Title: PSI 17/18 Front End - Lie Groups and Lie Algebras (Dupuis)

Date: Aug 17, 2017 10:30 AM

URL: http://pirsa.org/17080056

Abstract:

⚠ Tutorial 4 rescheduled tomorrow
3.45pm - 5.15pm
So no tutorial Today!

Today
- SU(2) recoupling theory.
  ↳ Clebsh-Gordan coef.
- From SU(2) to SU(N)
  ↳ Young tableaux

SU(2): spin j

$V_j = \{ |j\, m\rangle,\ m \in \{-j \dots j\} \}$ dim $= 2j+1$

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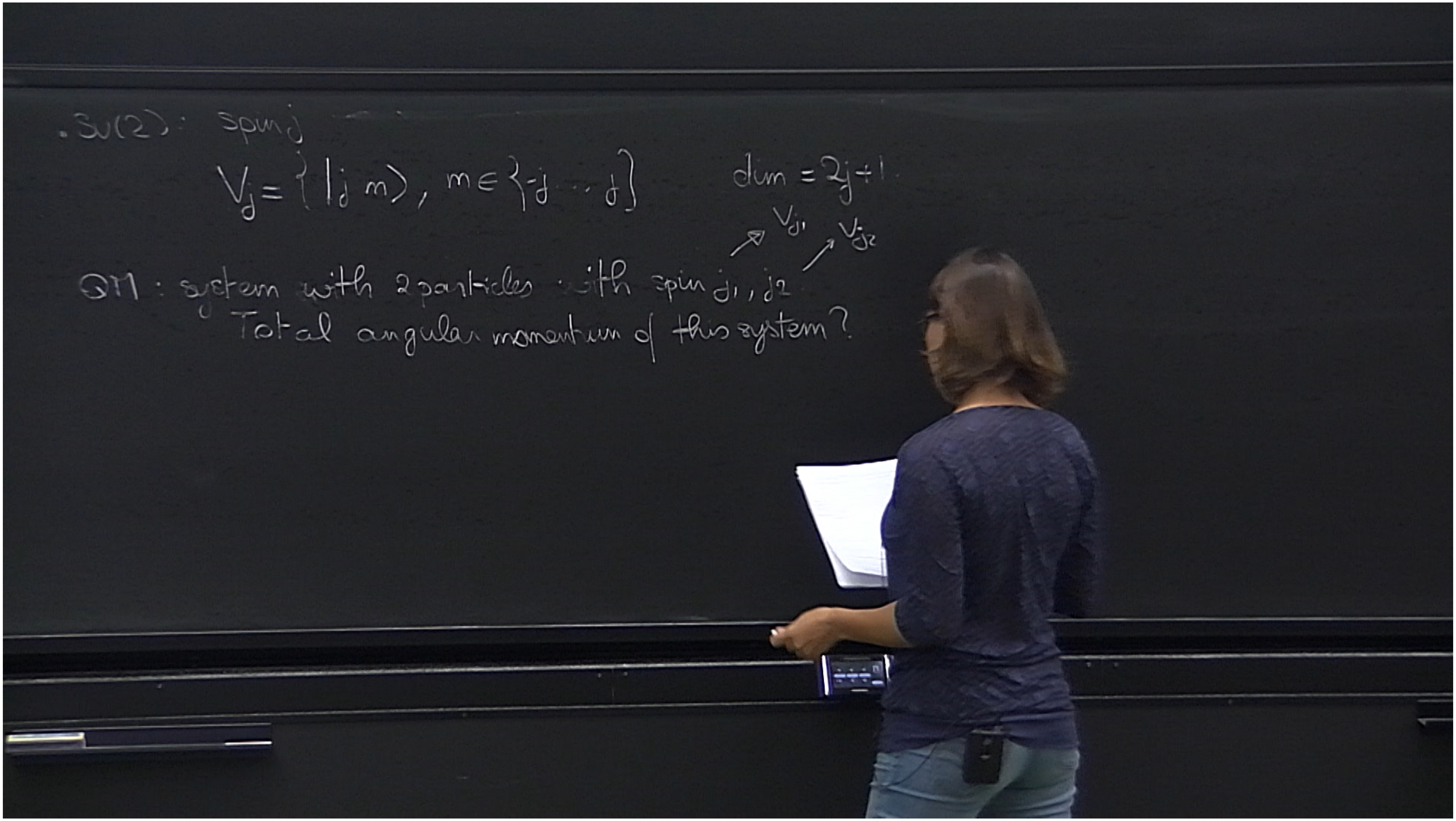

. SU(2): spin j
$V_j = \{ |j\, m\rangle,\ m \in \{-j \dots j\} \}$
dim = 2j+1
V_{j_1}
V_{j_2}
QM: system with 2 particles with spin j_1, j_2
Total angular momentum of this system?

• SU(2) : spin j

$$V_j = \{ |j\, m\rangle,\ m \in \{-j \dots j\} \}$$ dim $= 2j+1$

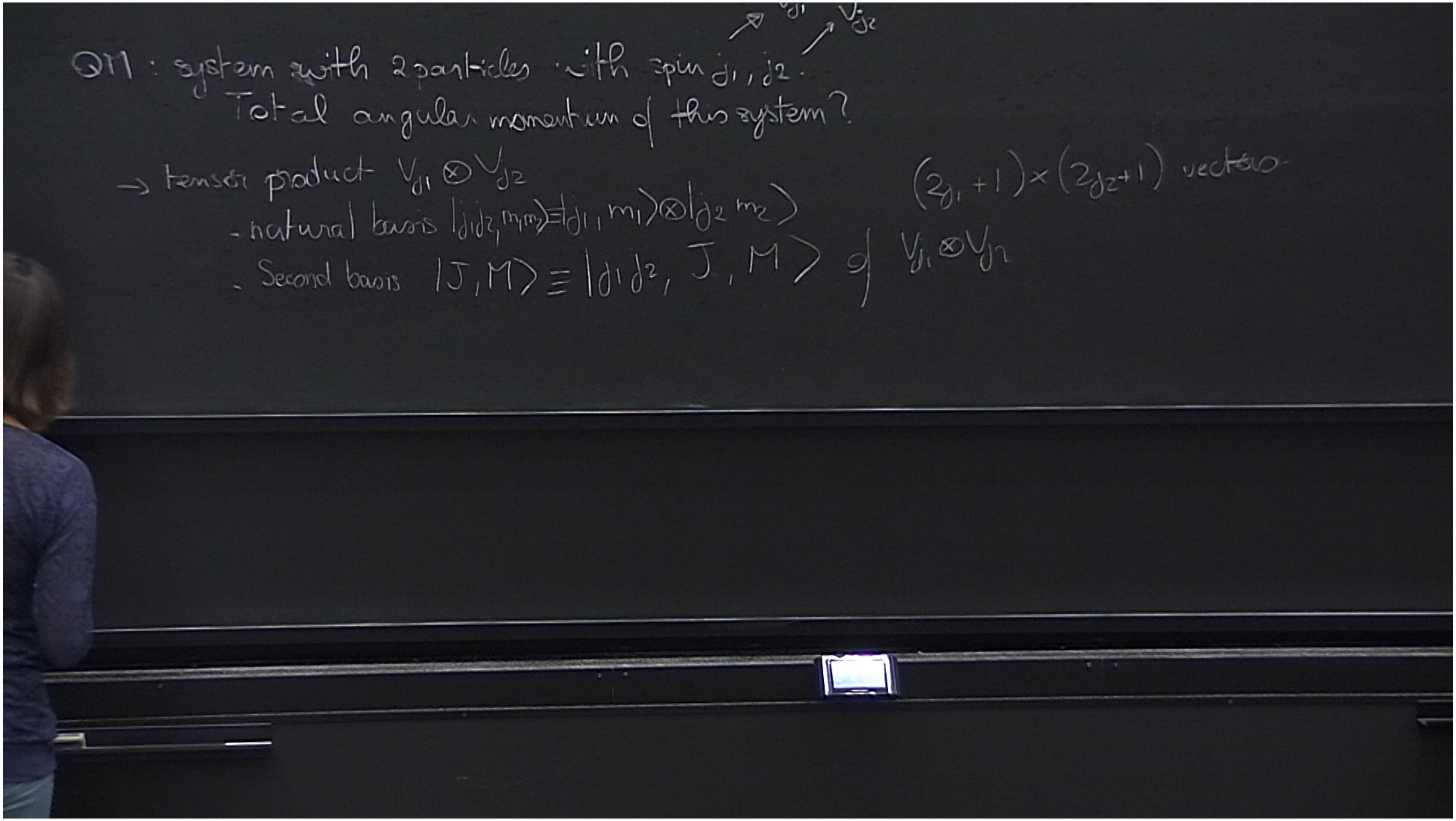
QM : system with 2 particles with spin j_1, j_2 (V_{j_1}, V_{j_2})

Total angular momentum of this system?

→ tensor product $V_{j_1} \otimes V_{j_2}$

natural basis $|j_1 j_2, m_1 m_2\rangle = |j_1, m_1\rangle \otimes |j_2\, m_2\rangle$

$(2j_1+1) \times (2j_2+1)$ vectors.


V_{j_1} V_{j_2}
QM : system with 2 particles with spin j_1, j_2.
Total angular momentum of this system?
→ tensor product $V_{j_1} \otimes V_{j_2}$
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- natural basis $|j_1 j_2, m_1 m_2\rangle = |j_1, m_1\rangle \otimes |j_2\, m_2\rangle$
- Second basis $|J, M\rangle \equiv |j_1 j_2, J, M\rangle$ of $V_{j_1} \otimes V_{j_2}$

V_{j_1} V_{j_2}

QM : system with 2 particles with spin j_1, j_2.

Total angular momentum of this system?

→ tensor product $V_{j_1} \otimes V_{j_2}$ $(2j_1 +$

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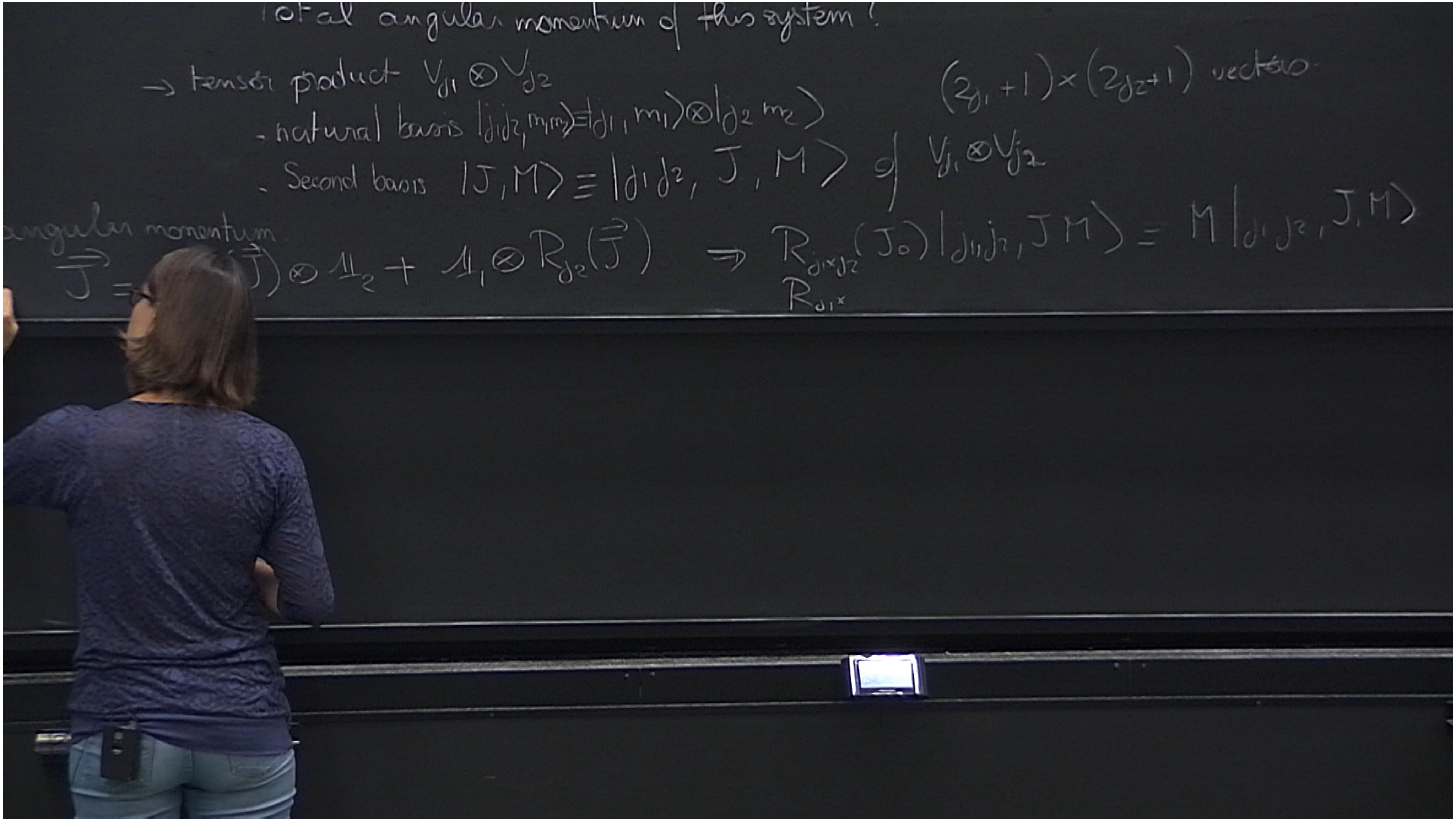
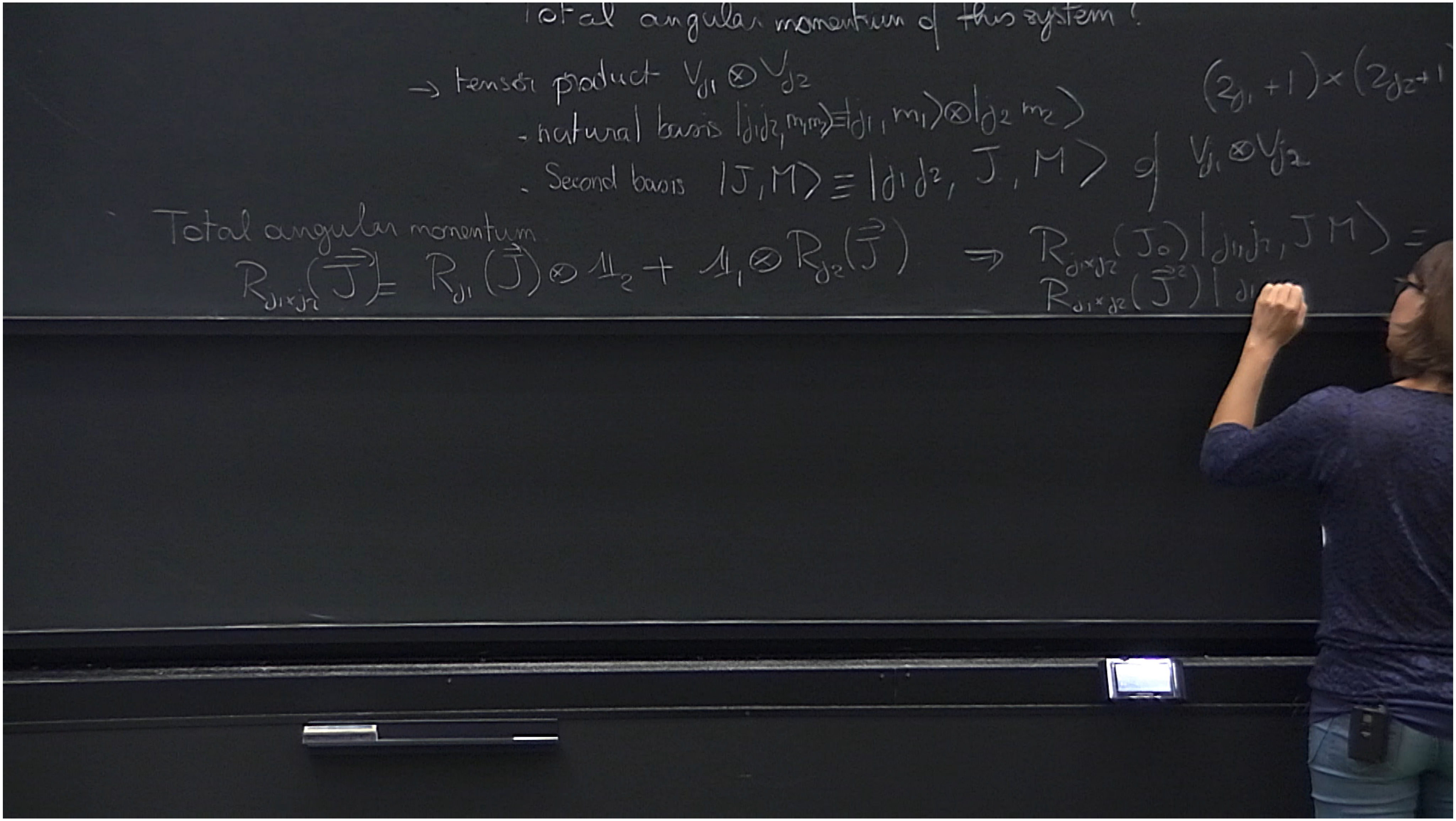
- Second basis $|J, M\rangle \equiv |j_1 j_2, J, M\rangle$ of $V_{j_1} \otimes V_{j_2}$

- Total angular momentum

$$\vec{J} = R_{j_1}(\vec{J}) \otimes \mathbb{1}_2 + \mathbb{1}_1 \otimes R_{j_2}(\vec{J})$$


Total angular momentum of this system?
→ tensor product V_j1 ⊗ V_j2
(2j1 + 1) × (2j2+1) vectors
- natural basis |j1j2, m1m2⟩ ≡ |j1, m1⟩ ⊗ |j2 m2⟩
- Second basis |J, M⟩ ≡ |j1 j2, J, M⟩ of V_j1 ⊗ V_j2
angular momentum
J = (J) ⊗ 1_2 + 1_1 ⊗ R_j2(J)
⇒ R_j1×j2 (J0) |j1, j2, J M⟩ = M |j1 j2, J, M⟩
R_j1×


Total angular momentum of this system ?
→ tensor product $V_{j_1} \otimes V_{j_2}$
$(2j_1+1)\times(2j_2+1)$
- natural basis $|j_1 j_2, m_1 m_2\rangle \equiv |j_1, m_1\rangle \otimes |j_2, m_2\rangle$
- Second basis $|J, M\rangle \equiv |j_1 j_2, J, M\rangle$ of $V_{j_1} \otimes V_{j_2}$
Total angular momentum
$R_{j_1 \times j_2}(\vec{J}) = R_{j_1}(\vec{J}) \otimes 1\!\!1_2 + 1\!\!1_1 \otimes R_{j_2}(\vec{J})$
$\Rightarrow R_{j_1 \times j_2}(J_0)|j_1 j_2, J M\rangle =$
$R_{j_1 \times j_2}(\vec{J}^2)|j_1$

Total angular momentum of this system?

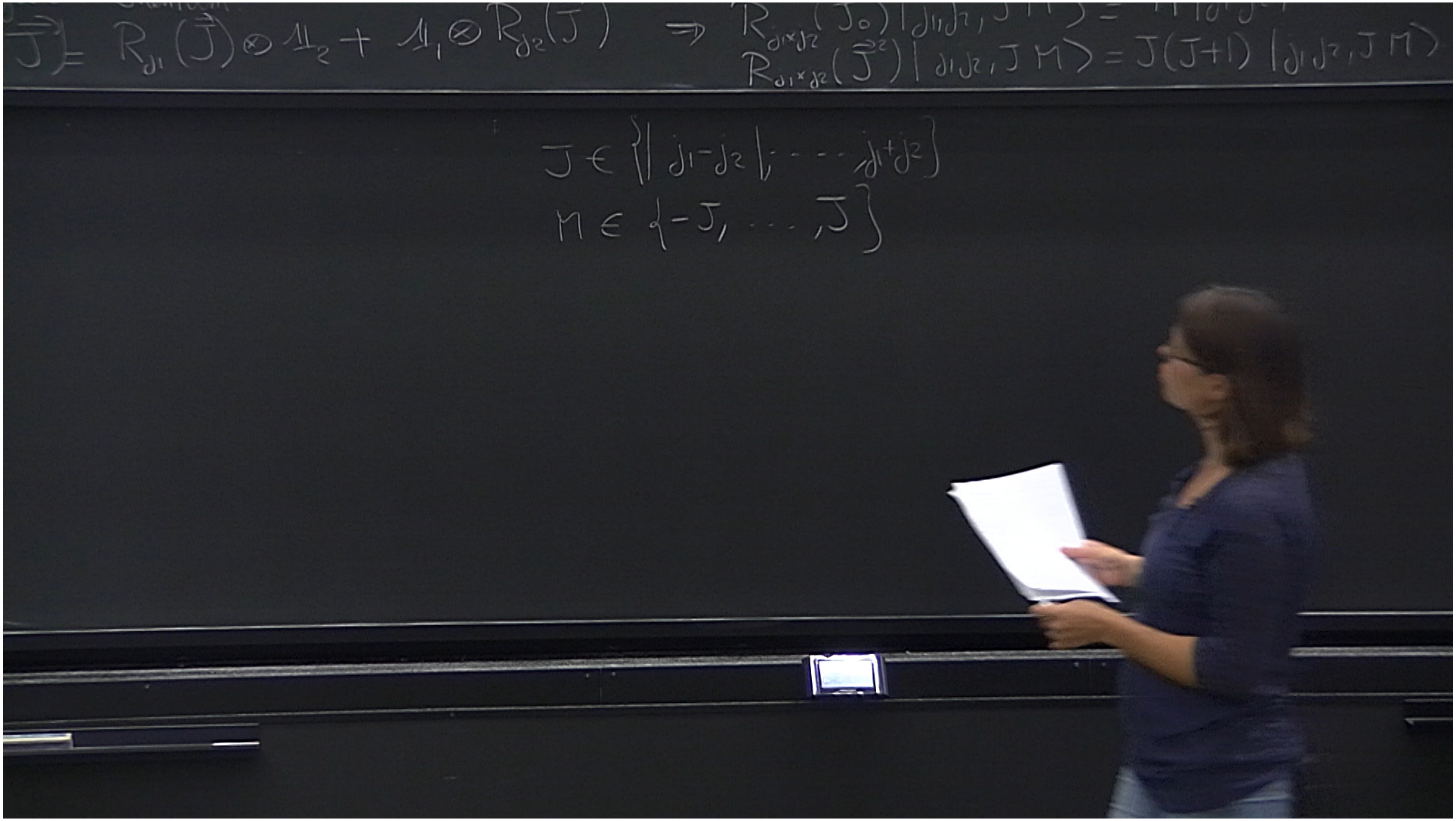
→ tensor product $V_{j_1} \otimes V_{j_2}$ $\quad (2j_1+1)\times(2j_2+1)$ vectors

- natural basis $|j_1 j_2, m_1 m_2\rangle \equiv |j_1, m_1\rangle \otimes |j_2\, m_2\rangle$
- Second basis $|J, M\rangle \equiv |j_1 j_2, J, M\rangle$ of $V_{j_1} \otimes V_{j_2}$

gular momentum

$$\vec{J}) = R_{j_1}(\vec{J}) \otimes 1\!\!1_2 + 1\!\!1_1 \otimes R_{j_2}(\vec{J}) \Rightarrow R_{j_1 \times j_2}(J_0)\,|j_1 j_2, J M\rangle = M\,|j_1 j_2, J, M\rangle$$

$$R_{j_1 \times j_2}(\vec{J}^2)\,|j_1 j_2, J M\rangle = J(J+1)\,|j_1 j_2, J M\rangle$$


$\vec{J} = R_{j_1}(\vec{J}) \otimes 1\!\!1_2 + 1\!\!1_1 \otimes R_{j_2}(\vec{J})$
$R_{j_1 \times j_2}(\vec{J}^2)\,|j_1 j_2, J M\rangle = J(J+1)\,|j_1 j_2, J M\rangle$
$J \in \{|j_1 - j_2|, \dots, j_1 + j_2\}$
$M \in \{-J, \dots, J\}$

- Second basis $|J, M\rangle \equiv |j_1 j_2, J, M\rangle$ of $\mathcal{H}_1 \otimes \mathcal{H}_2$

momentum

$$R_{j_1}(\vec{J}) \otimes \mathbb{1}_2 + \mathbb{1}_1 \otimes R_{j_2}(\vec{J}) \Rightarrow R_{j_1 \times j_2}(J_0)|j_1 j_2, J M\rangle = M |j_1 j_2, J, M\rangle$$

$$R_{j_1 \times j_2}(\vec{J}^2)|j_1 j_2, J M\rangle = J(J+1)|j_1 j_2, J M\rangle$$

$$J \in \{|j_1 - j_2|, \dots, j_1 + j_2\}$$

$$M \in \{-J, \dots, J\}$$

<u>Change of basis</u> (Clebsh-Gordan coefficients)

$$|j_1 j_2, J M\rangle = \sum_{m_1, m_2} |j_1 j_2\, m_1 m_2\rangle \underbrace{\langle j_1 j_2\, m_1 m_2 | J M\rangle}_{\equiv \langle j_1 j_2\, m_1 m_2 | j_1 j_2, J M\rangle}$$

Clebsh-Gordan coefficients

$$\mathcal{P}_{j_1 \times j_2}(\vec{J}^2)\,|j_1 j_2, J M\rangle = J(J+1)\,|j_1 j_2, J M\rangle$$

$\ldots\ldots j_1+j_2 \}$

$\ldots, J \}$

...n coefficients)

$$|j_1 j_2\, m_1 m_2\rangle \underbrace{\langle j_1 j_2\, m_1 m_2 | J M\rangle}_{\equiv \langle j_1 j_2\, m_1 m_2 | j_1 j_2, J M\rangle}$$

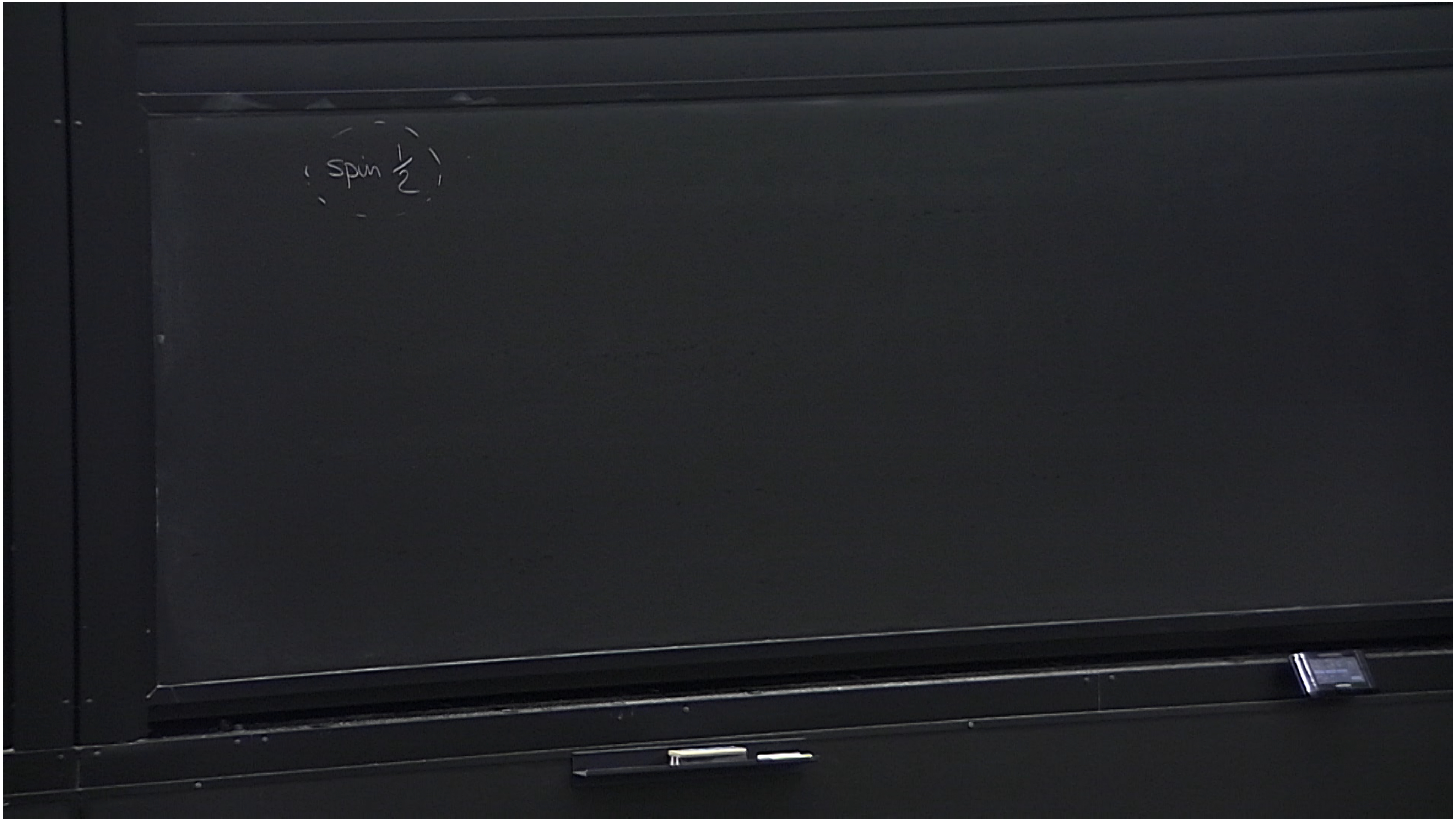
Clebsh-Gordan coefficients

vanish unless

$|j_1 - j_2| \leq J \leq j_1 + j_2$

$J - j_1 - j_2 \in \mathbb{Z}$

$m_1 + m_2 = M$


spin 1/2

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spin $\frac{1}{2}$

2. Irreducible representations of special unitary gp and Young tableaux.

(spin $\frac{1}{2}$)

2. Irreducible representations of special unitary gp and Young tableaux.

. Tensors in SU(N)

$$U \in SU(N) \;,\; U^{+}U = \mathbb{1} \;,\; \det U = 1$$

spin $\frac{1}{2}$

2. Irreducible representations of special unitary gp and Young tableaux.

- Tensors in SU(N)

- $U \in SU(N)$, $U^{\dagger}U = \mathbb{1}$, $\det U = 1$
- General state of SU(N) (quark) as vector $\psi = (\psi_1, \psi_2 \dots \psi_N)$

$$\psi'_i = U_{ij}\,\psi_j$$ U is a unitary matrix

tableaux.

$\psi_2 \ldots \psi_N)$.

is a unitary matrix

- Inner product $(\phi, \psi) = \phi_i^* \psi_i$

Inner product to remain invariant.

$\overline{\psi} \equiv \psi^*$ transforms as $\overline{\psi}'_i = U_{ij}^* \psi$.

$\overline{\psi}$ transforms in the anti-fundamental rep. of $SU(N)$.

spin $\frac{1}{2}$

2. Irreducible representations of special unitary gp and Young tableaux.

. Tensors in SU(N)

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$$\psi'_i = U_{ij}\,\psi_j$$ U is a unitary matrix

transforms under the fundamental rep. of SU(N) transforms under the

- Rank 2 tensor Ψ_{ij}

transformation property $\Psi'_{ij} = U_{ik} U_{jl} \Psi_{kl}$

transforms under the ...

. Rank 2 tensor Ψ_{ij}

transformation property $\Psi'_{ij} = U_{ik} U_{jl} \Psi_{kl}$

. Permutation $i \leftrightarrow j$ denoted P_{12} does not change transformation property

$$P_{12} \Psi'_{ij} = \Psi'_{ji}$$
$$= U_{jk} U_{il} \Psi_{kl}$$
$$= U_{jk} U_{il} P_{12} \Psi_{lk}$$

transforms under the ...

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$$P_{12} \Psi'_{ij} = \Psi'_{ji}$$

$$= U_{jk} U_{il} \Psi_{kl}$$

$$= U_{jl} U_{ik} P_{12} \Psi_{lk} = U_{jl} U_{ik} P_{12} \Psi_{kl}$$

$$= U_{ik} U_{jl} (P_{12} \Psi_{kl})$$

transforms under the ...

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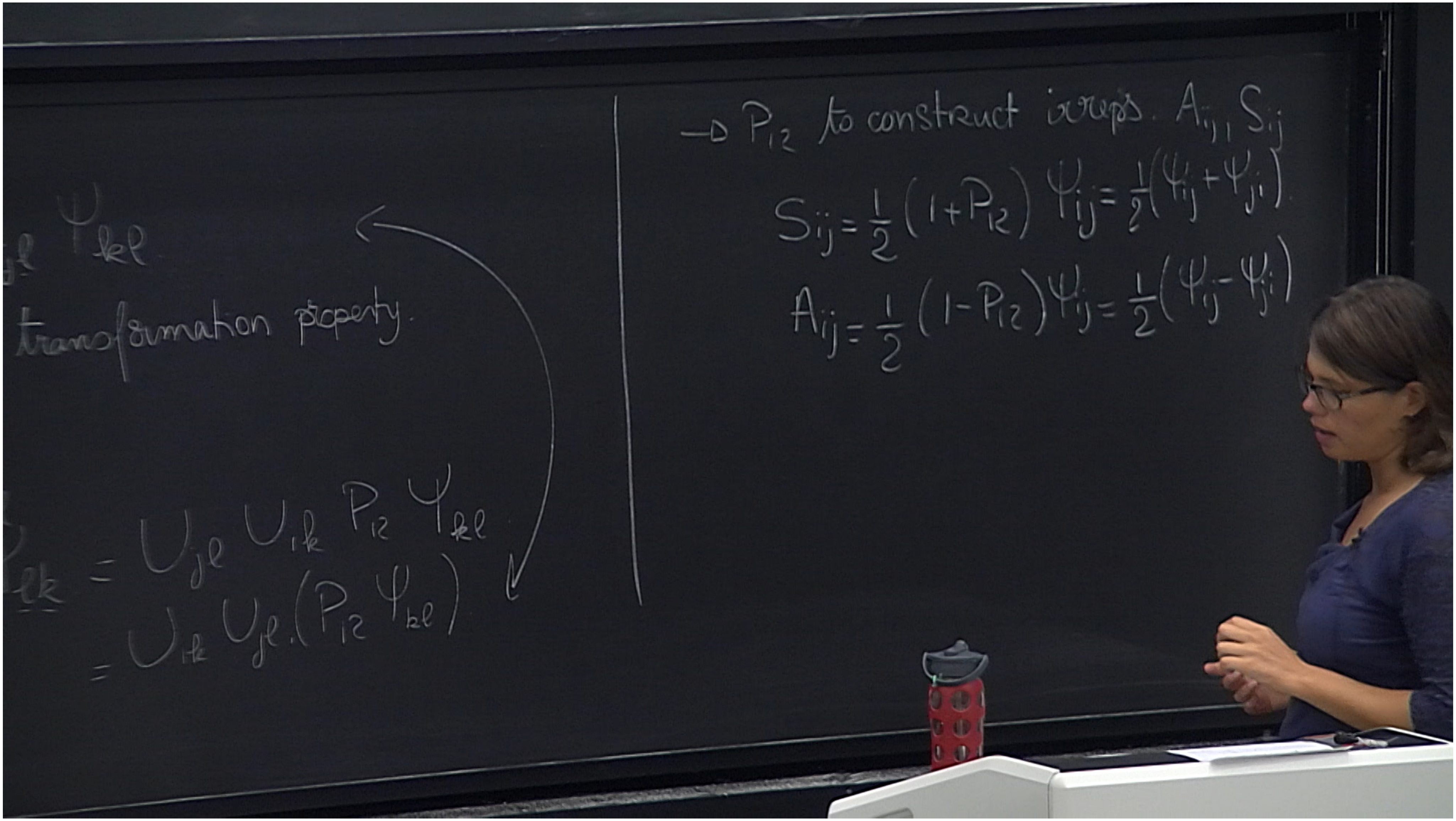
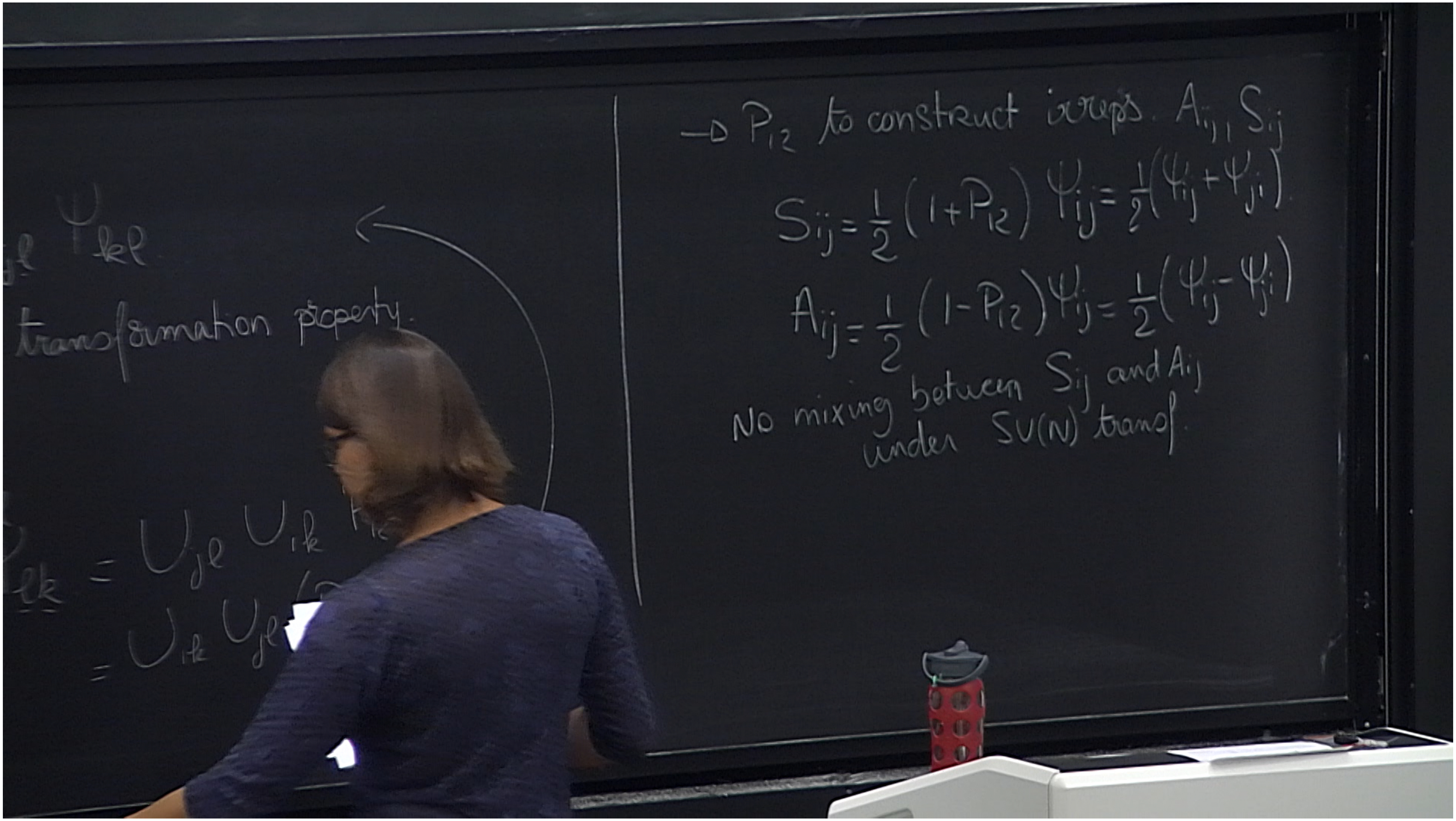
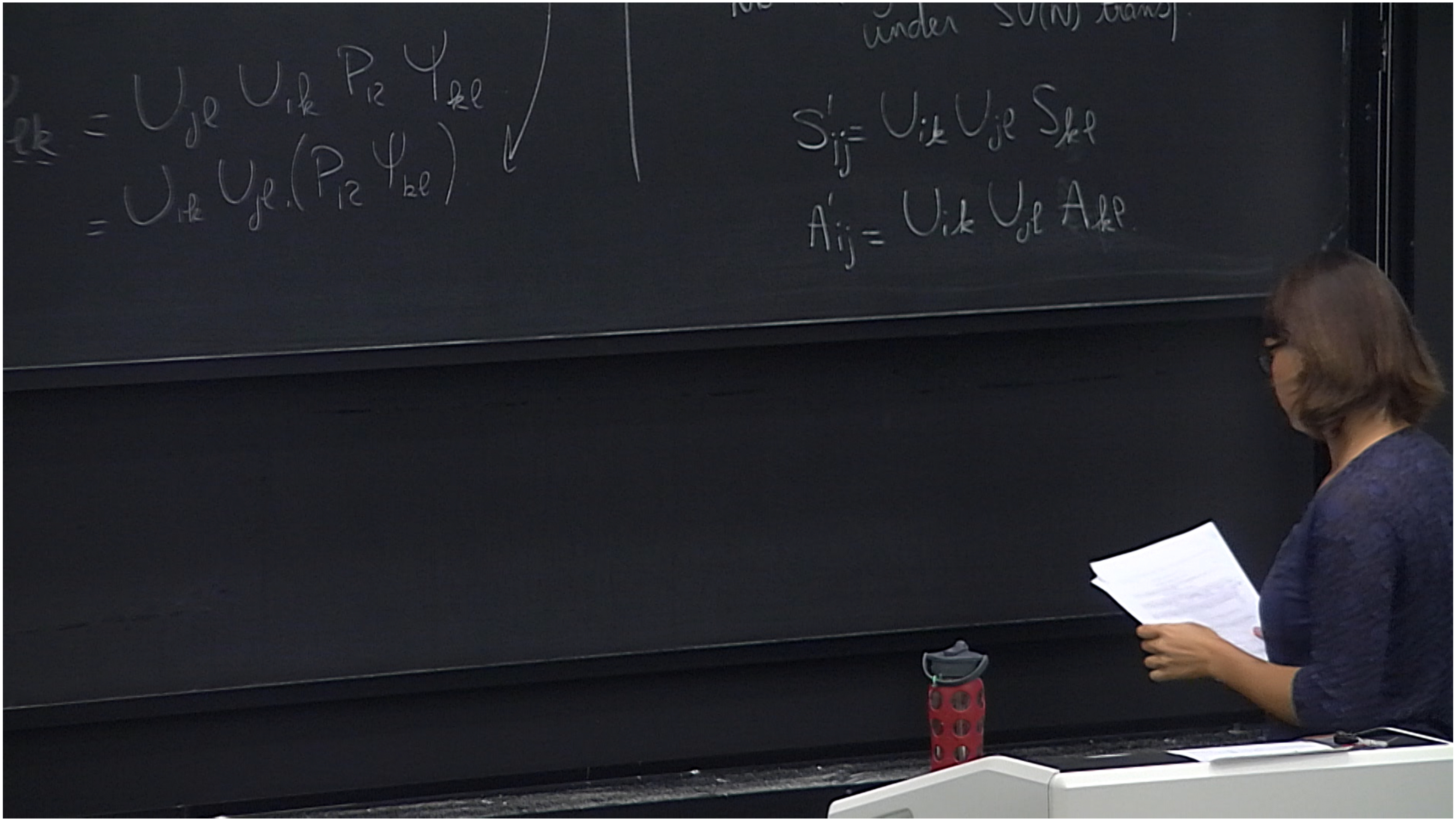


$\to P_{12}$ to construct irreps. A_{ij}, S_{ij}
$S_{ij} = \frac{1}{2}(1+P_{12})\psi_{ij} = \frac{1}{2}(\psi_{ij}+\psi_{ji})$
$A_{ij} = \frac{1}{2}(1-P_{12})\psi_{ij} = \frac{1}{2}(\psi_{ij}-\psi_{ji})$
ψ_{kl}
transformation property.
$= U_{jl} U_{ik} P_{12} \psi_{kl}$
$= U_{ik} U_{jl} (P_{12} \psi_{kl})$


→ P_{12} to construct irreps. A_{ij}, S_{ij}
$S_{ij} = \frac{1}{2}(1+P_{12})\,\psi_{ij} = \frac{1}{2}(\psi_{ij} + \psi_{ji})$
$A_{ij} = \frac{1}{2}(1-P_{12})\,\psi_{ij} = \frac{1}{2}(\psi_{ij} - \psi_{ji})$
No mixing between S_{ij} and A_{ij}
under SU(N) transf.
transformation property.
$= U_{j\ell}\, U_{ik}$
$= U_{ik}\, U_{j\ell}$

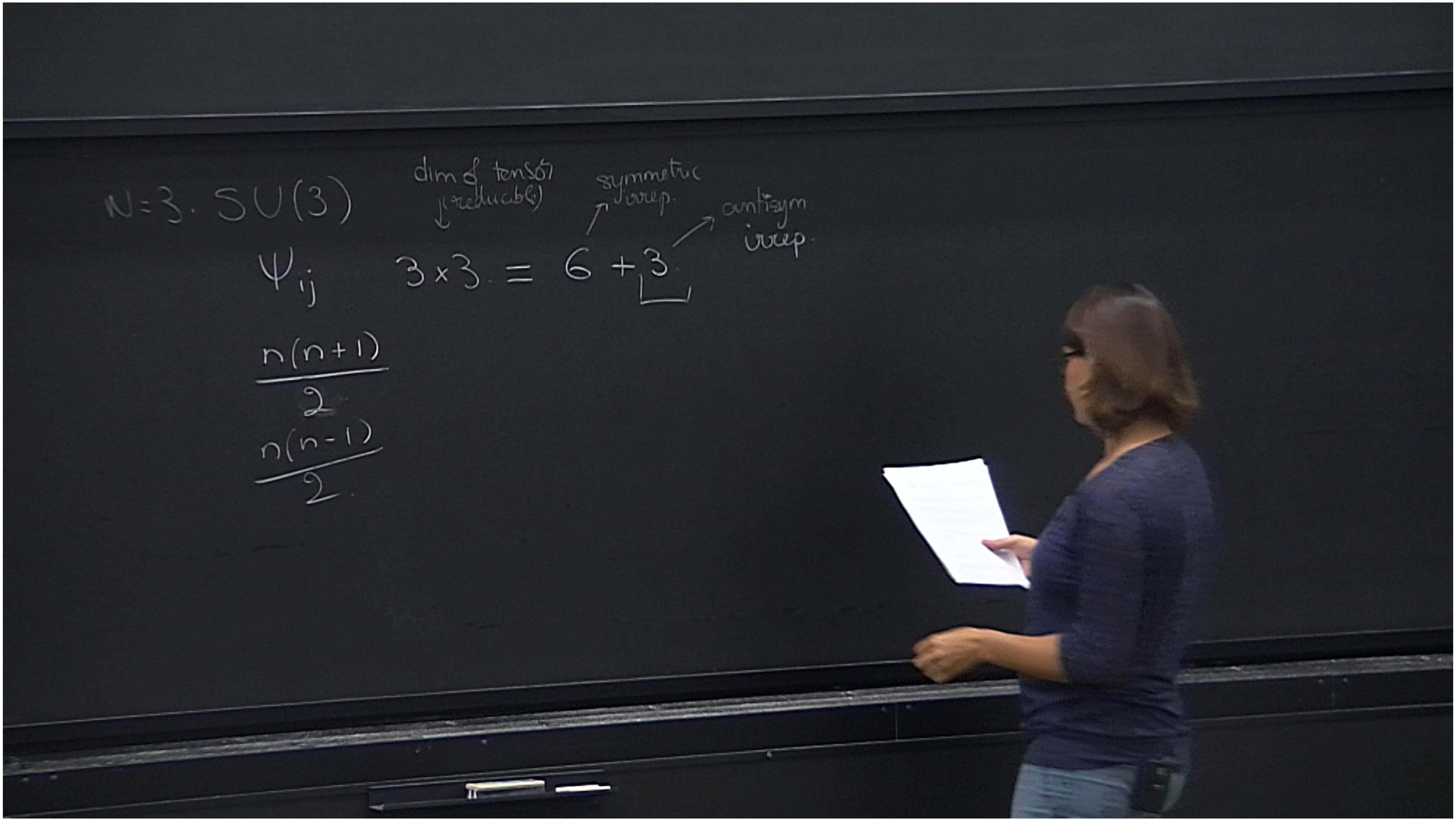

under SU(N) transf.
$= U_{jl} U_{ik} P_{12} \psi_{kl}$
$= U_{ik} U_{jl} (P_{12} \psi_{kl})$
$S'_{ij} = U_{ik} U_{jl} S_{kl}$
$A'_{ij} = U_{ik} U_{jl} A_{kl}$

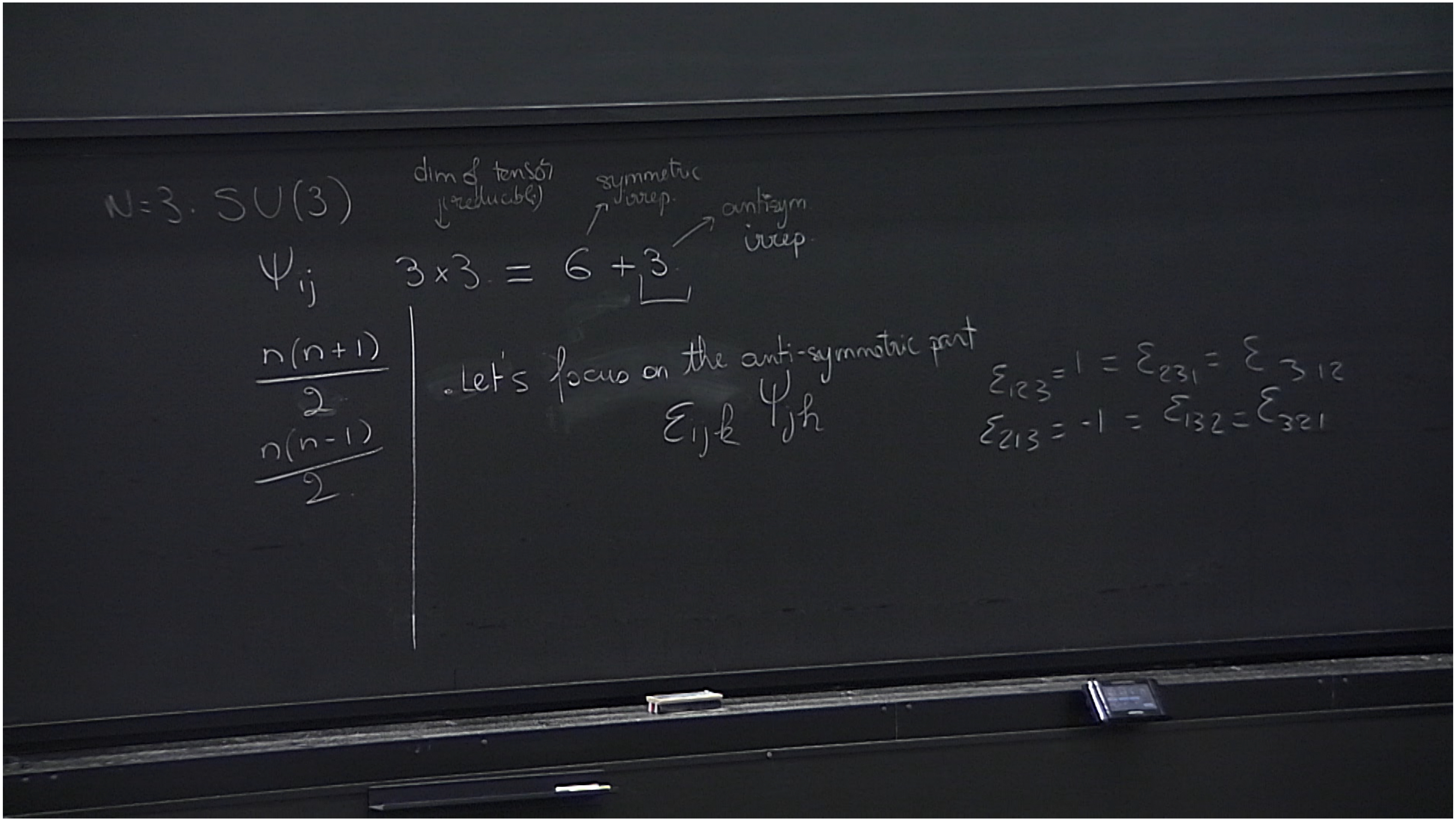
$N=3 \cdot SU(3)$

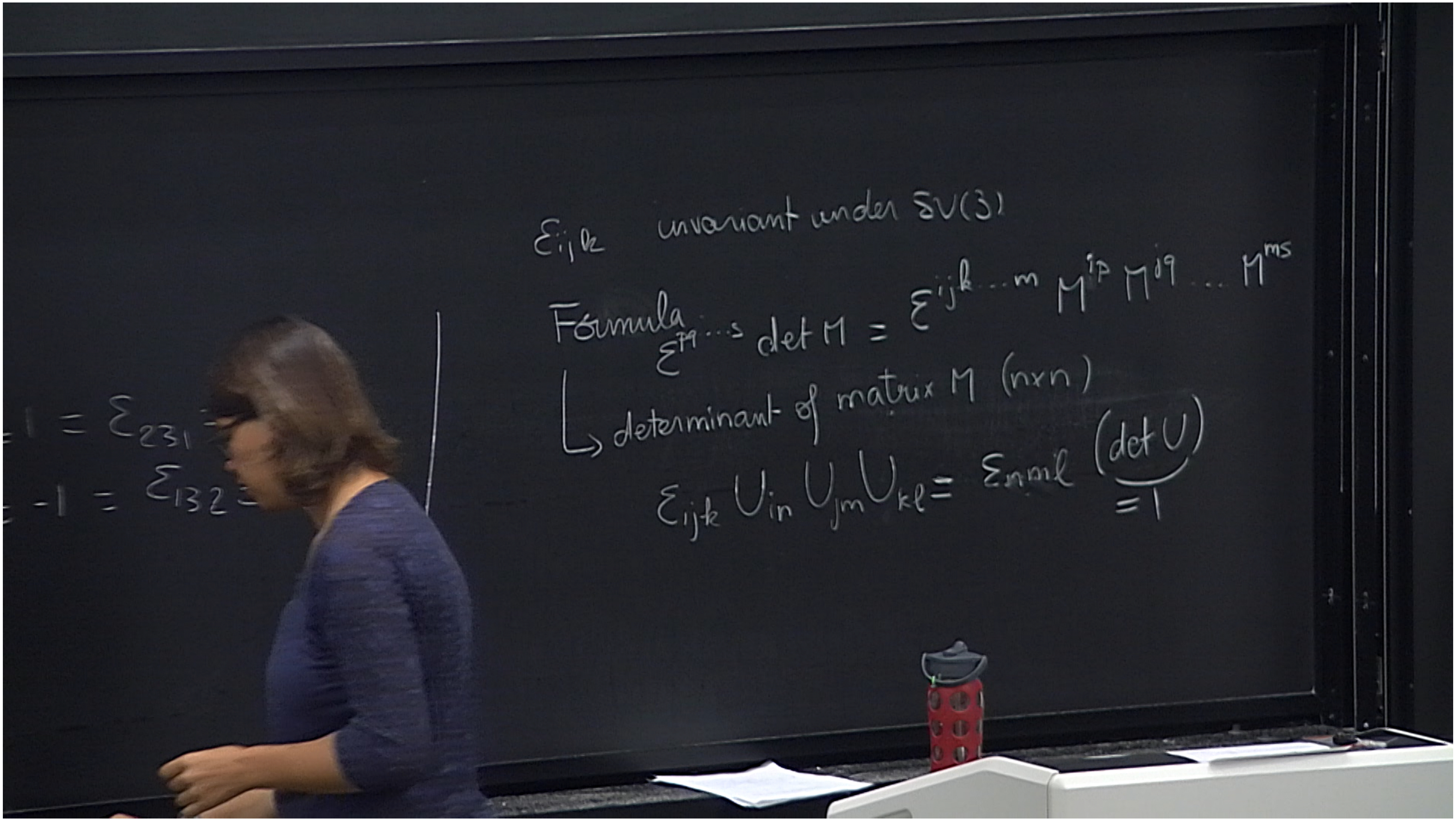
$\Psi_{ij} \quad 3 \times 3 = 6 + 3$

$$\frac{n(n+1)}{2}$$

$$\frac{n(n-1)}{2}$$


N=3. SU(3)
dim of tensor
(reducible)
symmetric
irrep.
antisym
irrep.
Ψij 3×3 = 6 + 3
n(n+1)/2
n(n-1)/2


N=3. SU(3)
dim of tensor (reducible)
symmetric irrep
antisym irrep
ψ_{ij} $3 \times 3 = 6 + 3$
$\frac{n(n+1)}{2}$
$\frac{n(n-1)}{2}$
. Let's focus on the anti-symmetric part
$\epsilon_{ijk} \psi_{jk}$
$\epsilon_{123} = 1 = \epsilon_{231} = \epsilon_{312}$
$\epsilon_{213} = -1 = \epsilon_{132} = \epsilon_{321}$


ε_{ijk} invariant under SU(3)
Formula $\varepsilon^{pq\dots s} \det M = \varepsilon^{ijk\dots m} M^{ip} M^{jq} \dots M^{ms}$
↳ determinant of matrix M (n×n)
$\varepsilon_{ijk} U_{in} U_{jm} U_{kl} = \varepsilon_{nml} (\det U)$
$=1$
$= 1 = \varepsilon_{231} =$
$= -1 = \varepsilon_{132} =$

$$U^{-1}_{lk} = U^{*}_{kl}$$

$$\varepsilon_{ijk} \, \underline{U_{il}} \, U_{jm} \, U_{kn} = \varepsilon_{lmn}$$

$$\varepsilon_{ijk} \, U_{jm} \, U_{km} = \varepsilon_{lmn} \, U^{*}_{il}$$

$$\varepsilon_{ijk} U_{il} U_{jm} U_{kn} = \varepsilon_{lmn}$$

$$U^{-1}_{lk} = U^{*}_{kl}$$

$$\varepsilon_{ijk} U_{jm} U_{km} = \varepsilon_{lmn} U^{*}_{il}$$

$$A_{ij} \to \varepsilon_{ijk} \Psi_{jk}$$

$$\varepsilon_{ijk} U_{il} U_{jm} U_{kn} = \varepsilon_{lmn}$$

$$U^{-1}_{lk} = U^{*}_{kl}$$

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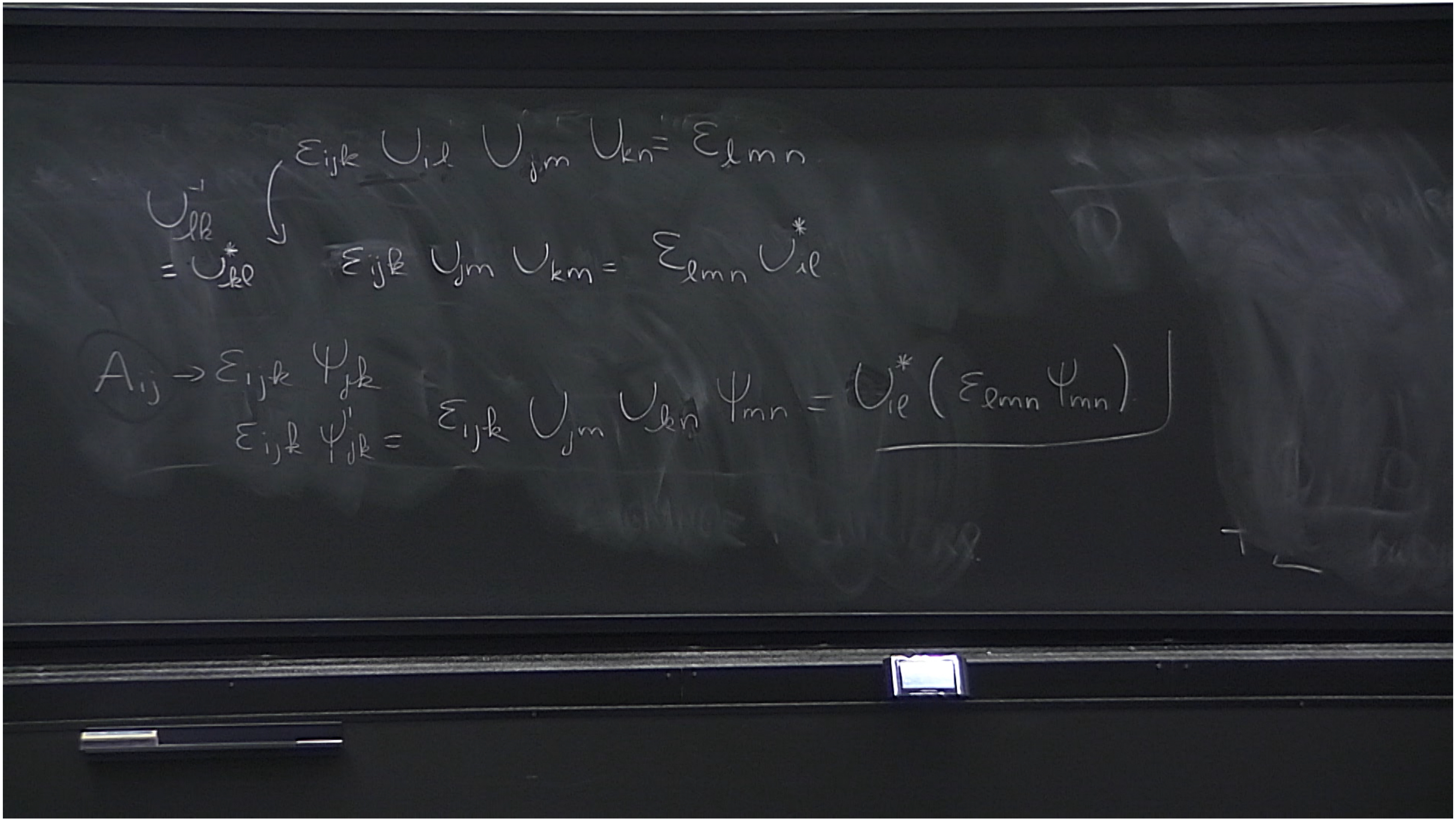
$$A_{ij} \to \varepsilon_{ijk} \psi_{jk}$$

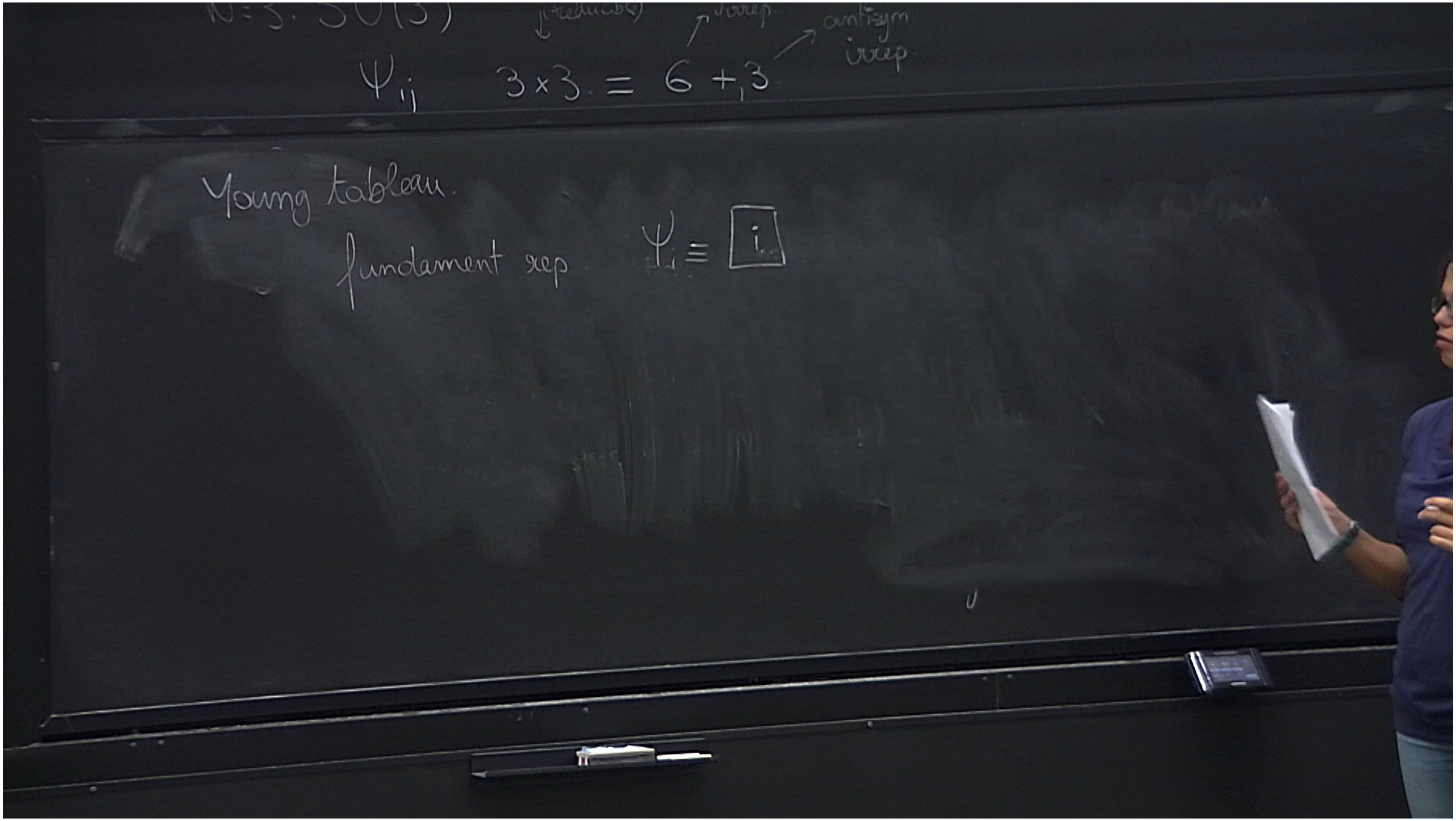
$$\varepsilon_{ijk} \psi'_{jk} = \varepsilon_{ijk} U_{jm} U_{kn} \psi_{mn} = U^{*}_{il} \left(\varepsilon_{lmn} \psi_{mn} \right)$$


$\varepsilon_{ijk} U_{il} U_{jm} U_{kn} = \varepsilon_{lmn}$
$U^{-1}_{lk} = U^{*}_{kl}$
$\varepsilon_{ijk} U_{jm} U_{km} = \varepsilon_{lmn} U^{*}_{il}$
$A_{ij} \to \varepsilon_{ijk} \psi_{jk}$
$\varepsilon_{ijk} \psi'_{jk} = \varepsilon_{ijk} U_{jm} U_{kn} \psi_{mn} = U^{*}_{il} (\varepsilon_{lmn} \psi_{mn})$

$$= U_{i\ell}^{*} \left(\varepsilon_{\ell m n} \psi_{mn} \right)$$

it transforms as $\overline{\psi}$, which is the anti-fundamental representation!

$$3 \times 3 = 6 + \overline{3}$$


irrep
antisym irrep
Ψij 3×3 = 6 + 3
Young tableau.
fundament rep Ψi ≡ i

$N=3: SU(3)$

(reducible) irrep antisym irrep

$\Psi_{ij} \quad 3\times 3 = 6 + 3$

Young tableau.

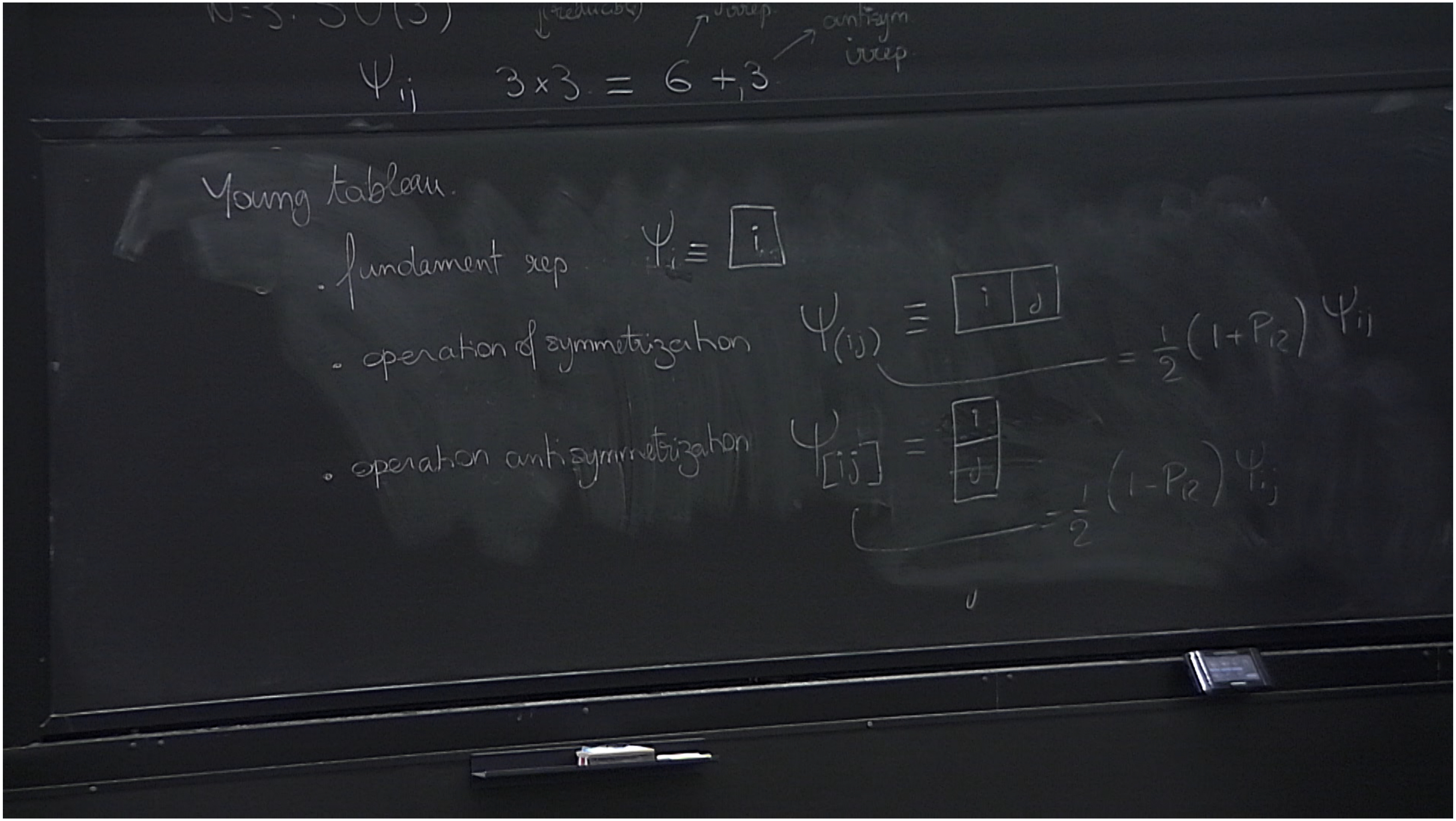
- fundament rep $\quad \Psi_i \equiv \boxed{i}$
- operation of symmetrization $\quad \Psi_{(ij)} \equiv \boxed{i \mid j}$

$N=3$: $SU(3)$ (reducible) irrep antisym irrep

$$\Psi_{ij} \qquad 3\times 3 = 6 + 3$$

Young tableau.

- fundament rep $\quad \Psi_i \equiv \boxed{i}$
- operation of symmetrization $\quad \Psi_{(ij)} \equiv \boxed{i \,|\, j} = \frac{1}{2}(1+P_{12})\Psi_{ij}$
- operation antisymmetrization $\quad \Psi_{[ij]} = \begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array}$


(reducible)
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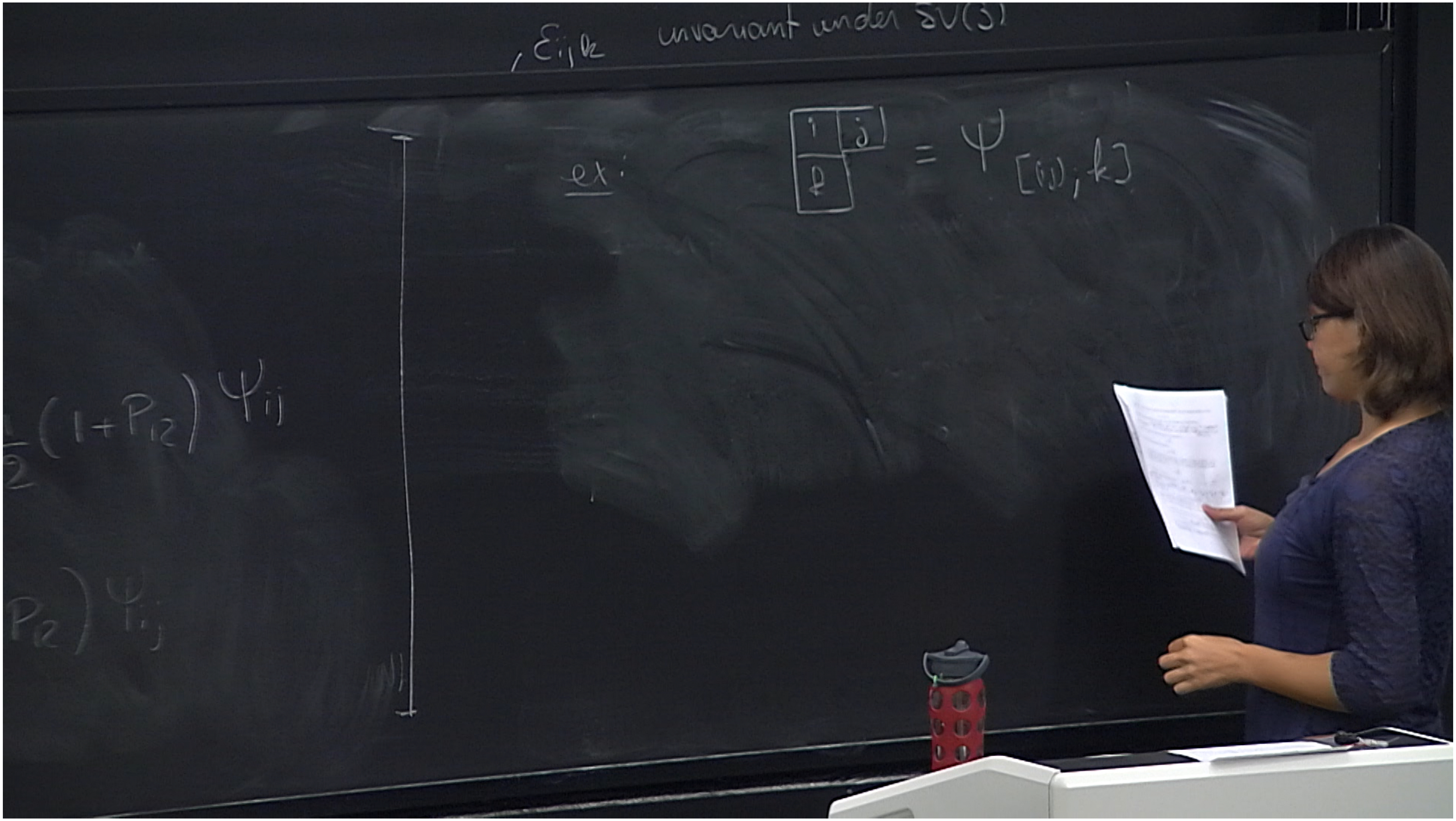
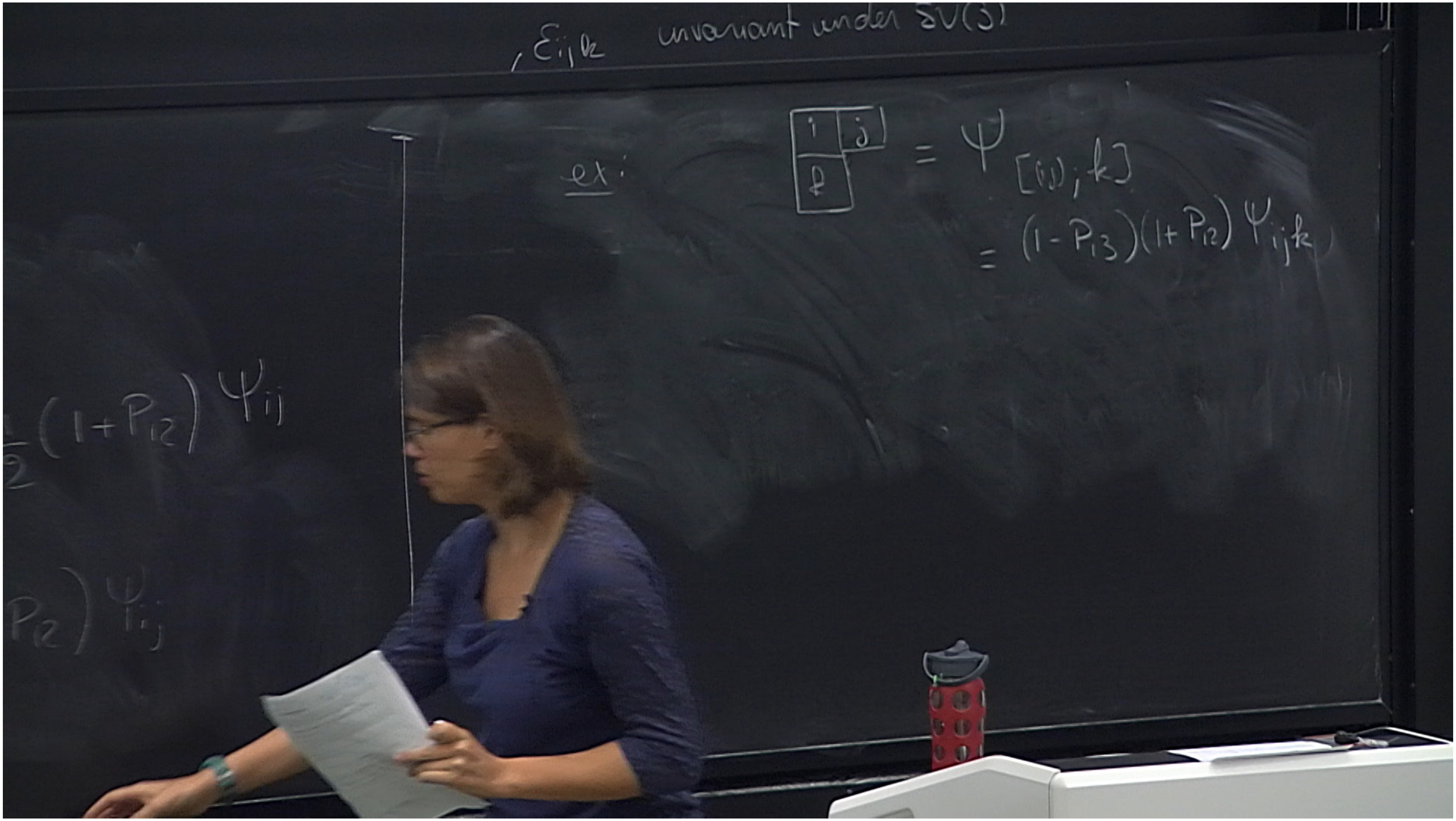
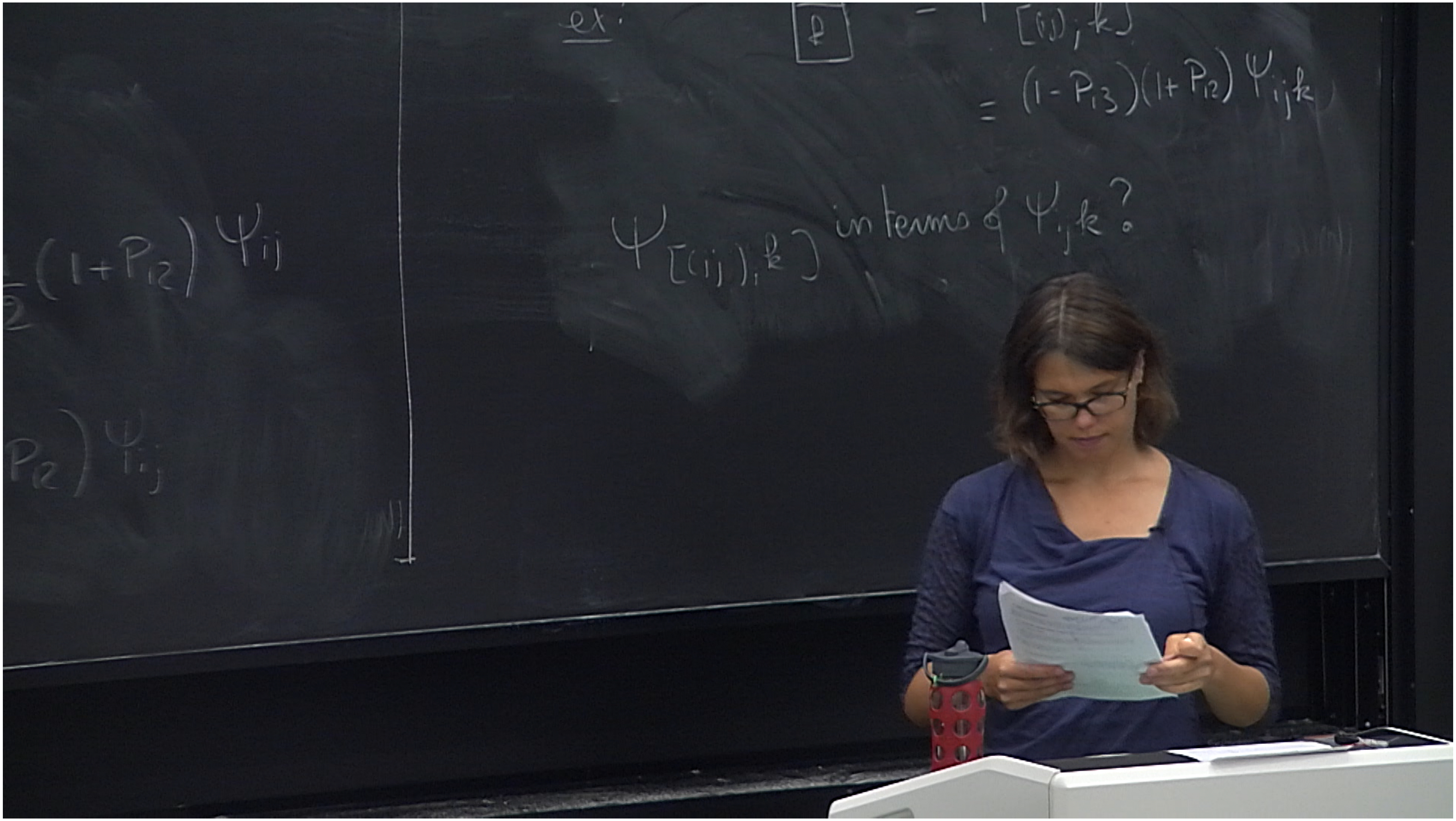
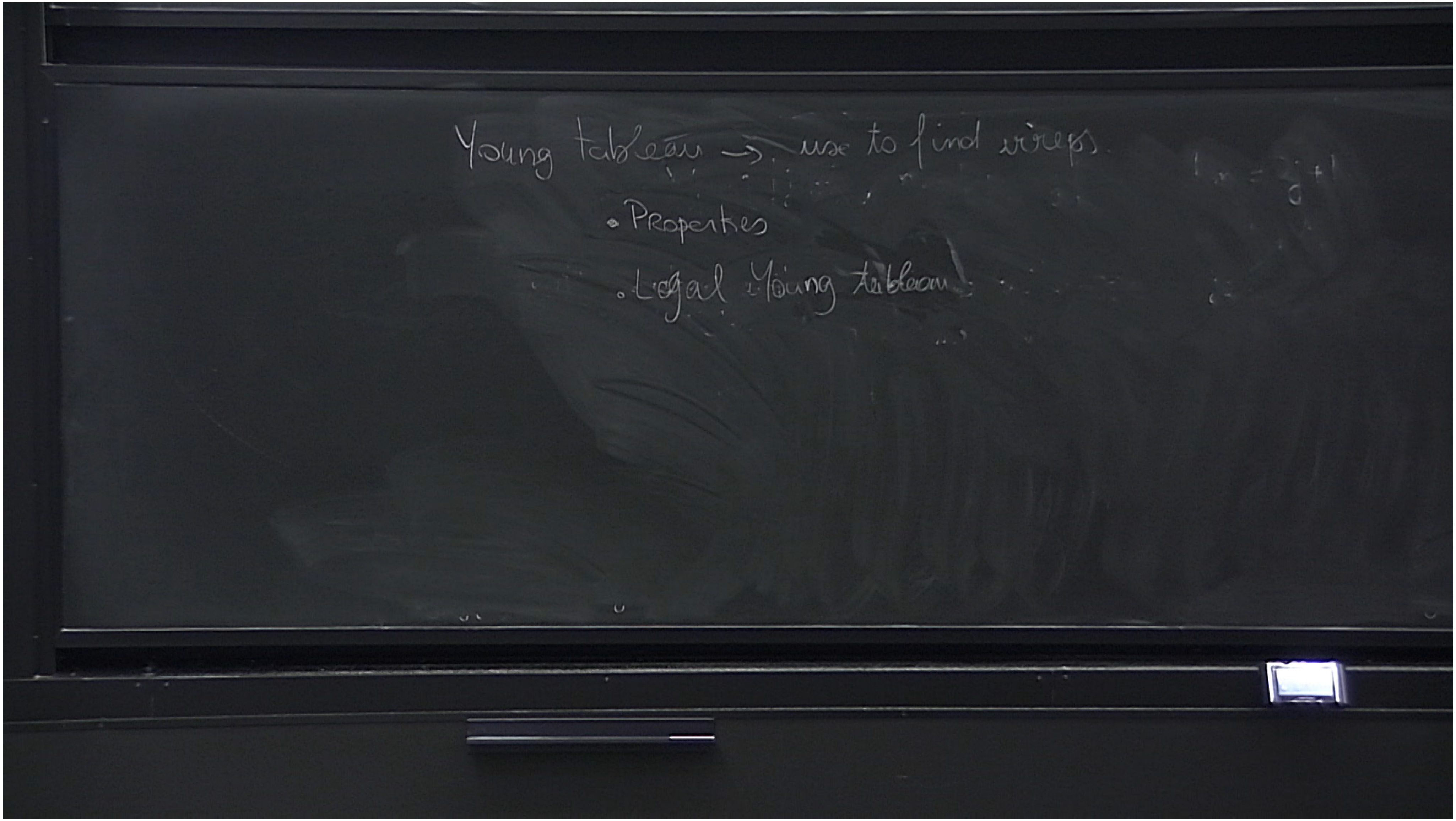
$\Psi_{ij} \quad 3\times 3 = 6 + 3$
Young tableau.
. fundament rep $\Psi_i \equiv \boxed{i}$
. operation of symmetrization $\Psi_{(ij)} \equiv \boxed{i \,|\, j} = \frac{1}{2}(1+P_{12})\Psi_{ij}$
. operation antisymmetrization $\Psi_{[ij]} = \frac{1}{2}(1-P_{12})\Psi_{ij}$

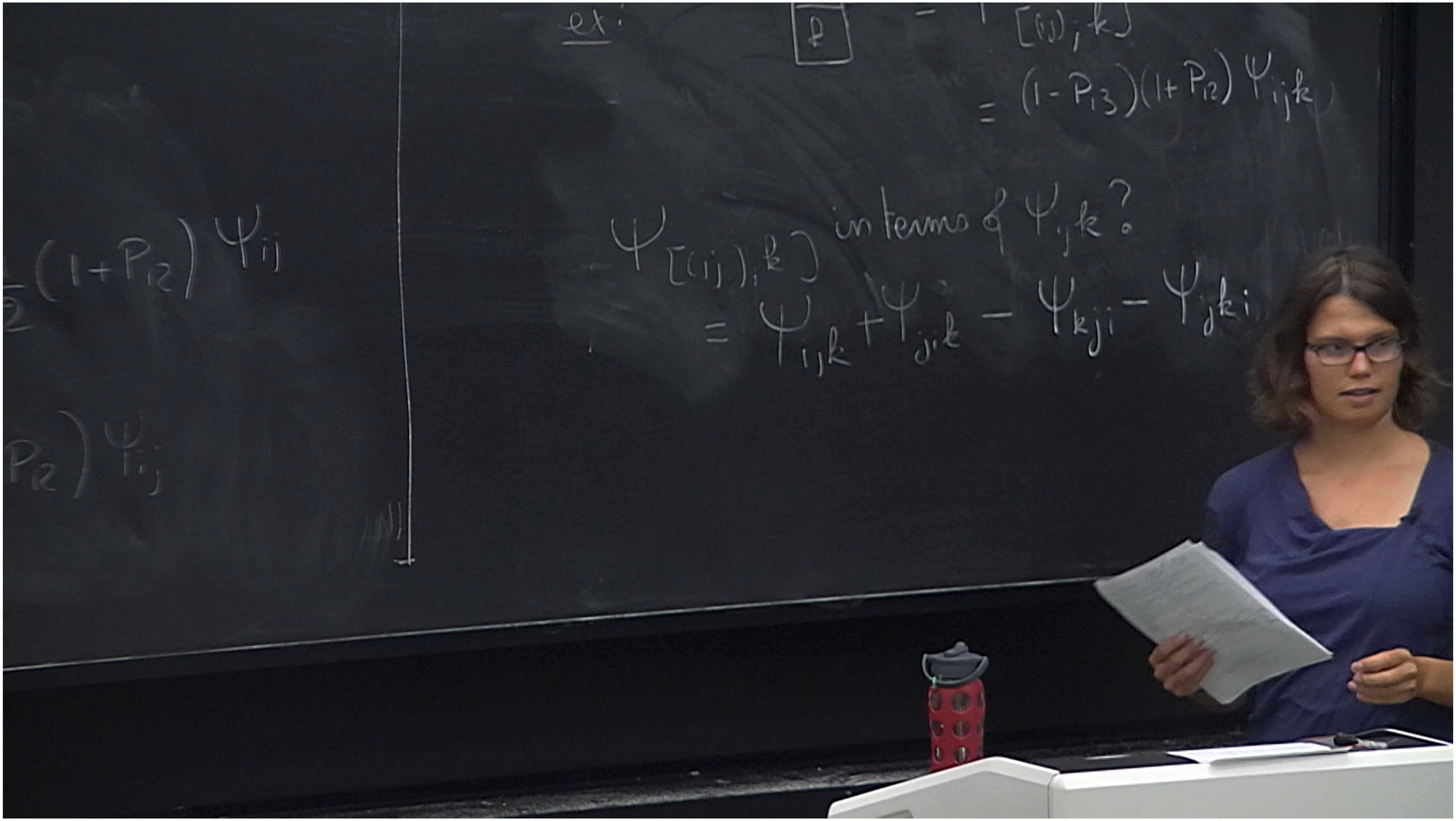

, εijk invariant under SU(3)
ex:
i j
k
= Ψ[(ij);k]
(1+P12) Ψij
P12) Ψij



, εijk invariant under SU(3)
ex:
= Ψ[(ij);k]
= (1-P13)(1+P12) Ψijk
(1+P12) Ψij
P12) Ψij


ex:
= (1 - P13)(1+ P12) Ψijk
Ψ[(ij);k] in terms of Ψijk?
(1+P12) Ψij
P12) Ψij


Young tableau → use to find irreps.
• Properties
• Legal Young tableau


ex:
$= (1 - P_{13})(1 + P_{12}) \Psi_{ijk}$
$\Psi_{[(ij);k]}$ in terms of Ψ_{ijk} ?
$= \Psi_{ijk} + \Psi_{jik} - \Psi_{kji} - \Psi_{jki}$

$\frac{}{2}(1+P_{12})\,\Psi_{ij}$

$P_{12})\,\Psi_{ij}$

ex:

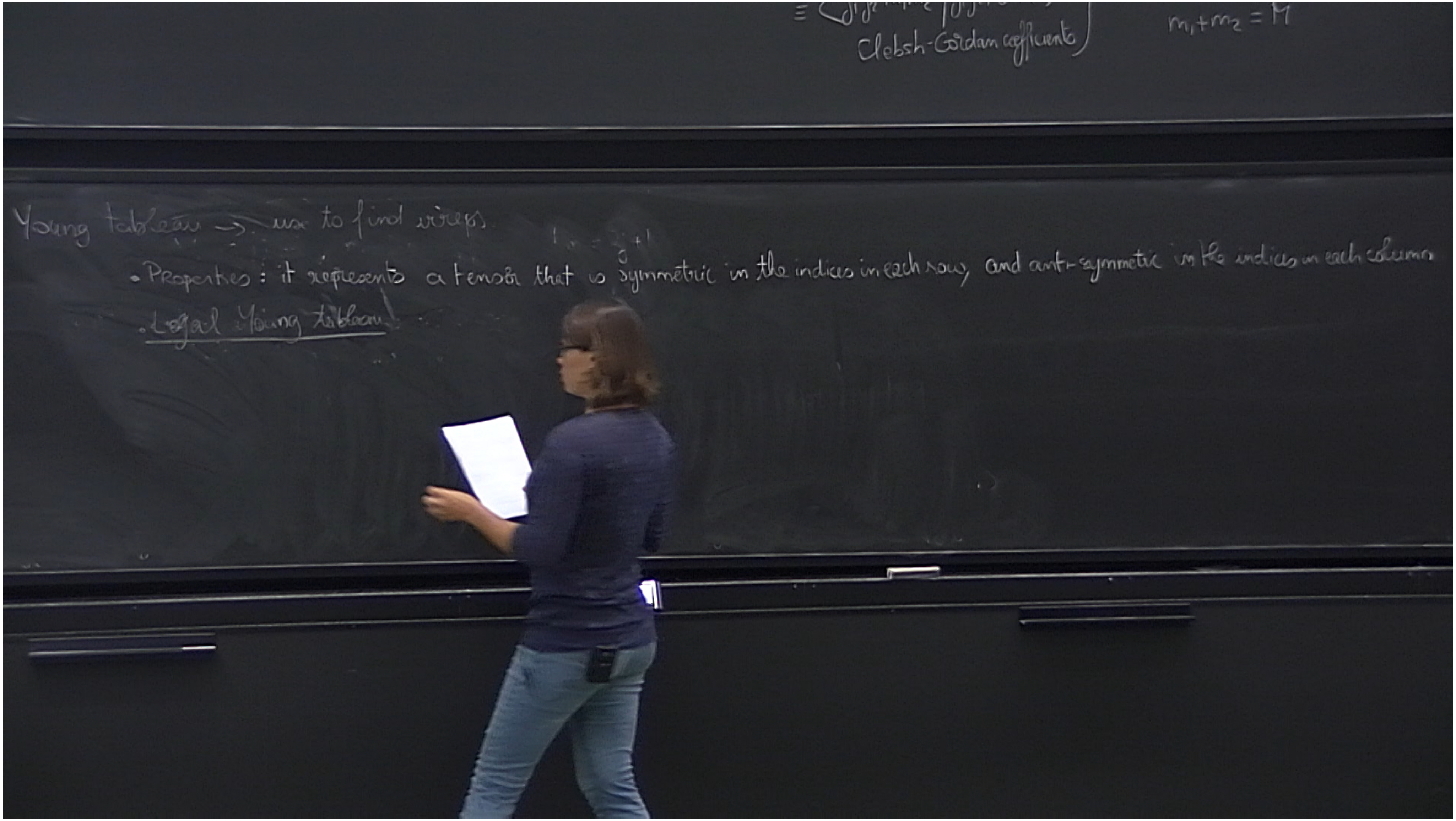
$$\begin{array}{|c|c|}\hline i & j \\ \hline k & \\ \cline{1-1}\end{array} = \Psi_{[(ij);k]}$$

$$= (1-P_{13})(1+P_{12})\,\Psi_{ijk}$$

$\Psi_{[(ij);k]}$ in terms of Ψ_{ijk}?

$$= \Psi_{ijk} + \Psi_{jik} - \Psi_{kji} - \Psi_{jki}$$

$$- \Psi_{kij}$$


Clebsh-Gordan coefficients
m1+m2 = M
Young tableau → use to find irreps
• Properties: it represents a tensor that is symmetric in the indices in each row, and anti-symmetric in the indices in each column
• Legal Young tableau

Young tableau → use to find irreps.

- Properties: it represents a tensor that is symmetric in the indices in each row, and
- Legal Young tableau:

  a) Lengths of rows do not increase from top to bottom

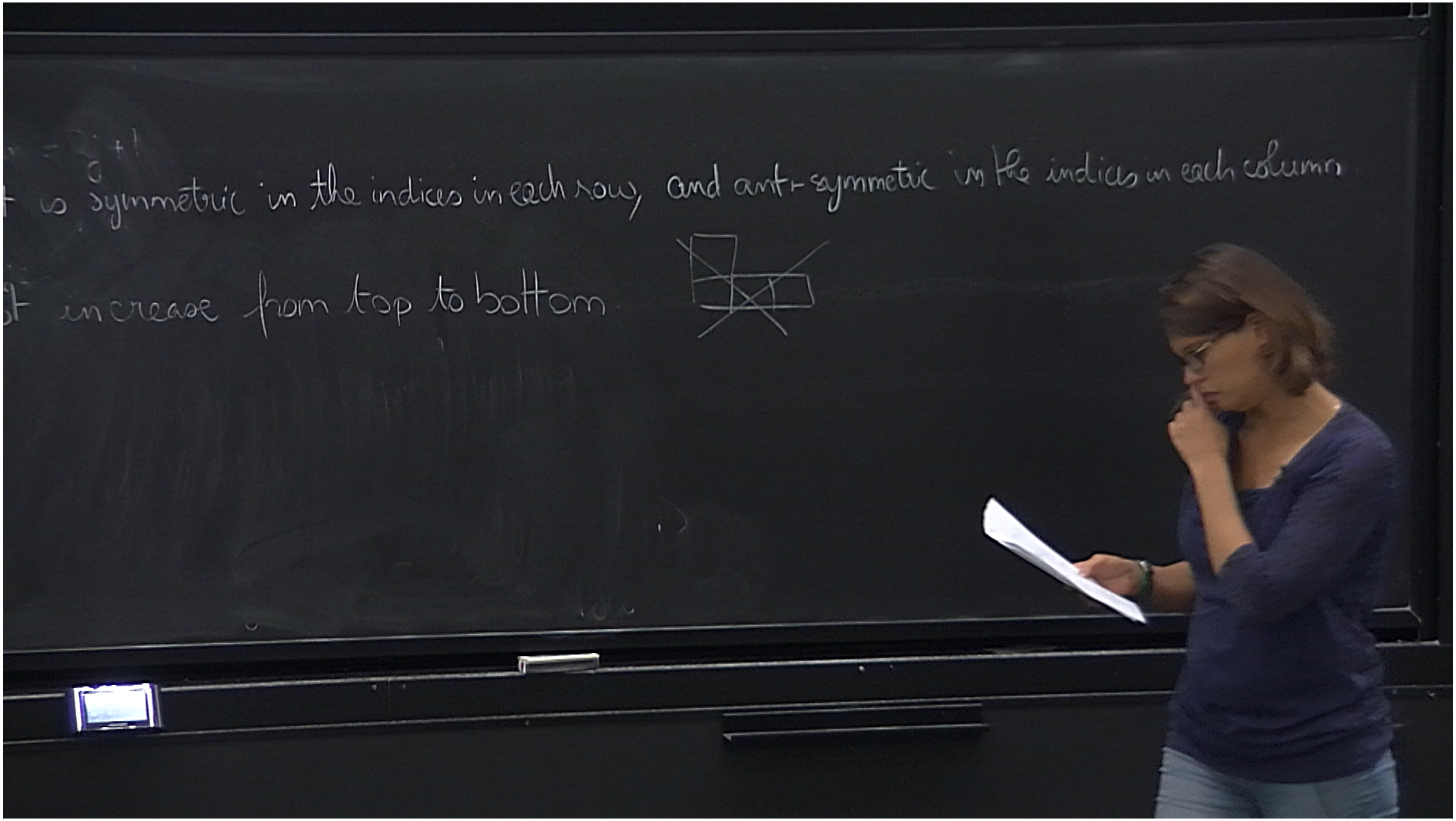

is symmetric in the indices in each row, and anti-symmetric in the indices in each column
increase from top to bottom

tableau → use to find irreps.

- Properties: it represents a tensor that is symmetric in the indices in each row, and anti-sym

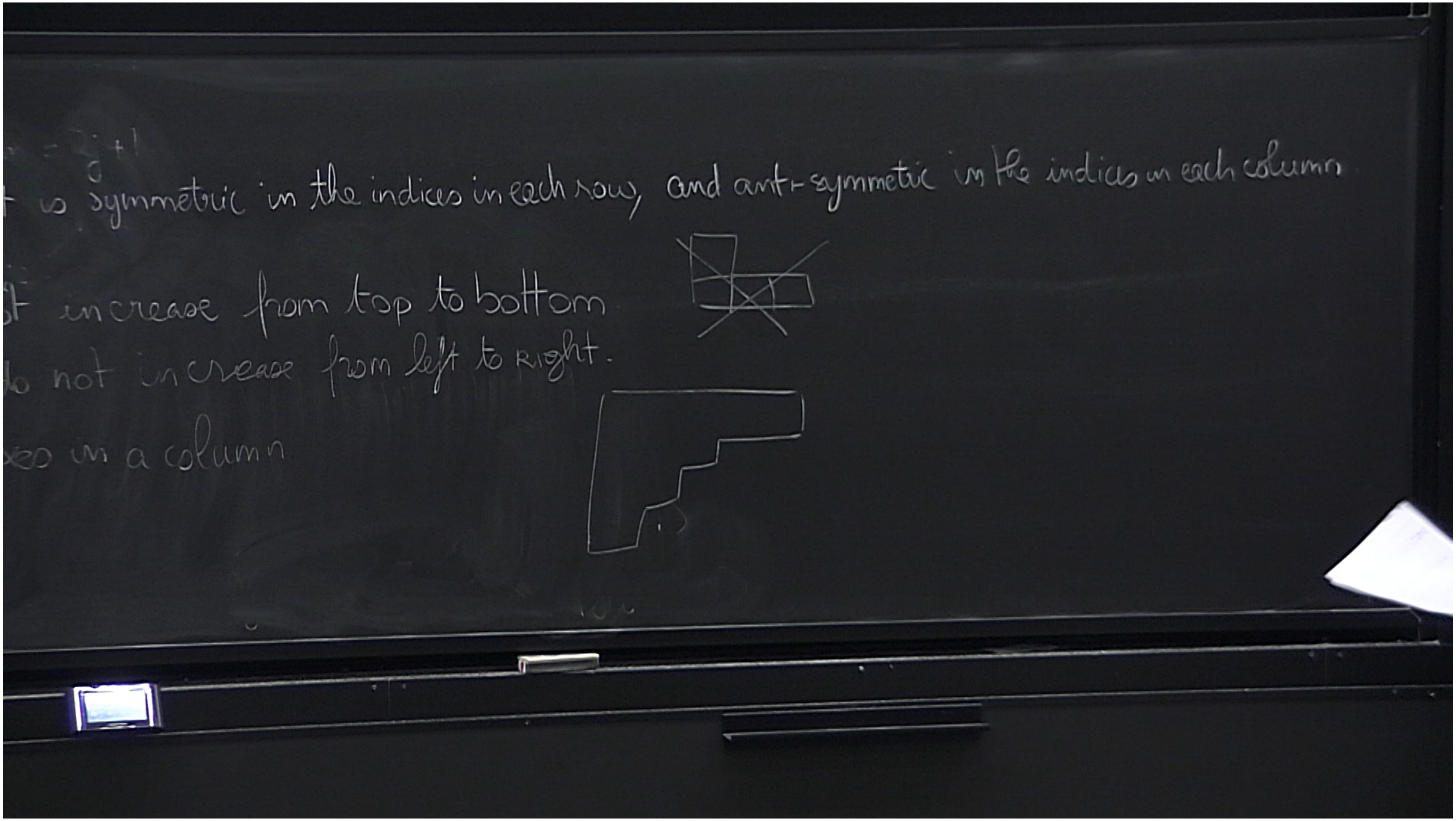
- Legal Young tableau:

SU(N)

a) Lengths of rows do not increase from top to bottom

b) Lengths of columns do not increase from left to right.

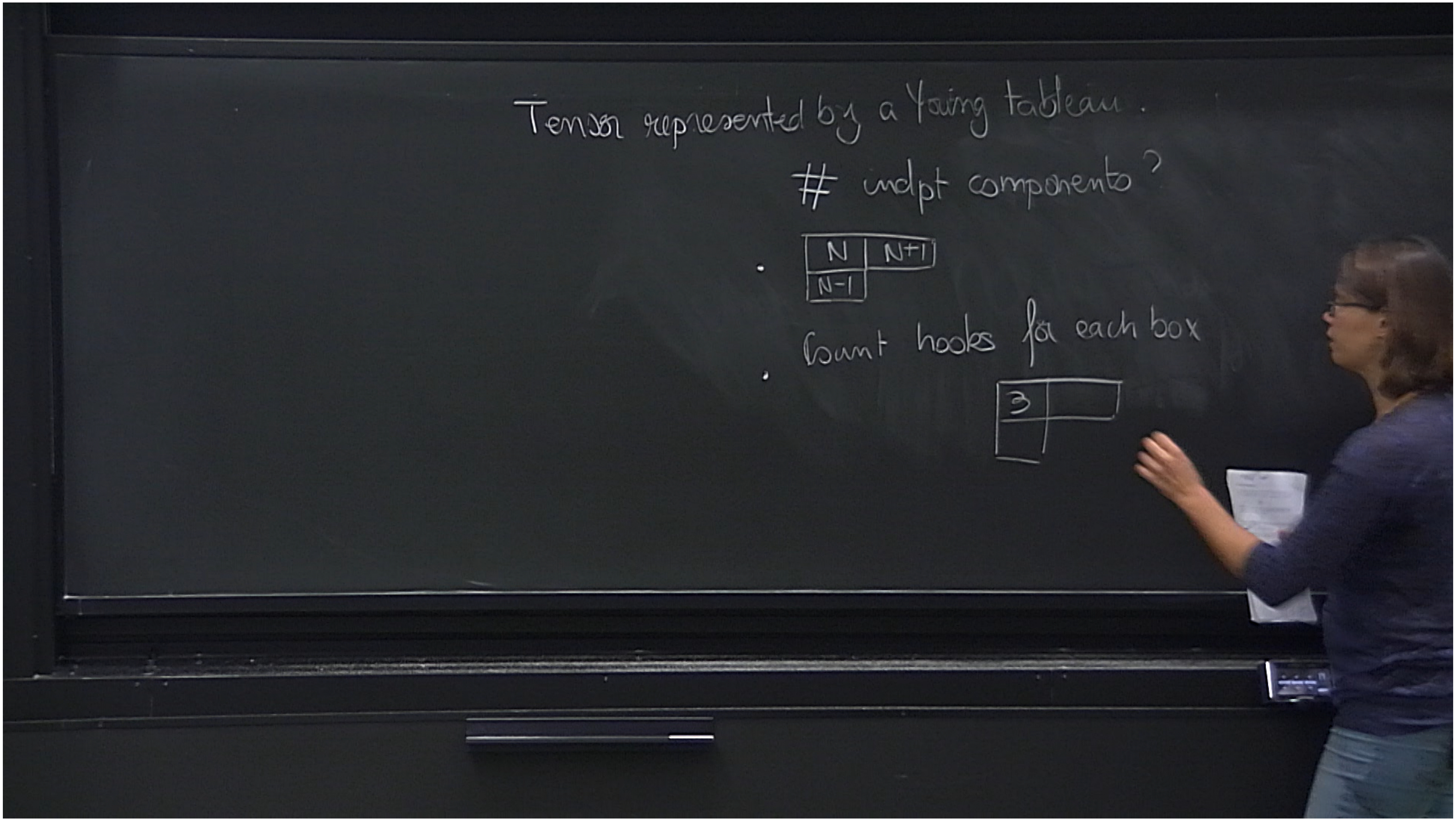
c) No more than N boxes in a column

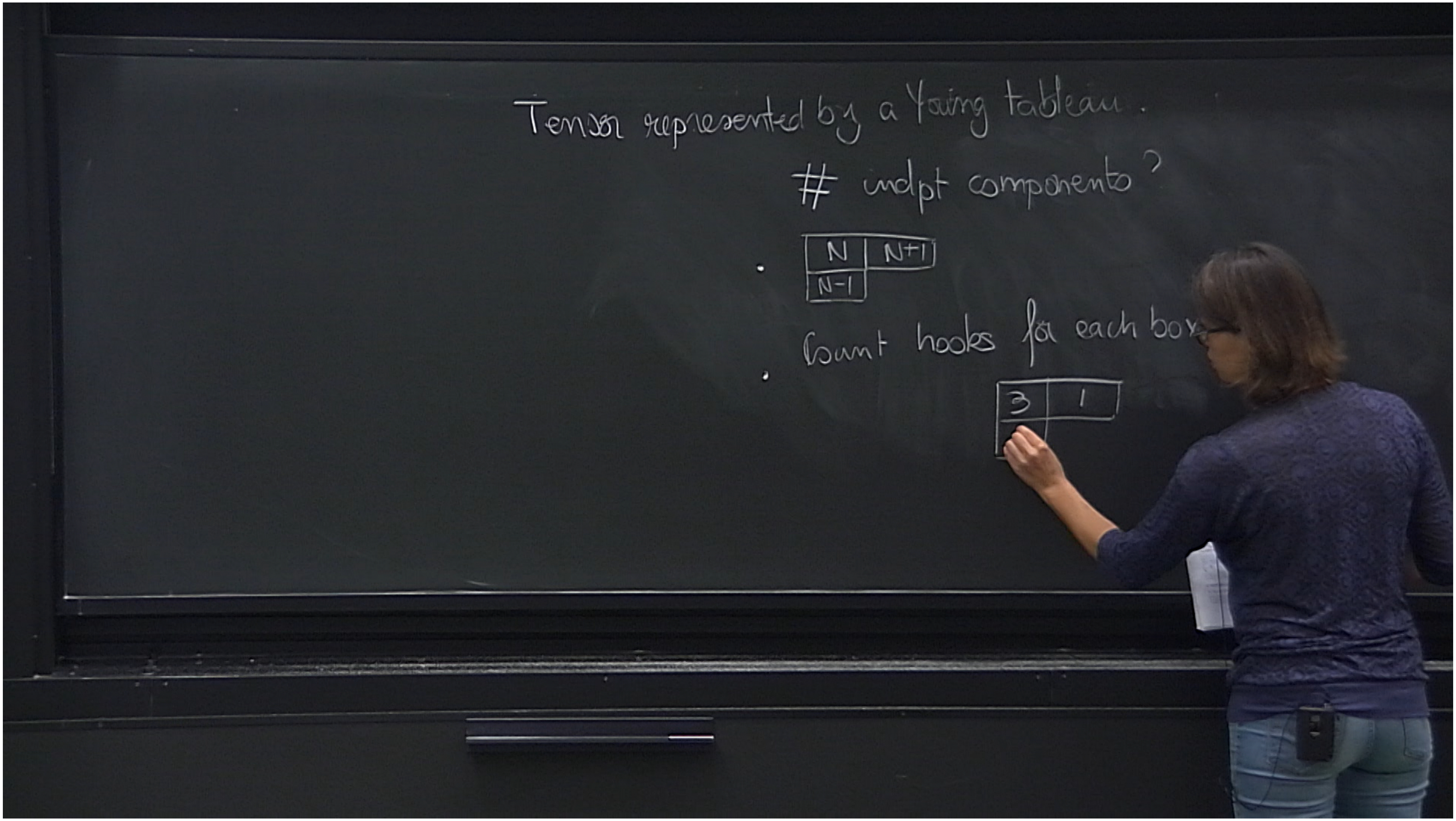

is symmetric in the indices in each row, and anti-symmetric in the indices in each column
increase from top to bottom
not increase from left to right.
in a column

Tensor represented by a Young tableau.

indpt components ?

N	N+1
N-1	


Tensor represented by a Young tableau.
indpt components?
N
N+1
N-1
Count hooks for each box
3


Tensor represented by a Young tableau.
indpt components?
N
N+1
N-1
Count hooks for each box
3
1

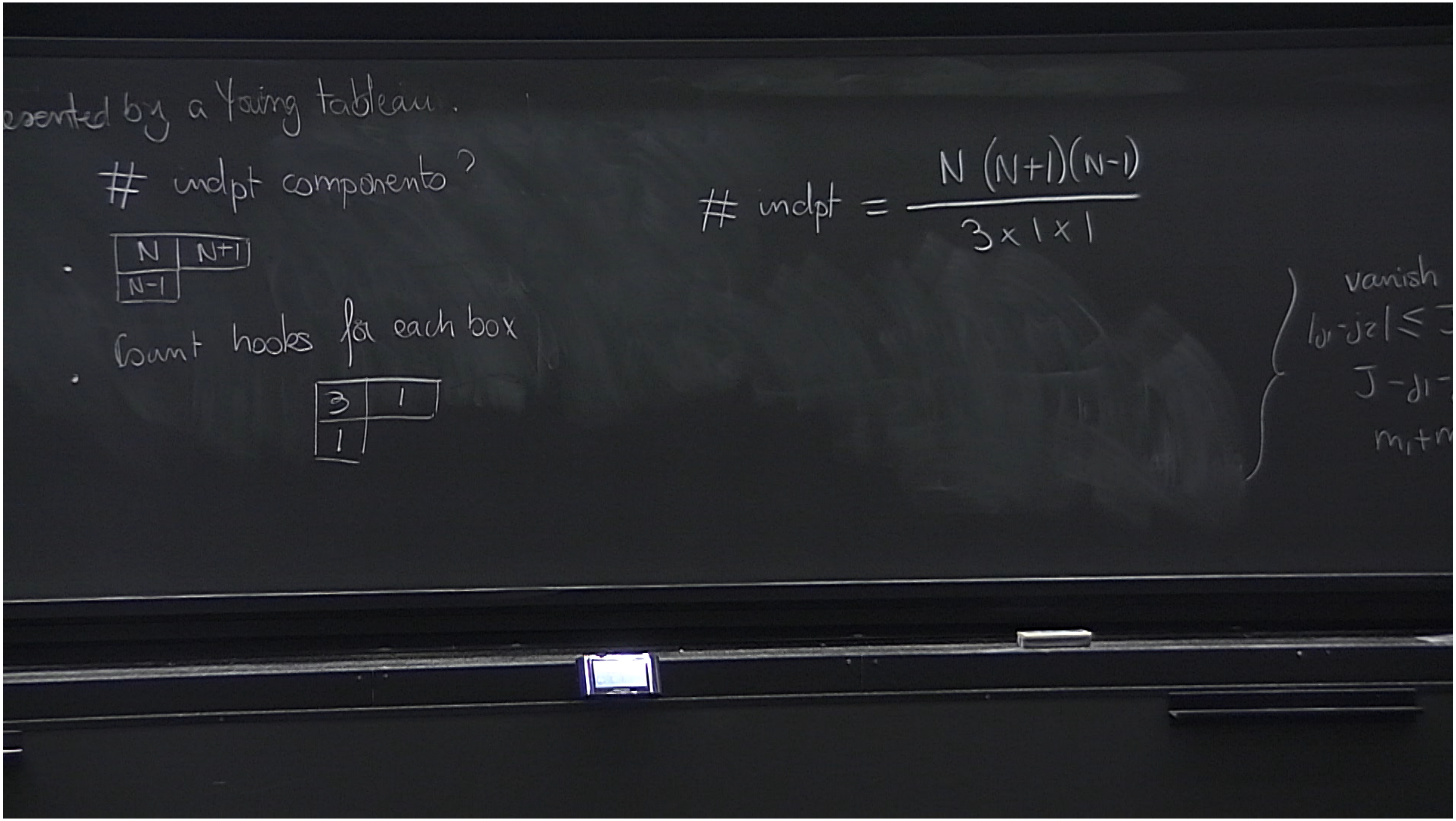
Tensor represented by a Young tableau.

\# indpt components?

- $\begin{array}{|c|c|} \hline N & N+1 \\ \hline N-1 & \\ \cline{1-1} \end{array}$

- Count hooks for each box

$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & \\ \cline{1-1} \end{array}$


esented by a Young tableau.
indpt components?
N | N+1
N-1
Count hooks for each box
3 | 1
1
indpt = N (N+1)(N-1) / 3 × 1 × 1
vanish

esented by a Young tableau.

$\#$ indpt components?

- $\begin{array}{|c|c|}\hline N & N+1 \\ \hline N-1 & \\ \cline{1-1}\end{array}$

- Count hooks for each box

$\begin{array}{|c|c|}\hline 3 & 1 \\ \hline 1 & \\ \cline{1-1}\end{array}$

$$\# \text{ indpt} = \frac{N(N+1)(N-1)}{3 \times 1 \times 1}$$

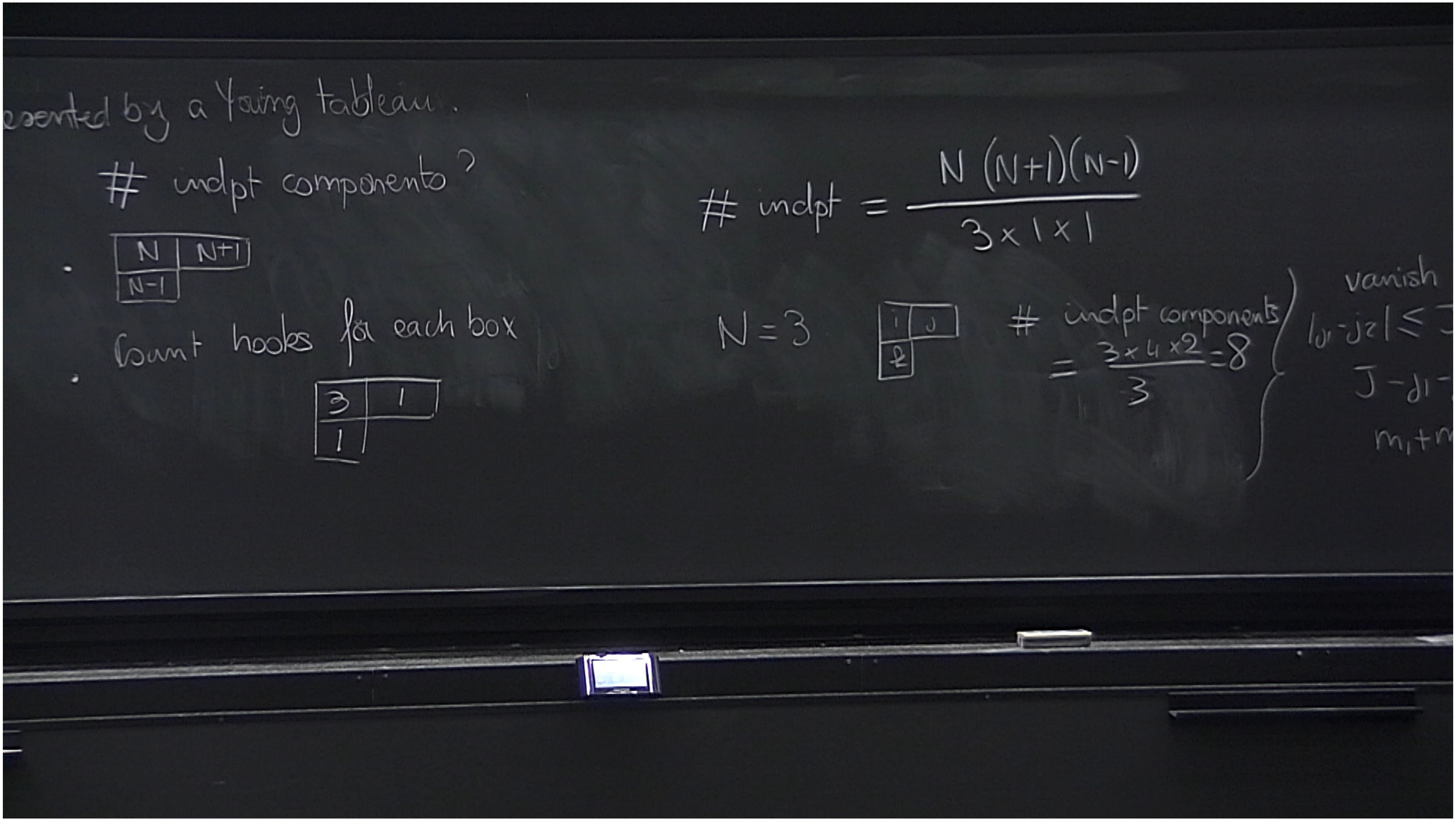
$N = 3$

vanish

$|j_1 - j_2| \leq J$

$J - j_1 -$

$m_1 + m$


esented by a Young tableau.
indpt components?
N | N+1
N-1
Count hooks for each box
3 | 1
1
# indpt = N (N+1)(N-1) / 3 × 1 × 1
N = 3
# indpt components = 3 × 4 × 2 / 3 = 8
vanish

...esented by a Young tableau.

indpt components?

-

N	$N+1$
$N-1$	

- Count hooks for each box

3	1
1	

$$\# \text{ indpt} = \frac{N(N+1)(N-1)}{3\times 1\times 1}$$

$N=3$

i	j
k	

$$\# \text{ indpt components} = \frac{3\times 4\times 2}{3} = 8$$

$= \text{dim of } SU(3)$

see tutorial

$$\# \text{ indpt} = \frac{N(N+1)(N-1)}{3\times 1\times 1}$$

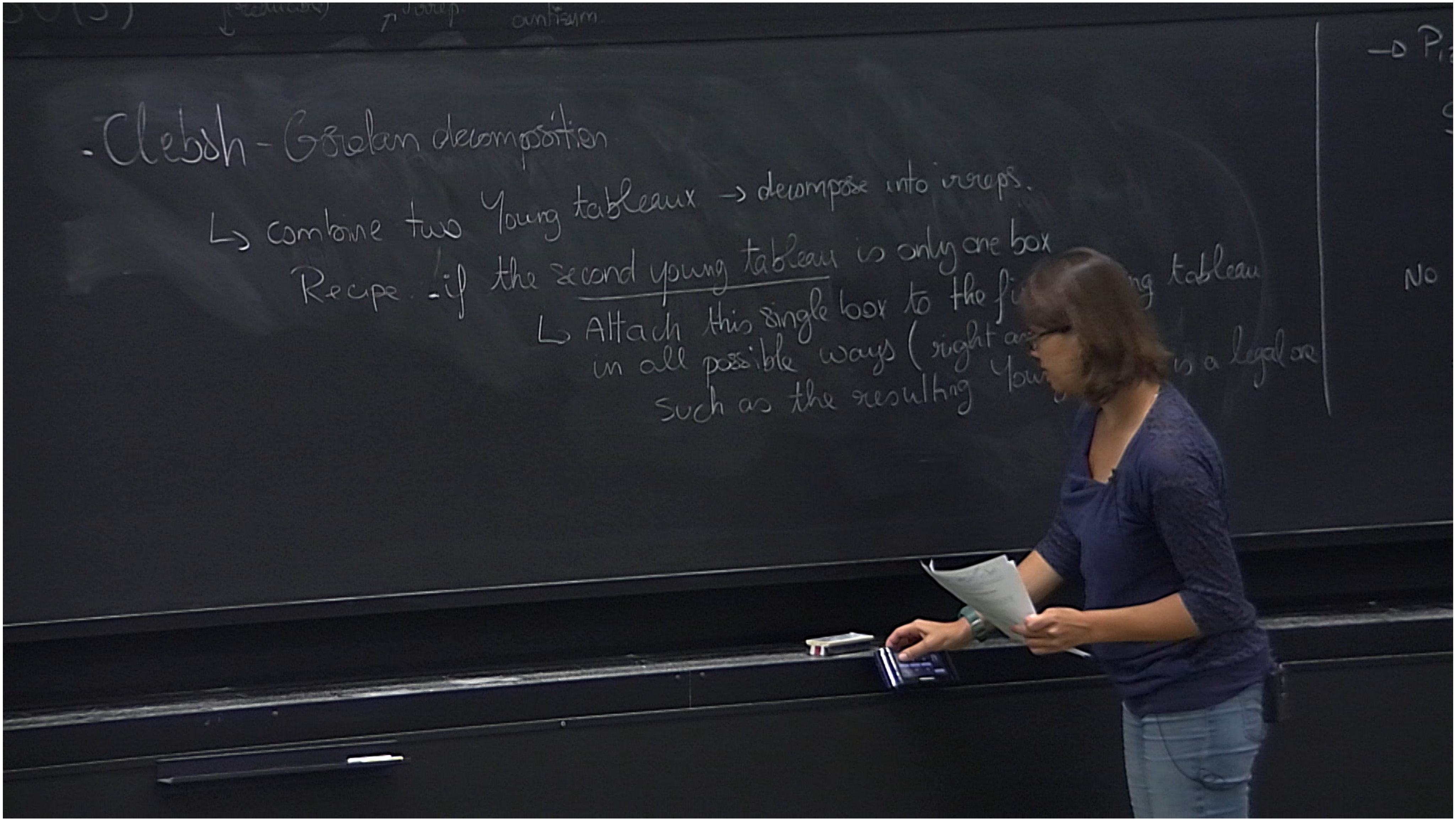
$N = 3$

$$\begin{array}{|c|c|}\hline i & j \\ \hline k & \\ \cline{1-1}\end{array}$$

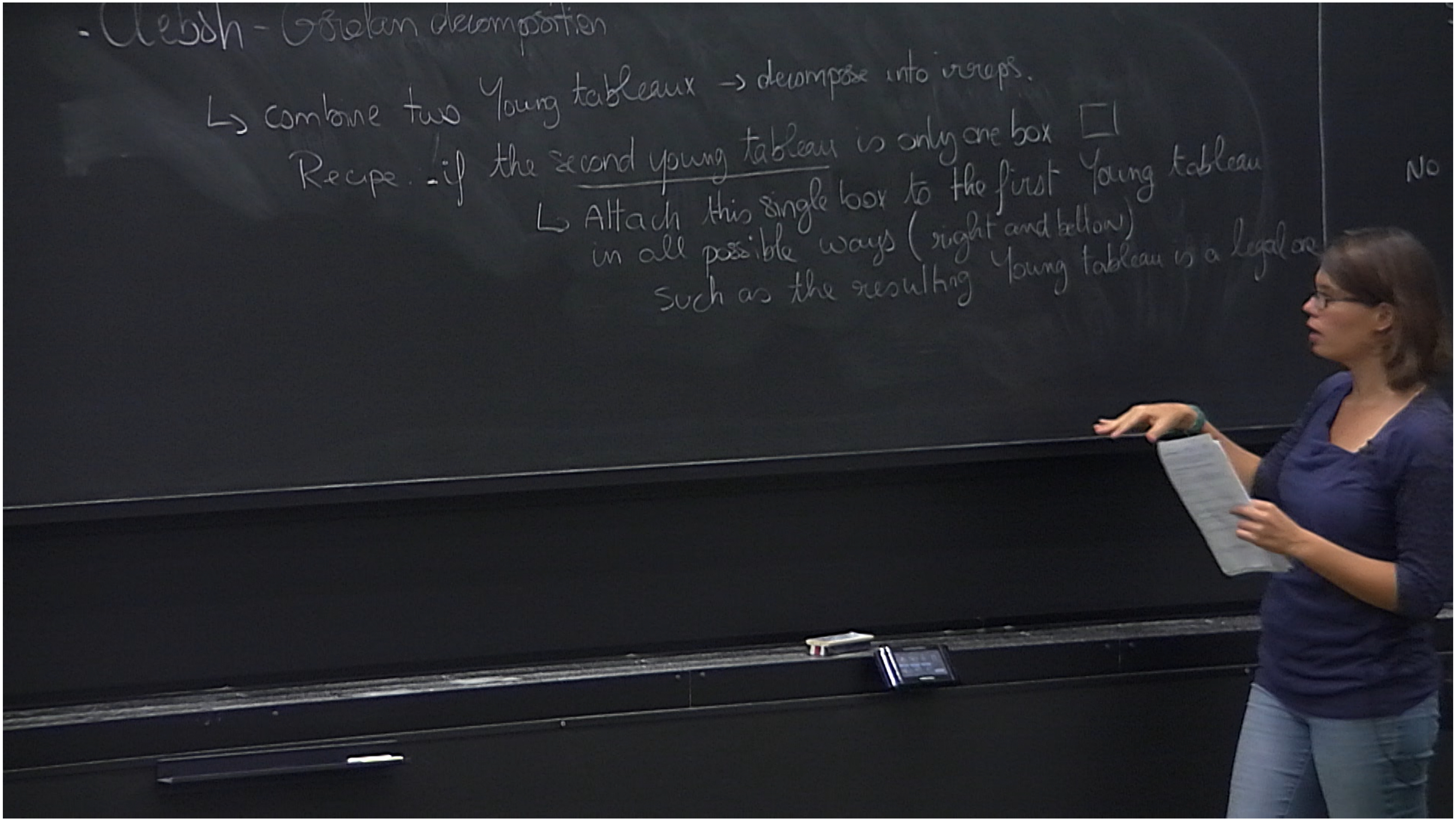
$\#$ indpt components

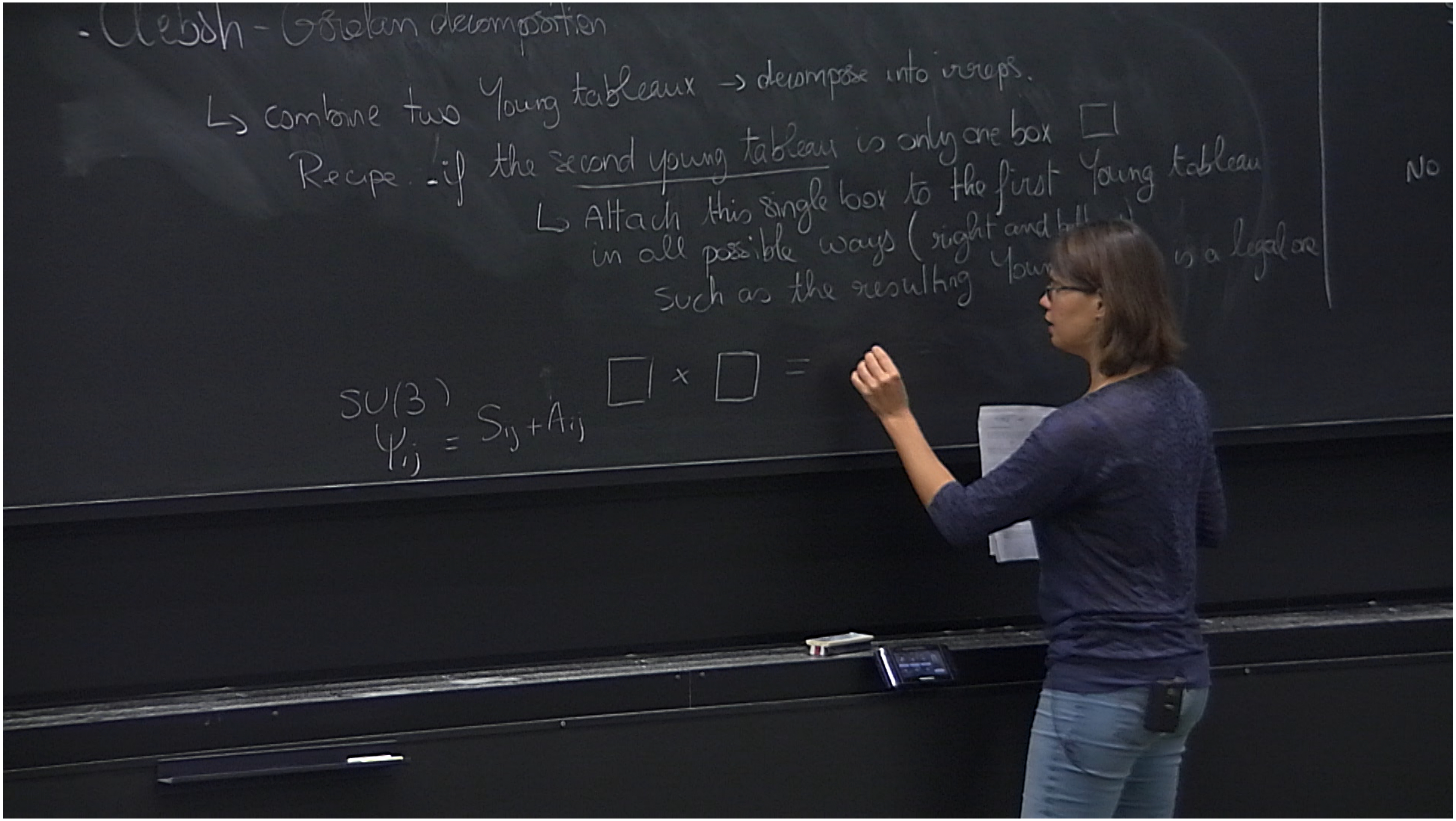
$$= \frac{3\times 4\times 2}{3} = 8$$

$= \dim \text{ of } SU(3)$



- Clebsh - Gordan decomposition
↳ combine two Young tableaux → decompose into irreps.
Recipe: - if the second young tableau is only one box
↳ Attach this single box to the
tableau
in all possible ways (right
is a legal one
such as the resulting
No


- Clebsh - Gordan decomposition
↳ combine two Young tableaux → decompose into irreps.
Recipe: - if the second young tableau is only one box □
↳ Attach this single box to the first Young tableau
in all possible ways (right and bellow)
such as the resulting Young tableau is a legal on
No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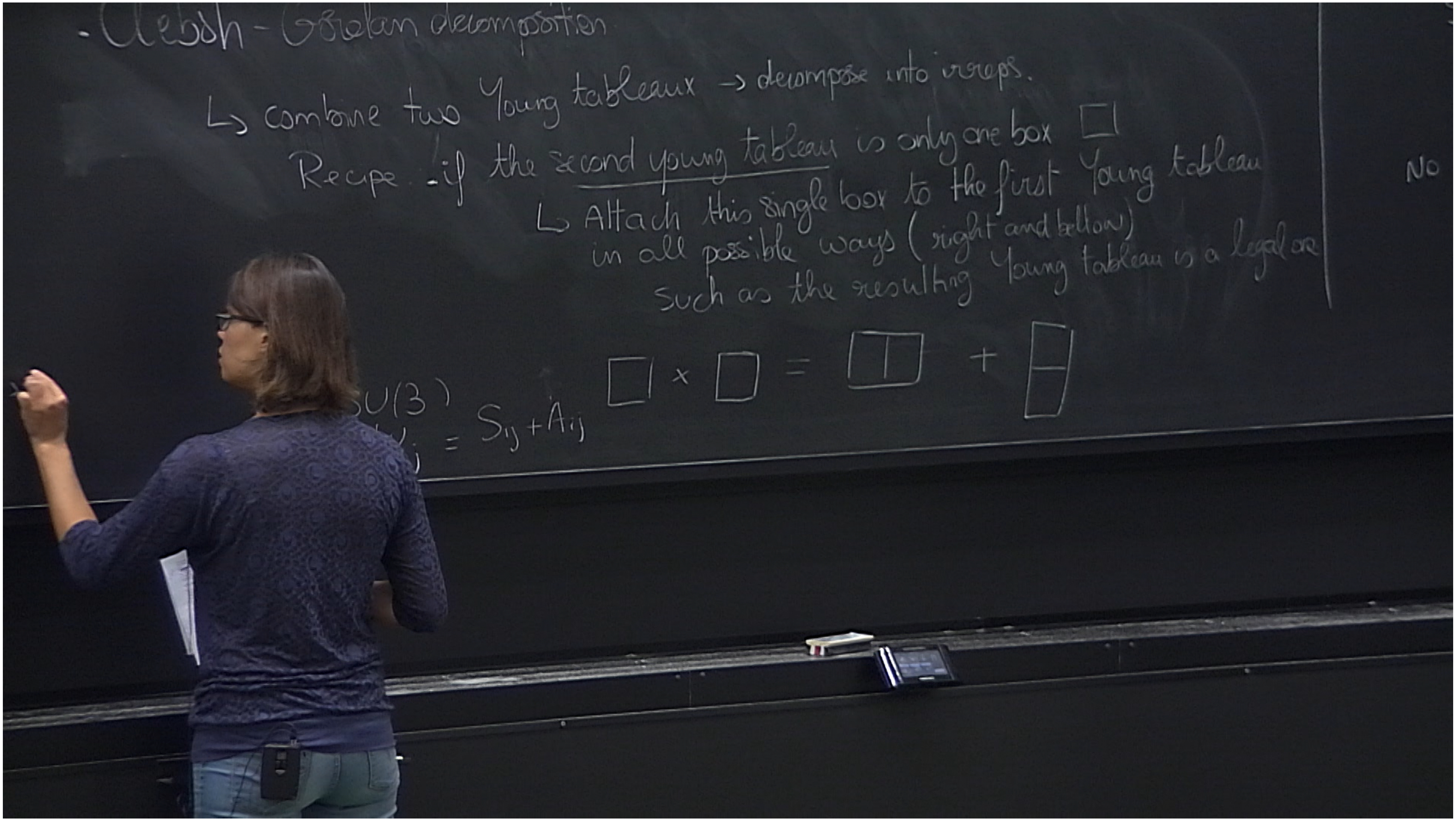
is a legal one
No
SU(3)
$\Psi_{ij} = S_{ij} + A_{ij}$


- Clebsh - Gordan decomposition
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Recipe. - if the second young tableau is only one box
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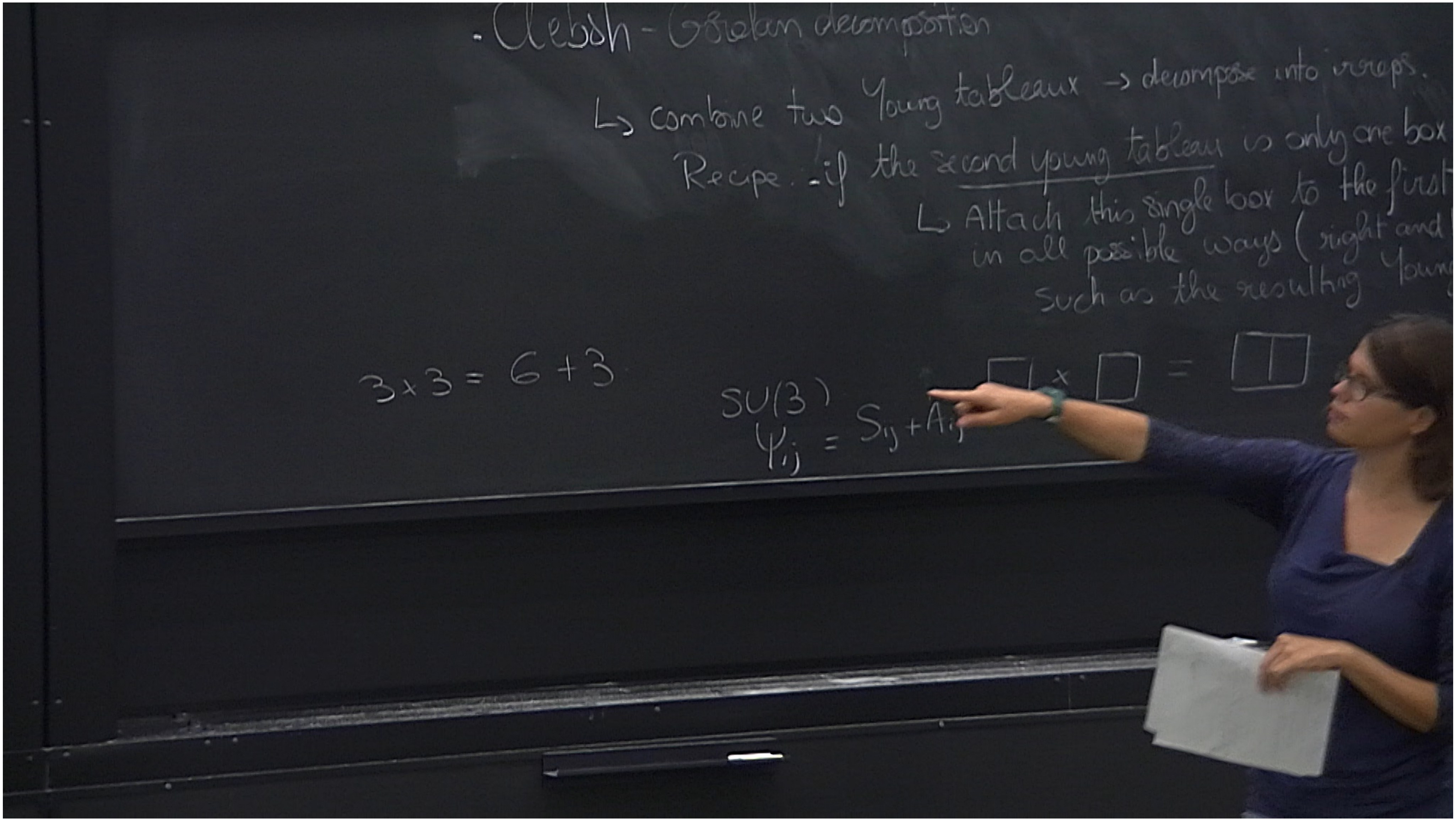
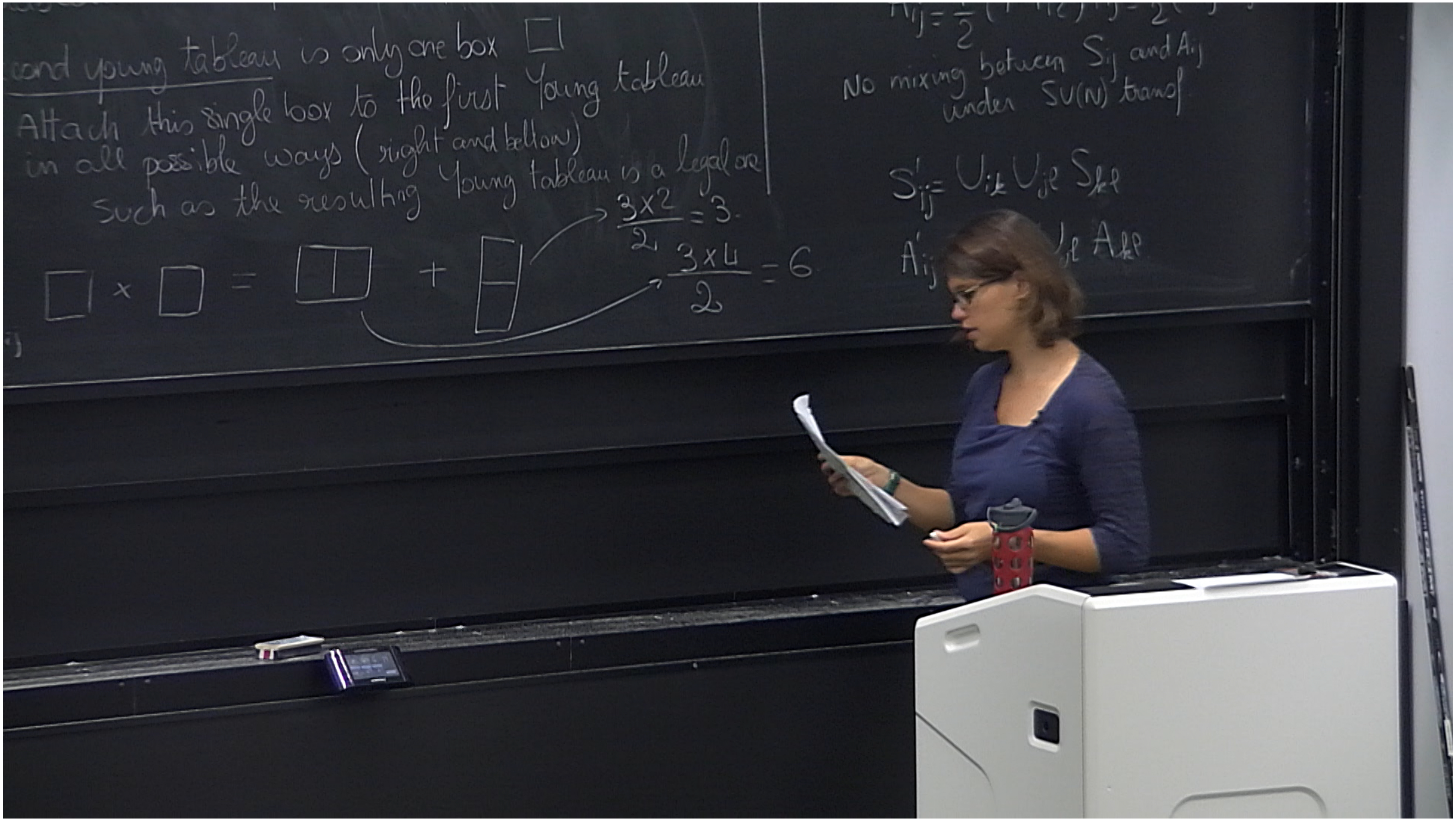
SU(3)
= S_ij + A_ij
No


Clebsh - Gordan decomposition
↳ combine two Young tableaux → decompose into irreps.
Recipe: - if the second young tableau is only one box
↳ Attach this single box to the first
in all possible ways (right and
such as the resulting
3×3 = 6 + 3
SU(3)
ψij = Sij + Aij



cond young tableau is only one box
Attach this single box to the first Young tableau
in all possible ways (right and bellow)
such as the resulting Young tableau is a legal one
3x2/2 = 3
3x4/2 = 6
No mixing between Sij and Aij
under SU(N) transf
S'ij = Uik Ujl Skl