Title: Entanglement and thermodynamics after a quantum quench in integrable systems

Date: Sep 22, 2017 11:00 AM

URL: http://pirsa.org/17080073

Abstract: <p>Entanglement and entropy are key concepts standing at the foundations of quantum and statistical mechanics, respectively. In the last decade the study of quantum quenches revealed that these two concepts are intricately intertwined. Although the unitary time evolution ensuing from a pure initial state maintains the system globally at zero entropy, at long time after the quench local properties are captured by an appropriate statistical ensemble with non zero thermodynamic entropy, which can be interpreted as the entanglement accumulated during the dynamics. Therefore, understanding the post-quench entanglement evolution unveils how thermodynamics emerges in isolated quantum systems. An exact computation of the entanglement dynamics has been provided only for non-interacting systems, and it was believed to be unfeasible for genuinely interacting models. Conversely, here we show that the standard quasiparticle picture of the entanglement evolution, complemented with integrability-based knowledge of the asymptotic state, leads to a complete analytical understanding of the entanglement dynamics in the space-time scaling limit. Our framework requires only knowledge about the steady state, and the velocities of the low-lying excitations around it. We provide a thorough check of our result focusing on the spin-1/2 Heisenberg XXZ chain, and considering quenches from several initial states. We compare our results with numerical simulations using both tDMRG and iTEBD, finding always perfect agreement.</p>

Entanglement spreading after a global quench in integrable models

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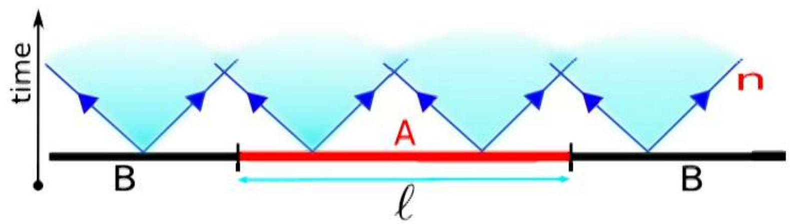
PI Waterloo, September 22, 2017



Outline

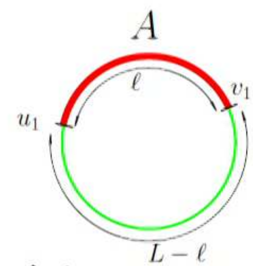
[V.A. and P. Calabrese, PNAS **114**, 7947 (2017)]

- **Entanglement** dynamics after quantum **quenches**.
- **Semiclassical** picture.
- **Bethe ansatz** treatment of quantum quenches in **integrable** models.
- **Complete** semiclassical description.
- The anisotropic spin-1/2 **Heisenberg chain**.
- Developments: **Rényi** diagonal entropies, **inhomogeneous** quenches.

Entanglement entropy in quantum systems

- Consider a quantum system in d dimensions in a **pure** state $|\Psi\rangle$

$$\rho \equiv |\Psi\rangle\langle\Psi|$$



- If the system is **bipartite**:

$$H = H_A \otimes H_B \to \rho_A = Tr_B \rho$$

- How to quantify the entanglement (**quantum** correlations) between A and B?
  - **von Neumann** entropy $S_A = -Tr\rho_A \log \rho_A = -\sum_i \lambda_i \log \lambda_i$
  - **Rényi** entropies $S_A^{(n)} = -\frac{1}{n-1}\log(Tr\rho_A^n) = -\frac{1}{n-1}\log(\sum_i \lambda_i^n)$

Quantum quenches in **isolated** many-body systems

Quantum quench protocol

- Initial state $|\Psi_0\rangle \Rightarrow$ **unitary** evolution under a many-body Hamiltonian $\mathcal{H}$

$\{|\psi_\alpha\rangle\}$ eigenstates of $\mathcal{H}$ $\quad |\Psi_0\rangle = \sum_\alpha c_\alpha |\psi_\alpha\rangle$ $\quad |\Psi(t)\rangle = \sum_\alpha e^{iE_\alpha t} c_\alpha |\psi_\alpha\rangle$

$c_\alpha \equiv \langle \Psi_0 | \psi_\alpha \rangle$

- For a generic observable $\widehat{\mathcal{O}}$:

$$\langle \Psi(t)|\widehat{\mathcal{O}}|\Psi(t)\rangle = \sum_{\alpha,\beta} e^{i(E_\alpha - E_\beta)t} c_\alpha^* c_\beta \widehat{\mathcal{O}}_{\alpha\beta}$$

- **Long time** $\Rightarrow$ **diagonal ensenble**.

$$\overline{\langle \Psi(t)|\widehat{\mathcal{O}}|\Psi(t)\rangle} = \langle \hat{\mathcal{O}} \rangle_{DE} = \sum_\alpha |\langle \Psi_0 | \psi_\alpha \rangle|^2 \widehat{\mathcal{O}}_{\alpha\alpha}$$

Integrable vs non integrable dynamics

- Generic (**non-integrable**) systems thermalize at long times.

$$\rho^{Gibbs} = \frac{1}{Z} \exp\Big(-\beta\mathcal{H} \Big)$$

- **Integrability** $\Rightarrow$ **Local** (quasi-local) conserved quantities $\boldsymbol{\mathcal{I}_j}$.

$$[\mathcal{H}, \mathcal{I}_j] = 0,\ \forall j \quad \text{and} \quad [\mathcal{I}_j, \mathcal{I}_k] = 0,\ \forall j, k \qquad \mathcal{I}_2 \equiv \mathcal{H}$$

- Include extra charges in Gibbs $\Rightarrow$ **Generalized Gibbs Ensemble** (GGE).
[Jaynes, 1957;Rigol,2008]

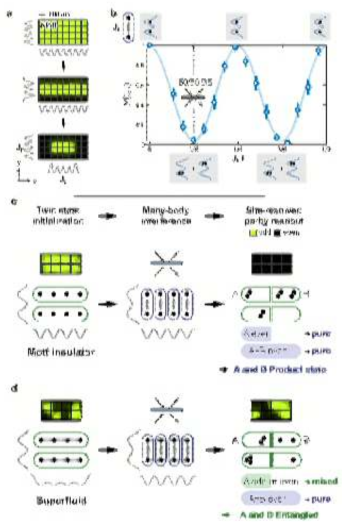
$$\rho^{GGE} = \frac{1}{Z} \exp\Big(\sum_j \beta_j \mathcal{I}_j \Big)$$

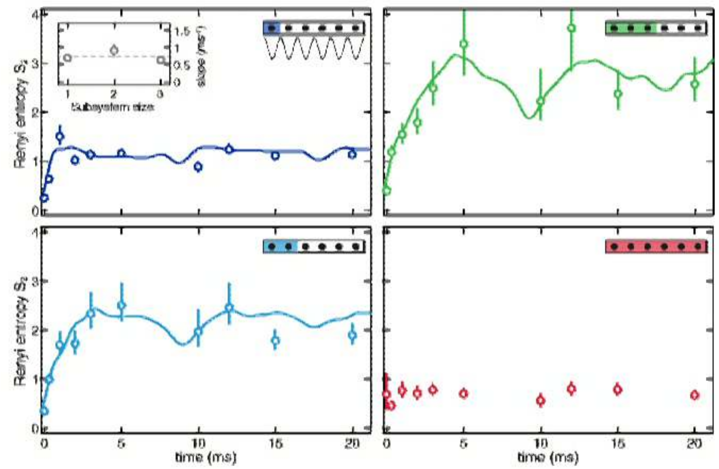
- **Main question**: Can we understand the out-of-equilibrium behavior of the entanglement entropy?

Entanglement dynamics: Experimental results

- Entanglement dynamics observed in cold atom experiments (Bose Hubbard model).

$$\mathcal{H} = -J\sum_{i=1}^{L}(b_i^\dagger b_{i+1} + h.c.) + \frac{U}{2}\sum_{i=1}^{L} n_i(n_{i+1} - 1)$$



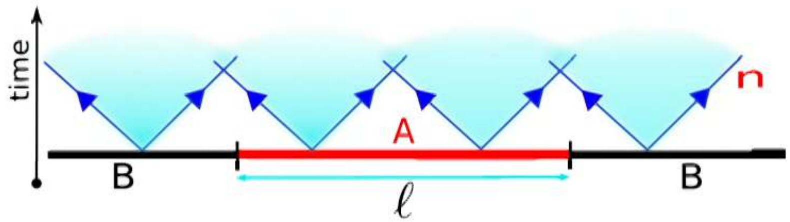


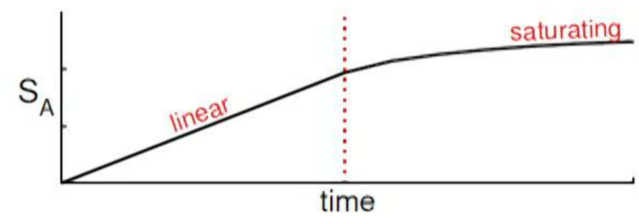
[Islam et al., Nature 528, 77 (2015)]
[Kaufman et al., 2016]

Entanglement dynamics: Semiclassical picture

- **Extensive** amount of energy $\Rightarrow$ **quasi-particles** produced uniformly in the initial state.

[Calabrese, Cardy, 2005]





$$S_A(t) \propto 2t \int_{2|v|t<\ell} d\lambda v(\lambda) f(\lambda) + \ell \int_{2|v|t>\ell} d\lambda f(\lambda)$$

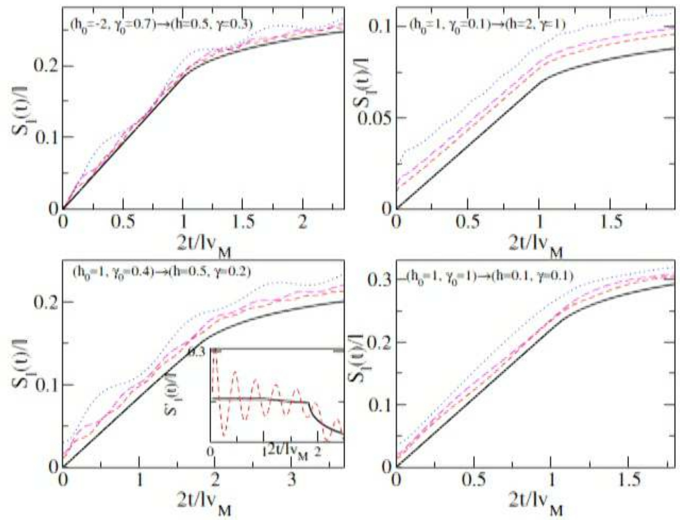
- Requires quasi-particles **group velocities** $v(\lambda)$
- $f(\lambda)$ **cross-section** for quasi-particle production.

Exact results in free models

- Analytical results available only for **free** models (XY chain).

[Fagotti and Calabrese, 2008]

$$\mathcal{H}(h,\gamma) = -\sum_{j=1}^{N}\left[\frac{1+\gamma}{4}\sigma_j^x\sigma_{j+1}^x + \frac{1-\gamma}{4}\sigma_j^y\sigma_{j+1}^y + \frac{h}{2}\sigma_j^z\right]$$



v_M maximum velocity

- Generic interaction quench $(h_0,\gamma_0)\to(h,\gamma)$.
- Semiclassical result is **exact** in the scaling limit.

$$t,\ell\to\infty,\ , t/\ell = cst$$

$$S_A(t) = t\int_{2|v(\varphi)|t<\ell}\frac{d\varphi}{2\pi}2|v(\varphi)|H(\cos\Delta_\varphi) + \ell\int_{2|v(\varphi)|t>\ell}\frac{d\varphi}{2\pi}H(\cos\Delta_\varphi)$$

Bethe ansatz for the spin-1/2 XXZ chain

- Bethe-Gaudin-Takahashi (BGT) equations:

$$L\vartheta_n(\boldsymbol{\lambda_{n;\gamma}}) = 2\pi \boldsymbol{J_{n;\gamma}} + \sum_{(m,\beta)\neq(n,\gamma)} \Theta_{m,n}(\boldsymbol{\lambda_{n;\gamma}} - \boldsymbol{\lambda_{m;\beta}})$$

$\boldsymbol{\lambda_{n;\gamma}}$ rapidities

$\boldsymbol{J_{n;\gamma}} \in \frac{1}{2}\mathbb{Z}$, B-T quantum numbers $n =$ size of multi-spin bound states

- Roots $\{\lambda_{n;\gamma}\} \Rightarrow XXZ$ chain eigenstates $|\{\lambda_{n;\gamma}\}\rangle$.

- Thermodynamic limit $\Rightarrow$ **macrostate** $\Rightarrow$ root and hole **densities**.

$\boldsymbol{\rho_{n,p}(\lambda)}$ $\boldsymbol{\rho_{n,h}(\lambda)}$ $\boldsymbol{\rho_{n,t}(\lambda) = \rho_{n,p}(\lambda) + \rho_{n,h}(\lambda)}$

- $\#$ equivalent microscopic eigenstates $\Rightarrow$ Yang-Yang **entropy**

$$S_{YY} \equiv \sum_n \int d\lambda [\rho_{n,t} \log \rho_{n,t} - \rho_{n,p} \log \rho_{n,p} - \rho_{n,h} \log \rho_{n,h}]$$

Quench Action method

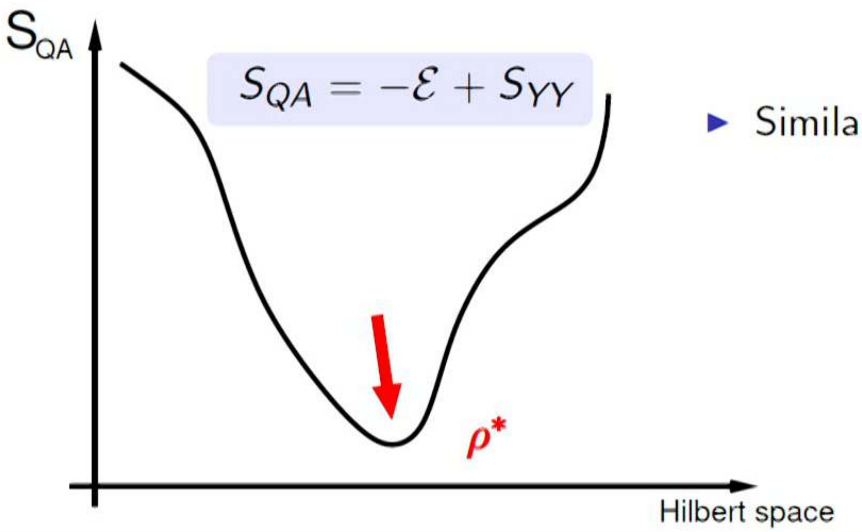
- Rewrite the diagonal ensemble average:

[Caux & Essler, 2013]

$$\sum_n |\langle\Psi_0|\Psi_n\rangle|^2 \mathcal{O}_{nn} \to \sum_\rho e^{-\mathcal{E}[\rho]+S_{YY}[\rho]}\langle\rho|\mathcal{O}|\rho\rangle$$

$$\mathcal{E} \equiv -2\ln|\langle\Psi_0|\Psi_n\rangle|$$

- **Saddle point** ρ^* from competition between $\mathcal{E}$ and S_{YY}.



- Similar to standard statistical physics

$$\mathcal{H} \leftrightarrow \mathcal{E}$$

Steady state: Quench Action results

- **Quench Action** $\Rightarrow$ steady state described by a **macrostate** $\rho^*_{n,p}$.

Néel quench, XXZ for $\Delta = 1$ [Caux and Essler, 2013] [Brockmann et al., 2014]

$$\rho_1^*(x) = \frac{8(4+x^2)}{\pi(19+3x^2)(1+6x^2+x^4)},$$

$$\rho_2^*(x) = \frac{8x^2(9+x^2)(4+3x^2)}{\pi(2+x^2)(16+14x^2+x^4)(256+132x^2+9x^4)},$$

$$\rho_3^*(x) = \frac{8(1+x^2)^2(5+x^2)(16+x^2)(21+x^2)}{\pi(19+3x^2)(9+624x^2+262x^4+32x^6+x^8)(509+5x^2(26+x^2))}.$$

- Associated Yang-Yang entropy:

$$S_{YY}[\rho^*] = s^*_{YY} L$$

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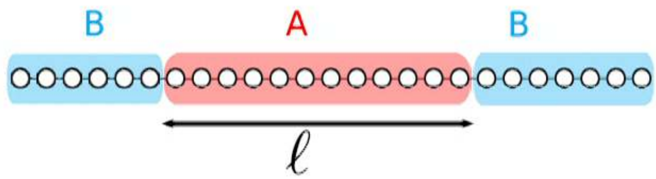
Steady state entanglement entropy

- Steady-state **entanglement** entropy density is the **Yang-Yang** entropy.



[Collura et al., 2014]

$$S_A/\ell = (\mathrm{Tr}\rho^{GGE} \log \rho^{GGE})/L = s^*_{YY}$$

- Cross section for quasi-particle production is fixed $f(\lambda) = s^*_{YY}(\lambda)$:

$$S_A(t) \xrightarrow{t\to\infty} \ell \sum_n \int d\lambda s^*_{YY}(n,\lambda)$$

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Entangling quasi-particles

- How to identify the **entangling** quasi-particles?
- **Local** observables $\Rightarrow$ long-time dynamics determined by **low-lying** excitations around steady state.

[Caux and Essler, 2013]

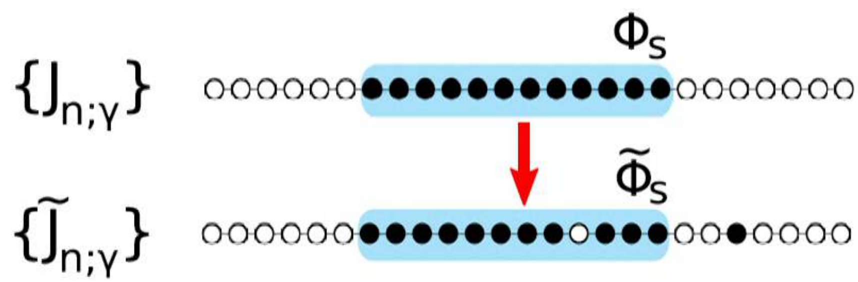
$$\mathcal{O}(t) = \lim_{L\to\infty} \sum_{\Phi} \left\{ \frac{\langle \Psi_0 | \Phi \rangle}{\langle \Psi_0 | \Phi_s \rangle} e^{i(E_\Phi - E_{\Phi_s})t} \frac{\langle \Phi | \mathcal{O} | \Phi_s \rangle}{2} + h.c. \right\}$$

$|\Phi_s\rangle$ Quench action representative state

- $t \to \infty \Rightarrow$ only $E_\Phi - E_{\Phi_s} = \mathcal{O}(1)$ are relevant.

Group velocities of entangling quasi-particles

- Entangling excitations $\Rightarrow$ are **particle-hole** excitations around $|\Phi_s\rangle$.



- **Dressed** energy and momentum.

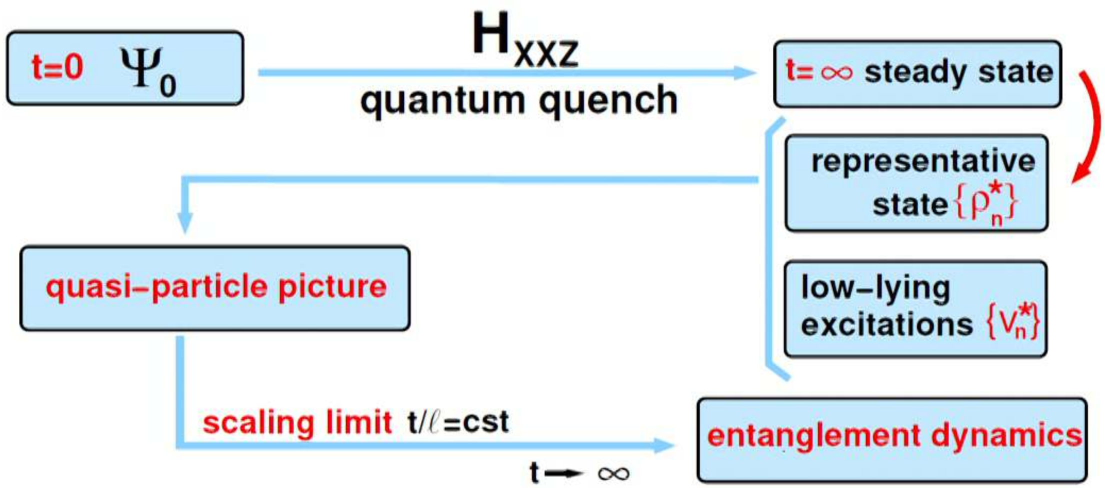
$$\delta E = E_{\tilde{\Phi}_s} - E_{\Phi_s} \qquad \delta P = P_{\tilde{\Phi}_s} - P_{\Phi_s}$$

- Group velocities $\boldsymbol{v_n^*}$

$$v_n^*(\lambda) = \frac{\delta E}{\delta P} = \frac{e_n'(\lambda)}{2\pi\rho^*_{n,t}(1+\rho^*_{n,h}/\rho^*_{n,p})}$$

- **Initial state** dependence.
- Dependence on bound state size.

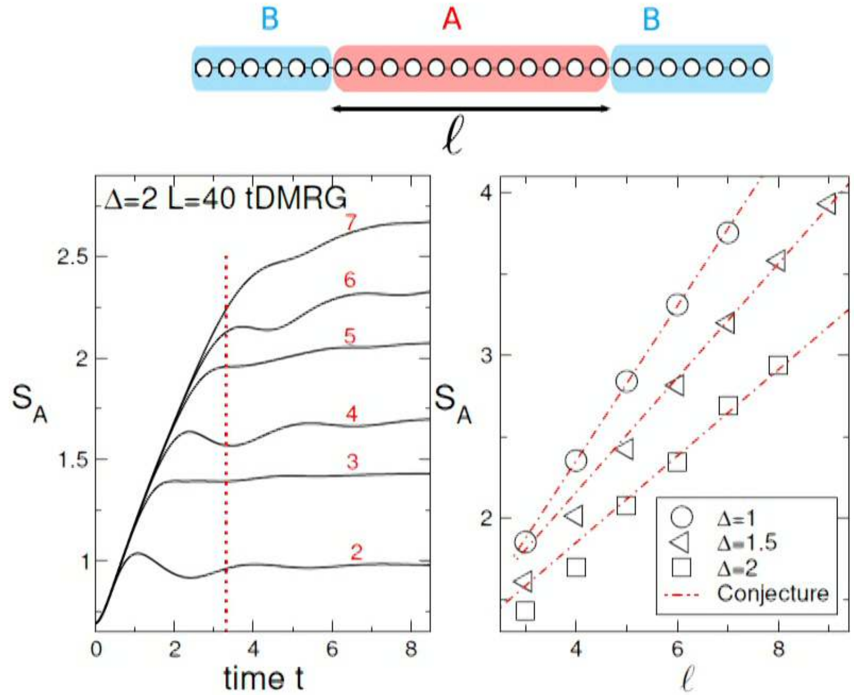
Theoretical program



$$S_A(t) \propto \sum_n \Big[t \int_{|v_n^*|t<\ell} d\lambda v_n^*(\lambda) s_{YY}^*(\lambda) + \ell \int_{|v_n^*|t>\ell} d\lambda s_{YY}^*(\lambda) \Big]$$

Numerical checks: Steady-state entanglement

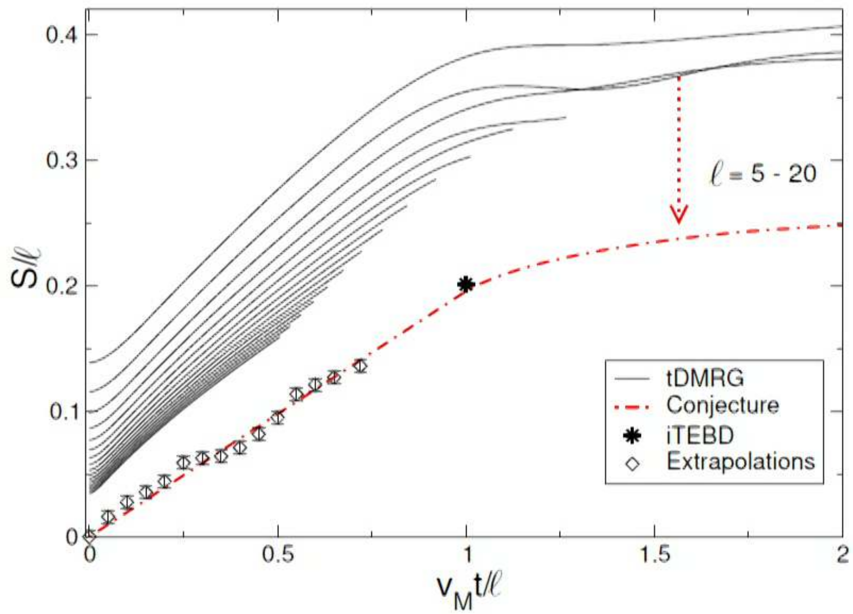
- *XXZ* chain: Quench from Néel state.



$$S_A(t) \propto \sum_n \Big[t \int_{|v_n^*|t<\ell} d\lambda v_n^*(\lambda) s_{YY}^*(\lambda) + \ell \int_{|v_n^*|t>\ell} d\lambda s_{YY}^*(\lambda) \Big]$$

Numerical checks: Full time evolution

- XXZ chain with $\Delta = 2$: Quench from Néel state.



- Fairly good agreement apart from finite size (time) corrections.

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$$S_A(t) \propto \sum_n \Big[\underset{|v_n^*|t<\ell}{t \int d\lambda v_n^*(\lambda) s_{YY}^*(\lambda)} + \underset{|v_n^*|t>\ell}{\ell \int d\lambda s_{YY}^*(\lambda)} \Big]$$

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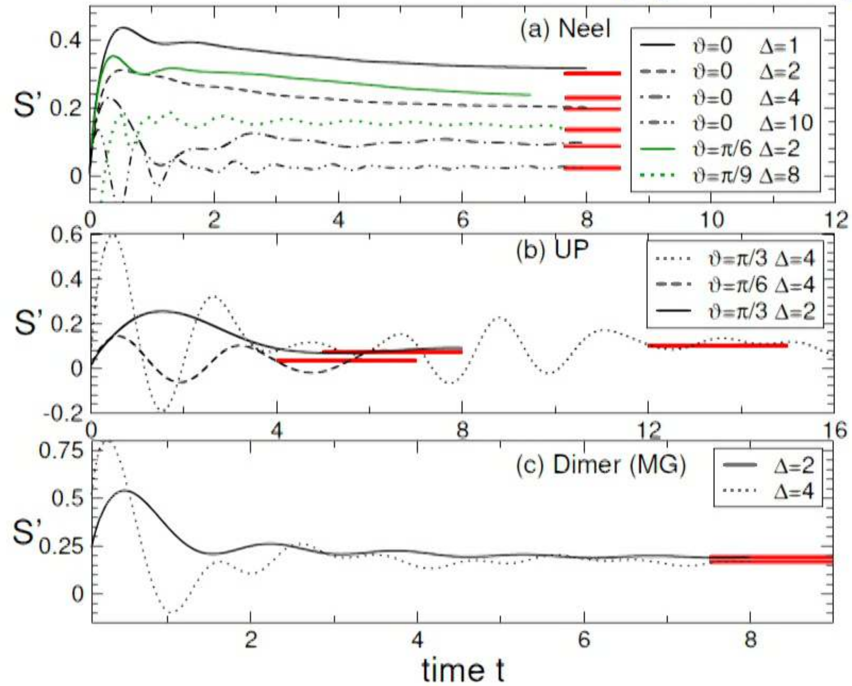
Numerical checks: Linear growth

- Quench in the XXZ chain.

iTEBD from [Fagotti et al., 2015]

$$S' \equiv \frac{dS(v_M t)}{d(v_M t)}$$



$$S_A(t) \propto \sum_n \Big[t \int_{|v_n^*|t<\ell} d\lambda v_n^*(\lambda) s^*_{YY}(\lambda) + \ell \int_{|v_n^*|t>\ell} d\lambda s^*_{YY}(\lambda) \Big]$$

Diagonal entropies

- From the **overlaps** with the initial state:

$$S_d^{(\alpha)} \equiv \frac{1}{1-n} \ln \sum_n |\langle \Psi_0 | n \rangle|^{2\alpha}$$

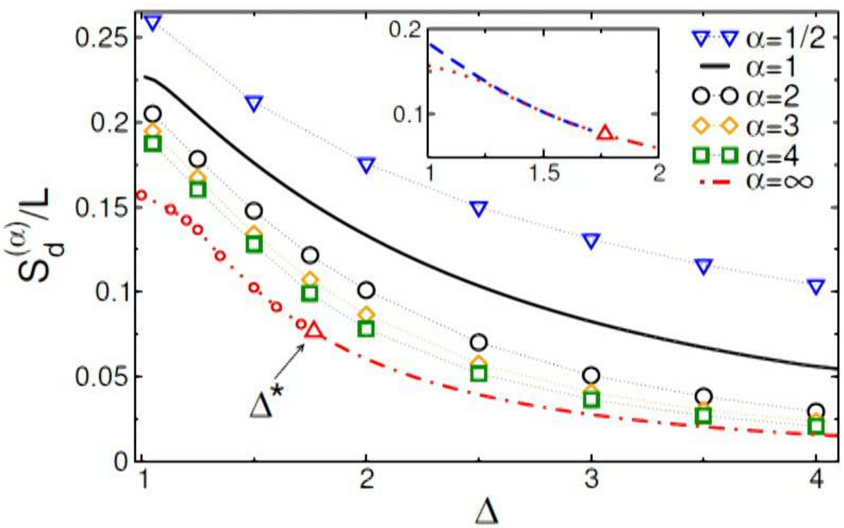
$$\lim_{\alpha\to 1} S_d^{(\alpha)} = S_d = -\sum_n \rho_{nn} \ln \rho_{nn}$$

with $\rho_{nn} \equiv |\langle \Psi_0 | n \rangle|^2$

- Why is S_d interesting? [Polkovnikov, 2008]
  + Generic quench $\to$ not conserved in time for the **full** system.
  + **Increasing** with time after the quench.
  + Conserved in **adiabatic** quenches.
  + Computed from the initial state.

Diagonal entropies in the steady state

- Quench from **Néel** state.



- **Remark**: Saddle point of Quench Action depends on α.

Diagonal versus entanglement entropies

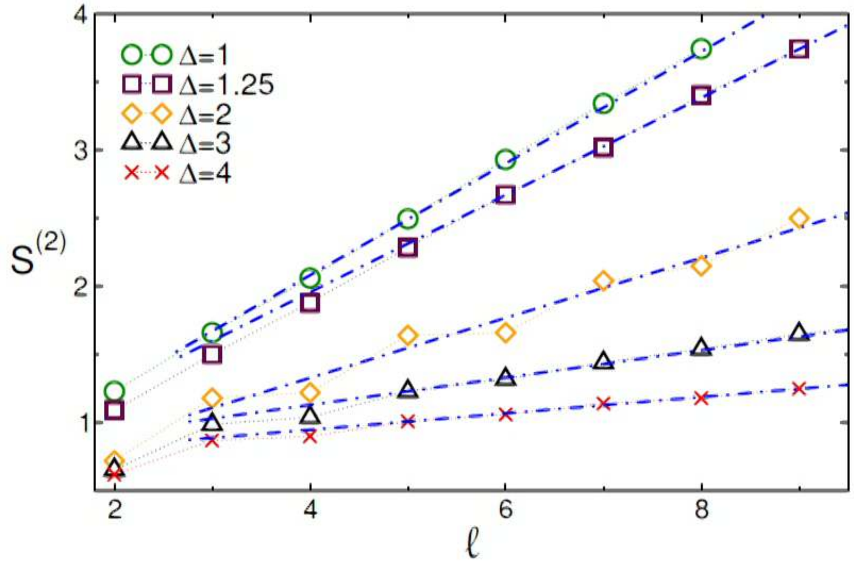
- **Conjecture**:

Proof in [V.A. and P. Calabrese, arXiv:1705.10765]

$$S_d^{(\alpha)} = \frac{1}{2} S^{(\alpha)}$$

[Gurarie, 2013]
[Fagotti, 2013]
[Collura, 2014]
[Piroli, 2016]

Néel quench

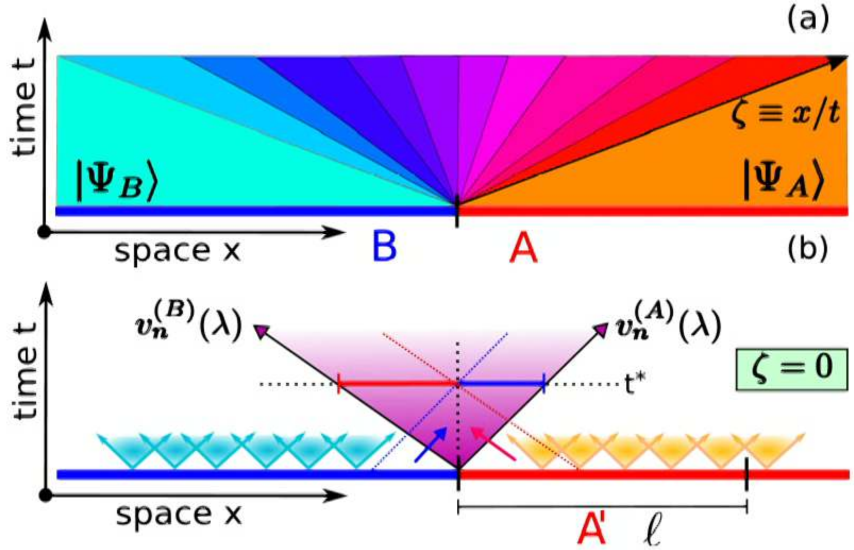


- Perfect agreement with DMRG data.

Inhomogeneous quenches

- Sudden junction of two **macroscopically** different states ⇒ steady **current**.
- **Integrability** ⇒ integrable **hydrodynamics**.

[Castro-Alvaredo et al., PRX 2016]
[Bertini et al., PRL 2016]



- **Semiclassical** picture for entanglement dynamics applies. [V.A., arXiv:1706.00020]

Conclusions

- **Entanglement** dynamics after quantum **quenches** in **integrable** models.
- Improved **Semiclassical** picture using **integrability**.
- Entanglement dynamics encoded in the **steady state** and **low-lying** excitations around it.
- Several promising developments: **Inhomogeneous** quenches, **Rényi** diagonal entropies.