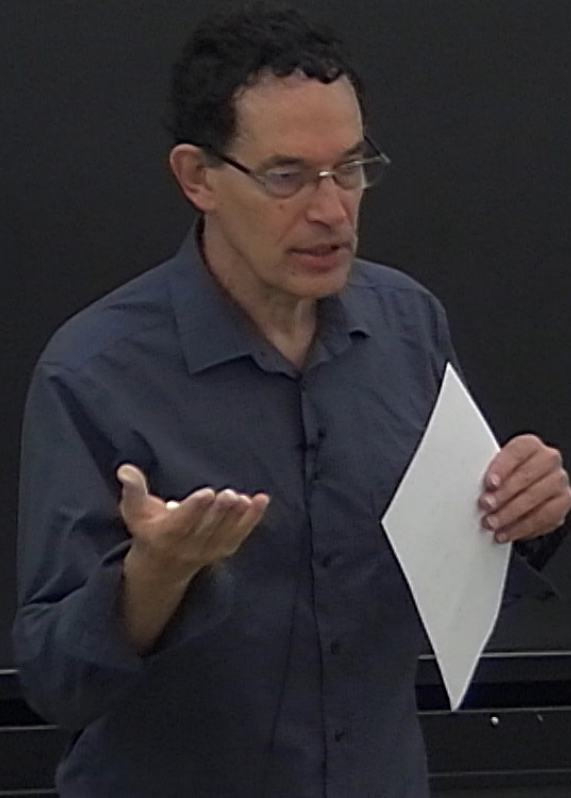
$x^\mu \rightarrow x'^\mu(x)$ arbitrary 1-1

$$g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

Gravity is encoded in $g_{\mu\nu}(x)$



(universal, i.e. objects of any mass or composition fall in same way).



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In fact, uniform gravitational field may be removed by a coordinate transformation!

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Free particle in Minkowski spacetime $\frac{d^2 x^\mu}{d\tau^2} = 0$

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may be ^{created} _{or} removed by a coordinate transformation!

$$\frac{d^2 x^\mu}{d\tau^2} = 0 \Rightarrow x^\mu = (\gamma, \gamma v, 0, 0) \tau$$

$$\left[\begin{array}{c} \text{timelike} \\ \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \Rightarrow \gamma = \frac{1}{\sqrt{1-v^2}} \end{array} \right]$$

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Look at it in new coordinates $x'^\mu(x') \leftrightarrow x^\mu(x^\mu)$

Gravity is encoded in $g_{\mu\nu}(x)$

(universal, i.e. objects of any mass or charge)

In fact, any gravitational field may be ^{created} removed by a coordinate transformation

Free fall in Minkowski spacetime

$$\frac{d^2 x^\mu}{d\tau^2} = 0 \Rightarrow x^\mu = (\gamma, \gamma v, 0, 0) \tau$$

timelike
 $\eta_{\mu\nu} \frac{dx^\mu}{d\tau}$

Locate it in new coordinates

$$x'^\mu(x') \Leftrightarrow x^\mu(x'')$$

$$0 = \frac{d^2 x^\alpha}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial x^\alpha}{\partial x'^\nu} \frac{dx'^\nu}{d\tau} \right) = \frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu} \frac{dx'^\mu}{d\tau} \frac{dx'^\nu}{d\tau} + \frac{\partial x^\alpha}{\partial x'^\nu} \frac{d^2 x'^\nu}{d\tau^2}$$

Gravity is encoded in $g_{\mu\nu}(x)$

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Free particle in Minkowski spacetime $\frac{d^2 X^\mu}{d\tau^2} = 0 \Rightarrow X^\mu = (\gamma, \gamma v, 0, 0) \tau$ [timelike $\eta_{\mu\nu} \frac{dX^\mu}{d\tau}$

Look at it in new coordinates $X'^\mu(x') \leftrightarrow X^\mu(x)$

$$0 = \frac{d^2 X^\alpha}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial X^\alpha}{\partial x'^\nu} \frac{dx'^\nu}{d\tau} \right) = \frac{\partial^2 X^\alpha}{\partial x'^\mu \partial x'^\nu} \frac{dx'^\mu}{d\tau} \frac{dx'^\nu}{d\tau} + \frac{\partial X^\alpha}{\partial x'^\nu} \frac{d^2 x'^\nu}{d\tau^2} \Rightarrow$$

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$$\frac{d^2 x^\mu}{d\tau^2} = 0 \Rightarrow x^\mu = (\gamma, \gamma v, 0, 0) \tau$$

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$x^\alpha(x'^\alpha)$

$$\frac{d^2 x^\alpha}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial x^\alpha}{\partial x'^\nu} \frac{dx'^\nu}{d\tau} \right) = \frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu} \frac{dx'^\mu}{d\tau} \frac{dx'^\nu}{d\tau} + \frac{\partial x^\alpha}{\partial x'^\nu} \frac{d^2 x'^\nu}{d\tau^2}$$

$$\Rightarrow \left(\frac{d^2 x'^\nu}{d\tau^2} + \frac{\partial x'^\nu}{\partial x^\mu} \frac{\partial^2 x^\mu}{\partial x'^\alpha \partial x'^\beta} \frac{dx'^\alpha}{d\tau} \frac{dx'^\beta}{d\tau} \right) = 0$$

(universal, i.e. objects of any mass or composition fall in same way).

may ^{created or} moved by a coordinate transformation!

$$\frac{d^2 x^\mu}{d\tau^2} = 0 \quad (\gamma, \gamma v, 0, 0) \tau$$

$$\left[\begin{array}{c} \text{timelike} \\ \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \Rightarrow \gamma = \frac{1}{\sqrt{1-v^2}} \end{array} \right]$$

$$\frac{d^2 x^\mu}{d\tau^2} = 0 \Rightarrow \frac{d^2 x^\mu}{d\tau^2} + \frac{\partial x^\mu}{\partial x'^\nu} \frac{\partial^2 x^\nu}{\partial x'^\alpha \partial x'^\beta} \frac{dx'^\alpha}{d\tau} \frac{dx'^\beta}{d\tau} = 0$$

Fictitious force.

ispractine $\frac{d^2 X^M}{d\tau^2} = 0 \Rightarrow X^M = (\gamma, \gamma v, 0, 0) \tau$

timelike $\eta_{\mu\nu} \frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} = -1 \Rightarrow \gamma = \frac{1}{\sqrt{1-v^2}}$

$X'^\alpha(x) \Leftrightarrow X^\alpha(x')$

$$0 = \frac{d^2 X^\alpha}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial X^\alpha}{\partial X'^\nu} \frac{dX'^\nu}{d\tau} \right) = \frac{\partial^2 X^\alpha}{\partial X'^\mu \partial X'^\nu} \frac{dX'^\mu}{d\tau} \frac{dX'^\nu}{d\tau} + \frac{\partial X^\alpha}{\partial X'^\nu} \frac{d^2 X'^\nu}{d\tau^2} \Rightarrow \boxed{\frac{d^2 X'^\nu}{d\tau^2} + \frac{\partial X'^\nu}{\partial X^\mu} \frac{\partial^2 X^\mu}{\partial X'^\mu \partial X'^\nu} \frac{dX'^\mu}{d\tau} \frac{dX'^\nu}{d\tau} = \text{Fictitious force}}$$

$\frac{1}{2}at^2$

$$(c=1)$$

$$x^\mu = (t, \underline{x})$$

$$\left(\frac{\partial}{\partial x^\alpha} \right) (f^\beta(x))$$

$$= \left. \frac{d}{dx^\alpha} f^\beta(x) \right|_{x^\mu \neq \alpha \text{ const}}$$

take

$$t' = t$$

$$x' = -at^2$$

$$dt^2$$

$$dt \left(\frac{\partial}{\partial x^\alpha} \right)$$

$$(c=1)$$

$$x^\mu = (t, \underline{x})$$

take $t' = t$
 $x' = x + \frac{1}{2}at^2$

$$\left(\frac{\partial}{\partial x^\mu} \right) \left(f^\mu(x^\mu(t)) \right) \left(\frac{\partial f}{\partial t}, \frac{\partial f}{\partial x}, \dots \right)$$

$$= \left. \frac{d}{dt} f^\mu(x) \right|_{x^\mu \neq x_{const}}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} \frac{dt}{dt} + \frac{\partial f}{\partial x} \frac{dx}{dt} + \dots$$

$$(c=1)$$

$$(t, x)$$

take $t' = t$
 $x' = x + \frac{1}{2}at^2$

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$$\frac{df}{dt} = \frac{\partial f}{\partial t} \frac{dt}{dt} + \frac{\partial f}{\partial x} \frac{dx}{dt} + \dots$$

$$dt^2 \quad d\left(\frac{\partial x^\mu}{\partial \tau}, \frac{dx^\mu}{d\tau}\right) \quad \frac{\partial}{\partial x^\mu} \left(\frac{\partial x^\mu}{\partial x'^\nu} \right) \frac{dx^\mu}{d\tau} + \frac{\partial x'^\nu}{\partial \tau^2}$$

$$t' = t$$

$$x' = x + \frac{1}{2}at^2$$

$$x^\mu = (\tau, \gamma\tau, 0, 0)$$

$$= (t, x)$$

$$t' = \gamma\tau$$

$$x' = \gamma\tau + \frac{1}{2}a(\gamma\tau)^2 = \gamma t' + \frac{1}{2}at'^2$$

$$y' = 0$$

$$z' = 0$$

$$\frac{\partial f}{\partial t} \frac{dt}{d\tau} + \frac{\partial f}{\partial x} \frac{dx}{d\tau} + \dots$$

$$dt^2 \quad d\left(\frac{\partial x^\mu}{\partial x^\nu}, \frac{dx^\nu}{d\tau}\right) \quad \frac{\partial}{\partial x^\mu} \left(\frac{\partial x^\mu}{\partial x^\nu} \right) \frac{dx^\nu}{d\tau} + \frac{\partial x^\nu}{\partial x^\mu} \frac{d^2 x^\mu}{d\tau^2}$$

$$t' = t$$

$$x' = x + \frac{1}{2}at^2$$

w/ $a = \text{const.}$

$$x^\mu = (t, \gamma t, 0, 0) \tau$$

$$= (t, x)$$

$$t' = \gamma \tau$$

$$x' = \gamma v \tau + \frac{1}{2}a(\gamma \tau)^2 = vt' + \frac{1}{2}at'^2$$

$$y' = 0$$

$$z' = 0$$

Uniform acceleration

$$\frac{\partial f}{\partial t} \frac{dt}{d\tau} + \frac{\partial f}{\partial x} \frac{dx}{d\tau} + \dots$$

$$dt^2 \quad d\tau \left(\frac{\partial x^\mu}{\partial x'^\alpha} \frac{dx^\mu}{d\tau} \right) \frac{dx'^\alpha}{d\tau} + \frac{\partial x'^\mu}{\partial x'^\nu} \frac{dx'^\nu}{d\tau} \frac{dx'^\mu}{d\tau}$$

$$t' = t$$

$$x' = x + \frac{1}{2}at^2$$

w/ $a = \text{const.}$

$$x^\mu = (t, x, 0, 0)$$

$$= (t, x)$$

Uniform acceleration

$$t' = \gamma \tau$$

$$x' = \gamma v \tau + \frac{1}{2}a(\gamma \tau)^2 = vt' + \frac{1}{2}at'^2$$

$$y' = 0$$

$$z' = 0$$

$$= \frac{\partial f}{\partial t} \frac{dt}{d\tau} + \frac{\partial f}{\partial x} \frac{dx}{d\tau} + \dots$$

$$-1 = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = \eta_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} \frac{dx'^\alpha}{d\tau} \frac{dx'^\beta}{d\tau}$$

$$d\tau^2 = dt^2 - \frac{1}{c^2} \left(\frac{dx}{dt} \right)^2 = dt^2 \left(1 - \frac{v^2}{c^2} \right)$$

$$\frac{\partial}{\partial x^\beta} \left(\frac{\partial x^\mu}{\partial x'^\nu} \right) \frac{dx^\beta}{d\tau}$$

fictitious force.

$$x^\mu = (t, x, 0, 0)$$

$$= (t, x)$$

Uniform acceleration

$$t' = \gamma \tau$$

$$x' = \gamma v \tau + \frac{1}{2} a (\gamma \tau)^2 = vt' + \frac{1}{2} a t'^2$$

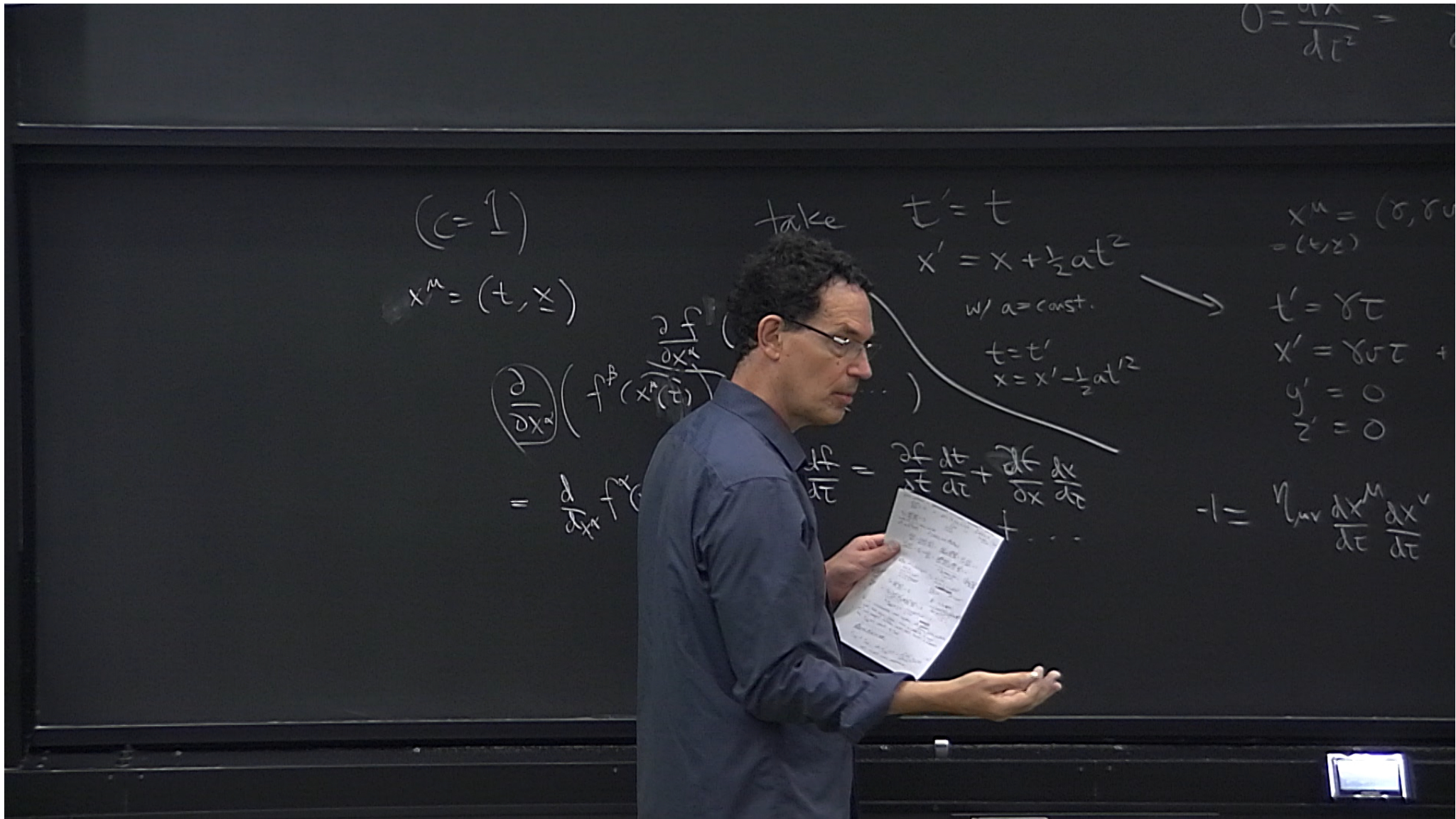
$$y' = 0$$

$$z' = 0$$

$$g'_{\alpha\beta}(x') =$$

$$-1 = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = \eta_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} \frac{dx'^\alpha}{d\tau} \frac{dx'^\beta}{d\tau}$$

$\underbrace{\hspace{10em}}_{g'_{\alpha\beta}(x')}$



$$0 = \frac{dx}{dt^2} =$$

$$(c=1)$$

$$x^\mu = (t, \underline{x})$$

take $t' = t$
 $x' = x + \frac{1}{2}at^2$

w/ $a = \text{const.}$

$$t = t'$$

$$x = x' - \frac{1}{2}at'^2$$

$$x^\mu = (t, \underline{x})$$

$$t' = \gamma \tau$$

$$x' = \gamma v \tau$$

$$y' = 0$$

$$z' = 0$$

$$\left(\frac{\partial}{\partial x^\mu} \right) \left(f^\mu(x^\nu(\tilde{t})) \right)$$

$$= \frac{d}{d\tilde{t}} f^\mu$$

$$\frac{df}{d\tau} = \frac{\partial f}{\partial t} \frac{dt}{d\tau} + \frac{\partial f}{\partial x} \frac{dx}{d\tau} + \dots$$

$$-1 = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$$

$d\tau^2$ $d\tau \left(\frac{\partial x^\mu}{\partial x'^\nu} \frac{dx^\mu}{d\tau} \right)$ $\frac{\partial}{\partial x^\mu} \left(\frac{\partial x^\mu}{\partial x'^\nu} \right) \frac{dx^\mu}{d\tau}$ $\frac{\partial x'^\mu}{\partial x^\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$ $\frac{\partial x'^\mu}{\partial x^\nu} \frac{dx^\mu}{d\tau}$

take $t' = t$
 $x' = x + \frac{1}{2}at^2$
 w/ $a = \text{const.}$
 $t = t'$
 $x = x' - \frac{1}{2}at'^2$

$x^\mu = (t, x, 0, 0)$
 $t' = \gamma\tau$
 $x' = \gamma v\tau + \frac{1}{2}a(\gamma\tau)^2 = vt' + \frac{1}{2}at'^2$
 $y' = 0$
 $z' = 0$

Uniform acceleration
 $\frac{\partial x^\mu}{\partial x'^\nu} \frac{\partial x^\mu}{\partial x'^\nu} + \frac{\partial x^\mu}{\partial x'^\nu} \frac{\partial x^\mu}{\partial x'^\nu}$
 -1 $(at')^2$

$\frac{d\tau}{d\tau} = \frac{\partial \tau}{\partial t} \frac{dt}{d\tau} + \frac{\partial \tau}{\partial x} \frac{dx}{d\tau} + \dots$

$-1 = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = \boxed{\eta_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} \frac{dx'^\alpha}{d\tau} \frac{dx'^\beta}{d\tau}} = g'_{\alpha\beta}(x')$

$$\frac{d}{dt} \left(\frac{\partial x}{\partial t} \right) \frac{dx^B}{dt} + \frac{\partial^2 x}{\partial x^A \partial x^B} \frac{dx^A}{dt} \frac{dx^B}{dt}$$

fictitious force.

$$a = (0, 0, 0, 0) \tau$$

Uniform acceleration

$$x^2 = vt' + \frac{1}{2}at'^2$$

$$00 \text{ compnt } -\frac{\partial x^0}{\partial x'^0} \frac{dx'^0}{dt'} + \frac{\partial x^0}{\partial x'^1} \frac{dx'^1}{dt'} = -1 + (at')^2$$

$$= \left[\eta_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} \right] \frac{dx'^\alpha}{dt'} \frac{dx'^\beta}{dt'}$$

$$g'_{\alpha\beta}(x')$$

$$g'_{\alpha\beta}(x') = \begin{pmatrix} -1 + (at')^2 & -at' & & \\ -at' & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$0 = \frac{dx}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial x}{\partial x'^\nu} \frac{dx'}{d\tau} \right) = \frac{\partial x}{\partial x'^\nu} \frac{d^2 x'}{d\tau^2} + \frac{\partial^2 x}{\partial x'^\nu \partial x'^\mu} \frac{dx'}{d\tau} \frac{dx'}{d\tau}$$

$$(c=1)$$

$$x^M = (t, x)$$

take $t' = t$
 $x' = x + \frac{1}{2}at^2$
 w/ $a = \text{const.}$

$$t = t'$$

$$x = x' - \frac{1}{2}at'^2$$

$$x^M = (0, \gamma\tau, 0, 0)\tau$$

$$= (t, x)$$

$$t' = \gamma\tau$$

$$x' = \gamma v \tau + \frac{1}{2}a(\gamma\tau)^2 = vt' + \frac{1}{2}$$

$$y' = 0$$

$$z' = 0$$

$$00 \text{ compnt } - \frac{\partial x^0}{\partial x'^0} \frac{\partial x^0}{\partial x'^0} = -1$$

$$\frac{df}{d\tau} = \frac{\partial f}{\partial t} \frac{dt}{d\tau} + \frac{\partial f}{\partial x} \frac{dx}{d\tau} + \dots$$

$$-1 = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = \boxed{\eta_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} \frac{dx'^\alpha}{d\tau} \frac{dx'^\beta}{d\tau}} = g'_{\alpha\beta}(x')$$

$$0 = \frac{dx}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial x}{\partial x'^\nu} \frac{dx'}{d\tau} \right) = \frac{\partial x}{\partial x'^\nu} \frac{d^2 x'}{d\tau^2} + \frac{\partial^2 x}{\partial x'^\nu \partial x'^\mu} \frac{dx'}{d\tau} \frac{dx'}{d\tau}$$

$$(c=1)$$

$$x^\mu = (t, \underline{x})$$

take $t' = t$

$$x' = x + \frac{1}{2}at^2$$

$y' = y$
 $z' = z$ w/ $a = \text{const.}$

$$t = t'$$

$$x = x' - \frac{1}{2}at'^2$$

$$x^\mu = (\gamma\tau, \gamma v\tau, 0, 0)$$

$$= (t, \underline{x})$$

$$t' = \gamma\tau$$

$$x' = \gamma v\tau + \frac{1}{2}a(\gamma\tau)^2 = vt' + \frac{1}{2}at'^2$$

$$y' = 0$$

$$z' = 0$$

so const $-\frac{\partial x^0}{\partial x'^0} \frac{\partial x^0}{\partial x'^0} = -1$

$$\left(\frac{\partial}{\partial x^\alpha} \right) \left(f^\beta(x^\mu(\tau)) \right) \left(\frac{\partial f}{\partial t}, \frac{\partial f}{\partial x}, \dots \right)$$

$$= \left. \frac{d}{dx^\alpha} f^\beta(x) \right|_{x^\mu = x'^\mu \text{ const}}$$

$$\frac{df}{d\tau} = \frac{\partial f}{\partial t} \frac{dt}{d\tau} + \frac{\partial f}{\partial x} \frac{dx}{d\tau} + \dots$$

$$-1 = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = \boxed{\eta_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} \frac{dx'^\alpha}{d\tau} \frac{dx'^\beta}{d\tau}} = g'_{\alpha\beta}(x')$$

$$\frac{d}{dt} \left(\frac{\partial x^k}{\partial x^v} \right) \frac{dx^v}{dt} + \frac{\partial x^k}{\partial x^v} \frac{d^2 x^v}{dt^2}$$

Fictitious force.

$$x = (0, vt, 0, 0) \tau$$

Uniform acceleration

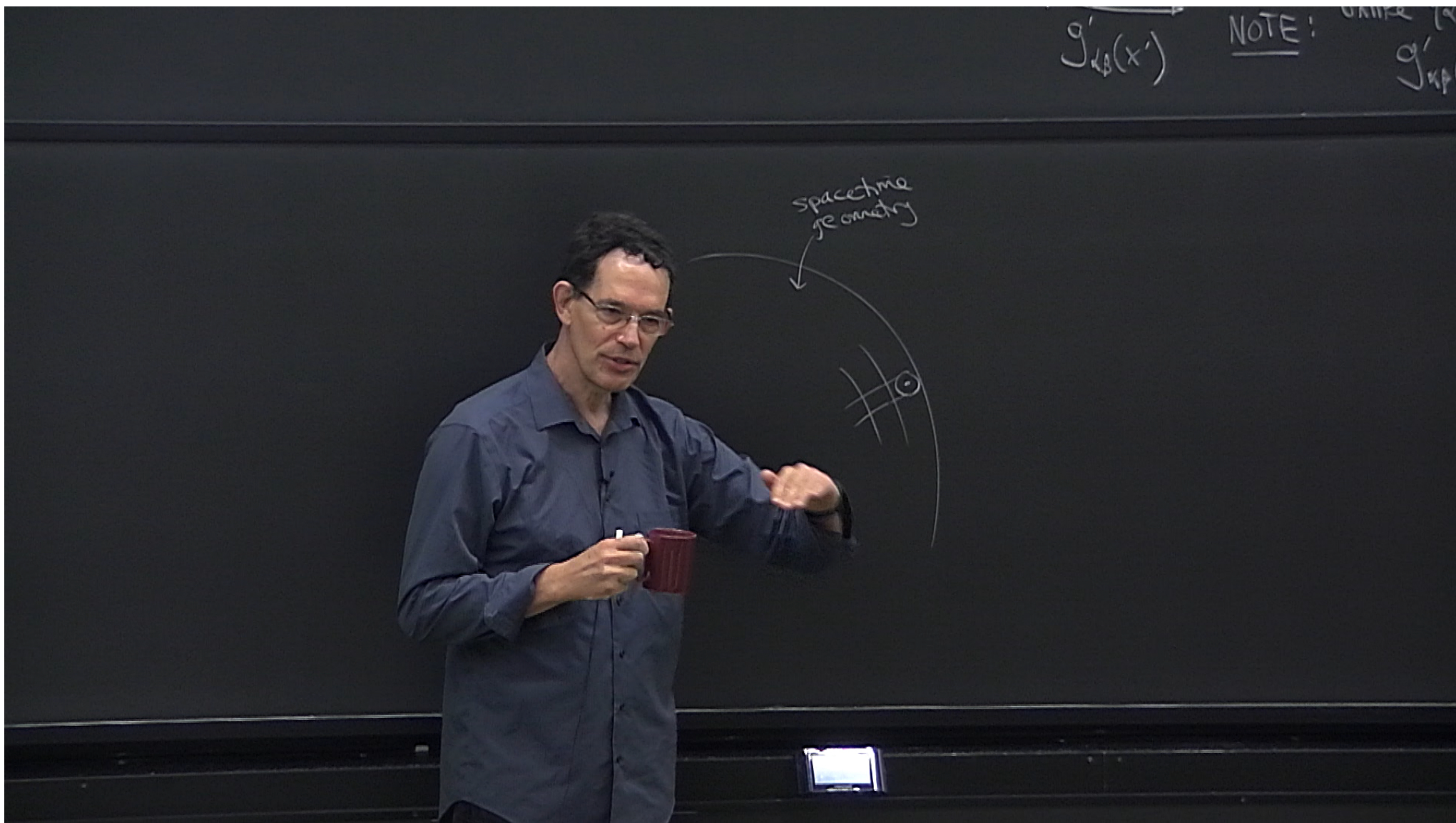
$$vt + \frac{1}{2} a(t\tau)^2 = vt' + \frac{1}{2} at'^2$$

$$00 \text{ comp} \quad -\frac{\partial x^0}{\partial x'^0} \frac{\partial x^0}{\partial x'^0} + \frac{\partial x^0}{\partial x'^1} \frac{\partial x^1}{\partial x'^0} = -1 + (at')^2$$

$$g'_{\mu\nu}(x') = \begin{pmatrix} -1 + (at')^2 & -at' & & \\ -at' & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$g'_{\mu\nu}(x') = \eta_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} \frac{dx'^\alpha}{d\tau} \frac{dx'^\beta}{d\tau}$$

NOTE: unlike $\eta_{\mu\nu}$, $g'_{\mu\nu}(x')$ NOT invariant under c.t.s.



$$J_{\alpha\beta}(x')$$

NOT

$g'_{\alpha\beta}(x')$ NOT invariant under c.t.s.

spacetime
geometry

every (sufficiently smooth) geometry is locally flat

$$J_{\mu\nu}(x')$$

NOT

$g'_{\mu\nu}(x')$ NOT invariant under c.t.s.

spacetime
geometry

every (sufficiently smooth) geometry is locally flat

laws of SR hold.