Title: PSI 17/18 Front End - Theoretical Mechanics (Kubiznak)

Date: Aug 17, 2017 09:00 AM

URL: http://pirsa.org/17080048

Abstract:

f) CONSTRAINTS

- LET NOT ALL q^i, p_i BE INDEPENDENT, THAT IS AT ANY INSTANT OF TIME WE HAVE A CONSTRAINT

$$\boxed{\phi(q^i, p_i) = 0}$$

- <u>REMARK</u>: ANY PHASE SPACE FUNCTION f CAN BE UNDERSTOOD AS "HAMILTONIAN" AND GENERATES THE CORRESPONDING "FLOW":

$$\frac{dq^i}{dt_f} = \{q^i, \overset{\text{"}H\text{"}}{f}\}$$

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$$0 = \{I, H\} = -\{H, I\}$$

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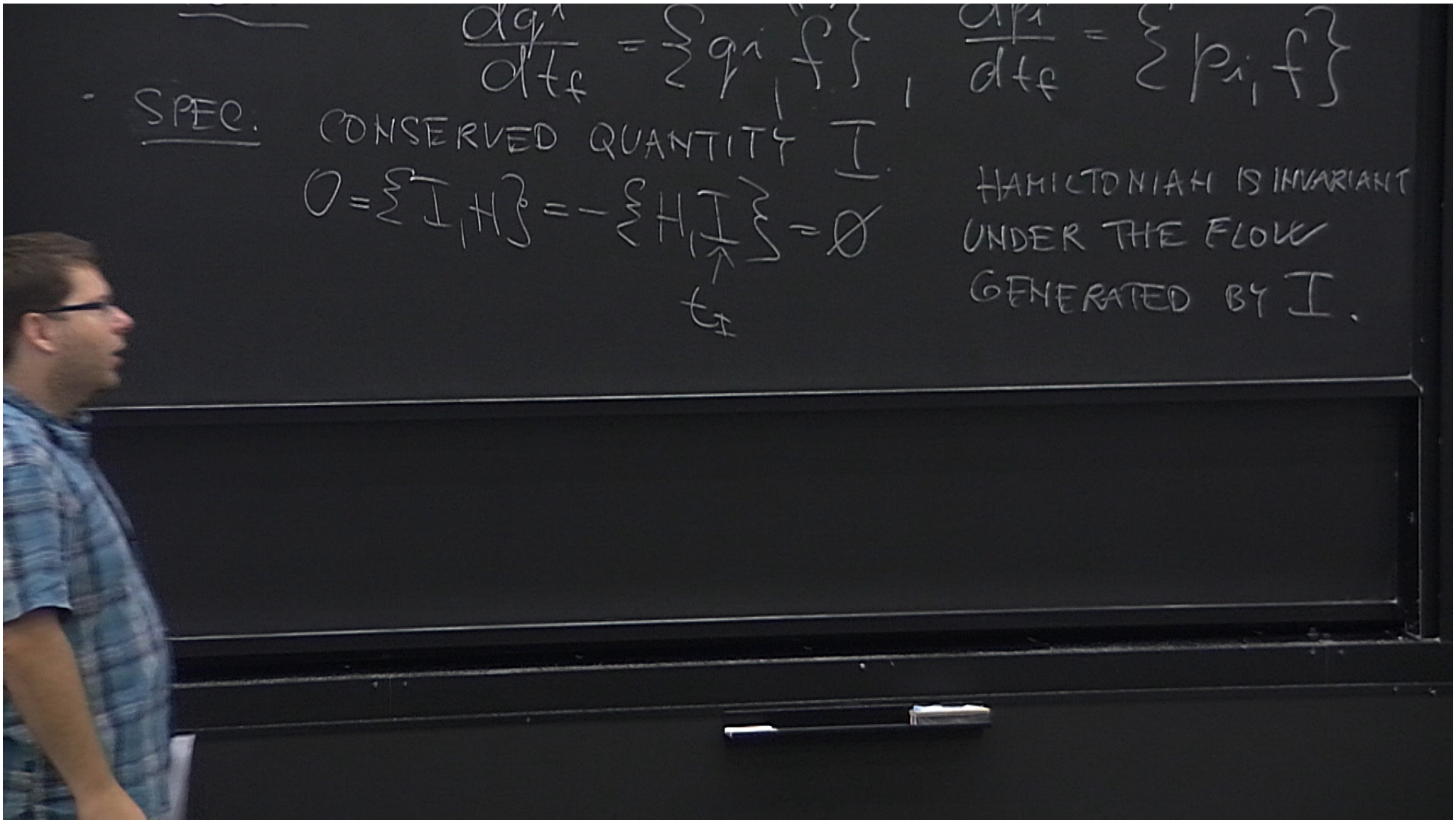
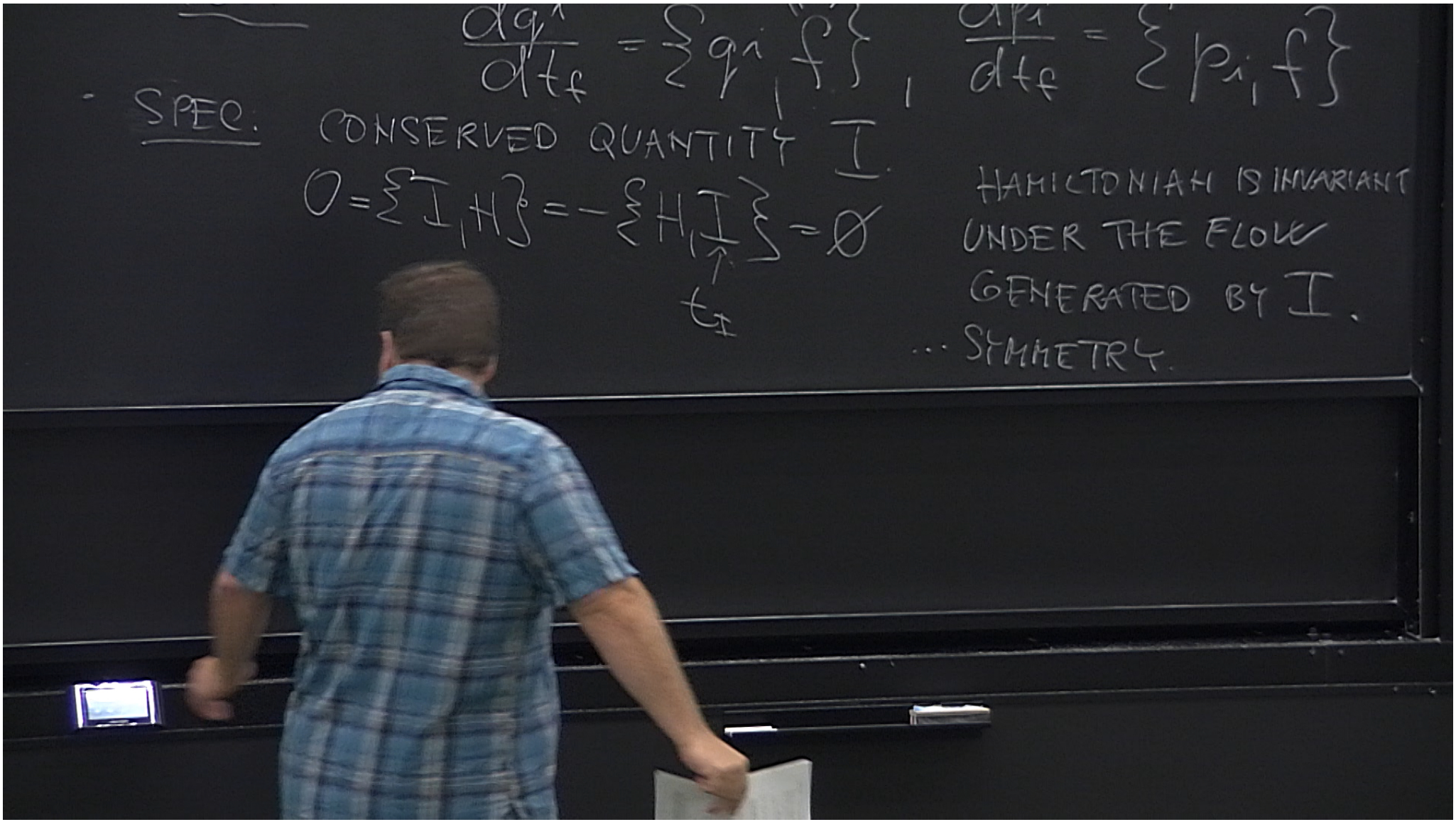
- SPEC. CONSERVED QUANTITY I.

$$0 = \{I, H\} = -\{H, \underset{t_I}{I}\} = \emptyset$$


$\frac{dq^i}{dt_f} = \{q^i, f\}$, $\frac{dp_i}{dt_f} = \{p_i, f\}$
SPEC. CONSERVED QUANTITY I.
$0 = \{I, H\} = -\{H, I\} = 0$
t_I
HAMILTONIAN IS INVARIANT UNDER THE FLOW GENERATED BY I.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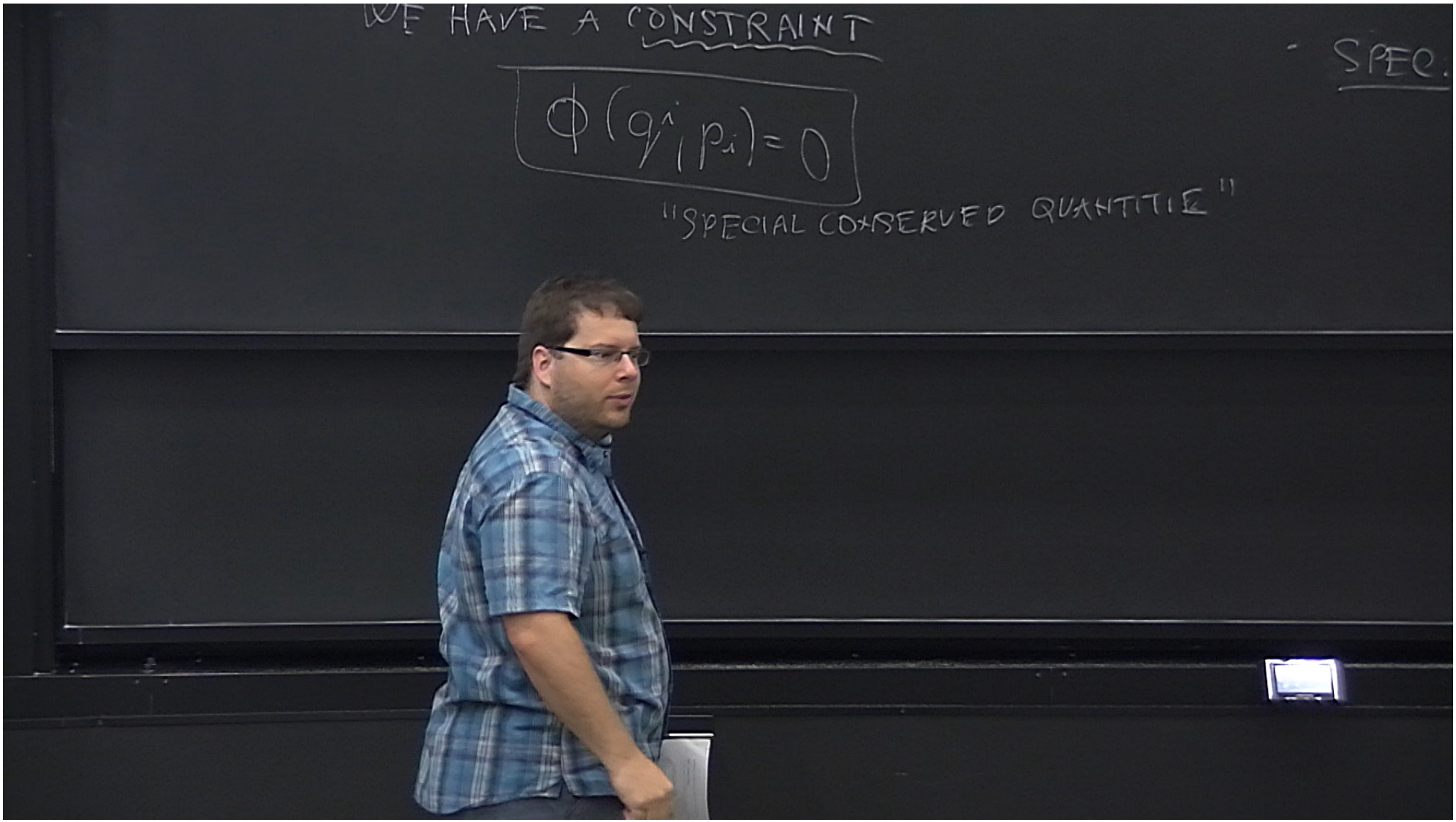
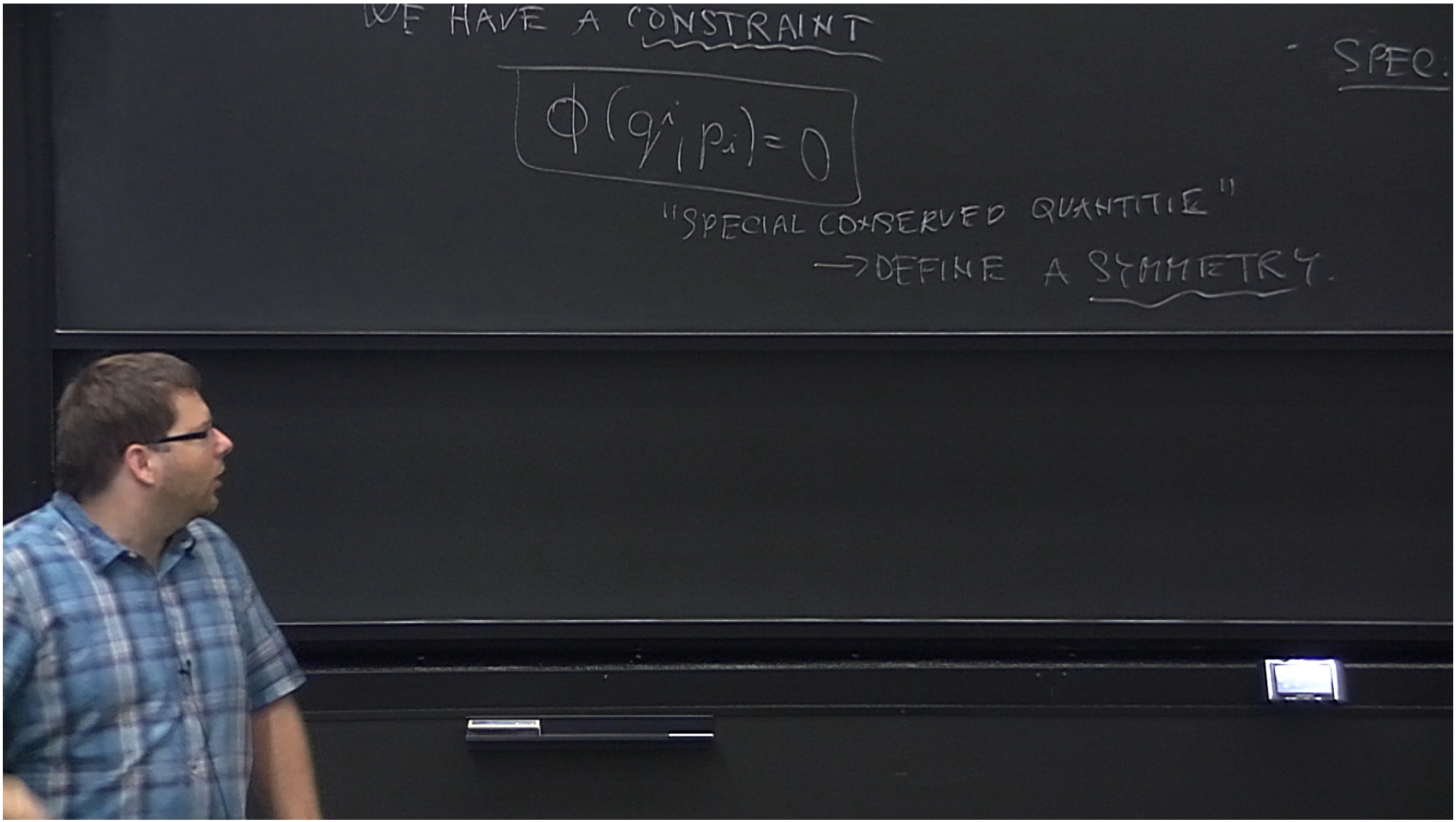
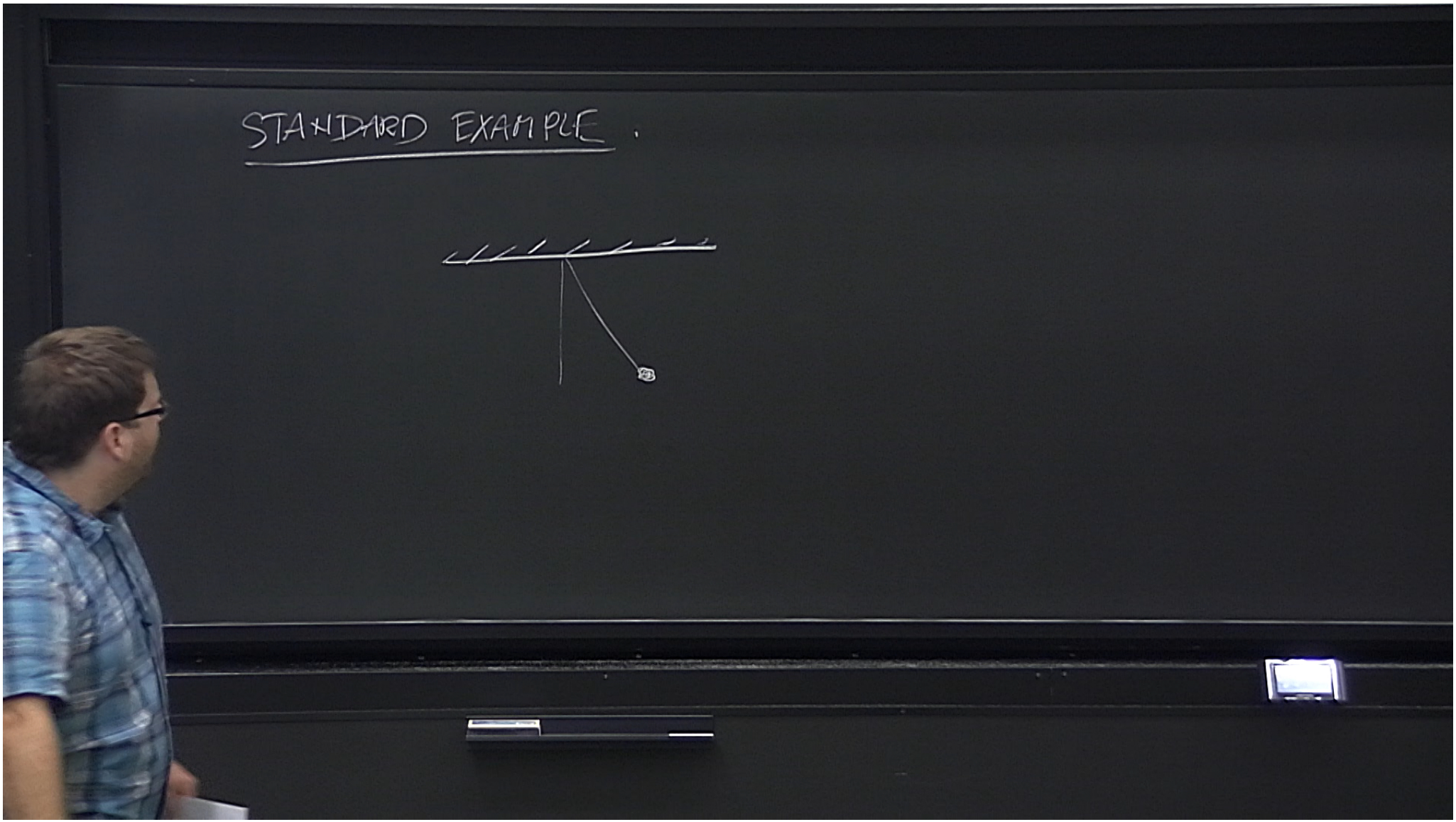
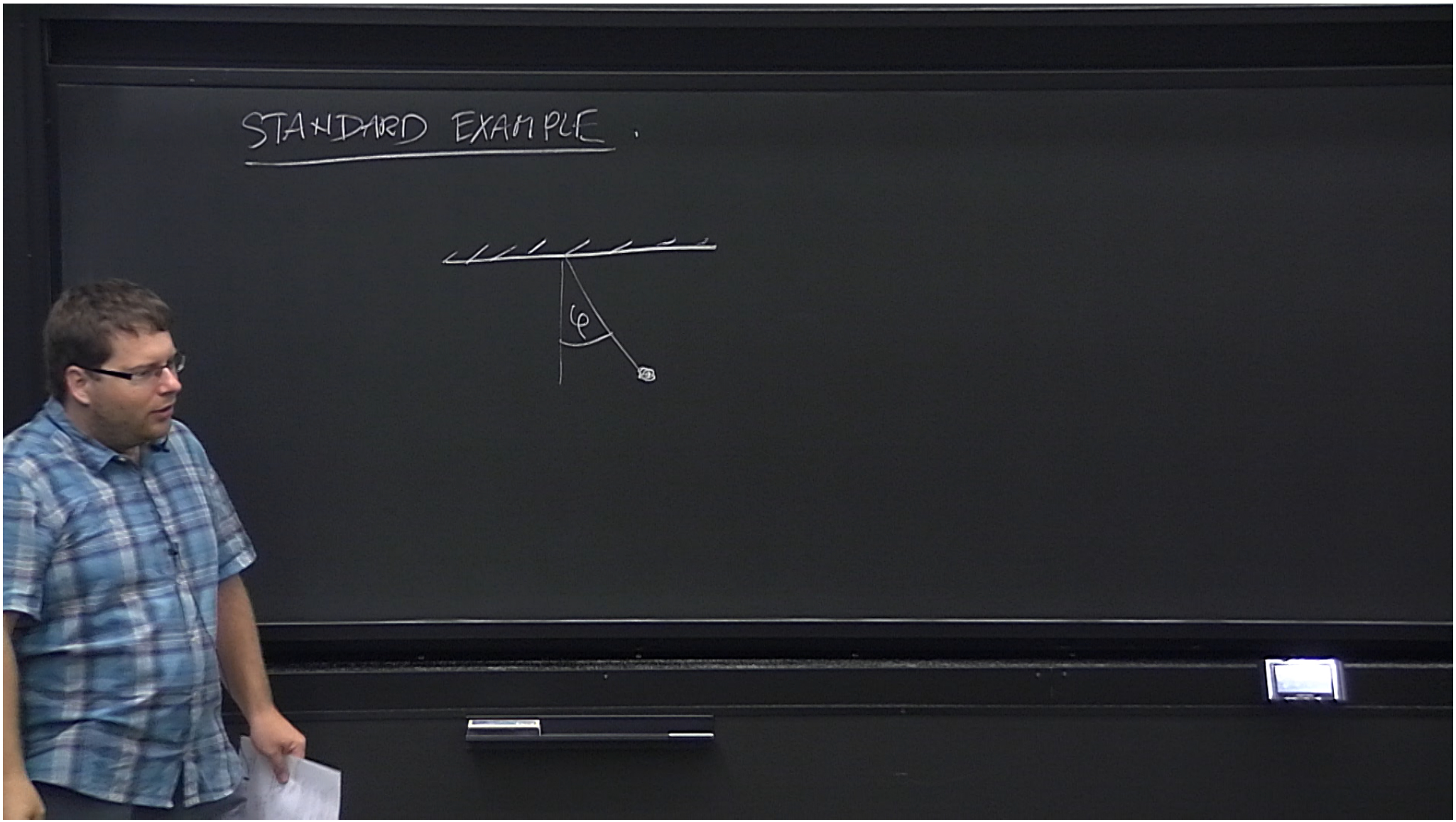
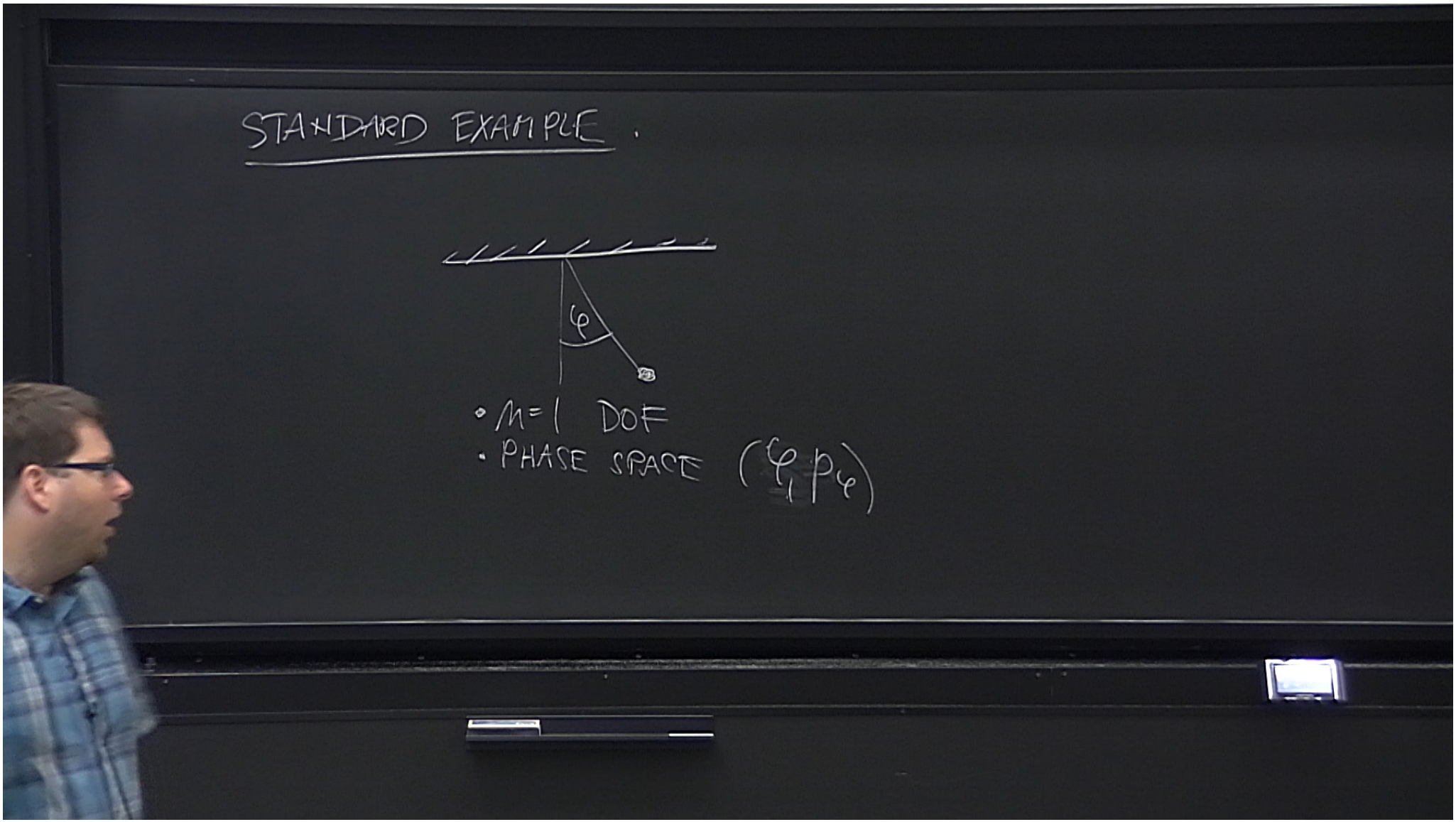
SPEC. CONSERVED QUANTITY I.
$0 = \{I, H\} = -\{H, I\} = 0$
t_I
HAMILTONIAN IS INVARIANT
UNDER THE FLOW
GENERATED BY I.
... SYMMETRY.

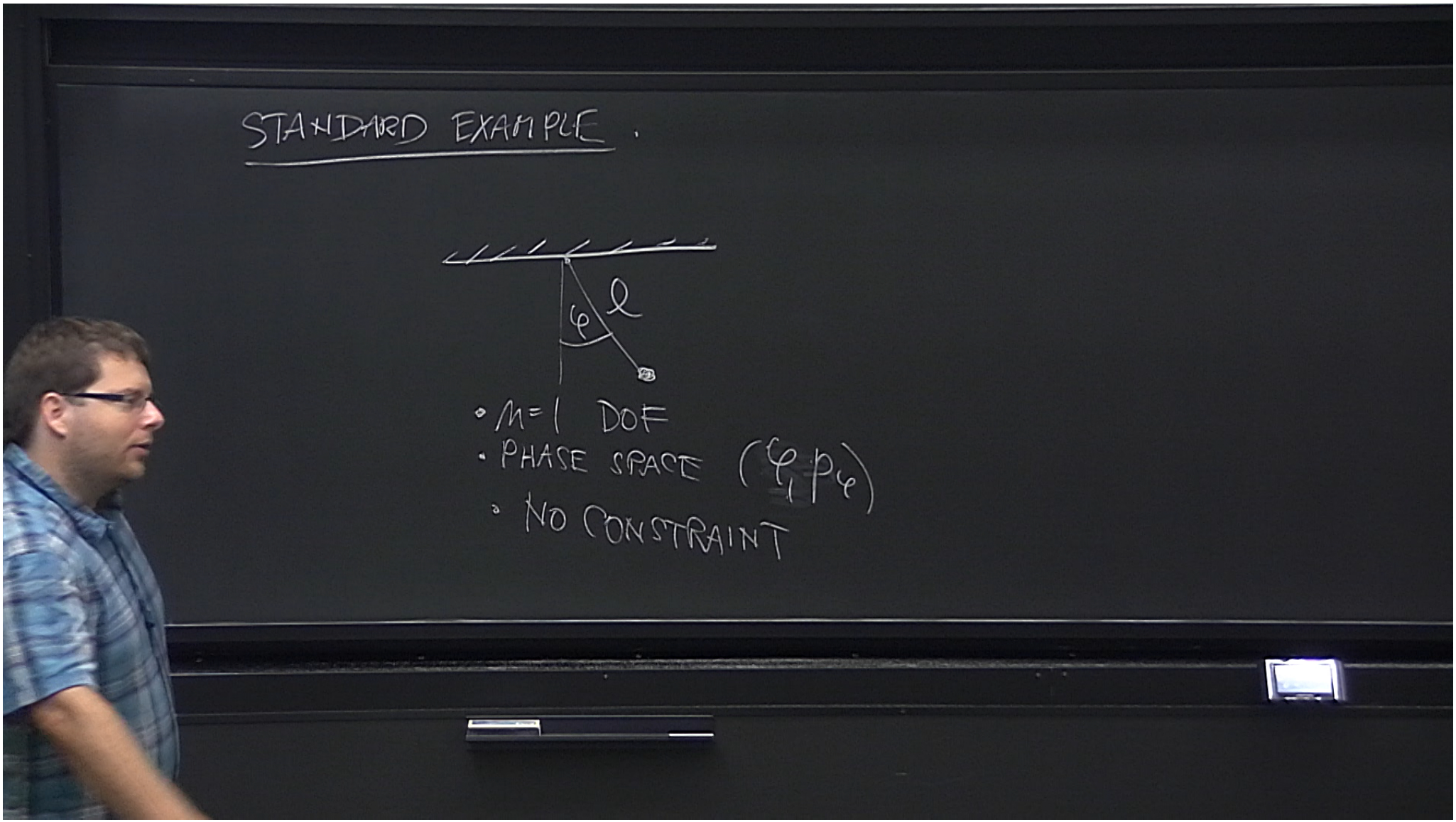

WE HAVE A CONSTRAINT
$\phi(q^i, p_i) = 0$
"SPECIAL CONSERVED QUANTITIE"
SPEC.

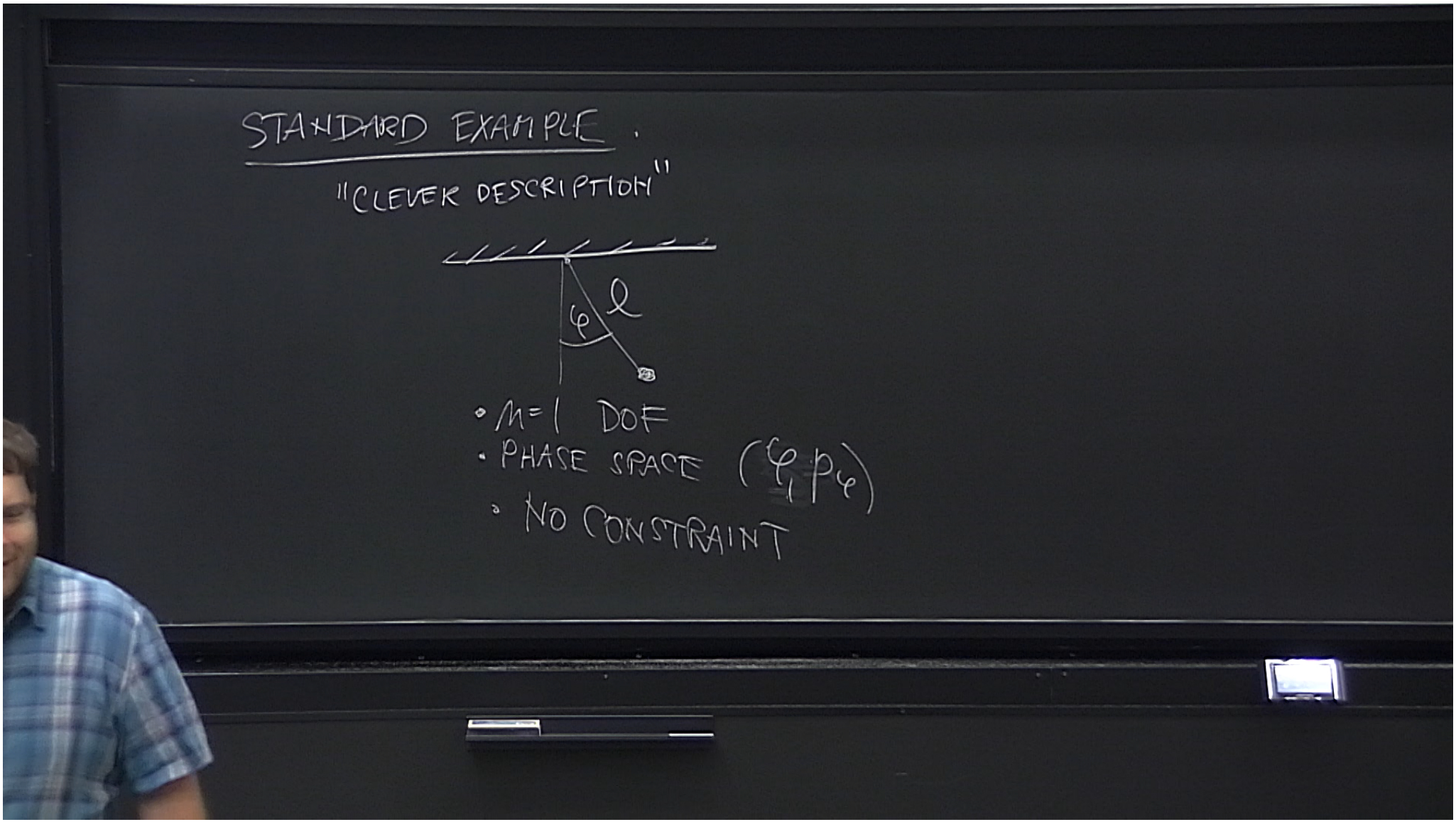

WE HAVE A CONSTRAINT
$\Phi(q^i, p_i) = 0$
"SPECIAL CONSERVED QUANTITIE"
→ DEFINE A SYMMETRY.
SPEC.

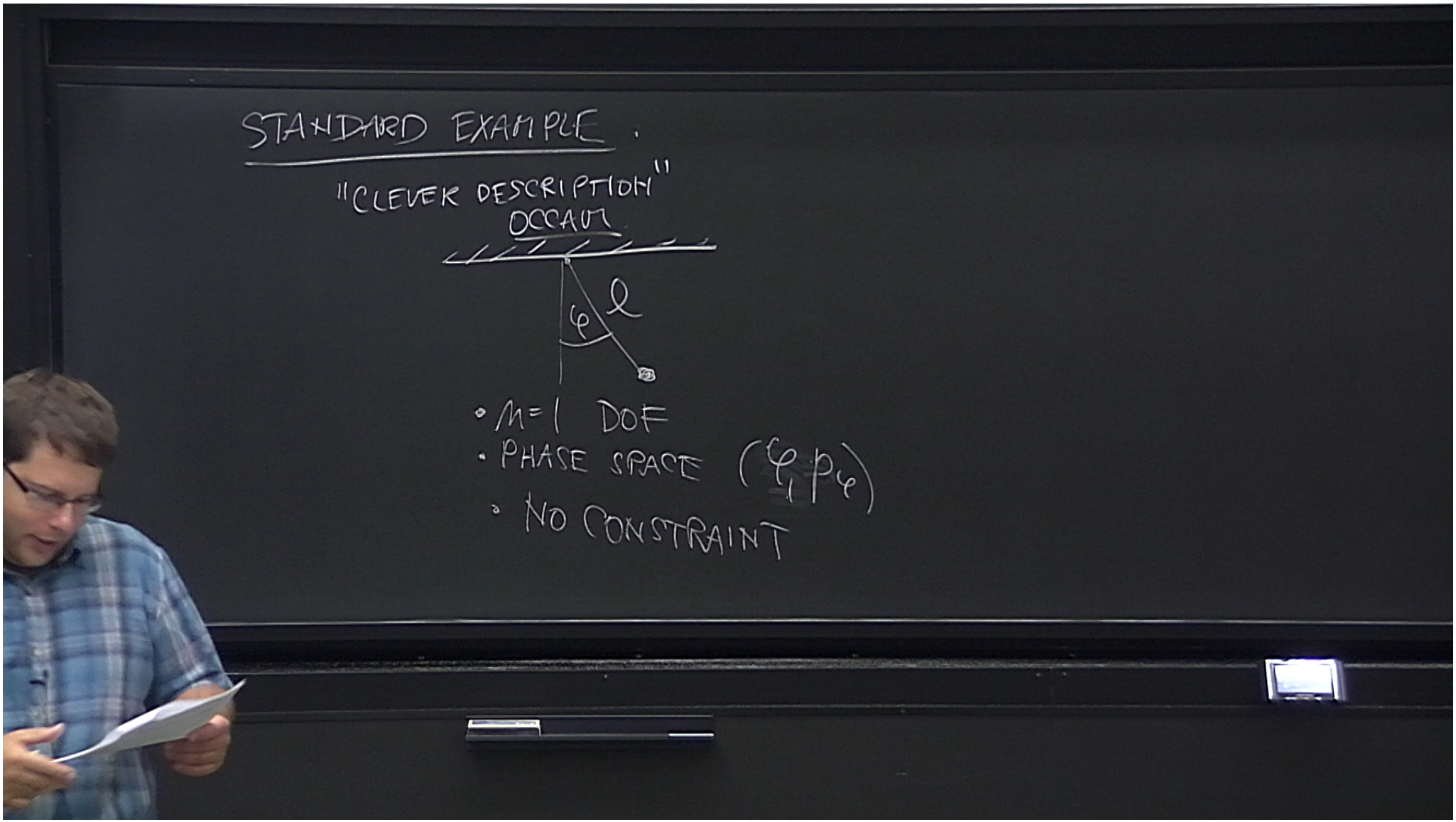

STANDARD EXAMPLE.


STANDARD EXAMPLE.
φ

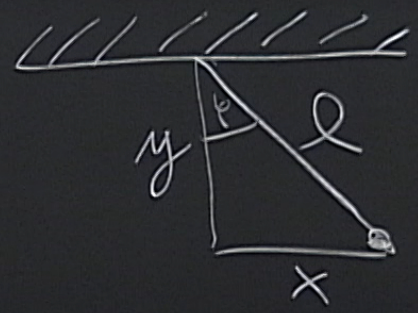

STANDARD EXAMPLE.
φ
• n = 1 DOF
• PHASE SPACE (φ, p_φ)


STANDARD EXAMPLE.
φ
ℓ
• $n=1$ DOF
• PHASE SPACE (φ, p_φ)
• NO CONSTRAINT


STANDARD EXAMPLE.
"CLEVER DESCRIPTION"
φ
ℓ
• $n=1$ DOF
• PHASE SPACE (φ, p_φ)
• NO CONSTRAINT

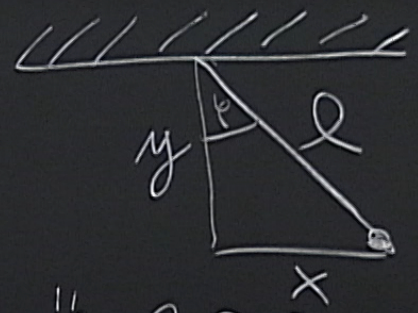

STANDARD EXAMPLE.
"CLEVER DESCRIPTION"
OCCAM
φ
ℓ
• $n=1$ DOF
• PHASE SPACE (φ, p_φ)
• NO CONSTRAINT

CONSTRAINT DESCRIPTION



VS

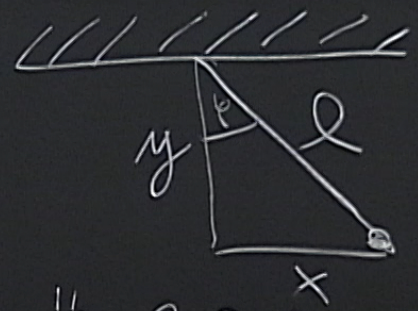
CONSTRAINT DESCRIPTION



VS

- "$n=2$ DOF"
- PHASE SPACE (x, y, p_x, p_y)

CONSTRAINT DESCRIPTION

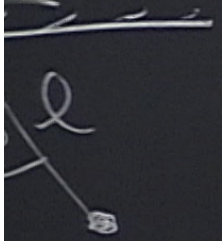


VS

- "$n = 2$ DOF"
- PHASE SPACE (x, y, p_x, p_y)
- CONSTRAINT

$$\underbrace{\phi = x^2 + y^2 - \ell^2 = 0}$$

TION"



DOF

SPACE (φ, p_φ)

CONSTRAINT

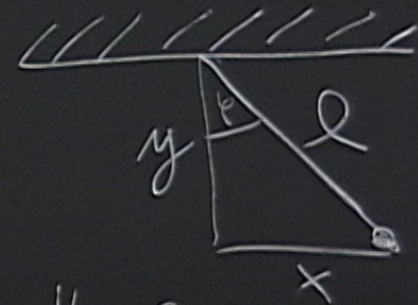
VS

ELIMINATE CONSTRAINT

$x = l\cos\varphi$

$y = l\sin\varphi$

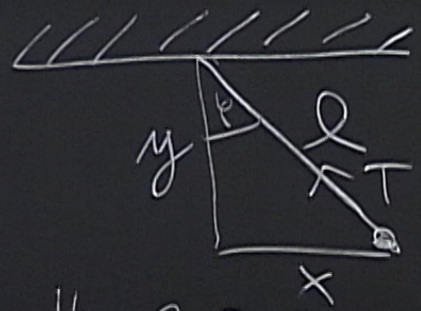
CONSTRAINT DESCRIPTION



- "$n = 2$ DOF"
- PHASE SPACE (x, y, p_x, p_y)
- CONSTRAINT

$$\underbrace{\phi = x^2 + y^2 - l^2 = 0}$$

CONSTRAINT DESCRIPTION



- "$n=2$ DOF"
- PHASE SPACE (x, y, p_x, p_y)
- CONSTRAINT

$$\underbrace{\phi = x^2 + y^2 - \ell^2 = 0}$$

- ADVANTAGE.
  - EXTRA SYMMETRY.
  - CAN CALCULATE "FORCE OF CONSTRAINT" ∴ TENSION OF STRING T.

CONSTRAINT

$\ell\cos\varphi$

$\ell\sin\varphi$

$y = \ell \sin \varphi$

- TO DEAL WITH CONSTRAINTS ONE USES A METHOD OF LAGRANGE MULTIPLIERS

$\Phi_j \ldots \quad j = 1, \ldots, m \ldots$ CONSTRAINTS.

$y = \ell \sin \varphi$

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$\phi_j \ldots \quad j = 1, \ldots, m \ldots$ CONSTRAINTS.

CONSIDER

$$\boxed{H_{TOT} = H + \sum_{j=1}^{m} \lambda_j(t)\, \phi_j}$$

$y = \ell \sin \varphi$

- TO DEAL WITH CONSTRAINTS ONE USES A METHOD OF LAGRANGE MULTIPLIERS

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LAGRANGE MULTIPLIERS

2 POSSIBILITIES

① ENLARGE PHASE SPACE EVEN FURTHER

... CONSIDER $\lambda_J(t)$ AS NEW COORDS.

NHTS.

TIPLIERS.

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... I HAVE TO VARY W.R.T. λ_J AS WELL

$(N+m)$ E-L ... EQUATIONS

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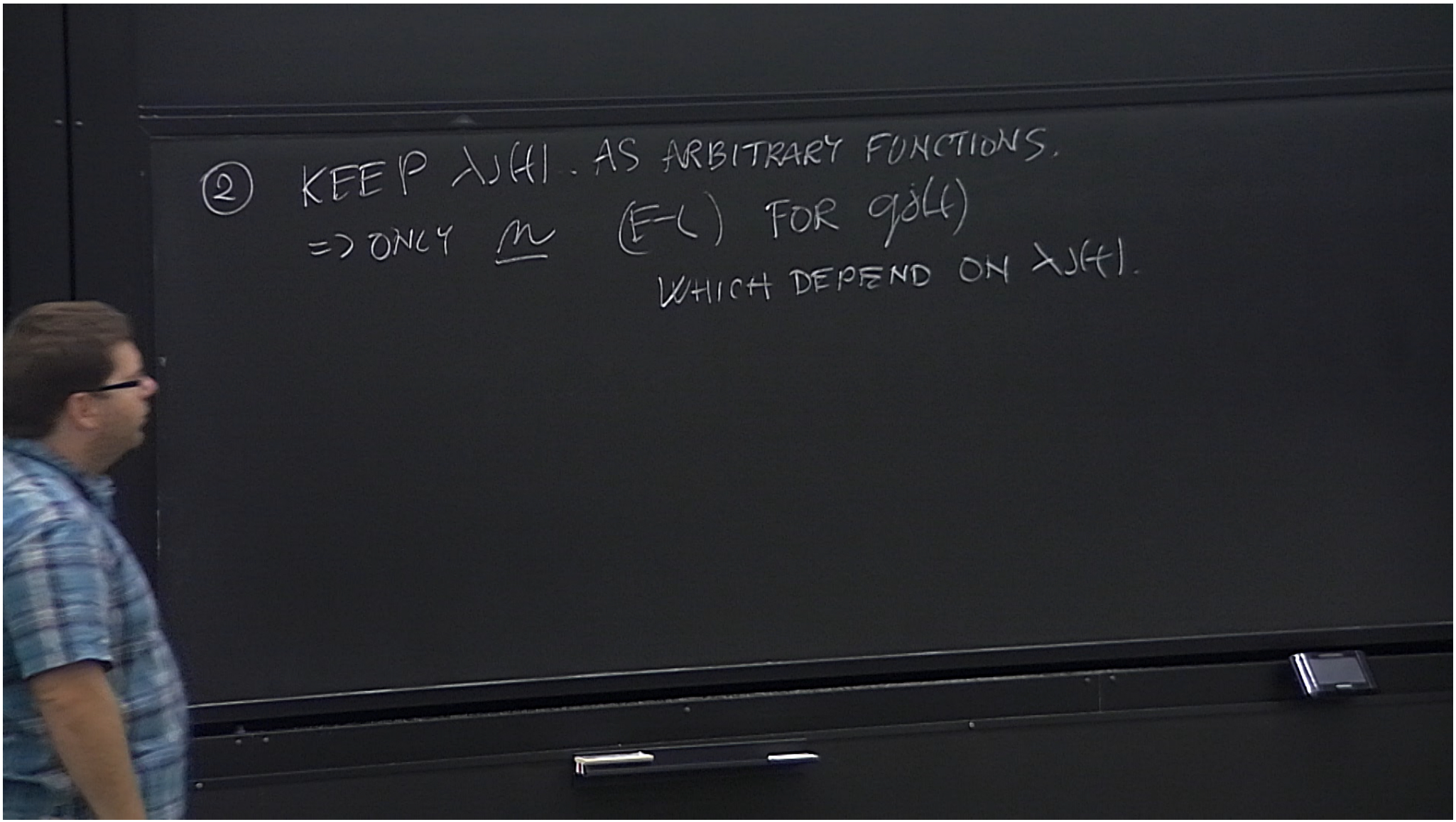
$(n+m)$ E-L ... EQUATIONS

$\underbrace{\phi_J = 0}$ ... $J = 1, \ldots, m$

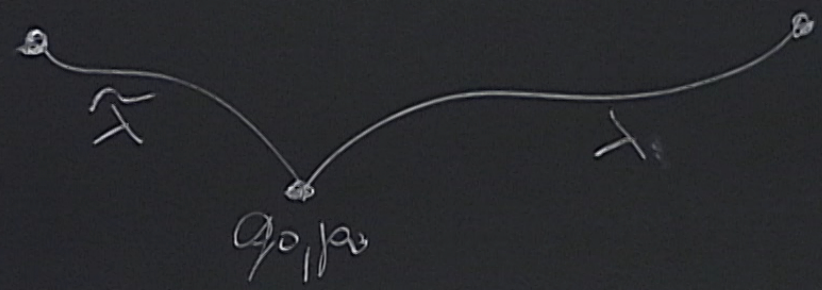
+ n .. MODIFIED (E-L) FOR $q^j(t)$

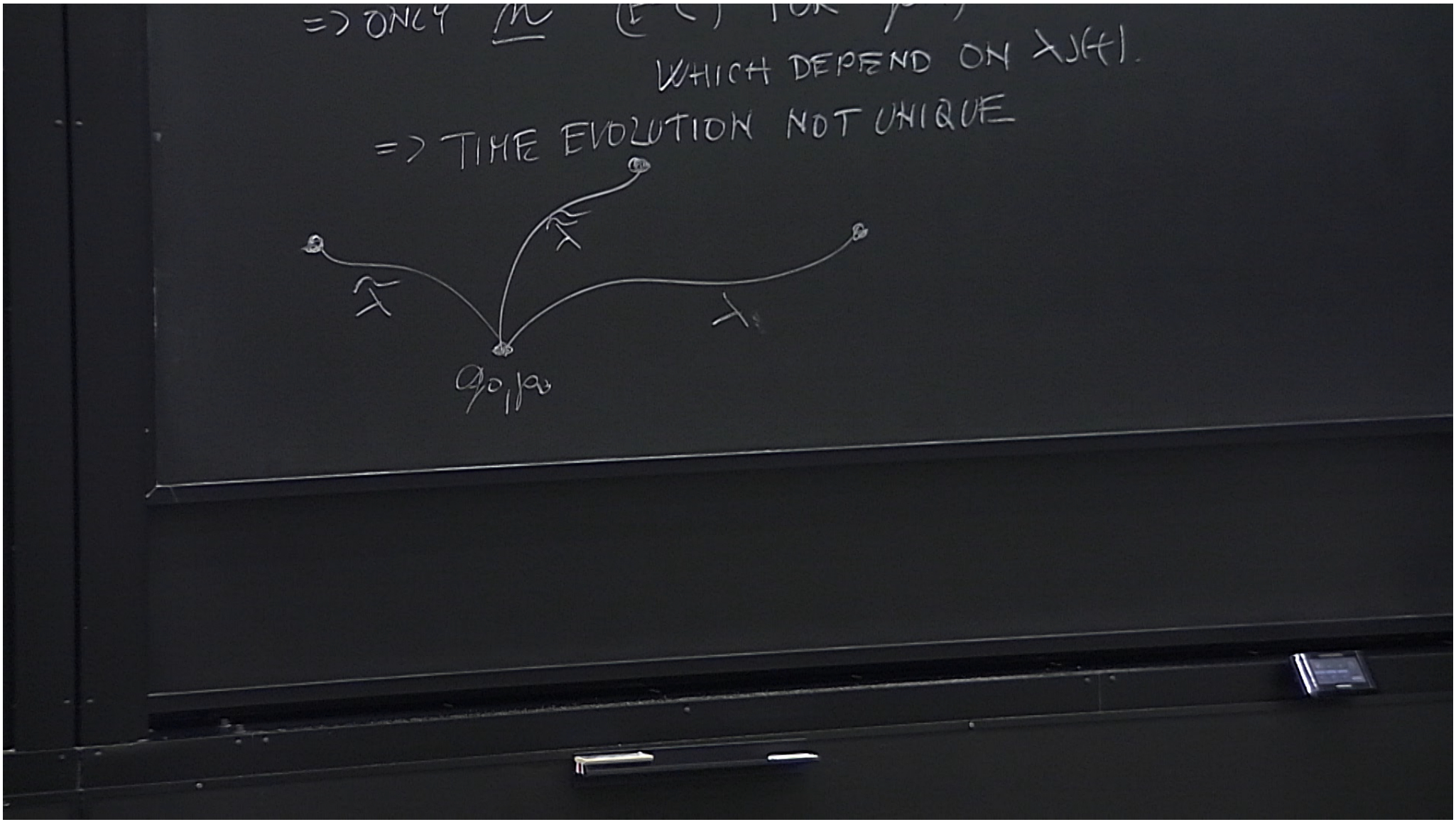

② KEEP λJ(t) AS ARBITRARY FUNCTIONS.
=> ONLY n (E-L) FOR qj(t)
WHICH DEPEND ON λJ(t).

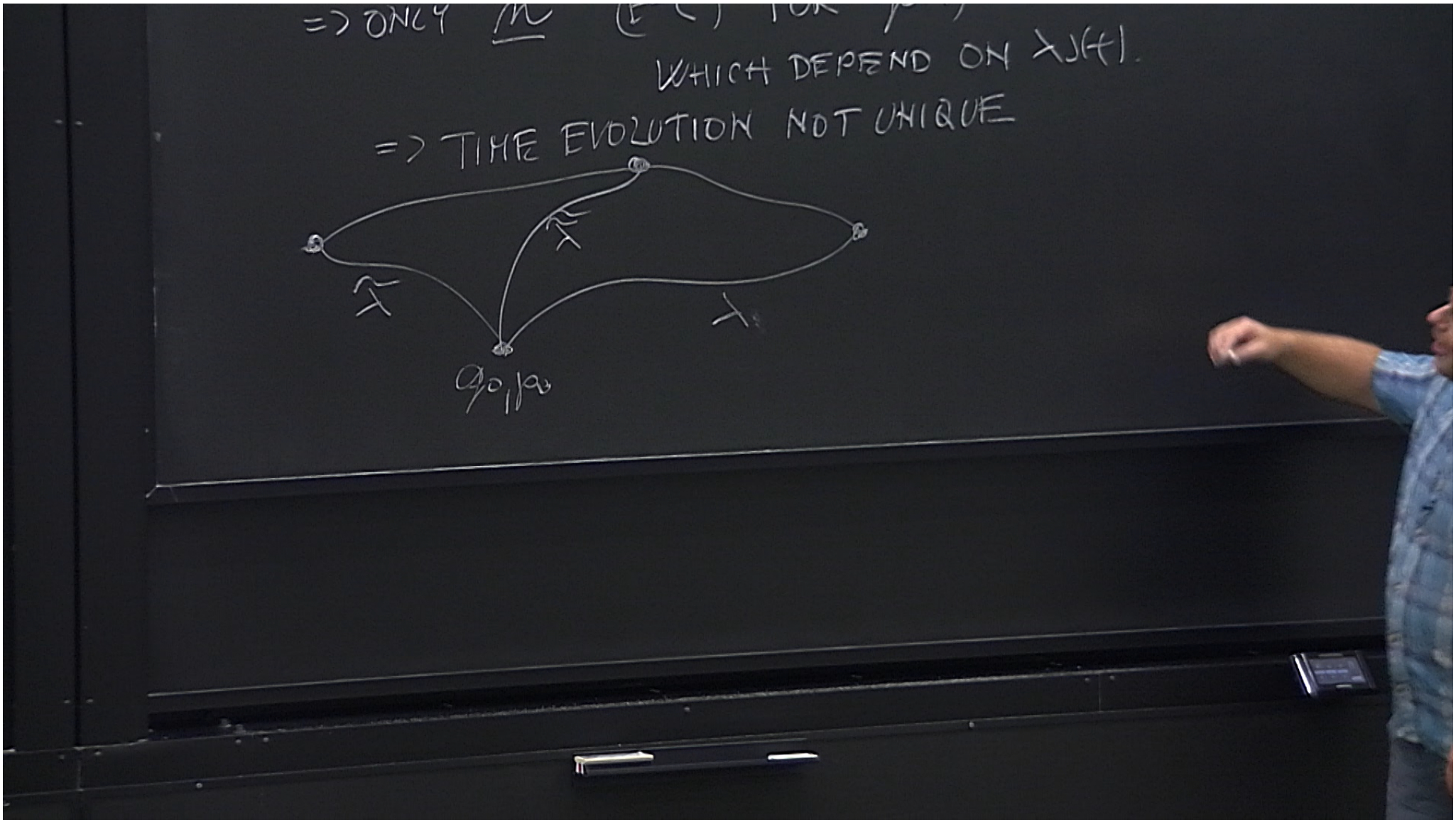
$\Rightarrow$ ONLY $\underline{\mathcal{H}}$ (E ( ) FOR ρ)

WHICH DEPEND ON $\lambda J(t)$.

$\Rightarrow$ TIME EVOLUTION NOT UNIQUE




=> ONLY
WHICH DEPEND ON λJ(t).
=> TIME EVOLUTION NOT UNIQUE
λ̃
λ̃
λ
q0, p0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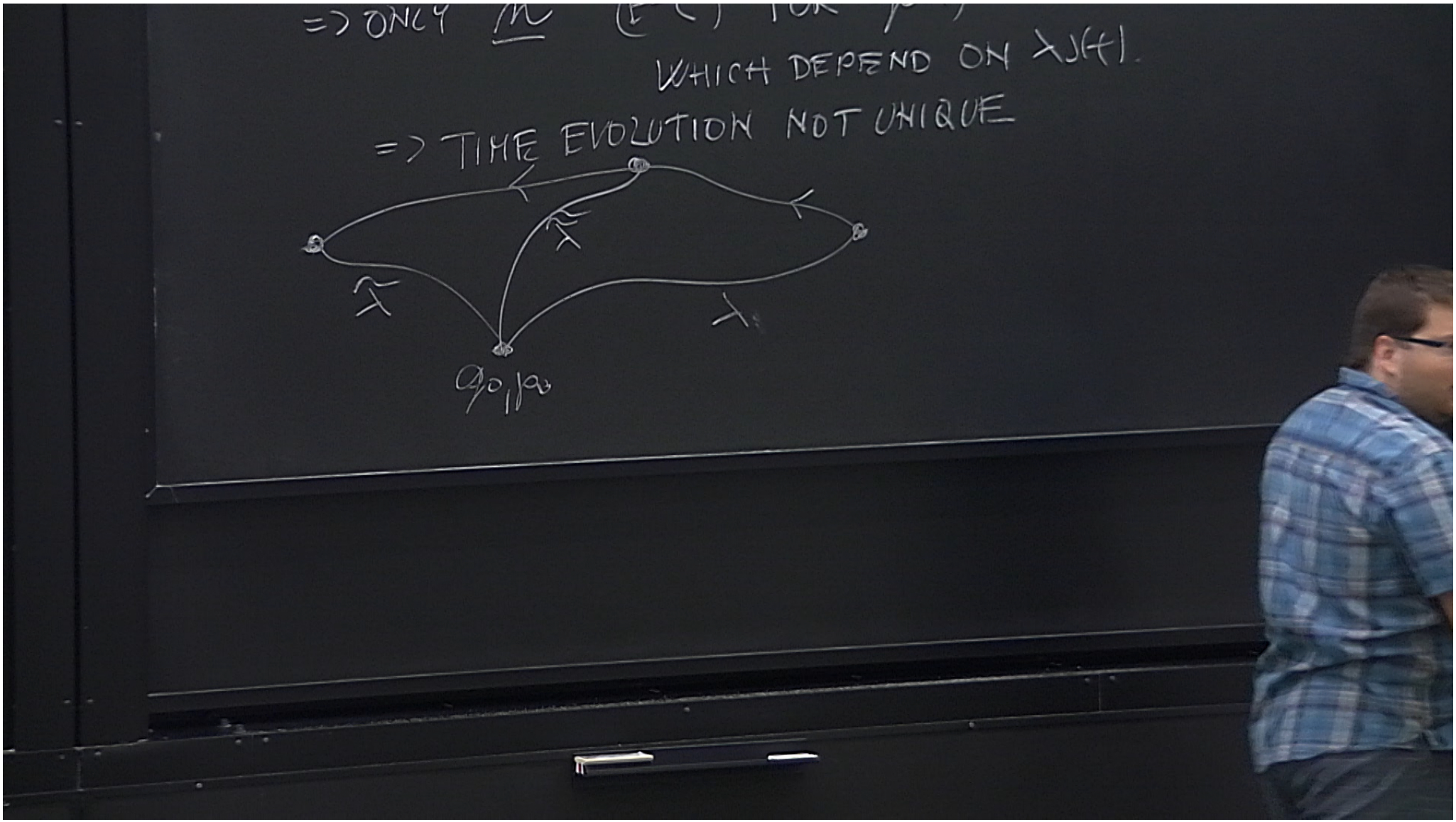
=> TIME EVOLUTION NOT UNIQUE
λ̃
λ̃
λ
q0, p0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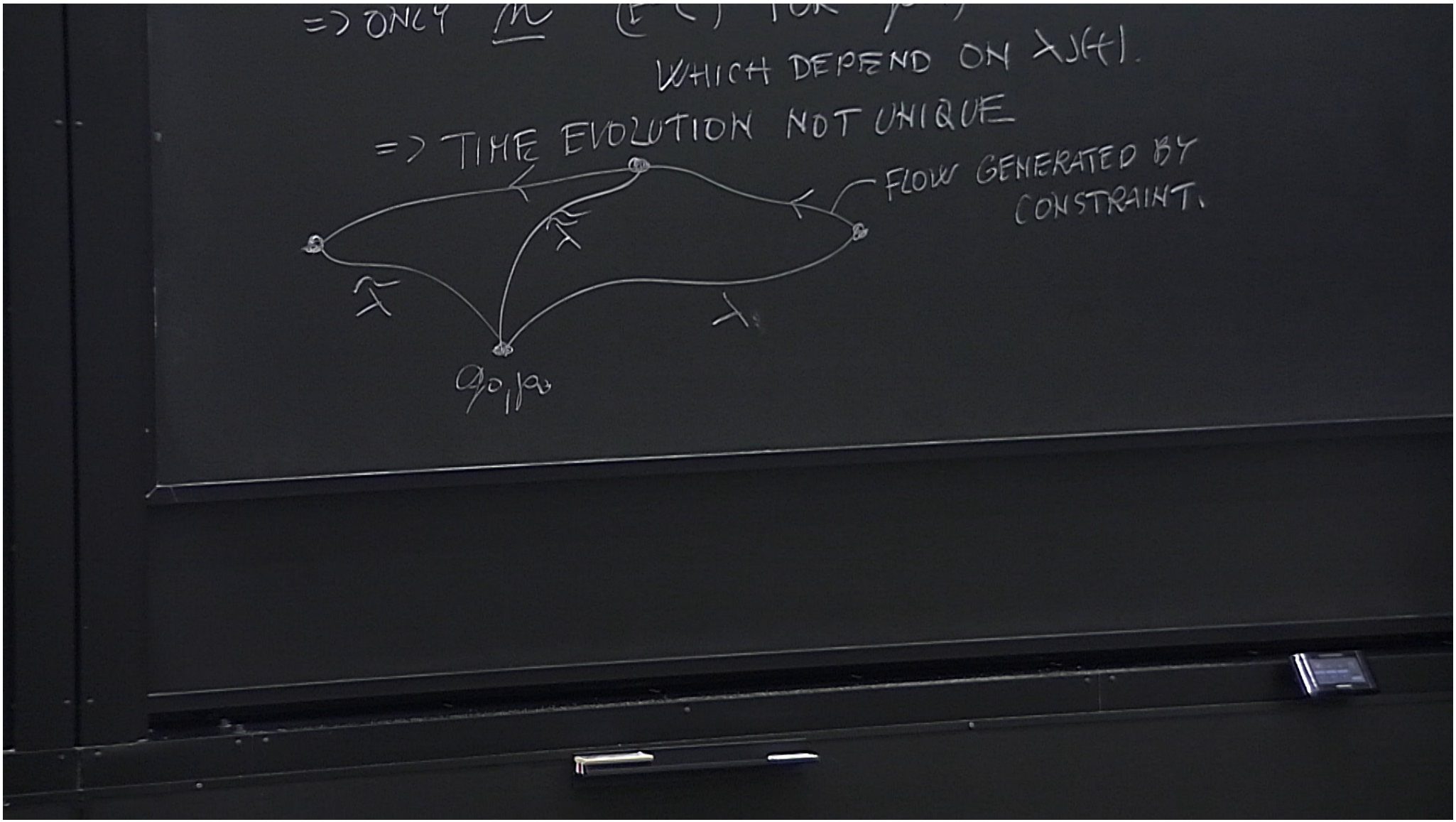
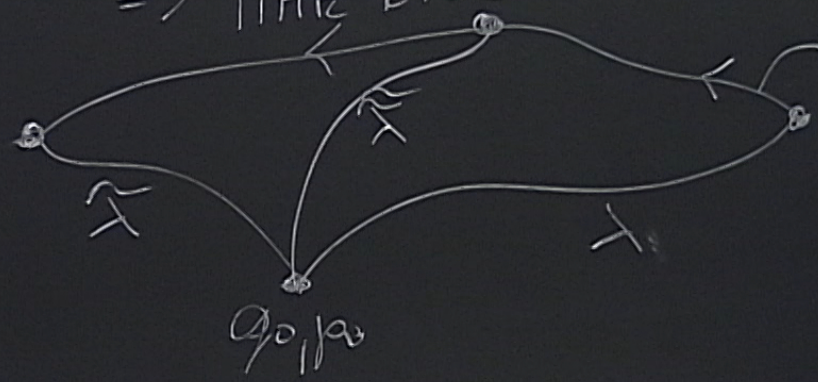
λ̃
λ̃
λ
q0, p0


=> ONLY
WHICH DEPEND ON λJ(t).
=> TIME EVOLUTION NOT UNIQUE
FLOW GENERATED BY CONSTRAINT.
λ̃
λ̃
λ
q0, p0

=> ONLY $\underline{\mathcal{H}}$ (E C) FOR

WHICH DEPEND ON $\lambda_J(t)$.

=> TIME EVOLUTION NOT UNIQUE



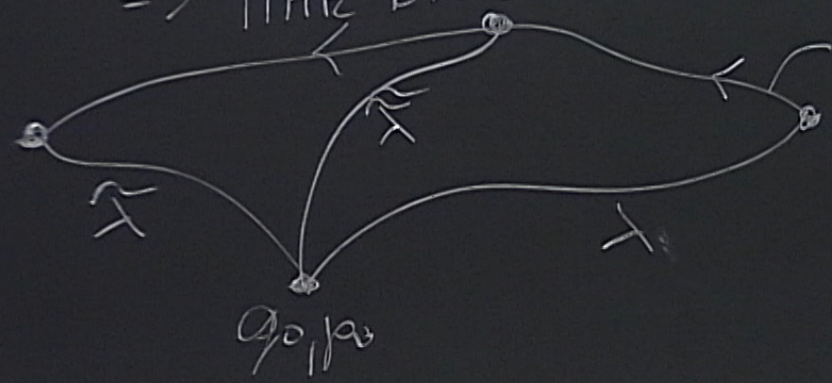
FLOW GENERATED BY CONSTRAINT.

ALL THE ENDPOINTS ARE PHYSI

=> ONLY $\underline{M}$ (E C) FOR

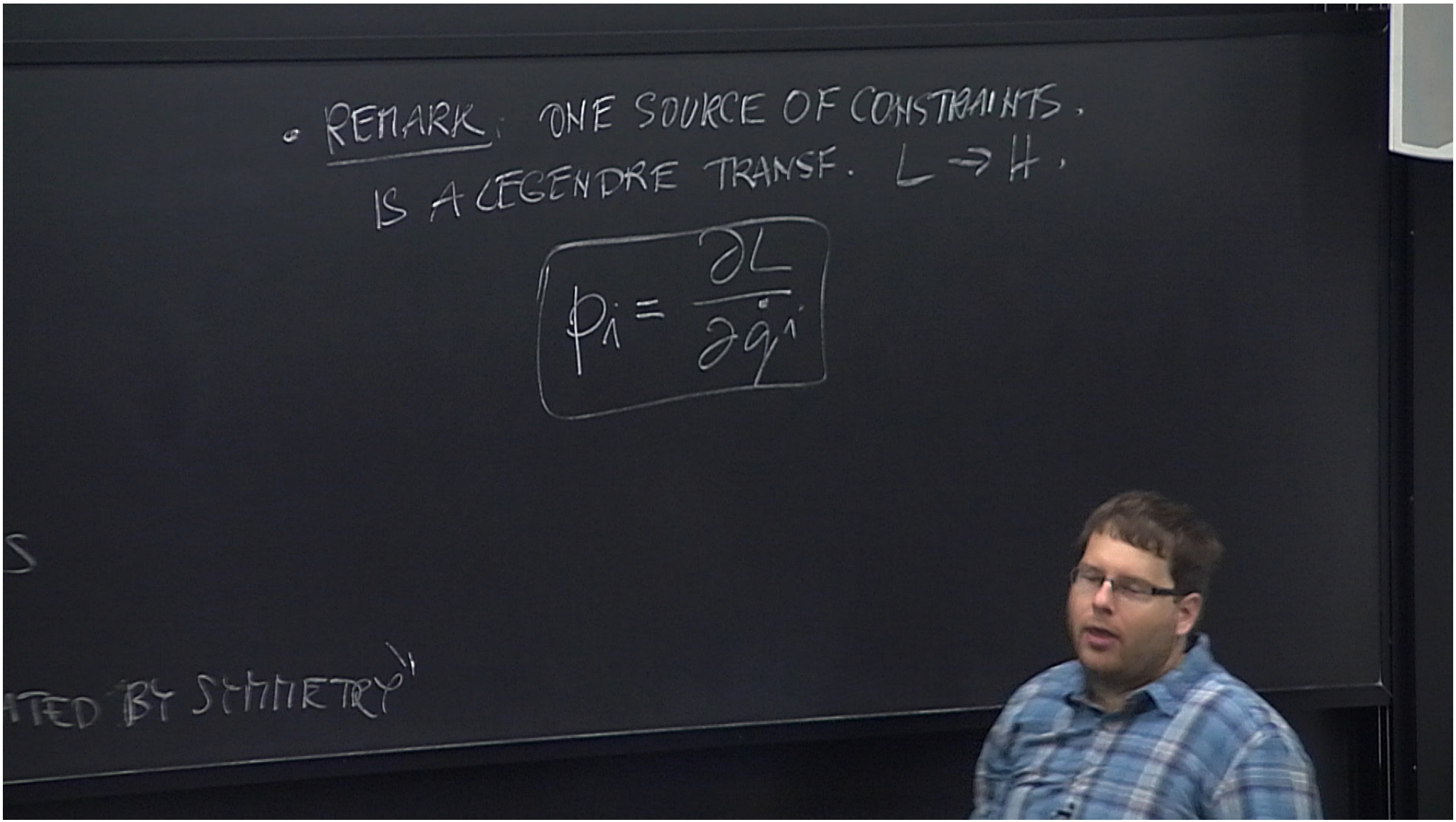
WHICH DEPEND ON $\lambda J(t)$.

=> TIME EVOLUTION NOT UNIQUE



FLOW GENERATED BY CONSTRAINT.

ALL THE ENDPOINTS ARE PHYSICALLY EQUIVALEN (RELATED BY SYMMETRY)


REMARK: ONE SOURCE OF CONSTRAINTS.
IS A LEGENDRE TRANSF. L → H.
$p_i = \frac{\partial L}{\partial \dot{q}^i}$
S
ATED BY SYMMETRY"

IS A LEGENDRE TRANSF.

$$\boxed{p_i = \frac{\partial L}{\partial \dot{q}^i}} \quad \xleftrightarrow{\text{INVERT}} \quad \dot{q}^i = \dot{q}^i(q^j, p_j)$$

S

ATED BY SYMMETRY"

IS A LEGENDRE TRANSF.

$$\boxed{p_i = \frac{\partial L}{\partial \dot{q}^i}} \xleftrightarrow{\text{INVERT}} \underline{\dot{q}^i = \dot{q}^i(q^j, p_j)}$$

MAY NOT EXIST.

S

ATED BY SYMMETRY"

$$\boxed{p_1 = \frac{\partial L}{\partial \dot{q}^1}} \longleftrightarrow \boxed{\dot{q}^1 = \ldots}$$

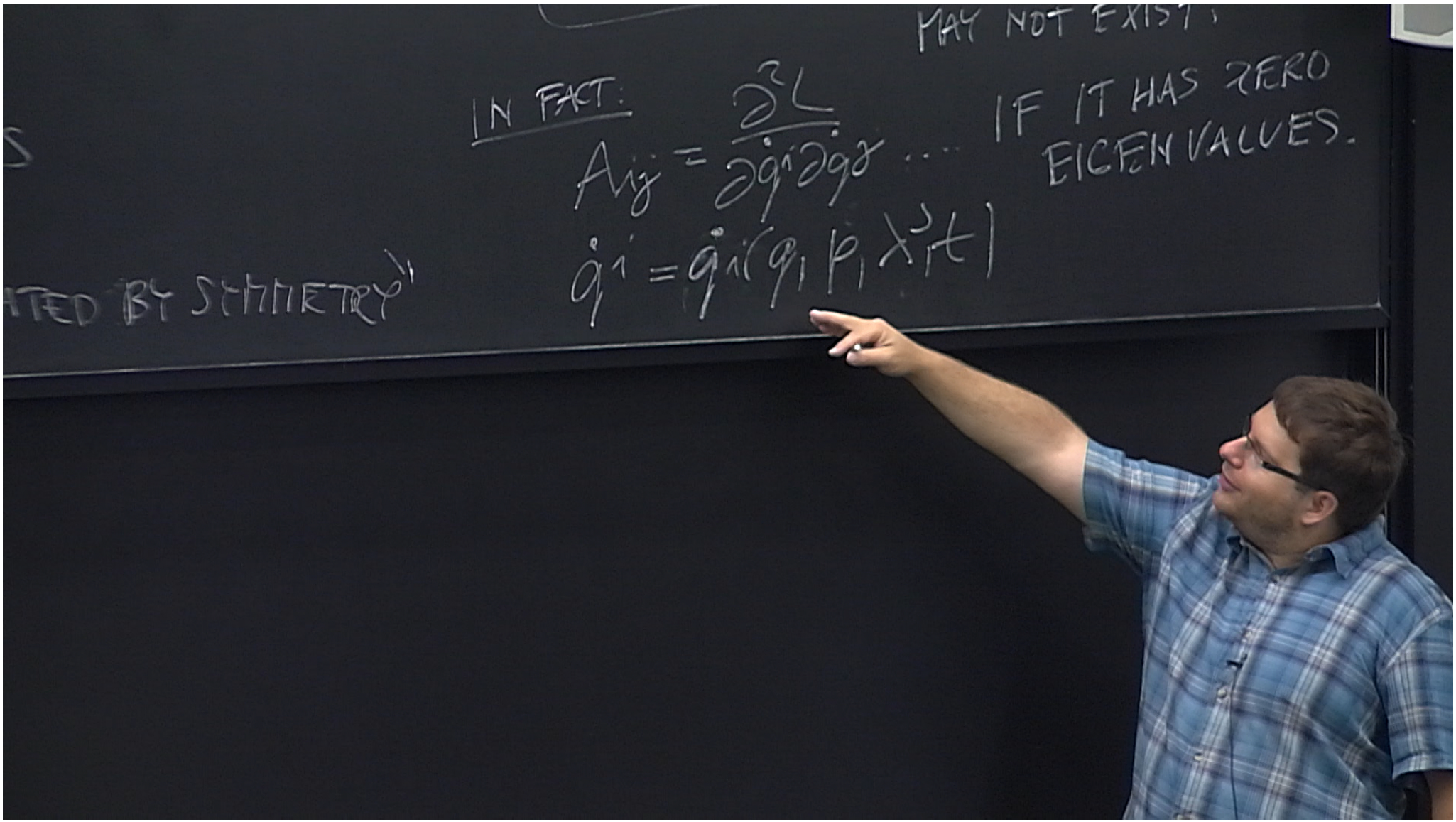
MAY NOT EXIST.

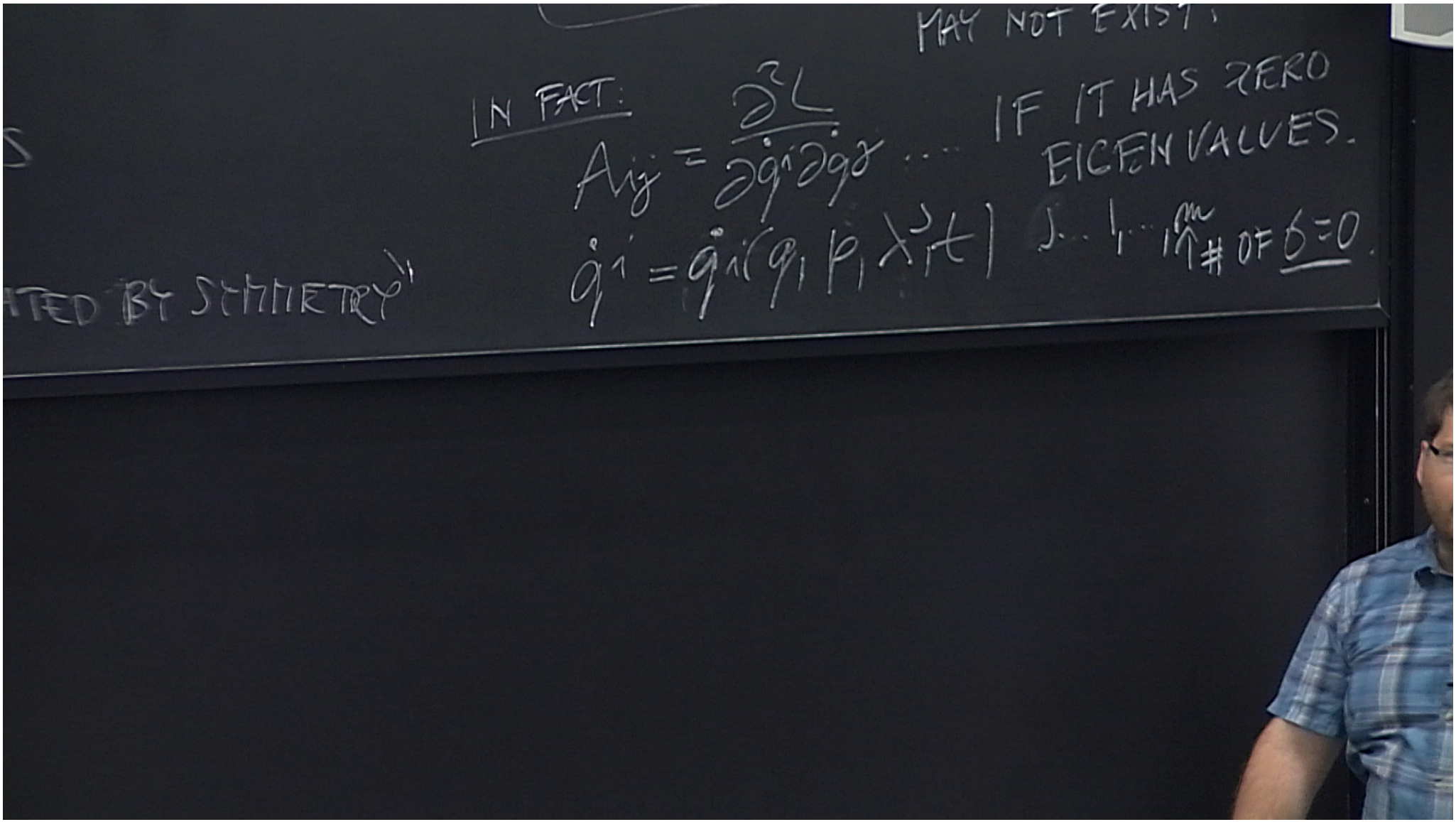
IN FACT:

$$A_{ij} = \frac{\partial^2 L}{\partial \dot{q}^i \partial \dot{q}^j}$$

S

ATED BY SYMMETRY"


MAY NOT EXIST.
IN FACT:
$A_{ij} = \frac{\partial^2 L}{\partial \dot{q}^i \partial \dot{q}^j}$ ....
IF IT HAS ZERO EIGENVALUES.
$\dot{q}^i = \dot{q}^i(q, p, \lambda^\nu, t)$
ATED BY SYMMETRY


MAY NOT EXIST.
IN FACT:
$A_{ij} = \frac{\partial^2 L}{\partial \dot{q}^i \partial \dot{q}^j}$ ....
IF IT HAS ZERO EIGENVALUES.
$\dot{q}^i = \dot{q}^i(q, p, \lambda^j, t)$
$j = 1 \ldots m$
OF $\sigma = 0$.
ATED BY SYMMETRY

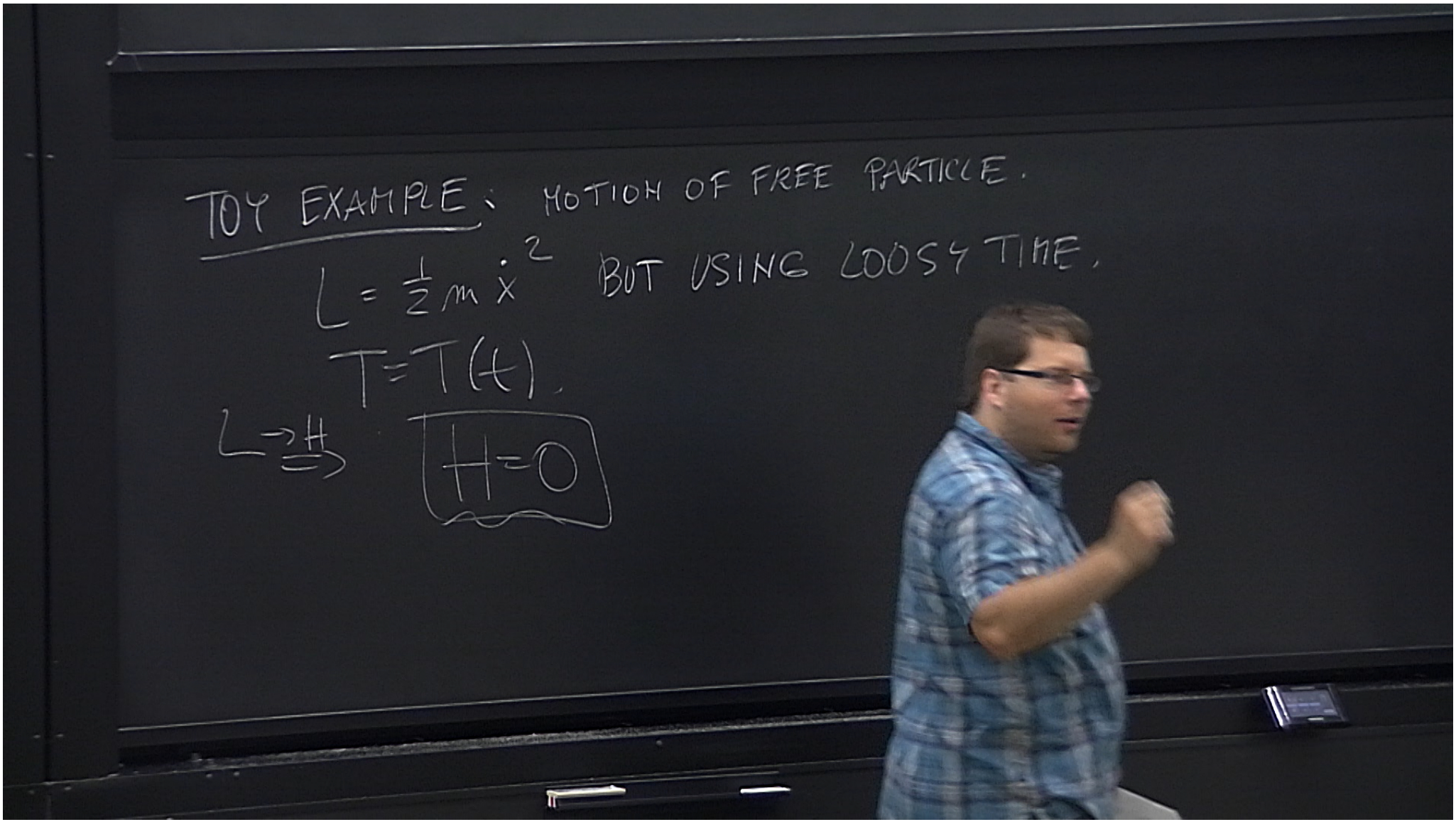
TOY EXAMPLE: MOTION OF FREE PARTICLE.

$$L = \frac{1}{2} m \dot{x}^2$$

TOY EXAMPLE: MOTION OF FREE PARTICLE.

$L = \frac{1}{2} m \dot{x}^2$ BUT USING LOOSY TIME.

$T = T(t)$.


TOY EXAMPLE: MOTION OF FREE PARTICLE.
$L = \frac{1}{2} m \dot{x}^2$ BUT USING LOOSY TIME.
$T = T(t)$.
$L \xrightarrow{H} \boxed{H = 0}$

TOY EXAMPLE: MOTION OF FREE PARTICLE.

$L = \frac{1}{2} m \dot{x}^2$ BUT USING LOOSY TIME.

$T = T(t)$.

$L \xrightarrow{\Rightarrow} H$ $\boxed{H = 0}$ + CONSTRAINT.

$H_{TOT} = \lambda(t)$ CONSTRAINT

TOY EXAMPLE: MOTION OF FREE PARTICLE.

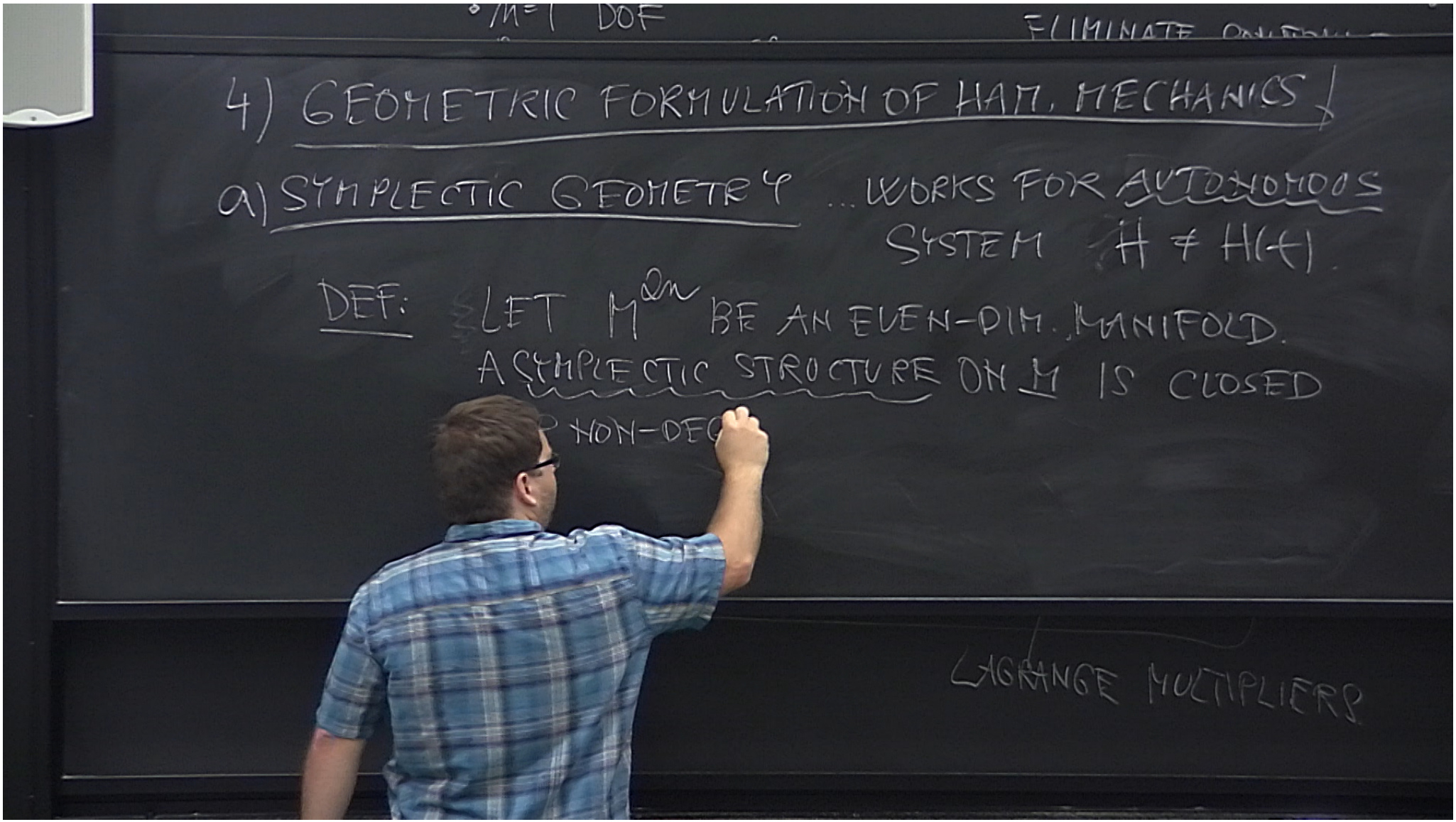
$L = \frac{1}{2} m \dot{x}^2$ BUT USING LOOSY TIME.

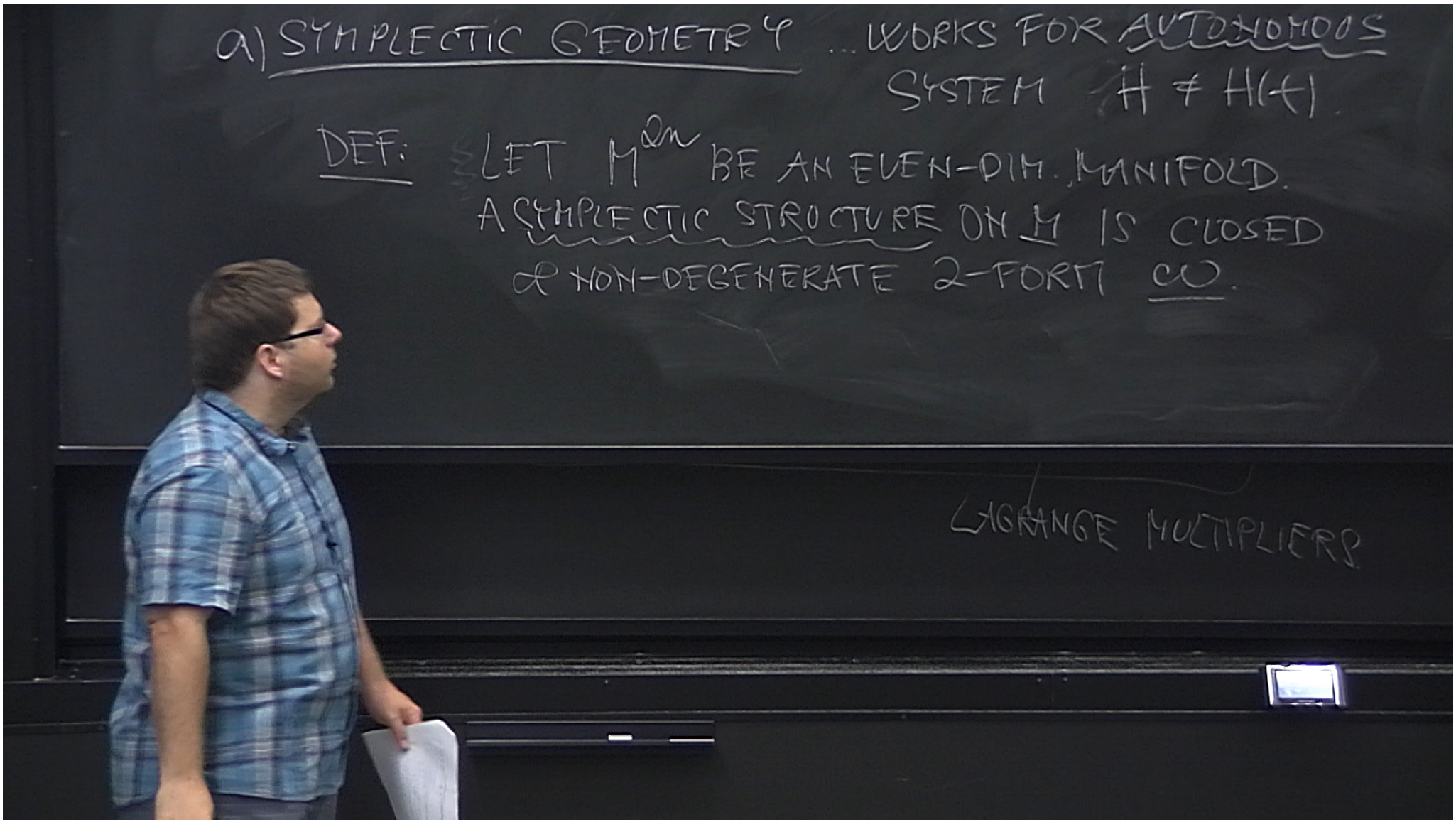
$T = T(t)$.

TOTALLY CONSTRAINT SYSTEM (GRAVITY)

$L \xrightarrow{\text{LT}} H$ $\boxed{H = 0}$ + CONSTRAINT.

$H_{TOT} = \lambda(t)$ CONSTRAINT


DOF
4) GEOMETRIC FORMULATION OF HAM. MECHANICS
a) SYMPLECTIC GEOMETRY ... WORKS FOR AUTONOMOUS SYSTEM H ≠ H(t)
DEF: LET M^2n BE AN EVEN-DIM. MANIFOLD.
A SYMPLECTIC STRUCTURE ON M IS CLOSED
NON-DEG
LAGRANGE MULTIPLIERS


a) SYMPLECTIC GEOMETRY ... WORKS FOR AUTONOMOUS SYSTEM $H \neq H(t)$
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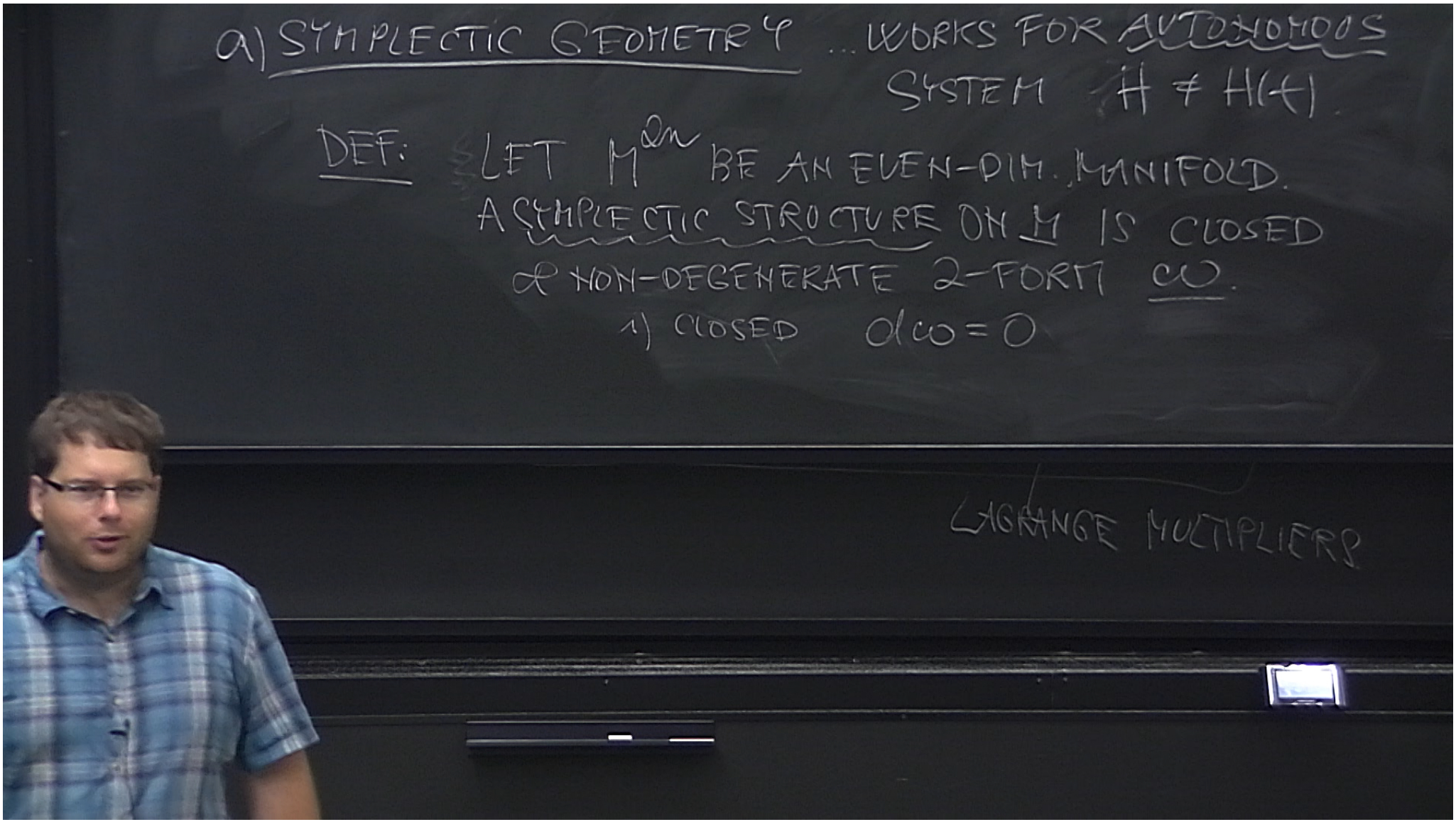
LAGRANGE MULTIPLIERS


a) SYMPLECTIC GEOMETRY ... WORKS FOR AUTONOMOUS SYSTEM $\dot{H} \neq H(t)$
DEF: LET M^{2n} BE AN EVEN-DIM. MANIFOLD.
A SYMPLECTIC STRUCTURE ON M IS CLOSED & NON-DEGENERATE 2-FORM ω.
1) CLOSED $d\omega = 0$
LAGRANGE MULTIPLIERS

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DEF: LET M^{2n} BE AN EVEN-DIM. MANIFOLD.
A SYMPLECTIC STRUCTURE ON M IS CLOSED & NON-DEGENERATE 2-FORM ω.

i) CLOSED $d\omega = 0$

ii) NON-DEG: $\forall X \neq 0 \ \exists Y \ \ \omega(X,Y) \neq 0$

LAGRANGE MULTIPLIERS

NICS

THE PAIR (M^{2n}, ω) .. SYMPLECTIC MANIFOLD,

DYNAMICS

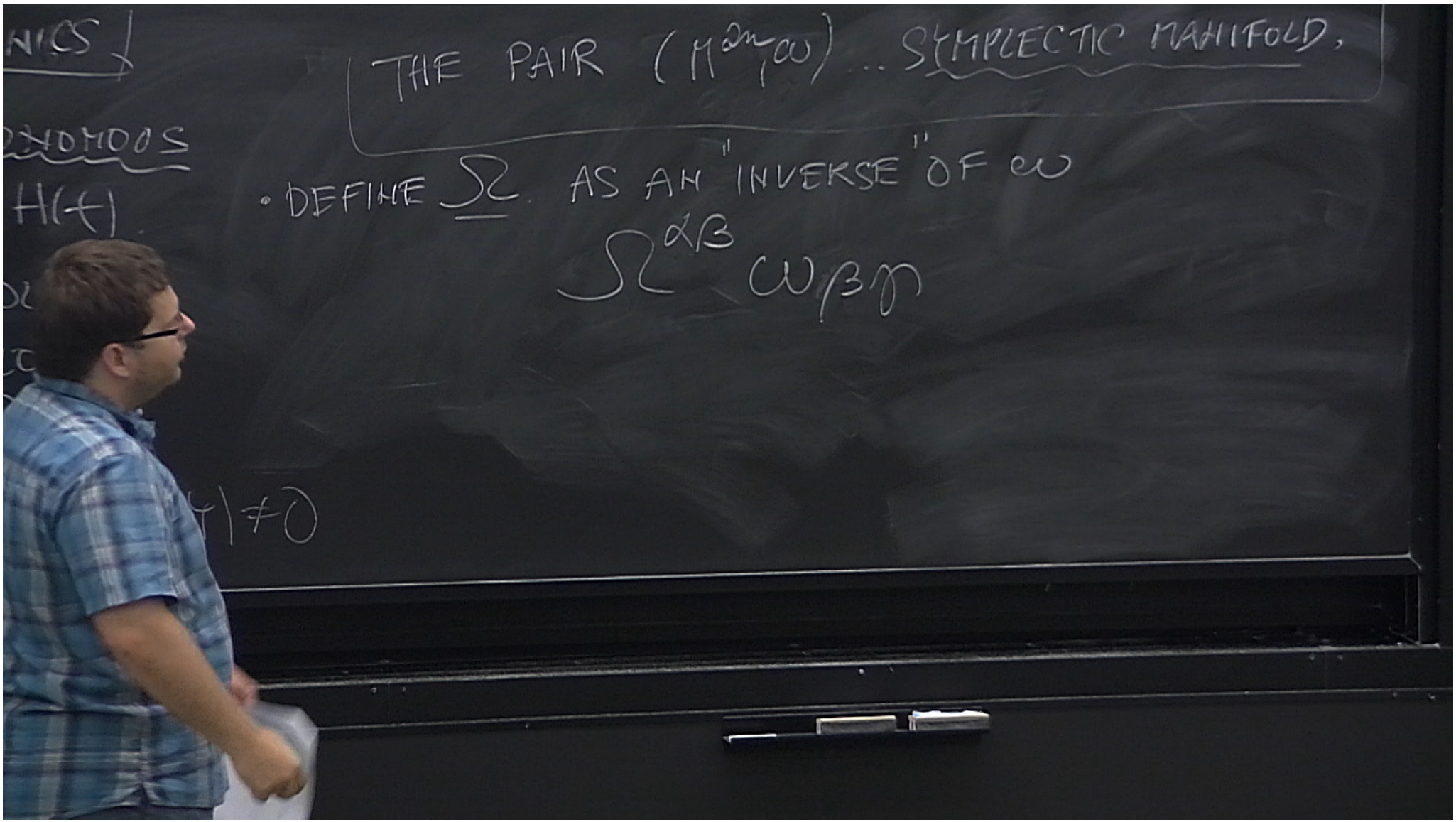
$H(t)$.

OLD.

LOSED

2.

$\omega(X,Y) \neq 0$


THE PAIR (M^{2n}, ω) .. SYMPLECTIC MANIFOLD,
• DEFINE Ω AS AN "INVERSE" OF ω
$\Omega^{\alpha\beta}\omega_{\beta\gamma}$

NICS

DYNOMOOS

$H(t)$

OLD.

LOSED

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$$\omega : TM \to T^*M \qquad V^{\alpha} \to V_{\beta} = V^{\alpha}\omega_{\alpha\beta}$$

$$\underbrace{\phi_J = 0} \quad \ldots \quad J = 1, \ldots, m$$

TIPLIERS

$+ \mu$ .. MODIFIED (E-L) FOR $q^{\alpha}(t)$

H(t)

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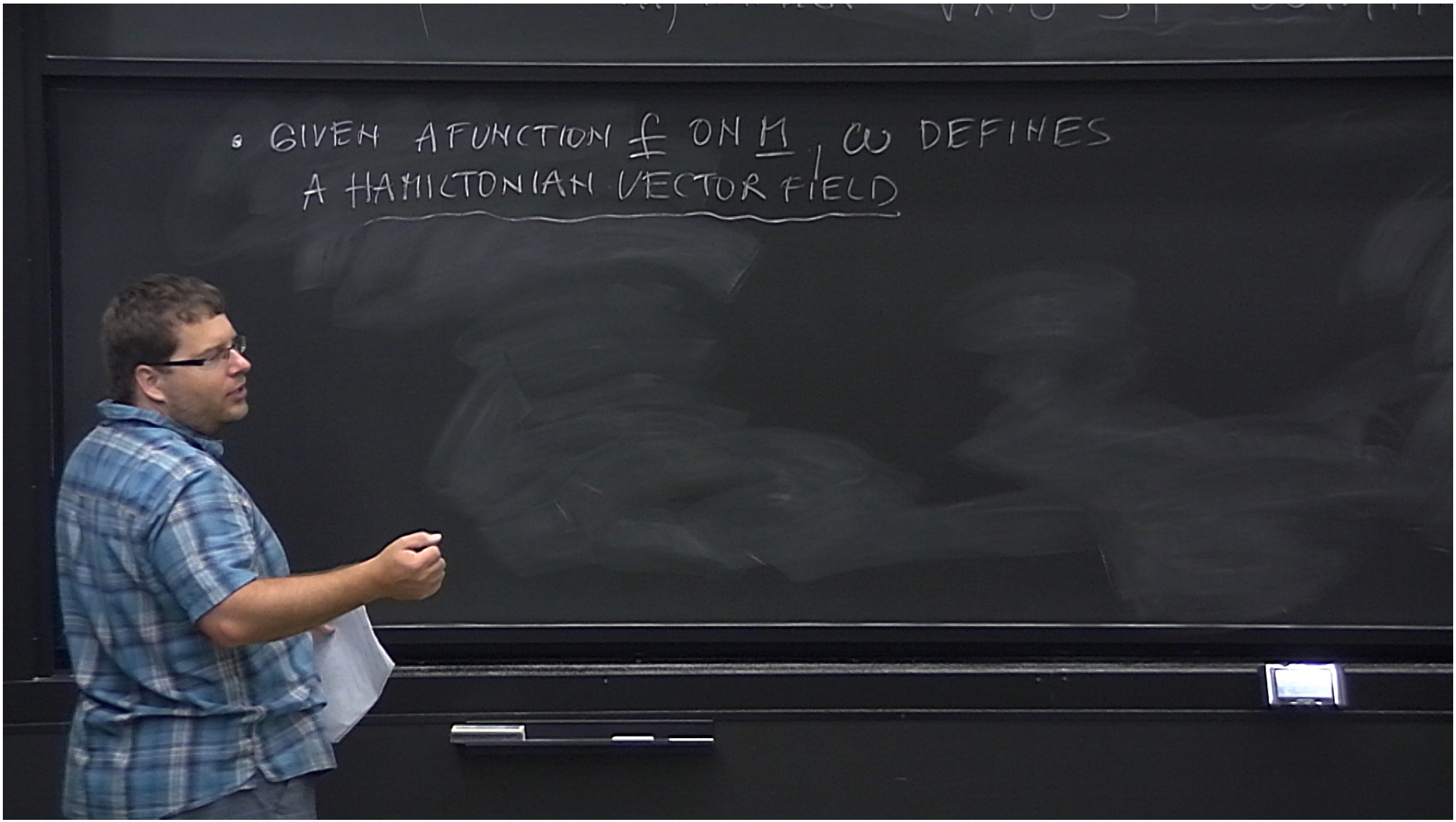
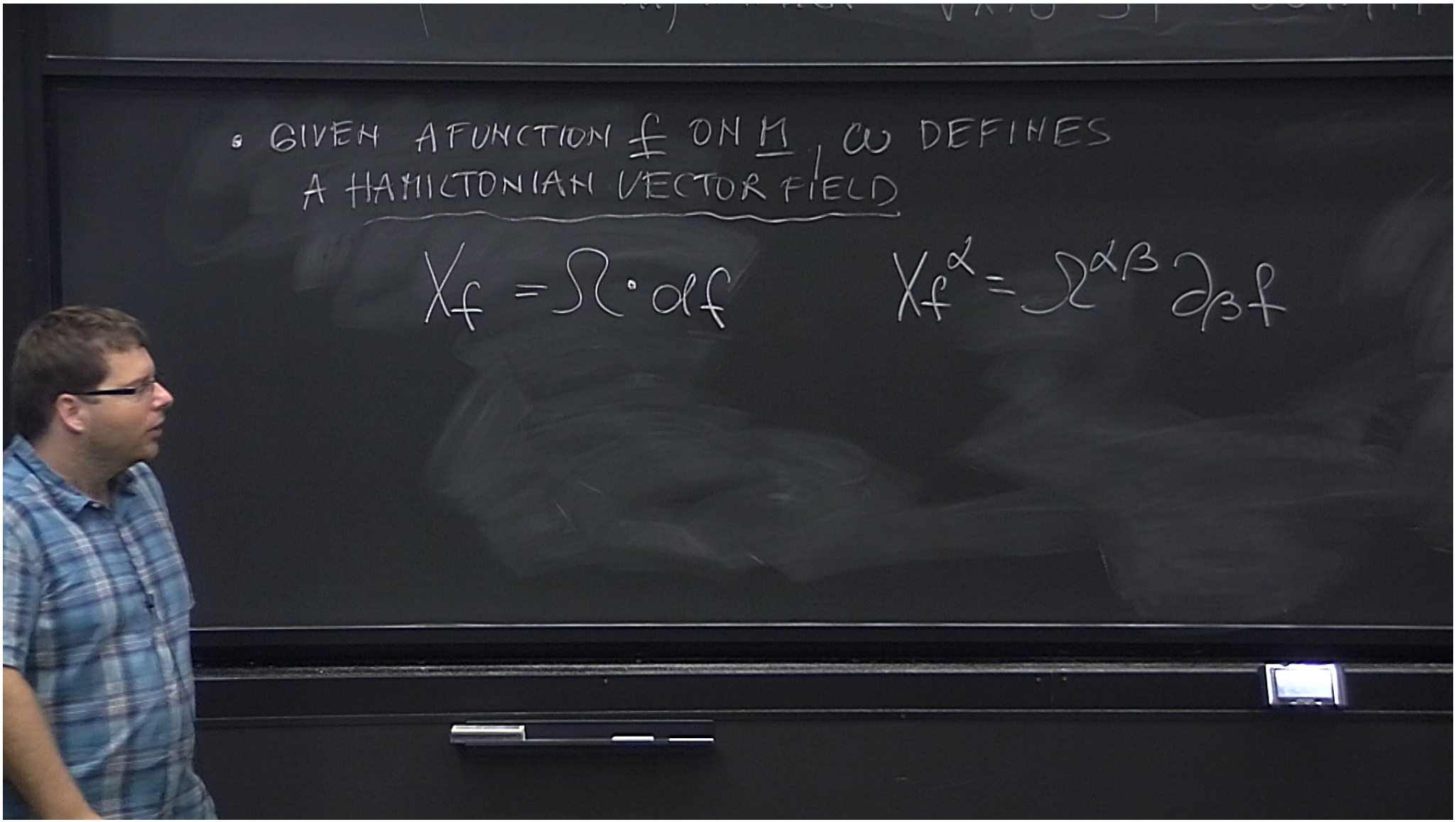

• GIVEN A FUNCTION f ON M, ω DEFINES
A HAMILTONIAN VECTOR FIELD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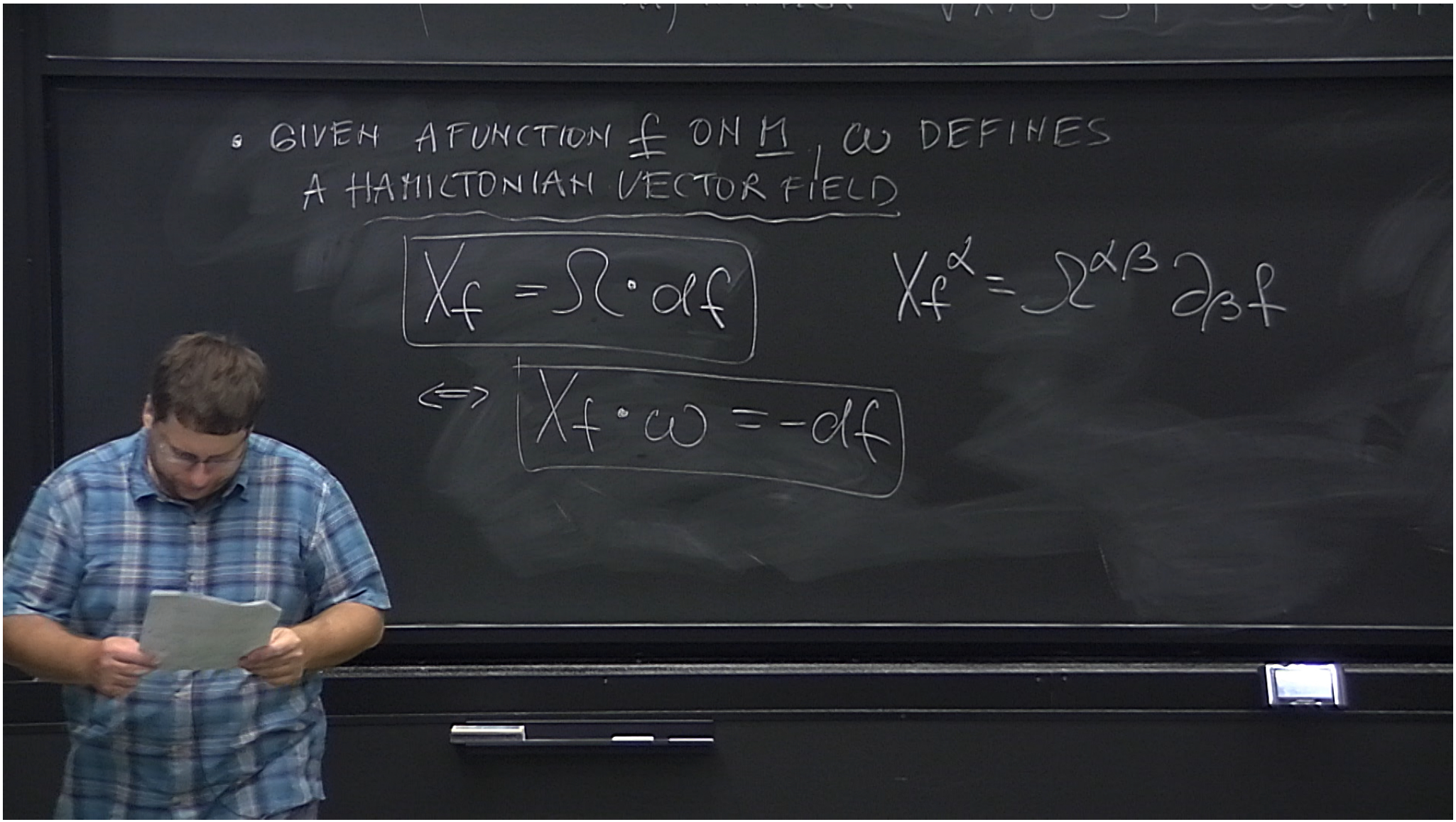
$X_f = \Omega \cdot df$
$X_f^\alpha = \Omega^{\alpha\beta} \partial_\beta f$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GIVEN A FUNCTION $\underline{f}$ ON $\underline{M}$, ω DEFINES A HAMILTONIAN VECTOR FIELD

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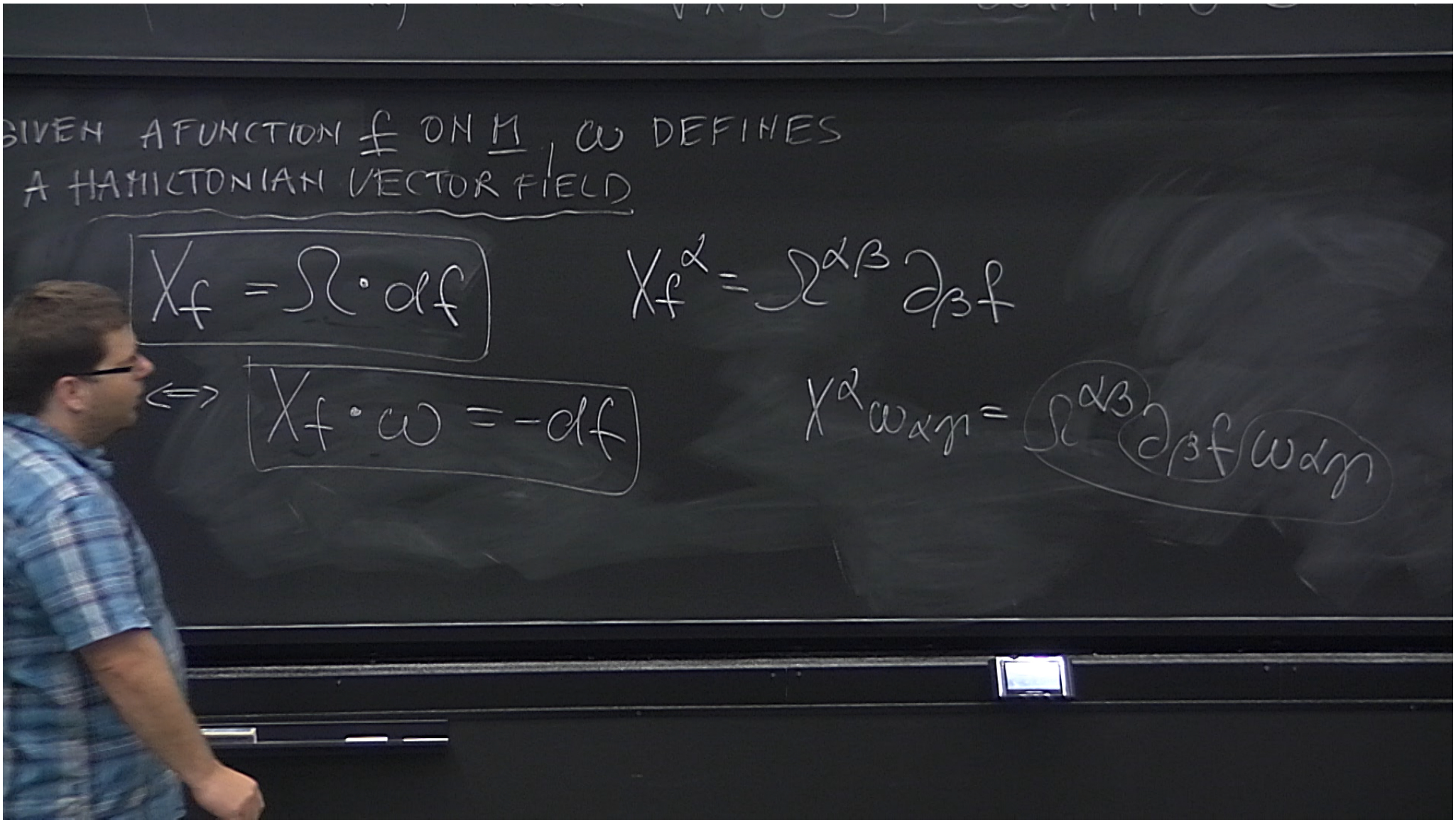
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ii) NON-DEG: $\forall X \neq 0 \ \exists Y \quad \omega(X,Y) \neq 0 \quad \Omega: T^*M \to TM$

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THEOREM: A HAMILTONIAN VECTOR FIELD PRESERVES SYMPLECTIC STRUCTURE

$$\mathcal{L}_{X_f}\,\omega = 0$$

PRESERVES SYMPLECTIC STRUCTURE

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PROOF: $\mathcal{L}_{X_f}\,\omega = X_f \cdot d\omega + d(X_f \cdot \omega)$

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$$\mathcal{L}_{X_f}\omega = X_f \cdot \underbrace{d\omega}_{0} + d(\underbrace{X_f \cdot \omega}_{=df}) = -d^2 f$$

[NOT = ANTI CONSTRAINT]

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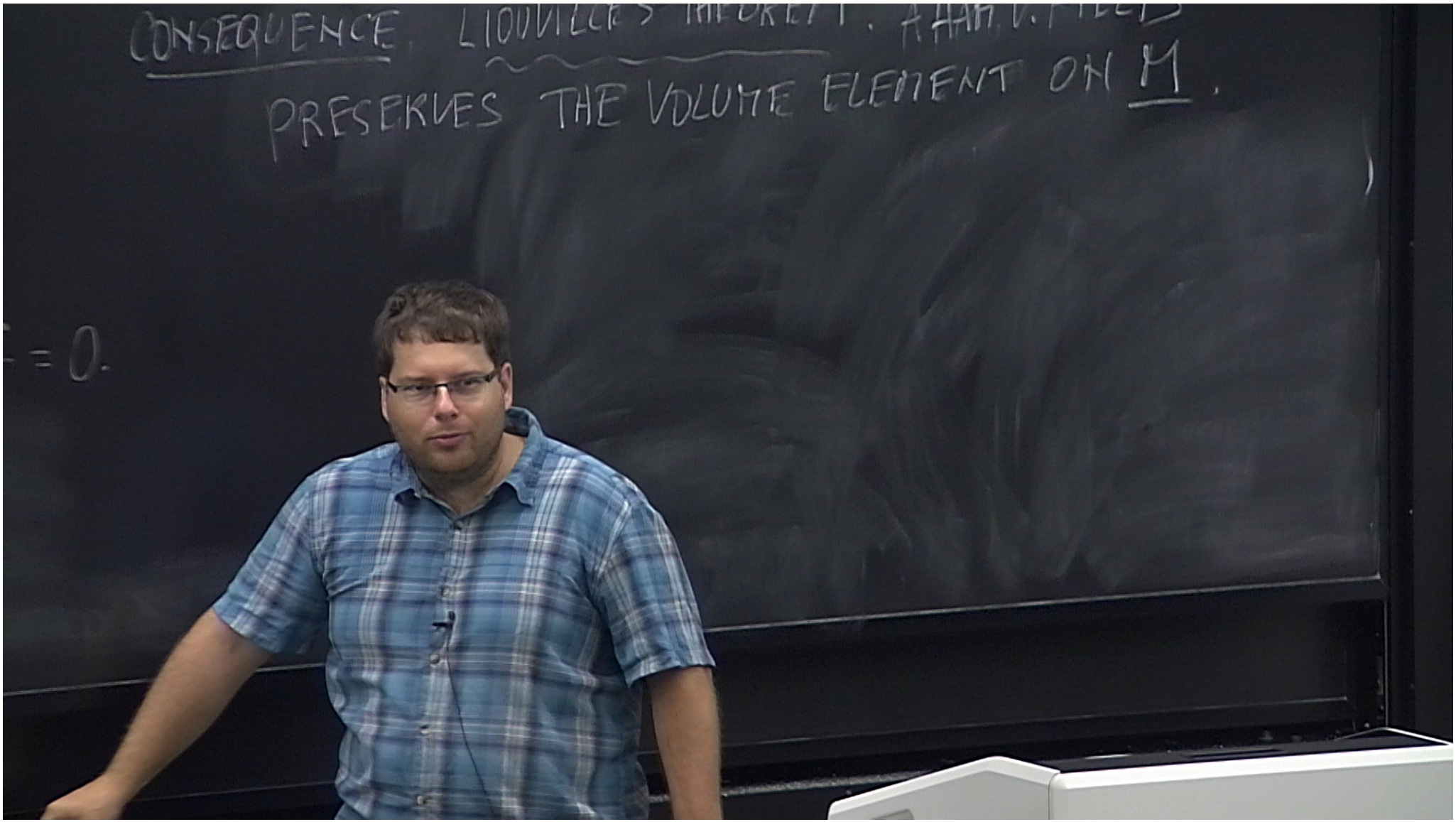
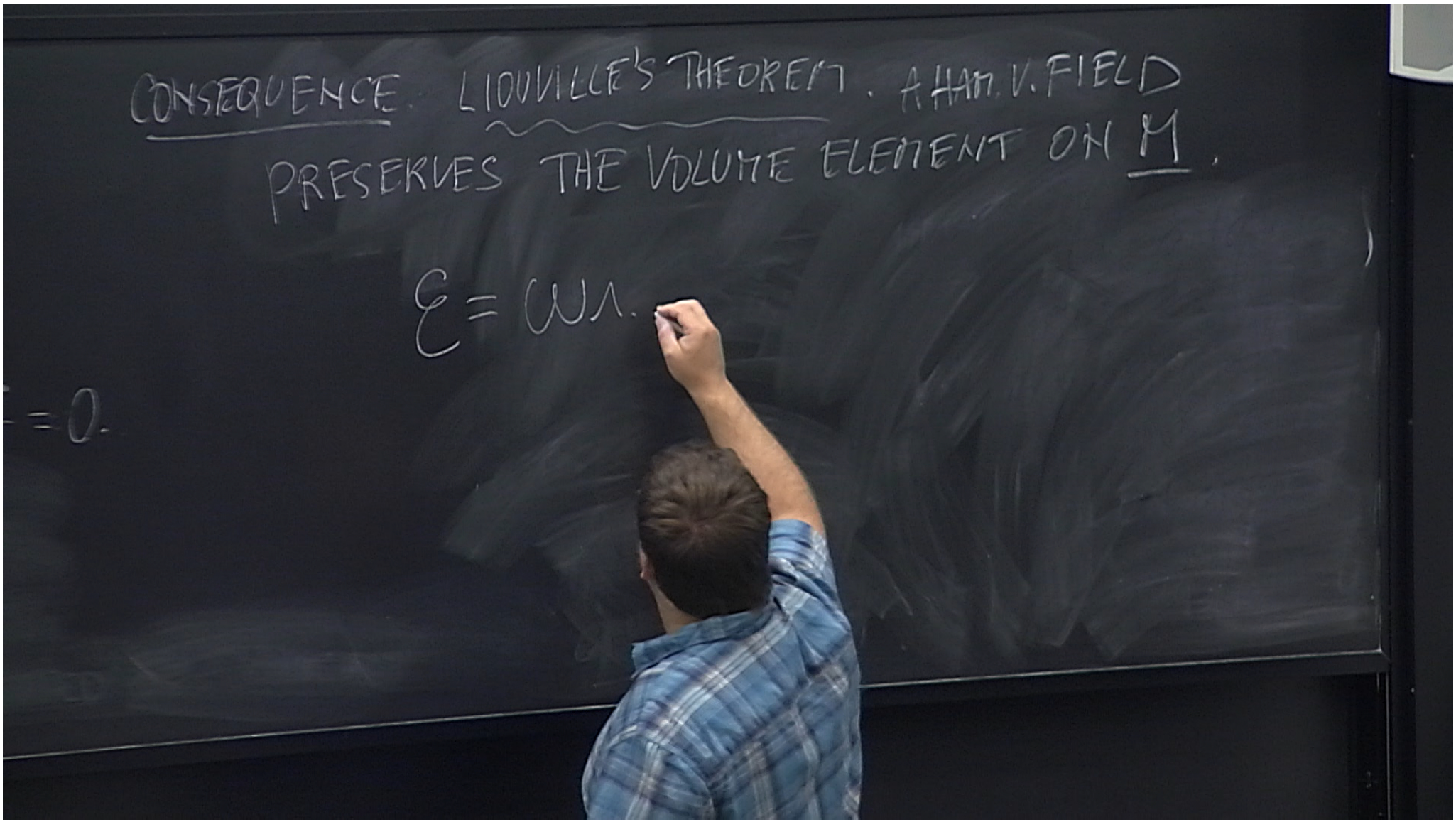
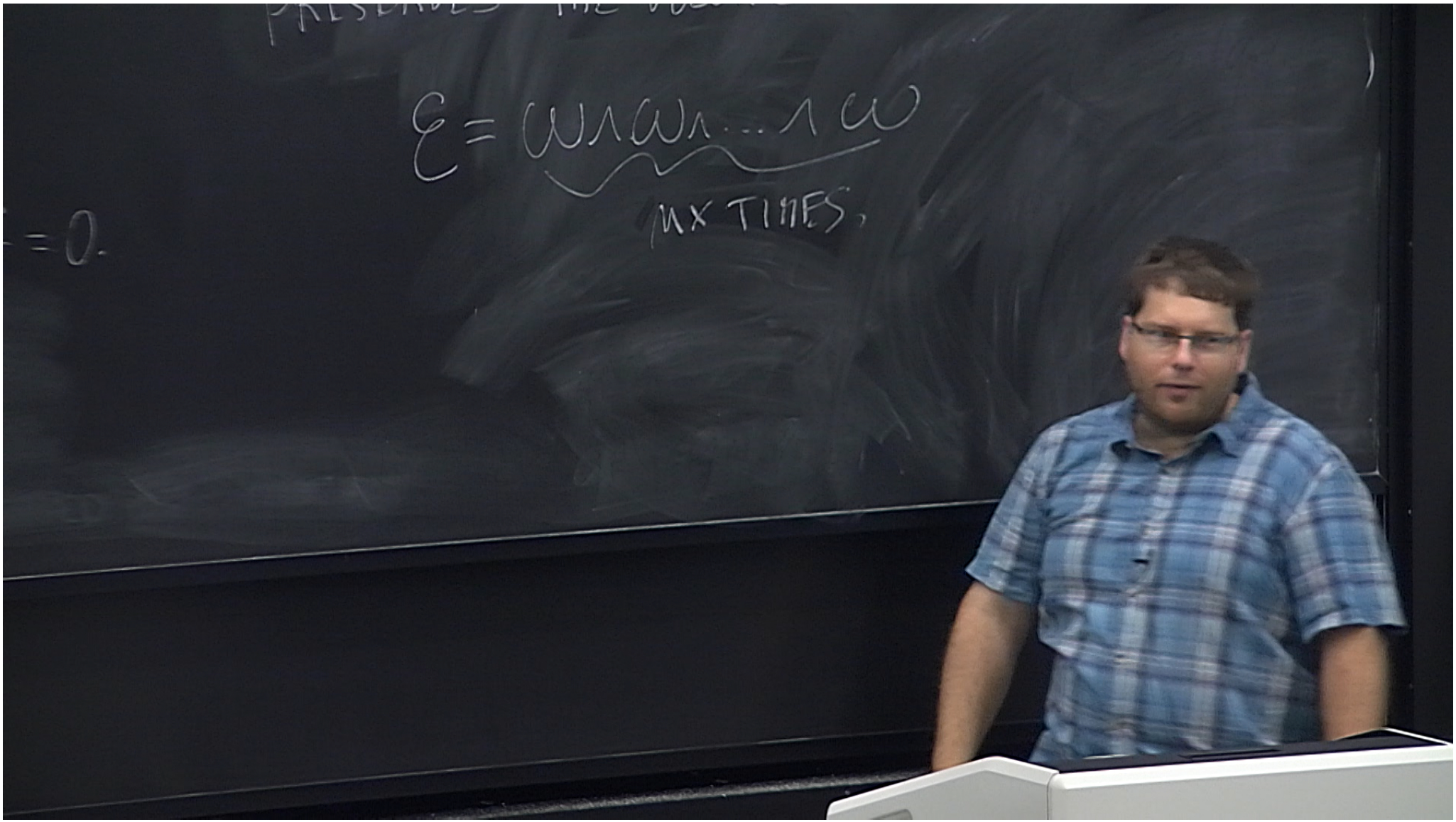
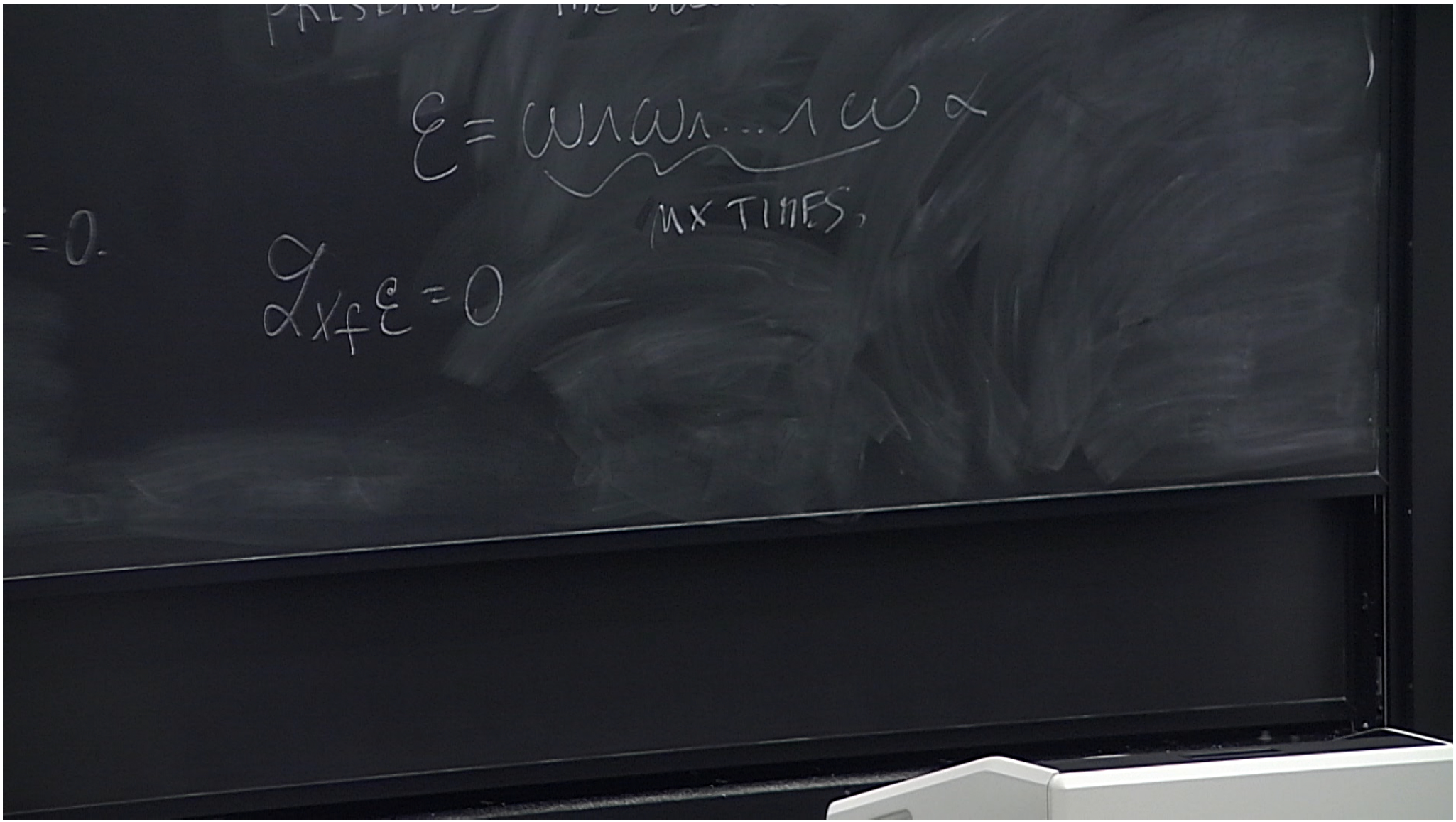
$H(q,p) = \lambda(t)$ (CONSTRAINT)


CONSEQUENCE
PRESERVES THE VOLUME ELEMENT ON M.


CONSEQUENCE. LIOUVILLE'S THEOREM. A HAM. V. FIELD
PRESERVES THE VOLUME ELEMENT ON M.
$\mathcal{E} = \omega \wedge$.
$= 0$.


$\mathcal{E} = \omega \wedge \omega \wedge \ldots \wedge \omega$
μ X TIMES.


$\mathcal{E} = \underbrace{\omega \wedge \omega \wedge \ldots \wedge \omega}_{n \times \text{TIMES}} \wedge \alpha$
$\mathcal{L}_{X_f} \mathcal{E} = 0$

PRESERVES THE VOLUME ELEMENT ON $\underline{M}$.

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$= 0.$

$$\boxed{\mathcal{L}_{X_f} \mathcal{E} = 0}.$$

- POISSON BRACKET .. TWO PHASE SPACE FUNCTIONS f, g.

$$\{f, g\} = df \cdot \Omega \cdot dg = -\omega(X_f, X_g)$$

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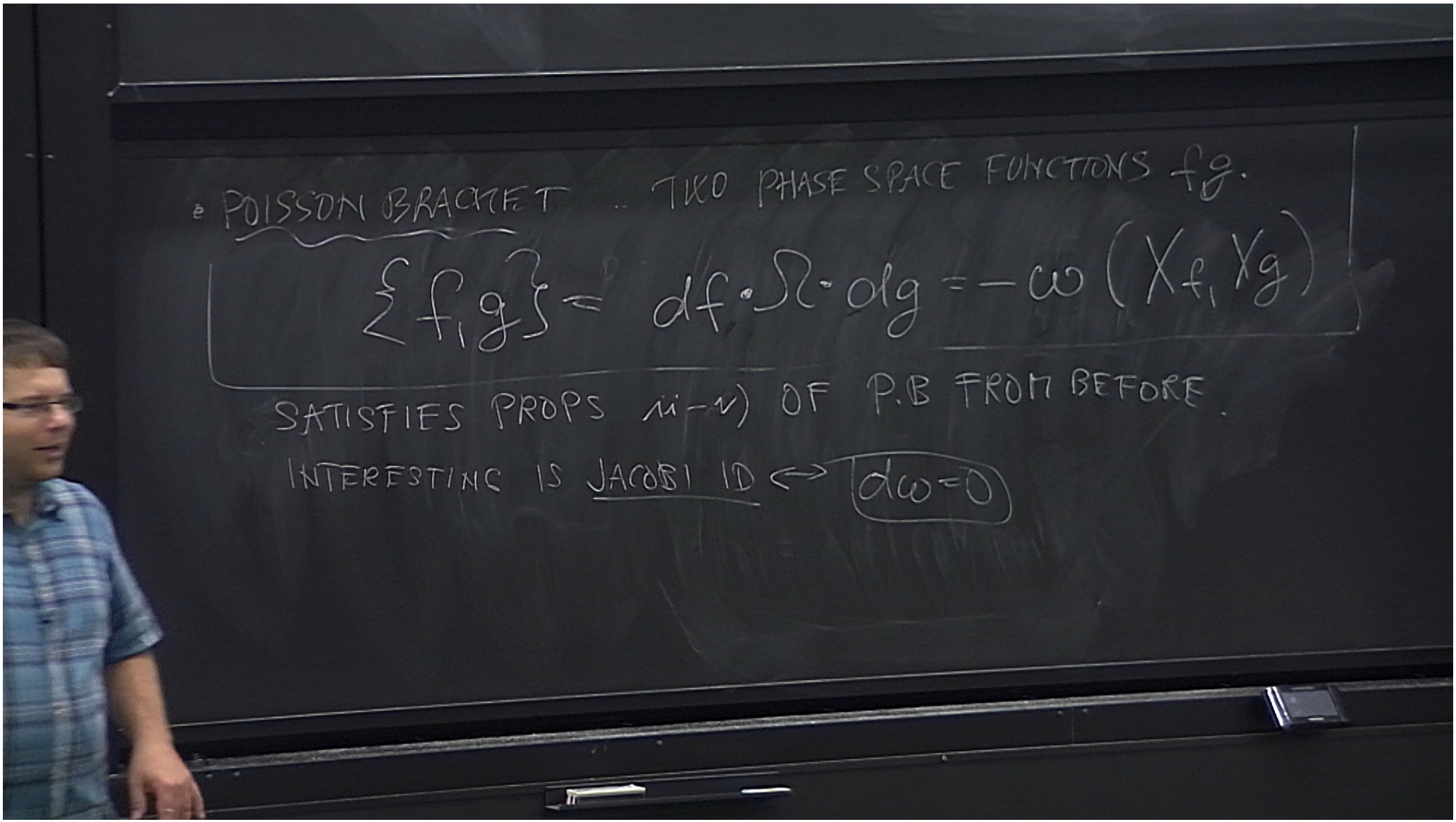
SATISFIES PROPS ii–v) OF P.B FROM BEFORE.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INTERESTING IS JACOBI ID $\Leftrightarrow$ $d\omega = 0$

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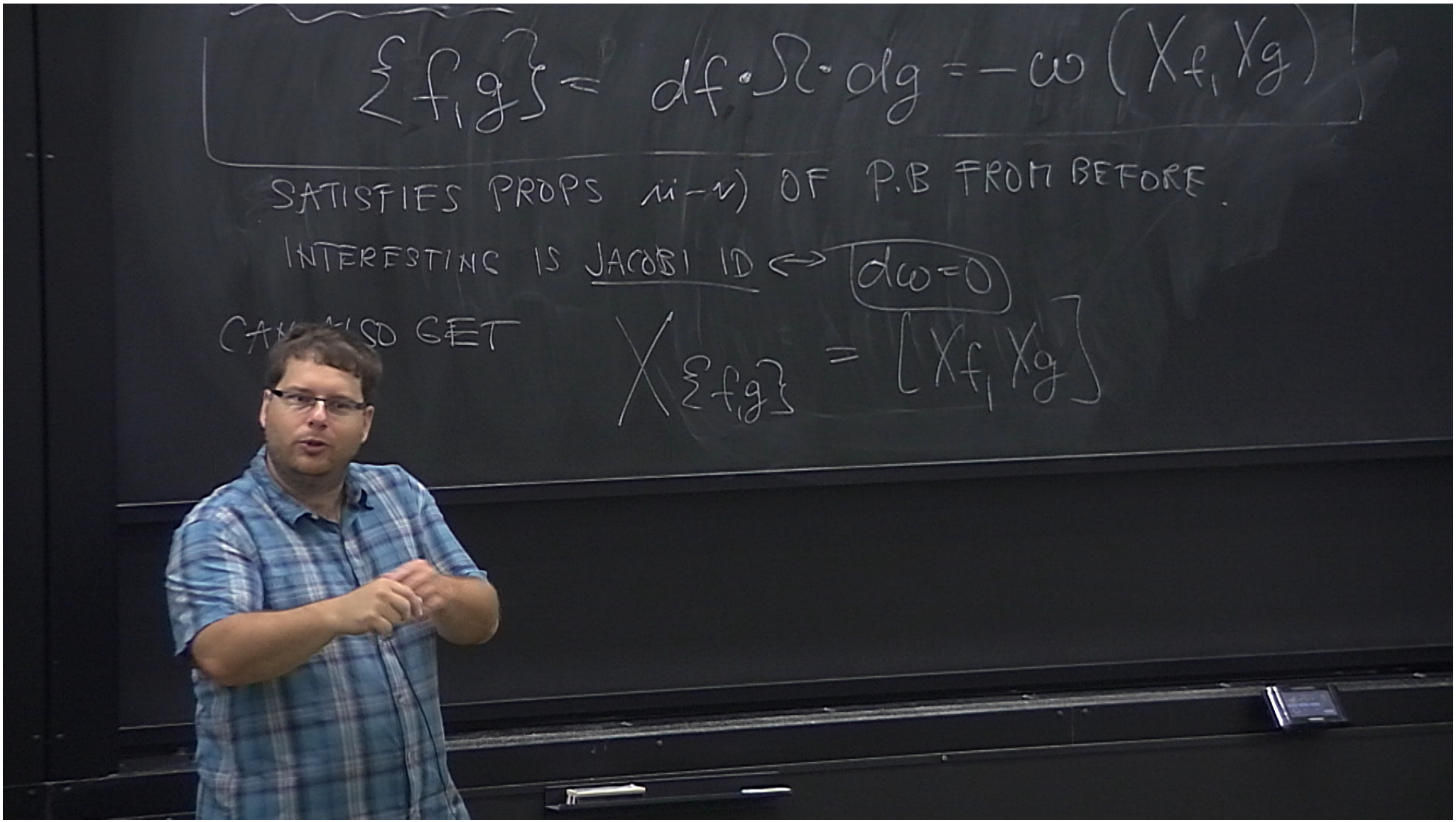
SATISFIES PROPS ii–v) OF P.B FROM BEFORE.

INTERESTING IS JACOBI ID $\Leftrightarrow$ $d\omega = 0$

CAN ALSO GET

$$X_{\{f,g\}}$$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CAN ALSO GET
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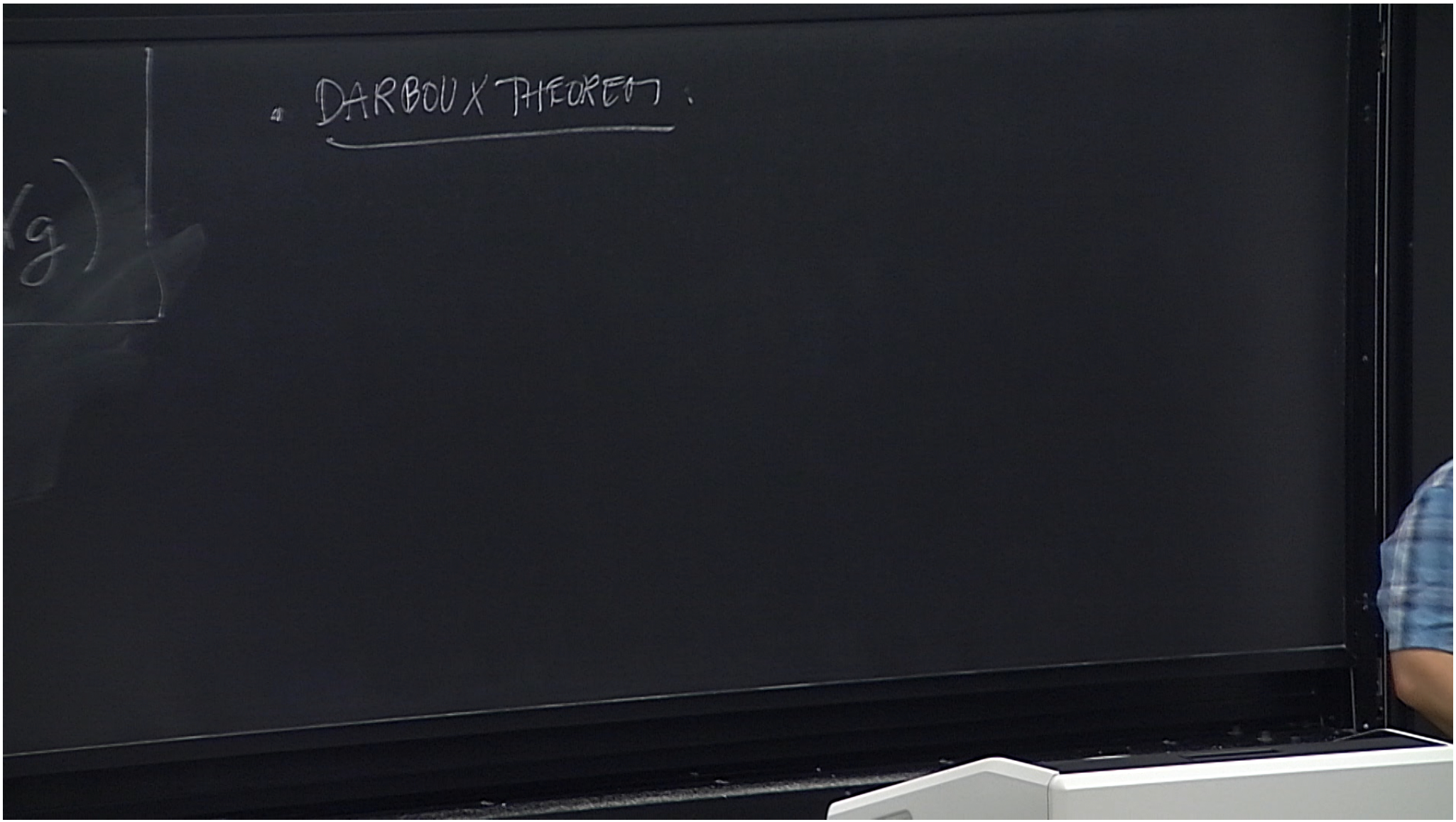
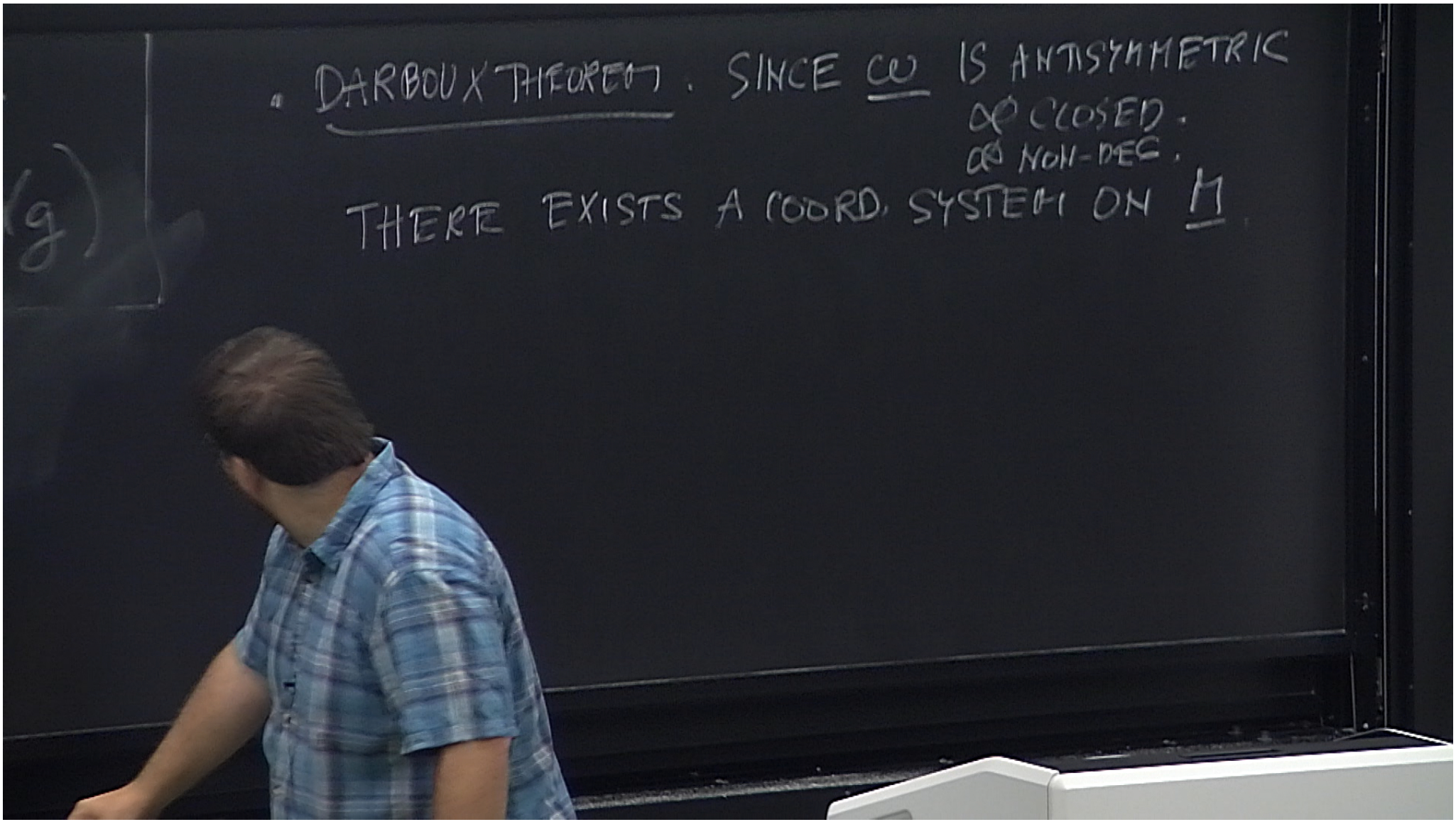

DARBOUX THEOREM

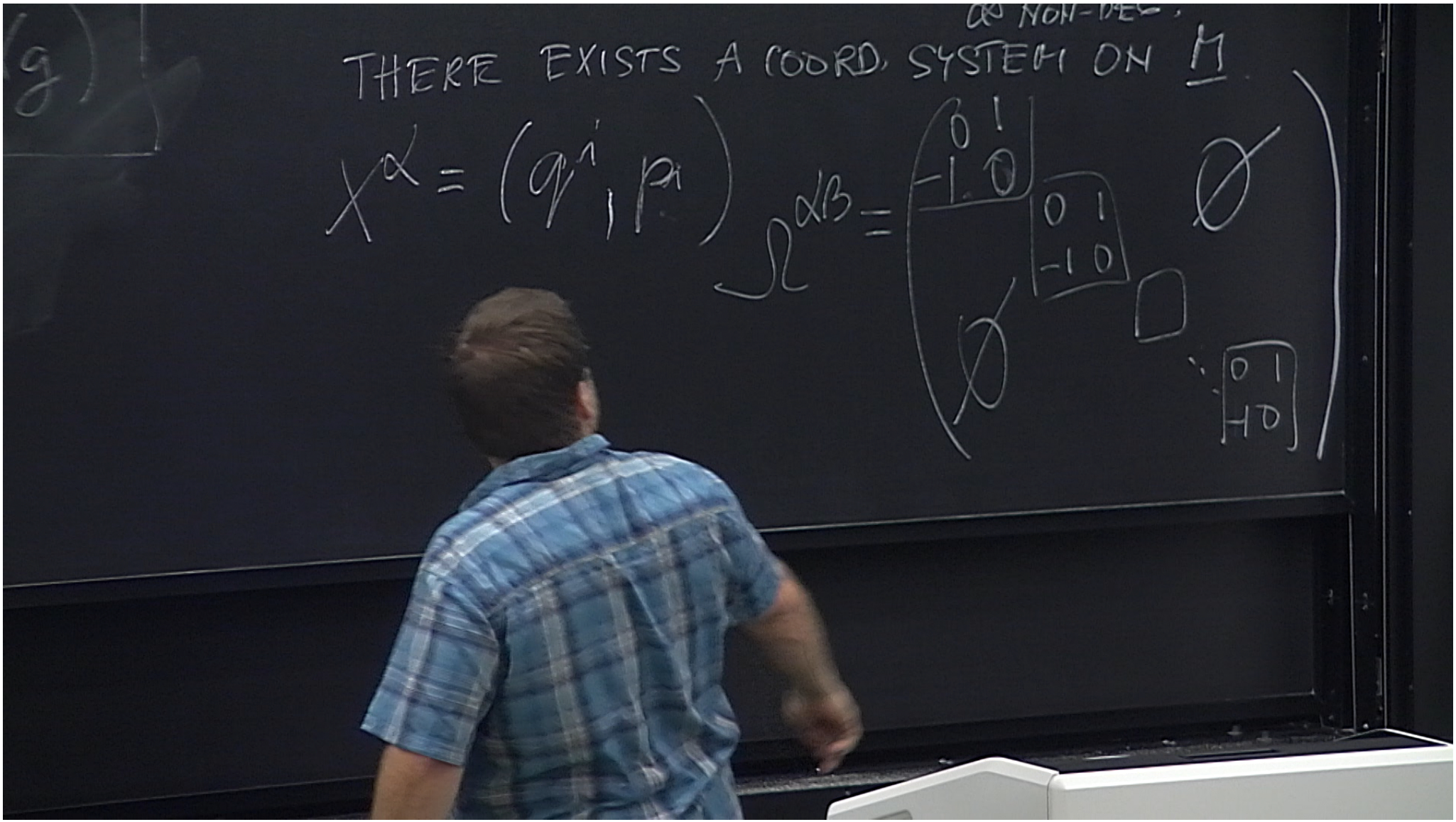

DARBOUX THEOREM. SINCE ω IS ANTISYMMETRIC
∞ CLOSED.
∞ NON-DEG.
THERE EXISTS A COORD. SYSTEM ON M

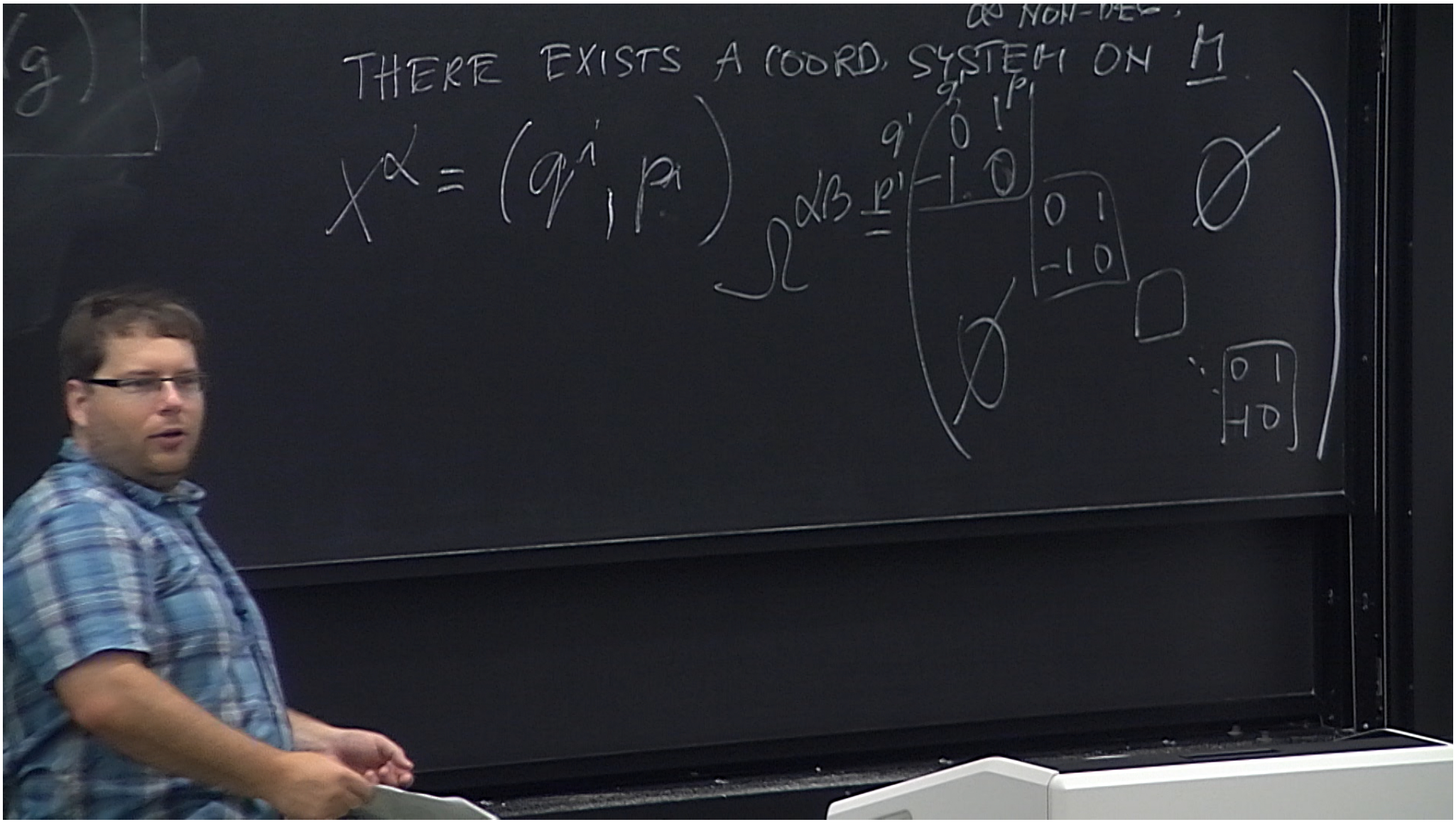
(g)

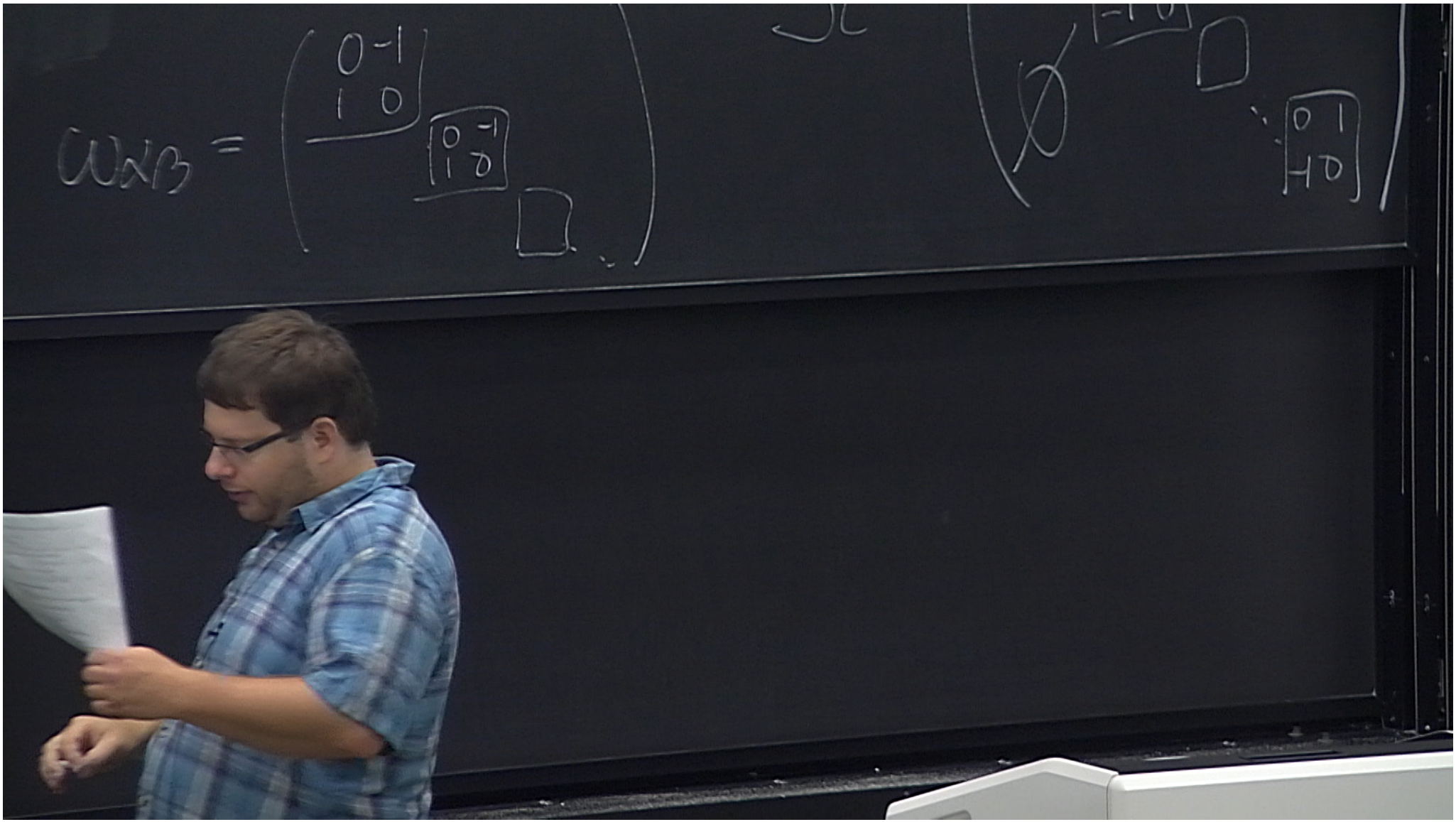
∞ NON-DEG.

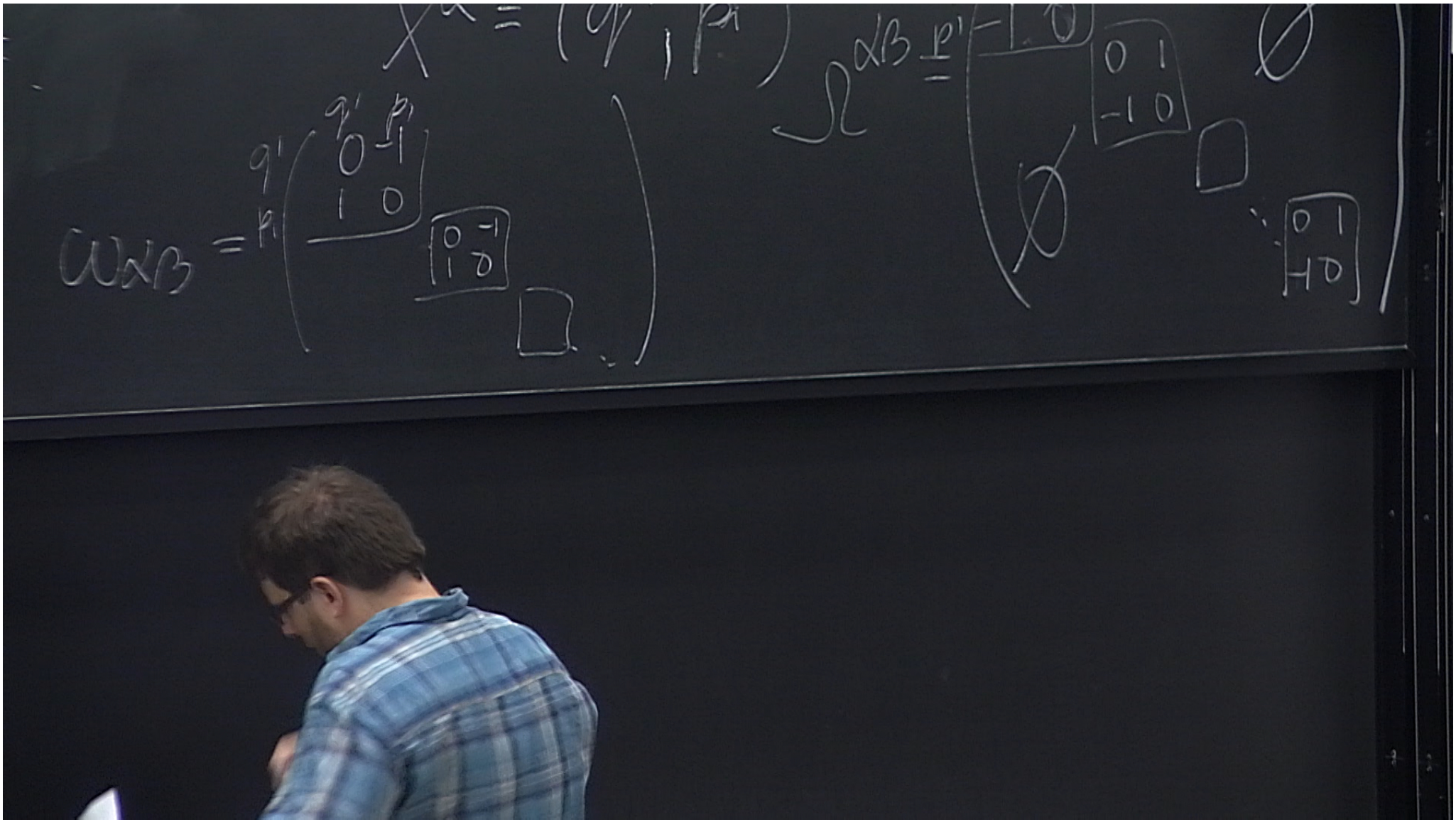
THERE EXISTS A COORD. SYSTEM ON $\underline{M}$

$$X^\alpha = (q^i, p_i) \qquad \Omega^{\alpha\beta} = \begin{pmatrix} \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & & \\ & \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & \\ & & \ddots \end{pmatrix}$$


THERE EXISTS A COORD. SYSTEM ON M
$X^\alpha = (q^i, p_i)$
$\Omega^{\alpha\beta} =$


THERE EXISTS A COORD. SYSTEM ON M
$X^\alpha = (q^i, p_i)$
$\Omega^{\alpha\beta} = \begin{pmatrix} \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & & & \emptyset \\ & \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & & \\ & & \ddots & \\ \emptyset & & & \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} \end{pmatrix}$


ωxB =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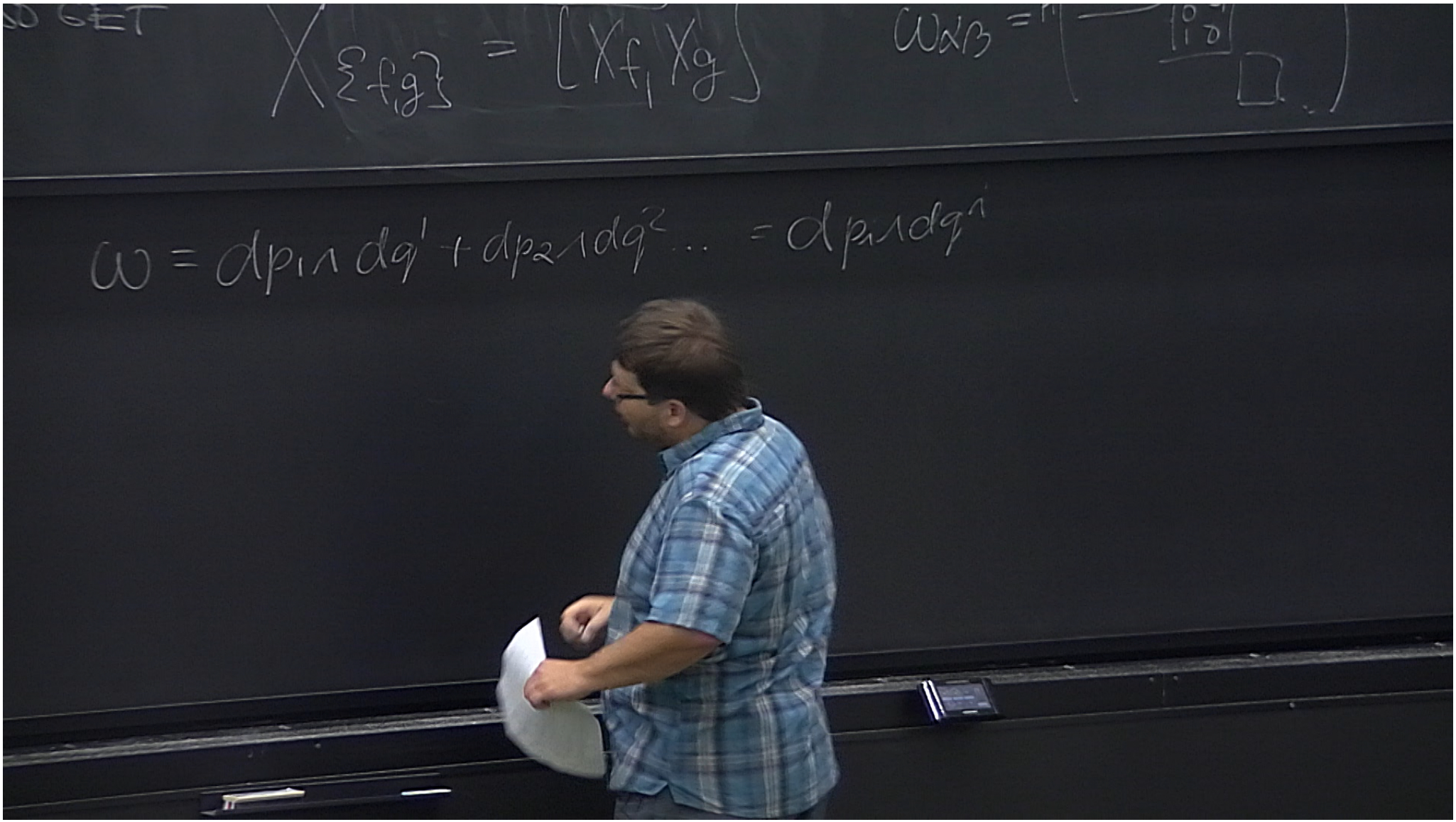
$X_{\{f,g\}} = [X_f, X_g]$
$\omega = dp_1 \wedge dq^1 + dp_2 \wedge dq^2 \ldots = dp_i \wedge dq^i$

CAN ALSO GET

$$X_{\{f,g\}} = [X_f, X_g]$$

$$\omega_{\alpha\beta} = \ldots$$

$$\omega = dp_1 \wedge dq^1 + dp_2 \wedge dq^2 \ldots = dp_i \wedge dq^i$$

CONSEQUENCE:

$$\mathcal{E} = \underbrace{\omega \wedge \ldots \wedge \omega}_{m\times}$$

N ALSO GET

$$X_{\{f,g\}} = [X_f, X_g]$$

$$\omega_{\alpha\beta} = \begin{pmatrix} & \\ & \end{pmatrix}$$

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CONSEQUENCE:

$$\varepsilon = \underbrace{\omega \wedge \ldots \wedge \omega}_{n\times} = dp_1 \wedge dq^1 \wedge dp_2 \wedge dq^2 \wedge \ldots \wedge dp_n \wedge dq^n$$

$$\int f = \int f \varepsilon = \int f \, dp_1 dq^1 \ldots dp_n dq^n.$$

$\{f, g\} = \partial_\alpha f \, \Omega^{\alpha\beta} \partial_\beta g$

dq^1

$dq^2 \wedge \ldots \wedge dp_n \wedge dq^n$

q^n.

$\ldots dq^1$

$\ldots q^2 \wedge \ldots \wedge dp_n \wedge dq^n$

$\ldots q^n.$

$$\{f,g\} = \partial_\alpha f \, \Omega^{\alpha\beta} \partial_\beta g$$

$$= \Omega^{\alpha\beta} \frac{\partial f}{\partial x^\alpha} \frac{\partial g}{\partial x^\beta} = \underbrace{\frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i}}$$

$$\omega = dp_1 \wedge dq^1 + dp_2 \wedge dq^2 \ldots = dp_i \wedge dq^i = dP_i \wedge dQ^i \quad \cdot \{f$$

$$= \Omega^{\ldots}$$

IQUENCE:

$$\varepsilon = \underbrace{\omega \wedge \ldots \wedge \omega}_{n\times} = dp_1 \wedge dq^1 \wedge dp_2 \wedge dq^2 \wedge \ldots \wedge dp_n \wedge dq^n$$

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