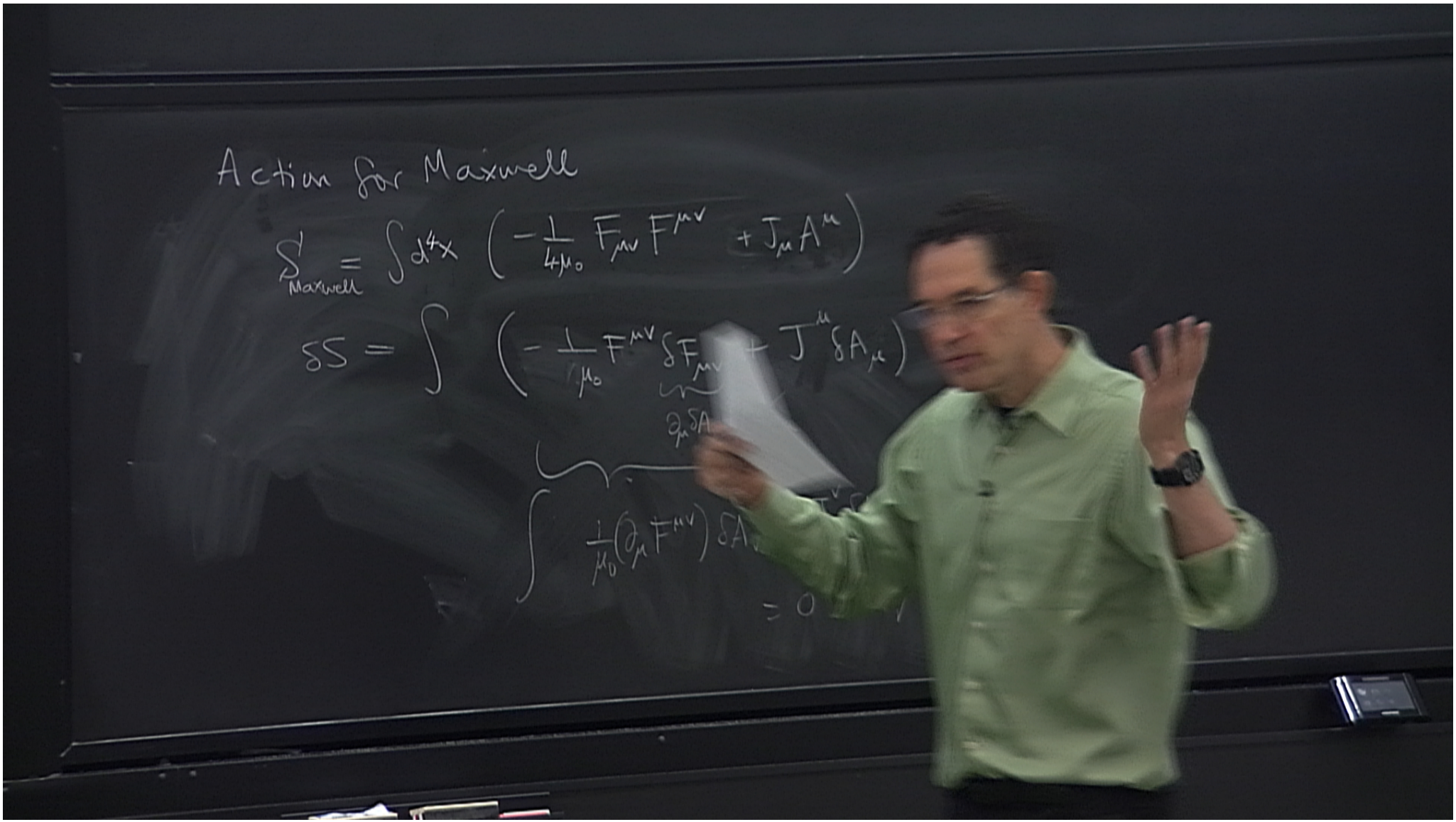
Title: PSI 17/18 - Relativity - Lecture 5

Date: Sep 07, 2017 10:45 AM

URL: http://pirsa.org/17090024

Abstract:


Action for Maxwell
$S_{Maxwell} = \int d^4x \left(-\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} + J_\mu A^\mu \right)$
$\delta S = \int \left(-\frac{1}{\mu_0} F^{\mu\nu} \delta F_{\mu\nu} + J^\mu \delta A_\mu \right)$
$\partial_\mu \delta A$
$\frac{1}{\mu_0} (\partial_\mu F^{\mu\nu}) \delta A$
$= 0$

(later, if time, get $T^{\mu\nu}_{em}$

Action for Maxwell

$$S_{Maxwell} = \int d^4x \left(-\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} + J_\mu A^\mu \right)$$

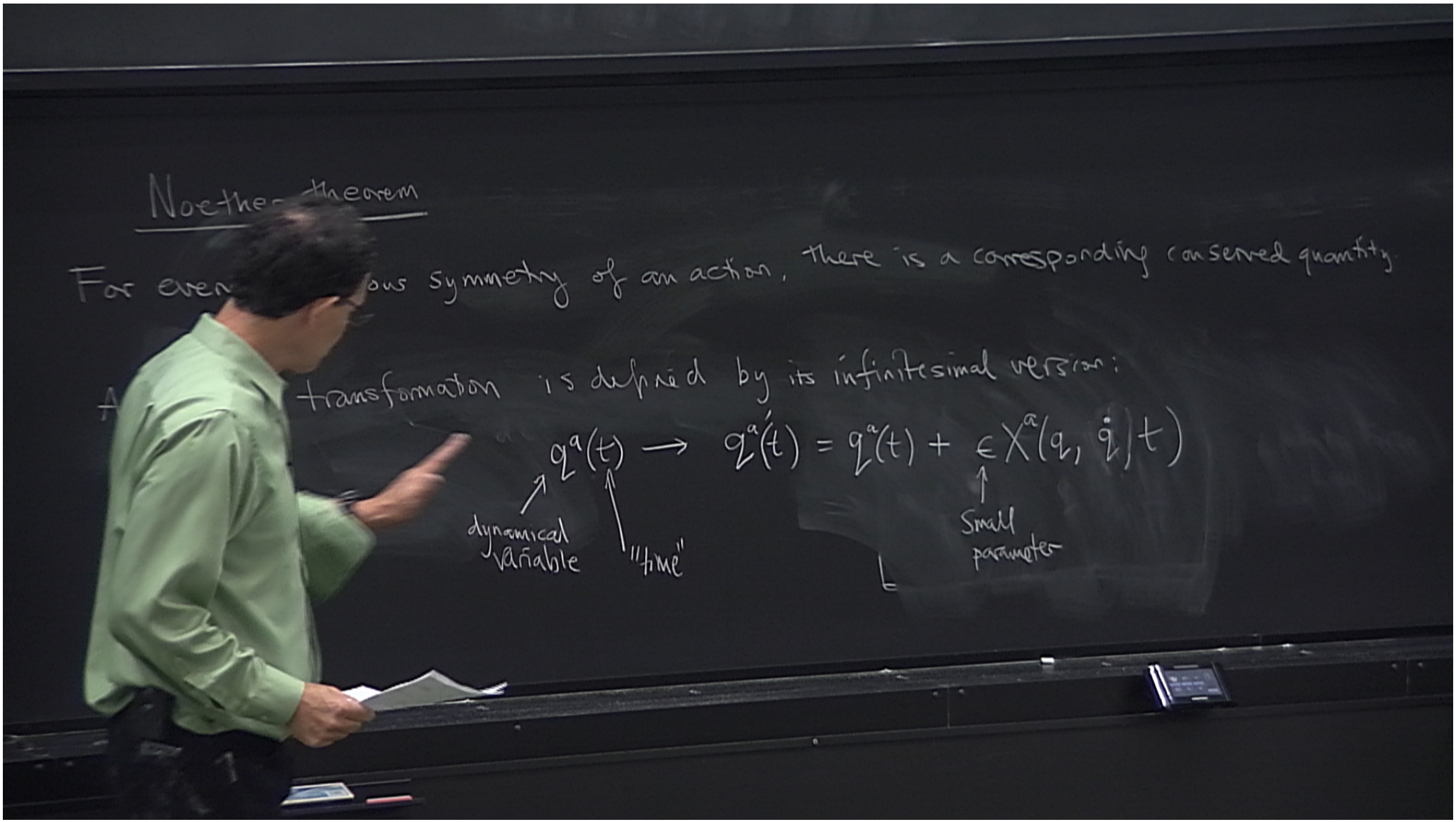
$$\delta S = \int \left(-\frac{1}{\mu_0} F^{\mu\nu} \underbrace{\delta F_{\mu\nu}}_{\partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu} + J^\mu \delta A_\mu \right)$$

$$\int \frac{1}{\mu_0} (\partial_\mu F^{\mu\nu}) \delta A_\nu + J^\nu \delta A_\nu = 0 \quad \forall \delta A_\nu \Rightarrow \boxed{\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu}$$

Noether theorem

For every continuous symmetry of an action, there is a corresponding conserved quantity

A continuous transformation


Noether theorem
For every continuous symmetry of an action, there is a corresponding conserved quantity
A transformation is defined by its infinitesimal version:
$q^a(t) \longrightarrow q'^a(t) = q^a(t) + \epsilon X^a(q, \dot{q}, t)$
dynamical variable
"time"
small parameter

λ

$S = \int d\tau$

free ptcle

coupling to electromagnetic fields

$A \sim \frac{1}{\lambda}$

$F_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$

$A_\mu \frac{dx^\mu}{d\lambda}$

We call this a symmetry of the action if, for constant ϵ, $S(q, \dot{q}$

magnetic fields $\quad A \sim \frac{1}{r}$

$$F_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0 \qquad A_\mu \frac{dx^\mu}{d\lambda}$$

if, for constant ϵ, $S'(q', \dot{q}', t) = S(q, \dot{q}, t)$ up to surface terms

$\longrightarrow \lambda$ $\quad S = \int d\tau$ $\quad$ free ptcle $\quad$ electromagnetic fields $\quad A \sim \frac{1}{\lambda}$

$$F_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0 \qquad A_\mu \frac{dx^\mu}{d\lambda}$$

call this a symmetry of the action if, for constant ϵ, $S(q', \dot{q}',$

Noether's proof: Consider making ϵ an (arbitrary) function of time

then

$\longrightarrow \lambda$ $\quad S = \int d\tau$ $\quad$ free ptcle $\quad$ electromagnetic fields $\quad A \sim \frac{1}{\lambda}$

$$F_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0 \qquad A_\mu \frac{dx^\mu}{d\lambda}$$

call this a symmetry of the action if, for constant ϵ, $S(q', \dot{q}')$

Noether's proof: Consider making ϵ an (arbitrary) function of time

then the variation (1) is a perfectly allowable variation

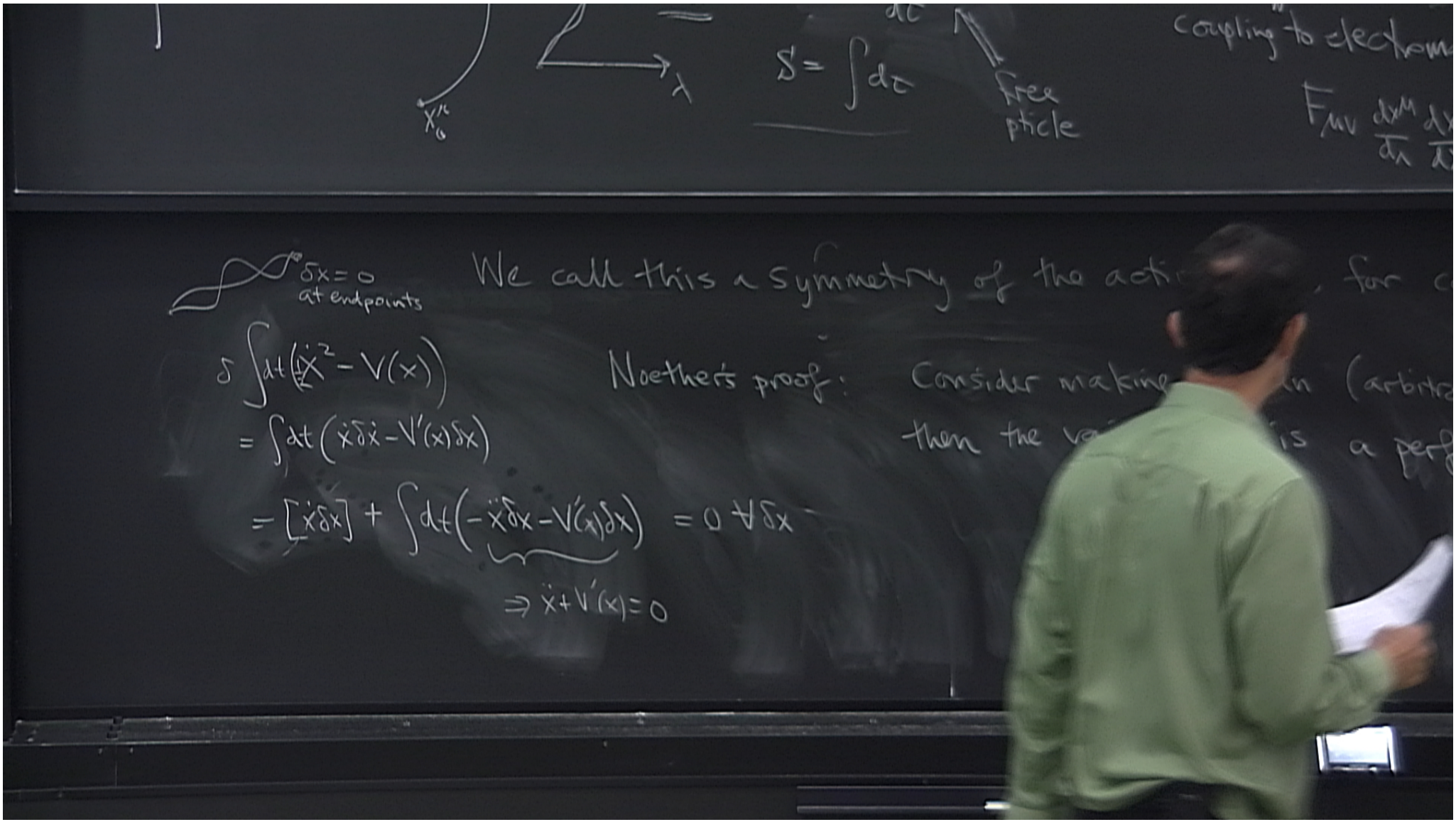
magnetic fields $\quad A \sim \frac{1}{r}$

$$\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0 \qquad A_\mu \frac{dx^\mu}{d\lambda}$$

for constant ϵ, $S'(q', \dot{q}', t) = S(q, \dot{q}, t)$ up to surface terms (to order ϵ)

(arbitrary) function of time

a perfectly allowable variation and hence we must have $\delta S' = 0$ up to surface terms using classical eq.'s of motion.


$S = \int d\tau$
free particle
coupling to electrom
$F_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx}{d\lambda}$
$\delta x = 0$ at endpoints
We call this a symmetry of the acti
for c
$\delta \int dt \left(\frac{1}{2}\dot{x}^2 - V(x)\right)$
$= \int dt \left(\dot{x}\delta\dot{x} - V'(x)\delta x\right)$
$= [\dot{x}\delta x] + \int dt \left(-\ddot{x}\delta x - V'(x)\delta x\right) = 0 \ \forall \delta x$
$\Rightarrow \ddot{x} + V'(x) = 0$
Noether's proof:
Consider makin
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then the va
is
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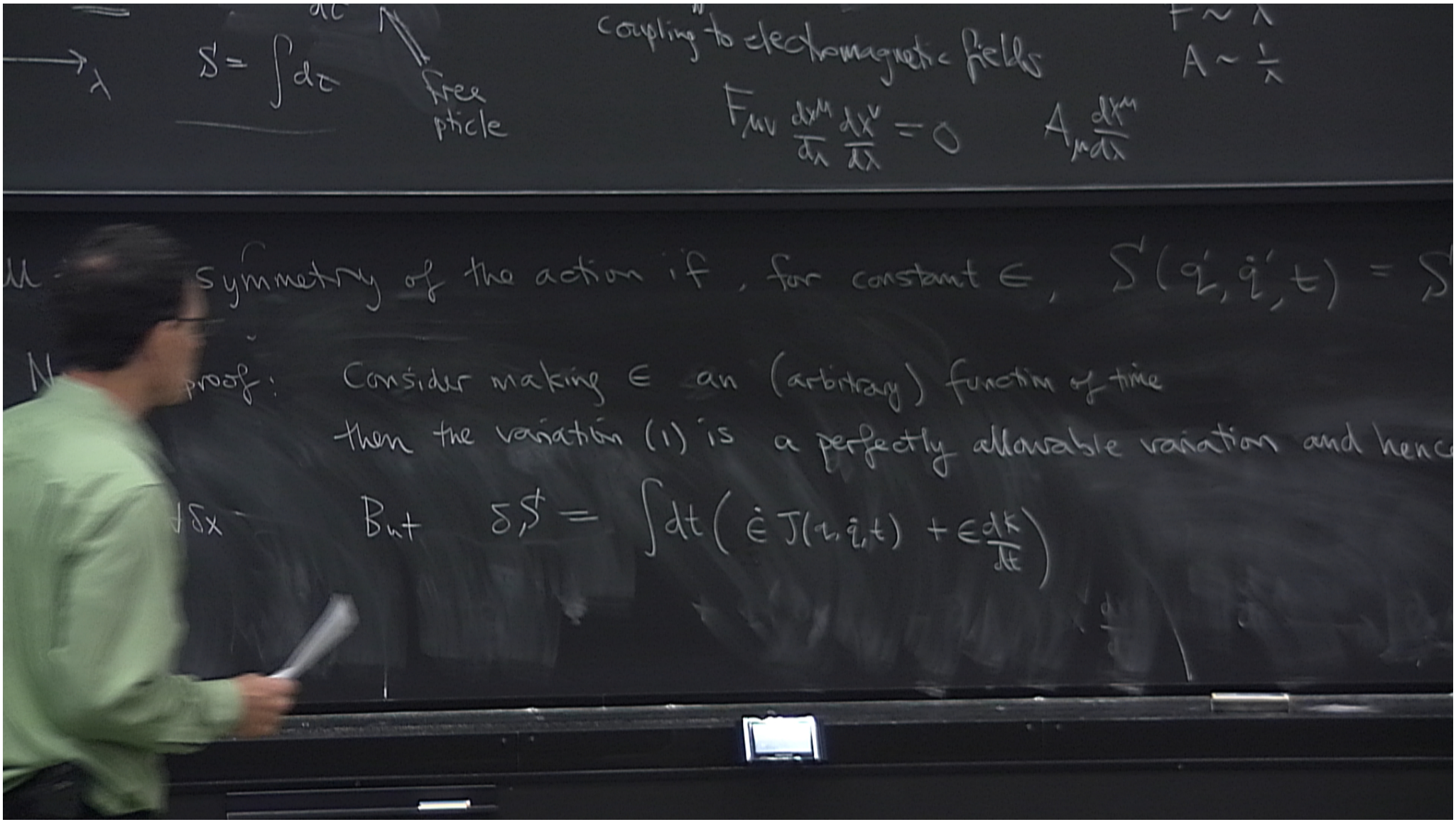
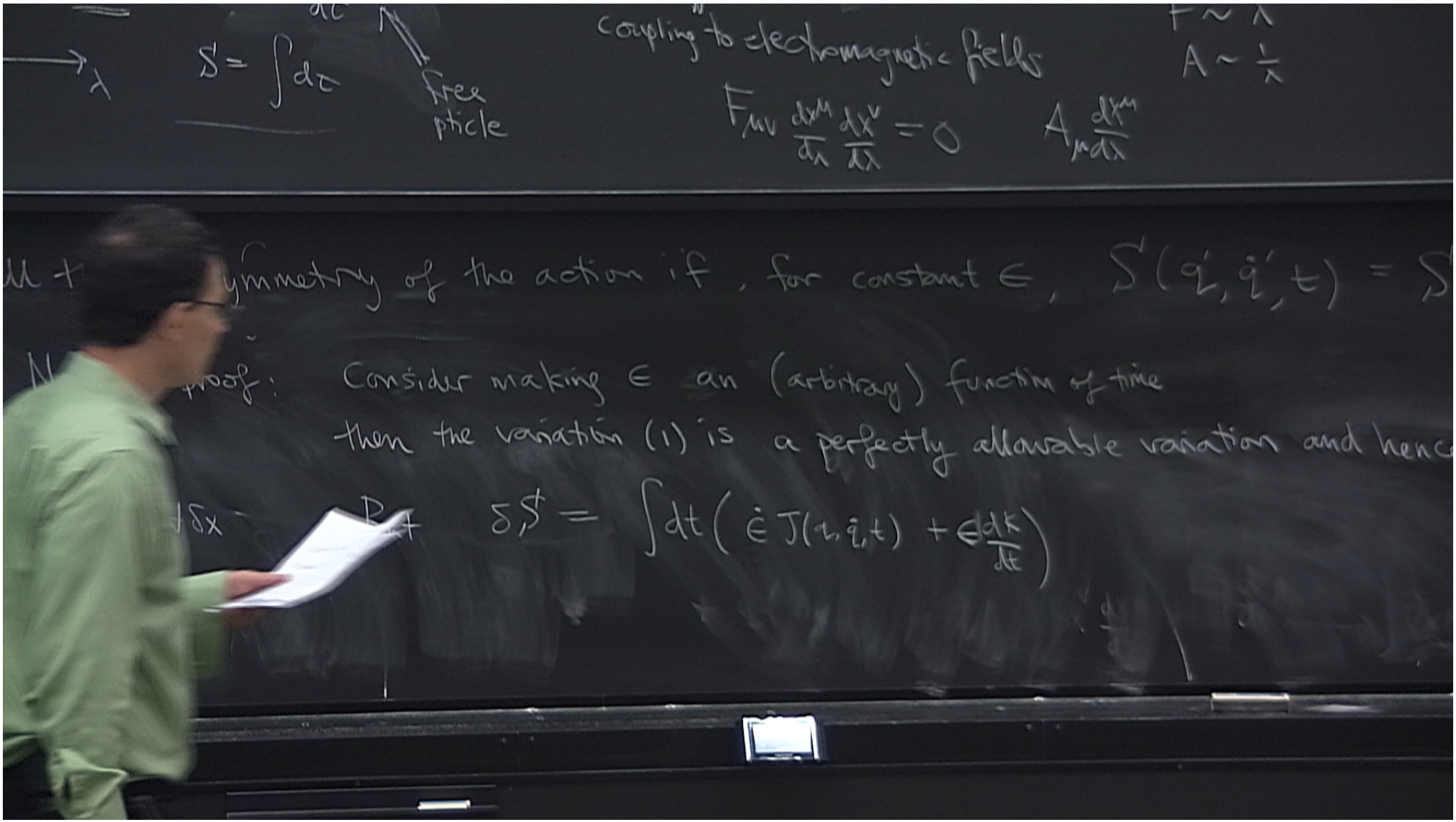

$S = \int d\tau$
free particle
coupling to electromagnetic fields
$F \sim \lambda$
$A \sim \frac{1}{\lambda}$
$F_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$
$A_\mu \frac{dx^\mu}{d\lambda}$
symmetry of the action if, for constant ϵ, $S(q', \dot{q}', t) = S$
proof: Consider making ϵ an (arbitrary) function of time
then the variation (1) is a perfectly allowable variation and henc
But $\delta S = \int dt \left(\dot{\epsilon} J(q, \dot{q}, t) + \epsilon \frac{dK}{dt} \right)$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Consider making ϵ an (arbitrary) function of time
then the variation (1) is a perfectly allowable variation and henc
$\delta S = \int dt \left(\dot{\epsilon} J(q, \dot{q}, t) + \epsilon \frac{dK}{dt} \right)$

$\Rightarrow E = p^0 c = \sqrt{\underline{p}^2 c^2 + m_0^2 c^4}$

Classical Mechanics

$$S = \int dt\, L(q, \dot{q}, t) \qquad \delta S = 0 \Rightarrow \quad \frac{\partial L}{\partial q^a} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)$$

$\forall \delta q$

$$\Rightarrow E = p^0 c = \sqrt{\underline{p}^2 c^2 + m_0^2 c^4}$$

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Noether for translations

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$$\forall \delta q$$

Noether for translations $\quad \delta q^a = \epsilon^a(t)$

$$\delta \dot{q}^a = \dot{\epsilon}^a(t) \quad (\text{its } t)$$

$$\delta S = \int \left(\frac{\partial L}{\partial q^a} \epsilon^a + \frac{\partial L}{\partial \dot{q}^a} \dot{\epsilon}^a \right) = 0 \quad \forall \epsilon^a(t)$$

$$\Rightarrow E = p^0 c = \sqrt{p^2 c^2 + m_0^2 c^4}$$

Classical Mechanics

$$S = \int dt\, L(q, \dot{q}, t) \qquad \delta S = 0 \Rightarrow \quad \frac{\partial L}{\partial q^a} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)$$

$\forall \delta q$

Noether for translations $\quad \delta q^a = \epsilon^a(t)$

$\delta \dot{q}^a = \dot{\epsilon}^a(t)$ (ints)

If L is translation invariant

$$\delta S = \int \left(\cancel{\frac{\partial L}{\partial q^a} \epsilon^a} + \frac{\partial L}{\partial \dot{q}^a} \dot{\epsilon}^a \right) = 0 \quad \forall \epsilon^a(t)$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right) = 0$$

$$= -\left(\frac{E}{c}\right)^2 + |\underline{p}|^2$$

more interesting : time translations $\frac{\partial L}{\partial t} = 0$: $q^a(t) \to q^a(t+\epsilon) = q^a(t) + \epsilon$

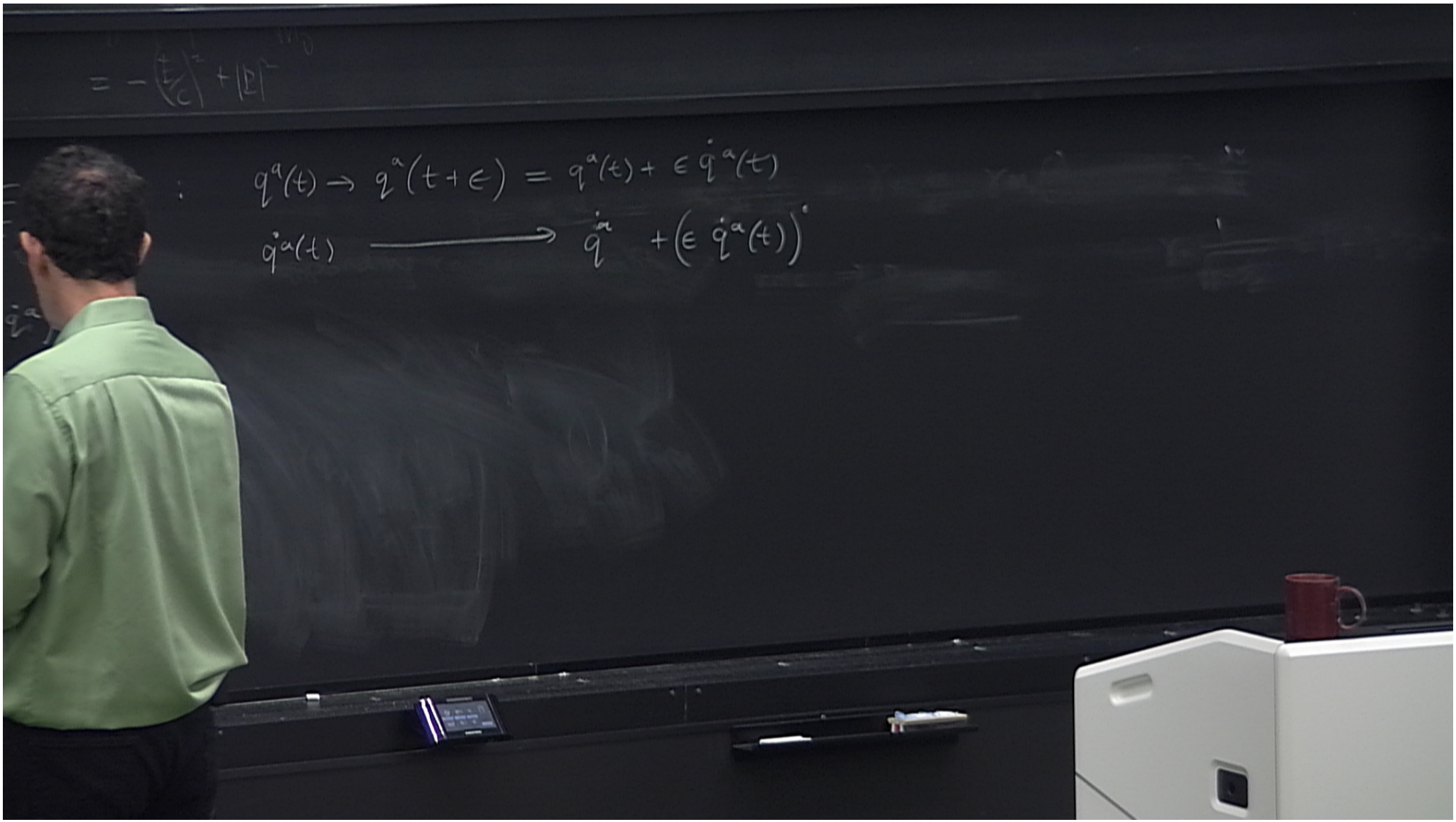
$$\delta S = \int \frac{\partial L}{\partial}$$

interesting : time translations $\frac{\partial L}{\partial t} = 0$:

$$q^a(t) \to q^a(t+\epsilon) = q^a(t) + \epsilon \dot{q}^a(t)$$

$$\dot{q}^a(t) \longrightarrow \dot{q}^a + \epsilon \ddot{q}^a(t)$$

$$\delta S = \int \left(\frac{\partial L}{\partial q^a} \epsilon \dot{q}^a + \frac{\partial L}{\partial \dot{q}^a} \right.$$


$q^a(t) \to q^a(t+\epsilon) = q^a(t) + \epsilon \dot{q}^a(t)$
$\dot{q}^a(t) \longrightarrow \dot{q}^a + (\epsilon \dot{q}^a(t))^{\cdot}$

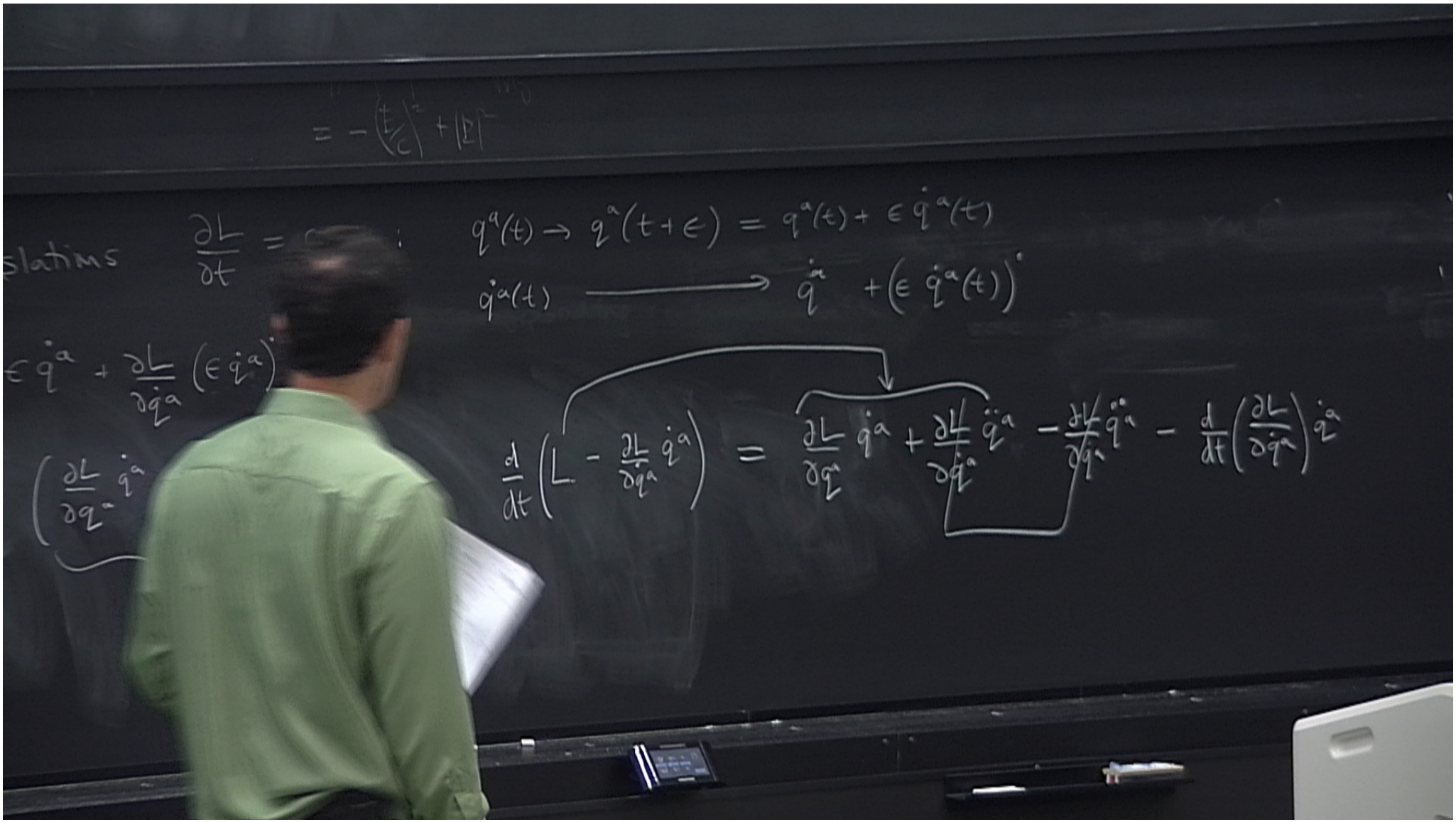
more interesting : time translations $\frac{\partial L}{\partial t} = 0$: $q^a(t) \to q^a(t+\epsilon) = q^a(t)$

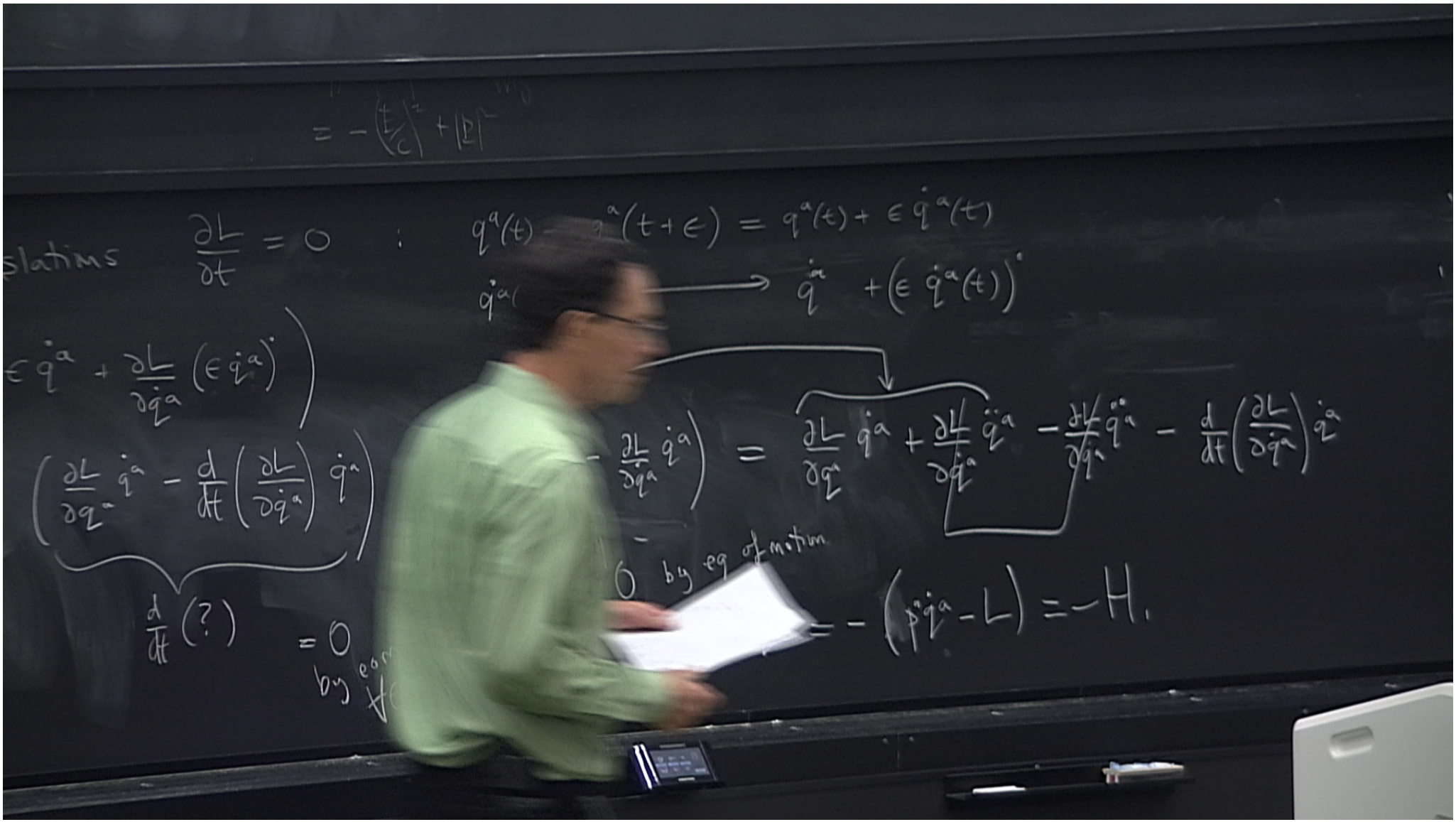
$\dot{q}^a(t) \longrightarrow \dot{q}^a$

$$\delta S = \int \left(\frac{\partial L}{\partial q^a} \epsilon \dot{q}^a + \frac{\partial L}{\partial \dot{q}^a} (\epsilon \dot{q}^a)^{\cdot} \right)$$

int by parts

$$= \int \epsilon \left(\frac{\partial L}{\partial q^a} \dot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a} \right) \dot{q}^a \right)$$


$q^a(t) \to q^a(t+\epsilon) = q^a(t) + \epsilon \dot{q}^a(t)$
$\dot{q}^a(t) \longrightarrow \dot{q}^a + (\epsilon \dot{q}^a(t))^{\cdot}$
$\frac{d}{dt}\left(L - \frac{\partial L}{\partial \dot{q}^a}\dot{q}^a\right) = \frac{\partial L}{\partial q^a}\dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}\ddot{q}^a - \frac{\partial L}{\partial q^a}\ddot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a$


$= -\left(\frac{t}{c}\right)^2 + |\underline{p}|^2$
slatims $\frac{\partial L}{\partial t} = 0$: $q^a(t) \to q^a(t+\epsilon) = q^a(t) + \epsilon \dot{q}^a(t)$
$\dot{q}^a(t) \longrightarrow \dot{q}^a + (\epsilon \dot{q}^a(t))^{\cdot}$
$\epsilon \dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}(\epsilon \dot{q}^a)^{\cdot}$
$\left(\frac{\partial L}{\partial q^a}\dot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a\right)$
$\frac{d}{dt}(?) = 0$ by eom
$\left(\frac{\partial L}{\partial q^a}\dot{q}^a - \frac{\partial L}{\partial \dot{q}^a}\dot{q}^a\right) = \frac{\partial L}{\partial q^a}\dot{q}^a + \frac{\partial L}{\partial \dot{q}^a}\ddot{q}^a - \frac{\partial L}{\partial q^a}\ddot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)\dot{q}^a$
0 by eq of motion
$= -(p^a\dot{q}^a - L) = -H.$

$$= -\left(\frac{E}{c}\right)^2 + |\underline{p}|^2$$

estry : time translations $\quad \frac{\partial L}{\partial t} = 0$:

$$q^a(t) \to q^a(t+\epsilon) = q^a(t) + \epsilon \dot{q}^a(t)$$

$$\dot{q}^a(t) \longrightarrow \dot{q}^a + (\epsilon \dot{q}^a(t))^{\cdot}$$

$$\delta S = \int \left(\frac{\partial L}{\partial q^a} \epsilon \dot{q}^a + \frac{\partial L}{\partial \dot{q}^a} (\epsilon \dot{q}^a)^{\cdot} \right)$$

$$= \int \epsilon \underbrace{\left(\frac{\partial L}{\partial q^a} \dot{q}^a - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right) \dot{q}^a \right)}$$

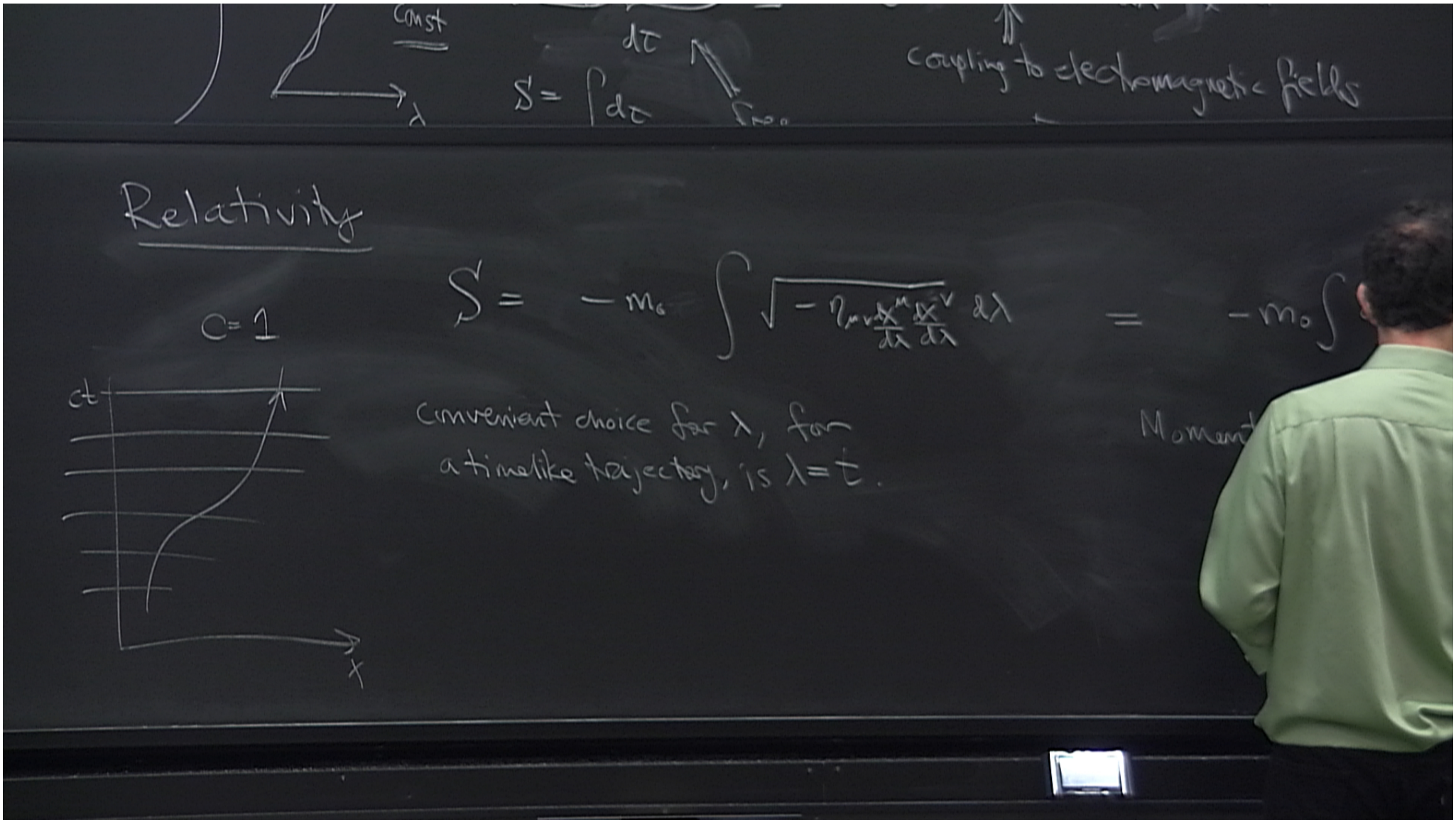
$$\frac{d}{dt} L(q,\dot{q}) \qquad \frac{d}{dt}(?) \qquad = 0$$

$$\frac{\partial L}{\partial q} \dot{q} + \frac{\partial L}{\partial \dot{q}} \ddot{q} \qquad \text{by eom } \forall \epsilon(t)$$

$$\frac{d}{dt}\left(L - \frac{\partial L}{\partial \dot{q}^a} \dot{q}^a\right) = \frac{\partial L}{\partial q^a} \dot{q}^a + \frac{\partial L}{\partial \dot{q}^a} \ddot{q}^a - \frac{\partial L}{\partial \dot{q}^a} \ddot{q}^a$$

$$= 0 \text{ by eq of motion}$$

$$L - p^a \dot{q}^a = -(p^a \dot{q}^a - L) = -H$$


const
λ
$S = \int d\tau$
$d\tau$
coupling to electromagnetic fields
Relativity
$c = 1$
$S = -m_0 \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, d\lambda = -m_0 \int$
ct
x
convenient choice for λ, for a timelike trajectory, is $\lambda = t$.
Moment

put c's back in by dimensions

$$E_{\text{particle}} = \overbrace{\gamma m_0}^{M} c^2$$

$$\underline{P}_{\text{particle}} = \underbrace{\gamma m_0}_{M} \underline{v}$$

$$\psi \to e^{i\theta} \psi$$

$$\int d^4x \underbrace{\left(F_{\mu\nu} F^{\mu\nu}\right.}_{} + L_0^4 \left(\underbrace{F_{\mu\nu} F^{\mu\nu}}_{L^{-8}}\right)^2 + \left.\right) \cdots$$

$$1a \longleftrightarrow ?$$

$$m_1 \ddot{\underline{x}} = \sum_j \frac{G \overset{m_1}{M} (\underline{x}_1 - \underline{x}_j)}{|\underline{x}_1 - \underline{x}_j|^3} + \underline{a}$$

A

$$\underset{\text{particle}}{E} = \gamma m_0 c^2$$

$$\underset{\text{particle}}{\underline{p}} = \underbrace{\gamma m_0}_{M} \underline{v}$$

15

Poincare $\Lambda^\mu{}_\nu$, a^ν

boosts 3

rot^ns 3 → $\underline{L}$

translⁿs in space + time → Momentum $\underline{p}$, Energy E

$$F^{\mu\nu} + L_0^4 \Big(\underbrace{F_{\mu\nu} F^{\mu\nu}}_{L^{-8}} + \Big) - \cdots$$

A

X^a

$$\underset{\text{particle}}{E} = \overbrace{\gamma m_0}^{M} c^2$$

$$\underset{\text{particle}}{\underline{p}} = \underbrace{\gamma m_0}_{M} \underline{v}$$

$$F^{\mu\nu} + L_0^4 \Big(\underbrace{F_{\mu\nu} F^{\mu\nu}}_{L^{-8}} + \Big) - \cdots$$

A

$\boxed{\text{SC group}}$

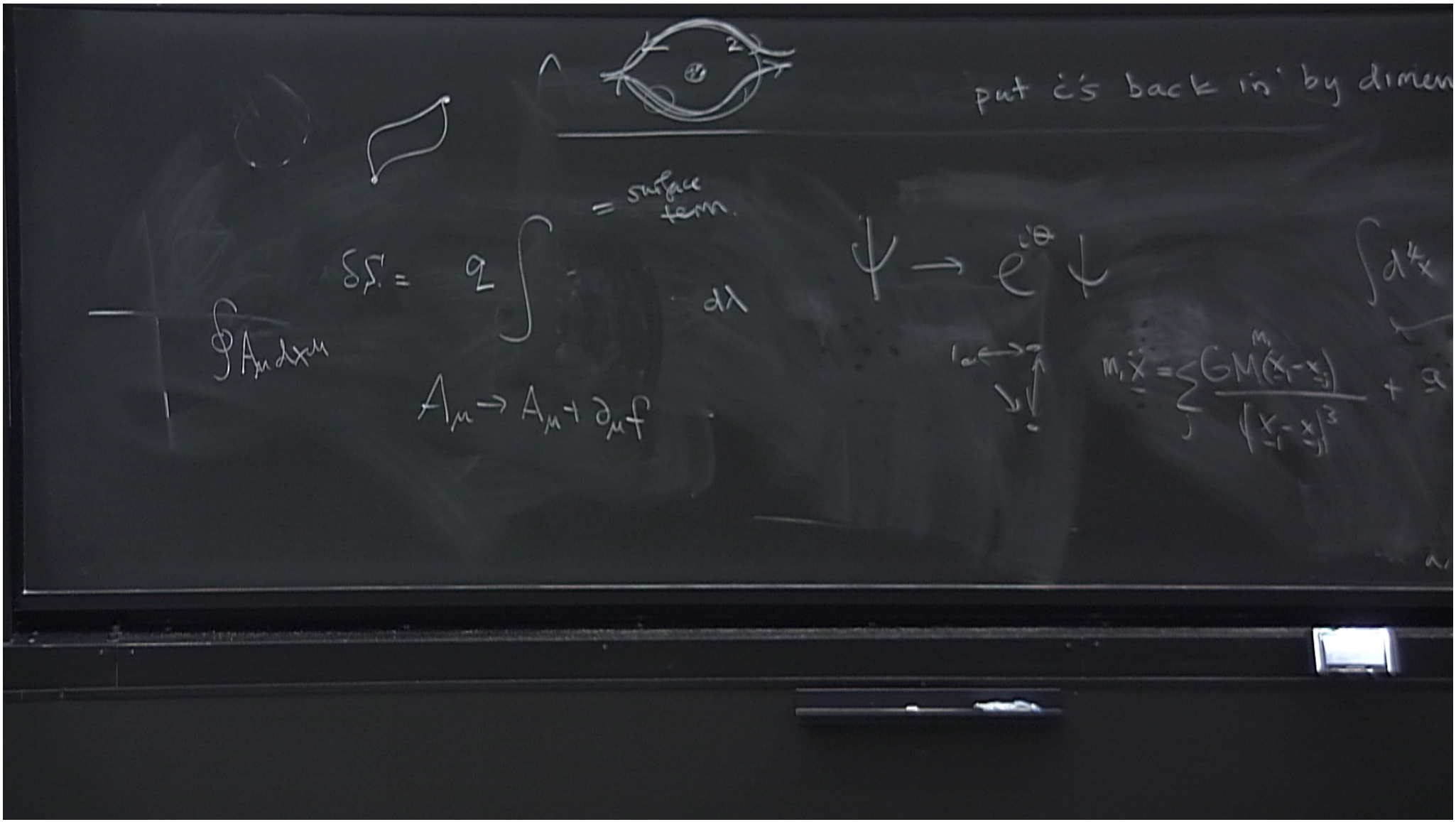
	4d	3d	2d
	15 generators	?	∞

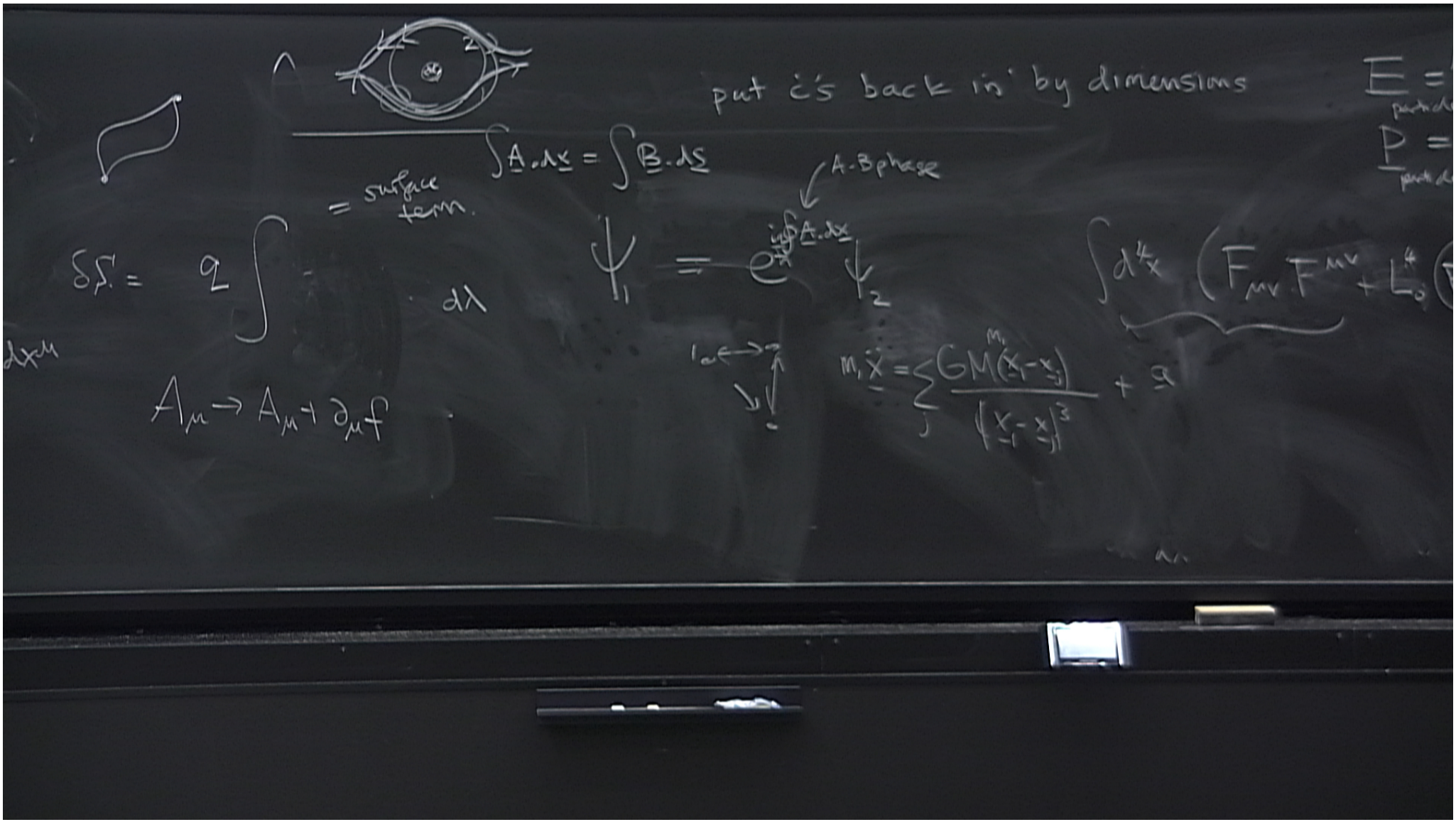
Poincare $\Lambda^\mu{}_\nu$, a^ν

$\Lambda^\mu{}_\nu \to$ boosts 3, rot'ns 3 $\to \underline{L}$

$a^\nu \to$ transl^{n}s in space + time $\to$ Momentum $\underline{p}$, Energy E

X^a


put ħ's back in by dimen
= surface term.
$\delta S = q \int \quad d\lambda$
$\oint A_\mu dx^\mu$
$A_\mu \to A_\mu + \partial_\mu f$
$\psi \to e^{i\theta} \psi$
$m_i \ddot{\underline{x}} = \sum_j \frac{G M^{m_i} (\underline{x}_i - \underline{x}_j)}{|\underline{x}_i - \underline{x}_j|^3} + q$
$\int d^4x$


put i's back in by dimensions
$\int \underline{A}\cdot d\underline{x} = \int \underline{B}\cdot d\underline{S}$
= surface term.
A-B phase
$\psi_1 = e^{iq\oint \underline{A}\cdot d\underline{x}}\psi_2$
$\delta S = q\int \quad d\lambda$
dx^μ
$A_\mu \to A_\mu + \partial_\mu f$
$m_1 \ddot{\underline{x}} = \sum_j \frac{GM(\underline{x}_1 - \underline{x}_j)}{|\underline{x}_1 - \underline{x}_j|^3} + q$
$\int d^4x \, (F_{\mu\nu}F^{\mu\nu} + L_0^*$